Homework 8

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a) The Nyquist rate is twice the highest frequency component : $f_s \geqslant 10\,kHz$

$$T_{max} = \frac{1}{10\,kHz} = \frac{1}{10\,000}\ sec$$

b)

$$\omega = \Omega T$$

$$\frac{\pi}{8} = \frac{1}{10000}\Omega$$

$$\Omega = \frac{10000\,\pi}{8} = 2\pi\, 625$$

$$f = 625\ Hz$$

c)

$$\omega = \Omega T$$

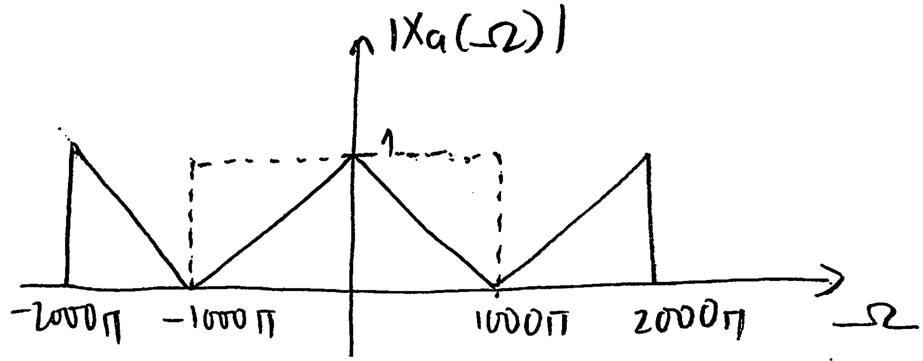
$$\frac{\pi}{8} = \frac{1}{20000}\Omega$$

$$\Omega = 2\pi\, 1250$$

$$f = 1250\,Hz$$

② a) $f_s = 2 f_{max} = 2 \times 1000 = 2000 \pi$.

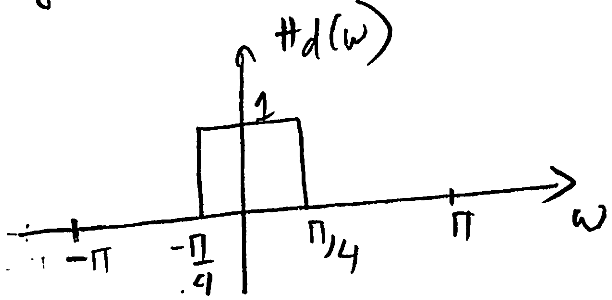
b)



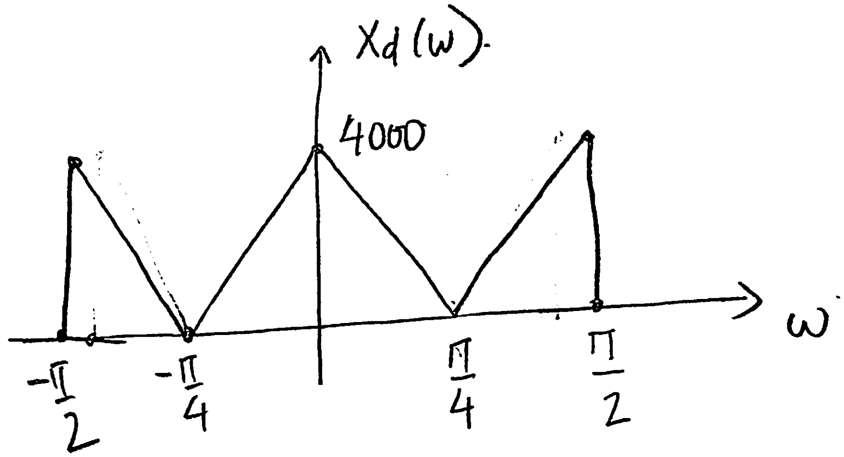
The smallest sampling rate such that we don't introduce aliasing into the spectrum of $X_a(\Omega)$ from -1000π to 1000π is $\Omega_s = 3000\pi$, or $f_s = \frac{\Omega_s}{2\pi} = 1500$ Hz.

