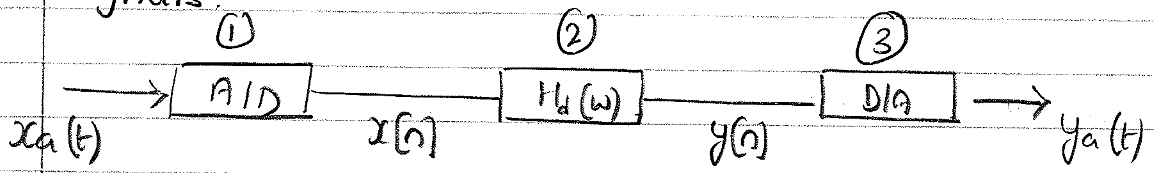
$$x(t) \longleftrightarrow X(j\omega) \Rightarrow x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$x(nT) \rightarrow x(n)$$

ECE 310 FA15: ~~Aa~~ ZOH.

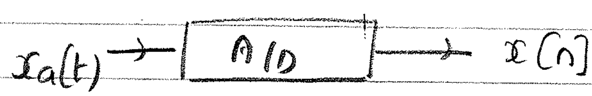
Last class we looked at DT processing of CT signals.

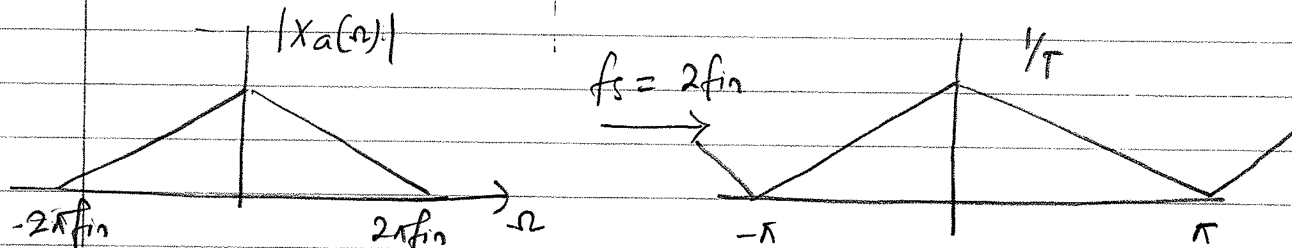


The DSP system above processes $x_a(t)$ to give $y_a(t)$, can be said to have an analog frequency Response,

$$H_c(\Omega) = \frac{Y_a(\Omega)}{X_a(\Omega)}$$

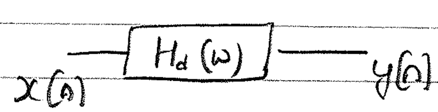
① A/D Converter:





$$X_d(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X_a\left(\frac{\omega + 2\pi n}{T}\right)$$

② DT LSI System:

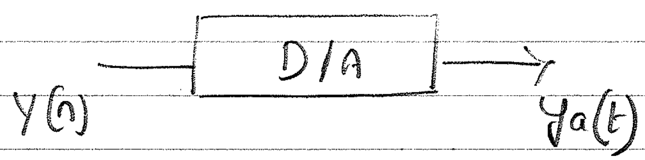


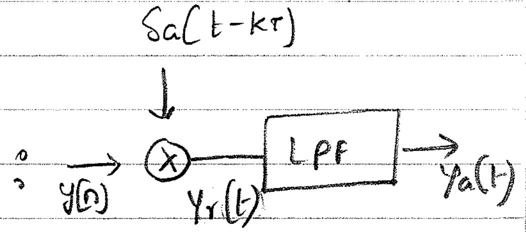
$$y[n] = \sum_m x[m]\, h[n-m]$$

$$Y_d(\omega) = X_d(\omega)\, H_d(\omega)$$

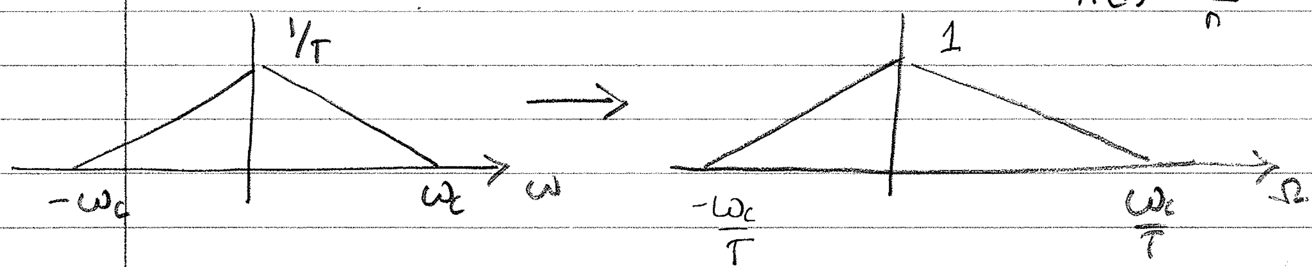
$$\Rightarrow \quad Y_d(\omega) = \frac{1}{T} H_d(\omega) \sum_{n=-\infty}^{\infty} X_a\left(\frac{\omega + 2\pi n}{T}\right)$$

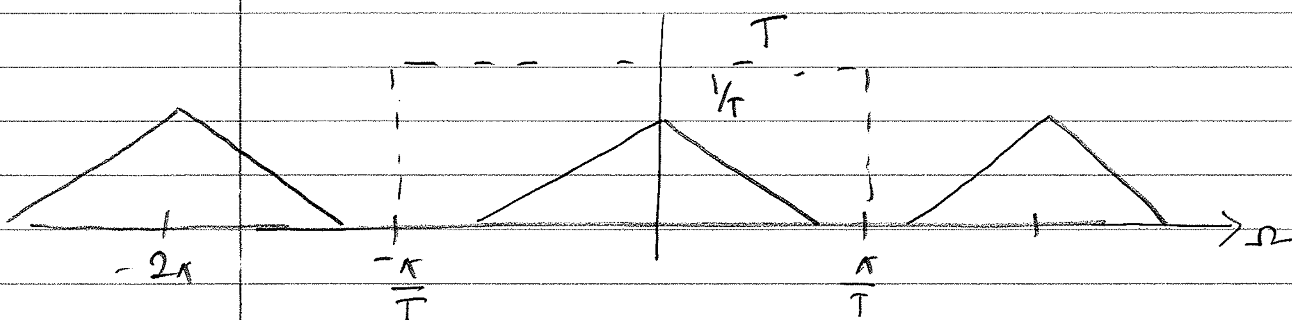
③ D/A Conversion :





$y_r(t) = \sum_n x[n]\delta(t-nT)$





$y_a(t) = \int y_r(\tau) g_a(t-\tau)d\tau$

$$y_a(t) = \sum_{n=-\infty}^{\infty} y(n)\, g_a(t-nT) \Rightarrow Y_a(\Omega) = Y_d(\Omega T) G_a(\Omega)$$

$= \int \sum x[n]\delta(\tau-nT) g_a(t-\tau)d\tau$

$$G_a(\Omega) = \begin{cases} T & |\Omega| \leq \pi/T \\ 0 & \text{else} \end{cases}$$

$= \sum x(n) \int \delta(\tau-nT) g_a(t-\tau)d\tau$

$\sum x(n)\, g_a(t-nT)$.

$$\Rightarrow \quad g_a(t) = \sin\left(\frac{\pi t}{T}\right)$$

$$\cancel{\Phi_{ee}(\Omega)} = \cancel{\int_{-\infty}^{\infty}}$$

$$\therefore \quad Y_a(\Omega) = \frac{1}{T} G_a(\Omega)\, H_d(\Omega T) \sum_{n=-\infty}^{\infty} X_a\left(\frac{\omega + 2\pi n}{T}\right)$$

