EEEN20030 Engineering Electromagnetics

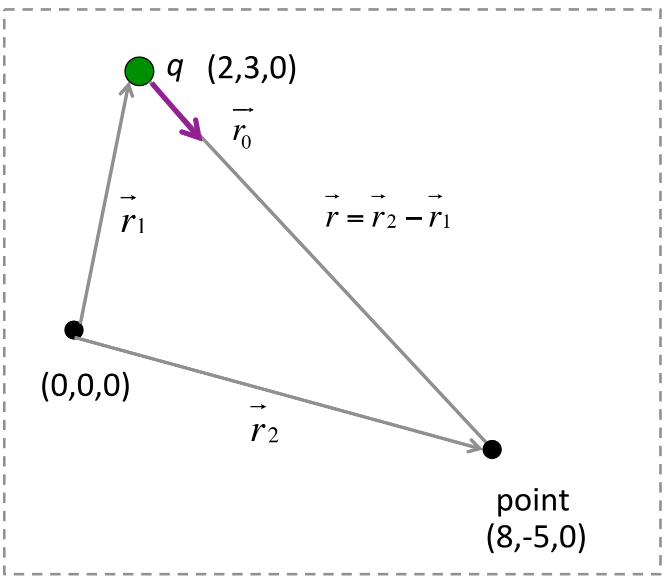
Semester 1, 2014/2015

Chapter 4

Notes and Examples

In-Class Example: Electric Field due to a Point Charge

A point charge q = 50μC is located at (2,3,0)m in free space. Find the electric field intensity (***E***) at point located at (8, -5, 0)m.



See page 71 --- the formula for the electric field due to a point charge at an arbitrary point.

Answer:

$$\vec{E} = E \cdot \vec{r}_0 =$$

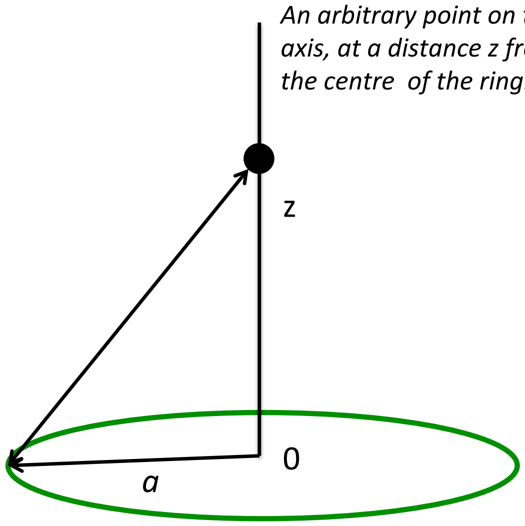
$$= 4.5 \cdot 10^3 (0.6\vec{i} - 0.8\vec{j}) \text{V/m}$$

In-Class Example: Electric Field due to a Line Charge Distribution

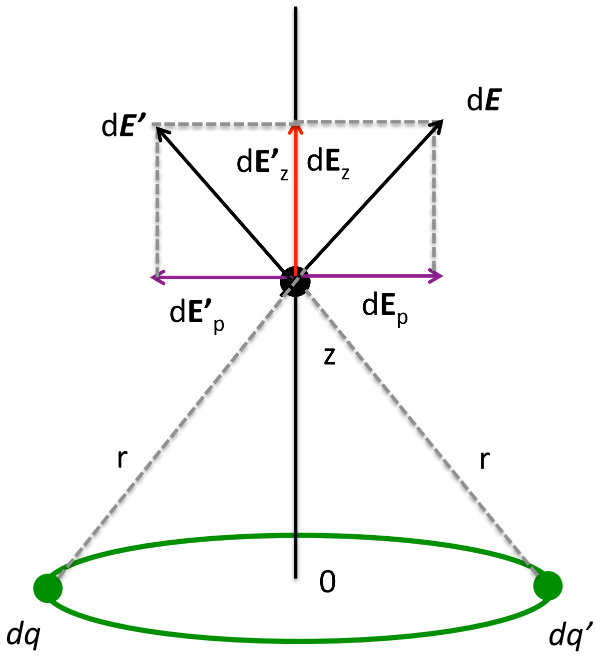
Find the electric field on the axis of a uniformly charged circular ring (see the figure).

