2.26

21 January 2015 09:12

Sample: 4, 2, 1, 3, 2, $\boxed{13}$, 1

outlier

$$\bar{x} = \frac{4+2+1+3+2+13+1}{7} \quad \left[= \frac{1}{n}\sum_{i=1}^{n} x_i \right]$$

$$\therefore \bar{x} = \frac{26}{7} = 3.7$$

Sample median:

Sort the data set

1. 1, 1, 2, $\boxed{2}$, 3, 4, 13
2. Sample Median: 2

Mode: Most Common value

$\therefore$ Sample mode = 1 and 2

2.33

21 January 2015 09:12

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

Use this formula for calculating by hand

Data Sample

5.7, 2.3, 6.2, 1.5, 4.0, 2.9

We want s^2 (Sample variance)

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}$$

1 2

$$\bar{x} = \frac{5.7 + 2.3 + 6.2 + \cdots + 2.9}{6} = \frac{22.6}{6} = 3.767$$

Long: 1

Long: 1

$$s^2 = \frac{(5.7-3.767)^2 + \cdots + (2.9-3.767)^2}{6-1}$$

$\Rightarrow$ long (and boring)

Quick: 2

$$\sum_{i=1}^{n} x_i^2 = 5.7^2 + 2.3^2 + \cdots + 2.9^2$$

$$= 102.88$$

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1} = \frac{102.88 - (6)(3.767)^2}{6-1}$$

$$= 3.548$$

Sample standard deviation:

$$s = \sqrt{3.548}$$

26 January 2015 09:18

Example: Loaded die

$\mathbb{P}(\text{die shows } i) \propto i$

$\Rightarrow \mathbb{P}(1) = p$

$\mathbb{P}(2) = 2p$

$\mathbb{P}(3) = 3p$

... etc.

From 3rd probability axiom:

$6p + 5p + 4p + 3p + 2p + p = 1$

$\Rightarrow p = \frac{1}{21}$

$\Rightarrow \mathbb{P}(1) = \frac{1}{21}, \mathbb{P}(2) = \frac{2}{21}, \ldots, \mathbb{P}(6) = \frac{6}{21}$

