University of Illinois
ECE 310

Spring 2015
Profs. Liang & Ravishankar

Exam IV

7:30-8:30pm, Thursday, April 30, 2015

Problem	Pts.	Score
1	10	
2	10	
3	10	
4	5	
5	23	
6	12	
7	20	
8	10	
Total	100	

Name Key

Section: 9:00 AM 3:00 PM

Score ____________

Please do not turn this page over until told to do so.

You may not use any books, calculators, or notes other than one <u>handwritten</u> two-sided sheets of 8.5" x 11" paper.

GOOD LUCK!

(10 Pts.)

1. Mark "True" or "False" for the following statements.
You will receive 2 or -1 point for a correct or incorrect answer, respectively. Negative cummulative scores of this problem will be rounded to zero.

   (a) FIR filters always have generalized linear phase. **True/False**

   (b) The BL transform $s = \alpha \frac{1-z^{-1}}{1+z^{-1}}$ can map a high-pass analog filter to a low-pass digital filter. **True/False**

   (c) In windowing design of FIR filters, the rectangular window gives a shorter transition band than the Hamming window. **True/False**

   (d) In windowing design of FIR filters, the Hamming window is sometime preferred over the rectangular window because it gives lower ripples in both pass and stop bands. **True/False**

   (e) A down-sampler as defined in class is always a linear system. **True/False**

(10 Pts.)

2. The unit pulse response of an FIR filter is given by
$\{h[n]\}_{n=0}^{81} = (-1)^n \frac{1}{4} \text{sinc} \frac{\pi}{4}(n - \frac{81}{2})(0.54 - 0.46\cos(2\pi n/81))$ with all other elements being zero.

   (a) Determine if it is a high-pass or a low-pass filter (*hint: no need to calculate the DTFT of* $h[n]$.)?

Anti-symmetric, length-82

$\longrightarrow$ High-pass filter

4pt

   (b) Let $H_d(\omega) = R(\omega)e^{j(\alpha - M\omega)}$ where $R(\omega)$ is a real-valued function. Determine α and M.

Type-2 GLP

$\alpha = \frac{\pi}{2}$ . . . 3pt

$M = \frac{81}{2}$ . . . 3pt

(10 Pts.)

3. Marie Threeten used the Frequency Sampling Design Method to design a length-60 generalized linear phase HPF with desired magnitude response

$$|D_d(\omega)| = \begin{cases} \frac{3(|\omega|-\pi/3)}{2\pi}, & \frac{\pi}{3} \le |\omega| \le \pi; \\ 0, & |\omega| < \frac{\pi}{3}. \end{cases}$$

The resulting filter coefficients h_n are expressed as

$$h_n = \frac{1}{60}\sum_{m=0}^{59} H[m]e^{j2\pi mn/60}, \qquad n = 0, 1, \cdots, 59$$

Find an expression for $H[m]$ for $m = 0, 1, \cdots, 59$.

HPF, even length $\longrightarrow$ type-2 GLP

(6pt)

$$G_d(w) = \begin{cases} \frac{3(w-\pi/3)}{2\pi} e^{j(\pi/2 - \frac{59}{2}w)}, & \frac{\pi}{3} \le w \le 4\pi/3 \\ 0, & 0 \le w < \pi/3,\ \frac{4\pi}{3} < w < 2\pi \end{cases}$$

(4pt)

$$H[m] = G_d(w)\Big|_{w=\frac{2\pi}{60}m} = G_d(\frac{\pi}{30}m), \quad m=0,1,\cdots,59$$

(5 Pts.)

4. The bilinear transformation (BLT) is used to design a digital filter. Let

$$H_L(s) = \frac{1}{1+5s},$$

and $H(z) = H_L(s)|_{s=\alpha\frac{1-z^{-1}}{1+z^{-1}}}$. It is desirable that the 3dB cutoff of $H(z)$ is at $\frac{\pi}{2}$ (i.e., $|H_d(\frac{\pi}{2})|^2 = \frac{1}{2}|H_d(0)|^2$). Which of the following choices represents the appropriate value for α?

(a) $\alpha = 1$

(b) $\alpha = \frac{1}{5}$

(c) $\alpha = 5$

(d) $\alpha = \frac{1}{\sqrt{5}}$

(e) None of the above

$H_a(\Omega) = \frac{1}{1+5j\Omega}$

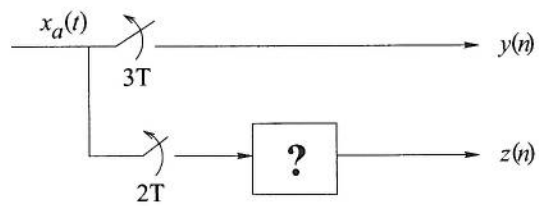
$|H_a(\Omega)|^2 = \frac{1}{1+25\Omega^2}$

