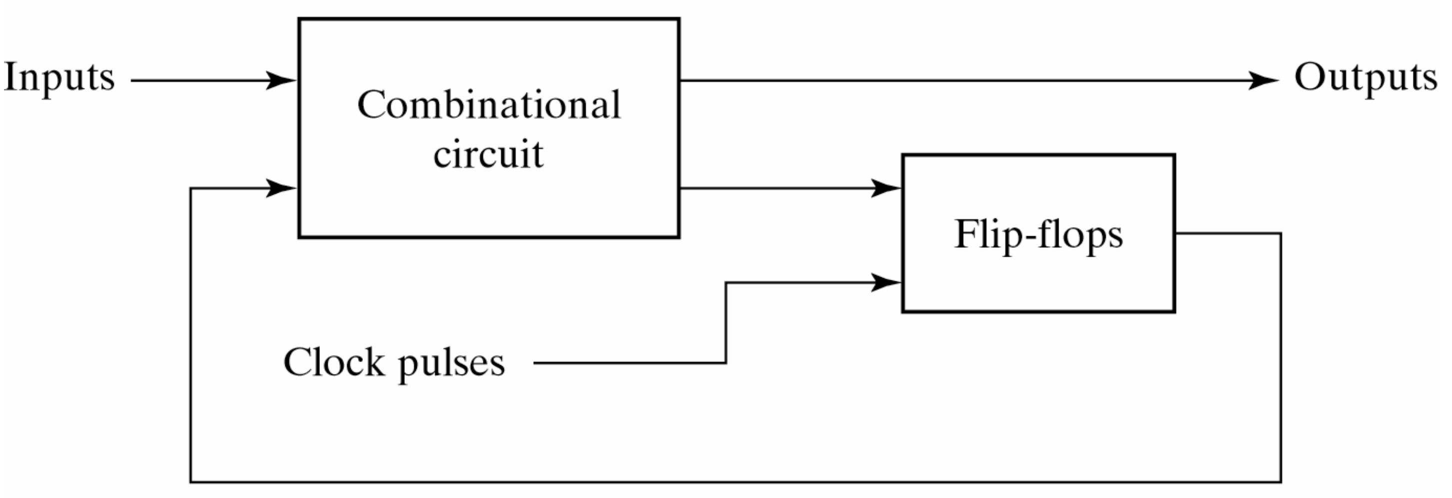
Sequential Logic II

Sequential Circuit Analysis

Sequential Logic

- Sequential logic contains both combinational logic circuits and memory storage elements (flip-flops).
- The value of the outputs of those systems is determined by external inputs and the present state of the storage elements.

