SEMESTER 1 EXAMINATION – 2014/2015

Module Code: EEEN30110, Module Title: Signals and Systems

Time Allowed: 2 hours

Answer all questions. The numbers in the right margin give an approximate indication of the relative importance in terms of grade steps of each part of a question. All rough work should be entered in your answer books

1. Find the first seven non-zero terms of the trigonometric Fourier series of

$$f(t) = \cos(t) \quad , \quad 0 \le t < \pi$$

where f is periodic with period π. Note that the period of the signal is ***not*** 2π so the signal is not simply a cosine. Hence determine the DC component, the fundamental, the second and third harmonics. **4**

- **N = 1024^2;**
- **tstep = pi/N;**
- **t = [0:tstep:pi – tstep];**
- **f = cos(t);**
- **FF = fft(f)/N;**
- **c_0 = FF(1);**
- **Alpha = 2*real(FF(2:14));**
- **Beta = -2*imag(FF(2:14));**
- $\boldsymbol{\omega_0 = 2\pi/N;}$

A "third order normalised Butterworth low-pass filter" has an input voltage e(t) and an output voltage $v_0(t)$. Because of the "normalisation" the filter is governed by the relatively simple ordinary differential equation:

$$\frac{d^3 v_0}{d t^3} + 2\frac{d^2 v_0}{d t^2} + 2\frac{d v_0}{d t} + v_0 = e(t)$$

Find the steady-state output of the filter when the input voltage $e(t)$ is given by:

$$e(t) = \sin^2(4t) \qquad \mathbf{1}$$

- **Find H(s) with initial conditions equal to zero**
- **Transform e(t) to s space by using identity $\boldsymbol{sin^2(t) = \frac{1}{2} - \frac{1}{2}cos(2t)}$**
- **Find convolution of H(s) and E(s) and inverse transform it**

and also when the input voltage $e(t)$ is equal to the signal $f(t)$ of the first part of question1. **2**

- **Find Laplace transform of all seven terms of the fourier series found in part 1 add magnitude and phase of transfer function**

2. Find the Fourier transform and the Fourier series of the following functions:

$$\sin(t), \quad 1 + \cos(3t), \quad \exp(jt) . \qquad \mathbf{1.5}$$

- $\boldsymbol{\mathcal{F}\left(Asin(\omega_0 t)\right) = \frac{A\pi}{j}\left(\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right)}$
- $\boldsymbol{\mathcal{F}\left(Acos(\omega_0 t)\right) = A\pi\left(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right)}$
- $\boldsymbol{\mathcal{F}(Aexp(j\omega)) = A2\pi\delta(\omega - \omega_0)}$
- $\boldsymbol{\mathcal{F}(1) = 2\pi\delta(\omega)}$

A “fifth order normalised Butterworth low-pass filter” has an input voltage e(t) and an output voltage $v_0(t)$. Because of the “normalisation” the filter is governed by the relatively simple ordinary differential equation:

$$\frac{d^5 v_0}{d t^5} + 3.236\frac{d^4 v_0}{d t^4} + 5.2359\frac{d^3 v_0}{d t^3} + 5.2359\frac{d^2 v_0}{d t^2} + 3.236\frac{d v_0}{d t} + v_0 = e(t)$$

Plot the "magnitude spectrum" of the filter and confirm its low-pass filtering properties. **1.5**

- **Make transfer function from given equation and vectors NUM and DEN**
- **Make frequency vector: w = [0:0.001:100]; (try different sizes and time steps based on plot**
- $\boldsymbol{H = polyval(NUM, i * w)./polyval(DEN, i * w);}$
- **plot(abs(H));**
- **Sketch plot**

To what extent is a signal of frequency 2 Hz suppressed by the filter? **0.5**

- **Sub in 2*2pi for $\boldsymbol{\omega}$ in the equation for $\boldsymbol{|H(j\omega)|}$**

Find a formula for the unit step response of the filter. **3.5**

- **Y(s) = H(s)*1/s**
- **Inverse Laplace of expression using partial fraction expansion**

3. A discrete-time system with input signal *q* and output signal *g* is described by the difference equation/recursion:

$$g(n) = \frac{2}{3}q(n) + \frac{1}{3}q(n-1) - g(n-1) - \frac{2}{9}g(n-2) \quad \text{for } n \geq 0.$$

Find the transfer function of the system. If the input signal is $q(n) = 3^{-n}u(n)$ (where $u(n)$ denotes the discrete-time unit step) and if the initial conditions are given by: $g(-1) = 0$, $g(-2) = 0.5$, find a formula for the output of the system for discrete time $n \geq 0$. **4**

- **Z transform the entire equation according to:**
  - $\boldsymbol{Z\{f(n-2)\} = z^{-2}F(Z) + z^{-1}f(-1) + f(-2)}$
- **This gives H(z)**
- **Now Z transform the source. These are the general transforms.**
  - $\boldsymbol{Z\{u(n)\} = \frac{1}{1-z^{-1}}}$
  - $\boldsymbol{Z\{\lambda^{-n}u(n)\} = \frac{1}{1-\lambda^{-1}z^{-1}}}$
  - $\boldsymbol{Z\{\delta(n)\} = 1}$

- $Z\{sin(\Omega_0 n)\} = \frac{sin(\Omega_0)z^{-1}}{1-2cos(\Omega_0)z^{-1}+z^{-2}}$
- $Z\{cos(\Omega_0 n)\} = \frac{1-cos(\Omega_0)z^{-1}}{1-2cos(\Omega_0)z^{-1}+z^{-2}}$

- **Convolution of Source and H(Z)**
- **Partial Fraction expansion, and perform inverse Z transform**
- **Inverse Z transforms are as follows:**
  - $Z^{-1}\{Az^{-k}\} = A\delta(n-k)$
  - $Z^{-1}\left\{\frac{A}{z^{-1}-\lambda}\right\} = \frac{A(-1)}{\lambda^{n+1}}u(n)$

Find the steady-state output signal of this system if the input signal $q(n)$ is the periodic signal of period 6 given by:

$$q(n) = 0.5 \quad \text{for } n = 0,1 \quad \text{and} \quad q(n) = -0.5 \quad \text{for } n = 2,3,4,5. \qquad \mathbf{3}$$

- **Create a vector of the input function q**
- **FF = fft(q)/6;**
- **c_0 = FF(1);**
- **Alpha = 2*real(FF(2:4));**
- **Beta = -2*imag(FF(2:4));**
- **Take transfer function NUM and DEN from previous part**
- **Create frequency vector: w = [0:3]*2*pi/6;**
- $H = polyval(NUM, exp(-i * w))./polyval(DEN, exp(-i * w));$
- **Abs = abs(H);**
- **Phase = angle(H);**
- **SS output =** $|H(1)| * c_0 + \sum\left|H\left(e^{j\Omega_k}\right)\right| * \alpha_k * cos(\Omega_k n) + \cdots$