CIRCUIT THEORY EEEN30020

Tutorial 2: Methods of Circuit Theory for Circuit Simulation

Problem Set, Useful Formulae and Workbook

The aim of this tutorial is to reinforce your knowledge on the methods and algorithms of Circuit Theory that form the core of simulators such as SPICE.

At the end of the tutorial, you will be given a short **test.** You must answer the test and return it to the Module Coordinator or Teaching Assistants. You can answer the test with your Homework/Lab team partner. Allow twenty minutes for the test. You can use any materials.

Problem Set

(1) Write down, but do not solve, the equations that SPICE would produce for the modified nodal analysis (MNA) of the circuit of Figure 1. What do you understand by the "stamp" of an element in MNA? Write the stamp of the conductance G_5, the current-controlled current source b i_1 and the voltage-controlled voltage source av_5 in the circuit of Figure 1.

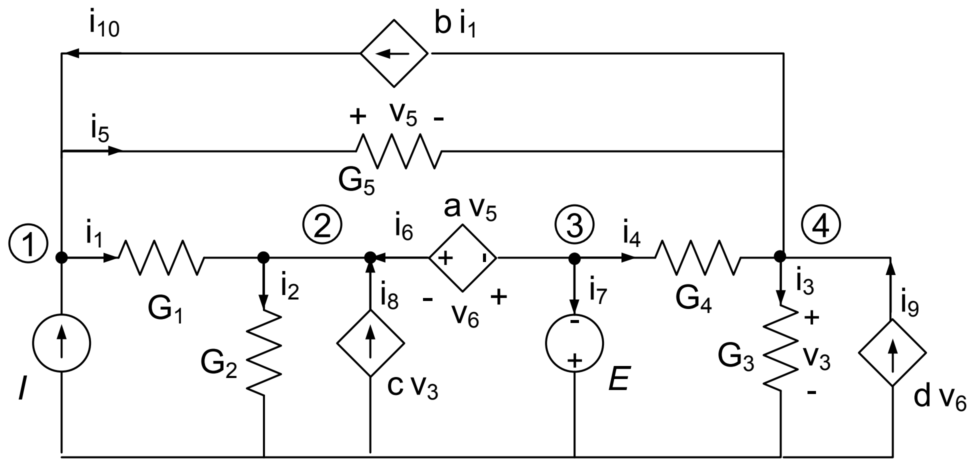


Figure 1

(2) Estimate the diode voltage in the circuit of Figure 2 using the Newton-Raphson algorithm. From an initial estimate of v = 0.7 V, run the algorithm for two iterations and show the successive estimates of the solution. The diode is modelled by the equation $i = 10^{-15}$ (exp (40v) –1).

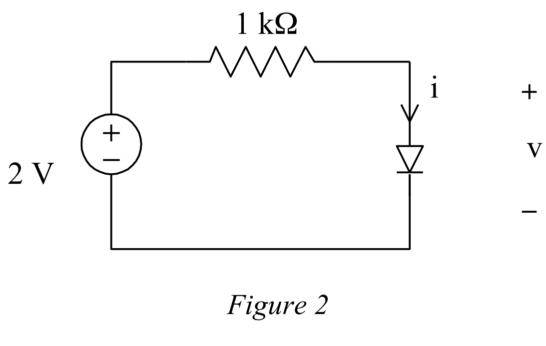


Figure 2

(3). Using the trapezoidal rule, estimate the capacitor voltage v(t) in the circuit of Figure 3 from t = 0 to t = 10 ms, taking a time step of 5 ms. The capacitor voltage at t = 0 is 1 V.

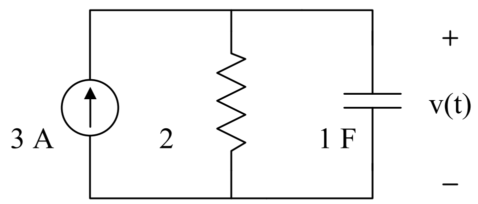


Figure 3

Useful Formulae and Other Notes

The ***Newton-Raphson algorithm*** estimates the solution to the nonlinear equation *F(v) = 0* by iterating the equation

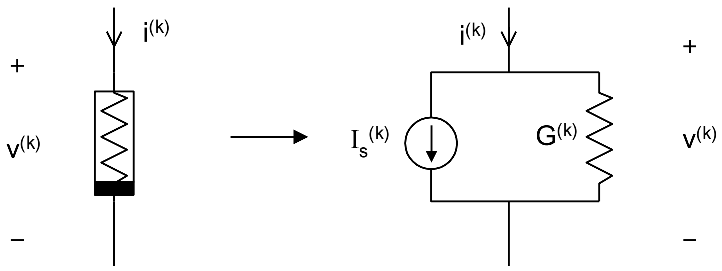
$$v^{(k+1)} = v^{(k)} - \frac{F(v^{(k)})}{F'(v^{(k)})}$$

from an initial estimate $v^{(0)}$.

