University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER I MIDTERM EXAMINATION 2015

School of Electrical and Electronic Engineering

EEEN 30020

Circuit Theory

Dr Elena Blokhina*

Time Allowed: 50 minutes

Instructions for Candidates

Answer **all questions**.

Solve all problems on rough-work paper, and include only either the answer, or outline solutions, in this answer book, as directed in each question part.
E.g. if equation used, write equation, and write solution, no need to show workings.

Instructions for Invigilators

Non-programmable calculators are permitted.

Rough-work paper will be provided.

This is a closed-book exam, you must not have any notes or materials pertaining to this exam in your possession.

Student Name ________________________________

Student Number________________________________

Question 1: Two-port circuits.

(i) State under what conditions the circuit of figure 1(a) can be considered as a two port. 5%

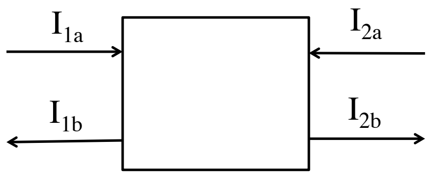


Figure 1(a)

Answer:

The current flowing into one terminal of a port must be the same as the current flowing out of the other terminal of that port. Therefore, it can be considered as a two port if

$$I_{1a} = I_{1b} \quad \text{and} \quad I_{2a} = I_{2b}$$

(ii) Explain what is the impedance matrix. How are the *z*-parameters defined? 7.5%

Answer:

The impedance matrix expresses V_1 *and* V_2 *in terms of* I_1 *and* I_2*. Since the two-port is linear, there is only one type of equations that can relate* V_1 *and* V_2 *to* I_1 *and* I_2*. In the matrix form, we obtain:*

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

where

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}, \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}, \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

(iii) A large circuit consists of two-ports that are interconnected in parallel. What matrix description would one use to describe such an interconnection? 5%

Answer:

If two-ports are connected in parallel, we can add the admittance matrices. Therefore, one would use the admittance matrix to describe this interconnection.

(iv) Find the impedance matrix of the circuit of figure 1(b). 25%

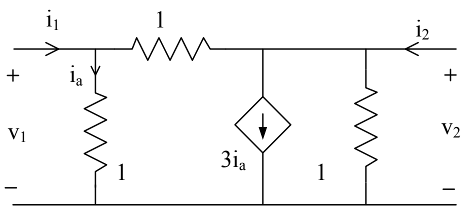


Figure 1(b)

Solution outline and answer:

Set $i_2 = 0$ *:*

KCL: $i_1 = i_a + i^{(1)}$ and $i^{(1)} = 3i_a + i^{(2)}$

where $i^{(1)}$ *and* $i^{(2)}$ *are the currents through the* 1Ω *and* 1Ω *resistors (in series) respectively.*

KVL: $v_1 = 1 \cdot i_a$ and $v_1 = 1 \cdot i^{(1)} + 1 \cdot i^{(2)}$

Resolving the four equations we obtain:

$$v_1 = \frac{1}{3} i_1 \text{ and } v_2 = -\frac{1}{3} i_1$$

Set $i_1 = 0$ *:*

KCL: $i_2 = i + 3i_a + i_a$

where i *is the currents through* 1Ω*. Note that the current* i_a *now flows through the two* 1Ω *resistors (in series).*

KVL: $v_2 = 1 \cdot i$ and $v_1 = 1 \cdot i_a + 1 \cdot i_a$

Resolving the three equations we obtain:

$$v_1 = \frac{1}{6} i_2 \text{ and } v_2 = \frac{1}{3} i_2$$

and so the matrix is $\underline{Z} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$

Question 2: Algorithms and methods of circuit simulation.

