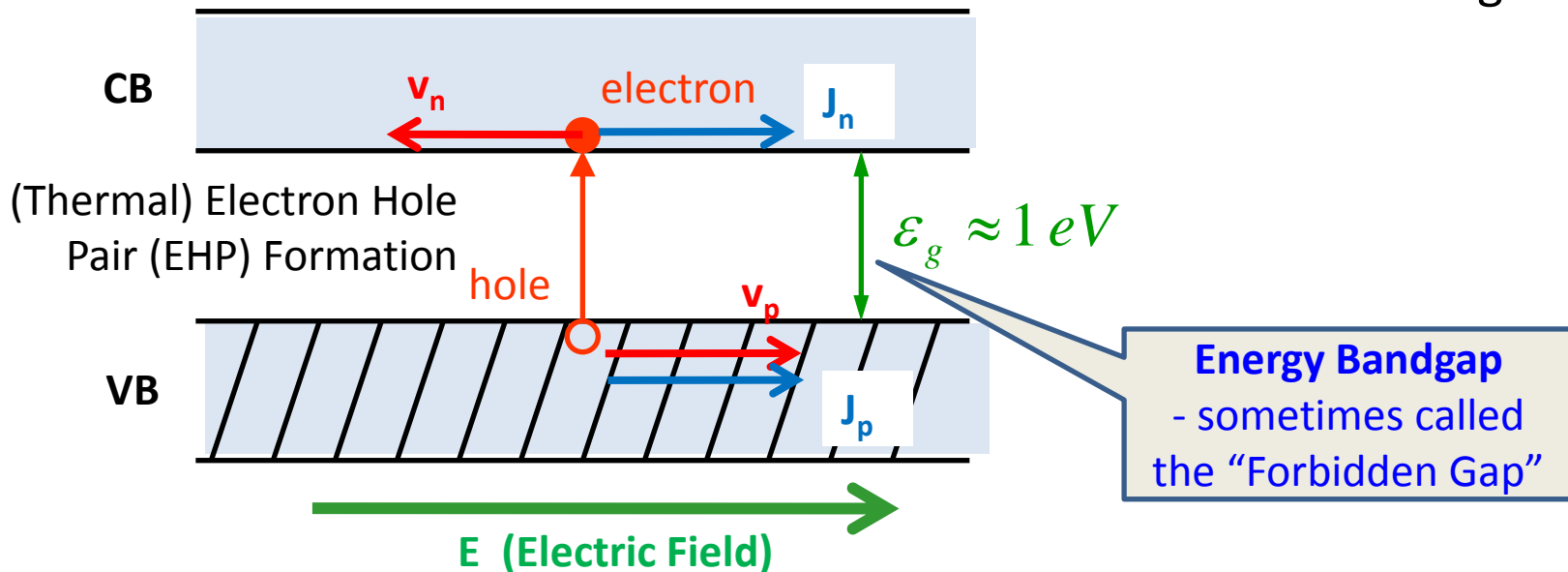


# Chapter 3

## Electrical Conduction in Pure and Doped Semiconductors

# Reminder: Energy Band Structure for Semiconductors

- A Conduction Band (CB) and a Valence Band (VB) exist separated by a ( $\approx$  small) *bandgap energy*  $\epsilon_g$



- The total current density is given by:

$$J = J_n + J_p = q \cdot v_n \cdot n + q \cdot v_p \cdot p$$

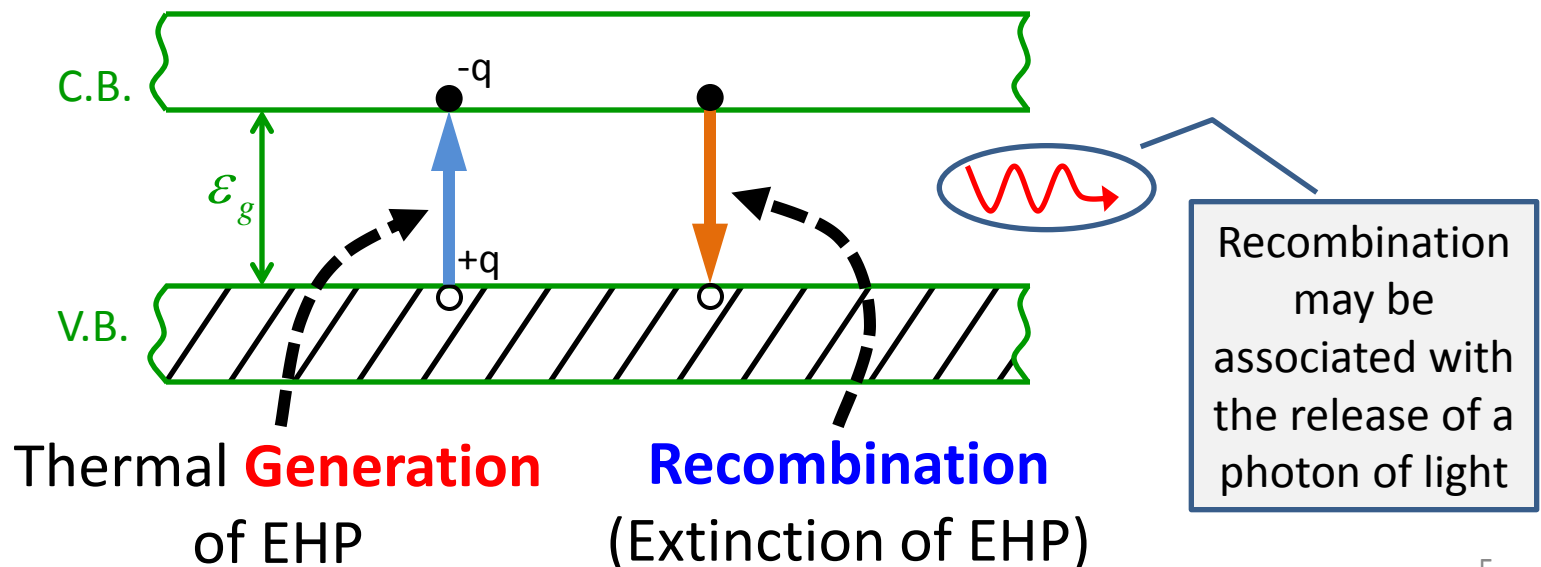
# Conductivity in Semiconductors: Differences Compared to Metals

- A first difference compared to metals is that both **electrons** (in the CB) and **holes** (in the VB) participate in electrical conduction in a semiconductor;
- A second major difference (at least in pure semiconductors), is that the electron (and hole) **concentrations** are vastly **lower** than in a metal, and also very temperature-sensitive;
- A less-obvious difference is that it turns out that we cannot use ordinary mass for either the electron or the hole when considering carrier transport in a semiconductor. Quantum mechanical analysis shows that an ***electron effective mass*** ( $m_e^*$ ) must be used for the electron and a ***hole effective mass*** ( $m_h^*$ ) must be used for the hole.

# Pure or “INTRINSIC” Semiconductors

# Pure or “Intrinsic” Semiconductor in Thermal Equilibrium

- The condition of “**thermal equilibrium**” in a semiconductor sample means that **no external stimulus** (light, voltage...) is applied to the sample and that all transients have died away – the sample is in **steady-state**;
- If  $T > 0K$ , Electron-Hole Pairs (EHPs) are continually being created due to thermal energy (“thermal generation”) but also being eliminated through a reverse process called **recombination**. In thermal equilibrium, these two processes exactly balance to provide a (statistically) constant electron and hole concentration ( $n$  and  $p$ , respectively)



# Carrier Concentrations in Semiconductors

**n** : (in general) this is the no. of CB electrons per unit volume (or the “**electron concentration**”) with units: ( $/\text{m}^3$ ) or ( $/\text{cm}^3$ )

**p** : (in general) this is the no. of VB holes per unit volume (or the “**hole concentration**”) with units: ( $/\text{m}^3$ ) or ( $/\text{cm}^3$ )

For a **pure** semiconductor crystal in thermal equilibrium, **these must clearly be equal** and this value is known as **the Intrinsic Concentration ( $n_i$ )**:

$$n = p = n_i$$

*Pure semiconductor only!*

We expect as  $\epsilon_g >$ ,  $n_i <<$  and as  $T >$ ,  $n_i >>$

E.g.: Measured intrinsic concentrations of common semiconductors at 300K:  
(compare with  $\approx 10^{22}$  CB electrons/ $\text{cm}^3$  in a good metallic conductor such as Cu)

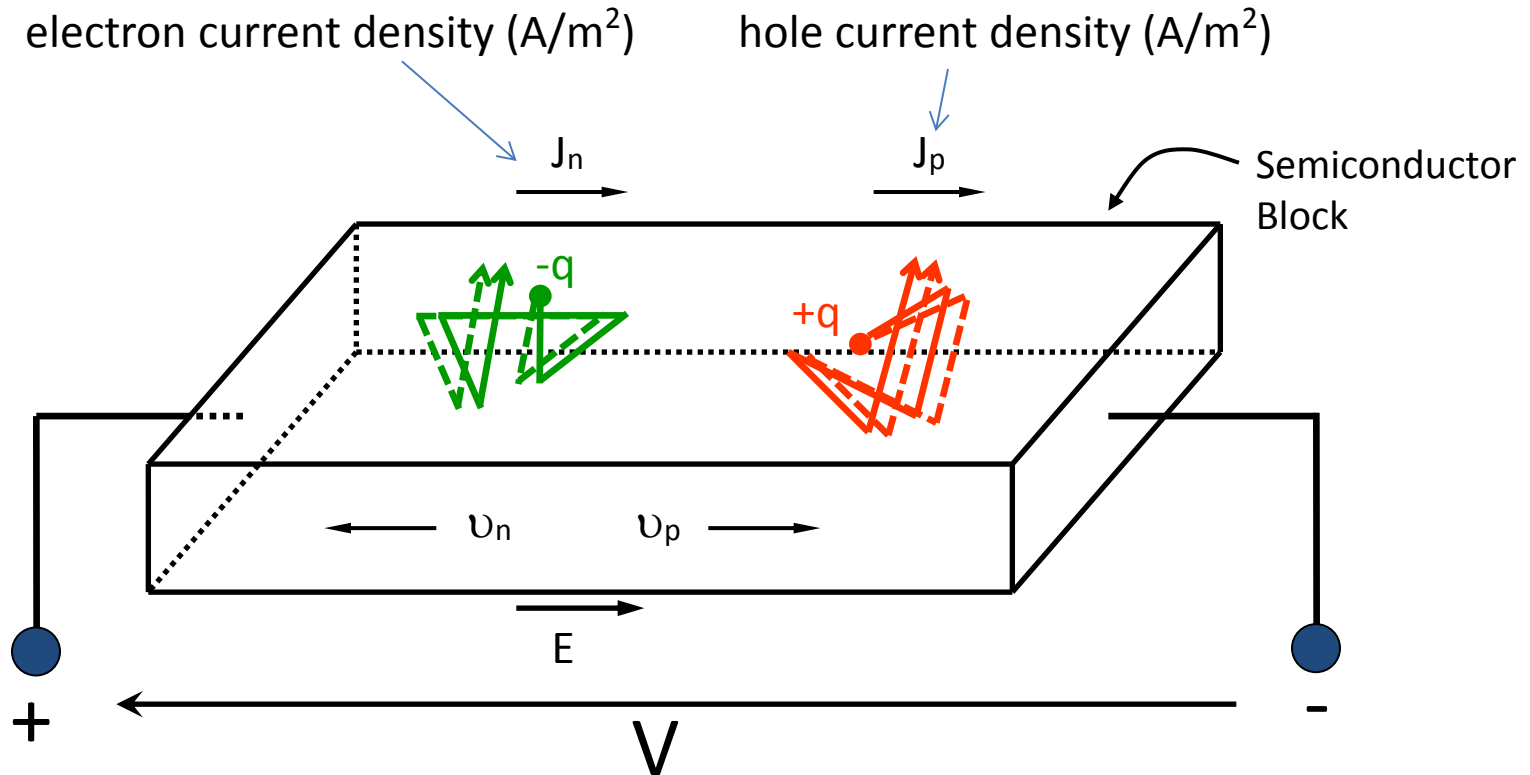
$$\text{SILICON : } Si (\epsilon_g = 1.1\text{eV}) : n_i = 1.5 \times 10^{10} / \text{cm}^3$$

$$\text{GALLIUM ARSENIDE : } GaAs (\epsilon_g = 1.43\text{eV}) : n_i = 2 \times 10^6 / \text{cm}^3$$

# Drift Current Flow in Semiconductors

- The process of current flow is similar to a metal : at  $T > 0K$ , electrons and holes are normally moving rapidly in the crystal with a random thermal velocity. If an electric field ( $E$ ) is applied, the carriers experience a force. This perturbs their velocities and both the electrons and holes acquire a drift velocity: an **electron drift velocity ( $v_n$ )** and a **hole drift velocity ( $v_p$ )** (these are not generally equal).
- The carriers drift in opposite directions, but contribute to current in the same direction.
- The current in a semiconductor caused by the movement or drift of charge carriers (electrons and holes) is sometimes called the “particle current” (as distinct from displacement or capacitive current).

