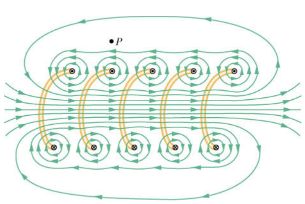
Magnetic Fields

Magnetic field vector, $\vec{B}$
Magnetic force on a moving charge,
Magnetic field lines
Motion of a moving charge particle in a uniform magnetic field
Magnetic force on a current-carrying wire $\vec{F}_B$
Magnetic torque on a wire loop
Magnetic dipole, magnetic dipole moment $\vec{\mu}$
Hall effect
Cyclotron particle accelerator

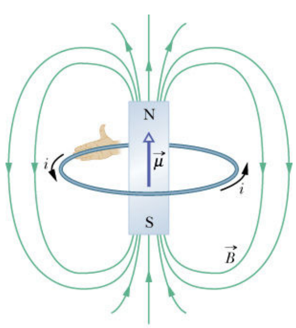


What Produces a Magnetic Field

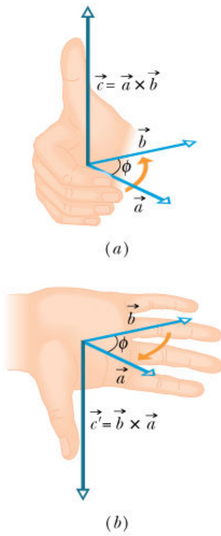
One can generate a magnetic field using one of the following methods:

Pass a current through a wire and thus form what is known as

Use



Empirically we know that both types of magnets attract small pieces of iron. Also, if suspended so that they can rotate freely they align themselves along the north-south direction. We can thus say that these magnets create in the surrounding space a "**magnetic field**" $\vec{B}$, which manifests itself by exerting a magnetic force $\vec{F}_B$.
We will use the magnetic force to define precisely the magnetic field vector $\vec{B}$.



The Vector Product of Two Vectors

The vector product $\vec{c} = \vec{a} \times \vec{b}$ of the vectors $\vec{a}$ and $\vec{b}$ is a vector $\vec{c}$.

The magnitude of $\vec{c}$ is given by the equation

$c = ab \sin \phi.$

The direction of $\vec{c}$ is perpendicular to the plane P defined by the vectors $\vec{a}$ and $\vec{b}$.

The sense of the vector $\vec{c}$ is given by the **right-hand rule**:

a. Place the vectors $\vec{a}$ and $\vec{b}$ tail to tail.

b. Rotate $\vec{a}$ in the plane P along the shortest angle so that it coincides with $\vec{b}$.

c. Rotate the fingers of the right hand in the same direction.

d. The thumb of the right hand gives the sense of $\vec{c}$.

The vector product of two vectors is also known as the "**cross**" product.

The Vector Product $\vec{c} = \vec{a} \times \vec{b}$ in Terms of Vector Components

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \ \vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}, \ \vec{c} = c_x\hat{i} + c_y\hat{j} + c_z\hat{k}$

The vector components of vector $\vec{c}$ are given by the equations

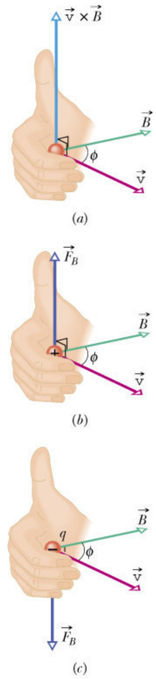
$c_x = a_y b_z - a_z b_y$, $c_y = a_z b_x - a_x b_z$, $c_z = a_x b_y - a_y b_x$

Note: Those familiar with the use of determinants can use the expression

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Note: The order of the two vectors in the cross product is important:

$\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right).$



$F_B = \boxed{\quad\quad}$ $\vec{F}_B = \boxed{\quad\quad}$

Definition of $\vec{B}$

The magnetic field vector is defined in terms of the force $\vec{F}_B$ it exerts on a charge q, which moves with velocity $\vec{v}$. We inject the charge q in a region where we wish to determine $\vec{B}$ at random directions, trying to scan all the possible directions. There is one direction for which the force $\vec{F}_B$ on q is zero. This direction is parallel with $\vec{B}$. For all other directions $\vec{F}_B$ is not zero, and its magnitude $F_B = \boxed{\quad\quad}$ where ϕ is the angle between $\vec{v}$ and $\vec{B}$. In addition, $\vec{F}_B$ is perpendicular to the plane defined by $\vec{v}$ and $\vec{B}$. The magnetic force vector is given by the equation $\vec{F}_B = \boxed{\quad\quad}$

SI unit of B: The defining equation is $F_B = \boxed{\quad\quad}$

If we shoot a particle with charge q = 1 C at right angles ($\phi = 90°$) to $\vec{B}$ with speed v = 1m/s and the magnetic force $F_B = 1$ N, then B = 1 tesla.