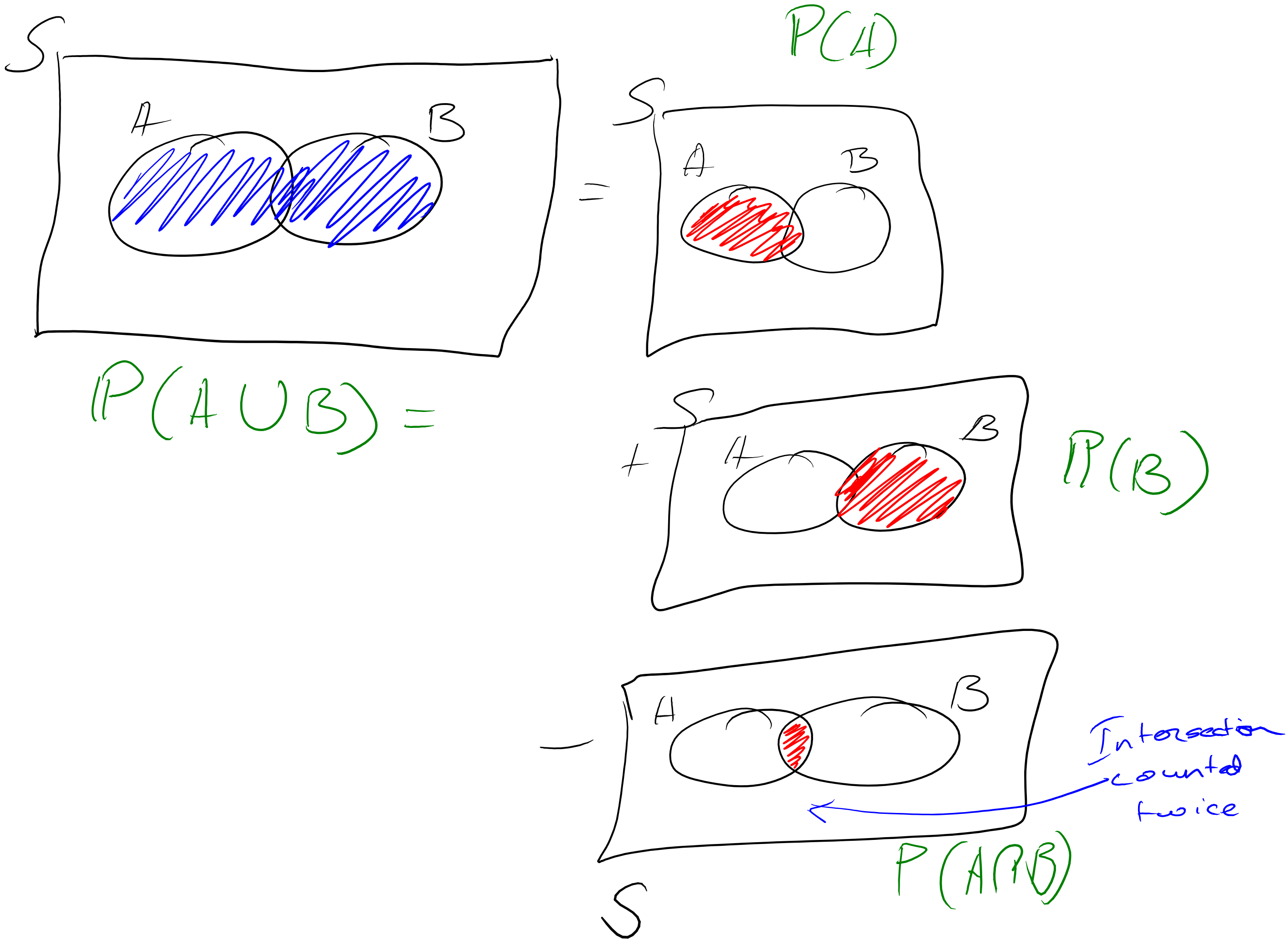
$A = \{2, 3\}$

$\mathbb{P}(A) = \mathbb{P}(2) + \mathbb{P}(3)$

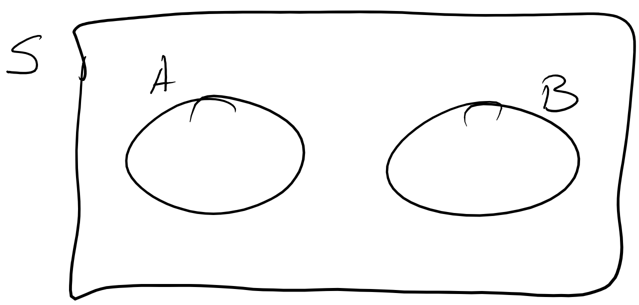
$= \frac{2}{21} + \frac{3}{21} = \frac{5}{21}$

3.8

26 January 2015 09:28



Mutually Exclusive:

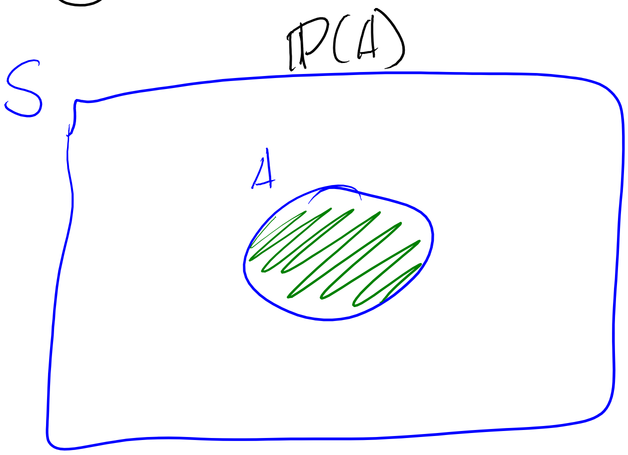


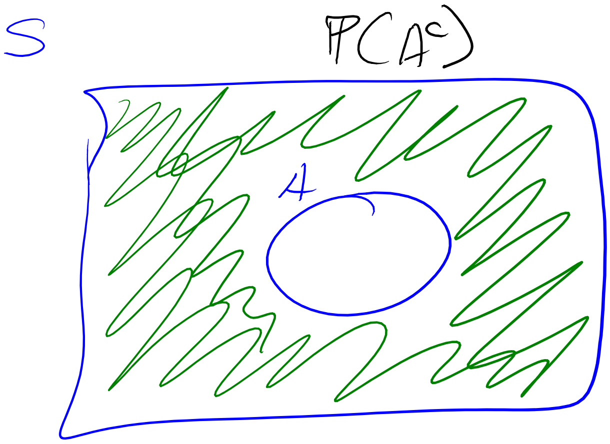
∴ No overlap

3.9

26 January 2015 09:32

Set Complement:





$$\Rightarrow \quad \mathbb{P}(S) = \mathbb{P}(A) + \mathbb{P}(A^c)$$

$$\therefore \quad \mathbb{P}(A) + \mathbb{P}(A^c) = 1$$

3.10

26 January 2015 09:36

Example: Roll 2 dice

A - Observe a 5

B - Dice sum to 7

$$\mathbb{P}(A) = \frac{11}{36} \qquad \mathbb{P}(B) = \frac{1}{6} = \left[\frac{6}{36}\right]$$

$$\Rightarrow \mathbb{P}(A \cap B) = \frac{2}{36}$$

$$\Rightarrow \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

$$= \frac{11}{36} + \frac{6}{36} - \frac{2}{36}$$

$$\therefore \mathbb{P}(A \cup B) = \frac{15}{36}$$

$$\Rightarrow \mathbb{P}(A^c) = 1 - \mathbb{P}(A)$$

$$= 1 - \frac{11}{36}$$

$$= \frac{25}{36}$$

3.12

26 January 2015 09:42

Example: Roulette

A : outcome is odd

B: outcome is red

C: outcome is in 1st dozen

$\therefore A \cap B = \{1, 3, 5, 7, 9, 19, 21, 23, 25, 27\}$

$\therefore A \cup B = \{1, 3, 5, 7, 9, 12, 14, 16, 18, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 30, 31, 32, 33, 34, 35, 36\}$

* $\mathbb{P}(A) = \frac{18}{38}$ $\quad \mathbb{P}(B) = \frac{18}{38}$

* $\mathbb{P}(A \cap B) = \frac{10}{38}$

* $\mathbb{P}(A \cup B) = \frac{26}{38}$

* $\mathbb{P}(C) = \frac{12}{38}$

---

(3) $P(A \cup B)$: $P(A) = P(B) = \frac{18}{38}$

$P(A \cap B) = \frac{10}{38}$

↳ Not Mutually exclusive! ↳ $P(A \cap B) \neq 0$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{18}{38} + \frac{18}{38} - \frac{10}{38}$$
$$= \frac{26}{38}$$

---

(4) $P(A \cap B \cap C) = \frac{5}{38}$

↳ There are 5 sample points in this event, each with a probability of $\frac{1}{38}$.

