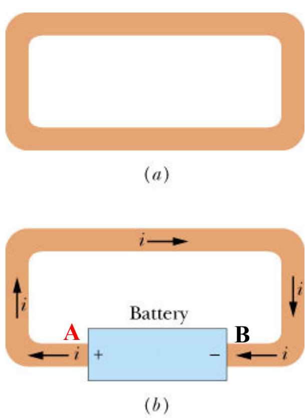
Current and Resistance

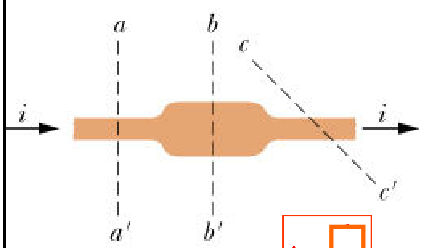
-Electric current (symbol i)
-Electric current density vector (symbol $\vec{J}$)
-Drift speed (symbol v_d)
-Resistance (symbol R) and resistivity (symbol ρ) of a conductor
-Ohmic and non-Ohmic conductors

-Ohm's law
-Power in electric circuits



Electric Current

Consider the conductor shown in fig. *a*. All the points inside the conductor and on its surface are at the same potential. The free electrons inside the conductor move in random directions and thus there is no net charge transport. We now make a break in the conductor and insert a battery as shown in fig. *b*. Points *A* and *B* are now at potentials V_A and V_B, respectively ($V_A - V_B = V$, the voltage of the battery). The situation is not static any more, but charges move inside the conductor so that there is a net charge flow in a particular direction. We define this net flow of electric charge as "electric current."



$i = \boxed{}$

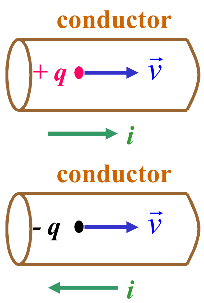
Consider the conductor shown in the figure. It is connected to a battery (not shown) and thus charges move through the conductor. Consider one of the cross sections through the conductor (aa' or bb' or cc').

The electric current i is defined as $i = \boxed{}$

Current = rate at which charge flows

Current SI Unit: C/s, known as the "ampere"

1.0 C is 6.2×10^{18} e



Current Direction

An electric current is represented by an arrow, which has the same direction as the charge velocity. The sense of the current arrow is defined as follows:

1. If the current is due to the motion of **positive** charges, the current arrow is **parallel** to the charge velocity $\vec{v}$.

2. If the current is due to the motion of **negative** charges, the current arrow is **antiparallel** to the charge velocity $\vec{v}$.

Q. The battery capacity of a lithium ion battery in a digital music player is 750 mA-h. The player can operate for eight hours if the battery is initially fully charged. Determine the number of electrons that flow through the player as you listen to songs for three hours.

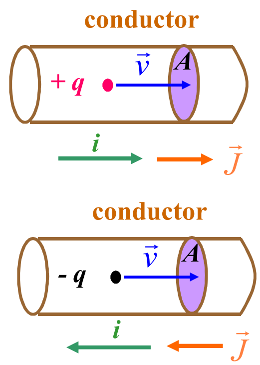
a) 6.2×10^{18} electrons

b) 1.0×10^{3} electrons

c) 2.4×10^{9} electrons

d) 6.3×10^{21} electrons

e) 8.1×10^{28} electrons



Current Density

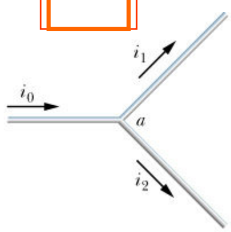
Current density is a vector that is defined as follows:

Its magnitude is SI unit for J: A/m^2

The direction of $\vec{J}$ is the same as that of the current.

The current through a conductor of cross-sectional area A is given by the equation $i = JA$ if the current density is constant.

If $\vec{J}$ is not constant, then $i = \int \vec{J} \cdot d\vec{A}$.



We note that even though the current density is a vector the electric current is not. This is illustrated in the figure to the left. An incoming current i_0 branches at point a into two currents, i_1 and i_2.

Current This equation expresses the conservation of charge at point a. Please note that we have not used vector addition.

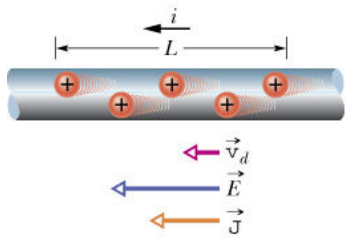
Q. When lightning strikes, the current flows from the ground upward to the clouds above. What is the direction of the electric field of the lightning?

a) upward

b) downward

c) perpendicular to the current at each location on the lightning bolt

d) parallel to the ground



$\boxed{J = nv_d e}$ $\boxed{\vec{J} = ne\vec{v}_d}$

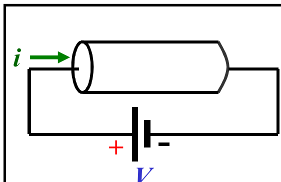
Drift Speed

When a current flows through a conductor the electric field causes the charges to move with a constant drift speed v_d. This drift speed is superimposed on the random motion of the charges.

