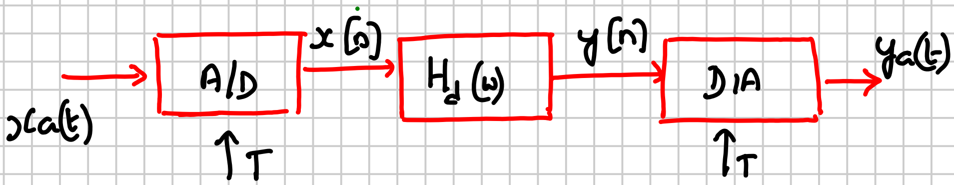
ECE 310 : Lecture 27 : Multi-Rate Signal processing

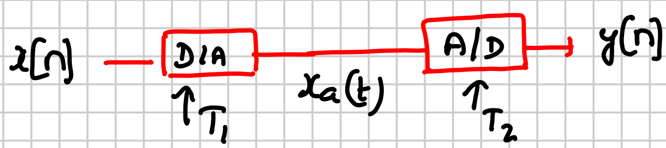
DSP system:



Note that the sampling rate across the system is same. Some applications require that a signal of sampled at rate T_1 be converted to an equivalent signal sampled at rate T_2.

SAMPLING RATE CONVERSION:

① Convert signal to continuous time and resample.



① T_1 and T_2 can be arbitrarily chosen

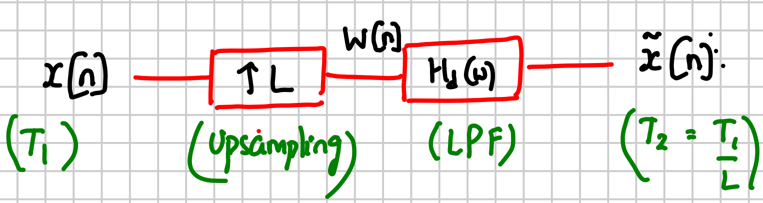
② Distortion in D/A and A/D converters

We will look at sampling rate conversion by an integer factor in the digital domain.

① Increasing Sampling Rate (Interpolation):

$$\boxed{T_2 = \frac{T_1}{L}}$$

Sampling Rate can be increased by UPSAMPLING followed by LOW PASS FILTERING



$\boxed{L = \text{integer}}$

Upsampling in Digital domain:

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow w[n]$$

$$w[n] = \begin{cases} x[n/L] & n = \text{multiple of } L \\ 0 & \text{otherwise} \end{cases}$$

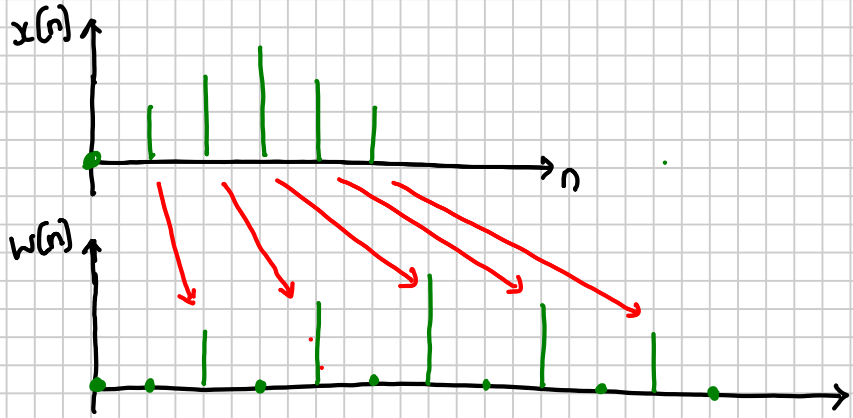
Note: Upsampler inserts $(L-1)$ zeros between two Samples of $x[n]$

Example: $L = 2$ upsampler

$$w[n] = \begin{cases} x[n/2] & n : \text{multiple of } 2 \\ 0 & \text{otherwise} \end{cases}$$

$w[0] = x[0]$ $w[1] = 0$ $w[2] = x[1]$

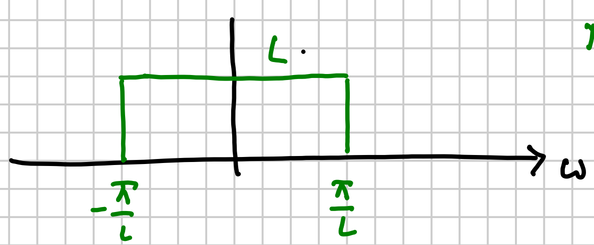
$w[3] = 0$ $w[4] = x[2]$



Step. 2: Convert the zero-valued samples into interpolated samples. This is achieved by a LPF at the output of upsampler.

$$x[n] \longrightarrow \boxed{\uparrow L} \xrightarrow{w[n]} \boxed{H_d(\omega)} \longrightarrow \tilde{x}[n]$$

$H_d(\omega)$ is an ideal LPF:



Note: $L = \frac{T_1}{T_2}$

To see the above let us look at the problem in Fourier domain.

$$\sum_{k=-\infty}^{\infty} w[n] e^{-j\omega n} = \sum_{\substack{n = \text{ mult of } L \\ (n = kL)}} w[n] e^{-j\omega n}$$

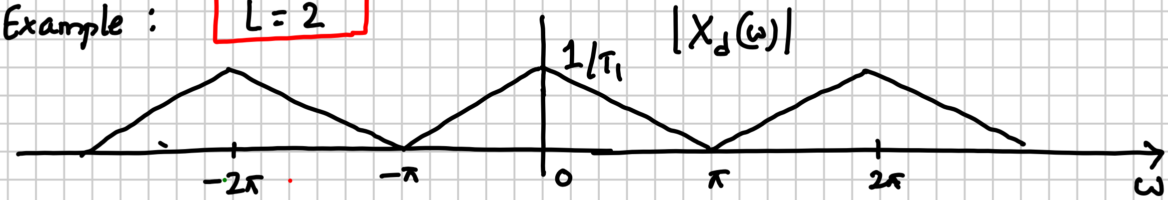
$$= \sum_{k=-\infty}^{\infty} w[kL] e^{-j\omega kL}$$

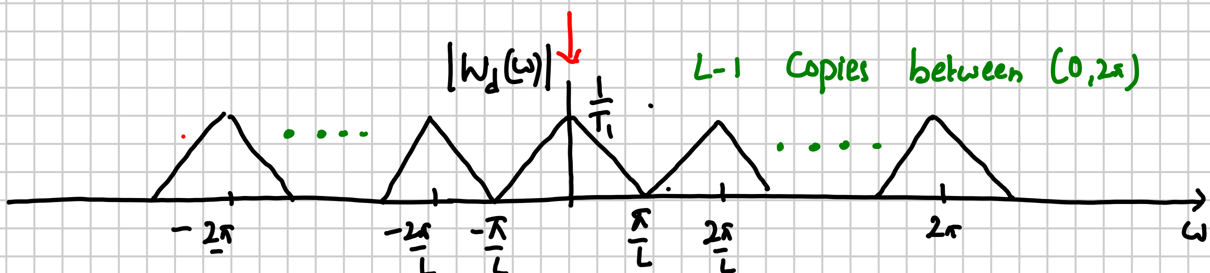
$$= \sum_{k=-\infty}^{\infty} x[k] e^{-j\omega L k}$$

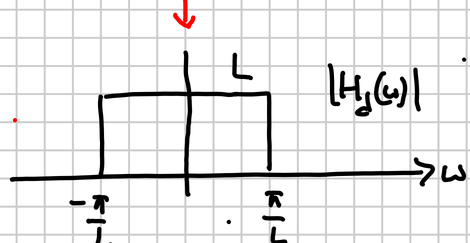
$$\Rightarrow \quad \boxed{W_d(\omega) = X_d(\omega L)}$$

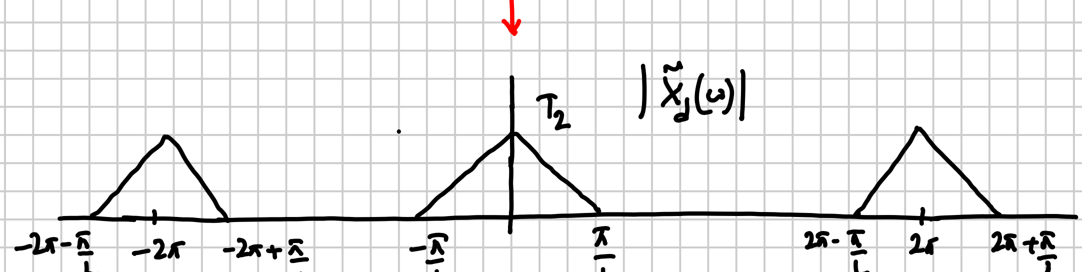
$\Rightarrow$ Spectrum of $w[n]$ is a L-fold compressed version of $x[n]$

Example : $\boxed{L=2}$

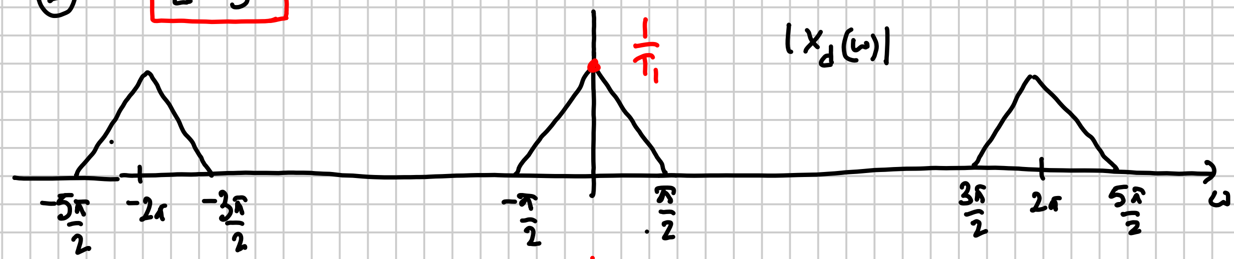


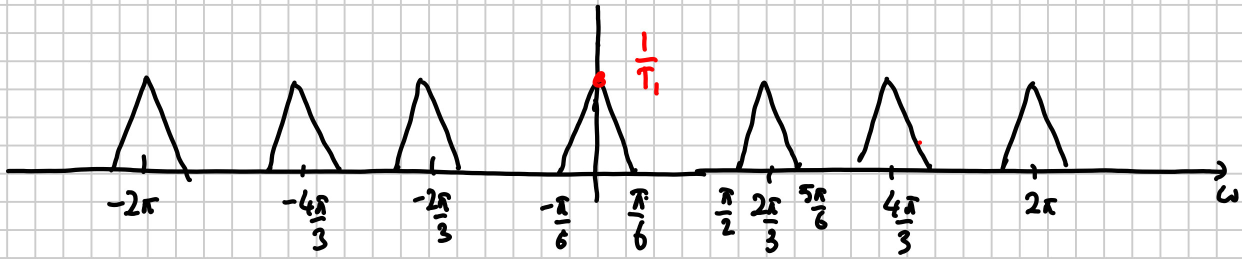


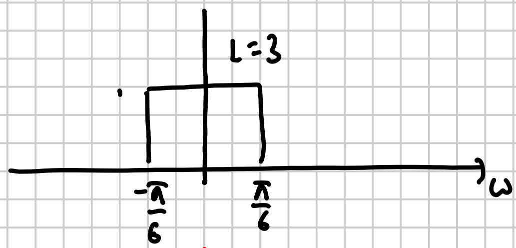


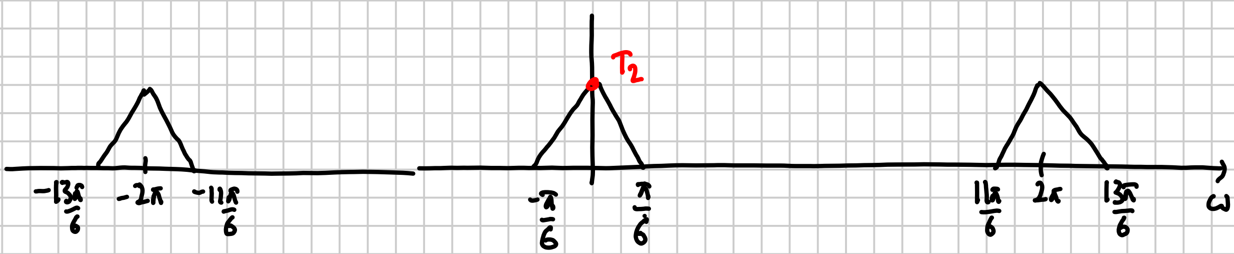


2 $\boxed{L = 3}$


$|X_d(\omega)|$
$\frac{1}{T_1}$
$-\frac{5\pi}{2}$ -2π $-\frac{3\pi}{2}$ $-\frac{\pi}{2}$ $\frac{\pi}{2}$ $\frac{3\pi}{2}$ 2π $\frac{5\pi}{2}$ ω


$\frac{1}{T_1}$
-2π $-\frac{4\pi}{3}$ $-\frac{2\pi}{3}$ $-\frac{\pi}{6}$ $\frac{\pi}{6}$ $\frac{\pi}{2}$ $\frac{2\pi}{3}$ $\frac{5\pi}{6}$ $\frac{4\pi}{3}$ 2π ω


$L=3$
$-\frac{\pi}{6}$ $\frac{\pi}{6}$ ω


T_2
$-\frac{13\pi}{6}$ -2π $-\frac{11\pi}{6}$ $-\frac{\pi}{6}$ $\frac{\pi}{6}$ $\frac{11\pi}{6}$ 2π $\frac{13\pi}{6}$ ω