2.26

21 January 2015 09:12

outlier

Sample: 4, 2, 1, 3, 2, $\boxed{13}$, 1

$$\bar{x} = \frac{4+2+1+3+2+13+1}{7} \quad \left[= \frac{1}{n}\sum_{i=1}^{n} x_i \right]$$

$$\therefore \bar{x} = \frac{26}{7} = 3.7$$

Sample median:

[Sort the data set]

1. 1, 1, 2, $\boxed{2}$, 3, 4, 13
2. Sample Median: 2

Mode: Most Common value

$\therefore$ Sample mode = 1 and 2

2.33

21 January 2015 09:12

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s^2 = \underbrace{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}$$

Use this formula for calculating by hand

Data Sample

5.7, 2.3, 6.2, 1.5, 4.0, 2.9

We want s^2 (Sample variance)

$$S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\underbrace{n-1}_{1}} = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{\underbrace{n-1}_{2}}$$

$$\bar{x} = \frac{5.7+2.3+6.2+\cdots+2.9}{6} = \frac{22.6}{6} = 3.767$$

Long: 1

Long: 1

$$s^2 = \frac{(5.7-3.767)^2 + \cdots + (2.9-3.767)^2}{6-1}$$

$\Rightarrow$ long (and boring)

Quick: 2

$$\sum_{i=1}^{n} x_i^2 = 5.7^2 + 2.3^2 + \cdots + 2.9^2$$

$$= 102.88$$

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1} = \frac{102.88 - (6)(3.767)^2}{6-1}$$

$$= 3.548$$

Sample standard deviation:

$$s = \sqrt{3.548}$$

3.6

26 January 2015 09:18

Example: Loaded die

$\mathbb{P}(\text{die shows } i) \propto i$

$\Rightarrow \mathbb{P}(1) = p$

$\mathbb{P}(2) = 2p$

$\mathbb{P}(3) = 3p$

... etc.

From 3rd probability axiom:

$6p + 5p + 4p + 3p + 2p + p = 1$

$\Rightarrow p = \frac{1}{21}$

$\Rightarrow \mathbb{P}(1) = \frac{1}{21}, \mathbb{P}(2) = \frac{2}{21}, \ldots, \mathbb{P}(6) = \frac{6}{21}$

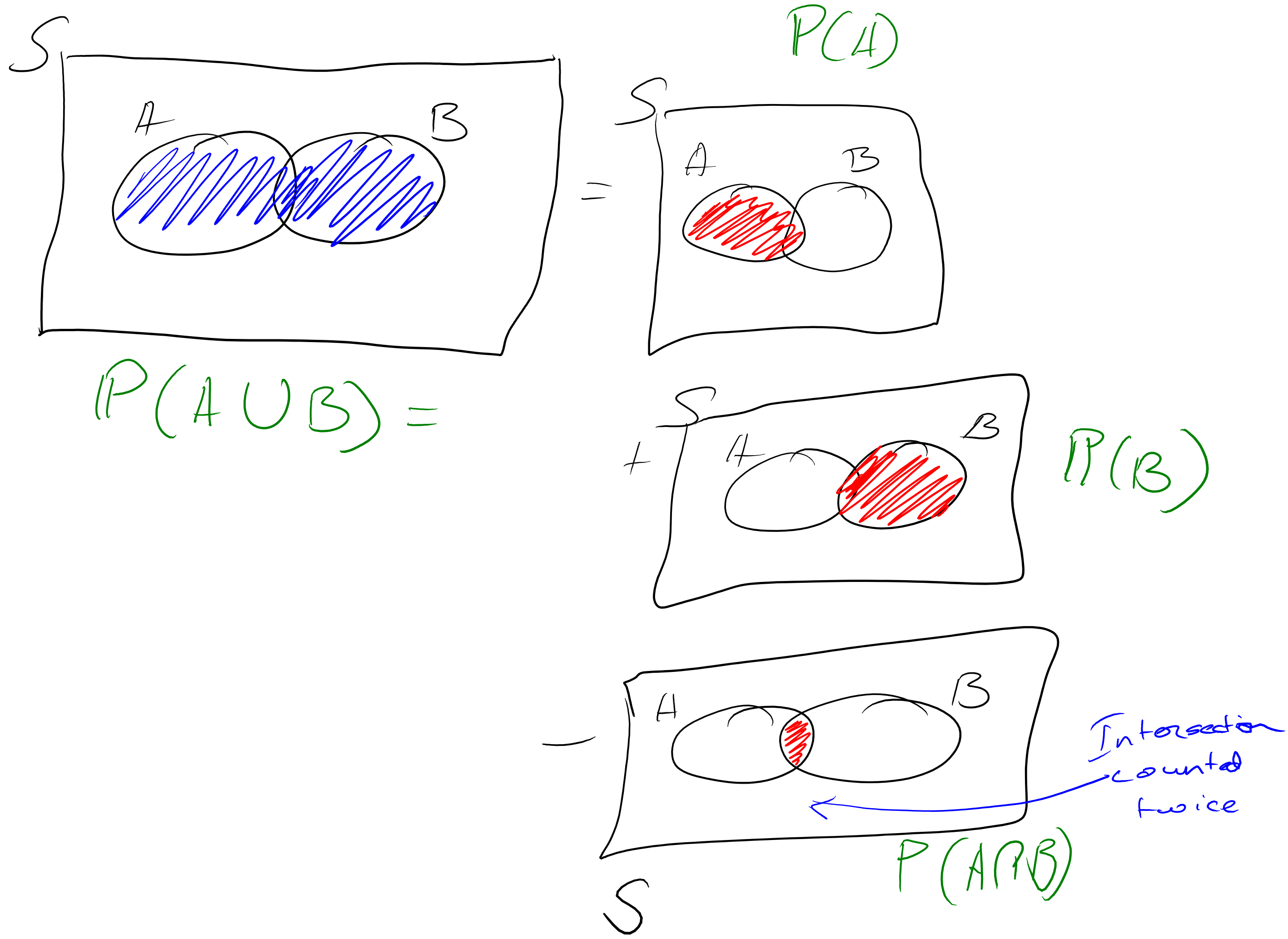
$A = \{2, 3\}$

$\mathbb{P}(A) = \mathbb{P}(2) + \mathbb{P}(3)$

$= \frac{2}{21} + \frac{3}{21} = \frac{5}{21}$

3.8

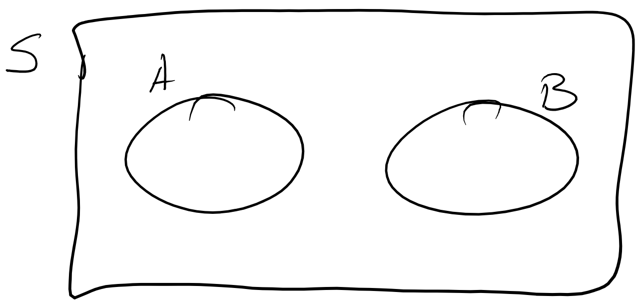
26 January 2015 09:28



$$\mathbb{P}(A \cup B) = P(A) + \mathbb{P}(B) - P(A \cap B)$$

Intersection counted twice

Mutually Exclusive:

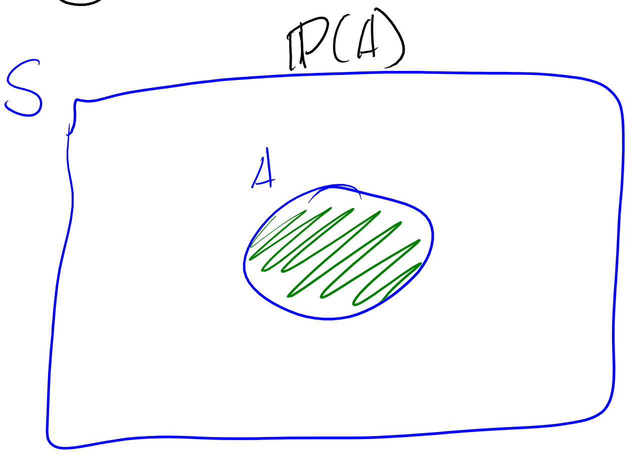


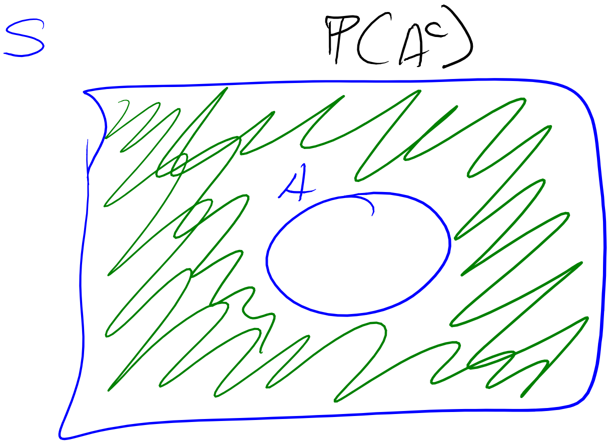
∴ No overlap

3.9

26 January 2015 09:32

Set Complement:





$$\Rightarrow \quad \mathbb{P}(S) = \mathbb{P}(A) + \mathbb{P}(A^c)$$

$$\therefore \quad \mathbb{P}(A) + \mathbb{P}(A^c) = 1$$

3.10

26 January 2015 09:36

Example: Roll 2 dice

A - Observe a 5

B - Dice sum to 7

$\mathbb{P}(A) = \frac{11}{36}$ $\quad \mathbb{P}(B) = \frac{1}{6} = \boxed{\frac{6}{36}}$