Q. A positively-charged particle is stationary in a constant magnetic field within a region of space. Which one of the following statements concerning the particle is true?

a) The particle will not move.

b) The particle will accelerate in the direction perpendicular to the field.

c) The particle will accelerate in the direction parallel to the field.

d) The particle will accelerate in the direction opposite to the field.

e) The particle will move with constant velocity in the direction of the field.

Q. An electron traveling due east in a region that contains only a magnetic field experiences a vertically *downward force,* toward the surface of the earth. What is the direction of the magnetic field?

a) upward, away from the earth

b) downward, toward the earth

c) due north

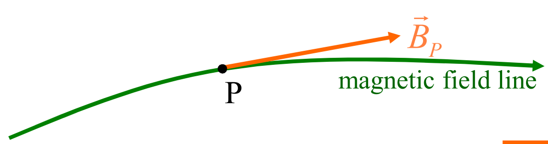
d) due west

e) due south

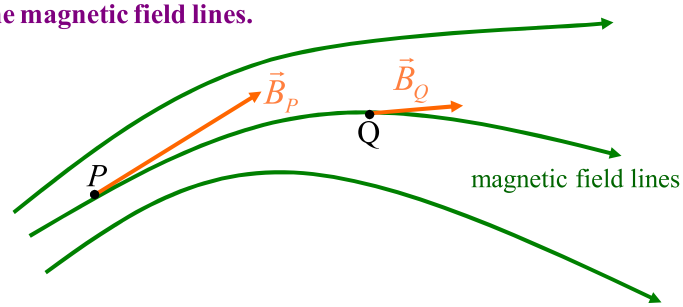
Magnetic Field Lines : In analogy with the electric field lines we introduce the concept of magnetic field lines, which help visualize the magnetic field vector $\vec{B}$ without using equations.

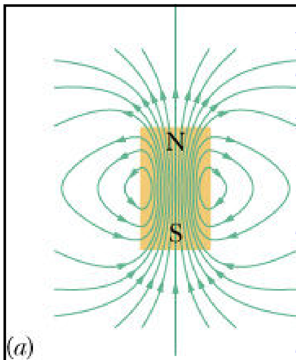
In the relation between the magnetic field lines and $\vec{B}$:

1. At any point *P* the magnetic field vector $\vec{B}$ is tangent to the magnetic field lines.



2. The magnitude of the magnetic field vector $\vec{B}$ is $\boxed{}$ to the density of the magnetic field lines.

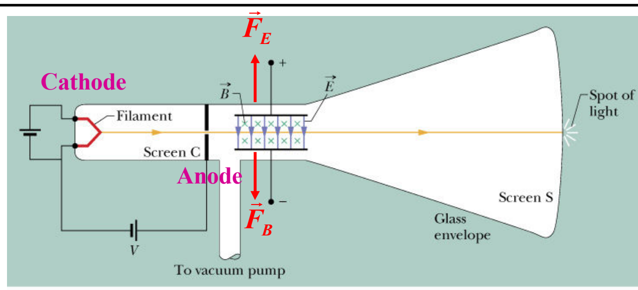




Magnetic Field Lines of a Permanent Magnet

The magnetic field lines of a permanent magnet are shown in the figure. The lines pass through the body of the magnet and form $\boxed{}$ This is in contrast to the electric field lines that originate and terminate on electric charges. The closed magnetic field lines enter one point of the magnet and exit at the other end. The end of the magnet from which the lines emerge is known as the ***north pole*** of the magnet. The other end where the lines enter is called the ***south pole*** of the magnet. The two poles of the magnet cannot be separated. Together they form what is known as a

