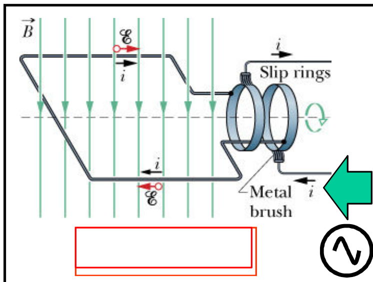
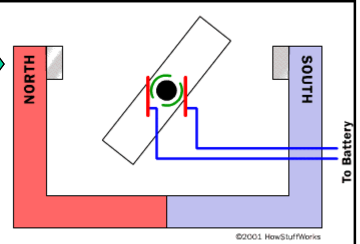
Alternating Current

-Electromagnetic oscillations in an *LC* circuit
-Alternating current (AC) circuits with capacitors
-Resonance in *RCL* circuits
-Power in AC circuits
-Transformers, AC power transmission



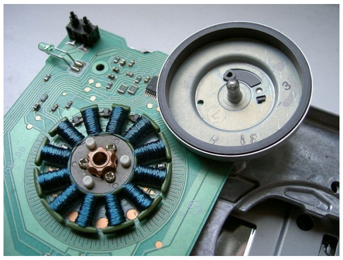
Rectified generator or DC motor

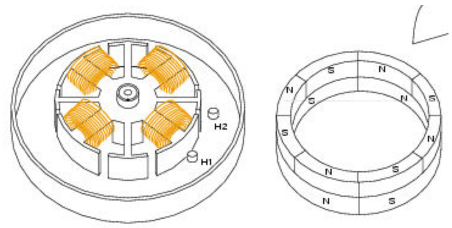
AC generator or motor

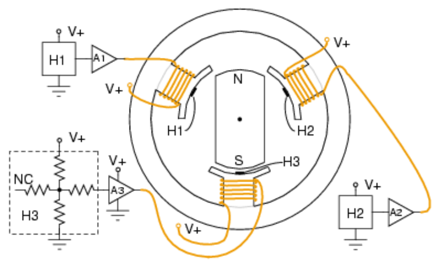


A is the area of the generator windings, N is the number of the windings, ω is the angular frequency of the rotation of the windings, and B is the magnetic field. This type of generator is known as "***alternating current***" or "**ac**" because the emf as well as the current change direction with a frequency $f = 2\pi\omega$. In the U.S. $f = 60$ Hz. Almost all commercial electrical power used today is ac even though the analysis of ac circuits is more complicated than that of dc circuits.

Emerging: brushless motor – models, bikes, next gen supercars







No making/breaking contacts – no sparking – higher powers – BUT needs high power transistor circuits

A Resistive Load

an ac generator connected to a resistor R.

From KLR we have:

The current amplitude is

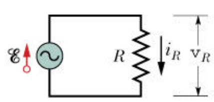
The voltage v_R across R is equal to $E_m \sin \omega t$.

The voltage amplitude is equal to E_m.

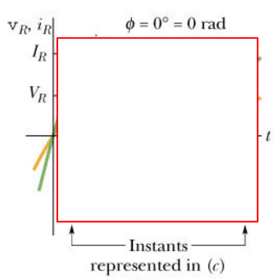
The relation between the voltage and current amplitudes is

In fig. b we plot the resistor current i_R and the resistor voltage v_R as functions of time t.

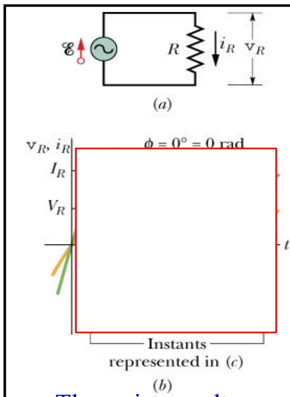
Both quantities reach their maximum values at the same time. We say that voltage and current are ***in phase***.

