

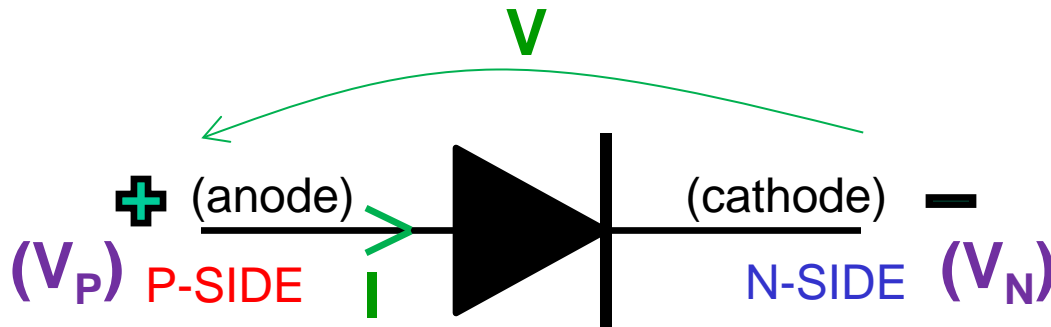
Chapter 8

Bias and Current Flow in the PN Junction

Biased PN Junction

- The term ‘bias’ in electronics refers to the application of one or more steady or “DC” voltages (or sometimes currents) to a semiconductor device;
- In the case of a PN junction which has just two terminals, we **forward bias** the junction when the potential on the P-side (anode) is higher than the potential on the N-side (cathode) (i.e. voltage across the device = $V > 0$)
- The opposite condition is referred to as **reverse bias**, where the potential on the anode is lower than that on the cathode (i.e. $V < 0$).
- Under conditions of **DC bias**, note that **thermal equilibrium no longer applies**.

Reference Senses and Notation



Forward Bias:

$V > 0, I > 0$

Reverse Bias:

$V < 0; I < 0$

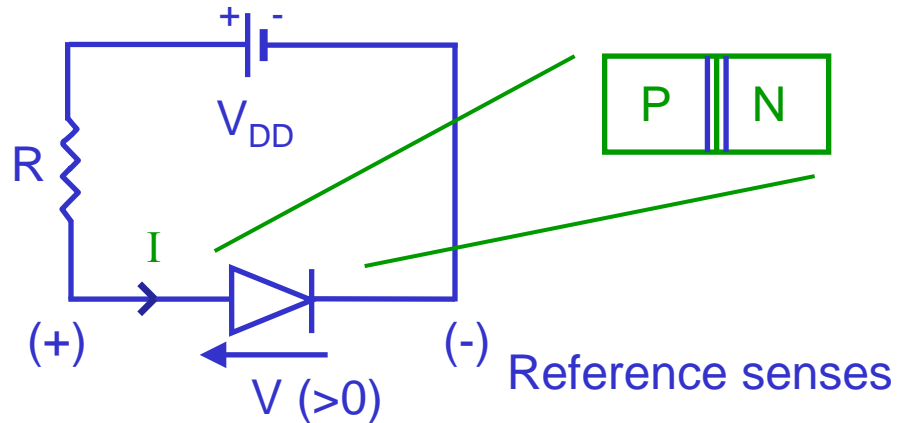
- We will adopt the notation that capital letters (perhaps with capital subscripts) always represent steady or DC voltages or currents
- For example, if V_p is the absolute DC voltage on the P-side (relative to 0V or Ground) and V_N is the equivalent DC voltage on the N-side, then from KVL above:

$$V = V_p - V_N$$

- Note that e.g. the current does not *have* to flow in the direction of the reference arrow, its just that *if it does* so then it is taken as a *positive* quantity.

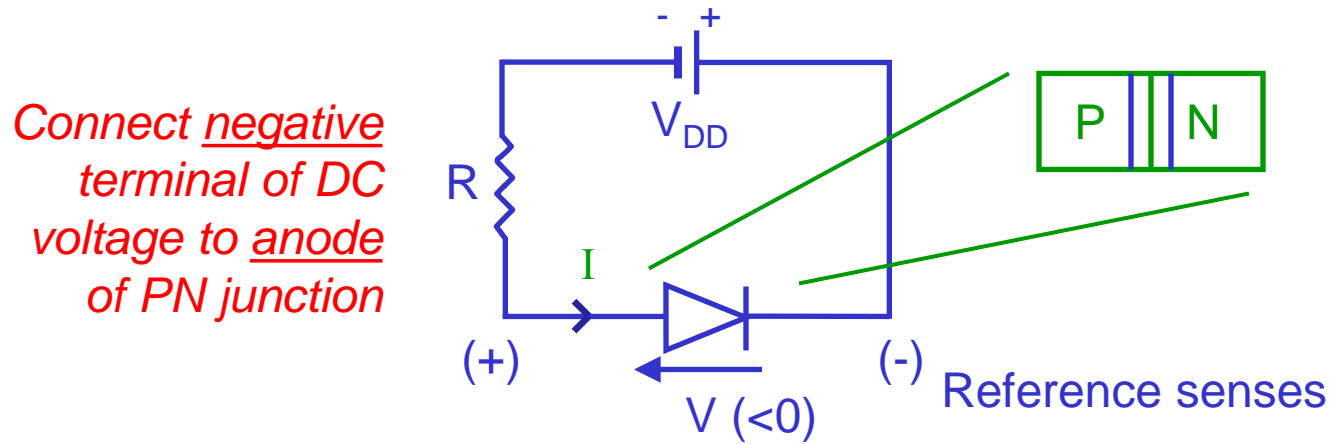
Forward Bias of the PN Junction

*Connect positive
terminal of DC
voltage to anode
of PN junction*



- In thermal equilibrium, the internal N-side is at a voltage (ϕ_i) higher than the P-side.
- As a result of applying the DC bias, a (positive) voltage V exists across the terminals of the device as shown in the diagram, and therefore the N-side becomes only ($\phi_i - V$) higher than the P-side, i.e. the potential barrier has been **reduced** by the forward bias.

Reverse Bias of the PN Junction



- Again, thermal equilibrium no longer applies;
- In this case the V shown in the diagram is negative. It thus adds in magnitude to the higher potential on the N-side in thermal equilibrium. The potential on the N-side now becomes $(\phi_i + |V|)$ higher than the P-side, i.e. the potential barrier has been **increased** by the **reverse bias**.

Extending from Thermal Equilibrium to the Non-equilibrium (Biased) Case

- We can simply re-use the formulas derived earlier that assumed thermal equilibrium, e.g. for the depletion layer width and penetration of the depletion layer into the P-side or N-side;
- All that needs to be done is everywhere to **replace (ϕ_i) with $(\phi_i - V)$** , where it is understood that $V > 0$ means forward bias and $V < 0$ means reverse bias. We should also remove any zero subscripts that indicated thermal equilibrium.

Depletion Layer Width with Bias

- For example, in thermal equilibrium we found previously that the equilibrium depletion layer width is given by:

$$W_o = \sqrt{\frac{2 \cdot \varepsilon \cdot \phi_i}{q} \cdot \left(\frac{N_A + N_D}{N_A \cdot N_D} \right)}$$

