Electrical Energy Systems

EEEN 20090

Review of Electromagnetics

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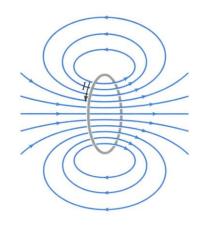
Overview

- Electromagnetics and Electrical (and Electronic) Engineering
 - Modelling
- Electromagnetics for Electrical Energy Systems
 - Lorentz Force Law
 - Ampere's Law
 - Materials
 - Magnetic Circuits
 - Faraday's Law
 - Inductors
 - The ideal transformer
- Electrical Energy Systems (future lectures)
 - Transformer
 - Synchronous Machines
 - Transmission Lines

Some Terminology

- Electrostatics
- Electrodynamics
- Magnetics
- Electromagnetics
- Electromechanics

Exercise: What is the difference and why?

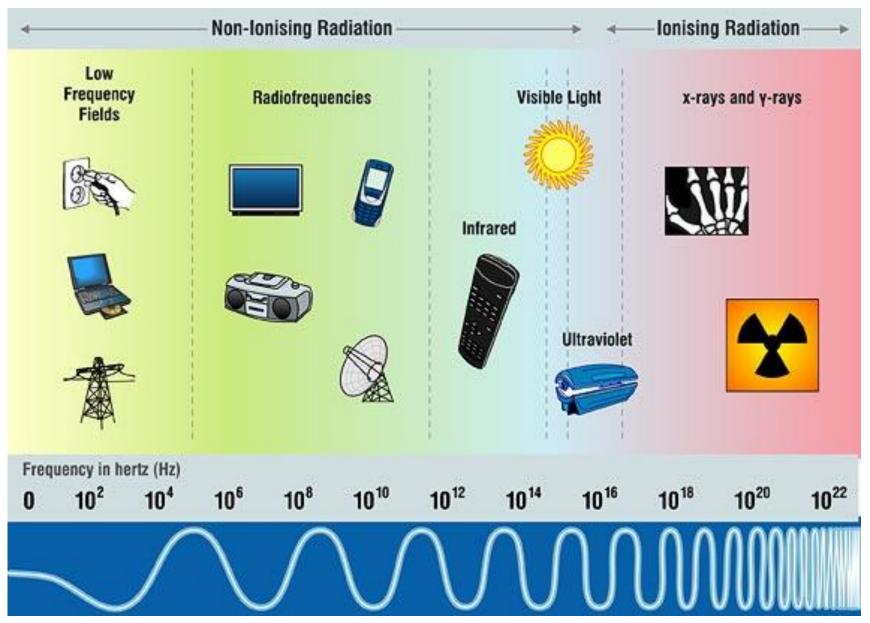


Electromagnetics in Electrical (and Electronic) Engineering

Electromagnetics is at the centre of applications in :

- Radar
- High-speed electronics
- Optics
- Laser design
- Imaging of the Human Body e.g. early detection of breast cancer
- Motors & generators that convert electrical ← mechanical power;
- Electromechanical transducers: loudspeakers, microphones, industrial and aerospace control systems, biomedical engineering applications;
- Magnetic recording: storing information on tape, computer disc, credit cards...

Frequency/Wavelength is important







Electromagnetics in electrical energy systems

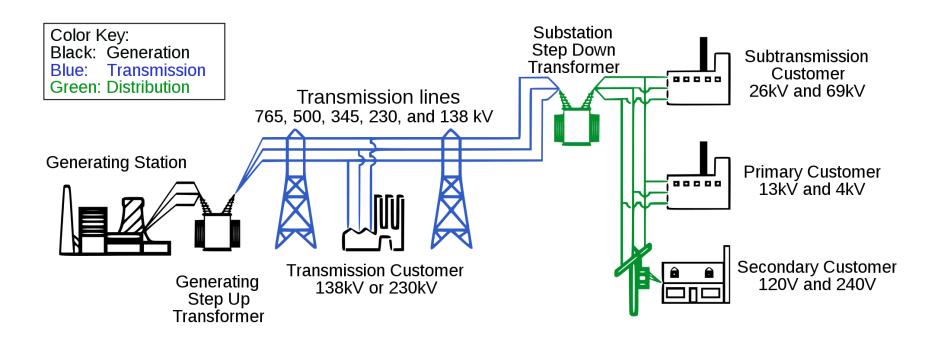
- A knowledge of electromagnetics is fundamental for studying electrical machines (generators, motors & transformers & transmission lines).
- It is fundamental to how we transform mechanical energy into electricity and back
- Magnetic materials form an essential part of an electromagnetic machine.







Basic Structure of the Electric System



U.S.-Canada Power System Outage Task Force – August 14th Blackout: Causes and Recommendations



Side bar: Modelling and simulation

Why do we need models and simulation

- Predict what will happen
 - design and analysis
- Different types
 - Physical models (e.g. scale models)
 - Mathematical models
- Different level of details required
 - For systems we want/need:
 - Simple model for each component
 - Only interested in system issues not in detail of each component
 - For components we may need
 - Detailed models





Engineering the art of approximation

- 'The art of being wise is the art of knowing what to overlook.' William James, American Philosopher and Psychologist, 1842 – 1910.
- When we represent a piece of the world in our minds, we discard many aspects – we make a model.
- An approximate model is often more useful than an exact one.
 - allows insight and intuition
 - pragmatically easier to work with
 - an approximate model is all that we can understand.
- Since every model is approximate, how do we choose useful approximations?
 - by knowing the details!







If I have seen a little further it is by standing on the shoulders of Giants

Examples

- To design a new car
- To study a new system of roads
- To design a better computer chip
- To analyse a chemical reaction at scale
- To study how electrical stimulation impacts muscles
- Predict how a new aircraft behaves before it flies
- Predict what will happen to the economy
- Plotting a new courses for spacecraft
- To design a new type of electrical generator
- To design an electrical energy system and to predict how it will behave

