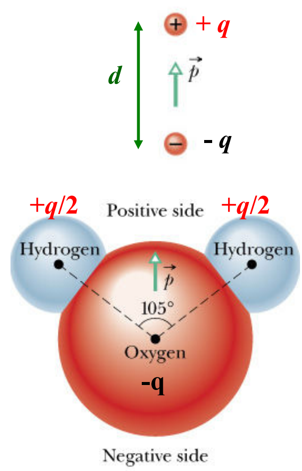


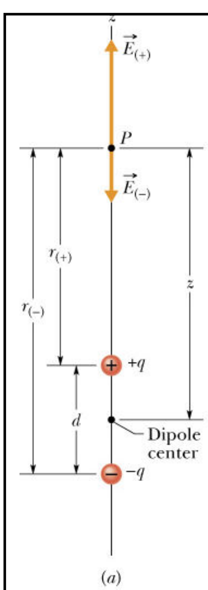
Electric Dipole

A system of two equal charges of opposite sign $(\pm q)$ placed at a distance d apart is known as an "electric dipole." For every electric dipole we associate a vector known as "the electric dipole moment" (symbol $\vec{p}$) defined as follows:

The magnitude $p = qd$

The direction of $\vec{p}$ is along the line that connects the two charges and points from $-q$ to $+q$.

Many molecules have a built-in electric dipole moment. An example is the water molecule (H_2O).



$$(1+x)^{-2} \approx 1-2x$$

Electric Field Generated by an Electric Dipole

The positive charge generates at P an electric field whose magnitude $E_{(+)} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+^2}$. The negative charge creates an electric field with magnitude $E_{(-)} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_-^2}$.

The net electric field at P is $E = E_{(+)} - E_{(-)}$.

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+^2} - \frac{q}{r_-^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(z-d/2)^2} - \frac{q}{(z+d/2)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$ We assume: $\dfrac{d}{2z} \ll 1$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1+\frac{d}{z}\right) - \left(1-\frac{d}{z}\right)\right] = \frac{qd}{2\pi\varepsilon_0 z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

22.9.1. A single, positive test charge is brought near a dipole. Under what circumstances will the force exerted on the test charge by the dipole be given by $F = \dfrac{de^2}{2\pi\varepsilon_0 z^3}$?

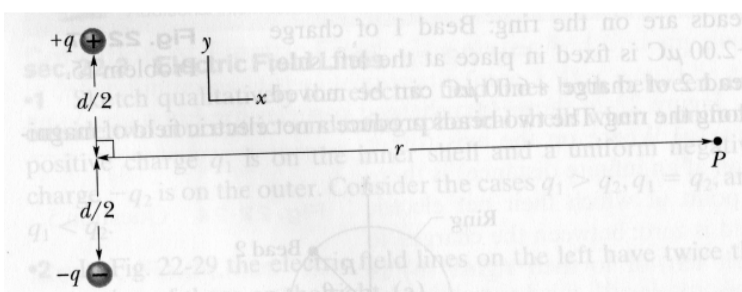
a) the test charge is a much greater charge than that of the dipole

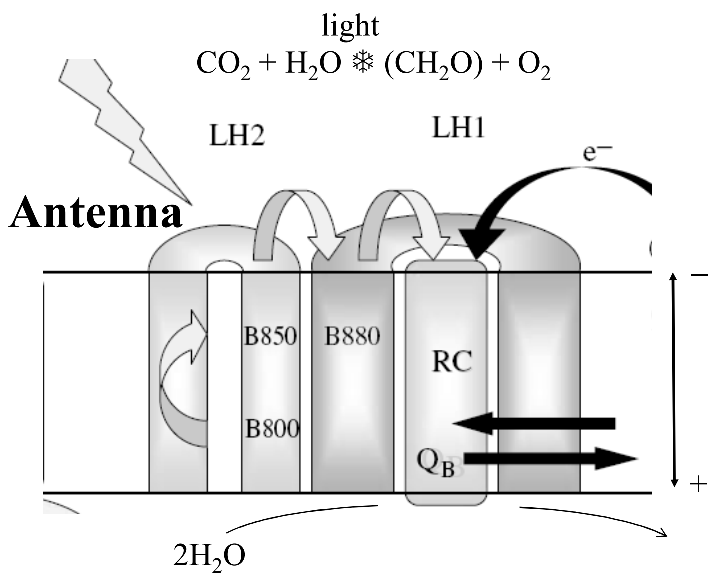
b) the test charge is a much smaller charge than that of the dipole

c) the test charge is very far from the dipole compared to the distance between the dipole charges

