Electrical Energy Systems

EEEN 20090

Transformers (Part 1)

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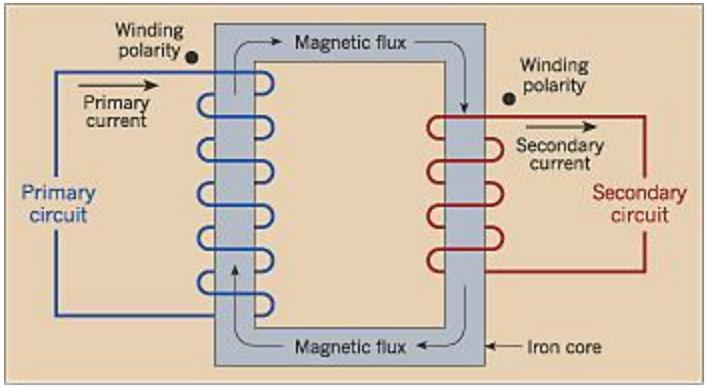
Overview

Part 1

- Why transformers – their place in the bigger system
- Ideal transformer
  - Impedance referral
  - Illustrate advantage of transformer – similar to the Edison illustration in Introduction lectures
- Some practical considerations, ratings, sizes, cooling

Part 2

- Model of “real transformer”
- Short and open circuit tests



Why Transformers

Why Important?

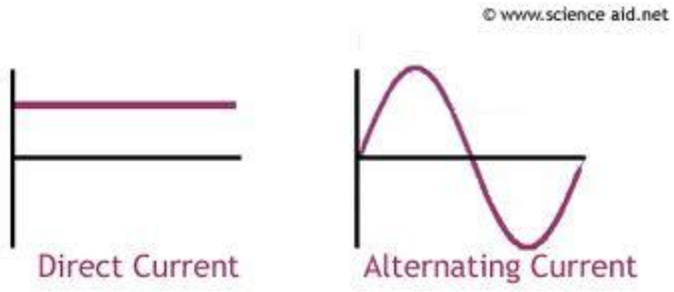
- Low Voltage => High Current=> High Losses

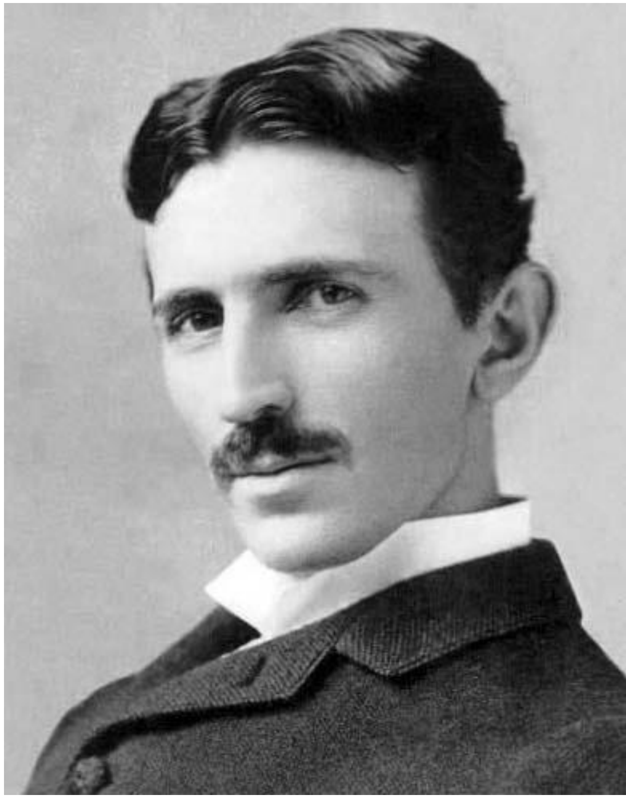
- High voltage => Low Current => Low Losses
  - But need insulation

- Transformer allows power transmission @ 220kV or 400kV in Ireland and up to 1MV elsewhere - China