$\Rightarrow P(A \cap B) = \frac{2}{36}$

$\Rightarrow P(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

$= \frac{11}{36} + \frac{6}{36} - \frac{2}{36}$

$\therefore P(A \cup B) = \frac{15}{36}$

$\Rightarrow P(A^c) = 1 - \mathbb{P}(A)$

$= 1 - \frac{11}{36}$

$= \frac{25}{36}$

3.12

26 January 2015 09:42

Example: Roulette

A : outcome is odd

B: outcome is red

C: outcome is in 1^{st} dozen

$\therefore A \cap B = \{1, 3, 5, 7, 9, 19, 21, 23, 25, 27\}$

$\therefore A \cup B = \{1, 3, 5, 7, 9, 12, 14, 16, 18, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 30, 31, 32, 33, 34, 35, 36\}$

* $P(A) = \frac{18}{38}$ $P(B) = \frac{18}{38}$
* $P(A \cap B) = \frac{10}{38}$
* $P(A \cup B) = \frac{26}{38}$
* $P(C) = \frac{12}{38}$

---

(3) $P(A \cup B)$: $P(A) = P(B) = \frac{18}{38}$

$P(A \cap B) = \frac{10}{38}$

↳ Not Mutually exclusive! ↳ $P(A \cap B) \neq 0$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{38} + \frac{18}{38} - \frac{10}{38}$

$= \frac{26}{38}$

---

(4) $P(A \cap B \cap C) = \frac{5}{38}$

↳ There are 5 sample points in this event, each with a probability of $\frac{1}{38}$.

3.15

28 January 2015 09:26

Illegal trading:

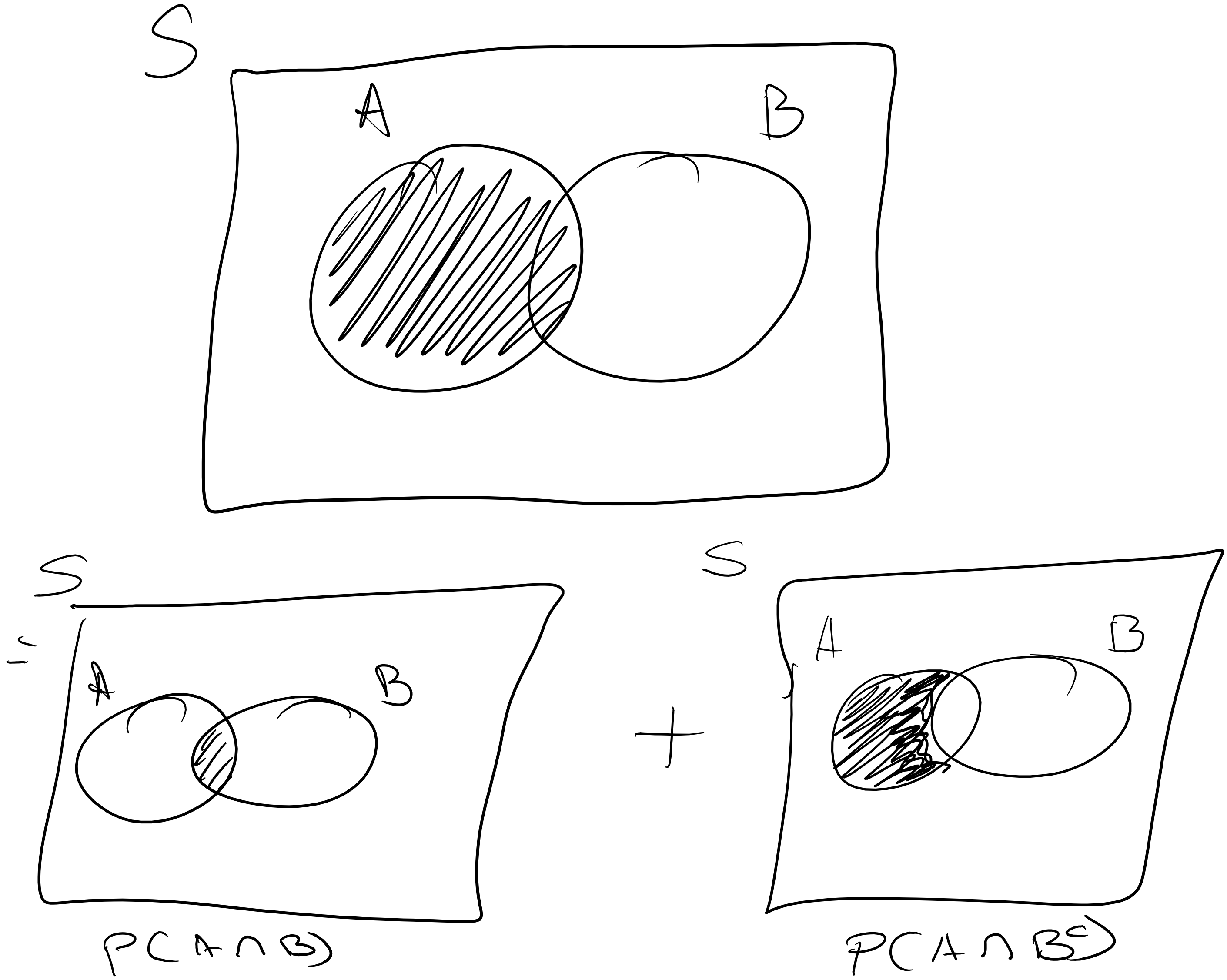
$$P(1^{st} \text{ illegal}) = \frac{3}{10}$$

$$P(2^{nd} \text{ illegal} \mid 1^{st} \text{ illegal}) = \frac{2}{9}$$

$$P(\text{both illegal}) = \left(\frac{3}{10}\right)\left(\frac{2}{9}\right) = \frac{6}{90}$$

3.16

28 January 2015 09:29



3.17

28 January 2015 09:34

Diagnostic Test

- D - the event that I have the disease
- A - the event that I test positive

We know:

$$P(A|D) = 0.95$$

$$P(A|D^c) = 0.01$$

$$P(D) = 0.001 \Rightarrow P(D^c) = 0.999 = 1 - P(D)$$

$$P(D|A) = \frac{P(A|D)P(D)}{P(A|D)P(D) + P(A|D^c)P(D^c)}$$

% have the disease after having tested positive

$$= \frac{(0.95)(0.001)}{(0.95)(0.001) + (0.01)(0.999)}$$

$$= \frac{0.00095}{0.01094}$$

$$= 0.087$$

3.18

28 January 2015 09:45

Independence:

Multip. rule:

$$P(B \cap A) = P(B \mid A)\,P(A)$$

but if A and B are independent

then $P(B \mid A) = P(B)$ so

$$P(B \cap A) = P(B)\,P(A)$$

3.19

28 January 2015 09:46

Tyres

- A := Tyre A passes test
- B := " B " "
- C := " C " "

$P(A) = 0.7$ $\Rightarrow$ $P(A^c) = 0.3$

$P(B) = 0.6$ $\Rightarrow$ $P(B^c) = 0.4$

$P(C) = 0.5$ $\Rightarrow$ $P(C^c) = 0.5$

- All fail

$$P(A^c \cap B^c \cap C^c) = P(A^c)\,P(B^c)\,P(C^c)$$

↑ independent!

$$= (0.3)(0.4)(0.5)$$
$$= 0.06$$

- $P(\text{at least } 1 \text{ pass}) = 1 - P(\underbrace{A^c \cap B^c \cap C^c}_{\text{all fail}})$

$$= 0.94$$

- $P(\text{Only } B \mid \text{At least } 1 \text{ Pass}) = \dfrac{\underbrace{P(\text{Only } B \cap \text{At least } 1 \text{ Pass})}_{P(B)}}{P(\text{At least } 1 \text{ Pass})}$

↑ only way this event can happen

A	✗	✗
B	✓	✓
C	✗	✓

A	✗	✗
B	✓	✓
C	✗	✓

$$\therefore\ P(\text{Only } B \mid \text{At least 1 Pass}) = \frac{P(\text{Only } B)}{P(\text{At least 1 Pass})}$$

$$= \frac{P(A^c)\,P(B)\,P(C^c)}{0.94}$$

$$= \frac{(0.3)(0.6)(0.5)}{0.94}$$

$$= 0.0957$$

3.20

02 February 2015 09:12

Corrosion

C - component is corroded

F - component is functioning

C and F are independent iff

$$P(C \cap F) = P(C)P(F)$$

- $P(C \cap F) = 0.2$ ... from table
- $P(C) = P(C \cap F) + P(C \cap F^c)$

$$= 0.2 + 0.4$$
$$= 0.6$$

- $P(F) = P(F \cap C) + P(F \cap C^c)$

$$= 0.2 + 0.3$$
$$= 0.5$$

Thus $P(C)P(F) = (0.6)(0.5) = 0.3 \neq 0.2$

$= P(C \cap F)$

$\therefore$ C and F are not independent

3.24

02 February 2015 09:23

Poker

- $\binom{52}{5} = 2,598,960$

- $\binom{13}{1} \times \binom{4}{3} \times \binom{12}{2} \times \binom{4}{1}^2$

Choose denomination

Choose 3 out 4 suits

Of remaining denominations Choose 2

different

[For those 2 denominations choose 1 out of 4 suits]

$= 54,912$

$$P(3 \text{ of a kind}) = \frac{54\ 912}{2,598,960} = 0.0211$$

- # Ways to get a full house

$\binom{13}{2} \times \binom{4}{3} \times \binom{4}{2} \times \binom{2}{1}$

Choose denomination

[out of 4 suits choose 3]

[out of 4 suits choose 2]

3A 2K or 3K 2A

[out of 2 denominations choose which has 3]

$= 3,744$

$$\Rightarrow P(\text{full house}) = \frac{3744}{2\ 598\ 960} = 0.0014$$

$\Rightarrow \quad P(\text{full house}) = \dfrac{-}{2{,}598{,}960} = 0.0014$

4.5

04 February 2015 09:06

Y - # heads observed in two coin tosses.