Take the radius of the ring a = 10 cm, the uniform line charge density $\lambda = 10^{-4}$ C/m and the distance z = 20 cm. What is the electric field at this point?

At what distance can you consider this charged ring as a point charge?



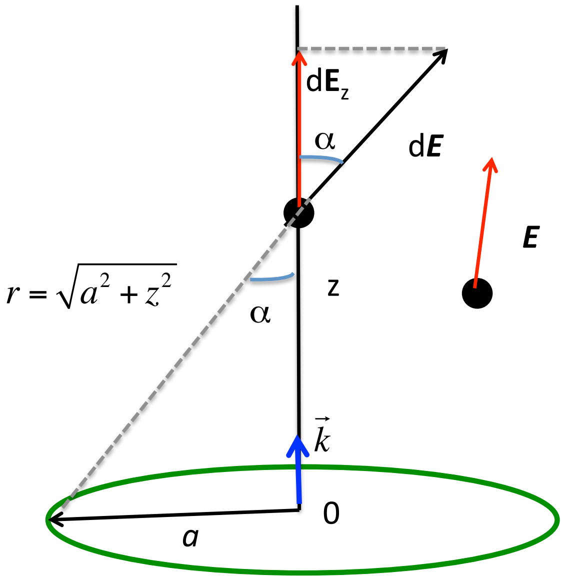
First, let us understand what is the direction of the electric field:



A portion of charge $dq=\lambda dl$ on the ring will create an electric field d**E** (vector) at the point z. We can present it in terms of its components $d\mathbf{E}_z$ (along the z-axis) and $d\mathbf{E}_p$ (in the plane of the ring – perpendicular to the z-axis)

This charge has its "counterpart" --- a charge *dq'* lying on the opposite side of the ring. This counterpart charge of the same amount *(dq'=dq)* will also create an electric field d**E'** at the point *z*. We can present it in terms of its components $d\mathbf{E'}_z$ and $d\mathbf{E'}_p$.

Because the system is symmetrical: dq = dq' and the distance is the same, we can state that $d\boldsymbol{E}_z = dE'_z$ and $d\boldsymbol{E}_p = -d\boldsymbol{E}'_p$. I.e., <u>all perpendicular components will cancel each other. Only the components along *z* will stay</u>.



Thus, the electric field due to this ring at an arbitrary point on the z-axis will be directed along the z axis. If we introduce a unit vector ***k***, we can write that

$$\vec{E}(z) = E_z \cdot \vec{k} \quad \text{(above the ring)}$$

This is the case only for the points on the z-axis. For an arbitrary point, the electric field will have both components, $\boldsymbol{E}_z$ and $\boldsymbol{E}_p$.

As soon as the direction of the electric field is found, we can determine its magnitude.

Page 76,
See "integrals"

Answers:

The electric field on the axis of the ring:

$$E(z) = \frac{1}{4\pi\varepsilon_0} \frac{Q\,z}{\left(a^2 + z^2\right)^{3/2}}$$

In the limiting case z>>a:

$$E(z) \approx \frac{1}{4\pi\varepsilon_0} \frac{Q}{z^2}$$

Now substituting numerical values:

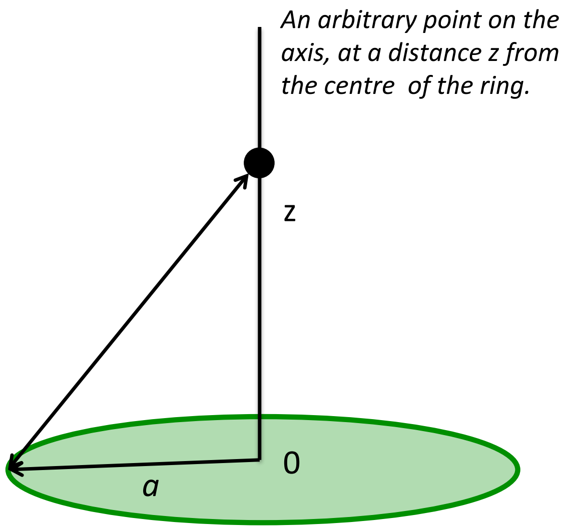
$$Q = 2\pi a \lambda = 6.3 \cdot 10^{-5} \quad \text{C}$$

$$E = \frac{1}{4 \cdot 3.14 \cdot 8.854 \cdot 10^{-12}} \frac{6.3 \cdot 10^{-5}\ 0.2}{\left(0.01 + 0.4\right)^{3/2}} \approx 1 \cdot 10^{7} \quad \text{V/m}$$