- Used widely across power systems and in other applications

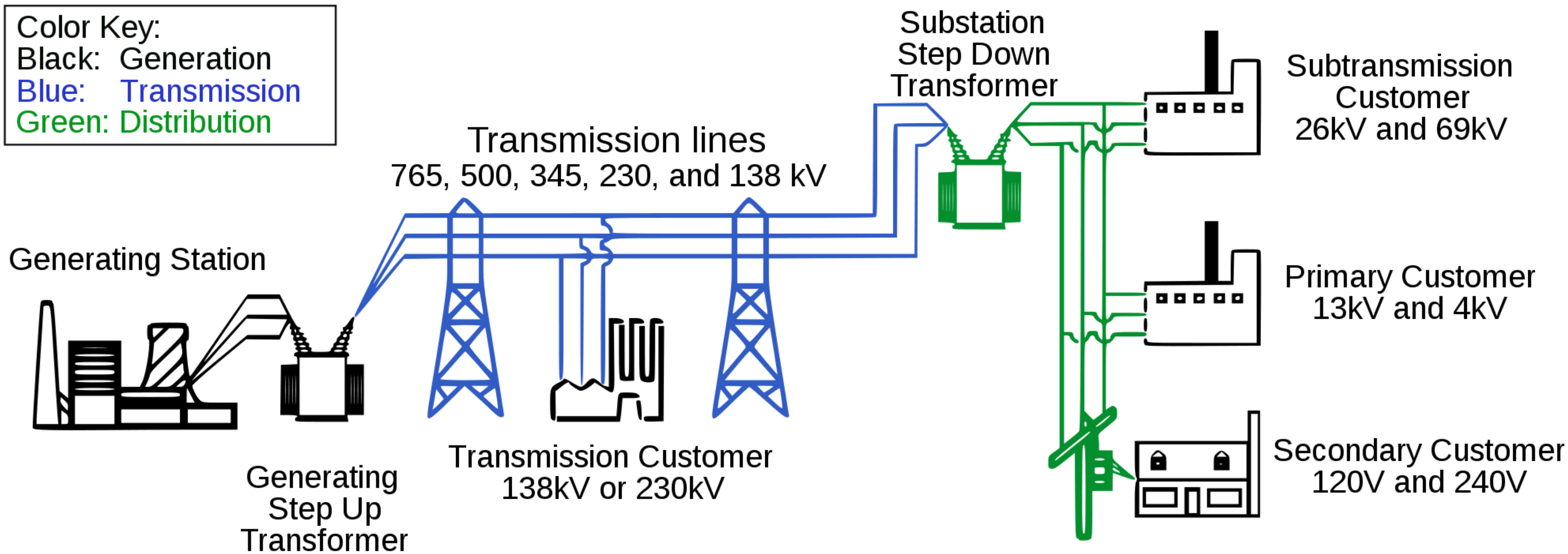
Edison and Tesla





The war of the currents

Basic Structure of the Electric System



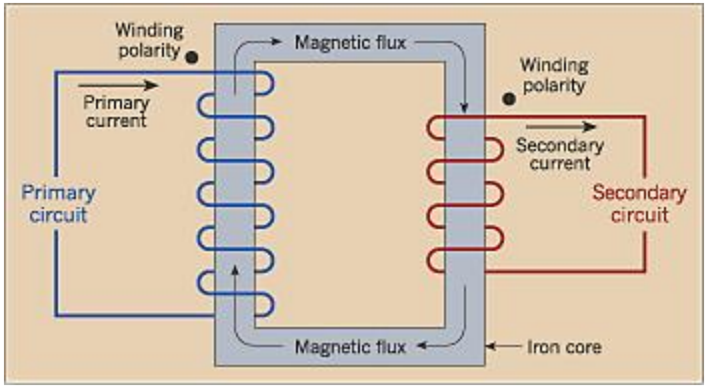
U.S.-Canada Power System Outage Task Force – August 14th Blackout: Causes and Recommendations

Transformers

Transformers

A **transformer** is a device for **transferring electric energy** from one circuit to another. Transformers are often used to **step up or down** supply voltages.

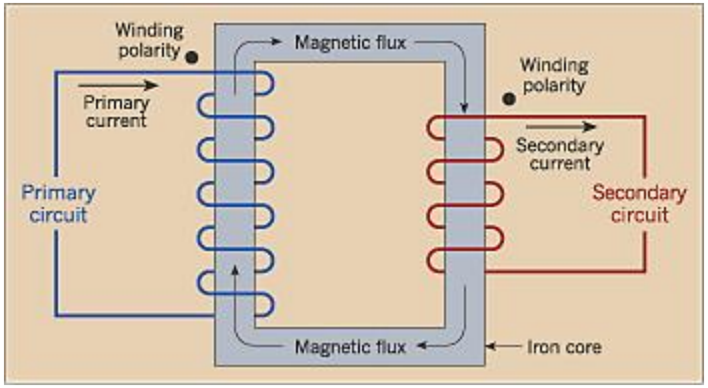
Exercise: Are there more step up transformers than step down ? Why?



Model of “ideal” Transformer

Transformers

A transformer is comprised of pair of magnetically-coupled coils, such that some of the magnetic flux produced by current in the primary coil links the turns of the secondary, and visa versa. The magnetic coupling can be improved by winding the coils on a common ferromagnetic (high μ) core. The coils are then known as 'windings' of the transformer.



Transformers

Since **magnetic induction** depends on the rate of change of the field with time, transformers are used with **periodically alternating voltages**.

Usually, there is **no electrical contact** between the two circuits, this is an important **safety feature**.

Most transformers are **highly efficient** and operate almost ideally.

Ideal transformer

- The Iron core has infinite permeability μ = infinity and has no losses
  - Infinitely small H gets infinite B
  - Reluctance is zero (equivalent to a short circuit in an electric circuit

- The windings have zero resistance

Applications: Transformers

Changing flux in a coil can also induce a *voltage* in a nearby coil:

To maximise the effect, place the coils on the same *iron core* – high permeability, so most of the flux stays in the iron, passes through both coils.

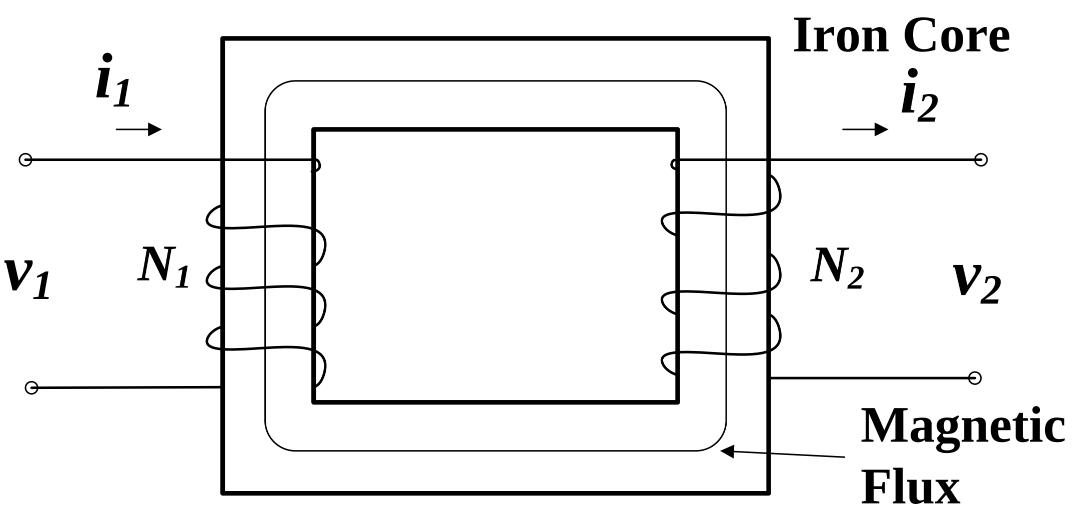
If same *flux*, $\phi(t)$, passes through both coils, then voltages:

$$v_1(t) = N_1 \frac{d\phi}{dt}$$

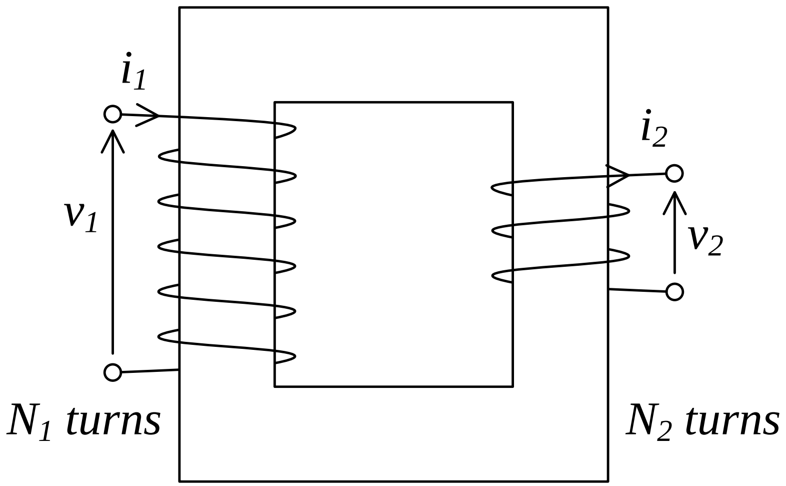
$$v_2(t) = N_2 \frac{d\phi}{dt}$$

Giving us

$$\boxed{\frac{v_2}{v_1} = \frac{N_2}{N_1}}$$



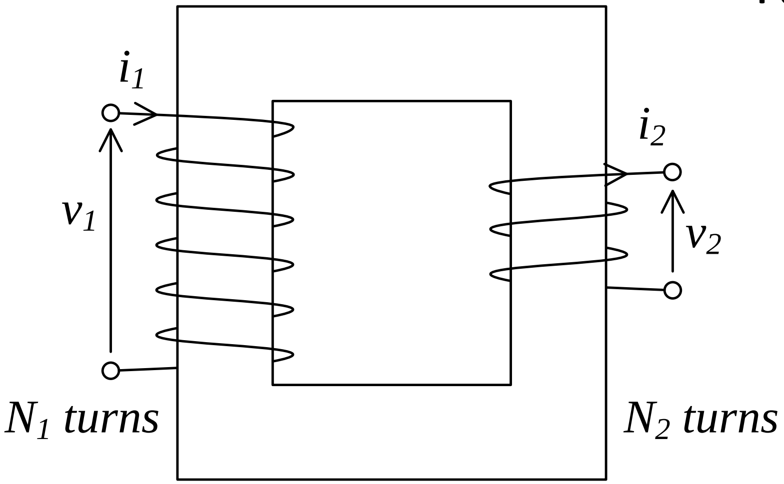
Ideal Transformer voltage relationship



$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

N_1/N_2 is the turns ratio secondary and primary

Ideal Transformer current relationship – power/energy considerations



No losses. Therefore,

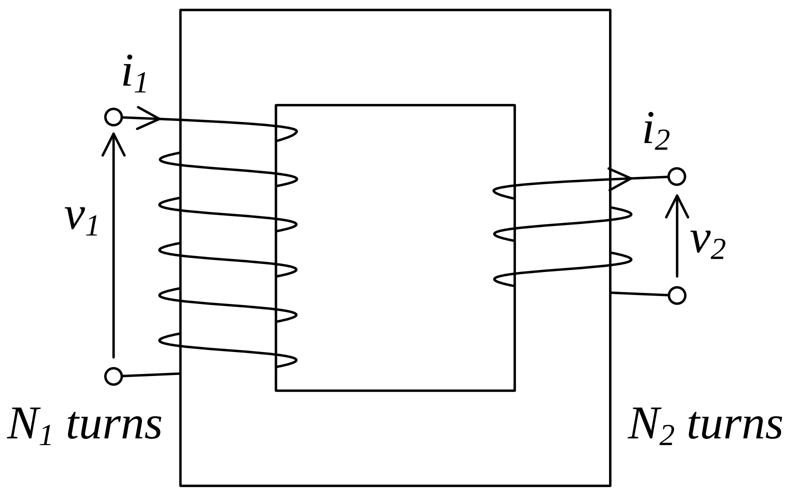
$$p_1 = p_2$$

$$v_1 i_1 = v_2 i_2$$

$$\frac{v_1}{v_2} = \frac{i_2}{i_1} = \frac{N_1}{N_2}$$

$$\boxed{N_1 i_1 = N_2 i_2}$$

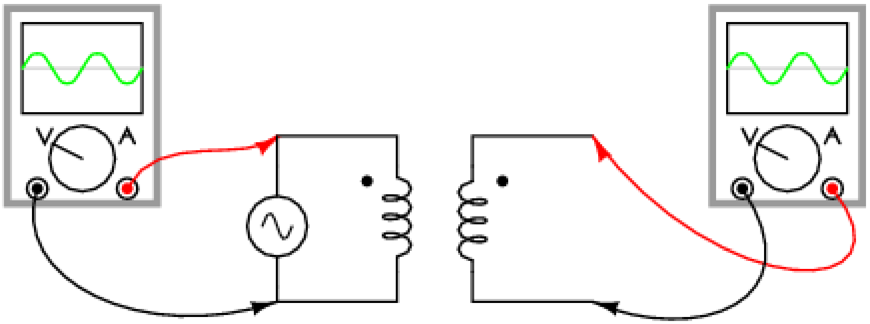
Ideal Transformer current relationship – magnetic circuits mmf1 = mmf2

