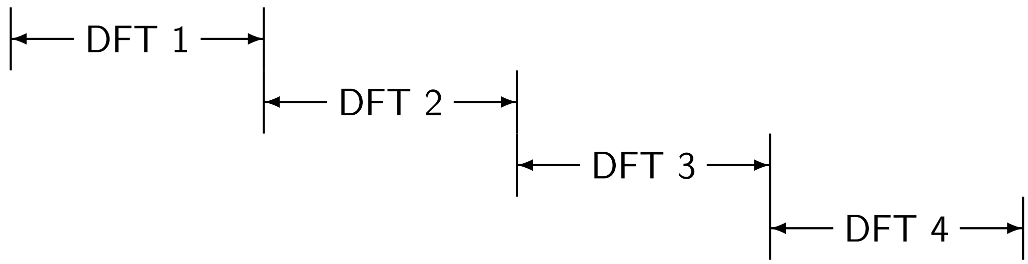
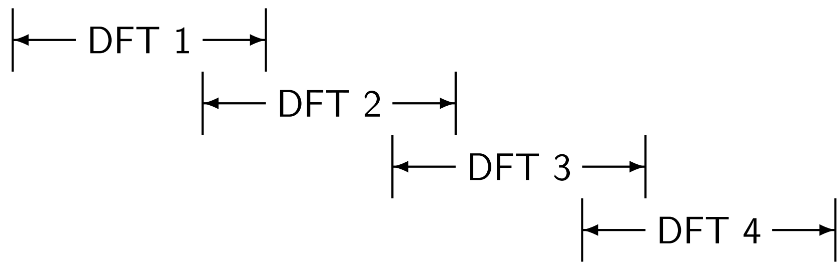
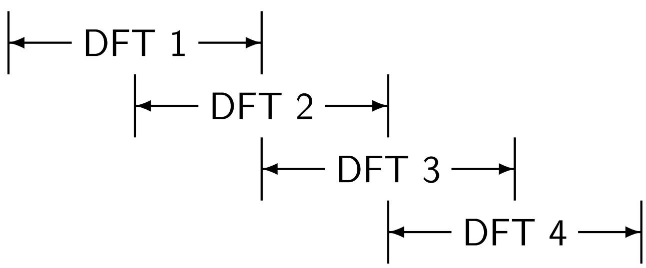
NO OVERLAP



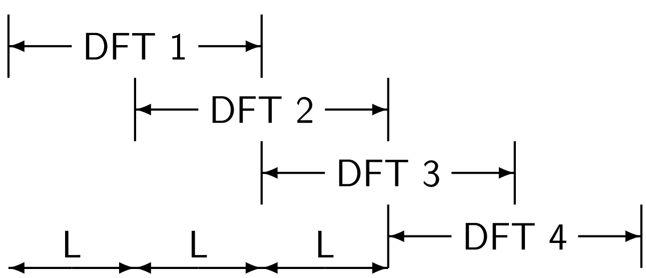
R/4 OVERLAP



R/2 OVERLAP



The parameter L



L is the number of samples between adjacent blocks.
This is sometimes called the hop size.

The short-time Fourier transform is defined as

$$\begin{aligned} X(\omega, m) &= \text{STFT}\{x(n)\} \\ &:= \text{DTFT}\{x(n+m)\,w(n)\} \\ &= \sum_{n=-\infty}^{\infty} x(n+m)\,w(n)\,e^{-j\omega n} \\ &= \sum_{n=0}^{R-1} x(n+m)\,w(n)\,e^{-j\omega n} \end{aligned}$$

where $w(n)$ is the window function of length R.

1. The STFT of a signal $x(n)$ is a function of two variables: time and frequency.
2. The block length is determined by the support of the window function $w(n)$.
3. A graphical display of the magnitude of the STFT, $|X(\omega, m)|$, is called the *spectrogram* of the signal. It is often used in speech processing.
4. The STFT of a signal is invertible.
5. One can choose the block length. A long block length will provide higher frequency resolution (because the main-lobe of the window function will be narrow). A short block length will provide higher time resolution because less averaging across samples is performed for each STFT value.
6. A *narrow-band* spectrogram is one computed using a relatively long block length R, (long window function).
7. A *wide-band* spectrogram is one computed using a relatively short block length R, (short window function).

