Difference Equations. (first-order.)

$$a_0 y(n) + a_1 y(n-1) = b_0 x(n) + b_1 x(n-1)$$

$$n \in \mathbb{Z}$$

$$x(n) \to \boxed{} \to y(n)$$

WLOG $a_0 = 1$

$$y(n) = b_0 x(n) + b_1 x(n-1) - a_1 y(n-1)$$

If $x(-1) = y(-1) = 0$, Then we can proceed.

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$ Transfer function

$$H^f(\omega) = \sum_n h(n) e^{-jn\omega} = \mathrm{DTFT}\{h\}$$

where h is the impulse response.

$$= H(e^{j\omega})$$ uses b_i & a_i.

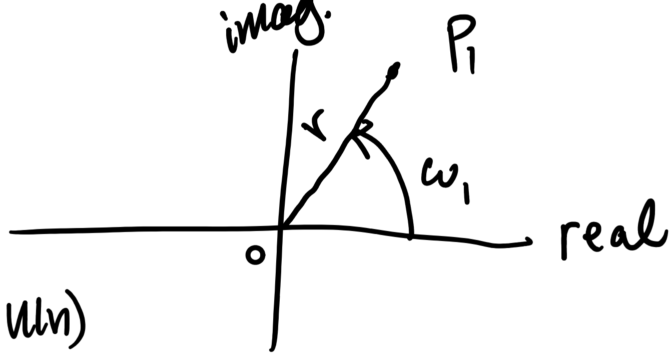
Second-order system.

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) - a_1 y(n-1) - a_2 y(n-2).$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{B(z)}{A(z)}$$

$$= \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{b_0 (z - z_1)(z - z_2)}{(z - p_1)(z - p_2)}$$

$$p_1 = r e^{j\omega_1}, \quad p_2 = r e^{-j\omega_1}$$



$$h(n) = A_0 \delta(n) + C_1 p_1^n u(n) + C_1^* (p_1^*)^n u(n)$$

u : step fcn.

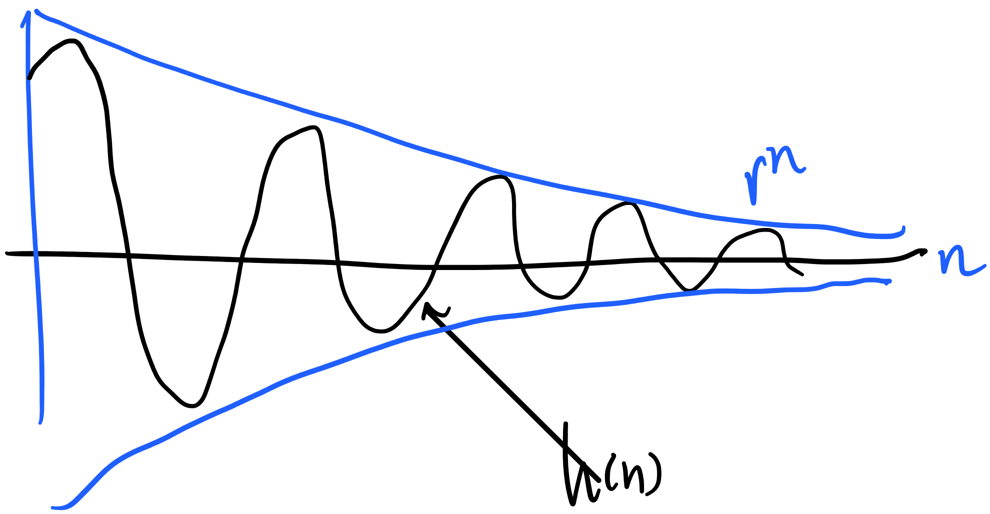
C_1 is determined via partial frac. expansion.

$$C_1 = R e^{j\theta}$$

$$h(n) = \left[R e^{j\theta} (r e^{j\omega_1})^n + R e^{-j\theta} (r e^{-j\omega_1})^n \right] u(n)$$

$$= R r^n \left[e^{j(\theta + \omega_1 n)} + e^{-j(\theta + \omega_1 n)} \right] u(n)$$

$$= 2R\, r^n \cos(\omega_1 n + \theta)\, u(n)$$



pole radius (r) and angle (ω_1) determines the behavior of the imp. resp.

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{z^2}{z^2 + a_1 z + a_2}$$

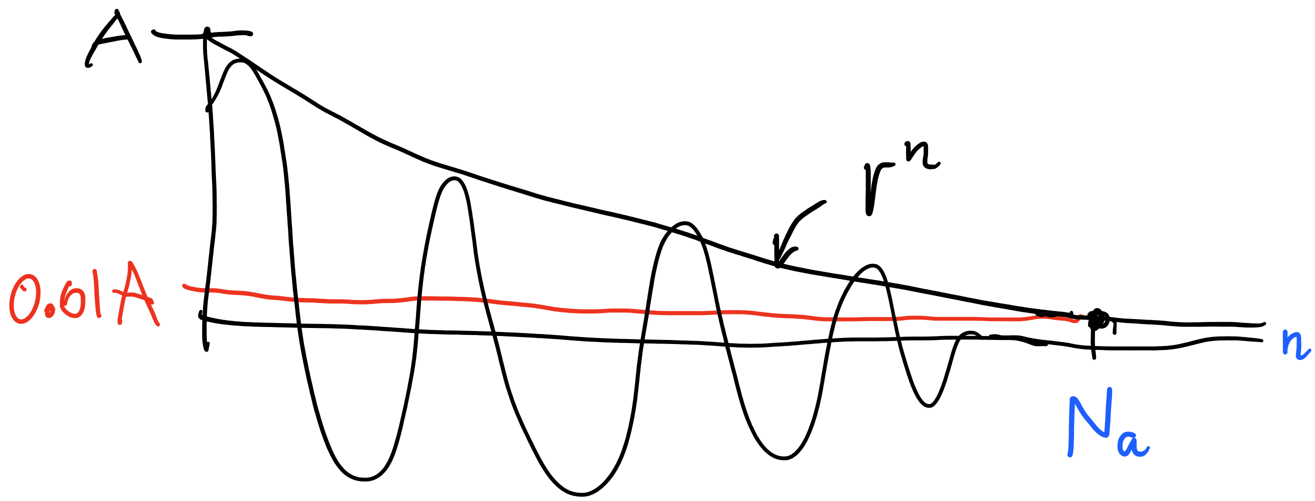
$$= \frac{z^2}{(z - re^{j\omega_1})(z - re^{-j\omega_1})} =$$

$$\rightarrow z^2 - z(re^{j\omega_1} + re^{-j\omega_1}) + r^2 e^{-j\omega_1} e^{+j\omega_1}$$

$$\rightarrow z^2 - z\, 2r\cos(\omega_1) + r^2$$

$$= z^2 + a_1 z + a_2$$

$$\boxed{a_1 = -2r\cos(\omega_1) \qquad a_2 = r^2}$$



Q: How to set r so that N_a has a presribed value?

"Given N_a, how should we set r?"

$$r^{N_a} = 0.01 \qquad r = (0.01)^{1/N_a}$$

$$\log r^{N_a} = \log 0.01$$

$$N_a \log r = \log 0.01$$

$$N_a = \frac{\log 0.01}{\log r}$$