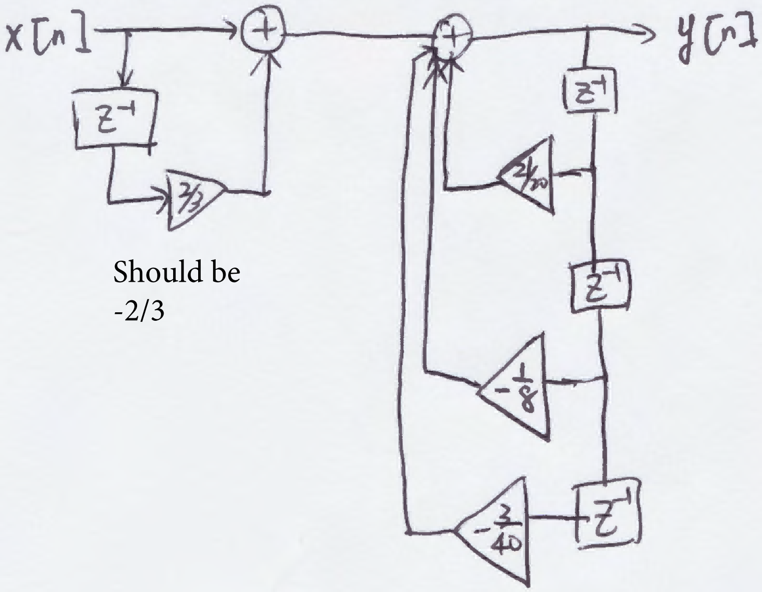
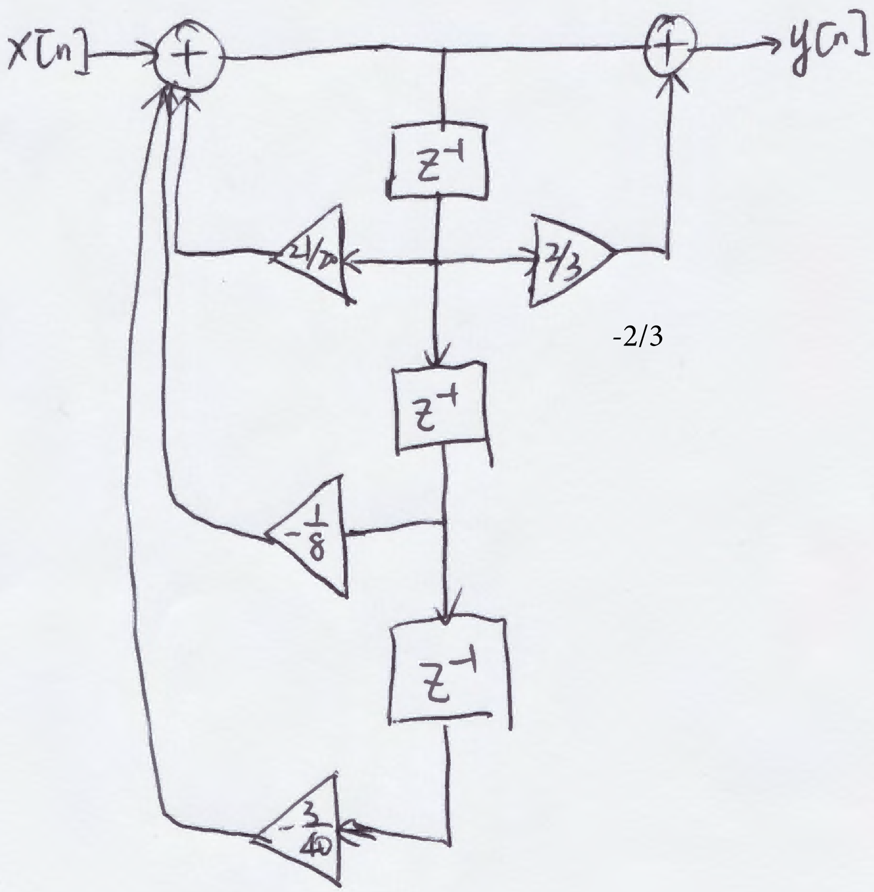
ECE 310 HW 9 Soln