# Current Flow in Semiconductors



$$J = J_n + J_p = q \cdot v_n \cdot n + q \cdot v_p \cdot p$$



# Electron and Hole Mobility

- A similar dynamic analysis can be carried out to that shown previously for metals. Provided the time scale is not very short, we can say that the electron drift velocity  $v_n$  reacts *instantly* to the electric field  $E$ , and at low electric field strengths, it is proportional to the electric field. The constant of proportionality is the **electron mobility**  $\mu_n$ :

$$\mu_n = \frac{q \cdot \bar{\tau}_e}{m_e^*}$$

$$v_n = \mu_n \cdot E$$

- A similar analysis can be carried out for holes, leading to a **hole mobility**  $\mu_p$ .

$$\mu_p = \frac{q \cdot \bar{\tau}_h}{m_h^*}$$

$$v_p = \mu_p \cdot E$$

- Where:  $\bar{\tau}_e$  ( $\bar{\tau}_h$ ) is the **mean free time between collisions** for electrons (holes)

# Effective Masses in Silicon

- Effective mass is actually a complex concept and more properly requires a tensor description for both electrons and holes;
- Different combinations of elements of the tensor are used in different contexts (e.g. *density-of-states effective mass*, *conductivity effective mass*...)
- In this course we will consider the effective masses as scalar constants for both electrons and holes;
- In the case of silicon (Si) we will use the following values where  $m_o$  (or “m”) is the electron rest mass:

$$\begin{aligned}m_e^* &= 1.08 \cdot m_o \\ m_h^* &= 0.556 \cdot m_o\end{aligned}$$

# Summary of Semiconductor Formulas

$J_n$  : electron current density (A/m<sup>2</sup>)

$J_p$  : hole current density (A/m<sup>2</sup>)

Total current density

$$J = J_n + J_p$$

## Electrons

$$v_n = \mu_n E$$

$\mu_n$  : **electron mobility**

$v_n$  : electron drift  
velocity

$$\mu_n = \frac{q \bar{\tau}_e}{m_e^*}$$

Note : use of electron  
effective mass here.

$$J_n = qn v_n = qn \mu_n E$$

## Holes

$$v_p = \mu_p E$$

$\mu_p$  : **hole mobility**

$v_p$  : hole drift  
velocity

$$\mu_p = \frac{q \bar{\tau}_h}{m_h^*}$$

Note : use of hole  
effective mass here.

$$J_p = qp v_p = qp \mu_p E$$

# GENERAL Formula for the Conductivity of a Semiconductor

- The total particle current in a semiconductor is given by:

$$J = J_n + J_p = q \cdot n \cdot \mu_n \cdot E + q \cdot p \cdot \mu_p \cdot E$$

$$J = (q \cdot n \cdot \mu_n + q \cdot p \cdot \mu_p) \cdot E$$

$$\text{or... } J = \sigma \cdot E$$

...where  $\sigma$  is the **semiconductor conductivity** (S/cm or S/m):

$$\sigma = q \cdot (n \cdot \mu_n + p \cdot \mu_p)$$

**\* Key  
Formula**

- Normally,  $\mu_n \neq \mu_p$ .
- In pure Silicon,  $\mu_n = 1300 \text{ cm}^2/\text{V}\cdot\text{sec}$   $\mu_p = 450 \text{ cm}^2/\text{V}\cdot\text{sec}$ . This difference in mobility has important consequences in electron devices: those relying mainly on electrons (N-channel FETs, NPN transistors..) tend to respond faster than those using holes

# Formula for the Conductivity of a PURE or INTRINSIC Semiconductor

- In a pure semiconductor material in thermal equilibrium, we have seen that the natural mechanism of thermal EHP formation must create *equal numbers* of electrons and holes.
- This concentration is a critical number for a particular semiconductor at a given temperature and is called the:

***intrinsic concentration  $n_i$***

- Using the previous general formula for semiconductor conductivity in the particular case of a pure semiconductor, we have  $n = p = n_i$  and the ***intrinsic conductivity*** is then given by:

$$\sigma_i = q \cdot (\mu_n + \mu_p) \cdot n_i$$

# Example 3.1

# Exercises

- Pure silicon at 300K has a band gap energy of 1.1eV and an intrinsic concentration of  $1.5 \times 10^{10} / \text{cm}^3$ . Calculate the intrinsic conductivity (in S/cm) and the intrinsic resistivity (in  $\Omega \cdot \text{cm}$ )

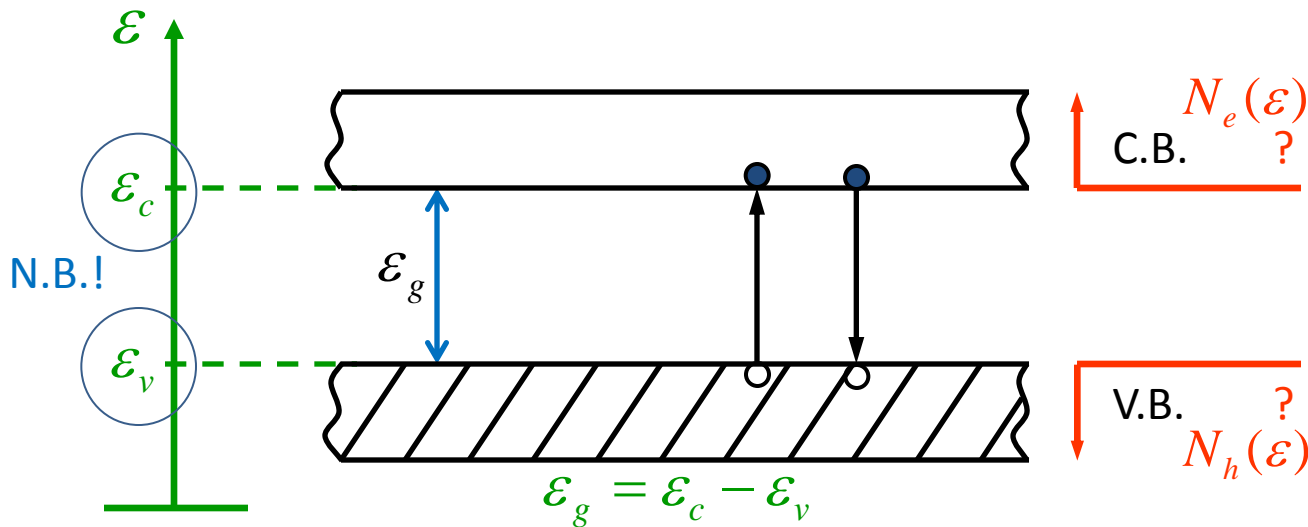
*Answers:  $\sigma_i (\text{Si}) = 4.2 \mu\text{S/cm}$ ;  $\rho_i = 238 \text{k}\Omega \cdot \text{cm}$*

- Repeat for pure gallium arsenide (GaAs) at 300K, for which the band gap energy is 1.43eV and the intrinsic concentration is  $2.0 \times 10^6 / \text{cm}^3$ . The electron mobility in GaAs is 8500  $\text{cm}^2/\text{V} \cdot \text{sec}$  while the hole mobility is 450  $\text{cm}^2/\text{V} \cdot \text{sec}$ .

*Answers:  $\sigma_i (\text{GaAs}) \approx 2.86 \text{nS/cm}$ ; or  $\rho_i \approx 349 \text{M}\Omega \cdot \text{cm}$*

*(Note the very high resistivity of pure GaAs. Sometimes it is described as semi-insulating (SI GaAs) rather than semiconducting for this reason)*

# Number Density Distributions of Electrons and Holes in Semiconductors



Propose that the **number density function** for **electrons** in the CB is:

$$N_e(\epsilon) = S_e(\epsilon) \cdot f_e(\epsilon) \quad (\text{only for } \epsilon \geq \epsilon_c)$$

$f_e(\epsilon)$  : probability of occupation function  
 $S_e(\epsilon)$  : density of states function

Propose that the **number density function** for **holes** in the VB is:

$$N_h(\epsilon) = S_h(\epsilon) \cdot [1 - f_e(\epsilon)] \quad (\text{only for } \epsilon \leq \epsilon_v)$$

probability of a hole =  
(1-probability of an electron)



# Number Density Distributions of Electrons and Holes in Semiconductors

By direct analogy with the analysis for metals (but using *effective mass* not rest mass):

$$S_e(\varepsilon) = \frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} \cdot (\varepsilon - \varepsilon_c)^{1/2}$$

$\varepsilon \geq \varepsilon_c$

$$S_h(\varepsilon) = \frac{8\sqrt{2}\pi(m_h^*)^{3/2}}{h^3} \cdot (\varepsilon_v - \varepsilon)^{1/2}$$

$\varepsilon \leq \varepsilon_v$

Use Fermi – Dirac distribution for  $f_e(\varepsilon)$  :

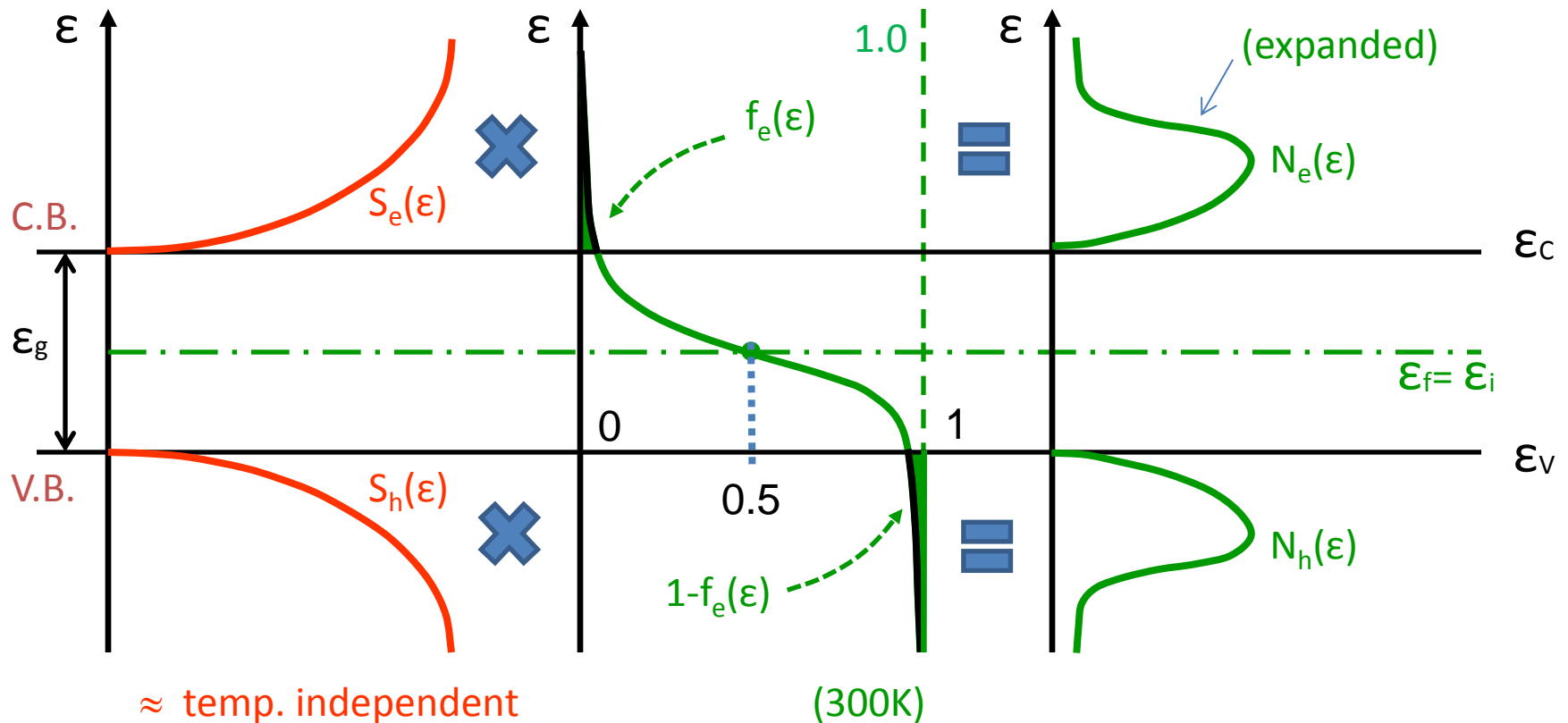
$$f(\varepsilon) = \frac{1}{1 + \exp\left[\frac{\varepsilon - \varepsilon_f}{kT}\right]}$$

Where is the Fermi Energy  $\varepsilon_f$  in a semiconductor?

