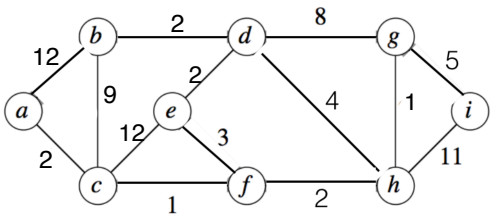
CS6033 Homework Assignment 12

Due Dec. 10th at 5:00 pm

1. Run Dijkstra's algorithm on the following graph where the source vertex is `a`.



2. Question 25.1-1 in CLRS on page 691.

3. Problem 25.2-1 in CLRS on page 699.

4. Suppose you are given a connected weighted undirected graph, G, with n vertices and m edges, such that the weight of each edge in G is an integer in the interval $[1, c]$, for a fixed constant $c > 0$. Show how to solve the single-source shortestpaths problem, for any given vertex v, in G, in time $O(n + m)$. Hint: Think about how to exploit the fact that the distance from v to any other vertex in G can be at most $O(cn) = O(n)$.

   This question is from Goodrich, Michael T.; Tamassia, Roberto. Algorithm Design and Applications

5. Dijkstra's algorithm is not used in graph's which contain negative weights since a negative edge can break the algorithm. A workaround to this is that when a graph contains negative weights, we increase all the edge weights by adding to each edge the absolute value of the lowest edge weight of the graph. This makes all the weights positive. For example, if the weights are $w_1 = -7, w_2 = -3, w_3 = 2$, we can ensure no edge has a negative weight by adding 7 to each edge. Thus $w_1 = 0, w_2 = 4, w_3 = 9$. As the weights are non-negative, we can now apply Dijkstra's.

   The transformation can be written as, $G = (V, E) \rightarrow G' = (V', E')$:
   $E' = E$
   $V' = V$
   $w'(u, v) = w(u, v) + |m|$ where m is the smallest weight of any edge in the graph

   - Does this approach give correct results?
   - If yes, what is the complexity of the transformation
   - If no, provide reasoning or counter-example why it doesn't work

6. Problem 25.2-6 in CLRS on page 700.

7. You are working for a ISP providing service in a relatively undeveloped country. A wealthy customer wants a highly reliable and stable connection from their house to the router that connects your network with the rest of the internet (the customer has been getting excessive ping in their online video games). In order to please this valued customer without spending money on expanding the ISP's infrastructure, your boss has tasked you with finding a dedicated path thru the network to route this customer's traffic. You, naturally, want this channel to have only the most reliable links. You have a directed graph, labeled with the chance of a given router (vertex) or link (edge), to go down on a given day. Design a algorithm so that you can quickly compute this and other reliable routes thru the network.