University of Illinois at Urbana-Champaign

ECE 310: Digital Signal Processing

PROBLEM SET 7: SOLUTIONS

Problem 1

1.

$$\begin{aligned} y[n] &= x[n] - x[n-10] \\ Y(z) &= (1 - z^{-10})X(z) \\ \therefore H(z) &= 1 - z^{-10} \\ H_d(\omega) &= 1 - e^{-j10\omega} = 2e^{-j5\omega} j \sin(5\omega) \\ |H_d(\omega)| &= 2|\sin(5\omega)| \\ \angle H_d(\omega) &= \begin{cases} -5\omega + \pi/2 & \text{for } \sin(5\omega) \geq 0 \\ -5\omega - \pi/2 & \text{for } \sin(5\omega) < 0 \end{cases} \end{aligned}$$

2. Note $h[n]$ is real since $H_d(\omega) = H_d^*(-\omega)$.

(a)

$$\begin{aligned} x[n] &= \cos\left(\frac{\pi}{10}n\right) + 3\sin\left(\frac{\pi}{3}n + \frac{\pi}{10}\right) \\ \therefore y[n] &= |H_d(\frac{\pi}{10})| \cos\left(\frac{\pi}{10}n + \angle H_d(\frac{\pi}{10})\right) + 3|H_d(\frac{\pi}{3})| \sin\left(\frac{\pi}{3}n + \frac{\pi}{10} + \angle H_d(\frac{\pi}{3})\right) \\ y[n] &= 2\cos\left(\frac{\pi}{10}n\right) + 3\sqrt{3}\sin\left(\frac{\pi}{3}n + \frac{\pi}{10} - \frac{\pi}{6}\right) \\ y[n] &= 3\sin\left(\frac{\pi}{3}n - \frac{\pi}{15}\right) \end{aligned}$$

(b)

$$\begin{aligned} x[n] &= 10 + 5\cos\left(\frac{2\pi}{5}n + \frac{\pi}{2}\right) \\ \therefore y[n] &= 10H_d(0) + 5|H_d(\frac{2\pi}{5})| \cos\left(\frac{2\pi}{5}n + \frac{\pi}{2} + \angle H_d(\frac{2\pi}{5})\right) \\ y[n] &= 0 + 0 = 0 \end{aligned}$$

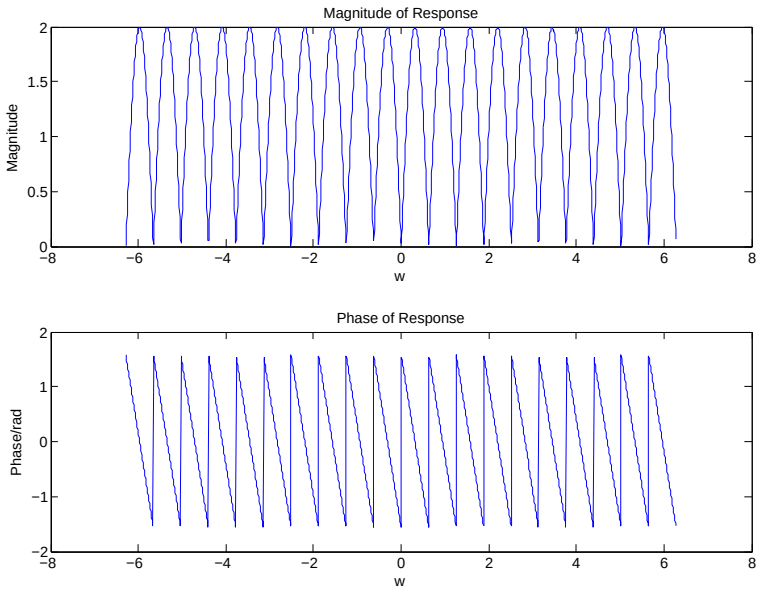


Figure 1: Magnitude and Phase for Problem 1

2. $x_1[n]=3$, $x_2[n]=\cos\left(\frac{\pi}{4}n+10^\circ\right)=\sin\left(\frac{\pi}{4}n+100^\circ\right)$, $x_3[n]=\sin\left(\frac{\pi}{3}n+25^\circ\right)$

$\Rightarrow$ $y_1[n]=9$. $y_2[n]=2\sin\left(\frac{\pi}{4}n+10^\circ\right)$, $y_3[n]=0$.

