MEEN10030 Semester I, December 2008

Solutions

QUESTION 1 – 20 marks (2 marks for each part)

(i) (b)
(ii) (a)
(iii) (c)
(iv) (c)
(v) (c)
(vi) (d)
(vii) (b)
(viii)(c)
(ix) (b)
(x) (a)

QUESTION 2 – (20 marks)

We must express the forces exerted on A by the two cables in terms of their components. We can then calculate the moment by the cross product

$$\mathbf{M_P} = \mathbf{r} \times \mathbf{F}$$

Let $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ be the forces exerted on A by the two cables. To express $\mathbf{F}_{AB}$ in terms of its components, we determine the position vector from A to B:

$$(0 - 4)\mathbf{i} + (4 - 0)\mathbf{j} + (8 - 6)\mathbf{k} = -4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \text{ (m)}.$$

We divide this by its magnitude to obtain a unit vector $\mathbf{e}_{AB}$ with the same direction as $\mathbf{F}_{AB}$. Noting that its magnitude is $(4^2 + 4^2 + 2^2)^{½} = 36^{½} = 6$, we can see that

$$\boldsymbol{\lambda}_{AB} = -\tfrac{2}{3}\mathbf{i} + \tfrac{2}{3}\mathbf{j} + \tfrac{1}{3}\mathbf{k}$$

This allows us to write the force $\mathbf{F}_{AB}$ in terms of its unit vector as $15\boldsymbol{\lambda}_{AB}$, i.e.,

$$\mathbf{F}_{AB} = -10\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$$

Similarly, we can do exactly the same for $\mathbf{F}_{AC}$ to obtain $\mathbf{F}_{AC} = 28\boldsymbol{\lambda}_{AC}$, i.e.,

$$\boldsymbol{\lambda}_{AC} = -(2/7)\mathbf{i} + (3/7)\mathbf{j} - (6/7)\mathbf{k}$$

$$\mathbf{F}_{AC} = 8\mathbf{i} + 12\mathbf{j} - 24\mathbf{k}$$

We next need to choose the vector **r**. Since the lines of action of both forces pass through point A, we can conveniently use the vector from O to A to determine the moments of both forces about O:

$$\mathbf{r}_{OA} = \mathbf{r} = 4\mathbf{i} + 6\mathbf{k} \text{ (m)}$$

Evaluating $\mathbf{r} \times \mathbf{F}$ we get $\Sigma\mathbf{M}_O = (\mathbf{r} \times \mathbf{F}_{AB}) + (\mathbf{r} \times \mathbf{F}_{AC})$:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 6 \\ -10 & 10 & 5 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 6 \\ 8 & 12 & -24 \end{vmatrix}$$

$$= (-60\mathbf{i} - 80\mathbf{j} + 40\mathbf{k}) + (-72\mathbf{i} + 144\mathbf{j} + 48\mathbf{k})$$

$$= \boxed{(132\mathbf{i} + 64\mathbf{j} + 88\mathbf{k}) \text{ (kN.m)}}$$

Alternatively, we could have used Varignon's theorem by summing the forces first:
$\Sigma\mathbf{M}_O = \mathbf{r} \times (\mathbf{F}_{AB} + \mathbf{F}_{AC})$:

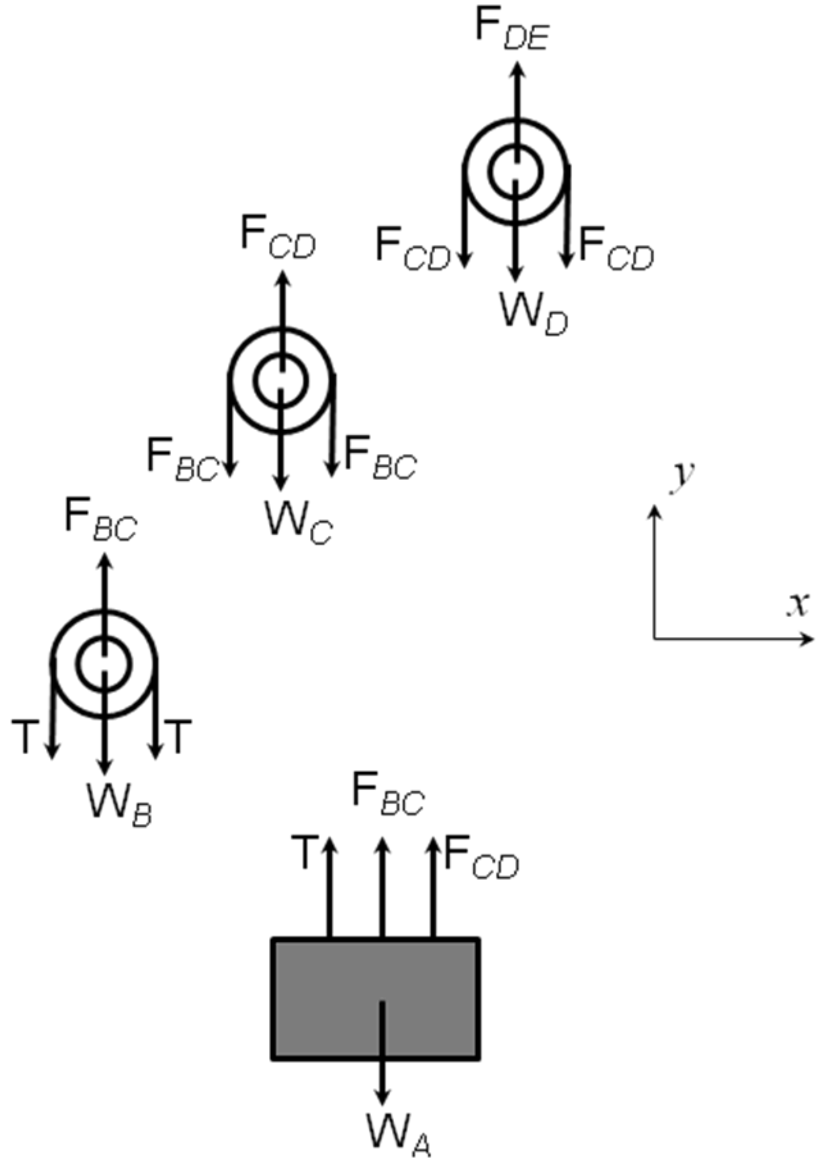
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 6 \\ -2 & 22 & -19 \end{vmatrix}$$

$$= \boxed{(-132\mathbf{i} + 64\mathbf{j} + 88\mathbf{k}) \text{ (kN.m)}}$$

$\boxed{\text{Note, the magnitude of the moment is } (132^2 + 64^2 + 88^2)^{½} = 171.06\text{kN.m}}$

QUESTION 3 – (20 marks)

PART (a) – 10 marks



IF THE PULLEYS, W_B, W_C AND W_D ALL WEIGHT SAME (CALL THIS "W"):

$F_{BC} = 2T + W$; $F_{CD} = 2F_{BC} + W$; $F_{DE} = 2F_{CD} + W$; $T = W_A - F_{BC} - F_{CD}$

$T = W_A - 2T - W - 2F_{BC} - W = W_A - 4W - 6T$

$\boxed{T = (W_A - 4W)/7}$

IF THE THREE PULLEYS ARE ASSUMED TO BE WEIGHTLESS:

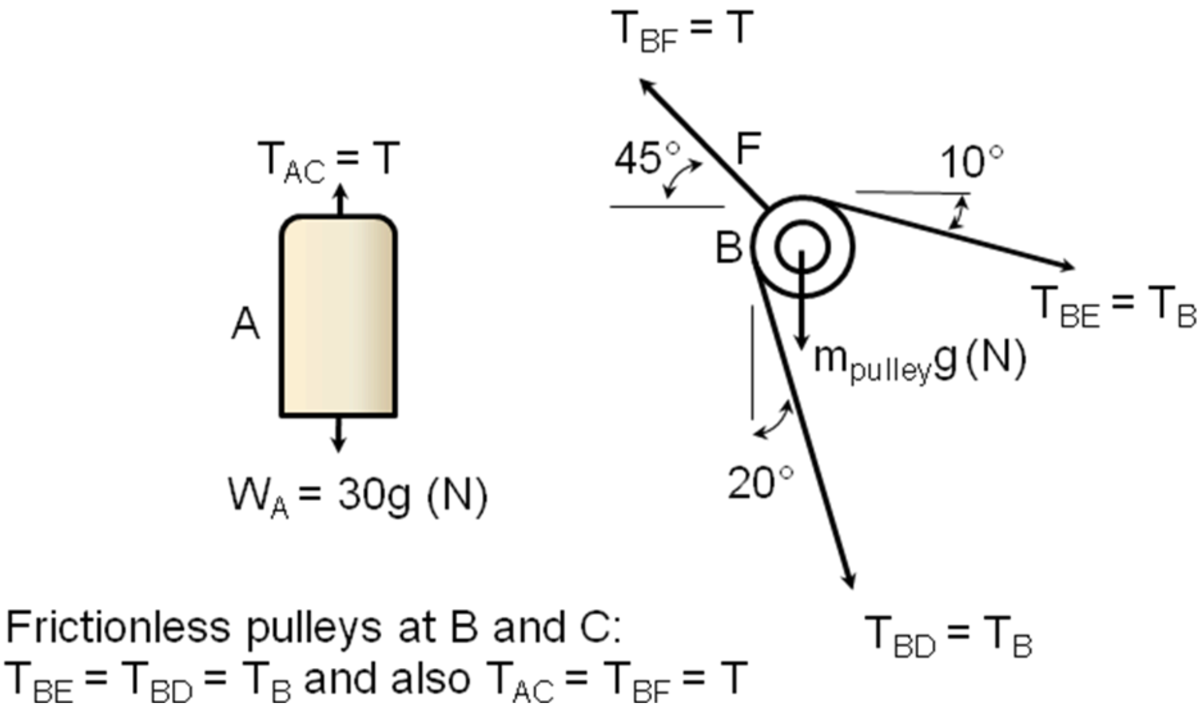
$T = \frac{1}{2} F_{BC} = \frac{1}{2} * \frac{1}{2} F_{CD} = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} F_{DE}$