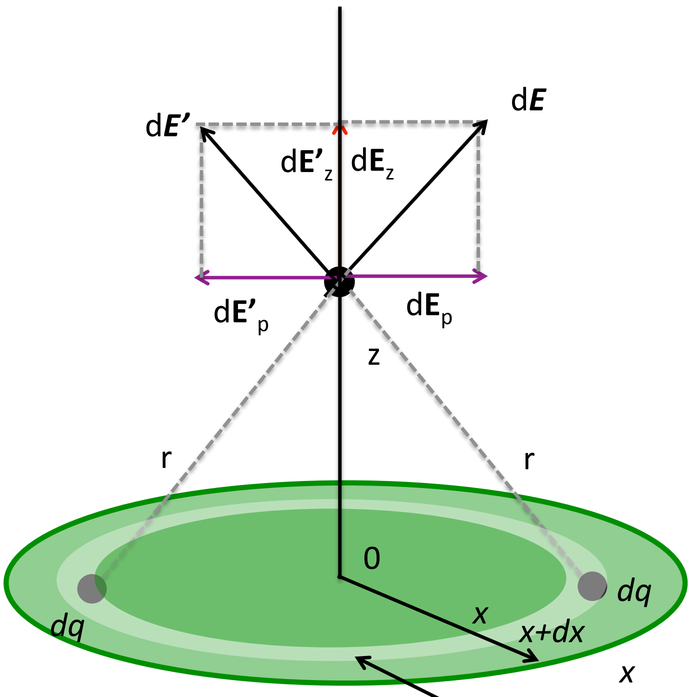
In-Class Example: Electric Field due to a Surface Charge Distribution

Find the electric field due to a uniformly charged circular disk (of radius *a* with surface charge density σ) along the axis *z* that passes through its center (see the figure).

Check that in the limiting case z>>a you will obtain the electric field due to a point charge whose charge is $Q=\sigma A_{disk}$



First, let us understand what is the direction of the electric field:



The systems is symmetrical, see the problem "Electric field due to a uniformly charge ring".

Along the z-axis: all perpendicular components of the electric field will cancel each other.

$$\vec{E}(z) = E_z \cdot \vec{k} \quad \text{(above the disk)}$$

The radius of the ring is *a*.

The area of this "belt" is $ds = 2\pi x\, dx$

Answers:

The electric field on the axis of the disk:

$$E(z) = \frac{1}{4\pi\varepsilon_0}\sigma 2\pi\left(1 - \frac{z}{\sqrt{z^2 + a^2}}\right)$$

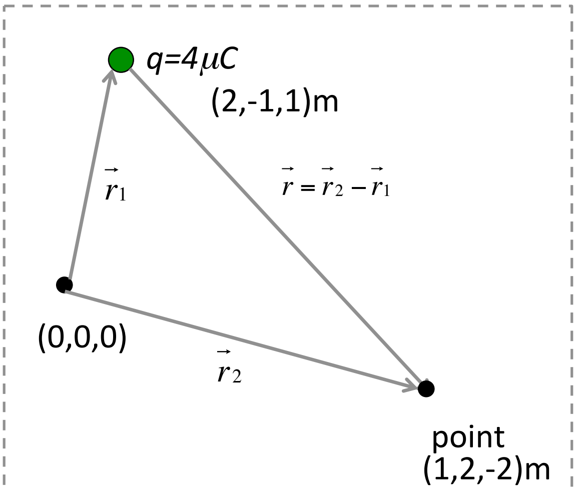
Check the limiting case z>>a:

$$E(z) \cong \frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi a^2}{z^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}$$

In-Class Example: Electric Field, Flux Density and Flux due to a Point Charge

Calculate the electric field at (1,2,-2)m due to a 4μC point charge at (2,-1,1)m. Using this value calculate the electric flux density at (1,2,-2) m.

Then calculate the total electric flux passing through a sphere of radius $\sqrt{19}$ m centered on the point charge. Assume ε=ε0.



