ECE 310: Lecture 22: Digital Filter Structures

$$x[n] \rightarrow \boxed{\begin{matrix}\text{LSI}\\ h[n]\end{matrix}} \rightarrow y[n]$$

So far: Looked at Causal LSI systems. LSI systems are characterized by impulse responsible $h(n)$.

$$\boxed{y(n) = \sum_{m=-\infty}^{\infty} h(m)\,x(n-m)}$$

Based on impulse response $h(n)$ LSI systems can be divided into two classes:

1. Finite-duration Impulse Response (FIR)
2. Infinite-duration impulse response (IIR)

Finite-Impulse Response (FIR):

$$h(n) = 0 \qquad n < 0 \text{ and } n \geq M$$

$$\Rightarrow \boxed{y(n) = \sum_{m=0}^{M-1} h(m)\,x(n-m)}$$

Difference Equation:

$$\boxed{y(n) = h(0)x(n) + h(1)x(n-1) + \dots + h(M-1)x[n-M+1]}$$

The Transfer function can be written as,

$$\boxed{H(z) = \sum_{n=0}^{M-1} h_n z^{-n}}$$

$\Rightarrow$ Transfer function is a polynomial in z^{-1} or z.

Infinite duration Impulse Response (IIR):

$h(n)$ has infinite duration

$$\Rightarrow \boxed{y(n) = \sum_{m=0}^{\infty} h(m)\,x(n-m)}$$

Example :

$$y(n) = \frac{1}{n+1} \sum_{k=0}^{n} x(k) \qquad n = 0, 1 \ldots$$

$$\Rightarrow \quad (n+1)\,y(n) = \sum_{k=0}^{n-1} x(k) + x(n)$$

$$\boxed{y(n) = \frac{n}{n+1}\, y(n-1) + \frac{1}{n+1}\, x(n)}$$

Difference Equation :

$$\boxed{\begin{aligned} y(n) = & -a_1 y(n-1) - a_2 y(n-2) + \ldots - a_k y(n-k) \\ & + b_0 x(n) + b_1 x(n-1) + \ldots + b_l x(n-l) \end{aligned}}$$

Transfer function :

$$\boxed{H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_l z^{-l}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_k z^{-k}}}$$

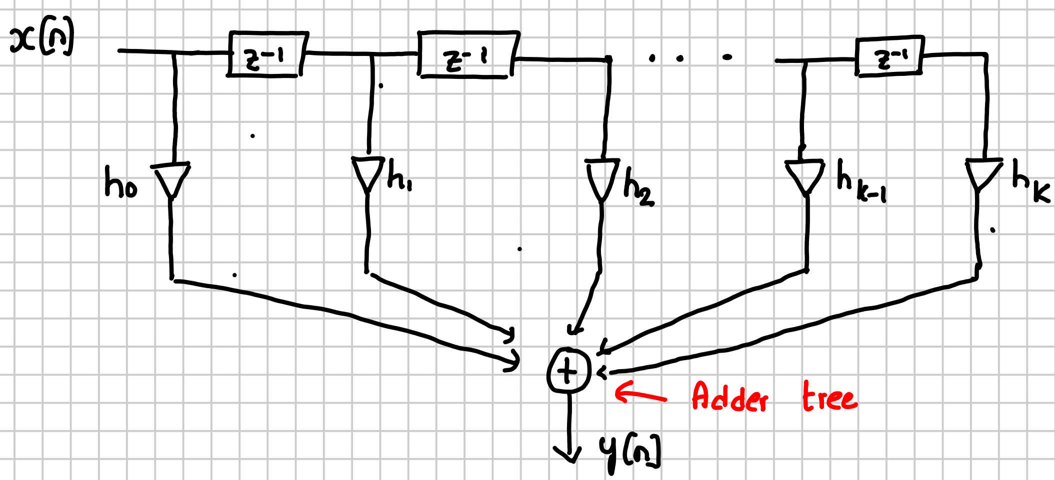
$H(z)$ is not a polynomial

Filter Structures : Implementation of Filters

FIR filter Structure :

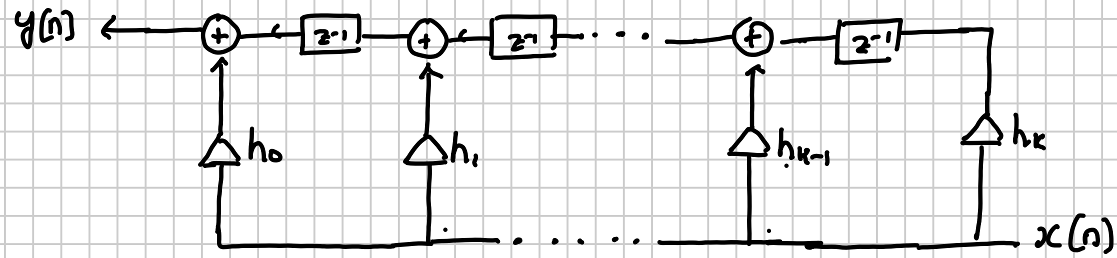
① Direct form Structure :