$Hd(0)=3$ $\quad Hd\left(\frac{\pi}{4}\right)=2e^{-90^\circ}$ $\quad Hd\left(\frac{\pi}{3}\right)=0$.

$\tilde{x}[n]=5+2\sin\left(\frac{\pi}{4}n+15^\circ\right)+10\cos\left(-\frac{\pi}{3}n+25^\circ\right)=5+2\sin\left(\frac{\pi}{4}n+15^\circ\right)+10\cos\left(\frac{\pi}{3}n-25^\circ\right)$

$\therefore \tilde{y}[n]=5Hd(0)+2|Hd\left(\frac{\pi}{4}\right)|\sin\left(\frac{\pi}{4}n+15^\circ+\angle Hd\left(\frac{\pi}{4}\right)\right)+10|Hd\left(\frac{\pi}{3}\right)|\cos\left(\frac{\pi}{3}n-25^\circ+\angle Hd\left(\frac{\pi}{3}\right)\right)$

$=15+4\sin\left(\frac{\pi}{4}n-75^\circ\right)$

3. a) Note $h[n]$ is NOT real since $Hd(\omega)\neq Hd^*(-\omega)$

$x[n]=5+10e^{j\left(\frac{\pi}{4}n+45^\circ\right)}+j^n$

$=5+10e^{j\left(\frac{\pi}{4}n+\frac{\pi}{4}\right)}+e^{j\frac{\pi}{2}n}$

$\therefore y[n]=5Hd(0)+10e^{j\left(\frac{\pi}{4}n+\frac{\pi}{4}\right)}Hd\left(\frac{\pi}{4}\right)+e^{j\frac{\pi}{2}n}Hd\left(\frac{\pi}{2}\right)$

$=0+10e^{j\left(\frac{\pi}{4}n+\frac{\pi}{4}\right)}\cdot\frac{\pi}{4}e^{j\frac{\pi}{2}}+e^{j\frac{\pi}{2}n}\cdot\frac{\pi}{2}e^{j}$

$=\frac{5\pi}{2}e^{j\left(\frac{\pi}{4}n+\frac{\pi}{4}+\frac{\sqrt{2}}{2}\right)}+\frac{\pi}{2}e^{j\left(\frac{\pi}{2}n+1\right)}$

b) $x[n]=5+10\cos\left(\frac{\pi}{4}n+45^\circ\right)+j^n$

$=5+5e^{j\left(\frac{\pi}{4}n+\frac{\pi}{4}\right)}+5e^{-j\left(\frac{\pi}{4}n+\frac{\pi}{4}\right)}+e^{j\frac{\pi}{2}n}$

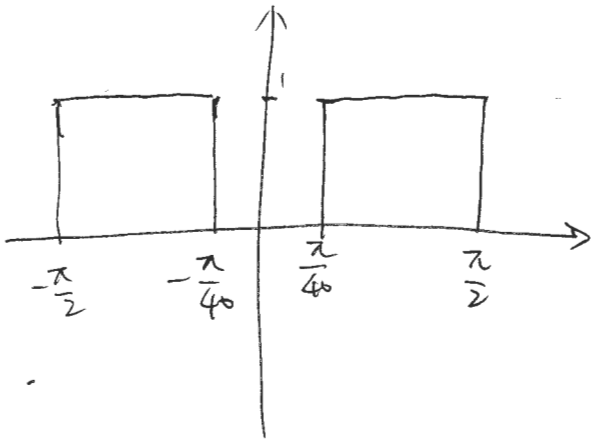
$y[n]=5Hd(0)+5e^{j\left(\frac{\pi}{4}n+\frac{\pi}{4}\right)}Hd\left(\frac{\pi}{4}\right)+5e^{-j\left(\frac{\pi}{4}n+\frac{\pi}{4}\right)}Hd\left(-\frac{\pi}{4}\right)+e^{j\frac{\pi}{2}n}Hd\left(\frac{\pi}{2}\right)$

$=\frac{5\pi}{2}\sin\left(\frac{\pi}{4}n+\frac{\pi}{4}+\frac{\sqrt{2}}{2}\right)+\frac{\pi}{2}e^{j\left(\frac{\pi}{2}n+1\right)}$