$T = W_A - F_{BC} - F_{CD} = W_A - (2T) - (4T) = W_A - 6T$

$\boxed{T = W_A/7}$

QUESTION 3 – (20 marks)

PART (b) – 10 marks



Equilibrium equations for the Cylinder:
$\Sigma F_y = 0$: $T - 30.g = 0$

Equilibrium equations for the Pulley:
$\Sigma F_x = 0$: $-T \cos 45° + T_B \cos 10° + T_B \sin 20° = 0$
$\Sigma F_y = 0$: $T \sin 45° - T_B \sin 10° - T_B \cos 20° - m_{pulley}.g = 0$

Solving these three equations gives the unknown mass of the pulley along with the tensions in the cables:

$T = 294.3$ N
$T_B = T_{BE} = T_{BD} = 156.8$ N
$\boxed{m_{pulley} = 3.41 \text{ kg}}$

QUESTION 4 – (20 marks)

PART (a) – 10 marks

The motion of Car A is characterised by:

v_A = 90kph in a southerly direction = -25m/s (i.e., ↓)

$a_A = 0$

$y_A = (y_A)_0 - 25.t = 0 - 25t$

Three seconds later, these have changed to:

$a_A = 0$

v_A = -25m/s (i.e., ↓)

y_A = -75m (i.e., ↓)

The motion of Car B is characterised by:

$a_B = -3m/s^2$

$v_B = (v_B)_0 + a_B.t = 0 - 3t$

$x_B = (x_B)_0 + (v_B)_0 t + ½.a_B.t^2$

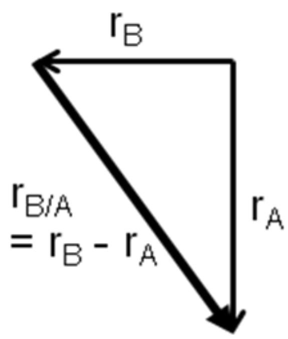
Three seconds later, these have changed to:

$a_B = -3m/s^2$

$v_B = -9m/s$

x_B = 50 – ½.3.9 = -36.5m/s (i.e., ←)

This allows us to construct position, velocity and acceleration vector diagrams and to calculate the position, velocity and acceleration of car B relative to those of car A:



$r_{B/A} = -36.5\mathbf{i} - 75\mathbf{j}$ (m)

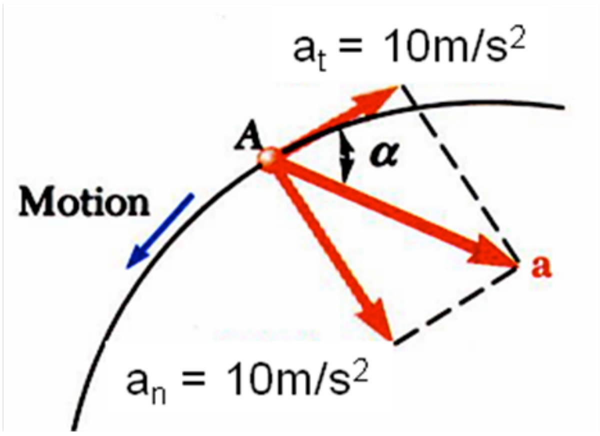
$v_{B/A} = -9\mathbf{i} -25\mathbf{j}$ (m/s)

$a_{B/A} = -3\mathbf{i}$ (m/s^2)

Can, of course, also express these in terms of a magnitude and an orientation.

