1. Consider the 3D vectors

$$\begin{aligned}\mathbf{A} &= 3\hat{x} - 3\hat{y} + 3\hat{z},\\ \mathbf{B} &= \hat{x} - 2\hat{y} - \hat{z},\\ \mathbf{C} &= -\hat{x} + 2\hat{y} + 4\hat{z},\end{aligned}$$

where $\hat{x} \equiv (1,0,0)$, $\hat{y} \equiv (0,1,0)$, and $\hat{z} \equiv (0,0,1)$ constitute an orthogonal set of unit vectors directed along the principal axes of a *right-handed* Cartesian coordinate system. Vectors can also be represented in component form — e.g., $\mathbf{A} = (3,-3,3)$, which is equivalent to $3\hat{x} - 3\hat{y} + 3\hat{z}$.

Determine:

a) The vector $\mathbf{A} - \mathbf{B} + 3\mathbf{C}$,

b) The vector *magnitude* $|\mathbf{A} - \mathbf{B} + 3\mathbf{C}|$.

c) The unit vector $\hat{u}$ along vector $\mathbf{A} + 2\mathbf{B} - \mathbf{C}$.

d) The *dot product* $\mathbf{A} \cdot \mathbf{B}$.

e) The *cross product* $\mathbf{B} \times \mathbf{C}$.

2. Charges $Q_1 = -8\pi\epsilon_o$ C and $Q_2 = Q_1/2$ are located at points P_1 and P_2 having the position vectors $\mathbf{r}_1 = -\hat{x} = -(1,0,0)$ m and $\mathbf{r}_2 = \hat{x} = (1,0,0)$ m, respectively. Determine the electric field vector $\mathbf{E}$ at points P_3, P_4, and P_5 having the position vectors $\mathbf{r}_3 = (0,0,0)$ m, $\mathbf{r}_4 = \hat{z} = (0,0,1)$ m, and $\mathbf{r_5} = -\hat{y} = (0,-1,0)$ m, respectively. Make sketches (3D perspective plots) showing the charge locations and the resulting electric field vectors — drawn coming out of points P_3, P_4, and P_5 respectively — in each case.

3. A particle with charge $q = 1$ C passing through the origin $\mathbf{r} = (x,y,z) = (0,0,0)$ of the lab frame is observed to accelerate with forces

$$\mathbf{F}_1 = 2\hat{y}, \quad \mathbf{F}_2 = 0, \quad \mathbf{F}_3 = 2\hat{y} - \hat{z}\,\mathrm{N}$$

when the velocity of the particle is

$$\mathbf{v}_1 = 0, \quad \mathbf{v}_2 = -2\hat{z}, \quad \mathbf{v}_3 = \hat{y}\,\frac{\mathrm{m}}{\mathrm{s}},$$

in turns. Use the Lorentz force equation $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ to determine the fields $\mathbf{E}$ and $\mathbf{B}$ at the origin. **Hint:** assume $\mathbf{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$ and then solve the three vector equations obtained from $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ for the unknowns B_x, B_y, B_z, and $\mathbf{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$.

4. Consider an infinite slab bounded between planes $z = 0$, $z = 1$, and extending to infinity along x and y. The medium inside the domain is free space. The electric and magnetic field inside the domain are given by

$$\mathbf{E} = \hat{y}60\pi\sin(\pi z)\cos(\omega t)\ \mathrm{V/m}$$

and

$$\mathbf{B} = \hat{x}(2\pi \times 10^{-7})\cos(\pi z)\sin(\omega t)\ \mathrm{T}$$

where $\omega = 5\pi \times 10^8$ rad/s. Assume that an electrically charged particle of charge $-1.602\,\mu$C is inserted in the aforementioned domain with initial velocity $\mathbf{v} = \hat{y}100$ m/s. The mass of the particle is 10^{-3} kg. Discuss the motion of the particle for the following two cases:

a) The particle is at position at $(x, y, z) = (0, 0, 0.5)$ m at $t = 0$.

b) The particle is at position at $(x, y, z) = (0, 0, 0.75)$ m at $t = 0$.

