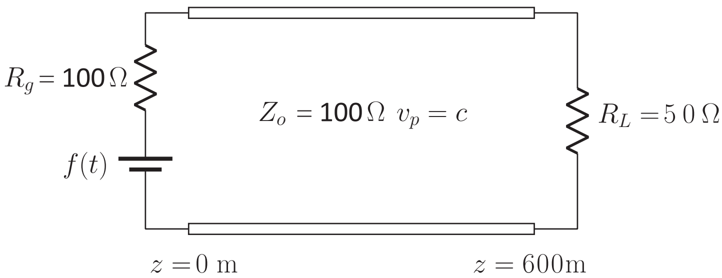
1. Let us consider a T.L. with characteristic impedance $Z_o = 100\,\Omega$, length $l = 600\,$m, and propagation velocity $v_p = c = 3\times10^8\,$m/s. A voltage source $f(t)$ with internal resistance $R_g = Z_o$ is connected at $z = 0$ and a load resistance $R_L = Z_o/2$ is placed at $z = l$ as shown in the following diagram.



a) For $f(t) = \delta(t)$, the initial voltage at the input of the T.L. is given by the voltage divider

$$\frac{Z_o}{R_g + Z_o}\delta(t) = \frac{1}{2}\delta(t).$$

In addition, the voltage reflection coefficients at the source-end and at the load-end are

$$\Gamma_{Vg} = \frac{R_g - Z_o}{R_g + Z_o} = \frac{100-100}{100+100} = 0 \quad \text{and} \quad \Gamma_{VL} = \frac{R_L - Z_o}{R_L + Z_o} = \frac{50-100}{50+100} = -\frac{1}{3},$$

respectively. The corresponding current reflection coefficients are

$$\Gamma_{Cg} = -\Gamma_{Vg} = 0 \quad \text{and} \quad \Gamma_{CL} = -\Gamma_{VL} = \frac{1}{3}.$$

Using these information, we can build the following "bounce diagrams" for the voltage $V(z,t)$ and current $I(z,t)$ on the transmission line.

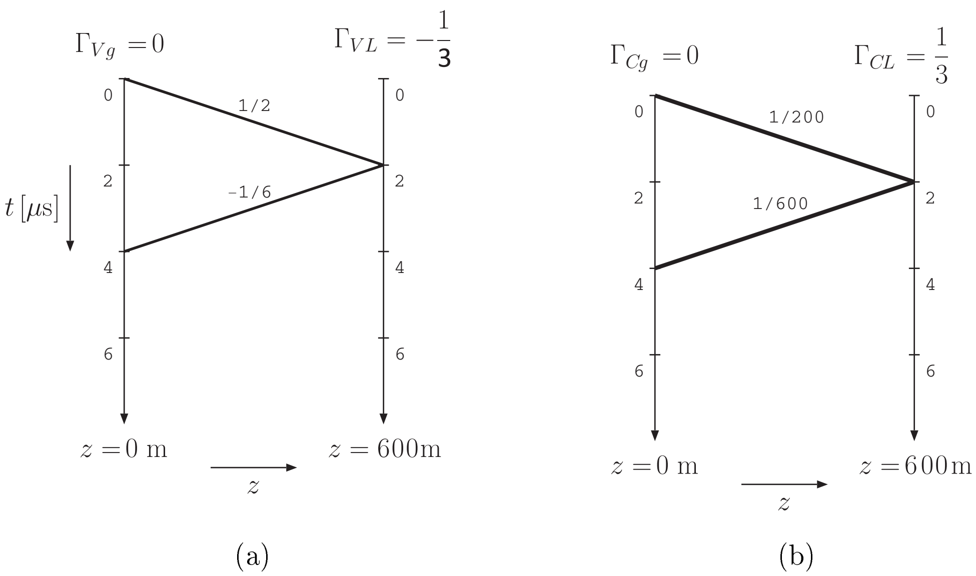


Figure 1: (a) Voltage and (b) current bounce diagrams for voltage source $f(t) = \delta(t)$.

b) The expressions for $V(z,t)$ and $I(z,t)$ are

$$V(z,t) = \frac{1}{2}\delta(t - \frac{z}{c}) - \frac{1}{6}\delta(t + \frac{z}{c} - 4\mu\text{s})$$
$$I(z,t) = \frac{1}{200}\delta(t - \frac{z}{c}) + \frac{1}{600}\delta(t + \frac{z}{c} - 4\mu\text{s}).$$