3.15

28 January 2015 09:26

Illegal trading:

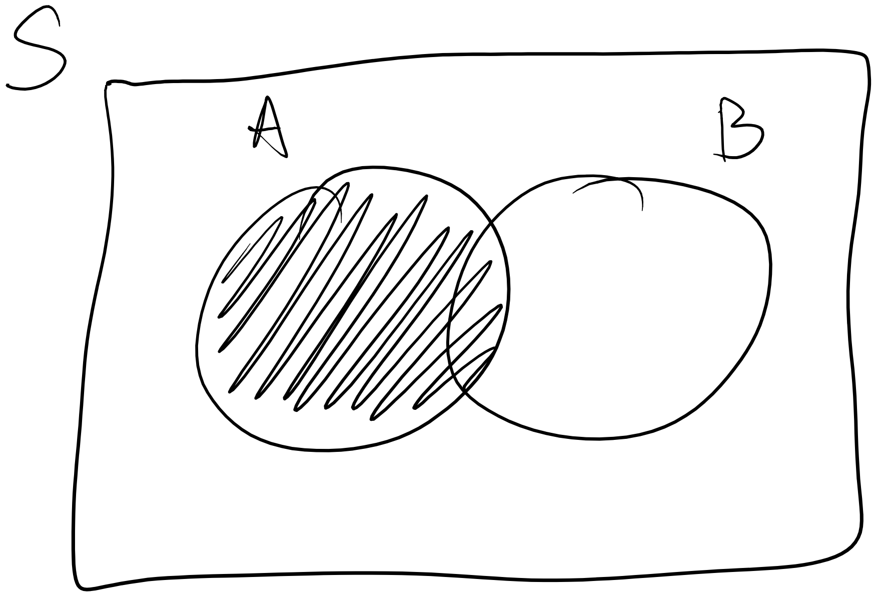
$$P(1^{st} \text{ illegal}) = \frac{3}{10}$$

$$P(2^{nd} \text{ illegal} \mid 1^{st} \text{ illegal}) = \frac{2}{9}$$

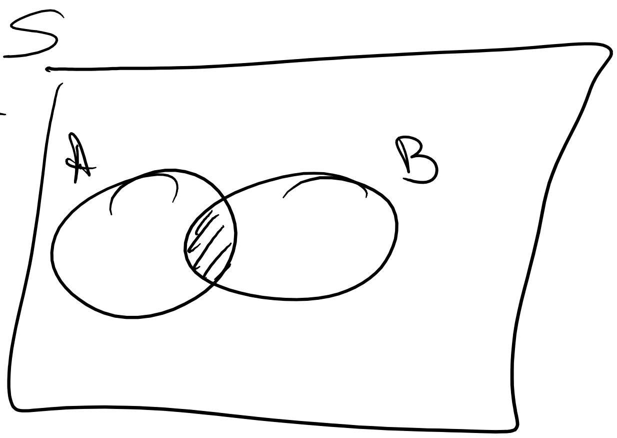
$$P(\text{both illegal}) = \left(\frac{3}{10}\right)\left(\frac{2}{9}\right) = \frac{6}{90}$$

3.16

28 January 2015 09:29

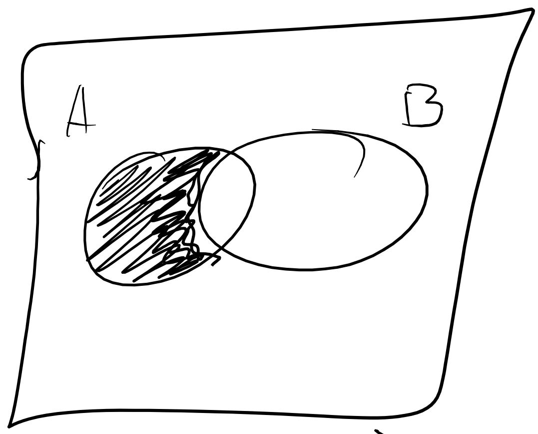


=



$P(A \cap B)$

+



$P(A \cap B^c)$

3.17

28 January 2015 09:34

Diagnostic Test

- D - the event that I have the disease
- A - the event that I test positive

We know:

$$P(A|D) = 0.95$$

$$P(A|D^c) = 0.01$$

$$P(D) = 0.001 \Rightarrow P(D^c) = 0.999 = 1 - P(D)$$

$$P(D|A) = \frac{P(A|D)P(D)}{P(A|D)P(D) + P(A|D^c)P(D^c)}$$

% have the disease after having tested positive

$$= \frac{(0.95)(0.001)}{(0.95)(0.001) + (0.01)(0.999)}$$

$$= \frac{0.00095}{0.01094}$$

$$= 0.087$$

3.18

28 January 2015 09:45

Independence:

Multip. rule:

$$P(B \cap A) = P(B \mid A)\, P(A)$$

but if A and B are independent

then $P(B \mid A) = P(B)$ so

$$P(B \cap A) = P(B)\, P(A)$$

3.19

28 January 2015 09:46

Tyres

- A = Tyre A passes test
- B = " B " "
- C = " C " "

$P(A) = 0.7$ $\qquad$ $P(A^c) = 0.3$

$P(B) = 0.6 \quad \Rightarrow \quad P(B^c) = 0.4$

$P(C) = 0.5$ $\qquad$ $P(C^c) = 0.5$

- All fail

$$P(A^c \cap B^c \cap C^c) = P(A^c)\,P(B^c)\,P(C^c)$$

independent!

