



University College Dublin  
An Coláiste Ollscoile, Baile Átha Cliath

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**SEMESTER II EXAMINATIONS - 2011/2012**

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**School of Mechanical and Materials Engineering**

**MEEN10050 Energy Engineering**

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# SOLUTIONS

**Time Allowed: 2 Hours**

**Instructions for Candidates**

**50 Marks (out of 100) are allocated for Section A (MCQ), based on all 20 Questions.**

- Questions A1 to A20 carry equal marks (+2.5 marks each).
- Negative marks apply for incorrect answers (-0.625 marks each).
- Zero mark applies for a blank answer on any question.

*Please complete **MULTIPLE CHOICE “ANSWER SHEET”** using **HP Pencil**.*

**50 Marks (out of 100) are allocated for Section B are based on any 2 Questions (25 marks each). Numbers in ( ) brackets indicate marks allocated to each part of a question.**

Fluid Property Data and Formulae are included at the end of this document.

**Question A1.**

A student is using a desk lamp with an incandescent bulb of 40 W for about 2000 hours per year. If the cost of electricity is 20 cents per kWh, how much money is he going to spend on artificial lighting during four years of study?

- (A) €6.40
- (B) €64.00
- (C) €640.00
- (D) €128.00

**Correct Answer**

**Solution**

$$\text{Cost} = \text{Power} \times \text{Hours} \times \text{Years} \times \text{Tariff} = 40 \times 2000 \times 4 \times 20 / 1000 = 64 \text{ €}$$

**Question A2.**

From what height does a mass of 42.6 kg have to drop into a swimming pool containing 10,000 kg of water to warm up the water by  $\Delta T = 0.001^\circ\text{C}$  (assuming that the mass eventually decelerates to zero velocity within the pool and that all of the kinetic energy is absorbed by the water).

- (A) 1.0 m
- (B) 10 m
- (C) 100 m
- (D) 1000 m

**Correct Answer**

**Solution**

$${}_1Q_2 = U_2 - U_1 + \frac{m(\bar{V}_2^2 - \bar{V}_1^2)}{2} + m \cdot g \cdot (Z_2 - Z_1) + {}_1W_2$$

$${}_1Q_2 = (U_2 - U_1)_{\text{falling\_mass}} + (U_2 - U_1)_{\text{water}} + \left(\frac{m(\bar{V}_2^2 - \bar{V}_1^2)}{2}\right)_{\text{falling\_mass}} + \left(\frac{m(\bar{V}_2^2 - \bar{V}_1^2)}{2}\right)_{\text{water}} + m_{\text{falling\_mass}} \cdot g \cdot (Z_2 - Z_1) + m_{\text{water}} \cdot g \cdot (Z_2 - Z_1) + {}_1W_2$$

$$0 = 0 + m_{\text{water}} \cdot C_p \cdot \Delta T_{\text{water}} + 0 + 0 + m_{\text{falling\_mass}} \cdot g \cdot (Z_2 - Z_1)_{\text{falling\_mass}} + 0 + 0$$

$$m_{\text{water}} \cdot C_p \cdot \Delta T = m_{\text{falling\_mass}} \cdot g \cdot Z_1$$

$$10,000 \times 4180 \times (0.001) = 42.6 \times 9.81 \times Z_1$$

$$Z_1 = 100 \text{ m (Answer)}$$

**Question A3.**

A triple glazed window consists of 3 clear glass panes of 3 mm thickness each, separated by air cavities of 12 mm each.

The window area is  $1.0 \text{ m}^2$ . If the thermal conductivity of the glass panes is  $0.8 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  and of the air cavities  $0.026 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  what is the total thermal resistance of the window?

(A)  $0.075 \text{ K}\cdot\text{W}^{-1}$

(B)  $0.934 \text{ K}\cdot\text{W}^{-1}$

**Correct Answer**

(C)  $7.5 \text{ K}\cdot\text{W}^{-1}$

(D)  $9 \text{ K}\cdot\text{W}^{-1}$

**Solution**

$$R_{\text{total}} = R_{\text{glass}} + R_{\text{air\_cavity}} + R_{\text{glass}} + R_{\text{air\_cavity}} + R_{\text{glass}}$$

$$= \frac{\Delta x_{\text{glass}}}{k_{\text{glass}}} + \frac{\Delta x_{\text{air\_cavity}}}{k_{\text{air}}} + \frac{\Delta x_{\text{glass}}}{k_{\text{glass}}} + \frac{\Delta x_{\text{air\_cavity}}}{k_{\text{air}}} + \frac{\Delta x_{\text{glass}}}{k_{\text{glass}}}$$

$$R_{\text{total}} = \frac{0.003}{0.8} + \frac{0.012}{0.026} + \frac{0.003}{0.8} + \frac{0.012}{0.026} + \frac{0.003}{0.8}$$

$$R_{\text{total}} = 0.934 \text{ K/W}$$

**Question A4.**

Considering heat losses from a building, which of the following is affected by changes in wind speed?

(A) Radiation heat losses from the roof

(B) Convection heat losses from windows

**Correct Answer**

(C) Conduction heat losses from the floor to the ground

(D) Radiation heat losses from the walls

**Question A5.**

The absorptivity of an opaque surface is 0.8. What is its reflectivity?

(A) 0.2

**Correct Answer**

(B) 0.8

(C) 0.0

(D) 1.8

**Solution**

$$\alpha + \rho = 1$$

$$\rho = 0.2$$

**Question A6.**

Two rooms in the top floor of a building are heated by the same heating system such that an internal temperature of  $+22^\circ\text{C}$  is maintained during a cold night when the ambient air temperature is  $-5^\circ\text{C}$ . The external surface temperature of the roof of room A was measured at  $-3^\circ\text{C}$  whilst that for room B was  $+5^\circ\text{C}$ . Which of the following applies?

