ECE 329 **Homework 8** **Due: Mar. 12, 2013, 5PM**

1. A z-polarized plane TEM wave, generated by a surface current on the $y = 0$ m plane, is propagating in vacuum (i.e., $v = c \approx 3 \times 10^8$ m/s and $\eta = \eta_o \approx 120\pi\,\Omega$) in the $-y$ direction. The electric field is observed to vary with time at $y = 0$ according to $E_z(0,t) = 4\triangle(\frac{t-t_1}{\tau}) - 2\triangle(\frac{2(t-t_2)}{\tau})$ V/m, where $t_1 = 1\ \mu$s, $t_2 = 2.5\ \mu$s, $\tau = 2\,\mu$s and $\triangle(\frac{t}{\tau})$ is the unit triangle function centered on the origin ($t = 0$) with width τ.

   a) Determine and plot the surface current density $\mathbf{J_s}(t)$ on the $y = 0$ plane which gives rise to the observed field.

   b) Determine the vector wavefield $\mathbf{E}(y,t)$ written explicitly in terms of all space and time variables y and t.

   c) Determine the accompanying wavefield $\mathbf{H}(y,t)$.

   d) Determine the maximum value of *Poynting vector* $\mathbf{E} \times \mathbf{H}$.

   e) What trajectory function $y = y(t)$ describes instantaneous locations of the peak of $\mathbf{E} \times \mathbf{H}$.

   f) Plot $E_z(y,t)$ vs t at $y = -1500$ m.

   g) Plot $H_x(y,t)$ vs y at $t = 12\,\mu$s.

2. In a homogeneous loss-less dielectric with $\epsilon = \epsilon_r \epsilon_o$ and $\mu = \mu_r \mu_o$ a plane TEM wave with the following components is observed:

$$\mathbf{E} = \hat{x}\frac{1}{2}u(t - \frac{z}{c/2}) + \hat{y}g(t - \frac{z}{c/2})\,\frac{\mathrm{V}}{\mathrm{m}}$$

   and

$$\mathbf{H} = \hat{x}(\frac{10z}{c} - 5t) + \hat{y}\frac{1}{120\pi}u(t - \frac{2z}{c})\,\frac{\mathrm{A}}{\mathrm{m}},$$

   where $u(t)$ denotes the unit-step function and c is the speed of light in free space. Using the above information,

   a) Determine the intrinsic impedance η for the medium.

   b) Determine the propagation velocity v.

   c) Determine ϵ_r and μ_r.

   d) Determine function $g(t)$.

3. We have on the $x = 0$ plane a pulse of sheet current $\mathbf{J}_s(t) = -\hat{z}\,2t\ \mathrm{rect}(\frac{t}{\tau})$ A/m, where $\tau = 2\,\mu$s (note that the current source is centered around $t = 0$). Regions adjacent to the current sheet are vacuum.

   a) Determine and plot $E_z(x,t)$ and $H_y(x,t)$ vs t for $x = -1200$ m.

   b) Determine and plot $E_z(x,t)$ and $H_y(x,t)$ vs x for $t = 5\,\mu$s.

   c) Determine and plot $E_z(x,t)$ and $H_y(x,t)$ vs x for $t = 1\,\mu$s.

   d) Determine the TEM wave energy radiated per unit area (in J/m^2 units) by the current pulse $\mathbf{J}_s(t)$.

   Hint: integrate the power injected per unit area, $-\mathbf{J}_s \cdot \mathbf{E}$, over the duration of pulse $\mathbf{J}_s(t)$.

4. For each of the four plane waves (in free space) described by

   a) $\mathbf{E}_1 = -4\cos(\omega t - \beta z)\hat{y}\,\mathrm{V/m}$
   b) $\mathbf{E}_2 = E_o\cos(\omega t - \beta x)\hat{y} - E_o\sin(\omega t - \beta x)\hat{z}$
   c) $\mathbf{H}_3 = \cos(\omega t + \beta z + \frac{\pi}{3})\hat{x} + \sin(\omega t + \beta z - \frac{\pi}{6})\hat{y}\,\mathrm{A/m}$
   d) $\mathbf{H}_4 = \cos(\omega t - \beta z - \frac{\pi}{2})\hat{x} + \sin(\omega t - \beta z)\hat{y}\,\mathrm{A/m}$:

      i. Determine the expression for $\mathbf{H}$ or $\mathbf{E}$ that accompanies the given wave field.
      ii. Find the instantaneous power that crosses a 1 m^2 area in the xy-plane from $-z$ to $+z$.
      iii. Find the time averaged power that crosses a 1 m^2 area in the xy-plane from $-z$ to $+z$.

5. An infinite plane current sheet of uniform, time-varying density $\mathbf{J_s}(t) = 2\cos(6\pi \times 10^8 t)\hat{y}$ A/m exists at the $x = 2$ m plane within a perfect dielectric medium having an electric permittivity of $\epsilon = \frac{9}{4}\epsilon_0$ and a magnetic permeability of $\mu = \mu_0$. Answer the following questions (using appropriate units) about the plane TEM waves which will propagate away from this surface current source.

   a) What is the magnitude and direction of the wave propagation velocity $\mathbf{v_p}$ of the TEM wave through the dielectric medium?
   b) What is the wave number β and wavelength λ of the TEM wave?
   c) What is the intrinsic impedance η of the dielectric medium?
   d) Write the expressions for wavefields $\mathbf{E}$ and $\tilde{\mathbf{E}}$ (the phasor of $\mathbf{E}$) and $\mathbf{H}$ and $\tilde{\mathbf{H}}$ (the phasor of $\tilde{\mathbf{H}}$) in terms of ω, β, and η for each of the regions $x > 2$ and $x < 2$ on either side of the current sheet source.
   e) Verify that the TEM wave satisfies the Poynting theorem, $\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \frac{\partial}{\partial t}(\frac{1}{2}\epsilon\mathbf{E} \cdot \mathbf{E} + \frac{1}{2}\mu\mathbf{H} \cdot \mathbf{H}) + \mathbf{J} \cdot \mathbf{E} = 0$, in the region $x > 2$. **Hint**: you should be able to prove this for a TEM wave propagating at an arbitrary frequency ω in an arbitrary perfect dielectric in terms of μ and ϵ in order to avoid explicit use of constants found in parts (a-c).
   f) What is the time-averaged power transported by the TEM wave across a square surface on the $x = 4$ m plane having area $A = 2\ \mathrm{m}^2$?