$$= (0.3)(0.4)(0.5)$$
$$= 0.06$$

- $P(\text{at least } 1 \text{ pass}) = 1 - \underbrace{P(A^c \cap B^c \cap C^c)}_{\text{all fail}}$

$$= 0.94$$

- $$P(\text{Only } B \mid \text{At least } 1 \text{ Pass}) = \frac{\boxed{P(\text{Only } B \cap \text{At least } 1 \text{ Pass})}}{P(\text{At least } 1 \text{ Pass})}$$

P(B)

only way this event can happen

A	✗	✗
B	✓	✓
C	✗	✓

	✓	
A	✗	✗
B	✓	✓
C	✗	✓

$$\therefore P(\text{Only } B \mid \text{At least 1 Pass}) = \frac{P(\text{Only } B)}{P(\text{At least 1 Pass})}$$

$$= \frac{P(A^c)\,P(B)\,P(C^c)}{0.94}$$

$$= \frac{(0.3)(0.6)(0.5)}{0.94}$$

$$= 0.0957$$

3.20

02 February 2015 09:12

Corrosion

C - component is corroded

F - component is functioning

C and F are independent iff

$$P(C \cap F) = P(C)P(F)$$

- $P(C \cap F) = 0.2$ ...from table
- $P(C) = P(C \cap F) + P(C \cap F^c)$

  $= 0.2 + 0.4$

  $= 0.6$
- $P(F) = P(F \cap C) + P(F \cap C^c)$

  $= 0.2 + 0.3$

  $= 0.5$

Thus $P(C)P(F) = (0.6)(0.5) = 0.3 \neq 0.2$

$= P(C \cap F)$

$\therefore$ C and F are not independent

3.24

02 February 2015 09:23

Poker

- $\binom{52}{5} = 2,598,960$

- $\binom{13}{1} \times \binom{4}{3} \times \binom{12}{2} \times \binom{4}{1}^2$

Choose denomination

Choose 3 out 4 suits

Of remaining denominations Choose 2

different

[For those 2 denominations choose 1 out of 4 suits]

$= 54,912$

$P(3 \text{ of a kind}) = \dfrac{54\ 912}{2,598,960} = 0.0211$

- # Ways to get a full house

$\binom{13}{2} \times \binom{4}{3} \times \binom{4}{2} \times \binom{2}{1}$

Choose denomination

[out of 4 suits choose 3]

[out of 4 suits choose 2]

3A 2K or 3K 2A

[out of 2 denominations choose which has 3]

$= 3,744$

$\Rightarrow P(\text{full house}) = \dfrac{3744}{2\ 598\ 960} = 0.0014$

$$\Rightarrow \quad P(\text{full house}) = \frac{}{2,598,960} = 0.0014$$

04 February 2015 09:06

Y - # heads observed in two coin tosses.

H - observe heads

T - observe tails

$P(Y=0) = P(T,T) = P(T)P(T)$ ... tosses independent

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

$P(Y=1) = P(T,H) + P(H,T)$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

$P(Y=2) = P(H,H) = P(H)P(H)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

4.9

04 February 2015 09:18

$$E(x) = 3.5$$

$$E(x^2) = 15.17$$

$$\sigma_x = \sqrt{E(x^2) - E(x)^2}$$

$$= \sqrt{15.17 - (3.5)^2}$$

$$= \sqrt{2.92}$$

$$= 1.7098$$

4.10

04 February 2015 09:21

Car Safety Ratings

Y - # stars obtained by randomly selected car [from this sample]

- $P(Y=1) = \frac{0}{98} = 0$
- $P(Y=2) = \frac{4}{98} = 0.0408$
- $P(Y=3) = \frac{17}{98} = 0.1735$
- $P(Y=4) = \frac{59}{98} = 0.602$
- $P(Y=5) = \frac{18}{98} = 0.1837$

[Y = 4 is the mode]

☆1	☆2	☆3	☆4	☆5
0	0.0408	0.1735	0.602	0.1837

(Note $\sum_x P(x) = 1$)

(2) $P(Y \leq 3) = P(Y=1) + P(Y=2)$

$+ P(Y=3)$

$= 0 + 0.0408 + 0.1735$

$= 0.2143$

3 $P(Y>3) = P(Y=4) + P(Y=5)$

$= 0.602 + 0.1837$

$= 1 - P(Y \leq 3)$

$= 0.7857$

4 $E(Y) = \sum_y y\,p(y)$

$= 1(0) + 2(0.0408) + 3(0.1735)$
$+ 4(0.602) + 5(0.1837)$

$= 3.9286$

5 $\sigma_x^2 = E(Y^2) - E(Y)^2$

$E(Y^2) = 1^2(0) + 2^2(0.0408) + 3^2(0.1735)$
$+ 4^2(0.602) + 5^2(0.1837)$

$= 15.9492$

$\therefore \sigma_x^2 = 15.9492 - (3.9286)^2$

$\therefore \quad = 0.5153$

$\sigma_x = \sqrt{0.5153} = 0.7178$

4.12

04 February 2015 09:38

Y – # trades which make a loss

$E(Y) = Var(Y) = 4$

X – Profit made per day

$X = 1000 - 10Y - 5Y^2$

$\therefore E(X) = E(1000 - 10Y - 5Y^2)$

$= E(1000) + E(-10Y) + E(-5Y^2)$

$= 1000 - 10E(Y) - 5E(Y^2)$

$= 1000 - 10E(Y) - 5[\sigma_y^2 + E(Y)^2]$

$\sigma_y^2 = E(Y^2) - E(Y)^2$

$\therefore E(Y^2) = \sigma_y^2 + E(\cancel{X})^2$

$= 4 + 16$

$= 20$

$\therefore E(X) = 1000 - 10(4) - 5(4 + 4^2)$

$= 860$

$P = 7X - 1.5[O] - 10$

$Var(P) = Var(7X - 1.5[O] - 10)$

$= 7^2 Var(X) + (-1.5)^2 Var[O] + 0$

$= 7^2[E(X^2) - E(X)^2] + (1.5)^2(50^2)$

$= 49[800{,}000 - (860)^2] + 1.5^2[50^2]$

$= 2{,}955{,}850$

$\therefore sd(P) = 1{,}719.2586$

4.13

09 February 2015 09:09

n trials

$$x \sim Bin(n, p)$$

distributed

binomial

probability of success on any trial.