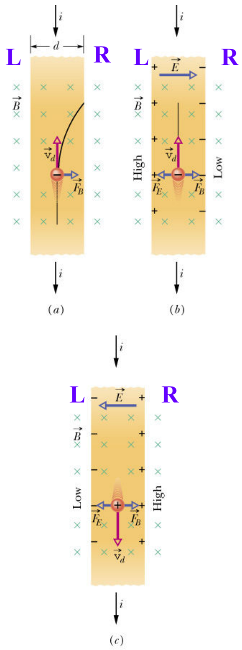
$\boxed{}$



$\vec{F}_E = \boxed{}$

$\vec{F}_B = \boxed{}$

Discovery of the Electron : A cathode ray tube is shown in the figure. Electrons are emitted from a hot filament known as the "cathode." They are accelerated by a voltage V applied between the cathode and a second electrode known as the "anode." The electrons pass through a hole in the anode and they form a narrow beam. They hit the fluorescent coating of the right wall of the cathode ray tube where they produce a spot of light. J.J. Thomson in 1897 used a version of this tube to investigate the nature of the particle beam that caused the fluorescent spot. He applied constant electric and magnetic fields in the tube region to the right of the anode. With the fields oriented as shown in the figure the electric force $\vec{F}_E$ and the magnetic force $\vec{F}_B$ have $\boxed{}$ directions. By adjusting B and E, Thomson was able to have $\boxed{}$ force.



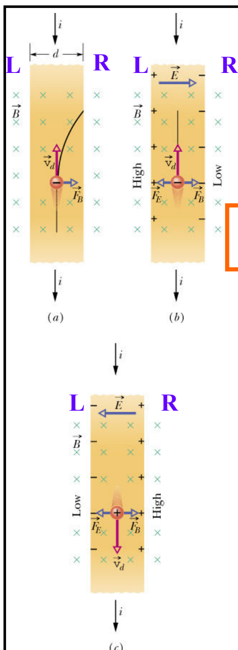
The Hall Effect

In 1879 Edwin Hall carried out an experiment in which he was able to determine that conduction in metals is due to the motion of $\boxed{\quad\quad}$ charges (electrons). He was also able to determine the concentration n of the electrons.

He used a strip of copper of width d and thickness ℓ. He passed a current i along the length of the strip and applied a magnetic field B perpendicular to the strip as shown in the figure. In the presence of $\vec{B}$ the electrons experience a magnetic force $\vec{F}_B$ that pushes them to the right (labeled "R") side of the strip. This accumulates negative charge on the R-side and leaves the left side (labeled "L") of the strip positively charged. As a result of the accumulated charge, an electric field $\vec{E}$ is generated as shown in the figure, so that the electric force balances the magnetic force on the moving charges: $F_E = \boxed{\quad} \rightarrow eE = \boxed{\quad} \rightarrow$

$E = \boxed{\quad}$ **(eq. 1)**. we have: $J = nev_d \rightarrow$

$$v_d = \frac{J}{ne} = \frac{i}{Ane} = \frac{i}{\ell dne} \quad \textbf{(eq. 2)}$$



$E = v_d B$ **(eq. 1)** $\quad v_d = i/\ell dne$ **(eq. 2)**

Hall measured the potential difference V between the left and the right side of the metal strip: $V = \boxed{\quad}$ (eq. 3).

We substitute E from eq. 3 and v_d from eq. 2 into eq. 1 and get:

$$\boxed{\quad} = B\frac{i}{\ell dne} \rightarrow \qquad n = \frac{Bi}{V\ell e} \quad \text{(eq. 4)}$$

By determining the polarity of the voltage that develops between the left-and right-hand sides of the strip, Hall was able to prove that

Current is as $\boxed{\quad\quad\quad\quad\quad\quad\quad\quad}$

Q. A charged particle is moving through a constant magnetic field. Does the magnetic field do work on the charged particle?

