UCD School of Electrical, Electronic
& Communications Engineering

EEEN30110 Signals & Systems

Lab 2 Signals and Systems Report

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Declaration:
I declare that the work described in this report was done by the person named above, and that the description and comments in this report are my own work, except where otherwise acknowledged. I have read and understand the consequences of plagiarism as discussed in the EECE School Policy on Plagiarism, the UCD Plagiarism Policy and the UCD Briefing Document on Academic Integrity and Plagiarism. I also understand the definition of plagiarism.

Signed: ______________________________

Date: ______________________________

Lab 2 Signals and Systems Report

Objective:

To investigate the trigonometric Fourier series.

Question 1

The signal $q(t)$ is periodic of period 3π. Over one period it is given by:

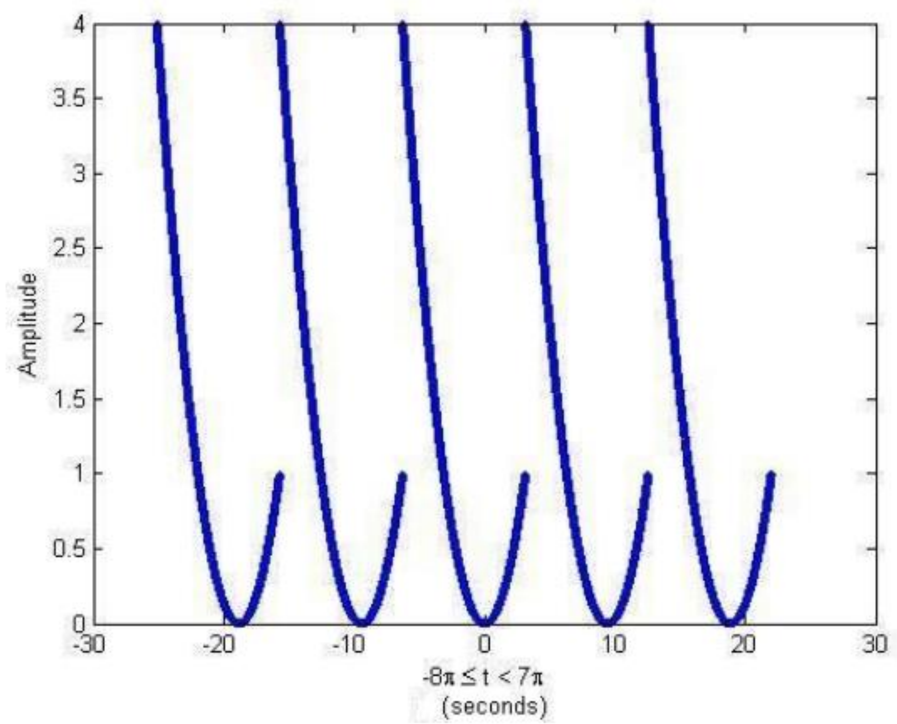
$$q(t) = \frac{t^2}{\pi^2} \qquad -2\pi \le t < \pi$$

Plot five cycles of the signal running over the time interval -8π to 7π.

We were given the signal which was periodic of period 3π:

$$q(t) = \frac{t^2}{\pi^2} \quad \text{for } -2\pi \le t < \pi$$

I utilized Matlab to plot the function for 5 cycles i.e. from -8π *to* 7π.



Question 2:

Analytically determine the trigonometric Fourier series of this signal $q(t)$.

To determine the trigonometric Fourier series of the signal $f(t)$ we can use definition 3:1 in the notes which states that any periodic signal f of period T the Fourier coefficients of f are the associated numbers:

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jnwt}dt$$

Then subbing in for our period T (3π) and our function f (q) to get:

$$c_n = \frac{1}{T}\int_{-2\pi}^{\pi} \frac{t^2}{\pi^2}e^{-jnwt}dt$$

Then using integration by parts I ended up with the following function:

$$c_n = \left(\frac{1}{3\pi}\right)\left(\frac{e^{-\frac{2}{3}jn\pi} - 9j + 6n\pi + 2j\pi^2n^2}{4n^3\pi^3} - \frac{e^{\frac{4}{3}jn\pi}(-9j + 2j\pi n + 2jn^2(2\pi)^2)}{4n^3\pi^2}\right)$$

Question 3:

Numerically determine the first seven Fourier coefficients of this signal $q(t)$.

