Gauss' Law

The flux (symbol Φ) of the electric field
Symmetry
Gauss' law

An infinite, uniformly charged insulating plane

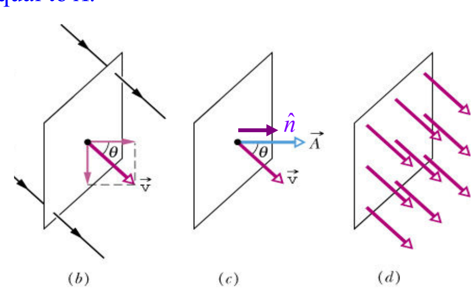
An infinite, uniformly charged insulating rod
A uniformly charged spherical shell
A uniform spherical charge distribution

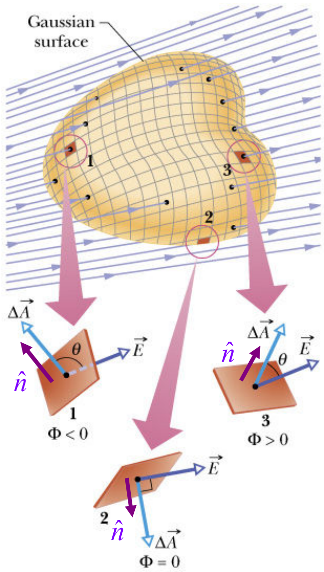
We will also apply Gauss' law to determine the electric field inside and outside charged conductors.

Flux of a Vector. Consider an airstream of velocity $\vec{v}$ that is aimed at a loop of area A . The velocity vector $\vec{v}$ is at angle θ with respect to the loop normal $\hat{n}$. The product $\Phi = vA\cos\theta$ is known as the **flux**. In this example the flux is equal to the volume flow rate through the loop (thus the name flux).

Note 1 : Φ depends on θ. It is maximum and equal to vA for $\theta = 0$ ($\vec{v}$ perpendicular to the loop plane). It is minimum and equal to zero for $\theta = 90°$ ($\vec{v}$ parallel to the loop plane).

Note 2 : $vA\cos\theta = \vec{v}\cdot\vec{A}$. The vector $\vec{A}$ is parallel to the loop normal and has magnitude equal to A.





$$\boxed{\Phi = \oint \vec{E} \cdot d\vec{A}}$$

Flux of the Electric Field.

Consider the closed surface shown in the figure.
In the vicinity of the surface assume that we have a known electric field $\vec{E}$. The flux Φ of the electric field through the surface is defined as follows:

1. Divide the surface into small "elements" of area ΔA.
2. For each element calculate the term $\vec{E} \cdot \Delta\vec{A} = EA\cos\theta$.
3. Form the sum $\Phi = \sum \vec{E} \cdot \Delta\vec{A}$.
4. Take the limit of the sum as the area $\Delta A \to 0$.

The limit of the sum becomes the integral:

$\Phi = \oint \vec{E} \cdot d\vec{A}$ Flux SI unit: $\text{N} \cdot \text{m}^2 / \text{C}$

Note 1: The circle on the integral sign indicates that the surface is closed. When we apply Gauss' law the surface is known as "Gaussian."

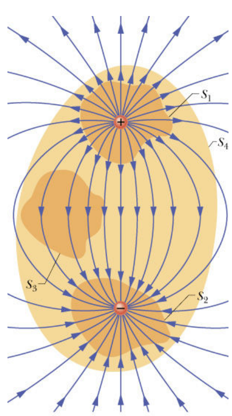
Note 2: Φ is proportional to the net number of electric field lines that pass through the surface.

Gauss' Law

Gauss' law can be formulated as follows:

The flux of $\vec{E}$ through any closed surface $\times\, \varepsilon_0$ = net charge q_{enc} enclosed by the surface.

In equation form: $\varepsilon_0 \Phi = q_{enc}$ Equivalently: $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$



$$\boxed{\varepsilon_0 \Phi = q_{enc}}$$

$$\boxed{\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}}$$

Note Gauss' law holds for **any** closed surface. Usually one particular surface makes the problem of determining the electric field very simple.

Note When applying Gauss' law for a closed surface we ignore the charges outside the surface no matter how large they are.

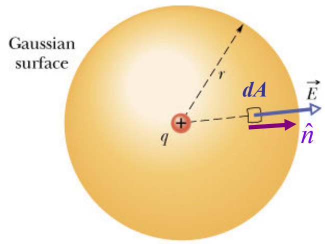
Example:

Surface S_1: $\varepsilon_0 \Phi_1 = +q$, Surface S_2: $\varepsilon_0 \Phi_2 = -q$

Surface S_3: $\varepsilon_0 \Phi_3 = 0$, Surface S_4: $\varepsilon_0 \Phi_4 = -q + q = 0$

Note: We refer to S_1, S_2, S_3, S_4 as "Gaussian surfaces."

Gauss' Law and Coulomb's Law



Here we will derive Coulomb's law from Gauss' law. Consider a point charge q. We will use Gauss' law to determine the electric field $\vec{E}$ generated at a point P at a distance r from q. We choose a Gaussian surface that is a sphere of radius r and has its center at q.

We divide the Gaussian surface into elements of area dA. The flux for each element is:

$d\Phi = EdA\cos 0 = EdA$ Total flux $\Phi = \oint EdA = E\oint dA = E\left(4\pi r^2\right)$

From Gauss' law we have: $\varepsilon_0 \Phi = q_{enc} = q \rightarrow 4\pi r^2 \varepsilon_0 E = q \rightarrow E = \dfrac{q}{4\pi r^2 \varepsilon_0}$

A spherical Gaussian surface of radius R is surrounding a particle with a net charge q. If the spherical Gaussian surface is replaced by a cube, under what conditions would the electric flux through the sides of the cube be the same as through the spherical surface?

a) under all conditions

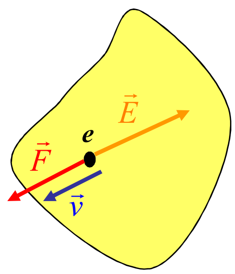
b) if the sides of the cube are of length R

c) if the sides of the cube are of length $2R$

d) if the diagonals of the cube are of length $2R$

e) under no conditions

The Electric Field Inside a Conductor vanishes



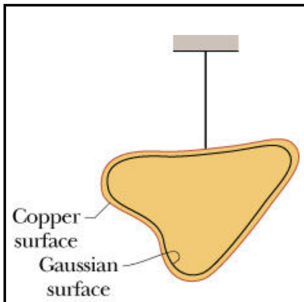
Consider the conductor shown in the figure to the left.

such an object contains negatively charged electrons, which are free to move inside the conductor. Let's assume for a moment that the electric field is not equal to zero. In such a case a nonvanishing force $\vec{F} = -e\vec{E}$ is exerted by the field on each electron. This force would result in a nonzero velocity $\vec{v}$, and the moving electrons would constitute an electric current.

