

5-001 Mid-Term Assignment on Page 20.

Let $\pi(\theta) = \frac{1}{C} \exp(-\frac{1}{2}(\theta + \cos \theta)^2)$ where $C = \int_{-\infty}^{\infty} \exp(-\frac{1}{2}(\theta + \cos \theta)^2) d\theta$.

$$\log(\pi(\theta)) = \log\left(\frac{1}{C}\right) + \log\left(\exp(-\frac{1}{2}(\theta + \cos \theta)^2)\right) = -\log C - \frac{1}{2}(\theta + \cos \theta)^2, m=0.$$

$$[\log(\pi(\theta))]' = -(\theta + \cos \theta) \cdot (1 + (-\sin \theta)) = -(\theta + \cos \theta)(1 - \sin \theta)$$

$$[\log(\pi(\theta))]' = -[(1 - \sin \theta)^2 + (\theta + \cos \theta)(-\cos \theta)] = -(1 - \sin \theta)^2 + \cos \theta(\theta + \cos \theta)$$

$$[\log(\pi(\theta))]' = -1 + 1 = 0.$$

Since we have that $[\log(\pi(\theta))]'|_{\theta=0} = 0$, we choose another θ by setting that $[\log(\pi(\theta))]' = 0$
 $\Leftrightarrow -(\theta + \cos \theta)(1 - \sin \theta) = 0 \Leftrightarrow \theta_1 = \frac{\pi}{2}, \theta_2 \approx -0.739$, since when $\theta_1 = \frac{\pi}{2}$ $[\log(\pi(\theta))]'|_{\theta=\frac{\pi}{2}} = 0$, we choose.
 $\theta = \theta_2 \approx -0.739$. $\therefore [\log(\pi(\theta))]'|_{\theta=-0.739} = -(1 - \sin(-0.739))^2 \approx -2.8$

$$\therefore \pi(\theta) \propto \exp(-1.4(\theta + 0.739)^2)$$

$$= -(1 + \sin(0.739))^2$$

$$\frac{f(y)}{M_{f(y)}} \leq 1$$

for all y

$$\int_{-\infty}^x f(y) dy$$

$$\int_{-\infty}^{+\infty} f(y) dy$$

density.