

# Chapter 10

## Transistors

### The JFET

# Transistors - General Comments

- Transistors are enormously useful in Electrical & Electronic Engineering. They are versatile and provide extremely valuable functions in electronic circuits;
- The PN junction has just 2 terminals, transistors generally have 3;
- The first transistor was invented in Bell Labs in 1947 and there are now numerous different kinds of transistor;
- Transistors tend to be used for two basic kinds of purpose:
  - As **signal amplifiers**: When suitable DC bias is applied, a transistor can be used to increase the amplitude of a time-varying signal (voltage or current), i.e. to magnify a signal. Often, the product of signal current and voltage (the signal power) is magnified, as in a *power amplifier*. With feedback, an amplifier can be converted to an *oscillator* - a free-running circuit producing a continuous (e.g.) sinusoidal signal from the DC power;
  - As **an electronic switch**: Transistors can use the voltage (or current) at one of their terminals to switch the flow of perhaps a much larger current through the other two. These two states 'ON' and 'OFF' of a transistor can also be used in digital electronics to represent the binary digits '0' and '1' enabling transistors to store and process digital signals.

# Transistors

FIELD EFFECT (FET)  
(Relying on just one  
type of carrier)

BIPOLAR  
(Relying on both electrons  
and holes)

Junction  
FET (JFET)

Metal  
semiconductor  
FET  
(MESFET)

Metal Oxide  
Semiconductor  
\*(MOSFET)\*

Bipolar  
Junction  
Transistor  
(BJT)

Heterojunction  
Bipolar  
Transistor  
(HBT)

NPN

PNP

NPN

PNP

Depletion type

\*Enhancement  
Type\*

N-channel

P-channel

N-channel

P-channel

Complementary MOS (CMOS)

# Field Effect Transistors (FETs)

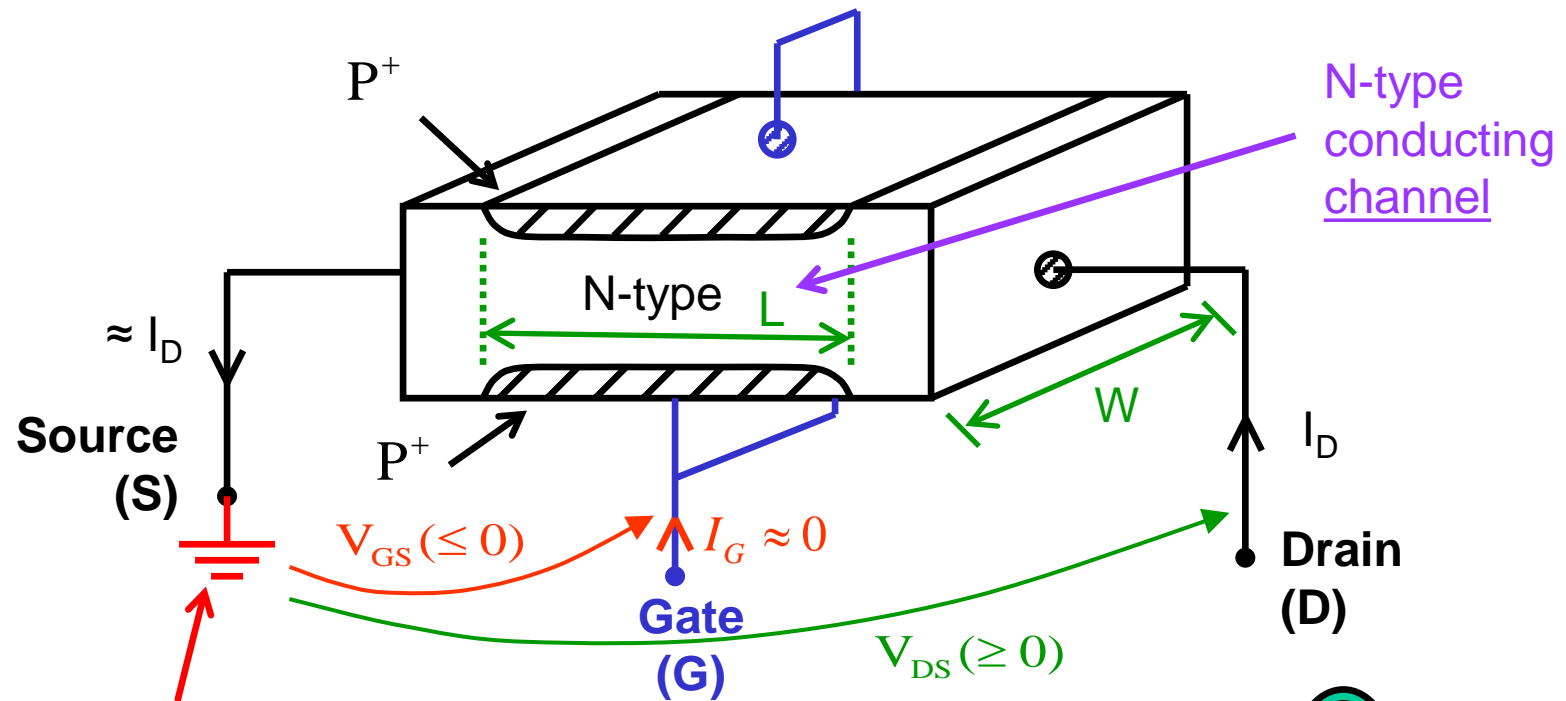
- This is the oldest transistor concept, originating in a patent by Lilienfeld in the 1920s, although practical realisation had to wait until the 1950s;
- All FETs share some common features: current may flow in a conducting **channel** connecting a **Source (S)** region to a **Drain (D)** region.
- The current flowing is controlled by the voltage (or electric field) applied to a third **Gate (G)** electrode, which under normal conditions is **electrically isolated** from the remainder of the structure (i.e. no DC current should normally flow through the Gate terminal). Exactly how this control is exercised varies between different kinds of FET;
- If the channel is naturally open-circuit, and the Gate voltage causes current to flow, the FET is said to be “**enhancement type**” (e.g. MOSFET)
- Conversely, if current flow tends to be suppressed by the Gate voltage the FET is described as “**depletion type**” (e.g. JFET)
- In an **N-channel** FET, **electrons** dominate the flow in the channel whereas **holes** dominate the flow in a **P-channel** FET.

