Eighth Edition

VECTOR MECHANICS FOR ENGINEERS:
STATICS

CHAPTER

8

Ferdinand P. Beer
E. Russell Johnston, Jr.

Lecture Notes:
J. Walt Oler
Texas Tech University

Friction

Contents

FRICTION

- In preceding chapters, it was assumed that surfaces in contact were either *frictionless* (surfaces could move freely with respect to each other) or *rough* (tangential forces prevent relative motion between surfaces).

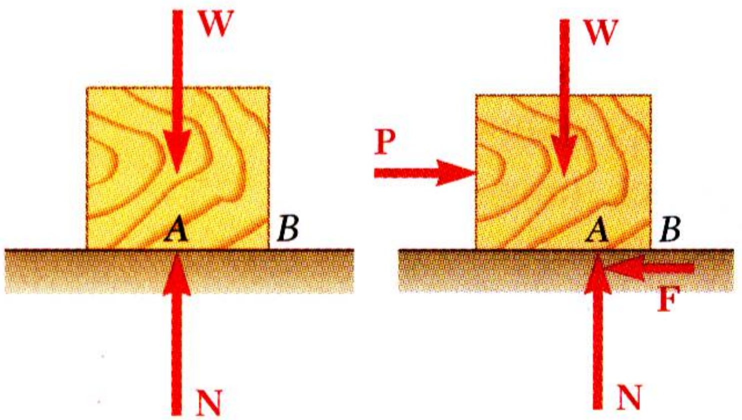
- Actually, no perfectly frictionless surface exists. For two surfaces in contact, tangential forces, called *friction forces*, will develop if one attempts to move one relative to the other.

- However, the friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.

- The distinction between frictionless and rough is, therefore, a matter of degree.

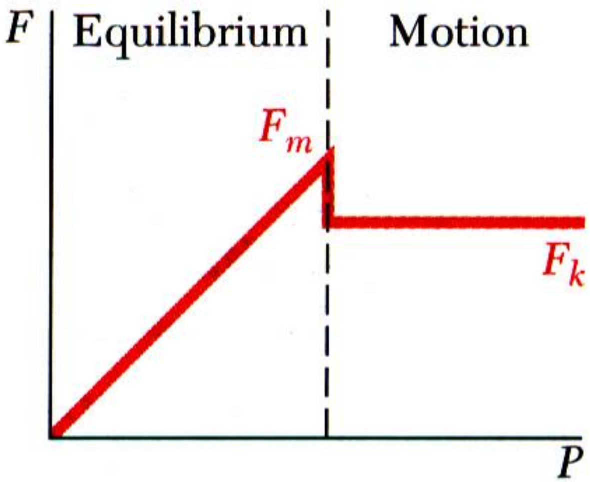
- There are two types of friction: *dry* or *Coulomb friction* and *fluid friction*. Fluid friction applies to lubricated mechanisms. The present discussion is limited to dry friction between nonlubricated surfaces.

The Laws of Dry Friction. Coefficients of Friction



- Block of weight W placed on horizontal surface. Forces acting on block are its weight and reaction of surface N.

- Small horizontal force P applied to block. For block to remain stationary, in equilibrium, a horizontal component F of the surface reaction is required. F is a *static-friction force*.



- As P increases, the static-friction force F also increases until it reaches a maximum value F_m.

$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as F drops to a smaller *kinetic-friction force* F_k. (**N** has not reached B)

$$F_k = \mu_k N$$

The Laws of Dry Friction. Coefficients of Friction

Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

- Maximum static-friction force:

$$F_m = \mu_s N$$

- Kinetic-friction force:

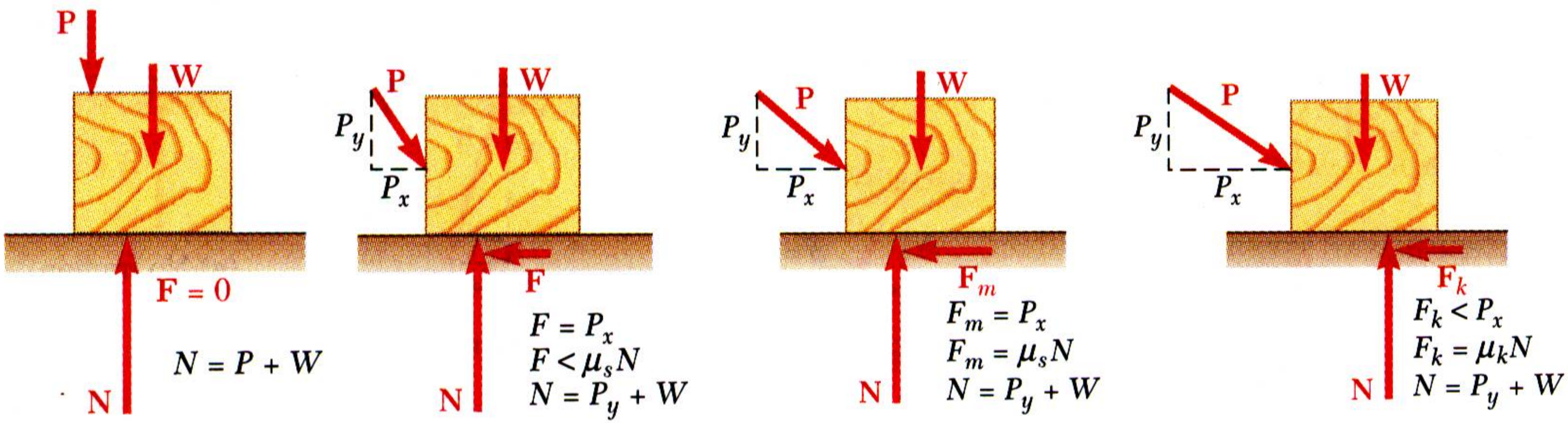
$$F_k = \mu_k N$$

$$\mu_k \cong 0.75\mu_s$$

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

The Laws of Dry Friction. Coefficients of Friction

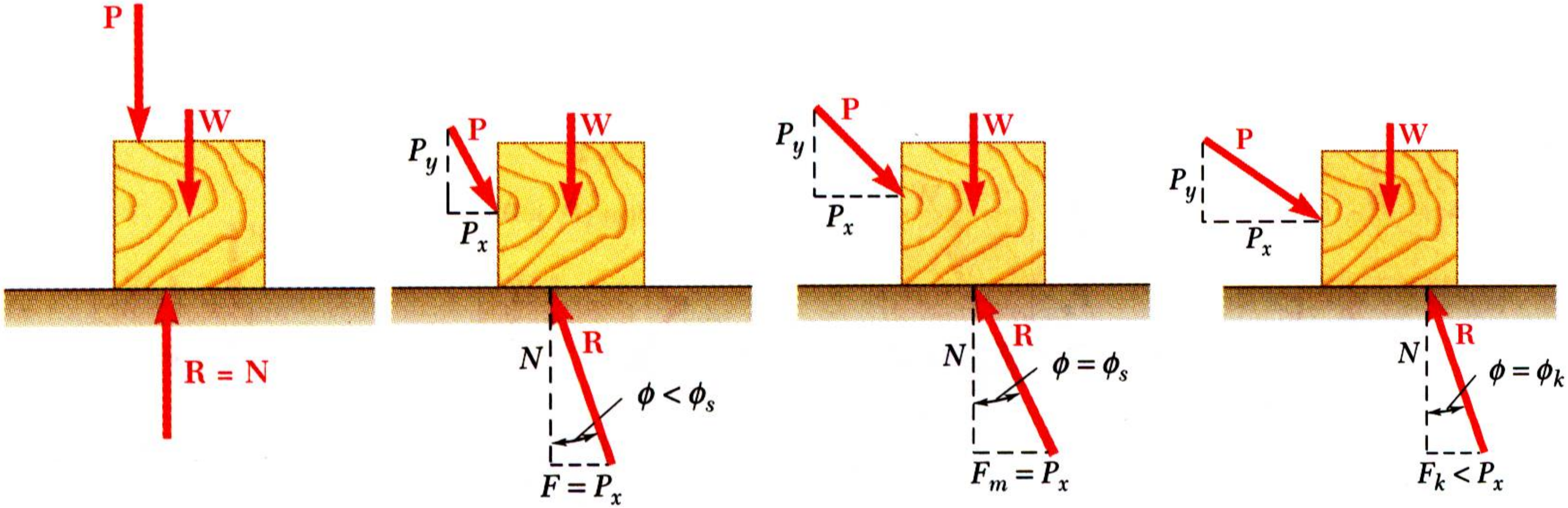
- Four situations can occur when a rigid body is in contact with a horizontal surface:



- No friction: ($P_x = 0$)
- No motion: ($P_x < F_m$)
- Motion "impending": ($P_x = F_m$)
- Motion: ($P_x > F_m$)

Angles of Friction

- It is sometimes convenient to replace normal force *N* and friction force *F* by their resultant ***R***:



- No friction
- No motion
- Motion impending

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

ϕ_s = "angle of static friction"

- Motion

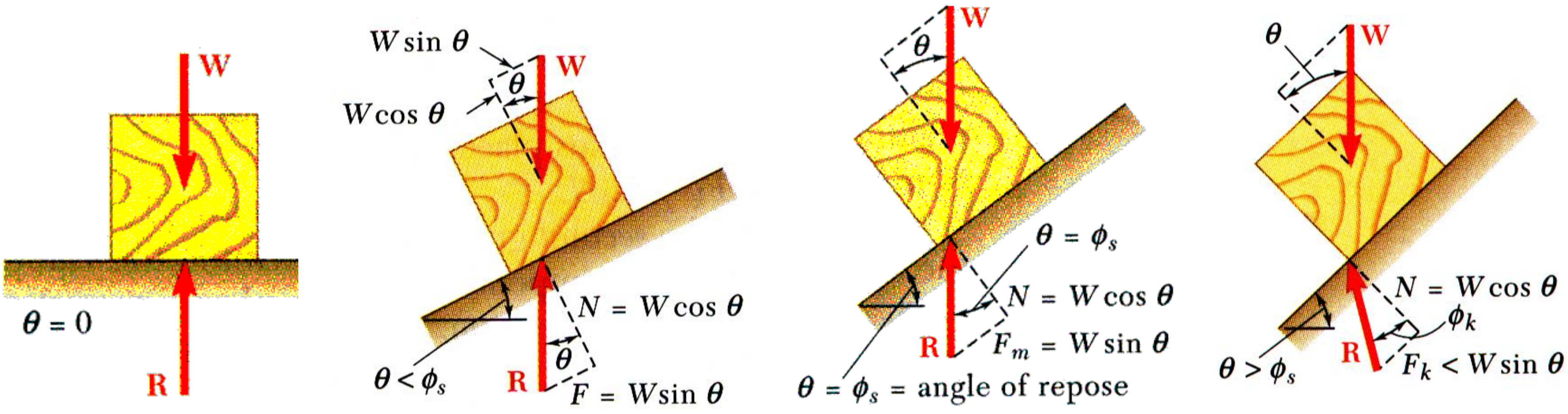
$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

ϕ_k = "angle…"

Angles of Friction

- Consider block of weight W resting on board with variable inclination angle θ.



- No friction
- No motion:

$$\theta < \phi_s$$

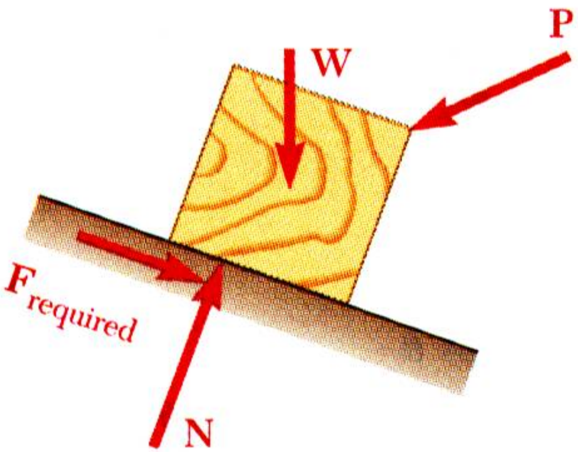
- Motion impending:

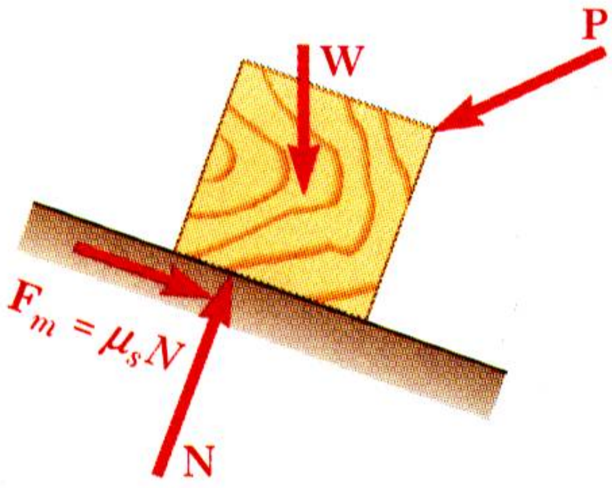
$$\theta = \phi_s$$

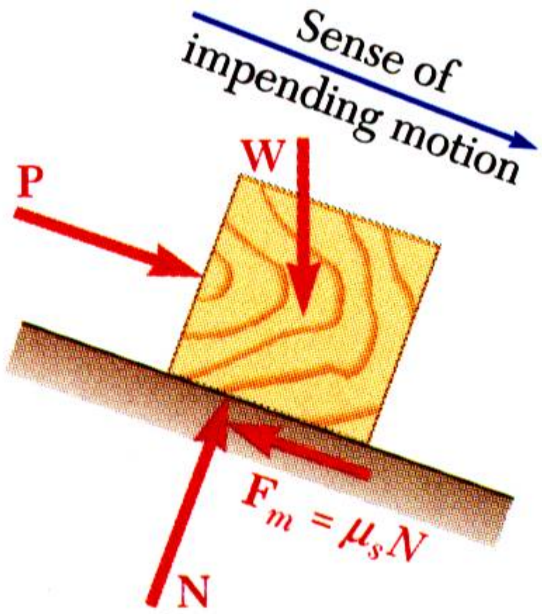
- Motion:

$$\theta > \phi_s$$

Problems Involving Dry Friction





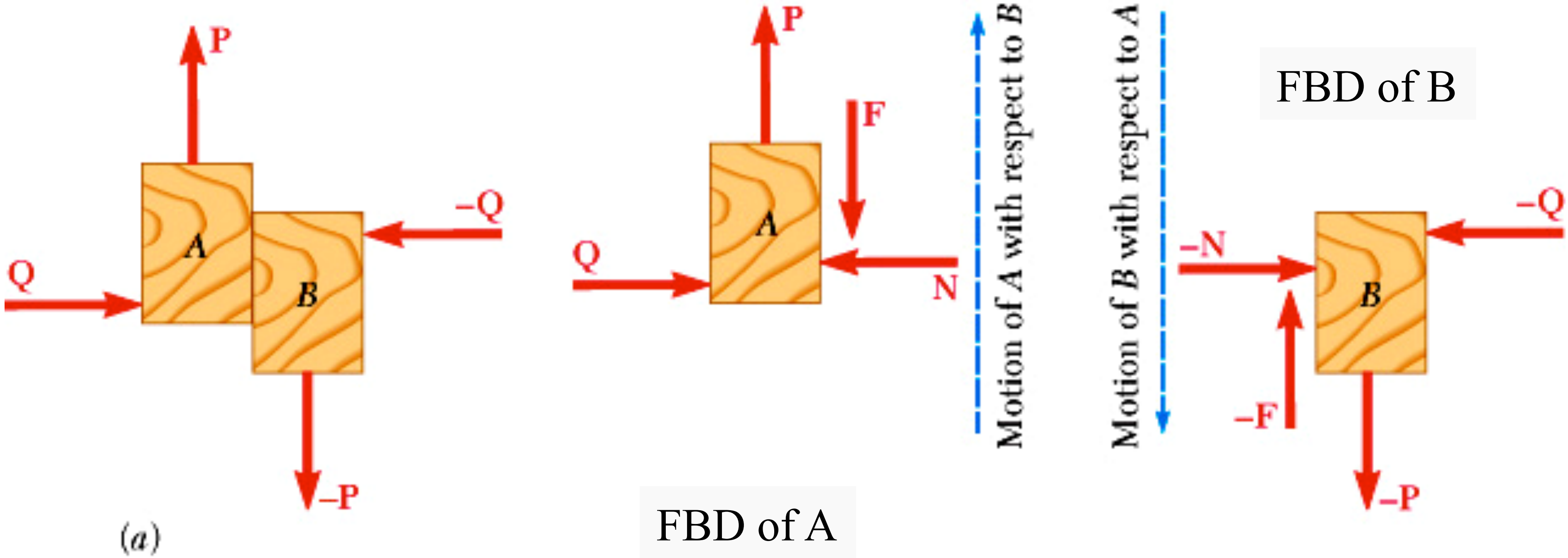


- All applied forces known
- Coefficient of static friction is known
- Determine whether body will remain at rest or slide

- All applied forces known
- Motion is impending
- Determine value of coefficient of static friction.

- Coefficient of static friction is known
- Motion is impending
- Determine magnitude or direction of one of the applied forces

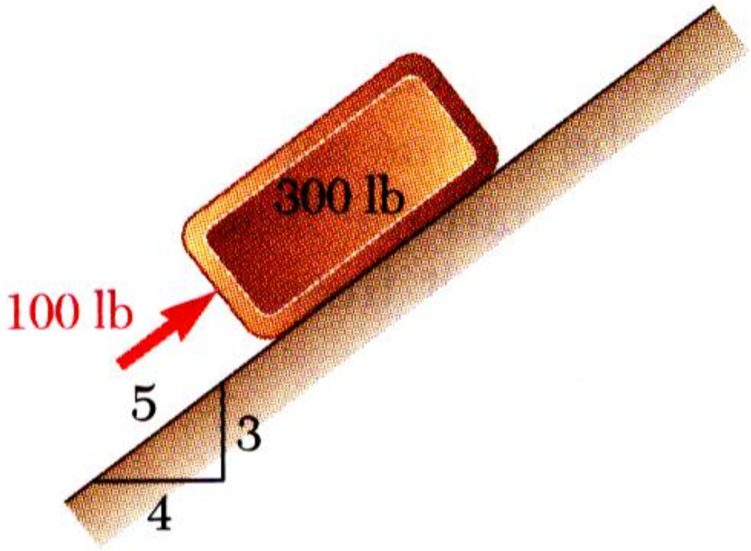
Problems Involving Dry Friction



- When two bodies *A* and *B* are in contact, the sense of the friction force acting on *A* is opposite to that of the motion (or impending motion) of *A* as observed from *B*. The sense of the friction force acting on *B* is opposite to that of the motion (or impending motion) of *B* as observed from *A*.

  e.g. if *A* is fixed and *B* moves down, *A* is moving up as observed from *B*;
  if both *A* and *B* are moving down but *B* is moving faster than *A*, *A* will be moving up as observed from *B*.