Consider the conductor of cross-sectional area A shown in the figure. We assume that the current in the conductor consists of positive charges. The total charge q within a length L is given by $q = (nAL)e$. This charge moves through area A in a time $t = \frac{L}{v_d}$. The current is $i = \boxed{} = nAv_d e$.

The current density is $J = \frac{i}{A} = \boxed{} = nv_d e$.

In vector form: $\vec{J} = ne\vec{v}_d$.



$\boxed{R = \frac{V}{i}}$

Resistance

If we apply a voltage V across a conductor (see figure) a current i will flow through the conductor.

We define the conductor resistance as the ratio $R = \frac{V}{i}$.

SI Unit for R: $\frac{V}{A} =$ the ohm (symbol Ω)

A conductor across which we apply a voltage $V = 1$ volt and results in a current $i = 1$ ampere is defined as having resistance of 1 Ω.

Q: Why not use the symbol "O" instead of "Ω"?

A: Suppose we had a 1000 Ω resistor.

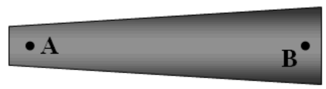
We would then write: 1000 O, which can easily be mistakenly read as 10000 Ω.

A conductor whose function is to provide a specified resistance is known as a "resistor."

The symbol is given to the left.

R

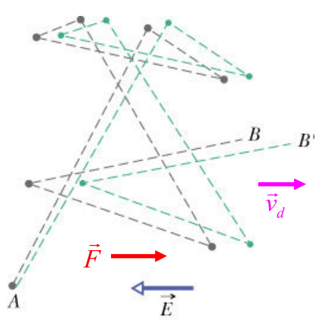
Q. A copper wire is fabricated that has a gradually increasing diameter along its length as shown. If an electric current is moving through the wire, how does the drift velocity of the electrons at point A compare with that at point B?



a) The drift velocity will be greater at point A than at point B.

b) The drift velocity will be the same at both points.

c) The drift velocity will be greater at point B than at point A.

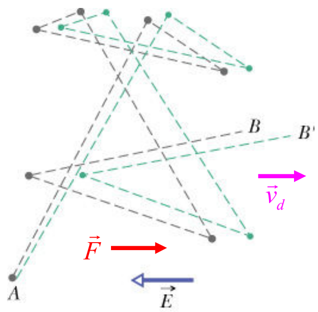


A Microscopic View of Ohm's Law

In order to understand why some materials such as metals obey Ohm's law, we must look into the details of the conduction process at the atomic level.

A schematic of an Ohmic conductor such as copper is shown in the figure. We assume that there are free electrons that move around in random directions with an effective speed $v_{eff} = 1.6\times\boxed{}$ m/s. The free electrons suffer collisions with the stationary copper atoms.

A schematic of a free electron path is shown in the figure using the dashed gray line. The electron starts at point A and ends at point B. We now assume that an electric field $\vec{E}$ is applied. The new electron path is indicated by the dashed green line. Under the action of the electric force the electron acquires a small drift speed v_d. The electron drifts to the right and ends at point B'.



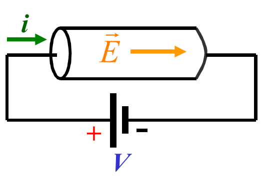
We assume that the average time between collisions with the copper atoms is equal to τ. The electric field exerts a force $F = eE$ on the electron, resulting in an acceleration $a = \frac{F}{m} = \frac{eE}{m}$. The drift speed is given by the equation $v_d = a\tau = \frac{eE\tau}{m}$ **(eq. 1)**.

We can also get v_d from the equation $J = nev_d \rightarrow v_d = \frac{J}{ne}$ **(eq. 2)**.

If we compare equations 1 and 2 we get: $v_d = \frac{J}{ne} = \frac{eE\tau}{m} \rightarrow E = \left(\frac{m}{ne^2\tau}\right)J$.

If we compare the last equation with $E = \rho J$ we conclude that

$\rho = \frac{m}{ne^2\tau}$. This is a statement of Ohm's law (the resistance of the conductor does not depend on voltage and thus E). This is because m, n, and e are constants. The time τ can also be considered to be independent of E since the drift speed v_d is so much smaller than v_{eff}.