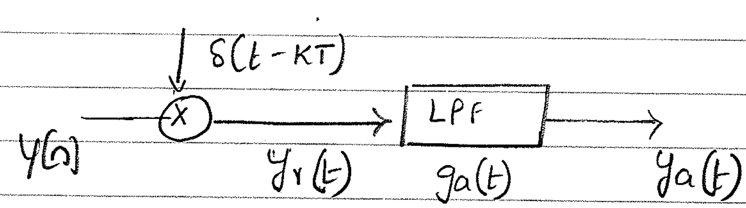
~~then~~ We can simplify the above eqn when:

① $|X_a(\Omega)| = 0 \quad$ for $|\Omega| > \frac{\pi}{T}$

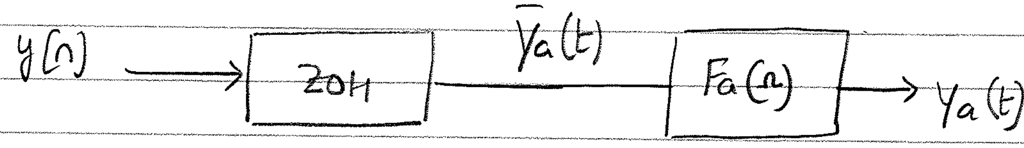
and ideal A/D & D/A Converters

The system then behaves as LTI.

<u>IMPLEMENTATION OF ~~ID~~ D/A : ZOH</u>



Band limited signal can be reconstructed using the system above. The above system is the ideal D/A Converter. A more practical D/A Converter is the ZOH shown below,

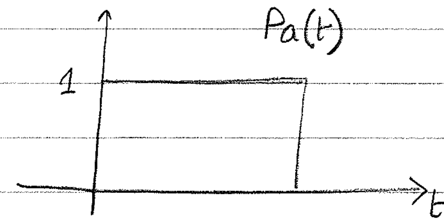


$\bar{y}_a(t)$ is the approximation to $y_a(t)$.

One can think of ZOH as a D/A that uses rectangular pulses

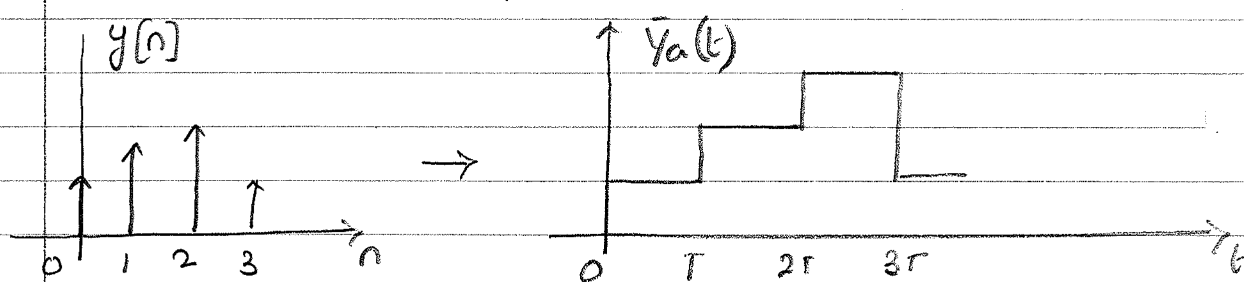
$$\bar{y}_a(t) = \sum_{n=-\infty}^{\infty} y(n)\, p_a(t-nT)$$

where $p_a(t)$:



$\cancel{\bar{y}_a(t) = \sum}$

$\therefore$ $\bar{y}_a(t) = \sum_{n=-\infty}^{\infty} y(n)\, p_a(t-nT)$ can be represented as,



Note : $\bar{y}_a(t)$ is a staircase approximation to $y_a(t)$

What happens in frequency domain?

$$P_a(\Omega) = \int_0^T 1 \cdot e^{-j\Omega t}\, dt$$

$$= \left.\frac{e^{-j\Omega t}}{-j\Omega}\right|_0^T$$

$$= \frac{e^{-j\Omega T} - 1}{-j\Omega} = \frac{1 - e^{-j\Omega T}}{j\Omega}$$

ECE 310 Lecture :