Sample Problem 8.1

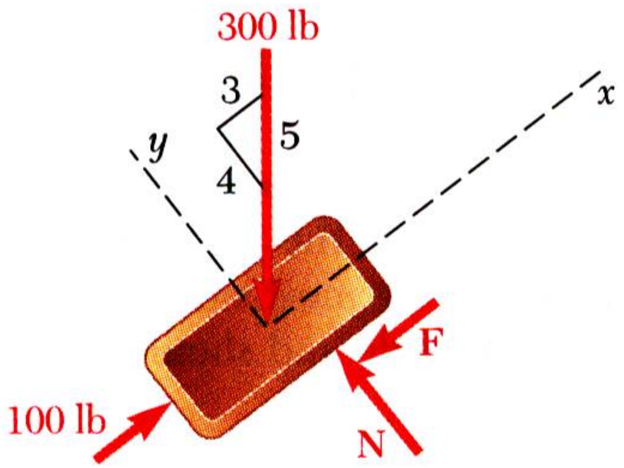


A 100 lb force acts as shown on a 300 lb block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force.

<u>SOLUTION</u>:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- If maximum friction force is less than friction force required for equilibrium, block will slide. In this case, calculate kinetic-friction force.

Sample Problem 8.1



SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

$$\sum F_x = 0: \qquad 100\text{ lb} - \tfrac{3}{5}(300\text{ lb}) - F = 0$$

$$F = -80\text{ lb} \qquad \text{(Reqd. for ≡bm)}$$

$$\sum F_y = 0: \qquad N - \tfrac{4}{5}(300\text{ lb}) = 0$$

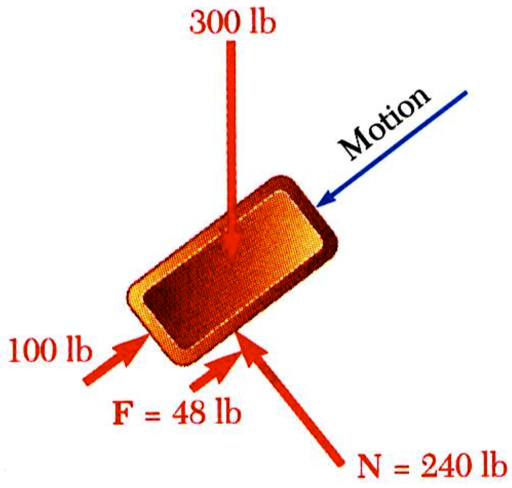
$$N = 240\text{ lb}$$

- Calculate maximum friction force and compare with friction force required for equilibrium. If it is less, block will slide.

$$F_{max} = \mu_s N \qquad F_{max} = 0.25(240\text{ lb}) = 60\text{ lb} \qquad F_{max} < F$$

$$\boxed{\textit{The block will slide down the plane.}}$$

Sample Problem 8.1



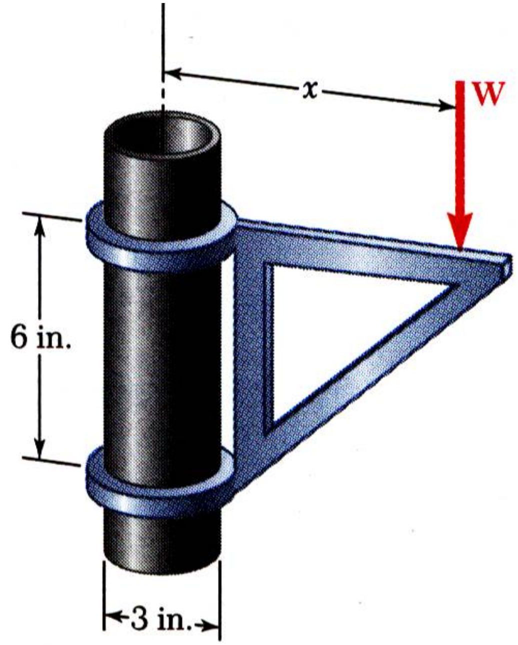
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k N$$
$$= 0.20(240\,\text{lb})$$

$$\boxed{F_{actual} = 48\,\text{lb}}$$

If 80lb force were applied in addition to the 100lb, we would have equilibrium!

Sample Problem 8.3

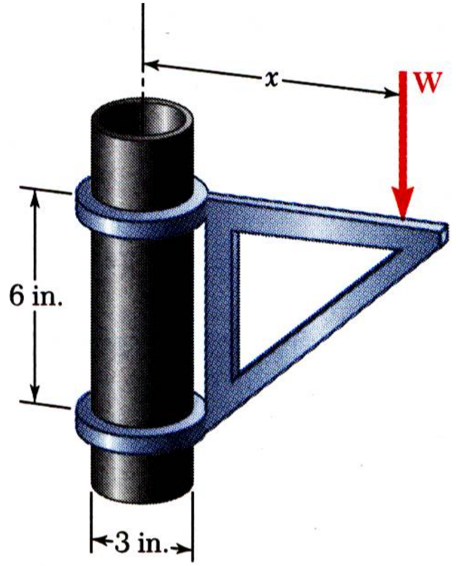


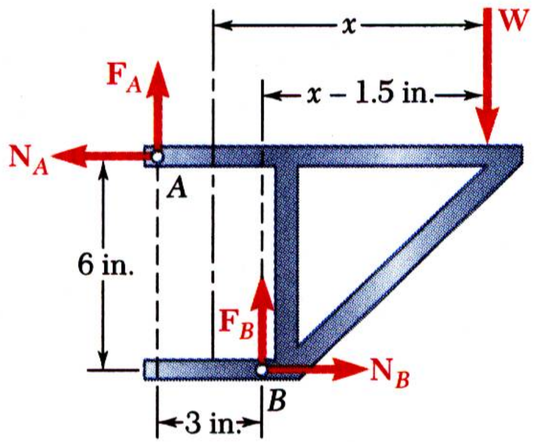
The moveable bracket shown may be placed at any height on the 3-in. diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance *x* at which the load can be supported. Neglect the weight of the bracket.

SOLUTION:

- When *W* is placed at minimum *x*, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.
- Apply conditions for static equilibrium to find minimum *x*.

