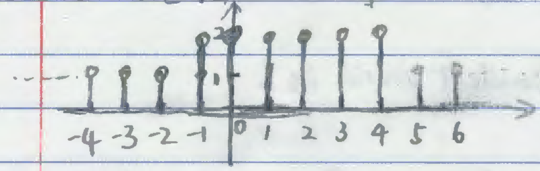
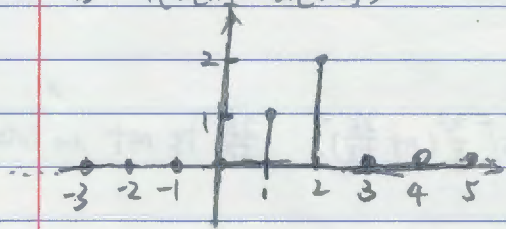
ECE 310 HW1 Soln

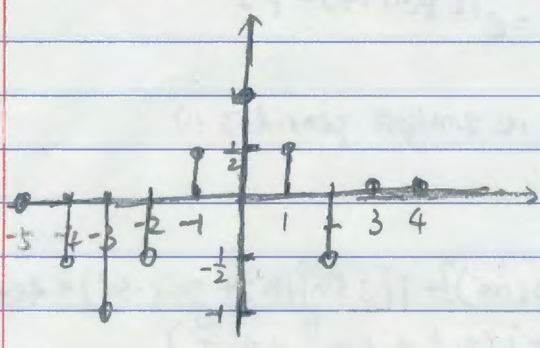
1) a) $u[n+1] + u[-n+4]$



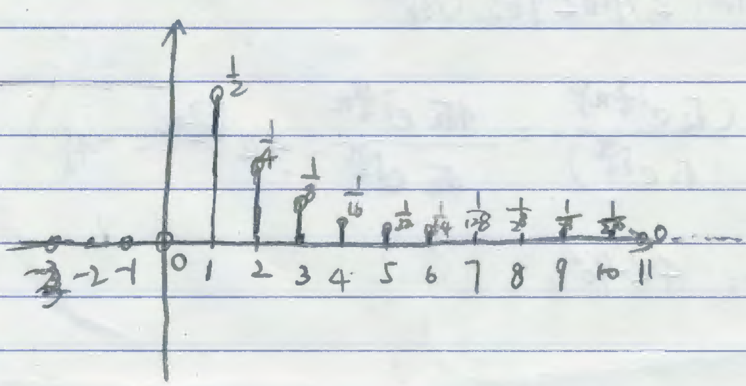
b) $n(u[n] - u[n-3])$



c) $\cos(n\pi/3)\,u[-n+2]\,u[n+4]$



d) $(\frac{1}{2})^n u[n-1]\, u[-n+10]$



2) a) $\sin[\frac{\pi n}{3}] = \sin[\frac{\pi n}{3} + 2k\pi] = \sin[\frac{\pi}{3}(n+6k)] \quad k \in \mathbb{Z}$

∴ It is a periodic signal, its smallest peorid is 6.

b) $\cos(\frac{2n}{3}) = \cos(\frac{2}{3}n + 2k\pi) = \cos[\frac{2}{3}(n + 3k\pi)]$ $3k\pi$ is not an integer.

∴ It is not a periodic signal

c) $\cos(\frac{\pi^2 n}{5}) = \cos(\frac{\pi^2 n}{5} + 2k\pi) = \cos[\frac{\pi^2}{5}(n + \frac{2k}{5\pi})]$ $\frac{2k}{5\pi}$ is not an integer

∴ It is not a periodic signal

d) $e^{i\pi(n-2)/5} = e^{i[\frac{\pi(n-2)}{5} + 2k\pi]} = e^{i[\frac{\pi}{5}(n+10k) - \frac{2\pi}{5}]}$

∴ It is a periodic signal, its smallost peorid is 10

3) a) $3\angle 150^\circ + 5\angle -60^\circ + 4\angle 120^\circ$

$= [3\cos(150^\circ) + 5\cos(-60^\circ) + 4\cos(120^\circ)] + i[3\sin(150^\circ) + 5\sin(-60^\circ) + 4\sin(120^\circ)]$

$= (-3 \cdot \frac{\sqrt{3}}{2} + 5 \cdot \frac{1}{2} + 4 \cdot -\frac{1}{2}) + i(3 \cdot \frac{1}{2} + 5 \cdot -\frac{\sqrt{3}}{2} + 4 \cdot \frac{\sqrt{3}}{2})$