Model Simulation



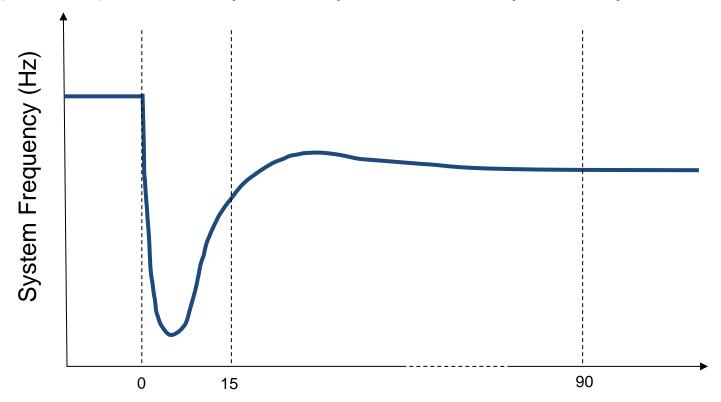






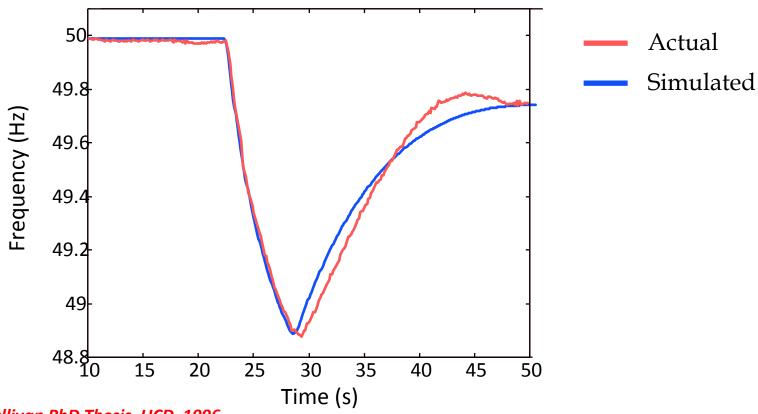
Rate of Change of Frequency

 The initial frequency slope - rate of change of frequency (ROCOF) - is an important parameter in power systems

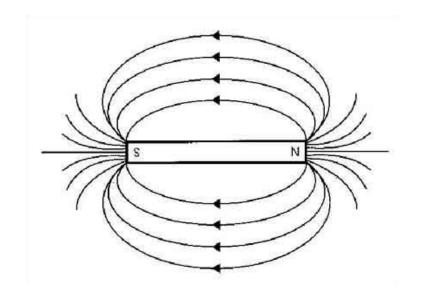


System Frequency Transient, Dr. Damian Flynn, Electrical Energy Systems

Simulation versus actual - validation



Dr. Jonathan O'Sullivan PhD Thesis, UCD, 1996



Electromagnetics for Electrical Energy Systems

Maxwell's equations



$$\begin{split} &\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0} \\ &\oint \mathbf{B} \cdot d\mathbf{A} = 0 \\ &\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_{\mathrm{B}}}{dt} \\ &\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt} + \mu_0 i_{enc} \end{split}$$

Formulated circa 1870, represent a fundamental unification of electric and magnetic fields predicting electromagnetic wave phenomena.

'the most outstanding achievement of 19th-century science'
Nobel Laureate Richard Feynman

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 An electric current produces a magnetic field (Ampere's Law)

 A changing magnetic field can produce an electric current and voltage (Faraday's Law)

 A charge moving in a magnetic field experiences a force (Lorentz force law)

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A charge moving in a magnetic field experiences a force (Lorentz Force Law)

Consider a charged particle, moving at velocity u, in a magnetic field of uniform flux density B.

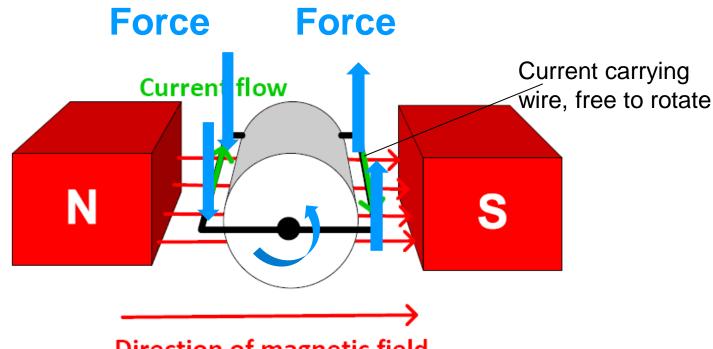
The particle experiences a force, F, due to the magnetic field, which is:

- proportional to its charge, Q
- proportional to its speed, u
- proportional to the component of the magnetic field perpendicular to its velocity
- in a direction perpendicular to both the plane of the velocity and the magnetic field

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Example: Electric motor

Force on a current carrying loop in a magnetic field...



Direction of magnetic field

...the wire loop will turn anti-clockwise.

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Ampere's Law

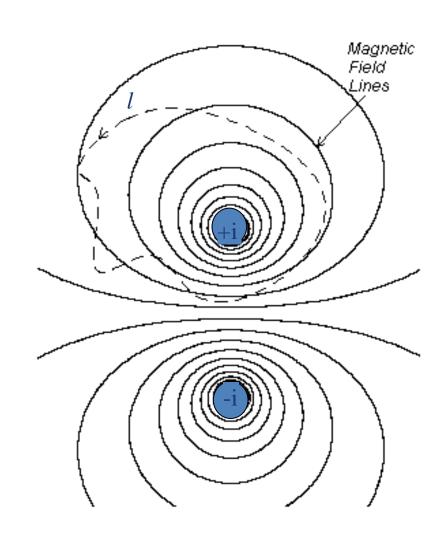
An electric current (or changing electric flux) through a surface produces a circulating magnetic field (H) around any path that bounds that surface.

Illustration of Ampere's Law

Two infinitely long conductors carrying a current *i* into and out of the screen. A magnetic field (H) results in the region around the conductors, and its **magnitude decreases** with distance from the conductors.

Taking an arbitrary closed path l, around the top conductor, the magnetic field, H, is related to the current in the conductor by *Ampere's Law: Units of H are* ampere-turn per metre (At/m)

$$\oint_{l} \vec{H}.dl = i_{enclosed} = +i$$



Magnetic Flux density and Magnetic Field

• The relationship between flux density (B) and magnetic field (H) is given by the material, whose properties we define in terms of magnetic permeability μ

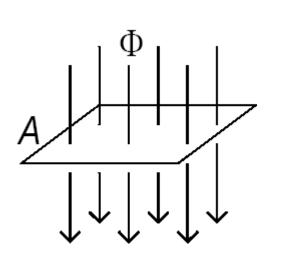
$$B = \mu H = \mu_0 \mu_r H$$

Magnetic Permeability of air, $\mu_0 = 4\pi \times 10^{-7}$ (H/m)

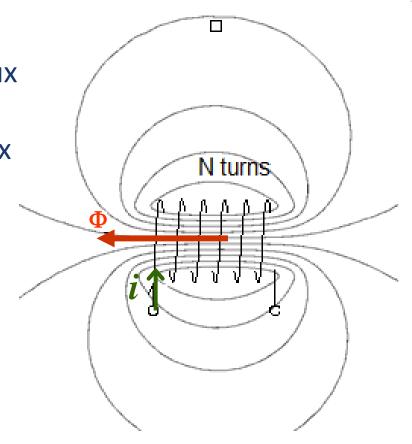
 μ_r is the relative permeability: examples, Air = 1, Iron ≈ 1000 , Perm. alloy ≈ 8000