Sample Problem 8.3





SOLUTION:

- When W is placed at minimum x, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.

$$F_A = \mu_s N_A = 0.25 N_A$$

$$F_B = \mu_s N_B = 0.25 N_B$$

- Apply conditions for static equilibrium to find minimum x.

$$\sum F_x = 0: \quad N_B - N_A = 0 \qquad N_B = N_A$$

$$\sum F_y = 0: \quad F_A + F_B - W = 0$$

$$0.25 N_A + 0.25 N_B - W = 0$$

$$0.5 N_A = W \qquad N_A = N_B = 2W$$

$$\sum M_B = 0: \quad N_A(6\text{ in.}) - F_A(3\text{ in.}) - W(x - 1.5\text{ in.}) = 0$$

$$6N_A - 3(0.25 N_A) - W(x - 1.5) = 0$$

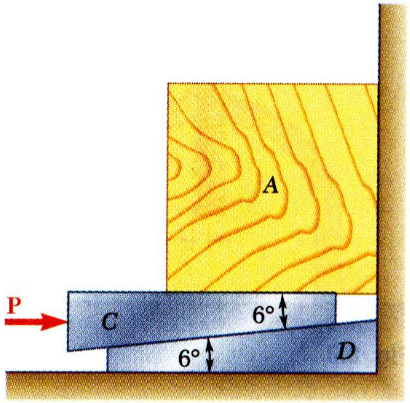
$$6(2W) - 0.75(2W) - W(x - 1.5) = 0$$

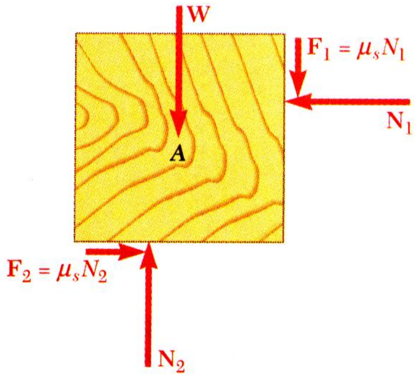
$$\boxed{x = 12\text{ in.}}$$

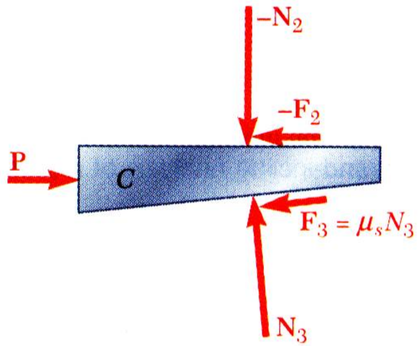
Blank

Wedges

Wedges







- *Wedges* - simple machines used to raise heavy loads.
- Force required to lift block is significantly less than block weight.
- Friction prevents wedge from sliding out.
- Want to find minimum force P to raise block.

- Block as free-body

$$\sum F_x = 0:$$
$$-N_1 + \mu_s N_2 = 0$$
$$\sum F_y = 0:$$
$$-W - \mu_s N_1 + N_2 = 0$$

or

$$\vec{R}_1 + \vec{R}_2 + \vec{W} = 0$$

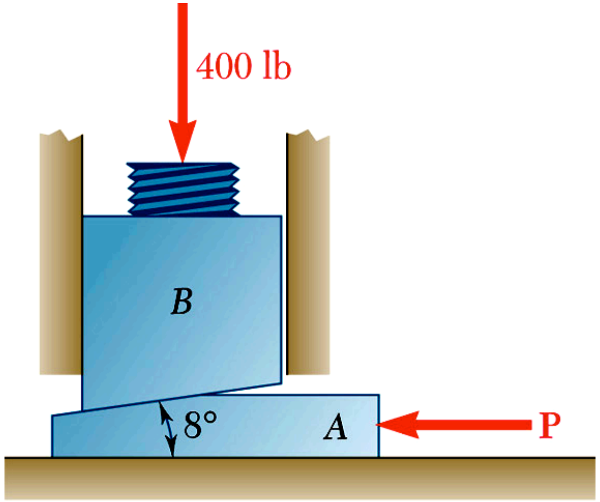
- Wedge as free-body

$$\sum F_x = 0:$$
$$-\mu_s N_2 - N_3\left(\mu_s \cos 6° - \sin 6°\right) + P = 0$$
$$\sum F_y = 0:$$
$$-N_2 + N_3\left(\cos 6° - \mu_s \sin 6°\right) = 0$$

or

$$\vec{P} - \vec{R}_2 + \vec{R}_3 = 0$$

Sample Problem 8.4



The position of the machine block *B* is adjusted by moving the wedge *A*. The coefficient of static friction is 0.35 between all surfaces of contact.

Determine the force ***P*** for which motion of block *B:*

(a)is impending upward,

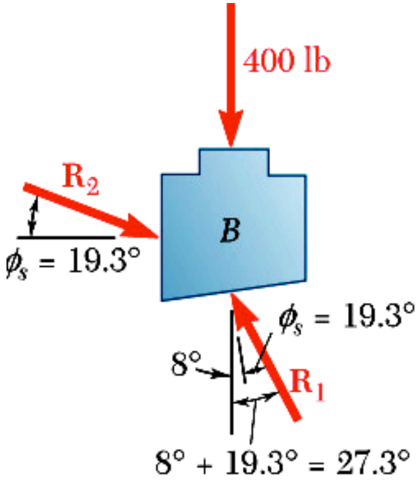
(b)is impending downward.

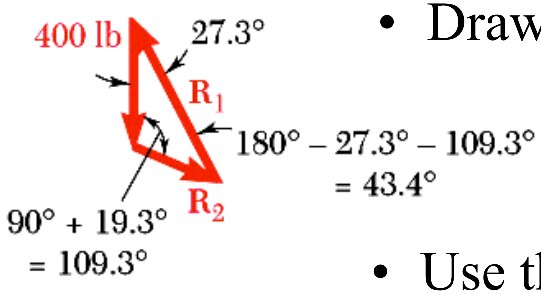
<u>SOLUTION</u>

- Draw the free-body diagrams of each part.
- With impending motion upwards, determine the direction and magnitude of reaction between the block *B* and the wedge *A* based on the free body of block *B*. Calculate the horizontal force ***P*** based on the free body of wedge *A*.

- With impending motion downwards, calculate force ***P*** in a similar way.

Sample Problem 8.4





SOLUTION (a): motion impending upward

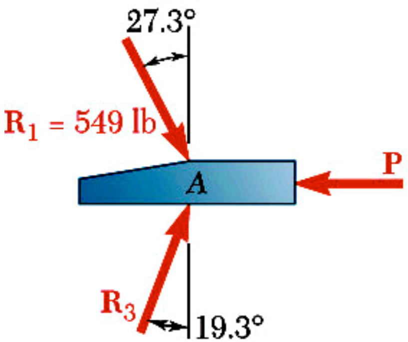
- Draw the free-body diagrams of block *B*.