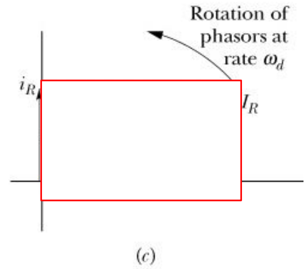


(*a*)



(*b*)





A convenient method for the representation of ac quantities is that of ***phasors.***

The resistor voltage v_R and the resistor current i_R are represented by rotating vectors known as phasors using the following conventions:

1. Phasors rotate in the counterclockwise direction with angular speed ω.

2. The length of each phasor is proportional to the ac quantity amplitude.

3. The projection of the phasor on the vertical axis gives the instantaneous value of the ac quantity.

4. The rotation angle for each phasor is equal to the phase of the ac quantity (ωt in this example).

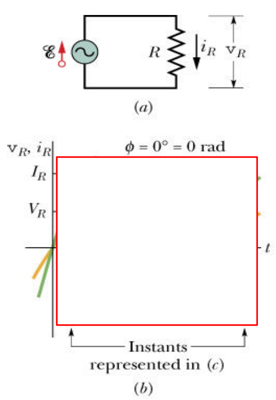
$$\langle P \rangle = \frac{\mathcal{E}_m^{\,2}}{\boxed{}} \qquad V_{\text{rms}} \equiv \frac{\mathcal{E}_m^{\,2}}{\boxed{}}$$

Average Power for R

$$\langle P \rangle = \frac{1}{T}\int_0^T P(t)\,dt \qquad P = \boxed{} \rightarrow$$

$$\langle P \rangle = \frac{\mathcal{E}_m^{\,2}}{R}\frac{1}{T}\int_0^T \sin^2 \omega t\,dt$$

$$\frac{1}{T}\int_0^T \sin^2 \omega t\,dt = \frac{1}{2} \quad \rightarrow \quad \langle P \rangle = \frac{\mathcal{E}_m^{\,2}}{\boxed{}}$$



We define the '$\boxed{}$' (rms) value of V as follows:

$$V_{\text{rms}} \equiv \frac{\mathcal{E}_m^{\,2}}{\boxed{}} \rightarrow \langle P \rangle = \frac{V_{\text{rms}}^2}{R}.$$

The equation looks the same as in the dc case. This power appears as heat on R.

(online)

Q. The graph shows the current as a function of time for an electrical device plugged into a outlet with an rms voltage of $\boxed{}$ V. What is the resistance of the device?