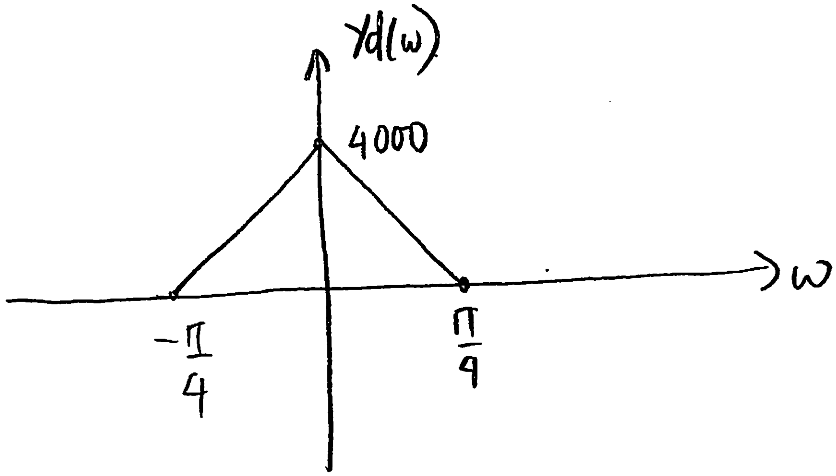
c) $f_s = \frac{1}{T} = \frac{1}{0.25 \text{ msec}} = 4000$ Hz

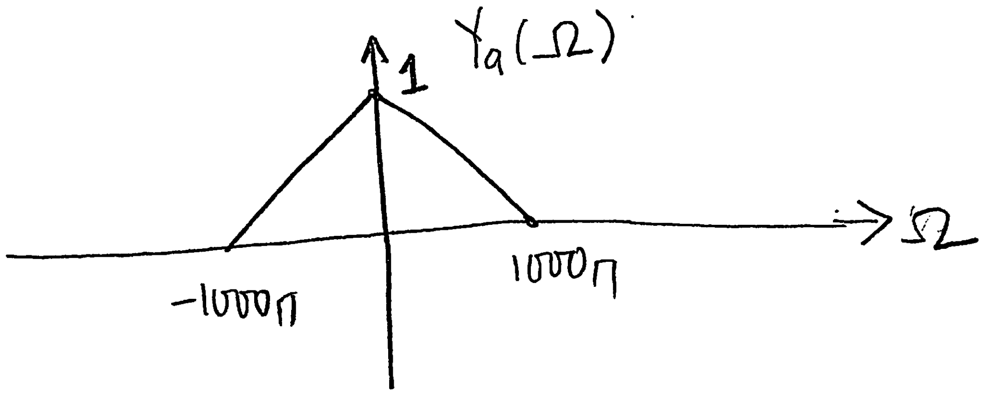
$$\omega = \frac{\Omega}{f_s} = \frac{1000\pi}{4000} = \frac{\pi}{4}.$$



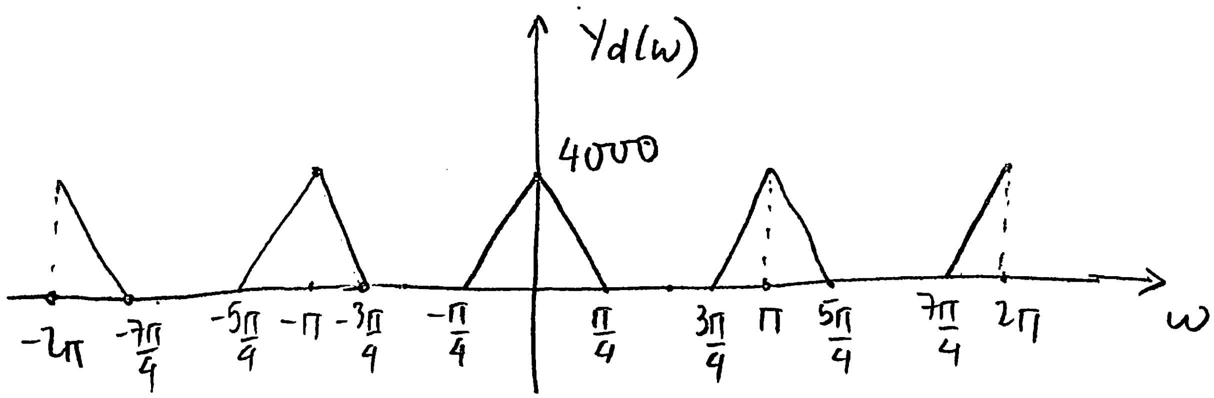
d)