$\Omega_c = \frac{1}{5}$

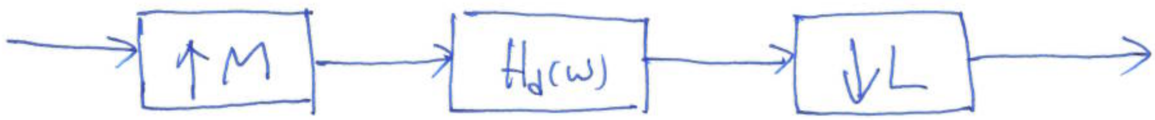
$\Omega_c = \alpha \tan\frac{w_c}{2} = \alpha\tan\frac{\pi}{4} = \alpha$

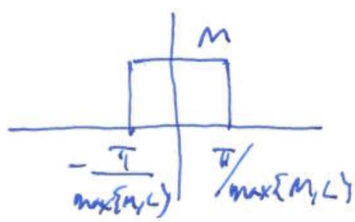
(23 Pts.)

5. Consider the following system consisting of two synchronized ideal A/D convertors. Assume that the input analog signal $x_a(t)$ is bandlimited to $\Omega_0 = \pi/(3T)$. Design a digital rate conversion subsystem marked with "?" using down-sampler(s), up-sampler(s), and digital filter(s) as necessary such that $y[n] = z[n]$. Draw a block diagram and determine all the essential parameters of the subsystem.



10pt





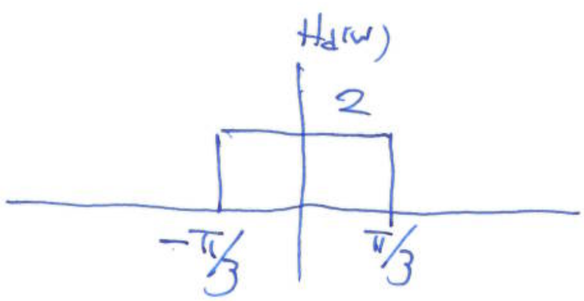
13pt

Note $T_1 = 3T$, $T_2 = 2T$

$$T_1 = \frac{3}{2} T_2$$

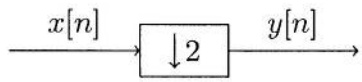
Therefore, $M = 2$

$L = 3$



(12 Pts.)

6. Consider the system shown below.



Let the DTFT of the input sequence $x[n]$ for one period be

$$X_d(\omega) = \begin{cases} 1 - \frac{2|\omega|}{\pi}, & |\omega| \le \pi/2; \\ 0, & \pi/2 < |\omega| \le \pi. \end{cases}$$

8 pt

(a) Find an expression for $Y_d(\omega)$ for $|\omega| < \pi$.

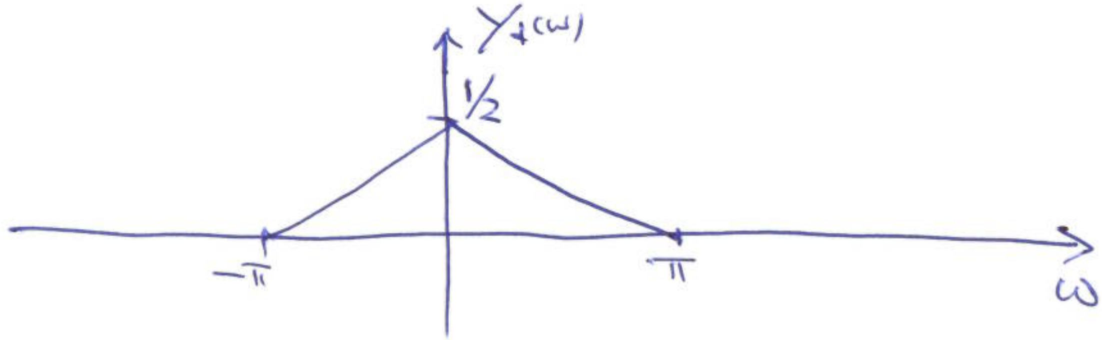
$$Y_d(\omega) = \frac{1}{2} \sum_{\ell=0}^{1} X_d\left(\frac{\omega - 2\ell\pi}{2}\right)$$

for $|\omega| < \pi$,

$$Y_d(\omega) = \frac{1}{2} X_d\left(\frac{\omega}{2}\right) = \frac{1}{2}\left(1 - |\omega|/\pi\right)$$

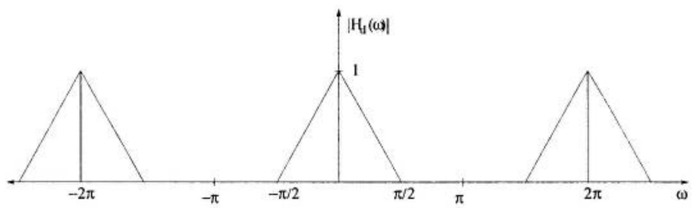
4 pt

(b) Sketch $Y_d(\omega)$ for $|\omega| < \pi$ (label your graph clearly.)



(20 Pts.)