(A) Roof of room A is better insulated

**Correct Answer**

(B) Roof of room B is better insulated

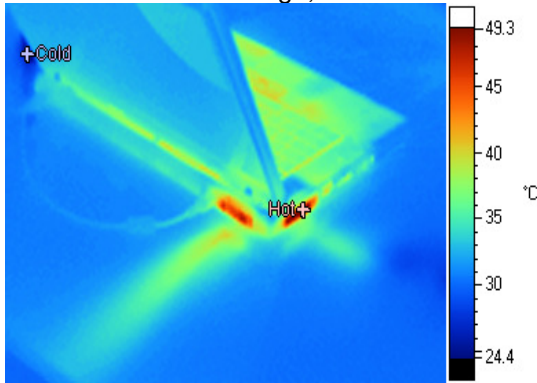
(C) Both roofs are equally well insulated

(D) It is not possible to determine which is better insulated

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**Question A7.**

The thermal image below refers to a laptop computer placed on top of a desk. Focussing on the “Hot+” area at the centre of the image, what is the main mechanism by which the laptop loses heat to the surroundings?



- (A) Conduction
- (B) Free Convection
- (C) Forced Convection
- (D) Radiation

**Correct Answer**

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**Question A8.**

Air is trapped in a rigid vessel, the internal volume of which remains constant. The initial temperature and pressure are 300°C and 300 kPa respectively. Heat loss occurs such that the air temperature reduces to 150°C. The new air pressure in the vessel will be

- (A) 406.5 kPa
- (B) 300 kPa
- (C) 150 kPa
- (D) 221.5 kPa

**Correct Answer**

**Solution to Question A8: Ideal Gas:**

$$\frac{P_1 \cdot V_1}{T_1} = \frac{P_2 \cdot V_2}{T_2} \text{ or, } \{ \text{when } V_1 = V_2 \}, \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ or } P_2 = P_1 \cdot \frac{T_2}{T_1}$$

$$P_2 = (300,000) \cdot \left( \frac{150 + 273.15}{300 + 273.15} \right) = (300,000) \cdot \left( \frac{423.15}{573.15} \right) = (300,000) \cdot (0.7329) = 221,486 = 221.5 \text{ kPa}$$

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**Question A9.**

A closed cylinder with an internal volume of 0.28 m<sup>3</sup> contains Nitrogen (N<sub>2</sub>) gas (M = 28.0 kg/kmol) at 28°C and at an absolute pressure of 280 kPa. Assuming ideal gas behaviour, what will be the mass of N<sub>2</sub>?

- (A) 0.28 kg
- (B) 1.0 kg
- (C) 9.48 kg
- (D) 0.877 kg

**Correct Answer**

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**Solution**

$$R_{\text{nitrogen}} = 296.8 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$R_{N_2} = \frac{R_u}{M_{CO_2}} = \frac{8314.5}{28.0} = 296.9 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$m = \frac{P.V}{R.T} = \frac{(280000).(0.28)}{(296.8).(28.0 + 273.15)} = \frac{(280000).(0.28)}{(296.8).(301.15)} = 0.877 \text{ kg}$$

#### Question A10.

An engine crankshaft rotates at  $6000 \text{ rev} \cdot \text{min}^{-1}$  and transmits a torque of  $31.83 \text{ N} \cdot \text{m}$  to its load. The engine power output is

- (A) 20.0 kW      **Correct Answer**  
 (B) 120 kW  
 (C) 191 kW  
 (D) 1200 kW

**Solution:**

$$\text{Power} = \frac{2.\pi.N.T}{60} (\text{watts}) = \frac{2.\pi.(6000).(31.83)}{60}$$

$$= 19,999 \text{ W} = 20.0 \text{ kW}$$

$N = \text{Rotational Speed (rev/min)}$

#### Question A11.

A tall storage tank contains liquid water at  $45.81^\circ\text{C}$  and is vented to atmosphere at the top (where atmospheric pressure =  $99.97 \text{ kPa}$ ). The distance from the bottom of the tank to the liquid surface level is  $4.72 \text{ m}$ . What will be the absolute pressure at the bottom of the tank?

- (A)  $145.81 \text{ kPa}$       **Correct Answer**  
 (B)  $109.97 \text{ kPa}$   
 (C)  $104.69 \text{ kPa}$   
 (D)  $45.81 \text{ kPa}$

**Solution**

Pressure	Saturation Temp. ( $^\circ\text{C}$ )	Specific Volume ( $\text{m}^3/\text{kg}$ )	
$P$	$T$	$v_f$	$v_g$
$10 \text{ kPa}$	$45.81$	$0.001010$	$14.67$

$$P = P_{atm} + \rho_{H_2O} \cdot g \cdot h = P_{atm} + \frac{g \cdot h}{v_f @ 45.81^\circ\text{C}}$$

$$= 99,965 + \frac{(9.81).(4.72)}{0.001010} = 99,965 + 45,845$$

$$= 145,810 \text{ Pa} = 145.81 \text{ kPa}$$

**Question A12.**

0.5 kg of water is contained in a rigid vessel at a temperature of 125°C and a pressure of 232.1 kPa. The specific enthalpy is 1050 kJ/kg. Which of the following could also be correct?

(A)  $v = 0.001065 \text{ m}^3/\text{kg}$

(B)  $v = 0.7706 \text{ m}^3/\text{kg}$

(B)  $x = 0.5$

(D)  $x = 0.24$

**Correct Answer**

**Solution**

Pressure	Saturation Temp. (°C)	Specific Volume (m <sup>3</sup> /kg)		Specific Enthalpy (kJ/kg)		
$P$	$T$	$v_f$	$v_g$	$h_f$	$h_{fg}$	$h_g$
232.1 kPa	125.00	0.001065	0.7706	524.99	2188.5	2713.5

$$h = h_f + x.h_{fg} = 524.99 + (0.24).(2188.5) = 1050 \text{ kJ/kg}$$

**Question A13.**

An ideal gas undergoes a polytropic compression process where the pressure increases from 125 kPa to 250 kPa whilst volume reduces from 1.0 m<sup>3</sup> to 0.57435 m<sup>3</sup>. What is the polytropic index  $n$ ?

