



University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATION 2010/2011

**MATH 10250
INTRODUCTION TO CALCULUS FOR ENGINEERS**

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Time Allowed: 2 hours

Instructions for Candidates

Full marks will be awarded for complete answers to **SIX** questions.

Details of calculations leading to your answers must be included.

No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Candidates are **NOT** allowed to use mathematical tables during this examination.

1. (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2 - x - 1$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = x - 3$.
Find $f \circ g$ and $g \circ f$.

- (b) By substituting $u = \sin x$ or otherwise, evaluate the definite integral

$$\int_0^{\pi/2} \sin^2 x \cos^3 x dx.$$

(Note that $\sin 0 = 0$, $\cos 0 = 1$ and $\sin(\pi/2) = 1$, $\cos(\pi/2) = 0$)

2. (a) Determine the regions of increase and decrease for

$$f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = (x^2 + 3x + 1)e^{-x}.$$

- (b) Solve for y the differential equation

$$\frac{dy}{dx} - 3y = x^2$$

given that $y = 1$ at $x = 0$.

3. (a) Use partial fractions to evaluate the definite integral

$$\int_3^4 \frac{x + 8}{x^2 + x - 6} dx.$$

- (b) Using L'Hopital's rule or otherwise, evaluate the following limit

$$\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^3 - 5x^2 + 8x - 4}.$$

4. (a) Define the hyperbolic functions $\sinh x$ and $\cosh x$, give rough sketches of their graphs and prove that

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}.$$

- (b) Find the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(2x)^{3n}}{n}.$$

[Questions continue on next page]

5. (a) Find the centroid of the region bounded by the curve $y = \sqrt{x}$ and the lines $y = 0$, $x = 1$ and $x = 4$.

- (b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - y = \cos(2x) - 3\sin(2x).$$

6. (a) Find the first five terms of the Maclaurin series for the function $f(x) = (x - 1)e^x$.

- (b) Use the Trapezoidal rule with four intervals of equal width to find an approximate value of the integral

$$\int_0^2 \sqrt{9 - x^2} dx.$$

7. (a) Find the length of the graph for the function

$$f : [2, 3] \rightarrow \mathbb{R}, \quad f(x) = \frac{1}{4}x^4 + \frac{1}{8x^2}.$$

- (b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = (8x + 4)e^x.$$

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