QUESTION 4 – (20 marks)

PART (b) – 10 marks



180kph = 50m/s
108kph = 30m/s

$a_t = \Delta v/\Delta t = (30 - 50)/2 = -10 m/s^2$

$a_n = v^2/\rho = 50^2/250 = 10 m/s^2$

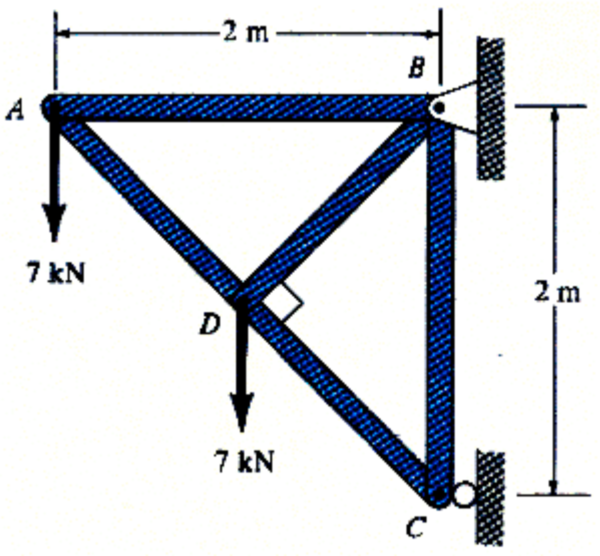
Alternatively, these vector components can be expressed as a magnitude and orientation:

$a = (a_t^2 + a_n^2)^{½} = 14.14 m/s^2$

$a = \tan^{-1}(a_n/a_t) = 45°$

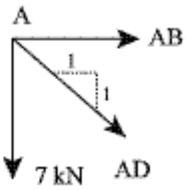
QUESTION 5 – (20 marks)

Determine the force in each member of the truss and state if the members are in tension or compression.



Solutions:

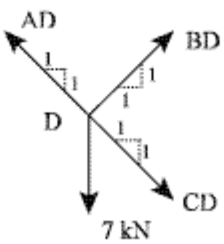
Begin at joint A since there are only two unknowns at this joint. The equilibrium equations are



$$\sum F_y = -7kN - \frac{1}{\sqrt{2}}AD = 0 \Rightarrow AD = -9.9kN$$

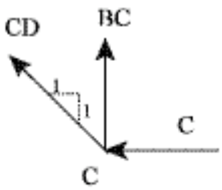
$$\sum F_x = AB + \frac{1}{\sqrt{2}}AD = 0 \Rightarrow AB = 7kN$$

Now move to joint D (two new unknowns)



$$\left.\begin{aligned}\sum F_x &= -\frac{1}{\sqrt{2}}AD + \frac{1}{\sqrt{2}}BD + \frac{1}{\sqrt{2}}CD = 0\\ \sum F_y &= \frac{1}{\sqrt{2}}AD + \frac{1}{\sqrt{2}}BD - \frac{1}{\sqrt{2}}CD = 0\end{aligned}\right. \Rightarrow \begin{aligned}BD &= 4.95kN\\ CD &= -14.85kN\end{aligned}$$

Finally move to joint C to find the last unknown force



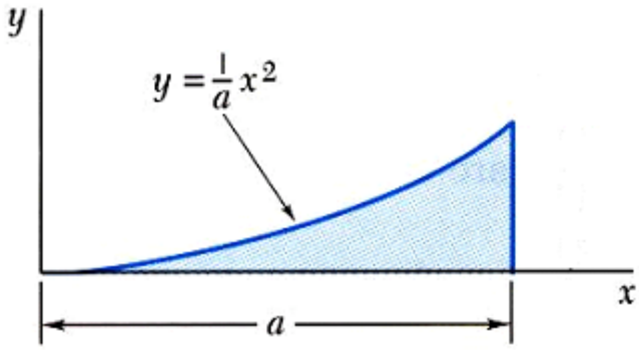
$$\sum F_y = \frac{1}{\sqrt{2}}CD + BC = 0 \Rightarrow BC = 10.5kN$$

In summary we have found (+means tension, - means compression)

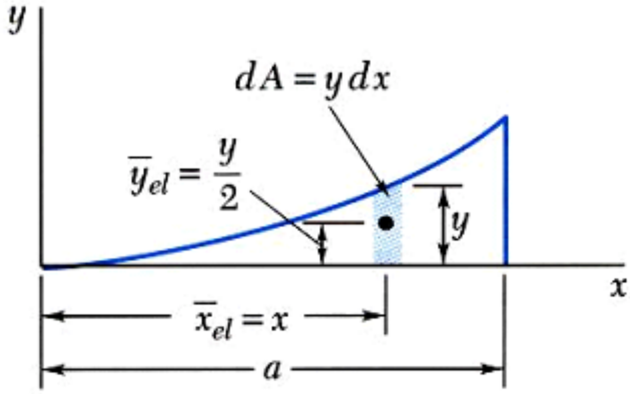
AB=7kN (tension); AD=9.90kN (compression); BD=4.95kN (tension);

CD=14.85kN (compression); BC=10.5kN (tension)

QUESTION 6 – (20 marks)
Determine by direct integration the location of the centroid of a parabolic spandrel.