(A) 1.0

(B) 1.25

**Correct Answer**

(C) 2.0

(D) 0.57435

**Solution**

$$P_1 V_1^n = P_2 V_2^n = (125).(1.0)^n = (250).(0.57435)^n$$

$$\left(\frac{P_1}{P_2}\right) = \left(\frac{V_2}{V_1}\right)^n \quad \text{or} \quad \left(\frac{125}{250}\right) = \left(\frac{0.57435}{1.0}\right)^n \quad \text{or} \quad 0.5 = (0.57435)^n$$

$$\ln(0.5) = n.(\ln(0.57435)) \quad \text{or} \quad (-0.69315) = n.(-0.55452)$$

$$n = \frac{-0.69315}{-0.55452} = 1.25$$

**Question A14.**

Heat is added to a closed cylinder of constant pressure containing 0.718 kg of air. What will be the heat input needed to increase the temperature of the air from 20°C to 120°C?

(A) 0.139 kJ

(B) 51.55 kJ

(C) 72.16 kJ

**Correct Answer**

(D) 100.5 kJ

**Solution**

$C_p$  for air

$$= 1005 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$$

$C_v$  for air

$$= 718 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$$

$$\begin{aligned} {}_1Q_2 &= U_2 - U_1 + {}_1W_2 = H_2 - H_1 \\ &= m \cdot C_{p(\text{CO}_2)} \cdot (T_2 - T_1) = (0.718) \cdot (1005) \cdot (120 - 20) \\ &= 72,159 \text{ J} = 72.16 \text{ kJ} \end{aligned}$$

**Question A15.**

The units of entropy S are

(A) J

(B)  $\text{J}\cdot\text{kg}^{-1}$

(C)  $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$

(D)  $\text{J}\cdot\text{K}^{-1}$

**Correct Answer**

**Question A16.**

A rigid stainless steel tank with an internal volume of 5 m<sup>3</sup> contains 2.5 kg of oxygen (O<sub>2</sub>) gas (M = 32.0 kg·kmol<sup>-1</sup>). The specific volume is

(A) 0.5 m<sup>3</sup>·kg

(B) 2.0 kg·m<sup>-3</sup>

(C) 2.0 m<sup>3</sup>·kg<sup>-1</sup>

**Correct Answer**

(D) 16.0 kmol·m<sup>-3</sup>

**Solution**

$$v = \frac{V}{m} = \frac{5 \text{ m}^3}{2.5 \text{ kg}} = 2.0 \text{ m}^3 \cdot \text{kg}^{-1}$$

**Question A17.**

A closed Thermodynamic System

(A) Always has a constant volume

(B) Always has a constant mass

**Correct Answer**

(C) Always contains heat

(D) Always contains work

**Question A18.**

Condensing of a refrigerant in the ideal vapour compression refrigeration cycle occurs at constant

- (A) Temperature
- (B) Pressure
- (C) Specific enthalpy
- (D) Specific entropy

**Correct Answer**

**Question A19.**

0.19444 kg of H<sub>2</sub>O is stored in a rigid vessel at 1.0 MPa, 200°C. Is this

- (A) Superheated vapour ?
- (B) Sub-cooled liquid ?
- (C) Saturated vapour ?
- (D) Saturated liquid ?

**Correct Answer**

Pressure	Satura- tion Temp. (°C)	Specific Volume (m <sup>3</sup> •kg <sup>-1</sup> )	
		$v_f$	$v_g$
1.0 MPa	179.91	0.001127	0.19444

At P = 1.0 MPa, T<sub>sat</sub> = 179.91 °C. At T = 200°C, H<sub>2</sub>O is superheated

**Question A20.**

Saturated liquid R134a refrigerant at 0.770 MPa, 30°C ( $h_f = 91.49 \text{ kJ}\cdot\text{kg}^{-1}$ ) is throttled to 0.20 MPa in a steady state, steady flow process. At this exit condition T<sub>sat</sub> = -10.09°C,  $h_f = 36.84 \text{ kJ}\cdot\text{kg}^{-1}$ ,  $h_g = 241.30 \text{ kJ}\cdot\text{kg}^{-1}$ . Is the dryness fraction at exit

- (A) 0.447
- (B) 0.379
- (C) 0.267
- (D) 0.226

**Correct Answer**

$$h_i = h_e = h_f + (x)(h_g - h_f) = 91.49 = 36.84 + (x)(241.3 - 36.84)$$

$$x = \frac{91.49 - 36.84}{204.46} = \frac{54.65}{204.46} = 0.267$$



## SECTION B (50 Marks) (please use an Answer Book)

Marks for Section B are based on any two Questions (25 marks each)

### Question B1 (25 Marks)

A room in an office building has a floor area of  $20 \text{ m}^2$ . It has an external wall consisting {from inside to outside} of 2 cm plaster ( $k = 0.8 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ ), 30 cm concrete ( $k = 2.0 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ ), 10 cm insulation ( $k = 0.04 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ ) and 2 cm plaster ( $k = 0.8 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ ). The total surface area of the wall is  $10 \text{ m}^2$ .

The interior convection coefficient is  $8 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ , the exterior convection coefficient is  $25 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ . Assume that the specific heat capacity of air  $1005 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$  and that the air density is  $1.3 \text{ kg}\cdot\text{m}^{-3}$ .

