

Chapter 4

Drift and Diffusion Currents
in a Semiconductor.

The Poisson Equation

Drift Current in a Doped Semiconductor

- We previously discussed current flow in a pure or intrinsic semiconductor in response to an electric field E due to the *carrier drift* of electrons and holes
- In fact, the key formulas are equally valid for a **doped** or **extrinsic** semiconductor, where of course n and p may now differ greatly in value
- In general, the total (drift) current density is given by:

$$J = J_n + J_p = q \cdot v_n \cdot n + q \cdot v_p \cdot p$$

... where v_n is the electron drift velocity and v_p is the hole drift velocity

Current Flow in a Semiconductor at Low-to-Moderate Electric Fields

- Provided the electric field (E) is not too high, we can use similar formulas to those earlier derived for a metal:

– Electrons: $V_n = \mu_n \cdot E$

where μ_n is the electron mobility given by:

$$\mu_n = (q \cdot \bar{\tau}_e / m_e^*)$$

– Holes: $V_p = \mu_p \cdot E$

where μ_p is the hole mobility given by:

$$\mu_p = (q \cdot \bar{\tau}_h / m_h^*)$$

Note that the ionised impurity atoms in extrinsic semiconductors cause extra scattering which reduces τ_e and τ_h and thus lowers the mobilities compared to the intrinsic case

'Ohm's Law' for Semiconductors

Substituting for the drift velocities, then at **low-to-moderate electric field** in a semiconductor (including the doped case) we can write the drift current density **J** flowing in response to an electric field **E** as:

$$\mathbf{J} = \mathbf{J}_n + \mathbf{J}_p = q \cdot n \cdot (\mu_n \mathbf{E}) + q \cdot p \cdot (\mu_p \mathbf{E})$$

Or:

$$\mathbf{J} = \mathbf{J}_n + \mathbf{J}_p = q \cdot (n \cdot \mu_n + p \cdot \mu_p) \cdot \mathbf{E} \Rightarrow \mathbf{J} = \boldsymbol{\sigma} \cdot \mathbf{E}$$

where σ is the conductivity of the semiconductor (its reciprocal is the resistivity ρ ($\sigma = 1/\rho$)):

$$\sigma = q \cdot (n \cdot \mu_n + p \cdot \mu_p)$$

Practical Conductivity Formulas

- The previous general formula for conductivity in a semiconductor can usually be simplified in practice;
- In an **N-type** semiconductor doped to a **donor concentration N_D** , provided: (1) the material is not too lightly doped, (2) the temperature is not too high and (3) the electric field is low-to-moderate:

$$\sigma_N = q \cdot N_D \cdot \mu_n$$

- In a **P-type** semiconductor doped to **an acceptor concentration N_A** , provided: (1) the material is not too lightly doped, (2) the temperature is not too high and (3) the electric field is low-to-moderate:

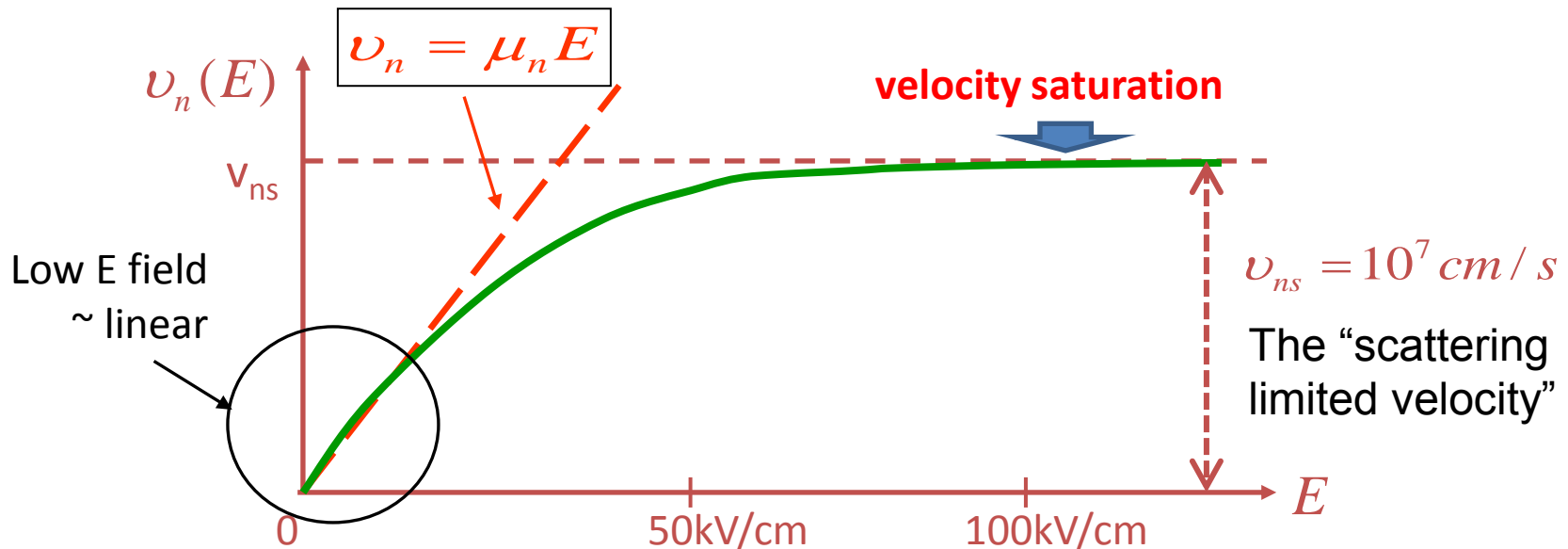
$$\sigma_P = q \cdot N_A \cdot \mu_p$$

Semiconductors at High Electric Fields

- Under low-to-moderate electric fields, the electrons (or holes) form a thermodynamic system with the same effective temperature as the lattice;
- The interaction between electrons (or hole) and the lattice is described in simple terms as occurring through “collisions” but more precisely the process involves the exchange of **quanta of lattice vibration energy** called **phonons** .
- At low electric fields, **acoustic phonons** are involved and the drift velocity is proportional to the field but at high electric fields **optical phonons** of essentially fixed energy are exchanged. The drift velocity then tends to saturate to a constant velocity called the **scattering limited velocity** or the **saturated velocity** (this happens eventually in all semiconductors, for both electrons and holes and typically $v_{\text{sat}} \approx 10^7$ cm/sec);
- At the same time, the temperature of the electron system increases relative to the lattice: this is called the “**hot electron**” effect.

Drift at High Fields – Velocity Saturation

- Example of drift velocity saturation at high electric fields (electrons in Si):



- At **low electric field**, the drift velocity is proportional to the field and essentially we can say that **Ohm's law is obeyed** at a constant temperature;
- At higher E field the drift velocity shows a **sub-linear** dependence on field and eventually it **saturates** to the scattering limited velocity;
- Even though voltages are usually small in semiconductors ($\sim V$), distances are tiny ($\sim 1 \mu\text{m}$) so the field may be very high $E = V/l \sim 1 \text{ MV/m}$ or $\sim 10 \text{ kV/cm}$

Expression for Velocity Saturation

- Consider the drift of electrons in a semiconductor. In many cases the dependence of the electron drift velocity v_n on the electric field E can be reasonably approximated as:

$$v_n(E) = \frac{\mu_n \cdot E}{1 + \left(\frac{\mu_n \cdot E}{v_{nsat}} \right)}$$

- We see that as $E \rightarrow 0$, $v_n(E) \approx \mu_n \cdot E$, whereas as $E \rightarrow \infty$, $v_n(E) \approx v_{nsat}$, as required. A similar expression may be used for holes in Si;
- Note that the drift velocity does not always follow this behaviour. For example, an important exception arises in the case of electrons in GaAs, as will be discussed further in SSE2.

Example 4.1

Compensation Doping

- In the practical manufacture of semiconductors it is often necessary to change the type of a particular region e.g. from N-type to P-type. This can be done in the latter case by introducing sufficient acceptors to more than cancel the existing donors. This process is called **compensation doping**.
- For example, detailed analysis shows that if N_A acceptors are added to a crystal where N_D donors already exist, then, provided $|N_A - N_D| \gg n_i$, the material now behaves as a P-type material with an **effective acceptor concentration of $(N_A - N_D)$** so that under normal conditions:

$$p_o = (N_A - N_D) \quad n_o = \frac{n_i^2}{(N_A - N_D)}$$

NB: this increases ionised impurity scattering!