Answers:

Direction vector:

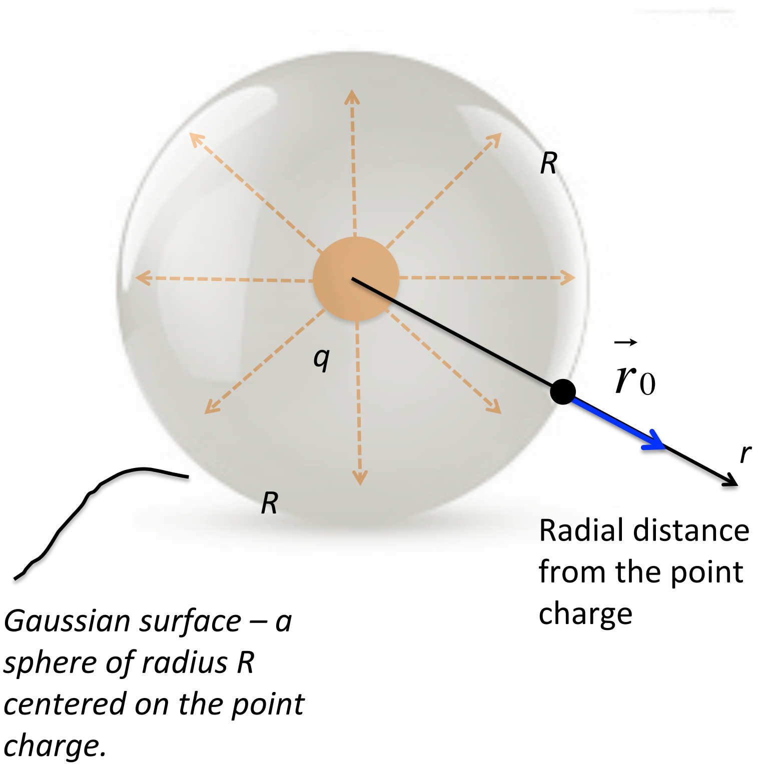
$$\vec{r}_0 = \vec{r}\,/\,r = \left(-\frac{1}{\sqrt{19}}, \frac{3}{\sqrt{19}}, -\frac{3}{\sqrt{19}}\right) m$$

$$\vec{E} = 1.89 \cdot 10^3\, \vec{r}_0 \,\mathrm{V/m}$$

$$\vec{D} = 1.675 \cdot 10^{-8}\, \vec{r}_0 \ \mathrm{C/m^2}$$

$$\psi = 4 \cdot 10^{-6}\ \mathrm{C}$$

In-Class Derivation: Proof of Gauss' Law for a Spherical Surface



The electric field E due to the point charge q *is:*

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\vec{r}_0$$

The surface of the sphere (in spherical coordinates) is given by equation r=R.

The electric field ***E*** due to the point charge *q is:*

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\vec{r}_0$$

On the surface of the sphere, *r=R,* and the electric field is

$$\vec{E}(r)\Big|_{r=R} = \frac{1}{4\pi\varepsilon}\frac{q}{R^2}\vec{r}_0$$

Let us now calculate the flux through the sphere (it is a closed surface)