Flux and Flux Density

A coil with N turns carrying a current i gives rise to a magnetic field (H) and flux density (B). The flux Φ (Webers, Wb) over a cross sectional area (A) is the flux density (Wb/m² or Tesla)



$$B = \frac{\Phi}{A}$$

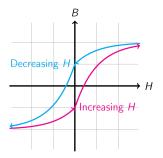


air-cored inductor

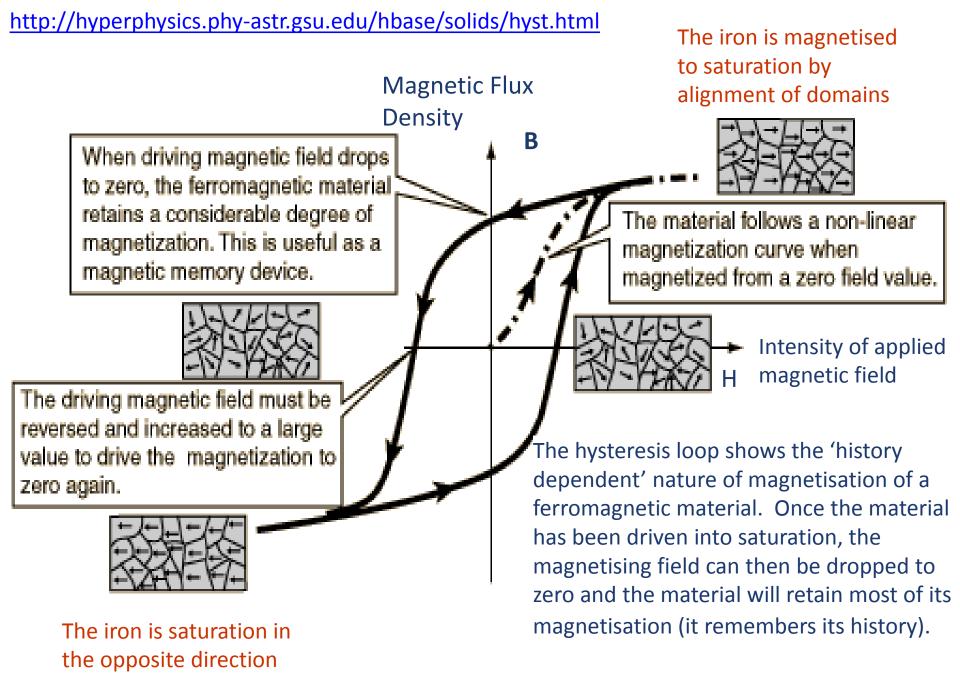
Nomenclature and Units

- H Magnetic field magnetic field strength magnetising field – magnetic field intensity
 Unit: Ampere-turns/meter (At/m)
- B Magnetic flux density magnetic induction magnetic field
 Unit: Tesla (T, Wb/m²)
- Φ Magnetic flux
 Unit: Weber (Wb)

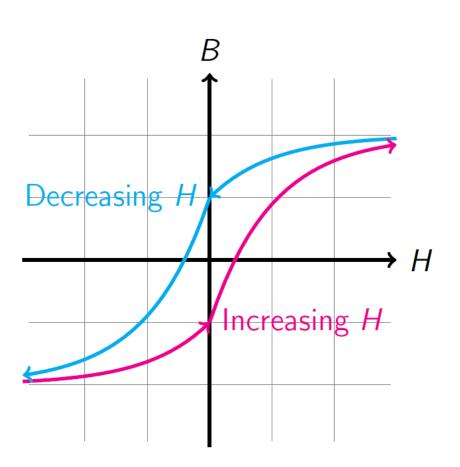
N.B.: Note the relationship between units:



Magnetic properties of materials



The BH curve for ferromagnetic materials



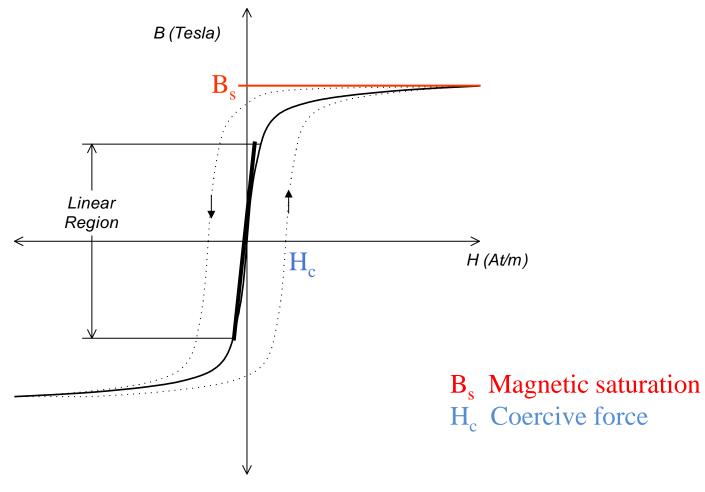
- non linear
- saturation
- hysteresis

To minimize losses and maintain performance, engineers will choose materials and designs to minimize hysteresis and avoid operating in the saturation region.

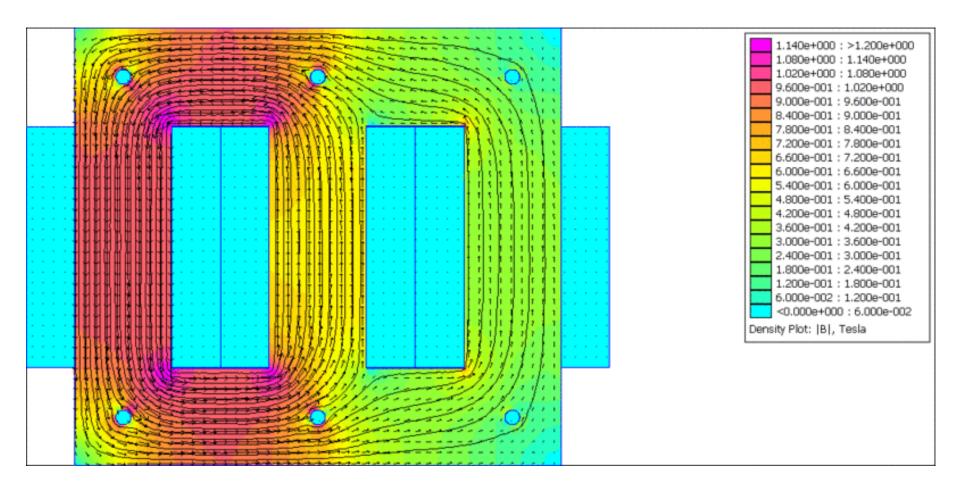
If you want high B (or Φ) you have to use ferromagnetic type materials. High Φ can also be got by large A.

