University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 1 EXAMINATIONS 2011/2012

MATH 10260

Linear Algebra for Engineers

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Time Allowed: 2 hours

Instructions for Candidates

Attempt **Question 1** (each part worth 5 marks) and
three other questions, each other question being worth 15 marks.

Details of calculations leading to your answers must be included.

No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Candidates are **not** allowed to use mathematical tables during this examination.

1. (a) The complex number $z = a + bi$, where a and b are real numbers, and $i^2 = -1$, satisfies the equation

$$\frac{z+i}{z-2} = 2 - 2i.$$

Find the values of a and b and find the modulus $|z|$ of z.

(b) Find the general solution of the following system of linear equations:

$$\begin{aligned} x + y + \ z &= 1 \\ x \quad\ + \ z &= -1 \\ x - y + 2z &= 0. \end{aligned}$$

(c) Find the eigenvalues of the matrix A, where

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 8 & 0 & 0 \end{pmatrix}.$$

(d) The 2×2 matrices A, B and C satisfy the equation $AC = B + B^2$. Find A explicitly, given that

$$B = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix}.$$

(e) Find the parametric equations of the line passing through the point $(0, 1, 2)$ that is perpendicular to the plane containing the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

2. (a) Let $z = a + bi$, where a and b are real numbers and $i^2 = -1$. Find the possible values of a and b, given that

$$z = 2|z| - 7 - 4i.$$

(b) Find the parametric equations of the line that passes through the point $(1, 1, -1)$ and is perpendicular to all lines with direction vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.

(c) Find the equation of the plane that contains the point $(2, -1, 1)$ and is perpendicular to the planes with equations $x - y + z = 1$ and $2x + y - 3z = 1$. Note that planes are perpendicular when their normal vectors are perpendicular.

3. (a) Find the value of c for which the following system of linear equations in x, y and z is consistent, and find the general solution of the system when c has this value:

$$\begin{aligned} x - 3y + 2z &= 4 \\ 2x + \ y - \ z &= 1 \\ 3x - 2y + \ z &= c. \end{aligned}$$

(b) Find the solution of the following system of three equations:

$$\begin{aligned} x + y - 3z + \ w &= 5 \\ 2x - y + \ z - 2w &= 2 \\ 7x + y - 7z + 3w &= 3. \end{aligned}$$

4. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 2 & 0 \\ -1 & 2 & 0 \end{pmatrix}.$$

(Note that 0 is an eigenvalue of A.) Hence find an invertible matrix which diagonalizes A.

(b) A 3×3 matrix B has trace equal to 4 and determinant equal to -8. Given that 2 is an eigenvalue of B, find the other two eigenvalues of B.

5. (a) Let c be a real number and let A be the matrix

$$A = \begin{pmatrix} 1 & c & 1 \\ c & 1 & c \\ 1 & c & c \end{pmatrix}.$$

Evaluate $\det A$ and find the values of c for which $\det A = 0$. Show that $\det A > 0$ when $c < -1$.

(b) Let B be the 3×3 matrix

$$\begin{pmatrix} 2 & 3 & 4 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}.$$

Find the inverse of B. Hence, or otherwise, find a 3×1 column vector v with

$$Bv = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$