H - observe heads

T - observe tails

$P(Y=0) = P(T,T) = P(T)P(T)$ ... tosses independent

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

$P(Y=1) = P(T,H) + P(H,T)$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

$P(Y=2) = P(H,H) = P(H)P(H)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

4.9

04 February 2015 09:18

$$E(x) = 3.5$$

$$E(x^2) = 15.17$$

$$\sigma_x = \sqrt{E(x^2) - E(x)^2}$$

$$= \sqrt{15.17 - (3.5)^2}$$

$$= \sqrt{2.92}$$

$$= 1.7098$$

4.10

04 February 2015 09:21

Car Safety Ratings

Y - # stars obtained by randomly selected car [from this sample]

- $P(Y=1) = \frac{0}{98} = 0$
- $P(Y=2) = \frac{4}{98} = 0.0408$
- $P(Y=3) = \frac{17}{98} = 0.1735$
- $P(Y=4) = \frac{59}{98} = 0.602$
- $P(Y=5) = \frac{18}{98} = 0.1837$

$\boxed{Y=4 \text{ is the mode}}$

☆1	☆2	☆3	☆4	☆5
0	0.0408	0.1735	0.602	0.1837

(Note $\sum_x p(x) = 1$)

② $P(Y \leq 3) = P(Y=1) + P(Y=3)$

$$+ P(Y=3)$$
$$= 0 + 0.0408 + 0.1735$$
$$= 0.2143$$

3) $P(Y>3) = P(Y=4) + P(Y=5)$
$$= 0.602 + 0.1837$$
$$= 1 - P(Y \leq 3)$$
$$= 0.7857$$

4) $E(Y) = \sum_{y} y\,p(y)$
$$= 1(0) + 2(0.0408) + 3(0.1735)$$
$$+ 4(0.602) + 5(0.1837)$$
$$= 3.9286$$

5) $\sigma_x^2 = E(Y^2) - E(Y)^2$

$E(Y^2) = 1^2(0) + 2^2(0.0408) + 3^2(0.1735)$
$$+ 4^2(0.602) + 5^2(0.1837)$$
$$= 15.9492$$

$\therefore \sigma_x^2 = 15.9492 - (3.9286)^2$
$\therefore \qquad = 0.5153$

$\sigma_x = \sqrt{0.5153} = 0.7178$

4.12

04 February 2015 09:38

Y – # trades which make a loss

$E(Y) = Var(Y) = 4$

X – Profit made per day

$X = 1000 - 10Y - 5Y^2$

$\therefore E(X) = E(1000 - 10Y - 5Y^2)$

$= E(1000) + E(-10Y) + E(-5Y^2)$

$= 1000 - 10E(Y) - 5E(Y^2)$

$= 1000 - 10E(Y) - 5[\sigma_y^2 + E(Y)^2]$

$$\left\{ \begin{aligned} \sigma_y^2 &= E(Y^2) - E(Y)^2 \\ \therefore E(Y^2) &= \sigma_y^2 + E(Y)^2 \\ &= 4 + 16 \\ &= 20 \end{aligned} \right\}$$

$\therefore E(X) = 1000 - 10(4) - 5(4 + 4^2)$

$= 860$

$P = 7X - 1.5[O] - 10$

$Var(P) = Var(7X - 1.5[O] - 10)$

$= 7^2 Var(X) + (-1.5)^2 Var[O] + 0$

$= 7^2[E(X^2) - E(X)^2] + (1.5)^2(50^2)$

$= 49[800,000 - (860)^2] + 1.5^2[50^2]$

$= 2,955,850$

$\therefore sd(P) = 1,719.2586$

4.13

09 February 2015 09:09

n trials

$$x \sim Bin(n, p)$$

distributed

binomial

probability of success on any trial.

4.17

09 February 2015 09:17

$$\boxed{P(x) = \binom{n}{x} p^x (1-p)^{n-x}}$$

- Broker sells 20 products
- $P(\text{Profit}) = 0.9$ for each product

X — # of products which make money

$X \sim Bin(20, 0.9)$

$\therefore \Rightarrow$ Slide 18 for failure

$$P(X \geq 18) = \sum_{y=18}^{20} \binom{20}{y} (0.9)^y (1-0.9)^{20-y}$$

$$= P(X=18) + P(X=19) + P(X=20)$$

$$= \binom{20}{18}(0.9)^{18}(1-0.9)^{20-18} + \binom{20}{19}(0.9)^{19}(0.1)^{20-19} + \binom{20}{20}(0.9)^{20}(0.1)^{0}$$

$$= 0.2852 + 0.2702 + 0.1216$$

$$= 0.677$$

$E(X) = np = (20)(0.9) = 18$

$Var(X) = npq = (20)(0.9)(1-0.9) = 1.8$

4.21

09 February 2015 09:33

Catalysts

X - # highly acidic catalysts in the sample

$X \sim \text{Hypergeometric}(N = 10, s = 4, n = 3)$

- $P(X=0) = \dfrac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = \dfrac{20}{120} = \dfrac{1}{6} = 0.1667$

- $P(X=1) = \dfrac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \dfrac{60}{120} = 0.5$

  ↑ 3 chosen simul...

- $E(X) = \dfrac{ns}{N} = \dfrac{(3)(4)}{10} = 1.2$

- $Var(X) = n\left(\dfrac{s}{N}\right)\left(\dfrac{N-s}{N}\right)\left(\dfrac{N-n}{N-1}\right)$

  $= 3\left(\dfrac{4}{10}\right)\left(\dfrac{10-4}{10}\right)\left(\dfrac{10-3}{10-1}\right)$

$$= 0.56$$

$$\therefore \quad \sigma = \sqrt{0.56} = 0.7483$$

4.26

11 February 2015 09:11

Cables

- X - # of faults in 100m of cable

λ - 2 [Rate Parameter] faults per 100m

$X \sim \text{Poisson}(\lambda = 2)$

$\boxed{1}$ $\mathbb{P}(X=0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{2^0 e^{-2}}{0!} = 0.1353$

$\boxed{2}$ $\mathbb{P}(X=1) = \dfrac{2^1 e^{-2}}{1!} = 0.2707$

$\boxed{3}$ $\mathbb{P}(X \geq 2) = 1 - \mathbb{P}(X<2)$

$= 1 - \mathbb{P}(X \leq 1)$

$= 1 - (\mathbb{P}(X=0) + \mathbb{P}(X=1))$

$= 0.59399$

$\boxed{4}$ 200m of cable

- X - # faults in 1st 100m $\rightarrow X \sim \text{Poisson}(\lambda_x = 2)$
- Y - " " " 2nd " $\rightarrow Y \sim \text{Poisson}(\lambda_y = 2)$

$\therefore$ # faults in 200m is $X+Y$

$\lambda_x + \lambda_y$

$\therefore X + Y \sim \text{Poisson}(2+2)$

$X + Y \sim \text{Poisson}(4)$

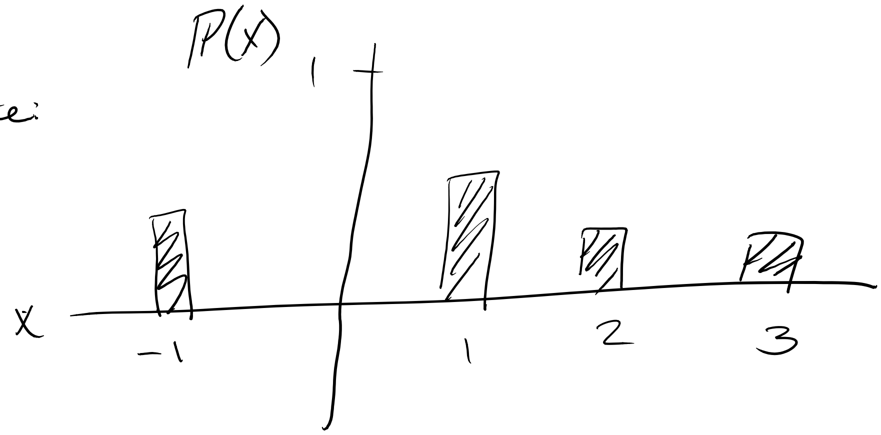
$\mathbb{P}([X+Y]=0) = \dfrac{4^0 e^{-4}}{0!} = 0.0183$

4,27

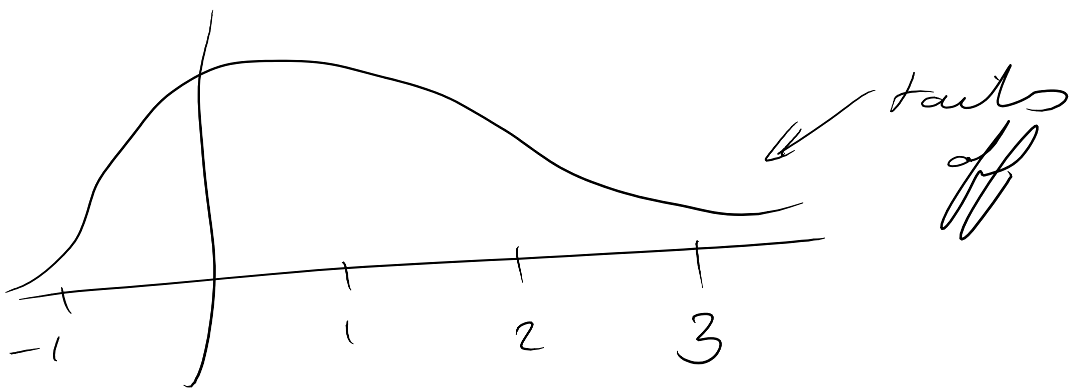
11 February 2015 09:24

Continuous Random Vars

Discrete:



Cont:



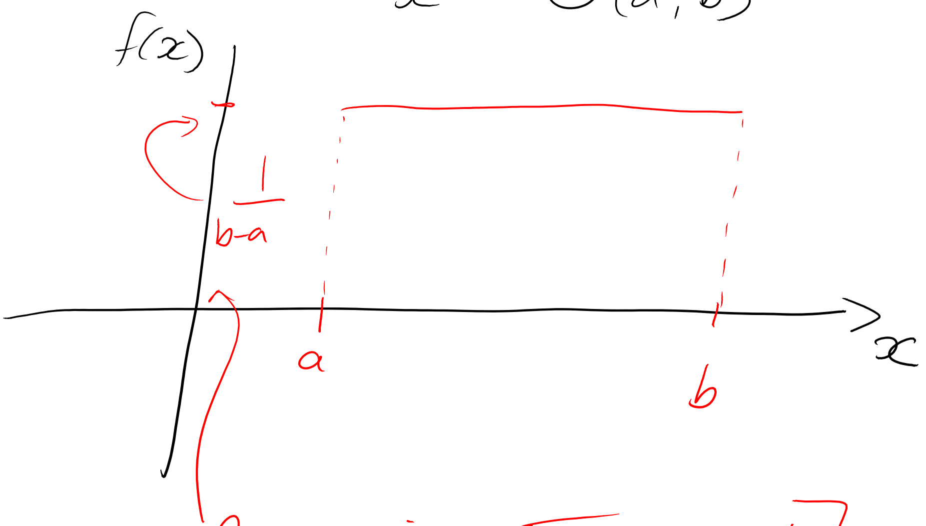
* Discrete: Heights Sum to 1
* Cont: Area under curve = 1

4.31

11 February 2015 09:34

Uniform distribution

$$x \sim U(a, b)$$



Normalise [Area = 1]

1. $$\mathbb{E}(x) = \int_a^b x \, f(x) \, dx$$

$$= \int_a^b \frac{x}{b-a} \, dx$$

$$= \left[\frac{x^2}{2(b-a)} \right] \Bigg|_a^b$$

$$= \frac{b^2}{2(b-a)} - \frac{a^2}{2(b-a)}$$

$$= \frac{b^2 - a^2}{2(b-a)}$$

$$= \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}}$$

$$= \frac{b+a}{2}$$

2 $$\mathbb{E}(x^2) = \int_a^b x^2 f(x)\,dx$$

$$= \int_a^b \frac{x^2}{b-a}\,dx$$

$$= \left[\frac{x^3}{3(b-a)}\right]\Bigg|_a^b$$

$$= \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{\cancel{(b-a)}(b^2+ba+a^2)}{3\cancel{(b-a)}}$$

$$= \frac{b^2+ba+a^2}{3}$$

$$\therefore \operatorname{Var}(x) = \mathbb{E}(x^2) - \mathbb{E}(x)^2$$

$$= \frac{b^2+ba+a^2}{3} - \frac{b^2+2ba+a^2}{4}$$

$$= \frac{4b^2+4ba+4a^2-3b^2-6ba-3a^2}{12}$$

$$= \frac{b^2 - 2ba + a^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

4.33

11 February 2015 09:46

Uniform

Y - thickness of sheets of steel

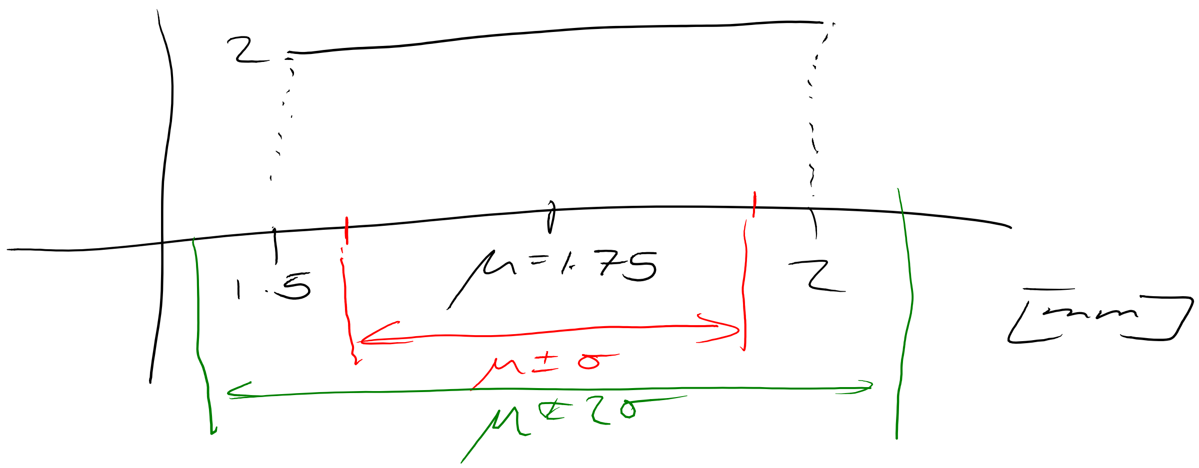
$Y \sim U(1.5, 2)$

- If $Y < 1.6$m then sheet is scrapped.

$\boxed{1}$ $E(Y) = \frac{a+b}{2} = \frac{1.5+2}{2} = 1.75$ mm

$$Var(Y) = \frac{(2-1.5)^2}{12} = 0.0208$$

$$s.d(Y) = \sqrt{0.0208} = 0.1443$$



- Sheet is scrapped if $Y < 1.6$mm

$$\mathbb{P}(Y < 1.6) = \frac{(1.6 - 1.5)}{(2 - 1.5)} = 0.2$$

4.35

16 February 2015 09:15

- $$\mathbb{E}(x) = \int_0^\infty x f(x)\,dx = \int_0^\infty x \lambda e^{-\lambda x} dx$$

$$= \lambda \int_0^\infty x e^{-\lambda x} dx$$

$$\Rightarrow u = x \qquad dv = e^{-\lambda x} dx$$

$$du = dx \qquad v = -\frac{1}{\lambda} e^{-\lambda x}$$

$$\therefore \int_0^\infty x e^{-\lambda x} dx = -\frac{x}{\lambda} e^{-\lambda x}\Big|_0^\infty + \int_0^\infty \frac{1}{\lambda} e^{-\lambda x} dx$$

$$\therefore -\frac{x}{\lambda} e^{-\lambda x}\Big|_0^\infty - \frac{1}{\lambda^2} e^{-\lambda x}\Big|_0^\infty$$

$$\therefore \left[0 + 0\right] - \left[0 - \frac{1}{\lambda^2}\right]$$

$$\therefore \lambda \int_0^\infty x e^{-\lambda x} dx = \lambda\left(\frac{1}{\lambda^2}\right) = \boxed{\frac{1}{\lambda}}$$

- $$F(x) = \int_0^x f(x)\,dx = \int_0^x \lambda e^{-\lambda x} dx$$

$$= \lambda\left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^x$$

$$= \boxed{1 - e^{-\lambda x}}$$

4.36

16 February 2015 09:24

Machine Breakdown

X - Length of time (days) between breakdowns

$X \sim \exp\left(\lambda = \frac{1}{18}\right)$ .... $\mathbb{E} = \frac{1}{\lambda} = 18$

$$\mathbb{P}(X > 21 \text{ days}) = 1 - \mathbb{P}(X \leq 21 \text{ days})$$

$$= 1 - \left[1 - e^{-\frac{21}{18}}\right]$$

$$= 0.3114$$

4.37

16 February 2015 09:31

Memoryless

$$\mathbb{P}(X > a+b \mid X > a) = \frac{\mathbb{P}(X > a+b \cap X > a)}{\mathbb{P}(X > a)}$$

$$= \frac{\mathbb{P}(X > a+b)}{\mathbb{P}(X > a)}$$

$$= \frac{1 - \mathbb{P}(X \leq a+b)}{1 - \mathbb{P}(X \leq a)}$$

$$= \frac{1 - [1 - e^{-\lambda(a+b)}]}{1 - [1 - e^{-\lambda a}]}$$

$$= \frac{e^{-\lambda a} e^{-\lambda b}}{e^{-\lambda a}}$$

$$= e^{-\lambda b}$$

$$= \mathbb{P}(X > b)$$

4.40

16 February 2015 09:39

Poisson & Exponential

- Y - # of ice-creams sold
- Y - Poisson ($\lambda = 20/\text{day}$)
- Day is 8 hrs $\Rightarrow \frac{1}{2}$ hour $= \frac{1}{16}$ day

$$\Rightarrow t = \frac{1}{16}$$

- X - time until 1st ice-cream sold

$$X \sim \exp(20)$$

$$\mathbb{P}(X < t) = 1 - e^{-20t} \quad \ldots \text{CDF of exponential}$$

$$\mathbb{P}(\cancel{X} < \tfrac{1}{16}\text{day}) = 1 - e^{-\frac{20}{16}}$$

$$= 0.7135$$

4.41

16 February 2015 09:46

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$$

let $\boxed{\alpha=1}$: $f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}$