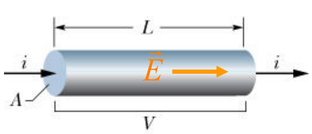
$\vec{E} = \rho\vec{J}$ $\vec{J} = \sigma\vec{E}$

Resistivity

Unlike the electrostatic case, the electric field in the conductor of the figure is not zero. We define as resistivity ρ of the conductor the ratio $\rho = \square$

In vector form: $\vec{E} = \rho\vec{J}$.

SI unit for ρ: $\frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\text{m} = \Omega \cdot \text{m}$



$R = \square$

The conductivity σ is defined as $\sigma = \square$

Using ρ, the previous equation takes the form: $\vec{J} = \sigma\vec{E}$.

The electric field inside the conductor is $E = \frac{V}{L}$. The current density is $J = \frac{i}{A}$. We substitute E and J into equation $\rho = \frac{E}{J}$ and get: $\rho = \frac{V/L}{i/A} = \frac{V}{i}\frac{A}{L} = R\frac{A}{L} \rightarrow R = \square$.

Q. When a potential difference is applied to a certain copper wire, a current of 1.5 A passes through the wire. If the wire was removed from the circuit and replaced with a copper wire of twice the diameter, what current would flow through the new wire? Assume the wires are identical in all other aspects.

a) 0.38 A

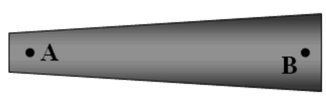
b) 0.75 A

c) 1.5 A

d) 3.0 A

e) 6.0 A

Q. A copper wire is fabricated that has a gradually increasing diameter along its length as shown. If an electric current is moving through the wire, which quantities vary along the length of the wire?



a) current only

b) current and current density only

c) current density and electric field only

d) resistivity and current only

e) current, resistivity, current density, and electric field

Q. How does the resistivity of a metal wire change if either the number of electrons per unit volume increases or the mean free time increases?

a) In both cases, the resistivity will increase.

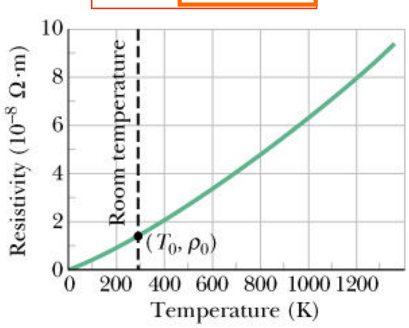
b) In both cases, the resistivity will decrease.

c) Increasing the number of electrons will increase the resistivity, but it will decrease if the mean free time increases.

d) Increasing the number of electrons will decrease the resistivity, but it will increase if the mean free time increases.

e) Too little information is given to make a determination.

$\rho - \rho_0 = \boxed{}$



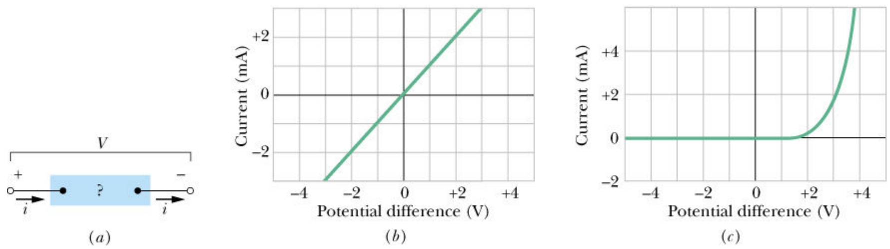
Variation of Resistivity with Temperature

In the figure we plot the resistivity ρ of copper as a function of temperature T. The dependence of ρ on T is almost linear. Similar dependence is observed in many conductors.

The following empirical equation is used for many practical applications:

$\rho - \rho_0 = \boxed{}$ The constant α is known as the "temperature coefficient of resistivity." The constant T_0 is a reference temperature usually taken to be room temperature ($T_0 = 293$ K), and ρ_0 is the resistivity at T_0. For copper, $\rho_o = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$.

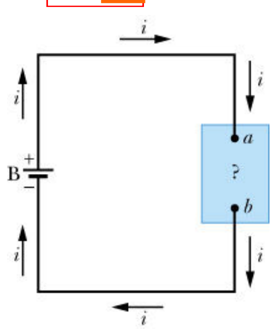
Note: Temperature enters the equation above as a difference $(T - T_0)$. Thus either the Celsius or the Kelvin temperature scale can be used.