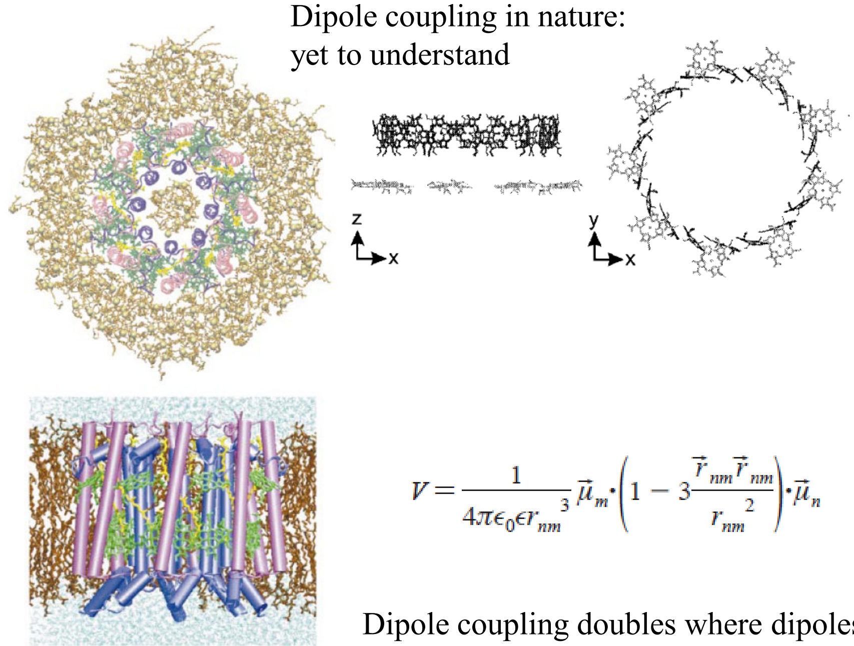
d) the test charge on a line is perpendicular to the dipole axis

What about perpendicular to the dipole axis?





Dipole coupling in nature:
yet to understand



$$V = \frac{1}{4\pi\epsilon_0\epsilon r_{nm}^{\ 3}}\vec{\mu}_m \cdot \left(1 - 3\frac{\vec{r}_{nm}\vec{r}_{nm}}{r_{nm}^{\ 2}}\right) \cdot \vec{\mu}_n$$

Dipole coupling doubles where dipoles are aligned

Q. Consider the two charges shown in the drawing. Which of the following statements correctly describes the direction of the electric force acting on the two charges?

$q_1 = +3.2\ \mu C$ $q_2 = -1.6\ \mu C$

a) The force on q_1 points to the left and the force on q_2 points to the left.

b) The force on q_1 points to the right and the force on q_2 points to the left.

c) The force on q_1 points to the left and the force on q_2 points to the right.

d) The force on q_1 points to the right and the force on q_2 points to the right.

Q. Consider the two charges shown in the drawing. Which of the following statements correctly describes the magnitude of the electric force acting on the two charges?

$q_1 = +3.2\ \mu C$ $q_2 = -1.6\ \mu C$

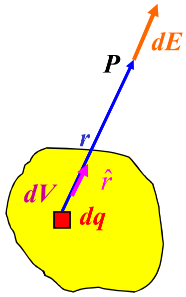
a) The force on q_1 has a magnitude that is twice that of the force on q_2.

b) The force on q_2 has a magnitude that is twice that of the force on q_1.

c) The force on q_1 has the same magnitude as that of the force on q_2.

d) The force on q_2 has a magnitude that is four times that of the force on q_1.

e) The force on q_1 has a magnitude that is four times that of the force on q_2.



Electric Field Generated by a Continuous Charge Distribution

Consider the continuous charge distribution shown in the figure. We assume that we know the volume density ρ of the electric charge. This is defined as $\rho = \boxed{}$ (Units: C/m^3).

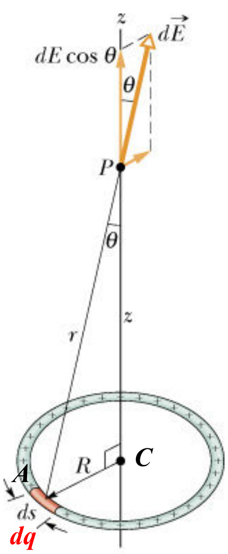
Our goal is to determine the electric field $d\vec{E}$ generated by the distribution at a given point P. This type of problem can be solved using the principle of superposition as described below.

