Electrical Energy Systems

EEEN 20090

Transformers (Part 2)

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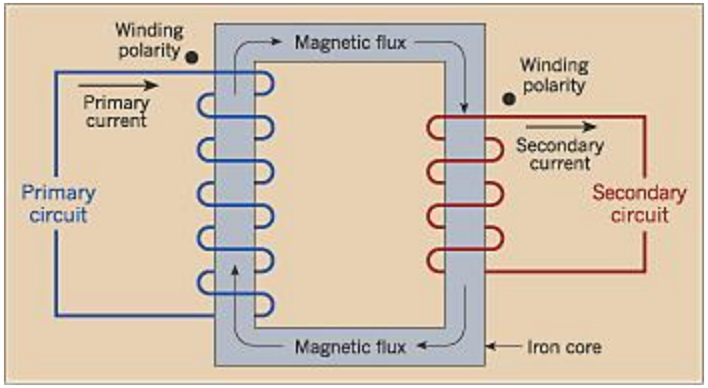
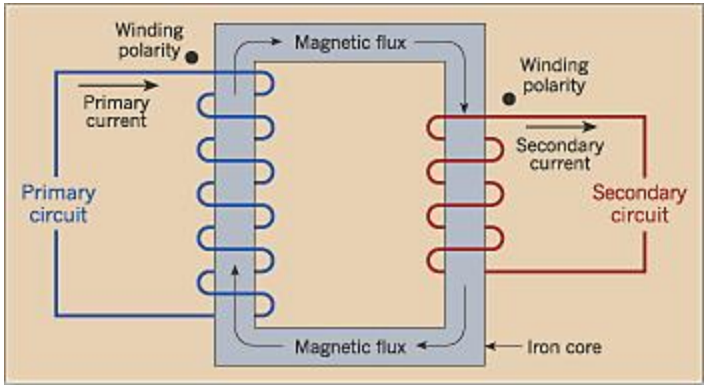
Feb 2015



Model of “real” transformer

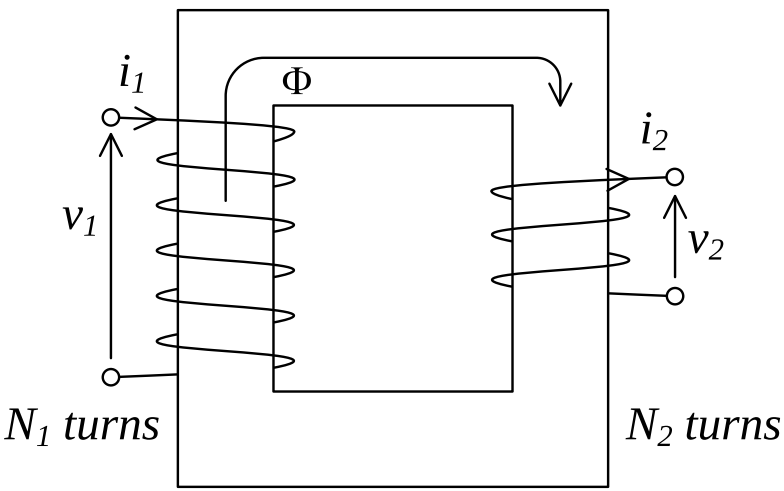
Real Transformer

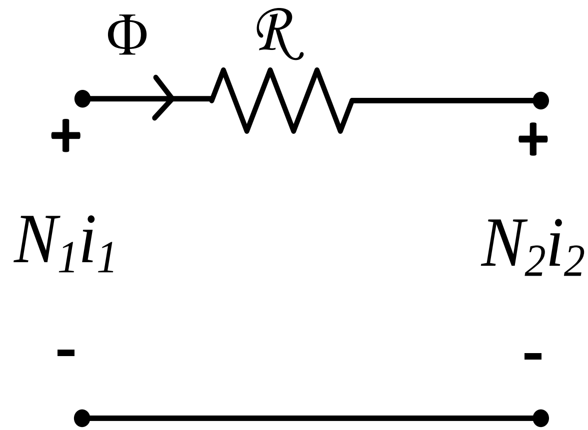
- Ideal model, is not a sufficiently accurate for many applications.
- A more accurate model must consider the following effects:
  - permeability μ is not infinity
  - Losses
    - Magnetic losses in the core
    - Non-zero winding resistance



Transformer with Magnetizing Inductance

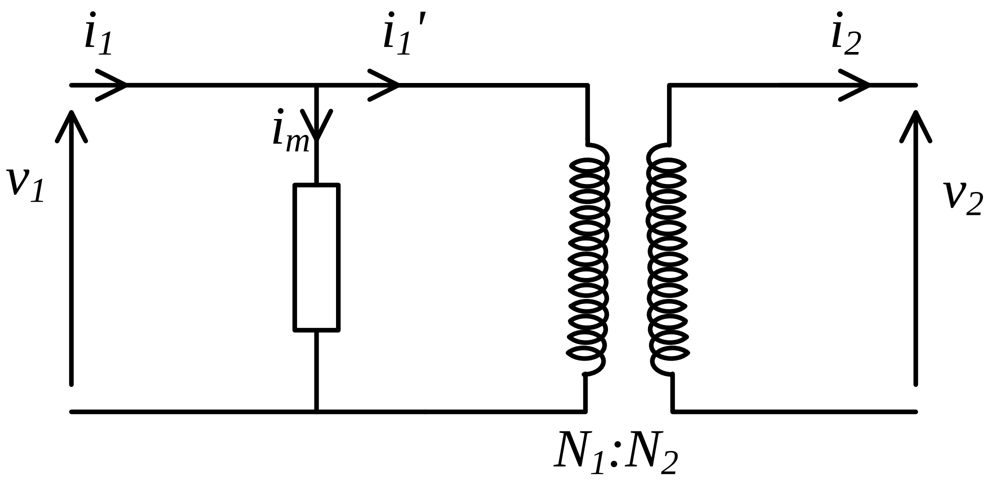
Magnetising Inductance





Ideal Transformer current relationship – magnetic circuits mmf1 = mmf2 but now there is an extra piece i.e Non Zero reluctance

Equivalent Circuit



$$N_1 i_1 - N_2 i_2 = \Phi \mathcal{R}$$

$$\therefore i_1 = \frac{N_2}{N_1} i_2 + \boxed{\frac{\Phi \mathcal{R}}{N_1}}$$

i_m=*Magnetising Current*

$$i_1 = i_1' + i_m$$

Exercise: What are the units of $\boxed{\frac{\Phi \mathcal{R}}{N_1}}$

Self Inductance

Apply Faraday Law to the terminals

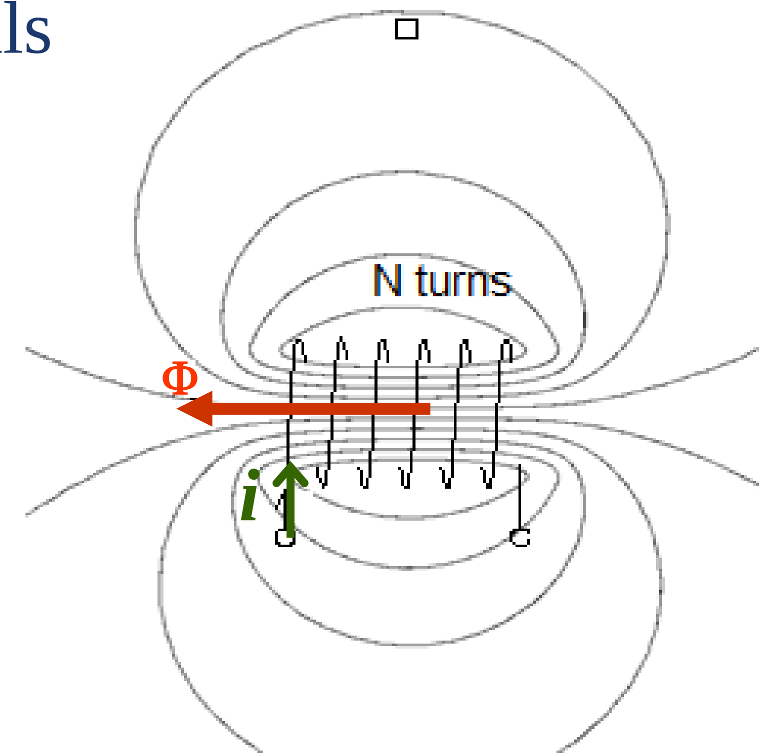
$$v(t) = N\frac{d\Phi}{dt} = \frac{d\psi}{dt}$$

$$= \frac{N^2 \mu A}{l}\frac{di}{dt} = L\frac{di}{dt}$$

L is known as the self inductance

units are H (Henry)

