1. Consider an infinite surface current density

$$\mathbf{J_s} = \hat{z} J_{so} \sin(\omega t)$$

flowing on $x = 0$ surface, where $J_{so} > 0$ is real-valued amplitude of the monochromatic surface current measured in A/m units. It is found that $\mathbf{J}_s$ injects field energy into propagating transverse electromagnetic (TEM) waves away from the $x = 0$ plane at an average rate of 8 W/m^2 — that is, the magnitude of the average Poynting vector $\langle \mathbf{S} \rangle$ is 4 W/m^2 for the waves excited on either side of the surface current.

   a) Denoting the TEM waves excited by $\mathbf{J}_s$ (above and below the $x = 0$ plane) as $\mathbf{E} = -\hat{z} E_o \sin(\omega t \mp \beta x)$ V/m and $\mathbf{H} = \pm \hat{y} H_o \sin(\omega t \mp \beta x)$ A/m, where wavenumber $\beta = \frac{\omega}{c}$, determine the numerical values of wave amplitudes E_o and H_o in V/m and A/m units (assuming wave propagation in free space).

   b) Determine the value of J_{so} in A/m units.

   c) Write down explicitly the phasors of $\mathbf{E}$ and $\mathbf{H}$ fields of part (a) (for regions above and below $x = 0$ plane) using the numerical values of E_o and H_o determined in that part.

   d) Write down the set of two, coupled, first-order differential equations whose solutions $\mathbf{E}(x, t)$ and $\mathbf{H}(x, t)$ are specified above.

2. For phasor $\tilde{\mathbf{E}} = (2\hat{x} - j2\hat{z})e^{-j\beta y}$:

   a) Does its $\hat{x}$ component *lead* or *lag* its $\hat{z}$ component by 90°? You can determine the answer by inspecting the locations of complex numbers $E_x \equiv \tilde{\mathbf{E}} \cdot \hat{x}$ and $E_z \equiv \tilde{\mathbf{E}} \cdot \hat{z}$ on the complex plane. The $\hat{x}$ component E_x *leads* the $\hat{z}$ component E_z by 90° if the angle of (complex number) E_x is greater than the angle of E_z by 90° (otherwise, it *lags*).

   b) Does the time-harmonic expression corresponding to phasor $\tilde{\mathbf{E}} = (2\hat{x} - j2\hat{z})e^{-j\beta y}$ describe a right-circular or left-circular wave?

Try understanding that in circularly polarized waves, the field vector rotates from that coordinate axis having the "leading phasor component" to the other axis having the "lagging phasor component" (by 90 degrees) just as illustrated by parts (a) and (b) above.

3. For each of the following plane TEM waves (in free space) described by

   a) $\mathbf{E}_1 = 4\cos(\omega t - \beta y)\hat{x} + 3\cos(\omega t - \beta y)\hat{z}$ V/m

   b) $\mathbf{E}_2 = 2\cos(\omega t + \beta y)\hat{x} + 2\sin(\omega t + \beta y)\hat{z}$ V/m

   c) $\mathbf{E}_3 = \cos(\omega t - \beta x - \frac{\pi}{2})\hat{y} + \sin(\omega t - \beta x)\hat{z}$ V/m

   d) $\mathbf{H}_4 = \cos(\omega t + \beta z + \frac{\pi}{3})\hat{x} + \sin(\omega t + \beta z - \frac{\pi}{6})\hat{y}$ A/m

   e) $\mathbf{H}_5 = 2\cos(\omega t + \beta x)\hat{z} - \sin(\omega t + \beta x)\hat{y}$ A/m

      i. Determine the corresponding $\mathbf{E}$ and $\mathbf{H}$ field phasors.

      ii. Determine the time-averaged Poynting vector $\mathbf{S}$, that is $\langle \mathbf{S} \rangle \equiv \langle \mathbf{E} \times \mathbf{H} \rangle$, by utilizing the phasor cross product $\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*$ appropriately.

      iii. Determine the time-averaged power that crosses a 1 m^2 area in the plane perpendicular to the direction of propagation in the direction of wave propagation by using the results of (ii).

iv. Determine the wave polarization using the following categories: linear polarized (state the polarization direction), circular polarized (state right- or left-circular), or neither (elliptical polarized).

v. Write the expression for the surface current $\mathbf{J_s}(t)$ that gives rise to each of the above TEM waves.

4. Two current sheets are fixed in free-space in the $x = 0$ and $x = \lambda/4$ planes having surface current densities $\mathbf{J}_s(t) = J_1 \cos(\omega t + \phi)\hat{z}$ and $\mathbf{J}_s(t) = J_2 \cos(\omega t)\hat{y}$, respectively, where J_1, J_2, and ϕ are real numbers. TEM waves will propagate away from each sheet and superpose with each other to create a wavefield whose polarization depends on the value of ϕ and the ratio of J_2 and J_1.

   a) Assuming that $J_2 = J_1$, determine the value(s) of ϕ for the superposed TEM waves propagating in the region $x > \lambda/4$ to have:

      i. right handed circular polarization,

      ii. left handed circular polarization,

      iii. linear polarization.

      Justify your choice of ϕ in each case by determining the electric field phasor produced for the region $x > \lambda/4$ with your choice of ϕ.

   b) Determine the time-averaged Poynting vector in the region $x > \lambda/4$ in (i) and (ii) above with $J_1 = J_2 = 1$ A/m.

   c) Repeat (b) if $J_1 = 1$ A/m and $J_2 = 0$.

   d) From the results of (b) and (c), what do you conclude about the relative power densities of linear and circular polarized TEM waves having equal instanteneous peak electric field magnitudes?

      Try understanding and remembering this result for future use.