1. A transmission-line has a load voltage of $V_L = j5\,\mathrm{V}$ and characteristic impedance of $150\,\Omega$ with the operating frequency as $300\,\mathrm{MHz}$. The wave velocity is $3 \times 10^8\,\mathrm{m/s}$.

   a) We are given the voltage at the load and the load resistance so the power dissipated in the load is easily calculated

$$P_L = \frac{1}{2}\mathrm{Re}\{V_L I_L^*\} = \frac{1}{2}Re\left\{V_L \cdot \frac{V_L^*}{Z_L^*}\right\} = \frac{1}{2}\cdot\frac{|V_L|^2}{R_L} = \frac{1}{2}\cdot\frac{5^2}{150} = \frac{1}{12}\,\mathrm{W}.$$

   b) Using our knowledge of $\lambda/2$ and $\lambda/4$ transformers. The length of the line is $\frac{3}{4}\lambda$, which can be divided into two sections of length $\frac{\lambda}{4}$ and $\frac{\lambda}{2}$. Lets take $\frac{\lambda}{2}$ length from the load end. The voltage at a distance $\frac{\lambda}{2}$ from the load will be

$$V(-\frac{\lambda}{2}) = -V_L = -j5\,\mathrm{V},$$

   and the current at the same location in the line will be

$$I(-\frac{\lambda}{2}) = -I_L = \frac{-j}{30}\,\mathrm{A}.$$

   Now, applyling $\frac{\lambda}{4}$ transformer concepts for the rest of the line, we will get

$$V_{in} = V(-\frac{3}{4}\lambda) = jI(-\frac{\lambda}{2})Z_o = 5\,\mathrm{V},$$

   and current at the input of the line will be

$$I_{in} = I(-\frac{3}{4}\lambda) = j\frac{V(-\frac{\lambda}{2})}{Z_o} = \frac{1}{30}\,\mathrm{A}.$$

   The current I_{in} is the same as the generator current I_g. The generator voltage can be found by

$$\frac{V_g - V_{in}}{Z_g} = I_g \quad \Rightarrow\ V_g = 6.25\,\mathrm{V}$$

   Then, we can find the power delivered by the generator

$$P_g = \frac{1}{2}Re\left\{V_g I_g^*\right\} = \frac{5}{48}\,\mathrm{W}.$$

2.

   a) Normalized load impedance $z_L = \frac{100}{25} = 4$

   Load reflection coefficient is $\Gamma_L = \frac{4-1}{4+1} = \frac{3}{5}\angle 0^o$. In SC: Γ_L is the vector from origin to the normalized impedance $z_L = 4$.

   Generalized reflection coefficient $\Gamma(d) = \Gamma_L e^{-j2\beta d}$ at $d = l = 0.15\lambda$ can be obtained in the SC starting from z_L and rotating clock-wise by 0.15λ. In degrees: $\theta = 0^o - \frac{4\pi}{\lambda}0.15\lambda = -\frac{3}{5}\pi\,\mathrm{rad} = -108^o$. Therefore, $\Gamma(l) = \frac{3}{5}\angle -108^o = \frac{3}{5}e^{-j\frac{3}{5}\pi}$.

   The normalized input impedance at the generator, can be directly read from the SC, and should be close to $z(l) = \frac{1+\Gamma(l)}{1-\Gamma(l)} = 0.37 - j0.664$.

   The input impedance at the generator is obtained by multiplying by $Z_o = 25\,\Omega$.

$$Z(l) = 25 \cdot z(l) = 9.25 - j16.6\,\Omega$$

   b) The obtained $Z(l)$ can be used to replace the effect of the TL in the circuit, and we calculate the Voltage phasor at that point using voltage division: $V(l) = V_g\frac{Z(l)}{Z_g+Z(l)} = 1.74 - j1.75\,\mathrm{V} = 2.468e^{-j0.251\pi}\,\mathrm{V} = 2.468\angle -45.16^o\,\mathrm{V}$

c) Using $V(l) = V^+ e^{j\beta d}\left(1 + \Gamma_L e^{-j2\beta d}\right) = V^+ e^{j\beta d}\left(1 + \Gamma(l)\right)$. Therefore,

