ECE 310: Lecture 28: Multirate Signal Processing

→ Reduction of sampling rate

Reduction of Sampling rate by integer factor:

Sampling rate can be reduced by using the system shown below,

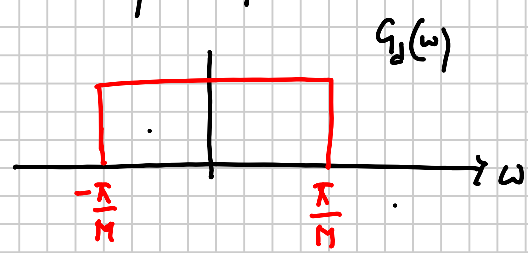
$$x(n) = x_a[nT_1]$$

$$\tilde{x}[n] = x_a[nT_2]$$

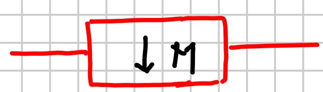
$\tilde{x}(n)$ is equivalent to sampling the continuous-time signal $x_a(t)$ at sampling rate T_2 given by,

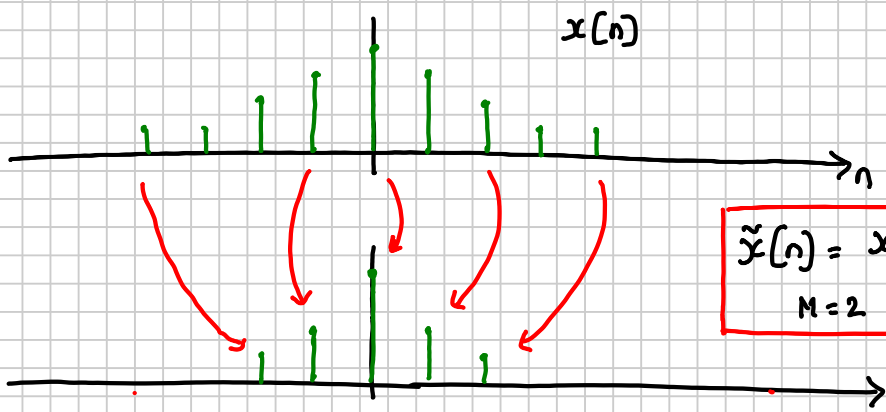
$\boxed{T_2 = MT_1}$ $\boxed{M = \text{Integer}}$

① $G_d(\omega)$: Low-pass filter



② Decimator:





$$\boxed{\begin{array}{c}\tilde{x}(n) = x(2n) \\ M = 2\end{array}}$$

In the frequency domain :

$$X_d(\omega) = \frac{1}{T_1} \sum_{k=-\infty}^{\infty} X_a\left[\frac{\omega}{T_1} - \frac{2\pi k}{T_1}\right]$$

$$\tilde{X}_d(\omega) = \frac{1}{T_2} \sum_{l=-\infty}^{\infty} X_a\left[\frac{\omega}{T_2} - \frac{2\pi l}{T_2}\right]$$

Now $T_2 = MT_1$

$$\tilde{X}_d(\omega) = \frac{1}{MT_1} \sum_{l=-\infty}^{\infty} X_a\left[\frac{\omega}{MT_1} - \frac{2\pi l}{MT_1}\right]$$

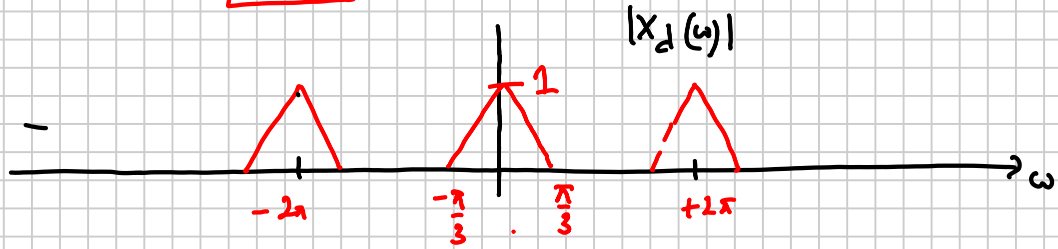
Note : $l = i + kM$, $-\infty < k < \infty$, $0 \leq i \leq M-1$

$$\therefore \quad \tilde{X}_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{T_1} \sum_{k=-\infty}^{\infty} \left(\frac{\omega}{MT_1} - \frac{2\pi i}{MT_1} - \frac{2\pi k}{T_1}\right]$$

