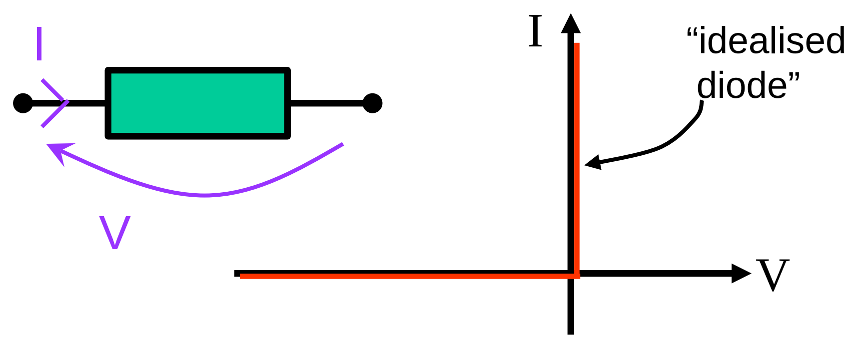
Chapter 7

The PN Junction; Electrostatic Analysis

The PN Junction

- A **PN Junction** is formed when a single crystal of semiconductor is doped P-type in one region and N-type in an immediately adjacent region: the PN junction is the interface surface separating the two;
- An idealised diode can be considered as a two-terminal circuit element with the following highly nonlinear I-V characteristic:



V >= 0: Perfect Short-Circuit (S/C)
V < 0: Perfect Open-Circuit (O/C)

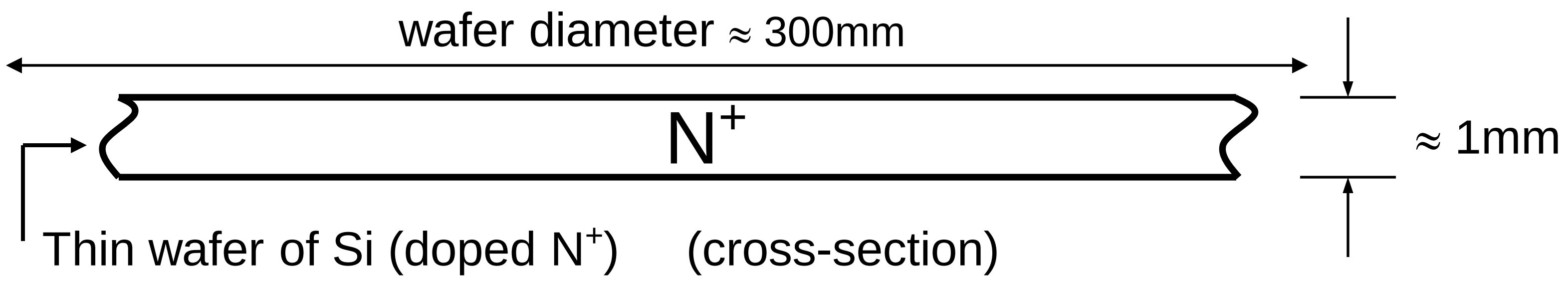
- The PN junction *approximates* this reasonably well, and so is often referred to as a *PN diode* or *junction diode*

Applications of the PN Junction

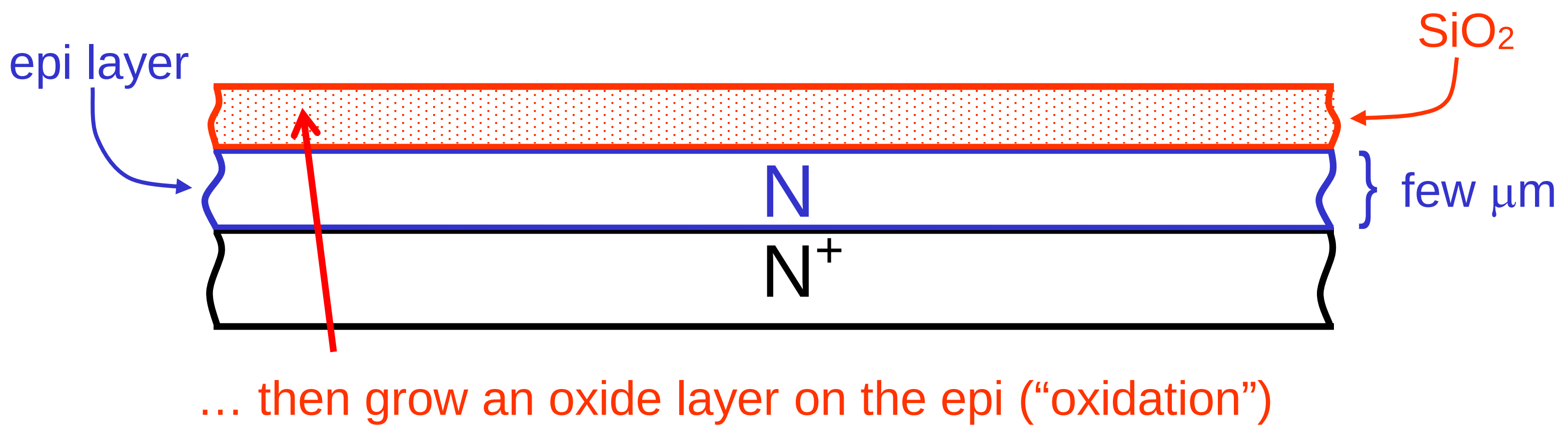
- The PN junction is a very useful component in electrical and electrical engineering. In its basic form it is always passive, that is, it cannot generate signal energy, but it finds many important uses, for example as an electronic switch, as a rectifier, in protection circuitry, for control circuits etc
- A thorough understanding of the PN junction is fundamental to the understanding of many other more advanced semiconductor structures, such as transistors

Fabrication of a (Discrete) PN Junction

(note that very many devices can be made simultaneously on a given wafer: study the "Processing Handout" for more background on the following)