(i) What currents are introduces to the Modified Nodal Analysis (MNA) equations as variables? 10%

Answer:

1) *A branch current is always introduced as an additional variable if it flows through a voltage source (independent or controlled).*

2) *In addition, if we are applying MNA in the sinusoidal steady state, we keep the branch current if it flows through an inductor.*

3) *A branch current is introduced if any circuit element is controlled by that current.*

4) *A branch current is introduced if it is requested as an output variable in a simulation environment).*

(ii) The trapezoidal rule for numerical integration uses the approximation 20%

$$x_{n+1} = x_n + \tfrac{1}{2}h\left(\dot{x}_n + \dot{x}_{n+1}\right)$$

where h is the time step. Draw the companion model of the circuit of figure 2(a) derived from the trapezoidal rule.

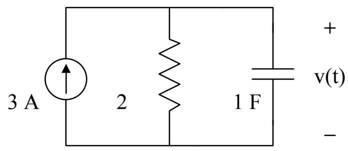
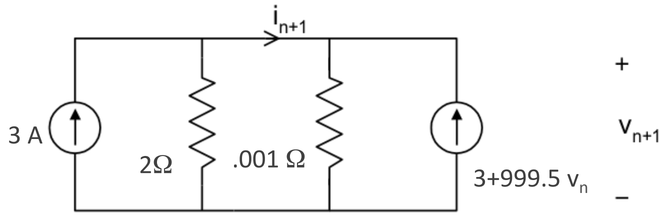


Figure 2(a)

Answer:



$$R_{eq} = \frac{1}{G_{eq}} = \frac{h}{2C} = \frac{0.002}{2}\Omega = 0.001\Omega \text{ and } I_s = i_n + (2C/h)v_n = i_n + 1000v_n$$

But from KCL, $3A = i_n + \dfrac{v_n}{2\Omega} \Rightarrow i_n = 3 - 0.5v_n$ *and so* $I_s = i_n + (2C/h)v_n = 3 + 999.5v_n$.

(iii) Using the companion model from part (ii), estimate the capacitor voltage *v(t)* in the circuit of figure 2(a) from t = 0 to t = 4 ms, taking a time step of 2 ms. The capacitor voltage at *t = 0* is 1 V. 20%

Solution outline and answer:

As explained in the in-class example, we can apply standard tools (KCL and KVL) to analyse the circuit. For example, from KCL:

$3A = i^{(1)} + i_{n+1}$ and $i_{n+1} = i^{(2)} - (3 + 999.5v_n)$ *where* $i^{(1)}$ *and* $i^{(2)}$ *are the currents through the 2Ω and 0.001Ω resistors. From Ohm's law (or by KVL),*

$$i^{(1)} = \frac{v_{n+1}}{2\Omega} \quad \text{and} \quad i^{(2)} = \frac{v_{n+1}}{0.001\Omega}$$

Resolving these equations (eliminating $i^{(1)}$, $i^{(2)}$ *and* i_{n+1}*), we obtain:*

$$v_{n+1} = (2\Omega \,||\, 0.001\Omega)(6 + 999.5v_n)$$

And so, starting the iterations in time:

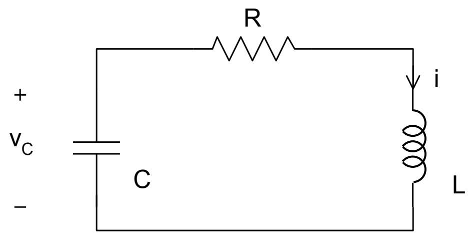
$t_0 = 0, \quad v_0 = 1V$

$t_1 = 0.002s, \quad v_1 = 1.005V$

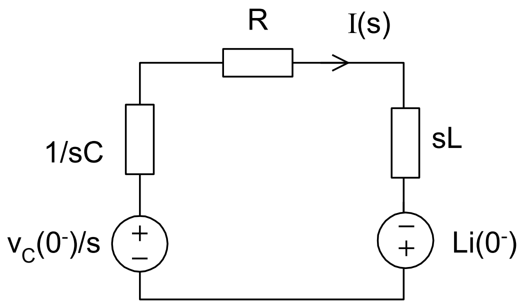
$t_2 = 0.004s, \quad v_2 = 1.00999V$

Question 3: The Laplace transform. 7.5%

(i) Transform the circuit of figure 3 to the Laplace domain:



Answer:



Rough work