UCD School of Electrical, Electronic
& Communications Engineering

EEEN30110 Signals & Systems

Lab 1 Signals and Systems Report

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Declaration:
I declare that the work described in this report was done by the person named above, and that the description and comments in this report are my own work, except where otherwise acknowledged. I have read and understand the consequences of plagiarism as discussed in the EECE School Policy on Plagiarism, the UCD Plagiarism Policy and the UCD Briefing Document on Academic Integrity and Plagiarism. I also understand the definition of plagiarism.

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Objective:

To investigate linear, constant coefficient ODEs and analysis of LTI systems. For our laboratory we will use an example of the dynamics of an aircraft.

Question 1

The longitudinal dynamics of an aircraft in steady flight are approximately described by the following equations where :

$$\alpha = angle\ of\ attack$$
$$\theta = pitching\ angle$$
$$\delta_e = elevator\ setting$$
$$\dot{\alpha} = \alpha_{11}\alpha + \alpha_{12}\dot{\theta} + b_1\delta_e$$
$$\ddot{\theta} = \alpha_{21}\alpha - \alpha_{22}\dot{\theta} + \alpha_{23}\dot{\alpha} + b_2\delta_e$$

For our investigations we will take the following values for our variables:

$$\alpha_{11} = -0.313$$
$$\alpha_{12} = 1$$
$$\alpha_{21} = -0.79$$
$$\alpha_{22} = -0.426$$
$$\alpha_{23} = 0$$
$$b_1 = 0.232$$

And for my report:

$$b_2 = 1.130$$

Therefore we will have the following equations for our system.

$$\dot{\alpha} = -0.313\alpha + \dot{\theta} + 0.232\delta_e$$

$$\ddot{\theta} = -0.79\alpha - 0.426\dot{\theta} + 1.13\delta_e$$

As I require a transfer function I will assume all initial conditions are zero.

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To find the transfer function $\frac{\Theta}{\Delta_e}$ I first transform the equations from the time-domain to the s-domain using the Laplace transform. The equations become the following:

$$sA = -0.313A + s\Theta + 0.232\Delta_e$$

$$s^2\Theta = -0.79A - 0.426s\Theta + 1.13\Delta_e$$

Rearranging these equations in terms of the variable alpha and then equating them we find:

$$A = \frac{s\Theta + 0.232\Delta_e}{s - 0.313} = \frac{s^2 + 0.426s\Theta - 1.13\Delta_e}{-0.79}$$

As our transfer function is equal to:

$$H(s) = \frac{\Theta}{\Delta_e}$$

We can rearrange our equations to find H(s)

$$H(s) = \frac{\Theta}{\Delta_e} = \frac{1.13s + 0.17041}{s^3 + 0.739s^2 + 0.923338s}$$

Question 2:

We are now looking for a step function where the elevator setting is 0 for all t < 0 and becomes 0.1 once t = 0

As all initial conditions are assumed to be equal to zero, the transfer function remains the same as in question 1.

$$u(t) = \begin{cases} 0 \ldots t < 0 \\ 0.1 \ldots t \geq 0 \end{cases}$$

$$\delta_e = (0.1)(u(t)) \qquad at \qquad t \geq 0$$

Then applying the formula:

$$\Theta(s) = X(s)H(s)$$

We now multiply our function by the step δ_e, which is the input step of 0.1, in order to find the step function of Θ.

$$\Theta(s) = \frac{0.1}{s} \frac{1.13s + 0.17041}{s^3 + 0.739s^2 + 0.923338s}$$

We now intend to convert back from the s-domain to the t-domain. To do this utilise Matlab to get the partial fraction expansion:

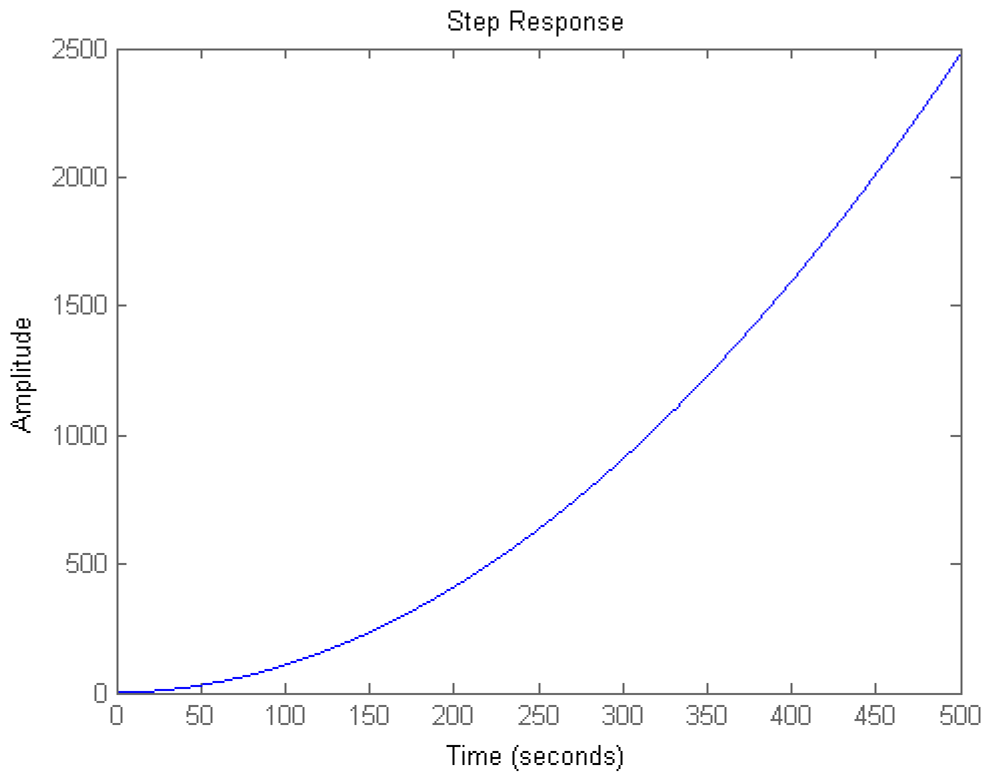
$$\Theta(s) = \frac{-0.05435 + 0.03304j}{s + 0.3695 - 0.8871j} + \frac{-0.5435 - 0.03304j}{s + 0.3695 + 0.8871j} + \frac{0.10869}{s} + \frac{0.01846}{s^2}$$

Then applying the Laplace transform to get the answer in the t-domain:

$$\theta(t) = 0.063604e^{j2.5953 - 0.3695t + j0.88702t} + 0.646e^{-j2.5953 - 0.3695t + j0.88702t} + 0.10869 + 0.01846t$$

However this answer is in the complex form and as the real answer is required we must convert it. Showing θ(t) in real form:

$$\theta(t) = 0.1272e^{-0.3695t}(\cos(0.88702t - 2.5953)) + 0.10869 + 0.01846t$$



This equation describes the systems response to the sudden change in the elevator setting, δ_e, from 0 (at t= 0). The equation is valid for all values t > 0. The pitching angle of the aircraft should be a real value so any answer with a complex component should be questioned.

Question 3:

The equation below is a very simple model of an actuator which controls the elevator setting on the aircraft. τ is called the time constant, a parameter of the actuator model, and u_e is the elevator control signal.

$$\tau\dot{\delta}_e + \delta_e = u_e$$

Using the Laplace transform to transform the equation from the time-domain to the s-domain we obtain:

$$\tau\Delta_e s + \Delta_e = U_e$$

Rearranging the equation I can get the equation in terms of U_e and Δ_e

$$\mathrm{U_e} = \Delta_\mathrm{e}(\tau\mathrm{s} + 1)$$

Therefore the transfer function of the system is:

$$\mathrm{H(s)} = \frac{\Delta_\mathrm{e}}{\mathrm{U_e}} = \frac{1}{\tau\mathrm{s} + 1} = \frac{1}{0.1\mathrm{s} + 1}$$

Question 4:

Taking the equations we found in questions 1 and 3 we can find our transfer function of our whole systems, as our transfer function is the output, pitching angle, over the input, elevator control signal. Multiplying the two we get $\frac{\Theta}{\mathrm{U_e}}$ which is the desired transfer function.

$$\mathrm{H(s)} = \frac{\Theta}{\mathrm{U_e}} = \frac{\Theta}{\Delta_\mathrm{e}}\frac{\Delta_\mathrm{e}}{\mathrm{U_e}}$$

$$\frac{\Theta}{\Delta_\mathrm{e}} = \frac{1.13\mathrm{s} + 0.17041}{\mathrm{s}^3 + 0.739\mathrm{s} + 0.9234\mathrm{s}}$$

$$\frac{\Delta_\mathrm{e}}{\mathrm{U_e}} = \frac{1}{0.1\mathrm{s} + 1}$$

$$\mathrm{H(s)} = \frac{1.13\mathrm{s} + 0.17041}{0.1\mathrm{s}^4 + 1.0739\mathrm{s}^3 + 0.8313338\mathrm{s}^2 + 0.9233383\mathrm{s}}$$

We now will find an equation which will describe the response of the system when the control is 0, at time is less than 0, and when it jumps to 0.1, once t is greater than or equal to 0. We then get the equation for the step function by multiplying it by the initial step of 0.1.

$$u(t) = \begin{cases} 0 \dots t < 0 \\ 0.1 \dots t \geq 0 \end{cases}$$

By applying the Laplace transform to this we have, $Ue = \frac{0.1}{s}$ and then we multiply both together to get the equation for our response in the s-domain.

$$\Theta(\mathrm{s}) = \frac{1.13\mathrm{s} + 0.17041}{0.1\mathrm{s}^4 + 1.0739\mathrm{s}^3 + 0.8314\mathrm{s}^2 + 0.9234\mathrm{s}}\frac{0.1}{\mathrm{s}}$$

$$\Theta(\mathrm{s}) = \frac{0.116\mathrm{s} + 0.0179}{0.1\mathrm{s}^5 + 1.0739\mathrm{s}^4 + 0.8314\mathrm{s}^3 + 0.9233383\mathrm{s}^2}$$

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We are now looking for an equation to describe the response of the system when the elevator control is 0 for all t < 0 and becomes 0.1 once t = 0 onwards.

Utilizing Matlab to obtain the partial fractions we get:

$$\Theta(s) = -\frac{0.00119}{s+10} + \frac{-0.05229 + 0.03889j}{s + 0.3695 - 0.88702j} + \frac{-0.05229 - 0.03889j}{s + 0.3695 + 0.88702j} + \frac{0.10577}{s} + \frac{0.01846}{s^2}$$

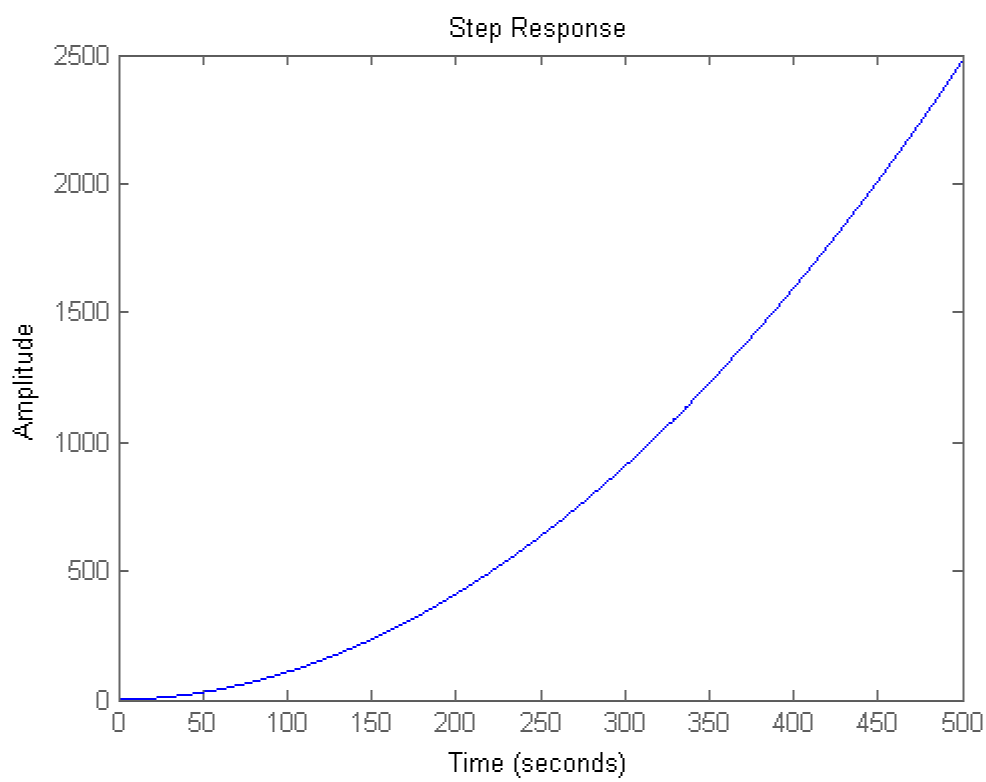
Then using the inverse Laplace transform to revert back to the time-domain:

$$\theta(t) = -0.00119e^{-10t} + 0.0652e^{j2.5021+j0.88702t-0.3695t} + 0.0652e^{j2.5021-j0.8870t-0.3695t} + 0.10577 + 0.01846t$$

Once again converting the equation to the real equivalent:

$$\theta(t) = 0.1304e^{-0.3695t}(\cos(0.8870t - 2.5021)) + 0.10577 + 0.01846t - 0.00119e^{10t}$$

Once t > 0 this equation will be valid. It describes the systems response to a sudden change in the elevator control from 0 (at t < 0) to 0.1 (at t >= 0).



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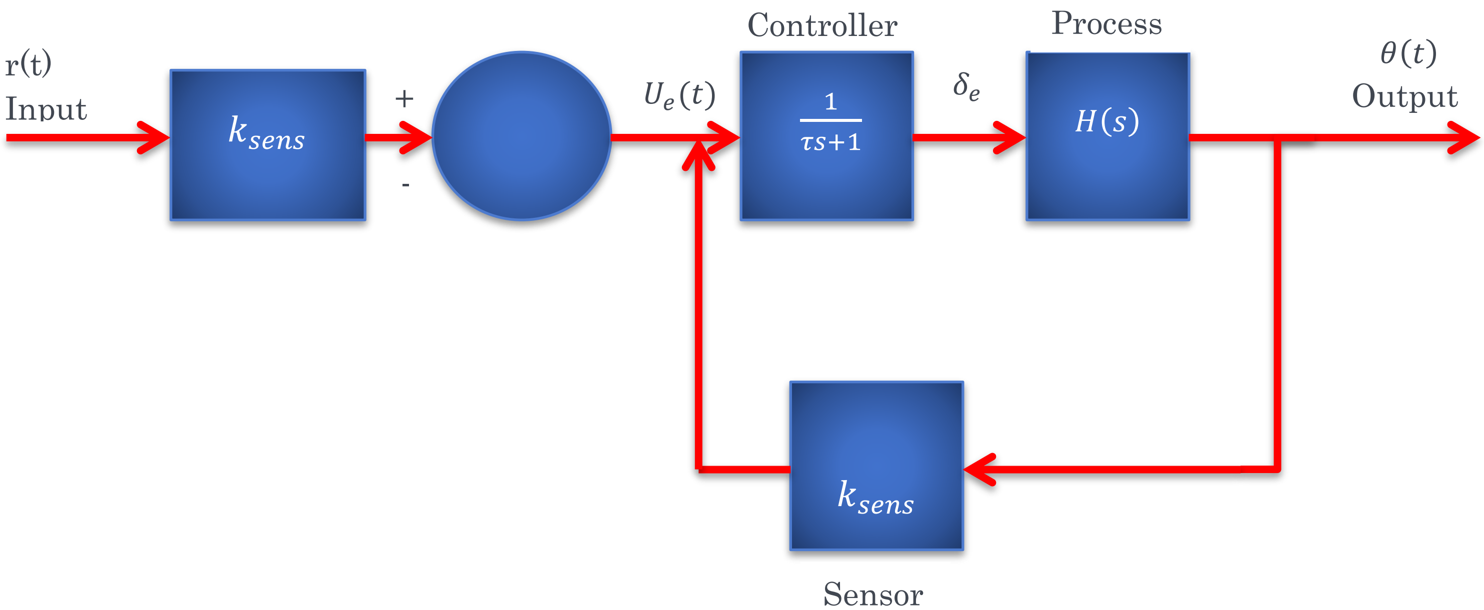
Question 5:

With the equation relating the input r to the elevator control signal,

$$u_e = k_{sens} r = \theta_{meas}$$

We can draw the control loop of the system. Where our transfer function:

$$H(s) = \frac{1.13s+0.17041}{s^3+0.739s^2+0.9233393s}$$



Question 6:

From question 1 we know that:

$$\frac{\Theta}{\Delta_e} = H(s)$$

Therefore:

$$\Theta = H(s)\Delta_e$$

Also from question 2:

$$\Delta_e = \left(\frac{1}{\tau s + 1}\right)(U_e)$$

Therefore if we combine these two equations we obtain the following:

$$\Theta = \left(\frac{H(s)}{\tau s + 1}\right)(U_e)$$

Furthermore we were given this function in question 5:

$$u_e = k_{sens}r - \theta_{meas}$$

Applying the Laplace transform to shift from the time-domain to the s-domain:

$$U_e = k_{sensR} - \theta_{meas}$$

Then combining our equations we can state:

$$\Theta = \left(\frac{H(s)}{\tau s + 1}\right)(R - \Theta)(k_{sens})$$

The input command r is zero for t < 0 and then jumps to 0.1 at t = 0 and remains at this value thereafter, so like before we are essentially looking for the step response.

Rearranging the function so that we get $\frac{\Theta}{R}$:

$$\Theta = \left(\frac{\left(\frac{Hk_{sens}}{\tau s + 1}\right)}{1 + \left(\frac{Hk_{sens}}{\tau s + 1}\right)}\right)(R)$$

So:

$$\frac{\Theta}{R} = \frac{H(s)k_{sens}}{\tau s + 1 + Hk_{sens}}$$

$$A = \text{top of } H(s) = 1.13s - 0.17041$$

$$B = \text{bottom of } H(s) = s^3 + 0.739s^2 - 0.923338s$$

Subbing in A and B for H(s) in our equation:

$$\frac{\Theta}{R} = \frac{Ak_{sens}}{\tau sB + B + ak_{sens}}$$

Subbing in the values for A and B:

$$\frac{\Theta}{R} = \frac{0.226s + 0.034082}{0.1s^4 + 1.0739s^3 + 0.83133383s^2 + 1.149338s + 0.0340825}$$

Now multiplying by $\frac{0.1}{s}$ to get the step function like in question 2:

$$\frac{\Theta}{R}\frac{0.1}{s} = \frac{0.0226s + 0.0034082}{0.1s^5 + 1.0739s^4 + 0.83133383s^3 + 1.149338s^2 + 0.0340825s}$$

Using Matlab to obtain the partial fractions in order to convert back to the t-domain:

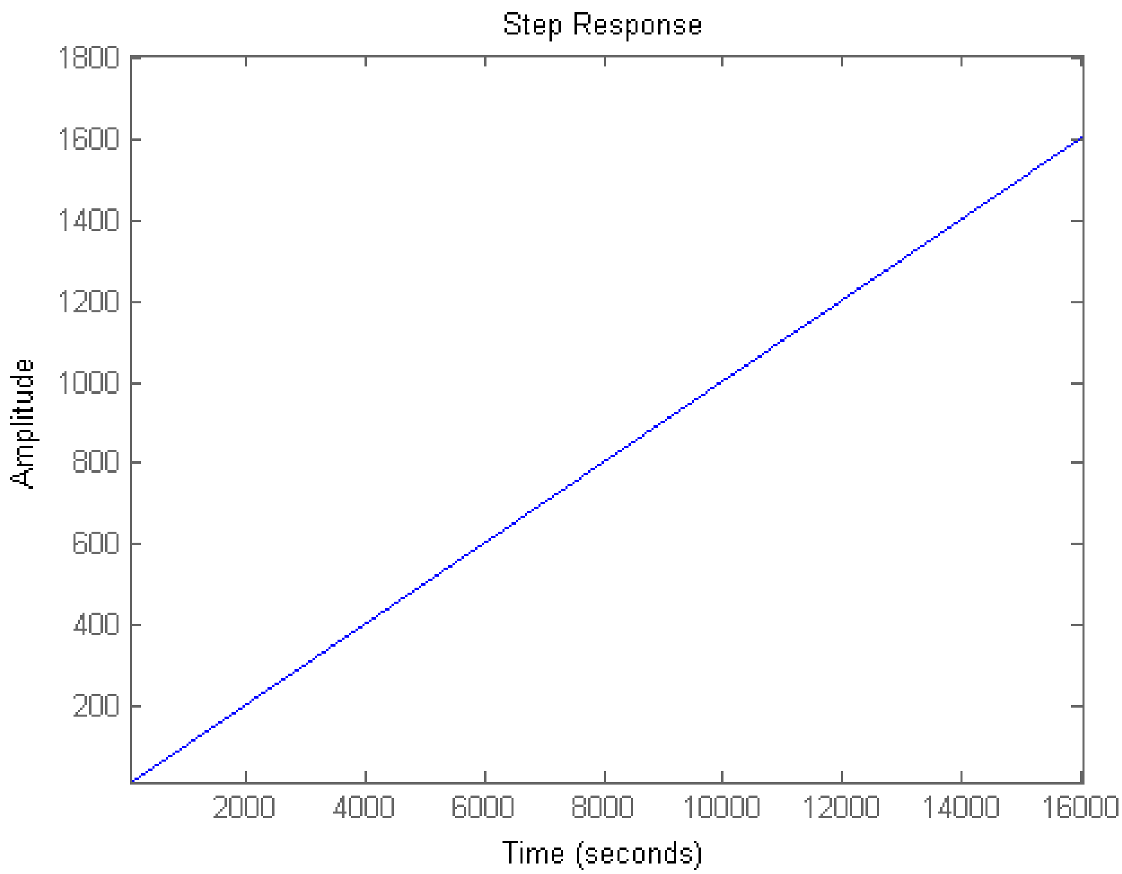
$$\Theta(s) = -\frac{0.0002}{s + 10.0237} + \frac{-0.0091 + 0.0055j}{s + 0.3425 - 1.0026j} + \frac{-0.0091 - 0.0055j}{s + 0.3425 + 1.0026j} + \frac{-0.0816}{s + 0.0303} + \frac{0.1}{s}$$

Now using the inverse Laplace transform to shift back from the s-domain to the time-domain.

$$\theta(t) = (-0.0002e^{-10.0237t}) + 0.0106e^{2.5962}(e^{-0.3425t})(e^{j1.0026t}) + 0.0106e^{2.5962}(e^{-0.3425t})(e^{-j1.0026t}) - 0.0816e^{-0.0303t} + 0.1$$

Now converting to the real form of our answer:

$$\Theta(t) = 0.0212e^{-0.3425t}(\cos(1.0026t - 2.5962)) - 0.0002e^{10.0237t} - 0.0816e^{-0.0303t} + 0.1$$



Question 7

The parameter k_p, the proportional gain, is introduced in order to help reduce the time taken for the pitch angle to reach the desired angle.

The new formula for u_e,

$$u_e = k_p(K_{sens}(r - \theta))$$

So similarly to question 6, we will attempt to find the step response for Θ by using the equation for u_e.

$$\Theta = \left(\frac{\left(\frac{H(s)}{\tau s + 1}\right) K_{sens}.k_p}{1 + k_p.K_{sens}(\frac{H(s)}{\tau s + 1})} \right)$$

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Therefore:

$$\Theta = \frac{H(s).K_{sens}.k_p}{\tau s + 1 + H(s).K_{sens}.k_p}$$

Where:

$$H(s) = \frac{1.13s + 0.17041}{s^3 + 0.739s^2 + 0.923338s}$$

$$K\_sens = 0.2$$

$$\tau = 0.1$$

As in question 6.

$$\Theta = \frac{(0.226s + 0.034082)K_p}{0.1s^4 + 0.0739s^3 + 0.8313338s^2 + 0.923338s + (0.226 + 0.034082)k_p}$$

Now I am going to vary k_p and attempt to find a quicker response time for the system.

I WAS UNAWARE AS TO HOW TO COMPLETE THE FINAL PART OF 7 USING MATLAB AND THIS IS WHY IT IS LEFT BLANK. I UNDERSTAND THAT I AM TO VARY k_p AND REMARK ON HOW THIS AFFECTS OUR SETTLING TIME USING THE IMPULSE() FUNCTION. I THEN LOOK FOR SHORTEST SETTLING TIME AND UTILIZE THIS TO FIND A SUITABLE VALUE FOR k_p AND USE THIS IN THE STEP RESPONSE FUNCTION TO OBTAIN OUR GRAPH.