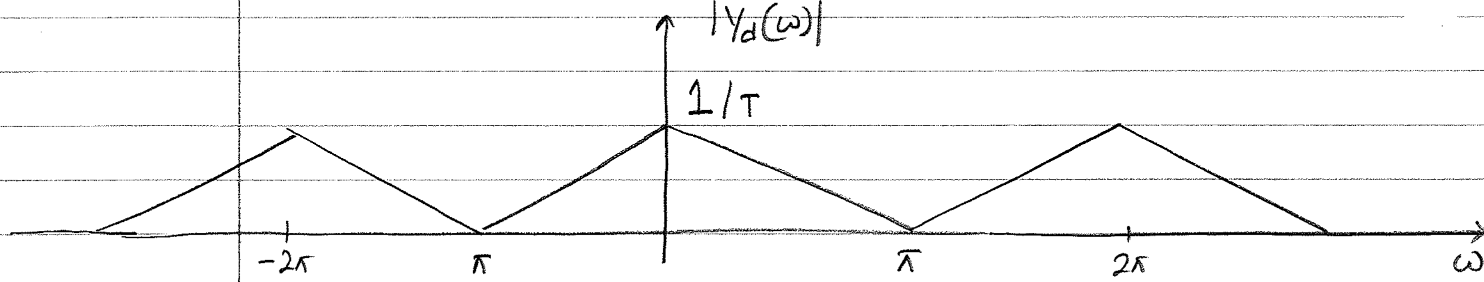
$$P_a(\Omega) = e^{-j\Omega T/2}\frac{\left(e^{j\Omega T/2} - e^{-j\Omega T/2}\right)}{j\Omega}$$

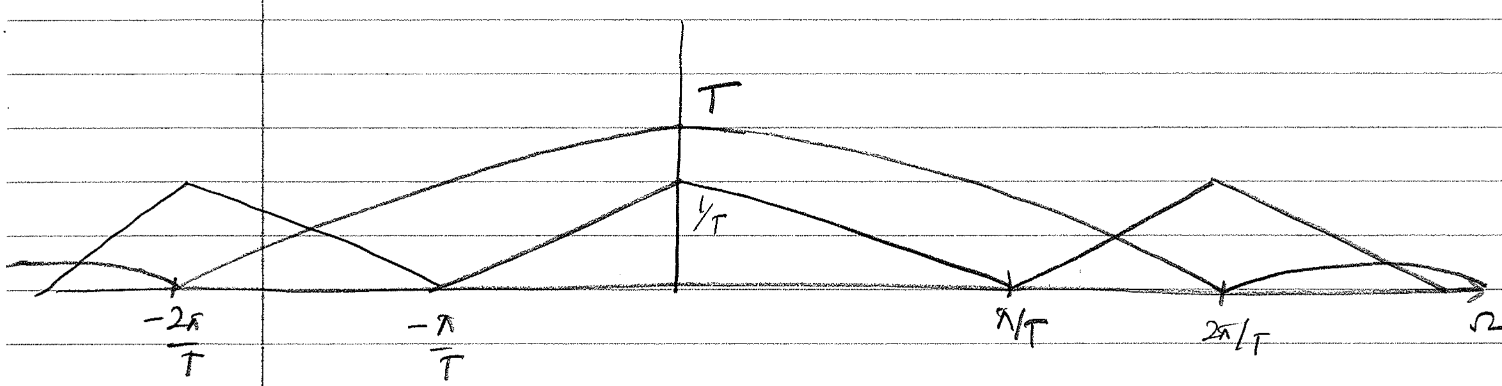
$$= \frac{2\sin\frac{\Omega T}{2}}{\Omega} \cdot e^{-j\Omega T/2}$$