$$L = \frac{N^2 \mu A}{l} \text{ and } \psi = Li$$

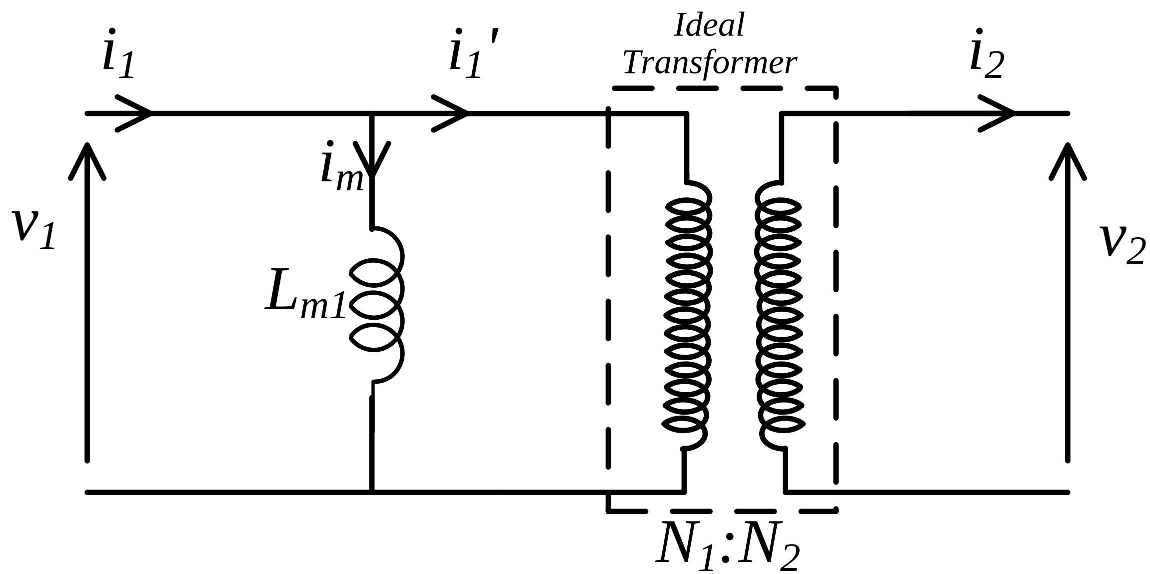


air-cored $\mu = \mu_0 \longrightarrow$ very small L

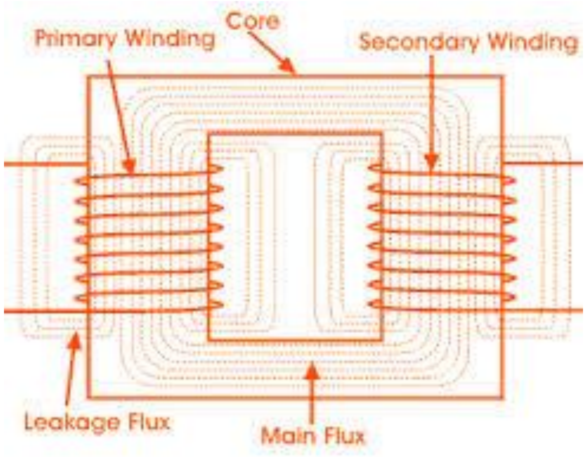
$\uparrow$ L by improving the permeability

iron-cored $\mu_I >> \mu_0 \longrightarrow \mu_I \simeq 1000\mu_0 \longrightarrow$ large L

Magnetising Inductance



$$\boxed{L_{m1} = \frac{\mu_0 \mu_r N_1{}^2 A}{l}}$$

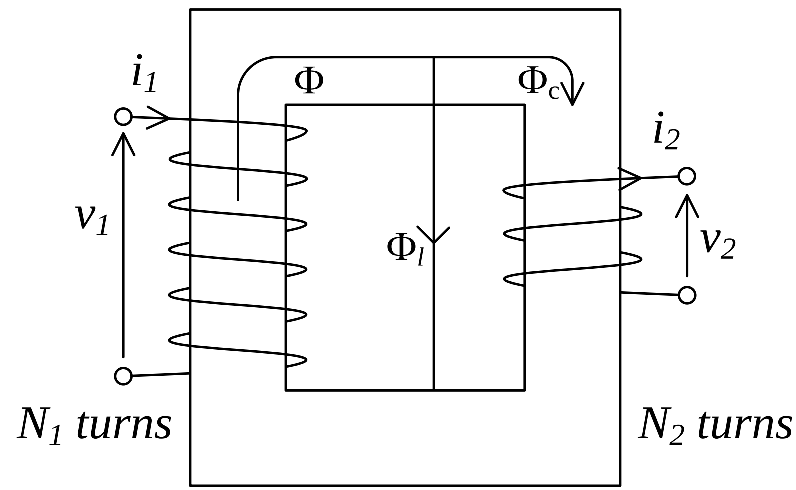


Leakage Flux

Leakage Flux

- As $\mu \neq \infty$ some Φ will *leak* into the surrounding material & hence will not link the secondary coil.

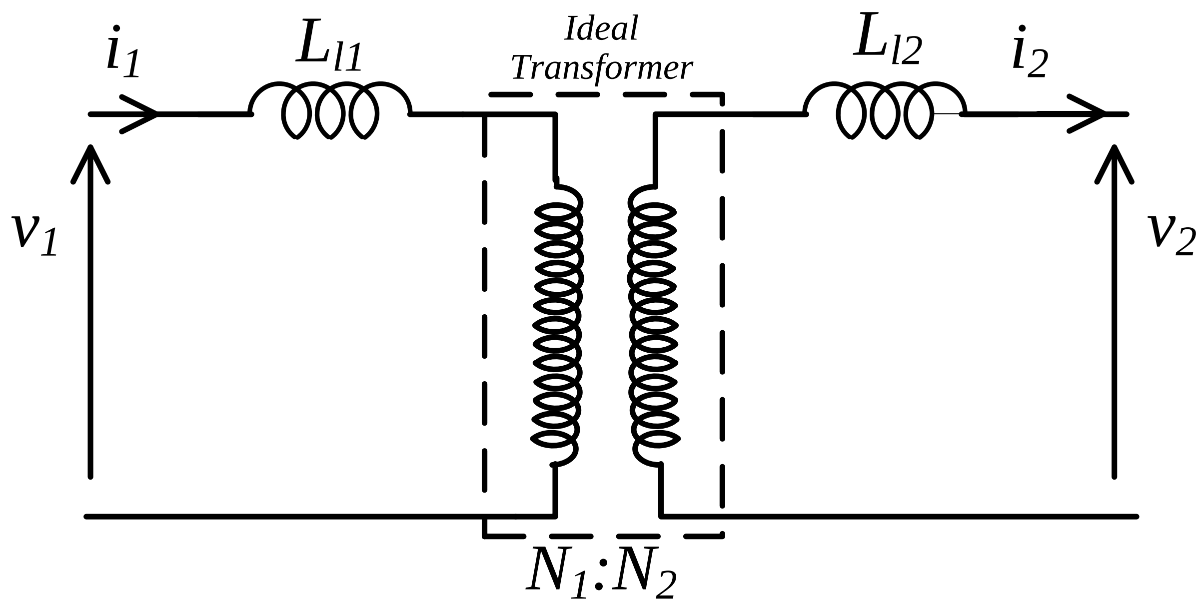
- The leakage flux is modelled in the equivalent electric circuit as an inductance L_{l1} in series with the primary of the ideal transformer.



$$\Phi = \Phi_c + \Phi_l$$

Φc is the flux in the core
Φ_l is the *leakage flux*

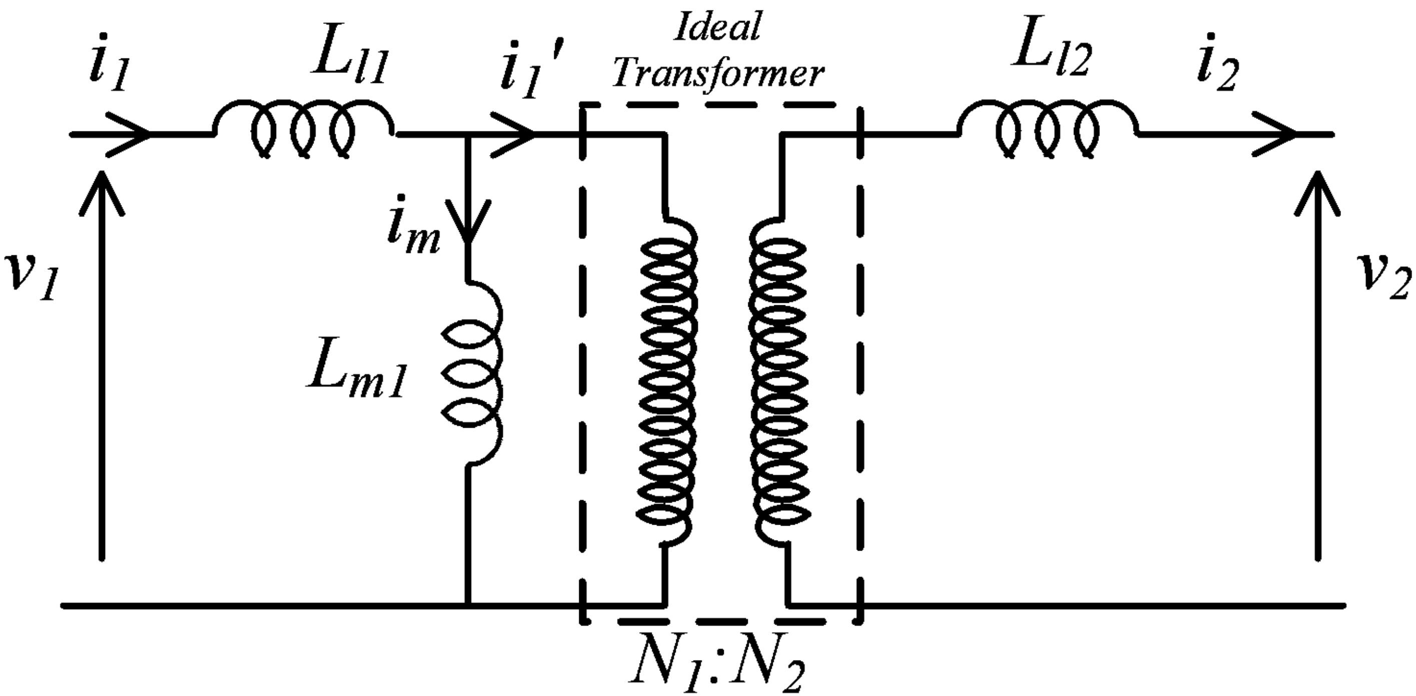
Equivalent Circuit



The flux produced by the current i_1 flowing in the inductance L_{l1} represents that portion of the primary flux which fails to link with the secondary.

