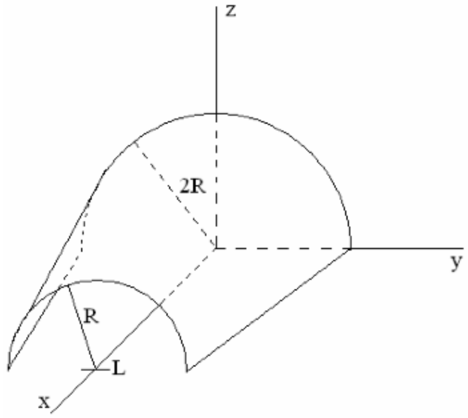
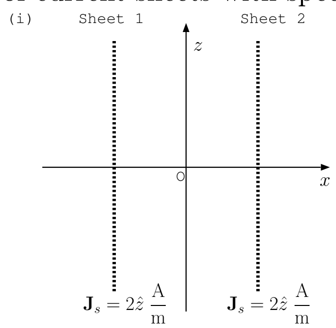
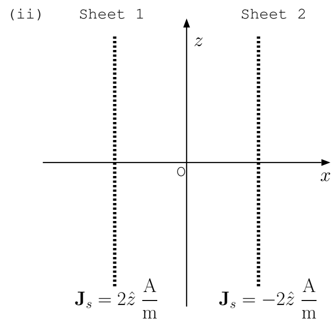
1. Given that $\mathbf{B} = 3\hat{x} + 2\hat{y} - \frac{4\pi R}{L}\hat{z}$, determine the magnetic flux $\int \mathbf{B} \cdot d\mathbf{S}$ through the partial cone surface shown in the figure below, where the $\mathbf{dS}$ vector points towards the bottom of the figure (i.e., it has a negative $\hat{z}$ component). **Hint**: Gauss's law for magnetic field $\mathbf{B}$ states that the surface integral $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$ over any closed surface S enclosing a volume V.



2. An infinite current sheet with a uniform current density $\mathbf{J}_s = J_s\hat{z}\ \frac{\mathrm{A}}{\mathrm{m}}$ produces magnetostatic fields $\mathbf{B}$ with $\frac{\mu_o J_s}{2}$ magnitude on both sides of the sheet and with opposing directions in consistency with the right-hand-rule and the Biot-Savart law.

   Determine the **magnetic field intensity** $\mathbf{H} = \frac{\mathbf{B}}{\mu_o}$ at origin O in the following diagrams due to a pair of current sheets with specified $\mathbf{J_s}$ vectors.





3. Consider an infinite slab (extending in y and z directions) of a finite width $W = 3$ m described by $-1 < x < 2$. The slab is electrically neutral but it conducts a uniform current density of $\mathbf{J} = 2\hat{y}$ A/m^2 (meaning that it contains equal densities of positive and negative charge carriers moving in opposite directions parallel to $\hat{y}$). Outside the slab, that is for $x > 2$ and $x < -1$, the charge and current densities are zero.

   a) Using the right-hand-rule and Biot-Savart law, discuss why the current slab should generate equal and opposite directed magnetic fields in $\pm\hat{z}$ directions in front of and behind the plane of symmetry of the slab.

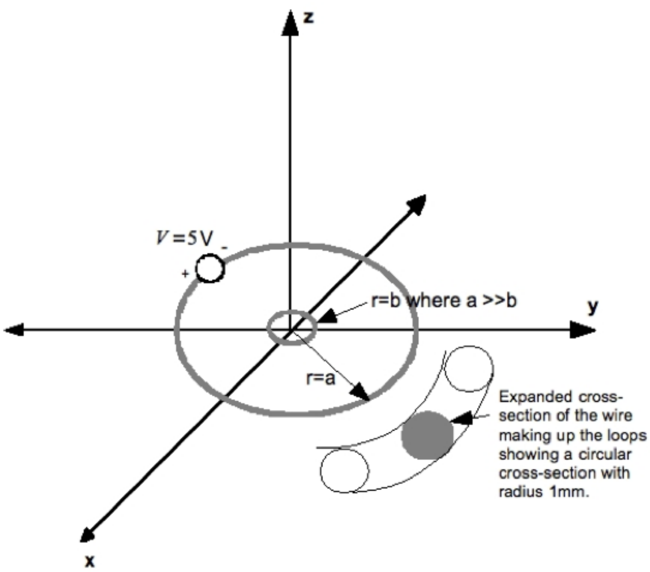
   b) Based on part (a), what is $\mathbf{B}$ on the $x = 0.5$ m plane? Briefly explain the reasoning behind your answer.

   c) Next, make use of the integral form of Ampere's law and the deductions of parts (a) and (b), to find $B_z(x)$ in the regions outside the slab. Hint: make use of a shifted coordinate system with its origin at the center of the slab.

d) Use Ampere's law to find $B_z(x)$ at a distance x within the current slab.

e) Plot B_z as a function of x over $-3 < x < 3$. Be sure to label all relevant values of B_z and x.

4. Consider two concentric circular wire loops of radii $a = 10$ cm and $b = 0.25$ cm placed on the x–y plane of the reference coordinate system with their centers at the origin. The medium is free space. The conductivity of the wire from which both loops are made is $\sigma = 4 \times 10^7$ S/m. The cross-section of the wire is circular with radius $r_w = 1$ mm and a 5 V battery is connected in the outer loop (see figure).



a) Calculate the current I_a that flows in the outer loop. **Hint:** resistance R of the outer loop can be calculated using the expression for R developed in Lecture 10 in terms of $d = 2\pi a$ and $A = \pi r_w^2$.

b) Derive an expression for the magnetic flux $\Psi_{a\to b}$ due to the current flowing in the outer loop that "links" the inner loop. Your expression should be in terms of the magnetic permeability of free space, the current I_a, and the radii of the two loops. **Hint:** Refer to the Lecture 13 notes for expressions for the magnetic flux density due to a current flowing in a circular wire loop. Also, take advantage of the fact that $b \ll a$ so that the magnetic field across the smaller loop can be considered nearly constant.

c) $L_{a\to b} \equiv \Psi_{a\to b}/I_a$ is *defined* to be the **mutual inductance** between the outer and inner loop. What is the numerical value of of the mutual inductance $L_{a\to b}$?