To numerically evaluate the STFT, we sample the frequency axis ω in N equally spaced samples from $\omega = 0$ to $\omega = 2\pi$.

$$\omega_k = \frac{2\pi}{N}\,k, \qquad 0 \le k \le N-1$$

We then have the discrete STFT,

$$\begin{aligned}
X^d(k,m) &:= X(\frac{2\pi}{N}k, m) \\
&= \sum_{n=0}^{R-1} x(n+m)\,w(n)\,e^{-j\frac{2\pi}{N}kn} \\
&= \sum_{n=0}^{R-1} x(n+m)\,w(n)\,W_N^{-kn} \\
&= \mathsf{DFT}_N\{\{x(n+m)\,w(n)\}_{n=0}^{R-1}, \underbrace{0,\dots,0}_{N-R}\}
\end{aligned}$$

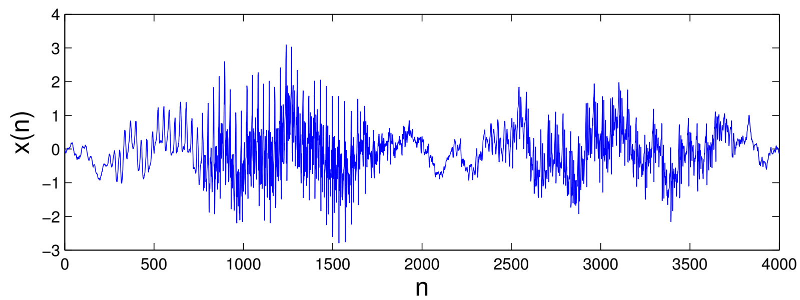
In this definition, the overlap between adjacent blocks is $R-1$. The signal is shifted along the window one sample at a time. That generates more points than is usually needed, so we also sample the STFT along the time direction. That means we usually evaluate

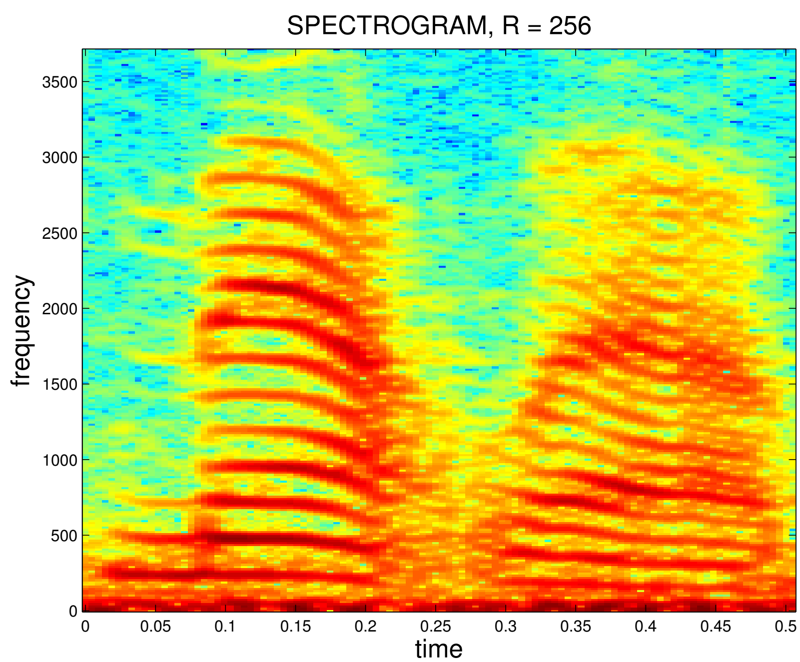
$$X^d(k, Lm)$$

where L is the time-skip. The relation between the time-skip, the number of overlapping samples, and the block length is