The thermostat of the room is set at  $22^\circ\text{C}$  and the ventilation rate is set at  $12 \text{ litres}\cdot\text{s}^{-1}$  per person. The following heat inputs are present:

Lap-top computer (40 W)

Lighting ( $10 \text{ W}\cdot\text{m}^{-2}$  of floor area)

1 person (120 W of sensible heat)

The minimum ambient air temperature is  $-5^\circ\text{C}$ .

- (a) What is the U-value of the wall? (10 marks)
- (b) What is the minimum capacity of a heater for that room? (10 marks)
- (c) What is the temperature of the external surface of the wall when the minimum temperature is observed? (5 marks)

## Solution

Equations;

Thermal Resistances (in units of  $\text{K}\cdot\text{W}^{-1}$ ):

$$R_{\text{cond}} = \frac{L}{k.A}$$

$$R_{\text{conv}} = \frac{1}{h.A}$$

Overall:

$$\dot{Q} = U.A.\Delta T_{\text{overall}}$$

$$\dot{Q} = \frac{\Delta T_{\text{overall}}}{R_{\text{total}}}$$

$$U.A = 1/R_{\text{total}}$$

$$R_{\text{total}} = \frac{1}{U_{\text{overall}}.A} = \text{K}\cdot\text{W}^{-1}$$

$$U_{\text{overall}} (\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}) = \frac{1}{R_{\text{total}} (\text{K}\cdot\text{W}^{-1}).A (\text{m}^2)}$$

Multi-Layered Plane Wall:

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{wall},1} + R_{\text{wall},2} + R_{\text{conv},2} = \frac{1}{h_1.A} + \frac{L_1}{k_1.A} + \frac{L_2}{k_2.A} + \frac{1}{h_2.A}$$

a)

Thermal resistances (in units of  $\text{K}\cdot\text{W}^{-1}$ ):

due to conduction

$$R_{\text{cond}} = \frac{L}{k.A} (\text{K}\cdot\text{W}^{-1})$$

and convection

$$R_{\text{conv}} = \frac{1}{h.A} (\text{K}\cdot\text{W}^{-1})$$

Total Thermal resistance of wall (in units of  $\text{K}\cdot\text{W}^{-1}$ ):

$$R_{\text{total}} = R_{\text{conv},i} + R_{\text{plast}} + R_{\text{concr}} + R_{\text{ins}} + R_{\text{plast}} + R_{\text{conv},o} = \frac{1}{h_i.A} + \frac{\Delta x_{\text{plast}}}{k_{\text{plast}}.A} + \frac{\Delta x_{\text{concr}}}{k_{\text{concr}}.A} + \frac{\Delta x_{\text{ins}}}{k_{\text{ins}}.A} + \frac{\Delta x_{\text{plast}}}{k_{\text{plast}}.A} + \frac{1}{h_o.A}$$

$$R_{total} = \left( \frac{1}{A} \right) \left( \frac{1}{h_i} + \frac{\Delta x_{plast}}{k_{plast}} + \frac{\Delta x_{concr}}{k_{concr}} + \frac{\Delta x_{ins}}{k_{ins}} + \frac{\Delta x_{plast}}{k_{plast}} + \frac{1}{h_o} \right) \quad (K.W^{-1})$$

$$R_{total} = \frac{1}{U_{overall} \cdot A} = \left( \frac{1}{10} \right) \left( \frac{1}{8} + \frac{0.02}{0.8} + \frac{0.3}{2} + \frac{0.10}{0.04} + \frac{0.02}{0.8} + \frac{1}{25} \right) = (0.10)(2.8625) = 0.28625 \text{ K.W}^{-1}$$

$$R_{total} = 0.28625 \text{ K.W}^{-1}$$

U-value of brick wall

$$U_{overall} \text{ (W.m}^{-2} \cdot \text{K}^{-1}) = \frac{1}{R_{total} \cdot A}$$

$$U = \frac{1}{(0.28625) \cdot (10)} = 0.3493 \text{ (W.m}^{-2} \cdot \text{K}^{-1})$$

$$U = 0.349 \text{ W/m}^2\text{K} \quad \textbf{Answer (a)}$$

(b)

$$\dot{Q} = \dot{Q}_{peopl} + \dot{Q}_{laptop} + \dot{Q}_{lights} - \dot{Q}_{vent} - \dot{Q}_{heatlosses}$$

$$\dot{Q} = \dot{Q}_{peopl} + \dot{Q}_{laptop} + \dot{q}_{lights} \cdot F - \dot{m}_{vent} \cdot c_p \cdot \Delta T - \frac{\Delta T}{R_{total}}$$

$$\dot{Q} = \dot{Q}_{peopl} + \dot{Q}_{laptop} + \dot{q}_{lights} \cdot F - \dot{V}_{vent} \cdot \rho \cdot c_p \cdot \Delta T - U \cdot A \cdot \Delta T = U \cdot A \cdot (T_i - T_o)$$

$$\dot{Q} = 120 + 40 + 10 \cdot 20 - 0.012 \cdot 1.3 \cdot 1005 \cdot 1 \cdot (22 - (-5)) - 0.349 \cdot 10 \cdot (22 - (-5))$$

$$\dot{Q} = 157 \text{ W}$$

**Answer (b)**

(c)

$$(T_1 - T_2) = \frac{\dot{Q}}{h_i \cdot A}$$

$$(T_1 - (-5)) = \frac{0.349 \cdot (22 - (-5))}{25 \cdot 10}$$

$$\text{or } T_1 = -4.62^\circ\text{C} \quad \textbf{Answer (c)}$$

### Question B2 (25 Marks)

During some actual expansion and compression processes in piston-cylinder devices, the gases have been observed to satisfy the relationship  $P.V^n = \text{constant}$ , where the polytropic index  $n$  is also constant.