# The Junction FET (JFET)

# The Junction FET

- The Junction FET or JFET is conceptually the simplest kind of transistor, although its usefulness in practical applications (in Silicon) is rather limited;
- The JFET is closely related to a family of advanced high-frequency microwave transistors including the MESFET (Metal-Semiconductor FET), often made with GaAs or even more elaborate material systems;
- In a JFET, the **control of the conducting channel is effected by using the voltage on the Gate to vary the width of a reverse-biased PN junction depletion layer**, thereby changing (reducing) the cross-sectional area of the channel . JFETs are thus *depletion mode FETs*

# Idealised Structure of an N-Channel JFET (Using Reverse-Biased Gate P<sup>+</sup>N Junction)

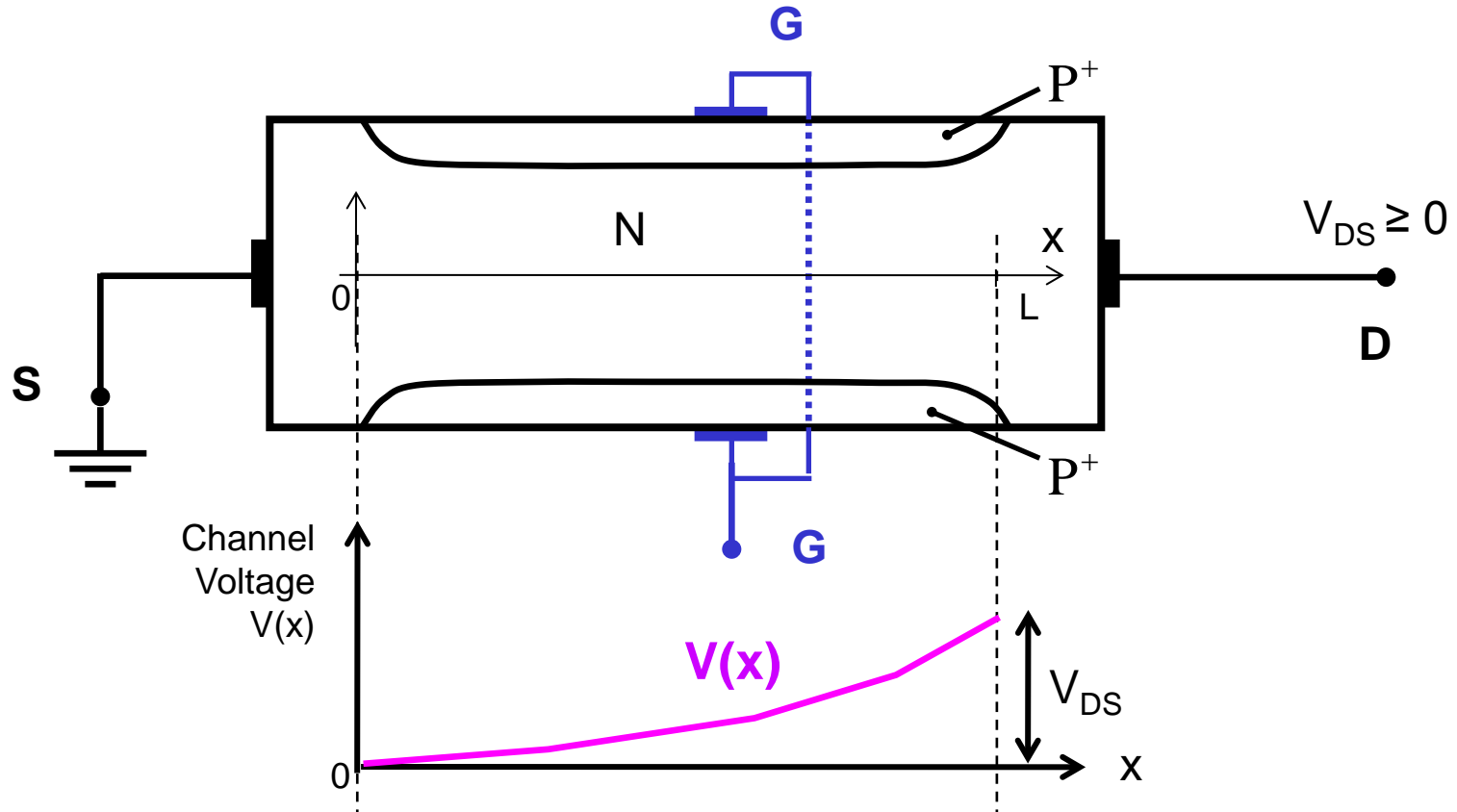


We will always assume that the Source is grounded here i.e. "Common Source" configuration

L : channel length  
W : channel width

\*\* Key FET Geometrical Parameters

## Assume $V_{DS} > 0$ : Channel Voltage $V(x)$

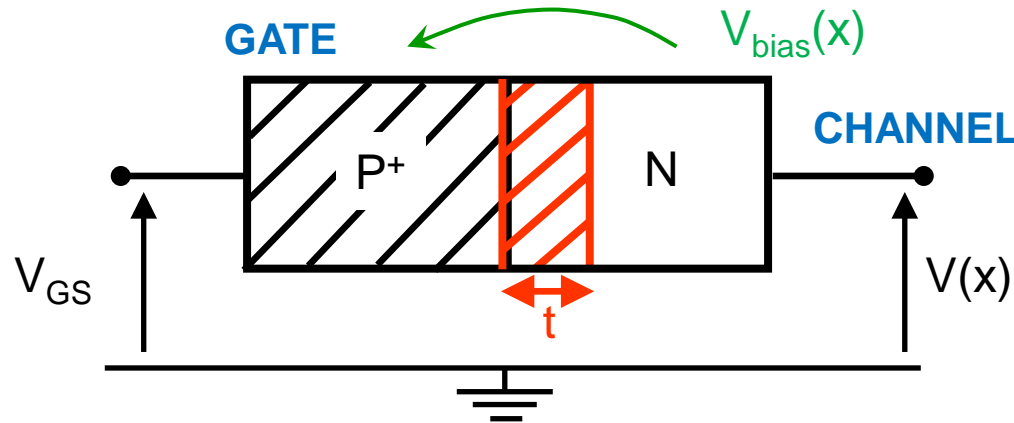


- The displacement along the centre-line of the channel is 'x', ranging between  $x=0$  and  $x=L$ . The **channel voltage** at any 'x' is  $V(x)$  and must vary from  $V(0) = 0$  to  $V(L) = V_{DS}$  ( $> 0$ , i.e. it is always positive, at any x)



# Shape of the Depletion Layer in a JFET

- If  $V_{GS}$  is  $< 0$  and  $V_{DS} > 0$ , then the **voltage  $V(x)$  along the channel** from S to D is **always positive**, and so each of the two (one-sided) P<sup>+</sup>N junctions is *continuously reverse-biased* along the channel.
- To determine how much reverse bias exists we can use the following picture at any channel position 'x':