$$\text{overlap} = R - L.$$

SPECTROGRAM EXAMPLE





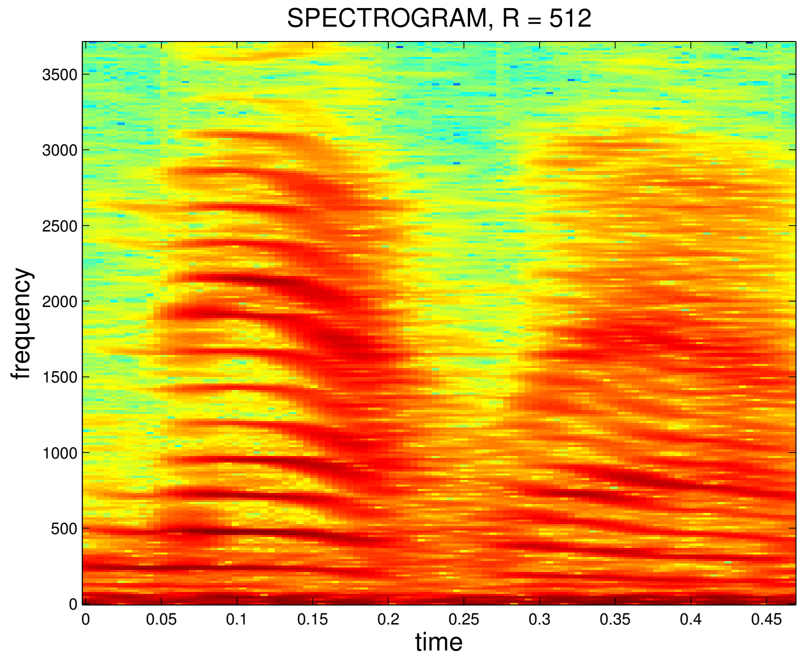
SPECTROGRAM EXAMPLE

The Matlab program for producing the figures on the previous page.

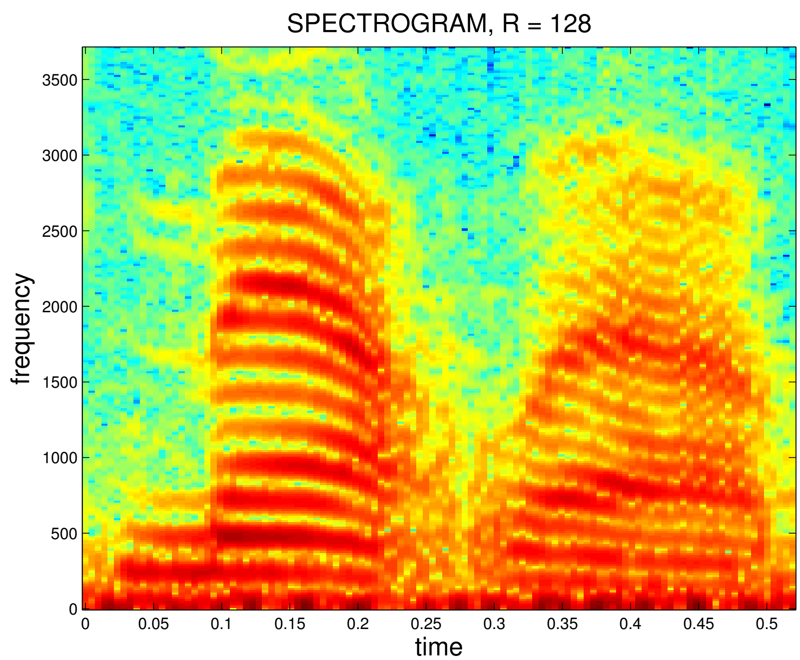
```
% ------ SPECTROGRAM EXAMPLE ------

% LOAD DATA
load mtlb;
x = mtlb;

figure(1), clf
plot(0:4000,x)
xlabel('n')
ylabel('x(n)')

% SET PARAMETERS
R = 256;                % R: block length
window = hamming(R);    % window function of length R
N = 512;                % N: frequency discretization
L = 35;                 % L: time lapse between blocks
fs = 7418;              % fs: sampling frequency
overlap = R - L;

% COMPUTE SPECTROGRAM
[B,f,t] = specgram(x,N,fs,window,overlap);

% MAKE PLOT
figure(2), clf
imagesc(t,f,log10(abs(B)));
colormap('jet')
axis xy
xlabel('time')
ylabel('frequency')
title('SPECTROGRAM, R = 256')
```