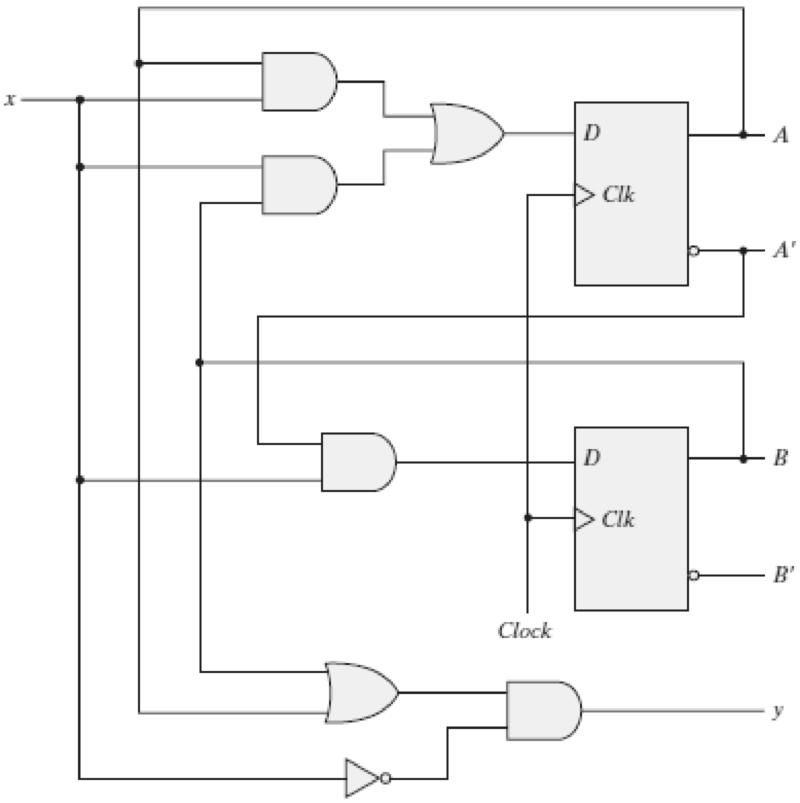


Sequential Circuit Analysis

- The behavior of a clocked sequential circuit is determined from the **inputs**, the **outputs** and the **state** of its flip-flops.
- The outputs and the next state are both a function of the inputs and the present state.
- The analysis of a sequential circuit consists of obtaining a function, a table or a diagram for the time sequence of inputs, outputs and internal states.
- There are three algebraic means for describing the behavior of sequential circuits: **State Equations**, **State Table** and **State Diagram**.

Sequential Circuit Analysis

- **State Equations**
  - Specify the next state as a function of the present state and inputs



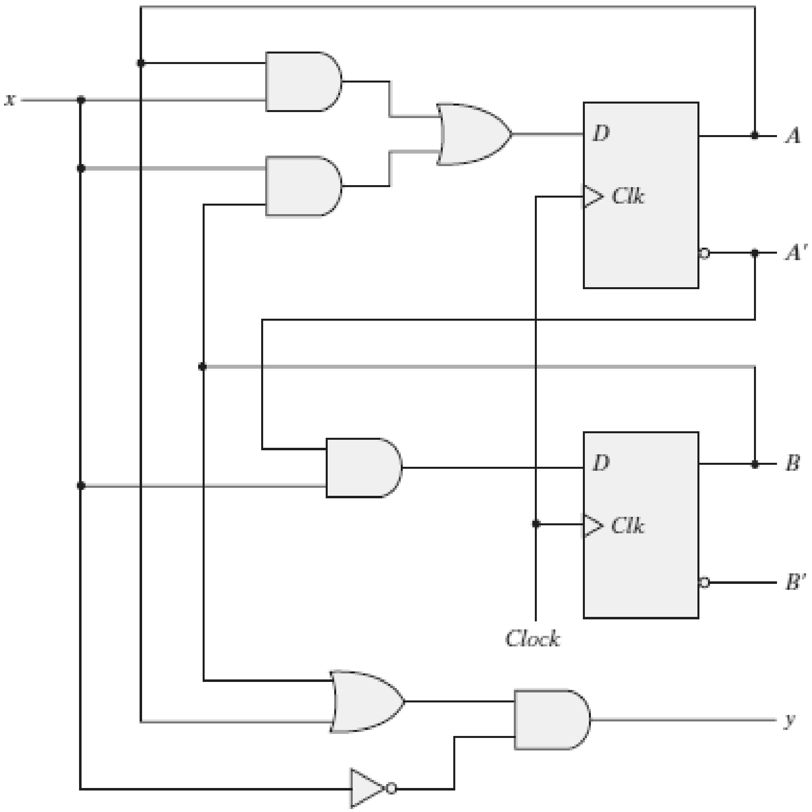
$A(t+1) = Ax + Bx$

$B(t+1) = A'x$

$y = (A+B)x'$

Sequential Circuit Analysis

- **State Table**
  - The time sequence of inputs, outputs and flip-flop states can be enumerated in a state table (sometimes called a transition table).



