MATH10260 Linear Algebra for Engineers
Problem Set 1: Numbers and Polynomials.

1. Write numbers represented by the following finite and periodic decimal expansions in the form of reduced fraction $\frac{n}{m}$, e.g.:

$$0.0123123123\ldots = \frac{123}{10000}\sum_{k=0}^{\infty}\frac{1}{1000^k} = \frac{123}{10000}\frac{1}{1-1/1000} = \frac{123}{9990} = \frac{41}{3330}$$

   a) $1.009009009\ldots$
   b) $0.245454545\ldots$
   c) 0.008 d) 0.125
   e) $0.142857142857\ldots$
   f) $0.012345679012\ldots$ (with 012345679 repeating)

2. Write the binomial expression for $(x+y)^5$ and use it to represent $(1+\sqrt{3})^5$ in the form $a + b\sqrt{3}$ with $a, b \in \mathbb{Q}$.

3. a) Prove the identity $(x-y)(x+y) = x^2 - y^2$.

   b) Show that $\frac{1}{1+\sqrt{5}} = -\frac{1}{4} + \frac{1}{4}\sqrt{5}$

   c) Let $a, b \in \mathbb{Q}$ be not both zero. Show that the multiplicative inverse of $a + b\sqrt{5}$ can be written in the form

$$\frac{1}{a+b\sqrt{5}} = c + d\sqrt{5}$$

   with $c, d \in \mathbb{Q}$. Give explicit expression for c and d in terms of a, b.

   d)* Check that the subset of real numbers

$$K = \{a + b\sqrt{5} : a, b \in \mathbb{Q}\}$$

   is a field. (Notice that $\mathbb{Q} \subset K \subset \mathbb{R}$.)

4. a) Check that 1 is a root of the polynomial

$$P(x) = x^5 - 3x^4 + 4x^3 - 4x^2 + 3x - 1.$$

   b) Find the multiplicity m of this root by evaluating subsequently derivatives of $P(x)$ at $x = 1$.

   c) Find the polynomial $Q(x)$ such that $P(x) = (x-1)^m \cdot Q(x)$. Does P have real roots other than $x = 1$?