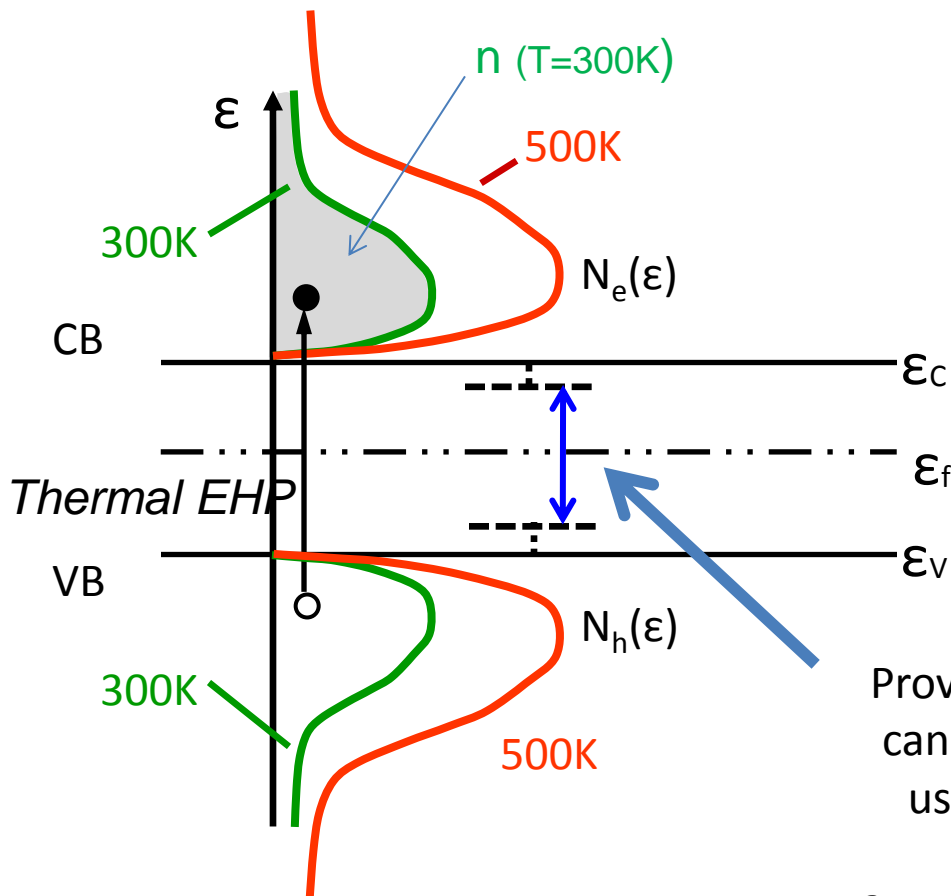
We will prove shortly that **in a pure semiconductor,  $\varepsilon_f$  is located close to the middle of the forbidden gap. It is then called the “intrinsic level”  $\varepsilon_i$**

# Number Density Distributions of Electrons and Holes in a Pure Semiconductor (at say $T=300\text{K}$ )

*\*\* assuming that  $\mathcal{E}_f = \mathcal{E}_i$  is located at the middle of the bandgap \*\**



# Expressions for the Number Density Distributions of Electrons and Holes in Semiconductors



Begin with **electrons**:

No. of electrons in C.B.  
per unit volume :

$$n = \int_{\epsilon_c}^{\infty} N_e(\epsilon) \cdot d\epsilon$$

$$= \int_{\epsilon_c}^{\infty} S_e(\epsilon) \cdot f_e(\epsilon) \cdot d\epsilon$$

Provided  $\epsilon_f$  is **below**  $\epsilon_c$  by at least a few  $kT$ , we can approximate the Fermi-Dirac distribution using the **Boltzmann approximation** in this integral

( $\epsilon_g \approx 1\text{eV}$     $kT \approx 0.0259\text{eV}$ ). Proof on next slide.

# Approximation of Fermi-Dirac Function by Boltzmann Distribution in CB

If:  $\frac{\varepsilon - \varepsilon_f}{kT} \gg 1$  in CB, then:  $e^{\frac{\varepsilon - \varepsilon_f}{kT}} \gg 1$

Hence:  $\frac{1}{1 + e^{\frac{\varepsilon - \varepsilon_f}{kT}}} \approx \underline{\underline{e^{-\left(\frac{\varepsilon - \varepsilon_f}{kT}\right)}}$

$$\Rightarrow n \approx \int_{\varepsilon_c}^{\infty} \underbrace{\frac{8\sqrt{2}\pi(m_e^*)^{3/2}}{h^3} \cdot (\varepsilon - \varepsilon_c)^{1/2}}_{S_e(\varepsilon)} \times \underbrace{e^{-\left(\frac{\varepsilon - \varepsilon_f}{kT}\right)}}_{\text{Boltzmann approx. to } f_e(\varepsilon)} \cdot d\varepsilon$$

Use the substitution:  $z = \frac{\varepsilon - \varepsilon_c}{kT}$

# Analysis for CB Electron Concentration in a Semiconductor ( $\varepsilon_f$ below $\varepsilon_c$ by at least a few kT)

Substitution for “z” gives :

$$n = \frac{4\pi}{h^3} \cdot (2m_e^* kT)^{3/2} \cdot e^{-\left(\frac{\varepsilon_c - \varepsilon_f}{kT}\right)} \cdot \int_0^\infty \sqrt{z} e^{-z} dz$$

$\longleftrightarrow = \frac{\sqrt{\pi}}{2}$  *(this value is found from tables of standard integrals)*

$$\Rightarrow \boxed{n = N_c \cdot e^{-\left(\frac{\varepsilon_c - \varepsilon_f}{kT}\right)}}$$

Where  $N_c$  is called **the effective density of states in the conduction band** and is given by:

$$N_c = 2 \cdot \frac{(2\pi \cdot m_e^* \cdot kT)^{3/2}}{h^3}$$

*Do not remember!*

## Analysis for VB Hole Concentration in a Semiconductor ( $\varepsilon_f$ above $\varepsilon_v$ by at least a few $kT$ )

Provided the Fermi level is located at least a few  $kT$  above the top edge of the Valence Band, then the Fermi-Dirac distribution can once be again approximated by the Boltzmann distribution. A similar analysis to that carried out for CB electrons will show that the hole concentration in the VB is given by:

$$p = N_v \cdot e^{-\left(\frac{\varepsilon_f - \varepsilon_v}{kT}\right)}$$

Where  $N_v$  is called **the effective density of states in the valence band** and is given by:

$$N_v = 2 \cdot \frac{(2\pi \cdot m_h^* \cdot kT)^{3/2}}{h^3}$$

## Example of Si at 300K

- Substituting the values for the effective masses given earlier, we find:


$$N_C = 2.82 \times 10^{19}/\text{cm}^3$$

$$N_V = 1.83 \times 10^{19}/\text{cm}^3$$

# Formula for the Intrinsic Concentration ( $n_i$ )

- In a **pure** semiconductor in thermal equilibrium, the intrinsic concentration  $n_i$  at a temperature  $T$  is the number of electrons in the CB per unit volume (= the number of holes in the VB per unit volume)
- The Fermi level in a pure semiconductor is sometimes called the **intrinsic level** (denoted by  $\mathcal{E}_i$ )
- Multiply the above formula for 'n' by the formula for 'p'. In a pure semiconductor this must give:  $n_i^2 = n \cdot p$

$$n_i^2 = N_c \exp\left[-\left(\frac{\mathcal{E}_c - \mathcal{E}_i}{kT}\right)\right] \cdot N_v \exp\left[-\left(\frac{\mathcal{E}_i - \mathcal{E}_v}{kT}\right)\right]$$



$$\Rightarrow n_i^2 = N_c \cdot N_v \cdot \exp\left[\frac{-\mathcal{E}_c + \mathcal{E}_i - \mathcal{E}_i + \mathcal{E}_v}{kT}\right]$$

$$\Rightarrow n_i^2 = N_c \cdot N_v \cdot \exp\left[-\frac{\mathcal{E}_g}{kT}\right] \quad \text{using } \mathcal{E}_g = (\mathcal{E}_c - \mathcal{E}_v)$$



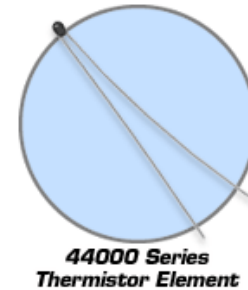
# Formula for the Intrinsic Concentration ( $n_i$ )

- Concentrating on the temperature dependence (and the bandgap dependence) of  $n_i$  we can extract the dependence on  $T$  from the earlier formulas for  $N_c$  and  $N_v$  to obtain the final result:

$$n_i = (N_c \cdot N_v)^{1/2} \cdot \exp\left[-\frac{\varepsilon_g}{2kT}\right] = \text{const} \cdot T^{3/2} \cdot \exp\left[-\frac{\varepsilon_g}{2kT}\right]$$

- This result shows that the intrinsic concentration is an extremely sensitive **increasing** function of **increasing temperature  $T$**  for a given energy bandgap ( $\varepsilon_g$ ).
- Equally, at a constant temperature, the intrinsic concentration is an extremely sensitive **decreasing** function of **increasing bandgap energy**
- The build-up of intrinsic carriers with temperature is a continuous background process in semiconductor electronic devices. It sets a fundamental limit on the maximum usable temperature of a semiconductor material (this limit is higher for wider bandgap semiconductors – e.g. From this point of view, Silicon (Si) with  $\varepsilon_g = 1.1\text{eV}$  is better than Germanium (Ge) with  $\varepsilon_g = 0.66\text{eV}$ , etc.)

# The Thermistor



- We have seen that the intrinsic carrier concentration of a semiconductor material increases very rapidly with temperature (T);
- This can be exploited in a specific kind of highly temperature-sensitive resistor made of semiconductor and called a “thermistor”;
- The intrinsic semiconductor conductivity as a function of T is:

$$\sigma_i(T) = q \cdot (\mu_n(T) + \mu_p(T)) \cdot n_i(T) = \text{const} \cdot (\mu_n(T) + \mu_p(T)) \cdot T^{3/2} \cdot \exp\left(-\frac{\mathcal{E}_g}{kT}\right)$$

- The mobility is found to decrease with T as  $\approx T^{-3/2}$  due to lattice vibration so that to a good approximation the resistance of the thermistor is given by:

$$R(T) = a \cdot \exp\left(+\frac{b}{T}\right) \quad \text{where } a \text{ and } b \text{ are constants}$$

- Thermistors are useful in safety applications: e.g. shutting down the power to an electric motor if the winding temperature gets too high...

# Position of the Intrinsic Level (i.e. Location of the Fermi Energy in a Pure Semiconductor)

- We obtained a useful formula for  $n_i$  by **multiplying** the formulas for  $n$  and  $p$  in a pure semiconductor. We find another useful result by **equating** them:

$$N_c \exp\left[-\left(\frac{\varepsilon_c - \varepsilon_i}{kT}\right)\right] = N_v \exp\left[-\left(\frac{\varepsilon_i - \varepsilon_v}{kT}\right)\right]$$




divide across by this and also cancel  
common terms in  $N_c$  and  $N_v$

$$\Rightarrow (m_e^*)^{3/2} \cdot \exp\left[\frac{-\varepsilon_c + \varepsilon_i + \varepsilon_i - \varepsilon_v}{kT}\right] = (m_h^*)^{3/2}$$

$$\Rightarrow -(\varepsilon_c + \varepsilon_v) + 2\varepsilon_i = kT \cdot \ln\left|\frac{(m_h^*)^{3/2}}{(m_e^*)^{3/2}}\right|$$

$$\Rightarrow \varepsilon_i = \frac{(\varepsilon_c + \varepsilon_v)}{2} + \frac{3kT}{4} \cdot \ln\left|\frac{(m_h^*)}{(m_e^*)}\right|$$

  
small

This proves that the  
intrinsic level is very close  
to the midpoint of the  
forbidden gap

## Example 3.2

# Doped or “EXTRINSIC” Semiconductors

# Extrinsic or 'Doped' Semiconductors

- If we could only use pure or intrinsic semiconductors, the engineering usefulness of these materials would be very limited.
- However by adding minute quantities of specially-selected other elements (“**impurities**”) in a deliberate and often uniform way to the native crystal, it turns out the electrical conductivity of semiconductor materials can be **increased by up to 8 or 9 orders of magnitude**. This process is called “**doping**” and the resulting semiconductor is described as “**extrinsic**”.
- Moreover, depending on the impurity used, it is possible to produce a material in which the concentration of CB electrons greatly exceeds that of VB holes (called an **N-type semiconductor**), or conversely a material with a much higher concentration of holes than electrons (called a **P-type semiconductor**)

# The Periodic Table of the Elements

Representative elements

Alkali metals  
↓  
Group 1A

Alkaline earth metals  
↓  
Group 2A

Period number

1  
2  
3  
4  
5  
6  
7

1 H 1.008  
2 He 4.003

3 Li 6.941  
4 Be 9.012

5 B 10.81  
6 C 12.01  
7 N 14.01  
8 O 16.00  
9 F 19.00  
10 Ne 20.18

11 Na 22.99  
12 Mg 24.31

13 Al 26.98  
14 Si 28.09  
15 P 30.97  
16 S 32.06  
17 Cl 35.45  
18 Ar 39.95

19 K 39.10  
20 Ca 40.08  
21 Sc 44.96  
22 Ti 47.88  
23 V 50.94  
24 Cr 52.00  
25 Mn 54.94  
26 Fe 55.85  
27 Co 58.93  
28 Ni 58.69  
29 Cu 63.55  
30 Zn 65.38

31 Ga 69.72  
32 Ge 72.59  
33 As 74.92  
34 Se 78.96  
35 Br 79.90  
36 Kr 83.80

37 Rb 85.47  
38 Sr 87.62  
39 Y 88.91  
40 Zr 91.22  
41 Nb 92.91  
42 Mo 95.94  
43 Tc 98  
44 Ru 101.1  
45 Rh 102.9  
46 Pd 106.4  
47 Ag 107.9  
48 Cd 112.4