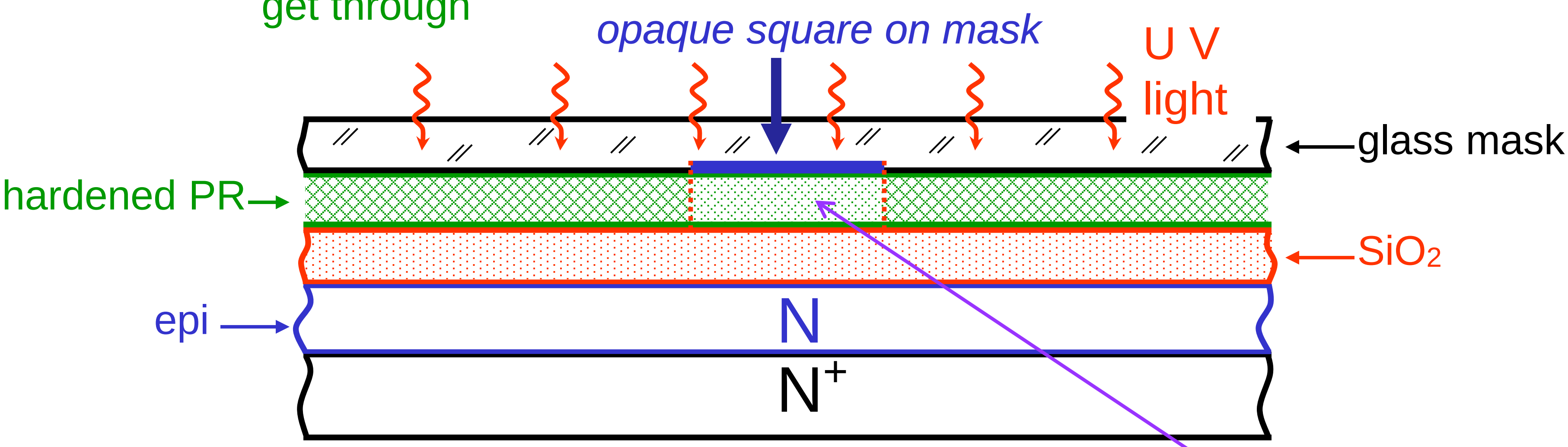


Grow a thin high quality crystal layer on the surface "epitaxial layer" (epi)…



Lithography

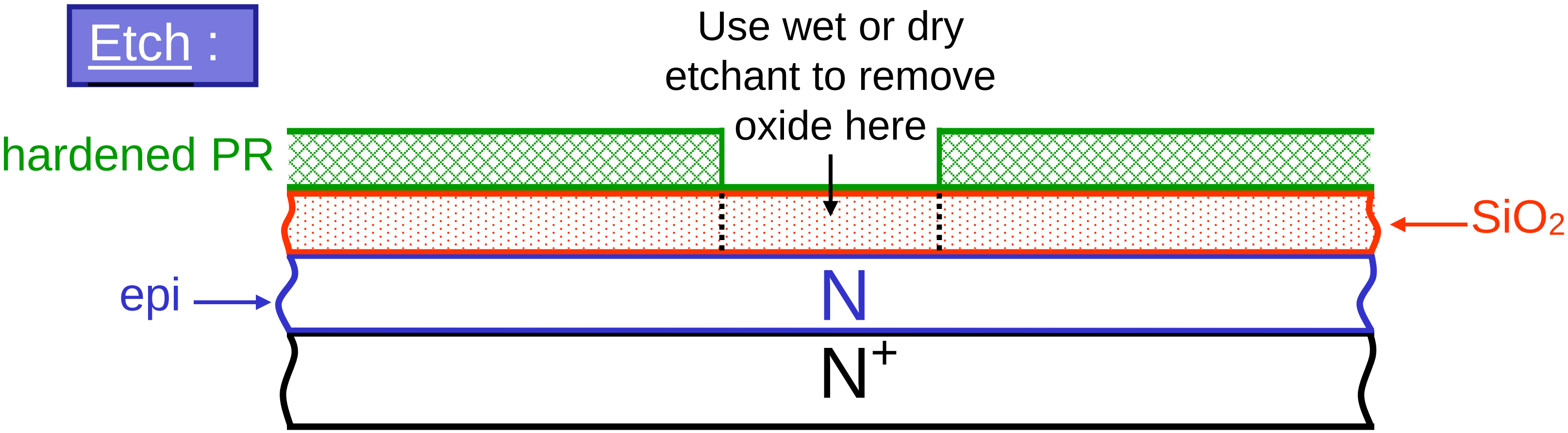
- Lithography involves the transfer of an opaque pattern from an otherwise clear mask to the oxide, i.e. the creation of a set of defined openings in the oxide (the diagram below shows *contact* lithography – *projection* lithography is now more common)
  - Coat surface of oxide with light-sensitive chemical Photoresist (PR)
  - Place mask on top of PR
  - Shine UV light through mask
  - PR hardens in the transparent regions of mask where UV light can get through



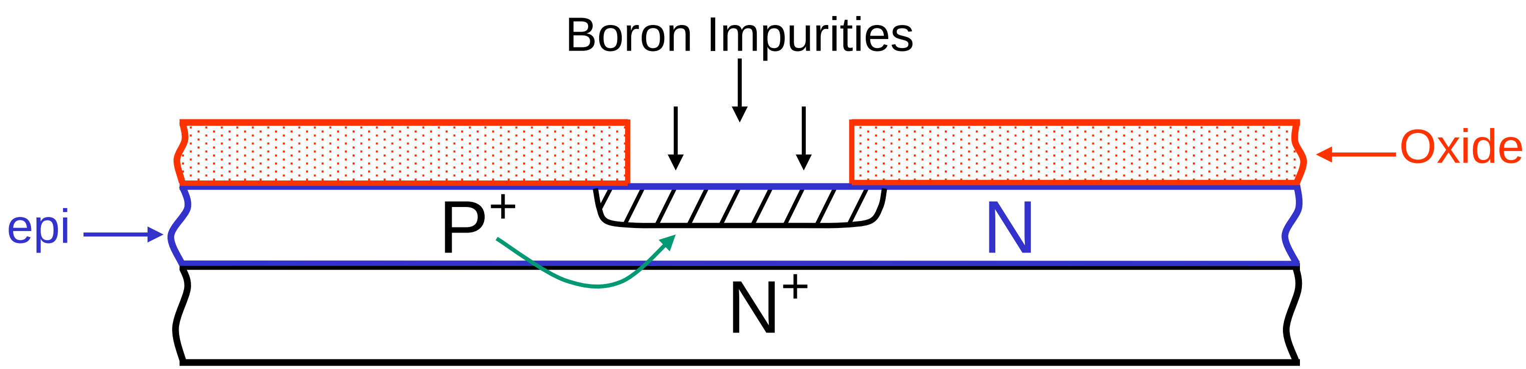
- Now remove mask. Use acid or dry etching to remove unhardened PR

Etching and Diffusion

Etch :

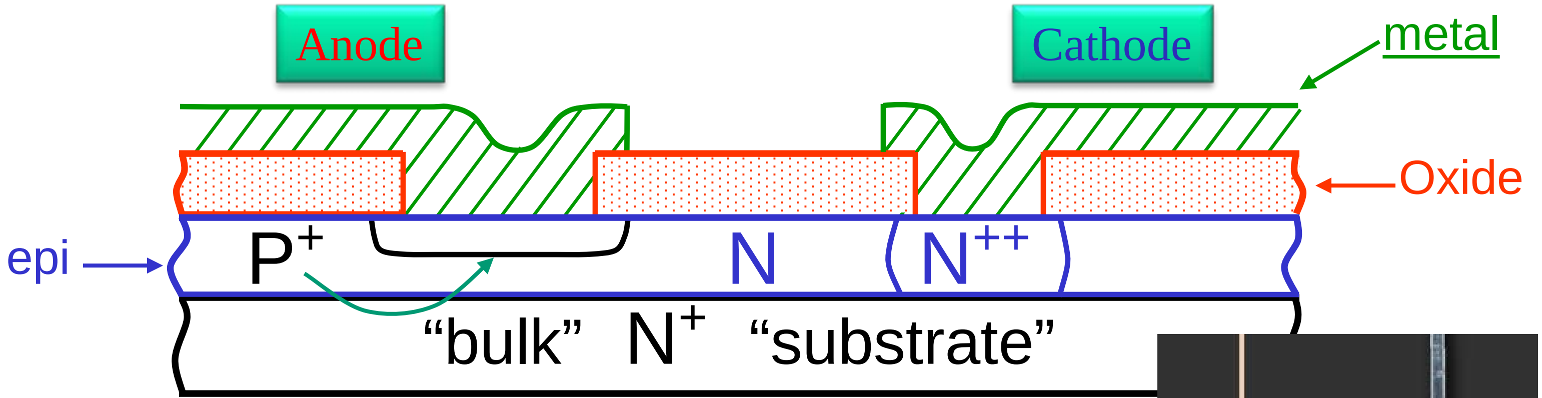


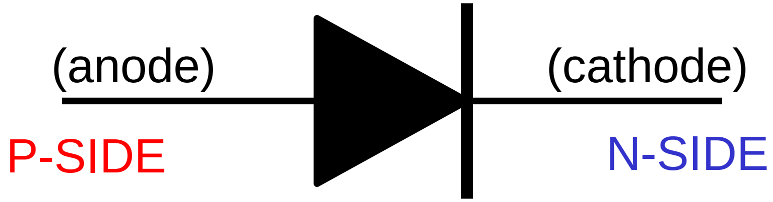
Diffusion : (or else Ion Implantation)



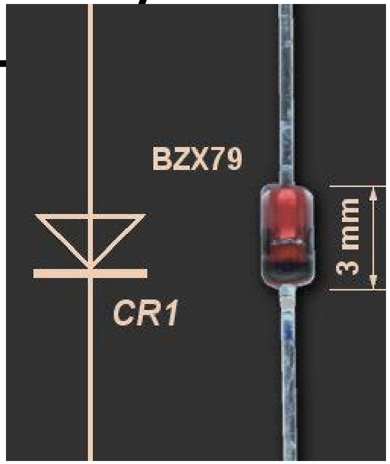
Metallisation and Finish

- Repeat the whole process to create a second diffused region (deep diffusion) to make good electrical contact from the top surface to the substrate
- Pattern metal on the surface to contact the P^+ and N^{++} regions
- Connect this metal to metal leads or wires within a package