$$\psi = \oiint_{Sphere} \varepsilon\vec{E}\cdot d\vec{s}$$

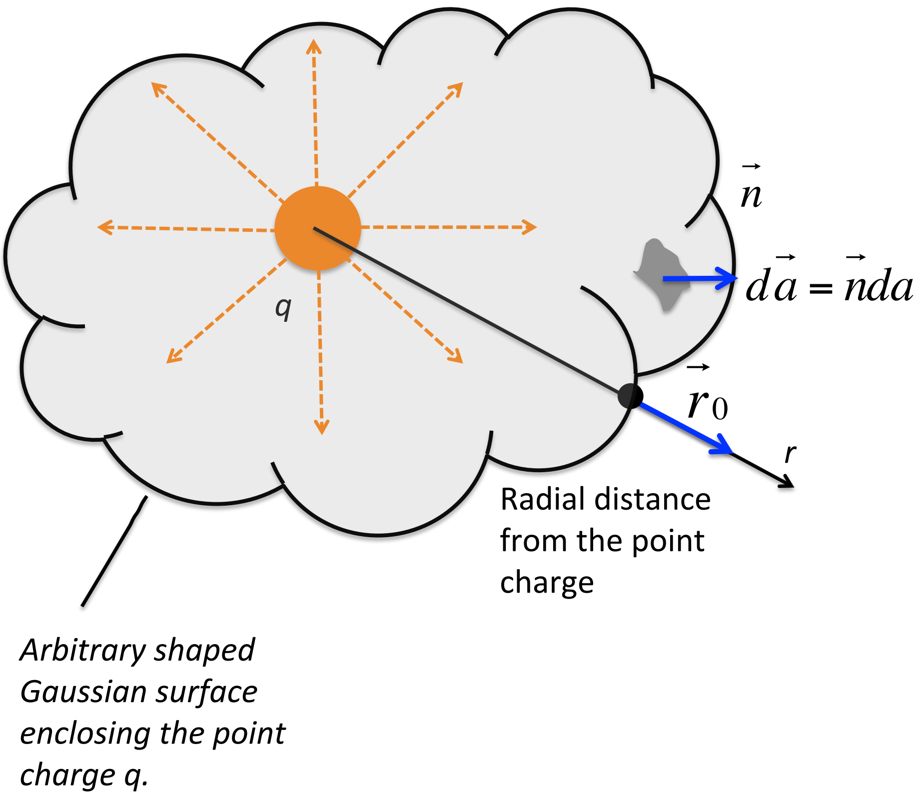
$$\psi = \oiint_{sphere} \underbrace{\varepsilon\ \vec{E}\cdot}_{\substack{\text{the same everywhere}\\ \text{on the surface of the sphere}}} d\vec{s} = \varepsilon E \oiint_{sphere} ds = \varepsilon EA =$$

$$= \varepsilon\frac{1}{4\pi\varepsilon}\frac{q}{R^2}4\pi R^2 = q$$

Result: the total flux over a spherical area centered on the point where the charge is located is equal to the magnitude of the charge.

This result is formalised by Gauss' law for electric fields.

In-Class Derivation: Proof of Gauss' Law for a General Closed Surface



$$\psi = \oiint_{surface} \vec{D} d\vec{a} = \oiint_{surface} \frac{q}{4\pi r^2} \vec{r}_0 d\vec{a}$$

Use the spherical coordinate system. In particular, see page 45 (lecture notes) for the expression of the infinitesimal area element ***da*** in spherical coordinates.

$$d\vec{a} = r^2 \sin\varphi d\varphi \, d\theta \, \vec{r}_0$$

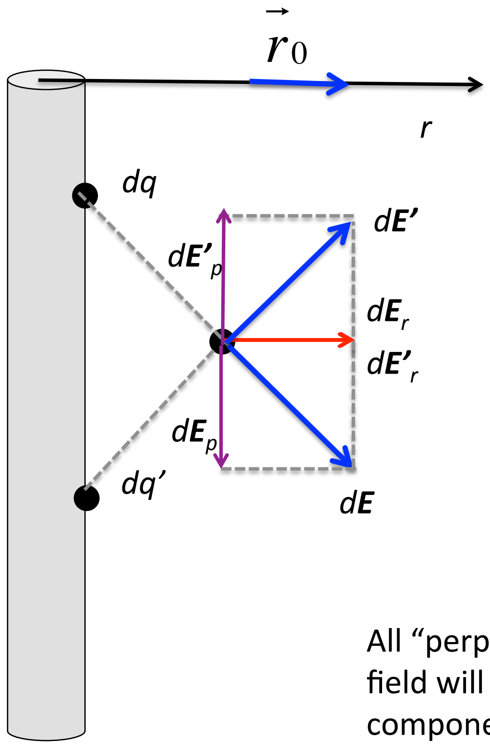
The flux:

$$\psi = \oiint_{surface} \frac{q}{4\pi r^2} \vec{r}_0 d\vec{a} \;= \frac{q}{4\pi} \oiint_{surface} \frac{1}{r^2} r^2 \sin\varphi \, d\varphi \, d\theta \;=$$

$$\frac{q}{4\pi} \int_0^{2\pi} \left[\int_0^{\pi} \sin\varphi d\varphi \right] d\theta = \frac{q}{4\pi} \theta \Big|_0^{2\pi} \cdot (-\cos\varphi)\Big|_0^{\pi} = \frac{q}{4\pi} 4\pi = q$$

In the case of two or more charges, superposition holds.

In-Class Example: Electric Field due to an Infinite Line of Charge (or a Charged Cylinder)



Infinite cylinder or infinite line of charge of density λ.