1. a) $$\frac{z^3 - \frac{2}{3}z^2}{z^3 - \frac{21}{20}z^2 + \frac{1}{8}z + \frac{3}{40}} = \frac{1 - \frac{2}{3}z^{-1}}{1 - \frac{21}{20}z^{-1} + \frac{1}{8}z^{-2} + \frac{3}{40}z^{-3}}$$



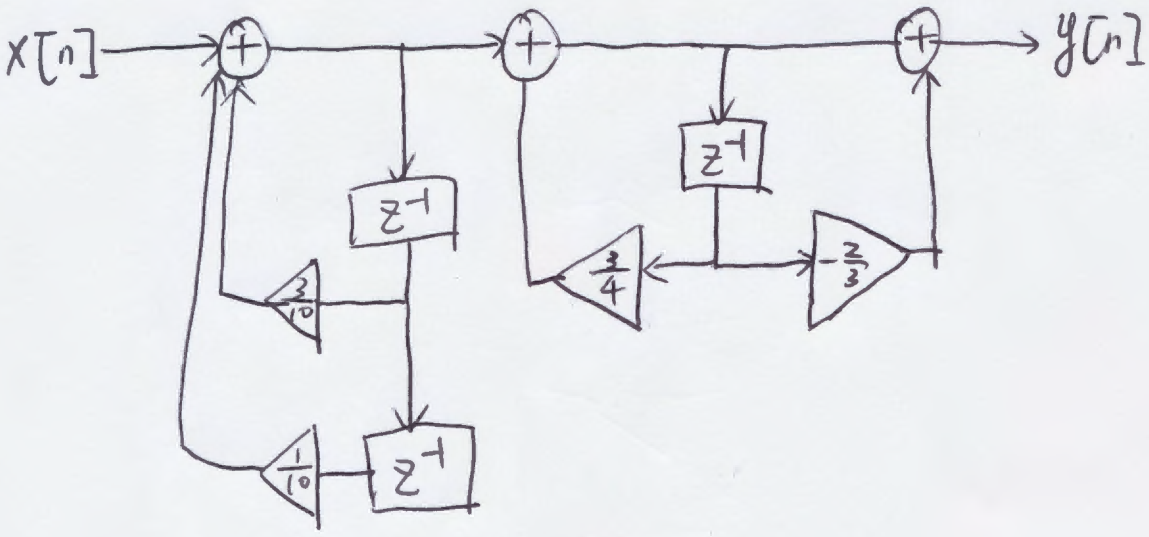
Should be
-2/3

b)

