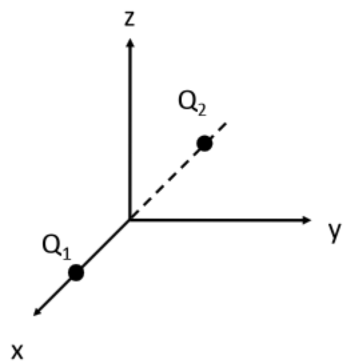
1. Gauss's law for electric field $\mathbf{E}$ states that $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_o} \int_V \rho dV$ over any closed surface S enclosing a volume V in which electric charge density is specified by $\rho(x, y, z)$ C/m^3.

   a) What is the *electric flux* $\oint_S \mathbf{E} \cdot d\mathbf{S}$ over the surface of a cube of volume $V = L^3$ centered about the origin, if $\rho(x, y, z) = -3$ C/m^3 within V and $L = 1$ cm?

   b) Repeat (a) for $\rho(x, y, z) = x^2 + y^2 + z^2$ C/m^3.

   c) What is the electric flux in part (b) for any one of the square surfaces of volume V?

2. Two unknown charges, Q_1 and Q_2 are located at $(x, y, z) = (1, 0, 0)$ and $(-1, 0, 0)$, respectively, as shown below. The displacement flux $\int_{yz-plane} \mathbf{D} \cdot \hat{x} dydz$ through the entire yz-plane (i.e., at $x = 0$) in the $+\hat{x}$ direction is 3 C. The flux through the plane $y = 1$ in the $+\hat{y}$ direction is 1 C. Determine Q_1 and Q_2 after writing a pair of algebraic equations relating the above displacement fluxes to Q_1 and Q_2.



Hint: What is the contribution of Q_1 to the flux $\int_{yz-plane} \mathbf{D} \cdot \hat{x} dydz$? See Example 5 in Lecture 3.

3. A particle of mass $m = 36\pi \times 10^{-3}$ kg and charge $Q = 2\,\mu$C is inserted at time $t = 0$ at a distance $d = 9.8$ m above a planar sheet charged uniformly with electric charge density of ρ_s C/m^2. The distance d is much smaller than the dimensions of the sheet so that, for all practical purposes the sheet can be assumed to be of infinite extent surrounded by free space.

   a) What should the sheet charge density be in order for the charge to levitate motionless at the position where it was placed at $t = 0$? Assume that the Coulomb electric field generated by charge Q is insignificant relative to the electric field generated by the charged sheet (i.e., treat Q as a test charge).

   b) Consider, next, the case where $\rho_s = 4.9$ μC/m^2. Describe the motion of the point charge Q for $t > 0$ by calculating its acceleration, $a(t)$, its velocity, $v(t)$, and its distance from the charged sheet. For your calculations, use as your reference coordinate system one with its plane $z = 0$ taken to coincide with the plane of the charged sheet.

4. Consider a three-slab geometry of three identical slabs of equal widths W in x-direction, and infinite extent in y and z directions. In slab 1 extending over $-W < x < 0$, charge density ρ is $5\rho_o$, where ρ_o measured in C/m^3 units is some positive number. In slab 2 extending over $0 < x < W$ the charge density is $-3\rho_o$. Finally, in slab 3 extending over $W < x < 2W$ the charge density is $-2\rho_o$. Charge density ρ is zero everywhere else. Determine and sketch $E_x(x)$ over the region $-3W < x < 3W$. Label both axes of your plot carefully, marking the field value at each break point.

Hint: use superposition of shifted and scaled versions of the static field configuration of a single charged slab.

5. For each of the displacement fields specified below, use the differential form of Gauss's law to determine the static charge density $\rho(x, y, z)$ C/m^3 that generates the field and describe the nature of the charge density "in words" (i.e. do we find a volumetric distribution of charge, or a surface charge, or a line charge?):

   a) The static displacement field in the region is specified as $\mathbf{D}(x, y, z) = 4\text{sgn}(z)\hat{z}$ C/m^2.

   b) The static displacement field in the region is specified as $\mathbf{D}(x, y, z) = -2\text{sgn}(x+3)\hat{x}$ C/m^2.

   c) The static displacement field in the region is specified as

$$\mathbf{D}(x,y,z) = \begin{cases} \hat{y}\,5\ \text{C/m}^2, & y < -5\,\text{m}, \\ -\hat{y}\,y\ \text{C/m}^2, & |y| < 5\,\text{m}, \\ -\hat{y}\,5\ \text{C/m}^2, & y > 5\,\text{m}. \end{cases}$$

   d) The static displacement field in the region is specified as

$$\mathbf{D}(x,y,z) = \begin{cases} \hat{x}\,2x\ \text{C/m}^2, & 0 < x < 2\,\text{m}, \\ -\hat{x}\,x\ \text{C/m}^2, & -4 < x < 0\,\text{m}, \\ 0\ \text{C/m}^2, & \textit{otherwise} \end{cases}$$

6. **Curl** and **divergence** exercises:

   a) On a 25-point graph consisting of x and y coordinates having the integer values $\{-2, -1, 0, 1, 2\}$ sketch the vector field $\mathbf{F} = -x\,\hat{x} - y\,\hat{y}$ and find $\nabla \times \mathbf{F}$ (curl of $\mathbf{F}$) and $\nabla \cdot \mathbf{F}$ (divergence of $\mathbf{F}$).

   b) Repeat (a) for $\mathbf{F} = -y\,\hat{x} + x\,\hat{y}$.

   c) Based on above results choose the correct answer in the statements below:

      i. $\nabla \times \mathbf{F} \neq 0$ implies the field strength varies (**along** or **across**) the direction of the field.

      ii. $\nabla \cdot \mathbf{F} \neq 0$ implies the field strength varies (**along** or **across**) the direction of the field.

7. Coulomb's field of a point charge Q stationed in a vacuum at the origin of a right-handed Cartesian coordinate system can be expressed as

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_o r^2}\hat{r} = \frac{Q}{4\pi\epsilon_o r^2}\frac{(x,y,z)}{r},$$

   where $r^2 \equiv x^2 + y^2 + z^2$ and $r \geq 0$.

   a) Verify that $\nabla \times \mathbf{E} = 0$ for $r > 0$ by showing that when $\nabla \times \mathbf{E}$ is expanded as usual, all of its components cancel out exactly.

   b) What is $\nabla \cdot \mathbf{E}$ for $r > 0$? **Hint**: you do not need to calculate this explicitly.

8. Given that

$$\mathbf{E} = \hat{y}\sin x + \hat{x}\cos y,$$

   a) determine $\nabla \times \mathbf{E}$ and $\nabla \times \nabla \times \mathbf{E}$,

   b) determine ρ such that Gauss's law is satisfied.