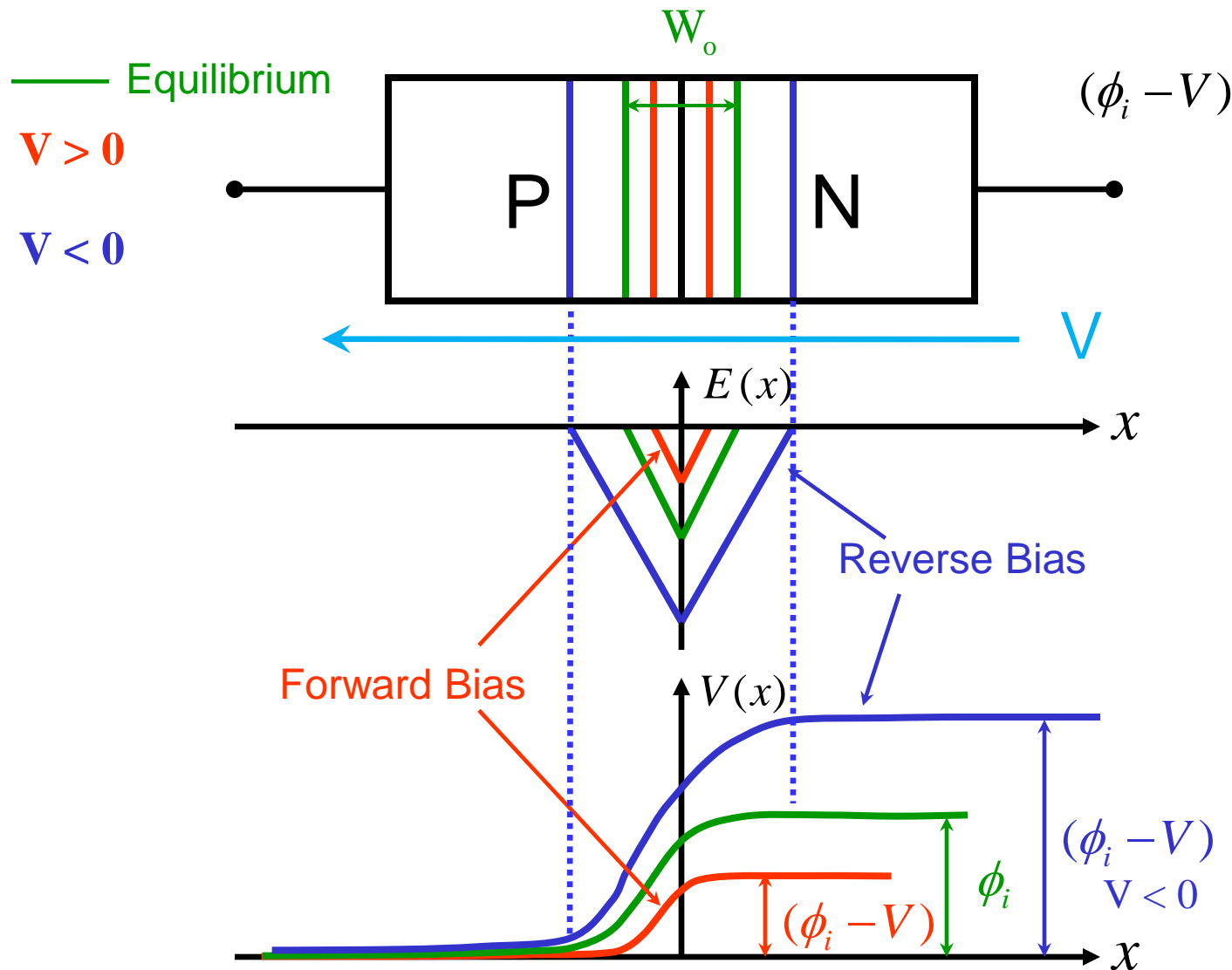
- Under biased conditions with a voltage V , this now reads:

$$W = \sqrt{\frac{2 \cdot \varepsilon \cdot (\phi_i - V)}{q} \cdot \left(\frac{N_A + N_D}{N_A \cdot N_D} \right)}$$

This is the version of the formula given in the Formula Sheet

- Similarly, we can modify the formulas for x_{no} and x_{po} to get general expressions for x_n and x_p under biased conditions.

Effect of DC Bias on Depletion Layer Width, Electric Field $E(x)$ and Potential $V(x)$



Example 8.1

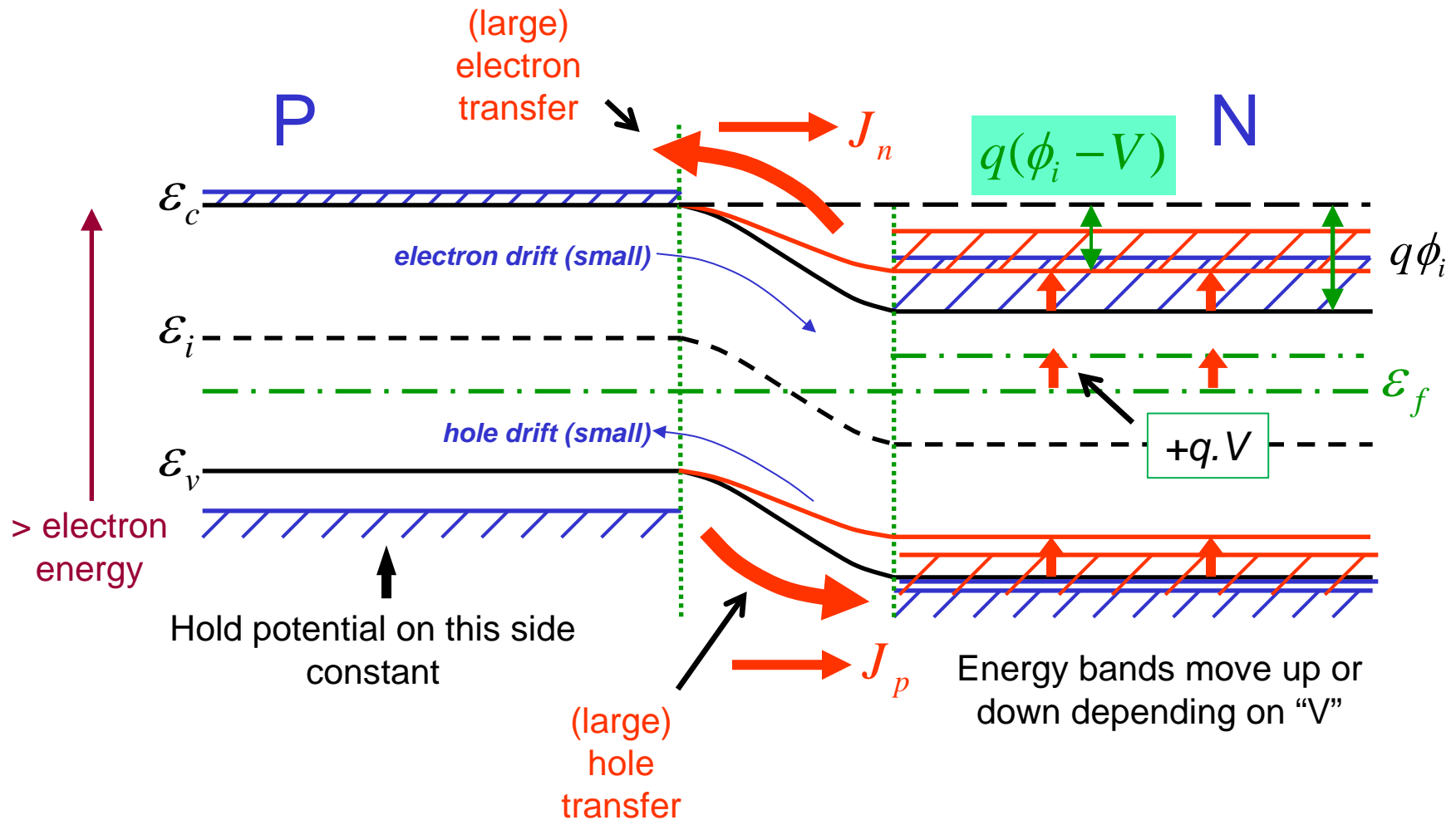
Current Flow in the PN Junction

- With a DC voltage V applied, it is possible for a current I to flow in the PN junction
- We now first consider *qualitatively* what happens within the structure under forward and reverse bias, respectively. This discussion is based on the energy band picture with the junction energy barrier modified from the value $q\phi_i$ in equilibrium to $q(\phi_i - V)$ with bias
- Following this, we use the basic semiconductor device equations to derive an *analytical formula* that allows us to predict I for any V in a PN junction, subject to certain approximations