$= (-\frac{3\sqrt{3}}{2} + \frac{1}{2}) + i(\frac{3}{2} - \frac{\sqrt{3}}{2})$

in polar form: $2.1918\angle 163.1868^\circ$

b) $\frac{(-1+i)^5}{(1+i)} = \frac{(\sqrt{2}e^{i\frac{3}{4}\pi})^5}{(\sqrt{2}e^{j\frac{\pi}{4}})} = \frac{4\sqrt{2}\, e^{j\frac{15}{4}\pi}}{\sqrt{2}\, e^{j\frac{\pi}{4}}} = 4e^{i\frac{7}{2}\pi} = -4j$

in polar form: $4\angle -90^\circ$

c) $\frac{5\angle 60^\circ}{2i} + \frac{\sqrt{2}\,e^{i\pi}}{2-i}$

$= \frac{5\cdot\frac{1}{2} + i5\frac{\sqrt{3}}{2}}{2i} + \frac{-\sqrt{2}}{2-i}$

$= \frac{-5i+5\sqrt{3}}{4} + \frac{-2\sqrt{2}-\sqrt{2}i}{5} = \left(-\frac{2\sqrt{2}}{5} + \frac{5\sqrt{3}}{4}\right) + i\left(-\frac{5}{4} - \frac{\sqrt{2}}{5}\right)$

polar form: $2.2453 \angle -43.7831$

d) $\left(\frac{-1+i3}{1-i} + \frac{3+i}{1+i2}\right)^n = \left[\frac{(-1+i3)(1+i2) + (3+i)(1-i)}{(1-i)(1+j2)}\right]^n = (-1)^n = e^{i\pi n}$

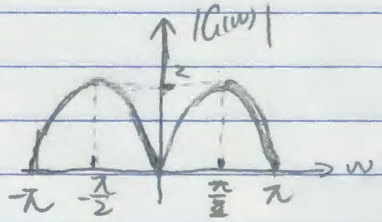
4) $G(w) = 1 - e^{-i2w} = e^{-iw}(e^{iw} - e^{-iw}) = e^{-iw}\left[\cos(w) + i\sin(w) - \cos(-w) - i\sin(-w)\right]$

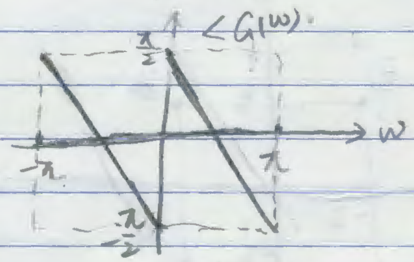
$= 2i\sin(w)\,e^{-iw} = 2\sin(w)\,e^{i(\frac{\pi}{2}-w)}$

$|G(w)| = 2\sin(w)$

$\angle G(w) = \begin{cases} e^{i(\frac{\pi}{2}-w)}, & \text{if } \sin(w) > 0 \\ e^{i(-\frac{\pi}{2}-w)}, & \text{if } \sin(w) < 0 \end{cases}$

$\therefore \angle G(w) = \begin{cases} \frac{\pi}{2} - w, & \text{if } 2n\pi \le w \le (2n+1)\pi \\ -\frac{\pi}{2} - w, & \text{otherwise} \end{cases} \quad n \in \mathbb{Z}$





5) a) $4z^4+1=0$

$z=|z|e^{i\angle z}$

$z^4=-\frac{1}{4}=|\frac{1}{4}|e^{i(\pi+2\pi k)}$

$\therefore z_n=(\frac{1}{4})^{\frac{1}{4}}e^{\frac{i(\pi+2\pi k)}{4}}$

$\therefore$ The roots are $z_0=\frac{1}{\sqrt{2}}e^{i\pi/4}$ $\quad z_1=\frac{1}{\sqrt{2}}e^{i3\pi/4}$

$z_2=\frac{1}{\sqrt{2}}e^{i5\pi/4}$ $\quad z_3=\frac{1}{\sqrt{2}}e^{i7\pi/4}$

b) $G(z)=(z-\frac{1}{\sqrt{2}}e^{i\pi/4})(z-\frac{1}{\sqrt{2}}e^{i3\pi/4})(z-\frac{1}{\sqrt{2}}e^{i5\pi/4})(z-\frac{1}{\sqrt{2}}e^{i7\pi/4})$