49 In 114.8  
50 Sn 118.7  
51 Sb 121.8  
52 Te 127.6  
53 I 126.9  
54 Xe 131.3

55 Cs 132.9  
56 Ba 137.3  
57\* La 138.9  
72 Hf 178.5  
73 Ta 180.9  
74 W 183.9  
75 Re 186.2  
76 Os 190.2  
77 Ir 192.2  
78 Pt 195.1  
79 Au 197.0  
80 Hg 200.6

81 Tl 204.4  
82 Pb 207.2  
83 Bi 209.0  
84 Po 209  
85 At 210  
86 Rn 222

87 Fr 223  
88 Ra 226  
89† Ac 227  
104 Rf 261  
105 Db 262  
106 Sg 263  
107 Bh 262  
108 Hs 265  
109 Mt 266  
110 — 269  
111 — 272  
112 — 277

113  
114  
115  
116  
117  
118

Transition elements

3B 4B 5B 6B 7B 8 9 10 11 12  
3B 4B 5B 6B 7B 8B 1B 2B

Halogens

Noble gases  
↓  
Group 8A

\*Lanthanides

†Actinides

58 Ce 140.1  
59 Pr 140.9  
60 Nd 144.2  
61 Pm 145  
62 Sm 150.4  
63 Eu 152.0  
64 Gd 157.3  
65 Tb 158.9  
66 Dy 162.5  
67 Ho 164.9  
68 Er 167.3  
69 Tm 168.9  
70 Yb 173.0  
71 Lu 175.0

90 Th 232.0  
91 Pa 231  
92 U 238.0  
93 Np 237  
94 Pu 244  
95 Am 243  
96 Cm 247  
97 Bk 247  
98 Cf 251  
99 Es 252  
100 Fm 257  
101 Md 258  
102 No 259  
103 Lr 260

Metals Metalloids Nonmetals

# Important Practical Example of an Extrinsic Semiconductor: Doped N-TYPE SILICON

- Like Carbon, pure Silicon (Si) can be formed into a crystal with tetravalent bonds (i.e. a diamond lattice) meaning that 4 bonding electrons are shared with 4 nearest neighbour atoms.
- Now replace one Si atom in the lattice with a **pentavalent** impurity atom with 5 valence electrons (from Group V of the Periodic Table e.g. *Phosphorous, Arsenic, Antimony...*)
- Only 4 of its 5 outermost electrons are then needed for crystal bonding. The 5<sup>th</sup> electron can be “donated” to the CB – this kind of impurity atom is called a **DONOR**.



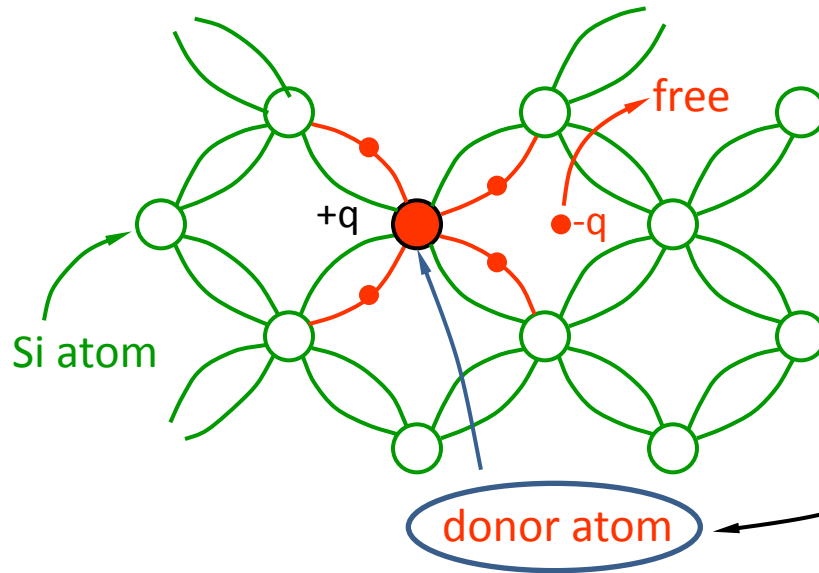
## Ionisation of Donor Atom

- In more detail, *provided* the temperature is not very low (i.e.  $< \approx 100\text{K}$ ), the 5<sup>th</sup> electron in each donor atom may be easily “ionised” by thermal energy to escape from the impurity atom and become a free electron (i.e. it enters the C.B.) but no VB hole is produced as a result of this process. Below  $\approx 100\text{K}$  the donor electron is not provided and this condition is referred to as “freeze-out”
- The crystal is said to have been doped **N-TYPE** with usually very many times more free electrons than holes;
- Note that as a result of the ionisation process, a charged (+q) ionised impurity atom is then left in a fixed location in the lattice corresponding to each donor site (this site will cause increased *ionised impurity scattering*)

# Donor Distribution in Lattice

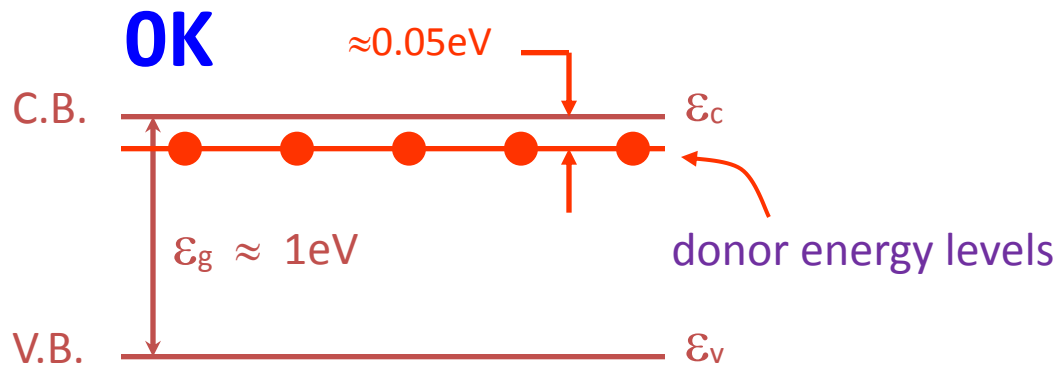
- In this Course, we will generally assume that these impurity **donor** atoms are uniformly distributed throughout the lattice, although non-uniform doping is also possible and quite common in practice.
- Typically, under moderate doping conditions, there is something like one impurity atom for about every million Si atoms;
- If the ionisation energy is small, then **at room temperature**, it is reasonable to assume that **every** impurity atom is ionised, and therefore that **each impurity atom contributes exactly one electron to the CB.**
- On an energy band diagram we can consider the 5<sup>th</sup> electron as occupying an energy level just below  $\mathcal{E}_c$ .

# Extrinsic Semiconductors – N-Type

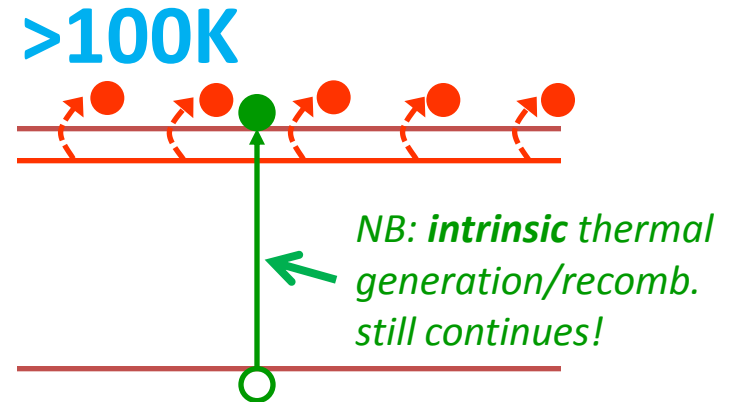


Crystal lattice

When it loses its electron this atom creates a fixed charged impurity site and becomes an **ionised donor**.



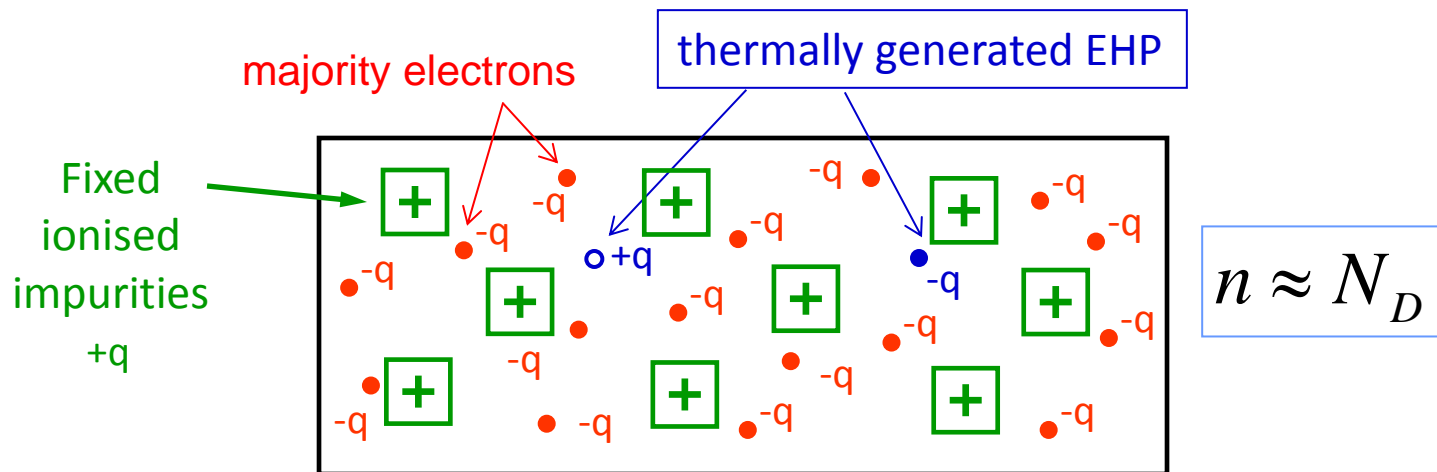
“freeze-out”



all donor electrons have transferred to CB

# N-Type Extrinsic Semiconductors

- In this case the semiconductor is doped with donors.
- **We define  $N_D$  as the Donor Concentration**, that is the number of donor atoms in the crystal per unit volume . (Units:  $/\text{m}^3$  or  $/\text{cm}^3$ )
- As we have seen, at room temperature (providing the donor concentration is not too low) then we can expect:  $n \sim N_D$  and  $n \gg p$ , so that electrons are called the majority carriers while holes are the minority carriers



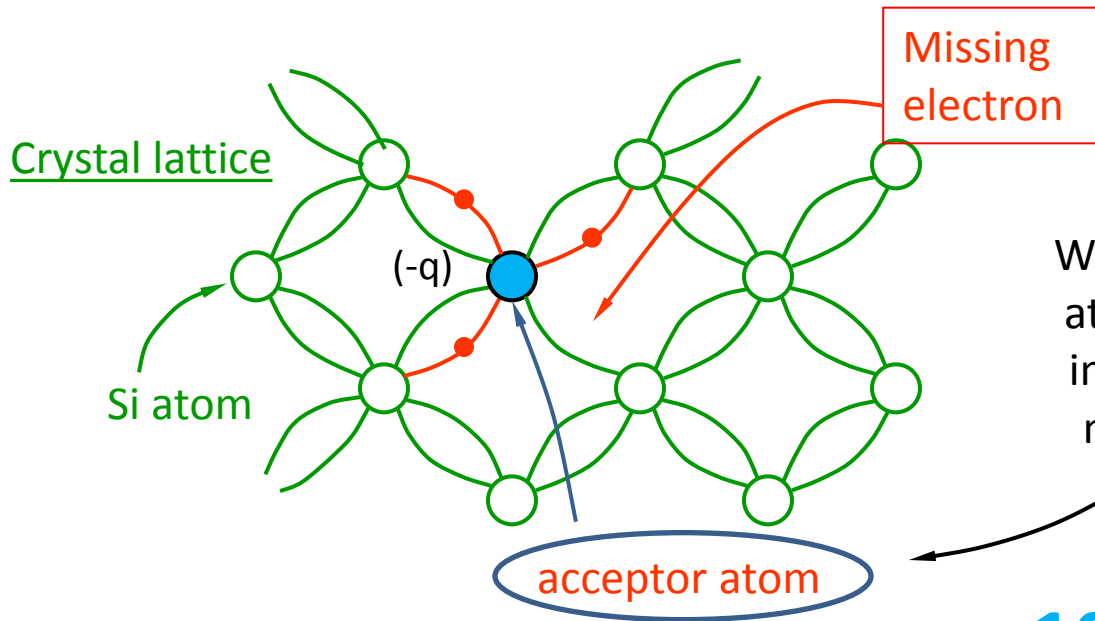
# Numerical Example of N-Type Extrinsic Semiconductors