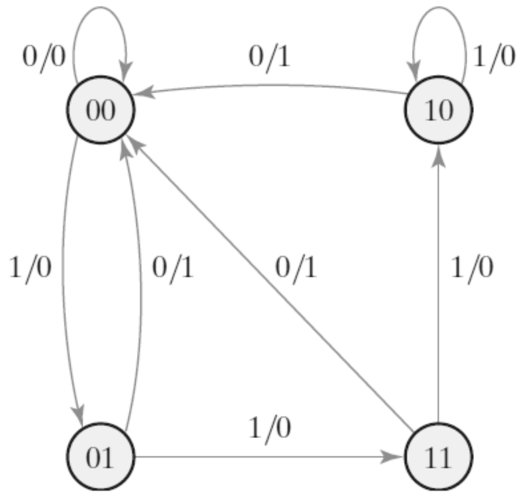
Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Sequential Circuit Analysis

- **State Diagram**
  - The information available in a state table can be represented graphically in the form of a state diagram.

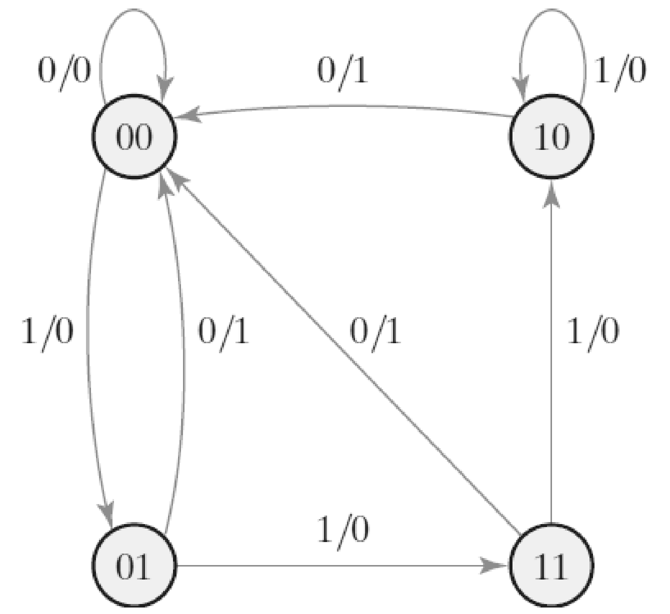
Second Form of the State Table

Present State		Next State				Output	
		$x = 0$		$x = 1$		$x = 0$	$x = 1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

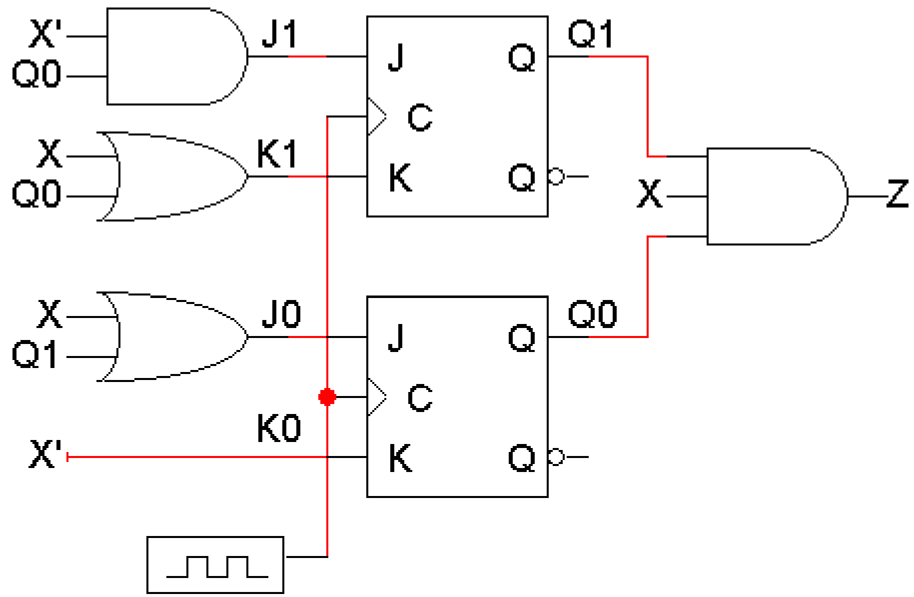


Sequential Circuit Analysis

- **State Diagram**
  - A state is represented by a circle, and the (clock-triggered) transitions between states are indicated by directed lines;
  - The binary number inside each circle indentifies the state of the flop-flops;
  - The directed lines are labelled with two binary numbers separated by a slash (input/output).