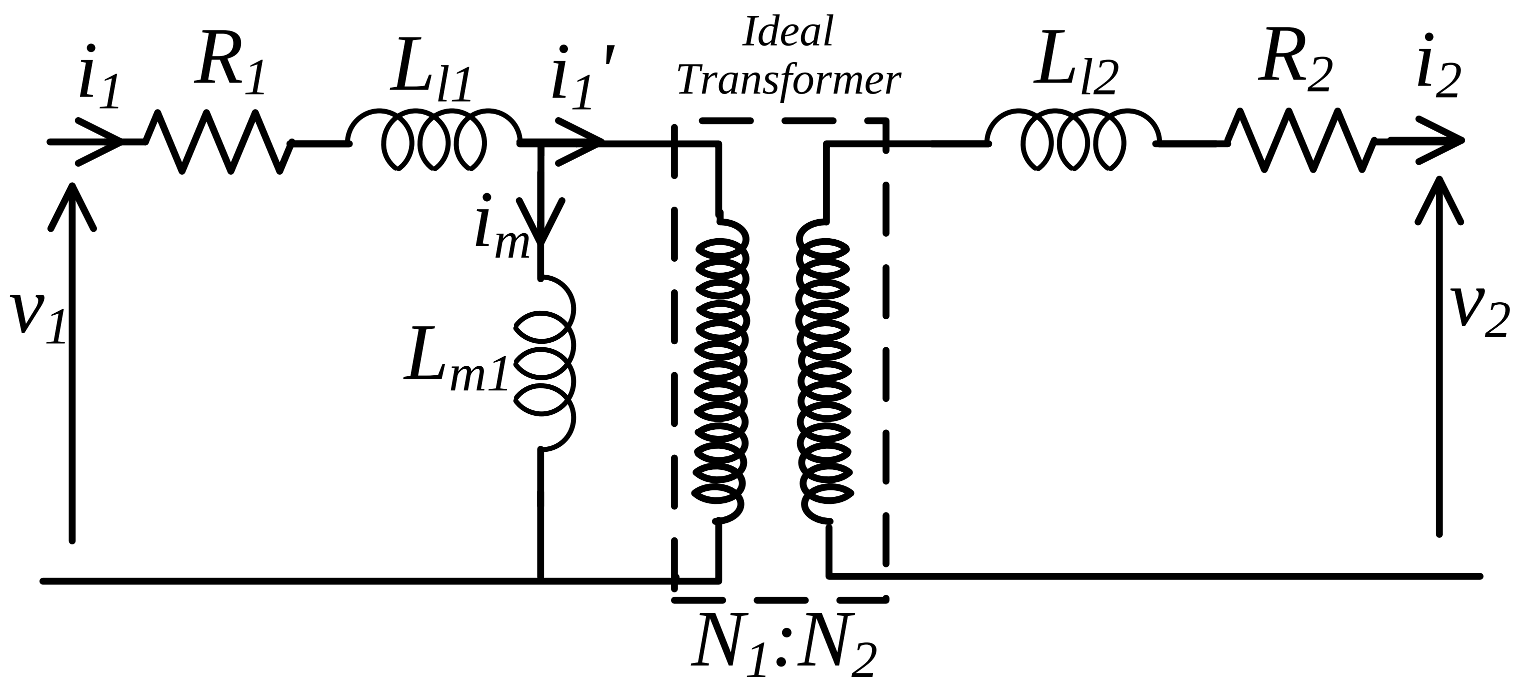
A similar leakage flux (modeled by L_{l2}) occurs on the secondary coil.

Inductances only no resistances

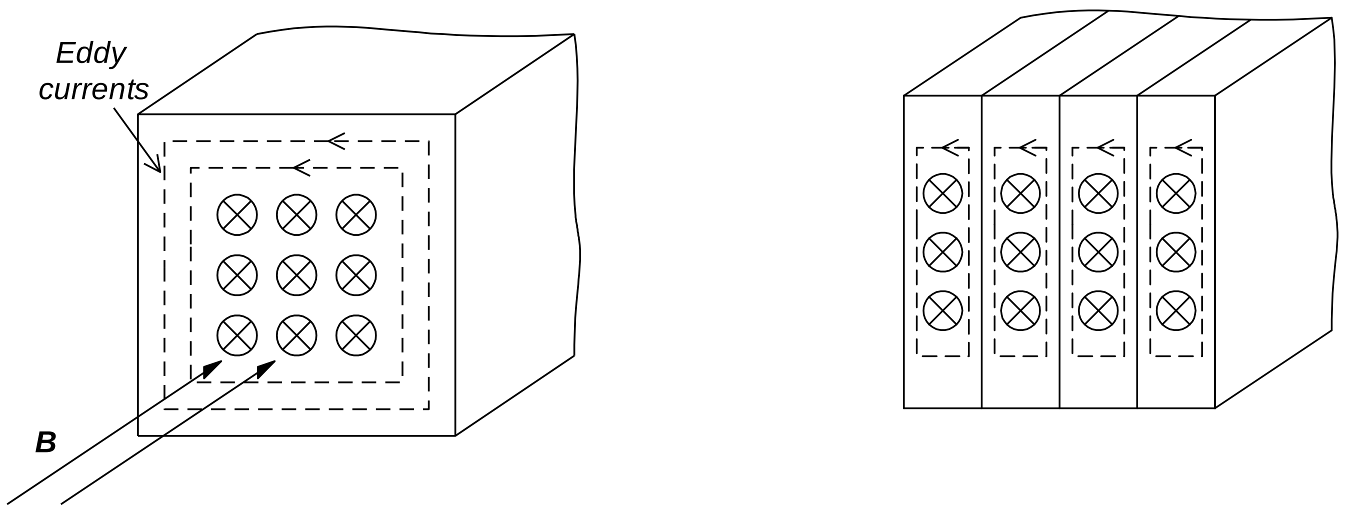


Losses

Now add coil resistances



Eddy Currents



(a) a solid conductor *(b)* a laminated conductor

Example 3.3: solution

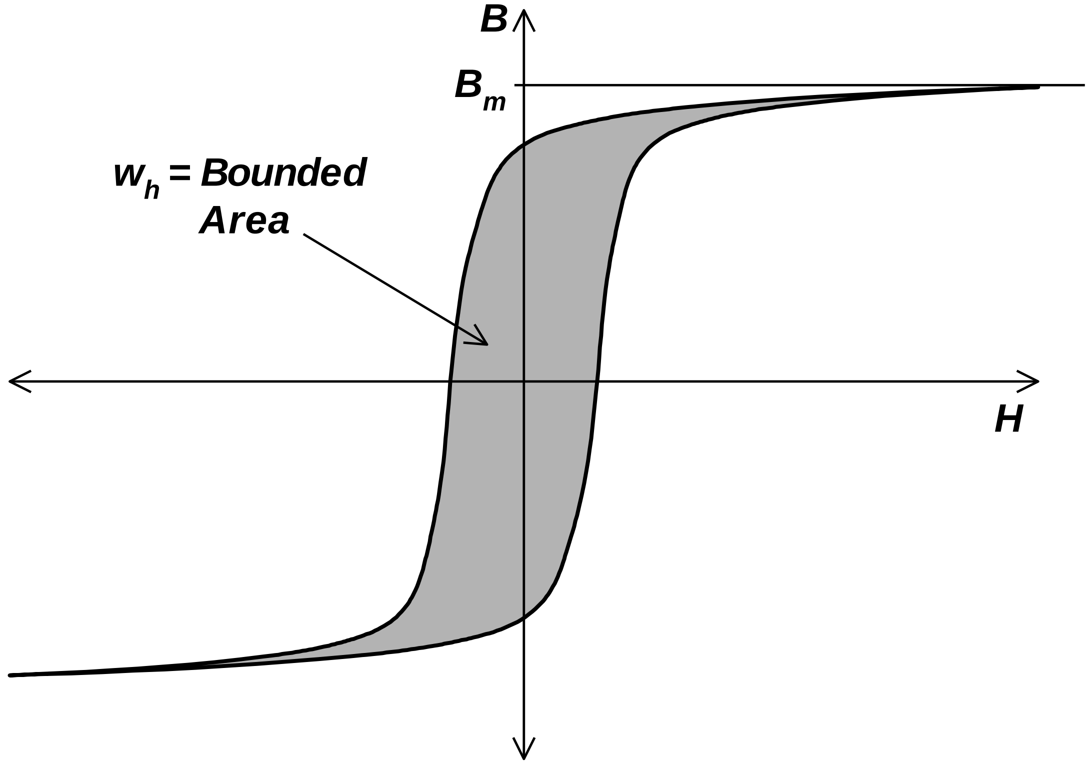
If d doubles and ρ increases by a factor of 5

$$k_e^{'} = K_e^{'}\left(2d\right)^2 / 5\rho = 0.8\left(K_e^{'} d^2 / \rho\right) = 0.8k_e$$

