ECE 310: FIR filter Design - Frequency Sampling Method, Park - McClellan Method

Frequency Sampling Method :

Given : $G_d(\omega)$: the desired frequency Response

Pick $H_d(\omega)$: so that $H_d(\omega)$ matches $G_d(\omega)$ at specific frequencies,

$$\boxed{\omega_m = \frac{2\pi m}{N}} \qquad 0 \le m \le N-1$$

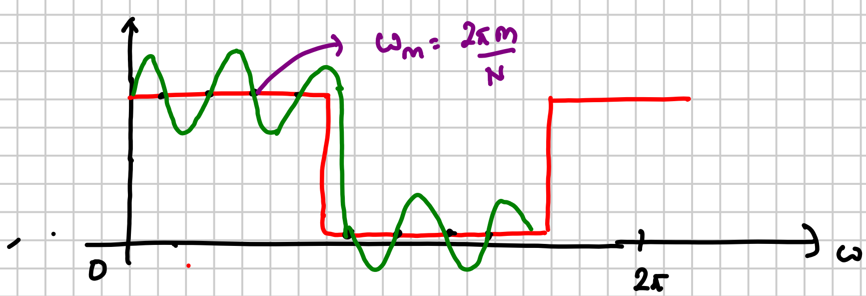
$h(n)$ is given by, $h(n) = \text{IDFT}\{D(\omega)\}$

where $D(\omega) = G_d(\omega) \times$ phase term

$$\Rightarrow \boxed{h(n) = \frac{1}{N}\sum_{m=0}^{N-1} D\left(\frac{2\pi m}{N}\right) e^{j\frac{2\pi mn}{N}}} \qquad 0 \le n \le N-1$$

This approach gives $|H_d(\omega)| = |G_d(\omega)|$ for $\omega = \frac{2\pi m}{N}$.

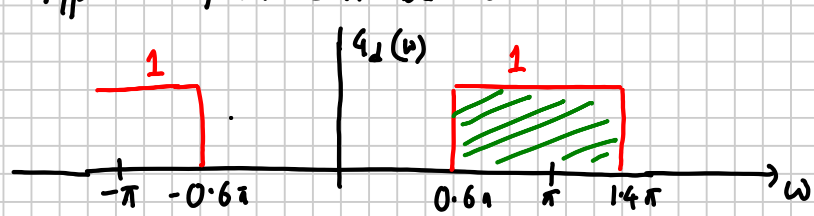
In general $|H_d(\omega)| \neq |G_d(\omega)|$ for $\omega = \frac{2\pi m}{N}$. The approach can give rise to large ripples at points where $\omega \neq \frac{2\pi m}{N}$



Example : HPF $\{h(n)\}_{n=0}^{60}$ with cut-off $\omega_c = 0.6\pi$

$\boxed{N = 61}$ $\qquad$ $\boxed{M = \frac{N-1}{2} = 30}$

Type I filter can be used.



$$\therefore \quad G_d(\omega) = \begin{cases} 0 & 0 \le \omega < 0.6\pi \\ 1 & 0.6\pi \le \omega \le 1.4\pi \\ 0 & 1.4\pi < \omega \le 2\pi \end{cases}$$

$$\Rightarrow \quad D(\omega) = \begin{cases} 0 & 0 \le \omega < 0.6\pi \\ e^{-j30\omega} & 0.6\pi \le \omega \le 1.4\pi \\ 0 & 1.4\pi < \omega \le 2\pi \end{cases}$$

Frequency Sample : $\omega_m = \frac{2\pi m}{61}$

$$\Rightarrow \quad 0.6\pi = \frac{2\pi m}{61} \Rightarrow m = \lfloor 18.3 \rfloor = 18$$

$$1.4\pi = \frac{2\pi m}{61} \Rightarrow m = \lfloor 42.7 \rfloor = 42$$

$$\therefore \quad D\left(\frac{2\pi m}{61}\right) = \begin{cases} 0 & 0 \le m \le 18 \\ e^{-j30 \cdot \frac{2\pi m}{61}} & 19 \le m \le 42 \\ 0 & 43 \le m \le 60 \end{cases}$$

