1. For a wave propagating in a vacuum in x < 0, we are given an incident electric field phasor

$$\tilde{\mathbf{E}_\mathbf{i}} = (j\hat{z} - \hat{y})e^{-j2\pi x} \text{ V/m}.$$

The wave encounters a boundary at x = 0 with $\mu = \mu_0$ and the reflected phasor is given by

$$\tilde{\mathbf{E}_\mathbf{r}} = -\frac{1}{2}(j\hat{z} - \hat{y})e^{j2\pi x} \text{ V/m}.$$

a) The incident wave is RHCP because $\hat{y}$ leads $\hat{z}$ and the wave is propagating in the $\hat{x}$ direction. The reflected wave is therefore LHCP.

b) The frequency can be calculated from $f = \frac{v_p \beta}{2\pi}$, where $\beta = 2\pi$ and $v_p = c$ in a vacuum. Thus, $f = 300$ MHz.

c) The permittivity of the dielectric can be calculated using the reflection coefficient, Γ.

$$\begin{aligned}
\Gamma &= \frac{\eta - \eta_0}{\eta + \eta_0} = -\frac{1}{2} \\
\eta &= \frac{1}{3}\eta_0 \\
\epsilon_r &= 9 \\
\epsilon &= 9\epsilon_0
\end{aligned}$$

d) The transmitted electric phasor can be derived from the incident electric phasor with updated β and τ. So, $1+\Gamma = \tau = \frac{1}{2}$ and $\beta = \frac{2\pi f}{v_p} = \frac{6\pi \text{E8}}{c/\sqrt{9}} = 6\pi$ and therefore $\tilde{\mathbf{E}_\mathbf{t}} = \frac{1}{2}(j\hat{z} - \hat{y})e^{-j6\pi x}$ V/m.

e) The ratio of time-averaged incident power to time-averaged transmitted power can be found using the conservation of enery: $\frac{\eta_0}{\eta}\tau^2 = 1 - \Gamma^2 = 75\%$.

2. A plane wave field

$$\mathbf{H}(y, t) = \hat{x} 5 \cos(\omega t + \beta y) \frac{A}{m}$$

is propagating in a dielectric in the region $y > 0$. Here, $\epsilon = 4\epsilon_0$ and thus $\eta = \frac{1}{2}\eta_0$. At $y = 0$, there is a boundary to a Perfect conductor where $\eta = 0$. Then the reflection and transmission coefficients can be found to be

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0 - \frac{1}{2}\eta_0}{0 + \frac{1}{2}\eta_0} = -1 \quad \text{and} \quad \tau = 1 + \Gamma = 0.$$

a) It can be shown that the incident, reflected, and transmitted electric fields are

$$\tilde{\mathbf{E}}_i = -\hat{z}\frac{5\eta_0}{2}e^{j\beta y}\,\frac{\text{V}}{\text{m}}$$

$$\tilde{\mathbf{E}}_r = -\hat{z}\Gamma\frac{5\eta_0}{2}e^{-j\beta y} = \hat{z}\frac{5\eta_0}{2}e^{-j\beta y}\,\frac{\text{V}}{\text{m}}$$

and

$$\tilde{\mathbf{E}}_t = -\hat{z}\tau\frac{5\eta_0}{2}e^{j\beta_{PEC} y} = 0\,\frac{\text{V}}{\text{m}}.$$

b) The associated incident, reflected, and transmitted magnetic fields are

$$\tilde{\mathbf{H}}_i = \hat{x}5e^{j\beta y}\,\frac{\text{A}}{\text{m}},$$

$$\tilde{\mathbf{H}}_r = -\Gamma\hat{x}5e^{-j\beta y} = \hat{x}5e^{-j\beta y}\,\frac{\text{A}}{\text{m}},$$

and

$$\tilde{\mathbf{H}}_t = 0\,\frac{\text{A}}{\text{m}}.$$

c) The vector current density on the surface of the PEC is found using boundary conditions:

$$\begin{aligned}
\mathbf{J_s}(t,y) &= \hat{n} \times (\mathbf{H}^{+} - \mathbf{H}^{-}) \\
&= \hat{n} \times (\mathbf{0} - \mathbf{H_i} + \mathbf{H_r}|_{y=0}) \\
&= -\hat{y} \times -(\hat{x}5\cos(\omega t) + \hat{x}5\cos(wt)) \\
&= -10\cos(\omega t)\hat{z}\ \frac{\mathrm{A}}{\mathrm{m}}.
\end{aligned}$$