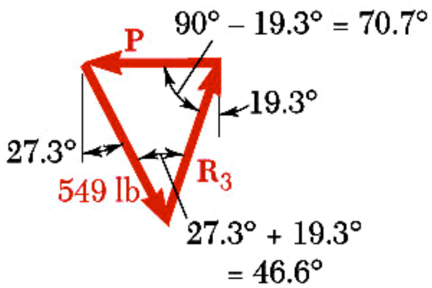
$$\phi_s = \tan^{-1} 0.35 = 19.3^\circ$$

- Use the law of sines.

$$\frac{R_1}{\sin 109.3^\circ} = \frac{400\text{ lb}}{\sin 43.4^\circ}$$

$$R_1 = 549\text{ lb}$$



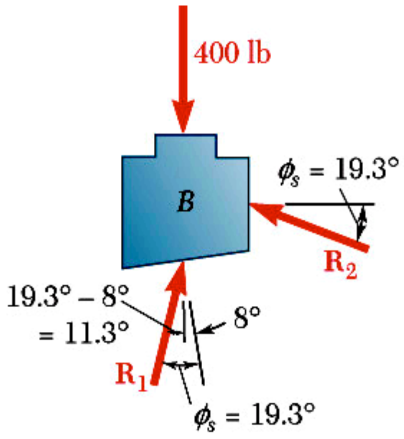


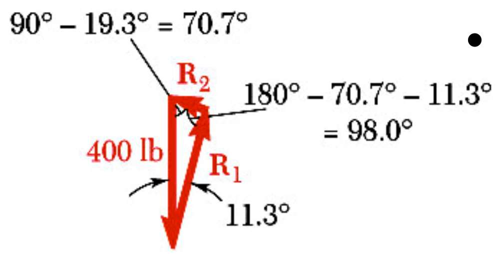
- Draw the free-body diagrams of wedge *A*.

$$\frac{P}{\sin 46.6^\circ} = \frac{549\text{ lb}}{\sin 70.7^\circ}$$

$$P = 423\text{ lb}$$

Sample Problem 8.4





SOLUTION (b): motion impending down

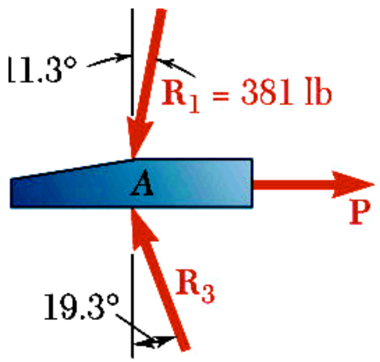
- Draw the free-body diagrams of block *B*.

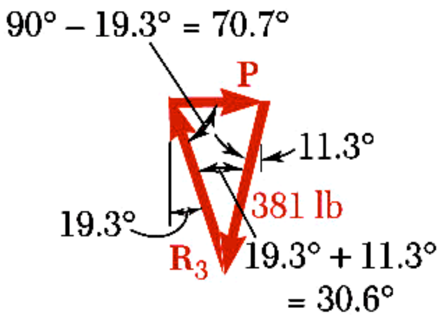
$$\phi_s = \tan^{-1} 0.35 = 19.3^\circ$$

- Use the law of sines.

$$\frac{R_1}{\sin 70.7^\circ} = \frac{400\,\text{lb}}{\sin 98.0^\circ}$$

$$R_1 = 381\,\text{lb}$$



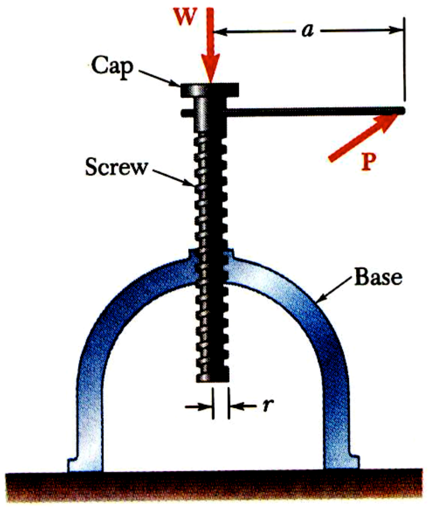


- Draw the free-body diagrams of wedge *A*.

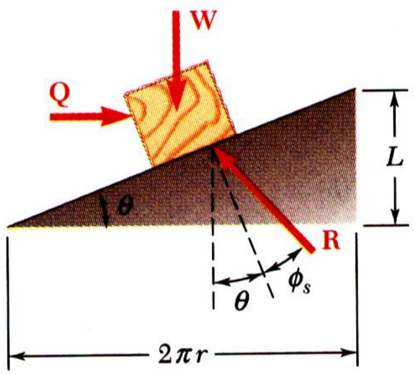
$$\frac{P}{\sin 30.6^\circ} = \frac{381\,\text{lb}}{\sin 70.7^\circ}$$

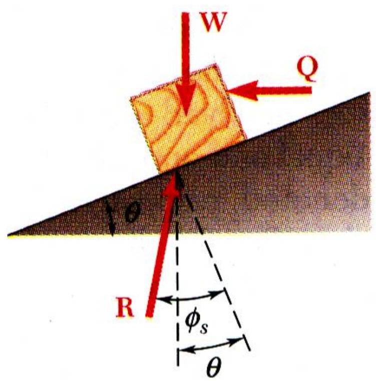
$$P = 206\,\text{lb}$$

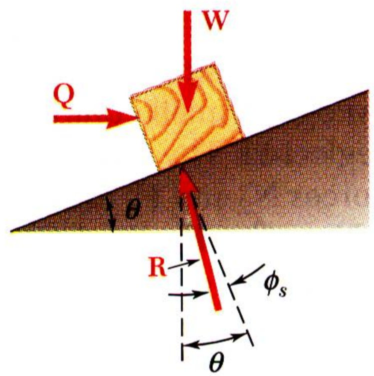
Square-Threaded Screws



- Square-threaded screws frequently used in jacks, presses, etc. Analysis similar to block on inclined plane. Recall friction force does not depend on area of contact.
- Thread of base can be "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.
- Moment of force Q is equal to moment of force P. $Q = Pa/r$

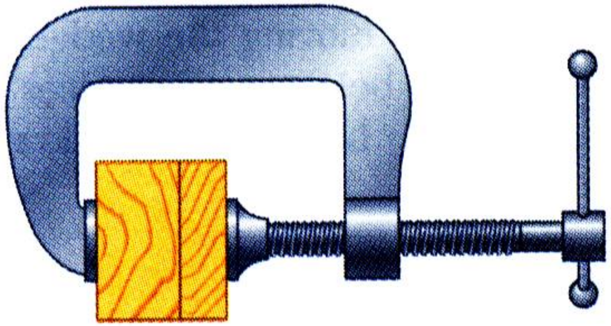






- Impending motion upwards. Solve for Q.
- $\phi_s > \theta$, Self-locking. Solve for Q to lower load.
- $\phi_s < \theta$, Non-locking. Solve for Q to hold load.