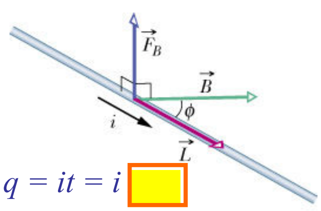
a) yes, because the force is acting as the particle is moving through some distance

b) no, because the magnetic force is always perpendicular to the velocity of the particle

c) no, because the magnetic field is a vector and work is a scalar quantity

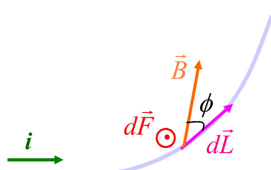
d) no, because the magnetic field is conservative

e) no, because the magnetic force is a velocity-dependent force



$q = it = i$ $\boxed{}$

$\vec{F}_B =$ $\boxed{}$



$d\vec{F}_B =$ $\boxed{}$

$\vec{F}_B =$ $\boxed{}$

Magnetic Force on a Straight Wire in a Uniform Magnetic Field

If we assume the more general case for which the magnetic field $\vec{B}$ forms an angle ϕ with the wire the magnetic force equation can be written in vector form as $\vec{F}_B =$ $\boxed{}$ Here $\vec{L}$ is a vector whose magnitude is equal to the wire length L and has a direction that coincides with that of the current. The magnetic force magnitude is $F_B =$ $\boxed{}$.

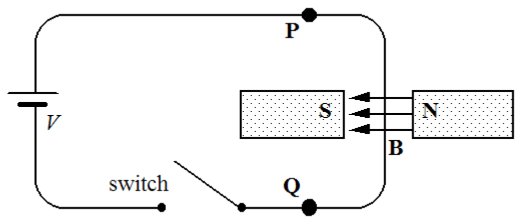
Magnetic Force on a Wire of Arbitrary Shape Placed in a Nonuniform Magnetic Field

In this case we divide the wire into elements of length dL, which can be considered as straight. The magnetic force on each element is $d\vec{F}_B = i d\vec{L} \times \vec{B}$. The net magnetic force on the wire is given by the integral $\vec{F}_B = i\int d\vec{L} \times \vec{B}$.

Q. A portion of a loop of wire passes between the poles of a magnet as shown. We are viewing the circuit from above. When the switch is closed and a current passes through the circuit, what is the movement, if any, of the wire between the poles of the magnet?

a) The wire moves toward the north pole of the magnet.

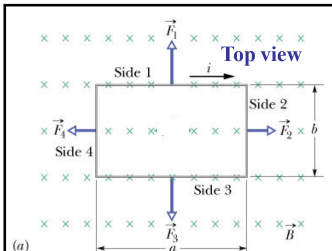
b) The wire moves toward the south pole of the magnet.



c) The wire moves upward (toward us).

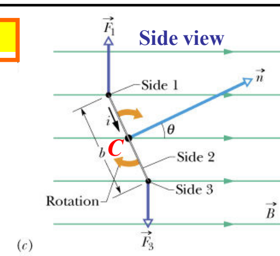
d) The wire moves downward (away from us).

e) The wire doesn't move.



$\tau_{net} = \boxed{}$

$F_{net} = 0$



Magnetic Torque on a Current Loop

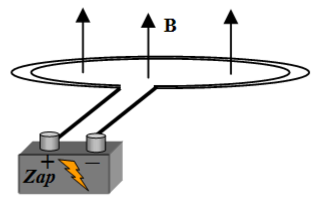
$F_1 = F_3 = \boxed{} = iaB.$

$F_2 = F_4 = ibB\sin(90-\theta) = ibB\boxed{}$. These forces cancel in pairs and thus $F_{net} = 0$.

The torque about the loop center C of F_2 and F_4 is zero because both forces pass through point C. The moment arm for F_1 and F_3 is equal to $\boxed{}\sin\theta$ - clockwise.

The net torque $\tau = \tau_1 + \tau_3 = \boxed{} = iabB\sin\theta = iAB\sin\theta.$ clockwise

Q. A circular loop of wire is placed in a magnetic field such that the plane of the loop is perpendicular to the magnetic field. The loop is then connected to a battery and a current then flows through the loop. Which one of the following statements concerning this situation is true?

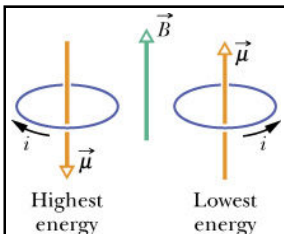


a) The magnetic force exerts a net torque on the loop.

b) The magnetic force exerts a net force on the loop.

c) The magnetic force exerts both a net force and a net torque on the loop.

d) The magnetic field has no affect on the loop.



$U = \mu B$ $\quad U = -\mu B$

Magnetic Dipole Moment

$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$

$\boxed{U = -\vec{\mu} \cdot \vec{B}}$

We define a new vector $\vec{\mu}$ associated with the coil, which is known as the magnetic dipole moment of the coil.