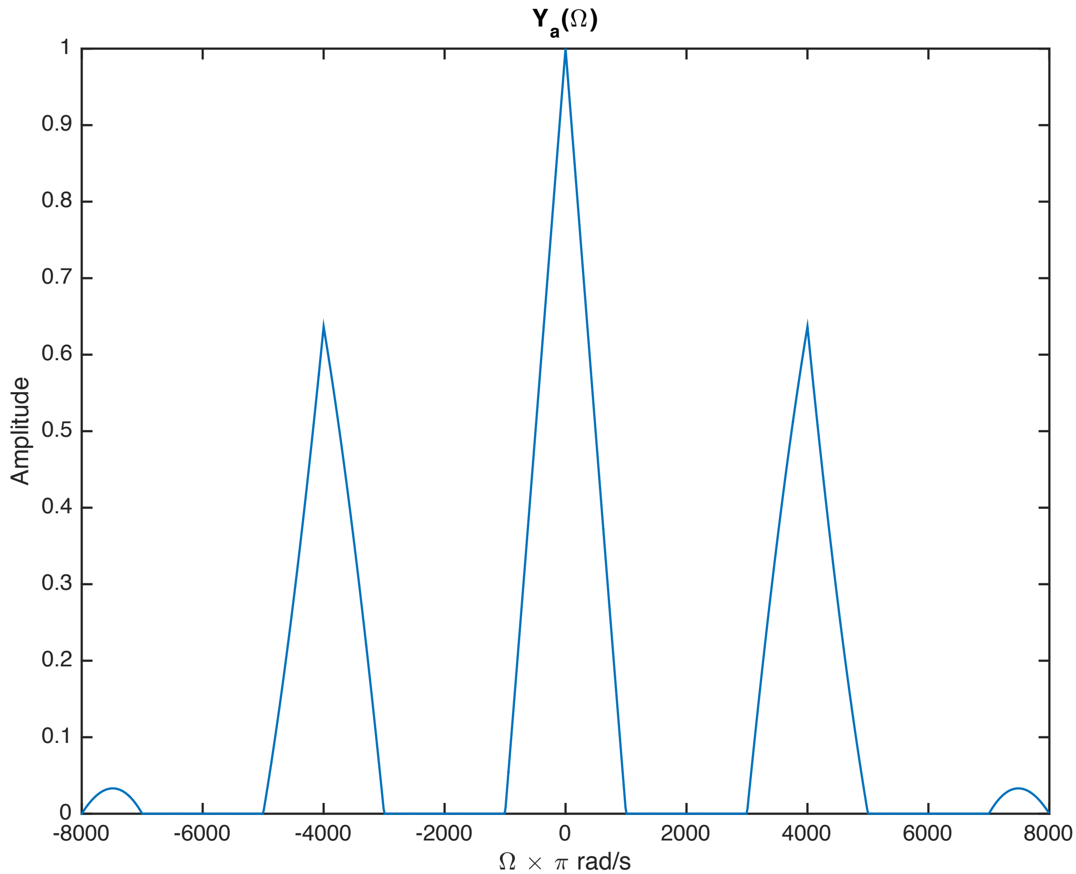

Xd(w)
4000
ω
-π/2
-π/4
π/4
π/2


Yd(w)
4000
ω
-π/4
π/4


Ya(Ω)
1
Ω
-1000π
1000π

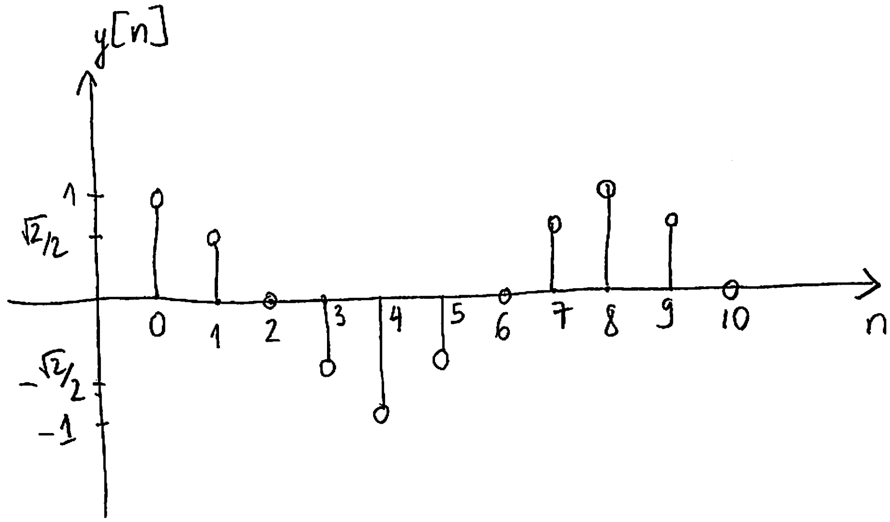
e)