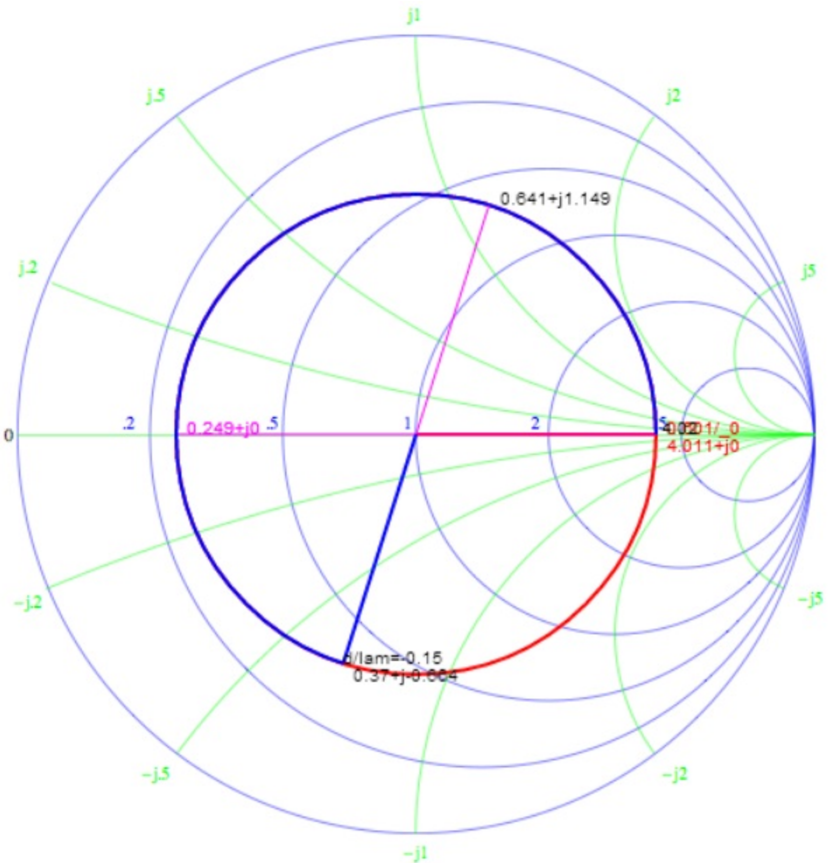
$$\begin{aligned} V(l) &= V^+ e^{j\beta d}\left(1 + \Gamma(l)\right) \\ V^+ &= \frac{V(l)}{1 + \Gamma(l)} e^{-j\beta d} = (1.554 + j1.988) e^{-j\frac{3}{10}\pi}\,\mathrm{V} = 1.08 - j2.22\,\mathrm{V}. \end{aligned}$$

d) Voltage phasor at the load

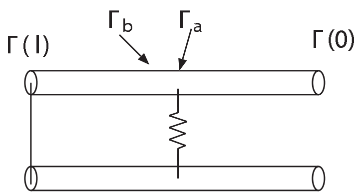
$$V(0) = V^+(1 + \Gamma_L) = V^+(1 + \frac{3}{5}) = 1.728 - j3.552\,\mathrm{V}$$

e) Current phasor at the load

$$I(0) = \frac{V(0)}{Z_L} = \frac{1.728 - j3.552}{100} = 0.0173 - j0.0355\,\mathrm{A}$$



3. Consider the following T.L. circuit.



Using the Smith chart and working in terms of impedances we find:

Impedance at the load $Z_L = \infty$

Normalized input impedance at the load $z_L = \frac{Z_L}{Z_o} = \infty$, now rotate from point z_L towards the generator (clockwise) by $d = 0.2\lambda$ until we reach A (at shunt connection)

Normalized input impedance A at shunt connection $z_a = -j0.327$

Normalized input admittance A at shunt connection $y_a = \frac{1}{z_a} = j3.055$

Normalized input admittance B at shunt connection $y_b = y_a + 1 = 1 + j3.055$

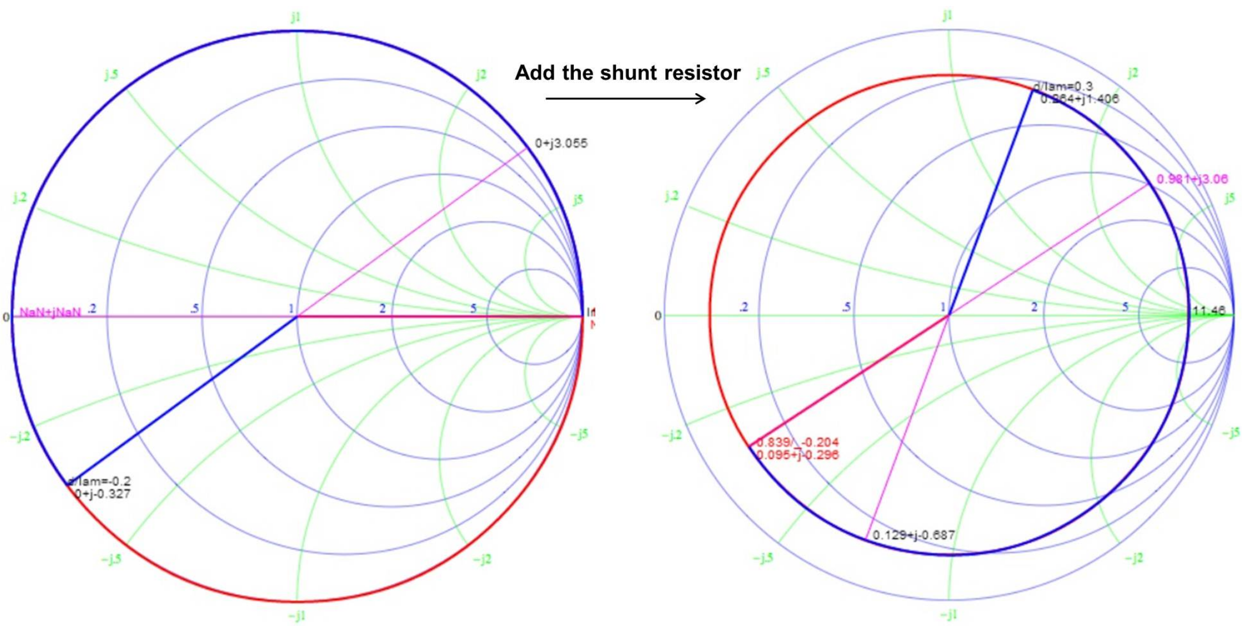
Normalized input impedance B at shunt connection $z_b = \frac{1}{1+y_a} = 0.097 - j0.296$, now rotate from point z_b towards the generator (clockwise) by $d = 0.2\lambda$ until we reach the generator

Normalized impedance at the generator $z(l) = 0.264 + j1.408$

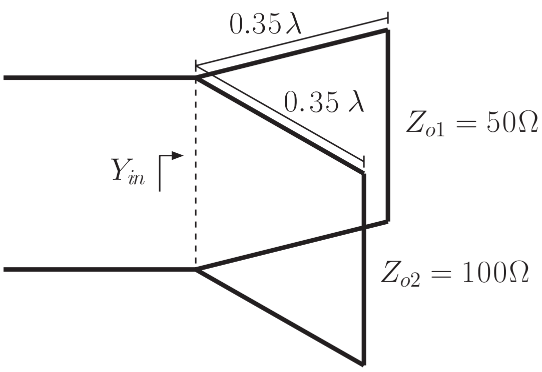
Impedance at the generator $Z(l) = 50 \cdot z(l) = 13.2 + j70.4\,\Omega$

Normalized admittance at the generator $y(l) = 1/z(l) = 0.129 - j0.687$

Admittance at the generator $Y(l) = \frac{1}{50} \cdot y(l) = 0.0026 - j0.014\,\Omega^{-1}$



4. Let consider two short-terminated T.L. stubs of length 0.35λ with characteristic impedances $Z_{o1} = 50\Omega$ and $Z_{o2} = 100\Omega$ connected in parallel as shown in the next diagram.



Using the Smith chart we find:

Normalized impedance at the load $z(0) = 0$

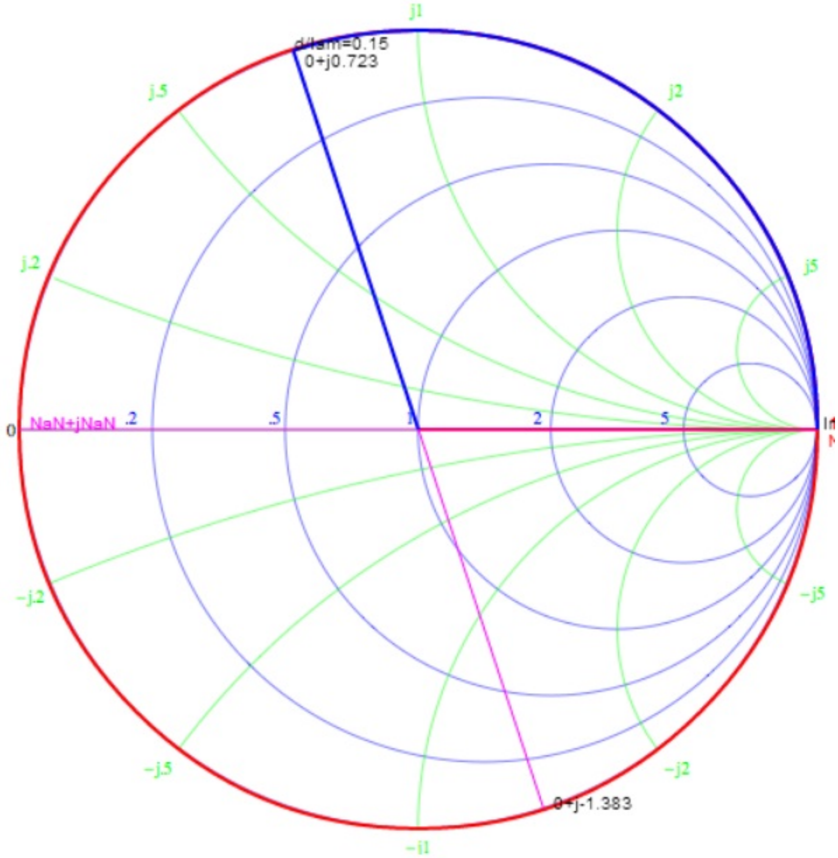
Normalized admittance at the load $y(0) = \infty$

Normalized admittance at $z = l$ (connection point) $y(l) = j0.723$

Admittance of the first stub $Y_1 = \frac{1}{Z_{o1}} y(l) = j0.0145\,\Omega^{-1}$

Admittance of the second stub $Y_2 = \frac{1}{Z_{o2}} y(l) = j0.00723\,\Omega^{-1}$

Admittance of the combined network $Y_{in} = Y_1 + Y_2 = j0.0217\,\Omega^{-1}$



5. We will solve as follows:

   a) The voltage standing wave ratio on the line is

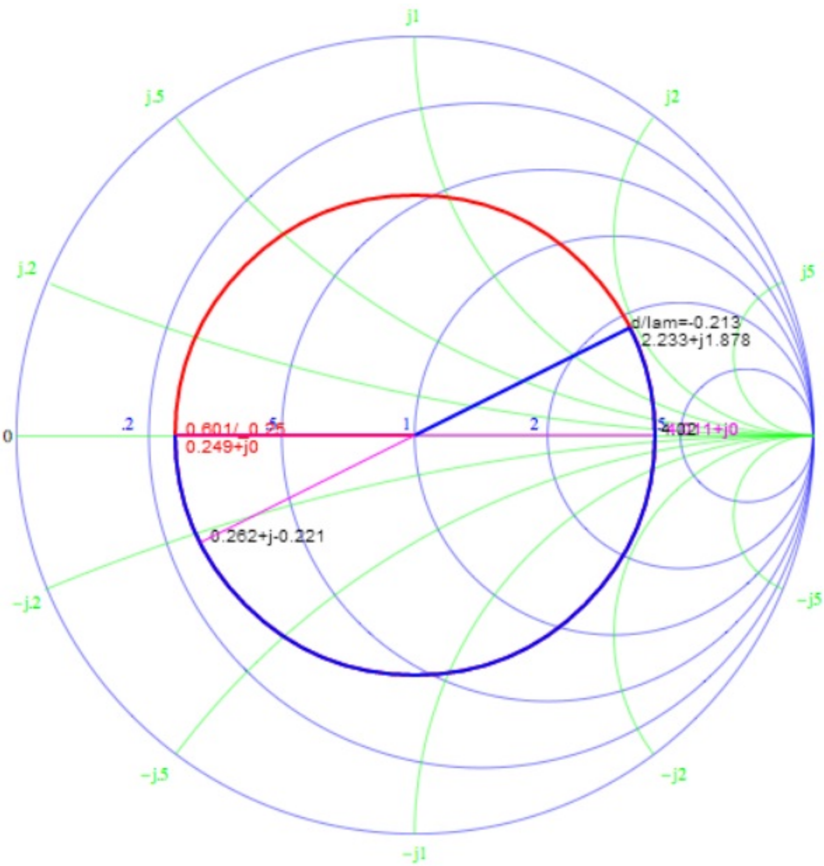
$$VSWR = \frac{|V|_{max}}{|V|_{min}} = \frac{7.2}{1.8} = 4.$$

   b) Adjacent voltage minimums are separated by a distance of 0.5λ, and $\frac{d}{0.5\lambda} = 4 + \frac{0.287\lambda}{0.5\lambda}$, therefore

$$d_{min} = 0.287\lambda.$$

   c) Draw the constant $|\Gamma|$ circle crossing the point $A: \; z =$ VSWR (corresponding to voltage maximum) on the real Γ axis. Then the other crossing point B (opposite to the point A) corresponds to the voltage minimum. By rotating from the point B to the load (counter-clockwisely) over a distance $d_{min} = 0.287\lambda$, we reach the point C which corresponds to the load. Then we can read from the Smith Chart $\Gamma_L = 0.6\angle\theta_L$, where $\theta_L = -180^\circ + \frac{0.287\lambda}{0.5\lambda} \times 360^\circ = 26.64^\circ$, i..e.

$$\Gamma_L = 0.6\angle 26.64^\circ.$$



d) At the point C we directly read $z_L = 2.233 + j1.878$ and Z_L can be determined as

$$Z_L = Z_o \cdot z_L = 112 + j94\,(\Omega).$$

e) From $|V^+| + |V^-| = |V|_{max} = 7.2\,(\mathrm{V})$ and $|V^+| - |V^-| = |V|_{min} = 1.8\,(\mathrm{V})$, we can obtain

$$|V^+| = 4.5\,\mathrm{V},$$

$$|V^-| = 2.7\,\mathrm{V}.$$

f) Instead of calculating the power at the load, we can calculate the power at some point of pure real impedance like: d_{max}. The normalize impedance at d_{max} is $z(d_{\mathrm{max}}) = \mathrm{VSWR} = 4$, then $Z(d_{\mathrm{max}}) = Z_o \cdot z(d_{\mathrm{max}}) = 200\,\Omega$. Therefore

$$P_{\mathrm{delivered}} = \frac{1}{2}\mathrm{Re}\left\{VI^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{|V|^2}{Z^*}\right\} = \frac{|V_{\mathrm{max}}|^2}{2Z(d_{\mathrm{max}})} = \frac{7.2^2}{2 \times 200} = 0.13\,(\mathrm{W}).$$

Since the line is lossless, $P_{\mathrm{delivered}}$ is exactly the same power delivered to the load.

6. In order to match a load Z_L to a T.L. with $Z_o = 25\,\Omega$, a quarter-wave transformer (Z_{qo}) is placed at a distance d_1 from Z_L.

a) To find d_1 using the Smith chart we can follow the next steps: (1) Locate the normalized impedance $z_L = Z_L/Z_o$ on the S.C.; (2) Draw a circle with radius equal to the distance from the center of the S.C. to z_L; (3) Find the point $z_L^{'}$ where the circle intersects the real axis (i.e. where the impedance is purely real); (4) Starting at z_L measure the angle you rotate (toward the generator) until reaching $z_L^{'}$; (5) This angle gives the distance d_1 in λ-units; (6) The impedance at this point is $Z(d_1) = Z_o \cdot z_L^{'}$.

i. If $Z_L = 100\,(\Omega)$, then $z_L = \frac{Z_L}{Z_o} = 4\,(\Omega)$. Since z_L is already on real axis, we do not have to find $z_L^{'}$. Thus, we have

$$d_1 = 0\lambda$$

and $Z(d_1) = Z_L = 100\,(\Omega)$. The characteristic impedance of the QWT is

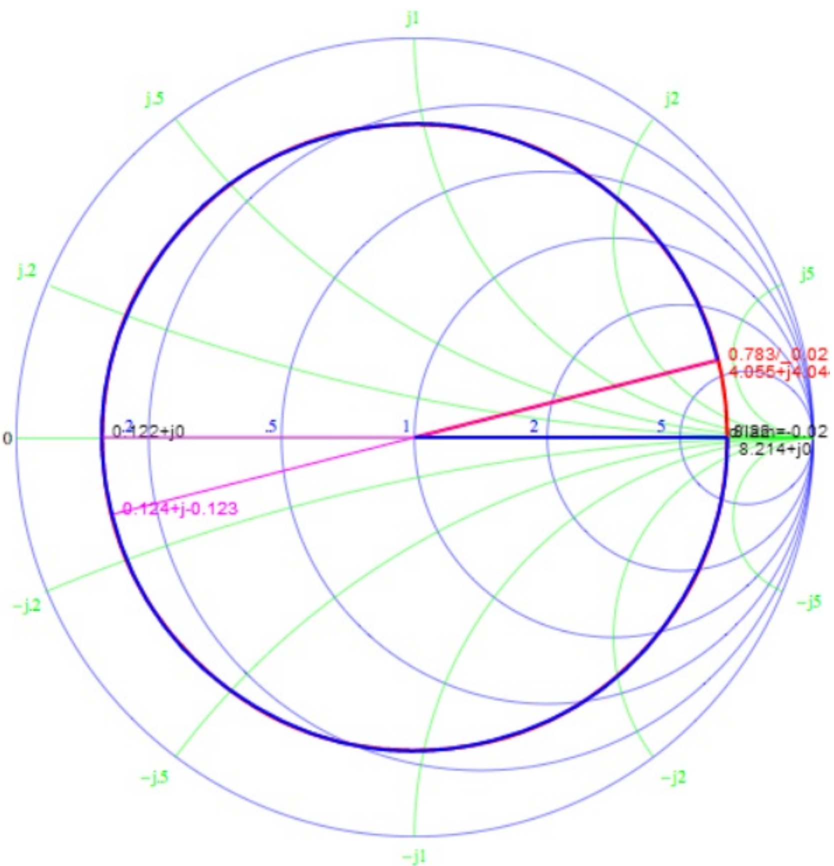
$$Z_{qo} = \sqrt{Z_o \cdot Z(d_1)} = \sqrt{25 \cdot 100} = 50\,(\Omega).$$

