

Algorithm Improvement Documentation

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1 Introduction

This document describes algorithm improvements. This serves the idea why I reason in the simulation of the jupyter notebook `new_algorithm.ipynb` that the `X_transformed` and `X_transformed_1` should be the same. We first show our scheme here:

$$\begin{aligned}\tilde{X} = H^* D^* X^* &= (H_k \otimes H_m) \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & D_n \end{bmatrix} \begin{bmatrix} X_{1,\text{loc}} \\ X_{2,\text{loc}} \\ \vdots \\ X_{k,\text{loc}} \end{bmatrix} \\ &= ((H_k I_k) \otimes (I_m H_m)) \begin{bmatrix} D_1 X_{1,\text{loc}} \\ D_2 X_{2,\text{loc}} \\ \vdots \\ D_k X_{k,\text{loc}} \end{bmatrix} = (H_k \otimes I_m) (I_k \otimes H_m) \begin{bmatrix} D_1 X_{1,\text{loc}} \\ D_2 X_{2,\text{loc}} \\ \vdots \\ D_k X_{k,\text{loc}} \end{bmatrix} \\ &= (H_k \otimes I_m) \begin{bmatrix} H_m & 0 & \cdots & 0 \\ 0 & H_m & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & H_m \end{bmatrix}_{k \text{ blocks}} \begin{bmatrix} D_1 X_{1,\text{loc}} \\ D_2 X_{2,\text{loc}} \\ \vdots \\ D_k X_{k,\text{loc}} \end{bmatrix} \\ &= (H_k \otimes I_m) \begin{bmatrix} H_m D_1 X_{1,\text{loc}} \\ H_m D_2 X_{2,\text{loc}} \\ \vdots \\ H_m D_k X_{k,\text{loc}} \end{bmatrix}\end{aligned}$$