$= \lambda e^{-\lambda x}$

where $\lambda = \frac{1}{\beta}$

$\therefore \quad X \sim \exp\left(\lambda = \frac{1}{\beta}\right)$

$$F(x) = \int_0^x f(x)\,dx = \int_0^x \left(\frac{\alpha}{\beta}\right)\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}\,dx$$

$*$ $u = x^{\alpha}$ then: $\frac{du}{dx} = \alpha x^{\alpha-1}$

$\Rightarrow dx = \frac{du}{\alpha x^{\alpha-1}}$

$$\therefore F(x) = \int_0^x \frac{\cancel{\alpha x^{\alpha-1}}}{\beta^{\alpha}} e^{-\frac{u}{\beta^{\alpha}}} \frac{du}{\cancel{\alpha x^{\alpha-1}}}$$

$$= \frac{1}{\beta^{\alpha}} \int_0^x e^{-\frac{u}{\beta^{\alpha}}}\,du$$

$$= \frac{1}{\cancel{\beta^{\alpha}}}\left[\cancel{-\beta^{\alpha}} e^{-\frac{u}{\beta^{\alpha}}}\right]_0^x$$

$$= \left[-e^{-\frac{x^{\alpha}}{\beta^{\alpha}}}\right]_0^x$$

$$= \left[-e^{-\frac{x^{\alpha}}{\beta^{\alpha}}}\right] + 1$$

$$= 1 - e^{-\frac{x^{\alpha}}{\beta^{\alpha}}}$$

$$= 1 - e^{-\frac{x^\alpha}{\beta^\alpha}}$$

4.43

18 February 2015 09:05

Weibull

- t - time until system installed

$t \sim \text{Weibull}(\alpha = \frac{1}{4}, \beta = 4)$

$$\mathbb{P}(t < 100) = 1 - e^{-\left(\frac{100}{4}\right)^{1/4}}$$

$$= 0.8931$$

$$\mathbb{P}(t \geq 100) = 1 - 0.8931$$
$$= 0.1069$$

C - Cost of installing system

- C is a discrete random variable which can take 3 values

$$C = \begin{cases} 10,000 & \text{if } t \leq 100 \\ 8,000 & \text{if } t > 100 \text{ NOT F} \\ 9,000 & \text{if } t > 100 \text{ YES F} \end{cases}$$

$$\mathbb{P}(t > 100 \text{ and not fined}) = \mathbb{P}(t > 100)\,\mathbb{P}(\text{Not fined})$$
$$= (0.1069)(0.4)$$
$$= 0.0428$$

$$\mathbb{P}(t > 100 \text{ and fined}) = (0.1069)(0.6)$$
$$= 0.0641$$

$$E(C) = \sum_{C} x\,p(x)$$

$$= 10{,}000(0.8931) + 8000(0.0428) + 9000(0.0641)$$

$$= 9850.3 \quad [€]$$

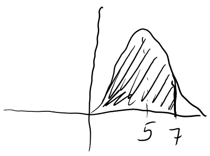
4.49

18 February 2015 09:33

Normal Distribution

- $X \sim N(\mu = 5, \sigma^2 = 3)$

$$P(X \leq 7) = P\left(\frac{X-5}{\sqrt{3}} \leq \frac{7-5}{\sqrt{3}}\right)$$



$$= P(Z \leq 1.1547)$$

$$= 0.8749 \text{ [From tables]}$$

- $P(X \geq 6) = 1 - P(X \leq 6)$

$$= 1 - P\left(\frac{X-5}{\sqrt{3}} \leq \frac{6-5}{\sqrt{3}}\right)$$

$$= 1 - P(Z \leq 0.5773)$$

$$= 1 - 0.7190$$

$$= 0.2810$$

- $P(X \leq 2) = P\left(\frac{X-5}{\sqrt{3}} \leq \frac{2-5}{\sqrt{3}}\right)$

$$= P(Z \leq -1.732)$$

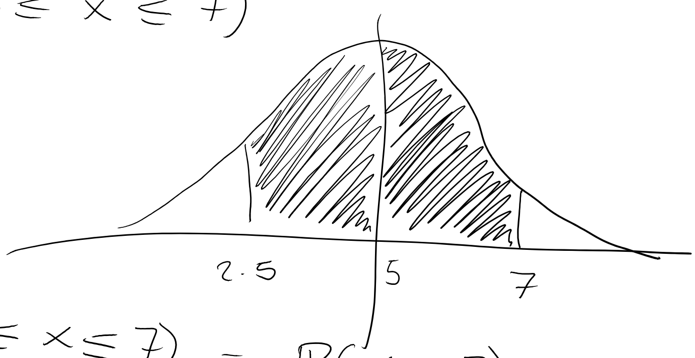
$$= P(Z \geq 1.732)$$

$$= 1 - P(Z \leq 1.732)$$

$$= 1 - 0.9582$$

$$= 0.0418$$

- $P(2.5 \leq X \leq 7)$



$$P(2.5 \leq X \leq 7) = P(X \leq 7) - P(X \leq 2.5)$$

$$= P\left(\frac{x-5}{\sqrt{3}} \leq \frac{7-5}{\sqrt{3}}\right) - P\left(\frac{x-5}{\sqrt{3}} \leq \frac{2.5-5}{\sqrt{3}}\right)$$

$$\mathbb{P}\left(\frac{\ }{\sqrt{3}} \le \frac{\ }{\sqrt{3}}\right) - \mathbb{P}\left(\frac{x-5}{\sqrt{3}} \le \frac{2.5-5}{\sqrt{3}}\right)$$

$$= \mathbb{P}(Z \le 1.1547) - \mathbb{P}(Z \le -1.44)$$

$$= 0.8749 - \mathbb{P}(Z \ge 1.44)$$

$$= 0.8749 - [1 - \mathbb{P}(Z \le 1.44)]$$

$$= 0.8749 - [1 - 0.9251]$$

$$= 0.8$$

4.50

23 February 2015 09:08

Paper Friction

F - Friction coefficient

$F \sim N(0.55, (0.013)^2)$

- $\mathbb{P}(0.53 \leq F \leq 0.56)$

$= \mathbb{P}(F \leq 0.56) - \mathbb{P}(F \leq 0.53)$

$= \mathbb{P}\left(\frac{F-0.55}{0.013} \leq \frac{0.56-0.55}{0.013}\right) - \mathbb{P}\left(\frac{F-0.55}{0.013} \leq \frac{0.53-0.55}{0.013}\right)$

$= \mathbb{P}(Z \leq 0.769) - \mathbb{P}(Z \leq -1.538)$

$= 0.7794 - \mathbb{P}(Z \geq 1.538)$

$= 0.7794 - [1 - \mathbb{P}(Z \leq 1.538)]$

$= 0.7794 - [1 - 0.9382]$

$= 0.7176$

- $\mathbb{P}(F \leq 0.52)$

$= \mathbb{P}\left(\frac{F-0.55}{0.013} \leq \frac{0.52-0.55}{0.013}\right)$

$= \mathbb{P}(Z \leq -2.3077)$

$= \mathbb{P}(Z \geq 2.3077)$

$= 1 - \mathbb{P}(Z \leq 2.3077)$

$= 1 - 0.98956$

$= 0.01044$

Not very likely. Only a 1% chance...

$\boxed{\leq 0.52}$

4.51

23 February 2015 09:20

Percentage Point Tables

$$\mathbb{P}(Z \geqslant a) = p$$

$$\mathbb{P}(Z \geqslant a) = 0.1$$

$$\Rightarrow a = 1.2816$$

4.52

23 February 2015 09:24

Exam Results

S - Score on exam

$S \sim N(78, 36)$

- $\mathbb{P}(S \geq C) = 0.1$

$\therefore \mathbb{P}\left(\frac{S-78}{6} \geq \frac{C-78}{6}\right) = 0.1$

$\therefore \mathbb{P}\left(Z \geq \frac{C-78}{6}\right) = 0.1$

$\therefore \frac{C-78}{6} = 1.2816$ ... Tables [P Point]

$\therefore C = 6(1.2816) + 78$

$= 85.6896$

- P - passing cutoff

$\mathbb{P}(S \leq P) = 0.25$

$\mathbb{P}\left(\frac{S-78}{6} \leq \frac{P-78}{6}\right) = 0.25$

$\therefore \mathbb{P}\left(Z \leq \frac{P-78}{6}\right) = 0.25$

$\mathbb{P}\left(Z \geq -\left(\frac{P-78}{6}\right)\right) = 0.25$ ... To use tables

$\therefore \frac{-P+78}{6} = 0.6745$

$$\therefore P = -\left[6(0.6745) - 78\right]$$

$$\therefore P = 78 - 6(0.6745)$$

$$\therefore P = 73.953$$

- $\mathbb{P}(S \geq 79)$

$$= \mathbb{P}\left(\frac{S-78}{6} \geq \frac{79-78}{6}\right)$$

$$= \mathbb{P}(Z \geq 0.1667)$$

$$= 1 - \mathbb{P}(Z \leq 0.1667)$$

$$= 0.4325$$

4.53

23 February 2015 09:37

$\mu = 50$
$\sigma = 6$

L - lifetime

$L \sim N(50, 36)$

- $\mathbb{P}(L \geq 55) = \mathbb{P}\left(\frac{L-50}{6} > \frac{55-50}{6}\right)$

$\therefore \mathbb{P}(Z > 0.83)$

$\therefore 1 - \mathbb{P}(Z < 0.83)$

$= 1 - 0.7967$

$= 0.2033$

- $\mathbb{P}(42 < L < 58)$

$= \mathbb{P}(L < 58) - \mathbb{P}(L < 42)$

$= \mathbb{P}\left(Z < \frac{58-55}{6}\right) - \mathbb{P}\left(Z < \frac{42-55}{6}\right)$