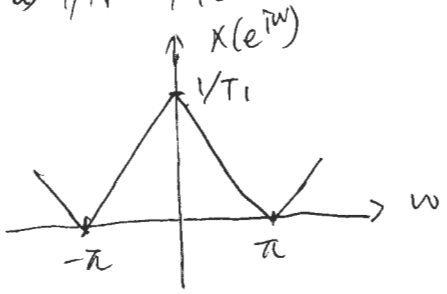
4. a) $f_N = 2 \cdot f_b = 24\,kHz$

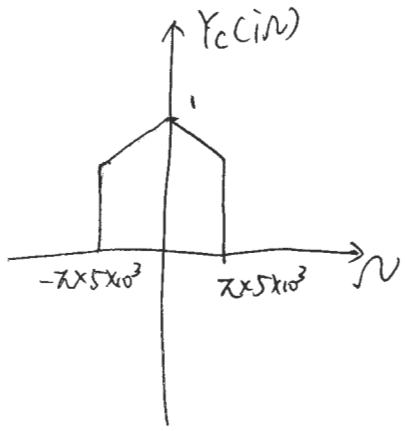
b) $w_{max} = T_N \cdot (2\pi) \cdot 6000 = \frac{\pi}{2}$

$w_{min} = T_N \cdot (2\pi) \cdot 300 = \frac{\pi}{40}$

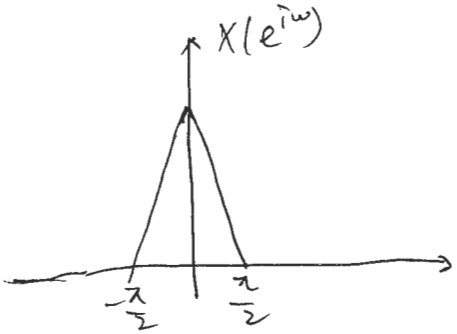


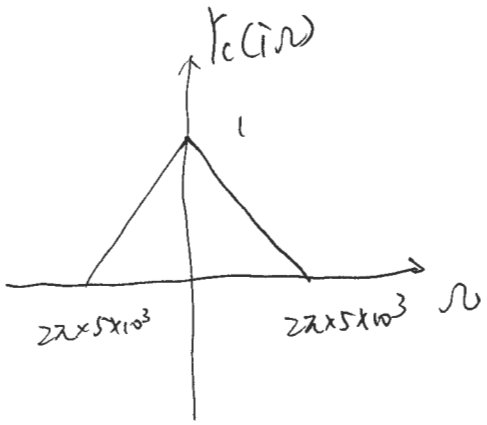
5. a) $1/T_1 = 1/T_2 = 10^4$



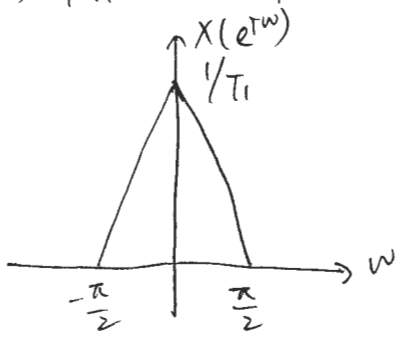


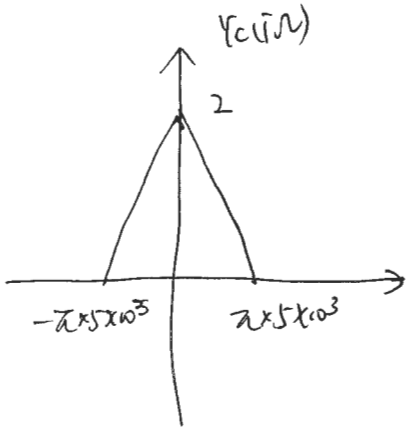
