SEMESTER 1 EXAMINATION – 20013/2014

Module Code: EEEN30110 , Module Title: Signals and Systems

Time Allowed: 2 hours

Answer all questions. The numbers in the right margin give an approximate indication of the relative importance in terms of grade steps of each part of a question. All rough work should be entered in your answer books

1. A method of transmission of digital information is binary FSK (frequency-shift keying). The method involves transmitting a certain number of cycles of a sinusoidal signal of one frequency ω_1 (rad/sec) to represent a binary 0 and a certain number of cycles of a sinusoidal signal of a second frequency ω_2 (rad/sec) to represent a binary 1. A signal $f(t)$ is a binary FSK signal corresponding to the periodic data stream 01010101 ... i.e. a 0 followed by a 1 followed by a 0 *etc*. Assume the frequency representing a binary 0 is 1kHz and that representing a binary 1 is 2kHz. In each case let the transmit times be 3msec, i.e. a 1 kHz sinusoid is transmitted for 3msec to represent a binary 0 and a 2kHz sinusoid is transmitted for 3msec to represent a binary 1. Select the phase so that there is no discontinuity in the signal at the end of each 3msec time block.

   By employing an intuitive argument determine what frequencies you expect to see present in the signal. **1**

   - **Expect to see cosines with odd multiples of the fundamental frequency to create the sharp rise at the centre of the period, and then sines with frequencies multiples of the frequency of each signal**

   Find the first few (three at least) non-zero terms of the trigonometric Fourier series of $f(t)$. **5**

   - **Create a time vector for the period 0 to $3ms$ and $3ms$ to $6ms$, with time-step that divides evenly into time period, so vectors can be added later**
   - $\boldsymbol{N = 1024^2}$
   - $\boldsymbol{tstep = \frac{1}{N}}$
   - **t = [0:tstep:0.5-tstep]*0.003; (create time vector of size N/2)**
   - **Create a vector of each function, and concatenate these vectors**
   - **FF = fft(f)/length(f);**
   - **C_0 = FF(1)**
   - **Alpha = 2*(FF(2:10)) (Vector of first 9 alphas)**
   - **Beta = -2*(FF(2:10)) (Vector of first 9 betas)**
   - **Open up variables in MATLAB to write the Fourier series**

   Determine the DC component, the fundamental and the second harmonic of $f(t)$. **1**

2. The part of the receiver for the binary FSK signal of problem 1 which is tasked with detecting that a binary 0 has been transmitted is a filter. This filter is designed to reject the low "switching frequency" corresponding to the switching between binary 0 and binary 1. The highest frequency

possible in this regard is where we alternate between binary 0 and binary 1 as above. The filter is also designed to reject the 2kHz sinusoid which represents a binary 1. On the other hand the filter should not reject, perhaps might even amplify the 1 kHz sinusoid which represents a binary 0. We achieve the required filter by employing the transfer function:

$$\frac{K s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

and selecting parameters K, ω_n and ζ such that the filter has a resonant frequency at about 1kHz, such that the gain at this frequency (i.e. the modulus of the transfer function at this frequency) is approximately unity and such that the gain at the frequencies 2kHz and the lower "switching frequency" are very small in comparison. Show that this can all be achieved if we take ω_n to be approximately 2000π (rad/sec) (i.e. 1kHz), ζ to be rather small (say 0.1) and K to be suitably large. **0.5**

- **Put transfer function in term of $j\omega$, do not leave it in s, and calculate gain as $|H(j\omega)|$**

Select K so that the gain at 1kHz is unity. **0.5**

Plot the magnitude of the frequency response vs frequency for the resulting filter and hence confirm that it has the required filtering properties. **1.5**

- **Create frequency vector in rad/s**
- $H = polyval(NUM, i * w)./polyval(DEN, i * w)$
- **plot(abs(H))**

By employing any method you see fit (Matlab's **lsim** command being probably the best) find the response over the time frame 0 to 12 msec of the filter when the input is the binary FSK signal of problem 1. **1.5**