Your *discussion* should be as quantitative as possible through the use of the Lorentz force equation and Newton's laws of motion.

5. A charge-neutral region of space contains stationary electrons (having charge $q = -e$, mass m_e, and number density N_e) and ions (having charge $q = +e$, mass $m_i \gg m_e$, and number density $N_i = N_e$). At time $t = 0$, a static and uniform electric field $\mathbf{E} = E_0\hat{z}$ V/m is applied, which generates an associated electric Lorentz force on the charges.

   a) Treating the charged particles as "test charges" (such that the Coulomb electric fields generated by the charges themselves can be neglected), what is the magnitude and direction (i.e., the unit vector) of the Lorentz accleration experienced by a single electron and ion, respectively?

   b) Under the influence of the Lorentz force, the charges move and thus constitute a current. At some time $t = t_0 > 0$, the electrons are moving at a velocity v_e m/s, while the ions are moving more slowly, at a velocity v_i m/s. What is the total volumetric current density $\mathbf{J}$ (in units of A/m^2) carried by both the ions and electrons in the region at this moment in time? **Hint:** Start by expressing the current density $\mathbf{J}$ in terms of charge density $\rho = qN$ C/m^3.

   c) At time t_0, a static and uniform magnetic field $\mathbf{B} = B_0\hat{x}$ Wb/m^2 is also applied. What is the magnitude and direction (i.e., the unit vector) of the acceleration of a single electron and ion, respectively, under both the electric and magnetic Lorentz forces? Does the presence of the magnetic field alter the total current density at $t = t_0$ found in part (b)? Explain.

   d) Repeat part (c) for an applied magnetic field $\mathbf{B} = B_0\hat{z}$ Wb/m^2.

6. Let $\mathbf{J} = x^2y(\hat{x}+\hat{y}+\hat{z})$ A/m^2 denote the electrical *current density* field — i.e., current flux per unit area — in a region of space represented in Cartesian coordinates. A current density of $\mathbf{J} = x^2y(\hat{x} + \hat{y} + \hat{z})$ A/m^2 implies the flow of electrical current in direction $\frac{\mathbf{J}}{|\mathbf{J}|} = \frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$ with a magnitude of $|\mathbf{J}| = x^2y\sqrt{3}$ amperes (A) per unit area.

   a) Calculate the total **current flux** $\oint_S \mathbf{J} \cdot d\mathbf{S}$ through a closed surface S enclosing a cubic volume $V = 1\,\mathrm{m}^3$ with vertices at $(x, y, z) = (0, 0, 0)$ and $(1, 1, 1)$ m.

      Hint: Surface S of cube V consists of six surfaces of square shapes having equal areas $S_i = 1\,\mathrm{m}^2$, $i = 1, 2, \cdots, 6$. The flux $\oint_S \mathbf{J} \cdot d\mathbf{S}$ is therefore the sum of six *surface integrals* $\int_{S_i} \mathbf{J} \cdot d\mathbf{S}$ taken over surfaces S_i, where the infinitesimal area vectors $d\mathbf{S}$ are, in turn, $\pm\hat{z}dxdy$, $\pm\hat{x}dydz$, and $\pm\hat{y}dzdx$ — by convention $d\mathbf{S}_i$ are taken as vectors pointing **away from** volume V (at each subsurface S_i) in flux calculations.

   b) Given the result of part (a), do you expect the total amount of electrical charge Q_V contained in volume V to increase or decrease? In answering this question assume that electric charge is *conserved* (as in real life!). Remember that electrical current represents electrical charges in motion.

   c) Given the result of part (b), do you expect the electric field flux $\oint_S \mathbf{E} \cdot d\mathbf{S}$ out of the same closed surface S to increase or decrease? Explain in terms of Gauss' law.