ECE 310 University of Illinois at Urbana-Champaign Fall 2015

Profs. Bresler & Radhakrishnan Homework 5 Due: Friday, October 9
Reading: Chapter 4.1-4.5, 4.8, 4.10-4.14

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1. Compute the convolution $x[n] * h[n]$ for the $x[n]$ and $h[n]$ given below *(arrow indicates n=0, and all elements of a sequence that are not listed are assumed to be zero)*:

   (a) $x[n] = \{1, \underset{\uparrow}{2}, 3\}$, $h[n] = \{\underset{\uparrow}{1}, 0, -1, 1, 2\}$

   (b) $x[n] = n^3(u[n+2] - u[n-10])$, $h[n] = \{\underset{\uparrow}{-1}, 2, -3\}$

   (c) $x[n] = n0.5^n u[n]$ and $h[n] = n(u[n] - u[n-3])$

   (d) $x[n] = (-1)^{-n}u[n]$ and $h[n] = e^{-n}u[n-2]$

2. Suppose $x[n] = n(u[n-1] - u[n-10]) + 0.5^n u[n-30]$, $h[n] = 0.3^n(u[n] - u[n-7])$, and let $y[n] = x[n] * h[n]$ be the convolution of $x[n]$ and $h[n]$. For what values of n is $y[n]$ *nonzero*?

3. Given the unit pulse response $h[n]$ and the other side information, determine for each of the following systems whether it is: (i) causal; (ii) non-causal; or (iii) causality can not be determined from the given information. Prove your answer.

   (a) $h[n] = (-0.3)^{n+1}u[n-1]$

   (b) $h[n] = (-0.3)^{n-3}u[n+2]$

   (c) $h[n] = (0.3)^n u[(n-3)^2]$

   (d) $h[n] = (0.3)^{n+1}u[n-1] + (-0.5)^{n-4}u[n]$, and the system is linear.

   (e) $h[n] = (-0.3)^{n+1}u[n-2]$, and the system is shift invariant.

   (f) $h[n] = (0.3)^{n+2}u[n]$, and the system is linear and shift invariant.

4. The response of an LSI system to input $x[n] = u[n-3]$ is $y[n] = 2^{-n}u[n-5]$. Use the system properties (linearity and shift-invariance) to find the system's impulse response h[n].

5. Find the (one-sided) z-transform (if it exists) and the corresponding region of convergence for each of the following signals. Simplify your expressions. (Recall that for real-valued signals, the transform should only have real-valued coefficients.) Also specify, in each case, whether the discrete-time Fourier transform of the sequence exists.

   (a) $x[n] = \begin{cases} [1, 0, -2, 3] & 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$

   (b) $x[n] = 3^n u[n] + 0.5^n u[n-3]$

   (c) $x[n] = \left(\frac{1}{2}\right)^{n-1} \sin(\frac{n\pi}{4} + \frac{\pi}{3})u[n-2]$

   (d) $x[n] = n\left(\frac{1}{2}\right)^n u[n-3]$

   (e) $x[n] = 2^n\,(u[n] - u[n-30])$

6. Determine the ROCs of the following one-sided z-transforms, and find their inverses. Simplify your expressions. (Recall that for transforms with real coefficients, the corresponding signals should be real.)

   (a) $\dfrac{z^{-3}}{1+az^{-1}}$

   (b) $\dfrac{z^2+z}{z^2-2z-3}$

   (c) $\dfrac{1}{(1+0.5z^{-1})(1-0.2z^{-1})}$

   (d) $\dfrac{1}{(z+0.5)^2(z-0.25)}$

   (e) $\dfrac{1}{z^3+1/8}$