The magnitude of the magnetic dipole moment is $\mu = \boxed{}$

where N is the number of wire turns on the coil

The sense of $\vec{\mu}$ is defined by the right-hand rule.

In vector form: $\vec{\tau} = \boxed{}$

The potential energy of the coil is: $U = -\mu B \cos\theta = \boxed{}$

U has a minimum value of $-\mu B$ for $\theta = 0$ (position of **stable** equilibrium).

U has a maximum value of μB for $\theta = 180°$ (position of **unstable** equilibrium).

Note : For both positions the net torque is $\tau = 0$.

Magnetic Fields Due to Currents

We will explore the relationship between an electric current and the magnetic field it generates in the space around it. We will follow a two-pronged approach, depending on the symmetry of the problem.

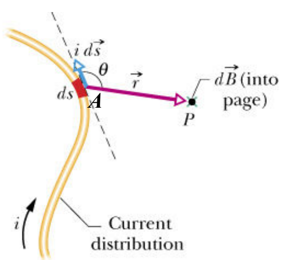
Focus to problems with high symmetry using Ampere's law (Law of Biot-Savart is non core)

Explore the magnetic field generated by currents (in straight wire, loop, solenoid).

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\boxed{}$$

The Law of Biot - Savart

This law gives the magnetic field $d\vec{B}$ generated by a wire segment of length ds that carries a current i.

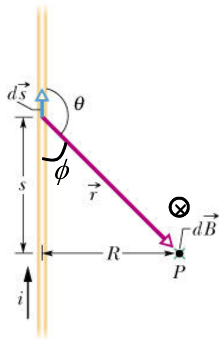


The magnetic field $d\vec{B}$ generated at point P by the element $d\vec{S}$ located at point A is given by the equation $d\vec{B} = \frac{\mu_0 i}{4\pi}\boxed{}$ Here $\vec{r}$ is the vector that connects point A (location of element ds) with point P at which we want to determine $d\vec{B}$. The constant $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A} = 1.26\times10^{-6}\ \text{T}\cdot\text{m/A}$ and is known as the "***permeability constant.***" The magnitude of $d\vec{B}$ is $dB = \frac{\mu_0 i}{4\pi}\frac{ds\sin\theta}{\boxed{}}$.
Here θ is the angle between $d\vec{s}$ and $\vec{r}$.

(will need in a minute)

$B =$

Long straight wire



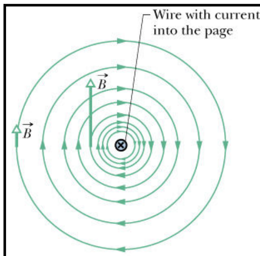
$dB = \frac{\mu_0 i}{4\pi} \frac{ds \sin\theta}{}$. Vector $d\vec{B}$ is pointing **into** the page. The magnetic field generated by the whole wire is found by integration:

$$B = \int_{-\infty}^{\infty} dB = 2\int_0^{\infty} dB = \frac{\mu_0 i}{2\pi}\int_0^{\infty}$$

$r = \sqrt{s^2 + R^2}$ $\qquad \sin\theta = \sin\phi = R/r = R/\sqrt{s^2+R^2}$

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty} \frac{R\,ds}{\left(s^2+R^2\right)^{3/2}} = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{\sqrt{s^2+R^2}}\right]_0^{\infty} =$$

$$\int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$



Magnetic Field Generated by a Long Straight Wire

$B =$ $\qquad$ $B =$ *(will need in a minute)*

The magnetic field lines form circles that have their centers at the wire. The magnetic field vector $\vec{B}$ is tangent to the magnetic field lines.

The direction of B is given by the standard screw rule

Q. The drawing shows two long, straight wires that are parallel to each other and carry a current of magnitude *i* toward you. The wires are separated by a distance *d*; and the centers of the wires are a distance *d* from the *y* axis. Which one of the following expressions correctly gives the magnitude of the total magnetic field at the origin of the *x, y* coordinate system?