Sample Problem 8.5



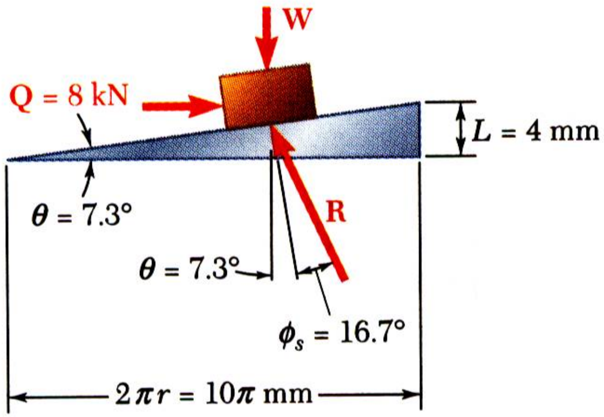
A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$.

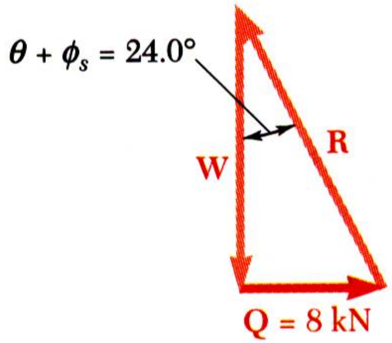
If a maximum torque of 40 N.m is applied in tightening the clamp, determine (a) the force exerted on the pieces of wood, and (b) the torque required to loosen the clamp.

SOLUTION

- Calculate lead angle and pitch angle.
- Using block and plane analogy with impending motion up the plane, calculate the clamping force with a force triangle.
- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

Sample Problem 8.5





SOLUTION

- Calculate lead angle and pitch angle. For the double threaded screw, the lead L is equal to twice the pitch.

$$\tan\theta = \frac{L}{2\pi r} = \frac{2(2\text{ mm})}{10\pi\text{ mm}} = 0.1273 \qquad \theta = 7.3°$$

$$\tan\phi_s = \mu_s = 0.30 \qquad \phi_s = 16.7°$$

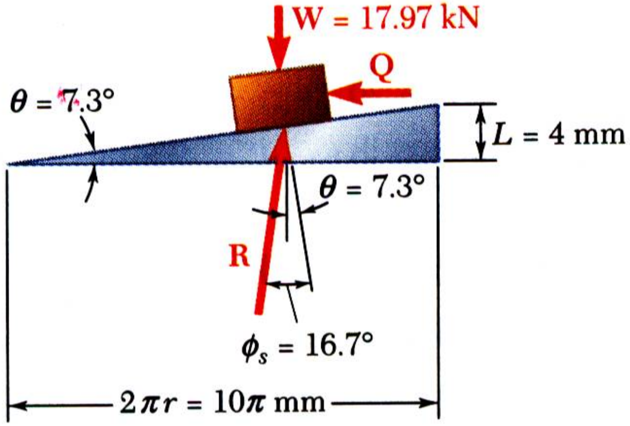
- Using block and plane analogy with impending motion up the plane, calculate clamping force with force triangle.

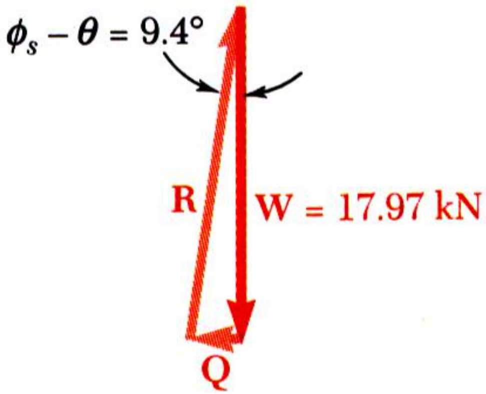
$$Qr = 40\text{ N}\cdot\text{m} \qquad Q = \frac{40\text{ N}\cdot\text{m}}{5\text{ mm}} = 8\text{ kN}$$

$$\tan(\theta + \phi_s) = \frac{Q}{W} \qquad W = \frac{8\text{ kN}}{\tan 24°}$$

$$\boxed{W = 17.97\text{ kN}}$$

Sample Problem 8.5





- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

$$\tan(\phi_s - \theta) = \frac{Q}{W} \qquad Q = (17.97\,\mathrm{kN})\tan 9.4°$$

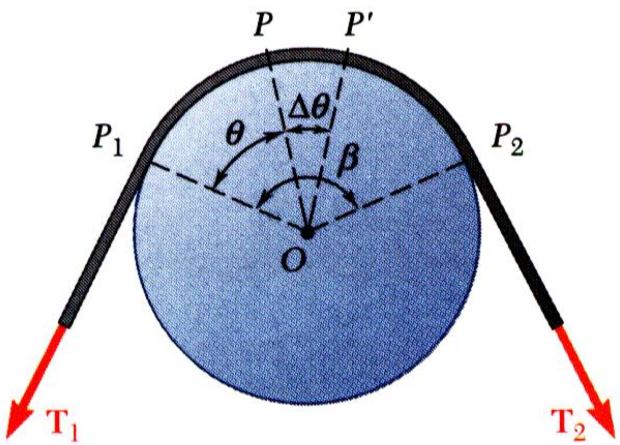
$$Q = 2.975\,\mathrm{kN}$$

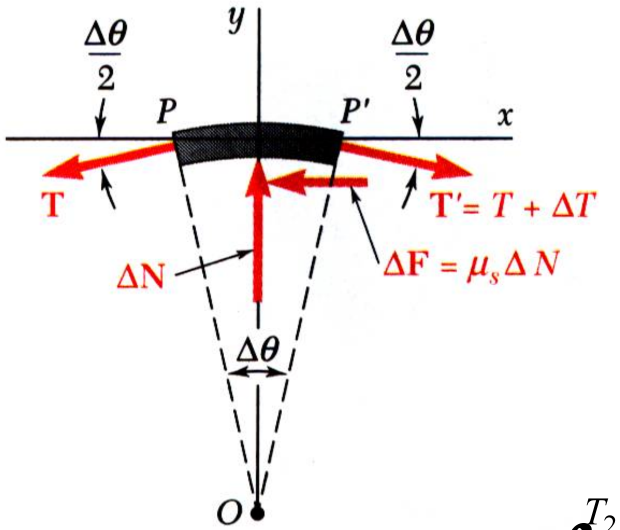
$$Torque = Q\,r = (2.975\,\mathrm{kN})(5\,\mathrm{mm})$$