b) $1/T_1 = 1/T_2 = 2 \times 10^4$



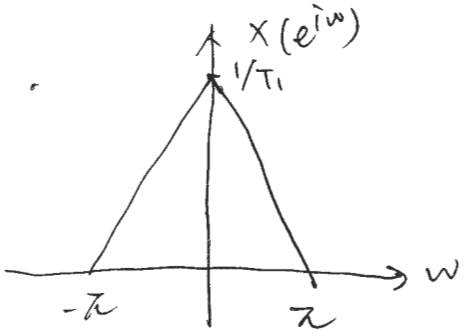


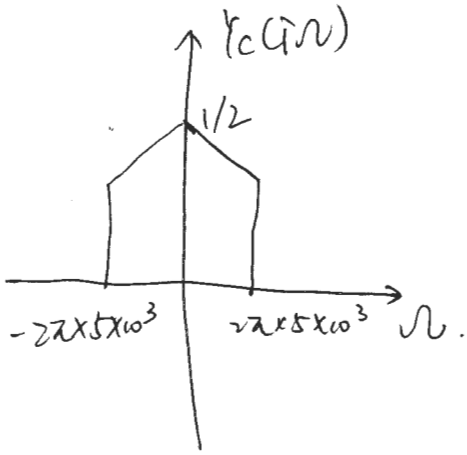
c) $1/T_1 = 2\times10^4$, $1/T_2 = 10^4$





d) $1/T_1 = 10^4$, $1/T_2 = 2\times10^4$





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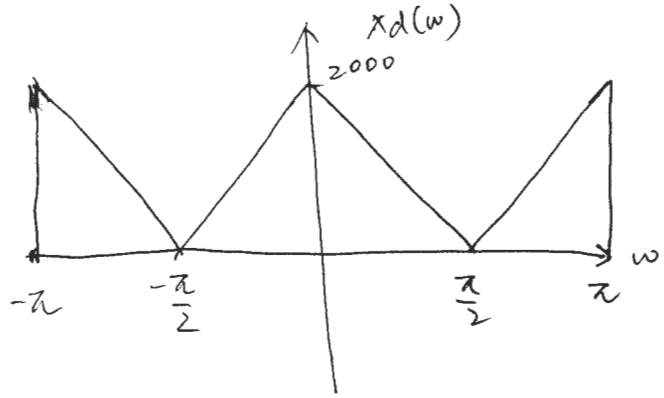

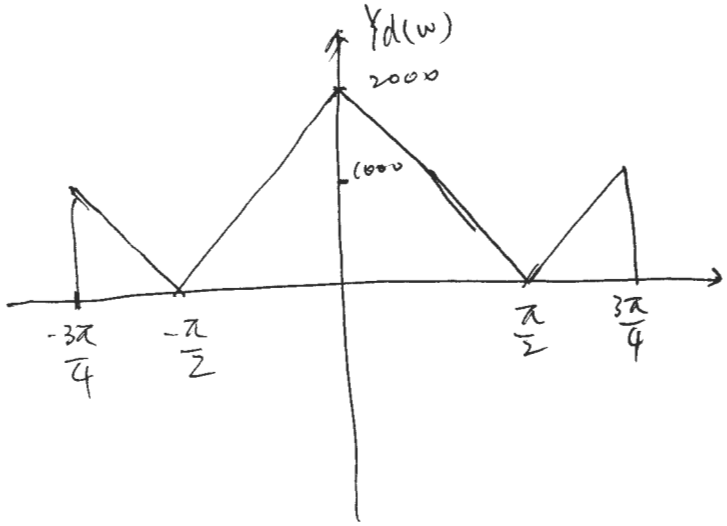

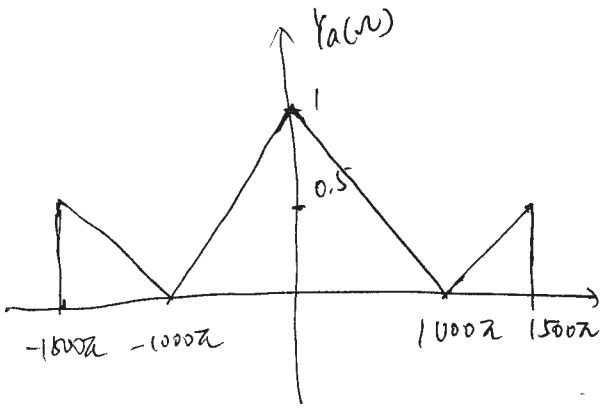
$Y_a(\nu)$
1
0.5
-1500π
-1000π
1000π
1500π