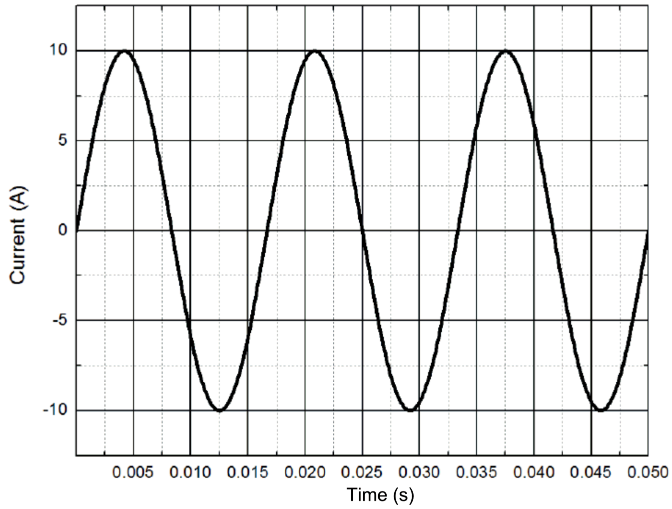
a) 24 Ω

b) 21 Ω

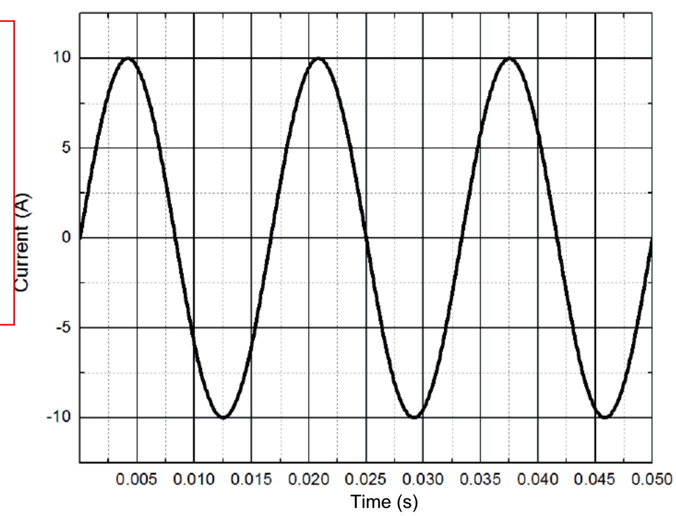
c) 17 Ω

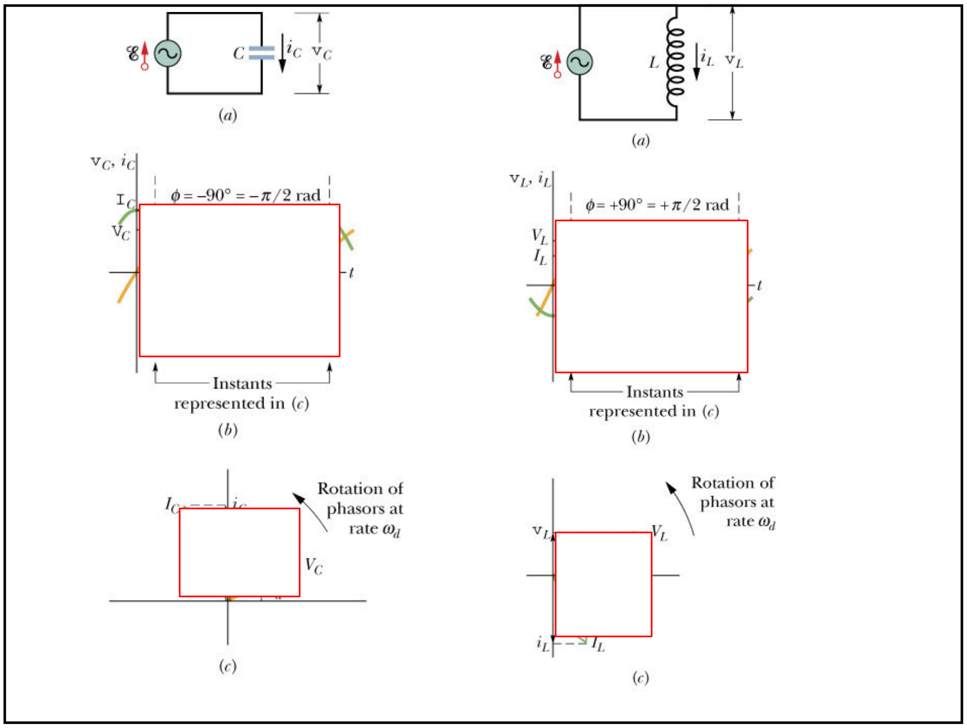
d) 14Ω

e) 12 Ω

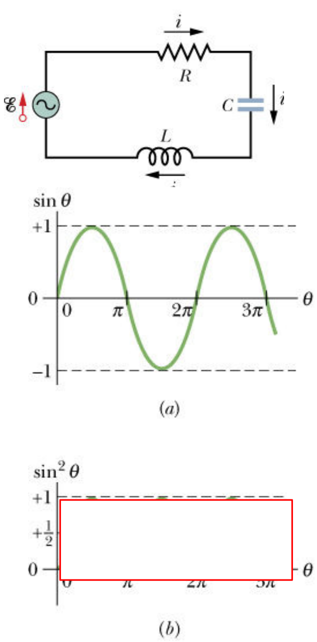


Q. The graph shows the current as a function of time for an electrical device plugged into a outlet with an rms voltage of $\boxed{}$ V. What is the resistance of the device?