Energy Bands Under DC Bias Conditions

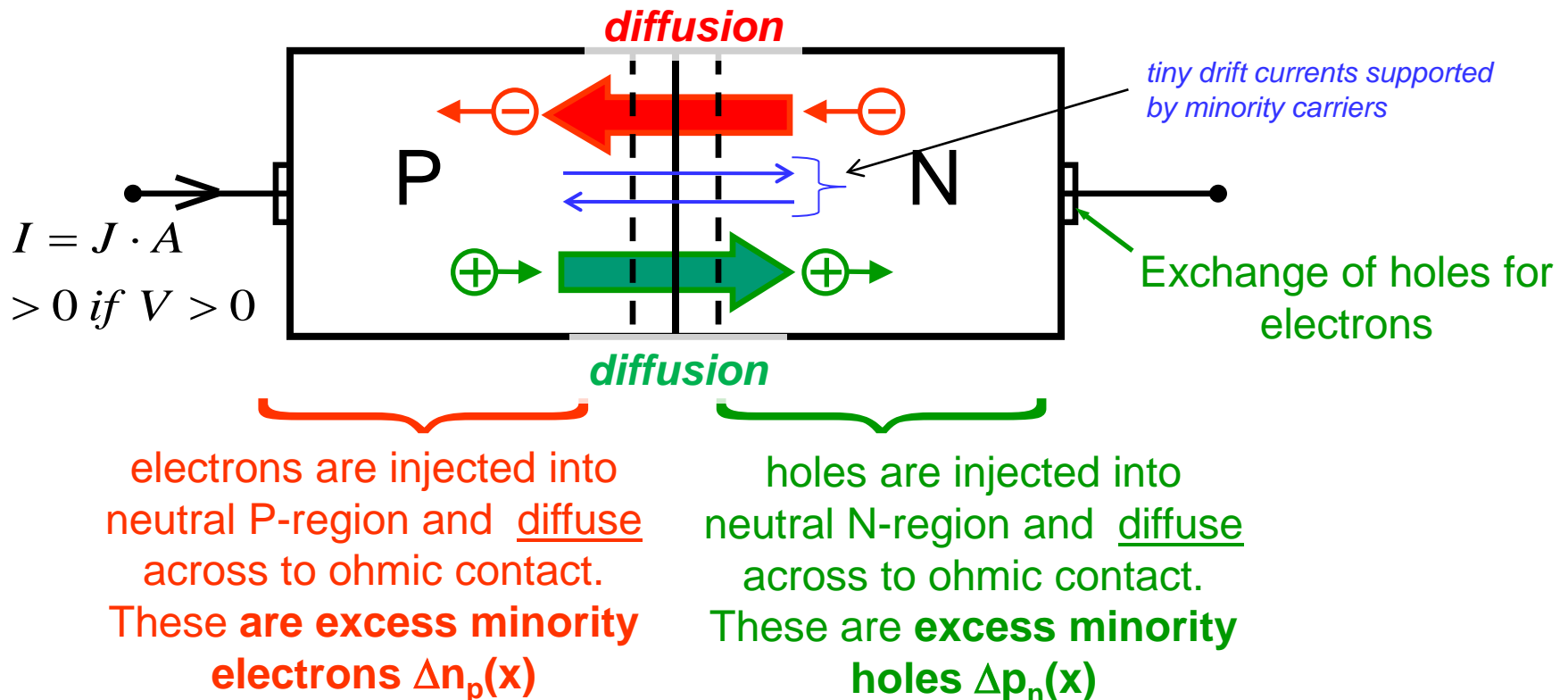
- Since thermal equilibrium no longer applies, there is no requirement for the Fermi energy to be everywhere uniform;
- Earlier we took the neutral P-region as our reference for potential. Hence, we can take account of the effect of an applied DC bias V (equivalent to an energy shift $q.V$) by **fixing or “pinning” the energy band diagram on the P-side** and simply **moving the band structure on the N-side**, either **up** (Forward Bias) or **down** (Reverse Bias) by $|q.V|$;
- Note that technically the depletion layer width should also change on the energy band diagrams as the bias V changes, but this makes the drawings very complex so we will ignore this effect for the moment;
- Under **Forward Bias**, the potential barrier opposing large-scale electron diffusion from N-side to the P-side is **reduced by $q.V$** . A large unbalanced electron diffusion current can then flow from $N \rightarrow P$.
- In exactly the same way, a large hole diffusion current can flow from P-side to N-side. Both of these current components add together.

DC Current Flow in Forward Bias



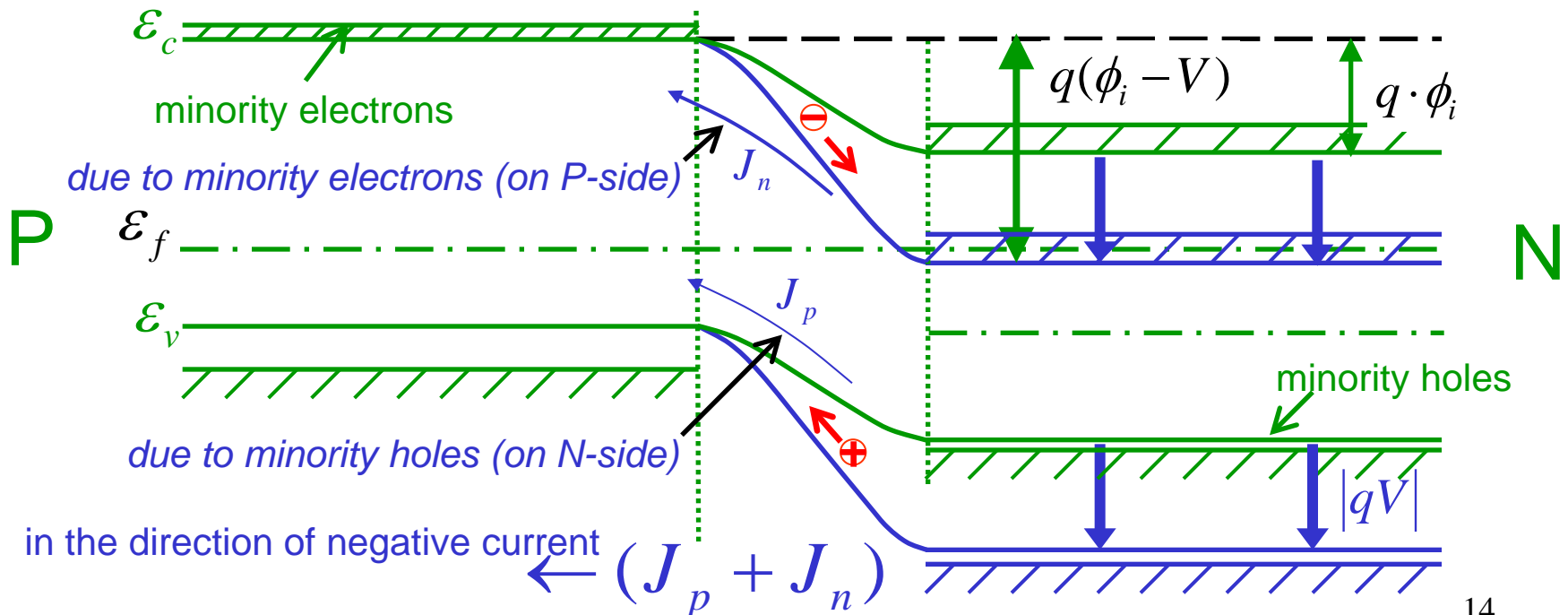
DC Current Flow in Forward Bias

- The result is that the total current density $J = J_p + J_n$ becomes large as V is made increasingly positive;
- Note that the small (now unbalanced) drift currents that oppose diffusion are \approx unaffected by the presence of the forward bias.



DC Current Flow in Reverse Bias

- Under reverse bias, the potential barrier opposing large-scale electron diffusion from N-side to the P-side is **increased by $q \cdot V$** . The result is that diffusion current is quickly killed off almost entirely
- All that is left is a **tiny drift current** of electrons from P \rightarrow N and a drift current holes from N \rightarrow P. This current is limited by the very small minority carrier concentrations and is largely **independent of V** .



Analytical Formula for Current Flow in a Biased PN Junction

- The previous qualitative discussion of the biased PN junction suggests that a **large** and rapidly growing current may be expected under **forward bias**, while a **very small** and essentially constant current will flow in the opposite direction under **reverse bias**.
- We now analyse the current flow in the PN junction making several important approximations:
 1. 1-D structure, with zero electric field in the neutral regions
 2. Uniformly doped, abrupt PN junction
 3. The excess carriers injected into both neutral regions under forward bias correspond to *low-level* injection conditions
 4. We neglect Generation and Recombination in the depletion layer

Consequence of Neglecting G-R in the Depletion Region

- The result of this assumption is that the DC electron current density $J_n(x)$ and the hole current density $J_p(x)$ must be **separately constant** across the depletion layer,
- Proof: use the continuity equation for holes in the region $x \in [-x_p, x_n]$:

$$\frac{\partial p(x, t)}{\partial t} = -\frac{1}{q} \cdot \frac{\partial J_p(x, t)}{\partial x} + G - R \quad \text{DC (static)} \Rightarrow \frac{\partial}{\partial t} = 0$$

$$0 = -\frac{1}{q} \frac{d J_p(x)}{dx} + \underbrace{G - R}_{\text{zero in depletion layer}} \Rightarrow \frac{d J_p(x)}{dx} = 0$$

$$\Rightarrow J_p(x) \text{ is constant, } \forall x \in [-x_p, x_n]$$

(Similar proof can be done for $J_n(x)$)

Minority Carrier Concentrations at the Edges of the Depletion Layer

Earlier (in developing an expression for ϕ_i) we proved that in thermal equilibrium :

$$\phi_i = \frac{kT}{q} \cdot \ln \left| \frac{p_{po}}{p_{no}} \right|$$

As before, we propose that in the case where a voltage “V” is applied (non-equilibrium conditions), this can simply be modified to :

$$(\phi_i - V) = \frac{kT}{q} \cdot \ln \left| \frac{p_p}{p_n} \right| = \frac{kT}{q} \cdot \ln \left| \frac{p_p(-x_p)}{p_n(x_n)} \right|$$

$$\Rightarrow p_n(x_n) = p_p(-x_p) \cdot \exp \left[-\frac{q(\phi_i - V)}{kT} \right]$$

Minority Carrier Concentrations at the Edges of the Depletion Layer

We can write, using the 'low-level' injection assumption:

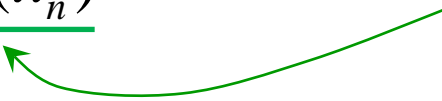
$$p_p(-x_p) = p_{po} + \Delta p_p(-x_p) \approx \underline{p_{po}}$$

(i.e. excess holes have negligible effect in P neutral region)

Also:

$$p_n(x_n) = p_{no} + \underline{\Delta p_n(x_n)}$$

excess (minority) holes
in neutral N-region
produced by "V"



$$\Rightarrow p_{no} + \Delta p_n(x_n) = \underline{p_{po} \cdot \exp\left[-\frac{q\phi_i}{kT}\right] \cdot \exp\left[\frac{qV}{kT}\right]}$$

||
 p_{no}

Minority Carrier Concentrations at the Edges of the Depletion Layer



“Boundary Condition on N-side”

$$\Delta p_n(x_n) = p_{no} \left[e^{qV/kT} - 1 \right]$$

This gives the minority hole concentration at the depletion layer edge of the neutral N-region as a function of bias “V”.