ii. If $Z_L = 100 + j100\,(\Omega)$, then $z_L = \frac{Z_L}{Z_o} = 4 + j4\,(\Omega)$. Now, we will draw a circle passing through z_L and this circle will intersects the real axis at $z_L^{'} = 8.214$. The angle rotated from z_L to $z_L^{'}$ gives us the distance as

$$d_1 = 0.02\lambda$$

and $Z(d_1) = Z_o \cdot z_L^{'} = 25 \cdot 8.214 = 205.35\,(\Omega)$. The characteristic impedance of the QWT is

$$Z_{qo} = \sqrt{Z_o \cdot Z(d_1)} = \sqrt{25 \cdot 205.35} = 71.65\,(\Omega).$$



b) For $Z_L = 100 + j100\,(\Omega)$, let us find VSWR in each region.

i. In the region $0 < d < d_1$, the reflection coefficient is $|\Gamma_L| = 0.78$. Therefore,

$$\text{VSWR}=\frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.783}{1-0.783} = 8.21.$$

ii. In the region $d_1 < d < d_1 + \frac{\lambda}{4}$, the reflection coefficient is $\Gamma(d_1) = \frac{Z(d_1)-Z_{qo}}{Z(d_1)+Z_{qo}} = \frac{205.35-71.65}{205.35+71.65} = 0.483$. Therefore,

$$\text{VSWR}=\frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.483}{1-0.483} = 2.868.$$

iii. In the region $d_1 + \frac{\lambda}{4} < d$, the reflection coefficient is 1 because of impedance match.

7. Single-stub tuner.

   a) Refering to Example 3 in course notes Lecture 37, d_1 needs to be selected such that $y(d_1) = 1+jb$, and then the shorted stub which has a purely imaginary impedance will provide $y_s = -jb$, so that $y(d_1) + y_s = 1 + j0$, which means that the impedance is matched at d_1.
   Now, $z_L = Z_L/Z_o = 0.5 + j0.5$ and $y_L = 1/z_L = 1 - j1$. We rotate from point y_L towards the generator (clockwise) until we reach the circle $g = 1$, and the intersection point reads $y(d_1) = 1 + j1$. The rotated distance is $d_1 = 0.323\lambda$, and the admittance of the stub is $y_s = 1 + j0 - y(d_1) = -j1$. To determine the length of the stub, we locate the short point $y = \infty$ and rotate towards the generator (clockwise) until we reach the point $y = -j1$. The rotation distance is the length of the stub: $d_s = 0.125\lambda$.
   (NOTE: Since the load admittance already has unit conductance, if you insert the stub right at the load, the required length of the stub to make the impedance matching will change from $0.125\lambda$to 0.375λ.)

   b) The procedure is the same as that in (a), except that now $Z_{os} = 50\,(\Omega) = 0.5Z_o$, or $Y_{os} = 2Y_o$. For this case, we still find that $y(d_1) = 1+j1$, or $Y(d_1) = \frac{1}{Z_o}(1+j1)$. Thus, the stub admittance should be $Y_s = \frac{1}{Z_o}(-j1) = \frac{1}{Z_{os}}(-j0.5) = -j0.5Y_{os}$, or $y_s = -j0.5$. By rotating from the short point to the y_s point, we obtain $d_s = 0.176\lambda$.