3. The inductance per unit length, capacitance per unit length, characteristic impedance, and propagation velocity for a transmission line are

$$\mathcal{L} = \frac{\mu}{\mathrm{GF}}, \quad \mathcal{C} = \epsilon\mathrm{GF}, \quad Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}, \quad \text{and} \quad v = \frac{1}{\sqrt{\mathcal{LC}}},$$

respectively, where GF is a geometrical factor. In the case of a coaxial cable with inner and outer conductor radii a and b, the geometrical factor is

$$\mathrm{GF} = \frac{2\pi}{\ln\left(\frac{b}{a}\right)}.$$

a) The RG-59 coax cable has radii $a = 0.016$ inches and $b = 0.056$ inches, and it is filled by a dielectric with $\epsilon = \epsilon_o$ and $\mu = \mu_o$. Since, the geometrical factor is

$$\mathrm{GF} = \frac{2\pi}{\ln\left(\frac{0.056}{0.016}\right)} = \frac{2\pi}{\ln(3.5)} = 5.015,$$

we can find that

$$\begin{aligned}
\mathcal{L} &= \frac{\mu_o}{\mathrm{GF}} = \frac{4\pi \times 10^{-7}}{2\pi/\ln(3.5)} = 2\ln(3.5) \times 10^{-7} = 2.506 \times 10^{-7}\,\frac{\mathrm{H}}{\mathrm{m}} = 250.6\,\frac{\mathrm{nH}}{\mathrm{m}}, \\
\mathcal{C} &= \epsilon_o\mathrm{GF} \approx \frac{10^{-9}}{36\pi}\frac{2\pi}{\ln(3.5)} = \frac{10^{-9}}{18\ln(3.5)} = 4.43 \times 10^{-11}\,\frac{\mathrm{F}}{\mathrm{m}} = 44.3\,\frac{\mathrm{pF}}{\mathrm{m}}, \\
Z_o &= \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu_o}{\epsilon_o}} \approx \frac{\ln(3.5)}{2\pi}120\pi = 60\ln(3.5) = 75.17\,\Omega, \\
v &= \frac{1}{\sqrt{\epsilon_o\mu_o}} = c \approx 3 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}.
\end{aligned}$$

b) In the case of the RG-58 coax cable, the radii are also $a = 0.016$ inches and $b = 0.056$ inches, but the dielectric filling it has $\epsilon = 2.25\epsilon_o$ and $\mu = \mu_o$. Since geometrical factor is

$$\mathrm{GF} = \frac{2\pi}{\ln\left(\frac{0.056}{0.016}\right)} = \frac{2\pi}{\ln(3.5)} = 5.015,$$

we can find that

$$\begin{aligned}
\mathcal{L} &= \frac{\mu_o}{\mathrm{GF}} = \frac{4\pi \times 10^{-7}}{2\pi/\ln(3.5)} = 2\ln(3.5) \times 10^{-7} = 2.506 \times 10^{-7}\,\frac{\mathrm{H}}{\mathrm{m}} = 250.6\,\frac{\mathrm{nH}}{\mathrm{m}}, \\
\mathcal{C} &= 2.25\epsilon_o\mathrm{GF} \approx \frac{2.25 \times 10^{-9}}{36\pi}\frac{2\pi}{\ln(3.5)} = \frac{1.25 \times 10^{-10}}{\ln(3.5)} = 9.98 \times 10^{-11}\,\frac{\mathrm{F}}{\mathrm{m}} = 99.8\,\frac{\mathrm{pF}}{\mathrm{m}}, \\
Z_o &= \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu_o}{2.25\epsilon_o}} \approx \frac{\ln(3.5)}{2\pi}\frac{120\pi}{\sqrt{2.25}} = 40\ln(3.5) = 50.11\,\Omega, \\
v &= \frac{1}{\sqrt{2.25\epsilon_o\mu_o}} = \frac{2}{3}c \approx 2 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}.
\end{aligned}$$

