University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 1 EXAMINATION 2010/2011

MATH 10250
INTRODUCTION TO CALCULUS FOR ENGINEERS

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Time Allowed: 2 hours

Instructions for Candidates

Full marks will be awarded for complete answers to **SIX** questions.
Details of calculations leading to your answers must be included.
No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.
Candidates are **NOT** allowed to use mathematical tables during this examination.
The use of graph paper in **NOT** required at this examination.

1. (a) Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 - x + 2$ and $g : \mathbb{R} \to \mathbb{R}$, $g(x) = x - 2$. Find $f \circ g$ and $g \circ f$.

   (b) By substituting $u = \cos x$ or otherwise, evaluate the definite integral

$$\int_0^{\pi/2} \sin^3 x \cos^2 x dx.$$

   (Note that $\sin 0 = 0, \cos 0 = 1$ and $\sin(\pi/2) = 1$, $\cos(\pi/2) = 0$)

2. (a) Determine the regions of increase and decrease for

$$f : \mathbb{R} \to \mathbb{R}\,, \quad f(x) = (x^2 - 2x - 1)e^{-2x}.$$

   (b) Solve for y the differential equation

$$\frac{dy}{dx} + 2y = x^2,$$

   given that $y = 0$ at $x = 0$.

3. (a) Use partial fractions to evaluate the definite integral

$$\int_2^3 \frac{x+1}{x^2 + x - 2} dx.$$

   (b) Using L'Hopital's rule or otherwise, evaluate the following limit

$$\lim_{x \to -2} \frac{x^3 + 3x^2 - 4}{x^3 + x^2 - 8x - 12}.$$

4. (a) Define the hyperbolic functions sinh$x$ and cosh$x$, give rough sketches of their graphs and prove that

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}}.$$

   (b) Use the ratio test to show that the series

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n}$$

   is convergent when $-1 < x < 1$. Decide, with mathematical reasons whether the series is convergent for $x = -1$ and $x = 1$.

5. (a) Find the centroid of the region bounded by the curve $y = 6 - x^2$ and the lines $y = 0$, $x = 1$ and $x = 2$.

   (b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = 2\cosh x.$$

6. (a) Find the first six terms of the Taylor series for the function $f(x) = x^2 \ln x$ centred at $a = 1$.

   (b) Use Simpson's formula with four intervals of equal width to find an approximate value of the integral

$$\int_0^2 \frac{x-1}{x+1} dx.$$

7. (a) Find the area of the surface obtained by rotating the curve $y = x^3$ lying between the lines $x = 0$ and $x = 1$ about the $x-$axis.
   (Hint: use the substitution $u = 1 + 9x^4$)

   (b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 2\cos(2x) - \sin(2x).$$

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