ECE 310 **University of Illinois at Urbana-Champaign** **Fall 2015**

Profs. Bresler & Radhakrishnan **Homework 3**

Due: Monday, September 21

Reading: Chapter 3

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1. Consider the length 4 sequence $x[n] = (1/2)^n$, $n = 0, 1, ..., 3$, and the sequences

$$v[n] = \begin{cases} x[n] & 0 \leq n \leq 3 \\ 0 & 4 \leq n \leq 7 \end{cases} \qquad w[n] = \begin{cases} 0 & 0 \leq n \leq 3 \\ x[n] & 4 \leq n \leq 7 \end{cases},$$

which correspond to zero-padding the sequence $\{x[n]\}_{n=0}^{n=3}$ on the right, and on the left, respectively.

   (a) Sketch the three sequence $x[n], v[n]$, and $w[n]$.

   (b) Derive a closed-form expression for the DTFT $Y_d(\omega)$ of the sequence $\{y[n]\}_{n=-\infty}^{n=\infty}$ obtained by zero padding the sequence $x[n]$ by an infinite number of zeros on both sides, and sketch the magnitude of $Y_d(\omega)$.

   (c) Determine closed-form expressions for the DFTs $X[k], k = 0, 1, ..., 3$, $V[k], k = 0, 1, ..., 7$, and $W[k], k = 0, 1, ..., 7$ of the sequences $x[n], v[n]$, and $w[n]$, respectively. How are these DFTs related to one another, and to $Y_d(\omega)$?

   (d) Sketch the magnitudes of $Y_d(\omega)$, and the DFTs $X[k], k = 0, 1, ..., 3$, $V[k], k = 0, 1, ..., 7$, and $W[k], k = 0, 1, ..., 7$. How are they related?

2. Evaluate the following expressions. In all cases, the first entry in the sequence in braces corresponds to $k = 0$.

   (a) Consider the sequence $X[k] = \left\{1, e^{-j3\pi/4}, 0, e^{j3\pi/4}\right\}$. Without actually evaluating the DFT, show that the inverse DFT of $X[k]$ is real.

   (b) Find the inverse DFT of the sequence $X[k]$ in Part (a). Your answer should contain no complex numbers.

   (c) Consider the sequence $Y[k] = \left\{1, e^{-j\pi/4}, 0, e^{j\pi/4}\right\}$. Without explicitly computing the inverse DFT sum, find the inverse DFT of the sequence $Y[k]$ using your answer in part (b). [*HINT: determine first the mathematical relationship between* $X[k]$ and $Y[k]$].

3. Let $X[k]$ be the N-point DFT of the sequence $x[n]$. For a given integer constant J, determine the N-point DFT of the sequence $y[n] = \sin(2\pi Jn/N)$, $0 \leq n \leq N-1$ in terms of $X[k]$.

4. Consider the real finite length sequence $\{x[n]\}_{n=0}^{4} = \{0, 1, 2, 3, 4\}$, and its DFT $\{X[k]\}_{n=0}^{4}$.

   (a) Let $y[n]$ be a finite length sequence whose DFT is $Y[k] = e^{-j4\pi k/5}X[k]$. Sketch $y[n]$.

   (b) Let $w[n]$ be a finite length sequence whose DFT is $W[k] = Im\,\{X[k]\}$. Sketch $w[n]$.

5. Let $x[n] = \cos(0.25\pi n)$ and $v[n]$ be the sequence obtained by applying a 28-point rectangular window to $x[n]$ before computing $V_d(\omega)$. Sketch $V_d(\omega)$ for $-\pi \leq \omega \leq \pi$, labeling the frequencies of all peaks and the first nulls on either side of the peak. In addition, label the amplitudes of the peaks and the strongest side lobe of each peak. If your sketch is approximate, indicate the approximation involved.

6. Repeat Problem 5 for the case when the rectangular window is replaced by a Hamming window.