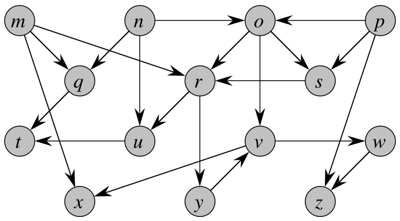
CS6033 Homework Assignment 8[*]

Due November 12th at 5:00 pm
No late assignments accepted

1. (10 points) What is the running time of DFS if the graph is given as an adjacency matrix? Justify your running time.

2. (10 points) Show the ordering of vertices produced by TOPOLOGICAL-SORT when it is run on the DAG below. Assume that the **for** loop of lines $5 - 7$ of the DFS procedure (page 604 in CLRS) considers the vertices in alphabetical order, and assume the adjacency list is ordered alphabetically.



3. (5 ponts) Explain how a vertex u of a directed graph can end up in a depth-first tree containing only u, even though u has both incoming and outgoing edges in G.

4. (15 points) Write a method that takes any two nodes u and v in a tree T, and quickly determines if the node u in the tree is a *descendant* or *ancestor* of node v.

   You may spend $O(n)$ time preprocessing the tree, where n is the number of nodes in the tree.

   Give the running time of your method and justify your running time.

5. (10 points) Given a directed graph G, design an algorithm to determine if there is a cycle in the graph. Determine the running time of your algorithm.

[*] Many of these questions came from outside sources.

6. (10 points) Bob loves foreign languages and wants to plan his course schedule to take the following nine language courses: LA15, LA16, LA22, LA31, LA32, LA126, LA127, LA141, and LA169.

   The course prerequisites are:

   (a) LA15: (none)
   (b) LA16: LA15
   (c) LA22: (none)
   (d) LA31: LA15
   (e) LA32: LA16, LA31
   (f) LA126: LA22, LA32
   (g) LA127: LA16
   (h) LA141: LA22, LA16
   (i) LA169: LA32.

   Find a sequence of courses that allows Bob to satisfy all the prerequisites. He will take only one course at a time.[1]

7. (15 points) In a game there are a large number of splits in the path in order to prevent the character from out-leveling the final boss. The game prevents the character from going back a floor once they have descended to the next floor, and there are a variety of (fixed) different options for a given floor in order to improve replay-ability and player choice. Given that you can never go back towards the entrance, a map of the entire dungeon, and a weighted value for the maximum possible rewards for any given floor, find an algorithm to determine the optimum (maximal) path through this dungeon.

8. Whenever groups of martians gather, they instinctively establish a *kissing order*. For any pair of martians, one martian always kisses the other, driving it away from food or potential mates. The same pair of martians always chooses the same kissing order, even after years of separation, no matter what other martian are around. Surprisingly, the overall kissing order can contain cycles – for example, martian A kisses martian B, which kisses martian C, which kisses martian A. (Hint: think about the different types of edges and how it relates to this problem. What does it mean in terms of who kisses who.)!

   (a) (15 POINTS) Prove that any finite set of martians can be arranged in a row from left to right so that every martian kisses the martian immediately to its left. Pretty please

[1] Question from Goodrich, Michael T.; Tamassia, Roberto. Algorithm Design and Applications

(b) (15 POINTS) Suppose you are given a directed graph representing the kissing relationships among a set of n martians. The graph contains one vertex per martian, and it contains an edge (i, j) *if and only if* martian i kisses martian j. Describe and analyze an algorithm to compute a kissing order for the martians, as guaranteed by part (a)!

9. (3 bonus points) Think of a good exam/homework question for the lecture 8.