

## EEEN20060 Communication Systems

## Link Layer Protocols – part 2

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Scoil na hInnealtóireachta  
Leictirí, Leictreonáil agus  
Cumarsáide UCD

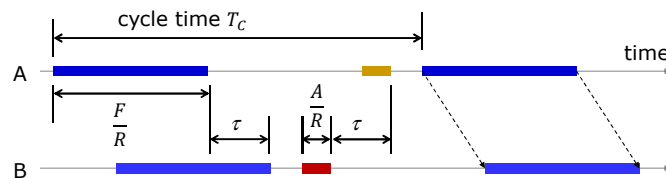
## Analysis of Stop and Wait

- How well does it perform?
  - calculate *throughput* or *efficiency*
  - **throughput**  $U$  = rate of transfer of useful bits (payload) (unit: bit/s)
  - **efficiency**  $\eta$  = throughput as fraction of physical layer bit rate
- Define some symbols
  - $D$  = maximum data bits in each frame
  - $H$  = overhead bits in frame (header + trailer)
  - $F = D + H$  = max. total bits in frame
  - $A$  = bits in acknowledgement (ACK or NAK)
  - $R$  = physical layer bit rate
  - $\tau$  = time delay through physical layer



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## Time Used by One Frame – Cycle Time



- With everything going well...

- |                                     |                               |
|-------------------------------------|-------------------------------|
| – time to transmit bits of frame    | $\frac{F}{R} = \frac{D+H}{R}$ |
| – delay before last bit is received | $\tau$                        |
| – processing delay at receiver      | ?                             |
| – time to transmit bits of ACK/NAK  | $\frac{A}{R}$                 |
| – delay before last bit is received | $\tau$                        |
| – processing delay at sender        | ?                             |



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## Assumptions

$$\text{Cycle Time } T_C = \frac{F}{R} + \tau + \frac{A}{R} + \tau = \frac{D+H+A+2\tau R}{R}$$

- To simplify analysis, assume
  - no processing delays at either end
    - ACK sent as soon as frame received, etc.
  - plenty of data to waiting to be sent
    - so all frames carry full payload of  $D$  data bits
    - no delay waiting for more data to become available
  - timeout set just longer than time needed
    - so even if no reply, cycle time remains ~same
  - no enquiries – just re-send data block
- Cycle time is time used in sending one frame
  - but does it transfer useful data?



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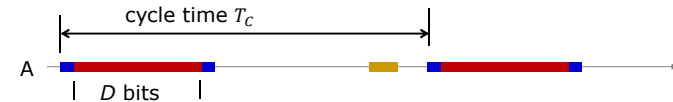
### Probability of Success

- Success = frame received with no errors
  - and ACK received with no errors
  - so no need to re-send data
- More definitions
  - $P_{SF}$  = probability of successful reception of frame
  - $P_{SA}$  = prob. of successful reception of ACK
  - $P_S = P_{SF} P_{SA}$  = overall probability of success
- Multiplication of probabilities
  - independent events
  - need both to happen for overall success



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### Throughput



- With our assumptions
  - send one frame in each cycle time
  - some succeed – fraction  $P_S$
  - if succeed, transfer  $D$  data bits
  - others fail, transfer nothing useful
- So throughput  $U = \frac{DP_S}{T_c} = \frac{DP_S R}{D+H+A+2\tau R}$ 
  - efficiency  $\eta = \frac{U}{R} = \frac{DP_S}{D+H+A+2\tau R}$



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### Efficiency Examples

- Frame: 2000 data bits, 50 overhead bits
  - physical layer 1 Mbit/s, so 2.05 ms to send
  - ACK or NAK: 50 bits, so 50  $\mu$ s to send
  - probability of success 0.9
- Short link – 200 m, propagation delay 1  $\mu$ s
  - cycle time 2.102 ms
  - throughput  $U = \frac{2000 \text{ bits}}{2.102 \text{ ms}} \cdot 0.9 \approx 856 \text{ kbit/s}$
  - efficiency 0.856
- Long link – 5000 km, propagation delay 25 ms
  - cycle time 52.1 ms
  - throughput 34.5 kbit/s, efficiency 0.0345