Therefore the eddy current loss p_e decreases by 20% as

$$p_e = k_e^{'} f^2 B_m^2 \text{ W/m}^3 = 0.8k_e f^2 B_m^2 \text{ W/m}^3$$

Hysteresis Loss



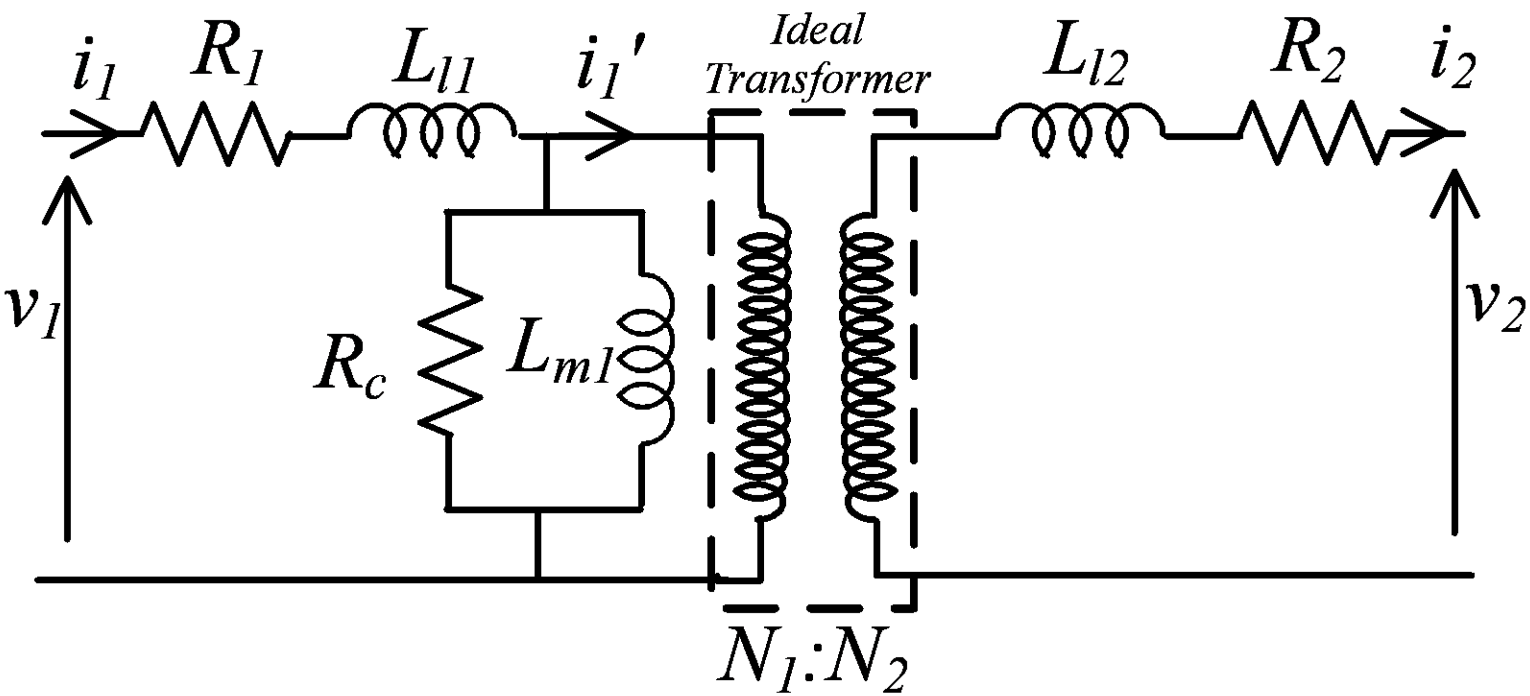
Energy is dissipated in the form of heat in a magnetic material which is taken through a magnetisation cycle.

Hysteresis Loss

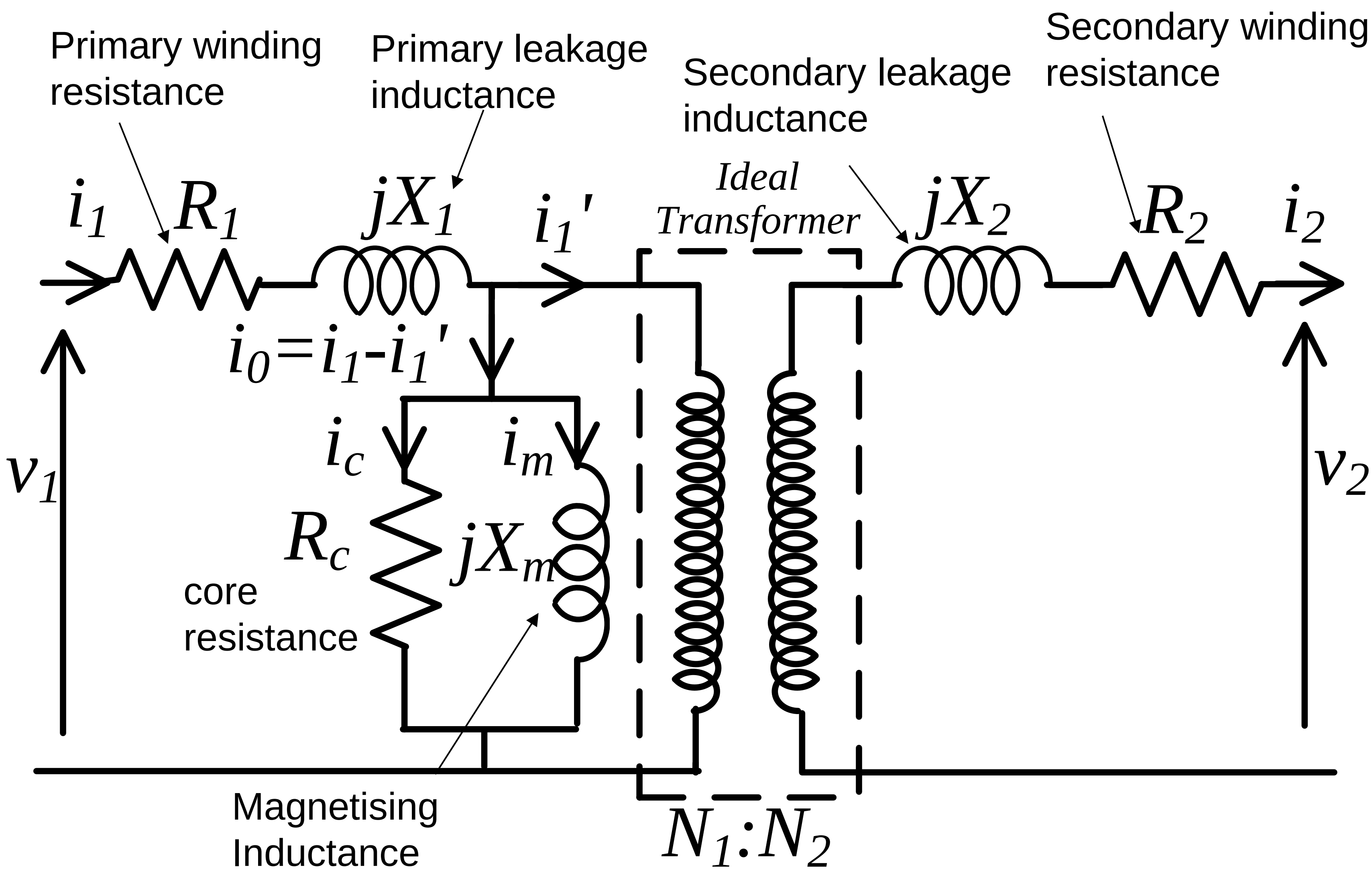
$$\int i(t)v(t) = \text{vol}(-A_1 + (A_2 + A_3) - A_3 + (A_4 + A_1))$$
$$= \text{vol}(A_2 + A_4)$$
$$= \text{vol}(\text{Area on } B, H \text{ curve inside hysteresis loop})$$

which is the energy from the electrical supply to the winding on one cycle. It is dissipated in the material. Since power is energy loss per second, this is equal to the volume times the area between the hysteresis curve times the frequency.

Now add core losses



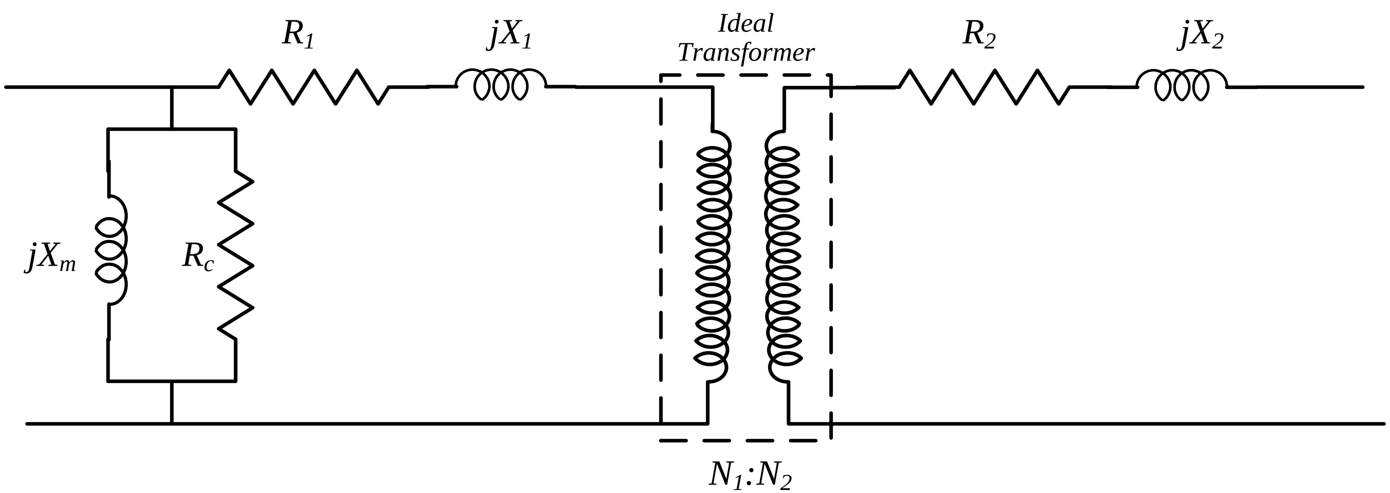
Single Phase Equivalent Circuit of the Transformer



