1.

a) I have a slab of some insulator with an unknown permittivity ϵ. To determine ϵ experimentally I go to the lab and insert the slab in between the plates of a capacitor whose plate spacing exactly matches the width of the slab. I observe that the time constant of exponential decay of the capacitor voltage in an RC circuit that I construct increases by 50% when the slab is inserted to replace the air spacing. Determine ϵ in terms of ϵ_o. Explain your reasoning carefully.

b) Repeat (a) if the slab width is only three quarters of the plate separation so that when the slab is inserted between the plates we still have 25% air filling.

c) I have a rod of some solid with an unknown permeability μ. To determine μ experimentally I go to the lab and insert the rod within a solenoid of many turns having a diameter exactly matching the diameter of the rod. I observe that the time constant of exponential decay of the solenoid current in an RL circuit that I construct decreases by 0.05% when the rod is inserted to replace the air core of the solenoid. Determine μ in terms of μ_o. Is the rod diamagnetic or paramagnetic? Explain.

2.

a) For current density $\mathbf{J} = (2y^2z^2\,\hat{x} + 3z\,\hat{y} + 4z(x-x_o)^2\,\hat{z})$ A/m^2, which is time independent, find the charge density $\rho(0,t)$ at the origin $(0,0,0)$ as a function of time t, if $\rho = 0$ at that location and time $t = 0$, $x_o = 3$ m, and coordinates x, y, and z are specified in meter units. **Hint:** use the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{J} = 0.$$

b) In part (a), deduce the physical units of the coefficients 2, 3, and 4 used in J_x, J_y, and J_z specifications, respectively, by applying dimensional analysis.

3.

a) Show that in a homogeneous conductor where $\mathbf{J} = \sigma\mathbf{E}$, Gauss's law $\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_o}$ and the continuity equation $\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{J} = 0$ can be used together to derive a differential equation

$$\frac{\partial\rho}{\partial t} + \frac{\sigma}{\epsilon_o}\rho = 0$$

for the charge density ρ.

b) Find the solution of the differential equation above for $t > 0$ if at $t = 0$ the charge density is $\rho(x,y,z,0) = \sin(40z)$ C/m^3 over all space.

c) According to the solution found in part (b), how long would it take for ρ to reduce to $0.01\sin(40z)$ C/m^3? Assume that $\sigma = 10^7$ S/m.

d) Discuss the energetics of the process examined above. Specifically, state whether the energy per unit volume is zero or non zero at $t = 0$ and as $t \to \infty$ and state what happens to any energy stored in the quasistatic field at $t = 0$.

4.

a) If

$$\mathbf{E} = \sin(\omega t - \beta y)\,\hat{z}\,\frac{\mathrm{V}}{\mathrm{m}},$$

$\frac{\omega}{\beta} = c$, $\sigma = 0$, and $\mu = \mu_o$, find the corresponding $\mathbf{H}$ by using Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Hint: find a suitable time-varying anti-derivative for $\nabla \times \mathbf{E}$.

b) If

$$\mathbf{H} = \cos(\omega t + \beta x)\,\hat{z}\,\frac{\mathrm{A}}{\mathrm{m}},$$

$\sigma = 0$, $\frac{\omega}{\beta} = \frac{2}{3}c$ and $\epsilon = 2.25\epsilon_o$, find the corresponding $\mathbf{E}$ by using Ampere's law

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

in which $\mathbf{J} = \sigma\mathbf{E}$ and $\mathbf{D} = \epsilon\mathbf{E}$.

5. Verify that vector identity

$$\mathbf{H} \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H} = \nabla \cdot (\mathbf{E} \times \mathbf{H})$$

holds for $\mathbf{E} = 4\hat{y}e^{-\alpha z}$ and $\mathbf{H} = 2\hat{x}e^{-\alpha z}$ by expanding both sides of the identity. Treat α as a real constant.

You should download the table of vector identities from ECE 329 web site and examine the list to familiarize yourself with the listed identities — they are widely employed in electromagnetics as well as in other branches of engineering such as fluid dynamics.