A similar analysis gives: :

“Boundary condition on P-side”

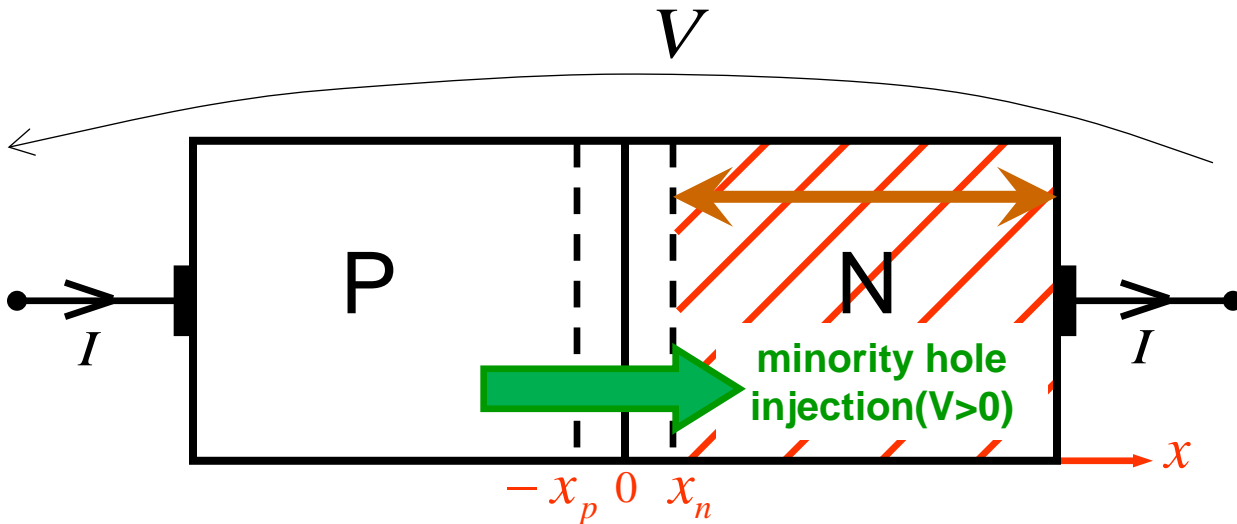
$$\Delta n_p(-x_p) = n_{po} \left[e^{qV/kT} - 1 \right]$$

No Need to Remember: These important Boundary Condition equations for the PN Junction are given in the Formula Sheet

Static Analysis of the PN Junction

We are looking for a formula of the kind $I = f(V)$.

1st consider neutral N-region:



Neutral region $x \in [x_n, \infty)$ "long-based" on N-side
:

We wish to solve for the minority hole concentration: $\Delta p_n(x)$

Solution for $\Delta p_n(x)$ in the Neutral N Region

2 basic equations

- Continuity : $\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q} \frac{\partial J_p(x,t)}{\partial x} + G - R$
- Current density : $J_p(x,t) = q p(x,t) \mu_p E - q D_p \frac{\partial p(x,t)}{\partial x}$

Can simplify these using the fact that this is DC $\Rightarrow \frac{\partial}{\partial t} = 0$ everywhere

The assumption of no electric field in neutral region means that there is no drift current carried so that current is carried entirely by diffusion :

$$0 = -\frac{1}{q} \cdot \frac{d J_p(x)}{dx} - \frac{\Delta p_n(x)}{\tau_p} \quad (1)$$

$$J_p(x) = -q D_p \cdot \frac{d p_n(x)}{dx} \quad (2)$$

Remember: τ_p is the **minority hole lifetime**: the average time an excess hole survives in the neutral N-region before being eliminated through recombination

Solution for $\Delta p_n(x)$ in the Neutral N Region

But $p_n(x) = \underbrace{p_{no}} + \Delta p_n(x)$ this does not depend on "x"
since the doping is uniform

$$\Rightarrow J_p(x) = -q D_p \frac{d \Delta p_n(x)}{dx} \quad (2)$$

Now substitute equation (2) into equation (1) on previous slide:

$$0 = -\frac{1}{q} \cdot \frac{d}{dx} \left(-q D_p \cdot \frac{d \Delta p_n(x)}{dx} \right) - \frac{\Delta p_n(x)}{\tau_p}$$

$$\Rightarrow 0 = D_p \cdot \frac{d^2 \Delta p_n(x)}{dx^2} - \frac{\Delta p_n(x)}{\tau_p}$$

Solution for $\Delta p_n(x)$ in the Neutral N Region

We re-write this equation as:
:

$$\frac{d^2 \Delta p_n(x)}{dx^2} = \frac{\Delta p_n(x)}{L_p^2}$$

...where: $L_p = \sqrt{D_p \tau_p} \quad (m)$

L_p is defined as the **hole diffusion length**

A solution to this second-order differential equation is :

$$\Delta p_n(x) = A \cdot \exp\left[\frac{x - x_n}{L_p}\right] + B \cdot \exp\left[-\left(\frac{x - x_n}{L_p}\right)\right]$$

Constants
of integration

(Exercise:
verify by substitution)

Boundary Conditions

- To eliminate the 2 unknown constants, A and B, we need 2 Boundary Conditions:

- We know from the earlier derivation of the minority carrier concentration at the edge of the depletion layer (on the N-side)

$$\Delta p_n(x_n) = p_{no} \cdot \left[e^{qV/kT} - 1 \right]$$

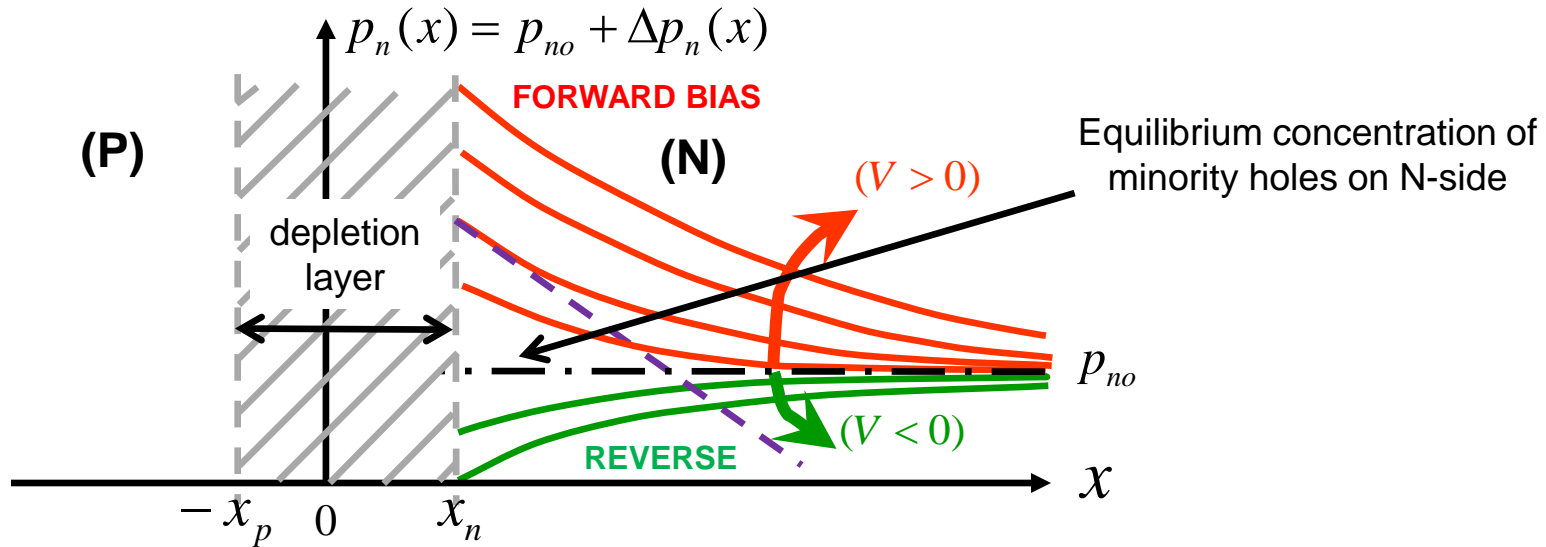
This means that $\Delta p_n(x_n) = A + B$

- But it is also true from physical arguments that $\Delta p_n(x)$ must $\rightarrow 0$ as $x \rightarrow \infty$. This forces $A = 0$. Thus $B = \Delta p_n(x_n)$.

