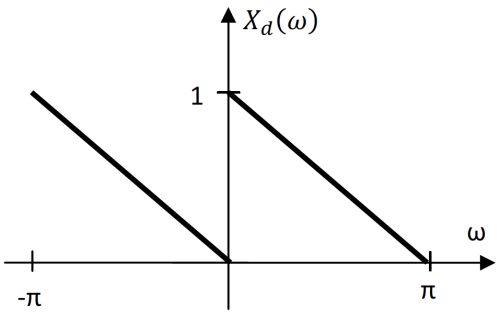
ECE 310 **University of Illinois at Urbana-Champaign** **Fall 2015**

Profs. Bresler & Radhakrishnan **Homework 2** **Due: Friday, September 11**
Reading: Chapter 2.1-2.6

General Instructions for HW

- Make sure you write your full name and section on your HW solution.
- All pages must be stapled together.
- Solutions to problems must appear in order.
- Unless the solution to a problem is very short, start the solution to each HW problem on a new page. This will help the TAs in their work, and ensure they do not miss solutions you have written.

1. Let $x[n]$ be a signal with DTFT as shown in the following figure. Determine and sketch the DTFT of $y[n] = x[n]\cos(\pi n/3)$.



2. Derive closed-form expressions (no sums) for the DTFT of the following sequences. Sketch the magnitude and phase for parts (a) and (b). [*HINT: do not evaluate any sums; use the properties of the DTFT!*]:

   (a) $x[n] = \delta[n+3] + \delta[n-3]$
   (b) $x[n] = u[n] - u[n-7]$
   (c) $x[n] = (\frac{1}{4})^n u[n]$
   (d) $x[n] = (\frac{1}{4})^n u[n+4]$
   (e) $x[n] = (\frac{1}{4})^n e^{j\pi n/3} u[n]$
   (f) $x[n] = (\frac{1}{4})^n \cos(\pi n/3) u[n-5]$
   (g) $x[n] = n(\frac{1}{4})^n u[n-3]$

3. In each case, determine whether the signal $x[n]$ whose DTFT is given below, is real-valued. Prove your answer. (Hint: you should not need to find $x[n]$.)

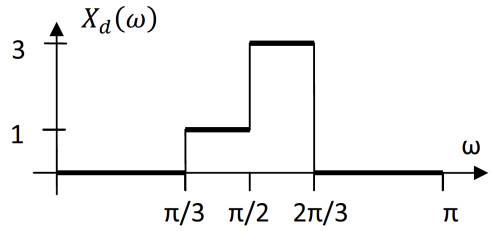
   (a) $X_d(\omega) = \sin^2(\omega) - j\sin(2\omega)$
   (b) $X_d(\omega) = e^{j\cos(\omega)} - e^{j\sin(\omega)}$
   (c) $X_d(\omega) = e^{\cos(\omega)} - e^{j\sin(\omega)}$

4. Let x[n] be an arbitrary signal, not necessarily real valued, with DTFT $X_d(\omega)$. In each case, express the DTFT of $y[n]$ in terms of $X_d(\omega)$.

   (a) $y[n] = x^*[n]$
   (b) $y[n] = x^*[-n+4]$
   (c) $y[n] = x[n-2]\cos(\pi n/5)$

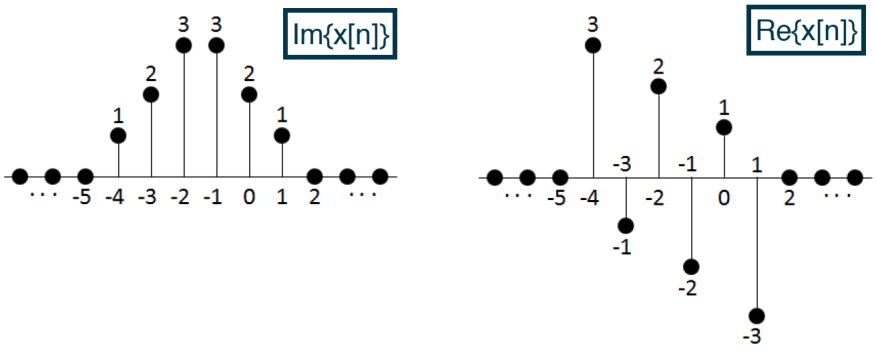
5. The DTFT of $x[n]$ is given below for $\omega \in [-\pi, \pi]$. Determine the signal $x[n]$ corresponding to each of the following cases:

   (a) $X_d(\omega) = 1 + 3e^{-j2\omega} - j3\sin(3\omega)$
   (b)



6. Let $X_d(\omega)$ denote the DTFT of the complex valued signal $x[n]$, where the real and imaginary parts of $x[n]$ are given below. Perform the following calculations without explicitly evaluating $X_d(\omega)$.

   (a) Evaluate $X_d(0)$
   (b) Evaluate $X_d(\pi)$
   (c) Evaluate $\int_{-\pi}^{\pi} X_d(\omega)d\omega$
   (d) Determine and sketch the signal whose DTFT is $X_d^*(-\omega)$



7. For each of the following finite length sequences, determine the corresponding DFT, $X[k]$. Sketch the magnitude and phase for parts (a) and (b).

   (a) $x[n] = \delta[n-4]$, $0 \le n \le 3$

   (b) $x[n] = \begin{cases} 3, & 0 \le n \le 3 \\ 0, & 4 \le n \le 7 \end{cases}$

   (c) $x[n] = \cos(\pi n/3)$, $0 \le n \le 5$

   (d) $x[n] = \begin{cases} 1, & n \text{ even}, \ 0 \le n \le 12 \\ 0, & n \text{ odd}, \ 0 \le n \le 12 \end{cases}$