1. Divide the charge distribution into "elements" of volume dV. Each element has charge $dq = \boxed{}$. We assume that point P is at a distance r from dq.

2. Determine the electric field $d\vec{E}$ generated by dq at point P.

The magnitude dE of $d\vec{E}$ is given by the equation $dE = \dfrac{dq}{4\pi\varepsilon_0 r^2}$.

3. Sum all the contributions: $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int \frac{\rho dV \hat{r}}{r^2}$.



Example : Determine the electric field $\vec{E}$ generated at point P by a uniformly charged ring of radius R and total charge q.

The distance between the element and point P is $r = \sqrt{z^2 + R^2}$.

$dE = \boxed{}$. The z-component of $d\vec{E}$ is given by

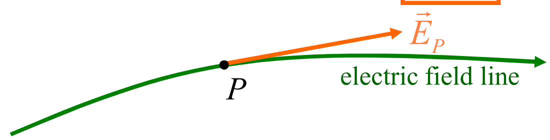
$dE_z = dE\boxed{}$ From triangle PAC we have: $\cos\theta = \boxed{}$

$\rightarrow dE_z = \boxed{} = \dfrac{zdq}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}$. $E_z = \displaystyle\int dE_z$

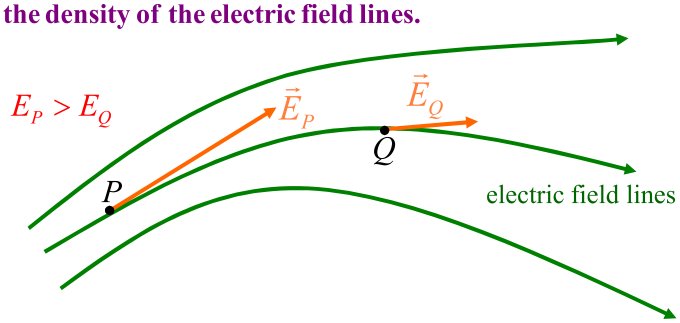
$$E_z = \frac{z}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\int dq = \frac{zq}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}$$

Electric Field Lines. In the 19th century Michael Faraday introduced the concept of electric field lines

1. **At any point P the electric field vector $\vec{E}$ is to the electric field lines.**



2. **The magnitude of the electric field vector $\vec{E}$ is to the density of the electric field lines.**



Example 2 : Electric field lines of an electric field generated by an infinitely large plane uniformly charged.

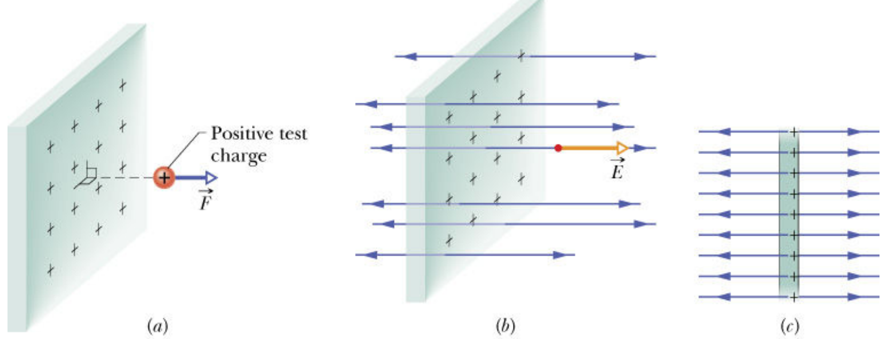
1. The electric field on either side of the plane has a

2. The electric field vector is to the charge plane.

3. The electric field vector $\vec{E}$ points from the plane.

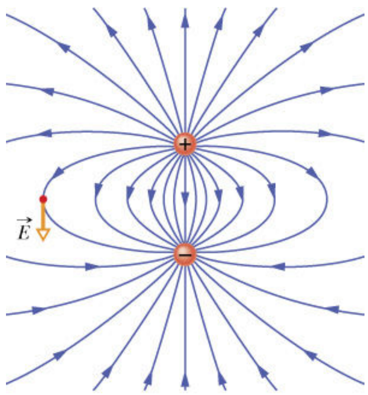
The corresponding electric field lines are given in fig. *c*.

Note : For a charged plane the electric field lines point inward.



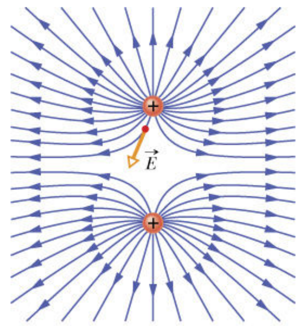
Example 3 :

Electric field lines generated by an electric positive and a negative point charge of the same size but of opposite sign)



Example 4 :

Electric field lines generated by two equal positive point charges



Q. A positively charged object is located to the left of a negatively charged object as shown. Electric field lines are shown connecting the two objects. The five points on the electric field lines are labeled A, B, C, D, and E. At which one of these points would a test charge experience the largest force?