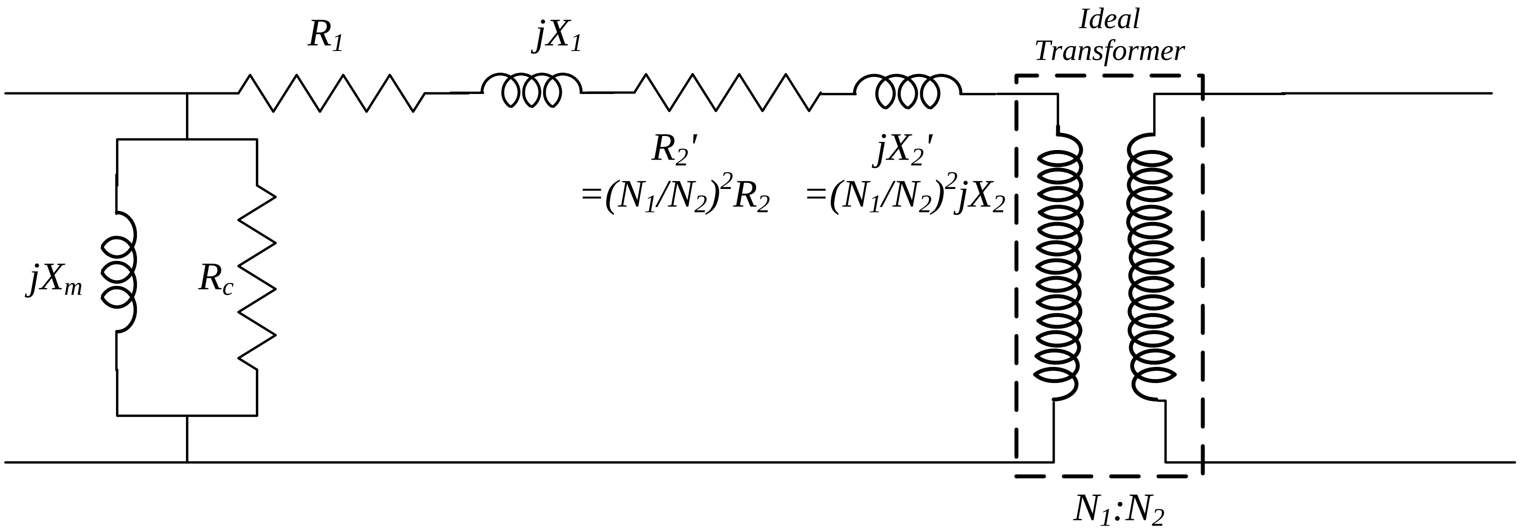
Simplified Equivalent Circuit

- Practical iron-cored transformers are usually designed so that under normal conditions, the voltage drop across (R_1+jX_1) is very small in comparison with v_1, and i_0 is small compared to the load current (i_1).
- We may therefore transfer the shunt impedance (R_c+jX_m) to the input terminals with very little loss of accuracy

Simplified Equivalent Circuit

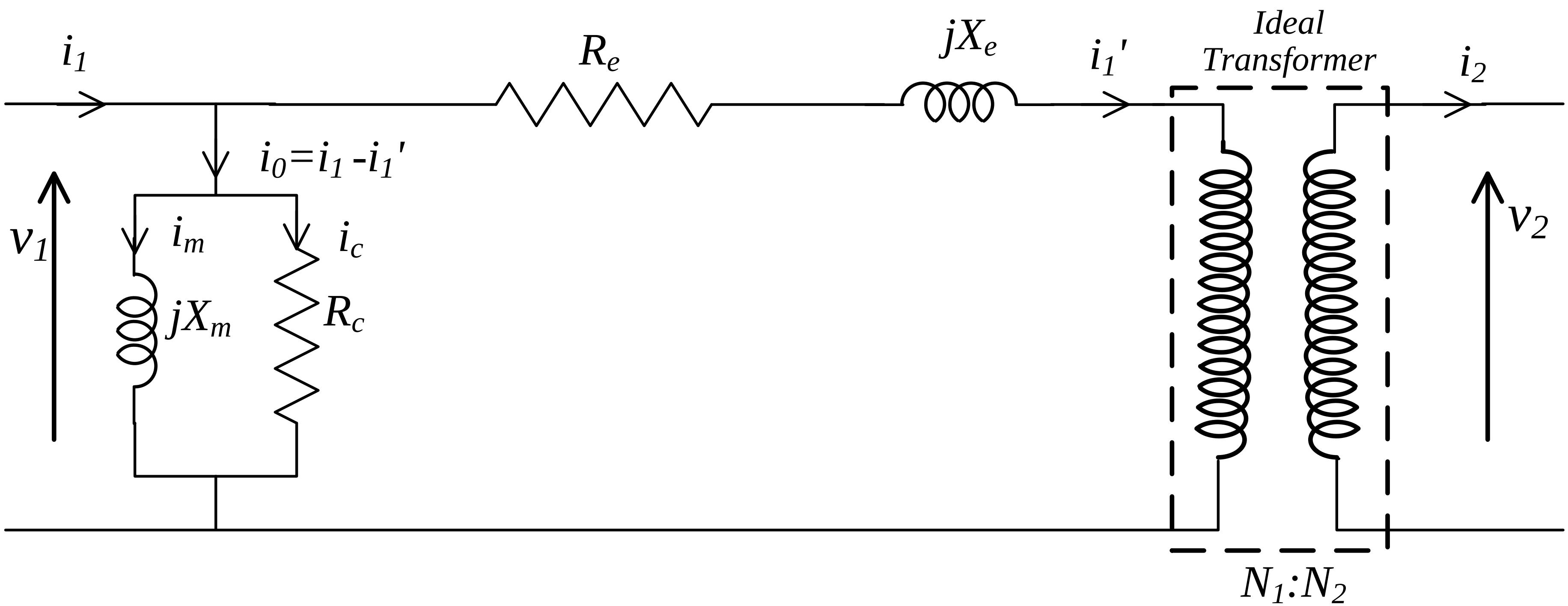


Simplified Equivalent Circuit



Refer all secondary components to primary side.

Simplified Equivalent Circuit



R_e = equivalent resistance = $R_1 + R_2$\`
X_e = equivalent reactance = $X_1 + X_2$\`