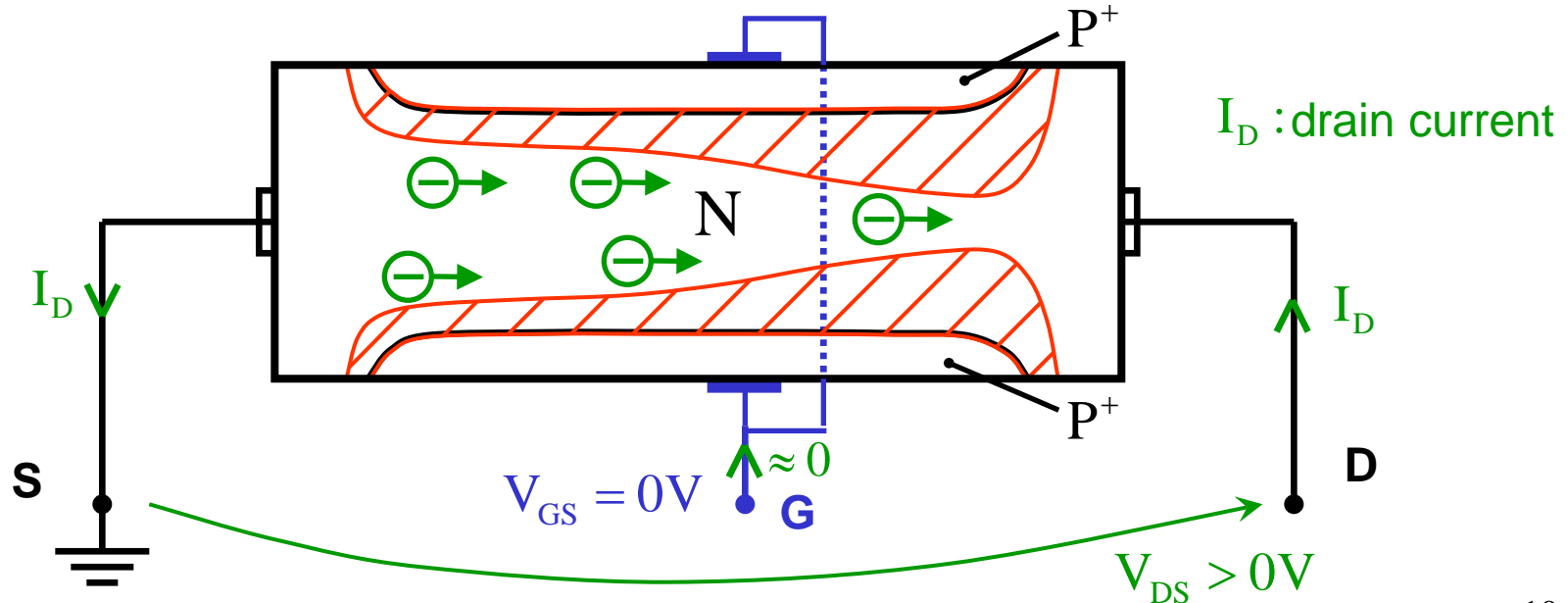
We will use "t" to represent depletion layer width here, to avoid confusion with the channel width "W"

- By KVL,  $V_{GS} = (V_{bias}(x) + V(x)) \Rightarrow V_{bias}(x) = (V_{GS} - V(x))$ , so that (1-sided):

$$t(x) = \sqrt{\frac{2 \epsilon (\phi_i - V_{bias})}{q N_D}} = \sqrt{\frac{2 \epsilon (\phi_i - V_{GS} + V(x))}{q N_D}} \Rightarrow \text{Depletion layer widens at drain end}$$

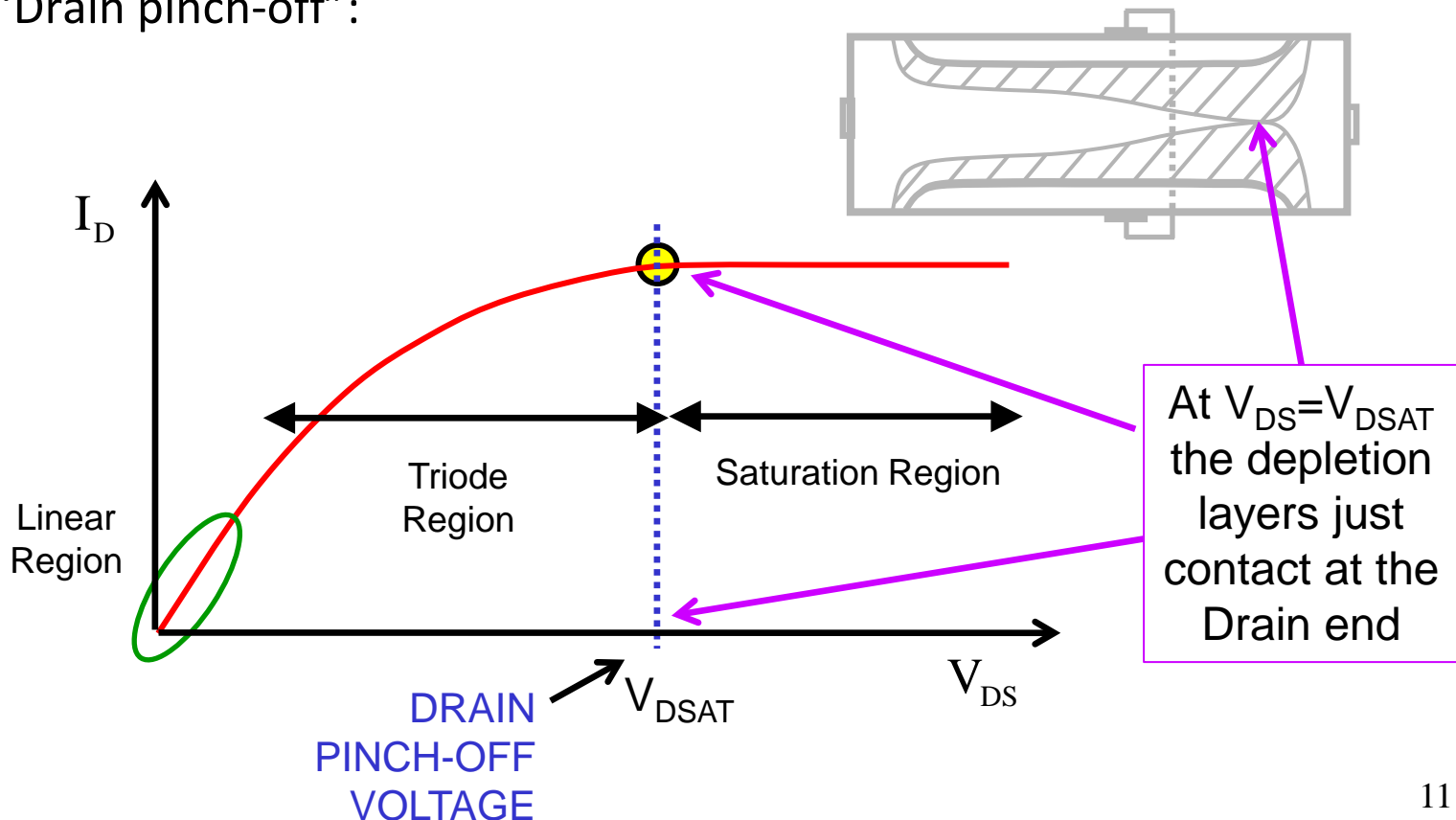
# Depletion Layers in N-channel JFET

- The channel voltage  $V(x)$  varies from  $V(x=0)=0$  (Source end) to  $V(x=L) = V_{DS}$  (Drain end). Therefore, the depletion layer width is **narrowest** at the **Source** end and **widest** at the **Drain** end;
- As  $V_{DS}$  is increased for a fixed (negative)  $V_{GS}$ , the *channel* tends to become narrower (or become “**pinched-off**”) towards the Drain end, causing the Drain current  $I_D$  to increase sub-linearly with  $V_{DS}$ :



