'6'

# Matrices

6- Lower Triangular matrix square matrix below diagonal non:

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 3 & 8 & 0 \\ 5 & 7 & -1 \end{bmatrix}$$

7 Transpose of a matrix for $A = [a_{ij}] = \begin{cases} 1 \le i \le m \\ 1 \le j \le n \end{cases}$

is

$$A^T = [a_{ji}]$$

Example

$$A = \begin{bmatrix} 7 & 9 & 1 \\ 0 & -2 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 7 & 0 \\ 9 & -2 \\ 1 & 3 \end{bmatrix}$$

8. The Symmetric Matrix

It is a square matrix with $[a_{ij}] = [a_{ji}]$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$ is Symmetric

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 7 & 5 & 6 \end{bmatrix}$$ is not Symmetric.

# Matrices

## Defn

A matrix is a grid of rows and columns i.e

$$A = [a_{ik}] \quad 1 \le i \le m ; \ 1 \le k \le n$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

## Types of Matrices :

1) Zero matrix all entries are zero $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2) Square matrix $m = n$ $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}_{2\times 2}$

3) Diagonal matrix all entries are zeros except diagonal :

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

4) identity matrix square matrix non diagonal zero's diagonal :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5) Upper triangular matrix square matrix below diagonal are ze

$$\begin{bmatrix} 2 & 4 & 8 \\ 0 & 7 & 9 \\ 0 & 0 & 6 \end{bmatrix}$$

## Matrix Arithemetic

1) Matrix equality

$$A = [a_{ij}] = B[b_{ij}] \quad \text{if} \quad b_{ij} = a_{ij} \begin{cases} 1 \le i \le m \\ 1 \le j \le n \end{cases}$$

2) Matrix addition

if $A = [a_{ij}] \begin{cases} 1 \le i \le m \\ 1 \le j \le n \end{cases}$ ; $B = b_{ij} \begin{cases} 1 \le i \le m \\ 1 \le j \le n \end{cases}$

$$C = A + B = [c_{ij}] \begin{cases} 1 \le i \le m \\ 1 \le j \le n \end{cases} \quad \text{and } c_{ij} = a_{ij} + b_{ij}$$

Example

$$A = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix}$$

3) Matrix Multiplication

Let $A_{m \times k}$ , $B_{k \times n}$

Then

$$C_{m \times n} = AB$$

Example

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 3 \end{bmatrix}_{2\times3} \qquad B = \begin{bmatrix} 2 & 3 \\ 6 & 3 \\ 2 & 1 \end{bmatrix}_{3\times2}$$

$$\underset{2\times2}{C} = AB = \begin{bmatrix} 14 & 11 \\ 20 & 12 \end{bmatrix}$$

4- Power of Matrix

Let $I_n$ is the identity matrix $n \times n$

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 \end{bmatrix}_{n\times n}$$

$$A^0 = I_n$$

$$A^r = \underbrace{AAA \cdots A}_{r \text{ times}}$$

Characteristics of Matrix Arithmetic

Let $A, B, C$ be matrices $Z_0$ is the zero matrix and $x$ is a constant.

1) $(A^T)^T = A$
2) $(A+B)^T = A^T + B^T$
3) $(AB)^T = B^T A^T$
4) $(A+B) = B+A$
5) $(A+B)+C = A+(B+C)$
6) $A + Z_0 = Z_0 + A = A$

7) $AB \neq BA$

8) $(AB)C = A(BC)$

9) $A(B+C) = AB + AC$

10) $(A+B)C = AC + BC$

11) $xA = Ax$

12) $(A^k)^p = A^{kp}$

## Zero-One Matrices ( Boolean Matrices)

A matrix with entries 0 or 1

Example

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Defn

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be $m \times n$ matrices. then

1) The join of A and B $A \vee B = \begin{cases} 1 & \text{if } a_{ij} = 1 \text{ or } b_{ij} = 1 \\ 0 & \text{otherwise} \end{cases}$

2) The meet of A and B is $A \wedge B = \begin{cases} 1 & \text{if } a_{ij} = b_{ij} = 1 \\ 0 & \text{otherwise} \end{cases}$

Exampk Find the join and meet of the zero one matrices

$$A = \underset{2\times3}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}} \qquad B = \underset{2\times3}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}}$$

The join of A and B $\Rightarrow A \vee B = \begin{bmatrix} 1 \vee 0 & 0 \vee 1 & 1 \vee 0 \\ 0 \vee 1 & 1 \vee 1 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

The meet of A and B $\Rightarrow A \wedge B = \begin{bmatrix} 1 \wedge 0 & 0 \wedge 1 & 1 \wedge 0 \\ 0 \wedge 1 & 1 \wedge 1 & 0 \wedge 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

## Defn

Let $A = [a_{ij}]$ be an $m \times k$ Zero-one matrix

$B = [b_{ij}]$ be a $k \times n$ Zero-one-matrix

Then

the Boolean product of A and B denoted by $A \odot B$ is the $m \times n$ matrix

$$c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \text{---} (a_{ik} \wedge b_{kj})$$

## Example

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Find the boolean product of A and B

$$A \odot B = \begin{bmatrix} (1\wedge 1)\vee(0\wedge 0) & (1\wedge 1)\vee(0\wedge 1) & (1\wedge 0)\vee(0\wedge 1) \\ (0\wedge 1)\vee(1\wedge 0) & (0\wedge 1)\vee(1\wedge 1) & (0\wedge 0)\vee(1\wedge 1) \\ (1\wedge 1)\vee(0\wedge 0) & (1\wedge 1)\vee(0\wedge 1) & (1\wedge 0)\vee(0\wedge 1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Defn:

Let $A$ be a square zero-one matrix, Let $r$ be a positive integer. The $r$-th boolean power of $A$

is $$A^{[r]} = \underbrace{A \odot A \odot A \cdots \odot A}_{r \text{ times}}$$

We also define

$A^{[0]}$ to be $I_n$

Example

Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ Find $A^{[n]}$ for all positive integers $n$

Soln

$$A^{[2]} = A \odot A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^{[3]} = A^{[2]} \odot A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^{[4]} = A^{[3]} \odot A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^{[5]} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore A^{[n]} = A^{[5]}$ for all positive integers $n \geq 5$