- The full solution for $\Delta p_n(x)$ in the range $[x_n, \infty[$ is thus:

$$\Delta p_n(x) = \underbrace{p_{no} \cdot \left[e^{qV/kT} - 1 \right]}_{= B} \cdot \exp \left[- \left(\frac{x - x_n}{L_p} \right) \right]$$

Minority Hole Distributions in Neutral N-Region



Knowing $\Delta p_n(x)$ we can find the current $J_p(x)$ using $J_p(x) = -qD_p \frac{d\Delta p_n(x)}{dx}$

$$\Rightarrow J_p(x) = \frac{qD_p}{L_p} \cdot p_{no} \left[e^{qV/kT} - 1 \right] \cdot \exp \left[- \left(\frac{x - x_n}{L_p} \right) \right]$$

Hence at $x = x_n$:

$$J_p(x_n) = \frac{qD_p}{L_p} \cdot p_{no} \cdot \left[e^{qV/kT} - 1 \right]$$

Final Analytical Result

A similar analysis can be carried out for minority electrons in the neutral P region giving the corresponding result for the electron current density at the opposite edge of the depletion layer:

$$J_n(-x_p) = \frac{qD_n}{L_n} \cdot n_{po} \cdot \left[e^{qV/kT} - 1 \right]$$

But since we assume no G-R in the depletion layer, we have seen that both $J_n(-x_p)$ and $J_p(x_n)$ must be constant within the depletion layer.

Hence the total current density (J) is :

$$\begin{aligned} J &= J_p(x_n) + J_n(-x_p) \\ &= \left[\frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n} \right] \times \left[e^{qV/kT} - 1 \right] \end{aligned}$$

'Ideal' PN Junction Equation for DC Current

- Using $I = J.A$, we thus obtain a key formula for the DC current in a PN junction as a function of the terminal voltage V (that is valid for both forward and reverse bias):

$$I = I_s \cdot \left[e^{qV/kT} - 1 \right]$$

- Where I_s is the **saturation current** or the **reverse leakage current**.

$$I_s = qA \left[\frac{D_p p_{no}}{L_p} + \frac{D_n n_{po}}{L_n} \right] \quad \underline{\underline{OR}} \quad I_s = qA \left[\frac{D_p n_i^2}{L_p N_D} + \frac{D_n n_i^2}{L_n N_A} \right]$$

Note :