Ohm's Law. A resistor was defined as a conductor whose resistance does not change with the voltage V applied across it. In fig. b we plot the current i through a resistor as a function of V. The plot (known as the "i-V curve") is a straight line that passes through the origin. Such a conductor is said to be "____ and it obeys Ohm's law, which states: The current i through a conductor is proportional to the voltage V applied across it. Not all conductors obey Ohm's law (these are known as ____). An example is given in fig. c where we plot i versus V for a semiconductor diode. The ratio V/i (and thus the resistance R) is not constant. As a matter of fact, the diode does not conduct for negative voltage values.

Note: Ohm's "law" is in reality a definition of Ohmic conductors (defined as the conductors that obey Ohm's law).

$P = \boxed{}$

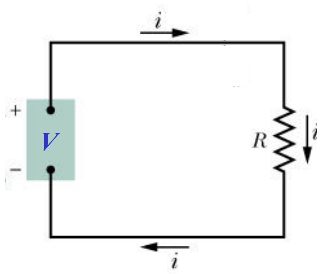


Power in Electric Circuits

Consider the circuit shown in the figure. A battery of voltage V is connected across the terminals a and b of a device. This can be a resistor, a motor, etc. The battery maintains a potential difference V between the terminals a and b and thus a current i flows in the circuit as shown in the figure. During the time interval dt a charge $dq = idt$ moves between the terminals. We note that $V_a > V_b$.

The potential energy of the charge decreases by an amount $dU = Vdq = Vidt$.
Using energy conservation we conclude that the lost energy has been transferred by the battery to the device and has been converted into some other form of energy.
The rate at which energy is transferred to the device is known as "**power**" and it is equal to $P = \frac{dU}{dt} = \boxed{}$

SI unit for P: $V \cdot A$ It is known as the "watt" (symbol W).



$P = \boxed{}$ $P = \boxed{}$

If the device connected to the battery is a resistor R then the energy transferred by the battery is converted as **heat** that appears on R. If we combine the equation $P = \boxed{}$ with Ohm's law $i = \dfrac{V}{R}$, we get the following two equivalent expressions for the rate at which heat is dissipated on R:

$P = i^2 R$ and $P = \dfrac{V^2}{R}$

Q. A wire is used as a heating element that has a resistance that is fairly independent of its temperature within its operating range. When a current i flows through the wire, the energy delivered by the heater each minute is E. For what amount of current will the energy delivered by the heater each minute be $4E$?

a) $2i$

b) $4i$

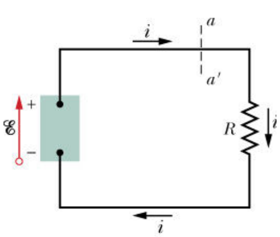
c) $0.5i$

d) $0.25i$

e) $8i$

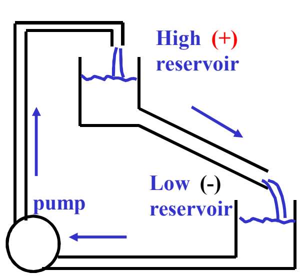
Circuits

- -Electromotive force (emf)
- -Ideal and real emf devices
- -Kirchhoff's loop rule
- -Kirchhoff's junction rule
- -Multiloop circuits
- -Resistors in series
- -Resistors in parallel
- -RC circuits, charging and discharging of a capacitor

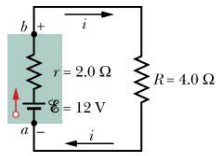


In order to create a current through a resistor, a potential difference must be created across its terminals. One way of doing this is to connect the resistor to a battery. A device that can maintain a potential difference between two terminals is called a **"seat of an emf "** or an **"emf device."** Here emf stands for electromotive force. Examples of emf devices are a battery, an electric generator, a solar cell, a fuel cell, etc.

These devices act like "charge pumps" in the sense that they move positive charges from the low-potential (negative) terminal to the high-potential (positive) terminal. A mechanical analog is given in the figure below.

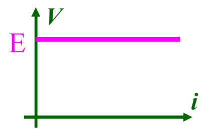


In this mechanical analog a water pump transfers water from the low to the high reservoir. The water returns from the high to the low reservoir through a pipe, which is the analog of the resistor.
The emf (symbol E) is defined as the potential difference between the terminals of the emf device when no current flows through it.



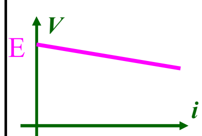
$\boxed{V = E}$

Ideal emf device



$\boxed{V = E - ir}$

Real emf device



Ideal and Real Emf Devices

An emf device is said to be **ideal** if the voltage V across its terminals a and b does **not** depend on the current i that flows through the emf device: $V = E$.

An emf device is said to be **real** if the voltage V across its terminals a and b **decreases** with current i according to the equation $V = \boxed{\quad}$
The parameter r is known as the "**internal resistance**" of the emf device.