Applied to a nonlinear circuit element described by the nonlinear current-voltage characteristic

$$i = g(v)$$

the Newton-Raphson algorithm results in the ***companion model*** or the ***linearised model*** for the nonlinear resistor (diode):



where

$$G^{(k)} = g'(v^{(k)}) \qquad \text{and} \qquad I_s^{(k)} = g(v^{(k)}) - g'(v^{(k)})v^{(k)}$$

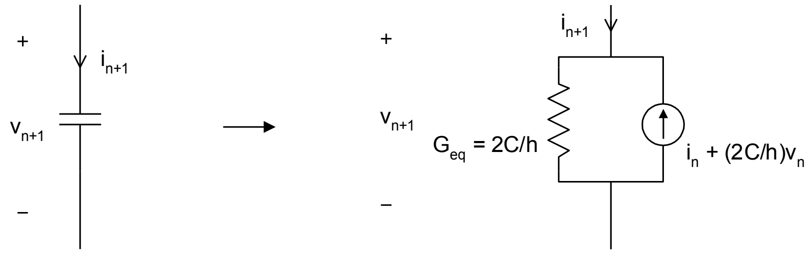
The ***trapezoidal rule*** is a numerical integration technique:

$$x_{n+1} = x_n + \frac{1}{2}h\left(\dot{x}_n + \dot{x}_{n+1}\right)$$

Applied to a capacitor

$$i = C\frac{dv}{dt}$$

it results in the ***companion model*** of the capacitor

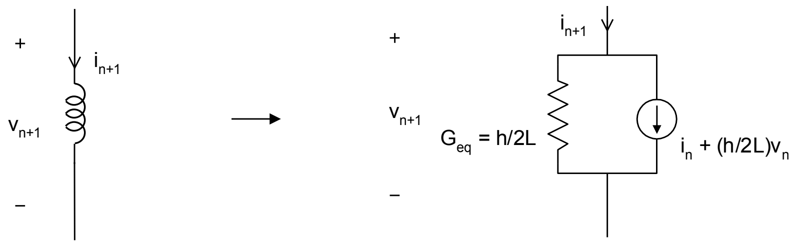


$$i_{n+1} = \frac{2C}{h}v_{n+1} - \left(i_n + \frac{2C}{h}v_n\right)$$

Applied to an inductor

$$v = L\frac{di}{dt}$$

it results in the ***companion model*** of the inductor



$$i_{n+1} = \frac{h}{2L}v_{n+1} + \left(i_n + \frac{h}{2L}v_n\right)$$

Workbook

(1) Write down, but do not solve, the equations that SPICE would produce for the modified nodal analysis (MNA) of the circuit of Figure 1. What do you understand by the "stamp" of an element in MNA? Write the stamp of the conductance G_5, the current-controlled current source b i_1 and the voltage-controlled voltage source av_5 in the circuit of Figure 1.