$$D_p = \frac{kT}{q} \mu_p$$

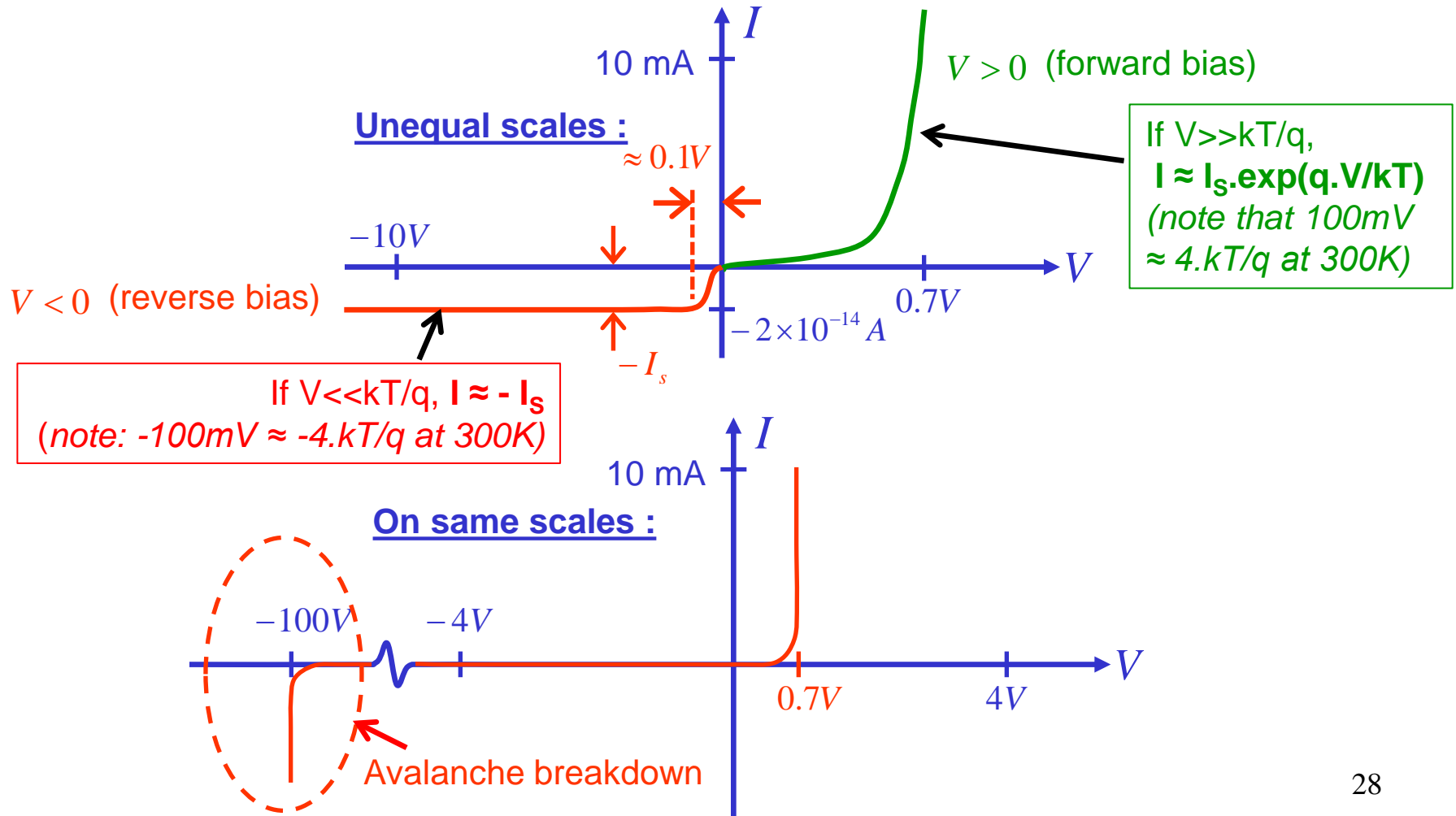
$$D_n = \frac{kT}{q} \mu_n$$

$$L_p = \sqrt{D_p \tau_p}$$

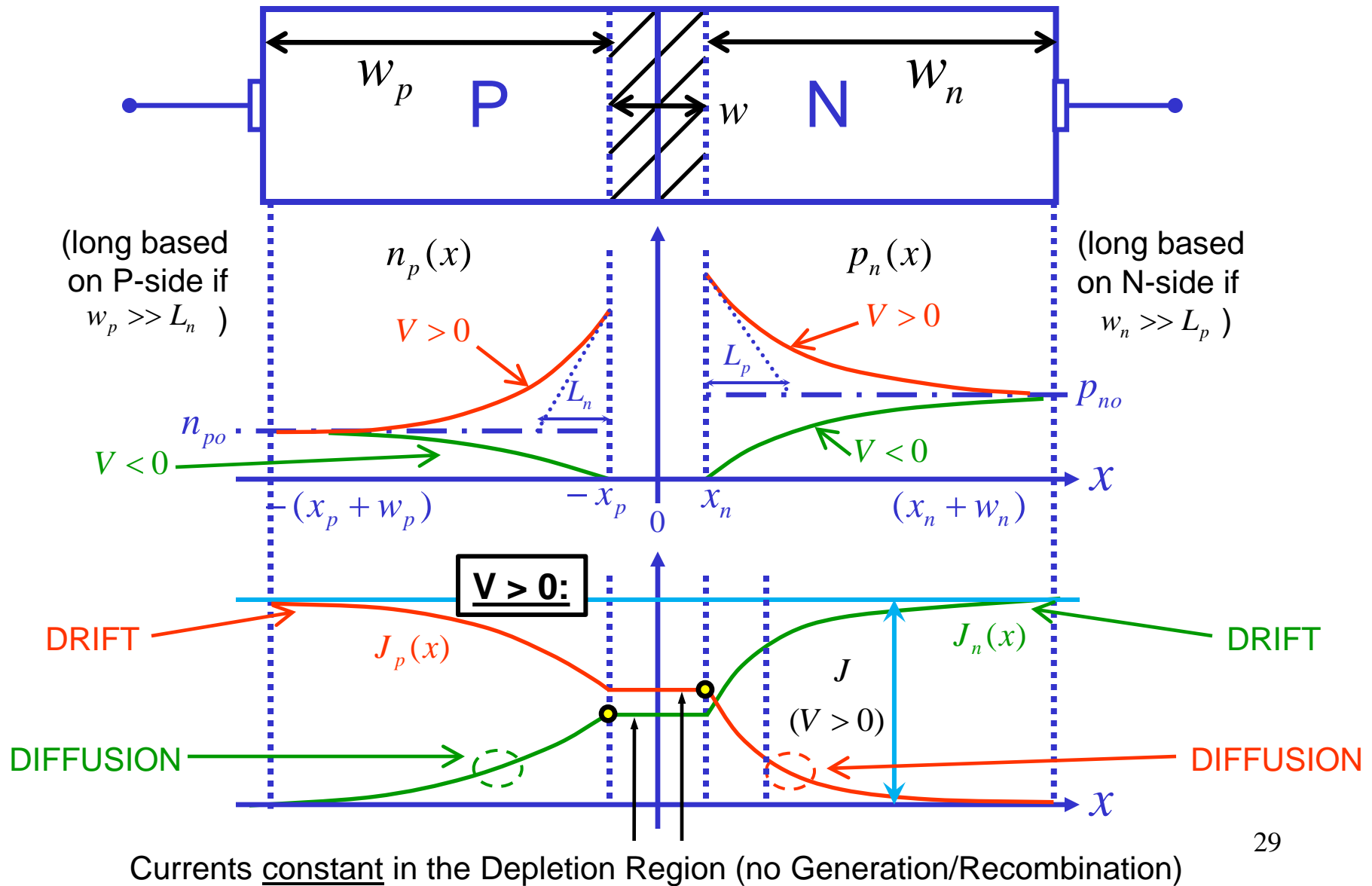
$$L_n = \sqrt{D_n \tau_n}$$

$$\frac{kT}{q} = 0.0259V \text{ at } T = 300K$$

DC Current-Voltage Characteristic of PN Junction (typical values, Si at 300K)



Internal Current Distributions in PN Junction



Temperature Dependence of Current in a PN Junction

- The current-voltage characteristic of a PN junction is quite sensitive to temperature
- This is obvious through the dependence on (qV/kT) but what is less obvious is the exponential increase of the reverse saturation current (I_s) with increasing temperature. This arises through the fact that I_s is directly proportional to the square of the intrinsic concentration n_i :

$$I = I_s \left[e^{qV/kT} - 1 \right]$$

Diagram illustrating the current-voltage characteristic equation of a PN junction. The equation is $I = I_s \left[e^{qV/kT} - 1 \right]$. Annotations include:

- A red arrow points from the text "strongly temp. dependent $\propto n_i^2$ " to the term I_s .
- A red arrow points from the text "obvious" to the term kT in the denominator of the exponent.

- Earlier we showed:

$$n_i = \text{const } T^{3/2} \exp \left[\frac{-\epsilon_g}{2kT} \right]$$

- Note also that I_s will be much smaller in wider bandgap materials

Long-Based/Narrow-Based PN Junction Analysis

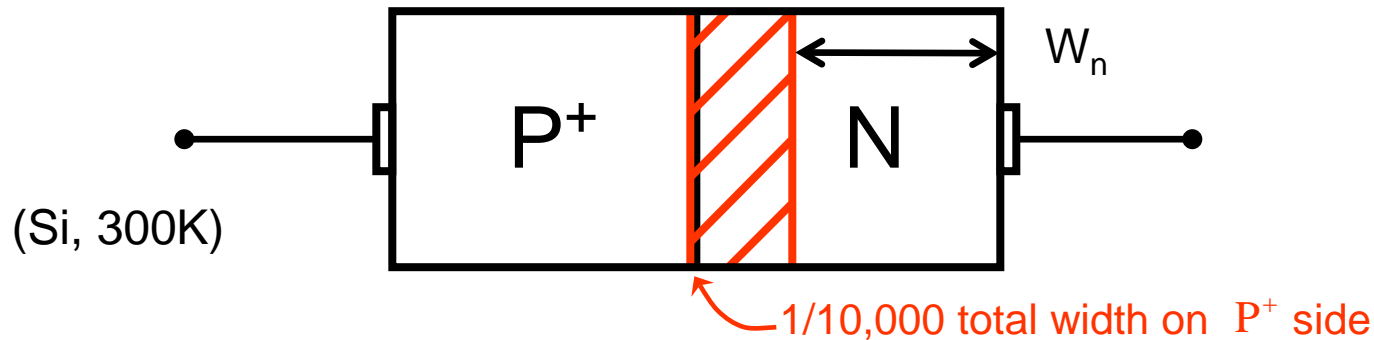
- One of the assumptions made in analysing the PN junction was the device could be considered “long-based” on both sides, meaning that the end-contacts were “far” from the depletion layer edges;
- More precisely, “long-based” means $W_p \gg L_n$ and $W_n \gg L_p$
- The opposite extreme is “narrow-based” whereby $W_p \ll L_n$ and $W_n \ll L_p$; It turns out that the analysis in this case leads to a simple result for I_s : just replace L_p by W_n and by L_n by W_p :

$$I_s = \left[\frac{q D_p p_{no}}{W_n} + \frac{q D_n n_{po}}{W_p} \right]$$

- The intermediate case leads to a much more complicated solution involving hyperbolic functions (cosh, sinh etc).

“One-sided” (Long-Based) PN Junction

- This situation often arises in practical PN structures e.g. we could have a P⁺N junction with $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{14}/\text{cm}^3$:



$$p_{po} = N_A = 10^{18} / \text{cm}^3; \quad n_{po} = \frac{n_i^2}{p_{po}} = \underline{225 / \text{cm}^3}$$

$$\underline{n_{no}} = N_D = 10^{14} / \text{cm}^3; \quad \underline{p_{no}} = \frac{n_i^2}{n_{no}} = \underline{2.25 \times 10^6 / \text{cm}^3}$$

majority

minority

i.e. simpler formula can be used

$$I_s = qA \left[\frac{D_p}{L_p} \underline{p_{no}} + \frac{D_n}{L_n} \underline{n_{po}} \right]$$

negligible

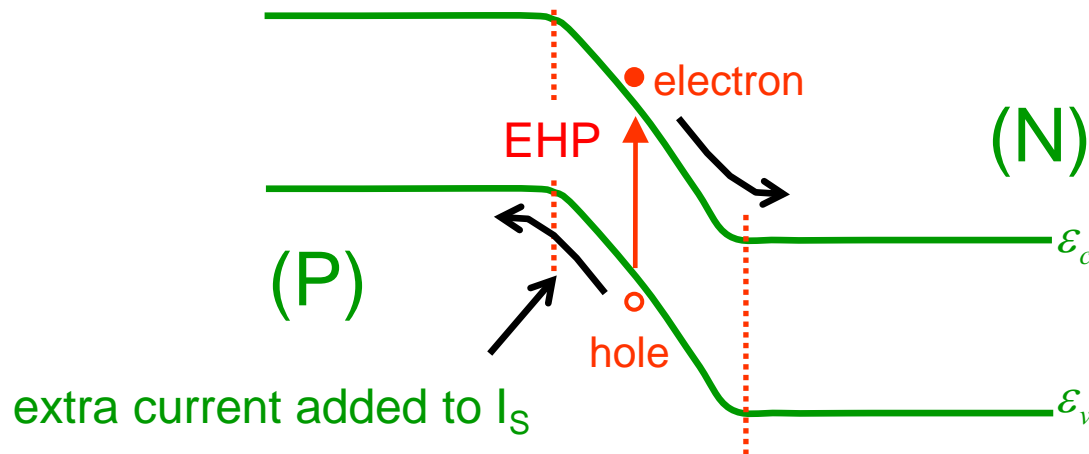
One-Sided (P^+N): $I_s = \frac{qAD_p p_{no}}{L_p} = \left(\frac{qAD_p n_i^2}{\underline{L_p} N_D} \right)$

If one-sided and narrow-based, replace L_p by W_n

EXAMPLE 8.2

Generation and Recombination in the Depletion Layer

- We now re-visit the assumption of zero G-R in the depletion layer. In practice, thermally generated EHPs may be produced in this region, and, because of the intense electric field, they are quickly separated: electrons to the N-side and holes to the P-side. This results in an extra current component added to the saturation current I_s .



- The earlier analysis may be extended to take this into account.

