EEEN 20060 – Communication Systems

Solutions to Problem Set 2

1. A transmitter outputs an analogue signal of bandwidth 1 MHz, centred on 1.8 MHz, with power 17 dBm. This signal travels on Cat5 cable (see lecture notes for attenuation graph). Write a mathematical expression for the signal power arriving at the far end of the cable, as a function of the length of the cable, *r*.

At 1.8 MHz, attenuation is 3 dB in a 100 m length of cable, so 0.03 dB/m (gain -0.03 dB/m). The power at distance *r* is therefore $17 - 0.03r$ dBm.

If you prefer to work with power in watts, the transmit power is 50 mW or 0.05 W. The attenuation factor for each metre length is 0.993116. The power at distance r is therefore 0.05×0.993116^r W.

2. In the system of problem 1, the receiver needs a signal-to-noise ratio of 20 dB, and generates noise power, in the relevant bandwidth, of 20 fW. What is the maximum cable length that will provide the required power at the receiver, without amplification? To simplify the calculations, you may assume that the cable contributes no noise or interference.

Noise power of 20 fW corresponds to -107 dBm. Signal power must be higher by 20 dB (factor of 100), so minimum receive power is -87 dBm (2 pW). This means maximum attenuation is 104 dB. At 0.03 dB/m, this corresponds to a cable length of 3467 m.

3. In the system of problems 1 and 2, propose a solution that will allow the signal to be received with a Cat5 cable of length of 5 km. Assume that any amplifier you use will generate, at its input, the same noise power as the receiver. Verify that your proposal will work.

Propose an amplifier at some point along the cable, well before the minimum signal-to-noise ratio is reached. For example, propose an amplifier at the mid-point of the cable. Attenuation on 2500 m of cable is 75 dB, so the amplifier could have gain of 75 dB, to bring the signal power back up to the transmit power level.

Verification: The signal power at the amplifier input will be -58 dBm, giving S/N of 49 dB. The signal power at the amplifier output will be 17 dBm, with the same S/N, so noise power at the amplifier output will be -32 dBm. At the receiver, the signal power will have been reduced to -58 dBm again. The noise from the amplifier will be at -107 dBm. This adds to the noise from the receiver, also -107 dBm. As these are independent noise sources, we can add the noise powers, giving total noise power at the receiver -104 dBm. This gives S/N 46 dB, well above the minimum required.

Note: In the general case, you would have to convert each noise power to watts to add them, but as the two noise powers are the same here, the sum will be twice one of them, and a factor of 2 corresponds to an increase of 3 dB.

4. In the system of problems 1 and 2, what is the maximum cable attenuation, in dB/km, that would allow operation over a length of 17.5 km without amplification?

As in problem 2, maximum attenuation is 104 dB. With length 17.5 km, this means maximum attenuation is 5.94 dB/km.

5. A radio transmitter outputs a signal of bandwidth 1 MHz, centred on 1.8 GHz, with power 17 dBm. The transmit antenna gain is 15 dB. The receiver needs a signal-to-noise ratio of 20 dB, and generates noise power, in the relevant bandwidth, of 20 fW. The receive antenna gain is 2.6 dB. What is the maximum distance at which the signal can be received correctly? You may assume *free-space propagation* – no obstacles or reflections along the path.

The basic equation is $P_{rx} = \frac{P_{tx}G_{tx}G_{rx}\lambda^2}{(4\pi)^2 r^2}$. Turning this around, $r^2 = \frac{P_{tx}G_{tx}G_{rx}}{P_{rx}}\frac{\lambda^2}{(4\pi)^2}$.

Thus $r = \sqrt{\frac{P_{tx}G_{tx}G_{rx}}{P_{rx}}}\frac{\lambda}{4\pi}$

Noise power of 20 fW corresponds to -107 dBm. Signal power must be higher, by 20 dB, so minimum receive power is -87 dBm. The first factor can now be calculated in dB: $\frac{P_{tx}G_{tx}G_{rx}}{P_{rx}} =$ 121.6 dB. Converting to a ratio and taking the square root gives $1.202\,264 \times 10^6$.

At 1.8 GHz, wavelength $\lambda = 0.1667$ m. This gives the second factor as 13.263×10^{-3}.

So maximum distance $r = 15\,945$ m.

6. Six devices share a channel, using fixed TDMA with a time frame of 10 ms. Their average transmission rates (including link-layer frame overhead) are shown in the table. Each transmission contains both source and destination addresses in the link-layer frame header. A guard time of 0.1 ms is needed between transmissions. Propose a suitable TDMA frame structure. Calculate the physical-layer bit rate required to support this.