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### Improving Efficiency

$$\eta = \frac{DP_S}{D + H + A + 2\tau R}$$

- Assume physical layer is fixed:  $\tau$ ,  $R$ , errors
  - we design link layer protocol
  - we control  $D$ ,  $H$ ,  $A$
- What should we do?
- Make  $H$ ,  $A$  small ?
  - but cannot reduce to zero...
- Make  $D$  very large ?
  - but  $P_S$  will depend on size of frame...
    - exact relationship depends on physical layer
  - longer frame more likely to have errors



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### Probability of Success – Example 1

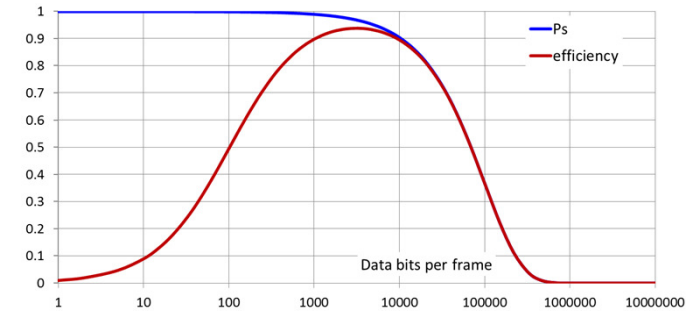
- One simple model of physical layer
  - independent decision on each bit (no bursts)
  - every bit has probability of error  $p$ 
    - so probability of good bit is  $1 - p$
- For successful reception of frame
  - need all bits good, prob.  $P_{SF} = (1 - p)^{(D+H)}$
  - similarly for ACK,  $P_{SA} = (1 - p)^A$
  - so prob. overall success  $P_S = (1 - p)^{(D+H+A)}$
- Example as earlier ( $D = 2000, H = 50, A = 50$ )
  - $p = 10^{-5}$  gives  $P_{SF} = 0.9797, P_{SA} = 0.9995$
  - overall prob. success  $P_S = 0.9792$
  - $p = 10^{-4}$  gives  $P_S = 0.81$
  - $p = 10^{-3}$  gives  $P_S = 0.122$



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### Example

$$\eta = \frac{D(1-p)^{(D+H+A)}}{D+H+A+2\tau R}$$

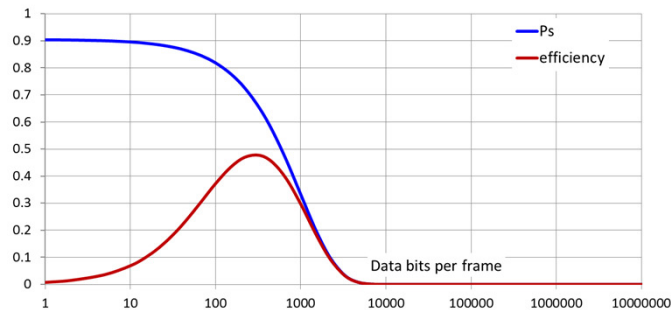


- Note broad peak – good for range of  $D$  values
  - graph is for  $H = A = 50$  bit,  
 $R = 1$  Mbit/s,  $\tau = 1 \mu s$ ,  $p = 10^{-5}$

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### Example

$$\eta = \frac{D(1-p)^{(D+H+A)}}{D+H+A+2\tau R}$$



- Choose  $D$  to suit physical layer!
  - graph is for  $H = A = 50$  bit,  
 $R = 1$  Mbit/s,  $\tau = 10 \mu s$ ,  $p = 10^{-3}$

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### Optimum Block Size

$$\eta = \frac{D(1-p)^{(D+H+A)}}{D+H+A+2\tau R}$$

- Differentiate with respect to  $D$ , set = 0 ?
  - $D$  is integer, but  $\eta$  is continuous function of  $D$
  - so OK to differentiate w.r.t.  $D$
- To simplify, replace constants
  - use  $B = H + A$ ,  $C = H + A + 2\tau R$ ,  $s = 1 - p$
  - then  $\eta = \frac{Ds^{(D+B)}}{D+C}$
  - so  $\frac{d\eta}{dD} = \frac{(D+C)[s^{(D+B)} + Ds^{(D+B)} \ln(s)] - Ds^{(D+B)}}{(D+C)^2} = 0$
  - $\Rightarrow (D+C)[1 + D \ln(s)] - D = 0$
  - $\Rightarrow D^2 \ln(s) + DC \ln(s) + C = 0$



