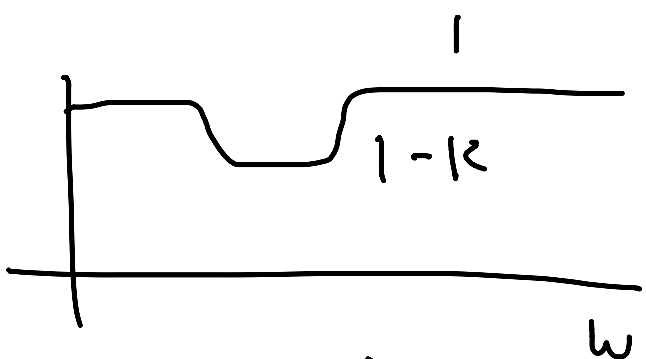
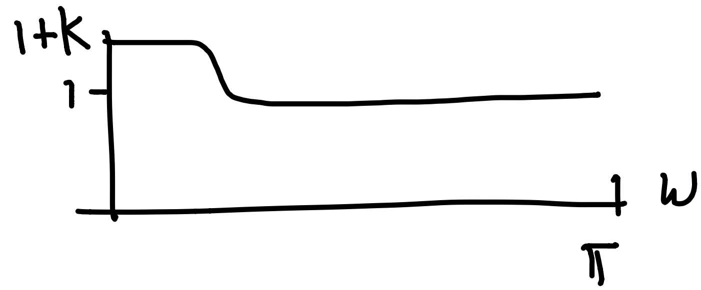
$\rightarrow H_1 \rightarrow H_2 \rightarrow H_3 \rightarrow$

different filters boost or attenuate different freq. bands.

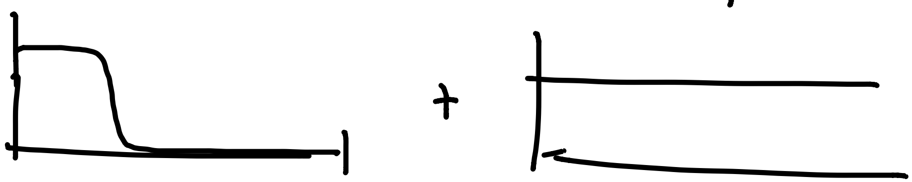


attenuates some freqs.

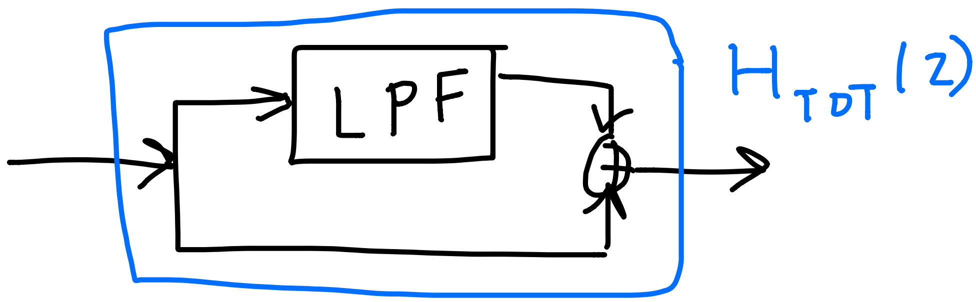
Shelving filter



$\overset{?}{=}$



NO

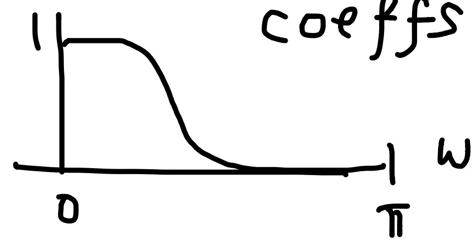


$$H_{TOT}(z) = 1 + H_{LPF}(z)$$

$$H^{f}_{TOT}(z) = 1 + H^{f}_{LPF}(\omega)$$

But H is complex-valued
So the sum does not give
the shelving filter that we
seek