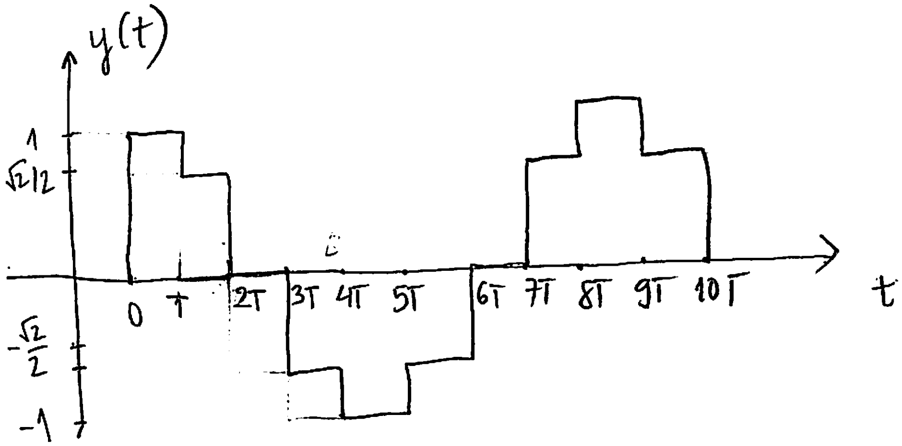

Yd(w)
4000
ω
-2π
-7π/4
-5π/4
-π
-3π/4
-π/4
π/4
3π/4
π
5π/4
7π/4
2π



$\mathbf{Y_a}(\Omega)$
Amplitude
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-8000
-6000
-4000
-2000
0
2000
4000
6000
8000
$\Omega \times \pi$ rad/s

(3) $y[n] = \cos\frac{\pi}{4}n.$





Problem 4. SOLUTION

Use the result in the notes (page 74). $h(t) = rect(\frac{t}{T} - \frac{1}{2})$.
The Fourier transform is $H(\Omega) = \frac{1-e^{-j\Omega T}}{j\Omega}$.
The DTFT for $y[n] = \cos(\frac{\pi}{4}n)$ is $Y_d(\omega) = \sum_{k=-\infty}^{\infty} \pi[\delta(\omega - \frac{\pi}{4} + 2k\pi) + \delta(\omega + \frac{\pi}{4} + 2k\pi)]$.
$y_{ZOH}(t) = \sum_{n=-\infty}^{\infty} y[n]h(t - nT)$. By the result in page 74, the Fourier transform

$$\begin{aligned} Y_{ZOH}(\Omega) &= H(\Omega)Y_d(\Omega T) \\ &= Te^{-j\Omega T/2}\text{sinc}(\frac{\Omega T}{2}) \sum_{k=-\infty}^{\infty} \pi[\delta(\Omega T - \frac{\pi}{4} + 2k\pi) + \delta(\Omega T + \frac{\pi}{4} + 2k\pi)] \\ &= \pi e^{-j\Omega T/2}\text{sinc}(\frac{\Omega T}{2}) \sum_{k=-\infty}^{\infty} [\delta(\Omega - \frac{\pi}{4T} + 2k\pi/T) + \delta(\Omega + \frac{\pi}{4T} + 2k\pi/T)] \end{aligned}$$

Solution 2

$T = \frac{1}{f} = \frac{1}{12}$s.
Define continuous time signal from the samples:
$y_s(t) = \sum_{n=-\infty}^{\infty} \cos(\frac{\pi}{4}n)\delta(t - nT) = \cos(3\pi t) \sum_{n=-\infty}^{\infty} \delta(t - \frac{1}{12}n)$.