# Drain Current Characteristic (fixed $V_{GS}$ )

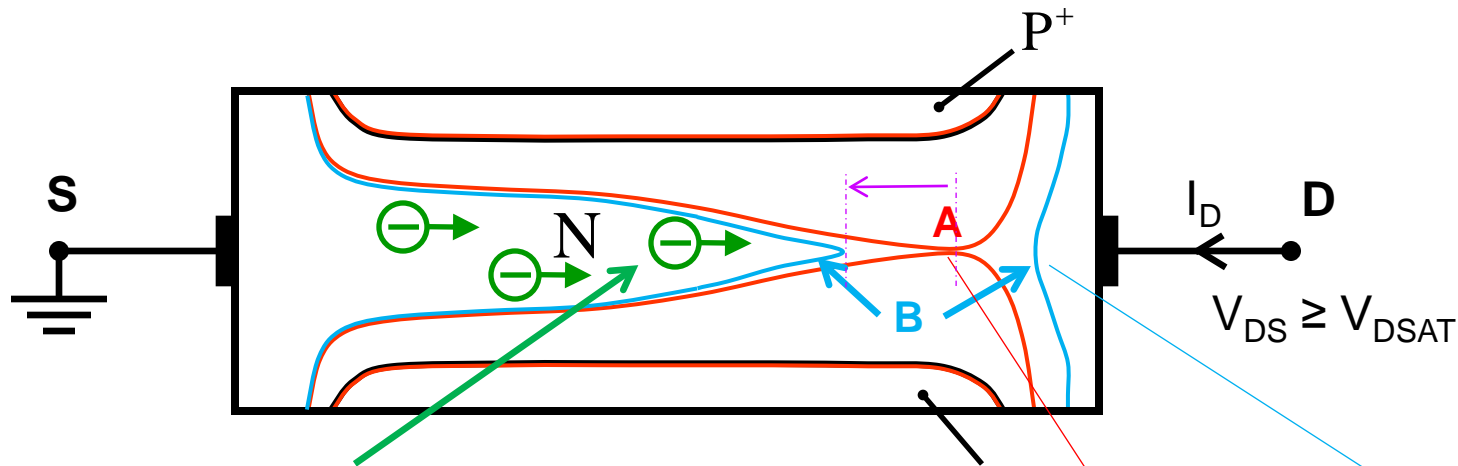
- For a fixed  $V_{GS} < 0$ , as  $V_{DS}$  is increased from zero, the drain current in the JFET ( $I_D$ ) varies with  $V_{DS}$  as follows, showing 3 general regions: **linear**, **triode** (or sub-linear) and **saturation**, the latter beginning at “Drain pinch-off”:



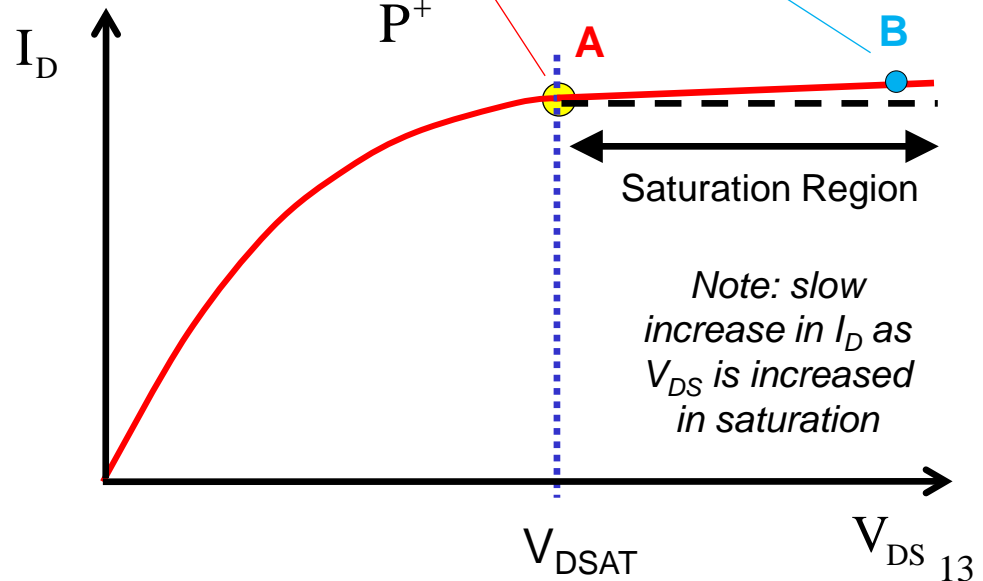
# Why Does Drain Current Not Switch to Zero for $V_{DS} \geq V_{DSAT}$ ?

- Behaviour in the saturation region is quite complex. Electrons are accelerated along conducting channel before the point at which the two depletion layers meet;
- These “hot” electrons are injected into the high electric field region of the (merged) depletion layers, where they drift at their *saturation* or *scattering limited* velocity ( $v_{ns}$ );
- The current density then “saturates” to an approximate value:  $J_{sat} \approx q.n.v_{ns}$ , leading to current saturation
- As  $V_{DS}$  is increased beyond  $V_{DSAT}$ , the merged depletion layer simply widens at the Drain end, although the fact that the initial point of contact moves slightly towards the Source, causes a small increase of the current in saturation