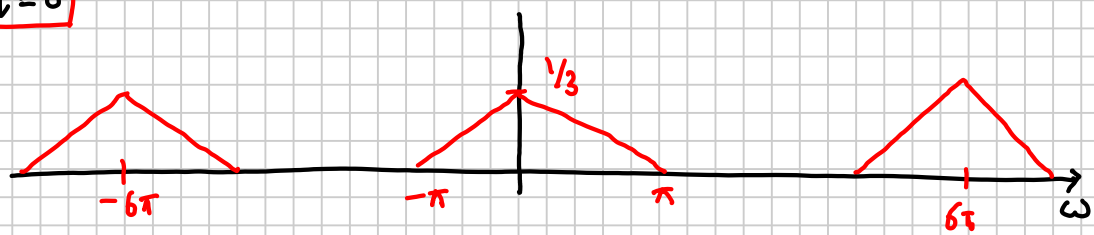
$$\Rightarrow \quad \tilde{X}_d(\omega) = \frac{1}{M} \sum_{i=0}^{M-1} X_d\left(\frac{\omega - 2\pi i}{MT_1} - \frac{2\pi k}{T_1}\right)$$

$$\Rightarrow \quad \boxed{\tilde{X}_d(\omega) = \frac{1}{M} X_d\left(\frac{\omega - 2\pi i}{M}\right)}$$

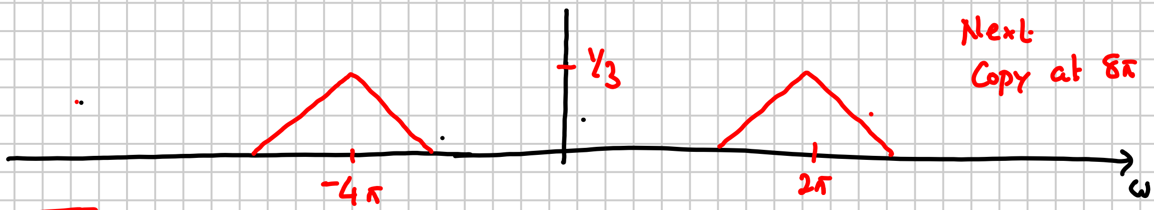
Example: $\boxed{M=3}$



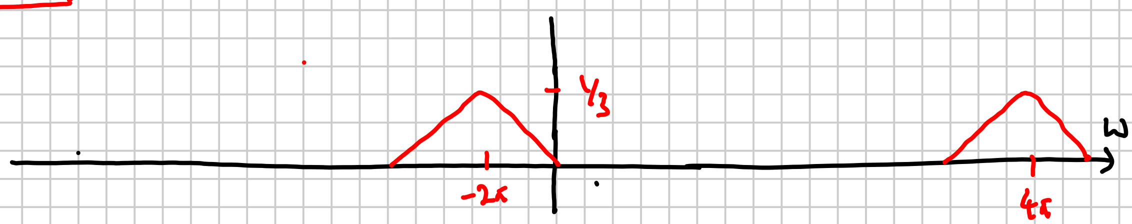
$\boxed{i=0}$



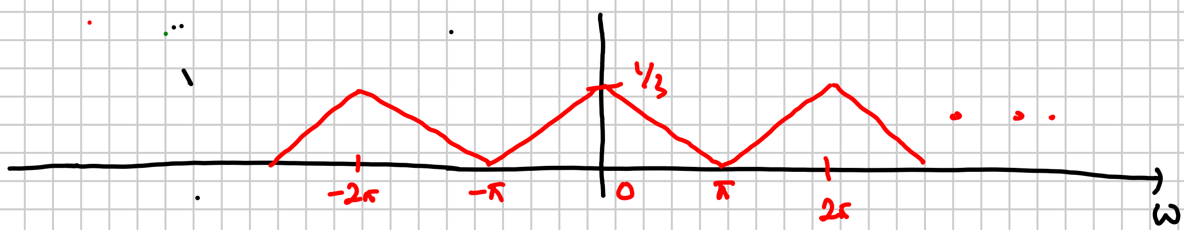
$\boxed{i=1}$



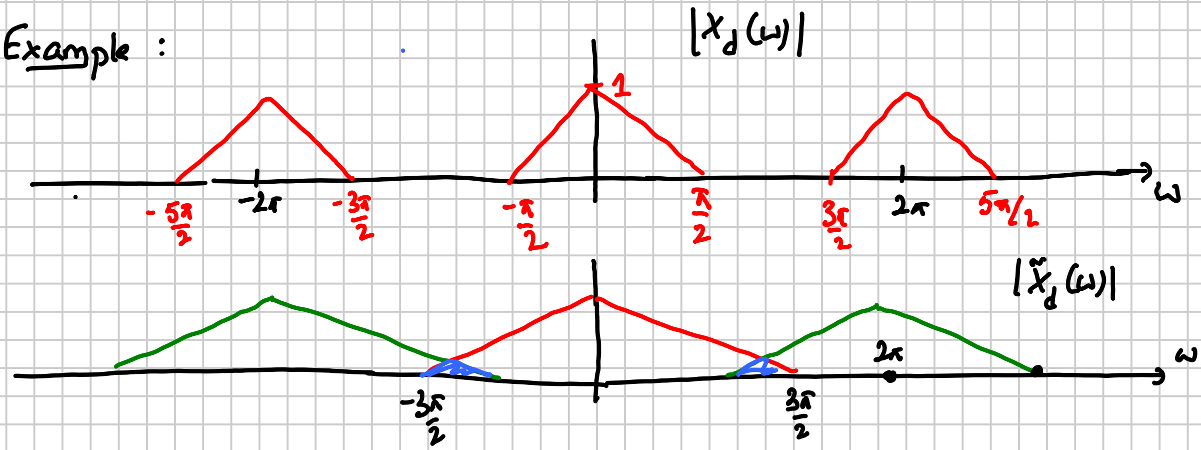
$\boxed{i=2}$



$|\tilde{X}_d(\omega)|$ is obtained by summing the above three waveforms



Example:



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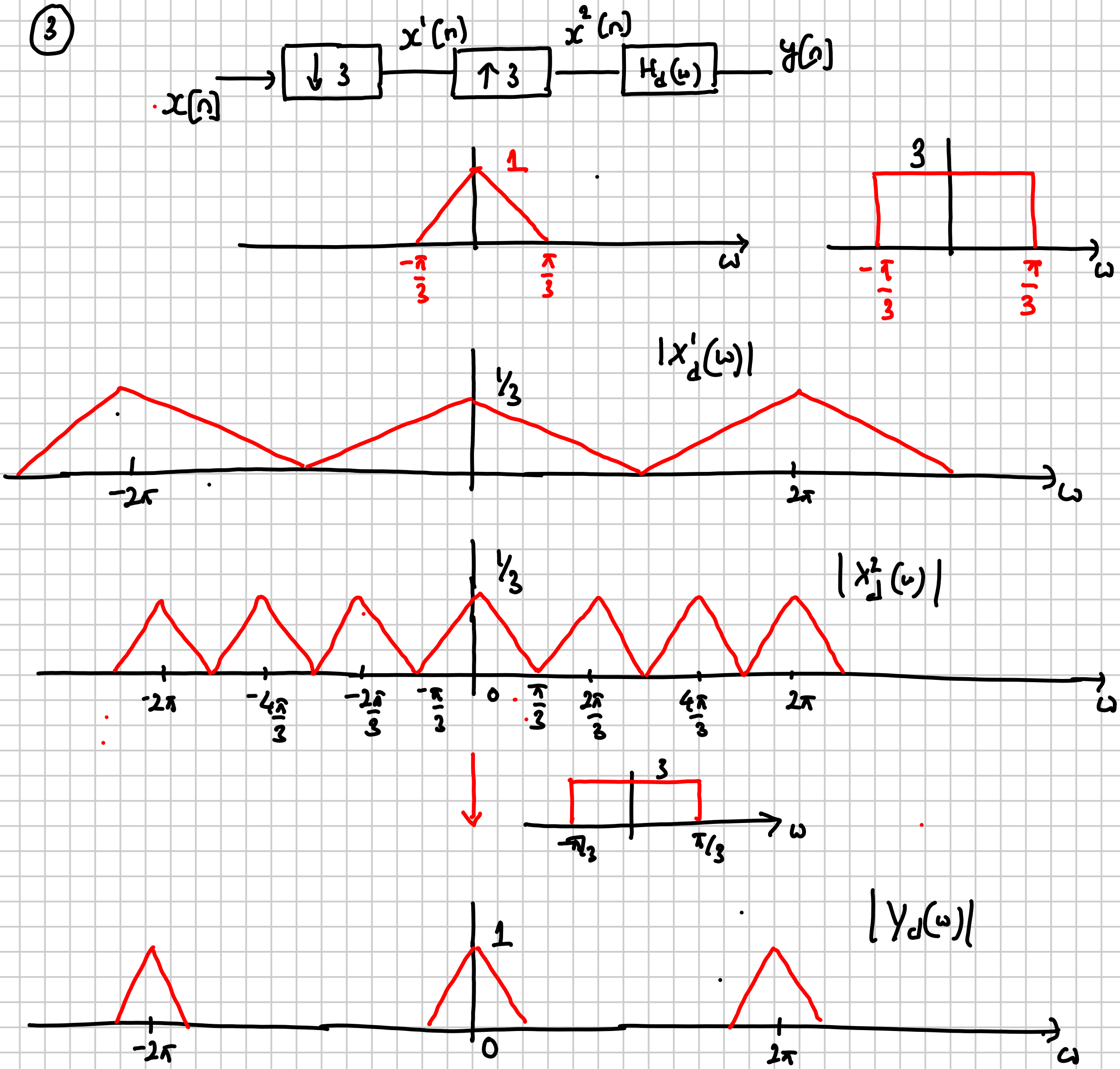

x[n]
↓ 3
x'[n]
↑ 3
x²[n]
H_d(ω)
y[n]
1
−π/3
π/3
ω
3
−π/3
π/3
ω
|X'_d(ω)|
1/3
−2π
2π
ω
1/3
|X²_d(ω)|
−2π
−4π/3
−2π/3
−π/3
0
π/3
2π/3
4π/3
2π
ω
3
−π/3
π/3
ω
|Y_d(ω)|
1
−2π
0
2π
ω