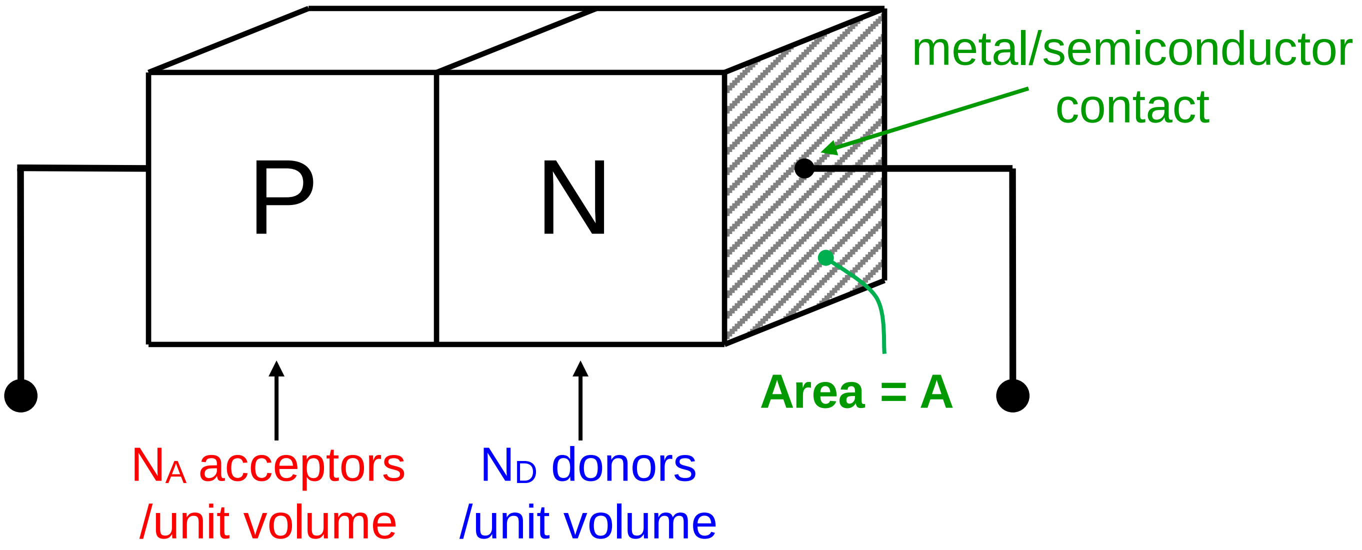




Circuit Schematic Symbol



Ideal 1-D PN Junction Structure



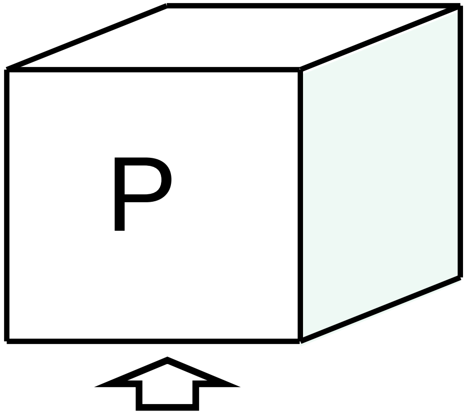
- This is an idealisation of the practical structure for analysis purposes;
- Assume uniform doping in each region (N_A in the P-region and N_D in the N);
- Assume that the junction is abrupt (i.e. the doping switches immediately from P-type to N-type across the junction)
- Assume metal-semiconductor contacts are "ohmic" with ≈ zero resistance
- Assume ohmic end contacts "far" from junction ("long base assumption")

First: Look at Two Separated Regions

Remember, in a semiconductor in thermal equilibrium: $n_o \cdot p_o = n_i^2$

N.B. notation: first "p" subscript indicates that this refers to P-type material

$$n_{po} \cdot p_{po} = n_i^2$$



N_A acceptors/unit volume

- At 300K, all acceptors are ionised.
- The majority HOLE conc. is:

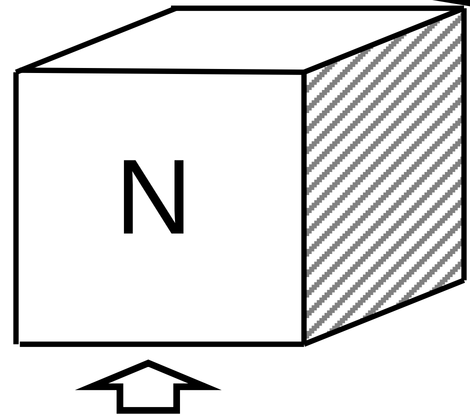
$$p_{po} = N_A$$

- The minority ELECTRON conc. is:

$$n_{po} = \left(\frac{{n_i}^2}{p_{po}}\right) = \left(\frac{{n_i}^2}{N_A}\right)$$

$$n_{no} \cdot p_{no} = n_i^2$$

first "n" subscript indicates that this refers to N-type material



N_D donors/unit volume

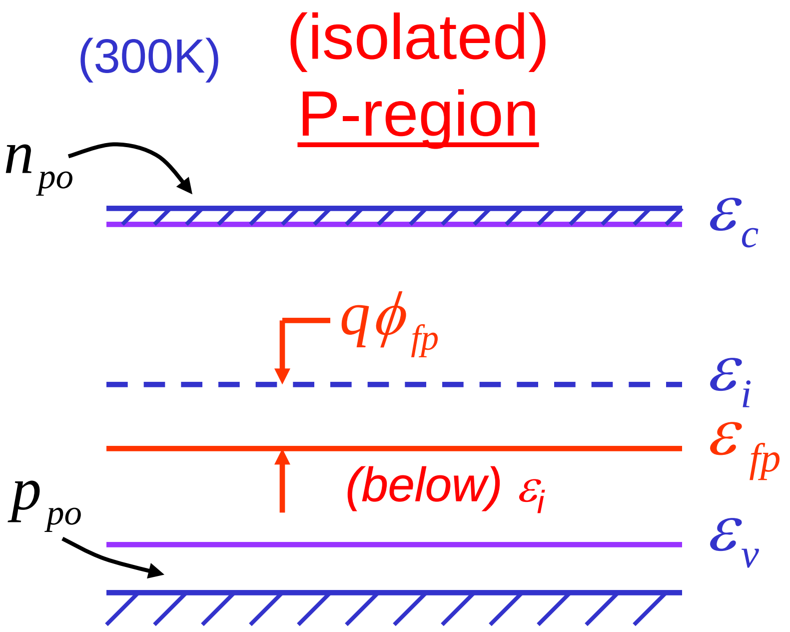
- At 300K, all donors are ionised.
- The majority ELECTRON conc. is:

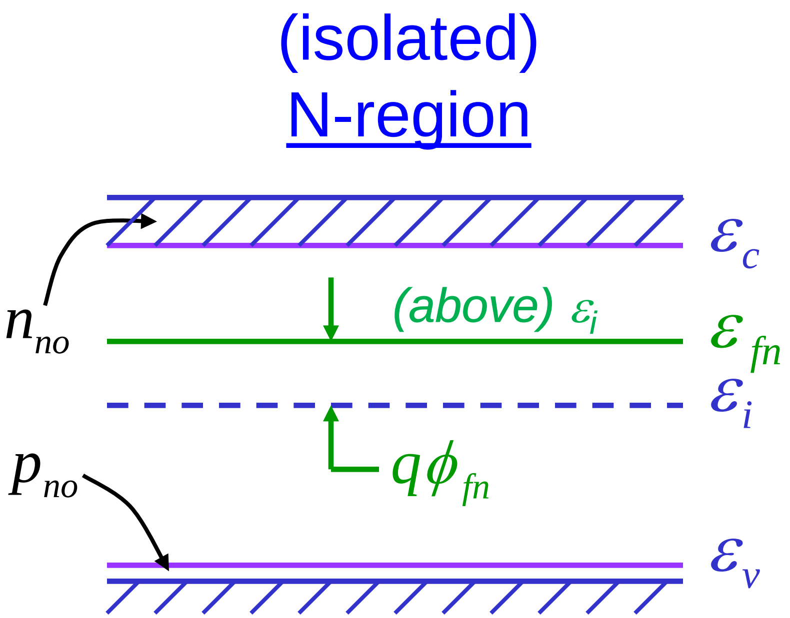
$$n_{no} = N_D$$

- The minority HOLE conc. is:

$$p_{no} = \left(\frac{{n_i}^2}{n_{no}}\right) = \left(\frac{{n_i}^2}{N_D}\right)$$

Isolated P and N in Thermal Equilibrium





p_{po}: (majority) hole concentration in P region in equilibrium

n_{po}: (minority) electron concentration in P region in equilibrium

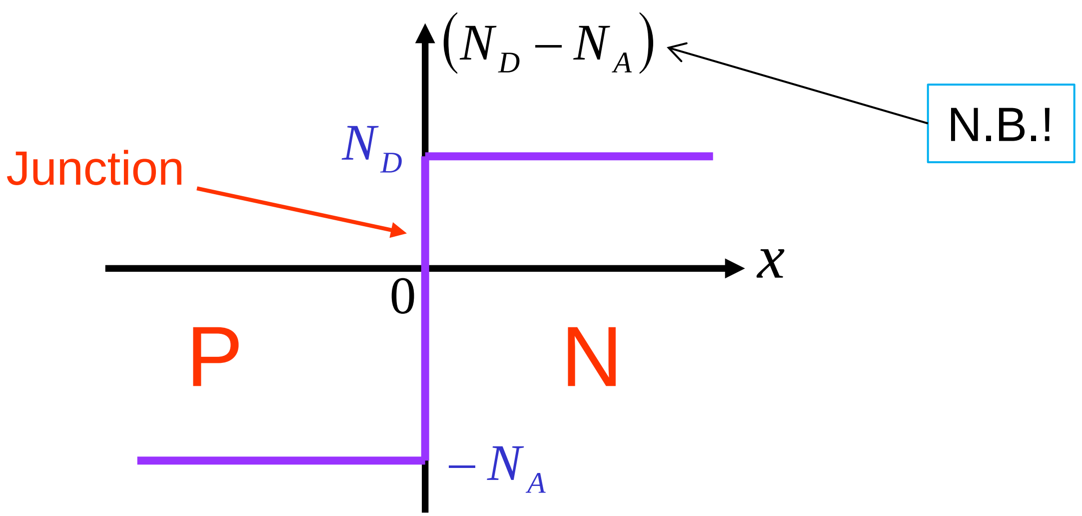
p_{no}: (minority) hole concentration in N region in equilibrium

