Gravitational Wave Event Rate Constraints On GRB Jet Angles

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Outline

1. Most short gamma-ray bursts (GRBs) are probably associated with compact binary coalescence
2. The observed rate of GRBs in the local Universe is a function of the geometry of the beamed jet and the rate of binary coalescence
3. Searches for gravitational waves (GWs) yield direct constraints on the binary coalescence rate
4. **We demonstrate a how to transform the posterior measurement of the binary coalescence rate measured from GW observations to a direct measurement of the GRB beaming angle**, while accounting for the uncertainty in the rate measurement and our ignorance in the details of the GRB progenitor model

GRB Beaming Angles From Coalescence Rate Measurements

Assuming that at least some fraction of sGRBs are due to compact binary coalescence, the observed rate of sGRBs may be written,

$$\mathcal{R}_{\rm grb} = \epsilon \mathcal{R}(1-\cos\theta), \qquad (1)$$

where:

- θ is the *mean* GRB jet opening angle, given a population of angles.
- $\mathcal{R}$ is the rate of GRB progenitor events (binary neutron star coalescence).
- ϵ is the (unknown) probability that any given coalescence will sucessfully generate a GRB, hereafter referred to as 'efficiency'. Note that the inclusion of the efficiency term ϵ allows for the possibility that not all GRB progenitors are BNS mergers.

Objective: determine the value of and uncertainty in the jet angle θ, given GW constraints on the binary coalescence rate $\mathcal{R}$ and a clearly stated level of ignorance on the GRB efficiency ϵ.

For the purposes of this study, we assume that the sGRB rate $\mathcal{R}_{\rm grb}$ in the local Universe is known to arbitrary accuracy and adopt a fiducial value of 10XXXXXXXXXXXXXXXX.

Robustness of Mean Angle Inference To Distribution Width

How robust is our inference on the *mean* GRB beaming angle to the *width* of the distribution of angles?

We conduct a simple Monte-Carlo experiment: simulate a population of binary inclinations and count how many of these events would be observed, given a certain jet angle distribution.

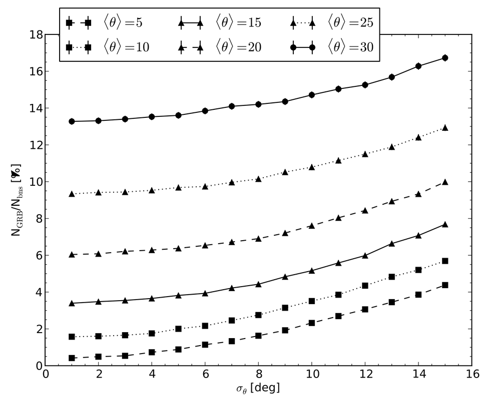


Figure 1: Expected relative numbers of observed GRBs and binary coalescences for different distributions on the GRB beaming angle. Lines in the figure correspond to jet angle population means, while the x-axis shows the width of the distribution. All distributions are Gaussian, truncated at $(0, 90]$ degrees.

Figure 1 shows that the relative numbers of observed GRBs to all binary mergers, for a given distribution mean, is quite insensitive to the distribution width - the relative numbers only change by a few %. Note, however, that the relative numbers of events are degenerate across a range of distribution means (e.g., the result for $p(\theta) = N(5, 10)$ is approximately the same as the result for $p(\theta) = N(10, 5)$).

Key finding: event rate-based inferences on θ are really *upper bounds* on the *mean* of the GRB jet angle population.