Due to the symmetry of the system:

$$d\mathbf{E}_p = -d\boldsymbol{E}'_p$$
$$d\mathbf{E}_r = d\boldsymbol{E}'_r$$

(see "Electric field due to a charged ring"

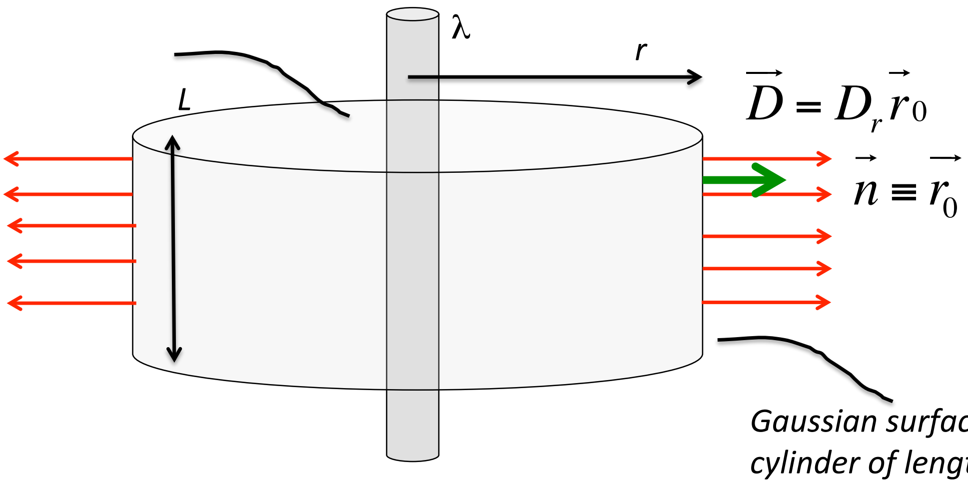
All "perpendicular components" of the electric field will cancel each other. Only the the "radial" components of the electric field $d\mathbf{E}_r$ will stay.

The direction of the electric field is:

$$\vec{E} = E_r \vec{r}_0$$

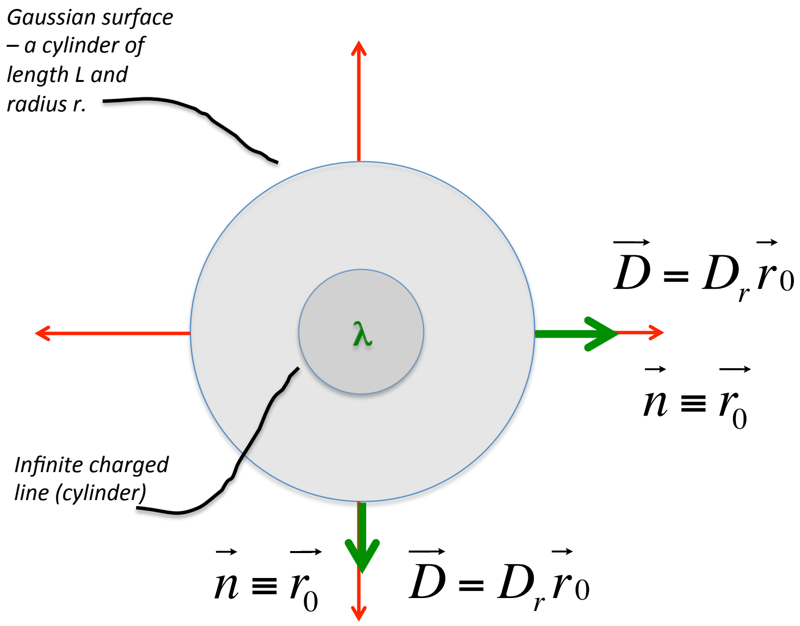
$$\vec{D} = D_r \vec{r}_0$$

No flux through the sides:



Gaussian surface – a cylinder of length L and radius r.

Cross-section view:



Answers:

The electric field is:

$$\vec{E}(r) = \frac{\lambda}{2\pi r \varepsilon} \vec{r}_0$$

The electric flux density is:

$$\vec{D}(r) = \frac{\lambda}{2\pi r} \vec{r}_0$$

In-Class Example: Electric Field due to an Infinite Line of Charge (Contd.)