Q. A brushless RC buggy LiPo battery has a 7.4-V emf and a contact (effective internal) resistance of 10 mΩ. To the nearest 0.1A, determine the terminal voltage when the current drawn from the battery is 150 A.

a) 3.0 V

b) 7.2 V

c) 5.9 V

d) 6.6 V

e) 5.8 V

Q. For this, determine the power going to contact heating.

a) 10 W

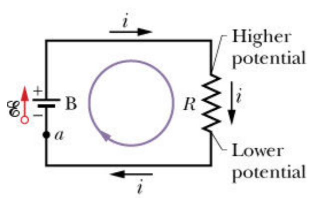
b) 22 W

c) 52 W

d) 225 W

e) 515 W

$\boxed{} = 0$



Current in a Single-Loop Circuit

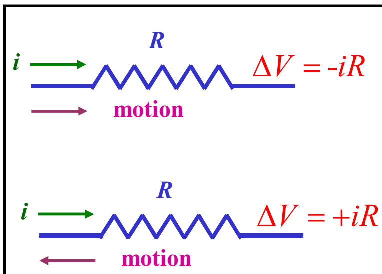
Consider the circuit shown in the figure.

In a time interval dt a charge $dq = i dt$ passes through the circuit. The battery is doing work $dW = \boxed{}$ Using energy conservation we can set this amount of work equal to the rate at which heat is generated on R: $\mathcal{E} i dt = R i^2 dt \rightarrow$
$\mathcal{E} = Ri \rightarrow \mathcal{E} - iR = 0.$

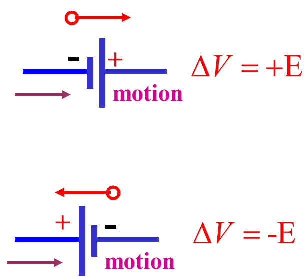
Kirchhoff put the equation above in the form of a rule known as Kirchhoff's loop rule (KLR for short).

KLR : The $\boxed{}$ of the changes in potential encountered in a complete traversal of any loop in a circuit is $\boxed{}$

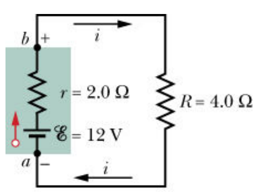
The rules that give us the algebraic sign of the charges in potential through a resistor and a battery are given on the next page.



Resistance Rule:
For a move through a resistance in the direction of the current, the change in the potential is $\Delta V = -iR$.
For a move through a resistance in the direction opposite to that of the current, the change in the potential is $\Delta V = +iR$.



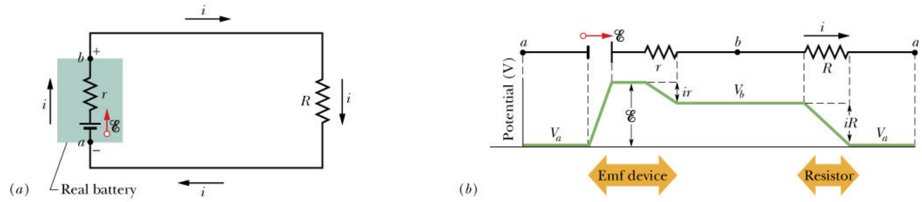
EMF Rule:
For a move through an ideal emf device in the direction of the emf arrow, the change in the potential is $\Delta V = +\mathrm{E}$.
For a move through an ideal emf device in a direction opposite to that of the emf arrow, the change in the potential is $\Delta V = -\mathrm{E}$..



Potential Difference Between Two Points:
Consider the circuit shown in the figure. We wish to calculate the potential difference $V_b - V_a$ between point *b* and point *a*.

$\boxed{V_b - V_a = \text{ sum of all potential changes } \Delta V \text{ along the path from point } a \text{ to point } b.}$

We choose a path in the loop that takes us from the initial point *a* to the final point *b*.
$V_f - V_i =$ sum of all potential changes ΔV along the path.
There are two possible paths: We will try them both.
Left path: $V_b - V_a = \mathrm{E} - ir$
Right path: $V_b - V_a = iR$
Note: The values of $V_b - V_a$ we get from the two paths are the same.

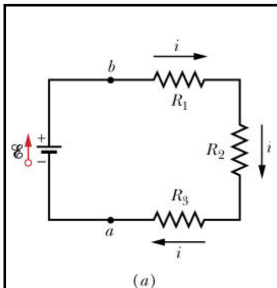


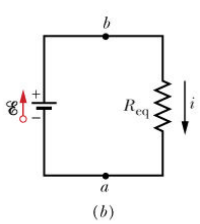
KLR example : Consider the circuit of fig. a. The battery is real with internal resistance r. We apply KLR for this loop starting at point a and going counterclockwise:

$\boxed{\qquad} = 0 \rightarrow i = \boxed{\qquad}$ We note that for an ideal battery, $r = 0$ and $i = \frac{\mathcal{E}}{R}$.