4.17

09 February 2015 09:17

$$\boxed{P(x) = \binom{n}{x} p^x (1-p)^{n-x}}$$

- Broker sells 20 products
- P(Profit) = 0.9 for each product

X – # of products which make money

$X \sim Bin(20, 0.9)$

∴ ⇒ Slide 18 for failure

$$P(X \geq 18) = \sum_{y=18}^{20} \binom{20}{y} (0.9)^y (1-0.9)^{20-y}$$

$$= P(X=18) + P(X=19) + P(X=20)$$

$$= \binom{20}{18}(0.9)^{18}(1-0.9)^{20-18} + \binom{20}{19}(0.9)^{19}(0.1)^{20-19} + \binom{20}{20}(0.9)^{20}(0.1)^{0}$$

$$= 0.2852 + 0.2702 + 0.1216$$

$$= 0.677$$

$$E(X) = np = (20)(0.9) = 18$$

$$Var(X) = npq = (20)(0.9)(1-0.9) = 1.8$$

4.21

09 February 2015 09:33

Catalysts

X - # highly acidic catalysts in the sample

$$X \sim \text{Hypergeometric}(N=10, s=4, n=3)$$

- $$P(X=0) = \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = \frac{20}{120} = \frac{1}{6} = 0.1667$$

- $$P(X=1) = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{60}{120} = 0.5$$

3 chosen simul...

- $$E(X) = \frac{ns}{N} = \frac{(3)(4)}{10} = 1.2$$

- $$Var(X) = n\left(\frac{s}{N}\right)\left(\frac{N-s}{N}\right)\left(\frac{N-n}{N-1}\right)$$

$$= 3\left(\frac{4}{10}\right)\left(\frac{10-4}{10}\right)\left(\frac{10-3}{10-1}\right)$$

$$= 0.56$$

$$\therefore \quad \sigma = \sqrt{0.56} = 0.7483$$

4.26

11 February 2015 09:11

Cables

- X - # of faults in 100m of cable

  λ - 2 [Rate Parameter]
  faults per 100m

  $X \sim Poisson(\lambda = 2)$

$\boxed{1}$ $\mathbb{P}(X=0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{2^0 e^{-2}}{0!} = 0.1353$

$\boxed{2}$ $\mathbb{P}(X=1) = \dfrac{2^1 e^{-2}}{1!} = 0.2707$

$\boxed{3}$ $\mathbb{P}(X \geq 2) = 1 - \mathbb{P}(X<2)$
$= 1 - \mathbb{P}(X \leq 1)$
$= 1 - (\mathbb{P}(X=0) + \mathbb{P}(X=1))$
$= 0.59399$

$\boxed{4}$ 200m of cable

- X - # faults in 1^{st} 100m -> $X \sim Poisson(\lambda_x = 2)$
- Y - " " " 2^{nd} " -> $Y \sim Poisson(\lambda_y = 2)$

$\therefore$ # faults in 200m is $X+Y$

$\therefore X+Y \sim Poisson(\underbrace{2+2}_{\lambda_x + \lambda_y})$

$X+Y \sim Poisson(4)$

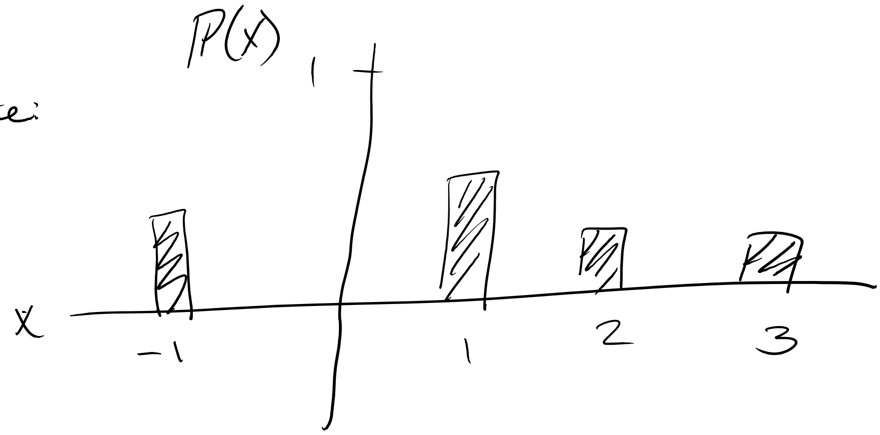
$\mathbb{P}([X+Y]=0) = \dfrac{4^0 e^{-4}}{0!} = 0.0183$

