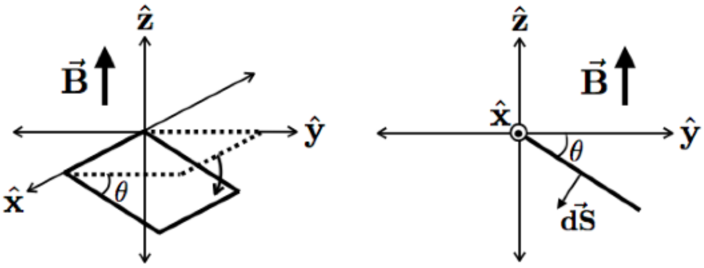
1. Given the time-varying magnetic field $\mathbf{B} = B_0(t\sin(\omega t)\ \hat{y} + \cos(\omega t)\ \hat{z})$ Wb/m^2, find the emf $\mathcal{E}$ for the following closed paths C:

   a) C is a rectangular path going from $(0,0,0)$ to $(1,0,0)$ to $(1,1,0)$ to $(0,1,0)$ back to $(0,0,0)$, with distance along the path measured in units of meters.

   b) C is a rectangular path having the same coordinates as defined above, but with the path direction reversed.

   c) C is a triangular path going from $(0,0,0)$ to $(1,0,0)$ to $(1,1,0)$ back to $(0,0,0)$, with distance along the path measured in units of meters.

2. A square loop of wire of some finite resistance R and 2 cm^2 surface area is located within a region of constant magnetic field $\mathbf{B} = 4\hat{z}$ Wb/m^2 as illustrated in the following diagrams (perspective and side views are shown).



   a) What is the magnetic flux Ψ through the loop when the orientation angle of the loop is $\theta = 0^o$? In your flux calculation make use of **dS** orientation shown in the diagram on the right.

   b) What is the flux Ψ as a function of angle θ (using the same sign convention as in part a)?

   c) Assuming that angle θ is time varying at a rate of $\frac{d\theta}{dt} = \pi\,\frac{\text{rad}}{\text{s}}$, what is the emf $\mathcal{E}$ around the loop at the instant when $\theta = 45^o$?

   d) In what direction will a positive induced current flow around the loop at the same instant? You may draw a picture to explain your answer. Be sure to justify your answer.

   e) What is the emf $\mathcal{E}$ derived from using the **opposite dS** orientation to that shown in the figure? In what direction will a postive induced current flow around the loop in this case?

3. A conducting wire loop of radius $r = 1$ m is moved with velocity $\mathbf{v} = 2\hat{x}$ m/s in a region where the background magnetostatic field is described by

$$\mathbf{B}(x,y,z) = \hat{z}\,25\times10^{-6}(1 + x/L)\,\text{T},$$

   where $L = 1000$ m. The center of the loop coincides with the origin $(x,y,z) = (0,0,0)$ at $t = 0$ and the plane of the loop coincides with $z = 0$ plane.

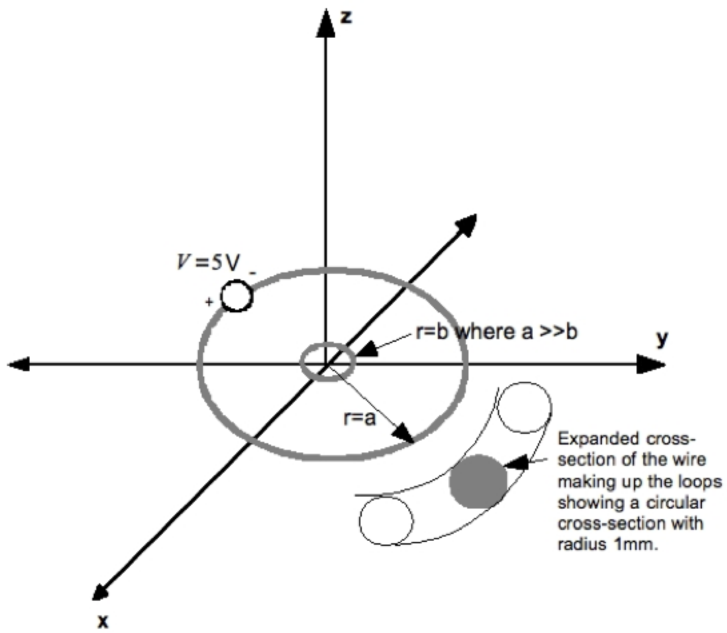
   a) Obtain an expression for the induced emf $\mathcal{E}(t)$ of the loop in motion for $t > 0$. Since $r \ll L$, the magnetic field across the loop can be considered nearly constant at each instant in time.

   b) What is the magnitude of the loop current if the loop resistance is $2\,\Omega$?