n_{no}: (majority) electron concentration in N region in equilibrium

Example 7.1

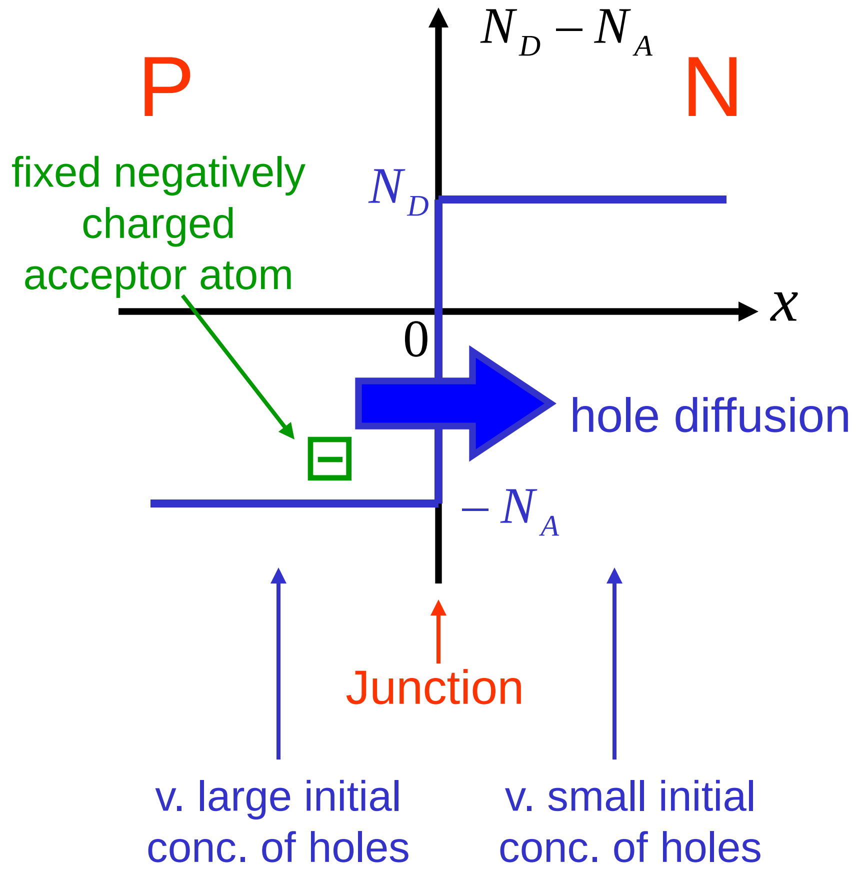
The PN Junction in Thermal Equilibrium

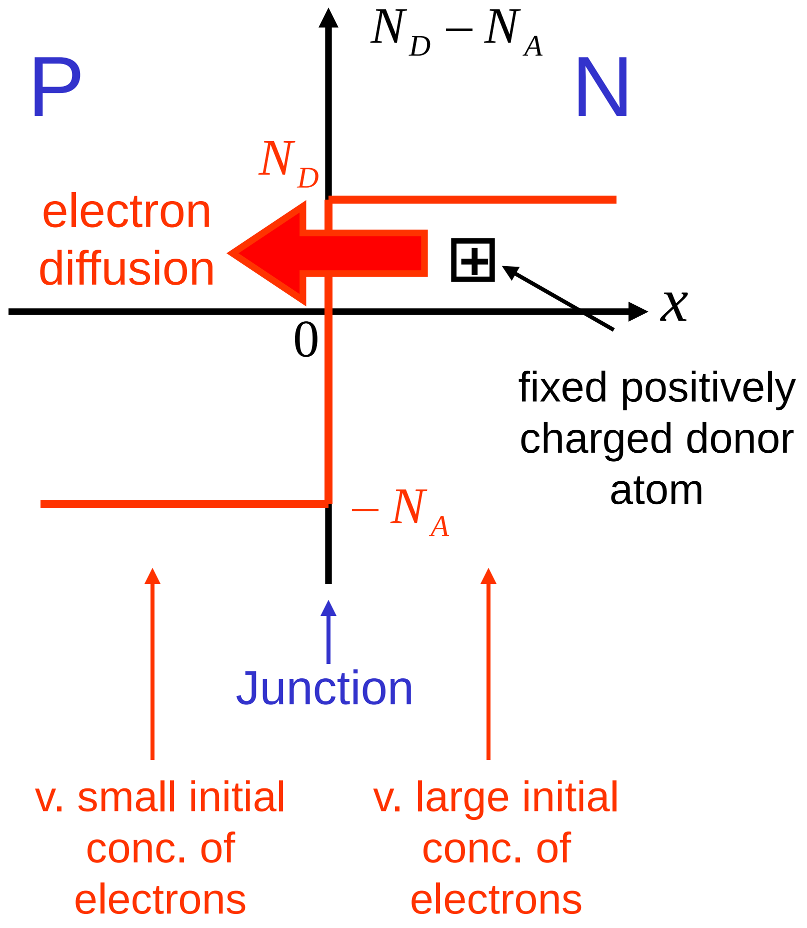
- Consider PN junction a moment after it is 'instantly formed' with the following doping profile at 300K:



- If every donor and acceptor atom were ionised, then there would be a huge concentration imbalance between electrons and holes on each side of the junction. This suggests diffusion would take place to equalise concentrations throughout the structure.
- But every hole that diffuses from P → N leaves behind an (unbalanced) negatively charged acceptor, and every electron going the other way leaves behind a positively charged donor on the N-side.