4. For twin-lead transmission lines, the geometrical factor is given by

$$\mathrm{GF} = \frac{\pi}{\cosh^{-1}\left(\frac{D}{2a}\right)}.$$

Since

$$Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu}{\epsilon}} = \frac{\cosh^{-1}\left(\frac{D}{2a}\right)}{\pi}\sqrt{\frac{\mu}{\epsilon}},$$

we can can find that

$$D = 2a\,\cosh\left(Z_o \pi\sqrt{\frac{\epsilon}{\mu}}\right).$$

Assuming $\epsilon = \epsilon_o$, $\mu = \mu_o$, and $a = 1\,\mathrm{mm} = 1\mathrm{E}-3\,\mathrm{m}$, let us calculate D as follows:

a) For $Z_o = 50\,\Omega$,

$$D = 2\dot{}10^{-3}\,\cosh\left(\frac{50\pi}{120\pi}\right) = 2.20\times10^{-3}\,\mathrm{m} = 2.2\,\mathrm{mm}.$$

b) For $Z_o = 300\,\Omega$,

$$D = 2\dot{}10^{-3}\,\cosh\left(\frac{300\pi}{120\pi}\right) = 12.3\times10^{-3}\,\mathrm{m} = 1.23\,\mathrm{cm}.$$

c) For $Z_o = 400\,\Omega$,

$$D = 2\dot{}10^{-3}\,\cosh\left(\frac{400\pi}{120\pi}\right) = 28.1\times10^{-3}\,\mathrm{m} = 2.81\,\mathrm{cm}.$$

5. The voltage and current waves $V(z,t)$ and $I(z,t)$ that propagate on a transmission line satisfy the following set of partial differential equations (PDE's)

$$-\frac{\partial V}{\partial z} = \mathcal{L}\frac{\partial I}{\partial t} \qquad \text{and} \qquad -\frac{\partial I}{\partial z} = \mathcal{C}\frac{\partial V}{\partial t}.$$

If $V(z,t) = 3\sin(\omega t + \beta z)$, we can use the first differential equation to find that

$$\frac{\partial I}{\partial t} = -\frac{1}{\mathcal{L}}\frac{\partial V}{\partial z} = -3\frac{\beta}{\mathcal{L}}\cos(\omega t + \beta z),$$

then, after integrating over time, we get

$$I(z,t) = -\frac{3\beta}{\omega\mathcal{L}}\sin(\omega t + \beta z).$$

Plugging this result into the second differential equation we have that

$$\frac{\partial V(z,t)}{\partial t} = -\frac{1}{\mathcal{C}}\frac{\partial I}{\partial z} = \frac{3\beta^2}{\omega\mathcal{L}\mathcal{C}}\cos(\omega t + \beta z),$$

and integrating over time, we finally obtain that

$$V(z,t) = \frac{3\beta^2}{\omega^2\mathcal{L}\mathcal{C}}\sin(\omega t + \beta z).$$

This result implies that

$$\frac{3\beta^2}{\omega^2\mathcal{L}\mathcal{C}} = 3, \qquad \text{then} \qquad \beta = \omega\sqrt{\mathcal{L}\mathcal{C}}.$$

In addition, the expression for the current becomes

$$I(z,t) = -3\sqrt{\frac{\mathcal{C}}{\mathcal{L}}}\sin(\omega t + \beta z).$$