$P_{avg} = I_{rms}^2 R$



Power in an *RCL* Circuit

We already have seen that the average power used by a capacitor and an inductor is equal to zero. The power on the average is consumed by the resistor.

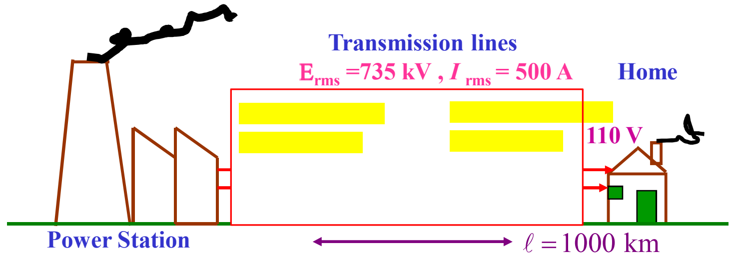
The instantaneous power $P = i^2 R = [\quad]^2\ R.$

The average power $P_{avg} = \frac{1}{T}\int_0^T P\,dt.$

$$P_{avg} = I^2 R\left[\frac{1}{T}\int_0^T \quad dt\right] = \frac{I^2 R}{2} = I_{rms}^2 R$$

$$P_{avg} = I_{rms} R I_{rms} =$$

The term $\cos\phi$ in the equation above is known as the "***power factor***" of the circuit. The average power consumed by the circuit is maximum when $\phi = 0$.



Energy Transmission Requirements

The resistance of the power line $R = \dfrac{\rho\ell}{A}$. R is fixed (220 Ω in our example).

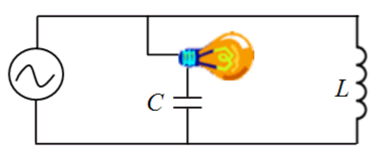
Heating of power lines $P_{heat} = I_{rms}^2 R$. This parameter is also fixed (55 MW in our example).

Power transmitted $P_{trans} = E_{rms} I_{rms}$ (368 MW in our example).

In our example P_{heat} is almost 15 % of P_{trans} and is acceptable.

To keep P_{heat} we must keep I_{rms} as low as possible. The only way to accomplish this is by **increasing** E_{rms}. In our example $E_{rms} = 735$ kV. To do that we need a device that can change the amplitude of any ac voltage (either increase or decrease).

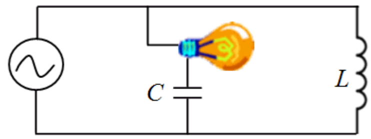
Q. A circuit contains an inductor, a capacitor, and a light bulb connected as shown. In which frequency limit is the light bulb the brightest?

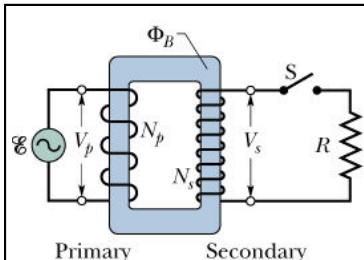


a) the low frequency limit

b) the high frequency limit

Q. A circuit contains an inductor, a capacitor, and a light bulb connected as shown. In which frequency limit is the light bulb the brightest?





The Transformer

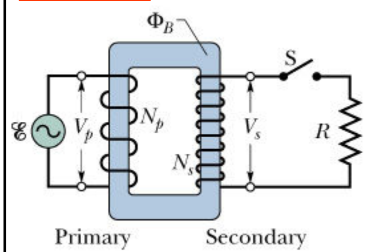
The transformer is a device that can change the voltage amplitude of any ac signal. It consists of two coils with a different number of turns wound around a common iron core.