Assuming such a process to be internally reversible, show that the work done (during a change of state from initial state 1 to final state 2) is given by;

$${}_1W_2 = \frac{P_2.V_2 - P_1.V_1}{1-n} \quad n \neq 1$$

(10 marks)

A mass of 0.12 kg of air undergoes a polytropic compression in a piston-cylinder assembly from an initial  $P_1 = 100 \text{ kPa}$  and temperature  $T_1 = 17^\circ\text{C}$  to a final pressure  $P_2 = 800 \text{ kPa}$ . Assuming ideal gas behaviour and a polytropic index  $n = 1.35$ , determine;

(1) the final volume, and

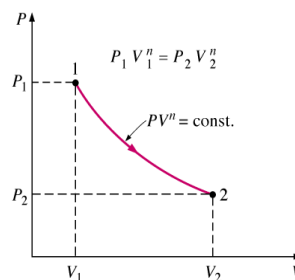
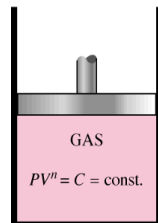
(8 marks)

(2) the final temperature ( $^\circ\text{C}$ )

(7 marks)

### SOLUTION B2

### Polytropic Process - Solution -



$${}_1W_2 = \int_1^2 P.dV \quad P.V^n = \text{Const}$$

$$P = \text{Const}.V^{-n} = P_1 V_1^n . V^{-n} = P_2 V_2^n . V^{-n}$$

$${}_1W_2 = \int_1^2 P.dV = \text{Const}. \int_1^2 V^{-n} dV = \text{Const}. \left[ \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} \right] \quad (n \neq 1)$$

$${}_1W_2 = \left[ \frac{P_2.V_2^n.V_2^{-n+1} - P_1.V_1^n.V_1^{-n+1}}{-n+1} \right] \quad \text{or} \quad {}_1W_2 = \left[ \frac{P_2.V_2 - P_1.V_1}{1-n} \right] \quad n \neq 1$$

A mass of 0.12 kg of air undergoes a polytropic compression in a piston-cylinder assembly from an initial  $P_1 = 100 \text{ kPa}$  and temperature  $T_1 = 17^\circ\text{C}$  to a final pressure  $P_2 = 800 \text{ kPa}$ . Assuming ideal gas behaviour and a polytropic index  $n = 1.35$ , determine;

(1) the final volume, and

(8 marks)

(2) the final temperature ( $^\circ\text{C}$ )

(7 marks)

Initially (at State "1"), the absolute Air pressure is;  $P_1 = 100 \text{ (kPa)}$

Treating Air as an ideal gas  $P.V = m.R.T$

$$\text{Initial Volume: } V_1 = \frac{m.R.T_1}{P_1} = \frac{(0.12).(287).(273+17)}{(100000)} = 0.0999 = 0.10 \text{ m}^3$$

$$\text{Volume at State "2" is given by; } V_2^{1.35} = V_1^{1.35} \left( \frac{P_1}{P_2} \right)$$

$$V_2 = V_1 \left( \frac{P_1}{P_2} \right)^{1/1.35} = V_1 \left( \frac{P_1}{P_2} \right)^{0.7407} = (0.10) \cdot \left( \frac{100}{800} \right)^{0.7407} = (0.10) \cdot (0.2143) = 0.02143 \text{ m}^3 \quad \text{(Answer (1))}$$

$$T_2 = \frac{P_2.V_2}{m.R} = \frac{(800000).(0.02143)}{(0.12).(287)} = 497.86 \text{ K} = 224.7^\circ\text{C}$$

**(Answer (2))**

Work Done (not required in this problem):

$${}_1W_2 = \left[ \frac{P_2.V_2 - P_1.V_1}{1-n} \right] = \left[ \frac{(800000).(0.02143) - (100000).(0.10)}{-0.35} \right] =$$

$$\left[ \frac{(17,144) - (10,000)}{-0.35} \right] = \left[ \frac{(7144)}{-0.35} \right] = -20,411 \text{ J} = -20.4 \text{ kJ}$$

### Question B3 (25 Marks)

A pumped domestic shower unit incorporates an electrical resistance heater to increase the temperature of liquid water before it is directed towards the shower head. The heater in a specific model draws 7.2 kW of electric power and operates in a steady state with an inlet water supply temperature of 5°C during cold weather.

- (A) What inlet water flow rate (in *litres per minute*) can be sustained if the outlet water temperature is to be 50°C at the exit from the heater? (12 marks)
- (B) What exit temperature (°C) would be achieved if this water flow rate were increased by 60%? (8 marks)

In working through your answers, write down any equation used, explain why you are using it and note any simplifying assumptions made in the course of your analysis.

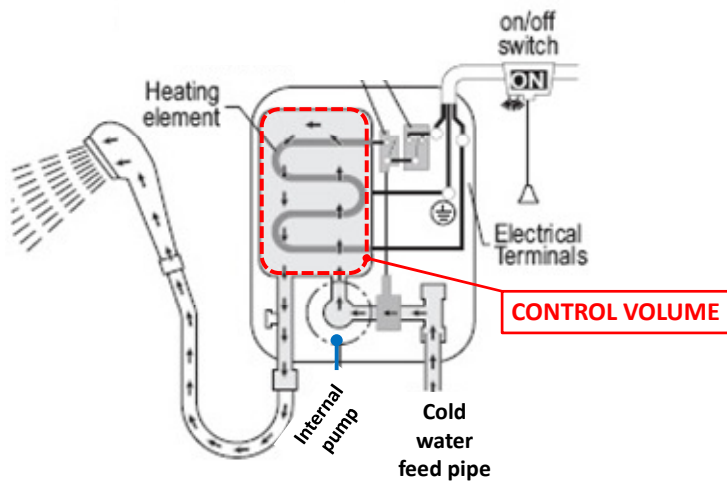
(5 marks)