- Consider an initially pure Si crystal . It will have  $\approx 10^{22}$  Si atoms/cm<sup>3</sup> and an intrinsic carrier concentration ( $n_i$ ) at 300K of about  $1.5 \times 10^{10}$ /cm<sup>3</sup> meaning that there are  $1.5 \times 10^{10}$  thermally generated electrons and  $1.5 \times 10^{10}$  thermally generated holes in each 1cm<sup>3</sup> .
- Suppose we dope the crystal to a level  $N_D = 10^{16}$  /cm<sup>3</sup> . This means that every millionth Si atom has been replaced by a donor atom;
- But every donor atom contributes 1 electron (assuming all are ionised) so that now  $n \approx 10^{16}$  /cm<sup>3</sup> . Compare with intrinsic case, where  $n_i \approx 1.5 \times 10^{10}$  /cm<sup>3</sup> – this means that the electrical conductivity of the doped crystal due to electrons has been increased by a factor of about 700,000
- Doping is a very powerful and flexible technique which allows us to ‘engineer’ the conductivity of a semiconductor over a huge range but still with relatively high precision

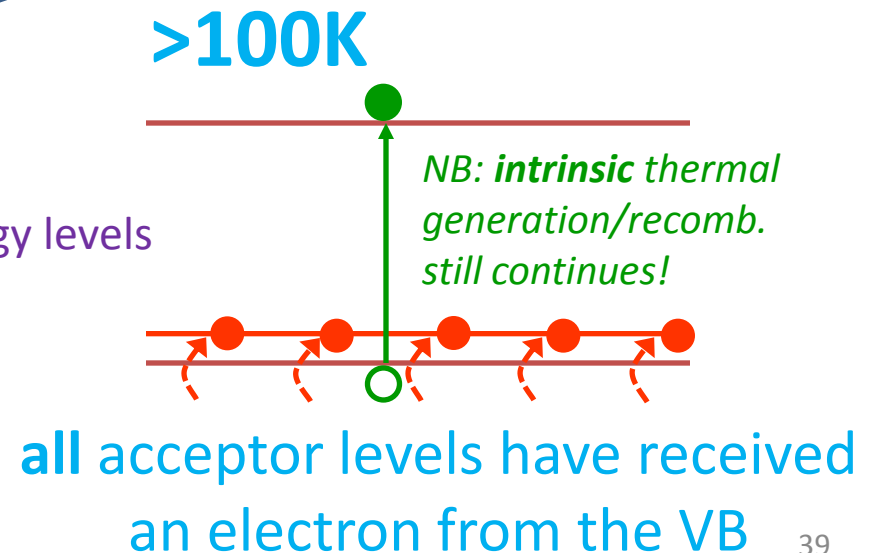
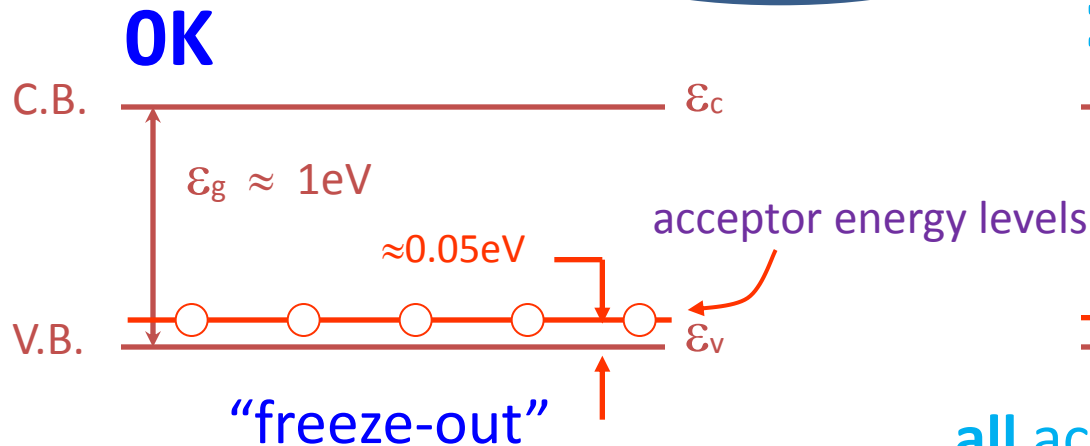
## Doped P-TYPE SILICON

- In this case we replace a small fraction of the host Si atoms with **trivalent** elements from **Group III** of the Periodic Table. These materials have just 3 valence electrons and examples include **Boron, Gallium or Aluminium**
- One of the 4 bonds around the impurity atom is then incomplete or missing an electron.
- If  $T > \sim 100\text{K}$ , another VB electron can easily acquire enough thermal energy to occupy this vacancy, thereby creating a Hole in the Valence Band.
- These impurity atoms are called **ACCEPTORS** and they become negatively charged (by  $-q$ ) when they “accept” an electron from the VB.
- At room temperature, it is reasonable to assume that every impurity acceptor atom has received an electron, and therefore that each impurity atom contributes exactly one hole to the VB.

# Extrinsic P-TYPE Semiconductor

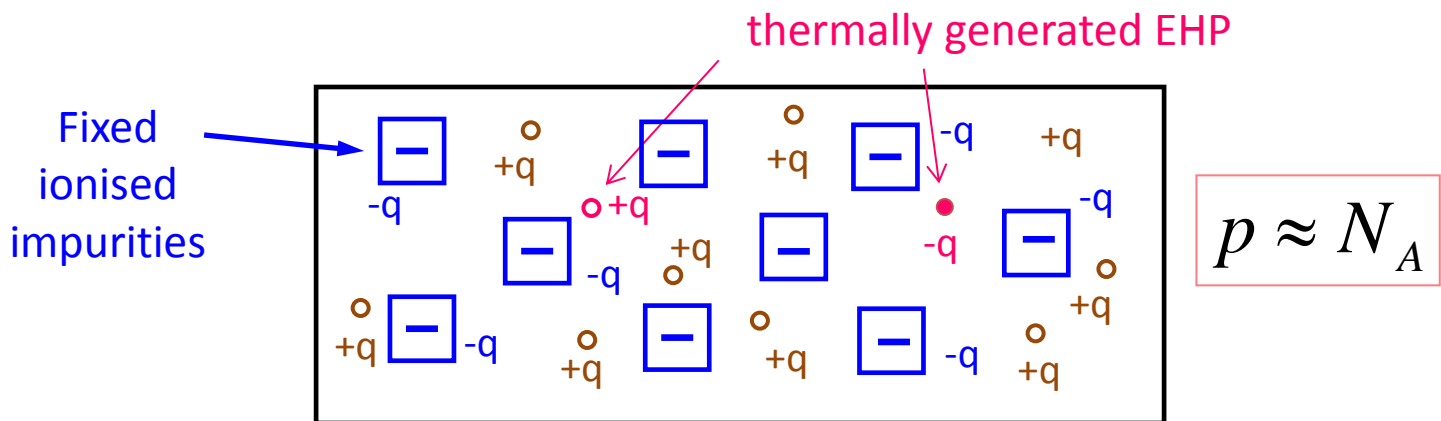


When it gains an electron this atom creates a fixed charged impurity site and becomes a negatively-charged **ionised acceptor**.



# P-Type Extrinsic Semiconductors

- In this case the semiconductor is doped with **acceptors**.
- **We define  $N_A$  as the Acceptor Concentration**, that is the number of acceptor atoms in the crystal per unit volume .  
(Units:  $/\text{m}^3$  or  $/\text{cm}^3$ )
- As we have seen, at room temperature (providing the donor concentration is not too low) then we can expect:  $p \sim N_A$  and  $p \gg n$ , so that holes are called the majority carriers while electrons are the minority carriers in this kind of material





# Carrier Concentrations in an Extrinsic Semiconductor in Thermal Equilibrium

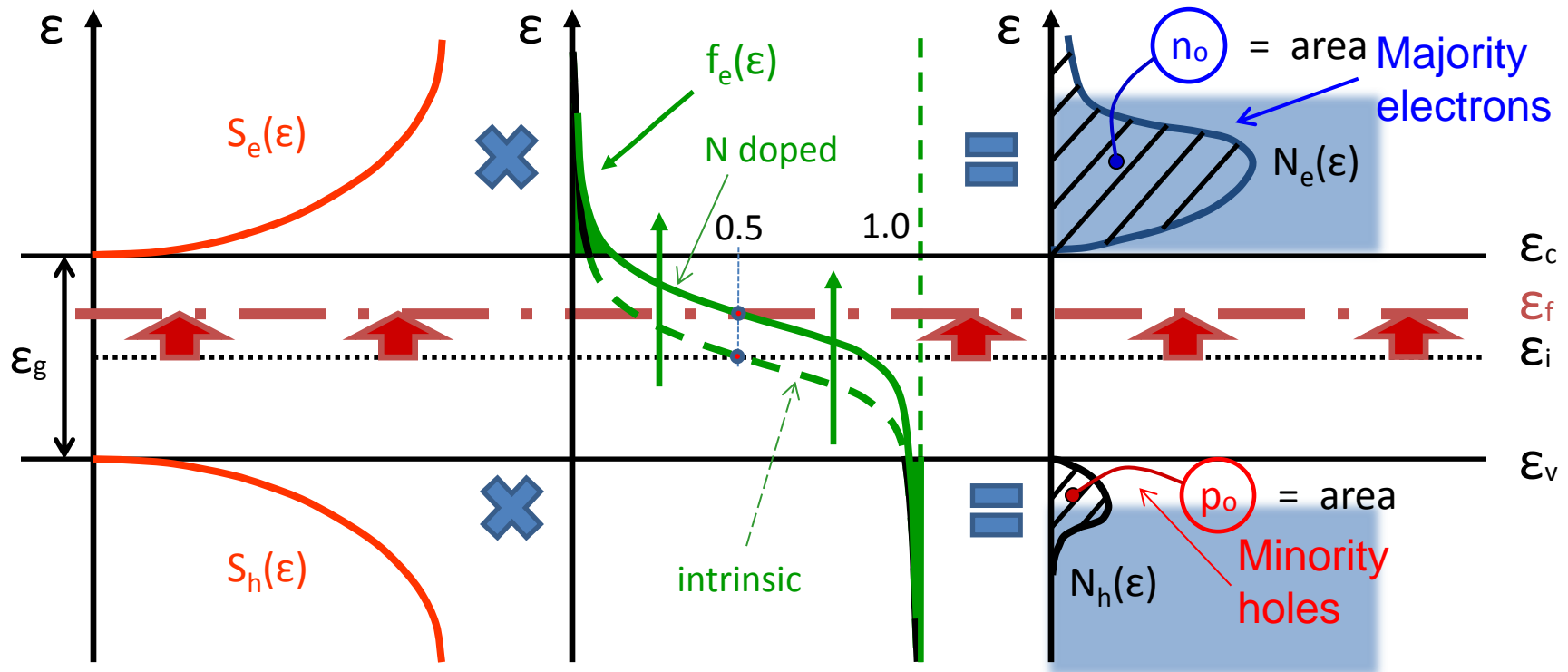
- We now develop some important basic relationships which apply to semiconductors under a wide range of conditions (intrinsic or extrinsic), provided *thermal equilibrium* applies;
- To emphasise the existence of thermal equilibrium, the subscript 'o' will be added to carrier concentration quantities:
  - $n_o$  = **conc. of C.B. electrons** in thermal equilibrium (i.e. no flows, no stimulus, steady-state...)
  - $p_o$  = **conc. of V.B. holes** in thermal equilibrium
- If a material is doped **N-Type**, the number distribution analysis carried out previously shows that the **only way in which we can have  $n_o \gg p_o$**  is for the **Fermi energy to move up** into the **UPPER HALF** of the forbidden gap (see next slide)
- If a material is doped **P-Type**, the number distribution analysis carried out previously shows that the **only way in which we can have  $p_o \gg n_o$**  is for the **Fermi energy to move down** into the **LOWER HALF** of the forbidden gap

# Number Density Distributions in an N-TYPE Extrinsic (or doped) semiconductor

N-TYPE semiconductor (300K)

$$n_o > p_o$$

Fermi energy ( $\mathcal{E}_f$ ) must lie in the upper half of the forbidden gap ( $\mathcal{E}_f > \mathcal{E}_i$ )