$$y(n) = h_0 x(n) + h_1 x(n-1) + \ldots + h_k x(n-k)$$



M-1 additions and M : multiplication

② Transpose Form : Reverse all flows

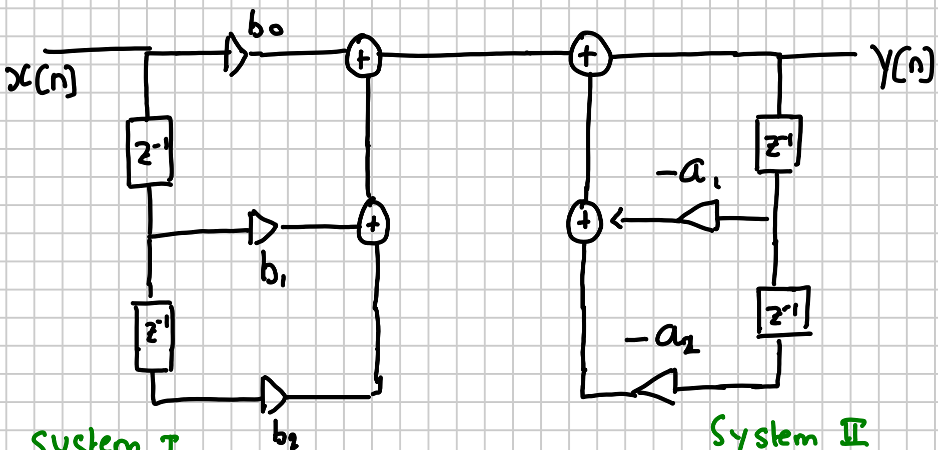


No need for an adder tree

IIR filter structures : We saw Direct form I and II earlier

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

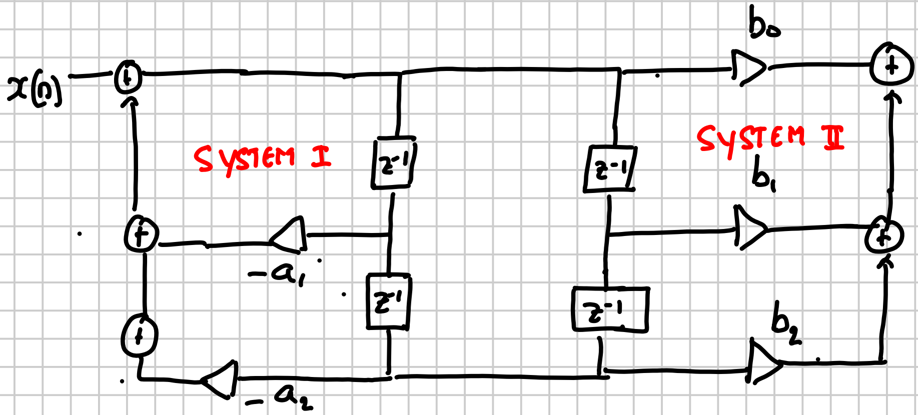
$$\Rightarrow \quad y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$



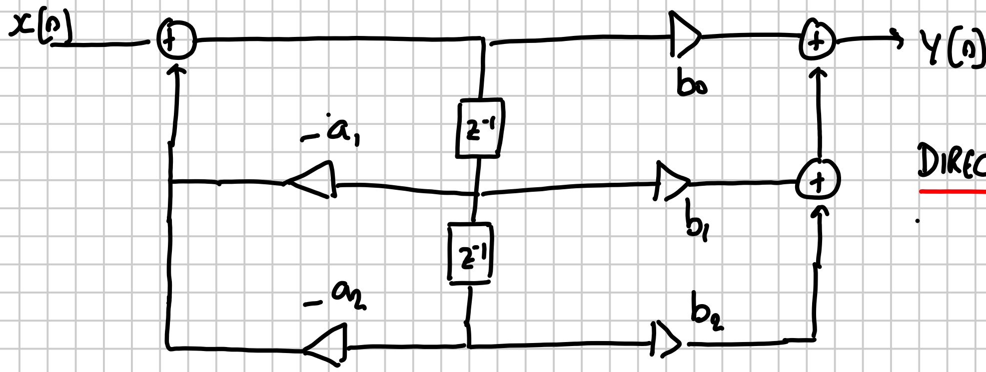
DIRECT FORM I

DIRECT FORM 2 :

Direct form I structure can be seen as a cascade of two LSI systems. The order can be interchanged.



