1. Consider a simplified model of a **vacuum diode** consisting of a **cathode** in the $x = 0$ plane and an **anode** in the $x = d$ plane, where the anode is held to a constant potential $V_a = 2$ V relative to the cathode. The region $0 < x < d$ between the cathode and the anode supports a charge density $\rho(x)$ accounting for the electrons in transit from the cathode (where they are emitted) to the anode. If the potential distribution in the region $0 < x < d$ is given by $V(x) = V_a(x/d)^{4/3}$, find the following:

   a) Electric field $\mathbf{E}$ at $x = d/4$,

   b) Volumetric charge density ρ at $x = d/2$,

   c) The surface charge density ρ_s on the anode.

2. Given that $V(x, y, z) = y^3 - xz$ and $\mathbf{E} = -\nabla V$, what is $\nabla \times \mathbf{E}$?

3. An important vector identity which is true for any vector field $\mathbf{A}(x, y, z)$ is

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A},$$

   where

$$\nabla^2 \mathbf{A} \equiv (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\mathbf{A}$$

   is *Laplacian* of $\mathbf{A}$ and $\nabla(\nabla \cdot \mathbf{A})$ is the gradient of the divergence of $\mathbf{A}$.

   Verify the identity for $\mathbf{A} = (z - y)\,\hat{y} + (z + y)\,\hat{z}$ by calculating each side of the identity and showing them to be the same.

4. Given that $\mathbf{E} = 2\,\hat{x} + 2y\,\hat{y} + z\,\hat{z}$ V/m, determine the electrostatic potential $V(3, 2, 1)$ if $V(0, 0, 0) = 0$.

5. Given the fields $\mathbf{E_1} = \hat{x}y + \hat{y}x$ V/m and $\mathbf{E_2} = \hat{x}y - \hat{y}x$ V/m, determine the circulation $\oint_C \mathbf{E} \cdot d\mathbf{l}$ for both $\mathbf{E_1}$ and $\mathbf{E_2}$ along a triangular path C traversing in order its vertices at $(x, y, z) = (-1, -1, 0)$, $(-1, 1, 0)$, and $(1, 1, 0)$ m.

   Hint: $d\mathbf{l} = (-\hat{x} - \hat{y})dx$ and $x = y$ on the slant edge of C.

6. Consider a static volumetric charge density $\rho(x, y, z) = 6\delta(z) + \rho_s\delta(z - 10)$ C/m^3 in a given region of free space (having permittivity ϵ_0), where the displacement field is $\mathbf{D} = \hat{x}\,2\epsilon_0 + \hat{z}\,4\epsilon_0$ C/m^2 for $0 < z < 10$ m and $D_z = 2\epsilon_0$ C/m^2 for $z > 10$ m. Furthermore, field $\mathbf{D}$ is uniform in each of regions $z < 0$, $0 < z < 10$ m, and $z > 10$ m.

   a) Determine ρ_s,

   b) Determine $\mathbf{D}$ for the region $z > 10$ m,

   c) Determine $\mathbf{D}$ for the region $z < 0$.

   d) Determine $\mathbf{E}$ in all three regions ($z < 0$, $0 < z < 10$, and $z > 10$)

   e) What is the voltage drop from the $z = 0$ m plane to the $z = 10$ m plane?

   f) Which of your answers above (parts a, b, c, d, or e) would change if the region from $0 < z < 10$ m were filled with a perfect dielectric having permittivity $\epsilon = 4\epsilon_0$ instead of a vacuum having ϵ_0? Explain.

7. The gap between a pair of parallel infinite copper plates extends from $z = 0$ to $z = W > 0$ and is initially occupied by vacuum (ϵ_o, μ_o). The plates carry equal and oppositely signed surface charge densities and as a consequence we have a constant electric field $\mathbf{E} = -3\hat{z}$ V/m in vacuum in the gap region and zero electric field elsewhere.

a) What are the corresponding displacement vector **D** and polarization vector **P** in the gap region?

b) What is the surface charge density ρ_s of the copper plate at $z = 0$?

c) Next, we fill the gap with a non-conducting fluid of permittivity $\epsilon = 81\epsilon_o$ without changing the surface charge densities of the copper plates. What are the new values of **E**, **D**, and **P** in the gap region?

d) What would be the new equilibrium values of **E**, **D**, and **P** in the gap region if some amount of salt were dissolved in the fluid in the gap (see part c) to raise its conductivity to $\sigma = 4$ S/m (conductivity of sea water)? State the values of **E**, **D**, and **P** after a steady-state equilibrium is reached and briefly explain your answer.

8. Consider the following spherically symmetric configuration of composite materials in *steady-state equilibrium*:
(i) The region defined by $r \leq a$, where $r = \sqrt{x^2 + y^2 + z^2}$ is the radial distance from the origin, has $\epsilon = \epsilon_o$, $\sigma = 10^6$ S/m, and holds a net charge of $Q = 2$ C.
(ii) A perfect dielectric shell with $\epsilon = 10\epsilon_o$ occupies the region $a < r < b$.
(iii) Region $b \leq r \leq c$ has the same material properties as region $r \leq a$ and holds a net charge of -4 C.
(iv) Region $r > c$ is occupied by free space.

Determine, in all four regions, (a) **D**, (b) **E**, and (c) **P**, *and* (d) the surface charge densities, in C/m^2 units, at each of the three material boundaries at $r = a$, b, and c.

Hint: Make use of Gauss's law in integral form, $\oint_S \mathbf{D} \cdot dS = \int_V \rho dV$, with $\mathbf{D} = \epsilon \mathbf{E} = \epsilon_o \mathbf{E} + \mathbf{P}$, and a crucial fact about steady-state fields within conducting materials.