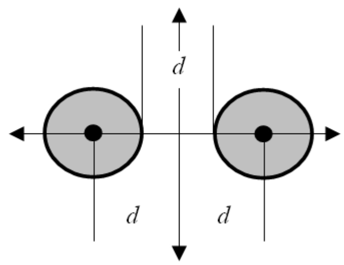
a) $\frac{\mu_0 i}{2d}$

b) $\frac{\mu_0 i}{\sqrt{2}d}$

c) $\frac{\mu_0 i}{2\pi d}$

d) $\frac{\mu_0 i}{\pi d}$

e) zero tesla



Q. The drawing shows two long, thin wires that carry currents in the positive *z* direction. Both wires are parallel to the *z* axis. The 50-A wire is in the *x-z* plane and is 5 m from the *z* axis. The 40-A wire is in the *y-z* plane and is 4 m from the *z* axis. What is the magnitude of the magnetic field at the origin?

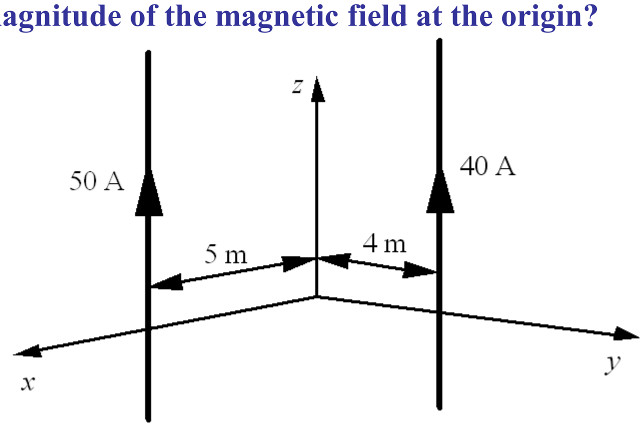
a) zero tesla

b) 1×10^{-6} T

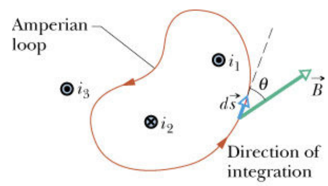
c) 2×10^{-6} T

d) 4×10^{-6} T

e) 7×10^{-6} T



Ampere's Law

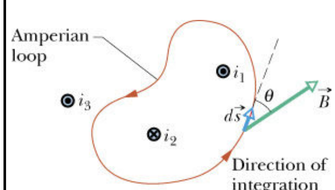


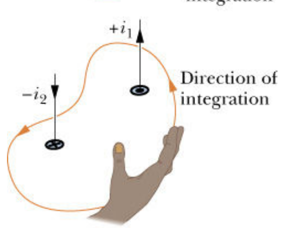
$\oint \vec{B} \cdot d\vec{s} =$

Ampere's law can be derived from the law of Biot-Savart, with which it is mathematically equivalent. Ampere's law is more suitable for advanced formulations of electromagnetism. It can be expressed as follows:

The line integral $\oint \vec{B} \cdot d\vec{s}$ of the magnetic field $\vec{B}$ along any closed path is equal to

$\oint \vec{B} \cdot d\vec{s} =$





Implementation of Ampere's Law :

1. Determination of $\oint$ The closed path is divided into n elements $\Delta\vec{s}_1, \Delta\vec{s}_2, ..., \Delta\vec{s}_n$.

We then form the sum:

$$\sum^{n} \vec{B}_i \cdot \Delta\vec{s}_i = \sum^{n} B_i \Delta s_i \cos\theta_i.$$

2. Calculation of

Account for directions of currents with signs

All currents outside the loop are not counted.

In this example : $i_{enc} = i_1 - i_2$.

Q. A copper cylinder has an outer radius 2*R* and an inner radius of *R* and carries a current *i*. Which one of the following statements concerning the magnetic field in the hollow region of the cylinder is true?

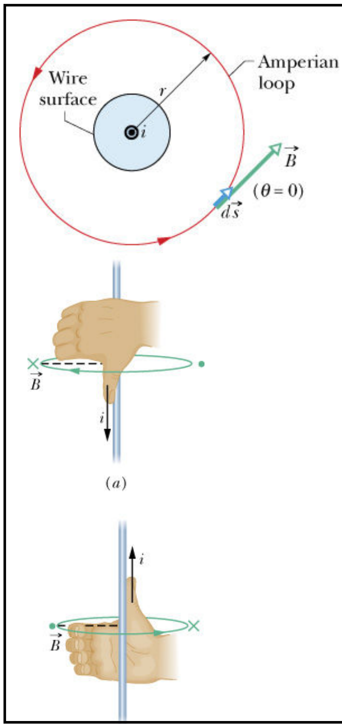
a) The magnetic field within the hollow region may be represented as concentric circles with the direction of the field being the same as that outside the cylinder.