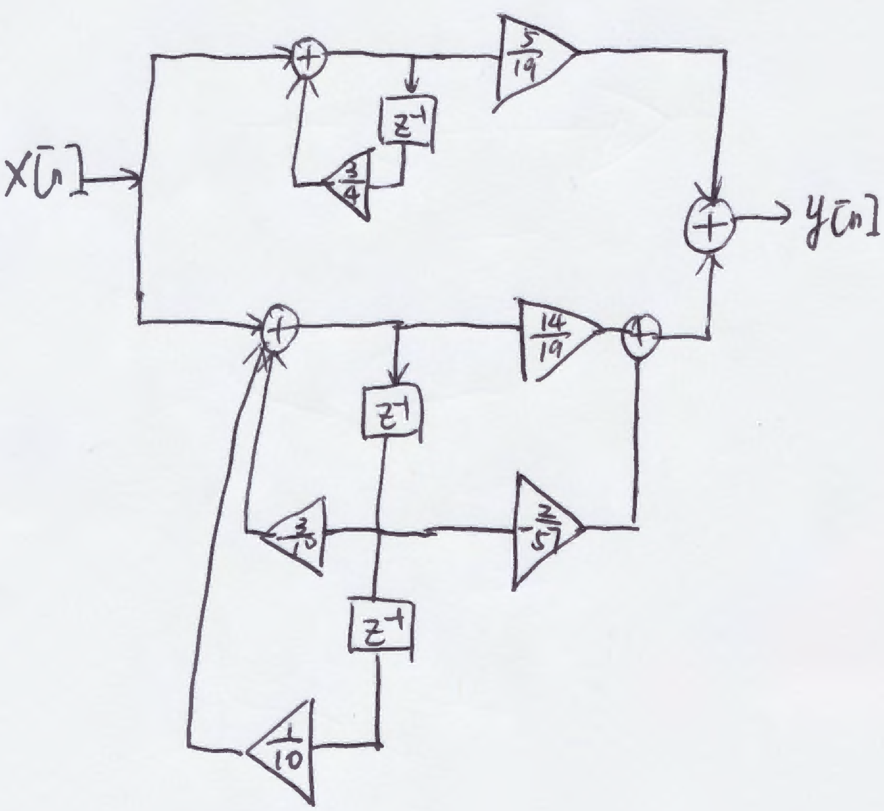


-2/3

c) $$H(z) = \left(\frac{1}{1-\frac{3}{10}z^{-1}-\frac{1}{10}z^{-2}}\right)\left(\frac{1-\frac{2}{3}z^{-1}}{1-\frac{3}{4}z^{-1}}\right)$$



d) $$H(z) = \frac{\frac{5}{19}}{1-\frac{3}{4}z^{-1}} + \frac{\frac{14}{19}-\frac{2}{57}z^{-1}}{1-\frac{3}{10}z^{-1}-\frac{1}{10}z^{-2}}$$



2. For any GLP filter, $H_d(\omega) = R(\omega)e^{i(\alpha - M\omega)}$, where $R(\omega)$ is real

1) $\{h_n\}_{n=0}^{2} = \{2, 1, 1\}$

since $h[n]$ has no symmetry, the filter is not GLP.

2) $\{h_n\}_{n=0}^{2} = \{1, 2, 3\}$

since $h[n]$ has no symmetry, the filter is not GLP.

3) $\{h_n\}_{n=0}^{2} = \{-1, 3, 1\}$

The unit-pulse response is asymmetric but the middle coefficient is nonzero, which prevents $H_d(\omega)$ from being expressed as $H_d(\omega) = R(\omega)e^{i(\alpha - M\omega)}$ where $R(\omega)$ is real. Therefore, the filter is not GLP

4) $\{h_n\}_{n=0}^{4} = \{1, 1, 1, -1, -1\}$

since $h[n]$ has no symmetry, the filter is not GLP

5) $\{h_n\}_{n=0}^{2} = \{1, 0, -1\}$

The given filter is asymmetric about its midpoint and the middle coefficient ($h[1] = 0$) is zero. Therefore, the filter is a type-3 GLP filter. Hence, $M = \frac{N-1}{2} = 1$. Following the same procedure in (a) to determine $R(\omega)$, which also will determine α

$$H_d(\omega) = 1 - e^{-i\omega \cdot 2} = e^{-i\omega}(e^{i\omega} - e^{-i\omega}) = e^{-i\omega}(2i\sin(\omega)) = e^{i(\frac{\pi}{2} - \omega)}(2\sin(\omega))$$

Therefore, $R(\omega) = 2\sin(\omega)$, $\alpha = \frac{\pi}{2}$, and M is verified. Taking a look at the phase of $H_d(\omega)$ to determine if the filter is Linear-phase:

$$\angle H_d(\omega) = \begin{cases} \frac{\pi}{2} - \omega, & 2\sin(\omega) > 0 \Rightarrow 0 < \omega < \pi \\ \frac{3\pi}{2} - \omega, & 2\sin(\omega) < 0 \Rightarrow -\pi < |\omega| < 0 \end{cases}$$

since $H_d(\omega)$ has a π jump at $\omega = 0$, the filter is not linear-phase.