(a) They $\boxed{}$ the conductor.
(b) They generate $\boxed{}$ around the conductor.
No such effects have ever been observed, thus the original assumption that there exists a nonzero electric field inside the conductor. We conclude that :

The electrostatic electric field $\vec{E}$ inside a conductor is equal to zero.

A Charged Isolated Conductor



Where is this charge located? To answer the question we will apply Gauss' law to the Gaussian surface shown in the figure, which is located just below the conductor surface. Inside the conductor the electric field $\vec{E} = 0$.

Thus $\Phi = \boxed{}$ (**eq. 1**).

From Gauss's law we have: $\Phi = \frac{q_{enc}}{\varepsilon_0}$ (**eq. 2**).

If we compare eq. 1 with eq. 2 we get $q_{enc} = 0$.

Thus no charge exists inside the conductor.

No electrostatic charges can exist inside a conductor.
All charges reside $\boxed{}$

When you calculate the electric flux through a Gaussian surface, of what are you determining the flow through the surface?

a) charge

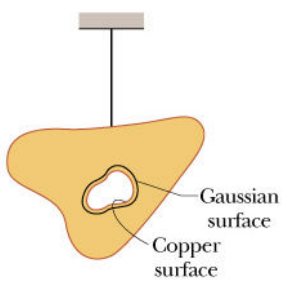
b) electric current

c) electric energy

d) electric field

e) None of the above answers are correct.

An Isolated Charged Conductor with a Cavity



We ask the question: Can charges reside on the walls of the cavity?

As before, Gauss's law provides the answer.

We will apply Gauss' law to the Gaussian surface shown in the figure, which is located just below the conductor surface. Inside the conductor the electric field $\vec{E} = 0$.

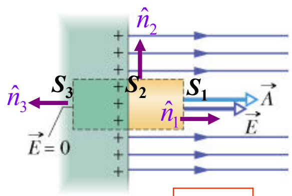
Thus $\Phi = \boxed{}$ (**eq. 1**).

From Gauss's law we have: $\Phi = \frac{q_{enc}}{\varepsilon_0}$ (**eq. 2**).

If we compare eq. 1 with eq. 2 we get $q_{enc} = \boxed{}$

Conclusion :

There is $\boxed{}$ on the cavity walls. All the excess charge q remains on the outer surface of the conductor.



$\boxed{E = \dfrac{\sigma}{\varepsilon_0}}$

The Electric Field Outside a Charged Conductor

The electric field inside a conductor is zero. This is not the case for the electric field outside. The electric field vector $\vec{E}$ is perpendicular to the conductor surface. If it were not, then $\vec{E}$ would have a component parallel $E_{\parallel}$ to the conductor surface.

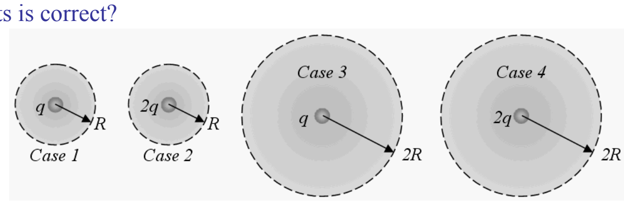
Since charges are free to move in the conductor, $E_{\parallel}$ would cause the free electrons to move, which is a contradiction to the assumption that we have stationary charges. We will apply Gauss' law using the cylindrical closed surface shown in the figure. The surface is further divided into three sections $S_1, S_2,$ and S_3 as shown in the figure. The net flux $\Phi = \Phi_1 + \Phi_2 + \Phi_3$.

$\Phi_1 = \boxed{\quad\quad}$ $\Phi_2 = \boxed{\quad\quad}$

$\Phi_3 = \boxed{\quad}$ (because the electric field inside the conductor is zero). $\Phi = \boxed{\quad\quad} \rightarrow$

$E = \boxed{\quad\quad}$ The ratio $\sigma = \dfrac{q_{enc}}{A}$ is known as surface charge density $\rightarrow E = \dfrac{\sigma}{\varepsilon_0}$.

Consider the five situations shown. Each one contains either a charge q or a charge $2q$. A Gaussian surface surrounds the charged particle in each case. Considering the electric flux through each of the Gaussian surfaces, which of the following comparative statements is correct?



a) $\Phi_2 = \Phi_4 > \Phi_1 = \Phi_3$

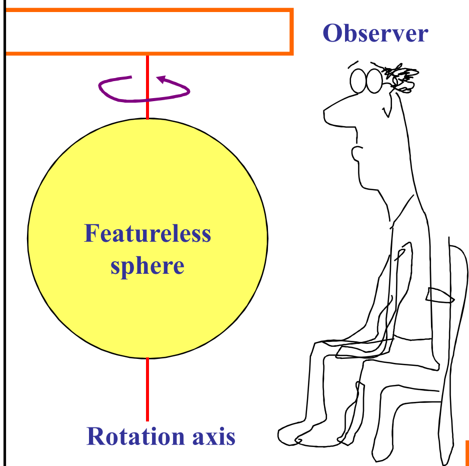
b) $\Phi_1 = \Phi_3 > \Phi_2 = \Phi_4$

c) $\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3$

d) $\Phi_3 = \Phi_4 > \Phi_2 = \Phi_1$

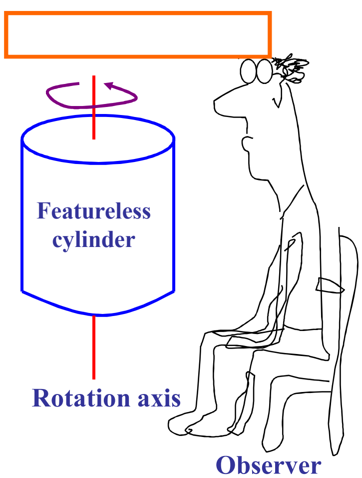
e) $\Phi_4 > \Phi_3 > \Phi_2 > \Phi_1$

Symmetry. We say that an object is symmetric under a particular mathematical operation (e.g., rotation, translation, …) if to an observer the object looks the same before and after the operation.
Note: Symmetry is a primitive notion and as such is very powerful.