4,27

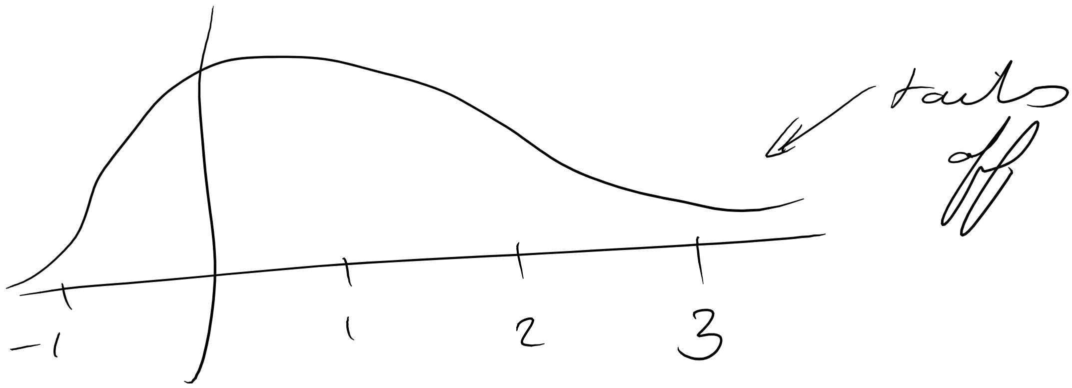
11 February 2015 09:24

Continuous Random Vars

Discrete:



Cont:



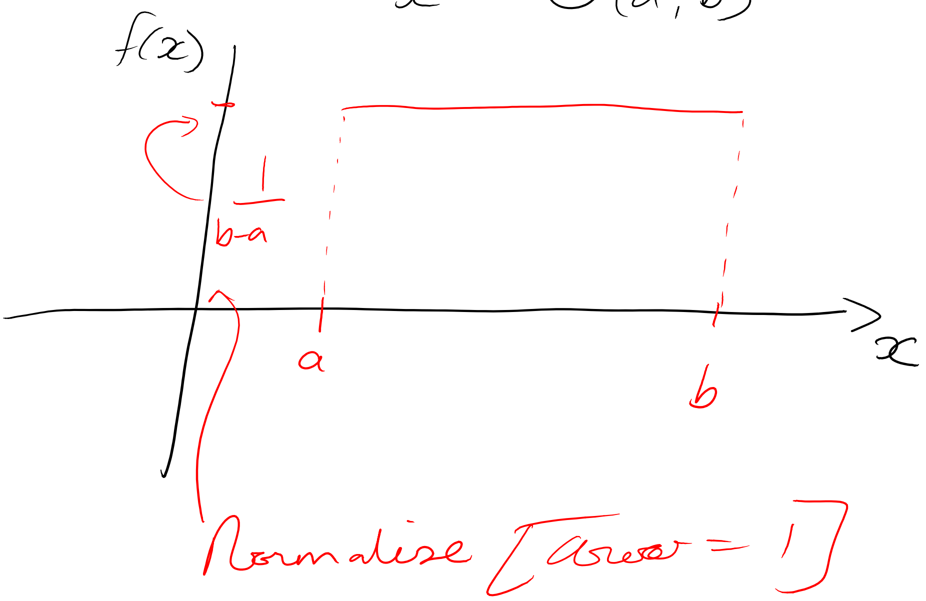
* Discrete: Heights Sum to 1
* Cont: Area under curve = 1

4.31

11 February 2015 09:34

Uniform distribution

$$x \sim U(a, b)$$



① $$\mathbb{E}(x) = \int_a^b x f(x)\, dx$$

$$= \int_a^b \frac{x}{b-a}\, dx$$

$$= \left[\frac{x^2}{2(b-a)} \right]\Bigg|_a^b$$

$$= \frac{b^2}{2(b-a)} - \frac{a^2}{2(b-a)}$$

$$= \frac{b^2 - a^2}{2(b-a)}$$

$$= \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}$$

$$= \frac{b+a}{2}$$

(2) $$\mathbb{E}(x^2) = \int_a^b x^2 f(x)\,dx$$

$$= \int_a^b \frac{x^2}{b-a}\,dx$$

$$= \left[\frac{x^3}{3(b-a)}\right]\Bigg|_a^b$$

$$= \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{\cancel{(b-a)}(b^2+ba+a^2)}{3\cancel{(b-a)}}$$

