5. The demo program filter_16.py implements a filter with transfer function

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}. \tag{1}$$

Modify the filter in the program so that the transfer function is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{2}$$

where $B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2}$ is set so that the impulse response is exactly

$$h(n) = r^n \cos(\omega_1 n)\, u(n) \tag{3}$$

where $u(n)$ is the unit step function. Note that the first value of this impulse response is 1, and that all other values are less than 1. That is $h(n) \leq 1$ for all n. To find b_n, you may consult a table of Z-transforms! How should **gain** be set to ensure the impulse response does not exceed the maximum allowed value of $2^{15} - 1$?

Write a Matlab program that calculates the impulse response using the **filter** function and plot the impulse response, to verify that the initial value is 1.

Implement the filter in real-time using Python/PyAudio (modify the demo program **filter_16.py**).

Python code: 5_my.py

```
#16 bit/sample
#y(n) = x(n) -a1 y(n-1) -a2 y(n-2)
from math import cos, pi
import struct
import pyaudio

Fs = 8000
T = 1
N = T * Fs
f = 800
om = 2.0 * pi * float(f) / Fs
r = 0.998

a0 = 1
a1 = -2 * r * cos(om)
a2 = r**2
b0 = 1
b1 = -r * cos(om)
```

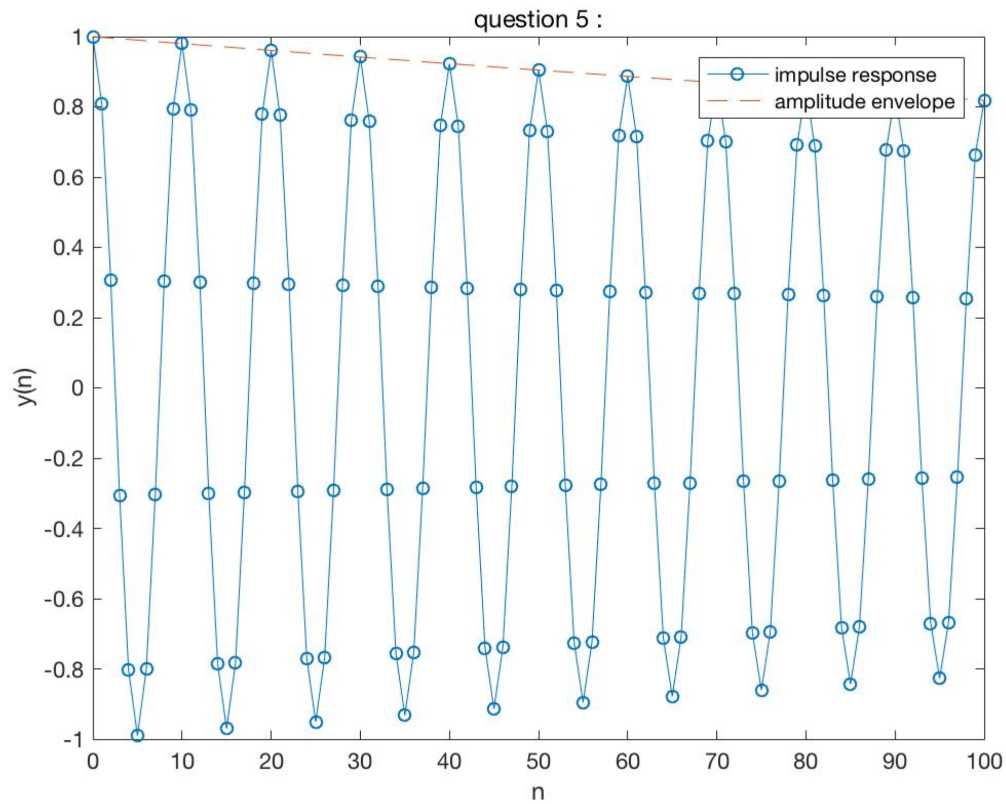
```
b2 = 0

x1 = 0.0
y1 = 0.0
y2 = 0.0
gain = 10000.0

p = pyaudio.PyAudio()
stream = p.open(format = pyaudio.paInt16,    #16 bits
                channels = 1,
                rate = Fs,
                input = False,
                output = True)

#
for n in range(0, N):
    if n == 0:
        x0 = 1.0
    else:
        x0 = 0.0

    y0 = x0 + b1 * x1 - a1 * y1 - a2 * y2

    #delays
    x1 = x0
    y2 = y1
    y1 = y0

    output_value = gain * y0
    if output_value > 2**15-1:
        output_value = 2**15-1
    elif output_value < -2**15:
        output_value = -2**15

    output_string = struct.pack('h', int(output_value))
    stream.write(output_string)

print(" * finished * ")
```

```
stream.stop_stream()
stream.close()

p.terminate()
```

Matlab code: q5.m

```
%% 5.Using the filter function and plot the impulse response
clc
clear all
f = 800;
Fs = 8000;
om = 2 * pi * f / Fs;
r = 0.998;

a0 = 1;
a1 = -2 * r * cos(om);
a2 = r^2;
b0 = 1;
b1 = -r * cos(om);
b2 = 0;

a = [a0 a1 a2]
b = [b0 b1 b2]

n = 0:100;
x = ( n==0 );
% make the filter
y = filter(b, a, x);
g = r.^n;

figure(1)
plot(n, y, 'o-', n, g, '--')
legend('impulse response', 'amplitude envelope')
xlabel('n')
ylabel('y(n)')
title('question 5 :')
```

Figure:



Comment:

I looked up the z-transform in the table and then found the coefficient of H(z). Then I assignment a and b to make them become z-transform of h(n). I assumed the frequency is 800 Hz and r is 0.998. Then I used filter to verify the figure, it was begin with 1 and then attenuated as 0.998^n. Since the figure is attenuation, gain should be set to ensure the initial output value less than 2^{15}-1.