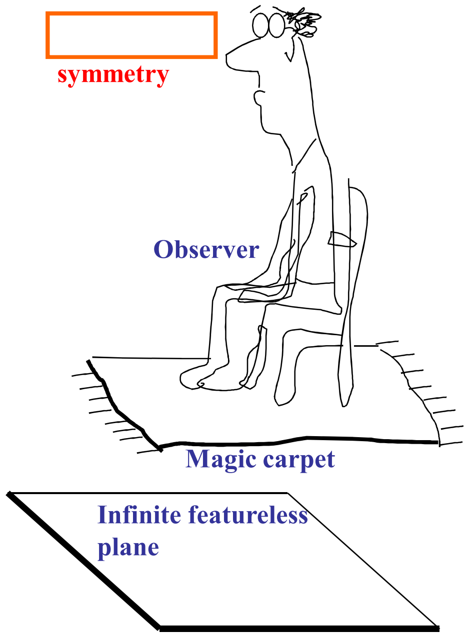
Example of Spherical Symmetry

Consider a featureless beach ball that can be rotated about a vertical axis that passes through its center. The observer closes his eyes and we rotate the sphere. When the observer opens his eyes, he **cannot** tell whether the sphere has been rotated or not. We conclude that the sphere has about the .



Consider a featureless cylinder that can rotate about its central axis as shown in the figure. The observer closes his eyes and we rotate the cylinder. When he opens his eyes, he **cannot** tell whether the cylinder has been rotated or not. We conclude that the cylinder has symmetry about the rotation axis.

Consider an infinite featureless plane. An observer takes a trip on a magic carpet that flies above the plane. The observer closes his eyes and we move the carpet around. When he opens his eyes the observer **cannot** tell whether he has moved or not. We conclude that the plane has ______ **symmetry.**



Recipe for Applying Gauss' Law

1. Make a sketch of the charge distribution.

2. Identify the ______ of the distribution and its effect on the electric field.

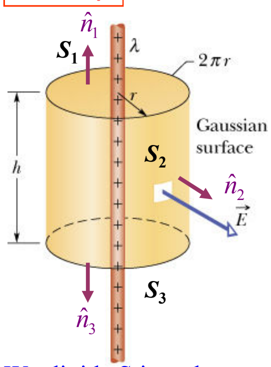
3. Gauss' law is true for **any** closed surface *S*. Choose one that makes the calculation of the flux Φ ______

4. Use Gauss' law to determine the electric field vector:

$$\Phi = \boxed{}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$



Electric Field Generated by a Long, Uniformly Charged Rod

Consider the long rod shown in the figure. It is uniformly charged with linear charge density λ.

Gaussian surface chosen to match symmetry of wire/fields. Makes for simpler calculations.

We divide S into three sections: Top flat section S_1, middle curved section S_2, and bottom flat section S_3. The net flux through S is $\Phi = \Phi_1 + \Phi_2 + \Phi_3$. Fluxes Φ_1 and Φ_2 vanish be ic field is at right angles with the normal to the surface:

$\Phi_3 = \quad = \quad \rightarrow \Phi = 2\pi rhE$. From Gauss's law we have: $\Phi = \frac{q_{\text{enc}}}{\varepsilon_0} =$

If we compare these two equations we get: $2\pi rhE = \frac{\lambda h}{\varepsilon_0} \rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$.

A straight, copper wire has a length of 0.50 m and an excess charge of -1.0×10^{-5} C distributed uniformly along its length. Find the magnitude of the electric field at a point located 7.5×10^{-3} m from the midpoint of the wire.

a) 1.9×10^{10} N/C

b) 7.3×10^{8} N/C

c) 6.1×10^{13} N/C

d) 1.5×10^{6} N/C

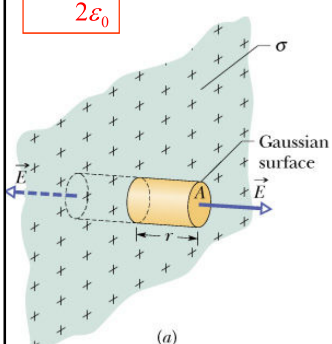
e) 4.8×10^{7} N/C

(a) A drum of a photocopying machine is of length 42 cm and radius 6 cm. What is the total charge on the drum where the electric field just above the drum's surface is 2.5 x 10^5 N/C, as is required for photocopying? You may approximate the drum to be a long, thin-walled metal tube. *(3 marks)*

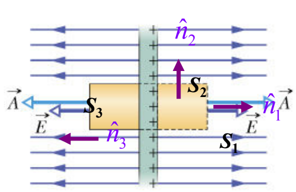
(b) What are the magnitudes of the electric fields at radial distances of 3 cm and 12 cm from the drum axis of rotation? Sketch E against r, for r = 0 to 12cm and clearly label your sketch with these values. *(4 marks)*

Speed of light in a vacuum, $c = 3.00 \times 10^8$ m s^{-1}.
Elementary charge, $e = 1.60 \times 10^{-19}$ C.
Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg.
Mass of proton, $m_p = 1.67 \times 10^{-27}$ kg.
Avogadro constant, $N_A = 6.02 \times 10^{23}$ mol^{-1}.
Planck constant, $h = 6.63 \times 10^{-34}$ J s.
Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J K^{-1} = 8.617×10^{-5} eV K^{-1}
Universal gas constant, $R = 8.31$ J mol^{-1} K^{-1}
Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.
Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.
Gravitational acceleration, $g = 9.80$ m s^{-2}.
Permeability constant, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}.
Permittivity constant, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}.
Density of mercury, $\rho = 13.6 \times 10^3$ kg m^{-3}.

$$E = \frac{\sigma}{2\varepsilon_0}$$



(a)



(b)

Electric Field Generated by a Thin, Infinite, Nonconducting Uniformly Charged Sheet

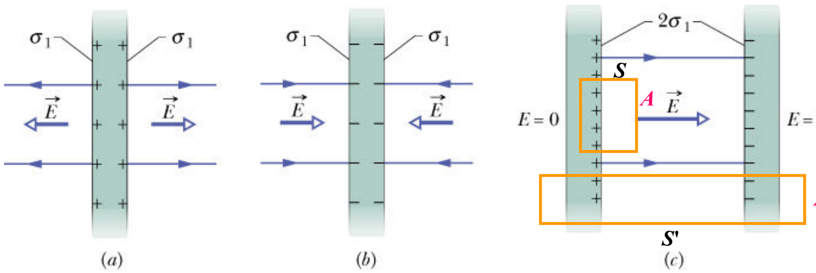
We assume that the sheet has a positive charge of surface density σ. From symmetry, the electric field

Gaussian surface chosen to match of arrangement/fields. Makes for simpler calculations.

