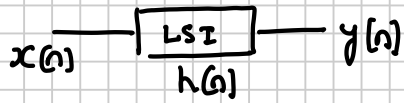
ECE 310 : Lecture 23: FIR filters - Generalized linear phase

Consider the following LSI system



The output can be written as

$$y(n) = \sum_{m=-\infty}^{\infty} h(m)\, x(n-m)$$

or

$$\boxed{Y_d(\omega) = H_d(\omega) X_d(\omega)}$$

$H_d(\omega)$: Frequency Response

Frequency Response can be written as

$$H_d(\omega) = |H_d(\omega)| \angle H_d(\omega)$$

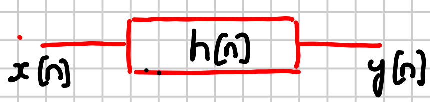
① Magnitude and phase response have the effect of altering the contributions of Complex exponential signals in input.
② Can be useful if the input is altered in a manner that is helpful.

Let $x(n) = \cos\omega_1 n + \cos\omega_2 n + \cos\omega_3 n$, $0 < \omega_1 < \omega_2 < \omega_3 < \pi$

Let, $\cos\omega_3 n \rightarrow$ noise : needs to be removed

Desired Signal:

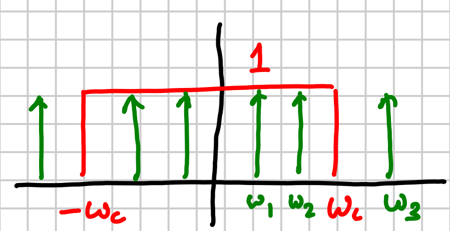
$$\boxed{y(n) = \cos\omega_1 n + \cos\omega_2 n}$$



$$\therefore \; Y_d(\omega) = |H_d(\omega_1)| \cos(\omega_1 n + \angle H_d(\omega_1))$$
$$+ |H_d(\omega_2)| \cos(\omega_2 n + \angle H_d(\omega_2))$$
$$+ |H_d(\omega_3)| \cos(\omega_3 n + \angle H_d(\omega_3))$$

Ideally we must have:

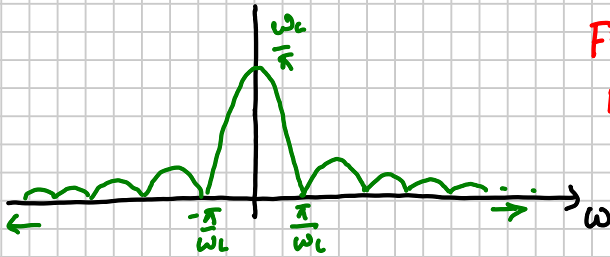
$$|H_d(\omega)| = \begin{cases} 1 & 0 \le \omega \le \omega_c \\ 0 & \text{else} \end{cases} \rightarrow$$



and

$$\angle H_d(\omega_1) = 0 \quad \& \quad \angle H_d(\omega_2) = 0$$

The ideal filter results in $h(n)$ shown below:



Filter not Causal

Need to truncate & Shift.

Will look at effect of Truncating / Windowing later. Look at shifting now.

Let us relax the requirement, we accept delayed version of the signal:

$$y[n] = \cos[\omega_1(n-M)] + \cos[\omega_2(n-M)] \qquad M>0$$

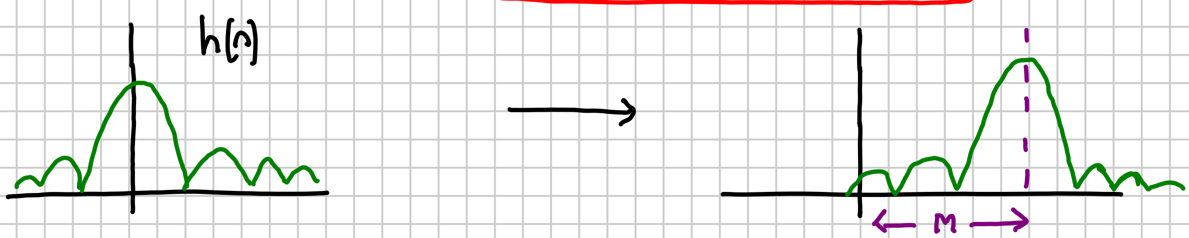
$$\therefore \quad \angle H_d(\omega_1) = -\omega_1 M$$

$$\angle H_d(\omega_2) = -\omega_2 M$$

or $\quad \angle H_d(\omega) = -\omega M$

$$\Rightarrow \boxed{H_d(\omega) = 1\, e^{-j\omega M}}$$

$$\Rightarrow \boxed{h(n) = \frac{1}{\pi} \operatorname{Sinc}[\omega_c(n-M)]}$$



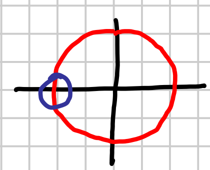
Shifting Impulse Response $\Rightarrow$ linear phase in frequency Response

i.e. $\quad H_d(\omega) = |H_d(\omega)|\, e^{-j\omega M}$

How realistic is Linear phase?

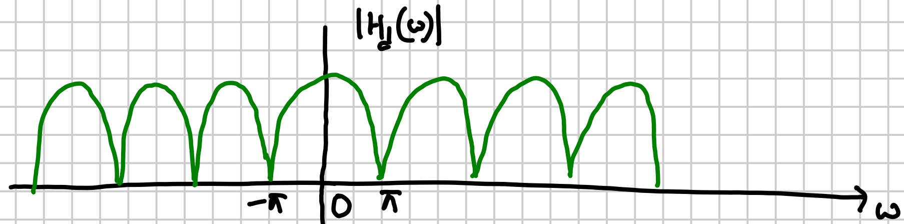
$$y(n) = \frac{x(n)}{2} + \frac{x(n-1)}{2}$$

$$\Rightarrow H(z) = \frac{1}{2}(1 + z^{-1})$$



$$\Rightarrow \quad H_d(\omega) = \frac{1}{2}\left(1 + e^{-j\omega}\right)$$

$$\Rightarrow \quad H_d(\omega) = \cos\frac{\omega}{2} \; e^{-j\omega/2}$$



$\Rightarrow$ Usually filter does not have exactly linear phase. It is enough to have linear phase in passband.