# MATH3018 Assignment 1, due on Jan 22

(1) Given the following data matrix $\mathbf{X}$ with 4 observations and 2 variables

$$\mathbf{X} = \begin{pmatrix} 1 & 3 \\ 7 & 6 \\ 5 & 4 \\ 7 & 3 \end{pmatrix}$$

Find the following:
(a) Sample mean vector $\overline{\mathbf{x}}$.
(b) Sample covariance matrix $\mathbf{S}$.
(c) $\mathbf{S}^{-1}$.
(d) Sample correlation matrix $\mathbf{R}$.

(2) Prove the following statements:

(a) The determinant of a $p \times p$ orthogonal matrix $\mathbf{Q}$ is either 1 or -1. (can use properties listed on page 24 of Chapter 2 slides directly)

(b) $\mathbf{A}$ is a symmetric matrix, and $\lambda_i$ and $\lambda_j$ are two distinct eigenvalues of $\mathbf{A}$, then the corresponding eigenvectors $\mathbf{u}_i$ and $\mathbf{u}_j$ must be orthogonal, i.e. $\mathbf{u}_i^T \mathbf{u}_j = 0$. (Hint: start from the definition of eigenvalue/eigenvector)

(c) For any matrix $\mathbf{A}$, $\mathbf{A}^\top \mathbf{A}$ is positive semi-definite.

(d) Sample covariance matrix is positive semi-definite.

(e) For any $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times m}$, $tr(\mathbf{AB}) = tr(\mathbf{BA})$.

(f) The trace of a symmetric $p \times p$ matrix $\mathbf{A}$ can be expressed as the sum of its eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_p$.

(3)

$$A = \begin{pmatrix} 4 & -\sqrt{2} \\ -\sqrt{2} & 3 \end{pmatrix}$$

(a) Calculate the eigenvalues $\lambda_1$ and $\lambda_2$ as well as the eigenvectors $\mathbf{u}_1$ and $\mathbf{u}_2$ of $A$ (ensure that $\mathbf{u}_1$ and $\mathbf{u}_2$ both of unit length). Write down the eigenvector matrix $U$ (of $A$ ) and calculate the inverse $U^{-1}$ using the property of orthogonality.

(b) Use the eigenvalue decomposition (EVD) to find $A^5$.

(c) Use the EVD to find $A^{1/2}$.