$\Phi_1 = \Phi_2 = EA\cos 0 = EA. \quad \Phi_3 = 0 \quad (\theta = 90°)$

$\rightarrow \Phi = 2EA.$ From Gauss's law we have: $\Phi = \frac{q_{enc}}{\varepsilon_0} =$

$\rightarrow \quad \rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}.$



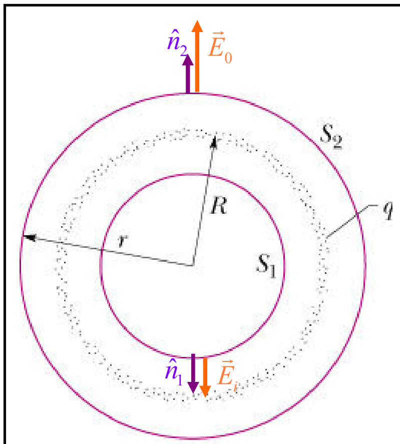
$E_i =$

$E_0 = 0$

The electric field generated by two parallel conducting infinite planes is charged with surface densities σ_1 and $-\sigma_1$. In figs. *a* and *b* we show the two plates isolated so that one does not influence the charge distribution of the other. The charge spreads out equally on both faces of each sheet. When the two plates are moved close to each other as shown in fig. *c*, then the charges on one plate attract those on the other. As a result the charges move on the inner faces of each plate. To find the field E_i between the plates we apply Gauss' law for the cylindrical surface S, which has caps of area A.

The net flux $\Phi = E_i A = \frac{q_{\text{enc}}}{\varepsilon_0} = \quad \rightarrow \quad E_i = \frac{2\sigma_1}{\varepsilon_0}$. To find the field E_0 outside the plates we apply Gauss' law for the cylindrical surface S', which has caps of area A.

The net flux $\Phi = E_0 A = \quad = \quad = 0 \rightarrow \quad E_0 = 0.$



$E_i = 0$

$E_0 = \frac{q}{4\pi\varepsilon_0 r^2}$

The Electric Field Generated by a Spherical Shell of Charge q and Radius R

Inside the shell :

The electric field flux $\Phi =$

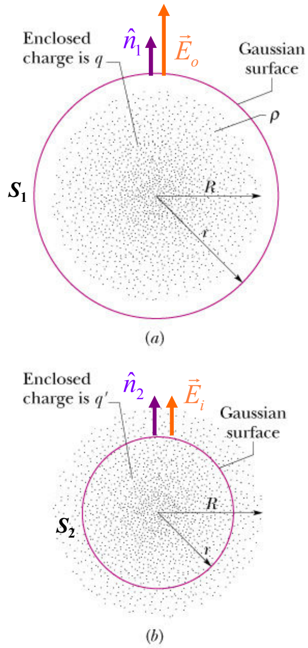
Thus $E_i = 0.$

Outside the shell :

The electric field flux $\Phi =$

Thus $E_0 = \frac{q}{4\pi\varepsilon_0 r^2}$.

Note : Outside the shell the electric field is the same as if all the charge of the shell were concentrated at the shell center.



Electric Field Generated by a Uniformly Charged Sphere of Radius R and Charge q

Outside the sphere :

The electric field flux $\Phi = \boxed{}$

Thus $E_0 = \dfrac{q}{4\pi\varepsilon_0 r^2}$.

Inside the sphere :

The electric field flux $\Phi = \boxed{}$

$$q_{\text{enc}} = \frac{4\pi / r^3}{4\pi / R^3} q = \frac{R^3}{r^3} q \to 4\pi r^2 E_i = \frac{R^3}{r^3}\frac{q}{\varepsilon_0}$$

Thus $E_i = \left(\dfrac{q}{4\pi\varepsilon_0 R^3}\right) r$.

note errors

A total charge of –6.50 μC is uniformly distributed within a sphere that has a radius of 0.150 m. What is the magnitude and direction of the electric field at 0.300 m from the surface of the sphere?

a) 2.89×10^5 N/C, radially inward

b) 9.38×10^5 N/C, radially outward

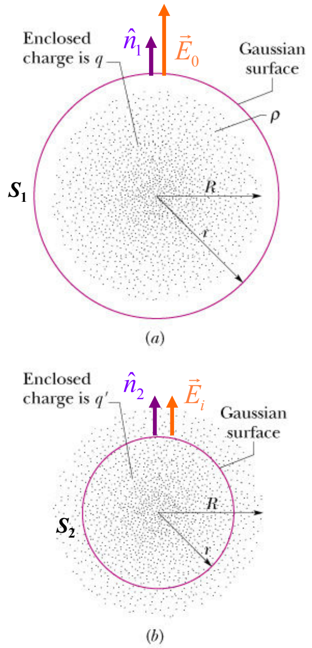
c) 1.30×10^6 N/C, radially inward

d) 6.49×10^5 N/C, radially outward

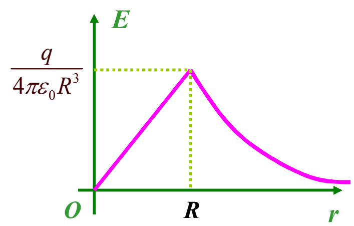
e) 4.69×10^5 N/C, radially inward

Electric Field Generated by a Uniformly Charged Sphere of Radius *R* and Charge *q*

Summary :



$E_0 =$

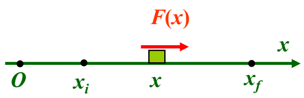


$E_i =$

Electric Potential

Calculate V if we know electric field.
Calculate the electric field if we know potential V.
Determine the potential V generated by a point charge.
Determine the potential V generated by charge distribution.
Determine the electric potential energy U of a system of charges.
Equipotential surface.
Relationship between equipotential surfaces and electric field lines.
Potential of a charged isolated conductor.

Determining Potential Energy Values :

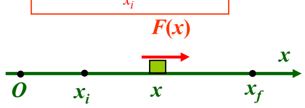


A conservative force F moves an object along the x-axis from an initial point x_i to a final point x_f. The work W that the force F does on the object is given by

$$W = \int_{x_i}^{x_f} \boxed{}\, dx.$$

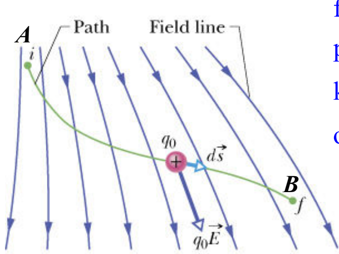
Therefore the expression for ΔU becomes $\boxed{\Delta U = -\int_{x_i}^{x_f} \boxed{}\, dx}$

$$\boxed{\Delta U = -\int_{x_i}^{x_f} F(x)dx}$$



Electric Potential Energy

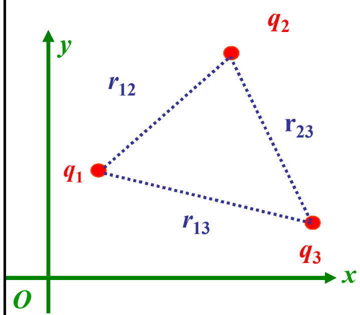
$$\boxed{\Delta U = -q_0 \int_i^f \vec{E} \cdot d\vec{s}}$$



$$\Delta U = U_f - U_i = -W = -\int_{x_i}^{x_f} F(x)dx$$

Consider an electric charge q_0 moving from an initial position at point A to a final position at point B under the influence of a known electric field $\vec{E}$. The force exerted on the charge is $\vec{F} = q_0 \vec{E}$.

$$\Delta U = -\int_i^f \vec{F} \cdot d\vec{s} = -q_0 \int_i^f \vec{E} \cdot d\vec{s}$$



Potential Energy U of a System of Point Charges

We define U as the work required to assemble the system of charges one by one, bringing each charge from infinity to its final position.