PN Junction in Thermal Equilibrium

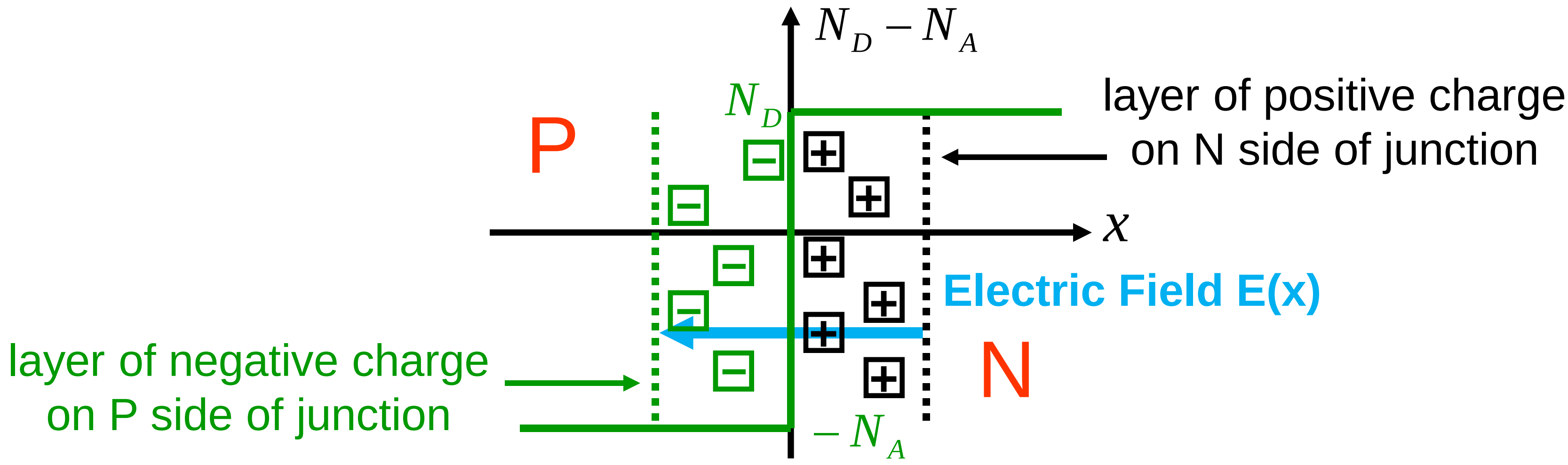


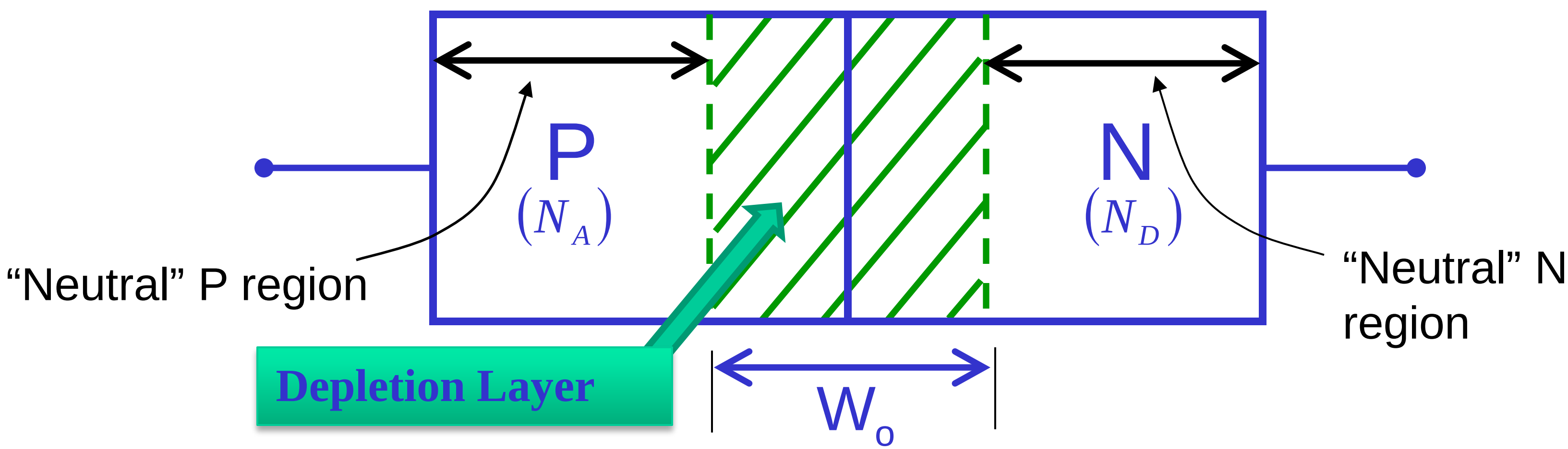


The Depletion Layer

- The result of the initial diffusion is to create a space charge of fixed, ionised impurity atoms near the junction: i.e. a layer of negatively charged ionised acceptors on the P-side and a layer of positively charged ionised donors on the N-side. Together, these are said to constitute the **depletion layer** (width = W)
- The charges set up an electric field between them, in such a direction that it tends to oppose further diffusion.
- In thermal equilibrium there can be no overall current, so the depletion layer grows until the electric field is large enough for the ***drift current*** to exactly cancel the ***diffusion current*** of both electrons and holes (therm. eqm $W = W_o$).

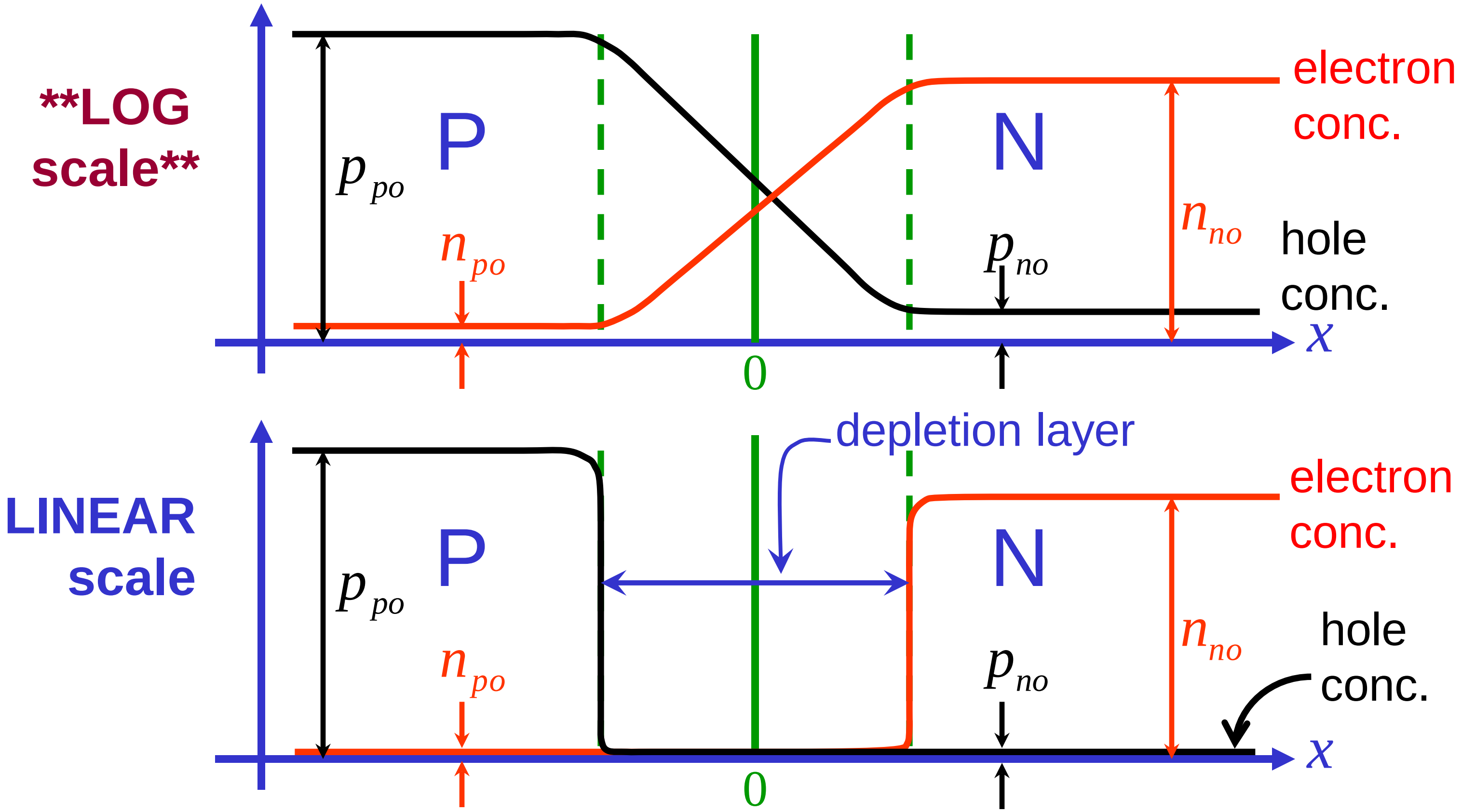
Formation of the Depletion Layer





The Depletion Approximation

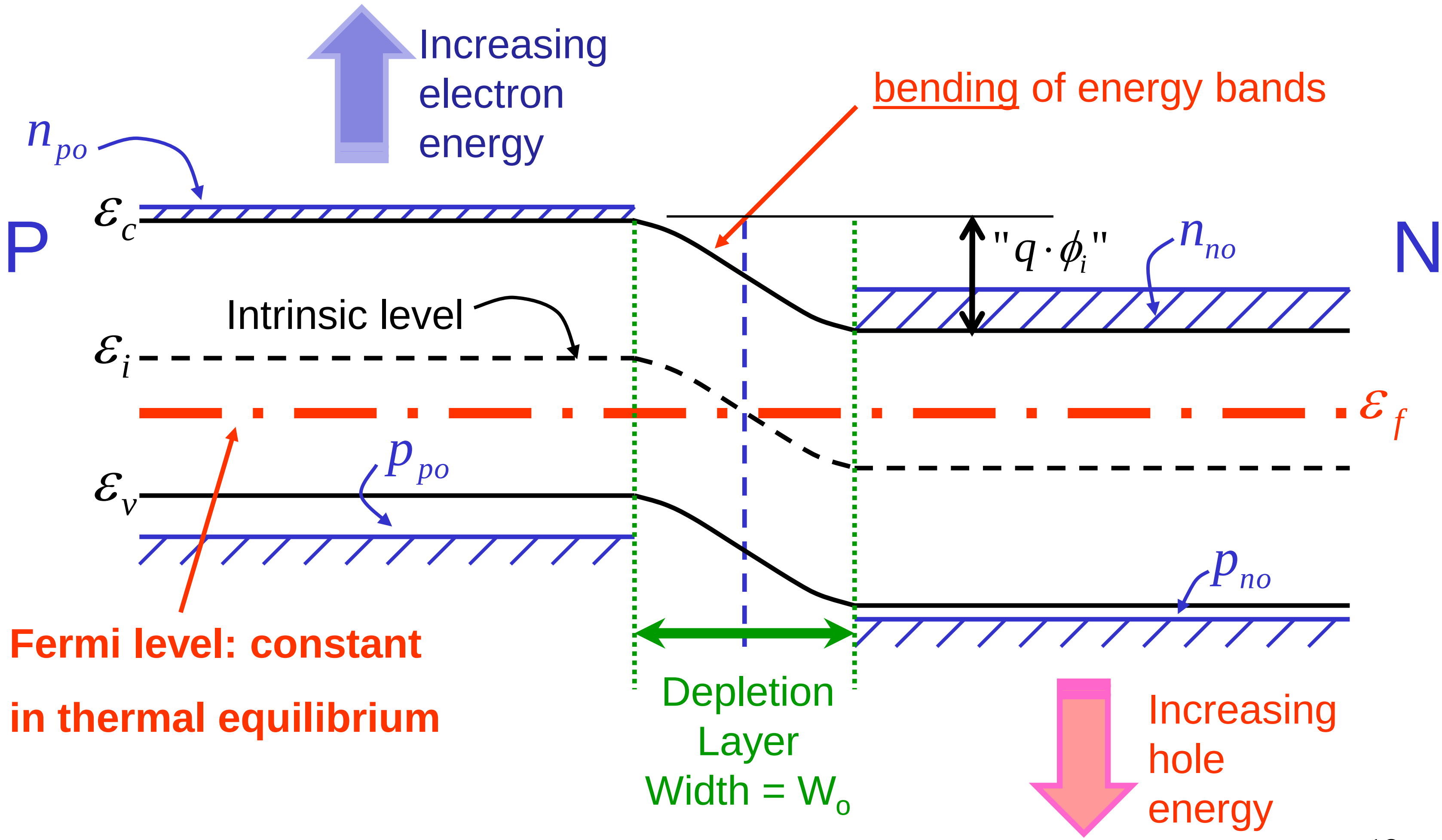
- In practice, the boundaries of the depletion layer are 'blurred' but we will assume that the edges of the layer are sharply defined: this is called the **depletion approximation**. One consequence is that the free carrier concentrations are assumed = 0 within the depletion layer.