After finding the formula used above I used Matlab's fft command and determined that the following first seven Fourier co-efficeints of the formula q(t):

Co-efficient	Value
C_0	1
C_1	-0.6415 – 0.1561j
C_2	0.149 + 0.2181j
C_3	0.0507 – 0.1592j
C_4	-0.1176 + 0.035j
C_5	0.0736 + 0.0636j
C_6	0.0127 – 0.0796j

Question 4:

Using this result write out the first thirteen terms of the trigonometric Fourier series of this signal $q(t)$.

As q(t) is a periodic function we are able to use the following formula to calculate the first thirteen terms of the Fourier series of q(t) is :

$$c_0 + \sum_{n=1}^{\infty} \alpha_n \cos(n\omega_0 t) + \beta_n \sin(n\omega_0 t)$$

Where:

$$c_0 = \frac{1}{T}\int_0^T q(t)dt$$

$$\alpha_n = 2Re(c_n) = \frac{2}{T}\int_0^T q(t)\cos(n\omega_0 T)\,dt$$

$$\beta_n = -2Im(c_n) = \frac{2}{T}\int_0^T q(t)\sin(n\omega_0 t)\,dt$$

Then taking the co-efficients I obtained in question 3 I found the following first thirteen terms of the Fourier series:

1
$-1.283\cos(\frac{2}{3}t) + 0.3122\sin(\frac{2}{3}t)$
$0.2994\cos(\frac{4}{3}t) - 0.4362\sin(\frac{4}{3}t)$
$0.1014\cos(2t) + 0.3184\sin(2t)$
$-0.2352\cos(\frac{8}{3}t) - 0.07\sin(\frac{8}{3}t)$
$0.1472\cos(\frac{10}{3}t) - 0.1272\sin(\frac{10}{3}t)$
$0.0254\cos(4t) + 0.1593\sin(4t)$

Question 5:

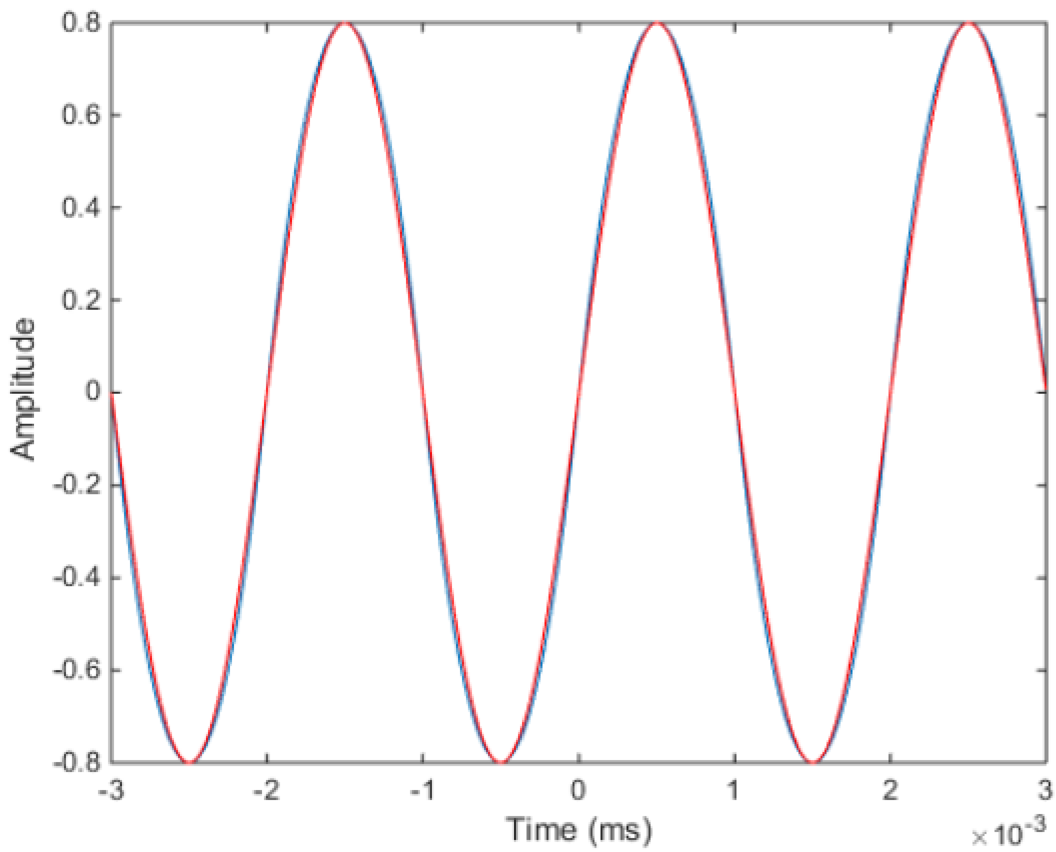
The voltage signal $f(t)$ is periodic and has frequency 500Hz. Over one period it is given by:

$$f(t)=\begin{cases}0.8(2000)^2 t(0.001-t) & 0\le t<0.001\\ 0.8(2000)^2 t(0.001+t) & -0.001\le t<0\end{cases}$$

Plot three cycles of the signal for t in the range -0.003 to 0.003. Confirm that the signal *looks like* a sinusoid of amplitude 0.8 V and frequency 500 Hz by plotting three cycles of a 0.8V amplitude, 500 Hz sinusoid in red or green *on the same axis*. You will need to use the **hold on** command to ensure that the sinusoid is directed to the same figure. You will also need to include the character string **'r'** or **'g'** to ensure that the data plots in red or green. As usual the command **help** will give further information.

The signal *f*(t) is a voltage signal with a frequency of 500Hz. We are told it is periodic and defined over one period as:

$$f(t)=\begin{cases}0.8(2000)^2 t(0.001-t) & 0\le t<0.001\\ 0.8(2000)^2 t(0.001+t) & -0.001\le t<0\end{cases}$$



By plotting both *f*(t) in blue and the 0.8V amplitude, 500Hz sinusoid signal in red on the same figure as instructed, it is clear to see that the signals appear to be almost identical.

Question 6:

Generate a vector of samples of the signal *f*(*t*) comprising 512 consecutive cycles. What is the time duration of this signal? If you have headphones (and if the version of Matlab which you are running supports it) you may listen to this signal using the **sound** command. The **help** command tells you how to do this. You will find that you need to give the **sound** command both the samples of the signal to be sounded and the frequency at which those samples were taken. This frequency is the reciprocal of the time-gap between your samples. Likewise generate the samples of 512 consecutive cycles of the 0.8 V amplitude 500 Hz sinusoid and listen to this, again using the sound command. You should find that not only does signal *f*(*t*) look like a 0.8 V amplitude, 500 Hz sinusoid, ***it also sounds like one***.

I generated a vector of samples comprising of 512 cycles of the signal *f*(t). I then got the period of this signal and used this to calculate that the signal is 1.024 seconds long as:

$$\omega = \frac{1}{T} = \frac{1}{500} = 0.002$$

After listening to the sounds I noticed that both sounded the same and concurrently our signal *f*(t) seems to be a 0.8V amplitude, 500Hz signal.

Question 7:

Numerically determine the first ten Fourier coefficients of the signal *f*(*t*) and confirm that, notwithstanding the facts that it looks and sounds like a sinusoid, ***it is not a sinusoid***.

Using the same process as in Question 3 and utilizing Matlab's fft function I obtained the following first ten Fourier co-efficients:

c_0	0+0j
c_1	$0 - 0.4128j$
c_2	0+0j
c_3	$0 - 0.0153j$
c_4	0+0j
c_5	$0 - 0.0033j$
c_6	0+0j
c_7	$0 - 0.0012j$
c_8	0+0j
c_9	$0 - 0.0006j$

Obviously $f(t)$ can't be a sinusoid as for a sinusoid the only term in the Fourier series which is able to equal the signal $f(t)$ itself is $A\sin(\omega t)$.

Question 8:

By looking at the relative values of the first few Fourier coefficients offer an explanation as to why the signal looks and sounds sinusoidal.