Combining delays:



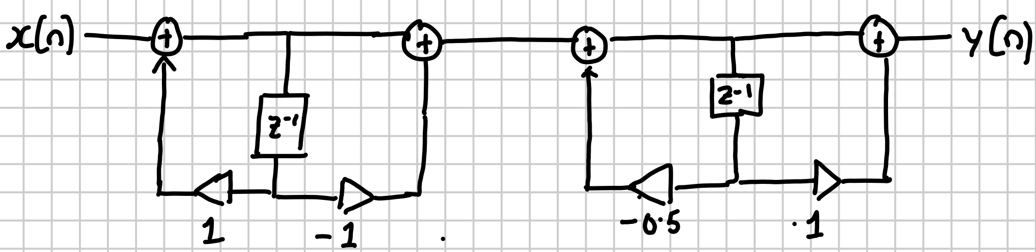
DIRECT FORM II

Cascade form structure:

Cascade form can be obtained from the system function H(z) by factorizing into first / second order terms can be implemented this way.

$$H(z) = \frac{1 - z^{-2}}{1 - 0.5z^{-1} - 0.5z^{-2}}$$

$$= \frac{(1-z^{-1})}{(1-z^{-1})} \cdot \frac{(1+z^{-1})}{(1+0.5z^{-1})}$$

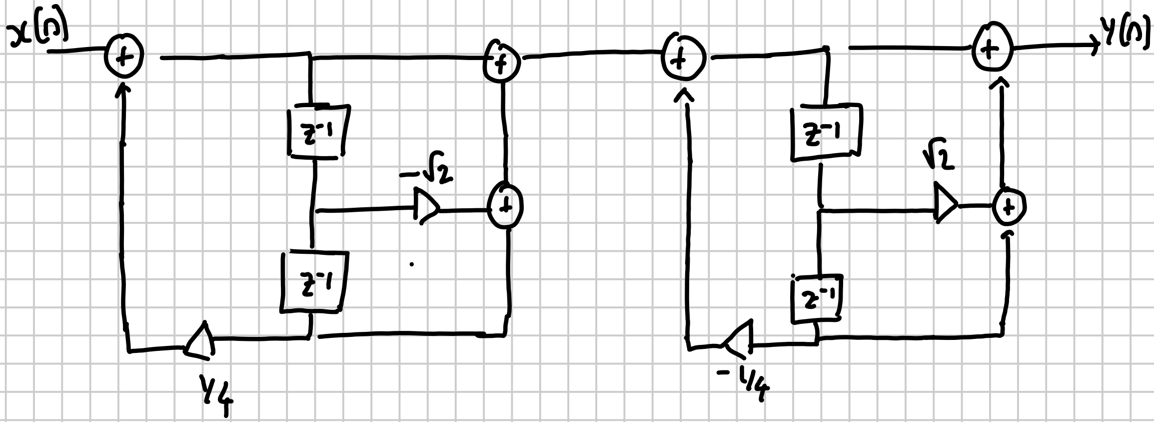


Second order terms are useful if one has Complex poles / zeros:

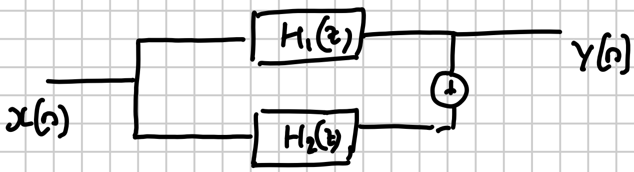
$$H(z) = \frac{z^4 + 1}{z^4 - 1/16}$$

$$= \frac{\underbrace{(z - e^{j\pi/4})(z - e^{-j\pi/4})}\ \overbrace{(z + e^{j\pi/4})(z + e^{-j\pi/4})}^{\text{Combine}}}{\underbrace{(z - \frac{1}{2})(z + \frac{1}{2})}\ \underbrace{(z - \frac{j}{2})(z + \frac{j}{2})}}$$

$$= \frac{(z^2 - \sqrt{2}z + 1)\ (z^2 - \sqrt{2}z + 1)}{(z^2 - \frac{1}{4})(z^2 + 1/4)}$$



Parallel form: Obtained by partial fraction expansion.



$$y(z) = H_1(z)\,X(z) + H_2(z)\,X(z)$$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = H_1(z) + H_2(z)$$

$$H(z) = \frac{10\left(1 - \frac{1}{2}z^{-1}\right)\cdot\left(1 - \frac{2}{3}z^{-1}\right)\left(1 + 2z^{-1}\right)}{\underbrace{\left(1 - \frac{3}{4}z^{-1}\right)}_{P_1}\underbrace{\left(1 - \frac{1}{8}z^{-1}\right)}_{P_2}\underbrace{\left(1 - \left(\frac{1}{2} + \frac{j}{2}\right)z^{-1}\right)}_{P_3}\underbrace{\left(1 - \left(\frac{1}{2} - \frac{j}{2}\right)z^{-1}\right)}_{P_4}}$$

$$H(z) = \frac{A_1}{P_1} + \frac{A_2}{P_2} + \frac{A_3}{P_3} + \frac{A_4}{P_4}$$

$A_1 = 2.93 \qquad A_2 = -17.68 \qquad A_3 = 12.25 - j14.57$

$A_4 = A_3^* = 12.25 + j14.57$

$$\therefore \quad H(z) = \frac{-14.75 - 12.90z^{-1}}{\underbrace{1 - \frac{7}{8}z^{-1} + \frac{3}{32}z^{-1}}_{H_1(z)}} + \frac{24.5 + 26.8\,z^{-1}}{\underbrace{1 - z^{-1} + 0.5z^{-2}}_{H_2(z)}}$$

$H_1(z)$ and $H_2(z)$ can be implemented as Direct form II Structures in parallel.