



University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATION 2011/2012

**MATH 10250
INTRODUCTION TO CALCULUS FOR ENGINEERS**

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Time Allowed: 2 hours

Instructions for Candidates

Full marks will be awarded for complete answers to **SIX** questions.
Details of calculations leading to your answers must be included.
No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.
Candidates are **NOT** allowed to use mathematical tables during this examination.
The use of graph paper is **NOT** required at this examination.

1. (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x - 3$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = x^2 + x - 2$.
Find $f \circ g$ and $g \circ f$.

- (b) By substituting $u = \sin x$ or otherwise, evaluate the definite integral

$$\int_0^{\frac{\pi}{2}} \cos^3 x \sin^2 x dx.$$

(Useful constants: $\sin 0 = 0$, $\cos 0 = 1$, $\sin(\pi/2) = 1$, $\cos(\pi/2) = 0$)

2. (a) Determine the regions of increase and decrease for

$$f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = (2x + 5)e^{-2x+1}.$$

- (b) Find the general solution of the differential equation

$$\frac{dy}{dx} - 2y = e^{-x}.$$

3. (a) Use partial fractions to evaluate the definite integral

$$\int_1^2 \frac{x+1}{x^2(x+2)} dx.$$

- (b) Evaluate the limit

$$\lim_{x \rightarrow 2} \frac{3 - \sqrt{2x+5}}{\sqrt{x+2} - 2}.$$

4. (a) Define the inverse hyperbolic functions $\sinh^{-1} x$ and $\cosh^{-1} x$ and prove that

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}.$$

- (b) Find the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n}.$$

5. (a) Find the volume of the solid obtained by rotating the graph of the function

$$f : [0, 1] \rightarrow \mathbb{R}, \quad f(x) = \ln x$$

about the x -axis.

- (b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2y = (2x + 1)e^{2x}.$$

6. (a) Find the first five terms of the Taylor series for the function $f(x) = (x + 1) \ln x$ about the point $a = 1$.

- (b) Use Simpson's rule with four intervals of equal width to find an approximate value of the integral

$$\int_0^2 \frac{x^2 + x}{x^2 + 1} dx.$$

7. (a) Find the centroid of the region in the plane bounded by the curve $y = \cos x$ and the lines $y = 0$, $x = 0$ and $x = \pi/2$.

- (b) The roots of the characteristic equation of a nonhomogeneous linear second order differential equation are $\lambda_1 = 3$, $\lambda_2 = -1$ and a particular solution is $y_p(x) = xe^{-x^2}$.

Find the associated differential equation and write down its general solution.

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