b) The magnetic field within the hollow region may be represented as concentric circles with the direction of the field being the opposite as that outside the cylinder.

c) The magnetic field within the hollow region is parallel to the axis of the cylinder and is directed in the same direction as the current.

d) The magnetic field within the hollow region is parallel to the axis of the cylinder and is directed in the opposite direction as the current.

e) The magnetic field within the hollow region is equal to zero tesla.

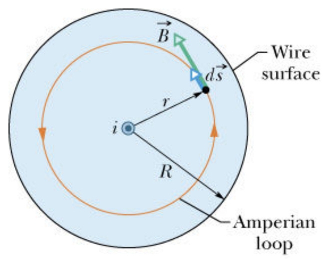


Magnetic Field Outside a Long Straight Wire

$$\oint \vec{B}\cdot d\vec{s} = \oint B\,ds\cos 0 = \boxed{\quad} = 2\pi r B = \mu_0 i_{\text{enc}} = \boxed{\quad}$$

$$\rightarrow B = \boxed{\quad}$$

Note: Ampere's law holds true for any closed path. We choose to use the path that makes the calculation of $\vec{B}$ as easy as possible.

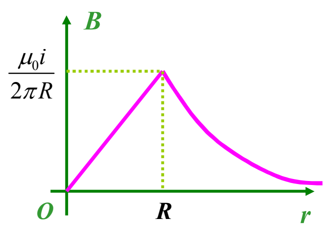


Magnetic Field Inside a Long Straight Wire

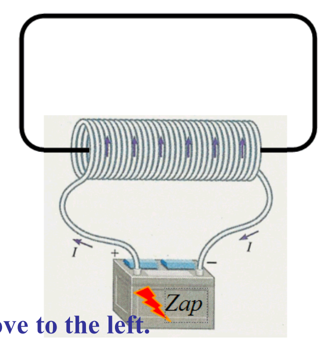
$$\oint \vec{B}\cdot d\vec{s} = \oint B\,ds\cos 0 = B\oint ds = 2\pi r B = \boxed{\quad}$$

$$i_{\text{enc}} = \boxed{\qquad\qquad}$$

$$2\pi r B = \mu_0 i\,\boxed{\quad} \rightarrow B = \left(\frac{\mu_0 i}{2\pi R^2}\right)\boxed{\quad}$$



Q. The drawing shows a rectangular wire loop that has one side passing through the center of a solenoid. Which one of the following statements describes the force, if any, that acts on the rectangular loop when a current is passing through the solenoid <u>and</u> through the loop wire?



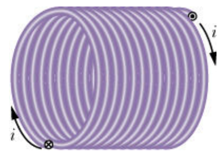
a) The magnetic force causes the loop to move upward.

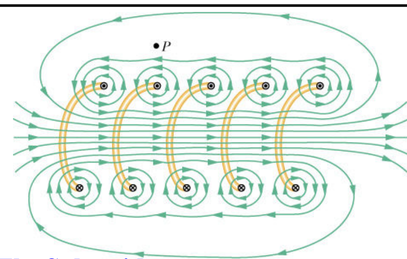
b) The magnetic force causes the loop to move downward.

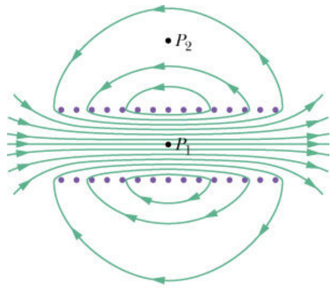
c) The magnetic force causes the loop to move to the right.

d) The magnetic force causes the loop to move to the left.

e) The loop is not affected by the current passing through the solenoid or the magnetic field resulting from it.