7. The bilinear transformation was used to design a low-pass digital filter. Let the magnitude frequency response $|H_d(\omega)|$ of the resulting digital filter be as shown in the following figure. Determine and sketch the magnitude frequency response $|H_a(\Omega)|$ of the analog prototype filter used in the transform (determine all the pertinent parameters for $|H_a(\Omega)|$ and the bilinear transform.)



$$|H_d(\omega)| = \begin{cases} 1 - \frac{2|\omega|}{\pi}, & |\omega| \leq \pi/2 \\ 0 & \pi/2 < |\omega| \leq \pi \end{cases}$$

10pt

Note that $H_d(\omega) = H_a(\alpha \tan \frac{\omega}{2})$

$\rightarrow H_a(\Omega) = H_d(2 \tan^{-1} \frac{\Omega}{\alpha})$

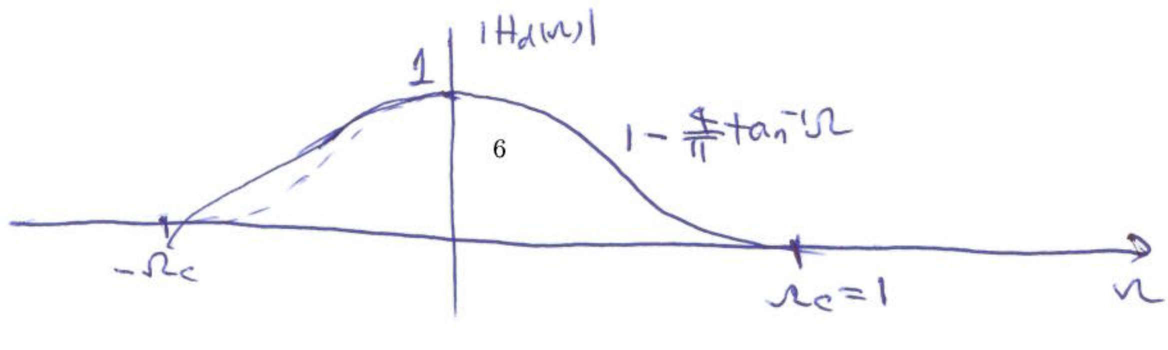
Therefore

$$|H_a(\Omega)| = |H_d(2 \tan^{-1} \frac{\Omega}{\alpha})|$$

Let $\alpha = 1$, $\Omega_c = \alpha \tan \frac{\omega_c}{2} = \tan \frac{\omega_c}{2}$

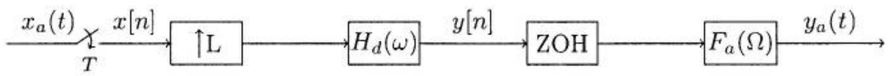
$\omega_c = \pi/2 \rightarrow \Omega_c = 1$

10pt

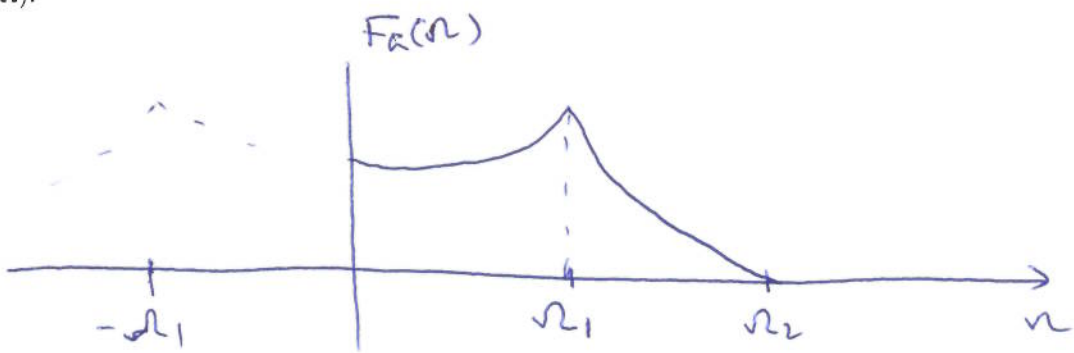


(10 Pts.)

8. Consider the following system, where $T = 0.01$ sec, and $H_d(\omega)$ is an ideal LPF with cutoff π/L.



$F_a(\Omega)$ is an analog compensation filter, picked such that the system from $y[n]$ to $y_a(t)$ functions as an ideal D/A, with any signal $x_a(t)$ bandlimited to 30π rad/sec. $F_a(\Omega)$ can have a transition band, in which its response can be arbitrary. Assuming $L = 3$, find the beginning and end of the transition band of $F_a(\Omega)$.



5 pt

$$\Omega_1 = 30\pi \text{ rad/sec} = \Omega_{max}$$

5 pt

$$\Omega_2 = \frac{2\pi}{\frac{T}{L}} - \Omega_{max} = \frac{L \cdot 2\pi}{T} - 30\pi$$

$$= \frac{6\pi}{10^{-2}} - 30 = 570\pi$$