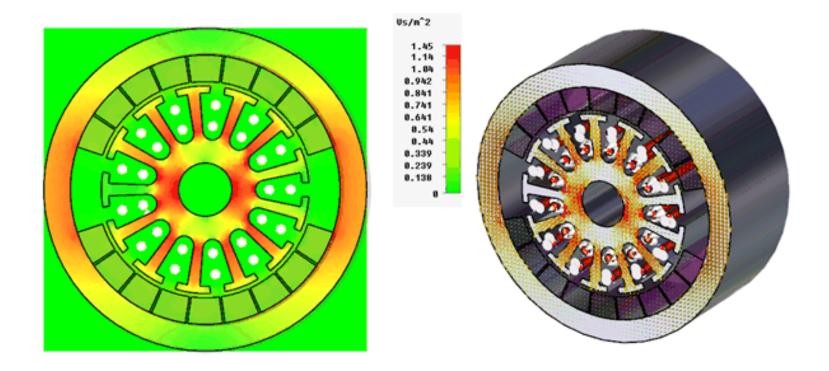
There are cost, performance trade-offs as there is with all engineering design.

Linearising the B-H curve for Iron

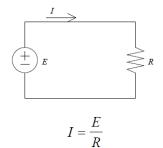


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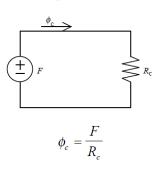




Electric Circuit

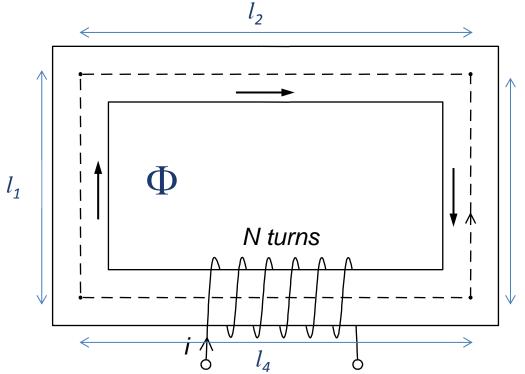


Magnetic Circuit



Magnetic circuits

Bringing it all together



Coil of N turns wrapped around an iron core So flux density in iron core >>> air Same cross sectional area (A) throughout So uniform B throughout the core Hence H is the same throughout What is the magnitude of the magnetic field?

$$\int_{l} \vec{H}.dl = i_{enclosed} = +i$$

$$H = Ni/l$$

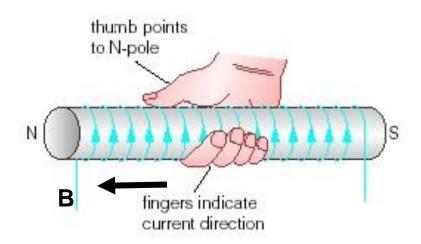
$$l = l_1 + l_2 + l_3 + l_4$$

= path length of the magnetic flux

H = magnitude of the magnetic field

N = total number of turns

i = current through the coil





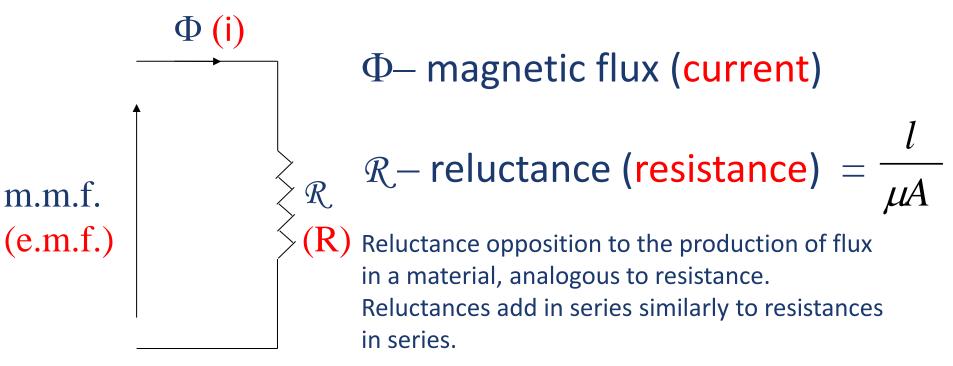
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Magnetic Circuits

- Analyse the operation of electromagnetic devices using magnetic equivalent circuits
 - An approximation

- Assumptions
 - The magnetic flux Φ is confined within the structure i.e. no leakage flux
 - The magnetic flux density (B) is approx. constant over the cross sectional area (A)

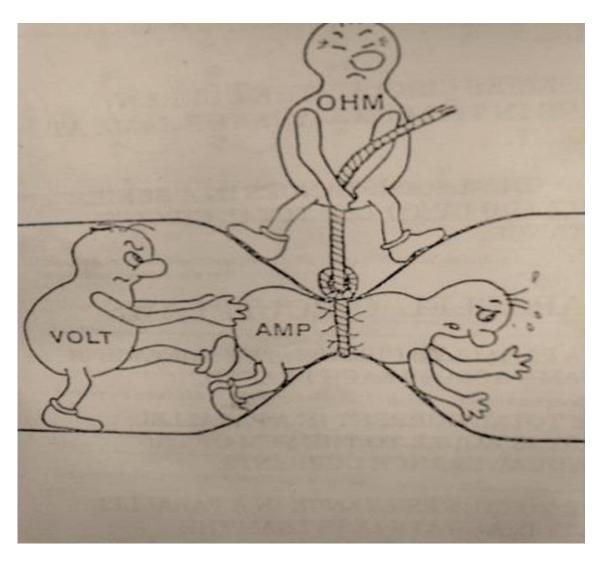
Magnetic Equivalent Circuit



m.m.f. – magnetomotive force is any physical driving (motive) force that produces magnetic flux (Note: equivalent to electromotive force, e.m.f.) The m.m.f. in an inductor or electromagnet consisting of a coil of wire is given by

m.m.f. (F) = Ni (Ampere-turns)

Ohm's law



Derivation of Reluctance R.

$$m.m.f. = Ni$$

$$\Rightarrow Ni = \frac{B}{\mu}l$$

$$\Phi = BA$$

$$\therefore Ni = \frac{BA}{\mu A}l = \Phi \frac{l}{\mu A}$$

$$m.m.f. = \Phi \mathcal{R}$$

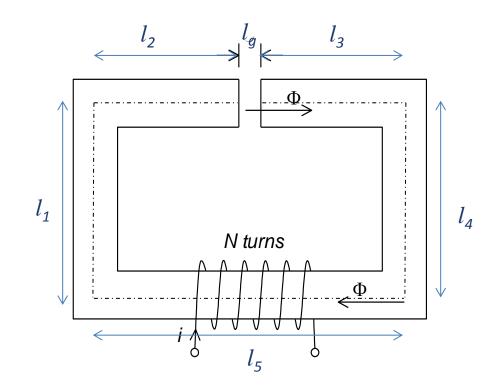
$$\mathcal{R} = \frac{l}{\mu A}$$

Calculate Reluctance, R.

