University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATIONS 2011/2012

MATH 10260

Linear Algebra for Engineers

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Time Allowed: 2 hours

Instructions for Candidates

Attempt **Question 1** (each part worth 5 marks) and **three** other questions, each other question being worth 15 marks.

Details of calculations leading to your answers must be included.

No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Candidates are **not** allowed to use mathematical tables during this examination.

1. (a) The complex number z satisfies the equation

$$z + \frac{1}{z} = \frac{8}{5}.$$

Find the two possible values of z and find the modulus $|z|$ of z.

(b) Find the general solution of the following system of linear equations:

$$\begin{aligned} x + y + z &= 4 \\ 3x + 2y - 2z &= 3 \\ -2x + 3y + 4z &= -1. \end{aligned}$$

(c) Find the eigenvalues of the matrix A, where

$$A = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}.$$

(d) The 2×2 matrices A, B and C satisfy the equation $CA = B^2 + B$. Find A explicitly, given that

$$B = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}.$$

(e) Find the parametric equations of the line passing through the point $(1, 1, 1)$ that is perpendicular to the plane containing the points $(1, 0, 0)$, $(0, -1, 0)$ and $(0, 0, 2)$.

2. (a) Let z and w be complex numbers and let A be the 2×2 complex matrix

$$\begin{pmatrix} z & w \\ -\overline{w} & \overline{z} \end{pmatrix},$$

where z and w denote the complex conjugates of z and w, respectively. Show that $\det A = |z|^2 + |w|^2$ and hence prove that A is invertible if z and w are not both 0.

(b) Find the equations of two different planes that are perpendicular to the line with parametric equations

$$x = 2 + 6t, \quad y = -2 + 2t, \quad z = 1 - 3t$$

and have the property that the shortest distance from the point $(0, 0, 0)$ to each plane is 2 units.

(c) Find the equation of the plane that contains the point $(1, 1, 1)$ and is perpendicular to the planes with equations $2x - y + z = 1$ and $3x + 2y - z = 4$.

3. (a) Find the value of c for which the following system of linear equations has *more than one* solution and find the general solution of the system when c has this value.

$$\begin{aligned} x + \ \ y + \ \ z &= 1 \\ x + 2y + 3z &= 1 \\ 3x + 2y + cz &= 3 \end{aligned}$$

(b) Find the solution of the following system of equations:

$$\begin{aligned} x + \ \ y + \ \ z + w &= 5 \\ -x - 2y + \ \ z + w &= 0 \\ -x - 2y - \ \ z + w &= 2 \\ 2x + 2y + 8z + w &= 0. \end{aligned}$$

4. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 2 & 0 \\ -1 & 2 & 0 \end{pmatrix}$$

(Note that $\det A = 0$.) Hence find an invertible matrix B which diagonalizes A.

(b) Find the eigenvalues of the matrix C, where

$$C = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}.$$

Deduce from your answer the eigenvalues of C^r for any positive integer r.

5. (a) The eigenvalues of the real symmetric matrix

$$S = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

are 1, 2 and 4. Find unit eigenvectors of S corresponding to these eigenvalues. Hence find a real orthogonal matrix which diagonalizes S.

(b) Show that the matrix

$$B = \begin{pmatrix} \cos t & \sin t \\ \sin t & -\cos t \end{pmatrix}$$

is orthogonal and satisfies $B^2 = I_2$.

6. (a) Find the values of x for which the determinant of the following matrix is 0:

$$A = \begin{pmatrix} x+1 & 2 & 3 \\ 2 & x+3 & 1 \\ 3 & 1 & x+2 \end{pmatrix}.$$

(b) Prove that the matrix

$$B = \begin{pmatrix} 2 & 1 & 1 \\ 1 & c & 1 \\ 4 & 1 & 2 \end{pmatrix}$$

has an inverse for all values of c, and find this inverse explicitly.