Device	Bit rate (kbit/s)
A	800
B	480
C	100
D	100
E	100
F	300

As source address is included in header, no extra marker is needed to identify which transmission is which. Also, devices can monitor all transmissions, and know when it is their turn to transmit.

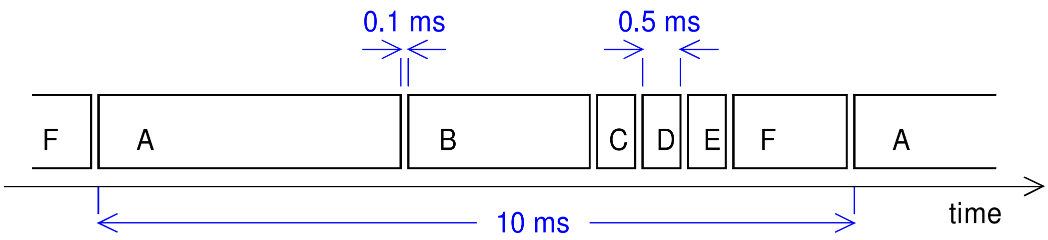
Proposal: devices will transmit in turn on the channel, in the order of the table. Each device will get a time allocation proportional to its need. Each transmission will be followed by a guard time of 0.1 ms.

Total bit rate is 1880 kbit/s, so in each 10 ms cycle, 18 800 bits must be transmitted. With six guard times in the cycle, the transmission time available in each cycle is 9.4 ms. Thus the physical-layer bit rate required is 2 Mbit/s.

The table below shows the allocation of time to each device, and how it is calculated.

Device	Bit rate (kbit/s)	Bits per cycle	Time slot needed (ms)	Guard time after (ms)
A	800	8000	4.0	0.1
B	480	4800	2.4	0.1
C	100	1000	0.5	0.1
D	100	1000	0.5	0.1
E	100	1000	0.5	0.1
F	300	3000	1.5	0.1
Total	1880	18800	9.4	0.6

The diagram shows the TDMA frame:



7. The six devices of problem 6 share a channel using token-passing. The device receiving the token may transmit one link-layer frame, 2000 bits long. The messages to be transmitted fit this frame exactly, so a device will either transmit a frame (if there is a message waiting), or not. The token is 50 bits long. The physical-layer bit rate is 4 Mbit/s and 0.1 ms must be allowed between transmissions from different devices (this includes propagation delay). Calculate the average cycle time and the link utilisation for device A. Sketch a typical cycle.

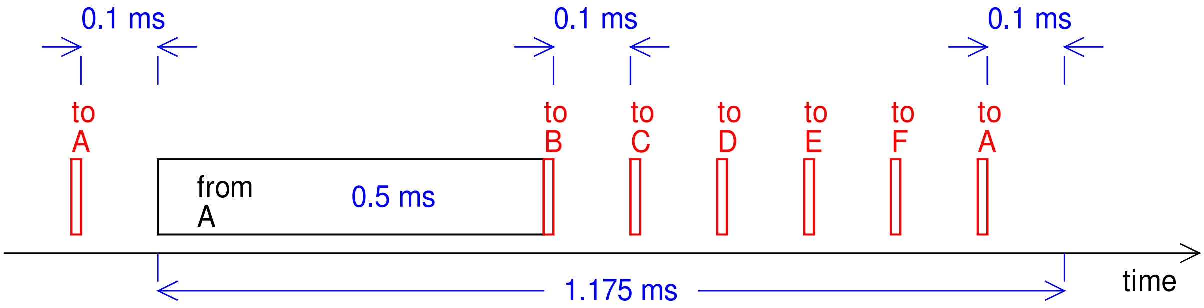
A frame of 2000 bits will be transmitted in $T_F = 0.5$ ms. A token of 50 bits will be transmitted in 12.5 µs. Latency is time to send six tokens, each followed by a gap of 0.1 ms, which gives $L = 0.675$ ms.

Device A transmits at 800 kbit/s, or $\lambda_A = 400$ message/s. If average cycle time is T_C, then average service rate is 1 message per cycle, or $\mu = \frac{1}{T_C}$. So link utilisation for device A is $\rho_A = \frac{\lambda_A}{\mu} = 400\ T_C$. This is also the probability of device A transmitting in any cycle.

A similar analysis for the other devices will give a total $\lambda = 940$ message/s, and average number of message transmissions per cycle $\rho = \sum \rho_i = 940\ T_C$.

Then average cycle time $T_C = L + T_F\rho = L + T_F\lambda T_C$. Algebra gives $T_C = \frac{L}{1 - T_F\lambda}$.
Substituting values gives $T_C = 1.2736$ ms, $\rho = 1.197$ and $\rho_A = 0.5094$.

