1. A monochromatic plane wave propagating in a vacuum in the region $x < 0$ has an electric field phasor given by

$$\tilde{\mathbf{E}}_{\mathbf{i}} = (j\hat{z} - \hat{y})e^{-j2\pi x} \text{ V/m}$$

The wave encounters a planar boundary at $x = 0$ which separates the vacuum from a perfect dielectric material in the region $x > 0$ which has magnetic permeability $\mu = \mu_0$ and electric permittivity $\epsilon > \epsilon_0$. The electric field phasor of the reflected wave in the $x < 0$ region is given by

$$\tilde{\mathbf{E}}_{\mathbf{r}} = -\frac{1}{2}(j\hat{z} - \hat{y})e^{j2\pi x} \text{ V/m}$$

   a) What are the polarizations of the incident and reflected waves?
   b) What is the linear frequency f of the wave?
   c) What is the permittivity ϵ of the dielectric material?
   d) Write the phasor expression for the electric field waveform that is transmitted into the dielectric medium.
   e) What percentage of the time-averaged incident power per unit area is transmitted into the dielectric medium?

2. A monochromatic plane wave described by

$$\mathbf{H}(y, t) = \hat{x}5\cos(\omega t + \beta y) \text{ A/m}$$

is propagating in the region $y > 0$, which is a non-magnetic perfect dielectric having permittivity $4\epsilon_0$. The wave is incident on the $y = 0$ plane which happens to be the boundary with a perfect conductor ($\sigma = \infty$) in the region $y < 0$.

   a) Write the phasor expressions for the incident, reflected, and transmitted electric fields $\tilde{\mathbf{E}}_{\mathbf{i}}$, $\tilde{\mathbf{E}}_{\mathbf{r}}$, and $\tilde{\mathbf{E}}_{\mathbf{t}}$.
   b) Write the phasor expressions for the incident, reflected, and transmitted magnetic fields $\tilde{\mathbf{H}}_{\mathbf{i}}$, $\tilde{\mathbf{H}}_{\mathbf{r}}$, and $\tilde{\mathbf{H}}_{\mathbf{t}}$.
   c) What is the vector current density $\mathbf{J}_{\mathbf{s}}(t)$ generated on the surface of the perfect conductor, i.e., at $y = 0$?

3. RG-58 is the coax cable that you have been using most frequently in your labs. It has the same geometrical dimensions as the RG-59 cable, but instead of having a dielectric filling with $\epsilon = \epsilon_o$, $\mu = \mu_o$ (like RG-59), it has $\epsilon = 2.25\epsilon_o$, $\mu = \mu_o$. The inner and outer conductor diameters for both types of cables are $2a = 0.032$ inches and $2b = 0.112$ inches, respectively. Given that

$$\mathcal{C} = \epsilon\text{GF}, \quad \mathcal{L} = \frac{\mu}{\text{GF}}, \quad Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}, \quad v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}},$$

and geometrical factor

$$\text{GF} = \frac{2\pi}{\ln\frac{b}{a}}$$

for a coax,

   a) Calculate, $\mathcal{L}$, $\mathcal{C}$, Z_o, v for the RG-59 coax cable.

b) Repeat (a) for RG-58.

4. $300\,\Omega$ twin-lead transmission lines (TL) are commonly used to connect TV sets and FM radios to their receiving antennas. For the twin-lead, the geometrical factor is

$$\text{GF} = \frac{\pi}{\cosh^{-1}\frac{D}{2a}},$$

where $2a$ is the diameter of each wire (cylindrical conductor) of the twin lead, and D is the distance between the centers of the wires. Assuming $\epsilon = \epsilon_o$, $\mu = \mu_o$, and $a = 1$ mm, calculate D for twin lead TL's having (a) $Z_o = 50\,\Omega$, (b) $Z_o = 300\,\Omega$, and (c) $Z_o = 400\,\Omega$.

5. Telegrapher's equations

$$\begin{aligned} -\frac{\partial V}{\partial z} &= \mathcal{L}\frac{\partial I}{\partial t} \\ -\frac{\partial I}{\partial z} &= \mathcal{C}\frac{\partial V}{\partial t} \end{aligned}$$

govern the voltage and current waves $V(z,t)$ and $I(z,t)$ that propagate on transmission line systems.

If $V(z,t) = 3\sin(\omega t + \beta z)$ on a TL, determine $I(z,t)$ and β (a positive number) by using the telegrapher's equations twice.

Hint: First use one of the telegrapher's equations to determine $I(z,t)$. Then use the other telegrapher's equation to determine $V(z,t)$ from $I(z,t)$ found in the first step. By requiring $V(z,t)$ found in step 2 to equal the original $V(z,t)$ you should be able to identify β.