Using the above definition we will prove that for a system of three point charges U is given by:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \boxed{} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$$

Note: Each pair of charges is counted only once.

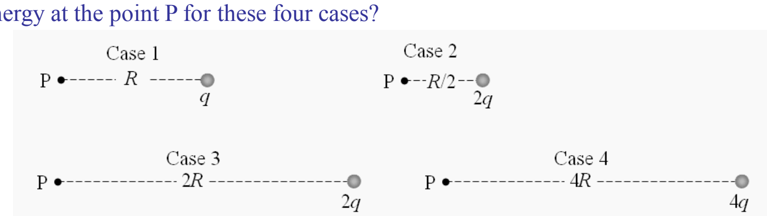
For a system of n point charges $\{q_i\}$ the potential energy U is given by:

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{i,j=1 \\ i<j}}^{n} \boxed{}.$$

Here r_{ij} is the separation between q_i and q_j.

The summation condition $i < j$ is imposed so that, as in the case of three point charges, each pair of charges is counted only once.

Consider the four cases shown of a charged particle located some distance from a point P. The quantity of charge and the distances vary. Note that the drawings are not necessarily drawn to scale. Which of the following expressions correctly ranks the electric potential energy at the point P for these four cases?



a) $U_2 > U_1 = U_3 = U_4$

b) $U_2 > U_1 > U_3 > U_4$

c) $U_2 > U_1 > U_3 = U_4$

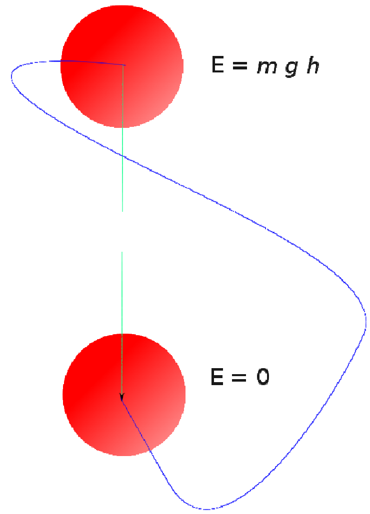
d) $U_2 = U_1 > U_3 = U_4$

e) $U_1 = U_3 = U_4 > U_2$

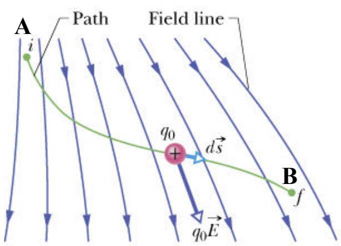
Under a force, the work done in moving a particle between two points is independent of the path taken

For a force, it is possible to define a numerical value of potential at every point in space

An electrostatic, magnetic or gravitational force is a force



The Electric Potential $V_P = -\int_{\infty}^{P}$



$\Delta U = \quad = -W = -q_0 \int_i^f \vec{E}\cdot d\vec{s}.$

ΔU depends on the value of q_0.

We define the **electric potential** V

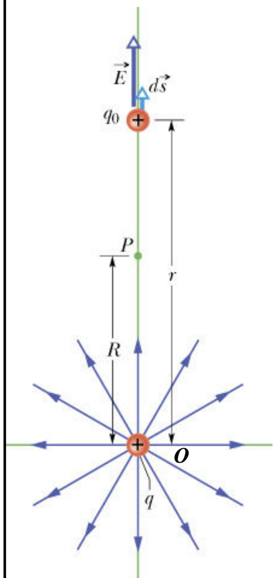
$\Delta V = \quad = -\frac{W}{q_0}$ Here $\Delta V = V_f - V_i$ $= -\int_i^f \vec{E}\cdot d\vec{s}.$

we can define arbitrarily the value of V at a reference point, which we choose to be at infinity:

$V_f =$ We take the initial position as the generic point P with potential V_P :

$V_P = -\int_{\infty}^{P} \vec{E}\cdot d\vec{s}.$ The potential V_P depends only on the coordinates of P and on $\vec{E}$.

$$V_P = \frac{1}{4\pi\varepsilon_0}\boxed{}$$



SI Units of V : Definition of voltage : $\Delta V = -\boxed{}$

Units of V: J/C, known as the volt

Potential Due to a Point Charge

Consider a point charge q placed at the origin.

$$V_P = -\int_{\infty}^{R} \vec{E}\cdot d\vec{s} = \int_{R}^{\infty} Edr\cos 0 = \int_{R}^{\infty} Edr$$

The electric field generated by q is:

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$V_P = \boxed{} \qquad \int \frac{dr}{x^2} = -\frac{1}{x}$$

$$\rightarrow V_P = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^{\infty} = \frac{1}{4\pi\varepsilon_0}\boxed{}$$

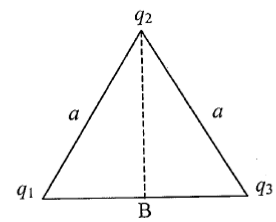
Consider two conducting spheres with one having a larger radius than the other. Both spheres carry the same amount of excess charge. Which one of the following statements concerning the potential energy of the two spheres is true?

a) The potential energy of the larger sphere is greater than that of the smaller sphere.

b) The potential energy of the larger sphere is the same as that of the smaller sphere.

c) The potential energy of the larger sphere is less than that of the smaller sphere.

Three points charges $q_1 = -1.0 \times 10^{-6}$ C, $q_2 = +1.0 \times 10^{-6}$ C, $q_3 = +2.0 \times 10^{-6}$ C are fixed in vacuum at the corners of an equilateral triangle as shown in the figure. Assume $a = 0.1$ m.



a) What is the resultant force on q_1?
b) What is the electric potential at the point B midway between q_1 and q_3?

Q. Why is an electrostatic force considered a conservative force?

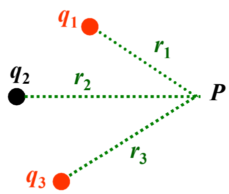
a) Charged particles do not experience friction (a non-conservative force).

b) The energy required to move a charged particle around a closed path is equal to zero joules.

c) The work required to move a charged particle from one point to another does not depend upon the path taken.

d) Answers (a) and (b) are both correct.

e) Answers (b) and (c) are both correct.