Note : The internal resistance r of the battery is an integral part of the battery's internal mechanism. There is no way to open the battery and remove r.

In fig. b we plot the potential V of every point in the loop as we start at point a and go around in the counterclockwise direction. The change ΔV in the battery is positive because we go from the negative to the positive terminal. The change ΔV across the two resistors is negative because we chose to traverse the loop in the direction of the current. The current flows from high to low potential.





$$R_{eq} = R_1 + R_2 + R_3$$

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

Resistors in Series e e

Consider the three resistors connected in series (one after the other) as shown in fig. a. These resistors have the **same current** i but different voltages V_1, V_2, and V_3. The net voltage across the combination is the sum $V_1 + V_2 + V_3$. We will apply KLR for the loop in fig. a starting at point a, and going around the loop in the counterclockwise direction:

$$\boxed{\qquad\qquad} = 0 \rightarrow i = \frac{\mathcal{E}}{R_1 + R_2 + R_3} \quad \textbf{(eq. 1)}$$

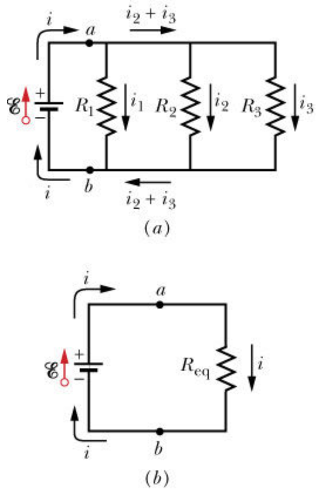
We will apply KLR for the loop in fig. b starting at point a, and going around the loop in the counterclockwise direction:

$$\boxed{\qquad} = 0 \rightarrow i = \frac{\mathcal{E}}{R_{eq}} \quad \textbf{(eq. 2)}$$

If we compare eq. 1 with eq. 2 we get: $R_{eq} = R_1 + R_2 + R_3$.

For n resistors connected in series, the equivalent resistance is:

$$R_{eq} = \sum_{i=1}^{n} R_i = R_1 + R_2 + \ldots + R_n$$



$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Resistors in Parallel

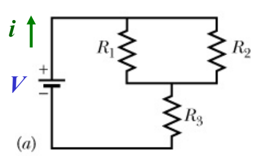
Consider the three resistors shown in the figure. "In parallel" means that the terminals of the resistors are connected together on both sides. Thus resistors in parallel have the **same potential** applied across them. In our circuit this potential is equal to the emf E of the battery. The three resistors have different currents flowing through them. The total current is the sum of the individual currents. We apply KJR at point *a*:

$i = \boxed{\quad\quad}$ $\quad i_1 = \frac{E}{R_1} \quad i_2 = \frac{E}{R_2} \quad i_3 = \frac{E}{R_3} \rightarrow$

$i = \boxed{\quad\quad\quad}$ (eq. 1). From fig. *b* we have:

$i = \boxed{\quad}$ (eq. 2). If we compare equations 1 and 2

we get: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

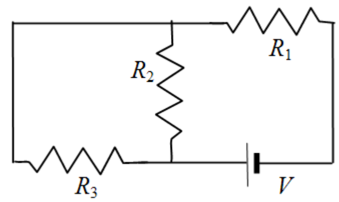


Equivalent Resistance

Consider the combination of resistors shown in the figure. We can substitute this combination of resistors with a single resistor R_{eq} that is "electrically equivalent" to the resistor group it substitutes.

This means that if we apply the same voltage V across the resistors in fig. *a* and across R_{eq}, the same current i is provided by the battery. Alternatively, if we pass the same current i through the circuit in fig. *a* and through the equivalent resistance R_{eq}, the voltage V across them is identical. This can be stated in the following manner: If we place the resistor combination and the equivalent resistor in separate black boxes, by doing electrical measurements we cannot distinguish between the two.

Q. Consider the three resistors and the battery in the circuit shown. Which resistors, if any, are connected in parallel?



a) R_1 and R_2

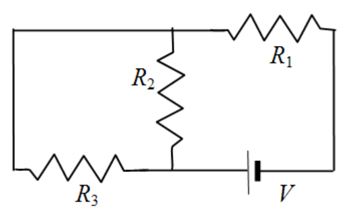
b) R_1 and R_3

c) R_2 and R_3

d) R_1 and R_2 and R_3

e) No resistors are connected in parallel.