$$= \left(2.975\times10^{3}\,\mathrm{N}\right)\left(5\times10^{-3}\,\mathrm{m}\right)$$

$$\boxed{Torque = 14.87\,\mathrm{N}\cdot\mathrm{m}}$$

Belt Friction





- Relate T_1 and T_2 when belt is about to slide to right.
- Draw free-body diagram for element of belt

$$\sum F_x = 0: \quad (T + \Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0$$

$$\sum F_y = 0: \quad \Delta N - (T + \Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0$$

- Combine to eliminate ΔN, divide through by $\Delta\theta$,

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s\left(T + \frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2} = 0$$

- In the limit as $\Delta\theta$ goes to zero,

$$\frac{dT}{d\theta} - \mu_s T = 0 \quad \Rightarrow \quad \frac{dT}{T} = \mu_s d\theta$$

- Separate variables and integrate from $\theta = 0$ to $\theta = \beta$

$$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^\beta \mu_s d\theta \quad \Rightarrow \quad \ln\frac{T_2}{T_1} = \mu_s\beta \quad \text{or} \quad \frac{T_2}{T_1} = e^{\mu_s\beta}$$

Belt Friction

By wrapping the rope around the bollard, the force exerted by the worker to pull the rope is much smaller than the tension in the taut portion of the rope.

Pull, bigger, direction of impending motion

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

Resist, smaller

$$\mu_s = 0.3$$

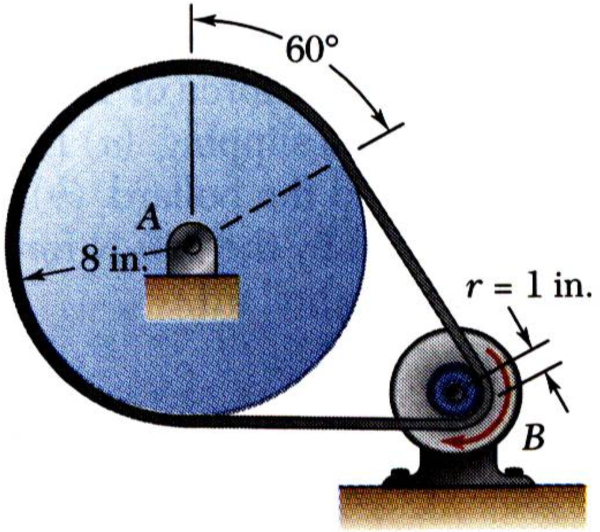
$$\beta = 2\pi \Rightarrow T_2 = 6.6T_1$$

$$\beta = 4\pi \Rightarrow T_2 = 43.4T_1$$

$$\beta = 6\pi \Rightarrow T_2 = 285.7T_1$$

In radians

Sample Problem 8.8



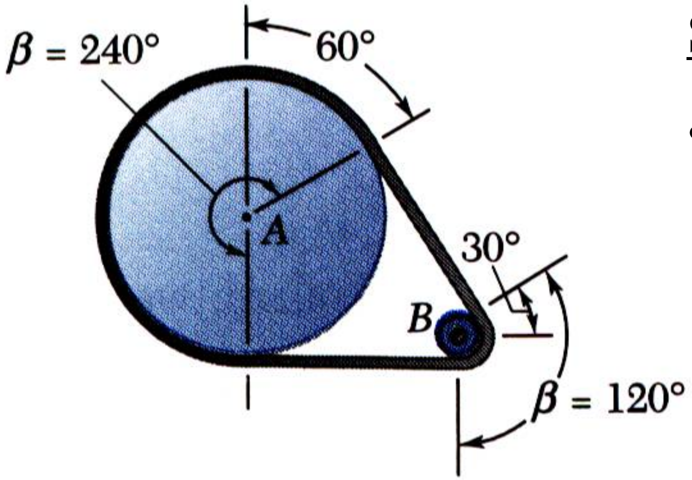
A flat belt connects pulley A to pulley B. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt.

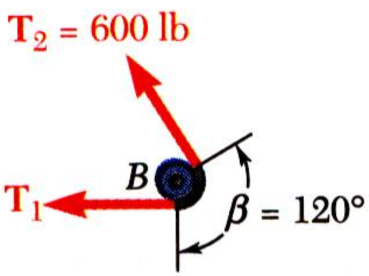
Knowing that the maximum allowable tension in the belt is 600 lb, determine the largest torque which can be exerted by the belt on pulley A.

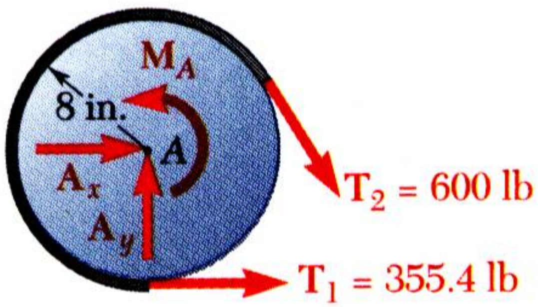
SOLUTION:

- Since angle of contact is smaller, slippage will occur on pulley B first. Determine belt tensions based on pulley B.
- Taking pulley A as a free-body, sum moments about pulley center to determine torque.

Sample Problem 8.8







SOLUTION:

- Since angle of contact is smaller, slippage will occur on pulley B first. Determine belt tensions based on pulley B.

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad \frac{600\,\text{lb}}{T_1} = e^{0.25(2\pi/3)} = 1.688$$

$$T_1 = \frac{600\,\text{lb}}{1.688} = 355.4\,\text{lb}$$

- Taking pulley A as free-body, sum moments about pulley center to determine torque.

$$\sum M_A = 0: \qquad M_A + (8\,\text{in.})(355.4\,\text{lb} - 600\,\text{lb}) = 0$$

$$\boxed{M_A = 163.1\,\text{lb}\cdot\text{ft}}$$