The coil on which we apply the voltage to be changed is called the "***primary***" and it has N_P turns. The transformer output appears on the second coil, which is known as the "secondary" and has N_S turns. The role of the iron core is to ensure that the magnetic field lines from one coil also pass through the second. We assume that if voltage equal to V_P is applied across the primary then a voltage V_S appears on the secondary coil. We also assume that the magnetic field through both coils is equal to B and that the iron core has cross-sectional area A. The magnetic flux through the primary $\Phi_P = N_P BA \rightarrow V_P =$ **(eq. 1)**.

The flux through the secondary $\Phi_S = N_S BA \rightarrow V_S =$ **(eq. 2)**.

$$\Phi_P = N_P BA \rightarrow V_P = -\frac{d\Phi_P}{dt} = -N_P A\frac{dB}{dt} \quad \textbf{(eq. 1)}$$

$$\Phi_S = N_S BA \rightarrow V_S = -\frac{d\Phi_S}{dt} = -N_S A\frac{dB}{dt} \quad \textbf{(eq. 2)}$$



If we divide equation 2 by equation 1 we get:

$\frac{V_S}{V_P} =$

The voltage on the secondary $V_S =$

If $N_S > N_P \rightarrow \frac{N_S}{N_P} > 1 \rightarrow V_S > V_P$, we have what is known as a "***step up***" transformer.

If $N_S < N_P \rightarrow \frac{N_S}{N_P} < 1 \rightarrow V_S < V_P$, we have what is known as a "***step down***" transformer.

Both types of transformers are used in the transport of electric power over large distances.

Q. The ac adapter for a laptop computer contains a transformer. The input of the adapter is the 120 volts from the ac wall outlet. The output from the transformer is 20 volts. What is the *turns ratio* of the transformer?

a)

b)

c)

d) This cannot be determined without knowing how many turns one of the coils in the transformer has.

Q. The ac adapter for a laptop computer contains a transformer. The input of the adapter is the 120 volts from the ac wall outlet. The output from the transformer is 20 volts. What is the *turns ratio* of the transformer?

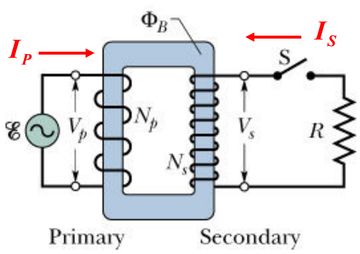
Q. Transformer A has a primary coil with 400 turns and a secondary coil with 200 turns. Transformer B has a primary coil with 400 turns and a secondary coil with 800 turns. The same current and voltage are delivered to the primary coil of both transformers. The secondary coils are connected to identical circuits. How does the power delivered by secondary coil A compare to the power delivered by secondary coil B?

b) Secondary coil A delivers one half the power delivered by secondary coil B

d) Secondary coil A delivers twice the power delivered by secondary coil B

Q. Transformer A has a primary coil with 400 turns and a secondary coil with 200 turns. Transformer B has a primary coil with 400 turns and a secondary coil with 800 turns. The same current and voltage are delivered to the primary coil of both transformers. The secondary coils are connected to identical circuits. How does the power delivered by secondary coil A compare to the power delivered by secondary coil B?

(in-class)



$\frac{V_S}{N_S} = \boxed{}$ $I_S N_S = \boxed{}$

We have that: $\frac{V_S}{N_S} = \boxed{}$

$\rightarrow V_S N_P = \boxed{}$ (eq. 1).

If we close switch S in the figure we have in addition to the primary current I_P a current I_S in the secondary coil. We assume that the transformer is "**ideal,**" i.e., it suffers no losses due to heating. Then we have: $V_P I_P = \boxed{}$ (eq. 2).