$$\begin{aligned} Y_s(\Omega) &= \frac{1}{2\pi}\pi[\delta(\Omega - 3\pi) + \delta(\Omega + 3\pi)] * (24\pi) \sum_{n=-\infty}^{\infty} \delta(\Omega - 24\pi n) \\ &= 12\pi \sum_{n=-\infty}^{\infty} [\delta(\Omega - 3\pi - 24\pi n) + \delta(\Omega + 3\pi - 24\pi n)]. \end{aligned}$$

ZOH corresponds to a filter $h_{ZOH}(t) = rect(\frac{t}{T} - \frac{1}{2})$.
$H_{ZOH}(\Omega) = \frac{1-e^{-j\frac{\Omega}{12}}}{j\Omega}$.
Therefore Fourier transform of the ZOH is

$$\begin{aligned} Y_{ZOH}(\Omega) &= Y_s(\Omega)H_{ZOH}(\Omega) \\ &= \pi e^{-j\Omega/24}\text{sinc}(\frac{\Omega}{24}) \sum_{n=-\infty}^{\infty} [\delta(\Omega - 3\pi - 24\pi n) + \delta(\Omega + 3\pi - 24\pi n)]. \end{aligned}$$

On the interval $0 \leq |\Omega| \leq 48\pi$, the components are

$$\begin{aligned} Y_{ZOH}(\Omega) =& 4\frac{1 - e^{-j\frac{\pi}{4}}}{j}\delta(\Omega - 3\pi) + 4\frac{1 - e^{j\frac{\pi}{4}}}{-j}\delta(\Omega + 3\pi) \\ &+ \frac{4}{7}\frac{1 - e^{-j\frac{7\pi}{4}}}{j}\delta(\Omega - 21\pi) + \frac{4}{7}\frac{1 - e^{j\frac{7\pi}{4}}}{-j}\delta(\Omega + 21\pi) \\ &+ \frac{4}{9}\frac{1 - e^{-j\frac{9\pi}{4}}}{j}\delta(\Omega - 27\pi) + \frac{4}{9}\frac{1 - e^{j\frac{9\pi}{4}}}{-j}\delta(\Omega + 27\pi) \\ &+ \frac{4}{15}\frac{1 - e^{-j\frac{15\pi}{4}}}{j}\delta(\Omega - 45\pi) + \frac{4}{15}\frac{1 - e^{j\frac{15\pi}{4}}}{-j}\delta(\Omega + 45\pi). \end{aligned}$$

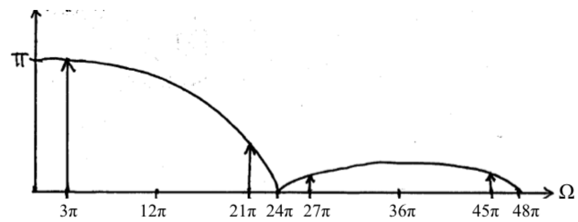


Figure 3: Problem 3: Magnitude of the Fourier transform of the ZOH output.

The magnitude is

$$\begin{aligned}|Y_{ZOH}(\Omega)| =& 3.0615\,\delta(\Omega - 3\pi) + 3.0615\,\delta(\Omega + 3\pi)\\ &+ 0.4374\,\delta(\Omega - 21\pi) + 0.4374\,\delta(\Omega + 21\pi)\\ &+ 0.3402\,\delta(\Omega - 27\pi) + 0.3402\,\delta(\Omega + 27\pi)\\ &+ 0.2041\,\delta(\Omega - 45\pi) + 0.2041\,\delta(\Omega + 45\pi)\end{aligned}$$

Output of and ideal D/A is $y_a(t) = \cos(3\pi t)$. The Fourier transform is

$$Y_a(\Omega) = \pi\delta(\Omega - 3\pi) + \pi\delta(\Omega + \pi).$$

Magnitude of the largest spurious component is 0.4374. The magnitude of the Fourier transform of the output of ZOH is shown in figure 3

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a) $H(z) = \frac{z^2+3z}{z^2+3z+2} = 1 - \frac{2}{z^2+3z+2}$

IIR

b) $H(z) = \frac{z+1}{z^2-\frac{z}{4}-\frac{1}{8}} = \frac{z+1}{(z-\frac{1}{2})(z+\frac{1}{4})}$

IIR

c) $H(z) = \frac{1}{3}(1-z^{-1}+z^{-2})$

FIR

d) $H(z) = \frac{1}{3}(1+z^{-1}+z^{-2})$

FIR