$$\Rightarrow \quad h[n] = \frac{1}{61} \sum_{m=0}^{60} D\left(\frac{2\pi m}{61}\right) e^{j\frac{2\pi n m}{61}} \qquad n = 0, .. 60$$

$$\Rightarrow \quad h[n] = \frac{1}{61} \sum_{m=19}^{42} e^{-j30 \cdot \frac{2\pi m}{61}} \cdot e^{j\frac{2\pi n m}{61}}$$

$$= \frac{1}{61} \sum_{m=19}^{42} e^{j\frac{2\pi}{61}(n-30)m}$$

put: $k = m - 19$

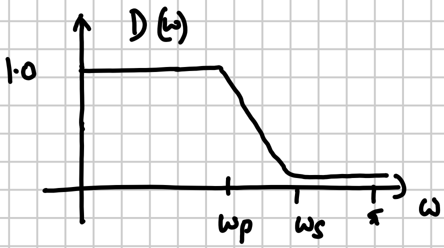
$$\Rightarrow \quad h[n] = \frac{1}{61} \sum_{k=0}^{23} \cdot e^{j\frac{2\pi}{61}(n-30)(k+19)}$$

$$\Rightarrow \quad h(n) = \frac{1}{61} e^{j\frac{2\pi}{61}(n-30)19} \underbrace{\sum_{k=0}^{23} e^{j\frac{2\pi}{61}(n-30)k}}_{\text{Geometric Series}}$$

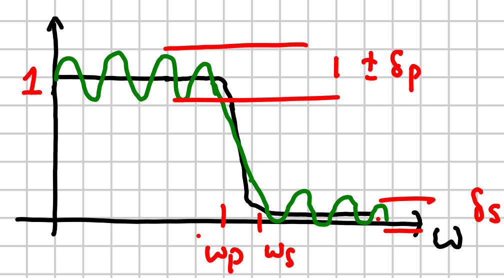
$$\therefore \quad h(n) = \frac{1}{61} e^{j\pi(n-30)} \frac{\sin\left(\frac{24\pi}{61}(n-30)\right)}{\sin\left(\frac{\pi}{61}(n-30)\right)} \qquad 0 \le n \le 60$$

Park-McClellan Filter :

Let $D(\omega)$: frequency Response of the filter to be designed given as a piece-wise linear function.



The designed approximates the desired frequency response with following tolerances :



Park-McClellan method is a iterative procedure in which the filter length N, ω_p, ω_s, and one of δ_p or δ_s are fixed, and δ_p or δ_s is a variable.

<u>Objective</u> : To determine filter coefficients $h(n)$ of a transfer function so that

$$\text{Error}, \quad \varepsilon(\omega) = W(\omega)\left[D(\omega) - H_d(\omega)\right]$$

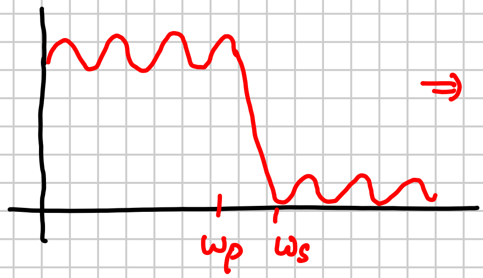
is minimized over sub intervals of $(0 \le \omega \le \pi)$

$W(\omega)$: Controls the relative sizes of peak errors in sub-bands.

Algorithm iteratively adjusts the coefficients until the peak absolute value of $\mathcal{E}(\omega)$ is minimized.

i.e $\quad \min \max_{\substack{0 \le \omega \le \omega_p \\ \omega_s \le \omega \le \pi}} |\mathcal{E}(\omega)|$

The designed filter has the following behavior



$\Rightarrow$ Equiripple Behavior

$$\omega_p \delta_p = \omega_s \delta_s$$

$$\Rightarrow \boxed{\frac{\delta_p}{\delta_s} = \frac{\omega_s}{\omega_p}}$$

Filter order is given by,

$$\boxed{N = \frac{-10 \log_{10}(\delta_p \delta_s) - 13}{2.324(\omega_s - \omega_p)}}$$