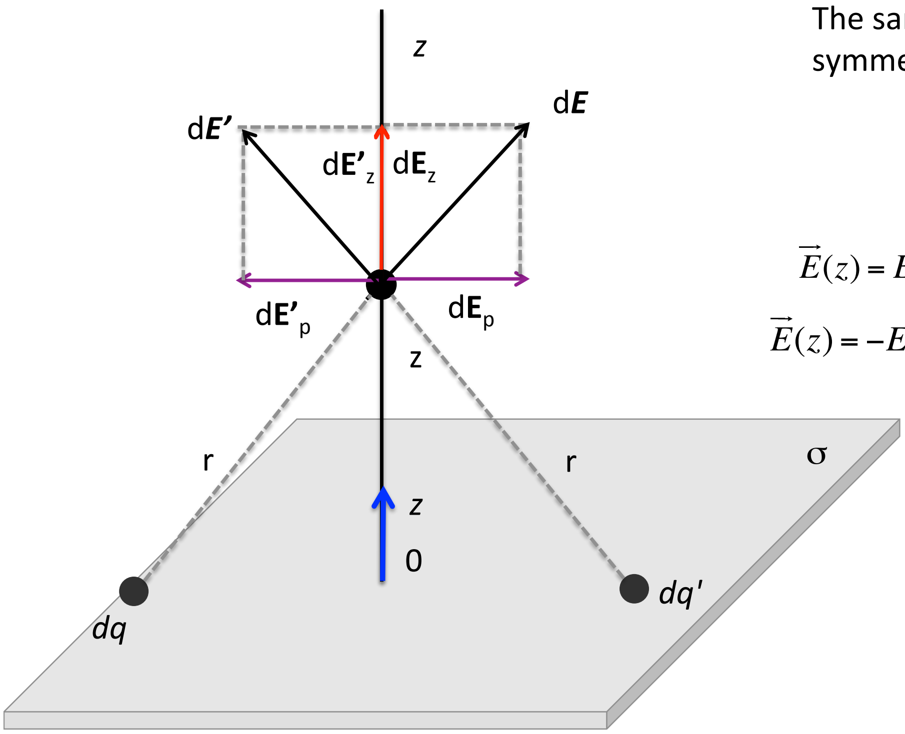
A uniform infinite line charge with density λ=20 nC/m lies in the *z*-axis. Find the electric field at (6,8,3)m. ε=ε0.

Answer:

$$\vec{E}(r) = 36\vec{r}_0 \quad \text{V/m}$$

In-Class Example: Electric Field due to an Infinite Charged Surface

Infinite charged surface with surface charge density σ.