Energy Band Picture of PN Junction

- It is very useful to draw an energy band diagram for the PN junction, which we will initially do assuming thermal equilibrium
- The guiding principle is that **the Fermi Level must be everywhere uniform in space in thermal equilibrium**
- This forces a *bending* of the energy bands to take place, which is consistent with the presence of an electric field E(x) within the depletion layer (which also means that the electrical potential V(x) and therefore the energy $\mathcal{E}(x)$ must change in passing through the depletion layer)
- The voltage developed across the depletion layer is called the built-in or contact potential, denoted here by ϕ_i.

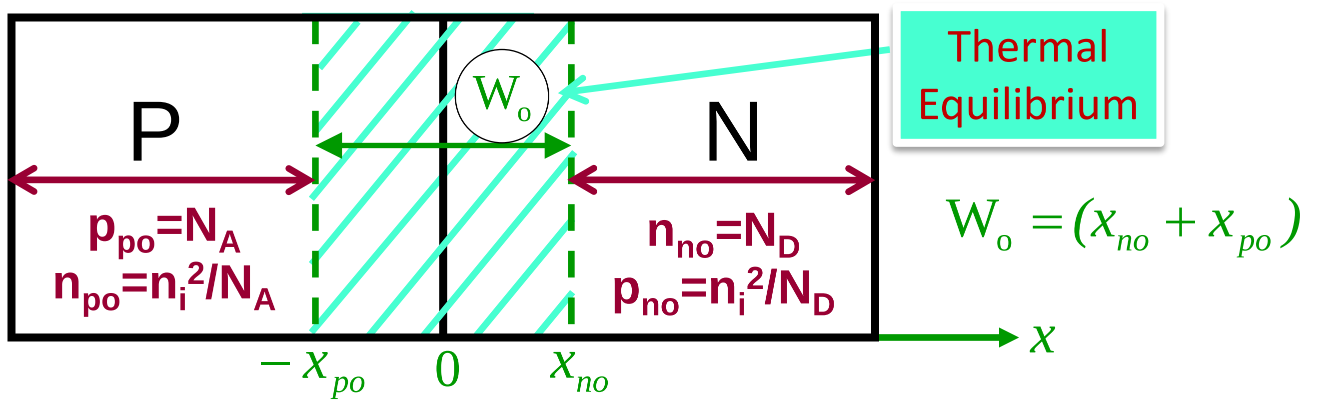
Energy Band Diagram for PN Junction in Thermal Equilibrium



See: http://jas.eng.buffalo.edu/education/pn/pnformation_B/index.html

Semiconductor Electrostatics

- This refers to the solution of Poisson's Equation in the depletion layer, leading to expressions for the electric field E(x) and electric potential V(x)



Poisson Equation: $$\frac{dE(x)}{dx} = \frac{\rho(x)}{\varepsilon}$$

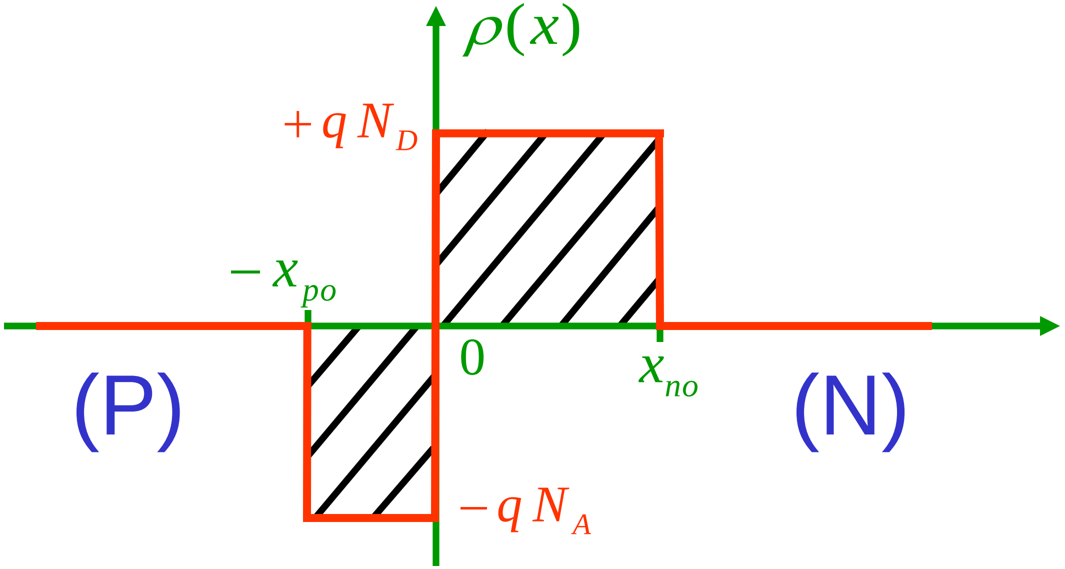
Where $\rho(x)$ = concentration of charge (Coulombs/m^3)

$\rho(x) = 0$ in the neutral regions $\Rightarrow E(x) = 0$ in the neutral regions

To solve for for $E(x), x \in [-x_{po}, x_{no}]$ we need to know $\rho(x)$

Solution for Electric Field E(x) on P-Side

The charge density function $\rho(x)$*:*



On P side : $x \in [-x_{po}, 0] \quad \rho(x) = -q\,N_A \quad Coul/m^3$

$$\boxed{\frac{dE(x)}{dx} = -\frac{q\,N_A}{\varepsilon}}$$

Poisson's Equation

Solution for Electric Field E(x) on P-Side

ε = permittivity = $\varepsilon_r \cdot \varepsilon_o$

ε_r = relative permittivity (dielectric constant) = 11.8(Si) 12.7(GaAs)

ε_o = permittivity of free space = $8.85 \times 10^{-12} F/m$

$$\int_{-x_{po}}^{x} \frac{dE(x)}{dx} dx = -\int_{-x_{po}}^{x} \frac{q N_A}{\varepsilon} dx$$

$$\Rightarrow E(x) - \underbrace{E(-x_{po})} = -\frac{q N_A}{\varepsilon}(x + x_{po})$$

= 0 at the boundary of depletion layer
(because it must be zero in the neutral region)

<u>P-side</u> : $E(x) = -\frac{q N_A}{\varepsilon}(x + x_{po})$ Linear function of position, negative in value, reaching maximum magnitude at x=0 (i.e. at the junction)

$$E_m = E(0) = -\frac{q N_A x_{po}}{\varepsilon}$$

Solution for Electric Field E(x) on N-side

On N-side : $x \in [0, x_{no}]$ $\rho(x) = +q\,N_D$

$$\int_0^x \frac{dE(x)}{dx}\,dx = +\int_0^x \frac{q\,N_D}{\varepsilon}\,dx$$

$$\Rightarrow E(x) - \underbrace{E(0)}_{=E_m} = \frac{q\,N_D}{\varepsilon}(x) \qquad E(x) = E_m + \frac{q\,N_D\,x}{\varepsilon}$$

But $E(x_{no})$ <u>must</u> = 0

$$0 = E_m + \frac{q\,N_D\,x_{no}}{\varepsilon} \qquad \Rightarrow 0 = -\frac{q\,N_A\,x_{po}}{\varepsilon} + \frac{q\,N_D\,x_{no}}{\varepsilon}$$