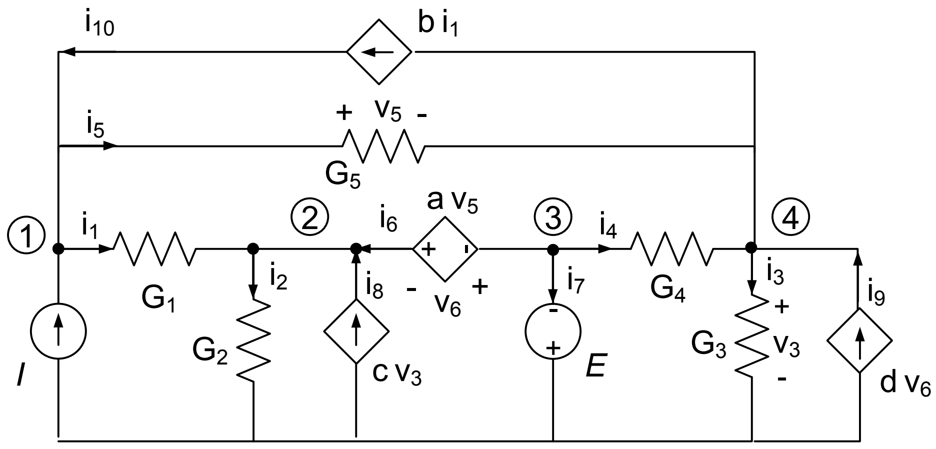


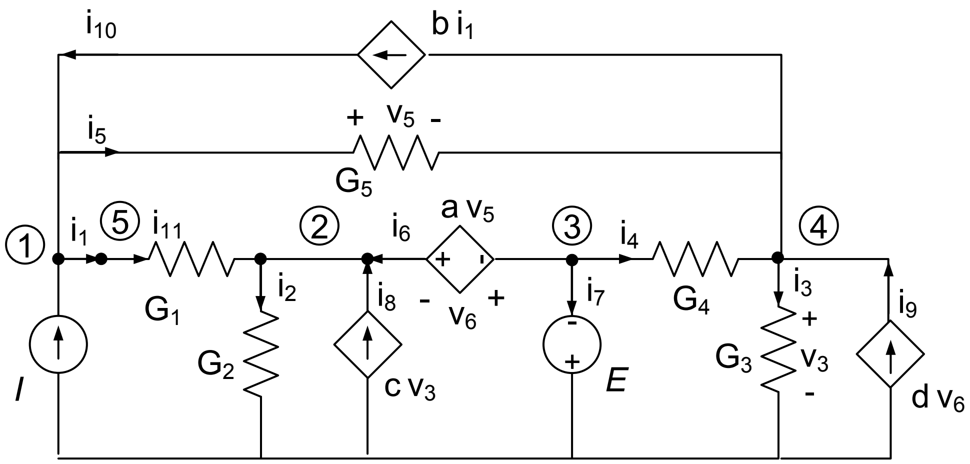
Figure 1

Solution steps:

(1) For the reference node and branch numbers see the circuit diagram. Note that in this circuit we have

(i) controlling current (i_1);
(ii) two voltage sources, controlled and independent;
(iii) current-controlled current source (CCCS).

These affect the number of variables that will appear in the MNA equations. To start to identify these variables, recall that any *current-controlled* current or voltage source requires a 0V-voltage source in SPICE inserted in the corresponding branch. We have the controlling current i_1 (used in the CCCS, bi_1) and so we add one more node:



Thus, above is the circuit to be analysed in this problem. The controlling current i_1 will appear as a variable in the MNA equations. List other variables of the MNA equations:

(2) Write the KCL equations in terms of the currents i_k ($k = 1\ldots11$):

Node 1:

Node 2:

Node 3:

Node 4:

Node 5:

(3) Write branch equations to eliminate the currents (except i_1, i_6 and i_7):

Branch 2:

Branch 3:

Branch 4:

Branch 5:

Branch 8:

Branch 9:

Branch 10:

Branch 11:

(4) Add the branch equation to branches 1, 6 and 7 (they will stay as the MNA equations together with the KCL equations for nodes 1-5):

Branch 1:

Branch 6:

Branch 7:

Assemble the MNA equations from step (2) and step (4) (using step (3) to eliminate branch currents):

Write the above equations in matrix form. Check the list of MNA stamps on pages 3.20 – 3.22 to identify the stamp of the conductance G_5, the current-controlled current source b i_1 and the voltage-controlled voltage source av_5.

(2) Estimate the diode voltage in the circuit of Figure 2 using the Newton-Raphson algorithm. From an initial estimate of v = 0.7 V, run the algorithm for two iterations and show the successive estimates of the solution. The diode is modelled by the equation $i = 10^{-15}(\exp(40v) - 1)$.