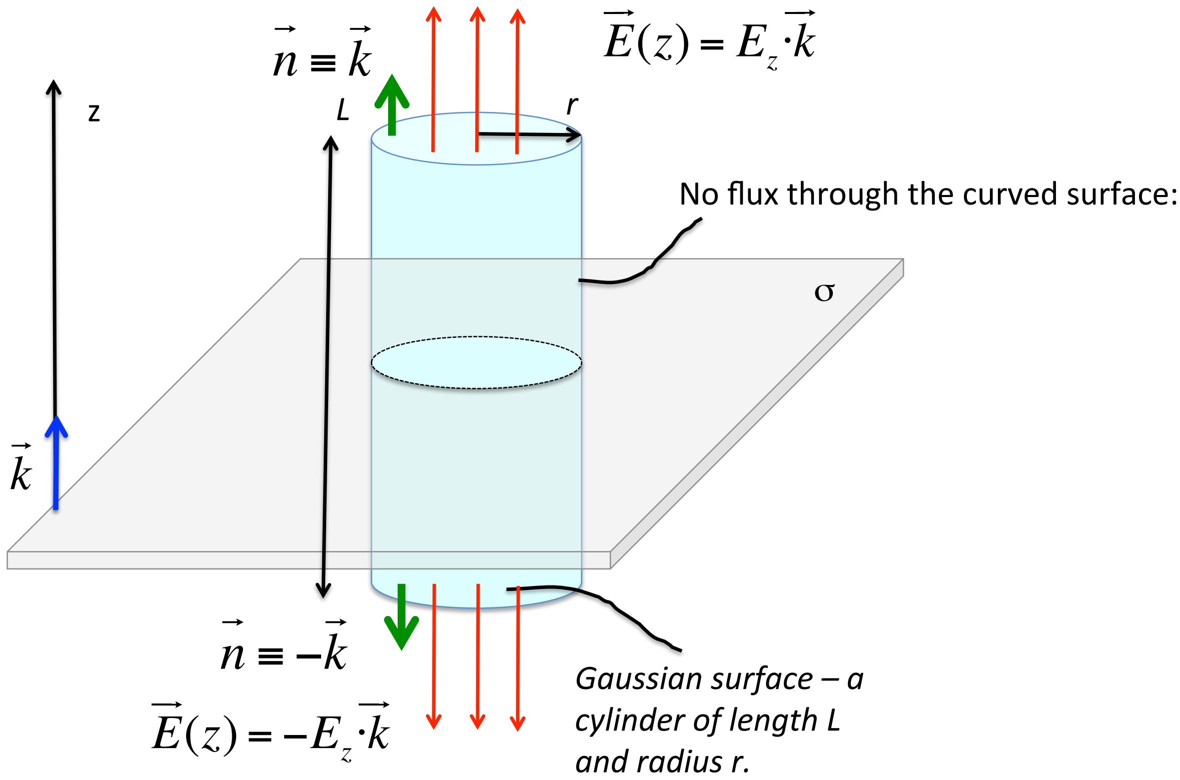


The same principle of symmetry is applied:

$$dE_p = -dE'_p$$
$$dE_z = dE'_z$$

$$\vec{E}(z) = E_z \cdot \vec{k} \quad \text{(above the plane)}$$

$$\vec{E}(z) = -E_z \cdot \vec{k} \quad \text{(below the plane)}$$



Answers:

Electric flux density and electric field due to an infinite charged plane:

$$\vec{D}(r) = \frac{\sigma}{2}\vec{k} \quad \text{(above the plane)}$$

$$\vec{E}(r) = \frac{\sigma}{2\varepsilon}\vec{k} \quad \text{(above the pane)}$$

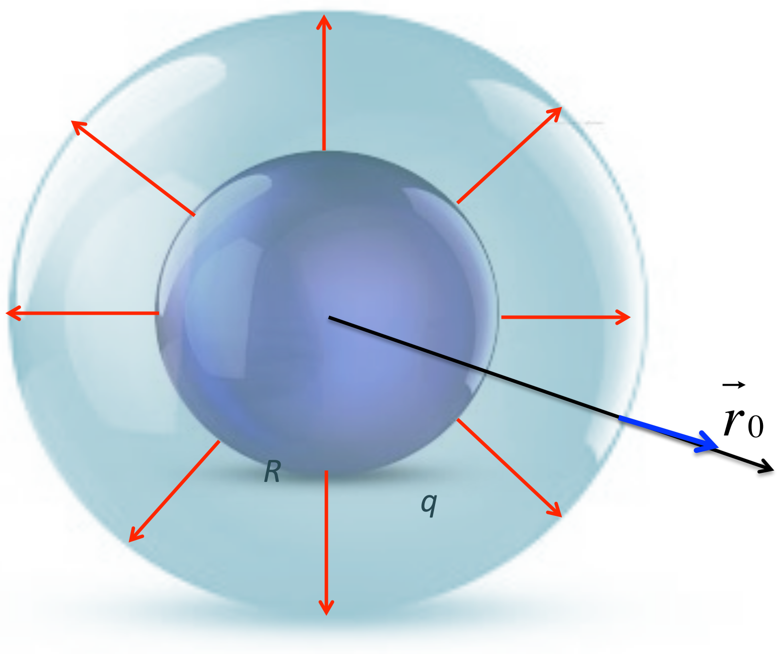
In-class example: Electric Field due to an Infinite Charged Surface (Contd.)

A uniform distribution of charge exists over the plane z=10cm with a density σ=1/(3π) nC/m^2. Find the electric field.

$$\vec{E}(z) = 6\vec{k} \quad \text{V/m} \quad \text{(above the plane)}$$

In-Class Example: Electric Field due to a Charged Sphere

Electric field due to a charged sphere with volume charge density ρ.



The electric field due to a charged sphere is:

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\vec{r_0}$$

$$\vec{D}(r) = \frac{1}{4\pi}\frac{q}{r^2}\vec{r_0}$$

Where $q = \rho\frac{4}{3}\pi R^3$

The same as the electric field due to a point charge.