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Отчёт
по лабораторной работе №1
по дисциплине «Методы оптимизации»
Вариант 2

Группа: Р3218

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1 Задание

Найти минимум функции на заданном интервале. Решить задачу методом половинного деления, методом золотого сечения и методом Ньютона; написать программу на языке Python. Параметры ε и δ принять равными 10^{-10} . Программа должна выполнить 25 итераций или остановиться при достижении критерия остановки итерационного процесса. Осуществлять вывод вычисленных значений на каждой итерации с точностью до 5 знаков после запятой.

Исходная функция: $f(x) = \ln(1 + x^2) - \sin x$

Интервал: $[a; b] = [0; \pi]$

2 График

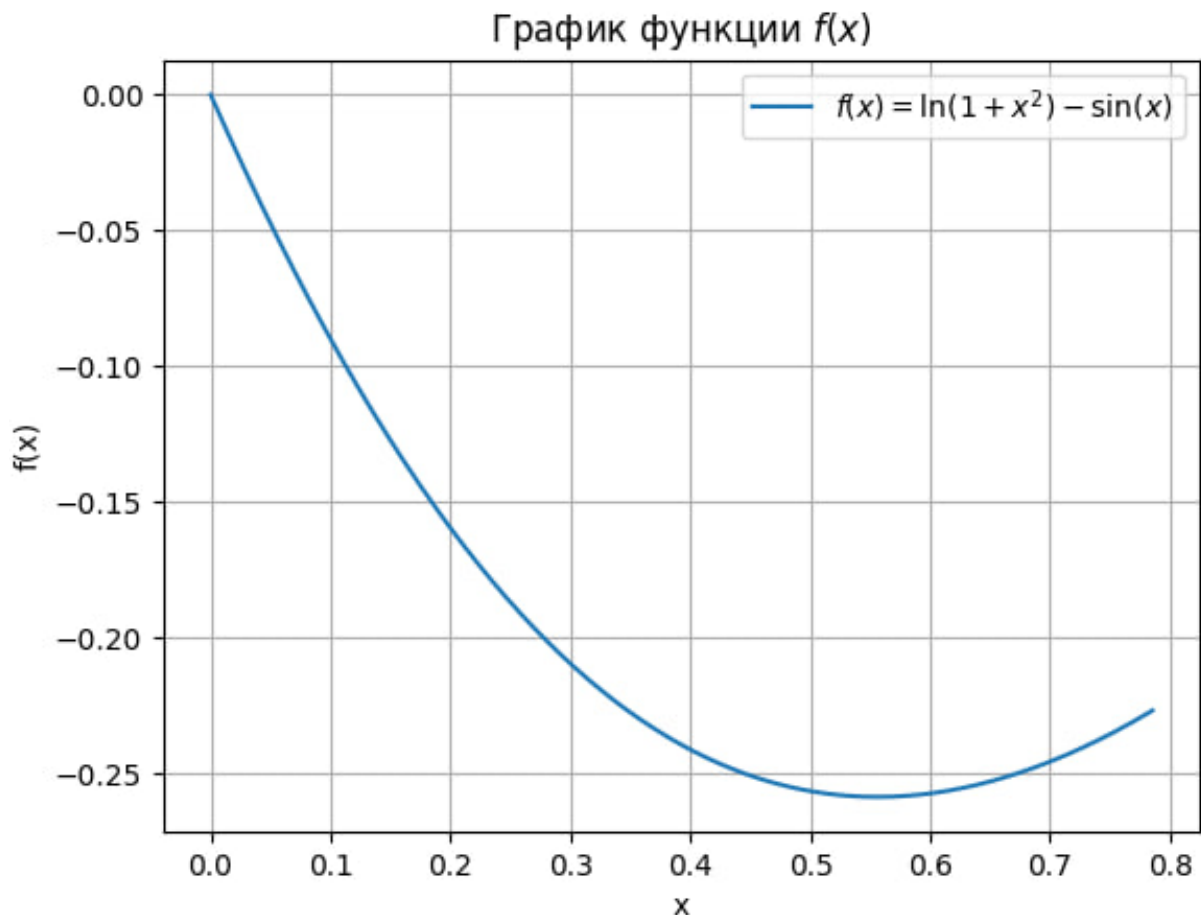


Рис. 1: Фрагмент графика в окрестности интересующего интервала

3 Исходный код реализованных методов

```
def f(x):
    return np.log(1 + x**2) - np.sin(x)
def bisection_method(a, b, epsilon, delta, iteration_limit):

    iteration_cnt = 0

    while iteration_limit > 0:
        x1 = (b + a - delta) / 2
        x2 = (b + a + delta) / 2
        f_x1 = f(x1)
        f_x2 = f(x2)

        if (f_x1 <= f_x2):
            b = x2
        else:
            a = x1

        epsilon_n = (b - a) / 2

        print(f"Итерация №{iteration_cnt}: a = {round(a, 5)},
              b = {round(b, 5)}, (b - a)/2 = {round(epsilon_n, 5)},
              x1 = {round(x1, 5)}, x2 = {round(x2, 5)},
              f(x1) = {round(f_x1, 5)}, f(x2) = {round(f_x2, 5)}",
              end=', ')

        if f_x1 < f_x2:
            print(f"f(x1) < f(x2)")
        else:
            print(f"f(x1) > f(x2)")

        if epsilon_n <= epsilon:
            break

        iteration_cnt += 1
        iteration_limit -= 1
    print(f"Лимит итераций исчерпан: x* = {round((a+b) / 2, 5)},
          f*(x) = {round(f((a+b) / 2), 5)}")
```

Листинг 1: Метод половинного деления

```

def golden_section_method(a, b, epsilon, delta, iteration_limit):
    iteration_cnt = 0