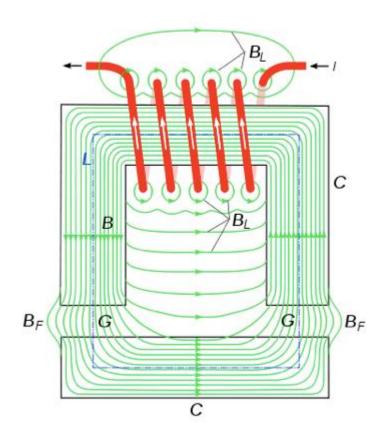
Example 2.3

Consider an iron core, with a gap of length g and an N-turn coil carrying current i wound about it as shown, giving rise to a magnetic field H and flux Φ in the core. (Air can be ignored but what about in the air gap ?)

- a) Calculate the reluctance, R.
 What affect does the air gap have on the properties of the core?
- b) Let $l_g = l / 100$ $l = l_1 + l_2 + l_3 + l_4 + l_5$ = Total Length of iron core Find the Reluctance \mathcal{R} total



Magnetic flux inside Iron is higher than in Air



Example 2.3 (a): Solution

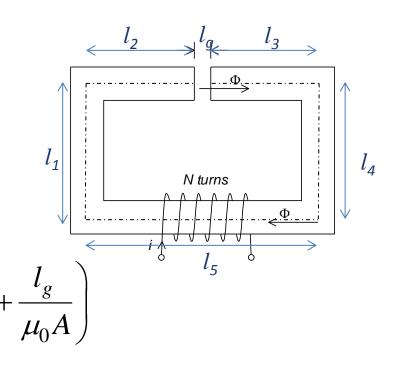
Applying Ampere's law around the circuit gives

$$Ni = \oint \vec{H}.d\vec{l} = \int \vec{H}.d\vec{l} + \int \vec{H}.d\vec{l}$$

$$m.m.f. = Ni = H_{iron}l + H_{air}l_{g}$$

$$= \frac{Bl}{\mu_{I}} + \frac{Bl_{g}}{\mu_{0}} = \frac{\Phi l}{\mu_{I}A} + \frac{\Phi l_{g}}{\mu_{0}A} = \Phi\left(\frac{l}{\mu_{I}A} + \frac{l_{g}}{\mu_{0}A}\right)$$

$$=\Phi\left(\mathcal{R}_{iron}\text{+}\mathcal{R}_{gap}\right)=\Phi\left(\mathcal{R}_{Total}\right)$$

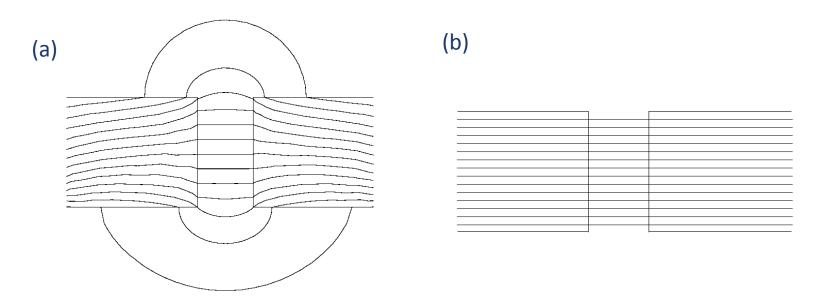


Therefore reluctances add in series, $\mathcal{R}_{Total} = \mathcal{R}_{iron} + \mathcal{R}_{gap}$

When a small gap is cut into the iron core, virtually all of the flux in the core passes through the gap.

Assume Constant Across Air Gap

If the gap is short, we can ignore the fringing field outside the air gap.



Magnetic flux pattern at the air gap (a) including and (b) neglecting fringing flux

Example 2.3 (b): Solution

$$\mathcal{R}_{air} = \frac{l_g}{\mu_0 A}$$

$$\mathcal{R}_{iron} = \frac{l}{\mu_I A}$$

$$l = 100 l_g$$

 μ_r , the relative permeability of Iron ≈ 1000

$$\mu_I = \mu_r \mu_0$$

$$\mu_I = 1000 \mu_0, (\mu_0 = 4\pi \times 10^{-7} \, H \, / \, m)$$

Example 2.3 (b): Solution cont.

$$\mathcal{R}_{Total} = \mathcal{R}_{iron} + \mathcal{R}_{air} = \frac{l_g}{10\mu_0 A} + \frac{l_g}{\mu_0 A}$$

$$\mathcal{R}_{Total} \approx \frac{l_g}{\mu_0 A}$$

Note:
$$\mathcal{R}_{air} >> \mathcal{R}_{iron}$$

 $Ni = \mathcal{R}_{Total} \Phi$ If \mathcal{R}_{Total} increases, Φ decreases Hence B decreases

Example 2.4

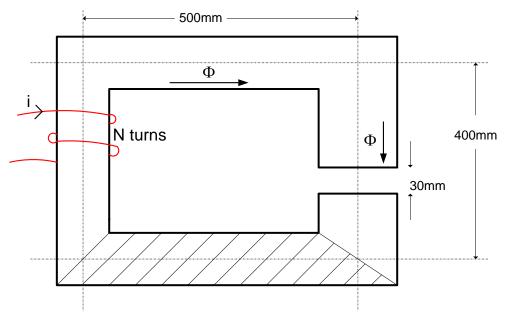
- The core cross sectional area is '1.6a' on the right hand side and 'a' on all other sides.
 - a) Calculate the m.m.f. so as the flux density in the air gap is 0.8T
 - b) The maximum m.m.f. such that the iron doesn't saturate
 - c) The maximum m.m.f. so all the core saturates