$= \mathbb{P}(Z < 1.33) - \mathbb{P}(Z < -1.33)$

$= \mathbb{P}(Z < 1.33) - \mathbb{P}(Z > 1.33)$

$= \mathbb{P}(Z < 1.33) - [1 - \mathbb{P}(Z < 1.33)]$

$= \mathbb{P}(Z < 1.33) - 1 + \mathbb{P}(Z < 1.33)$

$= 2\,\mathbb{P}(Z < 1.33) - 1$

$= \mathbb{P}(Z < 1.33) - 1 + \mathbb{P}(Z < 1.33)$

$= 2\,\mathbb{P}(Z < 1.33) - 1$

$= 2(0.9082) - 1$

$= 1.8164 - 1$

$= 0.8164 \implies 82\%$ in this range

- X - # joists that last less than 55 years [in this sample]

$$X \sim \text{Bin}(n = 4,\ p = 0.7967) \leftarrow \text{From first part}$$

$$\mathbb{P}(X = 2) = \binom{4}{2}(0.7967)^2(1 - 0.7967)^{4-2}$$

$$= 0.1574$$

5.11

25 February 2015 09:39

CD Manufacture

Y - # of defective CDs in sample

$Y \sim Bin(n_Y = 50, p)$

Then $Y = \sum_{i=1}^{50} X_i$ where $X_i \sim Bin(n_x = 1, p)$

- Thus $\mathbb{E}(X_i) = np = n_x p = p$
  and we know that $\bar{x}$ is an unbiased estimator for $\mathbb{E}(X_i)$
  so $\bar{x}$ is an unbiased estimator for p.

$$\hat{p} = \bar{x} = \frac{\sum_{i=1}^{50} x_i}{50} = \frac{10}{50} = 0.2$$

estimator

from question [10 defective CDs]

Aside: $X_i = \begin{cases} 1 & \text{if CD}_i \text{ is defective} \\ 0 & \text{otherwise} \end{cases}$

- Company claim 10% are defective
  i.e: $p = 0.1$

$$\mathbb{P}(Y > 9) = \mathbb{P}\left(\sum_{i=1}^{50} X_i > 9\right) = \mathbb{P}\left(\bar{X} > \frac{9}{50}\right)$$

By CLT: $\bar{X} \mathrel{\dot\sim} N\left(p, \frac{n_x p(1-p)}{50}\right)$

$$\bar{X} \mathrel{\dot\sim} N\left(0.1, \frac{(1)(0.1)(0.9)}{50}\right) \Leftarrow \text{Company claim}$$

$$\bar{X} \mathrel{\dot\sim} N(0.1, 0.0015)$$

Then $\mathbb{P}\left(\bar{X} > \frac{9}{50}\right) = \mathbb{P}\left(\frac{\bar{x} - 0.1}{\sqrt{0.0018}} > \frac{\frac{9}{50} - 0.1}{\sqrt{0.0018}}\right)$

$$= \mathbb{P}(Z > 1.8856)$$

$$= 1 - \mathbb{P}(Z \leq 1.8856)$$

$$= 1 - 0.9706$$

$$= 0.0294$$

- If the claim is true then we

- If the claim is true then we have observed an unlikely event

5.16

02 March 2015 09:46

Confidence Interval for non-normal population

- $n = 100$
- $\bar{x} = 83.2$
- $s = 6.4$

1) $\alpha = 0.05 \Rightarrow 95\%$ confidence interval

$\therefore \frac{\alpha}{2} = 0.025$

$\therefore Z_{\frac{\alpha}{2}} = 1.96$ ... from table

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$$83.2 \pm (1.96)\left(\frac{6.4}{10}\right)$$

$$= 83.2 \pm (1.96)(0.64)$$

$\Rightarrow (81.9456, 84.4544)$ ... interval

2) 99% C.I

$\Rightarrow \alpha = 0.01 \Rightarrow \frac{\alpha}{2} = 0.005$

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$$\bar{x} \pm Z_{0.005} \frac{s}{\sqrt{n}}$$

$\boxed{p = 0.5}$

$$83.2 \pm (2.5758)\left(\frac{6.4}{\sqrt{100}}\right)$$

2.5758

$$83.2 \pm 1.6485$$

$(81.5514, 84.8485)$ ... Wider than 95%

- We can see that the 99% C.I is wider than 95% ...

5.18

04 March 2015 09:12

CD Manufacture Revisited

- $\hat{p} = 0.2$, $n = 50$, $Z_{0.025} = 1.96$

95% C.I $\quad \hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$0.2 \pm (1.96)\sqrt{\frac{0.2(0.8)}{50}}$$

$$0.2 \pm (1.96)(0.0566)$$

$(0.089, 0.311)$ ... Very large C.I

$\Rightarrow$ More precise interval $\Rightarrow$ larger sample size

* 95% sure that the true population proportion of defective CDs produced by the company is between 9% and 31%

* Since the value claimed by the company (of 10%) is within this interval we can't say with 95% confidence that their claim is false

5.26

23 March 2015 09:22

Student Rent

$H_0: \mu = 5{,}000$ vs. $H_A: \mu > 5{,}000$

- $n = 50$ (students)
- $\bar{x} = 5200$
- $s = 735$

[Test Statistic]

$$\therefore Z_\mu = \frac{\bar{x} - \mu_0}{\left(s/\sqrt{n}\right)}$$

$$= \frac{5200 - 5000}{\left(735/\sqrt{50}\right)}$$

$$= 1.9241$$

[Rejection Region]

The rejection region for this test is where:

$$Z_\mu > Z_\alpha = Z_{0.05} = 1.6449$$

Since $1.9241 > Z_\alpha$ we reject the null hypothesis and conclude that the average rental income per room in student accomodations is greater than €5,000 per year.

room in student accom...
greater than €5,000 per year.

5.31

23 March 2015 09:35

Weathering Steel

- $n = 60$
- $\hat{p} = \frac{54}{60} = 0.9$

p here is the proportion of non-defective bridges

$\rightarrow H_0: p = 0.95$ vs. $H_A: p < 0.95$

$$\rightarrow Z_p = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.9 - 0.95}{\sqrt{\frac{0.95(1-0.95)}{60}}} = -1.777$$

Since $Z_p = -1.777 < -Z_\alpha = -1.6449$ We reject H_0 and conclude that more than 5% of these bridges have major corrosive damage.

5.27

23 March 2015 09:42

Road Usage

- $H_o = 71 \quad H_A > 71$
- $n = 50$
- $\bar{x} = 74.1$
- $s = 13.3$

$$\boxed{\alpha = 0.1}$$

$$Z_\mu = \frac{\bar{x} - \mu_o}{(s/\sqrt{n})} = \frac{74.1 - 71}{\left(\frac{13.3}{\sqrt{50}}\right)}$$

$$= 1.6481$$

$$Z_\alpha = Z_{0.1} = 1.2816$$

$$Z_\mu > Z_\alpha$$

Reject Null

$$\boxed{\alpha = 0.01}$$

$$Z_\alpha = 2.3263$$

$Z_\mu < Z_\alpha$

Cannot reject Null

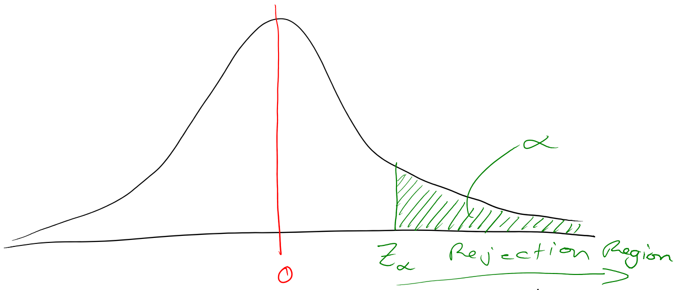
P-values + Rejection regions

- One tailed test

  e.g: $H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$

  Test statistic -> $Z_\mu = \dfrac{\bar{x} - \mu_0}{(s/\sqrt{n})}$

  If the null hypothesis is true
  ↳ $Z_\mu \sim N(0,1)$



→ Rejection region: We reject H_0 if $Z_\mu > Z_\alpha$ where Z_α is such that

$P(Z > Z_\alpha) = \alpha$ where $Z \sim N(0,1)$

- If Z_μ is in the rejection region we reject H_0, otherwise we fail to reject

- P-value: The probability of observing a value of the test statistic at least as extreme as the one observed [assuming the null is true]

- Here, if H_0 is true

  $Z_\mu \sim N(0,1)$

  $\therefore$ P-value $= \mathbb{P}(Z > Z_\mu)$ where $Z \sim N(0,1)$

- We reject the null if $p = \mathbb{P}(Z > Z_\mu) < \alpha$ (say $\alpha = 0.05$) and fail to reject if otherwise.