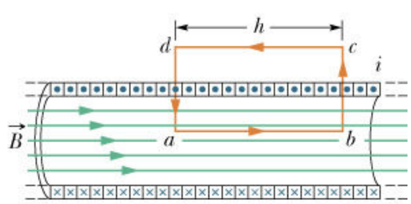




The Solenoid

The solenoid is a long, tightly wound helical wire coil in which the coil $\boxed{\quad}$ is much larger than the coil $\boxed{\quad}$ Viewing the solenoid as a collection of single circular loops, one can see that the magnetic field inside is approximately $\boxed{\quad}$

The magnetic field inside the solenoid is parallel to the solenoid axis. The sense of $\vec{B}$ can be determined using $\boxed{\quad}$ We curl the fingers of the right hand along the direction of the current in the coil windings. The thumb of the right hand points along $\vec{B}$. The magnetic field outside the solenoid is much weaker and can be taken to be $\boxed{\quad}$



$B = \boxed{\quad}$

We will use Ampere's law to determine the magnetic field inside a solenoid. We assume that the magnetic field is uniform inside the solenoid and zero outside. We assume that the solenoid has n turns per unit length.

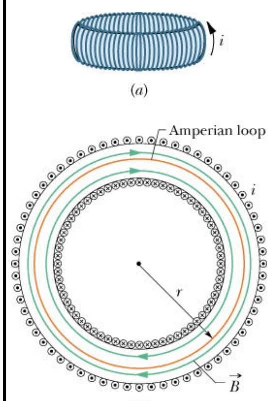
$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}$$

$$\int_a^b \vec{B}\cdot d\vec{s} = \int_a^b Bds\cos 0 = B\int_a^b ds = Bh \qquad \int_b^c \vec{B}\cdot d\vec{s} = \int_c^d \vec{B}\cdot d\vec{s} = \int_d^a \vec{B}\cdot d\vec{s} = \boxed{\quad}$$

$\rightarrow \oint \vec{B}\cdot d\vec{s} = \boxed{\quad}$ The enclosed current $i_{enc} = \boxed{\quad}$

$\oint \vec{B}\cdot d\vec{s} = \boxed{\quad} \rightarrow Bh = \boxed{\quad}$

$B = \boxed{\quad}$



(a)

(b)

Magnetic Field of a Toroid

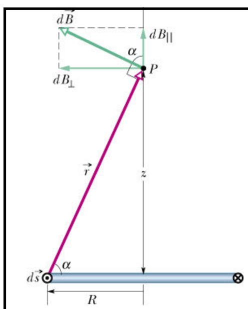
A toroid has the shape of a doughnut (see figure). We assume that the toroid carries a current i and that it has N windings. The magnetic field lines inside the toroid form circles that are concentric with the toroid center. The magnetic field vector is tangent to these lines. The sense of $\vec{B}$ can be found using the right-hand rule. We curl the fingers of the right hand along the direction of the current in the coil windings. The thumb of the right hand points along $\vec{B}$. The magnetic field outside the solenoid is approximately zero.

We use an Amperian loop that is a circle of radius r (orange circle in the figure):

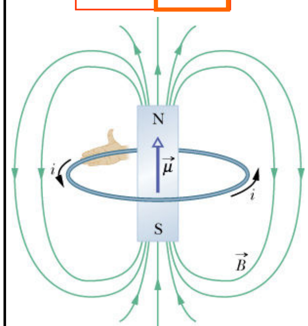
$\oint \vec{B} \cdot d\vec{s} = \oint B ds \cos 0 = B \oint ds = \boxed{\quad}$ The enclosed current $i_{enc} = \boxed{\quad}$

Thus: $2\pi r B = \boxed{\quad} \rightarrow B = \boxed{\quad}$

Note : The magnetic field inside a toroid is not uniform.



$\vec{B}(z) = \boxed{\quad}$



The Magnetic Field of a Magnetic Dipole.

Consider the magnetic field generated by a wire coil of radius R that carries a current i. The magnetic field at a point P on the z-axis is given by

$B = \dfrac{\mu_0 i R^2}{2\left(R^2 + z^2\right)^{3/2}}$. Here z is the distance between P and the coil center. For points far from the loop ($z \gg R$) we can use the approximation: $B \approx \boxed{\quad}$.

$B = \dfrac{\mu_0 i \pi R^2}{2 p z^3} = \dfrac{\mu_0 i A}{2\pi z^3} = \boxed{\quad}$. Here μ is the magnetic dipole moment of the loop. In vector form:

$\vec{B}(z) = \boxed{\quad}$

The loop generates a magnetic field that has the same form as the field generated by a bar magnet.