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## Solving...

$$D^2 \ln(s) + DC \ln(s) + C = 0$$

- quadratic equation:  $D_{opt} = -\frac{C}{2} + \sqrt{\left(\frac{C}{2}\right)^2 - \frac{C}{\ln(s)}}$
- $\ln(s) = \ln(1-p) \approx -p$  for  $p \ll 1$  (Taylor series)
- get  $D_{opt} \approx \sqrt{\left(\frac{C}{2}\right)^2 + \frac{C}{p}} - \frac{C}{2} = \frac{C}{2} \left[ \sqrt{1 + \frac{4}{pC}} - 1 \right]$
- note  $C = H + A + 2\tau R$  = wasted bit times/cycle  
• = 102 in our 1  $\mu$ s example
- with  $p = 10^{-5}$ , get  $D_{opt} \approx 3143$  bit
- in our 10  $\mu$ s example,  $C = 120$  bit times
- with  $p = 10^{-3}$ , get  $D_{opt} \approx 291$  bit



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## Probability of Success – Example 2

- Another simple model – burst errors
  - model error bursts as Poisson random process
  - average rate  $\lambda$  burst/s
- Poisson random process:
  - models occurrence of discrete events
  - no memory: events in non-overlapping time intervals are independent
  - in small time interval  $\Delta t$ , probability of exactly one event occurring  $\rightarrow \lambda \Delta t$  as  $\Delta t \rightarrow 0$
  - and probability of no event  $\rightarrow 1 - \lambda \Delta t$  as  $\Delta t \rightarrow 0$
- Results:
  - in longer interval  $T$ , expect  $\lambda T$  events
  - prob. no event in interval  $T$  is  $e^{-\lambda T}$



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## Success?

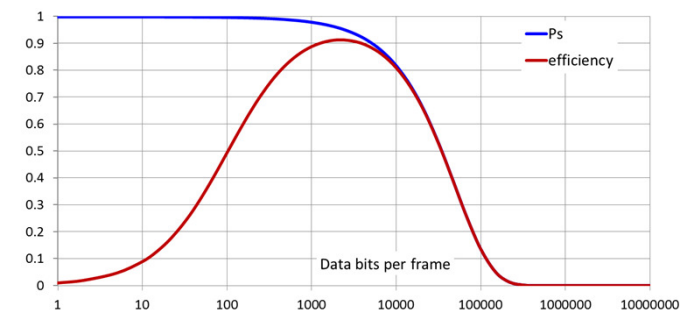
- Frame received successfully if no error burst
  - duration  $\frac{D+H}{R}$ , so  $P_{SF} = e^{-\lambda \frac{D+H}{R}}$
- ACK received successfully if no error burst
  - duration  $\frac{A}{R}$ , so  $P_{SA} = e^{-\lambda \frac{A}{R}}$
- Overall probability of success
  - $P_S = e^{-\lambda \frac{D+H}{R}} e^{-\lambda \frac{A}{R}} = e^{-\lambda \frac{D+H+A}{R}} = e^{-\frac{\lambda}{R}(D+H+A)}$
- Example as earlier (2000, 50, 50, 1 Mbit/s)
  - $\lambda = 2$  burst/s,  $P_{SF} = 0.9959$ ,  $P_{SA} = 0.9999$
  - overall prob. success  $P_S = 0.9958$
  - $\lambda = 20$  burst/s gives  $P_S = 0.959$
  - $\lambda = 200$  burst/s gives  $P_S = 0.657$



