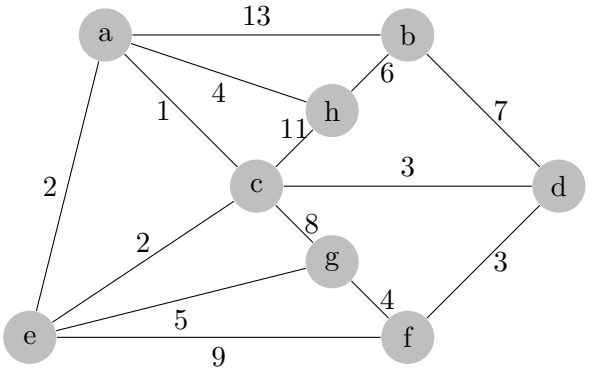
CS6033 Homework Assignment 10*

Due Nov 26th at 5:00 pm
No late assignments accepted

1. For the graph below:

   (a) What is the the cost of a *minimum spanning tree*?

   (b) How many minimum spanning trees does it have?



2. (10 points) For the graph above:

   (a) Run Prim's algorithm. Start at node **a**. Whenever there is a choice of nodes, always use alphabetic ordering. Draw a table showing the intermediate values of the cost array.

3. Perform the following operations using the UNIONFIND where you start from the singleton sets $\{1\}, ..., \{9\}$. Use path compression.

   - UNION$(1, 2)$
   - UNION$(3, 4)$
   - UNION$(5, 6)$

*Many of these questions came from outside sources. No answer will be posted.

- UNION$(7, 8)$
- UNION$(1, 4)$
- UNION$(5, 7)$
- UNION$(2, 9)$
- UNION$(9, 8)$
- FINDSET(1)
- FINDSET(5)

4. What is the best way to multiply a chain of matrices with dimensions that are $15 \times 3, 3 \times 7, 7 \times 27, 27 \times 10, 10 \times 5$, and 5×60? Show your work.

5. As stated, in dynamic programming we first solve the subproblems and then choose which of them to use in an optimal solution to the problem. Professor Capulet claims that we do not always need to solve all the subproblems in order to find an optimal solution. She suggests that we can find an optimal solution to the matrix-chain multiplication problem by always choosing the matrix A_k at which to split the subproduct $A_i A_{i+1} \cdots A_j$ (by selecting k to minimize the quantity $p_{i-1} p_k p_j$) before solving the subproblems. Find an instance of the matrix-chain multiplication problem for which this greedy approach yields a suboptimal solution.

   Question from Goodrich & Tamassia, *Algorithm Design and Applications*

6. Consider a modification of the rod-cutting problem in which, in addition to a price p_i for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts.

   Provide the recurrence formula that would determine the optimal profit for a rod of length n.

   Using your recurrence formula, design an algorithm to find the maximum profit the company can make for rods of length n.

   Create the subproblem graph when the rod length is 5.

   Showing the table(s) computed by your algorithm for a rod of length 5, when $c = 1$ and

length i	1	2	3	4	5
p_i	1	6	8	9	10

7. Imagine that you just joined a company, GT&T, which set up its computer network a year ago for linking together its n offices spread across the globe. You have reviewed the work done at that time, and you note that they modeled their network as a connected, undirected graph, G, with n vertices, one for each office, and m edges, one for each possible connection. Furthermore, you note that they gave a weight, $w(e)$, for each edge in G that was equal to the annual rent that it costs to use that

edge for communication purposes, and then they computed a minimum spanning tree, T, for G, to decide which of the m edges in G to lease. Suppose now that it is time renew the leases for connecting the vertices in G and you notice that the rent for one of the connections not used in T has gone down. That is, the weight, $w(e)$, of an edge in G that is not in T has been reduced. Describe an $O(n + m)$-time algorithm to update T to find a new minimum spanning, T, for G given the change in weight for the edge e.

Question from Goodrich & Tamassia, *Algorithm Design and Applications*

8. Suppose you are given a diagram of a telephone network, which is a graph G whose vertices represent switching centers, and whose edges represent communication lines between two centers. The edges are marked by their bandwidth, that is, the maximum speed, in bits per second, that information can be transmitted along that communication line. The bandwidth of a path in G is the bandwidth of its lowest-bandwidth edge. Give an algorithm that, given a diagram and two switching centers a and b, will output the maximum bandwidth of a path between a and b. What is the running time of your algorithm?

   Question from Goodrich & Tamassia, *Algorithm Design and Applications*

   Do not use any shortest path algorithms in your answer. (We haven't covered these algorithms yet in class.)

9. The holidays are here! You decide to travel to see your cousins in California

   Since you are on a budget, so you decide to drive to visit them. You choose your route so you pass by the most spectacular vistas on your route. Along your route are n hotels, at miles $m_1 < m_2 < \cdots < m_n$ from your starting location. The cost of the ith hotel is h_i.

   You won't drive more than 300 miles[1] per day for safety concerns.

   You want to minimize the cost of your travel.

   Provide a recurrence formula that would calculate the answer.

   Using your recurrence formula, design an algorithm to find the minimum cost.

10. (3 bonus points) Think of a good exam/homework question for the lecture 10.

[1] The hotels on your route are all within 300 miles of each other.