### Solution

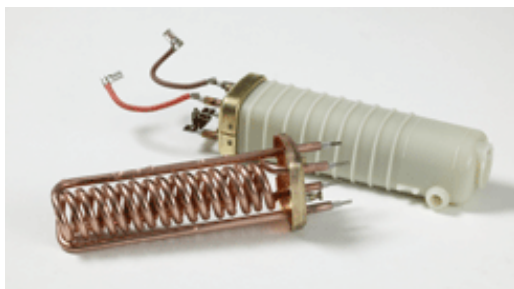
$$\dot{Q}_{cv} + \sum \dot{m}_i \left( h_i + \frac{\overline{V}_i^2}{2} + gZ_i \right) = \sum \dot{m}_e \left( h_e + \frac{\overline{V}_e^2}{2} + gZ_e \right) + \dot{W}_{cv}$$



[http://www.electricpoint.com.sg/images/water%20heater/joven%20EC630d\\_500.jpg](http://www.electricpoint.com.sg/images/water%20heater/joven%20EC630d_500.jpg)



[http://jimwolverson.files.wordpress.com/2013/09/shower\\_schematic01.jpg](http://jimwolverson.files.wordpress.com/2013/09/shower_schematic01.jpg)



**Typical electric shower heating element + casing**

Electrical energy input to the heating element (& therefore to the control volume, as described in the diagram above) is equivalent to work, not heat.

Therefore the power “output” from the control volume (which has a negative magnitude because it involves an input of power to the control volume) is:  $\dot{W}_{cv} = -7,200 \text{ watts}$

Neglecting any heat losses from the external surfaces of the heating chamber:

$$\dot{Q}_{cv} = 0 \quad (\text{i.e. assuming that electrical resistance heater is inside the control volume})$$

Assuming steady state, steady flow:  $\dot{m}_i = \dot{m}_e = \dot{m}$

Neglecting KE and PE changes:  $\frac{\bar{V}_i^2}{2} + gZ_i = \frac{\bar{V}_e^2}{2} + gZ_e$

First Law reduces to;  $\dot{m}h_i = \dot{m}h_e + \dot{W}_{cv}$  or  $-\dot{W}_{cv} = \dot{m}(h_e - h_i) = \dot{m}C_p(T_e - T_i)$

$$\dot{m} = \frac{-\dot{W}_{cv}}{(h_e - h_i)} = \frac{-\dot{W}_{cv}}{C_p(T_e - T_i)}$$

$$\dot{m} = \frac{7,200}{C_p(T_e - T_i)} = \frac{7.2 \times 10^3}{(4180)(50 - 5.0)} = \frac{7.2 \times 10^3}{(4180)(45.0)} = 0.03828 \text{ kg.s}^{-1} = 0.0383 \text{ kg.s}^{-1}$$

Volume Flow rate:  $Q \text{ (m}^3/\text{s)} = \dot{m} \text{ (kg/s)} \cdot v_f \text{ (m}^3/\text{kg)}$

Where the specific volume of the inlet water is best approximated by  $v_f @ 5^\circ\text{C} = 0.00100 \text{ m}^3.\text{kg}^{-1}$

(Strictly the pressure of the water at this point is likely to be above atmospheric so the water will be a sub-cooled liquid when at  $5^\circ\text{C}$ . In this case the thermodynamic properties are best approximated by looking at values in the tables for the correct temperature. Changes in pressure have a negligible effect on the specific volume or the specific enthalpy of a liquid, when compared with the effects of temperature change.)

$$Q \text{ (m}^3/\text{s)} = 0.03828 \text{ (kg/s)} \cdot (0.00100 \text{ m}^3/\text{kg}) = 0.00003828 \text{ m}^3/\text{s} = 3.83 \times 10^{-5} \text{ (m}^3/\text{s)}$$

$$Q \text{ (litres/min)} = 3.828 \times 10^{-2} \text{ (litres/s)} \cdot (60) = 2.297 \text{ litres/min} = 2.3 \text{ litres/min}$$

**(ANSWER)**

OR

$$h_{f\_50C} = 209.34 \text{ kJ/kg}$$

$$h_{f\_5C} = 21.02 \text{ kJ/kg}$$

$$\dot{m} = \frac{-\dot{W}_{cv}}{(h_e - h_i)} \quad \dot{m}_{H20} = \frac{7200}{(209.34 - 21.02)} = \frac{7200}{188.32 \times 10^3} = 0.03823 \text{ kg/s}$$

$$\dot{V}_{H20} = (0.03823 \text{ kg/s}) \cdot (0.001 \text{ m}^3/\text{kg}) = 0.0000382 \text{ m}^3/\text{s} = 0.0382 \text{ litres/s} = 2.294 \text{ litres/min}$$

(C) What exit temperature would be achieved if this water flow rate were increased by 60%?

$$\text{New mass flow rate} = (1.6) \cdot (0.0383) = 0.06128 \text{ kg/s}$$

$$T_e = 5.0 + \left( \frac{7,200}{(4,180)(1.6)(0.03828)} \right) = 5.0 + 28.12 = 33.12^\circ\text{C}$$