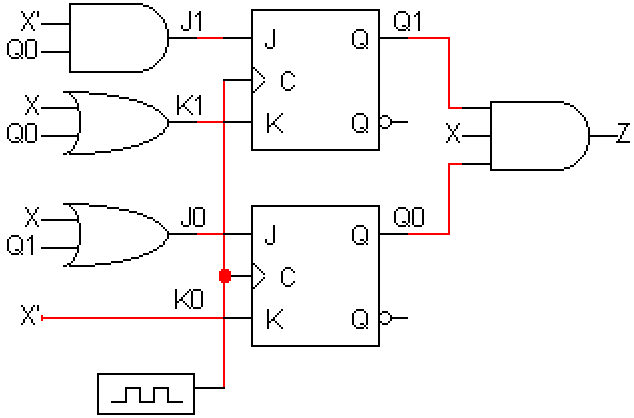
An example sequential circuit



- Here is a sequential circuit with two JK flip-flops. There is one input, X, and one output, Z.
- The values of the flip-flops (Q_1Q_0) form the state, or the memory, of the circuit.
- The flip-flop outputs also go back into the primitive gates on the left.

Analyzing the example circuit

- A basic state table for our example circuit is shown below.
- Remember that there is one input X, one output Z, and two flip-flops Q_1Q_0.
- The present state Q_1Q_0 and the input will determine the next state and the output.

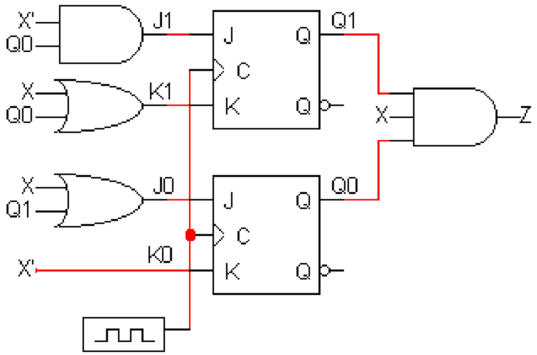


Present State Q_1	Present State Q_0	Inputs X	Next State Q_1	Next State Q_0	Outputs Z
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

The outputs are easy

- The output depends on the current state – Q0 and Q1 – as well as the inputs.
- From the diagram, you can see that

$Z = Q_1Q_0X$ Output at the current time



Present State		Inputs	Next State		Outputs
Q_1	Q_0	X	Q_1	Q_0	Z
0	0	0			0
0	0	1			0
0	1	0			0
0	1	1			0
1	0	0			0
1	0	1			0
1	1	0			0
1	1	1			1

Flip-flop input equations

- Finding the next states is harder. To do this, we have to figure out how the flip-flops are changing.

  Step 1:

  Find Boolean expressions for the flip-flop inputs.

  i.e. How do the inputs (say, J & K) to the flipflops depend on the current state and input

  Step 2:

  Use these expressions to find the actual flip-flop input values for each possible combination of present states and inputs.

  i.e. Fill in the state table (with new intermediate columns)

  Step 3:

  Use flip-flop characteristic tables or equations to find the next states, based on the flip-flop input values and the present states.

