Q6: Solution:

Theoretical Proof for Accept-Rejection Method: Let's consider the one dimensional case. For any $x \in R$, consider

$$P(X \leq x) = P\left(Y \leq x \,\middle|\, U \leq \frac{f(Y)}{Mg(Y)}\right) \quad \text{because } U \leq \frac{f(Y)}{Mg(Y)} \Rightarrow X = Y$$

$$= \frac{P\left(Y \leq x,\ U \leq \frac{f(Y)}{Mg(Y)}\right)}{P\left(U \leq \frac{f(Y)}{Mg(Y)}\right)}$$

$$= \frac{\int_{-\infty}^{x} \left(\int_{0}^{\frac{f(y)}{Mg(y)}} du \right) g(y)\, dy}{\int_{-\infty}^{+\infty} \left(\int_{0}^{\frac{f(y)}{Mg(y)}} du \right) g(y)\, dy} \quad \left(\because \frac{f(y)}{Mg(y)} \leq 1 \text{ for all } y \right)$$

$$= \frac{\int_{-\infty}^{x} \frac{f(y)}{Mg(y)} \cdot g(y)\, dy}{\int_{-\infty}^{+\infty} \frac{f(y)}{Mg(y)} \cdot g(y)\, dy}$$

$$= \frac{\int_{-\infty}^{x} f(y)\, dy}{\int_{-\infty}^{+\infty} f(y)\, dy.}$$

So. X has a distribution with the density f.