$$\Phi = BA$$

$$B = \mu_0 \mu_r H$$

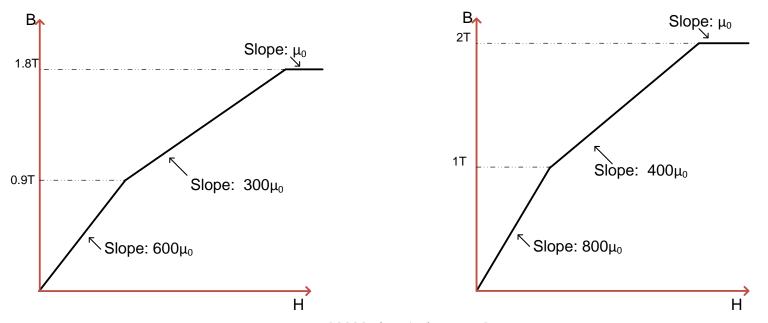
$$m.m.f. = Ni = \Phi \mathcal{R}$$

$$\mathcal{R} = \frac{l}{\mu \mathcal{A}}$$
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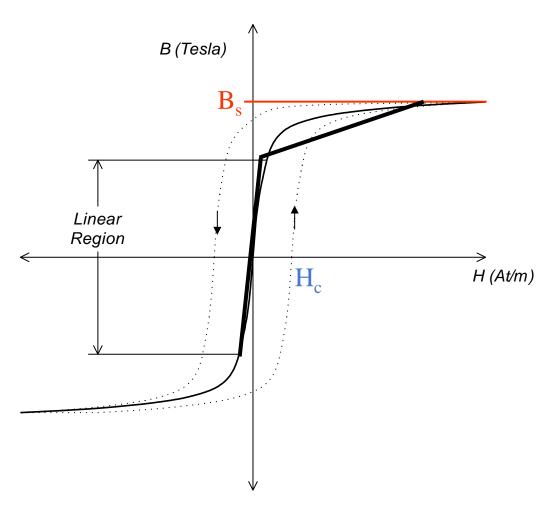
Graph 1: Un-shaded Area

Graph 2: Shaded Area



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Piecewise linear approximation:



Example 2.4 (a): Solution

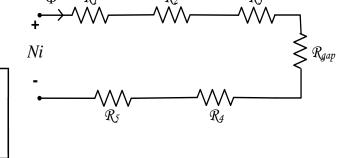
a) Calculate the m.m.f so as the flux density in the air gap

is 0.8T

$$Ni = \Phi[\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4 + \mathcal{R}_5 + \mathcal{R}_{gap}]$$

$$- \begin{bmatrix} l_1 & l_2 & l_4 & l_5 \\ l_4 & l_5 & l_4 \end{bmatrix}$$

$$=\Phi\left[\frac{l_{1}}{\mu_{1}A_{1}}+\frac{l_{2}}{\mu_{2}A_{2}}+\frac{l_{3}}{\mu_{3}A_{3}}+\frac{l_{4}}{\mu_{4}A_{4}}+\frac{l_{5}}{\mu_{5}A_{5}}+\frac{l_{g}}{\mu_{g}A_{g}}\right] \leftarrow \mathcal{R}_{S}$$



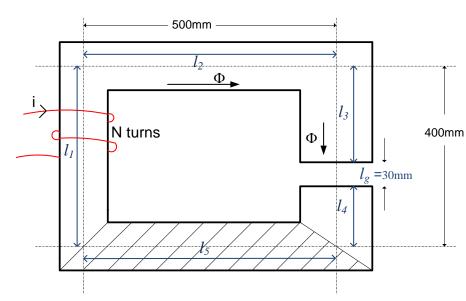
$$a = A_1 = A_2 = A_5$$
, $1.6a = A_3 = A_4 = A_g$

$$\Phi = B_{gap}A_g = 0.8(1.6a) = 1.28a Wb$$

Need to calculate $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_g$

$$\mu_g = \mu_0 = 4\pi \times 10^{-7} \,\text{H/m}$$

$$\mu_1 = \mu_2 = \mu_0 \mu_{r1} = B_1 / H_1 = \frac{\Phi / A_1}{H_1}$$



Example 2.4 (a): Solution cont.

From Graph 1,
$$\mu_1 = \mu_2 = \mu_0 \mu_{r1} = \frac{B_1}{H_1} = \frac{\Phi/a}{H_1} = \frac{1.28a/a}{H_1} = \frac{1.28a/a}{H_1} = \frac{1.28a/a}{600\mu_0} = \frac{1.28}{300\mu_0} = 462.65\mu_0$$

$$\therefore \mu_{r1} = 462.65$$

From Graph 1,
$$\mu_3 = \mu_4 = \mu_0 \mu_{r2} = \frac{B_2}{H_2} = \frac{\Phi/1.6a}{H_2} = \frac{1.28a/1.6a}{H_2} = \frac{0.8}{600 \,\mu_0} = 600 \,\mu_0$$

$$\mu_{r2} = 600$$

$$\mu_{r3} = 655.4$$

Example 2.4 (a): Solution cont.

$$Ni = \Phi \left[\frac{l_1 + l_2}{\mu_1 A_1} + \frac{l_3 + l_4}{\mu_3 A_3} + \frac{l_5}{\mu_5 A_5} + \frac{l_g}{\mu_g A_g} \right]$$

$$Ni = 1.28a \left[\frac{0.9}{462.65 \mu_0 a} + \frac{0.37}{600 \mu_0 1.6a} + \frac{0.5}{655.4 \mu_0 a} + \frac{0.03}{\mu_0 1.6a} \right]$$

$$Ni = \frac{1.28a}{\mu_0 a} [0.02184]$$

$$\therefore \boxed{Ni = 22,249 \text{At}}$$

Exercise Check the units

Note: Flux $,\Phi$, remains constant throughout the circuit, flux density, B, changes with changing cross sectional area.

Example 2.4 (b): Solution

b) The maximum m.m.f. such that the iron (graph 1) doesn't saturate

Iron saturates at 1.8T, $B_1 = 1.8$ T

$$\Rightarrow \Phi = BA = 1.8a$$

Recalculate μ_1, μ_3, μ_5

From Graph 1,
$$\mu_1 = \mu_2 = \mu_0 \mu_{r1} = \frac{B_1}{H_1} = \frac{\Phi/a}{H_1} = \frac{1.8a/a}{H_1} = \frac{1.8a/a}{\frac{0.9}{600\mu_0} + \frac{0.9}{300\mu_0}} = 400\mu_0$$

$$\mu_{r1} = 400$$

From Graph 1,
$$\mu_3 = \mu_4 = \mu_0 \mu_{r2} = \frac{B_2}{H_2} = \frac{\Phi/1.6a}{H_2} = \frac{1.8a/1.6a}{H_2} = \frac{1.125}{\frac{0.9}{600\mu_0} + \frac{0.225}{300\mu_0}} = 500\mu_0$$

$$\mu_{r2} = 500$$

Example 2.4 (b): Solution cont.
From Graph 2,
$$\mu_5 = \mu_0 \mu_{r3} = \frac{B_3}{H_3} = \frac{\Phi/a}{H_3} = \frac{1.8a/a}{H_3} = \frac{1.8a/a}{H_3} = \frac{1.8a/a}{1.800\mu_0} = 553.8\mu_0$$