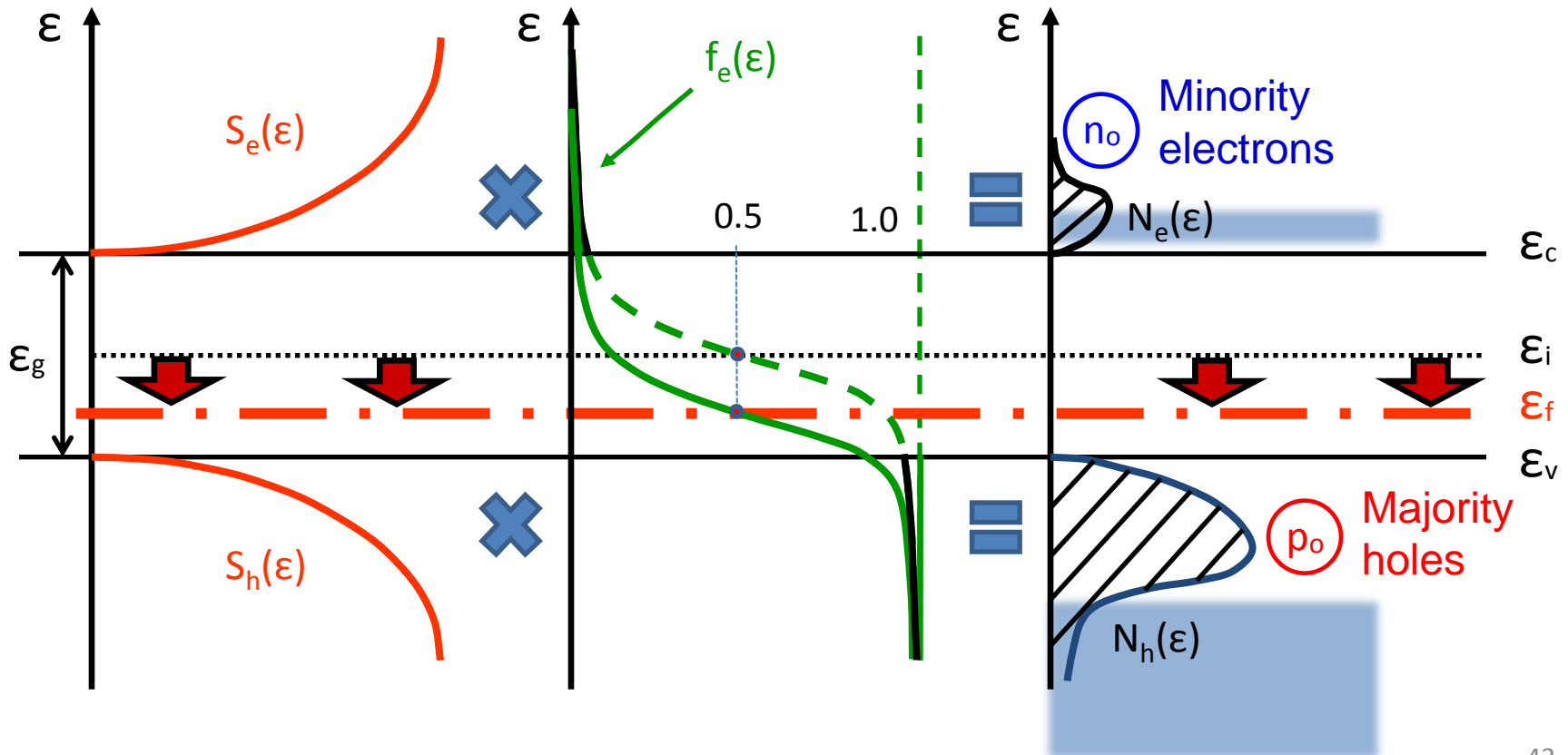


# Number Density Distributions in a P-TYPE Extrinsic (or doped) semiconductor

P-TYPE semiconductor (300K)

$$p_o > n_o$$

Fermi energy ( $\epsilon_f$ ) must lie in the lower half of the forbidden gap ( $\epsilon_f < \epsilon_i$ )

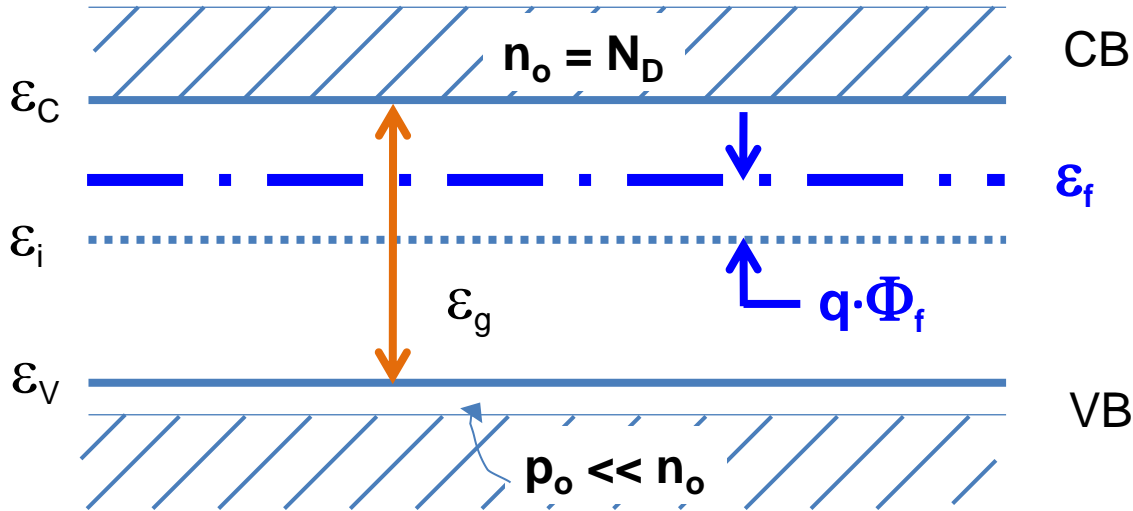


# Applet References

- For interactive applets illustrating the effects of altering various parameters on number distributions etc in semiconductors see:
- <http://jas.eng.buffalo.edu/education/semicon/fermi/functionAndStates/functionAndStates.html>
- <http://jas.eng.buffalo.edu/education/semicon/fermi/levelAndDOS/index.html>
- <http://jas.eng.buffalo.edu/education/semicon/fermi/bandAndLevel/fermi.html>

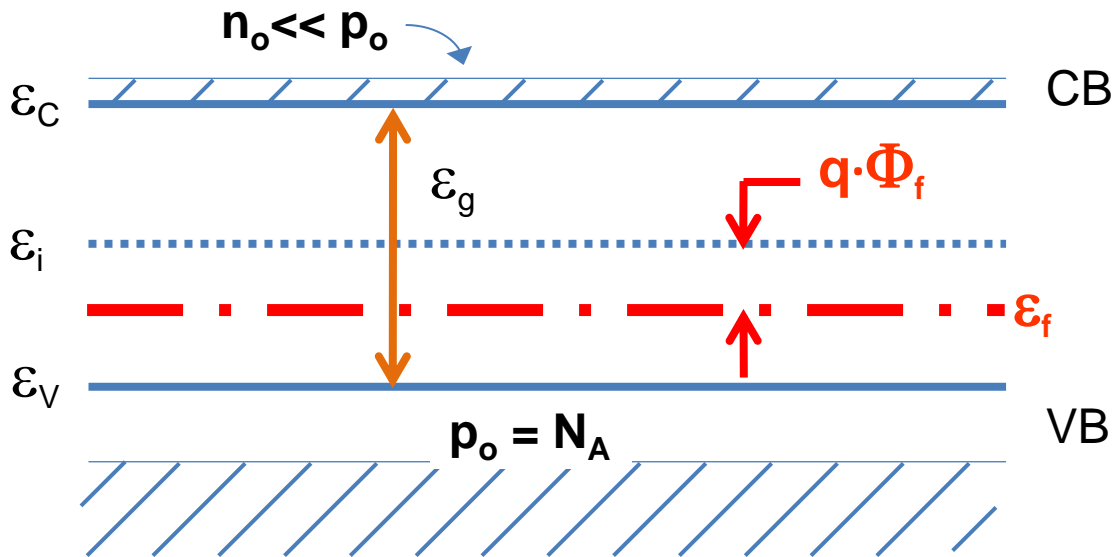
Parent Directory: <http://jas.eng.buffalo.edu/index.html>

# Summary of N-type Extrinsic Semiconductor



- Doped with **Donors** to a **donor concentration**  $N_D$
- Fermi level is in **upper half** of the forbidden gap
- Electron Concentration ( $n_o$ )  $\gg$  Hole Concentration ( $p_o$ ) unless very lightly doped
- If temperature  $T > \approx 100\text{K}$ ,  $n_o = N_D$
- Fermi level is offset from intrinsic level by an amount  $= q \cdot \Phi_f$

# Summary of **P-type** Extrinsic Semiconductor



- Doped with **Acceptors** to an **acceptor concentration**  $N_A$
- Fermi level is in **lower half** of the forbidden gap
- Hole Concentration ( $p_o$ )  $\gg$  Electron Concentration ( $n_o$ ) unless very lightly doped
- If temperature  $T > \approx 100\text{K}$ ,  $p_o = N_A$
- Fermi level is offset from intrinsic level by an amount  $= q \cdot \Phi_f$

## Example 3.3

# General Expressions for Carrier Concentrations

## $n_o$ and $p_o$ in Thermal Equilibrium

- We assume that the Fermi energy  $\mathcal{E}_f$  is located within the band gap and is separated from either the VB or the CB by at least a few  $kT$ , so that the **Boltzmann Approximation is valid**
- In the *intrinsic* case, we already derived expressions for  $n_o$  and  $p_o = n_i$
- These formulas can also be used directly for the *extrinsic* case but now **separately** for electrons and holes, as these concentrations are not equal in a doped semiconductor:

$$n_o = N_c \exp \left[ - \left( \frac{\mathcal{E}_c - \mathcal{E}_f}{kT} \right) \right]$$

$$p_o = N_v \exp \left[ - \left( \frac{\mathcal{E}_f - \mathcal{E}_v}{kT} \right) \right]$$



# Alternative Formula for Equilibrium Electron Concentration ( $n_o$ )

Consider the formula for  $n_o$  above:

... add in these cancelling terms, then  
split up the exponential into two parts

$$n_o = N_c \cdot \exp \left[ - \left( \frac{\varepsilon_c - \varepsilon_i + \varepsilon_i - \varepsilon_f}{kT} \right) \right] = N_c \cdot \exp \left[ - \left( \frac{\varepsilon_c - \varepsilon_i}{kT} \right) \right] \cdot \exp \left[ \frac{\varepsilon_f - \varepsilon_i}{kT} \right]$$

$$\Rightarrow n_o = n_i \cdot \exp \left[ \frac{\varepsilon_f - \varepsilon_i}{kT} \right]$$

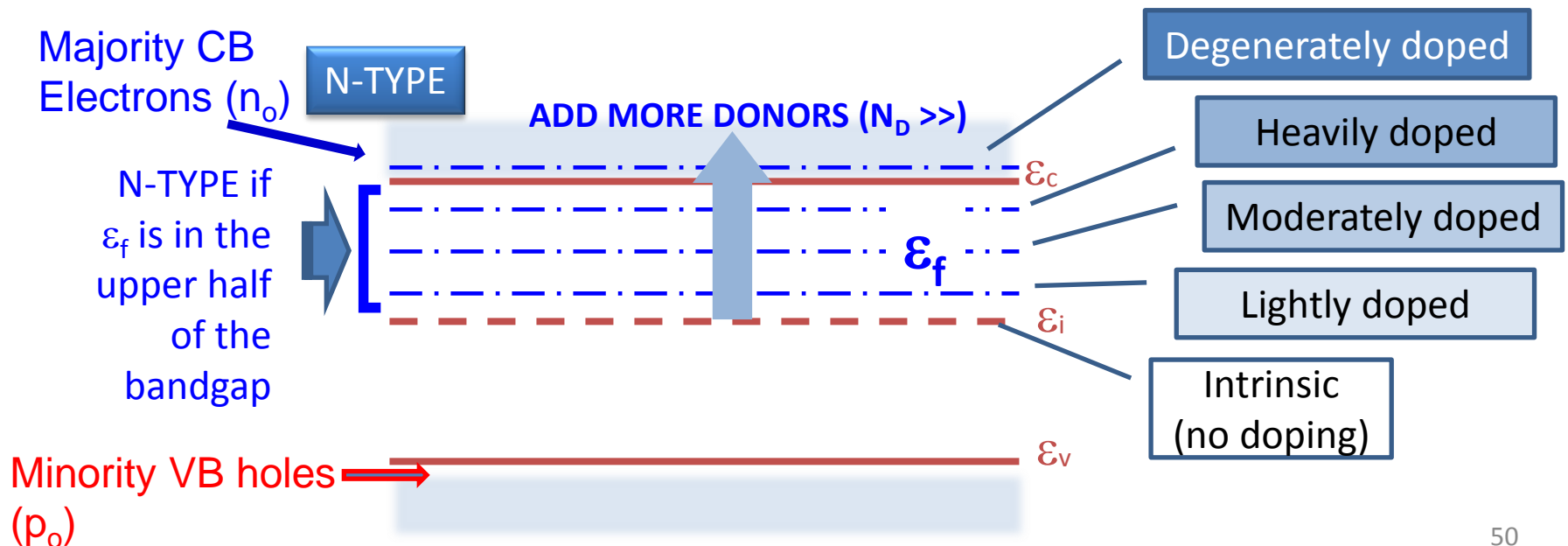
but this =  $n_i$

The “Boltzmann  
Approximation”

- This is a key result for the electron concentration in thermal equilibrium, valid under very general conditions, including for both N-type -and P-type doping
- Note that when  $\varepsilon_f = \varepsilon_i$ , then  $n_o = n_i$  as expected (intrinsic case)
- For  $n_o > n_i$ , we must have  $\varepsilon_f > \varepsilon_i$ , and the more  $\varepsilon_f$  moves above  $\varepsilon_i$  the larger  $n_o$  becomes (i.e. the material becomes more N-type)

