University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATIONS 2012/2013

MATH 10260

Linear Algebra for Engineers

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Time Allowed: 2 hours

Instructions for Candidates

Attempt **Question 1** (each part worth 5 marks) and **three** other questions, each other question being worth 15 marks.

Details of calculations leading to your answers must be included.

No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Candidates are **not** allowed to use mathematical tables during this examination.

1. (a) Let z be a complex number that satisfies

$$\frac{1}{z+1} = \frac{1}{z} + 1.$$

Find the two possible values of z, expressing your answers in the form $z = a + bi$, where a and b are real numbers, and $i^2 = -1$. Find also the modulus of each solution.

(b) Solve the following system of linear equations for x, y and z.

$$\begin{aligned} x - 2y - 2z &= 4 \\ 3x - y - z &= 7 \\ 6x + y + 7z &= 5. \end{aligned}$$

(c) Find the eigenvalues of the matrix A, where

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}.$$

(d) Let

$$M = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}.$$

If B is a 2×2 matrix satisfying

$$MB = I_2 + M^3,$$

where I_2 is the 2×2 identity matrix, find B explicitly.

(e) Find the parametric equations of the line passing through the point $(0, 1, 2)$ that is perpendicular to the plane containing the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

2. (a) Let $P(z) = z^3 - kz^2 + 22z - 20$, where k is a real number. If $3+i$ is a root of $P(z) = 0$, where $i^2 = -1$, find k and find the other two roots of $P(z) = 0$.

(b) Find the coordinates of the point of intersection of the lines with parametric equations

$$\begin{aligned} x &= 1 + 2t, \quad y = -1 + 3t, \quad z = 4t, \\ x &= 5 + 2s, \quad y = -1 - 3s, \quad z = -1 - 5s \end{aligned}$$

and find the equation of the plane that contains the two lines.

(c) Find the equation of the plane that contains the points $(1, 2, 5)$, $(5, -1, 0)$ and $(3, -1, -2)$, and determine the shortest distance from the point $(1, 1, 1)$ to this plane.

3. (a) Show that, whatever may be the value of the constant k, the following system of linear equations has a single solution for each of x, y and z, and find this solution:

$$\begin{aligned} -x + y + 2z &= -8 \\ 2x - 3y - 4z &= 16 \\ x + ky + 3z &= -7. \end{aligned}$$

(b) Find the general solution of the following system of equations:

$$\begin{aligned} x + y + z + w &= 2 \\ x + 2y - 3z + 2w &= 2 \\ 2x + 5y - 8z + 6w &= 5 \\ 3x + 4y - 5z + 2w &= 4. \end{aligned}$$

4. (a) Let A be the matrix

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}.$$

Given that 3 is an eigenvalue of A, find the other two eigenvalues of A and find eigenvectors of A corresponding to each of the three eigenvalues.

Hence find an invertible matrix B which diagonalizes A.

(b) Let C be the 2×2 matrix

$$C = \begin{pmatrix} 1 & x \\ 2 & 0 \end{pmatrix}.$$

where x is some number. If $C^2 + C$ has determinant 24, find the possible values of x.

5. (a) The eigenvalues of the real symmetric matrix

$$S = \begin{pmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

are 1, 3 and -4. Find unit eigenvectors of S corresponding to these eigenvalues. Hence find a real orthogonal matrix which diagonalizes S.

(b) Find values for the real numbers a and b so that the 2×2 matrix

$$B = \begin{pmatrix} a & -\frac{1}{2} \\ \frac{1}{2} & b \end{pmatrix}$$

is orthogonal.

6. (a) Let C be the matrix

$$\begin{pmatrix} 1 & 2 & x+1 \\ 1 & x & 3 \\ 1 & 3 & 3 \end{pmatrix}.$$

Find the values of x for which $\det C = 0$.

(b) The 3×3 matrices A and B satisfy the equation $AB = A + B$. Find A, given that

$$B = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 8 \\ 2 & 1 & 5 \end{pmatrix}.$$

Hint: $AB - A = A(B - I_3)$, where I_3 is the 3×3 identity matrix.