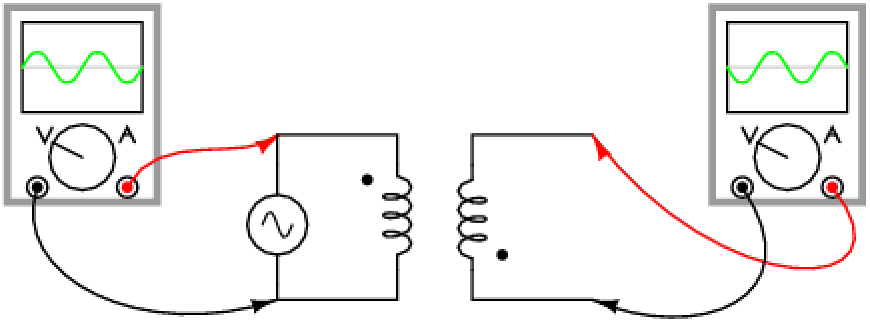


$$\boxed{N_1 i_1 = N_2 i_2}$$

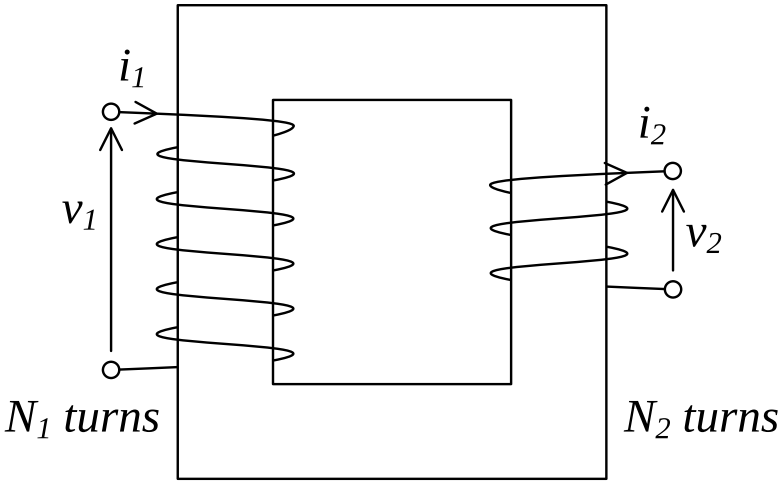
Zero reluctance so mmf1 = mmf2

Winding polarity



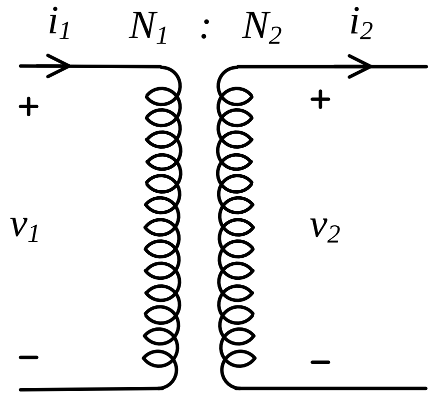


Ideal Transformer

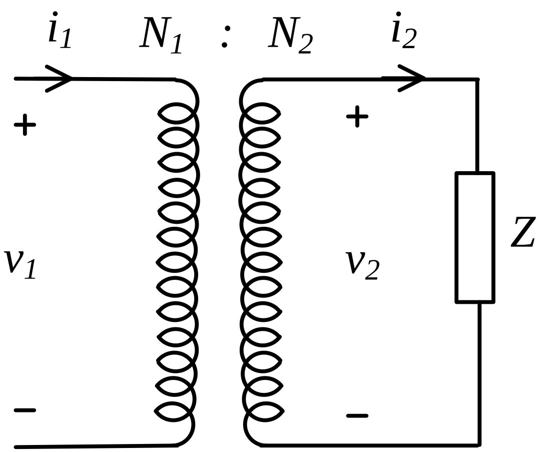


$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

$$\boxed{N_1 i_1 = N_2 i_2}$$



Referral of circuit element



$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

$$N_1 i_1 = N_2 i_2$$

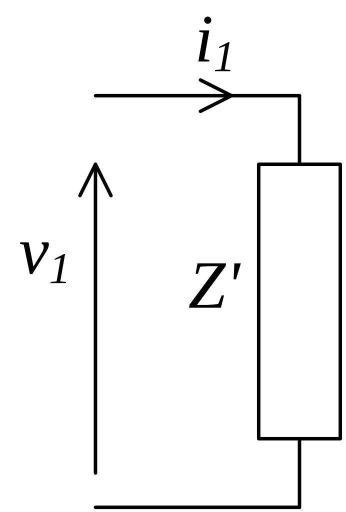
$$v_2 = Z i_2$$

$$= \left(\frac{N_2}{N_1}\right) v_1 = Z \left(\frac{N_1}{N_2}\right) i_1$$

$$\therefore v_1 = \left(\frac{N_1}{N_2}\right)^2 Z i_1$$

$$v_1 = Z' i_1$$

$$\boxed{Z' = \left(\frac{N_1}{N_2}\right)^2 Z}$$



Example 3.1:

A 230 V rms, 50 Hz supply is available to power a 10 kW, 50 V electric furnace. Assuming the required transformer is ideal and the electric arc furnace is a resistive load.

a) What should be its turns ratio?

b) Determine the primary and secondary current.

c) Determine the impedance as seen from the 230 V supply.

Note: rms

Example 3.1: Solution

a) Turns Ratio:

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{230}{50} = 4.6$$

$$\boxed{\frac{N_1}{N_2} = 4.6}$$

b) Primary & Secondary Current:

$$P = v_2 i_2 = v_1 i_1$$

$$\boxed{i_2 = \frac{10,000}{50} = 200A}$$

$$\boxed{i_1 = \left(\frac{N_2}{N_1}\right) i_2 = \frac{200}{4.6} = 43.48A}$$

Example 3.1: Solution cont.