# The Fermi Energy in an N-type Semiconductor

- The more we **dope** a semiconductor N-type, the more  $n_o$  becomes **greater** than  $n_i$ , and therefore the more the **Fermi energy** moves away from the intrinsic energy into the **UPPER HALF** of the Forbidden Gap;
- At extremely high doping levels,  $\mathcal{E}_f$  can actually begin to enter the CB and the material is then described as “**degenerately doped**” – the formula for  $n_o$  developed above becomes invalid in this case



# Degenerate Doping

- This refers to a condition of very high level of doping in a semiconductor sufficient to cause the Fermi level effectively to enter the CB (or VB in the P-type case);
- The Boltzmann approximation becomes increasingly invalid in this regime and more complex **Fermi-Dirac Integrals** are required;
- As a rule-of-thumb, degenerate conditions apply when the doping concentration rises to about 10% of the relevant density-of-states number (i.e. a doping level of about  $2 \times 10^{18}/\text{cm}^3$  in Si at 300K);
- For a good discussion of this phenomenon and a related applet, see:
- <http://jas.eng.buffalo.edu/education/semicon/fermi/heavyVSmode/index.html>

# Alternative Formula for Equilibrium Hole Concentration ( $p_o$ )

Consider the formula for  $p_o$  above:

Add in these cancelling terms, then split up the exponential into two parts

$$p_o = N_v \cdot \exp \left[ - \left( \frac{\mathcal{E}_f - \mathcal{E}_i + \mathcal{E}_i - \mathcal{E}_v}{kT} \right) \right] = N_v \cdot \exp \left[ - \left( \frac{\mathcal{E}_i - \mathcal{E}_v}{kT} \right) \right] \cdot \exp \left[ \frac{\mathcal{E}_i - \mathcal{E}_f}{kT} \right]$$

$$\Rightarrow p_o = n_i \cdot \exp \left[ \frac{\mathcal{E}_i - \mathcal{E}_f}{kT} \right]$$



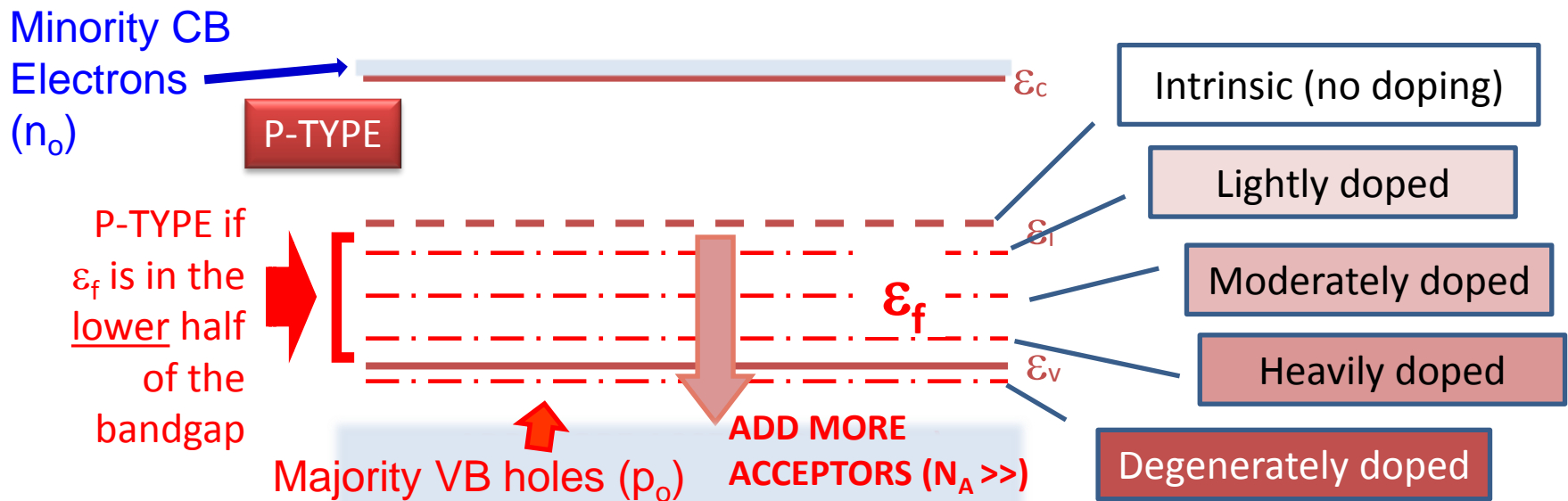
but this =  $n_i$

The “Boltzmann Approximation”

- This is also a key result, valid under very general conditions, including for both N-type -and P-type semiconductors, in thermal equilibrium
- Note that when  $\mathcal{E}_f = \mathcal{E}_i$ , then  $p_o = n_i$  as expected (intrinsic case)
- For  $p_o > n_i$ , we must have  $\mathcal{E}_f < \mathcal{E}_i$ , and the more  $\mathcal{E}_f$  moves below  $\mathcal{E}_i$  the larger  $p_o$  becomes (i.e. the material has become more P-type)

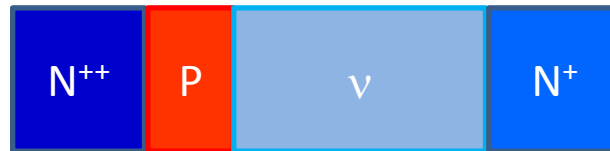
# The Fermi Energy in a P-type Semiconductor

- The more we **dope** a semiconductor **P-type**, the more  $p_o$  becomes **greater** than  $n_i$ , and therefore the more the **Fermi energy** moves away from the intrinsic energy into the **LOWER HALF** of the Forbidden Gap;
- At extremely high doping levels,  $\epsilon_f$  can actually begin to enter the VB and the material is then described as “**degenerately doped**” – the formula for  $p_o$  developed above becomes invalid in this case



# Notation to Indicate Level of Doping

- Over the years, a useful short-hand notation has been developed by the semiconductor industry to give an idea of the level of doping in a material :
  - Intrinsic**:- i.e. un-doped: '*i*';
  - N-TYPE**:- lightly-doped: *v*; moderately-doped: *N*; heavily doped *N<sup>+</sup>*; very heavily doped (or degenerate) *N<sup>++</sup>*
  - P-TYPE**:- lightly-doped: *π*; moderately-doped: *P*; heavily doped *P<sup>+</sup>*; very heavily doped (or degenerate) *P<sup>++</sup>*.
- Example: suppose we have a single crystal of Silicon with different regions with different doping:



- This could be denoted:  $N^{++} P v N^{+}$  (in fact this is a practical doping arrangement for an NPN bipolar transistor)

# Equilibrium Carrier Concentration Formulas based on the Boltzmann Approximation

- The Electron Concentration in the CB:  $n_o = n_i \cdot \exp\left[\frac{\varepsilon_f - \varepsilon_i}{kT}\right]$
- The Hole Concentration in the VB:  $p_o = n_i \cdot \exp\left[\frac{\varepsilon_i - \varepsilon_f}{kT}\right]$

- Multiply these together:

$$n_o \cdot p_o = n_i \cdot \exp\left[\frac{\varepsilon_f - \varepsilon_i}{kT}\right] \cdot n_i \cdot \exp\left[\frac{\varepsilon_i - \varepsilon_f}{kT}\right] = n_i^2 \cdot \exp\left[\frac{\varepsilon_f - \varepsilon_i + \varepsilon_i - \varepsilon_f}{kT}\right] = n_i^2$$

*these sum to zero*

- This leads to a simple and important formula valid for any semiconductor material in thermal eqm. (N-type or P-type):

$$n_o \cdot p_o = n_i^2$$

# Boltzmann Approximation –

## 3 Key Formulas

- These formulas allow us to connect **equilibrium electron or hole concentrations** to the **position of the Fermi energy** ( $\mathcal{E}_f$ ) in relation to the intrinsic energy ( $\mathcal{E}_i$ ) and to the intrinsic concentration ( $n_i$ ) for any non-degenerately-doped semiconductor:

$$n_o = n_i \cdot \exp \left[ \frac{\mathcal{E}_f - \mathcal{E}_i}{kT} \right]$$

$$p_o = n_i \cdot \exp \left[ \frac{\mathcal{E}_i - \mathcal{E}_f}{kT} \right]$$

$$n_i^2 = n_o \cdot p_o$$



# Conductivity of N-Type Extrinsic Semiconductor

- Assume operation at a temperature above “freeze-out” ( $T > \sim 100\text{K}$ ) but not so high that intrinsic thermal generation of EHPs dominates;
- Assume that the donor doping level is not too light so that  $N_D \gg n_i$  but equally not so high that the material is “degenerate”
- We can then say that  $n_o = N_D$ . And since  $n_o \cdot p_o = n_i^2$ , it follows that  $p_o = (n_i^2/N_D)$ . The conductivity is thus:

“normal conditions”

$$\sigma_{\text{N-type}} = q \cdot (n_o \cdot \mu_n + p_o \cdot \mu_p) = q \cdot [N_D \cdot \mu_n + (n_i^2/N_D) \cdot \mu_p]$$

$\approx 0$

- In practice the minority carrier contribution to conductivity (i.e. the second term) is usually negligible (see example further on), so that we can simply write:

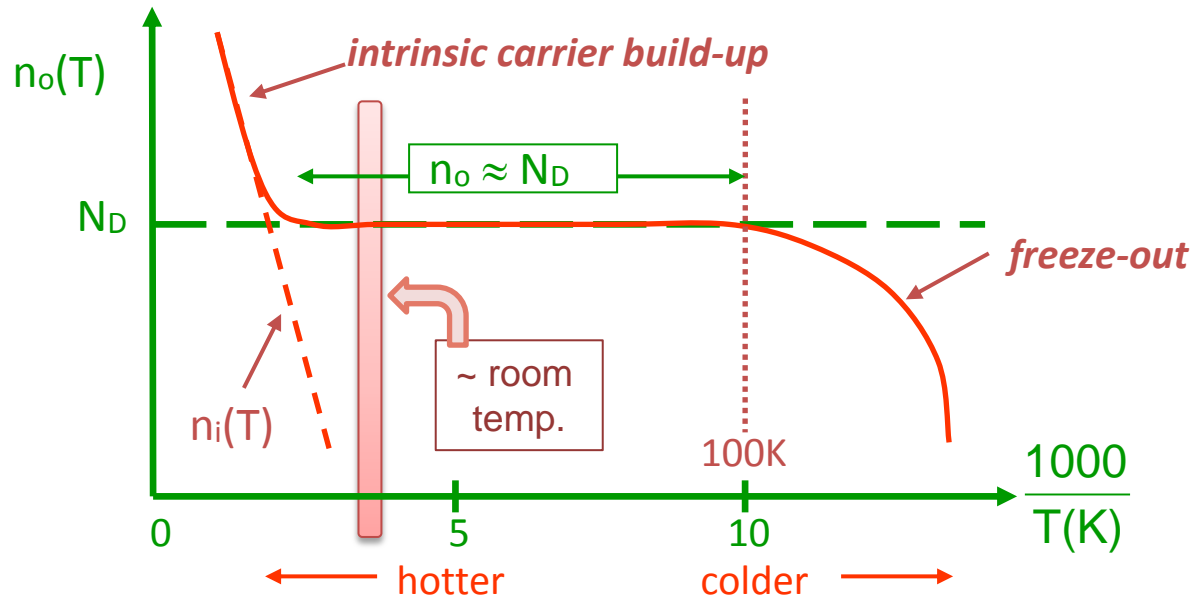
$$\sigma_{\text{N-type}} = q \cdot N_D \cdot \mu_n$$