Potential Due to a Group of Point Charges

$$\boxed{V = V_1 + V_2 + V_3}$$

1. We determine the potentials $V_1, V_2,$ and V_3 generated by each charge at point P:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}, \quad V_2 = \boxed{}, \quad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

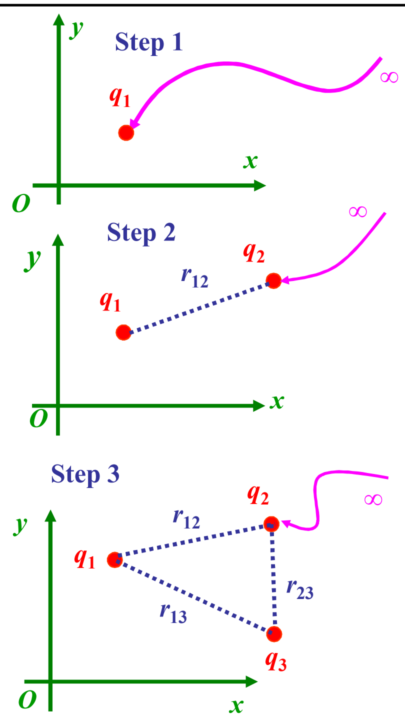
2. We add the three terms:

$$V = V_1 + V_2 + V_3$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \boxed{} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

Generally:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n} = \boxed{}$$



System potential energy for 3 charges

Step 1: Bring in q_1:

$W_1 = 0$

(no other charges around)

Step 2: Bring in q_2:

$W_2 = q_2 V(2)$

$$V(2) = \boxed{} \rightarrow W_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Step 3: Bring in q_3:

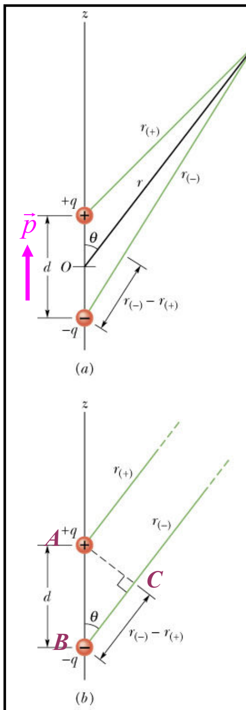
$W_3 = q_3 V(3)$

$$V(3) = \boxed{} \rightarrow$$

$$W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$W = W_1 + W_2 + W_3$$

$$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \boxed{} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$$



Example : Potential Due to an Electric Dipole

Point P is at a distance r from the center O of the dipole.
Line OP makes an angle θ with the dipole axis:

$$V = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\boxed{\quad\quad} = \frac{q}{4\pi\varepsilon_0}\boxed{\quad\quad}$$

We assume that $r \gg d$, where d is the charge separation.
From triangle ABC we have: $r_{(-)} - r_{(+)} \approx d\cos\theta$.

Also: $r_{(-)}r_{(+)} \approx r^2 \rightarrow V \approx \boxed{\quad\quad} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$,

where $p = qd =$ the electric dipole moment.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

The drawing shows four points surrounding an electric dipole. Which one of the following expressions best ranks the electric potential at these four locations?

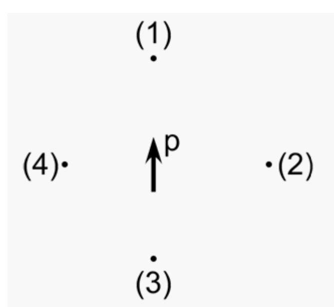
a) 1 > 2 > 3 > 4

b) 1 > 2 = 4 > 3

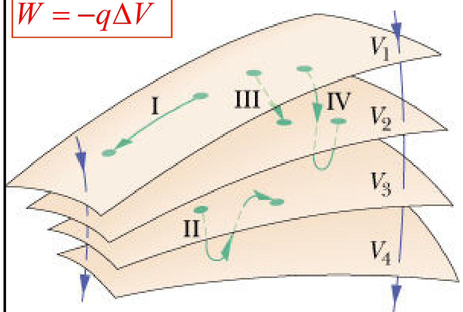
c) 3 > 2 > 4 > 1

d) 3 > 2 = 4 > 1

e) 2 = 4 > 1 = 3



$W = -q\Delta V$



Equipotential Surfaces

A collection of points that have the same potential is known as an equipotential surface. Four such surfaces are shown in the figure. The work done by $\vec{E}$ as it moves a charge q between two points that have a potential difference ΔV is given by

$W = -\boxed{}$

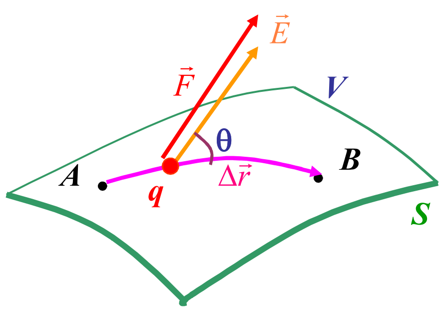
For path I: $W_{\text{I}} = \boxed{}$ because $\Delta V = 0$.

For path II: $W_{\text{II}} = \boxed{}$ because $\Delta V = 0$.

For path III: $W_{\text{III}} = \boxed{} = q\left(V_2 - V_1\right)$.

For path IV: $W_{\text{IV}} = \boxed{} = q\left(V_2 - V_1\right)$.

Note: When a charge is moved on an equipotential surface $\left(\Delta V = 0\right)$ the work done by the electric field is zero: $W = 0$.



The Electric Field $\vec{E}$ is Perpendicular to the Equipotential Surfaces

Consider the equipotential surface at potential V. A charge q is moved by an electric field $\vec{E}$ from point A to point B along a path $\Delta\vec{r}$. Points A and B and the path lie on S.

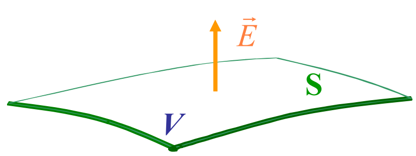
Let's assume that the electric field $\vec{E}$ forms an angle θ with the path $\Delta\vec{r}$.