c) Impedance seen from primary side ($Z^{'}$):

$$Z = \frac{v_2}{i_2} = 0.25\,\Omega$$

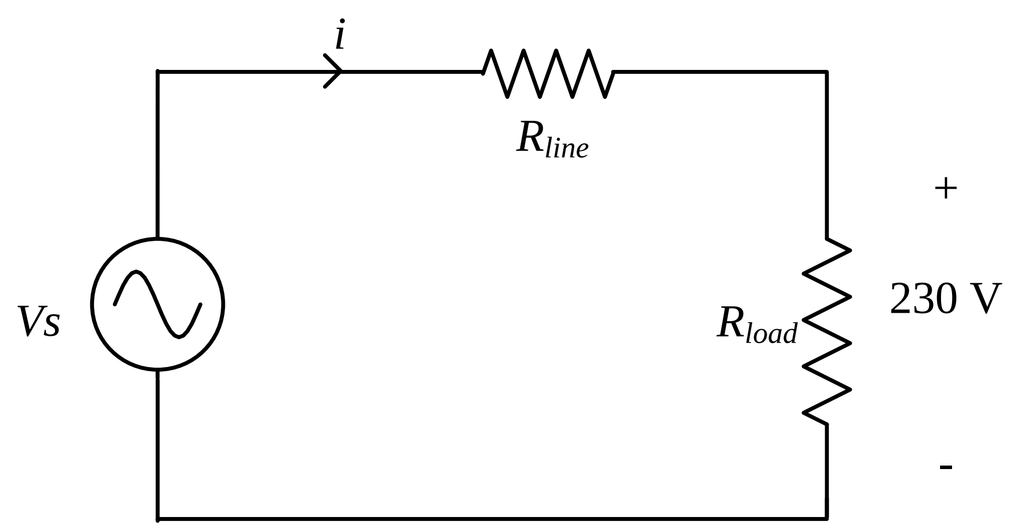
$$Z^{'} = \left(\frac{N_1}{N_2}\right)^2 Z$$

$$\boxed{Z^{'} = 4.6^2 Z = 5.29\,\Omega}$$

Example 3.2: Power Transfer

A large load centre consumes 1000 MW at 230 V rms.
The load is fed from a power station via a 100km resistive transmission line given by resistance R_{line} at an efficiency of $\eta = 0.95$.
The resistivity of the line is $\rho = 1.725 \times 10^{-8}\ \Omega m$.

a) What is the load resistance, R_{load}?
b) Find the required diameter of the line.



Example 3.2: Power Transfer-Solution

a) 230 V dropped across load

$$P = \frac{V^2}{R_{load}}$$

$$R_{load} = \frac{V^2}{P} = \frac{230^2}{1000 \times 10^6} = 52.9\,\mu\Omega$$

Example 3.2: Power Transfer-Solution cont.

b) Efficiency $\eta = \dfrac{P_{Load}}{P_{Load} + P_{Losses}} = \dfrac{R_{load} i^2}{R_{load} i^2 + R_{line} i^2}$

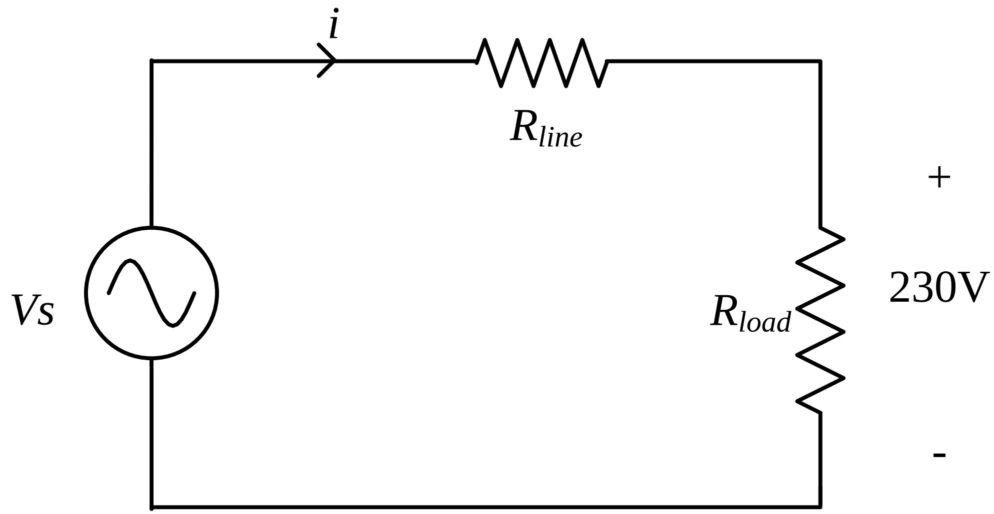
$$\therefore R_{line} = R_{load}\left(\frac{1-\eta}{\eta}\right) = 52.9\times10^{-6}\left(\frac{0.05}{0.95}\right)$$

