ECE 310 **University of Illinois at Urbana-Champaign** **Fall 2015**

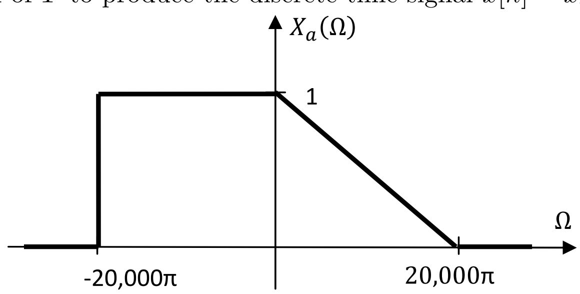
Profs. Bresler & Radhakrishnan **Homework 4** **Due: Friday, October 2**

Reading: Chapter 3,4

---

1. Let $x[n] = \cos(0.3\pi n)$ and $v[n]$ be the sequence obtained by applying a 40-point Hamming window to $x[n]$ before computing $V_d(\omega)$. Sketch $V_d(\omega)$ for $-\pi \leq \omega \leq \pi$, labeling the frequencies of the largest peaks and the first nulls on either side of the peak. In addition, label the amplitudes of the largest peaks and the strongest side lobe of each peak. Use a dB vertical scale for your plot. If your sketch is approximate, indicate the approximation involved. Recall that the DTFT of the Hamming window has mainlobe whose width is double that of the truncation (rectangular) window, and first sidelobe that is about -40dB relative to the mainlobe (see Ch. 11.5-11.6 for detailed sketches of the DTFTs of various windows.) *Hint: review Problem 5 from HW3 and its solution, where the same scenario is considered, but with a rectangular window.*

2. The continuous-time signal $x_a(t)$ has the Fourier transform shown below. The signal $x_a(t)$ is sampled with a sampling period of T to produce the discrete-time signal $x[n] = x_a(nT)$.



   (a) Sketch the DTFT ($X_d(\omega)$) of $x[n]$ for $|\omega| \leq 2\pi$ for the sampling frequencies F_s of (i) 20 kHz and (ii) 30 kHz.

   (b) What is the Nyquist rate (minimum sampling rate needed to avoid aliasing) for the signal $x_a(t)$?

3. A continuous-time signal $x_a(t) = \cos(\Omega_0 t)$ was sampled at a rate of 80 samples/sec to produce $x[n] = \cos(7\pi n/12)$. Find the three lowest possible different values of Ω_0 that could produce the sequence $x[n]$. Are there more possible values?

4. Let $x_a(t) = \cos(30\pi t)$ and $x[n] = x_a(n/40)$. Determine and sketch $|X[k]|, k = 0, ...15$, the length-16 DFT of the sequence $x[n], n = 0, 1, ...15$. Explain why your sketch does not look like a typical DFT spectrum of a sampled sinusoid.

5. In this problem we consider windowed DFT spectral analysis. N samples of a continuous-time signal $x_a(t)$ are acquired with period T to produce the sequence $x[n] = x_a(nT), n = 0, 1, ..., N-1$. After appropriate zero padding, a 1024-point DFT $V[k], k = 0, 1, ...1023$ is computed. Unless stated otherwise, $x_a(t) = \cos(118\pi t) + 0.5\cos(120\pi t)$.

   (a) What is the Nyquist rate for the signal $x_a(t)$?

   (b) If $N = 600$ and $T = 1/200$ s, which four DFT samples would have the greatest magnitude?

   (c) If $F_s = 200$Hz, (where F_s is the sampling frequency), what is the minimum value of N required in order to resolve the different frequency components in $x_a(t)$ (state and justify the criteria used for your answer)?

   (d) Consider input analog signals that contain analog frequencies no higher than those in the signal $x_a(t)$ specified above. Using the value of N that you calculated in part (c), what value of T (and what sampling frequency F_s) would give your spectral analysis method the highest resolution?

What would be the resolution (i.e., the minimum difference in analog frequencies that can be resolved) of this method?

(e) Faster is not always better. Explain intuitively (DO NOT GIVE AN EQUATION), why increasing the sampling frequency (F_s) *degrades* the resolution (i.e. the minimum resolvable frequency is increased)?

(f) Suppose now that $x_a(t) = \cos(118\pi t) + 0.5\cos(120\pi t) + 0.03\cos(110\pi t)$.

i. Suppose $N = 600$ and $T = 1/200$. Explain why the DFT may fail to reveal a peak corresponding to the third sinusoidal component. Will increasing N to 1024 overcome this problem? Why?

ii. Will multiplying $x[n], n = 0, 1, ...N - 1$ by a length-N Hamming window before zero padding and computing the DFT overcome this problem? Why? What other problem could arise in the spectral analysis of the given signal $x_a(t)$, when using a Hamming window as described?

iii. Repeat Part 5c above, assuming that a Hamming window has been used as above.

6. Determine whether each of the following systems characterized by the following input (x)-output (y) relations is, (i) linear or non-linear (ii) causal or non-causal (iii) shift-invariant or shift-varying. Assume that the input is zero before n = 0 and that the initial conditions of the systems are all set to zero. Justification is **not** needed.

(a) $y[n] = x[3 - n]$

(b) $y[n] = e^{j\pi n^2/4}x[n]$

(c) $y[n] = x[n^2]$

(d) $y[n] = x[2n]$

(e) $y[n] = x[n] + 2$

(f) $y[n] = \sum_{m=-\infty}^{n+3} x[m - 1]$

(g) $y[n] = \frac{x[n]}{x[5]}$ for $0 \leq n \leq 5$

(h) $y[n - 1] = x[n - 1] + \tan(4)x[n] - 2y[n]$

(i) $y[n] = y[n - 1] + |x[n]|$

7. Determine whether each of the systems characterized by the following relations is, with respect to the input, (i) linear or non-linear (ii) causal or non-causal (iii) shift-invariant or shift-varying. For part (a) assume that the input is zero before n = 0 and that the initial conditions of the systems are all set to zero. **Justify your answers with proofs or counter-examples**.

(a) $y[n] = 2y[n - 2] + x[n - 1]$

(b) $y[n] = y[n + 1] + x[n]$

(c) $y[n] = \sum_{m=-2}^{n} x[m - n]2^m$