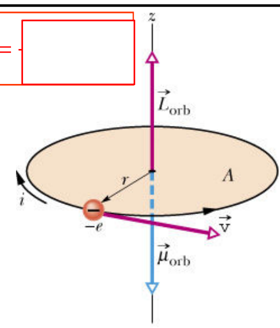
If we divide eq. 2 with eq. 1 we get: $\frac{V_P I_P}{V_P N_S} = \boxed{}$

$I_S = \boxed{}$

In a step-up transformer $(N_S > N_P)$ we have that $I_S < \boxed{}$

In a step-down transformer $(N_S < N_P)$ we have that $I_S > \boxed{}$

$\vec{\mu}_{\text{orb}} =$



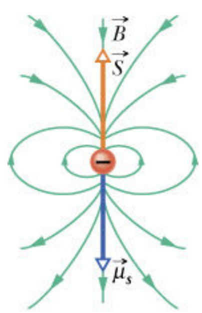
Magnetism and Electrons

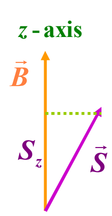
There are three ways in which electrons can generate a magnetic field. We have already encountered the first method. Moving electrons constitute a current, which according to Ampere's law generates a magnetic field in its vicinity. An electron can also generate a magnetic field because it acts as a magnetic dipole. There are two mechanisms involved.

Orbital Magnetic Dipole Moment. An electron in an atom moves around the nucleus as shown in the figure. For simplicity we assume a circular orbit of radius r with period T. This constitutes an electric current $i = \frac{e}{T} = \frac{e}{2\pi r / v} =$ The resulting magnetic dipole moment $\mu_{\text{orb}} = \pi r^2 i = \pi r^2 \frac{ev}{2\pi r} = \frac{evr}{2} = \frac{e(mvr)}{2m} =$

In vector form: $\vec{\mu}_{\text{orb}} =$ The negative sign is due to the negative charge of the electron.

$\vec{\mu}_S =$





Spin Magnetic Dipole Moment

In addition to the orbital angular momentum, an electron has what is known as "**intrinsic**" or "**spin**" angular momentum $\vec{S}$. Spin is a quantum relativistic effect. One can give a simple picture by viewing the electron as a spinning charge sphere. The corresponding magnetic dipole moment is given by the equation $\vec{\mu}_S =$

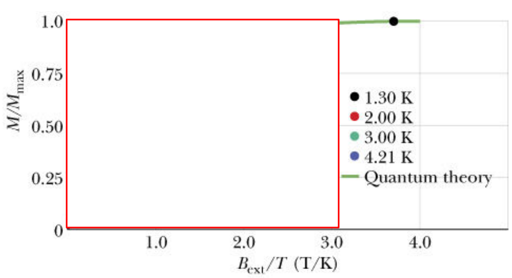
Spin Quantization. Unlike classical mechanics in which the angular momentum can take any value, spin $(\vec{S})$ and orbital $(\vec{L})$ angular momentum can only have certain discrete values. Furthermore, we cannot measure the vectors $\vec{S}$ or $\vec{L}$ but only their projections along an axis (in this case defined by $\vec{B}$). These apparently strange rules result from the fact that at the microscopic level classical mechanics do not apply and we must use **quantum mechanics**.

Magnetic Materials. Materials can be classified on the basis of their magnetic properties into three categories: **diamagnetic**, **paramagnetic**, and **ferromagnetic**. Below we discuss briefly each category. Magnetic materials are characterized by the magnetization vector $\vec{M}$, defined as the magnetic moment per unit volume.

$\vec{M} = \boxed{}$ **SI unit for M**: $\frac{\mathrm{A \cdot m^2}}{\mathrm{m^3}} = \frac{\mathrm{A}}{\mathrm{m}}$

Diamagnetism

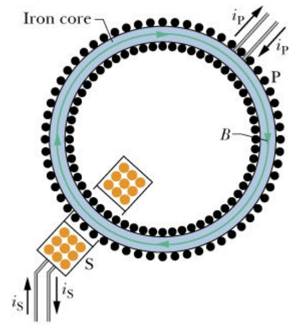
Diamagnetism occurs in materials composed of atoms that have electrons whose magnetic moments are antiparallel in pairs and thus result in a zero net magnetic moment. When we apply an external magnetic field $\vec{B}$, diamagnetic materials acquire a weak magnetic moment $\vec{\mu}$, which is directed opposite to $\vec{B}$. If $\vec{B}$ is inhomogeneous, the diamagnetic material is **repelled from regions of stronger field** to regions of weaker $\vec{B}$. All materials exhibit diamagnetism but in paramagnetic and ferromagnetic materials this weak diamagnetism is masked by the much stronger paramagnetism or ferromagnetism.



