



University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 1 EXAMINATION 2011/2012

**MATH 10250
INTRODUCTION TO CALCULUS FOR ENGINEERS**

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Time Allowed: 2 hours

Instructions for Candidates

Full marks will be awarded for complete answers to **SIX** questions.

Details of calculations leading to your answers must be included.

No credit will be given for unsubstantiated numerical answers.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Candidates are **NOT** allowed to use mathematical tables during this examination.

The use of graph paper is **NOT** required at this examination.

1. (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2 - 2x - 5$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = 2x - 1$.
Find $f \circ g$ and $g \circ f$.

- (b) By substituting $u = \sqrt{1 - x^2}$ or otherwise, evaluate the definite integral

$$\int_0^1 x^3 \sqrt{1 - x^2} dx.$$

2. (a) Determine the regions of increase and decrease for

$$f : \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = (2x + 1)e^{-x^2}.$$

- (b) Find the general solution of the differential equation

$$\frac{dy}{dx} - y = \cos(2x).$$

3. (a) Use partial fractions to evaluate the definite integral

$$\int_2^4 \frac{3x - 1}{x^2(x - 1)} dx.$$

- (b) Evaluate the limit

$$\lim_{x \rightarrow 2} \frac{2 - \sqrt{x + 2}}{3 - \sqrt{5 + 2x}}.$$

4. (a) Define the inverse trigonometric functions $\sin^{-1} x$ and $\cos^{-1} x$ and prove that

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}.$$

- (b) Use the ratio test to show that the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n2^{n+1}}$$

is convergent when $-2 < x < 2$. Decide, with mathematical reasons, whether the series is convergent for $x = -2$ and $x = 2$.

5. (a) Find the length of the graph of function

$$f : [0, 1] \rightarrow \mathbb{R}, \quad f(x) = e^x.$$

(*Hint:* use the substitution $u = \sqrt{1 + e^{2x}}$)

- (b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 9y = \cos(2x) - 2\sin(2x).$$

6. (a) Find the first five terms of the Maclaurin series for the function

$$f(x) = (x + 1)\cosh(2x).$$

- (b) Use Trapezoidal rule with four intervals of equal width to find an approximate value of the integral

$$\int_0^2 \frac{x^2 - 1}{x^2 + 1} dx.$$

7. (a) Find the volume of the solid obtained by rotating the graph of the function

$$f : [1, e] \rightarrow \mathbb{R}, \quad f(x) = x \ln x$$

about the x -axis.

- (b) The roots of the characteristic equation of a nonhomogeneous linear second order differential equation are $\lambda_1 = 1 + 2i$, $\lambda_2 = 1 - 2i$ and a particular solution is $y_p(x) = \sin(3x)$.

Find the associated differential equation and write down its general solution.

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