# Behaviour Beyond Drain Pinch-Off

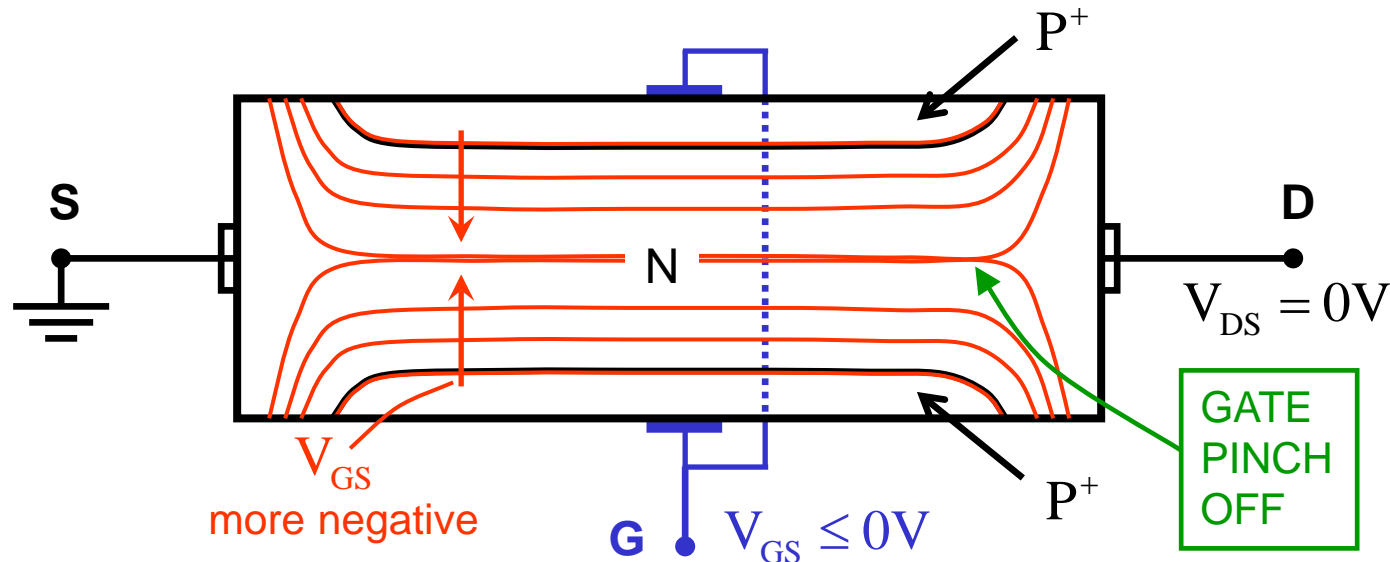


Hot electrons travelling at saturated velocity: can be injected across high field region at contact between two depletion layers.



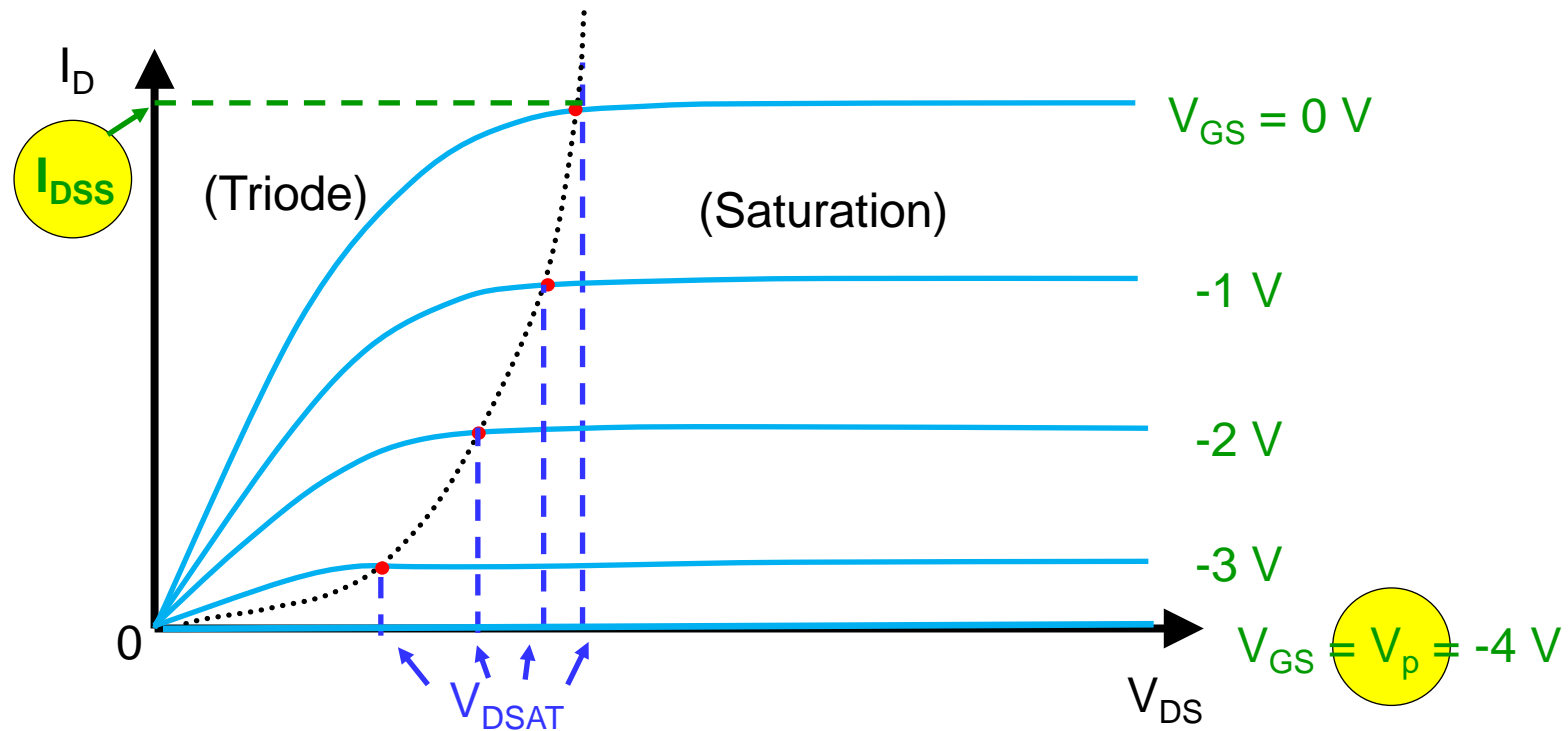
# Gate Pinch-Off and the Pinch-Off Voltage

- Now consider a different bias condition: apply **zero bias to the Drain** ( $V_{DS} = 0$ ) and systematically **make  $V_{GS}$  more negative** ( $V_{GS} \leq 0$ );
- Eventually, the depletion layers contact continuously along the channel: the device is then completely cut off;
- This condition is called **Gate pinch-off**, and the value of  $V_{GS}$  at which it occurs is called the **Pinch-Off Voltage  $V_{GS} = V_P$**