A cycle will contain an average of 1.197 message transmissions, so a typical cycle will have one or sometimes two messages. The diagram below shows a cycle with only one message, from device A (as is most likely). The rest of the cycle is occupied by tokens (shown as red boxes). Note no delay between message from A and token from A to B.



8. The six devices in problem 7 share a channel using Aloha. The arrival of messages at each device can be modelled as a Poisson random process, and the backoff algorithm ensures that the transmission attempts can also be modelled as a Poisson random process. The messages and link-layer frames are as in problem 7. The physical layer bit rate is 20 Mbit/s. Find the probability of collision.

As in the solution to problem 7, the average rate of transmission of messages in the whole network is λ = 940 message/s. Due to collisions, some messages will involve more than one attempt to transmit. The overall attempt rate is λ', and is related to λ by the probability of success (no collision): $P_S = \frac{\lambda}{\lambda'} \Rightarrow \lambda = \lambda' P_S$

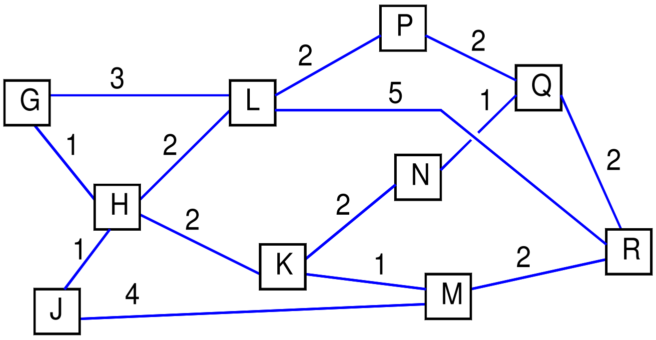
For a frame of 2000 bits at 20 Mbit/s, frame transmission time is T_F = 100 µs. With Aloha, success requires no other device attempting to transmit during an interval of length $2T_F$. With transmission attempts modelled as a Poisson random process, $P_S = e^{-\lambda' 2T_F}$.
Thus $\lambda = \lambda' e^{-\lambda' 2T_F}$. Substituting numbers: $940 = \lambda' e^{-2\times10^{-4}\lambda'}$.

To solve for λ' requires iteration: take an initial guess, calculate the value of λ that this would give, and use this to improve on the guess. For example, a guess of 1000 attempt/s will give a throughput of 818 message/s – too low. Trying 1200 attempt/s will give 944 message/s – too high, but much closer. Based on this, your next guess might be 1180 or 1190 attempt/s, both of which will give throughputs that are too low. Trying 1195 attempt/s gets quite close…

A value of 1193.4 attempt/s gives a throughput of almost exactly 940 message/s. Using this value, $P_S = \frac{\lambda}{\lambda'}$ = 0.78767, so probability of collision is 0.21233.

(A quicker calculation using a coarse approximation of 1200 attempt/s would give probability of collision 0.213.)

9. In the packet-switched network shown below, the cost of using each link is indicated beside the link. Draw up a routing table for use at node L, showing, for each destination, the next node to which a packet should be sent, and an alternative to be used in case of failure of the first choice.



Destination	Next Node	Alternative	Comment
G	G	H	Either G or H could be first choice
H	H	G	
J	H	G	
K	H	G	
M	H	R	G could also be alternative
N	P	H	
P	P	R	H could also be alternative
Q	P	R	H could also be alternative
R	R	P	

10. The network shown above uses distance-vector routing, with poisoned reverse – when generating a distance vector to send to one of its neighbours, a node ignores any routes where that neighbour is the next node. Write down the initial distance vector sent by node H to all of its neighbours. Draw up the distance table you would expect to find at node H when the network has been operating for some time. Write down the distance vector that node H would then send to node K.

Initial distance vector only includes known nodes, which are the neighbours. So vector is G:1, J:1, K:2, L:2.

After some time, table will include all nodes in network, with distance to each via each neighbour, as shown below.

Note that the distances in each column are based on information provided by that neighbour, and that neighbour is also applying the poisoned-reverse rule. The asterisks are not part of the table, but indicate where the distance would be different if the neighbour did not use the poisoned-reverse rule. The best route to each node is highlighted.

The distance vector sent to node K will give the lowest distance to each node, as in the table, except where that lowest distance is via node K – in those cases, the next lowest distance will be given: G:1, J:1, K:6, L:2, M:5, N:7, P:4, Q:6, R:7.

Destination	Dist. via G	Dist. via J	Dist. via K	Dist. via L
G	1	9 *	9 *	5
J	7 *	1	7 *	7 *
K	8 *	6 *	2	8 *
L	4	10 *	9 *	2
M	9 *	5	3	9 *
N	9 *	8 *	4	7
P	6	11 *	7	4
Q	8 *	9 *	5	6
R	9 *	7	5	7