Interesting facts: The strength of Earth's magnetic field is just about 25×10^{-6} T at equatorial latitudes. However, the scale length L associated with the spatial variation of Earth's magnetic field is much longer than 1000 m.

4. *Note: Parts (a-c) of this problem have already been worked out and submitted as a part of HW 5. You are asked to complete parts d and e in this assignment set.*

Consider two concentric circular wire loops of radii $a = 1$ m and $b = 0.05$ m placed on the x–y plane of the reference coordinate system with their centers at the origin. The medium is free space. The conductivity of the wire from which both loops are made is $\sigma = 4 \times 10^7$ S/m. The cross-section of the wire is circular with radius 1 mm. A 5 V battery is connected in the outer loop (see figure).



a) *Calculate the current I_a that flows in the outer loop.*

b) *Derive an expression for the magnetic flux $\Psi_{a\to b}$ that links the inner loop due to the current flowing in the outer loop. Your expression should be in terms of the magnetic permeability of free space, the current I_a, and the radii of the two loops.* ***Hint:*** *Refer to the Lecture 13 notes for expressions for the magnetic flux density due to a current flowing in a circular wire loop. Also, take advantage of the fact that $b \ll a$ so that the magnetic field across the smaller loop can be considered nearly constant.*

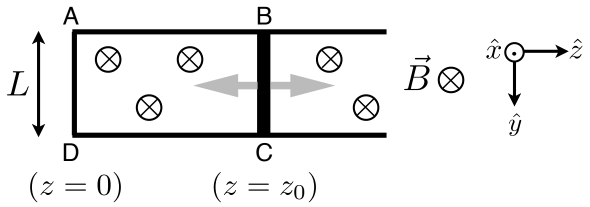
c) *Derive the mutual inductance $L_{a\to b} \equiv \Psi_{a\to b}/I_a$ between the outer and inner loop.*

d) Assume next that the inner loop is moving upwards, in the positive z direction, with speed 1 m/s. Calculate the *emf* induced in the inner loop *assuming* that the flux linking the loop due to the induced current is negligible.

e) You are given that the inductance of the inner loop is 0.25 μH. If the loop is moving upwards with speed 1 m/s, derive a differential equation for the induced current in the loop.

   Hint: the current flowing around the inner loop is the induced emf of the loop divided by the loop resistance while the emf is the negative of the time derivative of the total magnetic flux produced by the currents flowing in both loops — constructing an expression for the current amounts to finding the differential equation that is requested.

5. As shown in the diagram below, a pair of conducting rails separated by a distance L is connected at $z = 0$ to a fixed conducting rod (AD) and at $z = z_0$ to a conducting armature (BC) that can slide along the rail in the $\pm\hat{z}$ direction. A constant magnetic field $\mathbf{B} = -B_0\,\hat{x}$ exists in the region, which is shown pointing down into the page in the diagram. The armature is mechanically pulled in the $+\hat{z}$ direction at a constant velocity $\mathbf{v} = v_0\hat{z}$ m/s from its starting position at $z = z_0$, and the changing magnetic flux through the loop ABCD induces an electromotive force $\mathcal{E}$ and thus a current I_0 around the conducting loop.



a) What is the emf $\mathcal{E}$ induced in the loop ABCD?

b) What is the magnitude and direction of the induced current I_0 in terms of the resistance R of the conducting loop ABCD?

c) What is the magnitude and direction of the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ that is exerted on the armature by the magnetic field $\mathbf{B}$? **Hint**: express your answer in terms of the induced current I_0.

d) Now consider that the armature is no longer moved mechanically (though it is still free to move in response to Lorentz forces) and that a projectile of mass M is attached to it. A constant current of magnitude I' is introduced into the loop at point A such that it flows along the contour ABCD. The magnetic field generated by this current loop is negligibly small compared to the background magnetic field $\mathbf{B}$, and the mass of the armature is negligibly small compared to M. What is the magnitude and direction of the acceleration $\mathbf{a}$ of the projectile? **Hint**: $\mathbf{F} = M\mathbf{a}$.