$$\mu_{r3} = 553.8$$

$$Ni = \Phi \left[\frac{l_1 + l_2}{\mu_1 A_1} + \frac{l_3 + l_4}{\mu_3 A_3} + \frac{l_5}{\mu_5 A_5} + \frac{l_g}{\mu_g A_g} \right]$$

$$Ni = \frac{1.8a}{\mu_0 a} \left[\frac{0.9}{400} + \frac{0.37}{600(1.6)} + \frac{0.5}{553.8} + \frac{0.03}{1.6} \right]$$

$$Ni = \frac{1.8}{\mu_0} [0.022365]$$

$$Ni = 32,036At$$

Example 2.4 (c): Exercise

c) The maximum m.m.f. so all the core saturates



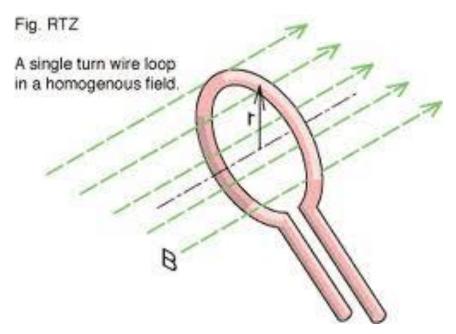
Faraday's Law

A changing magnetic field can produce (induce) an electric current and voltage

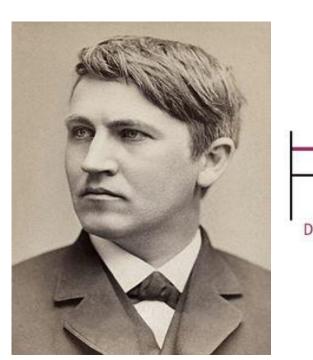
Faraday's Law

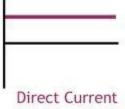
- Coil links a flux (B x cross sectional area)
- Voltage induced is equal the rate of change of flux

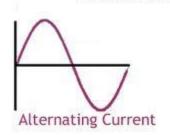
- Applications
 - Inductors
 - Induction machines
 - Inductive heating
 - etc.



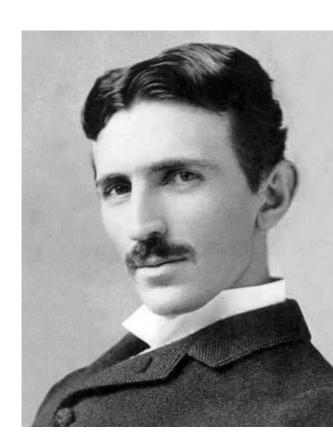
Edison and Tesla





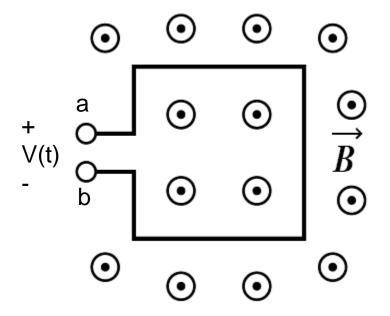


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The war of the currents

For example, consider the loop of wire shown. If the magnetic field B is changing a voltage will appear across the terminals a and b. If the loop is closed, a current will flow in it.



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If the circuit is a coil of *N turns*, each turn produces a voltage as above.

All the turns have the same flux through them, so all produce equal voltages. As the turns are connected in series, the voltages add.

So the total voltage between the ends of the coil is:

$$v(t) = N \frac{d\Phi}{dt} \qquad \circ \qquad \circ \vec{B}$$

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Flux linkage

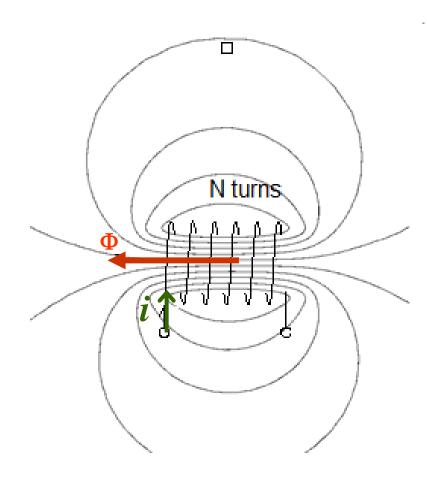
Each turn of the coil "loops" a flux Φ : we say that each turn *links* a flux Φ . The total flux *linked* by the coil (denoted Ψ) is

$$\Psi = N\Phi$$

$$= NBA = N\mu HA$$

$$= N\mu \frac{Ni}{l}A = \frac{N^2 \mu A}{l}i$$

What is the length *l* in this case?



Self Inductance

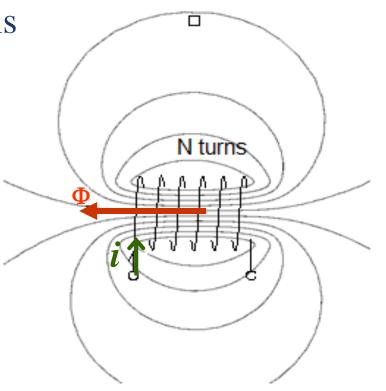
Apply Faraday Law to the terminals

$$v(t) = N \frac{d\Phi}{dt} = \frac{d\psi}{dt}$$

$$= \frac{N^2 \mu A}{l} \frac{di}{dt} = L \frac{di}{dt}$$

L is known as the self inductance units are H (Henry)

$$L = \frac{N^2 \mu A}{l} \text{ and } \psi = Li$$



air-cored
$$\mu = \mu_0 \longrightarrow \text{very small } L$$

† L by improving the permeability

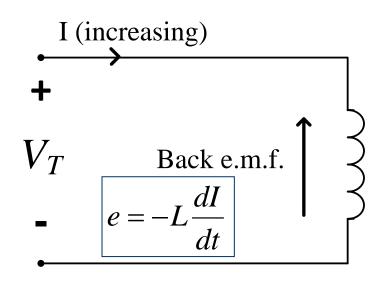
iron-cored
$$\mu_I >> \mu_0 \longrightarrow \mu_I \simeq 1000 \mu_0 \longrightarrow large L$$

Lenz's Law

 An induced voltage has a direction such that the magnetic field due to the current opposes the change in the magnetic field that induces the current.

Back e.m.f.