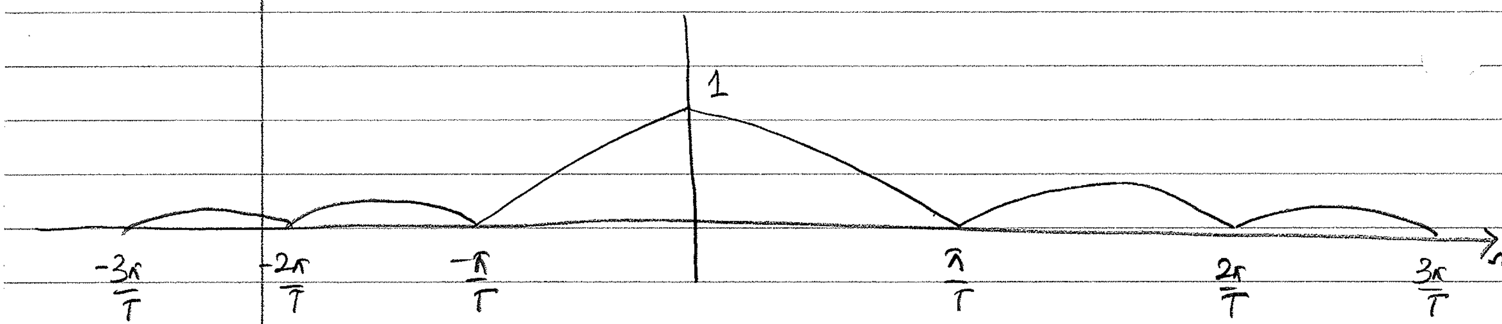
$$\Rightarrow P_a(\Omega) = \frac{\cancel{2}\, T \sin\frac{\Omega T}{2}}{\frac{\Omega T}{2}}\, e^{-j\Omega T/2}$$

$$P_a(\Omega) = T\,\text{Sinc}\,\frac{\Omega T}{2}\, e^{-j\Omega T/2}$$

$$\therefore \boxed{\bar{Y}_a(\Omega) = T e^{-j\Omega T/2}\,\text{Sinc}\,\frac{\Omega T}{2}\, Y_d(\Omega T)}$$







Note: Unlike $Y_a(\Omega)$ for the ideal D/A, $\bar{Y}_a(\Omega)$ for the ZOH has frequency content that extends all the way to $\Omega = \pm\infty$

Next step: Take $\bar{Y}_a(\Omega)$ from ZOH, and obtain $Y_a(\Omega)$ using $F_a(\Omega)$.

For ZOH system we have,

$$Y_a(\Omega) = F_a(\Omega)\,\bar{Y}_a(\Omega)$$

$$= F_a(\Omega)\; T e^{-j\Omega T/2}\, \mathrm{Sinc}\left(\frac{\Omega T}{2}\right) Y_d(\Omega T)$$

For the ideal D/A we have

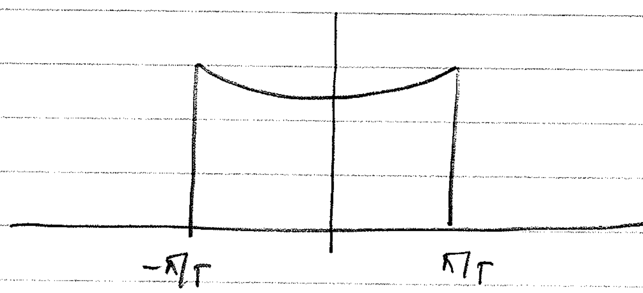
$$Y_a(\Omega) = \begin{cases} T\, Y_d(\Omega T) & |\Omega| \le \pi/T \\ 0 & \text{else} \end{cases}$$

We must have,

$$F_a(\Omega)\, T\, e^{-j\Omega T/2} \operatorname{Sinc}\left(\frac{\Omega T}{2}\right) Y_d(\Omega T) = \begin{cases} T\, Y_d(\Omega T) & |\Omega| \le \pi/T \\ 0 & \text{else} \end{cases}$$