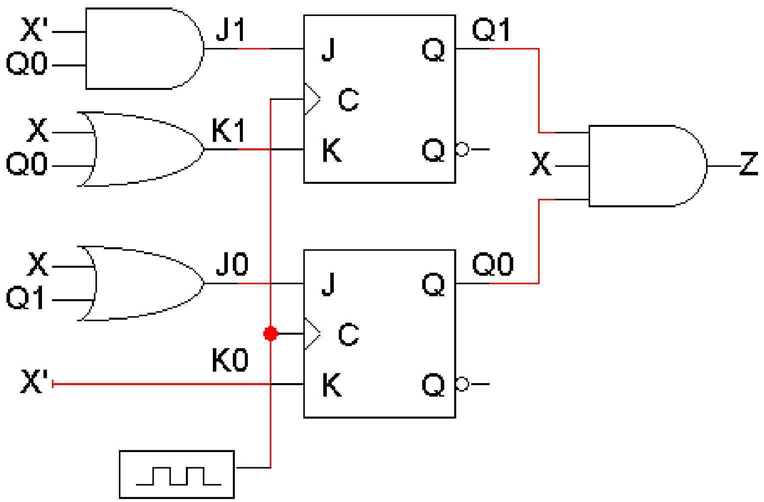
Step 1: Flip-flop input equations

- For our example, the flip-flop input equations are:

$$J_1 = X' Q_0$$
$$K_1 = X + Q_0$$

$$J_0 = X + Q_1$$
$$K_0 = X'$$

- JK flip-flops each have *two* inputs, J and K. (D and T flip-flops have one input each.)



Step 2: Flip-flop input values

- With these equations, we can make a table showing J_1, K_1, J_0 and K_0 for the different combinations of present state Q_1Q_0 and input X.

$$J_1 = X' Q_0 \qquad J_0 = X + Q_1$$
$$K_1 = X + Q_0 \qquad K_0 = X'$$

Present State		Inputs	Flip-flop Inputs			
Q_1	Q_0	X	J_1	K_1	J_0	K_0
0	0	0	0	0	0	1
0	0	1	0	1	1	0
0	1	0	1	1	0	1
0	1	1	0	1	1	0
1	0	0	0	0	1	1
1	0	1	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	1	1	0

Step 3: Find the next states

- Finally, use the JK flip-flop characteristic tables or equations to find the next state of *each* flip-flop, based on its present state and inputs.
- The general JK flip-flop characteristic equation is:

$$Q(t+1) = K'Q(t) + JQ'(t)$$

- In our example circuit, we have two JK flip-flops, so we have to apply this equation to *each* of them:

$$Q_1(t+1) = K_1'Q_1(t) + J_1Q_1'(t)$$
$$Q_0(t+1) = K_0'Q_0(t) + J_0Q_0'(t)$$

- We can also determine the next state for each input/current state combination directly from the characteristic table.

J	K	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

- Finally, here are the next states for Q_1 and Q_0, using these equations:

$$Q_1(t+1) = K_1'Q_1(t) + J_1Q_1'(t)$$
$$Q_0(t+1) = K_0'Q_0(t) + J_0Q_0'(t)$$

Present State		Inputs	FF Inputs				Next State	
Q_1	Q_0	X	J_1	K_1	J_0	K_0	Q_1	Q_0
0	0	0	0	0	0	1	0	0
0	0	1	0	1	1	0	0	1
0	1	0	1	1	0	1	1	0
0	1	1	0	1	1	0	0	1
1	0	0	0	0	1	1	1	1
1	0	1	0	1	1	0	0	1
1	1	0	1	1	1	1	0	0
1	1	1	0	1	1	0	0	1