    while iteration_limit > 0:

        x1 = a + (3 - np.sqrt(5)) / 2 * (b - a)
        x2 = a + (np.sqrt(5) - 1) / 2 * (b - a)
        f_x1 = f(x1)
        f_x2 = f(x2)
        T = ((np.sqrt(5) - 1)) / 2

        epsilon_n = (b - a) / 2
        if epsilon_n <= epsilon:
            break

        if f_x1 <= f_x2:
            b = x2
            x2 = x1
            f_x2 = f_x1
            x1 = b - T * (b - a)
            f_x1 = f(x1)
        else:
            a = x1
            x1 = x2
            f_x1 = f_x2
            x2 = a + T * (b - a)
            f_x2 = f(x2)
            epsilon_n *= T

        print(f"Итерация №{iteration_cnt + 1}: a = {round(a, 5)},
              b = {round(b, 5)}, epsilon_n = {round(epsilon_n, 5)},
              x1 = {round(x1, 5)}, x2 = {round(x2, 5)},
              f(x1) = {round(f_x1, 5)},
              f(x2) = {round(f_x2, 5)}", end=', ')
        if f_x1 < f_x2:
            print(f"f(x1) < f(x2)")
        else:
            print(f"f(x1) > f(x2)")
        iteration_cnt += 1
        iteration_limit -= 1
    print(f"Лимит итераций исчерпан: x* = {round(((a+b)/2), 5)},
          f*(x) = {round(f((a+b)/2), 5)}")

```

Листинг 2: Метод золотого сечения

```

def f(x):
    return np.log(1 + x**2) - np.sin(x)

def df(x):
    return ((2 * x) / (1 + x**2)) - np.cos(x)

def d2fdx(x):
    return ((2-2*x**2)/(1+x**2)**2+np.sin(x))

def newton_method(x0, epsilon, iteration_limit):
    iteration_cnt = 0
    while iteration_limit > 0:

        df_dx = df(x0)
        d2f_dx2 = d2fdx(x0)

        x1 = x0 - (df_dx/d2f_dx2)

        print(f"Итерация №{iteration_cnt + 1}:
              k = {iteration_cnt}, xk = {round(x1, 5)},
              f(xk) = {round(f(x1), 5)},
              f'(xk) = {round(df(x1), 5)}")

        if abs(df(x1)) <= epsilon:
            break

        x0 = x1

        iteration_cnt += 1
        iteration_limit -= 1
    print(f"Достигнут критерий epsilon: x* = {round((x1), 5)},
          f*(x) = {round(f(x1), 5)},
          f*'(x) = {df(x1)}")