b) $\{h_n\}_{n=0}^{3} = \{2,1,1,2\}$

The given filter is symmetric. Therefore, this filter is a type II GLP filter. Hence, $\alpha = 0$ and $M = \frac{N-1}{2} = \frac{3}{2}$. Following the same procedure in (a) to determine $R(\omega)$.

$$H_d(\omega) = 2 + e^{-j\omega} + e^{-j2\omega} + 2e^{-j3\omega}$$

$$= e^{-j\frac{3\omega}{2}}(2e^{j\frac{3\omega}{2}} + e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} + 2e^{-j\frac{3\omega}{2}})$$

$$= e^{-j\frac{3\omega}{2}}(4\cos(\tfrac{3\omega}{2}) + 2\cos(\tfrac{\omega}{2}))$$

$R(\omega) = 4\cos(\frac{3\omega}{2}) + 2\cos(\frac{\omega}{2})$

$\alpha = 0$ $M = \frac{3}{2}\pi$

$$\angle H_d(\omega) = \begin{cases} -\frac{3\omega}{2}, & R(\omega) > 0 \\ -\frac{3\omega}{2} + \pi, & R(\omega) < 0 \end{cases}$$

The filter is not linear phase since $R(\omega)$ changes sign at $\omega = \pm 0.41\pi$

3. i) $y[n] = \frac{2}{5}x[n] - x[n-1] + x[n-2] - \frac{2}{5}x[n-3]$

The unit pulse response is $h[n] = \frac{2}{5}\delta[n] - \delta[n-1] + \delta[n-2] - \frac{2}{5}\delta[n-3]$ This is FIR system

The unit pulse response $h[n]$ has even length and odd symmetry, thus it has Type II GLP.

Type IV

Further, $H_d(\omega) = \frac{2}{5} - e^{-j\omega} + e^{-j2\omega} - \frac{2}{5}e^{-j3\omega}$

$$= \frac{2}{5}e^{-j3\omega/2}(e^{j3\omega/2} - e^{-j3\omega/2}) - e^{-j3\omega/2}(e^{j\omega/2} - e^{-j\omega/2})$$

$$= \frac{2}{5}e^{-j3\omega/2}(2j\sin(3\omega/2)) - e^{-j3\omega/2}(2j\sin(\omega/2))$$

$$= e^{-j3\omega/2}(\tfrac{4}{5}j\sin(3\omega/2) - 2j\sin(\omega/2))$$

$$= e^{j(\frac{\pi}{2} - \frac{3\omega}{2})}(\tfrac{4}{5}\sin(3\omega/2) - 2\sin(\omega/2))$$

From the equation above, $R(\omega) = \frac{4}{5}\sin(\frac{3\omega}{2}) - 2\sin(\frac{\omega}{2})$, $\alpha = \frac{\pi}{2}$, $M = \frac{3}{2}$. Since $R(\omega)$ changes sign at $\omega = 0$, the filter does not have linear phase. (In general, filters with antisymmetric coefficients cannot have linear phase.)

ii) $y[n] = \frac{1}{3}x[n] + x[n-1] - x[n-2] - \frac{1}{3}x[n-3]$

a) $h[n] = \frac{1}{3}\delta[n] + \delta[n-1] - \delta[n-2] - \frac{1}{3}\delta[n-3] \Rightarrow$ FIR

b) $h[n]$ is even length

c) $h[n]$ has odd symmetry

d) $H_d(\omega) = \frac{1}{3} + e^{-j\omega} - e^{-j2\omega} - \frac{1}{3}e^{-j3\omega} = \frac{1}{3}e^{-j\frac{3\omega}{2}}(e^{j\frac{3\omega}{2}} - e^{-j\frac{3\omega}{2}}) + e^{-j\frac{3\omega}{2}}(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}})$