As we can see from the coefficients above in question seven the value for c_0 is equal to 0. Furthermore it is clear that for each coefficient $a_n$is also equal to 0. This reduces our original equation for our Fourier series from:

$$c_0 + \sum_{n=1}^{\infty} \alpha_n \cos(n\omega_0 t) + \beta_\mathrm{n} \sin(n\omega_0 t)$$

To:

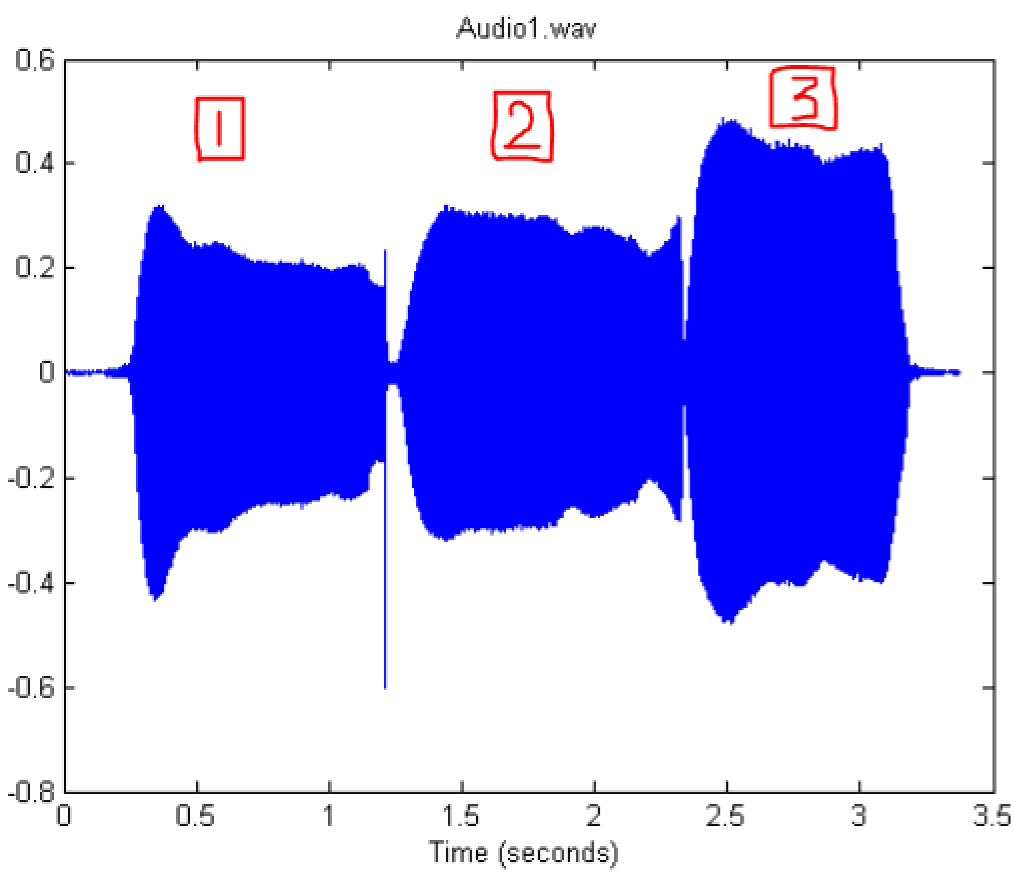
$$\sum_{n=1}^{\infty} \beta_\mathrm{n} \sin(n\omega_0 t)$$

Analysing our coefficients more closely it is clear to see that our second coefficient $c_1 = 0 - 0.4128j$ is much larger than the others. If we then calculate the magnitude of this trigonometric term we notice that it is 0.8256 which is quite close to the magnitude of our sinusoidal voltage wave (0.8V). We also note that as the signal is the sum of all the sinusoidal terms and as our dominating term is so close to 0.8V it is clear to see why it could be mistaken for our sinusoidal voltage signal of amplitude 0.8V and frequency 500Hz.

Question 9:

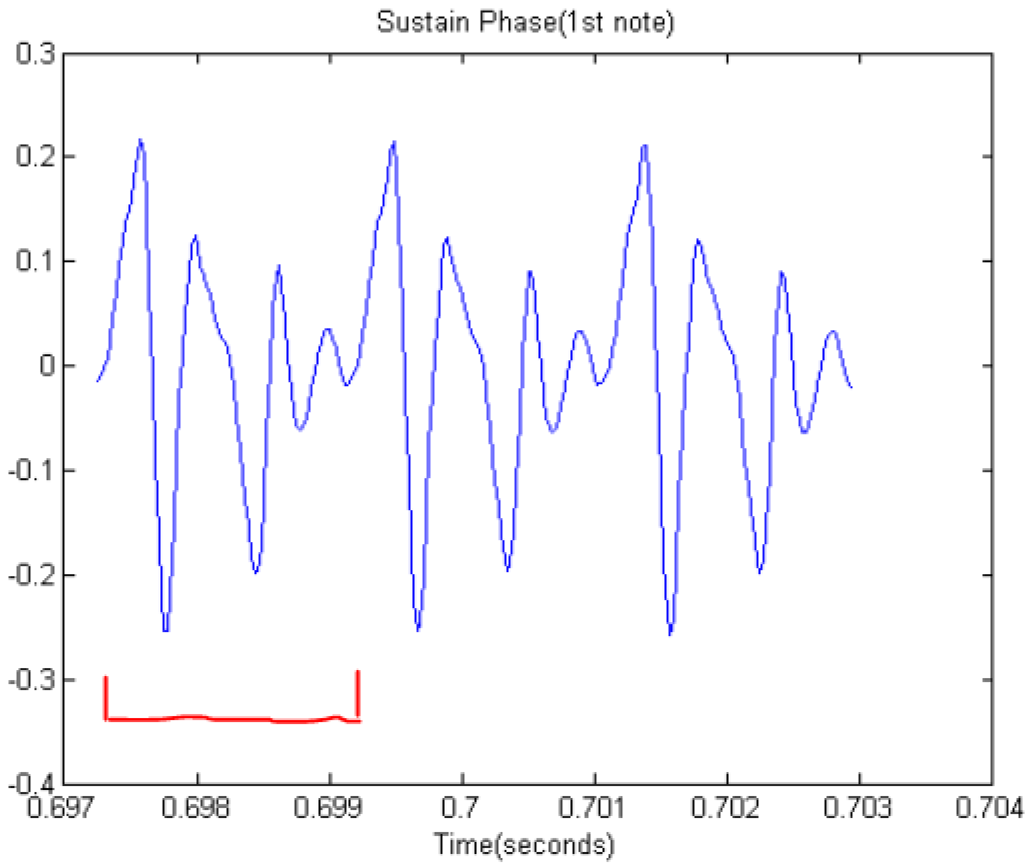
The audio file Audio1.wav comprises samples of three notes played on a Hohner harpsichord. Once you have downloaded the file Audio1.wav from Blackboard you should be able to use the **wavread** command to import the downloaded file into MATLAB from wherever you placed it. Subsequently you can play this file using the **sound** command (provided your version of MATLAB supports this command). Plot the samples of the downloaded signal and confirm that there are clearly three distinct audio signals in succession. The audio signal for each of the notes is very clearly divided into three phases, an early phase (the attack phase), a middle phase (the sustain phase) and an end phase (the decay phase). By plotting a suitable number of the samples from the sustain phase of the first note confirm that the signal is approximately *periodic* for the duration of this phase. Find the first five terms of the trigonometric Fourier series of this sustain-phase periodic signal. Find the fundamental frequency in hertz of this signal and hence determine what note was played (if you do not have a knowledge of music you may need to consult websites to find the mapping from notes to frequencies and *vice versa*).

Using Matlab to plot the signal I was able to obtain the following graph:



It is clear to see the three distinct audio signals from the graph. I have also denoted them 1, 2 & 3 in red ink with the signals lying below the respective numbers.

If we then plot the graph for the sustain phase of the fist note we see the signal is approximately periodic. An estimation for the period is around 1.9ms judging from the gaps in the peaks. The graph below shows three "periods". The length of a period if marked below in red.



Using the estimate of our period to be 1.9ms we can calculate a value for ω_0:

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1.9ms} = 3307Hz$$

Now using this I calculate the first 5 terms of our Fourier series using the formula:

$$c_0 + \sum_{n=1}^{\infty} \alpha_n \cos(n\omega_0 t) + \beta_n \sin(n\omega_0 t)$$

And we find:

0.0715
-0.0052cos(3307t) – 0.0008sin(3307t)
-0.0068cos(6614t)-0.0014sin(6614t)

So using my value for period time I calculated the frequency of the signal, and then applying the formula:

$$f = \frac{1}{T}$$

We get:

$$f = 526.3Hz$$

This corresponds on the frequency table to the musical note C_5.

Finally listening back to the audio file and comparing to an audio file of the note C_5 online I can conclude that they are at the very least extremely similar and to my ears the same.