**ECE 310** **University of Illinois at Urbana-Champaign** **Fall 2015**

**Profs. Bresler & Radhakrishnan** **Homework 1** **Due: Friday, September 4**

**Reading: Chapter 1 and Appendices A and D, Chapter 2 (2.1-2.5)**

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### General Instructions for HW

- Make sure you write your full name and section on your HW solution.
- All pages must be stapled together.
- Solutions to problems must appear in order.
- Unless the solution to a problem is very short, start the solution to each HW problem on a new page. This will help the graders in their work, and ensure they do not miss solutions you have written.

1. Sketch the following signals ($u[n]$ is the unit step function in the discrete-time variable $n$):

   (a) $u[n+1] + u[-n+4]$

   (b) $n(u[n] - u[n-3])$

   (c) $\cos(n\pi/3)u[-n+2]u[n+4]$

   (d) $(\frac{1}{2})^n u[n-1]u[-n+10]$

2. Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental (smallest) period.

   (a) $x[n] = \sin(\pi n/3)$

   (b) $x[n] = \cos(2n/3)$

   (c) $x[n] = \cos(\pi^2 n/5)$

   (d) $x[n] = e^{j\pi(n-2)/5}$

3. Evaluate and represent your final answer in both Cartesian and polar forms. Try to simplify as far as you can by hand, without using a calculator or computer.

   (a) $(3\angle 150°) - (5\angle -60°) + (4\angle 120°)$

   (b) $\dfrac{(-1+j)^5}{1+j}$

   (c) $\dfrac{5\angle 60°}{2j} + \dfrac{\sqrt{2}e^{j\pi}}{2-j}$

   (d) $(\dfrac{-1+j3}{1-j} + \dfrac{3+j}{1+j2})^n$

4. Derive close form expressions for the magnitude and phase of the function $G(\omega)$ of the real variable $\omega$, where $G(\omega) = 1 - e^{-j2\omega}$, and sketch (by hand) the magnitude and phase over the interval $\omega \in [-\pi, \pi]$. Label your plots.

5. Compute the following:

   (a) Determine the roots of the equation $4z^4 + 1 = 0$

   (b) Use the roots to factor the polynomial $G(z) = 4z^4 + 1$ as a product of the first order polynomials in z.

   (c) Express $G(z)$ as a product of first and second order factors with *real coefficients*.

   (d) Sketch the position of the roots in the complex plane.

6. Evaluate the following integrals, where $\delta(t)$ is the Dirac delta function and $u(t)$ is the unit step function:

   (a) $\int_{-\infty}^{\infty} (t^2 - 5t + 4)\delta(t)dt$

   (b) $\int_{-\infty}^{-3} (t^2 - 5t + 4)\delta(t)dt$

   (c) $\int_{-3}^{\infty} (t^2 - 5t + 4)\delta(t)dt$

   (d) $\int_{-\infty}^{\infty} (t^2 - 5t + 4)\delta(t - 3)dt$

   (e) $\int_{-\infty}^{\infty} (t^2 - 5t + 4)\delta(3t - 2)dt$

   (f) $[e^{-t}u(t)] * \delta(3t - 2)dt$, where $*$ is the convolution

7. Determine the Fourier transform of the following signals:

   (a) $\delta(2t - 3)$

   (b) $e^{-2\alpha t}u(t)$

   (c) $u(t) - u(t - T)$, where $T$ is a known real number.

   (d) $\sin(2\Omega_0 t + \phi)$, where $\Omega$ and $\phi$ are known real numbers.

   (e) $(u(t - 1) - u(t - 6))e^{j2\pi t}$