$$= \frac{b^2+ba+a^2}{3}$$

$$\therefore \operatorname{Var}(x) = \mathbb{E}(x^2) - \mathbb{E}(x)^2$$

$$= \frac{b^2+ba+a^2}{3} - \frac{b^2+2ba+a^2}{4}$$

$$= \frac{4b^2+4ba+4a^2-3b^2-6ba-3a^2}{12}$$

$$= \frac{b^2 - 2ba + a^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

4.33

11 February 2015 09:46

Uniform

Y - thickness of sheets of steel

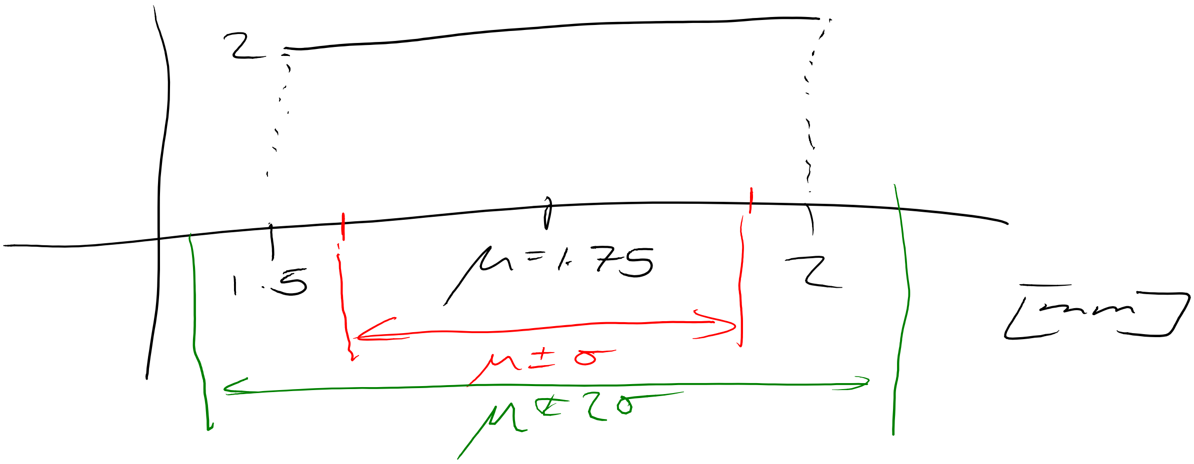
$Y \sim U(1.5, 2)$

- If $Y < 1.6$m then sheet is scrapped.

$\boxed{1}$ $E(Y) = \frac{a+b}{2} = \frac{1.5+2}{2} = 1.75$ mm

$$Var(Y) = \frac{(2-1.5)^2}{12} = 0.0208$$

$$s.d(Y) = \sqrt{0.0208} = 0.1443$$



- Sheet is scrapped if $Y < 1.6$mm

$$\mathbb{P}(Y < 1.6) = \frac{(1.6-1.5)}{(2-1.5)} = 0.2$$

4.35

16 February 2015 09:15

- $E(x) = \int_0^\infty x f(x)\,dx = \int_0^\infty x \lambda e^{-\lambda x} dx$

$= \lambda \int_0^\infty x e^{-\lambda x} dx$

$\Rightarrow u = x \qquad dv = e^{-\lambda x} dx$

$du = dx \qquad v = -\frac{1}{\lambda} e^{-\lambda x}$

$\therefore \int_0^\infty x e^{-\lambda x} dx = -\frac{x}{\lambda} e^{-\lambda x}\Big|_0^\infty + \int_0^\infty \frac{1}{\lambda} e^{-\lambda x} dx$

$\therefore -\frac{x}{\lambda} e^{-\lambda x}\Big|_0^\infty - \frac{1}{\lambda^2} e^{-\lambda x}\Big|_0^\infty$

$\therefore \left[0 + 0\right] - \left[0 - \frac{1}{\lambda^2}\right]$

$\therefore \lambda \int_0^\infty x e^{-\lambda x} dx = \lambda\left(\frac{1}{\lambda^2}\right) = \boxed{\frac{1}{\lambda}}$

- $F(x) = \int_0^x f(x)\,dx = \int_0^x \lambda e^{-\lambda x} dx$

$= \lambda\left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^x$

$= \boxed{1 - e^{-\lambda x}}$

4.36

16 February 2015 09:24

Machine Breakdown

X - Length of time (days) between breakdowns

$X \sim \exp\left(\lambda = \frac{1}{18}\right)$ .... $\mathbb{E} = \frac{1}{\lambda} = 18$

$$\mathbb{P}(X > 21 \text{ days}) = 1 - \mathbb{P}(X \leq 21 \text{ days})$$

$$= 1 - \left[1 - e^{-\frac{21}{18}}\right]$$

$$= 0.3114$$

4.37

16 February 2015 09:31

Memoryless

$$\mathbb{P}(X > a+b \mid X > a) = \frac{\mathbb{P}(X > a+b \cap X > a)}{\mathbb{P}(X > a)}$$

$$= \frac{\mathbb{P}(X > a+b)}{\mathbb{P}(X > a)}$$

$$= \frac{1 - \mathbb{P}(X \leq a+b)}{1 - \mathbb{P}(X \leq a)}$$

$$= \frac{1 - [1 - e^{-\lambda(a+b)}]}{1 - [1 - e^{-\lambda a}]}$$

$$= \frac{e^{-\lambda a} e^{-\lambda b}}{e^{-\lambda a}}$$

$$= e^{-\lambda b}$$

$$= \mathbb{P}(X > b)$$

4.40

16 February 2015 09:39

Poisson & Exponential

- X - # of ice-creams sold
- Y - Poisson ($\lambda = 20/\text{day}$)
- Day is 8 hrs $\Rightarrow \frac{1}{2}$ hour $= \frac{1}{16}$ day

$$\Rightarrow t = \frac{1}{16}$$

- X - time until 1st ice-cream sold

$$X \sim \exp(20)$$

$$P(X < t) = 1 - e^{-20t} \ \ldots \text{CDF of exponential}$$

$$P(\cancel{X} < \tfrac{1}{16}\text{day}) = 1 - e^{-\frac{20}{16}}$$

$$= 0.7135$$

4.41

16 February 2015 09:46

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$$

let $\boxed{\alpha = 1}$: $f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}$

$$= \lambda e^{-\lambda x}$$

where $\lambda = \frac{1}{\beta}$

$\therefore \quad X \sim \exp(\lambda = \frac{1}{\beta})$

$$F(x) = \int_0^x f(x)\,dx = \int_0^x \left(\frac{\alpha}{\beta}\right)\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} dx$$

let $u = x^{\alpha}$ then: $\frac{du}{dx} = \alpha x^{\alpha-1}$

$$\Rightarrow dx = \frac{du}{\alpha x^{\alpha-1}}$$

$$\therefore F(x) = \int_0^x \cancel{\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}}} e^{-\frac{u}{\beta^{\alpha}}} \cancel{\frac{du}{\alpha x^{\alpha-1}}}$$