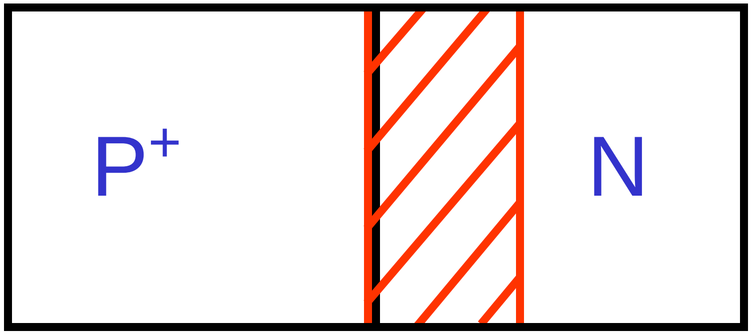
$$\Rightarrow \boxed{N_A\,x_{po} = N_D\,x_{no}}$$

i.e. the product of doping level by depletion layer penetration is equal on both sides of a PN junction. **Important Result.**

Example 7.2

Example of "1-Sided" PN Junction

$$N_A = 10^{18}/cm^3 \quad N_D = 10^{15}/cm^3$$



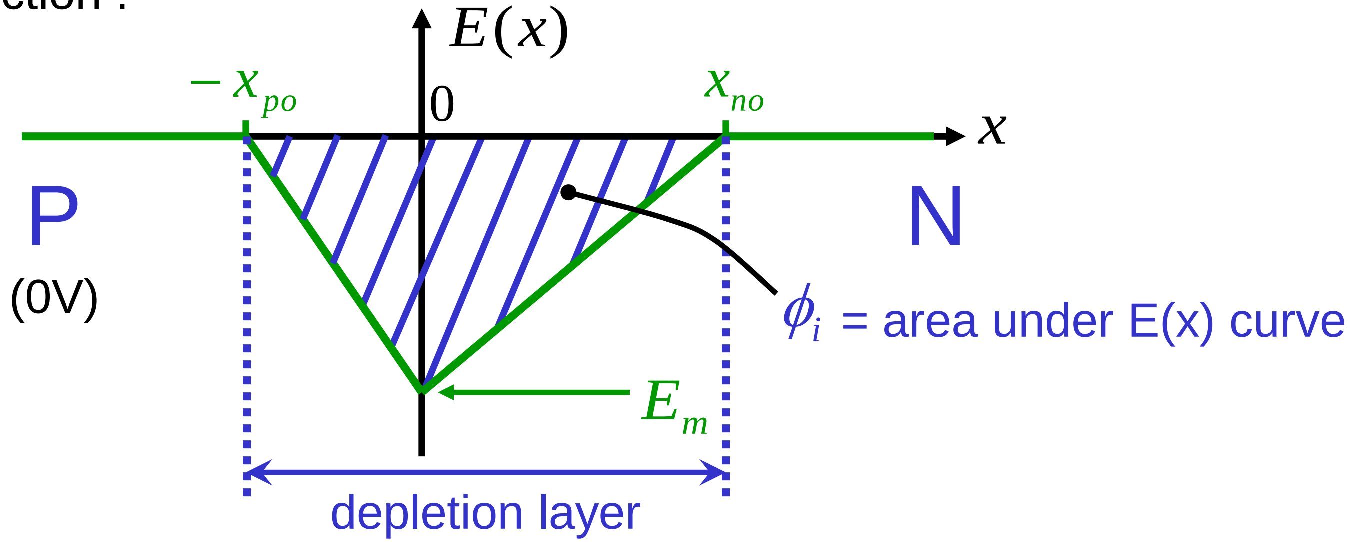
one-sided
(P^+N) junction

$$\Rightarrow x_{po} = \frac{1}{1000} x_{no} \ !$$

Note that the depletion layer is wider in the more lightly-doped region

E(x) and Solution for the Potential V(x)

- From earlier results we can sketch variation in Electric Field across junction :



Voltage is defined with reference to electric field

$$\boxed{-\frac{dV(x)}{dx} = E(x)}$$

V(x) is the electric potential or "voltage" at position "x"

As reference, we will take the potential of the P-side neutral region as "0V"

Solution for the Potential V(x) on P-Side

Note : $\phi_i \;=\; V(x_{no}) - V(-x_{po}) \;=\; -\int_{-x_{po}}^{x_{no}} E(x)dx$

i.e. area under E(x) curve

P-side : want to calculate $V(x),\ x \;\in\; [-x_{po}, 0]$

$$\frac{dV(x)}{dx} = -E(x)$$

$$\int_{-x_{po}}^{x} \frac{dV(x)}{dx} dx = -\int_{-x_{po}}^{x} E(x)dx$$

$$V(x) - \cancelto{0}{V(-x_{po})} \;=\; +\int_{-x_{po}}^{x} \frac{qN_A}{\varepsilon}(x + x_{po})\,dx$$

Show (exercise) : $V(x) \;=\; +\dfrac{qN_A(x + x_{po})^2}{2\varepsilon}$

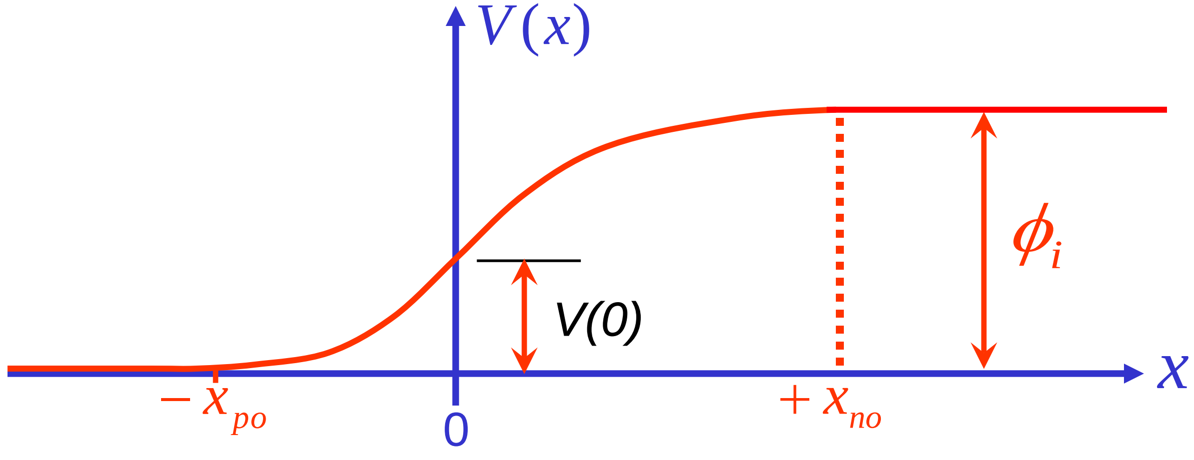
Solution for the Potential V(x) on N-Side

N-side : Want to calculate $V(x),\ x \in [0, x_{no}]$

$$\Rightarrow V(x) - V(0) = -\int_0^x E(x)\, dx$$

We found earlier that $E(x) = E_m + \dfrac{qN_D x}{\varepsilon}$ $\Rightarrow V(x) = V(0) - E_m x - \dfrac{qN_D x^2}{2\varepsilon}$

Where $V(0) = \dfrac{qN_A x_{po}{}^2}{2\varepsilon}$ (from P-side solution)



Formulas for Depletion Layer Penetration on P-Side and N-Side

We found earlier that : $E_m = -\dfrac{qN_A x_{po}}{\varepsilon} = -\dfrac{qN_D x_{no}}{\varepsilon}$ Hence, on the N-side, V(x) is:

$$V(x) = \underbrace{\frac{qN_A x_{po}^2}{2\varepsilon}}_{V(0)} + \underbrace{\frac{qN_D x_{no} \cdot x}{\varepsilon}}_{-E_m \cdot x} - \frac{qN_D x^2}{2\varepsilon}$$

But : $V(\underline{x_{no}}) = \phi_i$

equal

$$\Rightarrow \phi_i = \frac{qN_A^2 x_{po}^2}{2\varepsilon N_A} + \frac{qN_D x_{no}^2}{2\varepsilon}$$

$$\Rightarrow \phi_i = \underbrace{\frac{qN_D^2 x_{no}^2}{2\varepsilon N_A} + \frac{qN_D x_{no}^2}{2\varepsilon}}$$

$$\Rightarrow \underline{x_{no}} = \sqrt{\frac{2\varepsilon \phi_i N_A}{qN_D(N_A + N_D)}}$$

prove as (exercise)

We can then get x_{po} from : $\underline{x_{po} = \dfrac{N_D x_{no}}{N_A}}$

Formula for Total (Equilibrium) Depletion Layer Width

Just add formulas for x_{po} and x_{no} to get formula for W_o:

$$W_o = x_{po} + x_{no} \qquad\qquad W_o = x_{no}\left[\frac{N_D}{N_A} + 1\right]$$

