Newtons

F(x)= $x^2 + 3$ starting point 5

Solution:

نوجد المشتقة الاقتران $\acute{f}(x)$

$$f\acute{(x)} = 2x$$

نعوض في قانون نيوتن

$$x_{n+1} = x_n - \frac{f(x_n)}{\acute{f}(x_n)}$$

هي نقطة البداية في الاغلب بتكون (صفر) **n**

$$x_{0+1} = x_0 - \frac{f(5)}{\acute{f}(5)}$$

$$x_0 = 5$$

$$x_1 = 5 - \frac{f(5)}{\acute{f}(5)}$$

$$x_1 = 5 - \frac{x^2 + 3}{2x}$$

$$x_1 = 5 - \frac{5^2 + 3}{2 * 5}$$

$$x_1 = 5 - \frac{28}{10}$$

$$x_1 = 5 - 2.8$$

$$x_1 = 2.2$$

.....

$$x_{n+1} = x_n - \frac{f(x_n)}{\acute{f}(x_n)}$$

$$x_2 = x_1 - \frac{f(x_1)}{\acute{f}(x_1)}$$

$$x_2 = 2.2 - \frac{f(2.2)}{\acute{f}(2.2)}$$

$$x_2 = 2.2 - \frac{x^2 + 3}{2x}$$

$$x_2 = 2.2 - \frac{2.2^2 + 3}{2 * 2.2}$$

$$x_2 = 2.2 - \frac{7.84}{4.4}$$

$$x_2 = 2.2 - 1.782$$

$$x_2 = 0.418$$

Secant

F(x)= $x^2 + 3$ starting point [1,4]

Solution:

$$x_0 = 1$$
$$x_1 = 4$$
$$x_2 = x_1 - \frac{f(x_1) * (x_1 - x_0)}{f(x_1) - f(x_0)}$$
$$x_2 = 4 - \frac{f(4) * (4 - 1)}{f(4) - f(1)}$$
$$x_2 = 4 - \frac{(4^2 + 3) * (4 - 1)}{(4^2 + 3) - (1^2 + 3)}$$
$$x_2 = 4 - \frac{19 * 2}{19 - 4}$$
$$x_2 = 4 - \frac{38}{15}$$
$$x_2 = 4 - 2.53$$

$$x_2 = 1.47$$

.....

$$x_3 = x_2 - \frac{f(x_2) * (x_2 - x_1)}{f(x_2) - f(x_1)}$$
$$x_3 = 1.47 - \frac{f(1.47) * (1.47 - 4)}{f(1.47) - f(4)}$$
$$x_3 = 1.47 - \frac{(1.47^2 + 3) * (1.47 - 4)}{(1.47^2 + 3) - (4^2 + 3)}$$
$$x_3 = 1.47 - \frac{5.1609 * -2.53}{5.1609 - 19}$$
$$x_3 = 1.47 - \frac{-13.057}{-13.8391}$$
$$x_3 = 1.47 - 0.9434$$

$$x_3 =$$

Lagrange Interpolation Polynmial

	0	1	2	3
X	5	6	9	11
F(x)	12	13	14	16

Find f(7)

قانون الحل 😊

Lagrange = $L_0 + L_1 + L_2 + L_3$

$$L_0 = f(x_0) * \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$L_1 = f(x_1) * \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$L_2 = f(x_2) * \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$L_3 = f(x_3) * \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

الحل:

قيمة x حسب القيمة المعطاة بالسؤال وهي 7

نعوض في المعادلات السابقة على حسب ما ورد في الجدول

$$L_0 = f(x_0) * \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$L_0 = 12 * \frac{(7-6)(7-9)(7-11)}{(5-6)(5-9)(5-11)}$$

$$L_0 = 12 * \frac{(1)(-2)(-4)}{(-1)(-4)(-6)}$$

$$L_0 = 12 * \frac{8}{-24}$$

$$L_0 = 12 * -0.33$$

$$L_0 = -4$$

.....