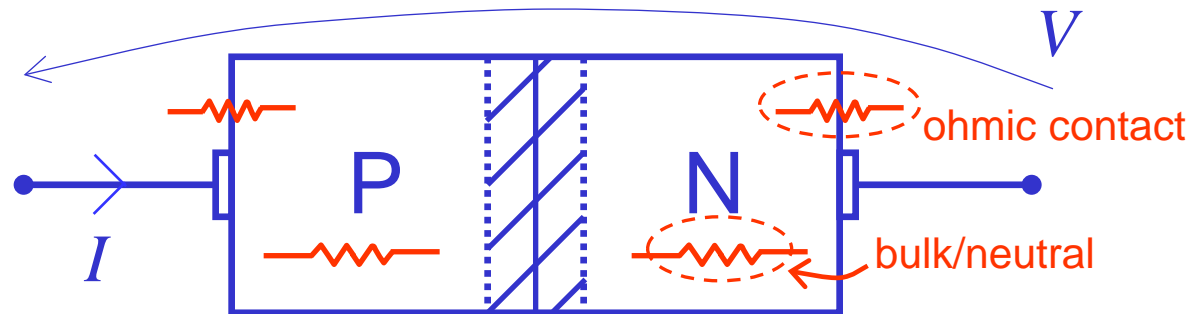
$$I = I_s \left[e^{qV/kT} - 1 \right] + I_{s1} \left[e^{qV/2kT} - 1 \right]$$

don't remember

Extra term due to G-R

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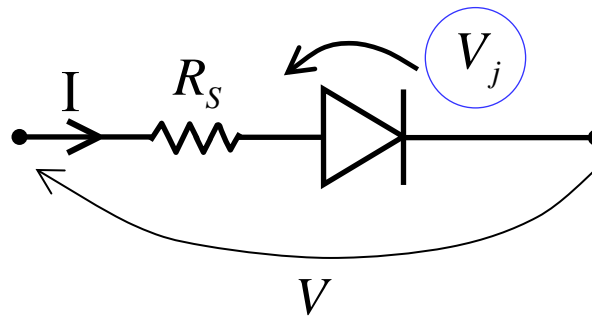
Effect of Parasitic Resistances



- Earlier we assumed zero electric field within each neutral region. In practice, there is 'ohmic' loss due to the finite conductivity of the semiconductor material in each region;
- In addition, the metal-semiconductor contacts are not perfect and this can be modelled by the introduction of a small series resistance at each contact;
- All of these can be lumped together into a single **parasitic resistance R_s** ($\sim 1\Omega$) in series with the 'ideal' PN structure.

DC Equivalent Circuit Model

- A simple **DC equivalent circuit** for the PN junction allowing for parasitic resistance effects can be constructed as follows:



- Although apparently a simple change, the introduction of R_s considerably complicates the task of solving this circuit (non-linear simultaneous algebraic equations involved):

$$I = I_s \cdot \left[e^{\frac{q V_j}{k T}} - 1 \right] \quad V_j = V - I \cdot R_s$$

The Ideality Factor (n)

- In practical PN junctions, it is found that the ideal PN junction equation does a good, but not perfect, job of fitting the DC characteristics;
- A better result is obtained by introducing an empirical 'ideality factor' 'n' (this is a dimensionless number ≥ 1):

$$I = I_S \cdot \left[e^{\left(\frac{qV}{n \cdot kT} \right)} - 1 \right]$$

- In a perfect PN junction, $n=1$, while in a practical well-made Si junction we might find $n = 1.04$, for example;
- The ideality factor n , and saturation current I_S can be determined experimentally by plotting the \log_e of the forward current against the voltage (i.e. $\ln|I|$ vs. V)

Determining 'n' and 'I_s'

- Assume a forward biased PN junction with $V \gg (kT/q)$.
Then we can use the approximation:

$$I \cong I_s \cdot e^{\left(\frac{qV}{nkT}\right)}$$

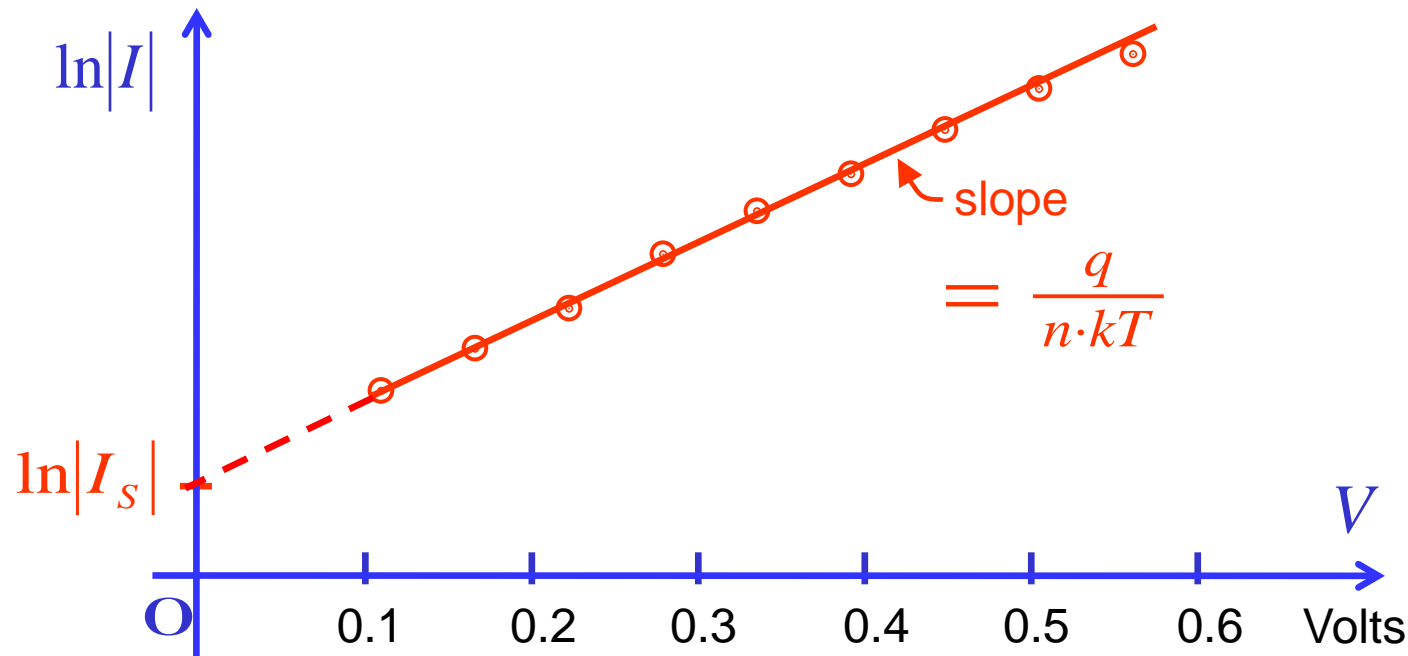
- Take the natural log of both sides:

$$\ln|I| = \ln|I_s| + \frac{qV}{nkT}$$

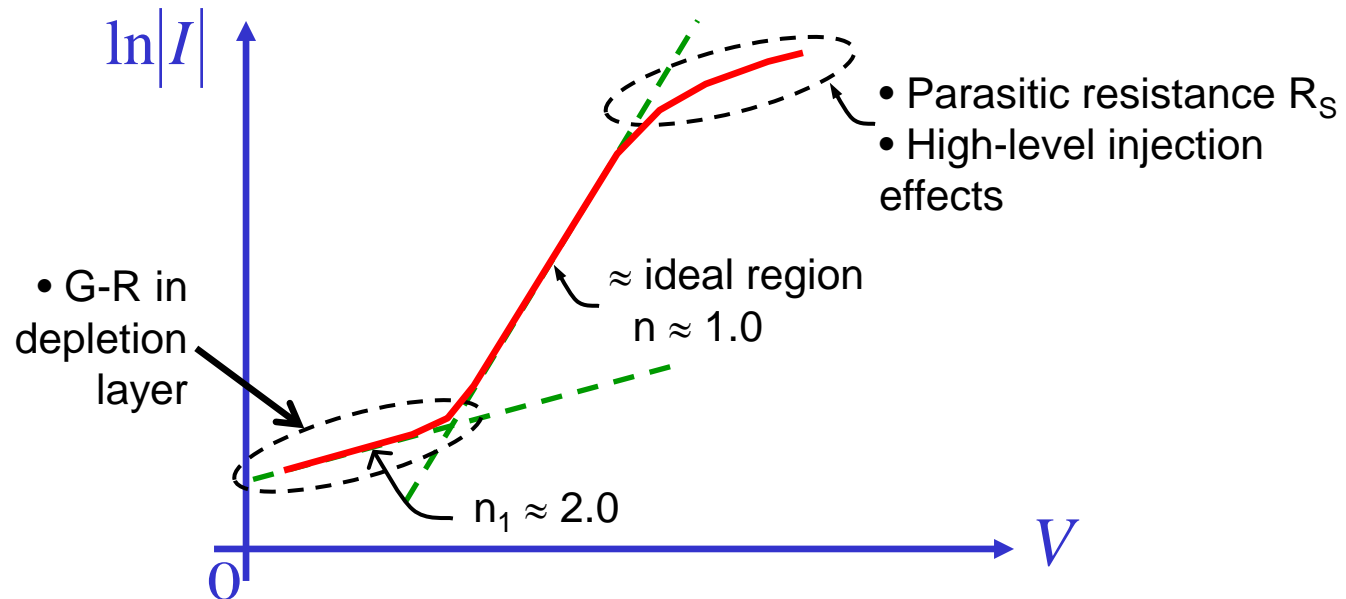
- This in the form of a straight line: “ $y = m.x + c$ ”
where “y” is $\ln|I|$ and “x” is V

Determining 'n' and 'I_s'

- By plotting $\ln|I|$ versus V (for $V > \sim 100\text{mV}$) we can find I_s from the y -axis intersection and then if the temperature T is known, we can estimate n from the slope.