- The converse is also possible. We could start with N_A acceptors and then compensation dope with donors to N-type (with $|N_D - N_A| \gg n_i$) so that:

$$n_o = (N_D - N_A) \quad p_o = \frac{n_i^2}{(N_D - N_A)}$$

Example 4.2

Diffusion Current

- We are familiar with the idea of **diffusion** from every-day experience: e.g. squirt some perfume from a spray at a location in a room – the smell is at first strong at that point, but come back an hour later and the smell has gone. Why? ... because the molecules in the spray have **diffused** away through random thermal motion in the air and have become uniformly distributed throughout the room.
- If a **non-uniform distribution of carriers exists in space** within a semiconductor crystal, a similar phenomenon occurs. The carriers **diffuse** so as to try to make the distribution uniform. This represents a net transfer of charge over time, and so constitutes an electrical current (separate to, and possibly additional to, drift current). We call this **diffusion current**, and it can exist for **both holes** and **electrons**.
- This current does not exist in metals because any non-uniformity of CB electrons in space is \approx instantly removed by the high conductivity.

Particle Current Flow in Semiconductors

- There are therefore actually **four** components of particle current flow in a semiconductor, in general.
- We have the **drift current** density for both electrons and holes, and also the **diffusion current** density for both electrons and holes:

$$\begin{aligned} J &= J_n + J_p \\ &= (J_n^{drift} + J_n^{diff}) + (J_p^{drift} + J_p^{diff}) \end{aligned}$$

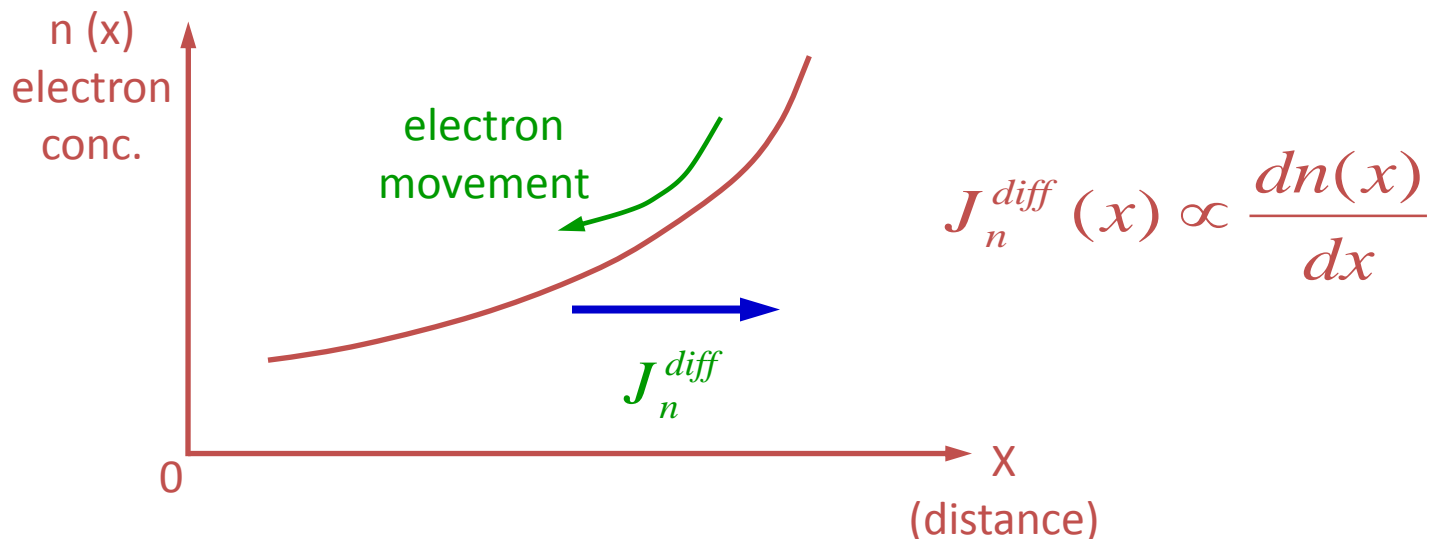
- Compare with a metal where we just have:

$$J = J_n = J_n^{drift}$$

Current Flow due to Electron Diffusion

(N.B. this is **additional** to the electron drift current in a semiconductor)

- Suppose an electron concentration is non-uniformly distributed in 1D space (x) according to $n(x)$;
- Because of diffusion, a movement of charge will take place from any high concentration region to a lower concentration region;
- We might expect the resulting **diffusion current** to be proportional to the slope of the carrier concentration:



Formula for Electron Diffusion Current

- This problem was addressed by Albert Einstein (though not for semiconductors!) in one of his famous 1905 papers
- A similar analysis to that given in his paper shows that we can write the 1D electron diffusion current as:

$$J_n^{diff}(x) = qD_n \cdot \frac{dn(x)}{dx}$$

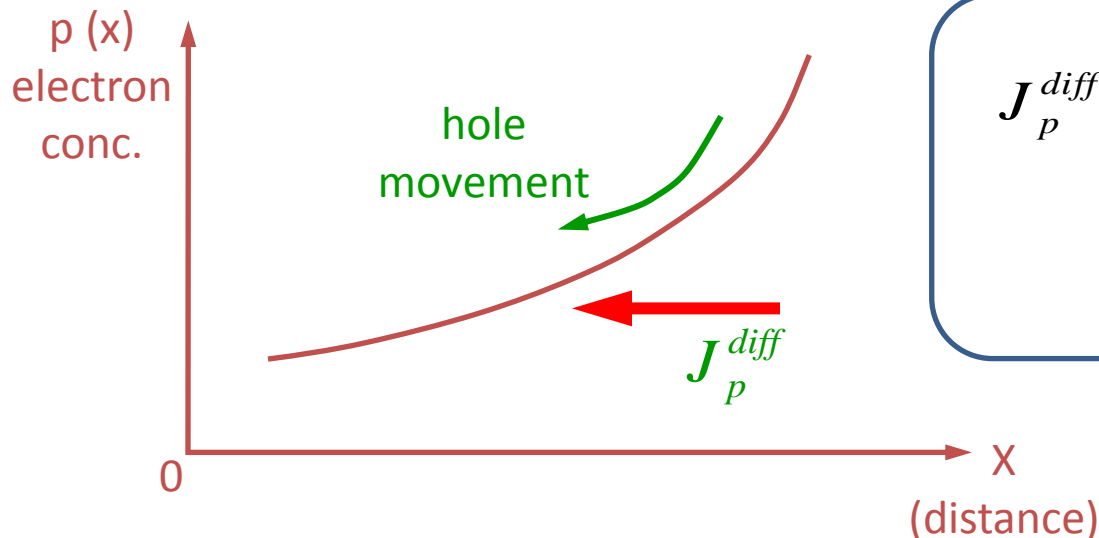
- Where D_n is the **electron diffusion coefficient** which is given in terms of the mobility μ_n by the following **Einstein Relationship**:

$$D_n = \frac{kT}{q} \cdot \mu_n$$

Current Flow due to Hole Diffusion

(additional to the drift of holes in a semiconductor)

- Again, suppose a hole concentration is non-uniformly distributed in 1D space (x) according to $p(x)$;
- Diffusion will once more cause an overall movement of charge from any high concentration region to a lower concentration region, producing a **hole diffusion current**;
- Note that for the same distribution, the **direction** of conventional current is **different** for hole diffusion than for electron diffusion:



N.B.

$$J_p^{diff}(x) = -q D_p \frac{dp(x)}{dx}$$

$$D_p = \frac{kT}{q} \cdot \mu_p$$

Einstein Relationship:
 D_p is the hole Diffusion Coefficient

EXAMPLE 4.3

Drift-Diffusion Currents for Electrons and Holes in a Semiconductor

- Putting all these results together, we arrive at two fundamental equations for semiconductor materials, under the framework of the "*drift-diffusion*" approximation.
- Time-Independent or Static Form** (low-to-moderate electric fields assumed):

$$J_n(x) = \underbrace{q \cdot n(x) \cdot \mu_n \cdot E(x)}_{J_n^{drift}} + \underbrace{q \cdot D_n \cdot \frac{dn(x)}{dx}}_{J_n^{diff}} \quad [1]$$

$$J_p(x) = \underbrace{q \cdot p(x) \cdot \mu_p \cdot E(x)}_{J_p^{drift}} - \underbrace{q \cdot D_p \cdot \frac{dp(x)}{dx}}_{J_p^{diff}} \quad [2]$$