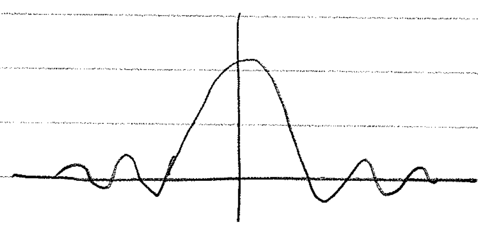
$$\text{or} \quad F_a(\Omega) = \begin{cases} \dfrac{e^{j\Omega T/2}}{\operatorname{Sinc}\left(\frac{\Omega T}{2}\right)} & |\Omega| \le \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

~~Best~~ ideal $|F_a(\Omega)|$ looks like

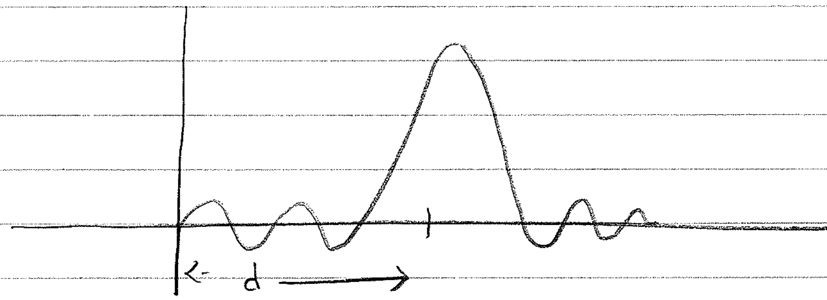


$F_a(\Omega)$ — LPF emphasizing higher frequencies in passband

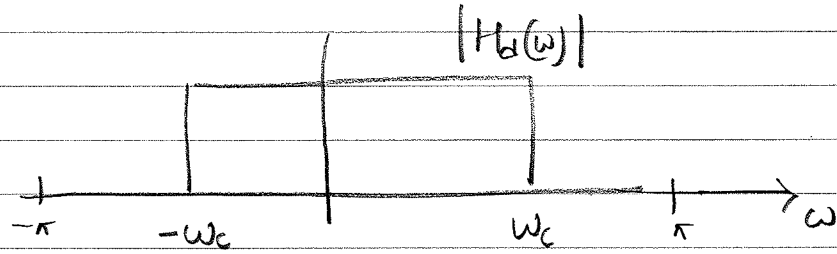
$F_a(\Omega)$ has finite support $\Rightarrow$ $f_a(t)$ has infinite support



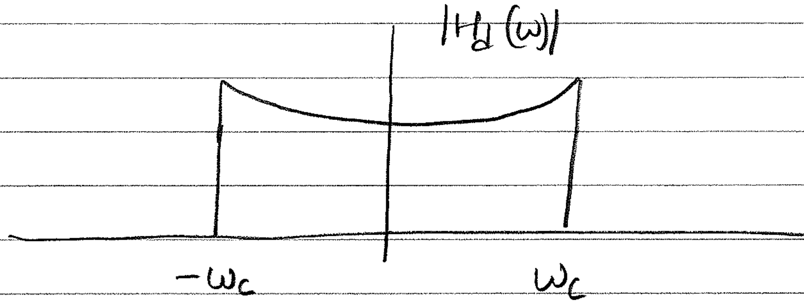
We usually use delayed & truncated version of $f_a(\Omega)$



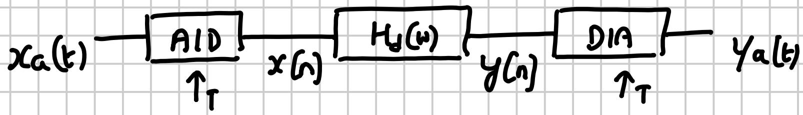
Also : The frequency emphasis within passband of $f_a(\Omega)$ can be performed digitally as a part of digital filter function.



we can design

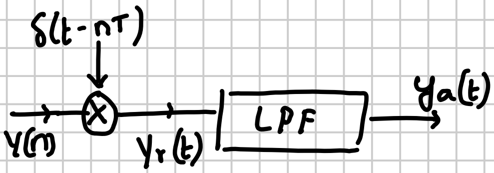


ECE 310: ZOH Examples

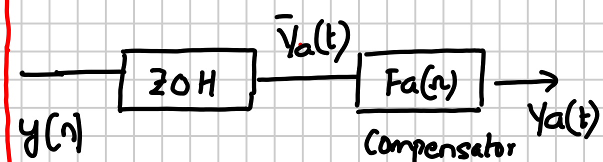


Last class we looked at D/A converters in detail.