$$R_{line} = 2.78\times10^{-6}\,\Omega$$

$$R_{line} = 2.78\times10^{-6}\,\Omega = \frac{\rho l}{\pi r^2}$$

$$r = 14.05\,\text{m}$$

$$\boxed{d = 28.10\,\text{m}}$$



Note: With the values for R_{load} and R_{line} now known, we can calculate Vs

$$\text{By voltage division}\quad \frac{R_{load}}{R_{load}+R_{line}}V_s = 230V$$

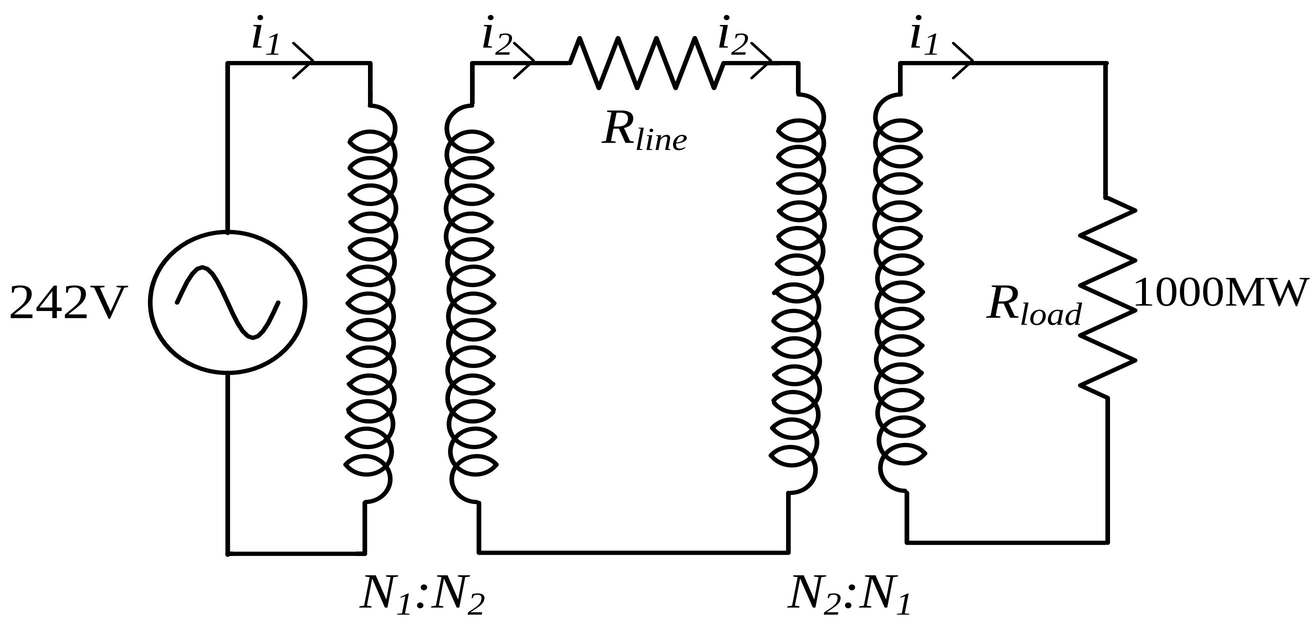
$$\therefore V_s = 230\left(\frac{R_{load}+R_{line}}{R_{load}}\right) = 230\left(\frac{52.9\times10^{-6}+2.78\times10^{-6}}{52.9\times10^{-6}}\right)$$

$$V_s = 242V$$

Example 3.2: Power Transfer

c) If a transformer is now employed to step up voltage at the power station and step it down again at load with a ratio, N_1:N_2 = 1:1000.

Find the required diameter of the line.



Example 3.2 (c): Power Transfer-Solution

c)

$$\eta = \frac{P_{Load}}{P_{Load} + P_{Losses}} = \frac{R_{load} i^2}{R_{load} i^2 + R_{line} i^2}$$

$$i_2 N_2 = i_1 N_1$$

$$i_2 = i_1 \frac{N_1}{N_2} = i_1 \frac{1}{1000}$$

$$\Rightarrow \eta = \frac{R_{load} i_1^{\,2}}{R_{load} i_1^{\,2} + R_{line} \left(i_1 \frac{N_1}{N_2} \right)^2}$$

$$R_{line} = \frac{R_{load} (1-\eta)}{\eta} \left(\frac{N_2}{N_1} \right)^2$$

Example 3.2 (c): Power Transfer-Solution cont.

$$R_{line} = \frac{52.9\times10^{-6}(1-0.95)}{0.95}\left(\frac{1000}{1}\right)^2$$

$$R_{line} = 2.78\,\Omega$$

$$R_{line} = 2.78 = \frac{\rho l}{\pi r^2}$$

$$\Rightarrow r^2 = 197.21\times10^{-6}$$

$$r = 14.05\,\text{mm}$$

$$\boxed{d = 28.10\,\text{mm}}$$

Stepping up voltage by a factor of 1000 has reduced required diameter by same factor

Practical Considerations

Practical Considerations

- permeability μ is not infinity
  - There is leakage flux
  - Some of the m.m.f (Ni) is needed to magnetise the core
- 95 % plus efficient but
  - non-zero winding resistance – i^2R losses
  - Magnetic losses in the core
    - eddy currents
    - hysteresis losses
- Oil cooled – force cooled
- Ratings MVA

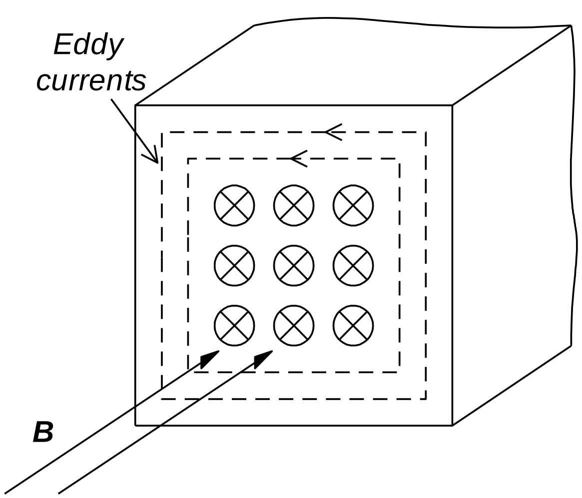
Eddy Losses

- If a **closed loop** of wire is placed in an **alternating magnetic field**, the induced voltage will cause **current** to flow around the loop.

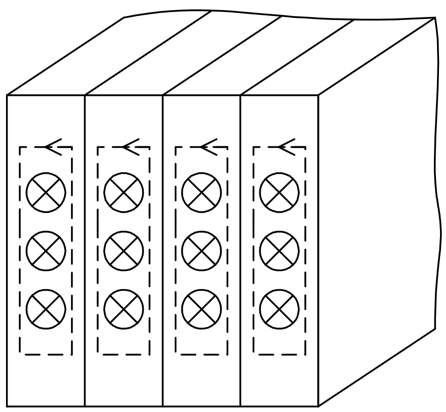
- In much the same way, a **solid block** of metal experiences currents induced in it by an alternating field. These are termed ***eddy currents***, and are a **source of energy loss (I^2R).**

- **Laminations** often used to **minimize** effect of eddy currents.