Let $H(z) = \frac{B(z)}{A(z)}$ be a LPF with real coeffs

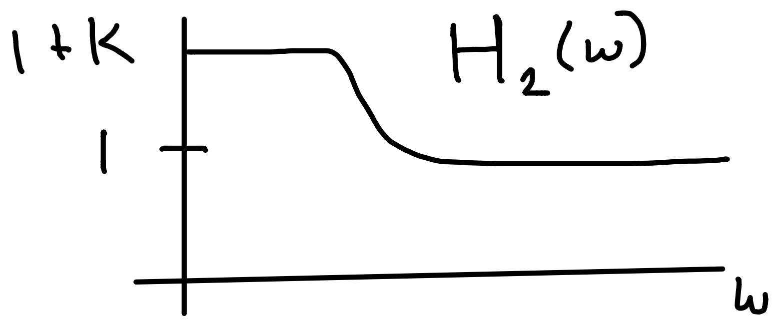


Define

$$P(z) = H(z)H(1/z)$$

P(z) is the product filter

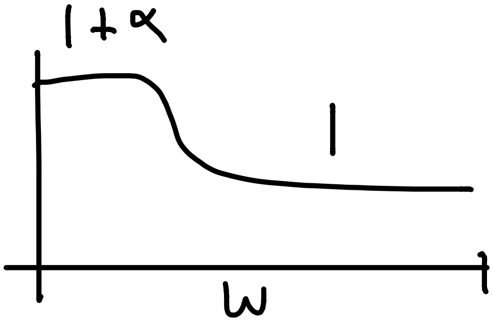
$$P^f(\omega) = H(e^{j\omega})H(e^{-j\omega})$$
$$= H^f(\omega)H^f(-\omega)$$
$$= H^f(\omega)[H^f(\omega)]^*$$
$$= |H^f(\omega)|^2 \quad \text{this real}$$



Suppose we define

$\alpha P(z) + 1 = G(z)$ has the form

$$G(z) = \alpha P(z) + 1 = \alpha H(z) H(1/z) + 1$$



$$= \alpha \frac{B(z) B(1/z)}{A(z) A(1/z)} + 1$$

$$= \frac{\alpha B(z) B(1/z) + A(z) A(1/z)}{A(z) A(1/z)}$$

Given $G(z)$ can we define

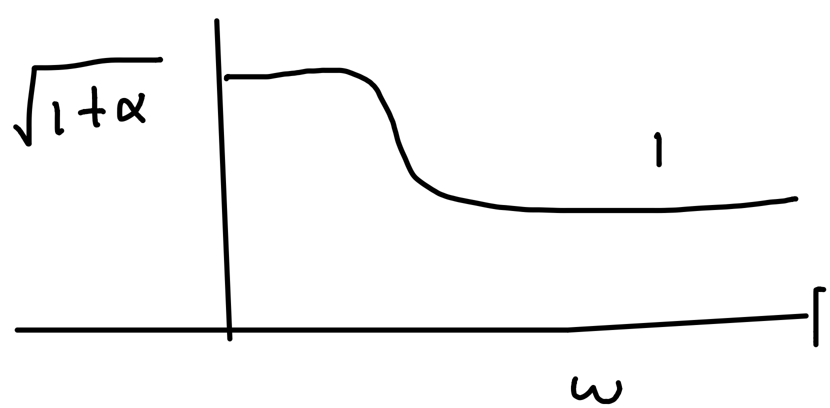
$H_2(z)$ via

$$G(z) = H_2(z) H_2(1/z) \; ?$$

If yes, then

$$G^f(\omega) = |H_2^f(\omega)|^2$$

$H_2(\omega)$



$$H_2(z) = \frac{?}{A(z)} = \frac{B_2(z)}{A(z)}$$

Find

$B_2(z)$ via

$$\alpha B(z) B(1/z) + A(z) A(1/z) = B_2(z) B_2(1/z)$$

Find $B_2(z)$ by Partitioning the roots of the Left-hand-side.