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## Example

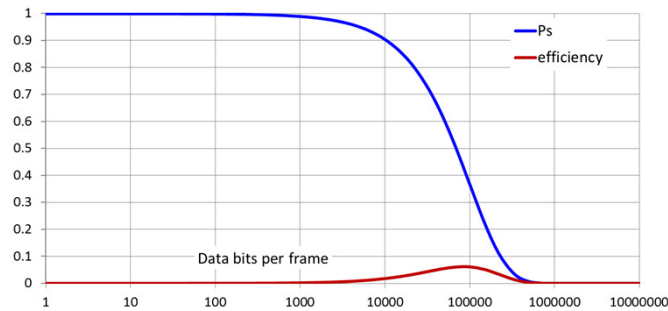
$$\eta = \frac{D e^{-\frac{\lambda}{R}(D+H+A)}}{D + H + A + 2\tau R}$$



- Function has exactly same form as before
  - graph is for  $H = A = 50$  bit,  
 $R = 1$  Mbit/s,  $\tau = 1 \mu$ s,  $\lambda = 20$  burst/s

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### Problems with Stop and Wait



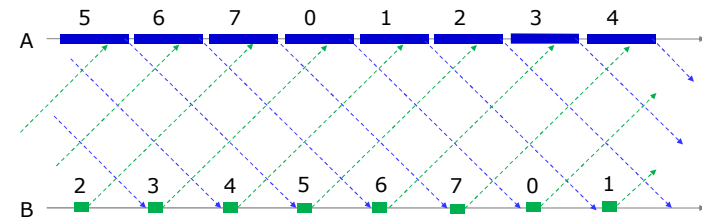
- Stop and Wait is OK for short links

- with long delay (or at very high bit rate), waiting time wastes too many bit times...
- example is satellite link,  $\tau = 250$  ms



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### Better Protocols



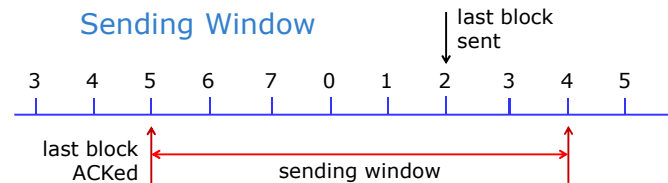
- Need to keep transmitting while waiting

- somehow keep track of data blocks and ACKs
  - need 2-way link, both directions at same time
- called *sliding window* protocols...
- *sending window* is range of sequence numbers that sender can send without getting ACK
- *slides* upwards as ACKs arrive for old blocks...



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### Sending Window

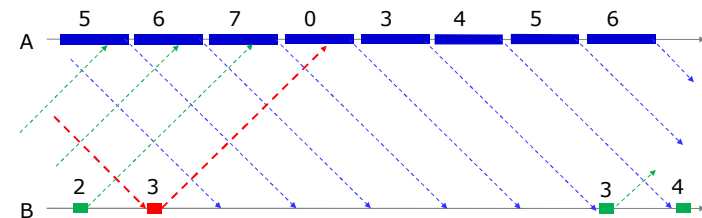


- Why limit number of un-ACKed data blocks?
- Limited storage to hold those data blocks
  - must be stored by link layer at sender
  - might be needed for re-sending if NAK arrives
- Limited range of sequence numbers
  - cannot have 2 un-ACKed blocks with same no.
- Design – choose limit large enough
  - so ACKs should arrive before limit is reached...



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### Receiver Behaviour ?



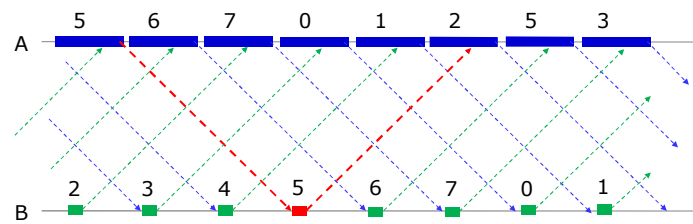
- Option 1 – no storage

- receiver only accepts data blocks in sequence
  - discards anything that it cannot use immediately
  - so after NAK, no more ACKs until missing block arrives
- when sender gets NAK
  - must go back and start again from bad block
- called “go back N” operation



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## Receiver Behaviour ?