$= \frac{2j}{3}e^{-j\frac{3\omega}{2}}\sin(\frac{3\omega}{2}) + 2je^{-j\frac{3\omega}{2}}\sin(\frac{\omega}{2}) = (\frac{2}{3}\sin(\frac{3\omega}{2}) + 2\sin(\frac{\omega}{2}))e^{j(\frac{\pi}{2} - \frac{3\omega}{2})}$

since $(\frac{2}{3}\sin(\frac{3\omega}{2}) + 2\sin(\frac{\omega}{2}))$ changes sign in the range $-\pi \leq \omega < \pi$, $H_d(\omega)$ does not have linear phase. Also, since $h[n]$ has odd symmetry, It is Type II GLP.

e) $R(\omega) = \frac{2}{3}\sin(\frac{3\omega}{2}) + 2\sin(\frac{\omega}{2})$

$\alpha = \frac{\pi}{2}$

$M = \frac{3}{2}$

iii) $y[n] = x[n] + x[n-2] + x[n-4]$

The unit pulse response is: $h[n] = \delta[n] + \delta[n-2] + \delta[n-4] \Rightarrow$ FIR system

since $h[n]$ has odd length and even symmetry, it has Type I GLP.

$H_d(\omega) = 1 + e^{-j2\omega} + e^{-j4\omega} = e^{-j2\omega}(2\cos(2\omega)+1)$

$\therefore R(\omega) = 2\cos(2\omega) + 1,\ M = 2,\ \alpha = 0$

since $R(\omega)$ changes sign in the range $-\pi \leq \omega < \pi$, the filter does not have linear phase.

iv) $y[n] = -x[n] + x[n-1] + \frac{1}{3}x[n-2]$

a) $h[n] = -\delta[n] + \delta[n-1] + \frac{1}{3}\delta[n-2] \Rightarrow$ FIR

b) $h[n]$ is odd-length

c) $h[n]$ has no symmetry

d) since $h[n]$ is a FIR filter and has no symmetry, $H_d(\omega)$ does not have linear phase. The same condition implies that $H_d(\omega)$ does not have Type I or Type II GLP.

e) N/A

v) $y[n] = x[n] - 0.76\, y[n-1]$

a) $H(z) = \frac{1}{1+0.76 z^{-1}}$ $H(z)$ is not a polynomial in z or z^{-1}, hence the system is IIR

b) N/A

c) N/A

d) since $h[n]$ is an IIR filter, $H_d(\omega)$ does not have linear phase. The same condition implies that $H_d(\omega)$ does not have type I or type II GLP.

4. From the difference equation:

$$y[n] = h_0 x[n] + h_1 x[n-1] + h_2 x[n-2]$$

$$\Rightarrow Y(z) = h_0 X(z) + h_1 z^{-1} X(z) + h_2 z^{-2} X(z)$$

$$\therefore H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2}$$

$$H_d(\omega) = h_0 + h_1 e^{-j\omega} + h_2 e^{-j2\omega}$$

From the problem, the filter is given as linear-phase FIR. Therefore, the FIR filter must be type-I or type-II generalized linear phase. Also, the problem asks you design a bandstop filter ($H_d(\frac{5}{6}\pi)=0$) since an FIR filter with even symmetry (type-I GLP) and N odd is the only type of FIR filter that can fulfil the bandstop and linear-phase requirements, $h_0 = h_2$ and

$$H_d(\omega) = h_0 + h_1 e^{-j\omega} + h_0 e^{-j2\omega}$$

Plugging in the two conditions that $H_d(0) = 1$ and $H_d(\frac{5\pi}{6}) = 0$, a system of equation is obtained:

$$1 = 2h_0 + h_1$$

$$0 = \frac{3}{2} h_0 - \frac{\sqrt{3}}{2} h_1 + j\left(\frac{\sqrt{3}}{2} h_0 - \frac{1}{2} h_1\right)$$

This system of equations is overdetermined (more equations than unknowns) since both the imaginary and real parts of the second equation must be zero. Solving this system of equations, it is obtained that

$$h_0 = 2 - \sqrt{3}$$
$$h_1 = 2\sqrt{3} - 3$$
$$h_2 = h_0 = 2 - \sqrt{3}$$