Exercise: Substitute for x_{no}. Show :

$$\boxed{W_o = \sqrt{\frac{2 \cdot \varepsilon \cdot \varphi_i}{q} \cdot \left(\frac{N_A + N_D}{N_A \cdot N_D}\right)}}$$

Do not remember.
Formula in handout sheet

Expression for Contact Potential ϕ_i

Use the fact that drift and diffusion currents must **cancel** in equilibrium . e.g:

$$J_{po}(x) = \underline{J_{po}^{drift}(x)} + J_{po}^{diff}(x) = 0 \quad \forall\, x \in [-x_{po},\ x_{no}]$$

$$\Rightarrow \underbrace{\cancel{q}\mu_p\, p(x) E(x)}_{\sigma_p} - \cancel{q} D_p \frac{dp(x)}{dx} = 0$$

$$\left(\text{Einstein}: \ D_p = \frac{kT}{q}\mu_p\right)$$

$$\Rightarrow E(x) = \left(\frac{D_p}{\mu_p}\right) \frac{1}{p(x)} \frac{dp(x)}{dx}$$

But $\phi_i = -\int_{-x_{po}}^{x_{no}} E(x)\,dx = -\left(\frac{kT}{q}\right) \int_{-x_{po}}^{x_{no}} \frac{1}{p(x)} \cdot \frac{dp(x)}{dx} dx$

$$= -\frac{kT}{q} \int_{p(-x_{po}) = p_{po}}^{p(x_{no}) = p_{no}} \frac{dp}{p}$$

Expression for Contact Potential ϕ_i

So $$\phi_i = -\frac{kT}{q}\int_{p_{po}}^{p_{no}}\frac{dp}{p} = -\frac{kT}{q}\left[\ln|p|\right]_{p_{po}}^{p_{no}}$$

$$= -\frac{kT}{q}\ln\left|\frac{p_{no}}{p_{po}}\right| = \frac{kT}{q}\ln\left|\frac{p_{po}}{p_{no}}\right|$$ ...using: -ln|x| = ln|1/x|

$$\left[\Rightarrow \quad p_{po} = p_{no}\exp\left[\frac{q\,\phi_i}{kT}\right] \quad \underline{\underline{or}} \quad \underline{p_{no} = p_{po}\exp\left[-\frac{q\,\phi_i}{kT}\right]}\right]$$

(Note : this is a useful result which we will use later)

Now :
$$\boxed{\phi_i = \frac{kT}{q}\ln\left|\frac{p_{po}}{p_{no}}\right|}$$

But :
$$\underline{p_{po} = N_A} \quad (T > 100K) \qquad \underline{p_{no} = \frac{n_i^2}{N_D}}$$

Expression for Contact Potential ϕ_i

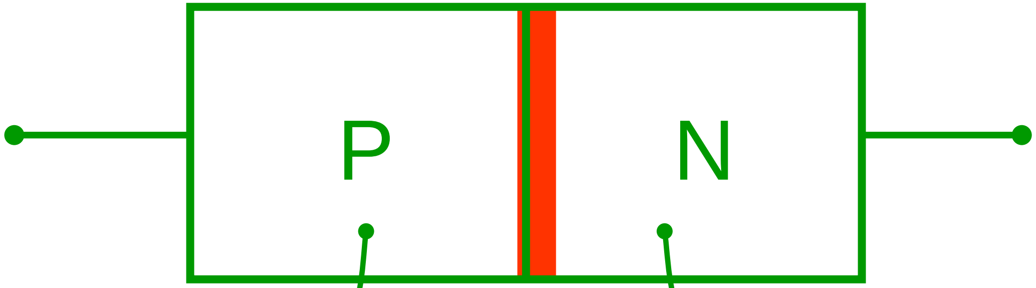
$$\Rightarrow \boxed{\phi_i = \frac{kT}{q} \ln\left|\frac{N_A \cdot N_D}{n_i^2}\right|}$$ (Units of Volts)

... hence, knowing the doping concentrations in the P- and N- regions of an ideal PN junction, we can calculate the *built-in* or *contact potential ϕ_i* at a given temperature T using this formula

For relevant applets, see:
http://jas.eng.buffalo.edu/education/pn/pnformation2/pnformation2.html
http://jas.eng.buffalo.edu/education/pn/pnformation3/index.html

Worked Example of PN Junction Analysis

Ideal Silicon PN junction at 300K:



$N_A = 10^{16} / cm^3$

$N_D = 10^{15} / cm^3$

What is the contact potential ? *using formula…*

$$\phi_i = 0.0259 \ln\left|\frac{10^{31}}{(1.5\times10^{10})^2}\right|(V) = \underline{0.66\,V}$$

Worked Example: cont'd

How far does depletion layer extend on N-side ?

$$x_{no} = \sqrt{\frac{2\varepsilon\,\phi_i\,N_A}{q\,N_D(N_A + N_D)}}$$

do not remember: this formula will be given if needed

* Warning : Be careful with units in these calculations. "ε" involves distance

Safest approach : convert all distances to metres.

$$x_{no} = \left[\frac{\overbrace{2\times\overset{\varepsilon_r}{11.8}\times\overset{\varepsilon_o}{8.85\times10^{-12}}}^{\varepsilon}\times\overset{N_A}{10^{22}}\times\overset{\phi_i}{0.66}}{\underset{q}{1.6\times10^{-19}}\times\underset{N_D}{10^{21}}\times\underbrace{1.1\times10^{22}}_{N_A+N_D}}\right]^{1/2}$$

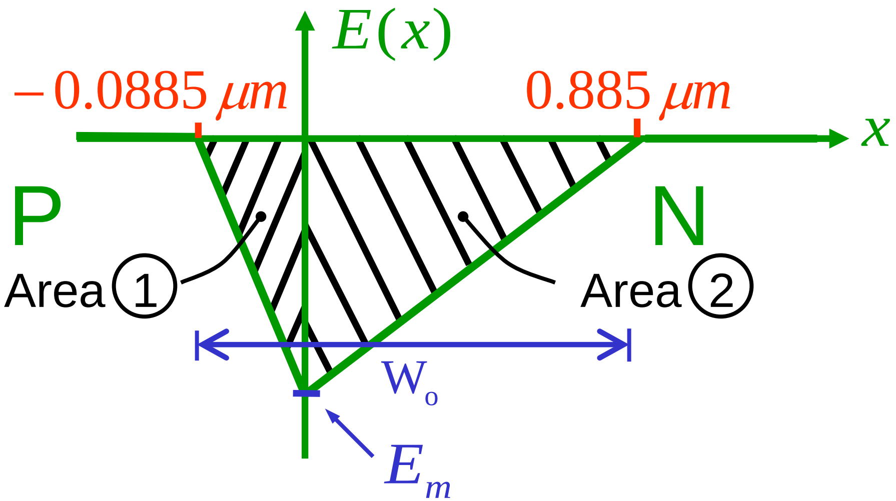
$$= \underline{0.885\,\mu m}$$

$\underline{\underline{or}}$ use cm with $\varepsilon_o = 8.85\times10^{-14}\ F/cm$

Worked Example: cont'd

How far does depletion layer extend on P-side ?

$$x_{po} = \frac{N_D}{N_A} \cdot x_{no} = \underline{0.0885 \mu m}$$



Exercise:

Use formula for W_o to show that

$$W_o = 0.973\,\mu m = x_{po} + x_{no}$$

√

What is maximum value of the electric field (E_m)?

$$E_m = -\frac{q N_A x_{po}}{\varepsilon}$$

Worked Example: cont'd

Use "cm" here for practice:

$$E_m = -\frac{1.6\times10^{-19}\times10^{16}\times0.0885\times10^{-4}}{11.8\times8.85\times10^{-14}} = \underline{-13.6\,kV/cm}$$

$$(or\ \ -1.36\,MV/m)$$

Note :

Area 1: $\frac{1}{2}\,|x_{po}|\,|E_m| = \underline{0.06\ V}$

Area 2: $\frac{1}{2}\,|x_{no}|\,|E_m| = \underline{0.60\ V}$

i.e. we can work out potentials from geometry, using areas under E(x) distributions

Add: 1 + 2 = total voltage across depletion layer = 0.66 V = ϕ_i

√ - *agrees with previous result*

Why is the Built-In Voltage ϕ_i Not Visible Externally?

- At first sight it seems that the existence of a potential difference (of ≈1V) across the depletion layer in equilibrium violates conservation of energy. Could we not use this ‘free’ voltage e.g. to light a bulb?
- In practice, if we try to make a circuit, small voltage drops are found to exist at both of the end metal-semiconductor contacts which exactly cancel out the built-in voltage.
- This hints that the detailed physics of metal-semiconductor contacts is quite complex in practice, but we will simply ignore these end-contact effects in our continuing discussion of the PN junction...