- **H = tf(NUM, DEN);**
- **t = [t1 t1+0.003 t1+0.006 t1+0.009];**
- **y = [f f];**
- **[yout, xout] = lsim(H, y, t);**
- **plot(t, yout);**
- **Sketch the plot of the output**

Again assuming that the input voltage to the filter system is the binary FSK signal of problem 1 find the first few terms of the trigonometric Fourier series of the steady-state output of the system. **2**

- **FF = fft(yout)/length(yout);**
- **c_0 = FF(1);**
- **Alpha = 2*real(FF(2:10));**
- **Beta = -2*imag(FF(2:10));**
- $\omega_0 = 2\pi/length(yout)$

Assume that the input to the filter is a binary FSK signal representing the binary sequence 0, i.e. a single binary 0. In other words the input is zero for all times except for the times from 0 to 3msec where it is a 1kHz sinusoid. Using Laplace transform methods find an analytic expression for the output of the filter in this event. Hint: recall the assertion in module lectures that you should transform all problems to problems involving exponentials and use the shifting property. **1**

- **Input for x<0 is simply zero so we get zero output as well**
- **Input from 0<x<0.003 is sin(2000pi)**
- **Laplace transform input and multiply by H(s) given**
- **Partial fraction expansion and inverse laplace to find function of time**
- **Find output y(t) at 0.003, and y'(t) at 0.003 to have initial condit for x>0.003**
- **Add in these as initial conditions for transfer func expression**
- **Find Y(s) with X(s) set to zero (just initial conditions)**
- **Inverse laplace transform, and a sub in (t-0.003) for t in expression**

3. An LTI, SISO, causal, discrete-time system with input $x(n)$ and output $y(n)$ is governed by the recursion/difference equation:

$$\begin{aligned} &y(n)-2.6y(n-1)+2.4325y(n-2)-0.914y(n-3)+0.1047y(n-4)= \\ &0.01x(n-1)+0.0155x(n-2)-0.0047x(n-3)-0.0003x(n-4) \end{aligned} \quad \text{for } n \geq 0$$

subject to zero initial conditions. Given an input $x(n)$ which is zero for n not equal to 0, 1 or 2, which is equal to 0.5 for n equal to 0 or 1 and which is equal to 0.25 for n equal to 2, find a formula for the resulting output of the system for $n \geq 0$.

- **Transform entire equation to Z space, and get transfer function. (Initial condit = 0)**
- **Transform input to Z domain: x(n) = $0.5\delta(n) + 0.5\delta(n-1) + 0.25\delta(n-2)$ =>**
- $\boldsymbol{X(z) = 0.5 + 0.5z^{-1} + 0.25z^{-2}}$
- **$Y(z)$ is convolution of H(z) and X(z)**
- **Partial Fraction expansion of Y(z) and inverse Z transform**
- $\boldsymbol{Z^{-1}\left\{\frac{A}{z^{-1}-\lambda}\right\} = \frac{A(-1)}{\lambda^{n+1}}}$ **(Keep (-1) separate when putting complex A into polar form)**
- $\boldsymbol{Z^{-1}\{Az^{-k}\} = A\delta(n-k)}$
- **This gives output in n, for n>=0**

Let the input to the system be a periodic signal of period 13. For n = 0 and 1 this signal equals 0.5 For n = 2 it equals 0.25. For n = 3, 4, … , 12 this signal is equal to 0. Find the resulting steady-state output of the system.

- **Create vector of one period input in MATLAB**
- $\boldsymbol{FF = fft(f)/13}$
- **Take c_0 and 2*(c_1 : c_6) for coefficients of fourier series**
- **Create a discrete frequency vector for multiples of $\frac{2\pi}{13}$**
- **Take NUM and DEN from H(z) in previous q**
- $\boldsymbol{H(e^{j\Omega})\text{: } H = polyval(NUM, exp(-i*w))./polyval(DEN, exp(-i*w))}$
- **Create a vector of $|H(e^{j\Omega})|$ with: $ABS = abs(H)$**
- **Create a vector of $\angle H(e^{j\Omega})$ with: $PHASE = angle(H)$**