$$L_1 = f(x_1) * \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}$$

$$L_1 = 13 * \frac{(7 - 5)(7 - 9)(7 - 11)}{(6 - 5)(6 - 9)(6 - 11)}$$

$$L_1 = 13 * \frac{(2)(-2)(-4)}{(1)(-3)(-5)}$$

$$L_1 = 13 * \frac{16}{15}$$

$$L_1 = 13 * 1.066$$

$$L_1 = 13.858$$

.....

$$L_2 = f(x_2) * \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

$$L_2 = 14 * \frac{(7 - 5)(7 - 6)(7 - 11)}{(9 - 5)(9 - 6)(9 - 11)}$$

$$L_2 = 14 * \frac{(2)(1)(-4)}{(4)(3)(-2)}$$

$$L_2 = 14 * \frac{-8}{24}$$

$$L_2 = 14 * -0.33$$

$$L_2 = -4.66$$

.....

$$L_3 = f(x_3) * \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$

$$L_3 = 16 * \frac{(7 - 5)(7 - 6)(7 - 9)}{(11 - 5)(11 - 6)(11 - 9)}$$

$$L_3 = 16 * \frac{(2)(1)(-2)}{(6)(5)(2)}$$

$$L_3 = 16 * \frac{-4}{60}$$

$$L_3 = 16 * -0.066$$

$$L_3 = -1.066$$

.....

Lagrange = $L_0 + L_1 + L_2 + L_3$

F(7) = $(-4) + 13.858 + (-4.66) + (-1.066)$

Lagrange = 4.132

Newton Divided Differences

القانون للحل 😊

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$b_3 = \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_3 - x_0}$$

$$f(x) = b_0 + b_1(x_1 - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$

	x_0	x_1	x_2	x_3
X	5	6	9	11
F(x)	12	13	14	16

Find f(7)

x	$f(x)$	$b1$	$b2$	$b3$
5	12	$\frac{13-12}{6-5} = 1$	$\frac{0.33-1}{9-5} = -0.1675$	
6	13			$\frac{0.134--0.1675}{11-5} = 0.05$
9	14	$\frac{14-13}{9-6} = 0.33$	$\frac{1-0.33}{11-6} = 0.134$	
11	16	$\frac{16-14}{11-9} = 1$		

7

نشوف السبعة وين رح تكون موجودة حتى نقدر نحدد b1,b2 المستخدمين

$$b_0 = 12$$

$$b_1 = 0.33$$

$$b_2 = 0.134$$

$$b_2 = 0.05$$

$$f(7) = 12 + 0.33(7 - 5) + 0.134(7 - 5)(7 - 6) + 0.05(7 - 5)(7 - 6)(7 - 9)$$

$$f(7) = 12 + 0.66 + 0.268 + (-0.2)$$

$$f(7) = 10.928$$

Newton Divided Differences

القانون للحل 😊

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$b_3 = \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_3 - x_0}$$

$$f(x) = b_0 + b_1(x_1 - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$

	x_0	x_1	x_2
X	0.160	0.161	0.162
F(x)	0.1593182066	0.1603053541	0.1612923412

١) Find f(0.1604)

٢) Find f(0.160)

0.1604

x	$f(x)$	$b1$	$b2$
0.160	0.1593182066	0.9871475	
0.161	0.1603053541	0.9869871	-0. 0802
0.162	0.1612923412		

١) نشوف 0.1604 وين رح تكون موجودة حتى نقدر نحدد b1,b2 المستخدمين

$$b_0 = 0.1593182066$$

$$b_1 = 0.9871475$$

$$b_2 = -0.0802$$

$$f(0.1604) = 0.1593182066 + 0.9871475(0.1604 - 0.160) + -0.0802(0.1604 - 0.160)(0.1604 - 0.161)$$

$$f(0.1604) = 0.1593182066 + 0.000394859 + 0.000000019248$$

$$f(0.1604) = 0.159713084$$