Lab Circuit

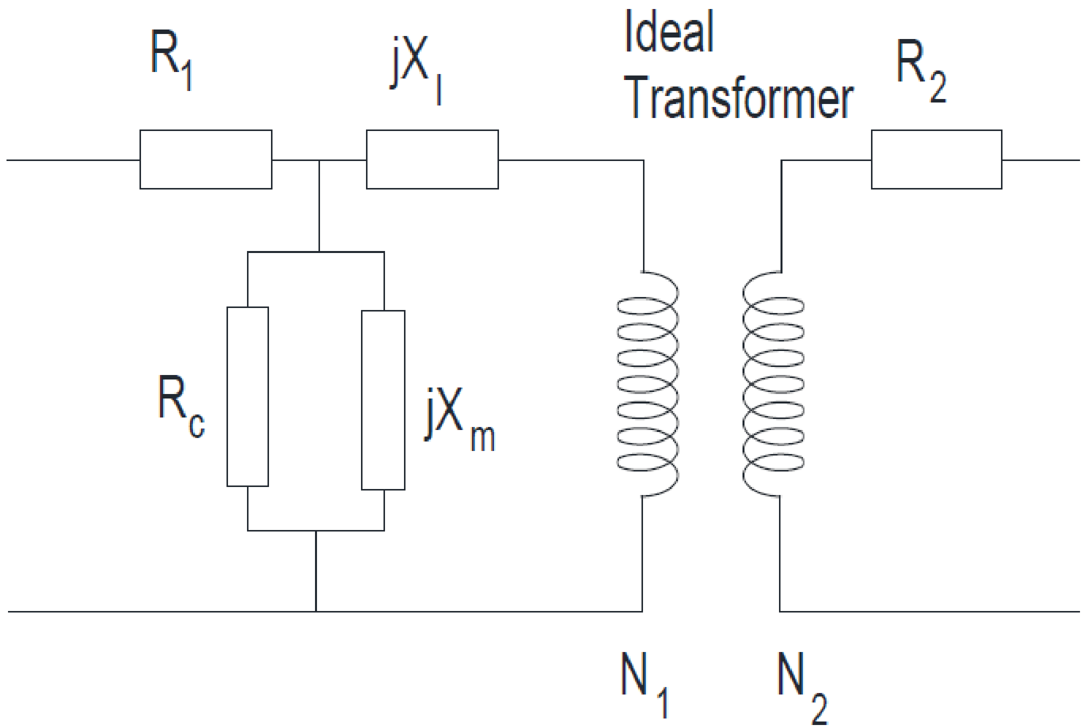


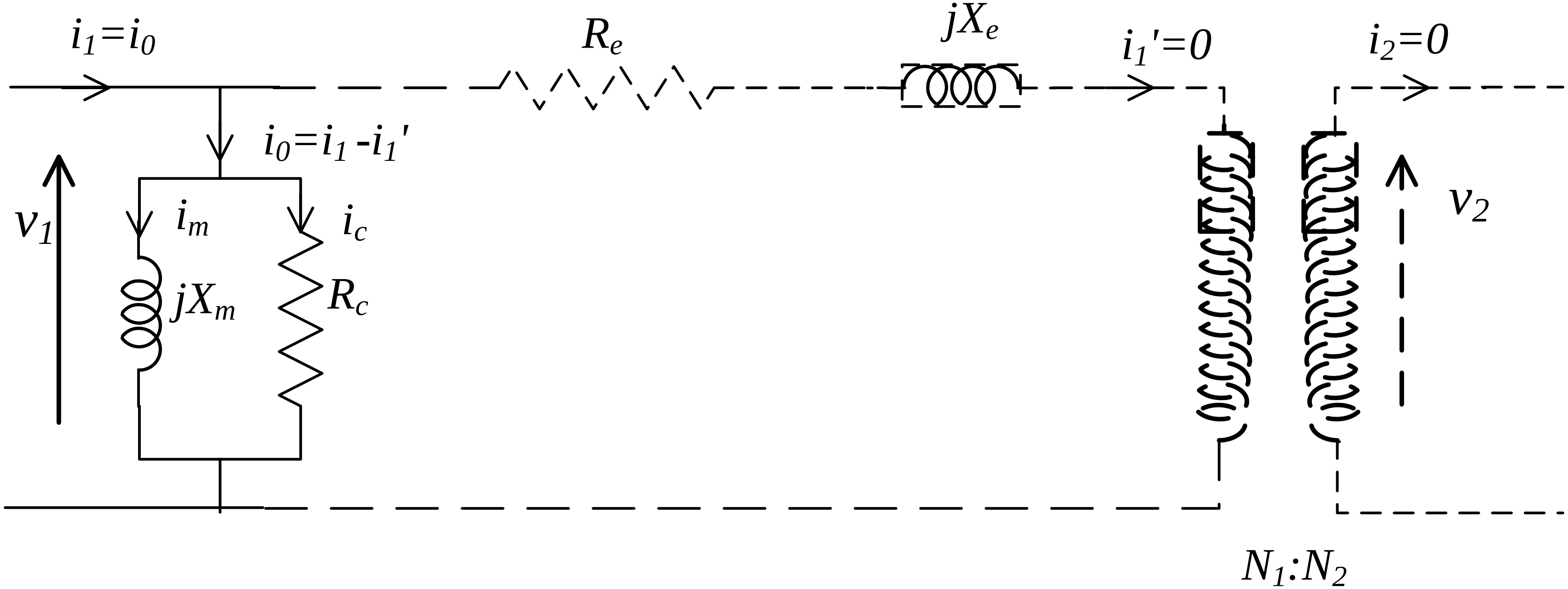
Fig 2 Power Transformer Equivalent Circuit

Transformer Tests

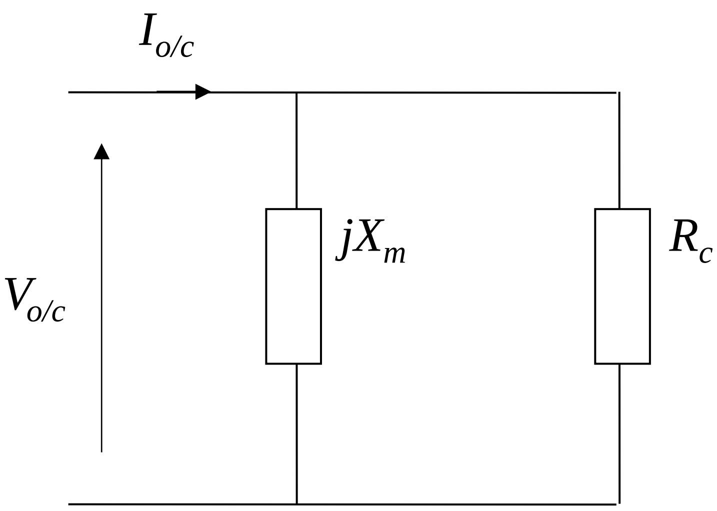
- Open Circuit (O/C) Test and Short Circuit (S/C) Test
- Apply O/C or S/C to windings (primary or secondary) of transformer
- Energise other side (secondary or primary) of transformer with rated voltage (O/C test) or current (S/C test)
- Measure resulting power, currents and/or voltages
- Use readings to determine transformer parameters
- Tests can be done on either side so circuits are referred to relevant side but typically O/C on low voltage (high current) side and S/C on high voltage (low current). Safety reasons.

Open-Circuit Test

One winding is open-circuited and the rated voltage applied to the other winding; only the small no-load current will be drawn from the supply. If the secondary winding is open-circuited, $i_2 = i_1` = 0$.



Open-Circuit Test



$$P_{o/c} = \frac{(V_{o/c})^2}{R_c}, \Rightarrow \boxed{R_c = \frac{(V_{o/c})^2}{P_{o/c}}}$$

(where $V_{o/c}$ is the RMS value)

$$I_{o/c} = \frac{V_{o/c}}{jX_m} + \frac{V_{o/c}}{R_c}$$

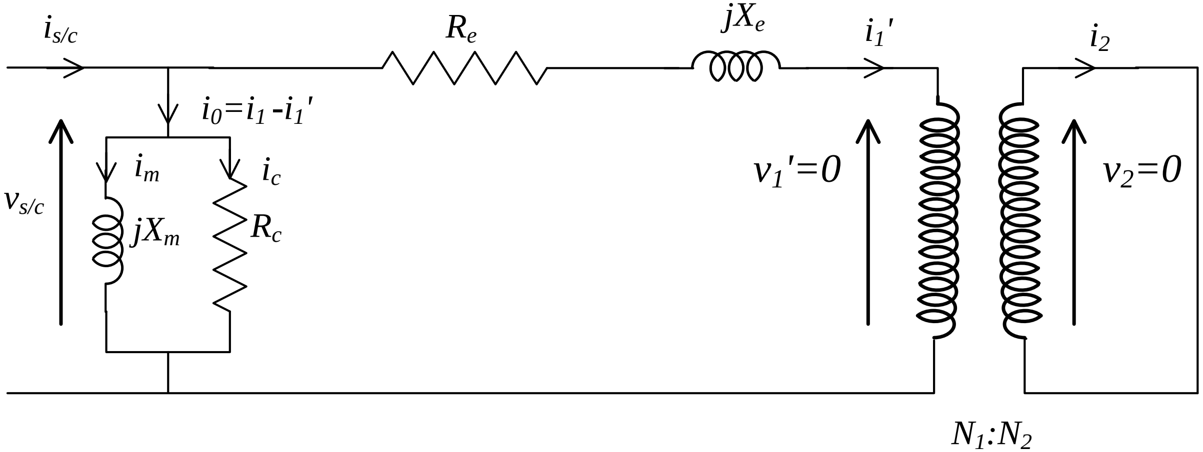
$$\left(\frac{I_{o/c}}{V_{o/c}}\right) = \sqrt{\frac{1}{X_m^{\ 2}} + \frac{1}{R_c^{\ 2}}}$$

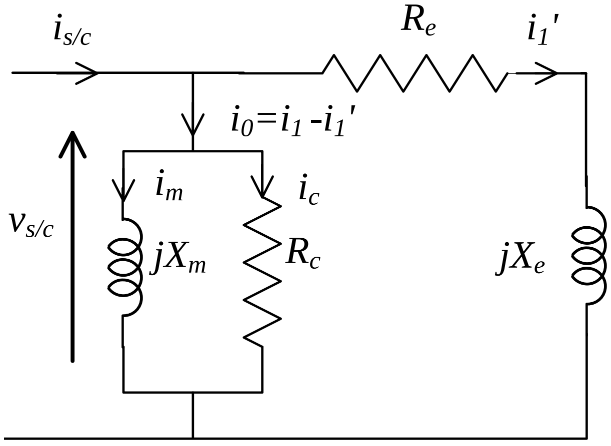
$$\Rightarrow \boxed{X_m = \frac{1}{\sqrt{\left(\frac{I_{o/c}}{V_{o/c}}\right)^2 - \frac{1}{R_c^{\ 2}}}}}$$

Measure $I_{o/c}$ and $P_{o/c}$, $V_{o/c}$ is rated value

Short-Circuit Test

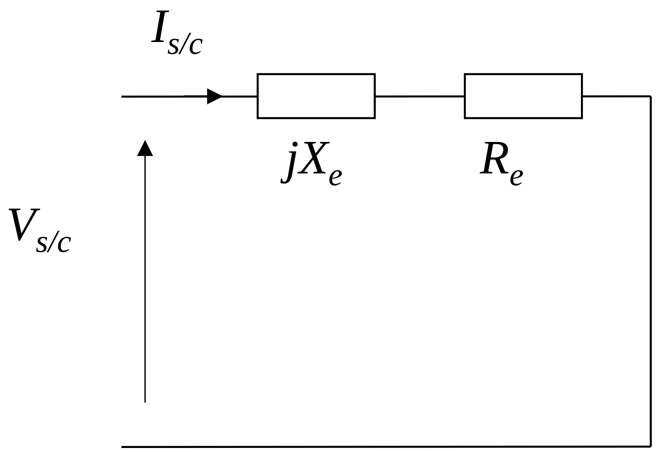
One winding is short circuited and the normal rated current is allowed to flow in the other winding by connecting it to an adjustable low-voltage source. Thus if the secondary is short circuited, a short circuit will be reflected in to the primary side of the equivalent circuit.