Q. Consider the three resistors and the battery in the circuit shown. Which resistors, if any, are connected in series?



a) R_1 and R_2

b) R_1 and R_3

c) R_2 and R_3

d) R_1 and R_2 and R_3

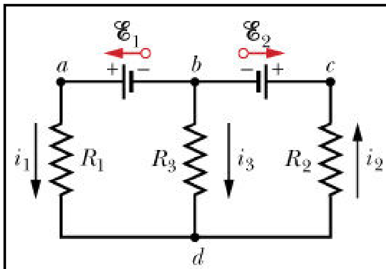
e) No resistors are connected in series.

Q. Two light bulbs, one "50 W" bulb and one "100 W" bulb, are connected in parallel with a standard 120 volt ac electrical outlet. The brightness of a light bulb is directly related to the power it dissipates. Therefore, the 100 W bulb appears brighter. How does the brightness of the two bulbs compare when these same bulbs are connected in series with the same outlet?

a) Both bulbs will be equally bright.

b) The "100 W" bulb will be brighter.

c) The "50 W" bulb will be brighter.

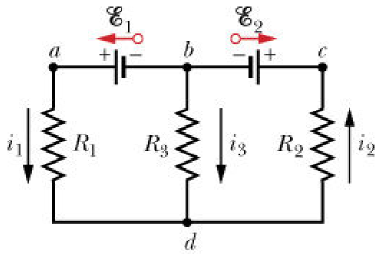


Multiloop Circuits

Consider the circuit shown in the figure. There are three branches in it: *bad*, *bcd*, and *bd*. We assign currents for each branch and define the current directions arbitrarily. The method is self-correcting. If we have made a mistake in the direction of a particular current, the calculation will yield a negative value and thus provide us with a warning.

We assign current i_1 for branch *bad*, current i_2 for branch *bcd*, and current i_3 for branch *bd*. Consider junction *d*. Currents i_1 and i_3 arrive, while i_2 leaves. Charge is conserved, thus we have: $i_1 + i_3 = i_2$. This equation can be formulated as a more general principle known as Kirchhoff's junction rule (KJR).

KJR : The sum of the currents entering any junction is equal to the sum of the currents leaving the junction.



In order to determine the currents $i_1, i_2,$ and i_3 in the circuit we need three equations. The first equation will come from KJR at point *d*:

KJR/junction d: $i_1 + i_3 = i_2$ **(eq. 1)**

The other two will come from KLR: If we traverse the left loop (*bad*) starting at *b* and going in the counterclockwise direction we get:

KLR/loop *bad*: $\boxed{} = 0$ (**eq. 2**). Now we go around the right loop (*bcd*) starting at point *b* and going in the counterclockwise direction:

KLR/loop *bcd*: $\boxed{} = 0$ (**eq. 3**)

We have a system of three equations (eqs.1, 2, and 3) and three unknowns $(i_1, i_2,$ and $i_3)$. If a numerical value for a particular current is negative, this means that the chosen direction for this current is wrong and that the current flows in the opposite direction. We can write a fourth equation (KLR for the outer loop *abcd*) but this equation does not provide any new information.

KLR/loop *abcd*: $\boxed{} = 0$ (**eq. 4**)

Q. What is the current through the 1-Ω resistor in this circuit?

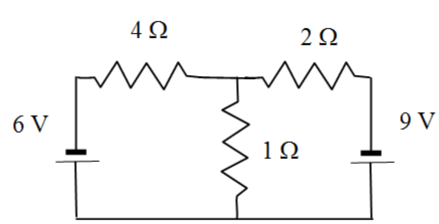
a) 2.8 A

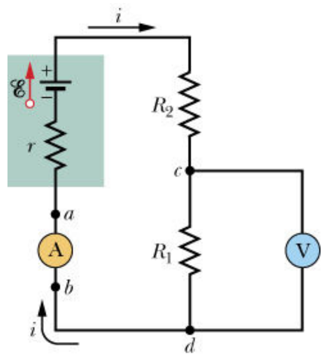
b) 3.0 A

c) 3.4 A

d) 4.3 A

e) 4.8 A

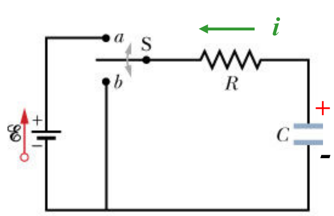




Ammeters and Voltmeters

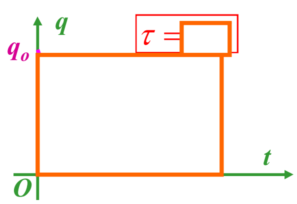
An ammeter is an instrument that measures current. In order to measure the current that flows through a conductor at a certain point we must cut the conductor at this point and connect the two ends of the conductor to the ammeter terminals so that the current can pass through the ammeter. An example is shown in the figure, where the ammeter has been inserted between points a and b.