# JFET DC Drain Characteristics

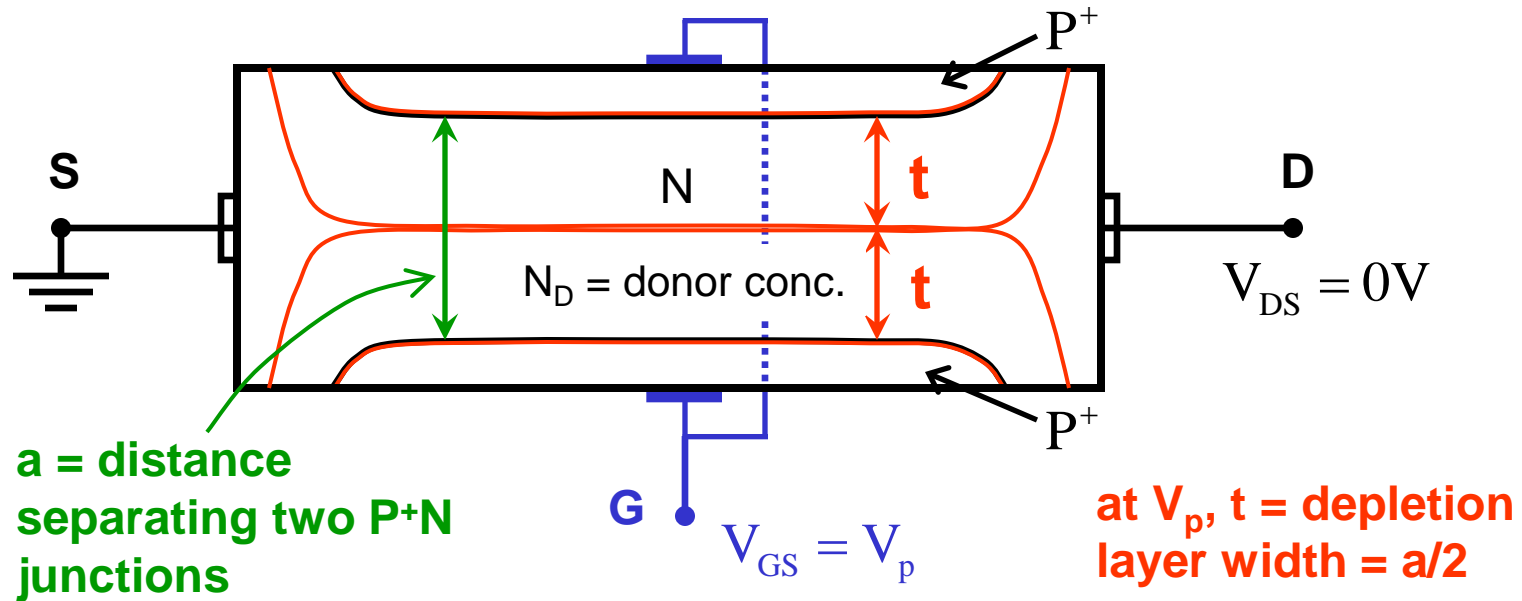
- Putting all this together, we find the following static Drain Characteristics for the N-Channel JFET (for illustration, a typical value is assumed for the Pinch-off Voltage:  $V_p = -4.0\text{V}$ )



**\*\*  $I_{DSS}$  and  $V_p$  are key *circuit-level* parameters for the JFET**

# Formula to Calculate Pinch-Off Voltage $V_p$

- Let the vertical displacement separating the two P+N junctions be “a”;
- It is clear that when  $V_{GS} = V_p$ , the depletion layer width “W” for each junction must be  $t = (a/2)$



$$t = (W) = \sqrt{\frac{2 \varepsilon \cdot (\phi_i - V_{GS} + V(x))}{q N_D}} \Rightarrow \frac{a}{2} = \sqrt{\frac{2 \varepsilon (\phi_i - V_p + 0)}{q N_D}}$$



# Expression for JFET Pinch-Off Voltage

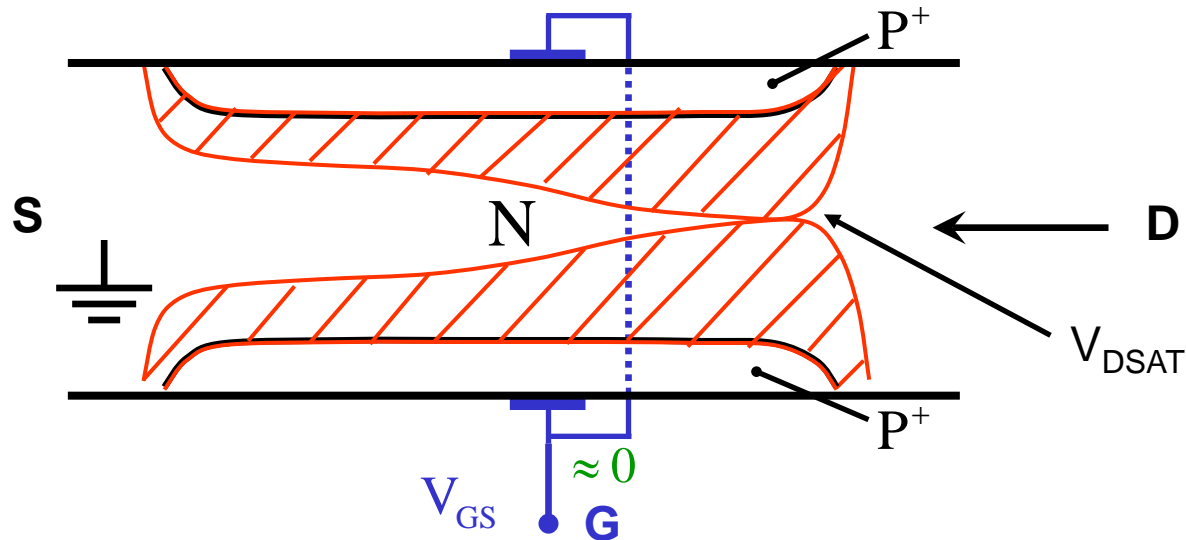
- Re-arranging the last equation, it is easy to derive the following expression for the Pinch-Off Voltage ( $V_P$ ) (prove this result as an exercise):

$$V_P = \varphi_i - \frac{q \cdot N_D \cdot a^2}{8 \cdot \epsilon}$$

*do not  
remember*

- For an **N-channel JFET**, the value calculated will always turn out to be **negative**, and will typically be a few V in magnitude.

## Relationship Between $V_{DSAT}$ and $V_P$



- Suppose  $V_{DS} = V_{DSAT}$  at some  $V_{GS}$ . Then at the point of contact between the 2 depletion layers:

$$t = \frac{a}{2} = \sqrt{\frac{2\epsilon(\phi_i - V_{GS} + V_{DSAT})}{qN_D}}$$

- Comparing with the earlier expression for  $V_P$ , we find a simple relationship connecting  $V_{DSAT}$  and  $V_P$  at a given  $V_{GS}$ :  $(V_{GS} - V_{DSAT}) = V_P$ , or