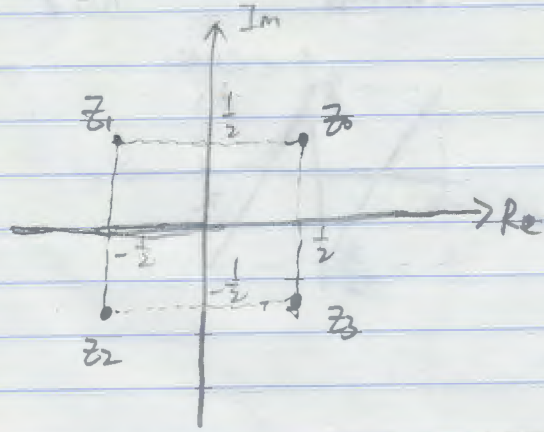
c) $z_3=\frac{1}{\sqrt{2}}e^{i7\pi/4}=\frac{1}{\sqrt{2}}e^{-i\pi/4}=z_0^*$

$z_2=\frac{1}{\sqrt{2}}e^{i5\pi/4}=\frac{1}{\sqrt{2}}e^{-i3\pi/4}=z_1^*$

$\because (z-z_n)(z-z_n^*)=z^2-2z\,Re\{z_n\}+|z_n|^2$

$\therefore G(z)=(z^2-z+\frac{1}{2})(z^2+z+\frac{1}{2})$

d)



6 a) $\int_{-\infty}^{\infty} (t^2-5t+4)\delta(t)\,dt = 4$

b) $\int_{-\infty}^{-3} (t^2-5t+4)\delta(t)\,dt = 0$

c) $\int_{-3}^{\infty} (t^2-5t+4)\delta(t)\,dt = 4$

d) $\int_{-\infty}^{\infty} (t^2-5t+4)\delta(t-3)\,dt = -2$

e) $\int_{-\infty}^{\infty} (t^2-5t+4)\delta(3t-2)\,dt = \int_{-\infty}^{\infty} (t^2-5t+4)\delta(3(t-\frac{2}{3}))\,dt = \frac{1}{3}\cdot\frac{10}{9} = \frac{10}{27}$

f) $[e^{-t}u(t)] * \delta(3t-2) = \frac{1}{3}e^{-(t-\frac{2}{3})}u(t-\frac{2}{3})$

7) a) $\delta(2t-3)$

$F[\delta(2t-3)] = \int_{-\infty}^{\infty} \delta(2t-3)e^{-j\Omega t}\,dt = \frac{1}{2}\cdot e^{-j\frac{3}{2}\Omega}$

b) $e^{2at}u(t)$

$F[e^{2at}u(t)] = \int_{-\infty}^{\infty} e^{-2at}u(t)e^{-j\Omega t}\,dt = \int_{0}^{\infty} e^{-2at}e^{-j\Omega t}\,dt = -\frac{1}{2a+j\Omega}$

c) $u(t)-u(t-T)$

$F[u(t)-u(t-T)] = \int_{-\infty}^{\infty} e^{-j\Omega t}[u(t)-u(t-T)]\,dt = \int_{0}^{T} e^{-j\Omega t}\,dt = Te^{-j\Omega\frac{T}{2}}\operatorname{sinc}(\frac{\Omega T}{2})$

d) $\sin(2\Omega_0 t+\phi)$

$F[\sin(2\Omega_0 t+\phi)] = F\left[\frac{e^{j(2\Omega_0 t+\phi)} - e^{-j(2\Omega_0 t+\phi)}}{2j}\right]$

$= \frac{1}{2j}\left[e^{j\phi}F[e^{j2\Omega_0 t}] - e^{-j\phi}F[e^{-j2\Omega_0 t}]\right]$

$= -\pi j\left[e^{j\phi}\delta(\Omega-2\Omega_0) - e^{-j\phi}\delta(\Omega+2\Omega_0)\right] = \pi j\left[e^{-j\phi}\delta(\Omega+2\Omega_0) - e^{j\phi}\delta(\Omega-2\Omega_0)\right]$

$= \pi\left[e^{j(\frac{\pi}{2}-\phi)}\delta(\Omega+2\Omega_0) - e^{j(\frac{\pi}{2}+\phi)}\delta(\Omega-2\Omega_0)\right]$

e) $(u(t-1)-u(t-6))e^{j2\pi t}$

Using the result from part c) with $T=5$, and using the time shift property

$$x(t-t_0) \leftrightarrow e^{-j\Omega t_0}$$

$$F[u(t-1)-u(t-6)] = e^{-j\Omega}(5e^{-j5\Omega/2}\,\mathrm{sinc}(5\Omega/2))$$

Because of modulation property

$$f(t)e^{j\Omega_0 t} \leftrightarrow F(\Omega-\Omega_0) \qquad \Omega_0 = 2\pi$$

$$\therefore F[(u(t-1)-u(t-6))e^{j2\pi t}] = 5e^{-j7(\Omega-2\pi)/2}\cdot\mathrm{sinc}((\Omega-2\pi)5/2)$$