MEEN10030 SOLUTIONS 2011-12

MECHANICS FOR ENGINEERS

Q1

(i) (a) Mass, Time, Space

(ii) (c) -0.535; 0.802; 0.267

(iii) (d) -2

(iv) (c) 359 N

(v) (b) $(6, 1.83)$ m

(vi) (a) $35°$

(vii) (d) 14 s

(viii) (a) $0.175\ m^4$

(ix) (c) $1.4F$; tension

(x) (b) at B

Q2

A vector analysis using $M_{AB} = \underline{u}_B \cdot (\underline{r} \times \underline{F})$ will be considered for the solution since the moment arm or perpendicular distance from the line of action of F to the axis AB will be difficult to determine.

Unit vector $\underline{u}_B$ defines the direction of the AB axis of the rod:

$$\underline{u}_B = \frac{\underline{r}_B}{|\underline{r}_B|} = \frac{0.4\underline{i} + 0.2\underline{j}}{\sqrt{0.4^2 + 0.2^2}} = 0.894\underline{i} + 0.447\underline{j}$$

Vector $\underline{r}$ is directed from any point on the AB axis to any point on the line of action of the force F. For example, position vectors $\underline{r}_C$ and $\underline{r}_D$ are suitable. For simplicity, choose $\underline{r}_D$

$$\underline{r}_D = 0.2\underline{j} \text{ m}$$

The force is:

$$\underline{F} = \{-600\underline{i} + 200\underline{j} - 300\underline{k}\} \text{ N}$$

We can now substitute these vectors into the determinant form of the mixed triple product

Q2 dd

$$\underline{M}_{AB} = \underline{u}_B \cdot (\underline{r}_D \times \underline{F}) = \begin{vmatrix} 0.894 & 0.447 & 0 \\ 0 & 0.2 & 0 \\ -600 & 200 & -300 \end{vmatrix}$$

$$= 0.894\,(0.2\,(-300)) - 0.447\,(0) + 0\,(\quad)$$

$$= -53.67 \text{ N.m}$$

The negative sign indicates that the sense of M_{AB} is opposite to that of $\underline{u}_B$.

Expressing $\underline{M}_{AB}$ as a Cartesian vector gives

$$\underline{M}_{AB} = |\underline{M}_{AB}|\,\underline{u}_B$$

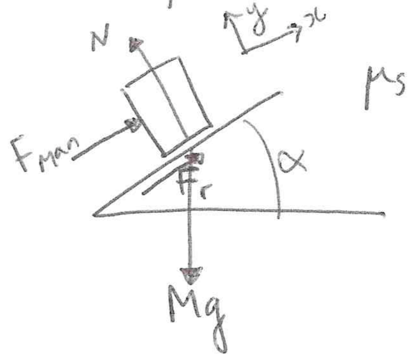
$$= (-53.67 \text{ N.m})\,(0.894\,\underline{i} + 0.447\,\underline{j})$$

$$= (-48.0\,\underline{i} - 24.0\,\underline{j}) \text{ N.m}$$

o ———— o

Q3

FBD for Box:



$\mu_s = 0.4$

$\sum F_x = 0$ gives:

$$F_{man} + F_r - Mg\sin\alpha = 0$$

$$F_{man} + 0.4N - Mg\sin\alpha = 0 \quad (1)$$

$\sum F_y = 0$ gives:

$$N - Mg\cos\alpha = 0$$

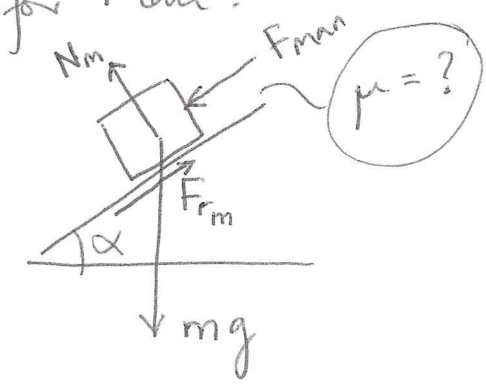
$$N = Mg\cos\alpha \quad (2)$$

Subsititute (2) into (1) gives

$$F_{man} + 0.4Mg\cos\alpha - Mg\sin\alpha = 0$$

$$\Rightarrow F_{man} = Mg(\sin\alpha - 0.4\cos\alpha) \quad (5)$$

FBD for Man:



$\sum F_x = 0$ gives:

$$F_{r_m} - F_{man} - mg\sin\alpha = 0$$

$$N_m \cdot \mu - F_{man} - mg\sin\alpha = 0 \quad (3)$$

$\sum F_y = 0$ gives:

$$N_m - mg\cos\alpha = 0$$

$$N_m = mg\cos\alpha \quad (4)$$

Substitute (4) into (3):

$$mg\cos\alpha \cdot \mu - F_{man} - mg\sin\alpha = 0$$

$$F_{man} = mg\cos\alpha \cdot \mu - mg\sin\alpha \quad (6)$$

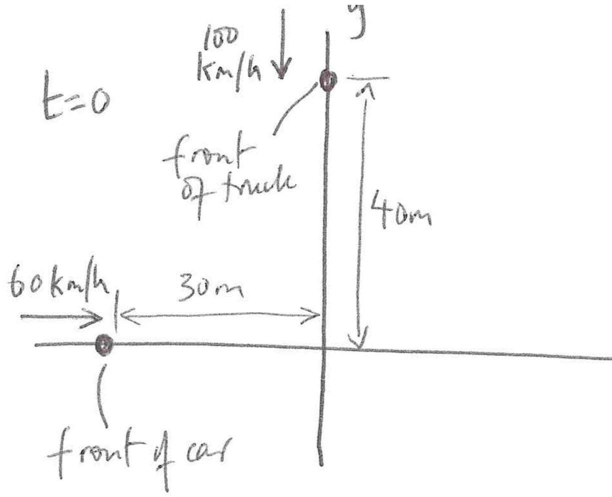
(5) = (6) $\Rightarrow$

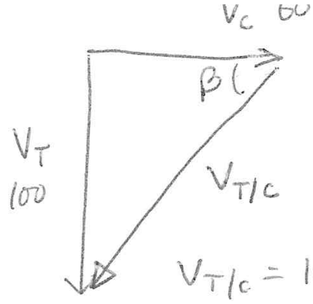
$$M\cancel{g}(\sin\alpha - 0.4\cos\alpha) = m\cancel{g}(\mu\cos\alpha - \sin\alpha)$$

$$\Rightarrow \quad \mu = \frac{\frac{M}{m}(\sin\alpha - 0.4\cos\alpha) + \sin\alpha}{\cos\alpha} = \boxed{0.8}$$

Note: this corresponds to v. high friction shoes!

Q4





$V_{T/C} = 117$ km/h $= \boxed{32.4 \text{ m/s}}$

(4 marks)

TRUCK reaches the crossing at:

$$\frac{40\text{m}}{100\text{ km/hr} \cdot \frac{1000\text{m}}{1\text{km}} \cdot \frac{1\text{hr}}{3600\text{s}}} = 1.44\text{s}$$

TRUCK passes the crossing completely at:

$$\frac{55\text{m}}{100\frac{\text{km}}{\text{h}} \cdot \frac{1000\text{m}}{1\text{km}} \cdot \frac{1\text{h}}{3600\text{s}}} = \boxed{1.98\text{s}}$$

CAR reaches the crossing at

$$X_c = X_{c_0} + V_{c_0} t - \frac{1}{2} 2\left(\frac{m}{s^2}\right) t^2$$

ASIDE: $\frac{60}{3.6} = 16.7$ m/s

ie $0 = -30\text{m} + 16.7\left(\frac{m}{s}\right)t - \frac{1}{2}2\left(\frac{m}{s^2}\right)t^2$

ie $t^2 - 16.7t + 30 = 0 \rightarrow t_{1,2} = \frac{16.7 \pm \sqrt{16.7^2 - 4 \cdot 30}}{2}$

$t_1 = 14.7\text{s}$; $\boxed{t_2 = 2.05\text{s}}$

corresponds to when car is going backwards (6 marks)

Clearly the car and truck **DO NOT CRASH** (barely) (1.98 V 2.05s)

Distance between front of car and back of truck just after the truck passes intersection completely is

$$X_c = -30\text{m} + 16.7\left(\frac{m}{s}\right) \cdot 1.98\,(s) - \frac{1}{2}2\left(\frac{m}{s^2}\right) \cdot 1.98^2 (s^2) = \boxed{-0.9\text{m}}$$ (5

Speed of car @ 1.98s is $V_c = \underset{16.7\text{ m/s}}{V_{c0}} - 2\left(\frac{m}{s^2}\right) \cdot 1.98\text{s} = 12.7\text{ m/s} \simeq \boxed{46 \text{ km/h}}$ (5

Q5

Divide the area into a rectangle (120 x 80 mm), a semicircle (80 mm diam) and a cutout circle (20 mm rad.).

Part No.	d_x (mm)	A (mm²)	I_y' (mm⁴)	$I_y = I_y' + d_x^2 A$ (mm
Rectangle	60	$120(80)$	$\frac{1}{12}(80)(120)^3$	4.608×10^7
Semicircle	$120 + \frac{4(40)}{3\pi}$	$\frac{1}{2}\pi(40)^2$	$\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(40)^4$	4.744×10^7
Circle	120	$\pi(20)^2$	$\frac{1}{4}\pi(20)^4$	1.822×10^7

Summing the results to get MoI of the composite area about the y-axis gives

$$I_y = I_y^{RECT} + I_y^{SEMIC.} - I_y^{CIRCLE} = \boxed{7.530 \cdot 10^7 \text{ mm}^4}$$

The total area is $A = A_{RECT} + A_{SEMIC} - A_{CIRCLE}$

$$= 1.086 \times 10^4 \text{ mm}^2$$

The radius of gyration about the y-axis is thus

$$k_y = \sqrt{\frac{I_y}{A}} = \boxed{83.3 \text{ mm}}$$