```

Листинг 3: Метод Ньютона

4 Вывод программы

```
Решение методом деления пополам:
Итерация №0: a = 0.3927, b = 0.7854, (b - a)/2 = 0.19635, x1 = 0.3927, x2 = 0.3927, f(x1) = -0.23927, f(x2) = -0.23927, f(x1) > f(x2)
Итерация №1: a = 0.3927, b = 0.58905, (b - a)/2 = 0.09817, x1 = 0.58905, x2 = 0.58905, f(x1) = -0.25771, f(x2) = -0.25771, f(x1) < f(x2)
Итерация №2: a = 0.49087, b = 0.58905, (b - a)/2 = 0.04909, x1 = 0.49087, x2 = 0.49087, f(x1) = -0.25551, f(x2) = -0.25551, f(x1) > f(x2)
Итерация №3: a = 0.53996, b = 0.58905, (b - a)/2 = 0.02454, x1 = 0.53996, x2 = 0.53996, f(x1) = -0.25825, f(x2) = -0.25825, f(x1) > f(x2)
Итерация №4: a = 0.53996, b = 0.5645, (b - a)/2 = 0.01227, x1 = 0.5645, x2 = 0.5645, f(x1) = -0.25838, f(x2) = -0.25838, f(x1) < f(x2)
Итерация №5: a = 0.55223, b = 0.5645, (b - a)/2 = 0.00614, x1 = 0.55223, x2 = 0.55223, f(x1) = -0.25842, f(x2) = -0.25842, f(x1) > f(x2)
Итерация №6: a = 0.55223, b = 0.55837, (b - a)/2 = 0.00307, x1 = 0.55837, x2 = 0.55837, f(x1) = -0.25842, f(x2) = -0.25842, f(x1) < f(x2)
Итерация №7: a = 0.5553, b = 0.55837, (b - a)/2 = 0.00153, x1 = 0.5553, x2 = 0.5553, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №8: a = 0.5553, b = 0.55684, (b - a)/2 = 0.00077, x1 = 0.55684, x2 = 0.55684, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №9: a = 0.5553, b = 0.55607, (b - a)/2 = 0.00038, x1 = 0.55607, x2 = 0.55607, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №10: a = 0.55568, b = 0.55607, (b - a)/2 = 0.00019, x1 = 0.55568, x2 = 0.55568, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №11: a = 0.55588, b = 0.55607, (b - a)/2 = 0.0001, x1 = 0.55588, x2 = 0.55588, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №12: a = 0.55588, b = 0.55597, (b - a)/2 = 5e-05, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №13: a = 0.55592, b = 0.55597, (b - a)/2 = 2e-05, x1 = 0.55592, x2 = 0.55592, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №14: a = 0.55595, b = 0.55597, (b - a)/2 = 1e-05, x1 = 0.55595, x2 = 0.55595, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №15: a = 0.55596, b = 0.55597, (b - a)/2 = 1e-05, x1 = 0.55596, x2 = 0.55596, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №16: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №17: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №18: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №19: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №20: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №21: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №22: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №23: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №24: a = 0.55597, b = 0.55597, (b - a)/2 = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Лимит итераций исчерпан: x* = 0.55597, f*(x) = -0.25843
```

Рис. 2: Вывод программы по методу половинного деления

Решение методом золотого сечения:

```
Итерация №1: a = 0.3, b = 0.7854, epsilon_n = 0.2427, x1 = 0.4854, x2 = 0.59999, f(x1) = -0.255, f(x2) = -0.25716, f(x1) > f(x2)
Итерация №2: a = 0.4854, b = 0.7854, epsilon_n = 0.15, x1 = 0.59999, x2 = 0.67081, f(x1) = -0.25716, f(x2) = -0.25007, f(x1) < f(x2)
Итерация №3: a = 0.4854, b = 0.67081, epsilon_n = 0.15, x1 = 0.55622, x2 = 0.59999, f(x1) = -0.25843, f(x2) = -0.25716, f(x1) < f(x2)
Итерация №4: a = 0.4854, b = 0.59999, epsilon_n = 0.0927, x1 = 0.52917, x2 = 0.55622, f(x1) = -0.25794, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №5: a = 0.52917, b = 0.59999, epsilon_n = 0.03541, x1 = 0.55622, x2 = 0.57294, f(x1) = -0.25843, f(x2) = -0.25823, f(x1) < f(x2)
Итерация №6: a = 0.52917, b = 0.57294, epsilon_n = 0.03541, x1 = 0.54589, x2 = 0.55622, f(x1) = -0.25836, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №7: a = 0.54589, b = 0.57294, epsilon_n = 0.01353, x1 = 0.55622, x2 = 0.56261, f(x1) = -0.25843, f(x2) = -0.2584, f(x1) < f(x2)
Итерация №8: a = 0.54589, b = 0.56261, epsilon_n = 0.01353, x1 = 0.55228, x2 = 0.55622, f(x1) = -0.25842, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №9: a = 0.55228, b = 0.56261, epsilon_n = 0.00517, x1 = 0.55622, x2 = 0.55866, f(x1) = -0.25843, f(x2) = -0.25842, f(x1) < f(x2)
Итерация №10: a = 0.55228, b = 0.55866, epsilon_n = 0.00517, x1 = 0.55471, x2 = 0.55622, f(x1) = -0.25842, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №11: a = 0.55471, b = 0.55866, epsilon_n = 0.00197, x1 = 0.55622, x2 = 0.55715, f(x1) = -0.25843, f(x2) = -0.25842, f(x1) < f(x2)
Итерация №12: a = 0.55471, b = 0.55715, epsilon_n = 0.00197, x1 = 0.55565, x2 = 0.55622, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №13: a = 0.55565, b = 0.55715, epsilon_n = 0.00075, x1 = 0.55622, x2 = 0.55658, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №14: a = 0.55565, b = 0.55658, epsilon_n = 0.00075, x1 = 0.556, x2 = 0.55622, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №15: a = 0.55565, b = 0.55622, epsilon_n = 0.00047, x1 = 0.55587, x2 = 0.556, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №16: a = 0.55587, b = 0.55622, epsilon_n = 0.00018, x1 = 0.556, x2 = 0.55609, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №17: a = 0.55587, b = 0.55609, epsilon_n = 0.00018, x1 = 0.55595, x2 = 0.556, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №18: a = 0.55587, b = 0.556, epsilon_n = 0.00011, x1 = 0.55592, x2 = 0.55595, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №19: a = 0.55592, b = 0.556, epsilon_n = 4e-05, x1 = 0.55595, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №20: a = 0.55595, b = 0.556, epsilon_n = 3e-05, x1 = 0.55597, x2 = 0.55598, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №21: a = 0.55595, b = 0.55598, epsilon_n = 3e-05, x1 = 0.55596, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №22: a = 0.55596, b = 0.55598, epsilon_n = 1e-05, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №23: a = 0.55596, b = 0.55597, epsilon_n = 1e-05, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) < f(x2)
Итерация №24: a = 0.55596, b = 0.55597, epsilon_n = 1e-05, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Итерация №25: a = 0.55597, b = 0.55597, epsilon_n = 0.0, x1 = 0.55597, x2 = 0.55597, f(x1) = -0.25843, f(x2) = -0.25843, f(x1) > f(x2)
Лимит итераций исчерпан: x* = 0.55597, f*(x) = -0.25843
```

Рис. 3: Вывод программы по методу золотого сечения

Решение методом Ньютона:

```
Итерация №1: k = 0, xk = 0.54001, f(xk) = -0.25825, f'(xk) = -0.02153
Итерация №2: k = 1, xk = 0.5558, f(xk) = -0.25843, f'(xk) = -0.00023
Итерация №3: k = 2, xk = 0.55597, f(xk) = -0.25843, f'(xk) = -0.0
Итерация №4: k = 3, xk = 0.55597, f(xk) = -0.25843, f'(xk) = -0.0
Достигнут критерий epsilon: x* = 0.55597, f*(x) = -0.25843, f*'(x) = -2.220446049250313e-16
```

Рис. 4: Вывод программы по методу Ньютона