# Temperature Dependence of Conductivity (1)

- Consider conductivity due to electrons in an N-type semiconductor:

$$\sigma_{Ne}(T) = q \cdot n_o(T) \cdot \mu_n(T)$$

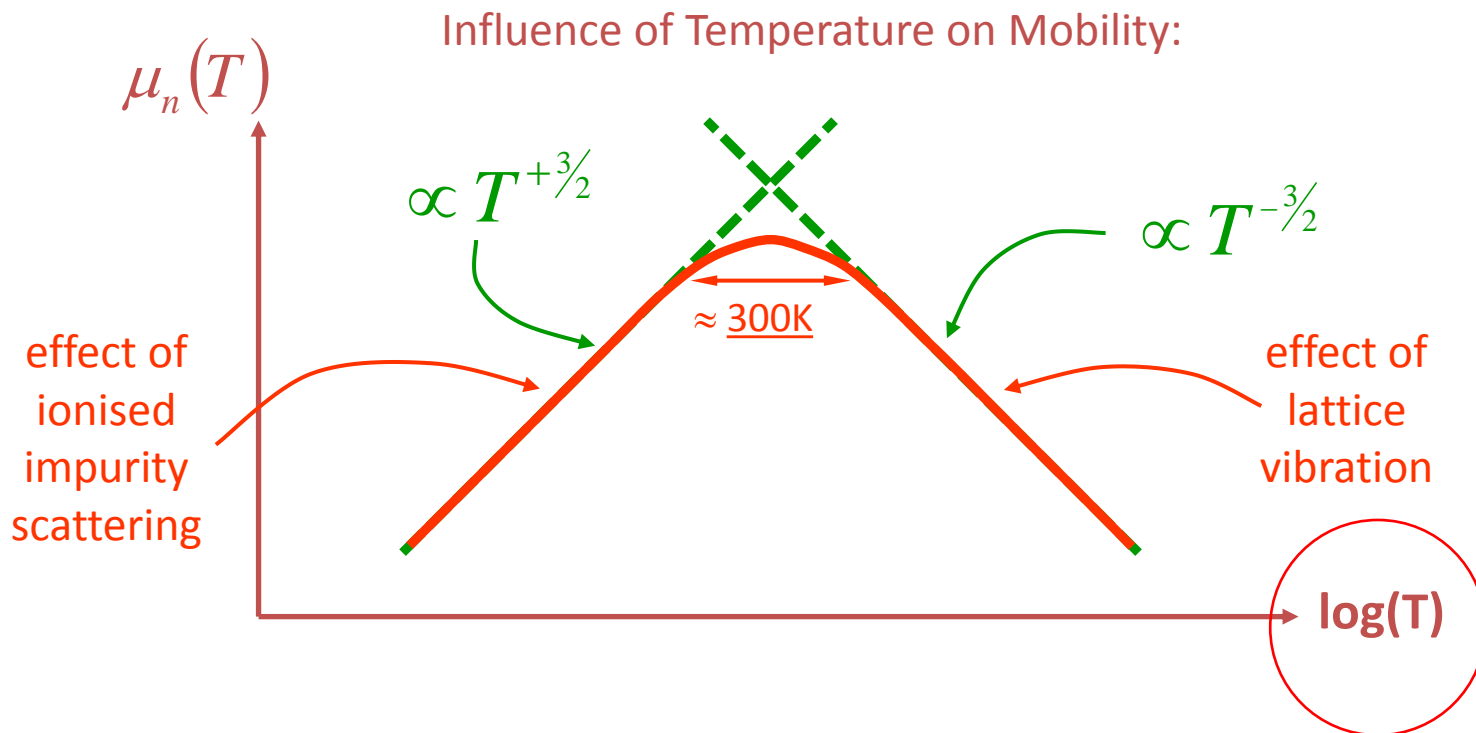
- Begin with variation of  $n_o(T)$ :





- Note that over a quite wide range of temperature we can say  $n_o = N_D$  but at high enough temperature, the exponential build-up of intrinsic carriers with  $> T$  eventually swamps the effect of the doping

## Temperature Dependence of Conductivity (2)

- Now consider temperature dependence of the electron mobility ( $\mu_n(T)$ ). This is shown on the following graph. (Note that a power law of the kind:  $\mu_n(T) = (\text{const. } T^\alpha)$  shows up as a straight line plot on a semi-log graph)



# Conductivity of P-Type Extrinsic Semiconductor

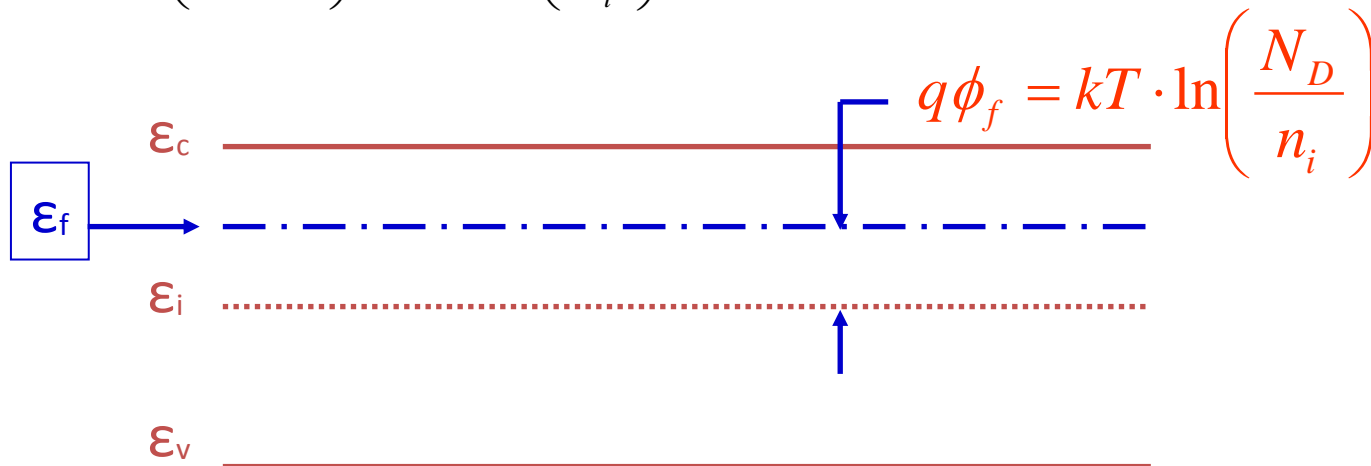
- Assume operation at a temperature above “freeze-out” ( $T > \sim 100\text{K}$ ) but not so high that intrinsic thermal generation of EHPs dominates;  “normal conditions”
  - Assume that the donor doping level is not too light so that  $N_A \gg n_i$  but equally not so high that the material is “degenerate”
  - We can then say that  $p_o = N_A$ . Since  $n_o \cdot p_o = n_i^2$ , it follows that  $n_o = (n_i^2/N_A)$ . The conductivity is thus: 
- $$\sigma_{\text{P-type}} = q \cdot (n_o \cdot \mu_n + p_o \cdot \mu_p) = q \cdot \left( \cancel{n_i^2 / N_A} \cdot \mu_n + N_A \cdot \mu_p \right)$$
- In practice the minority carrier contribution to conductivity (i.e. the first term) is generally negligible, so that we can simply write:

$$\sigma_{\text{P-type}} = q \cdot N_A \cdot \mu_p$$

# Relationship between Doping Level and Fermi Level Displacement: N-Type Case

- We have seen that there is clearly a close link between doping level and the position of the Fermi level: the greater the amount of doping the more  $\varepsilon_f$  is displaced from the intrinsic level  $\varepsilon_i$ .
- We can use the earlier carrier formulas based on the Boltzmann approximation to estimate this displacement. Assume “normal conditions” in an N-type semiconductor doped to a concentration  $N_D$ . Then  $n_o = N_D$

$$n_o = N_D = n_i \cdot \exp\left(\frac{\varepsilon_f - \varepsilon_i}{kT}\right) \Rightarrow \ln\left(\frac{N_D}{n_i}\right) = \frac{\varepsilon_f - \varepsilon_i}{kT} \Rightarrow (\varepsilon_f - \varepsilon_i) = q \cdot \phi_f = kT \cdot \ln\left(\frac{N_D}{n_i}\right)$$

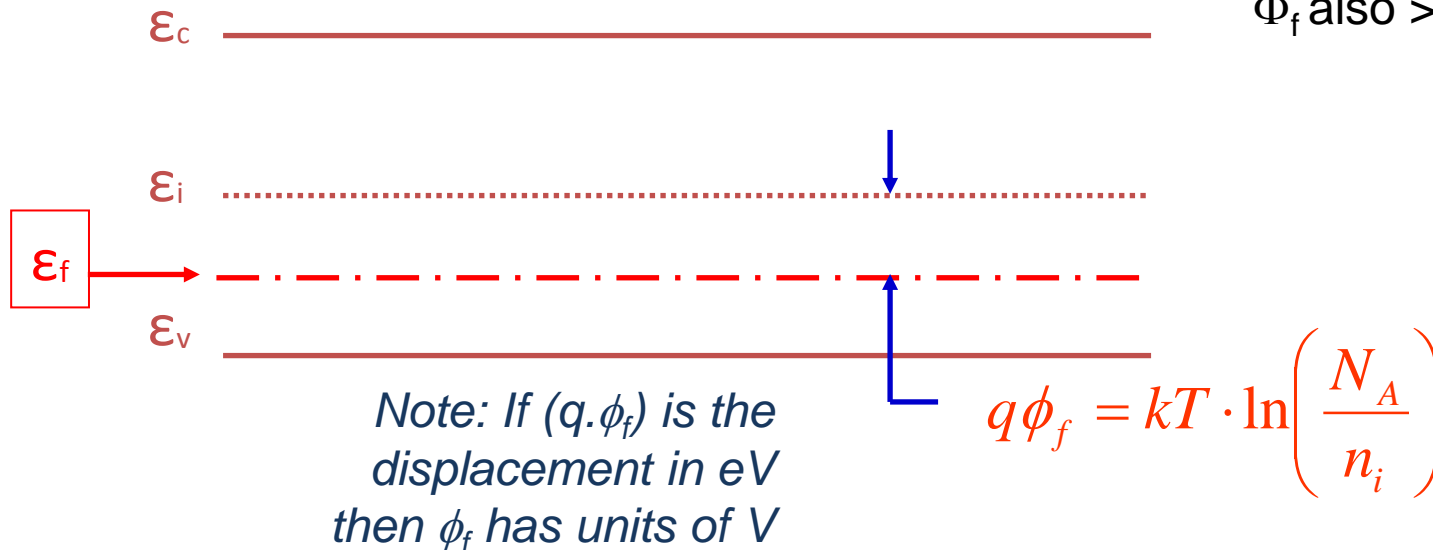


# Relationship between Doping Level and Fermi Level Displacement: P-Type Case

- A similar expression applies to the **P-type** case.
- Again, we assume “normal conditions” in a P-type semiconductor doped to a concentration  $N_A$ . Then using  $p_o = N_A$  a similar analysis gives:

$$(\varepsilon_i - \varepsilon_f) = q \cdot \phi_f = kT \cdot \ln\left(\frac{N_A}{n_i}\right)$$

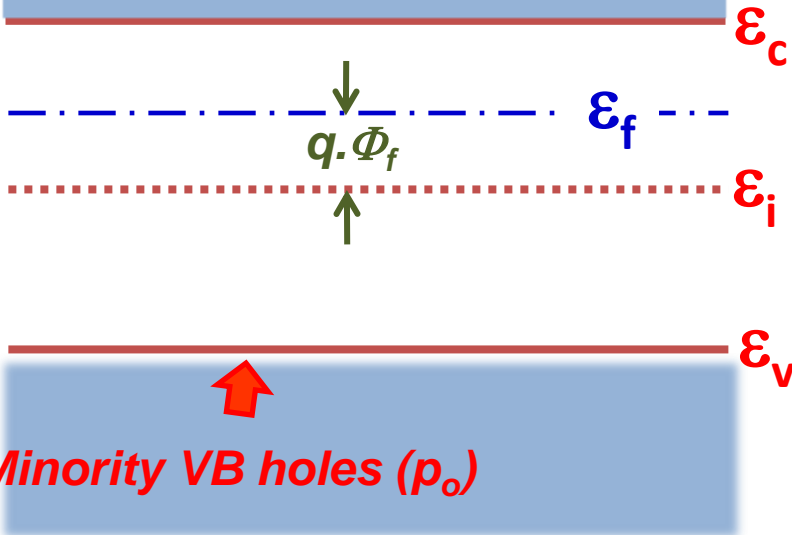
If  $N_A \gg$  then  $\Phi_f$  also  $>$



# SUMMARY

N-TYPE:  $N_D$  Donors

Majority CB electrons ( $n_o$ )



$$n_o = N_D$$

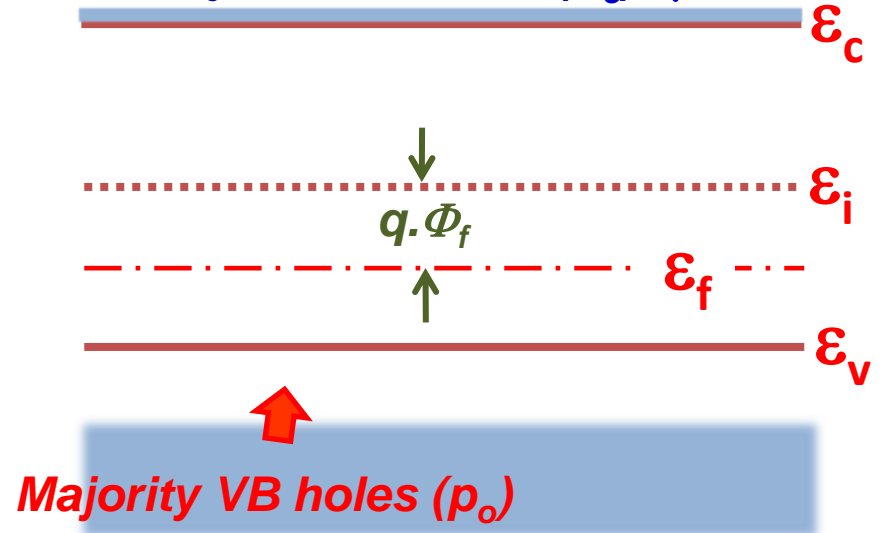
$$p_o = \frac{n_i^2}{N_D}$$

$$q \cdot \Phi_f = kT \cdot \ln \left| \frac{N_D}{n_i} \right|$$

$$\sigma_N = q \cdot N_D \cdot \mu_n$$

P-TYPE:  $N_A$  Acceptors

Minority CB electrons ( $n_o$ ) ↴



$$p_o = N_A$$

$$n_o = \frac{n_i^2}{N_A}$$

$$q \cdot \Phi_f = kT \cdot \ln \left| \frac{N_A}{n_i} \right|$$

$$\sigma_P = q \cdot N_A \cdot \mu_p$$

## EXAMPLE 3.4



## EXAMPLE 3.5