1. In general conducting media, plane TEM wave parameters $\gamma$ and $\eta$ satisfy

$$\gamma\eta = j\omega\mu \text{ and } \frac{\gamma}{\eta} = \sigma + j\omega\epsilon,$$

as well as

$$\mu = \frac{\gamma\eta}{j\omega}, \quad \sigma = \text{Re}\{\frac{\gamma}{\eta}\}, \quad \epsilon = \frac{1}{\omega}\text{Im}\{\frac{\gamma}{\eta}\}.$$

Using these relations, for a plane wave propagating in a non-magnetic material ($\mu = \mu_o$) with

$$\mathbf{H} = \hat{x}\, 25e^{-z}\cos(8\pi \cdot 10^6 t - \sqrt{3}z - \frac{\pi}{3})\,\frac{\text{A}}{\text{m}},$$

Determine:

a) The propagation constant $\gamma$ and intrinsic impedance $\eta$,

b) The permittivity $\epsilon$ and conductivity $\sigma$,

c) Phasor $\tilde{\mathbf{H}}$,

d) The corresponding phasor $\tilde{\mathbf{E}}$,

e) Time-averaged Poynting vector $\langle \mathbf{E} \times \mathbf{H} \rangle$,

f) The time averaged power dissipated in cubic volume bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$, all in meters.

2. A plane TEM wave propagating in a material medium has an electric field phasor

$$\tilde{\mathbf{E}} = 4e^{-(0.0001+j0.2)z}\hat{x}\,\frac{\text{V}}{\text{m}}.$$

Clearly, $\gamma = \alpha + j\beta = 0.0001 + j0.2$ 1/m, with $\alpha \ll \beta$.

When $\alpha \ll \beta$ is true, then all the wave parameters $\eta$, $\beta$, $v_p$, and $\lambda$ expressed in terms of $\mu$, $\epsilon$, $\omega$, and $f$ are identical to those for a perfect dielectric except that the attenuation constant $\alpha$ is not 0 but it equals $\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$.

If $\mu = 4\mu_o$ and $v_p = 10^8$ m/s, determine:

a) the **f**requency $f$ of the wave in Hertz,

b) the wavelength $\lambda$ of the wave,

c) the dielectric constant $\epsilon_r = \frac{\epsilon}{\epsilon_o}$ of the medium,

d) the conductivity $\sigma$ of the medium,

e) the location $z$ where the peak amplitude of the electric field is $1/e$ of the value at $z = 0$,

f) the magnetic field intensity phasor $\tilde{\mathbf{H}}(z)$.

3. Consider plane TEM wave propagation in a homogeneous material medium with $\mu = \mu_o$, $\epsilon = 81\epsilon_o$, and $\sigma = 4$ S/m appropriate for sea water:

a) Calculate the numerical values of the propagation constant $\gamma \equiv \alpha + j\beta$, wavelength $\lambda = \frac{2\pi}{\beta}$, penetration depth $\delta = \frac{1}{\alpha}$, and intrinsic impedance $\eta$ if the wave frequency 5 kHz — use approximations appropriate for the given $\frac{\sigma}{\omega\epsilon}$.

b) Repeat (a) if $f = \frac{\omega}{2\pi} = 5$ GHz.

c) A submarine submerged in the ocean wants to receive an electromagnetic signal from a ship located at the surface. If the ship is transmitting at 20 kHz, how close by must the submarine be located in order to receive at least 0.1% of the signal amplitude at the surface?

4. A TEM wave is propagating through a good conductor, with time-domain electric and magnetic fields given by:

$$\mathbf{E}(y,t) = 100\sqrt{2}e^{-10\pi y}\sin(4\pi \times 10^8 t - \beta y + \phi)\hat{z} \text{ V/m}$$

$$\mathbf{H}(y,t) = 50e^{-10\pi y}\sin(4\pi \times 10^8 t - \beta y - \frac{\pi}{3})\hat{h} \text{ A/m}$$

a) What is the complex propagation constant $\gamma$?

b) What is the magnitude and direction of the phase velocity $\mathbf{v_p}$ of the wave?

c) What is $\hat{h}$, the direction of the magnetic intensity field?

d) What is the complex impedance $\eta$ of the material?

e) What is the phase $\phi$ of the electric field?

5. A plane wave field

$$\mathbf{H} = 2\sin(\omega t + \beta x)\hat{y} - 2\cos(\omega t + \beta x)\hat{z}\,\frac{\text{A}}{\text{m}}$$

is propagating in vacuum in $-\hat{x}$ direction and is incident on $x = 0$ plane which happens to be the boundary of a perfect dielectric having permittivity $2.25\epsilon_o$ and permeability $\mu_o$ in the region $x < 0$.

a) What are the reflection and transmission coefficients $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ and $\tau = 1 + \Gamma$ for the described interface?

b) Write the phasor expressions for the incident, reflected and transmitted $\mathbf{E}$ fields, using the reflection and transmission coefficients found above.

c) Write the phasor expressions for the incident, reflected, and transmitted $\mathbf{H}$ fields in A/m units.

d) What are $\langle \mathbf{S_i} \rangle$, $\langle \mathbf{S_r} \rangle$, and $\langle \mathbf{S_t} \rangle$, the time-average power per unit area transported by the incident, reflected, and transmitted waves, respectively, in W/m$^2$ units? Are the values compatible with the conservation of energy? Explain.

e) What are the polarizations of the incident, reflected, and transmitted wave fields?

f) What is the *reflectance* $|\Gamma|^2$ expressed in dB units? What would be the physical significance of reflectance? **Hint:** think of it as a fraction.