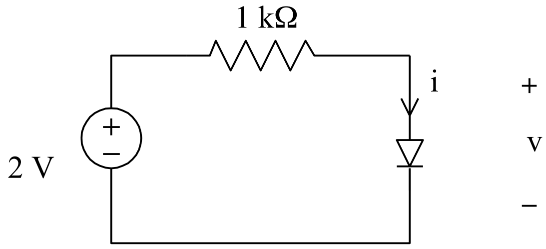


Figure 2

Solution steps: Check the companion model of the diode on page 2 of this Tutorial (or in the lecture book, page 3.41). Since the companion model employs a current source $I_s^{(k)}$, it is convenient to use the *Norton equivalent* circuit (see page 1-11 of the lecture book). Replace the series combination of the 2V voltage source and the 1kΩ resistor by the parallel combination of a current source and a resistor, where

$$I_{norton} = \frac{V_{th}}{R_{th}} =$$

$$R_{norton} = R_{th} =$$

Redraw the original circuit:

Write down the parameters of the companion model:

$$i = g(v) =$$

(1) $G^{(k)} = g'(v^{(k)}) =$

(2) $I_s^{(k)} = g(v^{(k)}) - g'(v^{(k)})v^{(k)} =$

Before you can proceed with the Newton-Raphson iterations, you must solve the companion circuit and find $v^{(k+1)}$:

Check whether or not you obtained the following answer: $v^{(k+1)} = \dfrac{2 \cdot 10^{-3} - I_s^{(k)}}{G^{(k)} + 10^{-3}}$ (3)

Use equation (3) with $G^{(k)}$ and $I_s^{(k)}$ computed from expressions (1) and (2).

Start with the initial guess $v^{(0)} = 0.7V$. For a given $v^{(0)}$, calculate $G^{(0)}$ and $I_s^{(0)}$:

$G^{(0)} =$

$I_s^{(0)} =$

Substitute their numerical values in (3) and calculate $v^{(1)}$:

$v^{(1)} =$

Use the numerical value $v^{(1)}$ you just found to calculate $G^{(1)}$ and $I_s^{(1)}$:

$G^{(1)} =$

$I_s^{(1)} =$

Substitute their numerical values in (3) and calculate $v^{(2)}$:

$v^{(2)} =$

(3) Using the trapezoidal rule, estimate the capacitor voltage v(t) in the circuit of Figure 3 from t = 0 to t = 10 ms, taking a time step of 5 ms. The capacitor voltage at t = 0 is 1 V.

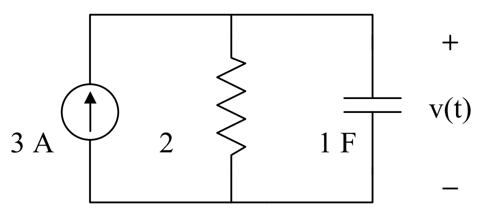


Figure 3

Solution steps:
Check the companion model of the capacitor on page 3 of this Tutorial (or in the lecture book, page 3.48). Calculate the parameters of the companion model:

$$G^{(eq)} = \frac{2C}{h} =$$

$$R^{(eq)} = \frac{1}{G^{(eq)}} = \frac{h}{2C} =$$

$$I_s^{(n)} = i_n + (2C/h)v_n =$$

Redraw the original circuit, replacing the capacitor by its companion model:

Firstly, we note that the current source in the companion model $I_s^{(n)} = i_n + (2C/h)v_n$ contains i_n. To simplify our analysis, we should express the current i_n in terms of the voltage v_n.

Write KCL for the first node of the circuit (where the current due to the independent 3A source, the current through 2Ω resistor and the current i_{n+1} join):

From this KCL equation, express i_n in terms of v_n:

Use this in the expression for $I_s^{(n)}$:

$I_s^{(n)} = i_n + (2C/h)v_n =$

Before you can proceed to the iterations of the trapezoidal rule, you must solve the companion circuit. Write

KCL node 1:

KCL node 2:

Ohm's law (2Ω resistor):

Ohm's law (companion model resistor):

Using these four equations, eliminate all currents and find the voltage v_{n+1} at the next time step in term of the voltage v_n at the current time step:

Check whether or not you obtained the following answer: $v_{n+1} = 0.00249688 \cdot (6+399.5 v_n)$ (1)

Using this equation, start the iterations of the trapezoidal rule and find the voltage at the next time steps. Take the initial value $v_0 = 1V$ at $t = 0$. Substitute this value to (1) to find the voltage at

$t_1 = 0.005s$: $v_1 =$

Take v_1 and substitute it to (1) again to find the voltage at

$t_1 = 0.01s$: $v_2 =$