Drift-Diffusion Currents for Electrons and Holes in a Semiconductor

- Under more general conditions, we must allow for *time-dependence*. We then have 5 basic functions of (1D) space and time *t* in a semiconductor: *n(x,t)*, *p(x,t)*, *J_n(x,t)*, *J_p(x,t)* and *E(x,t)*.
- The **Time-Dependent Form** of the drift-diffusion equations now reads (low-to-moderate electric fields assumed):

$$J_n(x,t) = \underbrace{q \cdot n(x,t) \cdot \mu_n \cdot E(x,t)}_{J_n^{drift}} + \underbrace{q \cdot D_n \cdot \frac{\partial n(x,t)}{\partial x}}_{J_n^{diff}} \quad [1]$$

$$J_p(x,t) = \underbrace{q \cdot p(x,t) \cdot \mu_p \cdot E(x,t)}_{J_p^{drift}} - \underbrace{q \cdot D_p \cdot \frac{\partial p(x,t)}{\partial x}}_{J_p^{diff}} \quad [2]$$

The Poisson Equation

- A third important equation in semiconductors relates (static) electric field $E(x)$ to charge density $\rho(x)$ which is the effective free charge per unit volume (Coul./m³). It is actually the same as Gauss' Law in electrostatics, but in semiconductor physics it tends to be called the *Poisson Equation*.

- Assuming 1D space and static variables, this equation reads:

$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon}$$

- Where ϵ is the **electrical permittivity** with units of Farads/m. This is a fundamental property of a given material, directly related to the refractive index or to the speed of light in the material (do not confuse with energy \mathcal{E} as used earlier)

The Electrical Permittivity

- The electrical permittivity ϵ can be written as the following product:

$$\epsilon = \epsilon_r \cdot \epsilon_o$$

... where ϵ_o is the **permittivity of free space**, and is a fundamental constant of Nature: $\epsilon_o = 8.85 \times 10^{-12}$ F/m

... and where ϵ_r is described as the **dielectric constant** (or the **relative permittivity**) of the given material

- Common values for dielectric constant (it is dimensionless):
 - Vacuum: $\epsilon_r = 1.0$ (the value for air is usually also taken as 1.0)
 - Silicon (Si): $\epsilon_r = 11.8$
 - Gallium Arsenide (GaAs): $\epsilon_r = 12.7$
 - Silicon Dioxide (SiO₂): $\epsilon_r = 3.82$

The Poisson Equation in Semiconductors

- We can easily derive the charge density function $\rho(x)$ once we know the carrier and (ionised) doping concentrations, by simply multiplying each by 'q', taking due account of the sign.
- A quite general (static) form of the Poisson Equation in semiconductors is therefore:

$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon} = \frac{q}{\epsilon} \cdot [p(x) - n(x) + N_D^+ - N_A^-] \quad [3]$$

.... where N_D^+ is the no. of ionised donors per unit volume ($\approx N_D$ above 100K) and N_A^- is the no. of ionised acceptors per unit volume ($\approx N_A$ above 100K)

The Poisson Equation in Semiconductors

- A more general 1D *time-dependent* form of the Poisson Equation can be written as:

$$\frac{\partial E(x,t)}{\partial x} = \frac{q}{\varepsilon} \cdot [p(x,t) - n(x,t) + N_D^+ - N_A^-] \quad [3]$$

- Other forms of the equation exist. For example, using the relationship between electric field and voltage $V(x,t)$:

$$E(x,t) = -\frac{\partial V(x,t)}{\partial x}$$

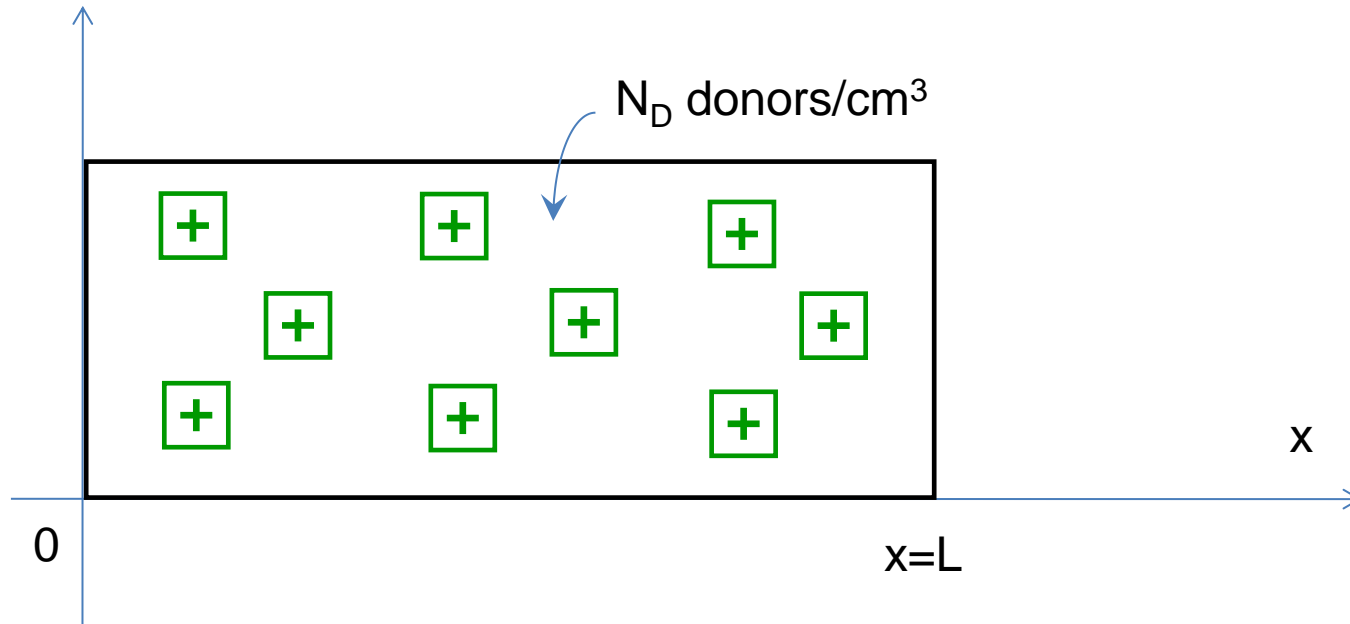
- We find:

$$\frac{\partial^2 V(x,t)}{\partial x^2} = -\frac{q}{\varepsilon} \cdot [p(x,t) - n(x,t) + N_D^+ - N_A^-]$$

- The special case where the RHS of this equation is zero is called *Laplace's Equation*

Poisson Equation: Worked Example

- A 1-D sample of Silicon at 300K has a length $L=3\mu\text{m}$ and is uniformly doped with 10^{15} **donors**/cm³. Assume that by some means all the free carriers have been removed. If the electric field E is zero at $x=0$, what is the electric field at $x=L$? What voltage drop is sustained across the sample?



- If each donor is ionised then each has a (positive) charge “q” and hence the charge density $\rho(x) = q \cdot N_D$ (Coul/m³ -a constant since uniformly doped)
- To find the electric field $E(x)$, use the Poisson Equation for this case:

$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon} = + \frac{q \cdot N_D}{\epsilon}$$

- Integrate both sides w.r.t. “x” to find $E(x)$:

$$\int_0^x \frac{dE(x)}{dx} \cdot dx = \int_0^x \frac{q \cdot N_D}{\epsilon} \cdot dx$$