EFFECT OF WINDOW LENGTH R

Narrow-band spectrogram: better frequency resolution

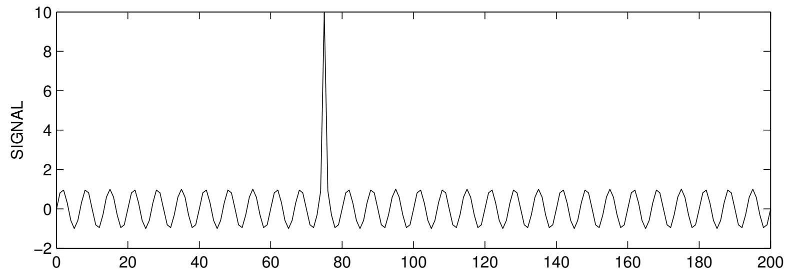


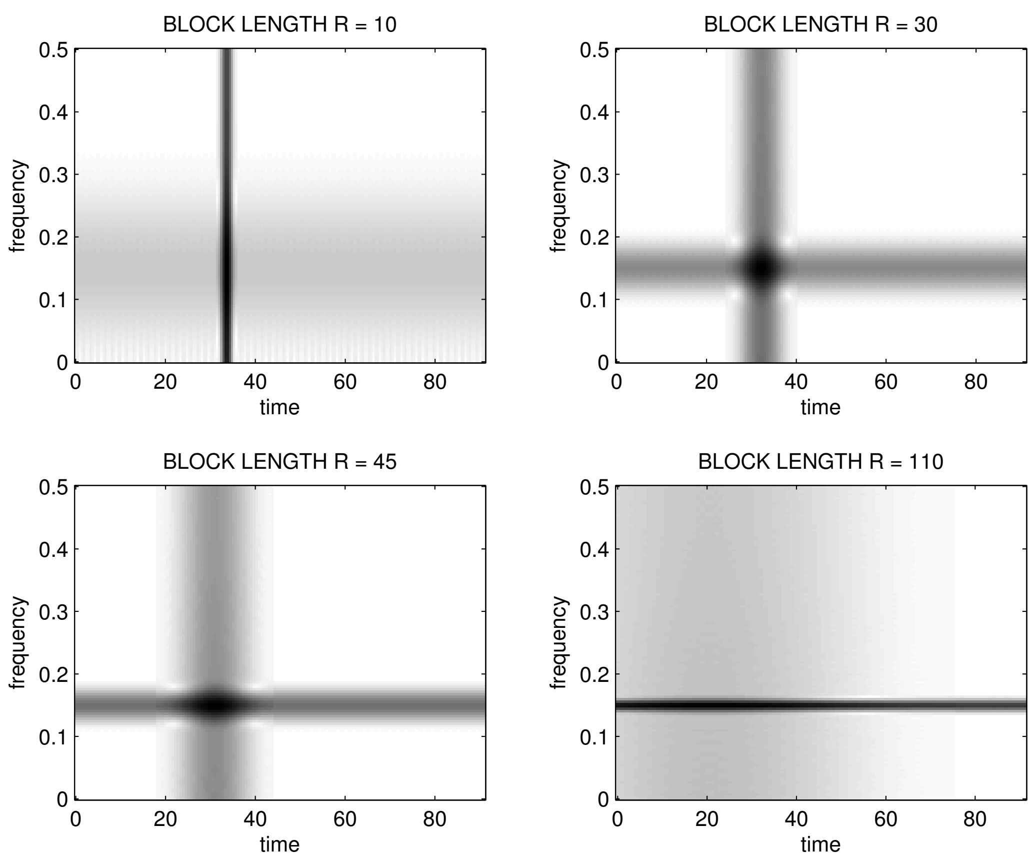
Wide-band spectrogram:: better time resolution



EFFECT OF WINDOW LENGTH R

Here is another example to illustrate the frequency/time resolution trade-off.



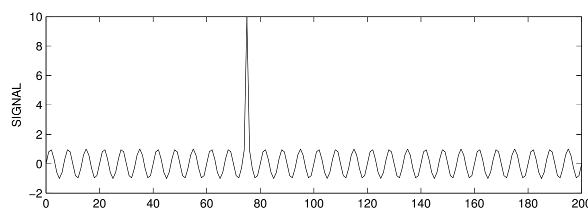


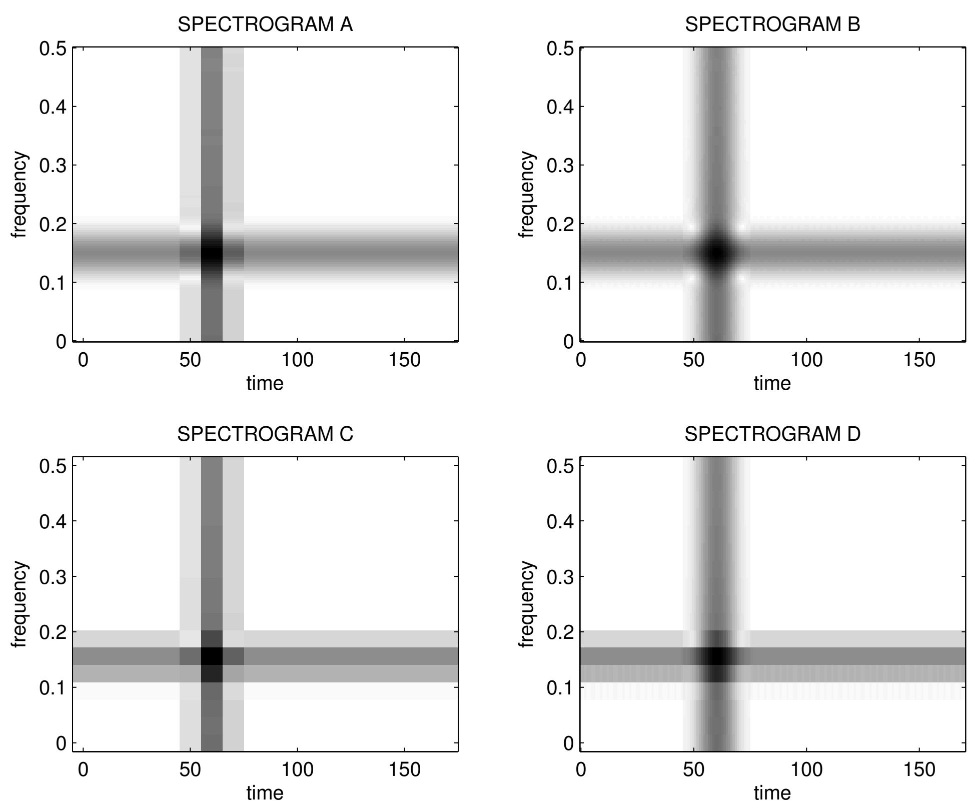
A spectrogram is computed with different parameters:

$$L \in \{1, 10\}, \qquad N \in \{32, 256\}$$

$L =$ time lapse between blocks.
$N =$ FFT length (Each block is zero-padded to length N.)
In each case, the block length is 30 samples.

For each of the four spectrograms, can you tell what L and N are?





L and N do not effect the time resolution or the frequency resolution. They only influence the 'pixelation'.