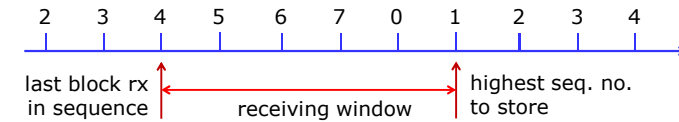
- Option 2 – with storage

- receiver accepts blocks out of sequence
  - may acknowledge them if good (option)
  - stores them until missing blocks arrive
- when sender gets NAK, resend rejected block
  - then resume where it was interrupted...
- called “selective reject” operation



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## Selective Reject – Receiving Window

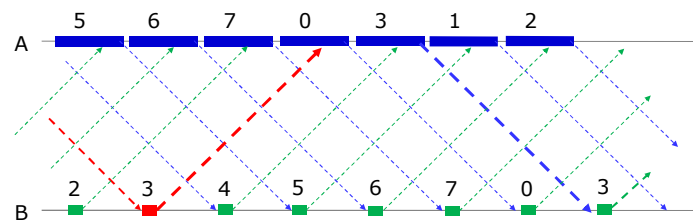


- Why limit number of blocks to receive out of sequence?
- Limited storage to hold those data blocks
  - must be stored by link layer at receiver
  - can only pass on to customer in sequence...
- Limited range of sequence numbers
  - cannot store 2 blocks with same number



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## Design – Selective Reject

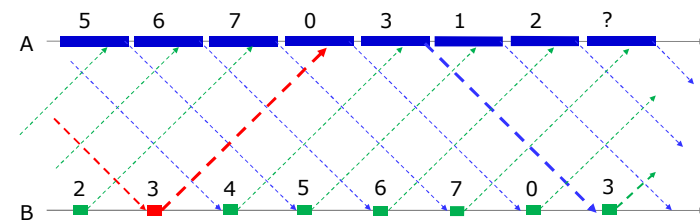


- How big should windows be?
- Depends on round-trip delay, frame time...
  - in example, delay rounds up to 6 frame times
    - e.g. from start of frame with block 5 to ACK 5 received
- If receiver sends NAK ?
  - cannot expect to get rejected data block again until 6 frame times have elapsed



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## Design – Selective Reject continued

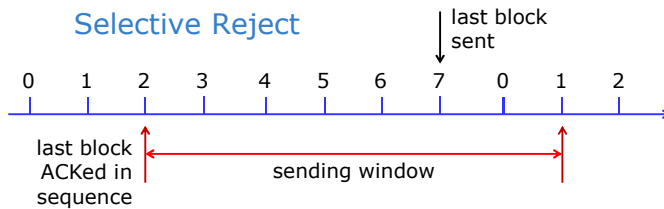


- So receiving window > round trip delay
  - delay measured in frame times, rounded up
  - to be efficient, must accept **all** blocks after NAK
- Sender sends block 3
  - cannot expect ACK for 6 frame times
  - but if get NAK instead, how long to get ACK?



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### Selective Reject

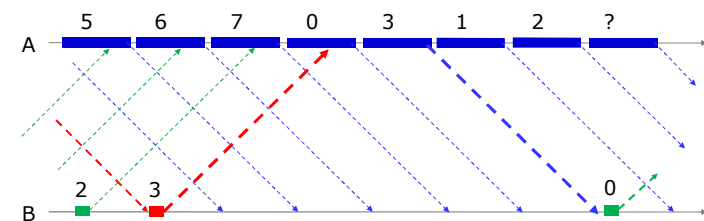


- Sending window > round trip delay
  - minimum, as in go back N
  - but will have to stop after NAK
- To continue non-stop after NAK
  - need > twice round trip delay
  - this is reasonable design criterion
    - allows continuous sending most of the time
    - could still fail if NAK, re-send, NAK again...



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### Selective Reject – Alternatives



- Some systems only ACK blocks in sequence
  - no ACKs for blocks received after NAK sent
  - ACK means “all up to this block received OK”
- HDLC (High-level Data Link Control)
  - bit-oriented protocol, basis for many others
  - ACK/NAK can travel in data frame...
  - ACK uses seq. no. of **Next** block expected...