Paramagnetism

The atoms of paramagnetic materials have a net magnetic dipole moment $\vec{\mu}$ in the absence of an external magnetic field. This moment is the vector sum of the electron magnetic moments.

In the presence of a magnetic field each dipole has energy $U = \boxed{}$. Here θ is the angle between $\vec{B}$ and $\vec{\mu}$. The potential energy U is minimum when $\theta = 0$. The magnetic field partially aligns the moment of each atom. Thermal motion opposes the alignment. The alignment improves when the temperature is lowered and/or when the magnetic field is large. The resulting magnetization $\vec{M}$ is parallel to the field $\vec{B}$. When a paramagnetic material is placed in an inhomogeneous field it moves in the region where $\vec{B}$ is stronger.



Ferromagnetism

Ferromagnetism is exhibited by iron, nickel, cobalt, gadolinium, dysprosium, and their alloys.

Ferromagnetism is observed even in the absence of a magnetic field (the familiar permanent magnets).

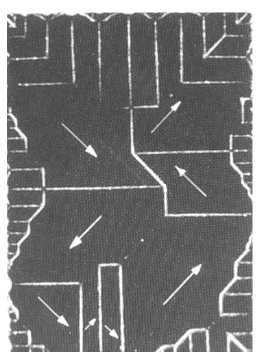
Ferromagnetism disappears when the temperature exceeds the Curie temperature of the material.

Above its Curie temperature a ferromagnetic material becomes paramagnetic.

Ferromagnetism is due to a quantum effect known as "exchange coupling," which tends to align the magnetic dipole moments of neighboring atoms.

The magnetization of a ferromagnetic material can be measured using a Rowland ring. The ring consists of two parts: a primary coil in the form of a toroid, which generates the external magnetic field B_0 and a secondary coil which measures the total magnetic fiel B. A magnetic material forms the core of the toroid. The net field $B =$

Here B_M is the contribution of the ferromagnetic core. B_M is proportional to the sample magnetization M.

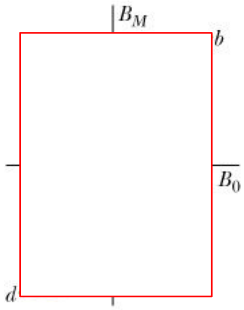


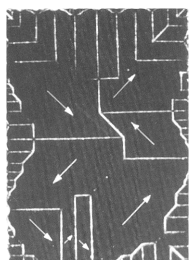
Magnetic Domains

Below the Curie temperature, all magnetic moments in a ferromagnetic material are perfectly aligned. Yet the magnetization is not saturated. The reason is that the ferromagnetic material contains regions called "**domains.**" The magnetization in each domain is saturated but the domains are aligned in such a way so as to have at best a small net magnetic moment. In the presence of an external magnetic field $\vec{B}_0$ two effects are observed:

1. The domains whose magnetization is aligned with $\vec{B}_0$ at the expense of those domains that are not aligned.

2. The magnetization of the nonaligned domains turns and $\vec{B}_0$.





Hysteresis

If we plot the net field B_M as a function of the applied field B_0 we get the loop shown in the figure known as a "**hysteresis**" loop. If we start with an unmagnetized ferromagnetic material, the curve follows the path from point a to point b, where the magnetization saturates. If we reduce B_0 the curve follows the path bc, which is different from the original path ab. Furthermore, even when B_0 is switched off, we have a nonzero magnetic field. Similar effects are observed if we reverse the direction of B_0. This is the familiar phenomenon of permanent magnetism and forms the basis of magnetic data recording. Hysteresis is due to the fact that the domain reorientation is not totally reversible and that the domains do not return completely to their original configuration.