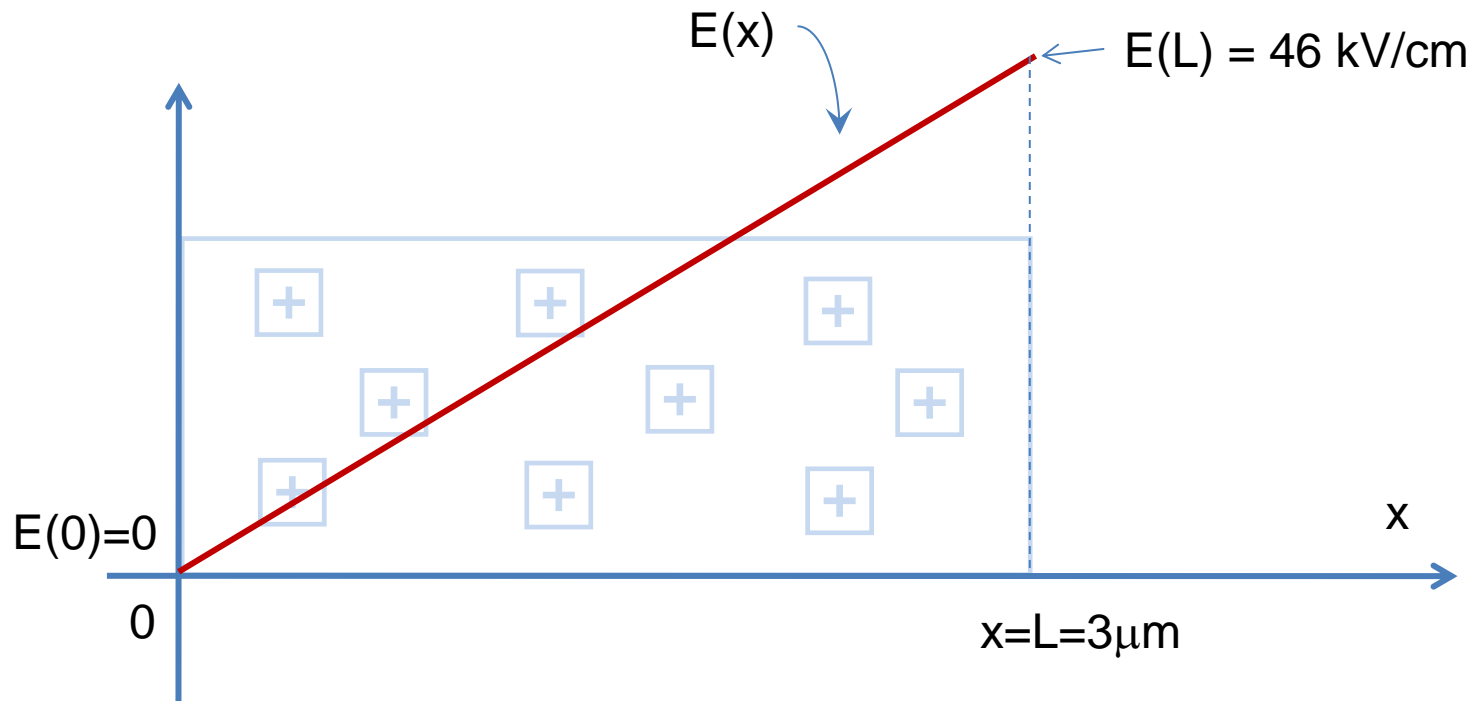
$$\Rightarrow E(x) - E(0) = \frac{q \cdot N_D \cdot x}{\epsilon}$$

- Hence the **electric field varies in a way that is linearly proportional to distance** when the charge density is **constant**. To find a numerical value for $E(L)$ we just substitute the given values (note: it is safer always to work in metres in problems involving permittivity!):

$$E(x) = \frac{q \cdot N_D \cdot x}{\epsilon} \Rightarrow E(L) = \frac{1.6 \times 10^{-19} \times 10^{21} \times 3 \times 10^{-6}}{11.8 \times 8.85 \times 10^{-12}} = 4.6 \text{ MV/m}$$

... or 46 kV/cm.

N.B. this = ϵ_r for Si.



- To find the **voltage dropped**, we can do it the hard way:

$$E(x) = -\frac{dV(x)}{dx}$$

... integrate both sides w.r.t. “x”, after substituting for “E(x)”:

$$\int_0^x \frac{dV(x)}{dx} \cdot dx = -\int_0^x E(x) \cdot dx = -\int_0^x \frac{q \cdot N_D \cdot x}{\epsilon} \cdot dx$$

$$\Rightarrow V(x) - V(0) = -\frac{q \cdot N_D \cdot x^2}{2 \cdot \epsilon}$$

- This shows that the voltage varies quadratically with “x” when the charge concentration is uniform. To find the voltage drop, one could substitute into the RHS with $x = L = 3\mu\text{m}$:

$$V(L) - V(0) = -\frac{q \cdot N_D \cdot L^2}{2 \cdot \epsilon} = -\frac{1.6 \times 10^{-19} \times 10^{21} \times (3 \times 10^{-6})^2}{2 \times 11.8 \times 8.85 \times 10^{-12}} = -6.9V$$

- There is, however, an easier way: since the voltage is the integral of the electric field, it can be found by just calculating the area under the graph of the electric field (taking account of the sign separately)...

$$V(L) - V(0) = -\frac{q \cdot N_D \cdot L^2}{2 \cdot \epsilon} = -\frac{1}{2} \cdot \left(\frac{q \cdot N_D \cdot L}{\epsilon} \right) \cdot L = -\frac{1}{2} \cdot E(L) \cdot L$$

... this is just the (negative) area under the triangle:

