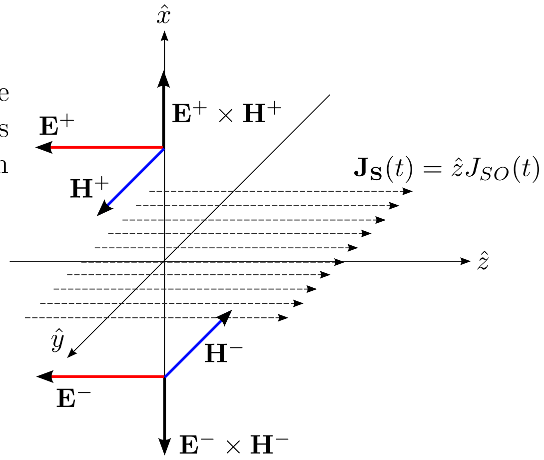
1.

A surface current $\mathbf{J_s} = \hat{z} J_{so} \sin(\omega t)$ flowing on the $y-z$ plane induces electromagnetic waves on both sides of the sheet as shown in the figure on the right. The induced fields are given by

$$\mathbf{E} = -\hat{z}\, E_o \sin(\omega t \mp \beta x)\,\mathrm{V/m} \quad for\, x \gtrless 0$$

$$\mathbf{H} = \pm\hat{y}\, H_o \sin(\omega t \mp \beta x)\,\mathrm{A/m} \quad for\, x \gtrless 0,$$

where $E_o = \eta_o H_o$.



a) The average Poynting vector for the wave propagating in $+x$ direction is $\langle \mathbf{S} \rangle_{x>0} = \frac{1}{2} E_o H_o\, \hat{x}$, while, for the wave propagating in $-x$ direction, we have $\langle \mathbf{S} \rangle_{x<0} = -\frac{1}{2} E_o H_o\, \hat{x}$. Since the average power density provided by the current sheet is $4\,\frac{\mathrm{W}}{\mathrm{m}^2}$, we can write that

$$\left|\langle \mathbf{S} \rangle_{x>0}\right| + \left|\langle \mathbf{S} \rangle_{x<0}\right| = E_o H_o = \frac{E_o^2}{\eta_o} = 8\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

Then, assuming $\eta_o = 120\pi\,\Omega$, we get

$$E_o = \sqrt{8\eta_o} = 54.92\,\frac{\mathrm{V}}{\mathrm{m}},$$

and also

$$H_o = \frac{E_o}{\eta_o} = 0.14\,\frac{\mathrm{A}}{\mathrm{m}}.$$

b) The amount of instantaneous electromagnetic density power injected by the current surface is

$$-\mathbf{J_s}(t) \cdot \mathbf{E}(0,t) = J_{so} E_o \sin^2(\omega t),$$

therefore, the average power density is

$$\langle -\mathbf{J_s}(t) \cdot \mathbf{E}(0,t) \rangle = \frac{1}{2} J_{so} E_o = 8\,\frac{\mathrm{W}}{\mathrm{m}^2},$$

which implies that

$$J_{so} = \frac{16}{E_o} = \frac{4}{\sqrt{\eta_o}} = 0.206\,\frac{\mathrm{A}}{\mathrm{m}}.$$

c) The phasors of $\mathbf{E}$ and $\mathbf{H}$ fields of part (a) are as follows:

$$\tilde{\mathbf{E}} = j E_o e^{\mp j\beta x}\,\hat{z} \qquad and \qquad \tilde{\mathbf{H}} = \mp j H_o e^{\mp j\beta x}\,\hat{y}.$$

For $x > 0$, the phasors will be

$$\tilde{\mathbf{E}} = j54.92 e^{-j\beta x}\,\hat{z}\,\frac{\mathrm{V}}{\mathrm{m}} \qquad and \qquad \tilde{\mathbf{H}} = -j0.14 e^{-j\beta x}\,\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}},$$

while for $x < 0$, the phasors will be

$$\tilde{\mathbf{E}} = j54.92 e^{j\beta x}\,\hat{z}\,\frac{\mathrm{V}}{\mathrm{m}} \qquad and \qquad \tilde{\mathbf{H}} = j0.14 e^{j\beta x}\,\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}.$$

d) From Faraday's Law and Ampere's Law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \nabla \times \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

we have two first-order differential equations

$$-\frac{\partial E_z(x,t)}{\partial x}\hat{y} = -\mu_0 \frac{\partial H_y(x,t)}{\partial t}\hat{y}$$

$$\mu_0 \frac{\partial H_y(x,t)}{\partial x}\hat{z} = \epsilon_0 \mu_0 \frac{\partial E_z(x,t)}{\partial t}\hat{z}$$

which can be further simplified as

$$\frac{\partial E_z(x,t)}{\partial x} = \mu_0 \frac{\partial H_y(x,t)}{\partial t}$$

$$\frac{\partial H_y(x,t)}{\partial x} = \epsilon_0 \frac{\partial E_z(x,t)}{\partial t}$$

for both $x > 0$ and $x < 0$ regions, where the fields can be represented in the form of

$$\begin{aligned} \mathbf{E}(x,t) &= E_z(x,t)\hat{z} \\ \mathbf{H}(x,t) &= H_y(x,t)\hat{y} \end{aligned}$$

2. Electric field phasor $\tilde{\mathbf{E}} = (2\hat{x} - j2\hat{z})\, e^{-j\beta y}$.

a) $E_x = \tilde{\mathbf{E}} \cdot \hat{x} = 2e^{-j\beta y}$, $E_z = \tilde{\mathbf{E}} \cdot \hat{z} = 2e^{-j\beta y - j\frac{\pi}{2}}$. The angle $\angle E_x = -\beta y$, while the angle of the $\hat{z}$ component is $\angle E_z = -\beta y - \frac{\pi}{2}$. We notice that $\angle E_x = \angle E_z + \frac{\pi}{2}$. Therefore we say that the $\hat{x}$ component **leads** the $\hat{z}$ component by 90^o.

b) $\mathbf{E}$ rotates in the direction that your left-hand fingers curl when the thumb is directed in propagation direction $\hat{y}$. Therefore we say that the wave is **left-hand circularly** polarized. We can also say that, $\mathbf{E}$ rotates as a function of time in anti-clockwise direction when viewed from the direction of propagation $\hat{y}$.

3. Let us consider the following five plane waves in free space,

$$\begin{aligned} \mathbf{E_1} &= 4\cos(\omega t - \beta y)\hat{x} + 3\cos(\omega t - \beta y)\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \mathbf{E_2} &= 2\cos(\omega t + \beta y)\hat{x} + 2\sin(\omega t + \beta y)\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \mathbf{E_3} &= \cos(\omega t - \beta x - \tfrac{\pi}{2})\hat{y} + \sin(\omega t - \beta x)\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \mathbf{H_4} &= \cos(\omega t + \beta z + \tfrac{\pi}{3})\hat{x} + \sin(\omega t + \beta z - \tfrac{\pi}{6})\hat{y}\ \tfrac{\mathrm{A}}{\mathrm{m}} \\ \mathbf{H_5} &= 2\cos(\omega t + \beta x)\hat{z} - \sin(\omega t + \beta x)\hat{y}\ \tfrac{\mathrm{A}}{\mathrm{m}} \end{aligned}$$

and the corresponding E and H fields in free space are

$$\begin{aligned} \mathbf{H_1} &= -\tfrac{4}{\eta_0}\cos(\omega t - \beta y)\hat{z} + \tfrac{3}{\eta_0}\cos(\omega t - \beta y)\hat{x}\ \tfrac{\mathrm{A}}{\mathrm{m}} \\ \mathbf{H_2} &= \tfrac{2}{\eta_0}\cos(\omega t + \beta y)\hat{z} - \tfrac{2}{\eta_0}\sin(\omega t + \beta y)\hat{x}\ \tfrac{\mathrm{A}}{\mathrm{m}} \\ \mathbf{H_3} &= \tfrac{1}{\eta_0}\cos(\omega t - \beta x - \tfrac{\pi}{2})\hat{z} - \tfrac{1}{\eta_0}\sin(\omega t - \beta x)\hat{y}\ \tfrac{\mathrm{A}}{\mathrm{m}} \\ \mathbf{E_4} &= \eta_0\cos(\omega t + \beta z + \tfrac{\pi}{3})\hat{y} - \eta_0\sin(\omega t + \beta z - \tfrac{\pi}{6})\hat{x}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \mathbf{E_5} &= -2\eta_0\cos(\omega t + \beta x)\hat{y} - \eta_0\sin(\omega t + \beta x)\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \end{aligned}$$

i. Phasors $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ are defined such that $\mathbf{E} = \mathrm{Re}\{\tilde{\mathbf{E}}e^{j\omega t}\}$ and $\mathbf{H} = \mathrm{Re}\{\tilde{\mathbf{H}}e^{j\omega t}\}$. For the fields above, we have

$$\begin{aligned} \tilde{\mathbf{E}}_\mathbf{1} &= 4e^{-j\beta y}\hat{x} + 3e^{-j\beta y}\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \tilde{\mathbf{E}}_\mathbf{2} &= 2e^{j\beta y}\hat{x} - j2e^{j\beta y}\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \tilde{\mathbf{E}}_\mathbf{3} &= -je^{-j\beta x}\hat{y} - je^{-j\beta x}\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \tilde{\mathbf{E}}_\mathbf{4} &= \eta_0 e^{j\frac{\pi}{3}} e^{j\beta z}\hat{y} - \eta_0 e^{j\frac{\pi}{3}} e^{j\beta z}\hat{x}\ \tfrac{\mathrm{V}}{\mathrm{m}} \\ \tilde{\mathbf{E}}_\mathbf{5} &= -2\eta_0 e^{j\beta x}\hat{y} + j\eta_0 e^{j\beta x}\hat{z}\ \tfrac{\mathrm{V}}{\mathrm{m}} \end{aligned}$$

and

$$\begin{aligned}
\tilde{\mathbf{H}}_\mathbf{1} &= -\tfrac{4}{\eta_0}e^{-j\beta y}\hat{z} + \tfrac{3}{\eta_0}e^{-j\beta y}\hat{x}\,\frac{\mathrm{A}}{\mathrm{m}} \\
\tilde{\mathbf{H}}_\mathbf{2} &= \tfrac{2}{\eta_0}e^{j\beta y}\hat{z} + j\tfrac{2}{\eta_0}e^{j\beta y}\hat{x}\,\frac{\mathrm{A}}{\mathrm{m}} \\
\tilde{\mathbf{H}}_\mathbf{3} &= -j\tfrac{1}{\eta_0}e^{-j\beta x}\hat{z} + j\tfrac{1}{\eta_0}e^{-j\beta x}\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}} \\
\tilde{\mathbf{H}}_\mathbf{4} &= e^{j\frac{\pi}{3}}e^{j\beta z}\hat{x} + e^{j\frac{\pi}{3}}e^{j\beta z}\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}} \\
\tilde{\mathbf{H}}_\mathbf{5} &= 2e^{j\beta x}\hat{z} + je^{j\beta x}\hat{y}\,\frac{\mathrm{A}}{\mathrm{m}}
\end{aligned}$$

ii. The time-averaged Poynting vector $\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$.

- For wave a), we have

$$\langle \mathbf{S_1} \rangle = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_\mathbf{1} \times \tilde{\mathbf{H}}_\mathbf{1}^*\right\} = \frac{1}{2}(\frac{16}{\eta_0} + \frac{9}{\eta_0})\hat{y} = \frac{25}{2\eta_o}\hat{y}\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

- For wave b), we have

$$\langle \mathbf{S_2} \rangle = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_\mathbf{2} \times \tilde{\mathbf{H}}_\mathbf{2}^*\right\} = \frac{1}{2}(\frac{4}{\eta_0} + \frac{4}{\eta_0})(-\hat{y}) = -\frac{4}{\eta_o}\hat{y}\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

- For wave c), we have

$$\langle \mathbf{S_3} \rangle = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_\mathbf{3} \times \tilde{\mathbf{H}}_\mathbf{3}^*\right\} = \frac{1}{2}(\frac{1}{\eta_0} + \frac{1}{\eta_0})\hat{x} = \frac{1}{\eta_o}\hat{x}\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

- For wave d), we have

$$\langle \mathbf{S_4} \rangle = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_\mathbf{4} \times \tilde{\mathbf{H}}_\mathbf{4}^*\right\} = \frac{1}{2}(\eta_0 + \eta_0)(-\hat{z}) = -\eta_0\hat{z}\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

- For wave e), we have

$$\langle \mathbf{S_5} \rangle = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}}_\mathbf{5} \times \tilde{\mathbf{H}}_\mathbf{5}^*\right\} = \frac{1}{2}(4\eta_0 + \eta_0)(-\hat{x}) = -\frac{5\eta_0}{2}\hat{x}\,\frac{\mathrm{W}}{\mathrm{m}^2}.$$

iii. The time averaged power flow density is given by the average Poynting vector $\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$. Therefore, the average power that crosses some surface A is given by $\langle P \rangle = \int_A \langle \mathbf{S} \rangle \cdot dS$. In the case of uniform plane waves, this expression simplifies to

$$\langle P \rangle = \langle \mathbf{S} \rangle \cdot \hat{n}\, A,$$

where $\hat{n}$ is the vector normal to the flat area A in the direction of propagation. Below, we are considering $A = 1\,\mathrm{m}^2$.

- For wave a), using part (ii)

$$\langle \mathbf{S_1} \rangle = \frac{25}{2\eta_o}\hat{y}\,\frac{\mathrm{W}}{\mathrm{m}^2},$$

  thus,

$$\langle P_1 \rangle = \frac{25}{2\eta_o}\,\mathrm{W}.$$

- For wave b), using part (ii)

$$\langle \mathbf{S_2} \rangle = -\frac{4}{\eta_o}\hat{y}\,\frac{\mathrm{W}}{\mathrm{m}^2},$$

  thus,

$$\langle P_2 \rangle = \frac{4}{\eta_o}\,\mathrm{W}.$$

- For wave c), using part (ii)

$$\langle \mathbf{S_3} \rangle = \frac{1}{\eta_o} \hat{x} \, \frac{\mathrm{W}}{\mathrm{m}^2},$$

thus,

$$\langle P_3 \rangle = \frac{1}{\eta_o} \, \mathrm{W}.$$

- For wave d), using part (ii)

$$\langle \mathbf{S_4} \rangle = -\eta_0 \hat{z} \, \frac{\mathrm{W}}{\mathrm{m}^2},$$

thus,

$$\langle P_4 \rangle = \eta_0 \, \mathrm{W}.$$

- For wave e), using part (ii)

$$\langle \mathbf{S_5} \rangle = -\frac{5\eta_0}{2} \hat{x} \, \frac{\mathrm{W}}{\mathrm{m}^2},$$

thus,

$$\langle P_5 \rangle = \frac{5\eta_0}{2} \, \mathrm{W}.$$

iv. To figure out the polarization of the wave, we need to focus on the phase difference of E-field components in different directions

- For wave a),

$$\tilde{\mathbf{E}}_\mathbf{1} = \quad 4e^{-j\beta y}\hat{x} + 3e^{-j\beta y}\hat{z} \quad = (4\hat{x} + 3\hat{y})e^{-j\beta y}$$

so, the wave is **linear polarized** in the direction of

$$\frac{4\hat{x} + 3\hat{y}}{5}$$

- For wave b),

$$\tilde{\mathbf{E}}_\mathbf{2} = 2e^{j\beta y}\hat{x} - j2e^{j\beta y}\hat{z} = (2\hat{x} - j2\hat{z})e^{j\beta y}$$

and the wave is propagating in $\hat{y}$, so the wave is **right-hand circular polarized**.

- For wave c),

$$\tilde{\mathbf{E}}_\mathbf{3} = -je^{-j\beta x}\hat{y} - je^{-j\beta x}\hat{z} = -j(\hat{y} + \hat{z})e^{-j\beta x}$$

so, the wave is **linear polarized** in the direction of

$$\frac{\hat{y} + \hat{z}}{\sqrt{2}}$$

- For wave d),

$$\tilde{\mathbf{E}}_\mathbf{4} = \eta_0 e^{j\frac{\pi}{3}} e^{j\beta z}\hat{y} - \eta_0 e^{j\frac{\pi}{3}} e^{j\beta z}\hat{x} = \eta_0(\hat{y} - \hat{x})e^{j\beta z + j\frac{\pi}{3}}$$

so, the wave is **linear polarized** in the direction of

$$\frac{-\hat{x} + \hat{y}}{\sqrt{2}}$$

- For wave e),

$$\tilde{\mathbf{E}}_\mathbf{5} = -2\eta_0 e^{j\beta x}\hat{y} + j\eta_0 e^{j\beta x}\hat{z} = \eta_0(-2\hat{y} + j\hat{z})e^{j\beta x}$$

so, the wave is **elliptical polarized**.

v. From Lecture 19, we know that for a current sheet$\mathbf{J}_s = \hat{x}f(t)$ at $z = 0$ plane, the E-field generated will have the form of

$$\vec{E} = -\hat{x}\frac{\eta_0 f(t \mp \frac{z}{c})}{2}$$

by the same analogy, since we know the E-field on one side of a currect sheet, we can get surrface current

- For wave a),

$$\mathbf{E_1} = 4\cos(\omega t - \beta y)\hat{x} + 3\cos(\omega t - \beta y)\hat{z}\,\frac{\text{V}}{\text{m}}$$

so,

$$\mathbf{J_1} = -\frac{8}{\eta_0}\cos(\omega t)\hat{x} - \frac{6}{\eta_0}\cos(\omega t)\hat{z}\,\frac{\text{A}}{\text{m}}$$

at $z = 0$ plane

- For wave b),

$$\mathbf{E_2} = 2\cos(\omega t + \beta y)\hat{x} + 2\sin(\omega t + \beta y)\hat{z}\,\frac{\text{V}}{\text{m}}$$

so,

$$\mathbf{J_2} = -\frac{4}{\eta_0}\cos(\omega t)\hat{x} - \frac{4}{\eta_0}\sin(\omega t)\hat{z}\,\frac{\text{A}}{\text{m}}$$

at $y = 0$ plane

- For wave c),

$$\mathbf{E_3} = \cos(\omega t - \beta x - \frac{\pi}{2})\hat{y} + \sin(\omega t - \beta x)\hat{z}\,\frac{\text{V}}{\text{m}}$$

so,

$$\mathbf{J_3} = -\frac{2}{\eta_0}\cos(\omega t - \frac{\pi}{2})\hat{y} - \frac{2}{\eta_0}\sin(\omega t)\hat{z}\,\frac{\text{A}}{\text{m}}$$

at $y = 0$ plane

- For wave d),

$$\mathbf{E_4} = \eta_0\cos(\omega t + \beta z + \frac{\pi}{3})\hat{y} - \eta_0\sin(\omega t + \beta z - \frac{\pi}{6})\hat{x}\,\frac{\text{V}}{\text{m}}$$

so,

$$\mathbf{J_4} = -2\cos(\omega t + \frac{\pi}{3})\hat{y} + 2\sin(\omega t - \frac{\pi}{6})\hat{x}\,\frac{\text{A}}{\text{m}}$$

at $z = 0$ plane

- For wave e),

$$\mathbf{E_5} = -2\eta_0\cos(\omega t + \beta x)\hat{y} - \eta_0\sin(\omega t + \beta x)\hat{z}\,\frac{\text{V}}{\text{m}}$$

so,

$$\mathbf{J_5} = 4\cos(\omega t)\hat{y} + 2\sin(\omega t)\hat{z}\,\frac{\text{A}}{\text{m}}$$

at $x = 0$ plane

4. The total field is $\tilde{\mathbf{E}} = \frac{\eta_o}{2}e^{-j\beta x}\left[\hat{z}J_1 e^{j(\phi+\frac{\pi}{2})} + \hat{y}J_2\right]$. The $\frac{\pi}{2}$ offset is from the $\frac{\lambda}{4}$ distance between the surfaces

   a) Let $J = J_1 = J_2$.

      i. The direction of propagation is $\hat{x}$ and to have a right handed circular polarization, the $\hat{y}$ component needs to lead by 90^o the $\hat{z}$ component. Therefore

$$\begin{aligned} -\frac{\pi}{2} &= \phi + \frac{\pi}{2} + 2n\pi, \\ \phi &= -\pi - 2n\pi = (2n-1)\pi \end{aligned}$$

      where n is an arbitrary integer. The electric field phasor for the region will be $\tilde{\mathbf{E}} = \frac{\eta_o}{2}e^{-j\beta x}\left[\hat{z}J_1 e^{j(\phi+\frac{\pi}{2})} + \hat{y}J_2\right] = \frac{\eta_o J}{2}e^{-j\beta x}\left[-j\hat{z} + \hat{y}\right]\frac{\text{V}}{\text{m}}$.

      ii. To have left handed circular polarization, the $\hat{z}$ component needs to lead by 90^o the $\hat{y}$ component. Therefore

$$\begin{aligned} \frac{\pi}{2} &= \phi + \frac{\pi}{2} + 2n\pi, \\ \phi &= 0 - 2n\pi = 2n\pi \end{aligned}$$

      where n is an arbitrary integer. The electric field phasor for the region will be $\tilde{\mathbf{E}} = \frac{\eta_o}{2}e^{-j\beta x}\left[\hat{z}J_1 e^{j(\phi+\frac{\pi}{2})} + \hat{y}J_2\right] = \frac{\eta_o J}{2}e^{-j\beta x}\left[j\hat{z} + \hat{y}\right]\frac{\text{V}}{\text{m}}$.

iii. To have linear polarization, the $\hat{z}$ component needs to be in phase with the $\hat{y}$ component or off by 180^o. Thus

$$\begin{aligned} 0 &= \phi + \frac{\pi}{2} + n\pi, \\ \phi &= -\frac{\pi}{2} - n\pi = (n - \frac{1}{2})\pi \end{aligned}$$

where n is an arbitrary integer. The electric field phasor for the region will be $\tilde{\mathbf{E}} = \frac{\eta_o}{2} e^{-j\beta x} \left[\hat{z} J_1 e^{j(\phi + \frac{\pi}{2})} + \hat{y} J_2 \right] = \frac{\eta_o J}{2} e^{-j\beta x} \left[\hat{z} + \hat{y} \right] \frac{\mathrm{V}}{\mathrm{m}}$.

b) The corresponding magnetic field of (iii) is

$$\tilde{\mathbf{H}} = \frac{J}{2} e^{-j\beta x} \left[\hat{y} - \hat{z} \right] \frac{\mathrm{A}}{\mathrm{m}}.$$

Therefore the time-averaged Poynting vector is

$$\begin{aligned} \langle \mathbf{S} \rangle &= \frac{1}{2} \mathrm{Re} \left\{ \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right\} \\ &= \hat{x} \frac{1}{2} (\eta_0 \frac{J_1^2}{4} + \eta_0 \frac{J_2^2}{4}) \\ &= \hat{x} \frac{\eta_0}{8} (J_1^2 + J_2^2) \frac{\mathrm{W}}{\mathrm{m}^2}. \end{aligned}$$

This result does not depend on the angle ϕ, therefore the time-averaged Poynting vector will be the same for cases (i) , (ii) and (iii). If $J_1 = J_2 = 1 \frac{\mathrm{A}}{\mathrm{m}}$.:

$$\langle \mathbf{S} \rangle = \hat{x}\, 30\pi \frac{\mathrm{W}}{\mathrm{m}^2}.$$

c) If J_2 =0, then $\tilde{\mathbf{E}}_\mathbf{2} = 0$, and $\tilde{\mathbf{H}}_\mathbf{2} = 0$. Therefore the time-averaged Poynting vector is still

$$\begin{aligned} \langle \mathbf{S} \rangle &= \frac{1}{2} \mathrm{Re} \left\{ \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right\} \\ &= \hat{x} \frac{\eta_0}{8} (J_1^2 + J_2^2) \\ &= \hat{x} \frac{\eta_0}{8} J_1^2 \\ &= \hat{x} 15\pi \frac{\mathrm{W}}{\mathrm{m}^2} \end{aligned}$$

d) From the results of (b) and (c), we can see that in case of circularly polarized waves the power content is twice that of a linearly polarized wave field of an equal instantaneous peak electric field magnitudes.