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### Example – HDLC Frame

Header			Data	Trailer	
Flag	Address	Control	Data (if any)	Frame Check Sequence (CRC) 16 bit	Flag
01111110	8 bit	8 bit			01111110

0	Sequence Number	P/F	Next
---	-----------------	-----	------

0 = Information frame, with data and seq. no., also carries ACK...

1	0	Type	P/F	Next
---	---	------	-----	------

10 = Supervisory frame, no data. Type: Receiver Ready, Receiver not Ready, Reject (NAK), Selective Reject.

1	1	Type	P/F	Modifier
---	---	------	-----	----------

11 = Un-numbered frame, no data. Control messages: reset, disconnect, change mode, etc.

Option for 7-bit seq. no.s, adds another byte to header...

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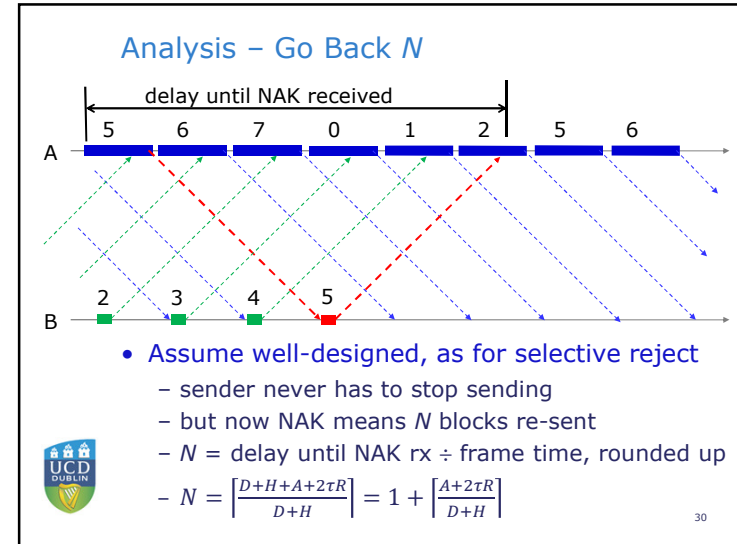
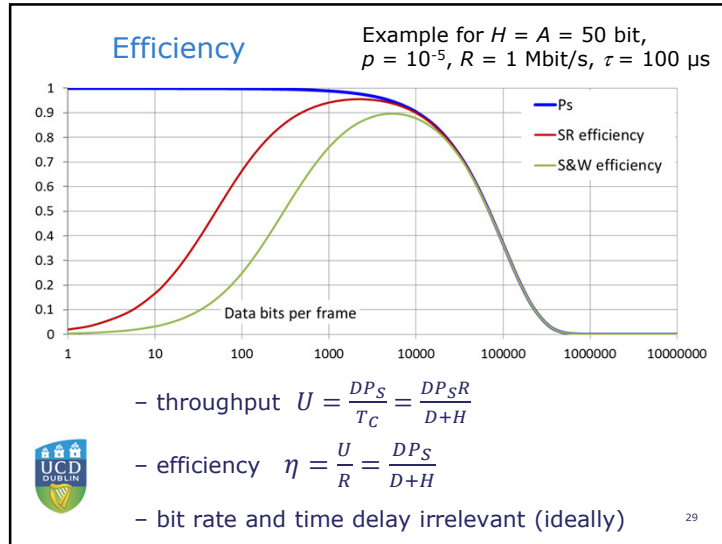
### Analysis – Selective Reject



- Assume well designed...
  - sender never has to stop?
    - stops with very low probability, which we ignore
  - only reason to re-send block is error
- Cycle time is now frame transmit time
  - send one frame in each cycle time  $T_c = \frac{D+H}{R}$
  - some succeed – fraction  $P_S$
  - if succeed, transfer  $D$  data bits
  - others fail (NAK), transfer nothing useful