R_c *and* jX_m are significantly larger than R_e *and* jX_e and can be neglected

Short-Circuit Test



$$V_{s/c} = I_{s/c}(R_e + jX_e)$$

$$\frac{V_{s/c}}{I_{s/c}} = R_e + jX_e$$

$$\left(\frac{V_{s/c}}{I_{s/c}}\right)^2 = R_e^{\,2} + X_e^{\,2}$$

$$\boxed{X_e = \sqrt{\left(\frac{V_{s/c}}{I_{s/c}}\right)^2 - R_e^{\,2}}}$$

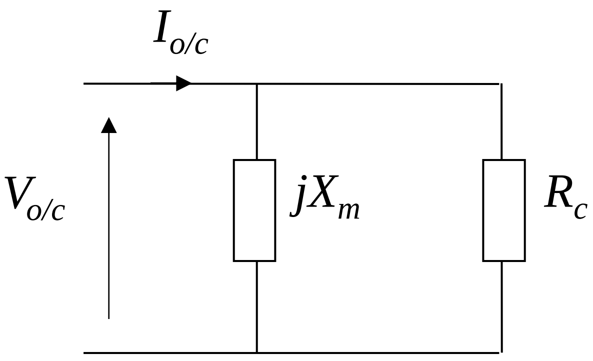
Measure $V_{s/c}$ and $P_{s/c}$, $I_{s/c}$ is rated value

$$P_{s/c} = R_e (I_{s/c})^2$$

$$\Rightarrow \boxed{R_e = \frac{P_{s/c}}{(I_{s/c})^2}}$$

(where $I_{s/c}$ is the RMS value)

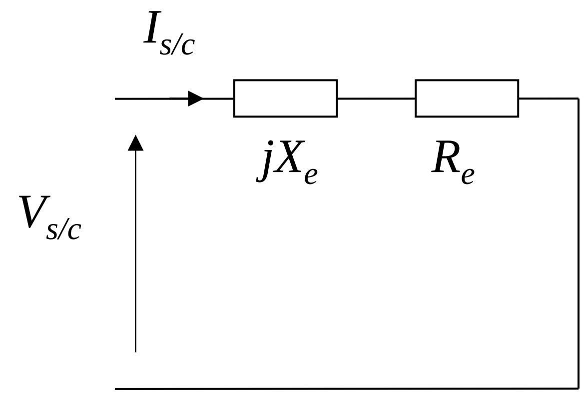
O/C S/C Tests Summary



Measure $I_{o/c}$ and $P_{o/c}$, $V_{o/c}$ is rated value

$$R_c = \frac{(V_{o/c})^2}{P_{o/c}}$$

$$X_m = \frac{1}{\sqrt{\left(\frac{I_{o/c}}{V_{o/c}}\right)^2 - \frac{1}{R_c^{\,2}}}}$$



Measure $V_{s/c}$ and $P_{s/c}$, $I_{s/c}$ is rated value

$$R_e = \frac{P_{s/c}}{(I_{s/c})^2}$$

$$X_e = \sqrt{\left(\frac{V_{s/c}}{I_{s/c}}\right)^2 - R_e^{\,2}}$$

Example 3.4:

A single phase, 1000 V/100 V, 1kVA transformer gave the following test results at 50 Hz:

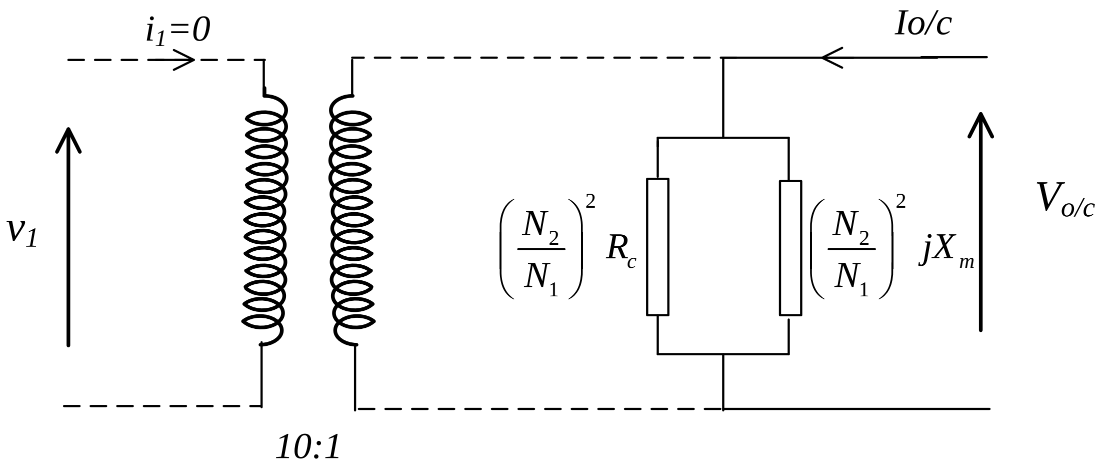
For an open circuit test on the low voltage side the measured current and power were $\sqrt{2}/10$ A and 10 W respectively

A short circuit test performed on the high voltage side gave a voltage measurement of $10\sqrt{2}$ V and a power measurement of 10 W.

a) Find the approximate equivalent circuit for this transformer applied to the primary side.

b) Find the current in a $1/20\pi$ F capacitor placed between the terminals of the secondary side with a rated voltage applied to the primary side of frequency 100 Hz. What happens to the core losses.

Example 3.4 (a): Solution o/c test



Turns Ratio:

$$\frac{N_1}{N_2}=\frac{1000}{100}=10$$

o/c Test on LV side:

$$V_{o/c}=100\ V,$$

$$P_{o/c}=10\text{ W},$$

$$I_{o/c}=\frac{\sqrt{2}}{10}A.$$

$$\left(\frac{N_2}{N_1}\right)^2 R_c=\frac{(V_{o/c})^2}{P}=\frac{100^2}{10}=1000\,\Omega$$

$$R_c=\left(\frac{N_1}{N_2}\right)^2 1000=(10)^2 1000=100k\Omega$$

Example 3.4 (a): Solution o/c test cont.

$$\frac{I_{o/c}}{V_{o/c}} = \frac{\sqrt{2}/10}{100} = \frac{\sqrt{2}}{1000}\,\Omega$$

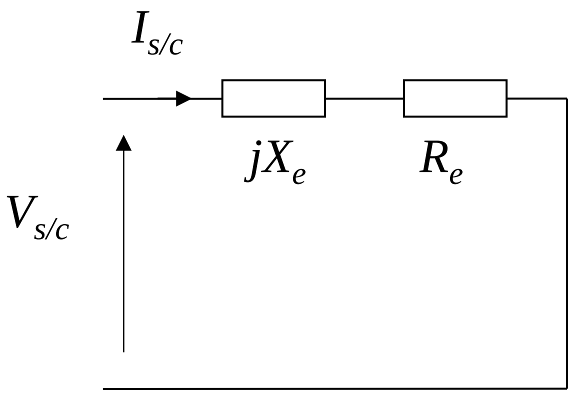
$$\left(\frac{N_2}{N_1}\right)^2 X_m = \frac{1}{\sqrt{\left(\frac{I_{o/c}}{V_{o/c}}\right)^2 - \frac{1}{\left(\left(\frac{N_2}{N_1}\right)^2 R_c\right)^2}}}$$

$$\Rightarrow \left(\frac{N_2}{N_1}\right)^2 X_m = \frac{1}{\sqrt{\left(\frac{\sqrt{2}}{1000}\right)^2 - \frac{1}{1000^2}}} = 1000\,\Omega$$

$$X_m = \left(\frac{N_1}{N_2}\right)^2 1000 = (10)^2 1000 = 100k\Omega$$

$$\boxed{\begin{aligned} jX_m &= j\omega L_m \\ L_m &= \frac{X_m}{\omega} \\ L_m &= \frac{100,000}{100\pi} \\ L_m &= \frac{1000}{\pi} H \end{aligned}}$$

Example 3.4 (a): Solution s/c test



$$R_e = \frac{P_{s/c}}{(I_{s/c})^2}$$

$$I_{s/c} = I_{Rated} = \frac{1\,\text{kVA}}{1000} = 1\,\text{A}$$

$$\Rightarrow \boxed{R_e = \frac{10}{1^2} = 10\,\Omega}$$

Example 3.4 (a): Solution s/c test cont.

$$\frac{V_{s/c}}{I_{s/c}} = \frac{10\sqrt{2}}{1} = 10\sqrt{2}\ \Omega$$

$$X_e = \sqrt{\left(\frac{V_{s/c}}{I_{s/c}}\right)^2 - R_e^{\ 2}}$$

$$X_e = \sqrt{\left(10\sqrt{2}\right)^2 - 10^2}$$

$$\boxed{X_e = 10\ \Omega}$$

$$jX_e = j\omega L_e$$

$$\boxed{L_e = \frac{10}{100\pi} = \frac{1}{10\pi}\ \mathrm{H}}$$

Example 3.4:

A single phase, 1000 V/100 V, 1kVA transformer gave the following test results at 50 Hz:

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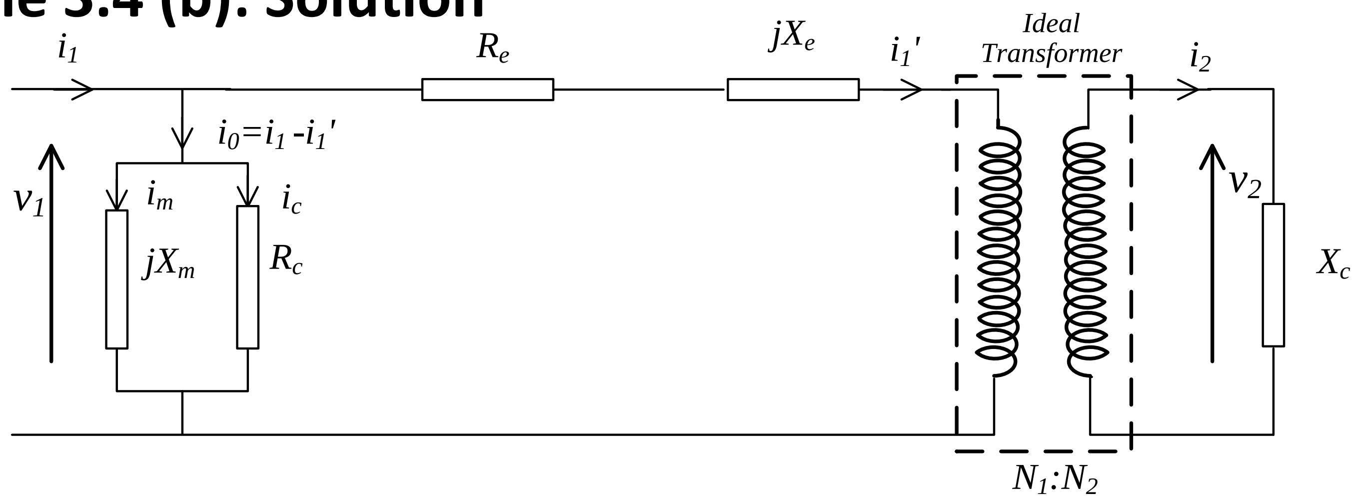
Example 3.4 (b): Solution



$$v_1 = 1000 \text{ V}, \; f = 100 \text{ Hz}, i_2 = ?$$

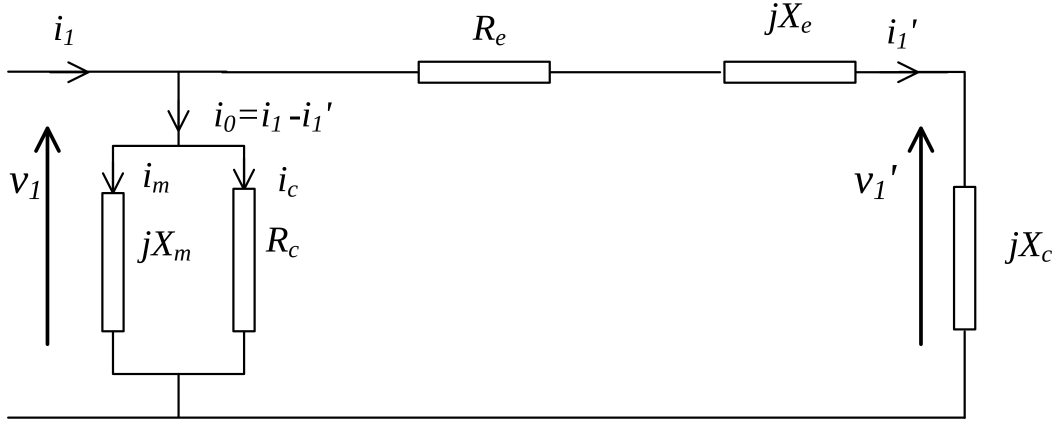
X_e & X_m vary with freq.

$$jX_e = j\omega L_e = j200\pi\left(\frac{1}{10\pi}\right) = j20\,\Omega$$

$$jX_m = j\omega L_m = j200\pi\left(\frac{1000}{\pi}\right) = j200\;k\Omega$$

$$jX_c = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-20\pi j}{200\pi} = \frac{1}{j10} - j0.1\,\Omega$$

Example 3.4 (b): Solution cont.

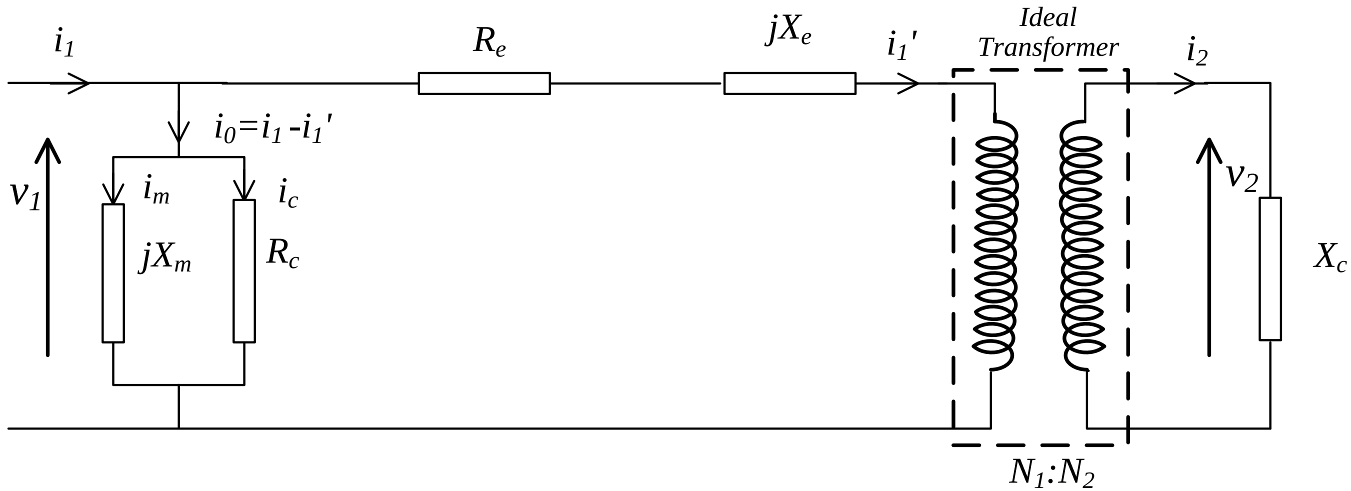


$$X'_c = \left(\frac{N_1}{N_2}\right)^2 X_c = 100(-0.1) = -10$$

$$\text{Voltage Division: } v_1' = v_1\left(\frac{jX'_c}{R_e + jX_e + jX'_c}\right) = v_1\left(\frac{-10j}{10+20j-10j}\right)$$

$$v_1' = 1000\left(\frac{-j}{1+j}\right) = -500 - j500 \text{ V}$$

Example 3.4 (b): Solution cont.

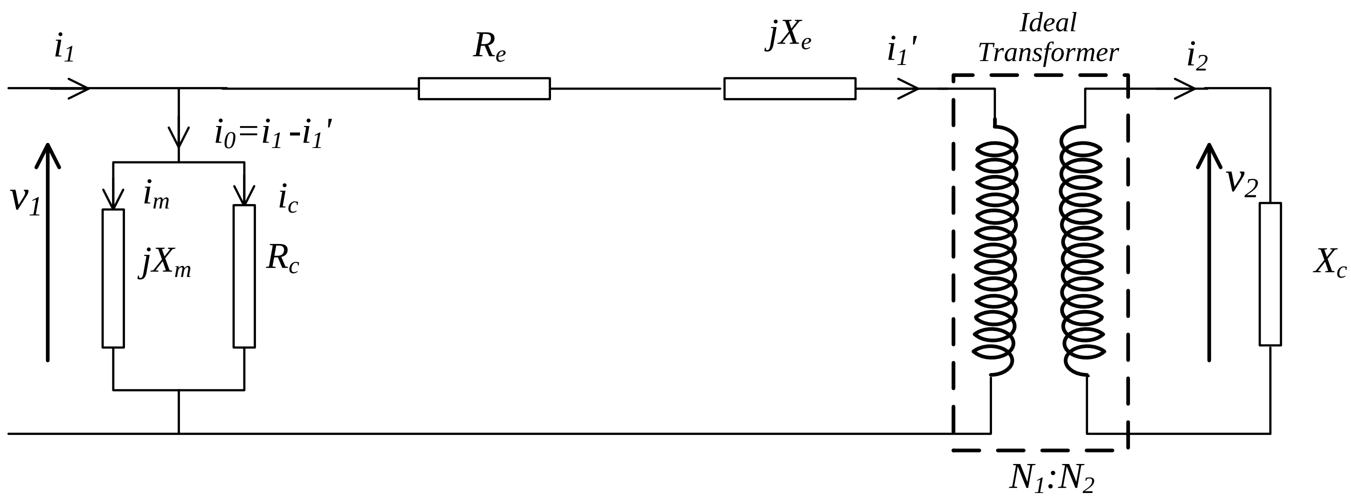


$$\text{Refer to Secondary Side: } v_2 = \left(\frac{N_2}{N_1}\right) v_1^{'} = \frac{1}{10}\left(-500 - j500\right) = -50 - 50j \text{ V}$$

$$v_2 = i_2\, jX_c \Rightarrow i_2 = \frac{v_2}{jX_c} = \frac{-50 - j50}{\frac{1}{j10}}$$

$$\boxed{I_2 = 500 - j500 \text{ A}}$$

Example 3.4 (b): Solution what happens to core losses



Example 3.3: solution

If d doubles and ρ increases by a factor of 5

$$k_e^{'} = K_e^{'}\left(2d\right)^2 / 5\rho = 0.8\left(K_e^{'} d^2 / \rho\right) = 0.8k_e$$

Therefore the eddy current loss p_e decreases by 20% as

$$p_e = k_e^{'} f^2 B_m^2 \text{ W/m}^3 = 0.8k_e f^2 B_m^2 \text{ W/m}^3$$

Hysteresis Loss

$$\int i(t)v(t) = \text{vol}(-A_1 + (A_2 + A_3) - A_3 + (A_4 + A_1))$$
$$= \text{vol}(A_2 + A_4)$$
$$= \text{vol}(\text{Area on } B, H \text{ curve inside hysteresis loop})$$

which is the energy from the electrical supply to the winding on one cycle. It is dissipated in the material. Since power is energy loss per second, this is equal to the volume times the area between the hysteresis curve times the frequency.

Example 3.4 (b): Solution what happens to core losses con.

$$P_{o/c} = \frac{(V_{o/c})^2}{R_c}$$

$$hence$$

R_c decreases to $\mathrm{R_c}/4$ due to eddy currents

R_c decreases to $\mathrm{R_c}/2$ due to hystersis !