The complete state table

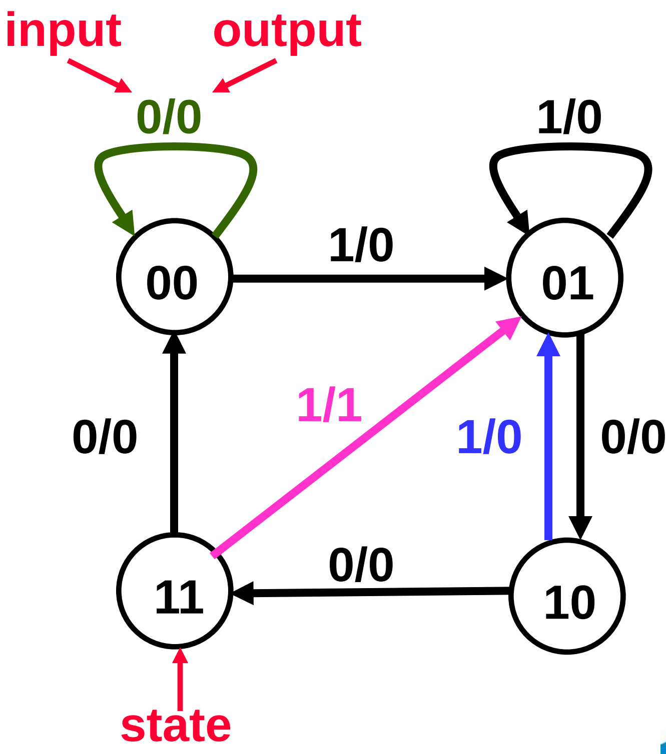
- The table starts with Present State and Inputs.
  - Present State and Inputs yield FF Inputs.
  - Present State and FF Inputs yield Next State, based on the flip-flop characteristic tables.
  - Present State and Inputs yield Output.

Present State		Inputs	FF Inputs				Next State		Outputs
Q_1	Q_0	X	J_1	K_1	J_0	K_0	Q_1	Q_0	Z
0	0	0	0	0	0	1	0	0	0
0	0	1	0	1	1	0	0	1	0
0	1	0	1	1	0	1	1	0	0
0	1	1	0	1	1	0	0	1	0
1	0	0	0	0	1	1	1	1	0
1	0	1	0	1	1	0	0	1	0
1	1	0	1	1	1	1	0	0	0
1	1	1	0	1	1	0	0	1	1

State Diagram

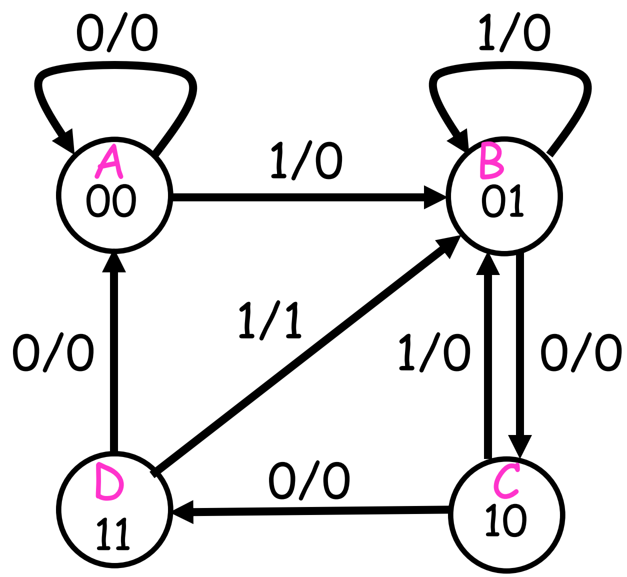
- We can also represent the state table graphically with a state diagram.
- A diagram corresponding to our example state table is shown below.

Present State Q_1	Present State Q_0	Inputs X	Next State Q_1	Next State Q_0	Outputs Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	1	1



- Always check the size of your state diagrams.
  - If there are *n* flip-flops, there should be 2^n nodes in the diagram.
  - If there are *m* inputs, then each node will have 2^m outgoing arrows.
    - From each state
- In our example,
  - We have two flip-flops, and thus four states or nodes.
  - There is one input, so each node has two outgoing arrows.

Present State Q_1	Present State Q_0	Inputs X	Next State Q_1	Next State Q_0	Outputs Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	1	1



Sequential Circuit Analysis Summary

- To analyze sequential circuits, you have to:
  - Find Boolean expressions for the outputs of the circuit and the flip-flop inputs.
  - Use these expressions to fill in the output and flip-flop input columns in the state table.
  - Finally, use the characteristic equation or characteristic table of the flip-flop to fill in the next state columns.

- The result of sequential circuit analysis is a state table or a state diagram describing the circuit.