University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 2 EXAMINATIONS 2010/2011

MATH 10260

Linear Algebra for Engineers

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Time Allowed: 2 hours

Instructions for Candidates

Attempt **Question 1** (each part worth 5 marks) and
three other questions, each other question being worth 15 marks.

Details of calculations leading to your answers must be included.

No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Candidates are **not** allowed to use mathematical tables during this examination.

1. (a) The complex number $z = a + bi$, where a and b are real numbers, and $i^2 = -1$, satisfies the equation

$$(3 + 4i)^2 = z - 2\overline{z}.$$

Find a and b. Note: $\overline{z}$ denotes the complex conjugate of z.

(b) Find the general solution of the following system of linear equations:

$$\begin{aligned} x + 2y + 3z &= 0 \\ 2x + \ y + \ z &= 2 \\ 4x - \ y + 2z &= 1. \end{aligned}$$

(c) Find the eigenvalues of the matrix A, where

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 1 & 0 \end{pmatrix}.$$

(d) The 2×2 matrices A, B and C satisfy the equation $CA = B + B^{-1}$. Find A explicitly, given that

$$B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}.$$

(e) Find the parametric equations of the line passing through the point $(0, 1, 2)$ that is perpendicular to the plane containing the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

2. (a) Let $z = a + bi$, where a and b are real numbers and $i^2 = -1$. Find a and b, given that

$$z = |z| - 1 + 3i.$$

(b) Find the parametric equations of the line of intersection of the planes

$$x + y + 2z = 3 \quad \text{and} \quad x - y - z = 1.$$

(c) Find the equation of the plane that contains the point $(0, 0, 0)$ and the line with parametric equations

$$x = 4s + 3, \quad y = s + 4, \quad z = 3s + 5.$$

3. (a) Find the value of c for which the following system of linear equations has *more than one* solution. Find also the solution of the system when c does *not* have this value.

(Question 3 continues overleaf.)

$$\begin{aligned} x - 2y - 2z &= 4 \\ 3x - y - z &= 7 \\ 6x + y + cz &= 11 \end{aligned}$$

(b) Find the solution of the following system of four equations:

$$\begin{aligned} x + 2y + 3z + w &= 5 \\ 2x + y + z + w &= 3 \\ x + 2y + z + 3w &= 1 \\ 4x + y + z + 2w &= 4. \end{aligned}$$

4. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ -1 & 1 & 3 \end{pmatrix}.$$

Hence find an invertible matrix B which diagonalizes A.

(b) The 3×3 matrix C has eigenvalues 1, 2 and -2. Calculate the trace and determinant of C^4.

5. (a) Find the values of x for which the determinant of the following matrix is 0:

$$A = \begin{pmatrix} 1 & 1 & x \\ 2 & 3 & x^2 \\ 4 & 9 & x^3 \end{pmatrix}.$$

(b) Let B be the 3×3 matrix

$$\begin{pmatrix} 1 & 1 & 2 \\ 3 & k & 8 \\ 2 & 1 & 4 \end{pmatrix}.$$

Show that B has an inverse for all values of k and find this inverse explicitly.