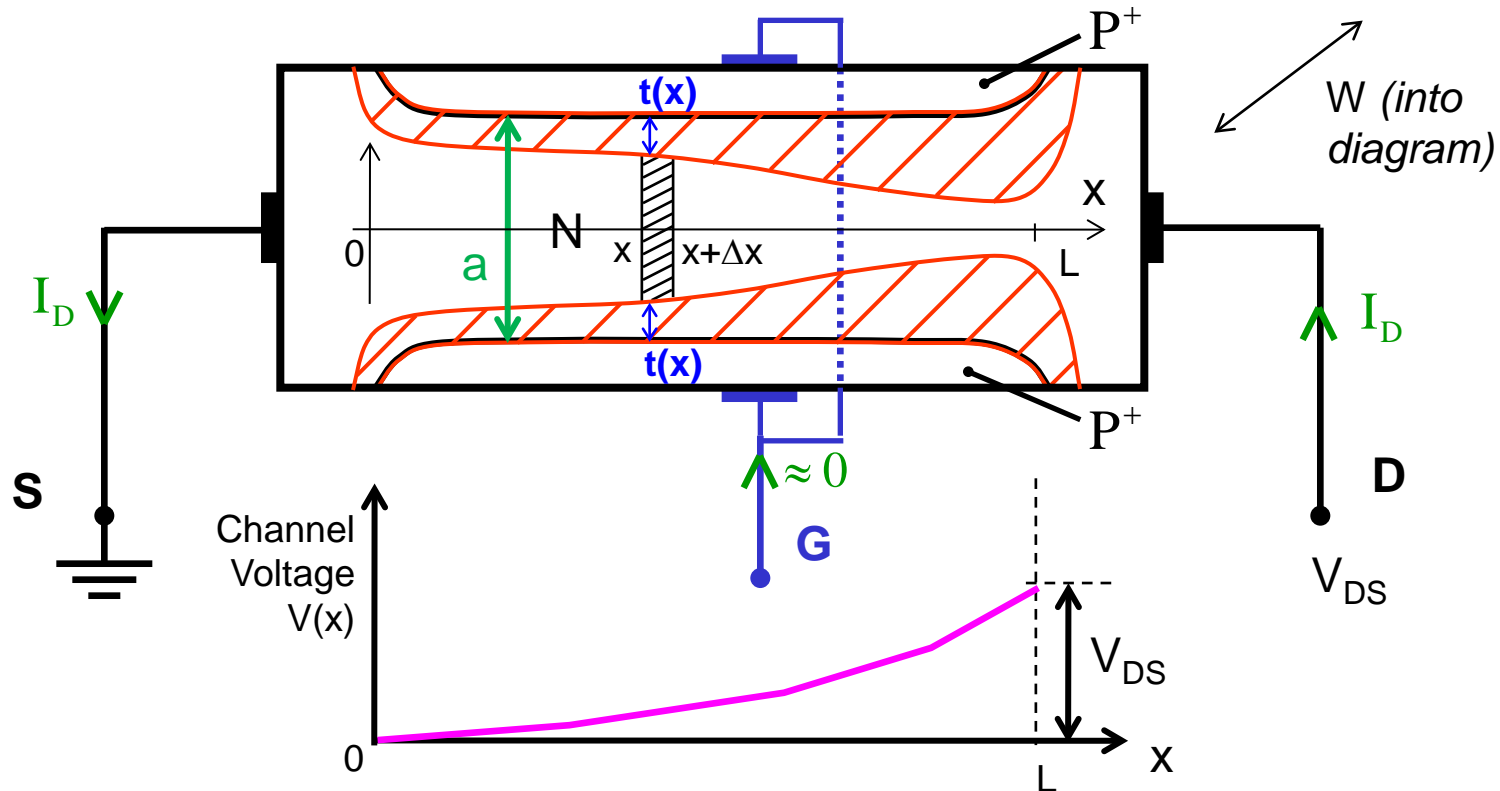
$$V_{DSAT} = (V_{GS} - V_P)$$

$$[\text{e.g. } V_P = -4V; V_{GS} = -1V \Rightarrow V_{DSAT} = +3V]$$

# Example 10.1

# Analysis for JFET DC Current in Triode Region

- We restrict the bias voltages such that:  $V_p \leq V_{GS} \leq 0$  and  $0 \leq V_{DS} \leq V_{DSAT}$ . This means the device is operated in the triode region



# Assumptions

- Assume current flow in channel is essentially 1-D (valid if  $L \gg a$ ). This is called the “Gradual Channel Approximation” (originally made by Schockley);
- Neglect diffusion and assume constant mobility in channel;
- Note that  $I_D$  is independent of ‘x’. Thus we can then apply Ohm’s Law to the shaded elemental section of the channel between “x” and “(x+Δx)”:

$$\Delta V(x) = I_D \cdot \Delta R(x)$$

- Where  $\Delta R(x)$  is the resistance of the elemental section.  
Use:

$$R = \frac{\rho \cdot l}{A} = \frac{l}{\sigma \cdot A} = \frac{l}{(q \cdot N_D \cdot \mu_n) \cdot A}$$

## Analysis in Triode Region (Study Optional)

- Substitute for length:  $l = \Delta x$  and area:  $A = (a - 2t(x)) \cdot W$  to obtain:

$$\Delta R(x) = \frac{\Delta x}{qN_D \mu_n (a - 2t(x)) \cdot W}$$

$$\Rightarrow \Delta V(x) = \frac{I_D \cdot \Delta x}{qN_D \mu_n (a - 2t(x)) \cdot W}$$

- Now divide across by  $\Delta x$  and take limit as  $\Delta x \rightarrow 0$ :

$$I_D = qN_D \mu_n (a - 2t(x)) \cdot W \cdot \frac{dV}{dx}$$

...where:

$$t(x) = \sqrt{\frac{2\varepsilon(\phi_i - V_{GS} + V(x))}{qN_D}}$$

# Analysis in Triode Region (Study Optional)

- Substitute for  $t(x)$ :

$$\Rightarrow I_D = qN_D\mu_n \left( a - 2\sqrt{\frac{2\varepsilon(\phi_i - V_{GS} + V(x))}{qN_D}} \right) \cdot W \frac{dV(x)}{dx}$$

- Now integrate both sides wrt “x” between the limits of 0 and L and divide by L:

$$\Rightarrow \frac{1}{L} \cdot \int_0^L I_D \cdot dx = I_D = \frac{qN_D\mu_n}{L} \int_0^L \left( a - 2\sqrt{\frac{2\varepsilon(\phi_i - V_{GS} + V(x))}{qN_D}} \right) \cdot W \cdot \frac{dV(x)}{dx} \cdot dx$$

- Change variable of integration on RHS from “x” to “V”:

$$\Rightarrow I_D = \frac{qN_D\mu_n W}{L} \cdot \int_{V(0)}^{V(L)} \left( a - 2\sqrt{\frac{2\varepsilon \cdot (\phi_i - V_{GS} + V)}{qN_D}} \right) \cdot dV$$

- But  $V(0)=0$  and  $V(L)=V_{DS}$ :

$$\Rightarrow I_D = \frac{qN_D\mu_n W}{L} \cdot \int_0^{V_{DS}} \left( a - \sqrt{\frac{8\varepsilon}{qN_D}} \cdot \sqrt{(\phi_i - V_{GS} + V)} \right) \cdot dV$$

# Final Result for Static Current

- This can be integrated (optional exercise) to give the final result:

*do not remember*

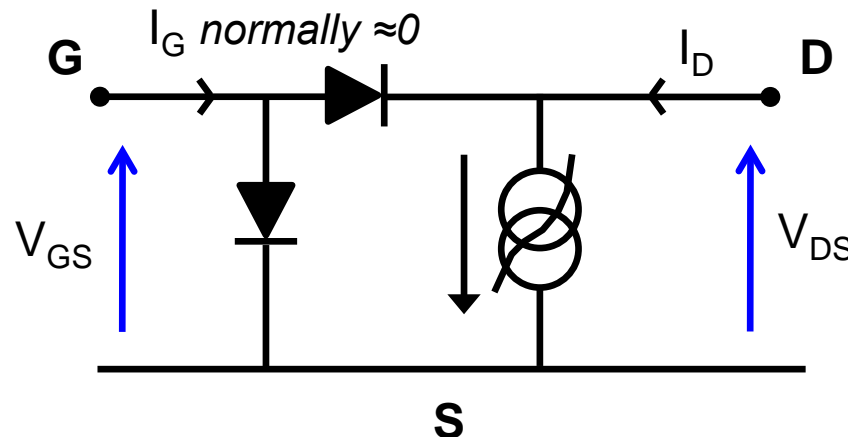
$$I_D = \frac{qN_D\mu_n Wa}{L} \cdot \left[ V_{DS} - \frac{2}{3} \sqrt{\frac{8\epsilon}{qN_D a^2}} \cdot \left\{ (\phi_i + V_{DS} - V_{GS})^{3/2} - (\phi_i - V_{GS})^{3/2} \right\} \right]$$

- This equation can be used for  $V_{DS}$  up to  $V_{GS} - V_P (=V_{DSAT})$ . Beyond this point, the saturation current can be simply approximated as a constant equal to the value at  $V_{DS} = V_{DSAT}$ :

*do not remember*

$$I_D^{(sat)} = \frac{qN_D\mu_n Wa}{L} \cdot \left[ V_{DSAT} - \frac{2}{3} \sqrt{\frac{8\epsilon}{qN_D a^2}} \cdot \left\{ (\phi_i + V_{DSAT} - V_{GS})^{3/2} - (\phi_i - V_{GS})^{3/2} \right\} \right]$$

DC Equivalent  
Circuit Model  
for JFET





# Small-Signal JFET Analysis

- Generalise the DC result for channel-current to *time-varying signals*:

$$i_D(t) = f(v_{GS}(t), v_{DS}(t))$$

- Replace each signal with the sum of a DC part and a small-signal part:

$$\cancel{I_D} + \text{Im}\{I_d \cdot e^{j\omega t}\} = f(V_{GS} + \text{Im}\{V_{gs} \cdot e^{j\omega t}\}, V_{DS} + \text{Im}\{V_{ds} \cdot e^{j\omega t}\})$$

$$\cong f(\cancel{V_{GS}}, V_{DS}) + \text{Im}\left\{V_{gs} \cdot \left.\frac{\partial f}{\partial v_{GS}}\right|_{DC} \cdot e^{j\omega t}\right\} + \text{Im}\left\{V_{ds} \cdot \left.\frac{\partial f}{\partial v_{DS}}\right|_{DC} \cdot e^{j\omega t}\right\}$$

$$\Rightarrow I_d = g_m \cdot V_{gs} + g_{ds} \cdot V_{ds}$$

- Where:  $g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{DC}$  and:  $g_{ds} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{DC}$

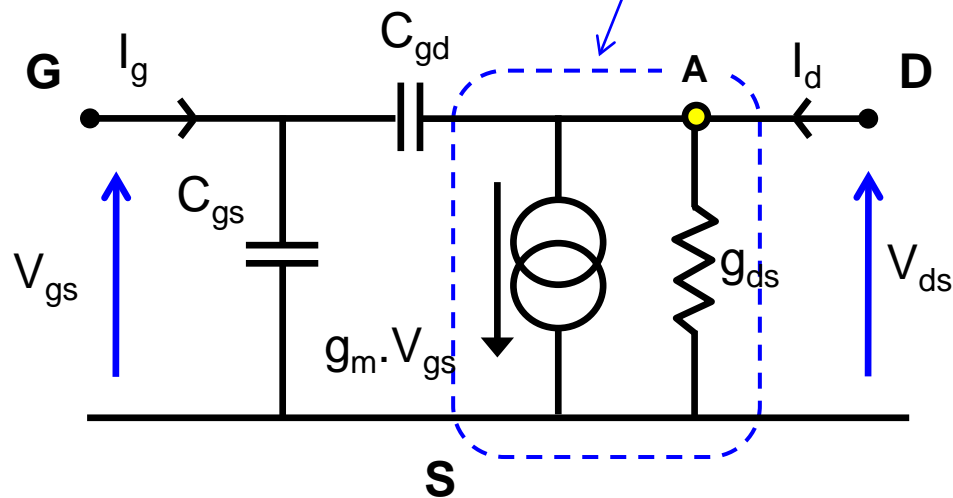
transconductance

drain-source conductance

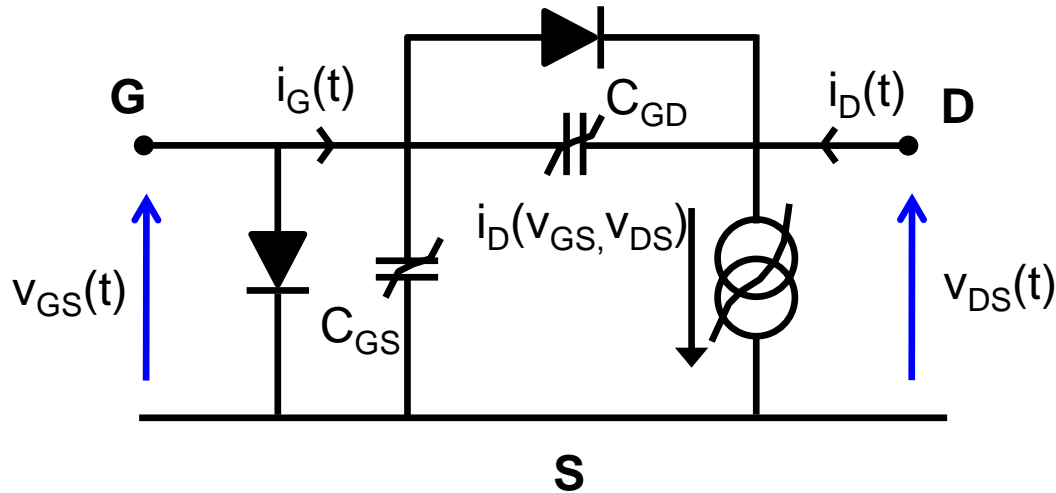
2-D First-Order  
Taylor Series

# Small-Signal JFET Equivalent Circuit

- The result from the previous slide:  $I_d = (g_m \cdot V_{gs} + g_{ds} \cdot V_{ds})$  can be seen as an instance of KCL at node “A” below, leading to a small-signal equivalent circuit for the drain-source channel as a **linear voltage-controlled current source** in shunt with a **linear conductance**;
- Adding in capacitances to represent the depletion layer capacitances (and ignoring parasitic resistances);, we arrive at the following **small-signal equivalent circuit for the JFET**:



# Large-Signal JFET Equivalent Circuit



## Note:

- JFETs have inferior analogue properties to bipolar transistors and inferior digital properties to those of MOS transistors.
- Their very high impedance at DC/low-frequencies is sometimes useful in circuit design, and they also tend to have better linearity performance at radio frequencies (RF)