Evaluating these expressions at $z = \frac{l}{4} = 150\,\text{m}$, we have

$$V(\frac{l}{4}, t) = \frac{1}{2}\delta(t - 0.5\mu\text{s}) - \frac{1}{6}\delta(t - 3.5\mu\text{s})$$
$$I(\frac{l}{4}, t) = \frac{1}{200}\delta(t - 0.5\mu\text{s}) + \frac{1}{600}\delta(t - 3.5\mu\text{s}).$$

c) According to the previous result, the impulse voltage response at $z = \frac{l}{4}$ is

$$h_v(t) = \frac{1}{2}\delta(t - 0.5\mu\text{s}) - \frac{1}{6}\delta(t - 3.5\mu\text{s}).$$

Therefore, if the source voltage is $f(t) = 10u(t)\,\text{V}$, we can compute $V(\frac{l}{4}, t)$ as follows

$$V(\frac{l}{4}, t) = f(t) * h_v(t) = 5u(t - 0.5\mu\text{s}) - \frac{5}{3}u(t - 3.5\mu\text{s})\,\text{V}.$$

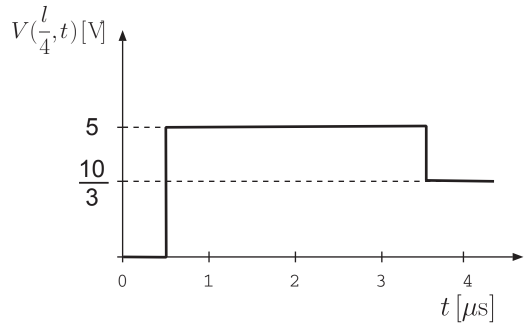


Figure 2: Voltage as function of time at $z = \frac{l}{4}$ for voltage source $f(t) = 30u(t)$.

d) We know that

$$V(z, t) = \frac{1}{2}\delta(t - \frac{z}{c}) - \frac{1}{6}\delta(t + \frac{z}{c} - 4\mu\text{s})$$
$$I(z, t) = \frac{1}{200}\delta(t - \frac{z}{c}) + \frac{1}{600}\delta(t + \frac{z}{c} - 4\mu\text{s}).$$

Now, if the source voltage is $f(t) = 10u(t)\,\text{V}$, we can compute $V(z, t)$ and $I(z, t)$ as follows

$$V(z, t) = 5u(t - \frac{z}{c}) - \frac{5}{3}u(t + \frac{z}{c} - 4\mu\text{s})\,\text{V}$$

$$I(z, t) = \frac{1}{20}u(t - \frac{z}{c}) + \frac{1}{60}u(t + \frac{z}{c} - 4\mu\text{s})\,\text{A}.$$

Therefore, by plotting $V(z, t)$ and $I(z, t)$ as a function of time, we can easily find the steady state values as

$$V(z, t) = 5 - \frac{5}{3} = \frac{10}{3}\text{V} \quad \textit{and} \quad I(z, t) = \frac{1}{20} + \frac{1}{60} = \frac{1}{15}\text{A}.$$

2. Let us consider the following circuit diagram where Z_o, R_L, and L are unknowns.

a) According to the voltage waveform plotted above, the incident pulse has an amplitude of 60 V. Since the ratio between this amplitude and the source voltage is given by the voltage divider formula $\tau_g = \frac{Z_o}{Z_o + 50} = \frac{60}{90} = \frac{2}{3}$, we can find that $Z_o = 100\,\Omega$.

