1. A transmission line of length $L = 5\,\mathrm{m}$ and $v = \frac{1}{3}c = 10^8\,\mathrm{m/s}$ is open circuited at ends $z = 0$ and $z = L$.

   a) In general, the phasor voltage on the T.L. has the following form

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

and the corresponding phasor current is given by

$$I(z) = \frac{V^+}{Z_o} e^{-j\beta z} - \frac{V^-}{Z_o} e^{j\beta z}.$$

Because the ends of the T.L. are open circuited, the current phasors at these locations must be zero. Applying this condition at $z = 0$ (i.e., $I(0) = 0$), we find that

$$V^+ = V^-.$$

At the other end, we have that $I(L) = 0$, and thus

$$V^+ e^{-j\beta L} - V^+ e^{j\beta L} = -V^+ 2j \sin(\beta L) = 0.$$

The the non-trivial solutions of the previous equation (i.e., for $V^+ \neq 0$) must satisfy $\sin(\beta L) = 0$, and therefore

$$\beta = \frac{n\pi}{L}$$

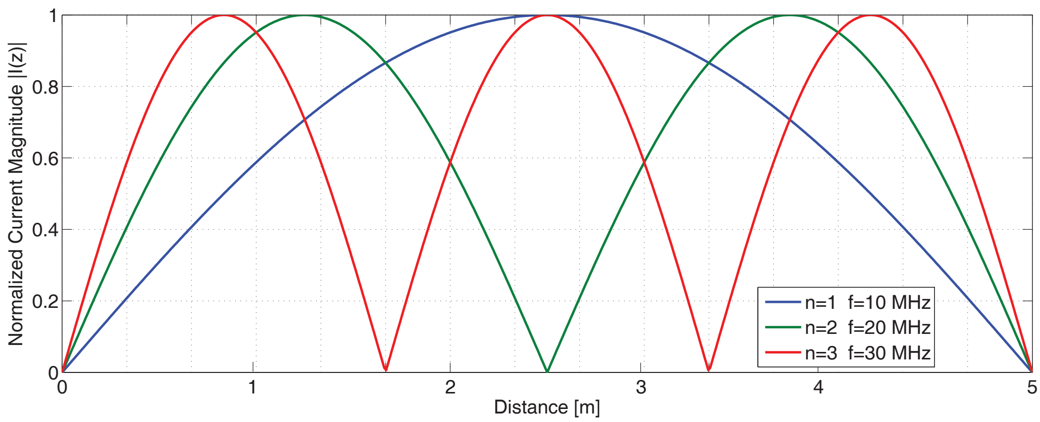
for any integer n. As a result the resonant frequencies f_n are simply

$$f_n = \frac{v}{\lambda} = \frac{\beta}{2\pi} v = \frac{n}{2l} v = n \frac{10^8}{2 \times 5} = 10 \cdot n\,\mathrm{MHz}.$$

   b) The current on the T.L. for the n-th resonant frequency f_n is given by

$$I(z) = -2j \frac{V^+}{Z_o} \sin\left(\frac{n\pi}{L} z\right)\,\mathrm{A}.$$

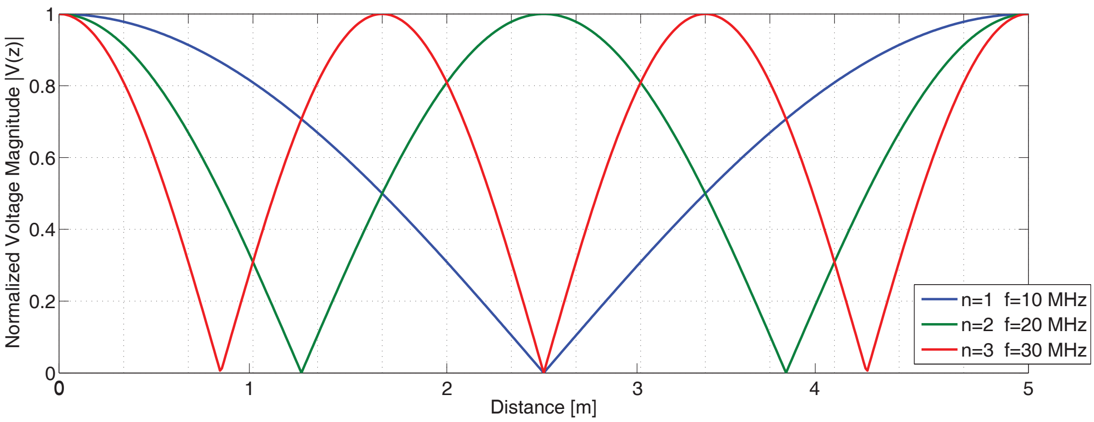
The maximum magnitude of the current will be $\frac{2V^+}{Z_0}$, and the normalized figure :



   c) We can also find that the voltage on the T.L. for the n-th resonant frequency f_n is given by

$$V(d) = 2V^+ \cos\left(\frac{n\pi}{L} z\right)\,\mathrm{V}.$$

The maximum magnitude of the current will be $2V^+$, and the normalized figure :



2. By the steady state analysis, we can regard all components as general impedance. The impedance of the capacitor is

$$Z_C = \frac{1}{j\omega C}.$$

   (a) The input impedance of an open T.L. stub is

$$\begin{aligned} Z_{in} &= -jZ_o \cot(\beta l) \\ &= j5.255\,\Omega \end{aligned}$$