**(ANSWER)**

## Properties of H<sub>2</sub>O

**Table 1: Saturated Water & Steam**

Pressure	Satura- tion Temp. (°C)	Specific Volume (m <sup>3</sup> •kg <sup>-1</sup> )		Specific Internal Energy (kJ•kg <sup>-1</sup> )			Specific Enthalpy (kJ•kg <sup>-1</sup> )			Specific Entropy (kJ•kg <sup>-1</sup> •K <sup>-1</sup> )		
$P$	$T_{sat.}$	$v_f$	$v_g$	$u_f$	$u_{fg}$	$u_g$	$h_f$	$h_{fg}$	$h_g$	$s_f$	$s_{fg}$	$s_g$
0.87 kPa	5.00	0.001000	147.01	21.0	2360.8	2381.8	21.02	2489.1	2510.1	0.0763	8.9585	9.0248
2.0 kPa	17.50	0.001001	67.00	73.48	2326.0	2399.5	73.48	2460.0	2533.5	0.2607	8.4629	8.7237
10 kPa	45.81	0.001010	14.67	191.8	2246.1	2437.9	191.8	2392.8	2584.7	0.6493	7.5009	8.1502
12.35 kPa	50.0	0.001012	12.03	209.3	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
20 kPa	60.06	0.001017	7.649	251.38	2205.4	2456.7	251.40	2358.3	2609.7	0.8320	7.0766	7.9085
100 kPa	99.63	0.001043	1.6940	417.4	2088.7	2506.1	417.5	2258.0	2675.5	1.3026	6.0568	7.7394
101.325 kPa	100.00	0.001044	1.6729	418.94	2087.6	2506.5	419.04	2257.0	2676.1	1.3069	6.0480	7.3549
200 kPa	120.23	0.001061	0.8857	504.49	2025.0	2529.5	504.7	2201.9	2706.7	1.5301	5.5970	7.1271
232.1 kPa	125.00	0.001065	0.7706	524.74	2009.9	2534.6	524.99	2188.5	2713.5	1.5813	5.4962	7.0775
500 kPa	151.86	0.001093	0.3749	639.68	1921.6	2561.2	640.23	2108.5	2748.7	1.8607	4.9606	6.8213
1.0 MPa	179.91	0.001127	0.19444	761.7	1822.0	2583.6	762.8	2015.3	2778.1	2.1387	4.4478	6.5865
10 MPa	311.06	0.001452	0.018026	1393.0	1151.4	2544.4	1407.7	1317.1	2724.7	3.3596	2.2544	5.6141
14 MPa	336.75	0.001611	0.011485	1548.6	928.2	2476.8	1571.1	1066.5	2637.6	3.6232	1.7485	5.3717
15 MPa	342.24	0.001658	0.010337	1585.6	869.8	2455.5	1610.5	1000.0	2610.5	3.6848	1.6249	5.3098

**Table 2: Superheated Steam**

Pressure	Temp. (°C)	Specific Volume (m <sup>3</sup> •kg <sup>-1</sup> )	Specific Internal Energy (kJ•kg <sup>-1</sup> )	Specific Enthalpy (kJ•kg <sup>-1</sup> )	Specific Entropy (kJ•kg <sup>-1</sup> •K <sup>-1</sup> )
$P$	$T$	$v$	$u$	$h$	$s$
10 kPa	500	35.679	3132.3	3489.1	9.8978
100 kPa	500	3.565	3131.6	3488.1	8.8342
200 kPa	500	1.7814	3130.8	3487.1	8.5133
1.0 MPa	500	0.3541	3124.4	3478.5	7.7622
10 MPa	500	0.03279	3045.8	3373.7	6.5966
15.0 MPa	500	0.02080	2996.6	3308.6	6.3443

## MEEN 10050 - Energy Engineering Formulae

### Heat Transfer

Conduction:  $\dot{Q}_{\text{conduction}} = -k.A.\frac{dT}{dX}$

Convection:  $\dot{Q}_{\text{conv}} = h.A.(T_{\text{surface}} - T_{\text{fluid}})$

Radiation:  $\dot{Q}_{\text{radiation-net}} = \epsilon_s \cdot \sigma \cdot A_s \cdot (T_s^4 - T_{\text{surr}}^4)$

Stefan-Boltzmann Const.  $\sigma = 5.67 \times 10^{-8} \text{ W.m}^{-2}\text{K}^{-4}$

Thermal Resistances ( $\text{K} \cdot \text{W}^{-1}$ ):  $R_{\text{cond}} = \frac{L}{k.A}$

$R_{\text{conv}} = \frac{1}{h.A}$

Resistances in Series:  $R_{\text{total}} = R_1 + R_2 = \sum R$

In Parallel:  $\frac{1}{R_{\text{total}}} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{R_1 \cdot R_2}{R_1 + R_2}$

Overall:  $\dot{Q} = U.A.\Delta T_{\text{overall}}$

$\dot{Q} = \frac{\Delta T_{\text{overall}}}{R_{\text{total}}}$

$U.A = 1/R_{\text{total}}$

Single-Layer Plane Wall:

$\dot{Q} = \frac{(T_i - T_o)}{\left( \frac{1}{h_i.A} + \frac{\Delta x}{k_w.A} + \frac{1}{h_o.A} \right)}$

$\frac{1}{U.A} = \frac{1}{h_i} + \frac{\Delta x}{k_w} + \frac{1}{h_o}$

Multi-Layered Plane Wall:

$R_{\text{total}} = R_{\text{conv},1} + R_{\text{wall},1} + R_{\text{wall},2} + R_{\text{conv},2} = \frac{1}{h_1.A} + \frac{L_1}{k_1.A} + \frac{L_2}{k_2.A} + \frac{1}{h_2.A}$

### Ideal Gases:

$P.v = R.T$

$P.V = m.R.T$

$C_{P_0} = C_{v_0} + R$

$du = C_{v_0}.dT$

$dh = C_{P_0}.dT$

$u = \int_0^T C_{v_0} dT = C_{v_0}.T$

### Specific Heats:

$C_{v_0} = \left( \frac{\partial u}{\partial T} \right)_{v=\text{const}}$

$C_{P_0} = \left( \frac{\partial h}{\partial T} \right)_{P=\text{const}}$

### Mixtures of Liquid & Vapour:

Specific Volume

$v_{\text{mix}} = v_f + x.v_{fg}$

$v_{fg} = v_g - v_f$

$V = m.v$

Specific Internal Energy

$u_{\text{mix}} = u_f + x.u_{fg}$

$u_{fg} = u_g - u_f$

$U = m.u$

Specific Enthalpy  $h = u + P.v$

$h_{\text{mix}} = h_f + x.h_{fg}$

$h_{fg} = h_g - h_f$

$H = m.h$

Specific Entropy

$s_{\text{mix}} = s_f + x.s_{fg}$

$s_{fg} = s_g - s_f$

$S = m.s$

$dS = \left( \frac{\delta Q}{T} \right)_{\text{int. rev}}$

$x = (m_{\text{vapour}})/(m_{\text{total}})$



**Work:**  $\delta W = \int P.dV$  (rev)  ${}_1W_2 = \int_1^2 \delta W = \int_1^2 P.dV$   ${}_1W_2 = P(V_2 - V_1)$  {constant P}