- •An inductor reacts to the constantly changing current in an AC circuit by producing an "back" e.m.f. that opposes the changing current. (Lenz's law)
- •If the current is increasing a "back" e.m.f is produced which tries to stop the increase.



$$V_T + e = 0$$
 Kirchhoff Voltage Law $V_T - L \frac{di}{dt} = 0$ $V_T = L \frac{di}{dt}$

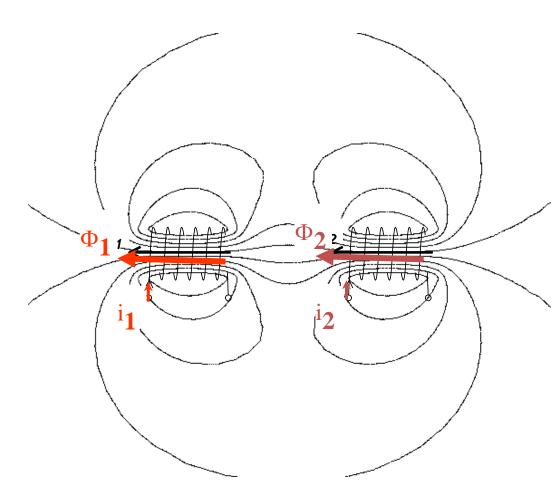
Mutual Inductance

Consider two current-carrying coils close together as shown. The flux linked by each coil depends not only on its current, but also on the current and geometry of its neighbour. That is:

$$\Psi_1 = L_1 i_1 + M_{12} i_2$$

$$\Psi_2 = L_2 i_2 + M_{21} i_1$$

The inductances M_{12} and M_{21} are known as the *mutual inductances* of the coils with respect to each other.



Mutual inductance explained

 Mutual inductance describes the voltage induced in one electrical circuit by the rate of change of electric current in another circuit.

 The voltage induced in electrical circuit by the rate of change of electric current in the same circuit, which we have been studying so far, is sometimes called the **self inductance** as both self and mutual inductance may be relevant when analysing a circuit.

Applications: Transformers

Changing flux in a coil can also induce a voltage in a nearby coil:

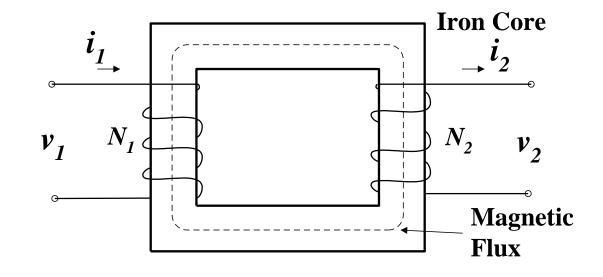
To maximise the effect, place the coils on the same *iron core* – high permeability, so most of the flux stays in the iron, passes through both coils.

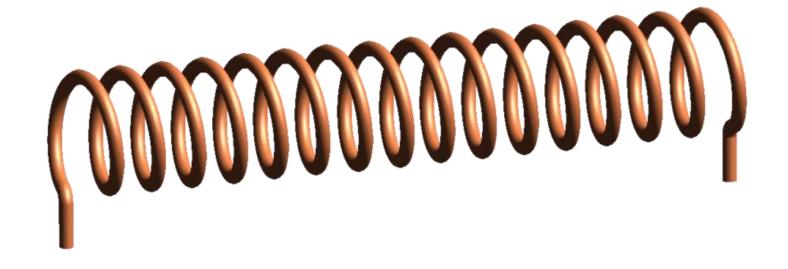
If same flux, $\phi(t)$, passes through both coils, then voltages:

$$v_1(t) = N_1 \frac{d\phi}{dt}$$
$$v_2(t) = N_2 \frac{d\phi}{dt}$$

Giving us

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$





Inductor as a storage device

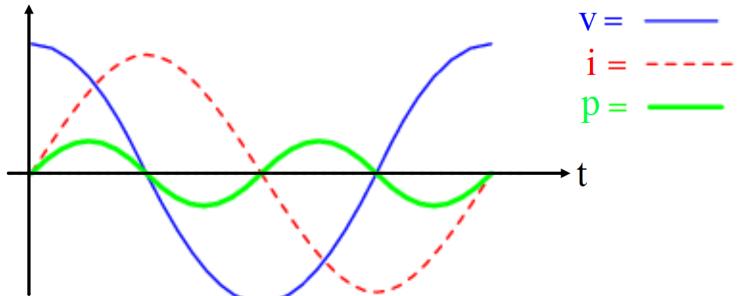
Energy storage

If an inductor, L, has a current, $i=I_m sin(\omega t+\theta)$ flowing through it, the voltage across the inductor is

$$v = L\frac{di}{dt} = L\frac{d}{dt} \left[I_m \sin(\omega t + \theta) \right] = \omega L I_m \cos(\omega t + \theta) = V_m \cos(\omega t + \theta)$$

The power, p, is given by

$$p = vi = V_m I_m \sin(\omega t + \theta) \cos(\omega t + \theta) = \frac{V_m I_m}{2} \sin(2\omega t + 2\theta)$$

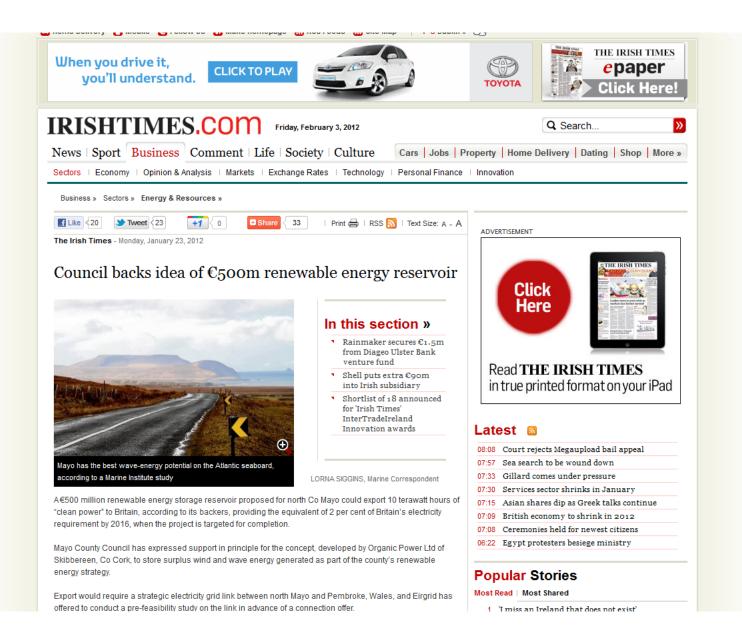


Energy storage

- Current lags voltage in an inductor by 90° due to the opposition an inductor shows to change in current.
- The power is sinusoidal. Being sinusoidal, it's average value is zero
- In terms of energy, at the times when *p* is positive, an inductor absorbs energy. At the times when *p* is negative, an inductor returns energy to the circuit and acts as a source ("reactive power").



Beware





Key takeaways

What is important

- Electromagnetics is important
- For systems we need to simplify it
- Lorentz force law electricity back to mechanical energy
 synchronous machines induction machines etc.
- Ampere's law
- Material properties
- Magnetic circuits
- Faraday's law inductors
- Transformers