It is essential that the ammeter resistance R_A be much smaller than the other resistors in the circuit. In our example: $R_A \ll R_1$ and $R_A \ll R_2$.
A voltmeter is an instrument that measures the potential difference between two points in a circuit. In the example of the figure we use a voltmeter to measure the potential across R_1. The voltmeter terminals are connected to the two points c and d.
It is essential that the voltmeter resistance R_V be much larger than the other resistors in the circuit. In our example: $R_V \gg R_1$ and $R_V \gg R_2$.



RC Circuits : Discharging of a Capacitor

Consider the circuit shown in the figure.

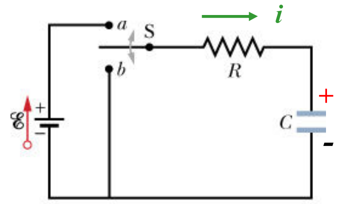


We will write KLR starting at point b and going in the counterclockwise direction: $\boxed{\quad} = 0$.

Taking into account that $i = \frac{dq}{dt}$ we get: $\boxed{\quad} = 0$.

This is a homogeneous, first order, linear differential equation with initial condition $q(0) = q_0$ The solution is: $q = q_0 e^{-t/\tau}$, where $\tau = RC$. If we plot q versus t we get a decaying exponential. The charge becomes zero at $t = \infty$. In practical terms we only have to wait a few time constants:

$q(\tau) = (0.368)q_0, \quad q(3\tau) = (0.049)q_0, \quad q(5\tau) = (0.007)q_0.$

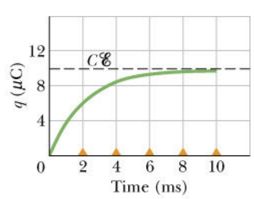


RC Circuits : Charging of a Capacitor

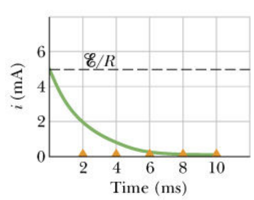
Consider the circuit shown in the figure. We assume that the capacitor is initially uncharged and that at $t = 0$ we throw the switch S from the middle position to position a. The battery will charge the capacitor C through the resistor R.

Our objective is to examine the charging process as a function of time.
We will write KLR starting at point b and going in the counterclockwise direction:

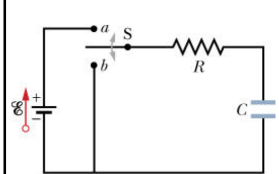
$\mathrm{E} - iR - \dfrac{q}{C} = 0.$ The current $i = \dfrac{dq}{dt} \rightarrow \boxed{} = 0.$ If we rearrange the terms we have: $\boxed{} = \mathrm{E}.$ This is an inhomogeneous, first order, linear differential equation with initial condition $q(0) = 0.$ This condition expresses the fact that at $t = 0$ the capacitor is uncharged.



(*a*)



(*b*)



Differential equation: $\boxed{} = \mathrm{E}$ $\boxed{\tau = RC}$

Initial condition: $q(0) = 0$

Solution: $q = C\mathrm{E}\left(1 - e^{-t/\tau}\right)$ Here: $\tau = RC$

The constant τ is known as the "time constant" of the circuit. If we plot q versus t we see that q does not reach its terminal value CE but instead increases from its initial value and reaches the terminal value at $t = \infty$. Do we have to wait for an eternity to charge the capacitor? In practice, no.

$q(t = \tau) = (0.632)C\mathrm{E}$

$q(t = 3\tau) = (0.950)C\mathrm{E}$

$q(t = 5\tau) = (0.993)C\mathrm{E}$

If we wait only a few time constants the charge, for all practical purposes, has reached its terminal value CE.

The current $i = \dfrac{dq}{dt} = \boxed{}$ If we plot i versus t we get a decaying exponential (see fig. b).

Q. What effect, if any, does increasing the battery emf in an RC circuit have on the time to charge the capacitor?

a) The charging time will decrease because the rate of charge flowing to the plates will increase.

b) The charging time will decrease because the rate of charge flowing to the plates will decrease.

c) The charging time will not change because the charging time does not depend on the battery emf.

d) The charging time will increase because the emf is increased.

e) The charging time will decrease because potential difference across the plates will be larger.

Q. The resistance in an RC circuit is comprised of a 1.5-MΩ resistor in parallel with a 2.0-MΩ resistor. What is the time constant for this circuit if the capacitance is 2.5 μF?

a) 2 s

b) 7.0 ms

c) 5.0 μs

d) 120 s

e) 4000 s