$$= \frac{1}{\beta^{\alpha}} \int_0^x e^{-\frac{u}{\beta^{\alpha}}} du$$

$$= \cancel{\frac{1}{\beta^{\alpha}}} \left[\cancel{-\beta^{\alpha}} e^{-\frac{u}{\beta^{\alpha}}} \right]_0^x$$

$$= \left[-e^{-\frac{x^{\alpha}}{\beta^{\alpha}}} \right]_0^x$$

$$= \left[-e^{-\frac{x^{\alpha}}{\beta^{\alpha}}} \right] + 1$$

$$= 1 - e^{-\frac{x^{\alpha}}{\beta^{\alpha}}}$$

$$= 1 - e^{-\frac{x^{\alpha}}{\beta^{\alpha}}}$$

4.43

18 February 2015 09:05

Weibull

- t - time until system installed

$t \sim \text{Weibull}(\alpha = \frac{1}{4}, \beta = 4)$

$$\mathbb{P}(t < 100) = 1 - e^{-\left(\frac{100}{4}\right)^{1/4}}$$

$$= 0.8931$$

$$\mathbb{P}(t \geq 100) = 1 - 0.8931$$
$$= 0.1069$$

C - Cost of installing system

- C is a discrete random variable which can take 3 values

$$C = \begin{cases} 10{,}000 & \text{if } t \leq 100 \\ 8{,}000 & \text{if } t > 100 \text{ NOT F} \\ 9{,}000 & \text{if } t > 100 \text{ YES F} \end{cases}$$

$$\mathbb{P}(t > 100 \text{ and not fined}) = \mathbb{P}(t > 100)\,\mathbb{P}(\text{Not fined})$$
$$= (0.1069)(0.4)$$
$$= 0.0428$$

$$\mathbb{P}(t > 100 \text{ and fined}) = (0.1069)(0.6)$$
$$= 0.0641$$

$$\mathbb{E}(c) = \sum_{c} x\,p(x)$$

$$= 10{,}000(0.8931) + 8000(0.0428) + 9000(0.0641)$$

$$= 9850.3 \quad [€]$$

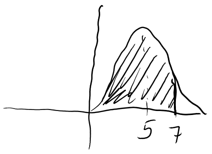
4.49

18 February 2015 09:33

Normal Distribution

- $X \sim N(\mu = 5, \sigma^2 = 3)$

$$\mathbb{P}(X \leq 7) = \mathbb{P}\left(\frac{X-5}{\sqrt{3}} \leq \frac{7-5}{\sqrt{3}}\right)$$



$$= \mathbb{P}(Z \leq 1.1547)$$

$$= 0.8749 \text{ [From tables]}$$

- $\mathbb{P}(X \geq 6) = 1 - \mathbb{P}(X \leq 6)$

$$= 1 - \mathbb{P}\left(\frac{X-5}{\sqrt{3}} \leq \frac{6-5}{\sqrt{3}}\right)$$

$$= 1 - \mathbb{P}(Z \leq 0.5773)$$

$$= 1 - 0.7190$$

$$= 0.2810$$

- $\mathbb{P}(X \leq 2) = \mathbb{P}\left(\frac{X-5}{\sqrt{3}} \leq \frac{2-5}{\sqrt{3}}\right)$

$$= \mathbb{P}(Z \leq -1.732)$$

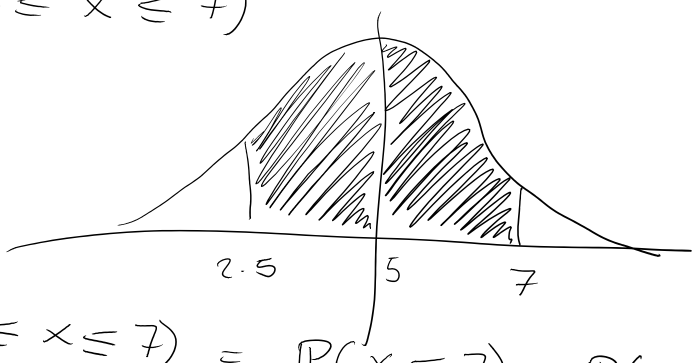
$$= \mathbb{P}(Z \geq 1.732)$$

$$= 1 - \mathbb{P}(Z \leq 1.732)$$

$$= 1 - 0.9582$$

$$= 0.0418$$

- $\mathbb{P}(2.5 \leq X \leq 7)$



$$\mathbb{P}(2.5 \leq X \leq 7) = \mathbb{P}(X \leq 7) - \mathbb{P}(X \leq 2.5)$$

$$= \mathbb{P}\left(\frac{x-5}{\sqrt{3}} \leq \frac{7-5}{\sqrt{3}}\right) - \mathbb{P}\left(\frac{x-5}{\sqrt{3}} \leq \frac{2.5-5}{\sqrt{3}}\right)$$

$=$

$$\mathbb{P}\left(\frac{\quad}{\sqrt{3}} \le \frac{\quad}{\sqrt{3}}\right) - \mathbb{P}\left(\frac{x-5}{\sqrt{3}} \le \frac{2.5-5}{\sqrt{3}}\right)$$

$$= \mathbb{P}(Z \le 1.1547) - \mathbb{P}(Z \le -1.44)$$

$$= 0.8749 - \mathbb{P}(Z \ge 1.44)$$

$$= 0.8749 - [1 - \mathbb{P}(Z \le 1.44)]$$

$$= 0.8749 - [1 - 0.9251]$$

$$= 0.8$$