Wide-Range $\ln |I|$ versus V Plot



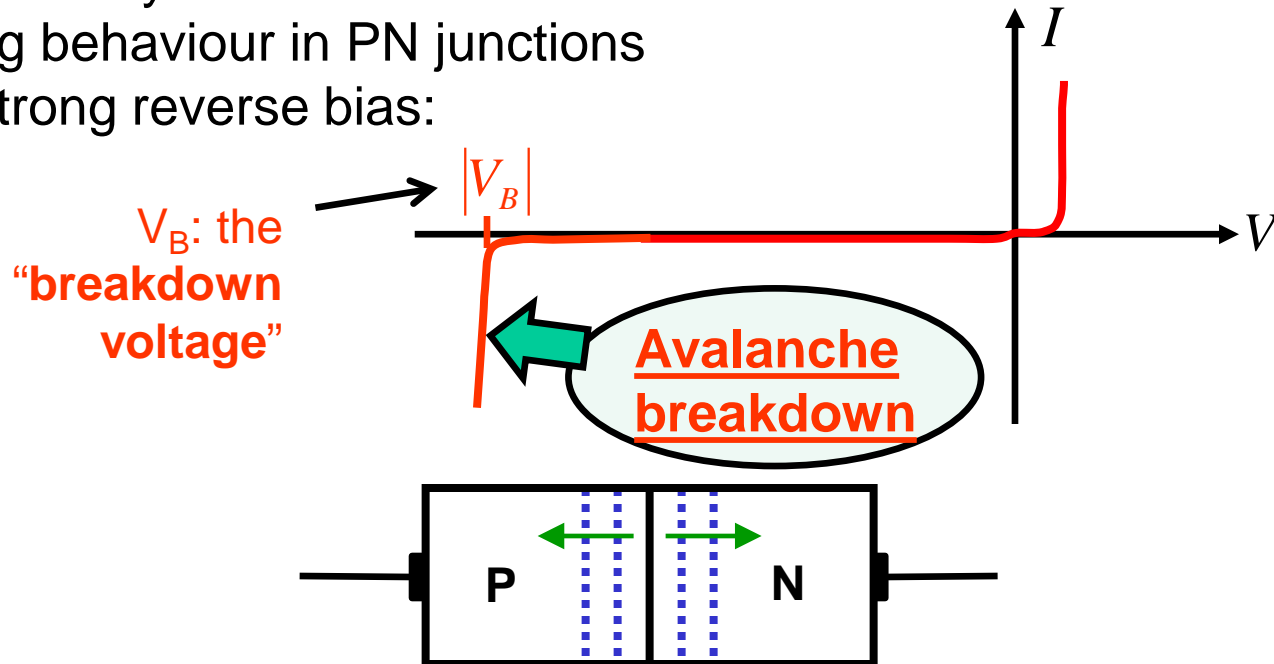
$$I = I_s \cdot \left[e^{\frac{qV_j}{n \cdot kT}} - 1 \right] + I_{s1} \cdot \left[e^{\frac{qV_j}{n_1 \cdot kT}} - 1 \right] \quad V_j = V - I \cdot R_s$$

- Graphical plots like this are very useful for experimental **parameter extraction**, i.e. using measurements to identify critical parameters of device model: the above plot could be used to determine I_s , n , I_{s1} , n_1 , R_s .

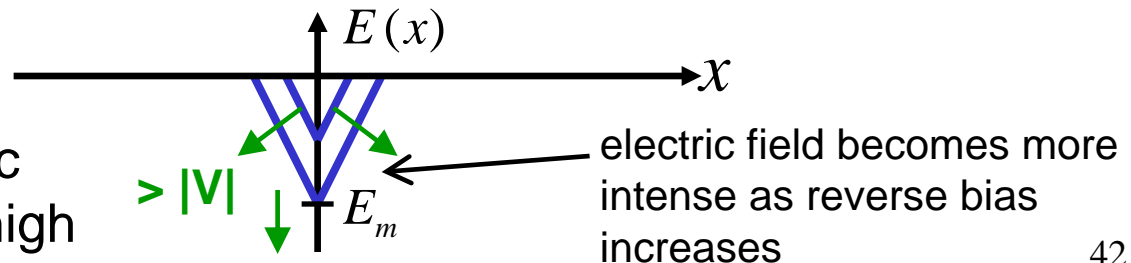
Example 8.3

Impact Ionisation and Avalanche Breakdown in PN Junctions

Experimentally we observe the following behaviour in PN junctions under strong reverse bias:

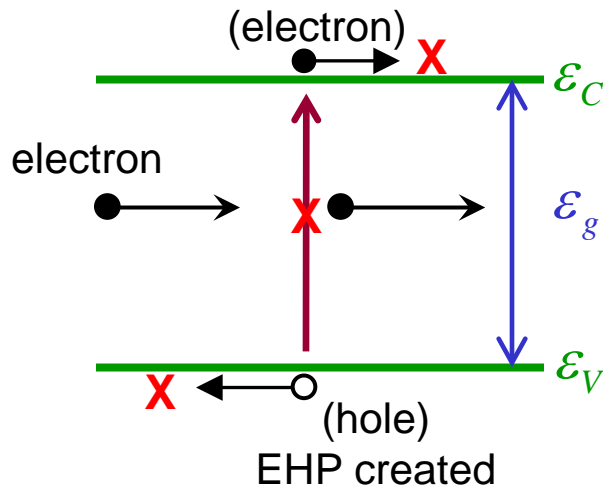


As $|V|$ increases under reverse bias, the maximum electric field becomes very high

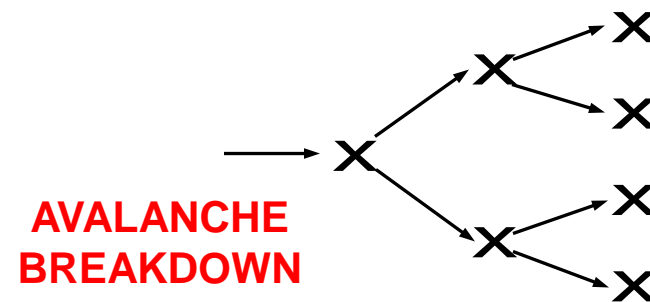


Impact Ionisation and Avalanche Breakdown

- At very high electric fields, the energy acquired by electrons between collisions can become so high that a collision with the lattice can transfer an electron from VB to CB (i.e. an EHP created). This is called **impact ionisation**
- The electron and hole thereby created are also accelerated by the field, and can acquire enough energy to create further EHPs – a chain reaction can then occur leading to a large increase in (reverse) current – **avalanche breakdown**

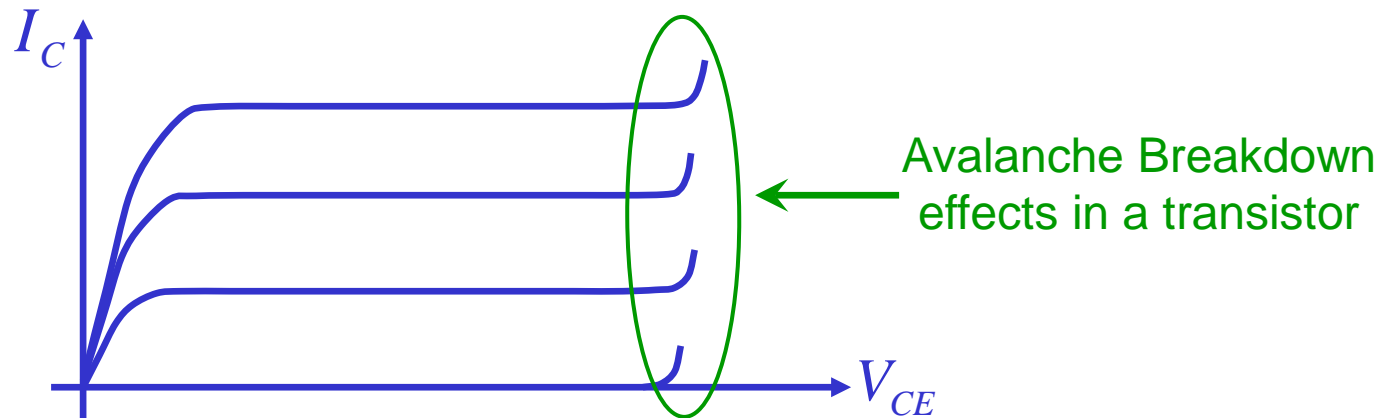


Chain reaction :



Avalanche Breakdown

- A sudden increase in current at high voltages due to avalanche breakdown is very commonly observed in semiconductor devices, e.g. transistor characteristics:



- Although it sounds catastrophic, it need not be if handled carefully and some kinds of device are even operated for years in continuous avalanche breakdown!

The Zener Diode

- When a PN junction is heavily doped on both sides (P^+N^+), the breakdown voltage can become quite low;
- Diodes specifically designed to produce a stable, carefully-controlled reverse breakdown voltage often go under the general name of “Zener Diodes”
- Zener diodes are widely used as voltage references in electronic circuits