~~Bust~~

$\mathbb{P}(Z > Z_\alpha) = \alpha$ so if $\mathbb{P}(Z > Z_\mu) < \alpha$

$\mathbb{P}(Z > Z_\alpha) = \alpha$ so if $\mathbb{P}(Z > Z_\mu) < \alpha$
then we must have that

$$Z_\mu > Z_\alpha \quad [\text{think about probs}]$$

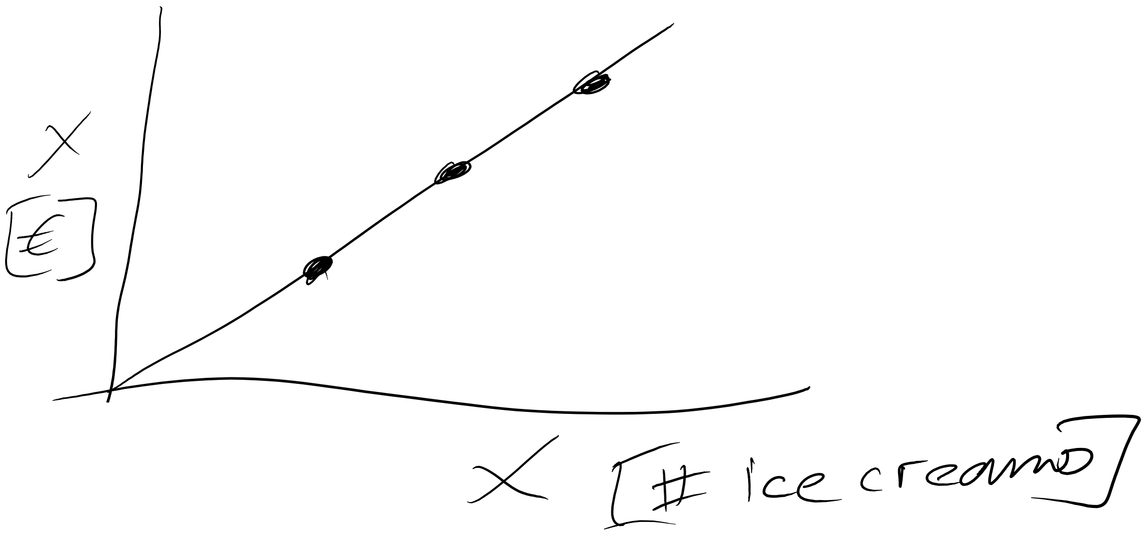
i.e. Z_μ is in the rejection region

$$\therefore p < \alpha \iff Z_\mu > Z_\alpha$$

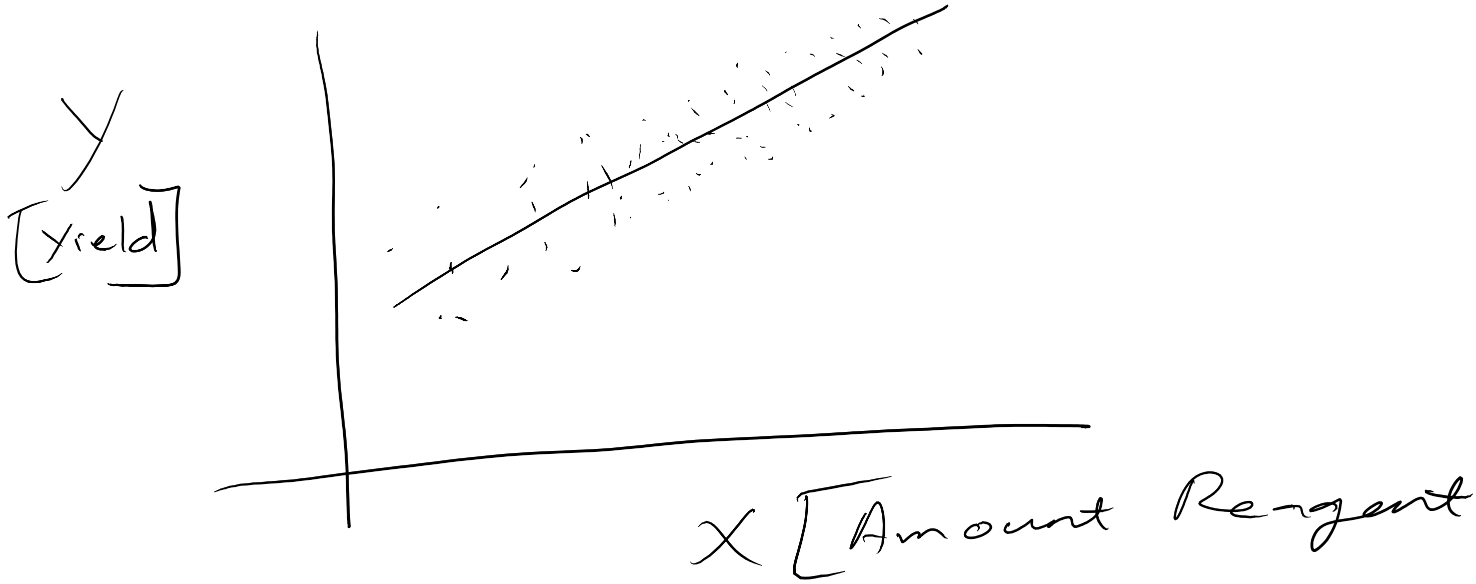
6.2

30 March 2015 09:22

Deterministic Relationship



Probabilistic Relationship



6.9

30 March 2015 09:37

$$SS_E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$= \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\frac{\partial (SS_E)}{\partial \hat{\beta}_1} = -2 \sum_{i=1}^{n} (y_i - \beta_0 - \hat{\beta}_1 x_i)^2 \, x_i$$

$$\frac{\partial (SS_E)}{\partial \hat{\beta}_0} = -2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$$

Maximise these constraints

$\rightarrow$

6.11

30 March 2015 09:43

Spectrometric Oil Analysis

→ Regression Coefficients

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum_{i=1}^{n} x_i y_i - N\bar{x}\bar{y}}{\sum x_i^2 - N\bar{x}^2}$$

$$= \frac{1{,}304{,}050 - (10)(2778.6)(39.4)}{97685656 - (10)(2778.6)^2}$$

$$= 0.0102$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 39.4 - (0.0102)(2778.6)$$

$$= 11.0052$$

So the estimated regression equation:

$$\hat{Y}_i = 11.0052 - 0.0102 X_i$$

Regression Coeffs estimate

$$SS_E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$= \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\frac{\partial SS_E}{\partial \hat{\beta}_0} = -2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) \qquad (1)$$

$$\frac{\partial SS_E}{\partial \hat{\beta}_1} = -2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) x_i \qquad (2)$$

(1) to 0 ...

$$-2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

$$\therefore \sum_{i=1}^{n} y_i - n\hat{\beta}_0 - \hat{\beta}_1 \sum_{i=1}^{n} x_i = 0$$

6.24

08 April 2015 10:09

$$\begin{cases} \hat{\beta}_1 = 0.0102 \\ MS_E = 5.466 \\ S_{xx} = 20{,}479{,}476 \\ n = 10 \end{cases}$$

$$90\% \text{ CI} : \hat{\beta}_1 \pm t_{\underset{n-2}{8}, 0.05} \sqrt{\frac{MS_E}{S_{xx}}}$$

From table

$$t_{8, 0.05} = 1.86$$

$$90\% \text{ CI} : 0.0102 \pm (1.86)\sqrt{\frac{5.466}{20{,}479{,}476}}$$

$$= 0.0102 \pm 0.00096$$

$$(0.0092, 0.0112)$$

6.26

08 April 2015 10:16

Spectrometric Oil Analysis

$$X^* = 2800$$

$$\hat{Y}^* = \hat{\beta}_0 + \hat{\beta}_1 X^*$$

$$= 11.0052 + (0.0102)(2800)$$

$$= 39.5625$$

$\therefore$ We estimate that there will be 39.5625 ppm of iron (on average) in the oil of a car that has driven 2800 miles since the last oil change.

6.29

08 April 2015 10:23

$$\begin{cases} S_{xx} = 20,479,476 \\ S_{xy} = 209,281.6 \\ S_{yy} = 2178.4 \end{cases}$$

$$R^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = \frac{(209,281.6)^2}{(20,479,476)(2178.4)}$$

$$= 0.9818$$

$\Rightarrow$ 98% of the variation in iron detected in the engine oil is due to the distance travelled since the last oil change

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \sqrt{R^2} = \sqrt{0.9818} = 0.9908$$

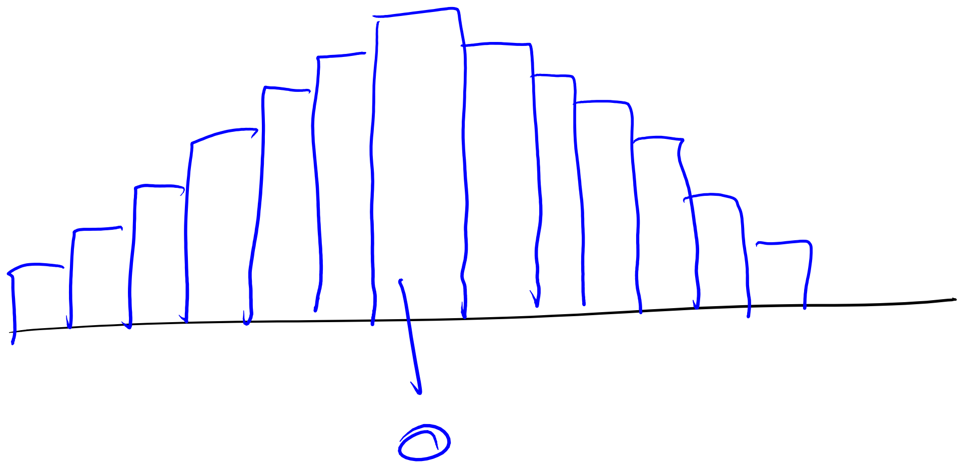
Amount of iron detected in the oil is highly correlated with the distance travelled since last oil change.