Eddy Currents

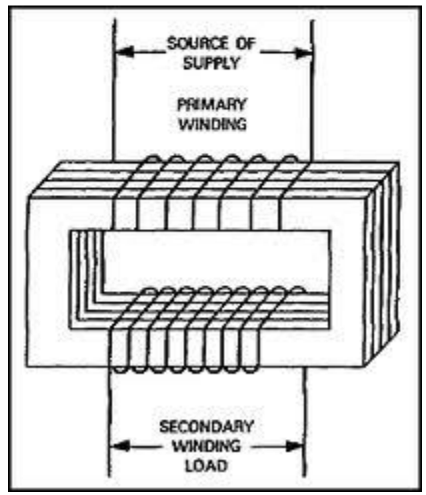


(a) a solid conductor



(b) a laminated conductor

Laminated cores



Example 3.3:

The eddy current loss in a material operating at frequency, f can be expressed as

$$p_e = k_e f^2 B_m^2 \text{ W/m}^3$$

where B_m is the maximum flux density and k_e, the characteristic constant of the material can be expressed as

$$k_e = K_e' d^2 / \rho$$

d being the thickness of the lamination and ρ the resistivity of the material.

If the resistivity is increased by a factor of 5 and the width of the material is doubled what would happen to the eddy current loss?

Example 3.3: solution

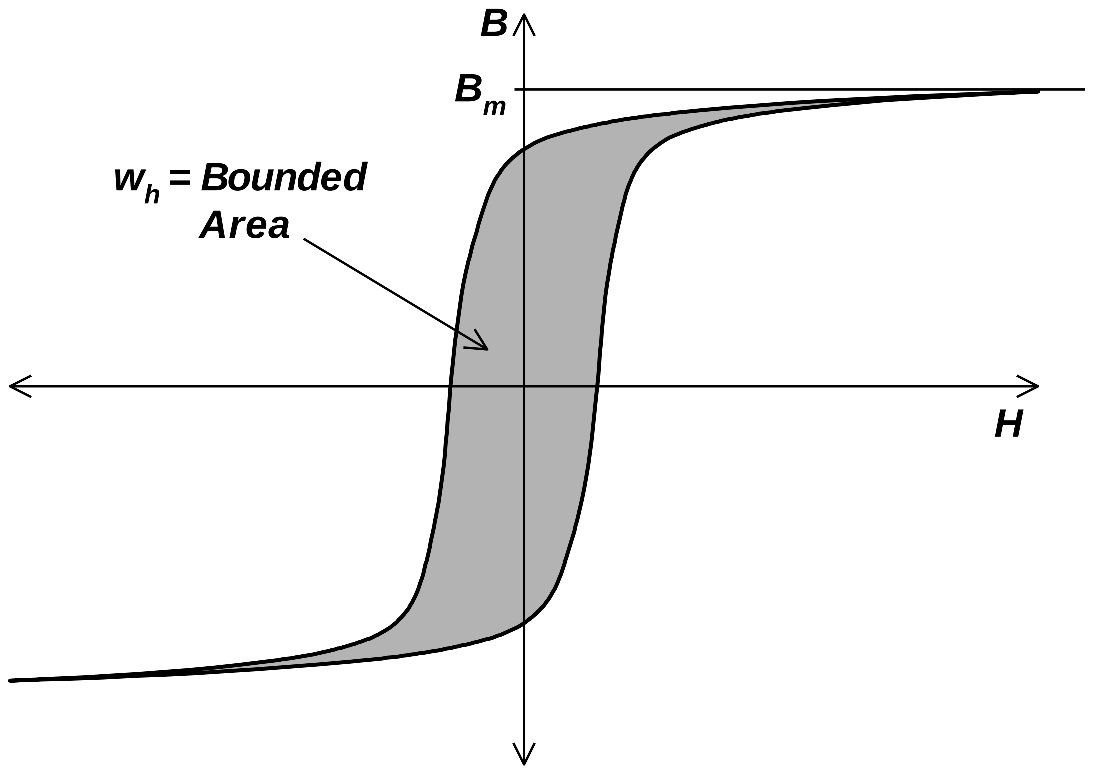
If d doubles and ρ increases by a factor of 5

$$k_e^{'} = K_e^{'}\left(2d\right)^2 / 5\rho = 0.8\left(K_e^{'} d^2 / \rho\right) = 0.8k_e$$

Therefore the eddy current loss p_e decreases by 20% as

$$p_e = k_e^{'} f^2 B_m^2 \text{ W/m}^3 = 0.8k_e f^2 B_m^2 \text{ W/m}^3$$

Hysteresis Loss



Energy is dissipated in the form of heat in a magnetic material which is taken through a magnetisation cycle.

Hysteresis Loss

- $B=f(H)$
- For a given H, the value of B actually depends on the history of how B and H have varied.
- If B is varying sinusoidally, the relationship settles into the familiar cycle (Anti Clockwise)
- Dependent on frequency, i.e. 50 Hz sinusoid means that it is traced out 50 times per second
- Losses proportional to the area of BH curve

Hysteresis Loss

Consider the behaviour of iron over one cycle. First apply Faraday's law for the voltage *v* across the coil.

$$\begin{aligned} v &= N\frac{\mathrm{d}\phi}{\mathrm{d}t} \\ &= N\frac{\mathrm{d}}{\mathrm{d}t}(BA) \\ &= NA\frac{\mathrm{d}B}{\mathrm{d}t} \end{aligned}$$

Apply Ampere's law for the current *i*.

