ECE 310 **University of Illinois at Urbana-Champaign** **Fall 2015**

Profs. Bresler & Radhakrishnan **Homework 12** **Due: Wednesday Dec 9**

Reading: Chapter 14

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1. You are to compute by hand the DFT of the following sequence:

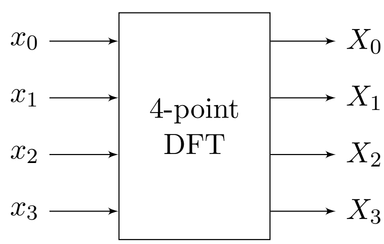
$$\mathbf{x} = [1 \quad 2 \quad 3 \quad 4]$$

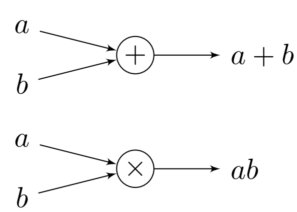
(a) Compute the DFT of $\mathbf{x}$ using the definition of the DFT (do not use an FFT). Show your work and give exact answers.

(b) Use a decimation-in-time radix-2 FFT to compute the DFT of $\mathbf{x}$. Neatly draw and label the flow graph used to compute your answer.

(c) Repeat part (b) using decimation-in-frequency.

2. Suppose that you wish to design a circuit to perform a length-8 DFT. You are given pair of chips that compute the DFT of a length-4 complex input sequence. The inputs and the outputs of this chip are all complex numbers. You also have access to complex multiplication and addition circuits, each of which has two complex inputs and one complex output.





(a) Your goal is to use as few complex multiplication circuits as possible. Fortunately, you don't need to use a multiplier circuit to multiply by $+1$, -1, $+j$, or $-j$. Explain why these are trivial multiplications.

(b) Show how you would connect two chips and the multiplication and addition circuits to compute a length-8 decimation-in-time FFT. Explain your solution. How many nontrivial multiplications are required?

(c) Repeat part (b) using decimation-in-frequency.

3. In this problem, you will derive a radix-3 decimation-in-frequency FFT. Consider a length-N sequence $x[0], \ldots, x[N-1]$, where N is a power of 3. Denote the DFT by $X[0], \ldots, X[N-1]$.

(a) Derive an expression for $X[3r]$, that is, for $X[0], X[3], X[6], \ldots$ as $\frac{N}{3}$- point DFT of time-aliased samples.

(b) Derive an expression for $X[3r+1]$ that is, for $X[1], X[4], X[7], \ldots$ as $\frac{N}{3}$- point DFT of aliased and modulated samples. Repeat for the remaining outputs, $X[3r+2]$

(c) Draw a flow graph for the radix-3 decimation-in frequency FFT of length-9 sequence. To save space, you may represent each length-3 DFT as a block with three inputs and three outputs and then draw a second diagram showing the length-3 DFT.

4. You are given two sequences $\{x[n]\}_{n=0}^{n=349}$ and $\{h[n]\}_{n=0}^{n=63}$ and are asked to compute their linear convolution $y[n] = x[n] * h[n]$. You decide to use the DFT to speed up the computation.

(a) What is the length of the sequence $y[n]$?

(b) Find the smallest number of zeros that should be padded to each sequence so that the linear convolution can be computed using the DFT.

(c) To further speed computation, you decide to use a radix-2 FFT to compute the DFT. How should the sequences be padded so that their linear convolution can be computed using the smallest possible radix-2 FFT?

5. In high performance DSP applications, we must deal with very high sample rates and large filters. WIth so much data, computational efficiency is critical. Consider a vector $\mathbf{x}$ of 40,000 samples that is to be filtered with a length 256 FIR filter $\mathbf{h}$.

(a) First, consider the old-fashioned way: direct linear convolution. Determine the number of real multiplications and additions required to convolve $\mathbf{x}$ with $\mathbf{h}$.

(b) Next, consider "fast convolution" using two complex-valued radix-2 FFTs and one inverse FFT. The transform length is chosen as an appropriate power of two. Assume reduced butterflies with one complex multiplication per butterfly. Determine the number of real multiplications and additions required to implement this convolution.

(c) Since the data vector is much longer than the filter length, the overlap-add algorithm yields a more efficient solution. Determine the size of the power-of-two length FFT that minimizes the total number of multiplies per output sample using the overlap-add method of fast convolution. Estimate the number of real multiplications required by the overlap-add method. Justify your answer.

(d) Compare the computational efficiency of these three methods of convolution. Note that multiplication is generally much more expensive than addition.