Ideal



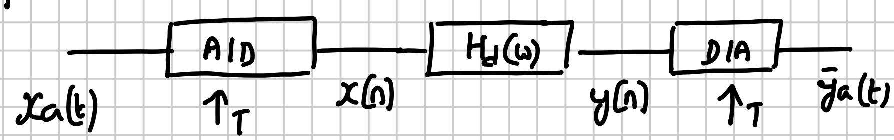
ZOH



$\bar{y}_a(t)$ = staircase approximation to $y_a(t)$

Examples:

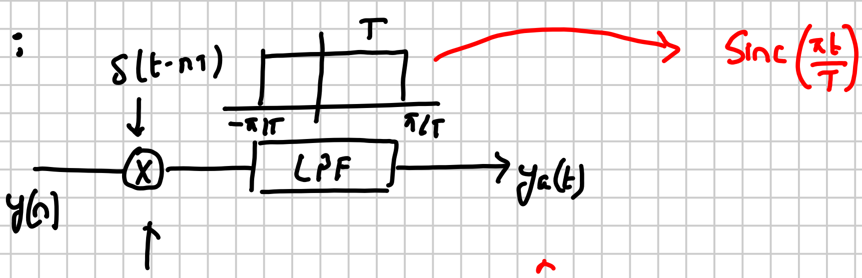
(1)

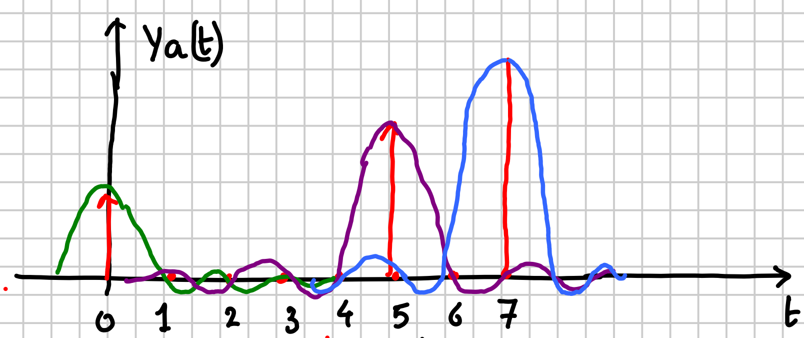


Let $y(n)' = \delta[n] + 3\delta(n-5) + 5\delta[n-7]$

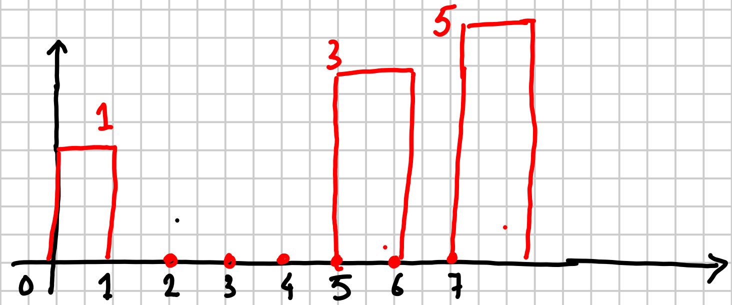
Sketch $y_a(t)$ for an (1) ideal (2) ZOH D/A

ideal D/A:



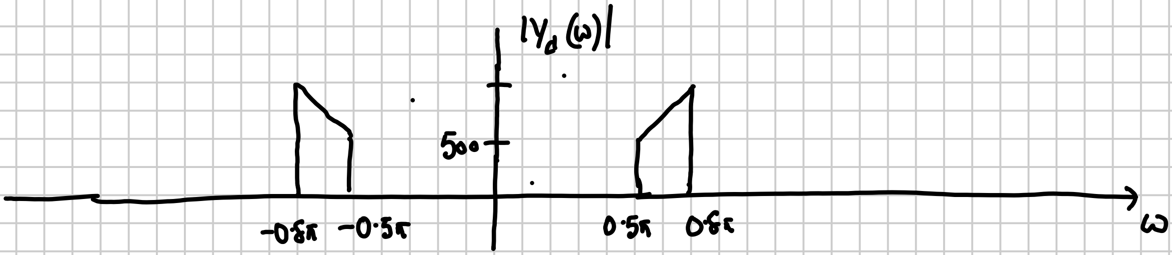


ZOH:

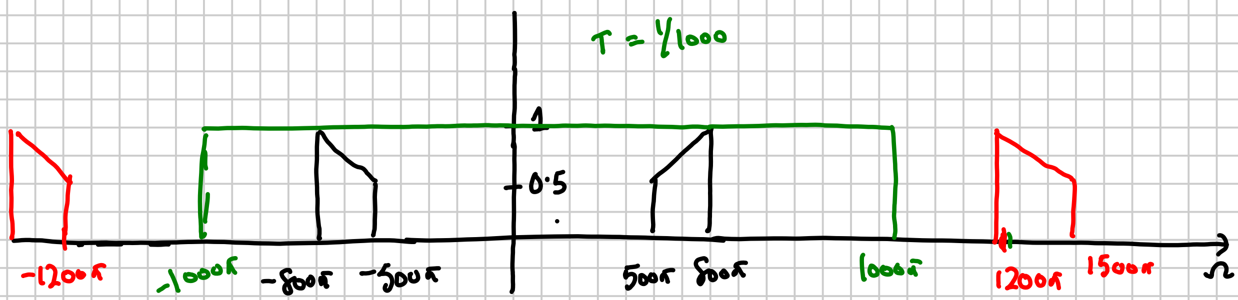


② $y(n) \longrightarrow \boxed{D/A} \longrightarrow y_a(t)$ $\quad T = \frac{1}{1000}$

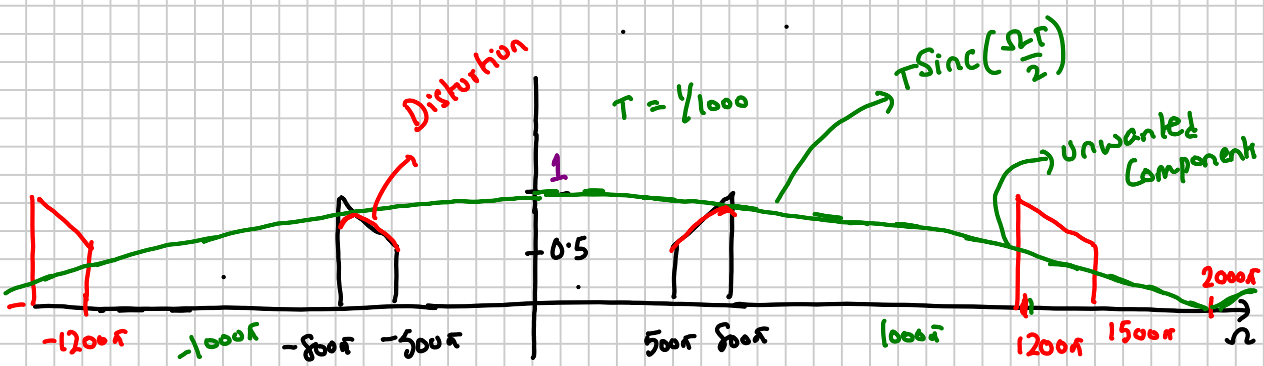
$\uparrow T$



ⓐ Sketch $Y_a(\Omega)$ for ideal D/A:



ⓑ Sketch output of ZOH:



First Unwanted frequency, $\omega = 1200\pi$

Highest amplitude of unwanted frequency

$= 1 \times \mathrm{Sinc}\left(\frac{\Omega T}{2}\right)$ at $\Omega = \underline{1200\pi}$