$$\oint \vec{H}\cdot \mathrm{d}\vec{l} = i_{\mathrm{encl}} = Ni$$

$$H \cdot l = Ni$$

$$H = \frac{Ni}{l}$$

$$i = \frac{l}{N}\cdot H$$

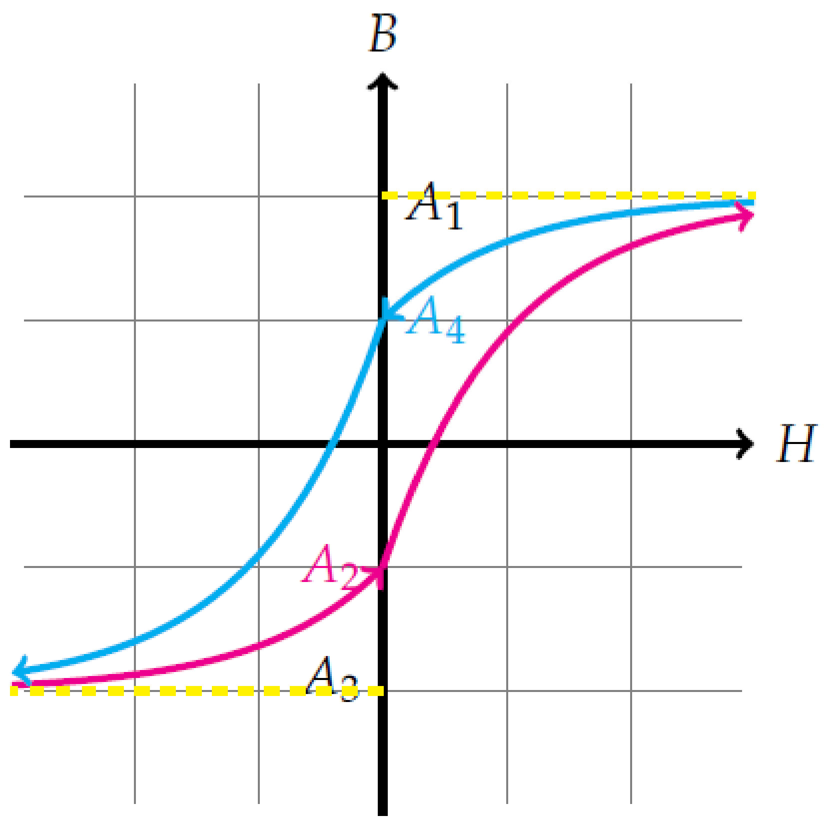
Hysteresis Loss

These expressions allow us to work out the power P, from which we get the energy W:

$$
\begin{aligned}
P(t) &= v(t)i(t) \\
W &= \int v(t)i(t)\,\mathrm{d}t \\
&= \int NA\tfrac{\mathrm{d}B}{\mathrm{d}t}\cdot\frac{l}{N}H\,\mathrm{d}t \\
&= \int A\tfrac{\mathrm{d}B}{\mathrm{d}t}\cdot lH\,\mathrm{d}t \\
&= lA\int H\tfrac{\mathrm{d}B}{\mathrm{d}t}\cdot\mathrm{d}t \\
&= lA\int H\cdot\mathrm{d}B \\
&= \text{volume}\int H\cdot\mathrm{d}B
\end{aligned}
$$

Hysteresis Loss

Divide the hysteresis curve into A_1, A_2, A_3, A_4

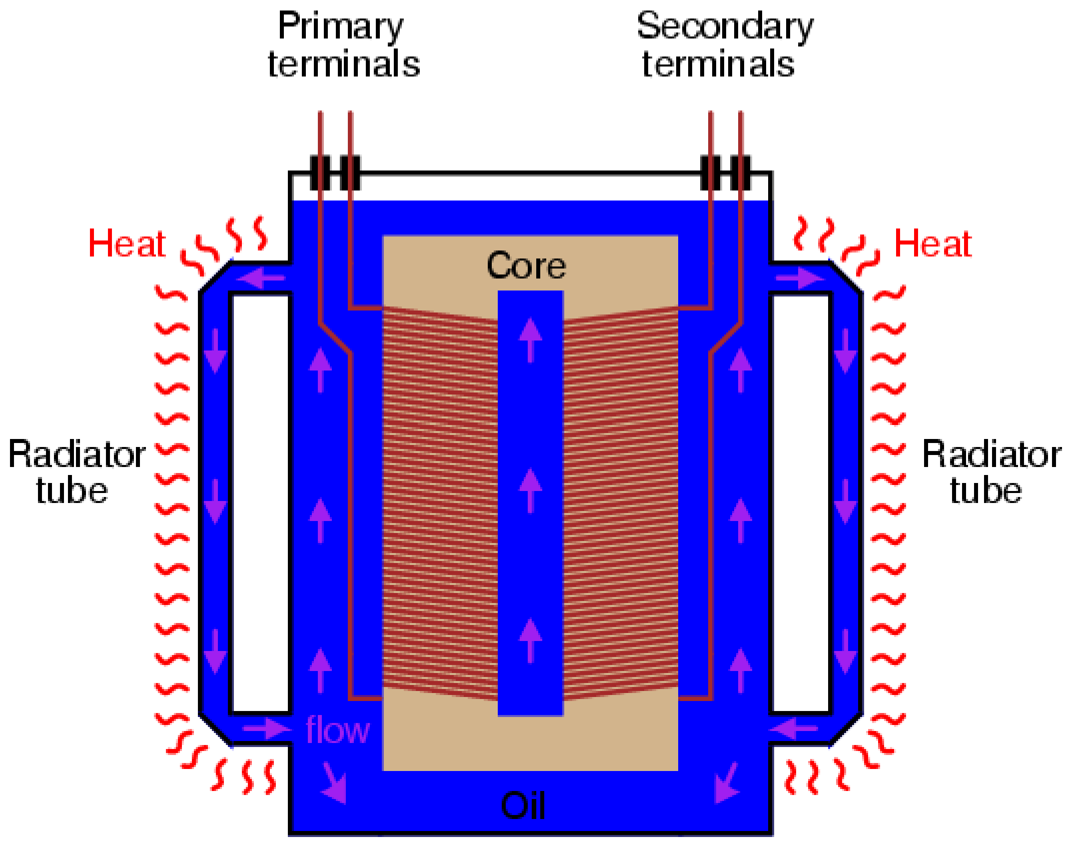


Hysteresis Loss

$$\begin{aligned}\int i(t)v(t) &= \mathrm{vol}(-A_1 + (A_2 + A_3) - A_3 + (A_4 + A_1)) \\ &= \mathrm{vol}(A_2 + A_4) \\ &= \mathrm{vol}(\text{Area on } B, H \text{ curve inside hysteresis loop})\end{aligned}$$

which is the energy from the electrical supply to the winding on one cycle. It is dissipated in the material. Since power is energy loss per second, this is equal to the volume times the area between the hysteresis curve times the frequency.

Cooling Transformers



Ratings

- VA volt amps , kVA, MVA etc.
- Not kW, MW etc.
- Voltage rating and a current rating