${}_1W_2 = \frac{P_2.V_2 - P_1.V_1}{1-n}$  {polytropic:  $P.V^n = \text{constant}$ ,  $n \neq 1.0$ }  ${}_1W_2 = P_1.V_1 \cdot \ln\left(\frac{V_2}{V_1}\right)$  {P.V=constant}

$W_{\text{net cycle}} = \oint_{\text{cycle}} P.dV$   $P = \omega.T = 2.\pi.N.T$

**First Law:**  $\oint \delta Q = \oint \delta W$  (cycle)  $\delta Q = dE + \delta W$  (process)

**First Law - System:**  ${}_1Q_2 = U_2 - U_1 + \frac{m(\bar{V}_2^2 - \bar{V}_1^2)}{2} + m.g.(Z_2 - Z_1) + {}_1W_2$   $\dot{Q} = \frac{dE}{dt} + \dot{W}$

**First Law - Control Volume:**

$$\dot{Q}_{cv} + \sum \dot{m}_i \left( h_i + \frac{\bar{V}_i^2}{2} + gZ_i \right) = \sum \dot{m}_e \left( h_e + \frac{\bar{V}_e^2}{2} + gZ_e \right) + \dot{W}_{cv} + \frac{d}{dt} \left( U + m.\bar{V}^2/2 + m.g.Z \right)$$

**SSSF:**  $\dot{Q}_{cv} + \sum \dot{m}_i \left( h_i + \frac{\bar{V}_i^2}{2} + gZ_i \right) = \sum \dot{m}_e \left( h_e + \frac{\bar{V}_e^2}{2} + gZ_e \right) + \dot{W}_{cv}$

$\dot{m} = \rho.A.\bar{V}_{\text{average}}$  **SSSF**  $q_{cv} + h_i + \frac{\bar{V}_i^2}{2} + gZ_i = h_e + \frac{\bar{V}_e^2}{2} + gZ_e + w_{cv}$

**Heat Engines:** 1<sup>st</sup> Law:  $W_{\text{net,out}} = Q_{\text{in}} - Q_{\text{out}}$  **Efficiency:**  $\eta_{th} = \frac{\dot{W}_{\text{net,out}}}{\dot{Q}_{\text{in}}} = \frac{W_{\text{net,out}}}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}}$

**I.C. Engine**  $\dot{W} = \omega.T = 2.\pi.N.T$  {N in rev.s<sup>-1</sup>}  $i.m.e.p = \frac{\oint_{\text{cycle}} P.dV}{V_d}$   $i.m.e.p = \frac{P_i}{\left( \frac{N}{n_r} \right) V_d}$

**Refrigeration:**  $C.O.P._R = \frac{\dot{Q}_L}{\dot{W}_{in}}$  **Heat Pump:**  $C.O.P._{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}}$

**Additional Data** which may or may not be required. (Sections A and B)

0°C	= 273.15 K
R <sub>u</sub>	= 8314.5 J.kg <sup>-1</sup> .K <sup>-1</sup>
R <sub>air</sub>	= 287.0 J.kg <sup>-1</sup> .K <sup>-1</sup>
R <sub>nitrogen</sub>	= 296.8 J.kg <sup>-1</sup> .K <sup>-1</sup>
R <sub>water vapour</sub>	= 461.9 J.kg <sup>-1</sup> .K <sup>-1</sup>
R <sub>carbon dioxide (CO2)</sub>	= 189 J.kg <sup>-1</sup> .K <sup>-1</sup>
M <sub>air</sub>	= 28.96 kg.kmol <sup>-1</sup>
M <sub>oxygen (O2)</sub>	= 32.0 kg.kmol <sup>-1</sup>
M <sub>nitrogen (N2)</sub>	= 28.0 kg.kmol <sup>-1</sup>
M <sub>carbon dioxide (CO2)</sub>	= 44.0 kg.kmol <sup>-1</sup>
M <sub>water vapour (H2O)</sub>	= 18.0 kg.kmol <sup>-1</sup>

C <sub>p</sub> for air	= 1005 J.kg <sup>-1</sup> .K <sup>-1</sup>
C <sub>v</sub> for air	= 718 J.kg <sup>-1</sup> .K <sup>-1</sup>
C <sub>p</sub> for liquid water	= 4180 J.kg <sup>-1</sup> .K <sup>-1</sup>
C <sub>p</sub> for nitrogen	= 1039 J.kg <sup>-1</sup> .K <sup>-1</sup>
C <sub>v</sub> for nitrogen	= 743 J.kg <sup>-1</sup> .K <sup>-1</sup>
C <sub>p</sub> for carbon dioxide	= 846 J.kg <sup>-1</sup> .K <sup>-1</sup>
C <sub>v</sub> for carbon dioxide	= 657 J.kg <sup>-1</sup> .K <sup>-1</sup>
Acceleration due to gravity g	= 9.81 m.s <sup>-2</sup>
Standard Atmospheric Press.	= 101.325 kPa (760 mm Hg)
Approx. composition of air:	
	3.76 kmol N <sub>2</sub> : 1 kmol O <sub>2</sub>