The work done by the electric field is: $W = \boxed{} = \boxed{} = \boxed{}$

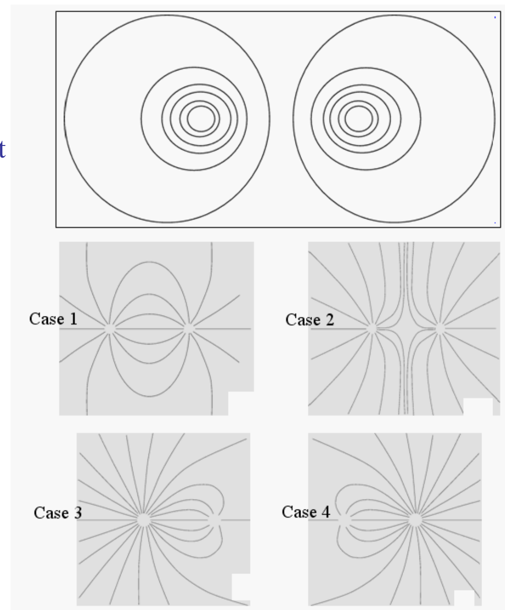
We also know that $W = 0$. Thus: $qE\Delta r\cos\theta = 0$, where

$q \neq 0, \quad E \neq 0, \quad \Delta r \neq 0$. Thus: $\cos\theta = \boxed{} \rightarrow \boxed{}$

The correct picture is shown in the figure below.



Consider the equipotential lines shown in the box. The labeled cases indicate electric field line drawings. Which of these cases best matches the equipotential lines shown?



a) 1

b) 2

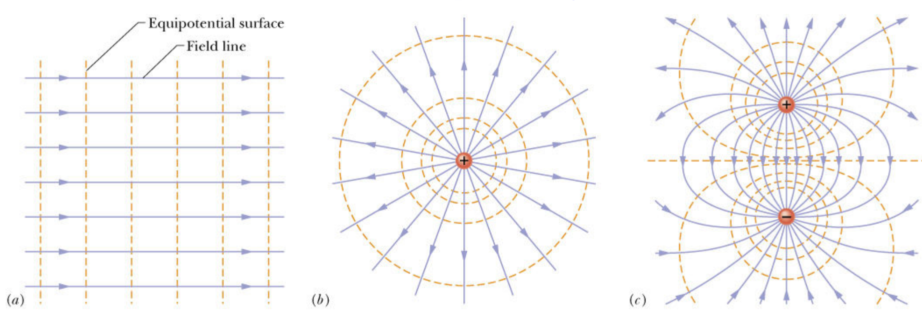
c) 3

d) 4

e) None of these cases match the equipotential lines shown.

Examples of Equipotential Surfaces and the Corresponding Electric Field Lines

Uniform electric field | Isolated point charge | Electric dipole



Equipotential surfaces for a point charge q :

$V = \dfrac{q}{4\pi\varepsilon_0 r}$. Assume that V is constant $\rightarrow r =$ $\boxed{}$

Thus the equipotential surfaces are spheres with their center at the point charge and radius $r =$ $\boxed{}$

Consider the equipotential lines shown in the box. The labeled cases indicate electric field line drawings. Which of these cases best matches the equipotential lines shown?

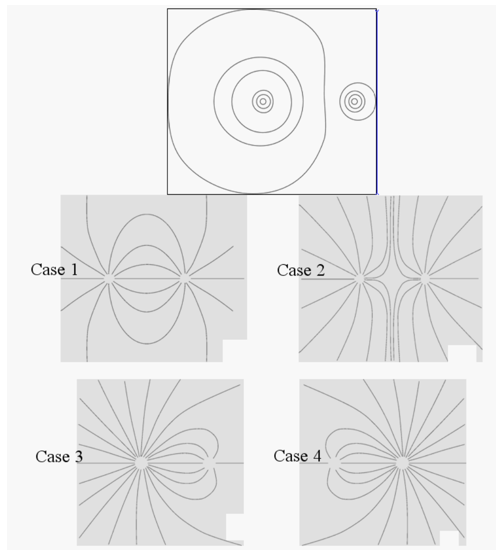
a) 1

b) 2

c) 3

d) 4

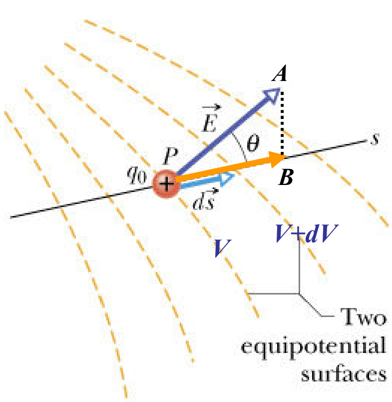
e) None of these cases match the equipotential lines shown.



Calculating the Electric Field $\vec{E}$ from the Potential V

Consider two equipotential surfaces that correspond to the values V and $V + dV$

Consider an arbitrary direction represented by the vector $d\vec{s}$.



The work done by the electric field is given by:

$W = -\boxed{}$ (eq. 1).

Also $W = Fds\cos\theta = Eq_0 ds\cos\theta$ (eq. 2)

If we compare these two equations we have:

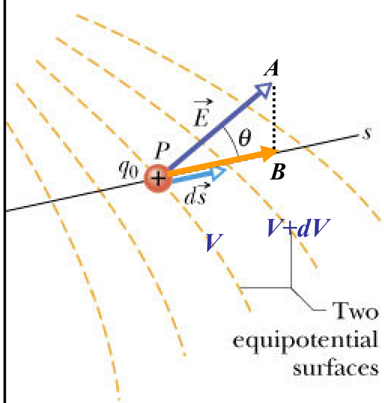
$\boxed{} \rightarrow E\cos\theta = -\frac{dV}{ds}$.

From triangle PAB we see that $E\cos\theta$ is the component E_s of $\vec{E}$ along the direction s.

Thus: $E_s = -\boxed{}$ $\quad E_s = \boxed{}$

$E_s = -\boxed{}$

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in this direction.



If we take s to be the x- , y-, and z-axes we get:

$E_x = -\boxed{}$

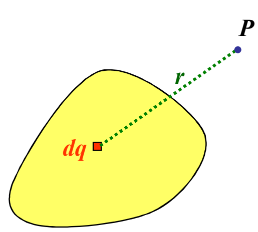
$E_y = -\boxed{}$

$E_z = -\boxed{}$

If we know the function $V(x, y, z)$ we can determine the components of $\vec{E}$ and thus the vector $\vec{E}$ itself :

$\vec{E} = \boxed{}$

Potential Due to a Continuous Charge Distribution



$V = \dfrac{1}{4\pi\varepsilon_0}\boxed{}$

1. We divide the distribution into elements of charge dq.
For a volume charge distribution, $dq = \boxed{}$.
For a surface charge distribution, $dq = \boxed{}$
For a linear charge distribution, $dq = \boxed{}$

2. We determine the potential dV created by dq at P: $dV = \dfrac{1}{4\pi\varepsilon_0}\boxed{}$

3. We sum all the contributions in the form of the integral: $V = \dfrac{1}{4\pi\varepsilon_0}\boxed{}$.

Note 1 : The integral is taken over the whole charge distribution.
Note 2 : The integral involves only scalar quantities.