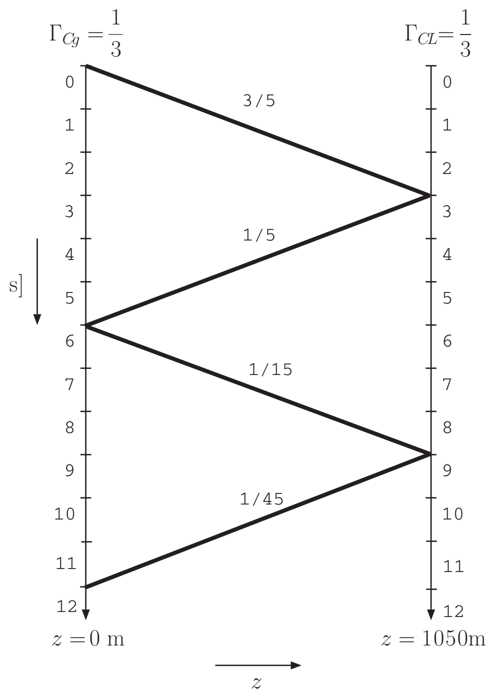
b) The amplitude of the first reflected pulse is $-20\,\text{V}$, therefore, the reflection coefficient at the load is $\Gamma_L = -\frac{20}{60} = -\frac{1}{3}$. Next, given that $\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o}$, we can find that $R_L = \frac{Z_o}{2} = 50\,\Omega$.

c) The time interval between the incident pulse and the second reflected pulse is $6\,\mu$s, since this time is equal to the two-way travel time, we have that the time it takes the pulse to travel from one end of the line to the other is $T = \tau_L = 3\,\mu$s.

d) Since at $z = 300\,$m, the incident pulse is delayed by $2\,\mu$s, the propagation speed is $v_p = \frac{300\,\text{m}}{2\,\mu\text{S}} = 150 \times 10^6\,$m/s. Then, we can find that the length of the line is $L = v_p \times \tau_L = 150 \times 3 = 450\,$m.

e) The reflection coefficient at the source is $\Gamma_g = -\frac{1}{3}$. Therefore, the next two voltage impulses are $-\frac{20}{9}\delta(t-10)$ V and $\frac{20}{27}\delta(t-14)$ V.

f) Bounce diagram for the current waveform $I(z,t)$.



g) The expression for the current waveform for $0 < z < L$ and $0 < t < 12\,\mu$s is

$$I(z,t) = \frac{3}{5}\delta(t - \frac{z}{v_p}) + \frac{1}{5}\delta(t + \frac{z}{v_p} - 6\,\mu\text{s}) + \frac{1}{15}\delta(t - \frac{z}{v_p} - 6\,\mu\text{s}) + \frac{1}{45}\delta(t + \frac{z}{v_p} - 12\,\mu\text{s})\,\text{A}.$$

3. A system of two connected transmission lines is shown in the figure below. A switch is closed at $t = 0$ and the positive voltages are measured for $5\,\mu s$ giving the bounce diagram in the figure. Using these figures, we can identify and compute the following parameters of the circuit.

a) Transmission time $T_1 = 2\,\mu$s and $T_2 = 3\,\mu$s.

b) $v_{p1} = \frac{400\,\text{m}}{2\,\mu\text{S}} = 200 \times 10^6\,$m/s and $v_{p2} = \frac{300\,\text{m}}{3\,\mu\text{S}} = 100 \times 10^6\,$m/s.

c) Since there is no reflection at the load R_L, the characteristic impedance of line 2 is $Z_2 = R_L = 60\,\Omega$.

d) At the interface, note that $1 + \Gamma_{12} = \frac{40}{60} = \frac{2}{3}$, thus the reflection coefficient from 1 to 2 is $\Gamma_{12} = -\frac{1}{3}$.

e) Since $\Gamma_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = -\frac{1}{3}$, the characteristic impedance of line 1 is $Z_1 = 2Z_2 = 120\,\Omega$.

f) The reflection coefficient at the source is $\Gamma_S = \frac{12}{-20} = -\frac{3}{5}$. Since $\Gamma_S = \frac{R_g - Z_1}{R_g + Z_1}$, the source resistance is $R_g = \frac{1}{4}Z_1 = 30\,\Omega$.

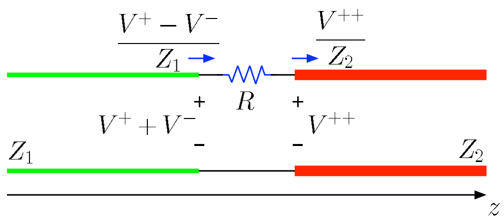
g) The source voltage is $V_o = 60\frac{R_g + Z_1}{Z_1} = 60\frac{30+120}{120} = 75\,$V.

h) Reflected voltage $V^{-+-} = -4\,$V.

i) Since, as $t \to \infty$, transmission lines become ordinary wires, the steady-state voltage on line 1 is $V_1 = V_o \frac{R_L}{R_g + R_L} = 75 \frac{60}{30+60} = 50\,\text{V}$.

j) Same as above, the steady-state voltage on line 2 is $V_2 = 50\,\text{V}$.