Solutions:



Using a vertical strip, calculate the total area

$$A = \int dA = \int y\,dx = \int_0^a \frac{1}{a} x^2\,dx = \left[\frac{1}{a}\frac{x^3}{3}\right]_0^a = \frac{a^2}{3}$$

Using the same vertical strip, perform a single integration to find the first moments

$$Q_y = \int \bar{x}_{el}\,dA = \int xy\,dx = \int_0^a x\left(\frac{1}{a}x^2\right)dx = \left[\frac{1}{a}\frac{x^4}{4}\right]_0^a = \frac{a^3}{4}$$

$$Q_x = \int \bar{y}_{el}\,dA = \int \frac{y}{2}\,y\,dx = \int_0^a \frac{1}{2}\left(\frac{1}{a}x^2\right)^2 dx = \left[\frac{1}{2a^2}\frac{x^5}{5}\right]_0^a = \frac{a^3}{10}$$

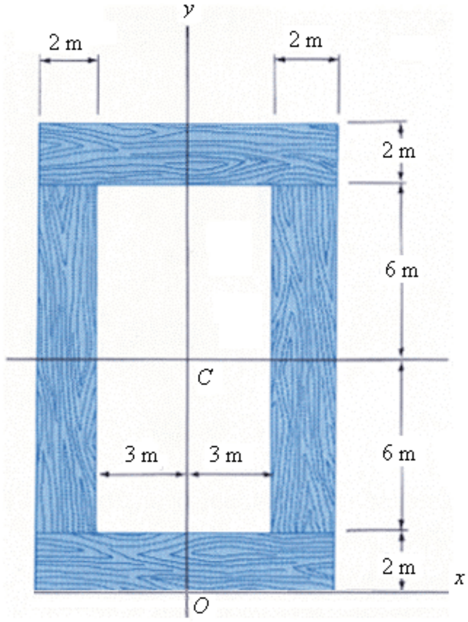
Calculate the centroid coordinates.

$$\bar{x}A = Q_y \Rightarrow \bar{x}\frac{a^2}{3} = \frac{a^3}{4} \Rightarrow \bar{x} = \frac{3}{4}a$$

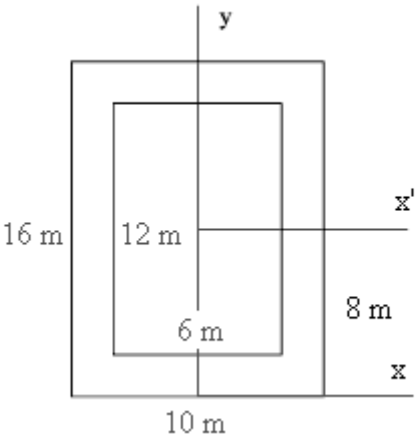
$$\bar{y}A = Q_x \Rightarrow \bar{y}\frac{a^2}{3} = \frac{a^3}{10} \Rightarrow \bar{y} = \frac{3}{10}a$$

QUESTION 7 – (20 marks)

Determine the moment of inertia I_x and I_y of the beam's cross-sectional area with respect to the x axis and y axis as shown.



Solutions:



Consider the cross-section to be a rectangle with a rectangular hole

For the rectangle:

$$I_x^a = I_{x'}^a + A^a d^2 = \frac{1}{12}(10m)(16m)^3 + (10m)(16m)(8m)^2 = 3413m^4 + 1280m^4 = 4693m^4$$

For the rectangular hole:

$$I_x^b = I_{x'}^b + A^b d^2 = \frac{1}{12}(6m)(12m)^3 + (6m)(12m)(8m)^2 = 864m^4 + 576m^4 = 1440m^4$$

For the beam:

$$I_x = I_x^1 - I_x^2 = 3253m^4$$

$$I_y = \frac{1}{12}(16m)(10m)^3 - \frac{1}{12}(12m)(6m)^3 = 1333m^4 - 216m^4 = 1117m^4$$