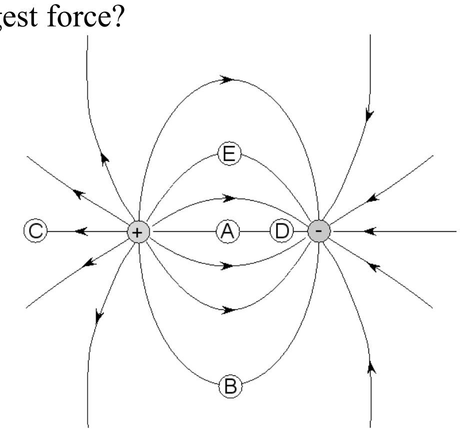
a) A

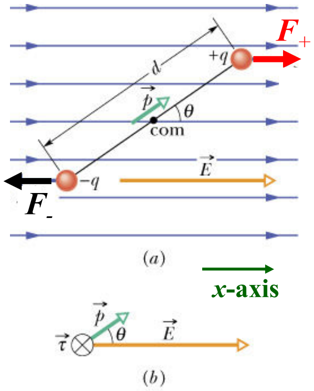
b) B

c) C

d) D

e) E





Forces and Torques Exerted on Electric Dipoles by a Uniform Electric Field

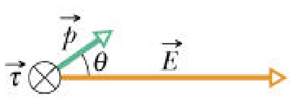
The electric field exerts a force $F_+ = \boxed{\quad}$ on the positive charge and a force $F_- = \boxed{\quad}$ on the negative charge. The net force on the dipole is $F_{\text{net}} = \boxed{\quad}$

The net torque generated by F_+ and F_- about the dipole center is

$$\tau = \tau_+ + \tau_- = \boxed{\quad} - |F_-|\frac{d}{2}\sin\theta = \boxed{\quad} = -pE\sin\theta$$

In vector form: $\boxed{\quad}$

The electric dipole in a uniform electric field does not move but can rotate about its center. $\boxed{\quad}$



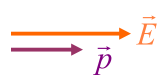
Potential Energy of an Electric Dipole in a Uniform Electric Field

$$U = -\int_{90^\circ}^{\theta} \boxed{\quad} = -\int_{90^\circ}^{\theta} pE\sin\theta' d\theta'$$

$$U = -pE\int_{90^\circ}^{\theta} \sin\theta' d\theta' = -pE\cos\theta = \boxed{\quad}$$

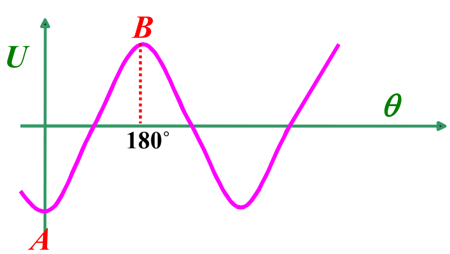
$U = \boxed{\quad}$

$U = \boxed{\quad}$



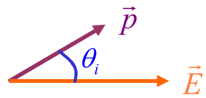
At point A $(\theta = 0)$, U has a minimum value $U_{\min} = \boxed{\quad}$

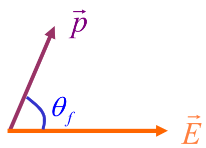
It is a position of stable equilibrium.



At point B $(\theta = 180^\circ)$, U has a maximum value $U_{\max} = \boxed{\quad}$

It is a position of unstable equilibrium.





Work Done by an External Agent to Rotate an Electric Dipole in a Uniform Electric Field

Consider the electric dipole in fig. *a*. It has an electric dipole moment $\vec{p}$ and is positioned so that $\vec{p}$ is at an angle θ_i with respect to a uniform electric field $\vec{E}$.

An external agent rotates the electric dipole and brings it to its final position shown in fig. *b*. In this position $\vec{p}$ is at an angle θ_f with respect to $\vec{E}$.

The work W done by the external agent on the dipole is equal to the difference between the initial and final potential energy of the dipole:

$$W = \boxed{\qquad} = -\boxed{\quad}\cos\theta_f - \left(-\boxed{\quad}\cos\theta_i\right)$$

$$W = \boxed{\quad}\left(\cos\theta_i - \cos\theta_f\right)$$