$\Rightarrow$ Positive square root as $S_{xy} > 0$

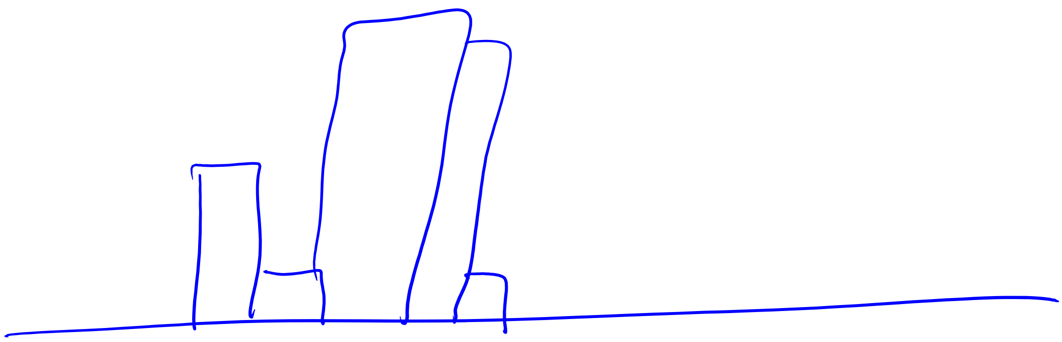
6.31

08 April 2015 10:31

Normality



Good

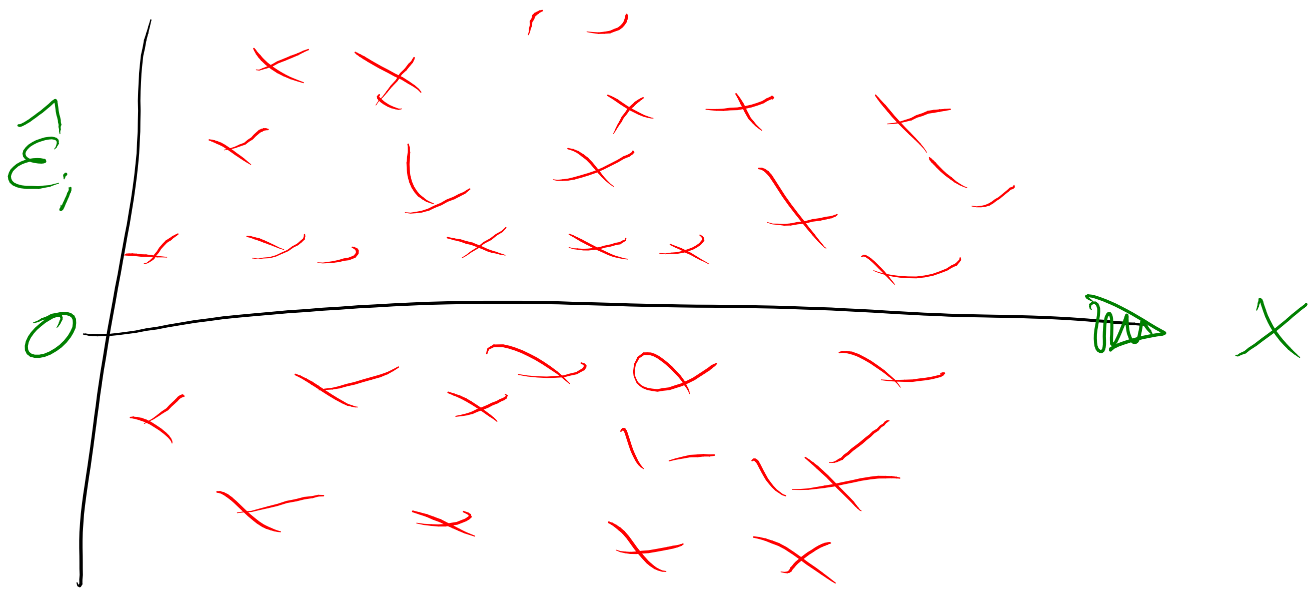


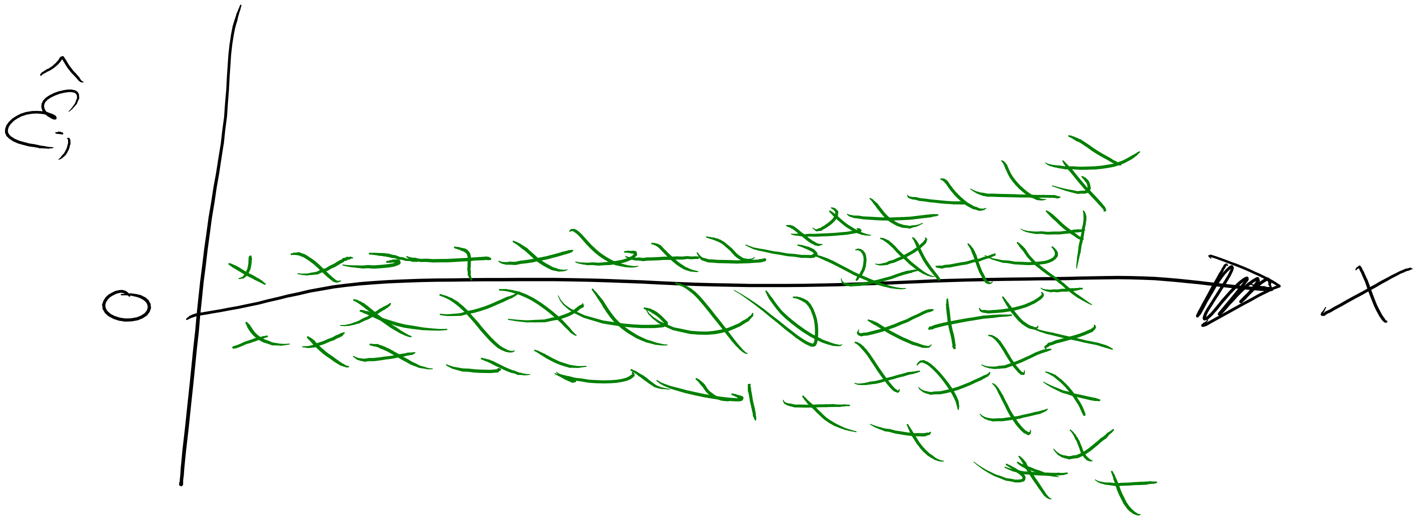
Bad!

6.32

08 April 2015 10:34

Constant Variance





6.35

08 April 2015 10:38

Summary Statistics

$\sum x_i^2 = 6,966$ $\qquad$ $\sum y_i^2 = 334,385$

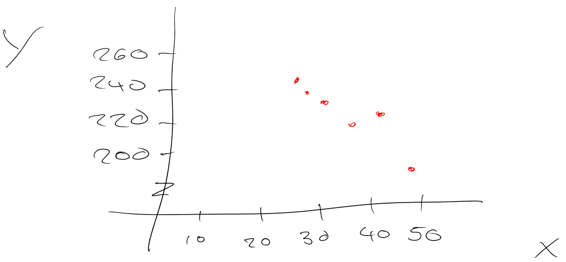
$\sum x_i = 200$ $\qquad$ $\sum y_i = 1411$

$n = 6$ $\qquad$ $\bar{y} = 235.1667$

$\bar{x} = 33.333$ $\qquad$ $\sum x_i y_i = 46,197$

X - Price $\qquad$ Y - Units Sold

Plot



$\rightarrow$ Linear model looks appropriate

Parameter estimates

$\neq \sum x_i \sum y_i$

$$S_{xy} = \sum x_i y_i - n\bar{x}\bar{y}$$

$$= 46,197 - (6)(33.333)(235.1667)$$

$$= -835.8697$$

$$S_{xx} = \sum x_i^2 - n\bar{x}^2$$

$$= 6,966 - (6)(33.333)^2$$

$$= 299.4667$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{-835.8697}{299.4667} = -2.7912$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 235.1667 - (-2.7912)(33.333)^2$$

$$= 328.2056$$

Test Model Utility

$$t = \frac{\hat{\beta}_1}{\sqrt{\frac{MSE}{S_{xx}}}}$$

$$SS_E = S_{yy} - \hat{\beta}_1 S_{xy}$$

$$= \left[334{,}385 - 6(235.1667)^2\right] - (-2.7912)(-835.8697)$$

$$= 231.8269$$

$$MS_E = \frac{231.8269}{6-2} = 57.9567$$

$H_0: \beta_1 = 0$ vs $H_A: \beta_1 \neq 0$ $(\alpha = 0.05)$

$$t = \frac{-2.7912}{\sqrt{\frac{57.9567}{299.4667}}} = -6.3447$$

$$t_{n, \frac{\alpha}{2}} = t_{4, 0.025} = 2.776 \text{ (Tables)}$$

Rejection region:

$$|t| > 2.776$$

Since $|t| > 2.776$ we reject H_0 and conclude that the number of units sold is related to the price per unit

---

90% Confidence Interval for β_1:

$$\hat{\beta}_1 \pm t_{\frac{\alpha}{2}, n-2} \sqrt{\frac{MS_E}{S_{xx}}}$$

$$\pm t_{0.05, 4} \sqrt{\frac{MSE}{S_{xx}}}$$

$$-2.7912 \pm 2.132 \sqrt{\frac{57.9567}{299.4667}}$$

(,) Interval

Prediction: Estimate the avg.

# of units sold when the price is €32.50

$\hat{y}^* = 328.2056 - 2.7912 x^*$

$= 328.2056 - 2.7912(32.50)$

$= 237.492$

Coefficient of determination / correlation

↳ Correlation: $r = \dfrac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

$= \dfrac{-835.8697}{\sqrt{(299.4667)(2564.7393)}}$

$= -0.9538$

- r is close to -1 which indicates a strong negative relationship between price and the number sold.
- $R^2 = (-0.9538)^2 = 0.9097$

⇒ 91% of the variation in the number of units sold is due to its linear relationship.