Q. The electric potential of a charged, spherical conductor with a radius of 0.1 m is 100 V. What is the electric potential at a point located 2 m from the center of the sphere?

a) zero volts

b) 2.5 V

c) 5 V

d) 10 V

e) 20 V

Q. The electric potential of an uncharged, spherical conductor with a radius of 0.1 m is 10 V. If the sphere is located in a region of space resulting in no electric fields, what is the electric potential at a point located 2 m from the center of the sphere in that region?

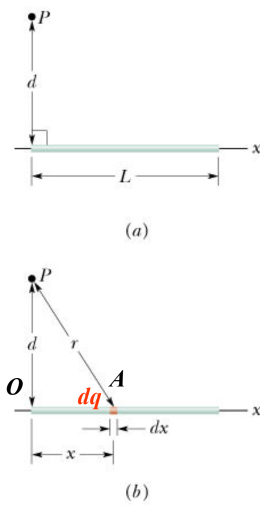
a) zero volts

b) 2.5 V

c) 5 V

d) 10 V

e) 20 V



Example : Potential created by a line of charge of length L and uniform linear charge density λ at point P.
Consider the charge element $dq = \lambda dx$ at point A, a distance x from O. From triangle OAP we have:
$r = \sqrt{d^2 + x^2}$. Here d is the distance OP.
The potential dV created by dq at P is:

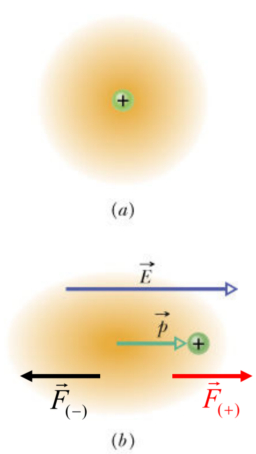
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\boxed{\quad}$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \boxed{\quad}$$

$$\int \frac{dx}{\sqrt{d^2+x^2}} = \ln\left(x + \sqrt{d^2+x^2}\right)$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + \sqrt{d^2+x^2}\right)\right]_0^L$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\boxed{\quad}\right]$$



Induced Dipole Moment

Many molecules such as H_2O have a permanent electric dipole moment. These are known as $\boxed{\quad}$ molecules. For others, such as O_2 , N_2, etc. the electric dipole moment is zero. These are known as $\boxed{\quad}$ molecules. One such molecule is shown in fig. *a*. The electric dipole moment $\vec{p}$ is zero because the center of the positive charge coincides with the center of the negative charge. In fig. *b* we show what happens when an electric field $\vec{E}$ is applied to a $\boxed{\quad}$ molecule. The electric forces on the positive and negative charges are equal in magnitude but opposite in direction.

As a result the centers of the positive and negative charges move in opposite directions and do not coincide. Thus a nonzero electric dipole moment $\vec{p}$ appears. This is known as "induced" electric dipole moment, and the molecule is said to be $\boxed{\quad}$ When the electric field is removed $\vec{p}$ disappears.

Q. Which one of the following statements concerning electrostatic situations is false?

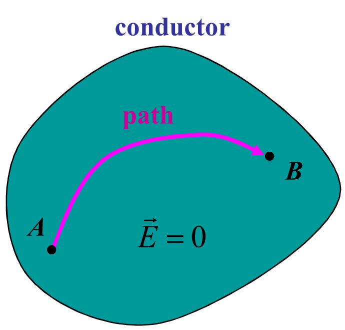
a) No work is required to move a charge along an equipotential surface.

b) If the electric potential with a region of space is zero volts, the electric field within that region must also be zero V/m.

c) If a charge is moved along an equipotential surface, there is no component of the force acting along the charge's path.

d) The electric field is always perpendicular to equipotential surfaces.

e) The electric field is zero V/m everywhere inside a conductor.



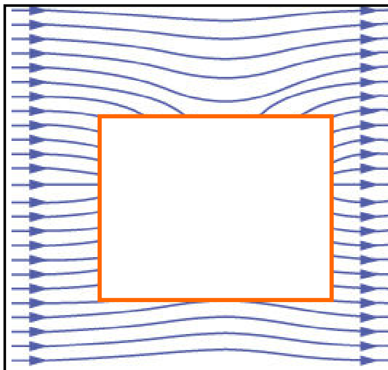
Potential of an Isolated Conductor

A conductor is $\boxed{}$

Consider two points A and B on or inside a conductor. The potential difference $V_B - V_A$ between these two points is given by the equation:

$$V_B - V_A = -\int_A^B \boxed{}$$

We already know that the electrostatic field $\boxed{}$ inside a conductor is zero. Thus the integral above $\boxed{}$ for any two points on or inside the conductor.



Isolated Conductor in an External Electric Field

We already know that the surface of a conductor is an equipotential surface. We also know that the electric field lines are perpendicular to the equipotential surfaces.

From these two statements it follows that the electric field vector $\vec{E}$ is _______ to the conductor surface, as shown in the figure.
All the charges of the conductor reside on the surface and arrange themselves in such a way so that the net electric field inside the conductor _______

The electric field just outside the conductor is: _______

Electric Field and Potential in and around a Charged Conductor : A Summary

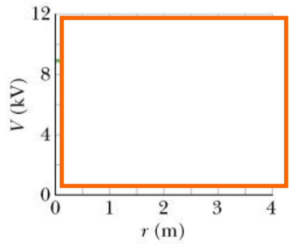
1. All the charges reside on the conductor surface.

2. The electric field inside the conductor is zero: $E_{\text{in}} = 0.$

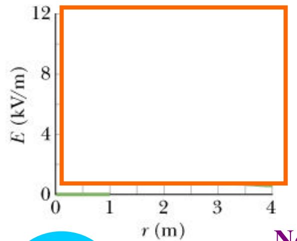
3. The electric field just outside the conductor is: $E_{\text{out}} = \dfrac{\sigma}{\varepsilon_0}.$

4. The electric field just outside the conductor is perpendicular to the conductor surface.

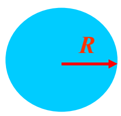
5. All the points on the surface and inside the conductor have the same potential. The conductor is an equipotential surface.



(a)



(b)



Electric Field and Electric Potential for a Spherical Conductor of Radius *R* and Charge *q*

For $r < R$, $V =$

For $r > R$, $V =$

For $r < R$,

For $r > R$, $E =$

Note : Outside the spherical conductor, the electric field and the electric potential are identical to that of charge equal to the net conductor charge and placed at the