   (b) According to the voltage split,

$$V_L = V_g \frac{R_L}{R_L + R_g + Z_{in} + Z_C}.$$

   Since $V_L = \frac{1}{2}V_g$ and $R_L = R_g$, we get $Z_{in} + Z_C = 0$. Therefore,

$$C = -\frac{1}{\omega Z_o \cot(\beta l)} = 606\,\text{pF}.$$

3. For this two TL case:
   a) For half-wave transform, $V_{L1} = -V_{in} = -j10\,\text{V}$.
   b) For quarter-wave transform, $V_{in} = jI_{L2}Z_o = j\frac{V_{L2}}{Z_{L2}}Z_o$. So $V_{L2} = \frac{Z_{L2}}{Z_o}(-j)(j10) = 5\,\text{V}$.
   c) $I_{L1} = \frac{V_{L1}}{Z_{L1}} = \frac{-j10}{200} = -j0.05\text{A}$.
   d) $I_{L2} = \frac{V_{L2}}{Z_{L2}} = \frac{5}{50} = 0.1\text{A}$.
   e) For half-wave transform: $Z_{in1} = Z_{L1} = 200\,\Omega$; For quarter-wave transform: $Z_{in2} = \frac{Z_o^2}{Z_{L2}} = 200\,\Omega$. Combine in parallel:
   $Z_{in} = (\frac{1}{Z_{in1}} + \frac{1}{Z_{in2}})^{-1} = 100\,\Omega$.
   f) $P = P_1 + P_2 = \frac{1}{2}\text{Re}\{V_{L1}I_{L1}^*\} + \frac{1}{2}\text{Re}\{V_{L2}I_{L2}^*\} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\,\text{W}$.

4. The input impedance of a quarter-wavelength transformer is $Z_{in} = \frac{Z_o^2}{Z_L}$.
   a) Given that $I_{in} = \frac{V_{in}}{Z_{in}} = \frac{Z_L}{Z_o^2}V_{in}$ and $V_{in} = j\frac{V_L}{Z_L}Z_o$, $V_L = -jI_{in}Z_o = -j50$ V.
   b) If the load end is short then $V_L = 0$. This implies that the current input must also be zero, and therefore, $I_{in} = 0.5\angle 0°$ A is not a possibility in this situation.
   c) Given that $I_{in} = \frac{V_{in}}{Z_{in}} = \frac{Z_L}{Z_o^2}V_{in}$ and considering $Z_L = 50\,\Omega$, yields $V_{in} = 100\,\text{V}$.

5. We will use our knowledge of $\lambda/2$ and $\lambda/4$ transformers to solve this problem. Since the line connected to the generator is one wavelength long (i.e. two $\lambda/2$ lines combined), the voltage and current at the junction of the three transmission lines are equal to the voltage and current at the generator ($V_{in} = V_g$).

a) The voltage phasor at load R_{L1} will be

$$V_{L1} = -V_g = -100 = 100e^{j\pi}\,\text{V},$$

and thus, the current phasor at load R_{L1} will be

$$I_{L1} = \frac{V_{L1}}{R_{L1}} = 2e^{j\pi}\,\text{A}.$$

We know from problem 3 that load current in case of $\lambda/4$ transformer is $I_L = -j\frac{V_{in}}{Z_o}$, thus the voltage phasor at load R_{L2} will be

$$V_{L2} = I_{L2}Z_{L2} = -j\frac{V_{in}}{Z_{o2}}R_{L2} = \frac{100}{50}50e^{-j\frac{\pi}{2}} = 100e^{-j\frac{\pi}{2}}\,\text{V},$$

and thus, the current phasor at load R_{L2} will be

$$I_{L2} = \frac{V_{L2}}{R_{L2}} = 2e^{-j\frac{\pi}{2}}\,\text{A}.$$

b) The power dissipated in resistor R_{L1} is

$$P_{L1} = \frac{1}{2}\text{Re}\left\{V_{L1}I_{L1}^*\right\} = \frac{1}{2}\text{Re}\left\{\frac{|V_{L1}|^2}{R_{L1}^*}\right\} = 100\,\text{W}.$$

Then examining the second line, we know that the $\lambda/4$ transformer has a load current $I_L = -j\frac{V_{in}}{Z_o}$. Thus, the expression for the time average dissipated power in resistor R_{L2} is

$$P_{L2} = \frac{1}{2}\text{Re}\left\{V_{L2}I_{L2}^*\right\} = \frac{1}{2}\text{Re}\left\{100e^{-j\frac{\pi}{2}}\cdot 2e^{j\frac{\pi}{2}}\right\} = 100\,\text{W}.$$

6. The lossless TL has $l = \lambda$and $Z_0 = 50\,\Omega$

a) $I_L = \frac{V_L}{Z_L} = \frac{-j10}{150} = -j\frac{1}{15}$ A.

b) $Z(d) = Z_L$ for every $\frac{\lambda}{2}$, so at $d = \frac{\lambda}{2}$ and $d = \lambda$.

c) For quarter-wave transform, $V(\frac{\lambda}{4}) = jI_LZ_o = \frac{10}{3}$V, and $I(\frac{\lambda}{4}) = j\frac{V_L}{Z_o} = \frac{1}{5}$A.

d) $Z(\frac{\lambda}{4}) = \frac{V(\frac{\lambda}{4})}{I(\frac{\lambda}{4})} = \frac{50}{3}\,\Omega$.

e) For half-wave transform, $V(\frac{\lambda}{2}) = -V_L = j10$ V, and $I(\frac{\lambda}{2}) = -I_L = j\frac{1}{15}$ A.

f) Since $l = \lambda$, $V_{in} = V_L = -j10$ V, and $Z_{in} = Z_L = 150\,\Omega$;

$$\begin{aligned}\frac{V_{in}}{V_g} &= \frac{Z_{in}}{Z_{in}+R_g} = \frac{-j10}{-j30} = \frac{1}{3}\\ R_g &= 2Z_{in} = 300\,\Omega.\end{aligned}$$