SEMESTER 1 EXAMINATION – 2014/2015

Module Code: EEN30110, Module Title: Signals and Systems

Time Allowed: 2 hours

Answer all questions. The numbers in the right margin give an approximate indication of the relative importance in terms of grade steps of each part of a question. All rough work should be entered in your answer books

1. Find the first seven non-zero terms of the trigonometric Fourier series of

$$f(t) = \cos(t) \quad , \quad 0 \le t < \pi$$

where f is periodic with period π. Note that the period of the signal is ***not*** 2π so the signal is not simply a cosine. Hence determine the DC component, the fundamental, the second and third harmonics. **4**

A "third order normalised Butterworth low-pass filter" has an input voltage e(t) and an output voltage $v_0(t)$. Because of the "normalisation" the filter is governed by the relatively simple ordinary differential equation:

$$\frac{d^3 v_0}{d\,t^3} + 2\frac{d^2 v_0}{d\,t^2} + 2\frac{d\,v_0}{d\,t} + v_0 = e(t)$$

Find the steady-state output of the filter when the input voltage $e(t)$ is given by:

$$e(t) = \sin^2(4t)$$ **1**

and also when the input voltage $e(t)$ is equal to the signal $f(t)$ of the first part of question1. **2**

2. Find the Fourier transform and the Fourier series of the following functions:

$$\sin(t)\,, \quad 1+\cos(3t)\,, \quad \exp(jt)\ . \qquad \mathbf{1.5}$$

A "fifth order normalised Butterworth low-pass filter" has an input voltage e(t) and an output voltage $v_0(t)$. Because of the "normalisation" the filter is governed by the relatively simple ordinary differential equation:

$$\frac{d^5 v_0}{d\,t^5}+3.236\frac{d^4 v_0}{d\,t^4}+5.2359\frac{d^3 v_0}{d\,t^3}+5.2359\frac{d^2 v_0}{d\,t^2}+3.236\frac{d\,v_0}{d\,t}+v_0=e(t)$$

Plot the "magnitude spectrum" of the filter and confirm its low-pass filtering properties. **1.5**

To what extent is a signal of frequency 2 Hz suppressed by the filter? **0.5**

Find a formula for the unit step response of the filter. **3.5**

3. A discrete-time system with input signal q and output signal g is described by the difference equation/recursion:

$$g(n)=\frac{2}{3}q(n)+\frac{1}{3}q(n-1)-g(n-1)-\frac{2}{9}g(n-2) \quad \text{for } n\geq 0.$$

Find the transfer function of the system. If the input signal is $q(n)=3^{-n}u(n)$ (where $u(n)$ denotes the discrete-time unit step) and if the initial conditions are given by: $g(-1) = 0$, $g(-2) = 0.5$, find a formula for the output of the system for discrete time $n\geq 0$. **4**

Find the steady-state output signal of this system if the input signal $q(n)$ is the periodic signal of period 6 given by:

$$q(n)=0.5 \quad \text{for } n=0,1 \quad \text{and} \quad q(n)=-0.5 \quad \text{for } n=2,3,4,5. \qquad \mathbf{3}$$