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**Analysis – Go Back N**

- Still send one frame in time  $T_C = \frac{D+H}{R}$
- How many frames to send one data block?
  - if succeed on first attempt, use 1 frame
    - probability  $P_S$
  - if succeed on second try, use  $N + 1$  frames
    - fail once, then succeed, probability  $(1 - P_S)P_S = P_F P_S$
  - if succeed on third try, use  $2N + 1$  frames
    - fail twice, then succeed, probability  $P_F^2 P_S$

average =  $\sum_{k=1}^{\infty} \{(k-1)N + 1\} P_F^{k-1} P_S$

$$= P_S \sum_{k=0}^{\infty} (kN + 1) P_F^k = P_S \left[ N \sum_{k=0}^{\infty} k P_F^k + \sum_{k=0}^{\infty} P_F^k \right]$$

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**Analysis...**

- Geometric series:
 
$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad \text{for } |r| < 1, \quad \text{so } \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

differentiating this:  $\sum_{n=0}^{\infty} nr^{n-1} = \frac{1}{(1-r)^2}$
- So average no. frames to send one block
 
$$= P_S \left[ NP_F \frac{1}{(1-P_F)^2} + \frac{1}{1-P_F} \right] = \frac{N(1-P_S) + P_S}{P_S}$$
- So fraction of frames carrying useful data
 
$$= \frac{P_S}{P_S + N(1-P_S)}$$

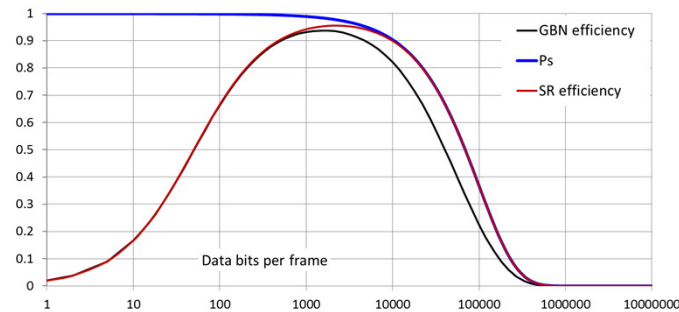
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## Efficiency

Example for  $H = A = 50$  bit,  
 $p = 10^{-5}$ ,  $R = 1$  Mbit/s,  $\tau = 100$   $\mu$ s

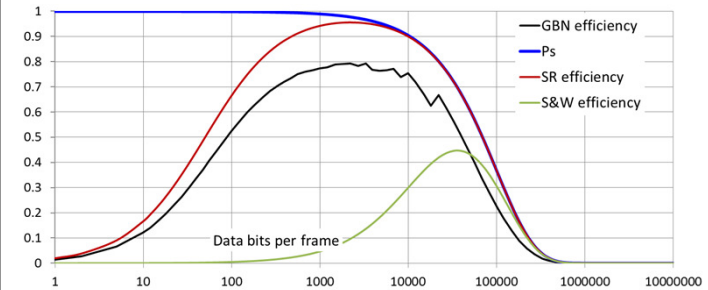


- throughput  $U = \frac{D}{T_C} \frac{P_S}{P_S + N(1-P_S)} = \frac{DR}{D+H} \frac{P_S}{P_S + N(1-P_S)}$
- efficiency  $\eta = \frac{U}{R} = \frac{DP_S}{(D+H)\{P_S + N(1-P_S)\}}$

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## Comparison

Example for  $H = A = 50$  bit,  
 $p = 10^{-5}$ ,  $R = 1$  Mbit/s,  $\tau = 10$  ms



- efficiency falls with longer delay, larger  $N$ 
  - small fall if probability of success high...
  - jagged curve due to step changes in  $N$
- still better than stop and wait...

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## Summary

- Stop & Wait
  - simple, can use "one way at a time" channel
  - OK if waiting time short, relative to frame
    - poor with long link, high bit rate
- Sliding Window, Go Back N
  - need storage at sender, but simple receiver
  - need bi-directional channel
  - OK with short-medium delay
    - or longer delay if probability of success is high
- Sliding Window, Selective Reject
  - need storage at sender and receiver
  - if well designed, windows large enough...
    - performance ~independent of delay or bit rate



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