ECE 310 **University of Illinois at Urbana-Champaign** **Fall 2015**

Profs. Bresler & Radhakrishnan **Homework 6** **Due: Friday, October 19**
Reading: Ch. 5

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1. Use the unilateral z-transform to determine the convolution of the signals $x[n] = (2)^{-n}u[n-2]$, $y[n] = (1 + 3^{-n})u[n]$.

2. A second-order causal, linear, shift invariant system is described by the following LCCDE, for input $x[n]$ and output $y[n]$,

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n] - 0.5x[n-1]$$

   (a) Use the z-transform to find the impulse response, $h[n]$, of this system.

   (b) Use the z-transform to find the system response to the input $x[n] = 3^{-n}u[n]$ when the system is initially at rest, i.e. zero initial conditions.

   (c) Draw a Direct Form II realization of the system.

3. Determine whether or not the following systems, with input $x[n]$ and output $y[n]$, are bounded input, bounded output (BIBO) stable.

   (a) $y[n] = x^5[n] + n3^{-n}$

   (b) $y[n] = \tan(x[n])$

   (c) $y[n] = n\cos(x[n])$

   (d) $y[n] = x[n] * h[n]$ where $h[n] = \begin{cases} 0 & \text{if } n < 0 \\ 10^{100} & 0 \le n \le 10^{10} \\ e^{-0.01n} & 10^{10} < n < \infty \end{cases}$

4. Determine whether or not each of the following system functions represents that of a BIBO stable system. Assume that each of these system functions is the one-sided z-transform of the impulse response for a causal LSI system.

   (a) $H(z) = \dfrac{z+10}{z^2+1/4}$

   (b) $H(z) = \dfrac{z+10}{z^2-1.5z+0.5}$

   (c) $H(z) = \dfrac{z-10}{z+3}$

   (d) $H(z) = \dfrac{z+1}{z^2+j}$

   For each case above in which the system is determined to be BIBO unstable, find a bounded real-valued input that produces an unbounded output.

5. A causal LSI system is such that the input $x[n] = 3^{-n}(0.5u[n] - u[n-1])$ gives an output $y[n] = 2^{-n}u[n]$.

   (a) Find the impulse response $h[n]$ of the system. Is the impulse response unique?

   (b) Determine whether or not the system is BIBO stable. Where are the poles of the transfer function located?

   (c) Find the LCCDE that characterizes this system. Show the Direct Form I and Direct Form II realizations of the system.