4. Let us consider two transmission lines with characteristic impedances Z_1 and Z_2 joined at a junction that includes a series resistance R.



a) At the junction, on the side of the first line, the total voltage is equal to the sum of the incident voltage V^+ plus the reflected voltage V^-. This total voltage is equal to the sum of the voltage across the resistor (V_R) plus the voltage transmitted to the second line (V^{++}). Thus, the KVL equation at the junction is

$$V^+ + V^- = V_R + V^{++}.$$

On the other hand, the current on the first line is equal to the sum of the incident current V^+/Z_1 plus the reflected current $-V^-/Z_1$, therefore, the total current is $(V^+ - V^-)/Z_1$. This current is equal to the current flowing through the resistor (V_R/R) and it is also equal to the current transmitted to the second line (V^{++}/Z_2). Thus, the KCL equation at the junction is

$$\frac{V^+}{Z_1} - \frac{V^-}{Z_1} = \frac{V_R}{R} = \frac{V^{++}}{Z_2}.$$

b) Combining the previous equations by eliminating $V^{++}$and V_R we have

$$\frac{V^+}{Z_1} - \frac{V^-}{Z_1} = \frac{V^+ + V^-}{R + Z_2} \quad \to \quad \left(\frac{1}{Z_1} - \frac{1}{Z_{eq}}\right) V^+ = \left(\frac{1}{Z_1} + \frac{1}{Z_{eq}}\right) V^-,$$

where $Z_{eq} = R + Z_2$. Thus, the reflection coefficient is

$$\Gamma_{12} = \frac{V^-}{V^+} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1},$$

Similarly, but eliminating V^- and v_R, we have

$$\frac{V^+}{Z_1} - \frac{V^{++}(R + Z_2)/Z_2 - v^+}{Z_1} = \frac{V^{++}}{Z_2} \quad \to \quad \frac{2}{Z_1} V^+ = \left(\frac{R + Z_2}{Z_1 Z_2} + \frac{1}{Z_2}\right) V^{++}.$$

Therefore, the transmitted coefficient is

$$\tau_{12} = \frac{V^{++}}{V^+} = \frac{2 Z_2}{R + Z_1 + Z_2}.$$

c) Considering $Z_1 = 50\,\Omega$, $Z_2 = 25\,\Omega$, and $R = 100\,\Omega$, we can find that $Z_{eq} = 125\,\Omega$. Then, we calculate

$$\Gamma_{12} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1} = \frac{125 - 50}{125 + 50} = \frac{3}{7},$$

and

$$\tau_{12} = \frac{2 Z_2}{R + Z_1 + Z_2} = \frac{2 \times 25}{100 + 50 + 25} = \frac{2}{7}.$$

Note that in this case $1 + \Gamma_{12} \neq \tau_{12}$.

5. We will solve as follows:

a) The injection coefficient can be read directly from the plot as $\tau_g = 0.5$ (because $V_{input}(t) = \tau_g u(t) \Rightarrow V_{input}(t) = \tau_g$). Also, we know

$$\tau_g = \frac{Z_o}{R_g + Z_o}.$$

Thus, the characteristic impedane of the T.L. $Z_o = R_g = 50\,\mathrm{Ohm}$.

b) The plot has a jump at $t = 2\,\mathrm{ns}$, and we can infer that it takes the wave $2\,\mathrm{ns}$ to start from the source, then get reflected by the defect, finally go back to the source. Based on this, we have

$$\frac{2d}{v} = 2\,\mathrm{ns}.$$

Therefore, $d = 0.2\,\mathrm{m}$.

c) Again, from the plot we know the reflection coefficient at the defect is $\Gamma_d = 1$ $((1 - 0.5)/0.5)$. On the other hand, we have

$$\Gamma_d = \frac{(R_d + Z_o) - Z_o}{(R_d + Z_o) + Z_o} = \frac{R_d}{R_d + 2Z_o}.$$

where R_d denotes the effective series resistance of the defect. We see R_d should be very/infinitely large. So the defect is actually an open circuit.

d) Since all the incident wave get reflected at the defect, there will be no voltage response at the load.