ECE 310 **University of Illinois at Urbana-Champaign** **Fall 2015**

Profs. Bresler & Radhakrishnan **Homework 8** **Due: Friday, October 30**

Reading: Chapter 9

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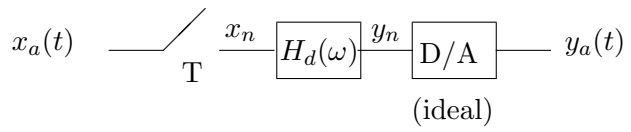
1. Consider the following system with uniform sampling



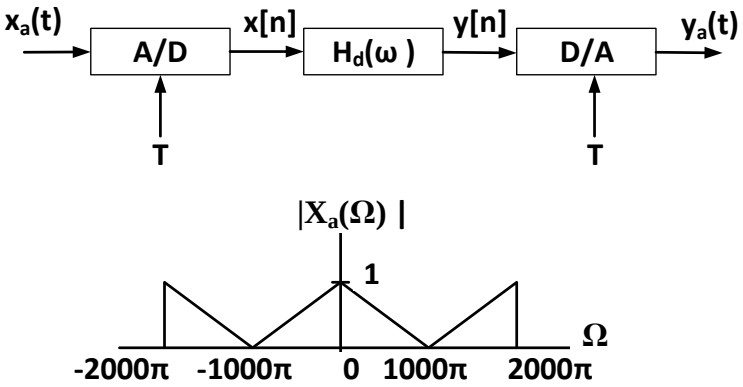
The discrete-time system $H_d(\omega)$ is an ideal low-pass filter with cutoff frequency $\frac{\pi}{8}$.

(a) If $x_a(t)$ is bandlimited to 5 kHz, what is the maximum value of T that will avoid aliasing in the A/D converter?

(b) If $\frac{1}{T} = 10$ kHz and $x_a(t)$ is sufficiently bandlimited such that the overall system from $x_a(t)$ to $y_a(t)$ behaves as an LSI system, what will the cutoff frequency of the effective continuous-time filter be?

(c) Repeat part (b) for $\frac{1}{T} = 20$ kHz.

2. The system shown below with input $x_a(t)$ is used to produce an output $y_a(t)$ such that $Y_a(\Omega) = X_a(\Omega)H_a(\Omega)$ where $H_a(\Omega)$ corresponds to an ideal low-pass filter with cut-off frequency of $\Omega_c = 1000\pi$ rad/s.



(a) Assuming ideal A/D and D/A converters, find Nyquist Sampling Rate for the signal $x_a(t)$.

(b) Determine the largest T possible such that $Y_a(\Omega) = X_a(\Omega)H_a(\Omega)$.

(c) Determine the required $H_d(\omega)$ for sampling period $T = 0.25$ ms.

(d) Assuming $T = 0.25$ms, sketch $X_d(\omega)$, $Y_d(\omega)$, and $Y_a(\Omega)$, for an ideal D/A.

(e) Suppose the ideal D/A is now replaced by a zero-order hold, using the pulse

$$p_a(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{else.} \end{cases}$$

Sketch $Y_a(\Omega)$ for $|\Omega| \le 8000\pi$. Find the amplitude of the largest unwanted (out of the band $|\Omega| \le \frac{\pi}{T}$) component of $Y_a(\Omega)$, due to the nonideal D/A.

3. Let the input to a ZOH operating with period T be $y[n] = cos(n\pi/4)$. Sketch the output of the ZOH by hand for $0 < t < 10T$.

4. Sketch by hand the Fourier transform of the output of a ZOH operating at 12 Hz for an input $y[n] = cos(n\pi/4)$. Do the sketch for $0 \le |\Omega| \le 48\pi$ Determine the magnitude of the largest unwanted component at the output.

5. The transfer functions of three LSI systems are given below. For each system, determine whether it is an FIR or IIR filter.

   (a) $H(z) = \frac{z^2+3z}{z^2+3z+2}$

   (b) $H(z) = \frac{z+1}{z^2-\frac{z}{4}-\frac{1}{8}}$

   (c) $H(z) = \frac{1}{3} \times \left(1 - z^{-1} + z^{-2}\right)$

   (d) $H(z) = \frac{1}{3} \times \left(1 + z^{-1} + z^{-2}\right)$