Posterior Inferences On The Jet Angle

Given equation **??** one may construct a jet angle posterior via transformation of the rate posterior:

1. Construct the joint-PDF on the jet angle θ and efficiency ϵ from the joint-PDF on the rate and the efficiency,

$$p(\theta, \epsilon) = p(\mathcal{R}, \epsilon) \left|\left| \frac{\partial(\mathcal{R}, \epsilon)}{\partial(\theta, \epsilon)} \right|\right|, \qquad (2)$$

   where the second term is the Jacobian determinant for the transformation and we assume that the rate posterior and the prior PDF on the efficiency are logically independent, such that: $p(\mathcal{R}, \epsilon) = p(\epsilon|\mathcal{R})p(\mathcal{R}) = p(\epsilon)p(\mathcal{R})$

2. Inferences on the jet angle are then arrived at by marginalising the joint posterior $p(\theta, \epsilon)$ over the unknown efficency:

$$p(\theta) = \int_\epsilon p(\theta, \epsilon)\, \mathrm{d}\epsilon = \int_\epsilon \mathrm{d}\epsilon p(\mathcal{R})(\epsilon) \left|\left| \frac{\partial(\mathcal{R}, \epsilon)}{\partial(\theta, \epsilon)} \right|\right|, \qquad (3)$$

Priors On GRB 'Efficiency' ϵ

We present results for the following:

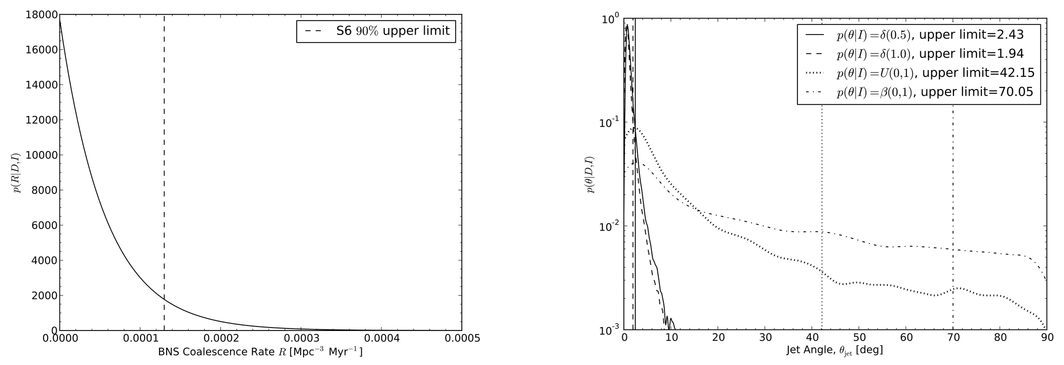
- $p(\epsilon|I) = \delta(\epsilon - 1.0)$: Efficiency known; all mergers yield GRBs
- $p(\epsilon|I) = \delta(\epsilon - 0.5)$: Efficiency known; 50% of mergers yield GRBs
- $p(\epsilon|I) = U(0, 1])$: Unknown efficiency, equal probability in $(0, 1]$
- $p(\epsilon|I) = \beta(0, 1])$: Unknown efficiency, Jeffrey's prior for Bernoulli trial (success=GRB!)

Finally, the measurement aim to transform is the binary coalescence rate posterior. We consider two possibilities:

1. The final binary neutron star coalescence rate posterior from observing runs in the initial-detector era: the S6/vSR2,3 loudest event rate posterior, constructed via the formalism of [**?**], and using the results from [**?**].
2.

The results presented here are, therefore, concerned with the hypothesis that BNS are GRB progenitors. The result, however, is quite general and trivially extended to rate posteriors for other sources.

Constraints From The Initial Detector Era



(a) Rate Posterior (b) Jet Posterior

Figure 2: S6 results

Constraints In The Advanced Detector Era

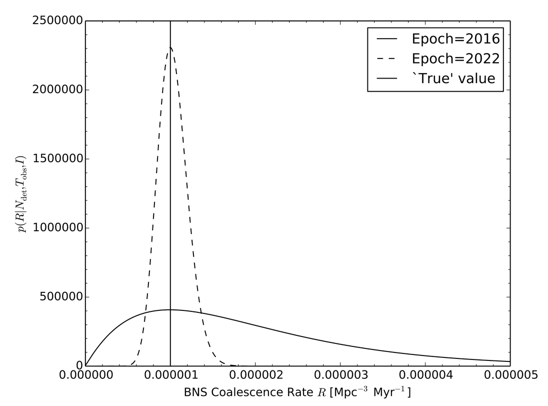
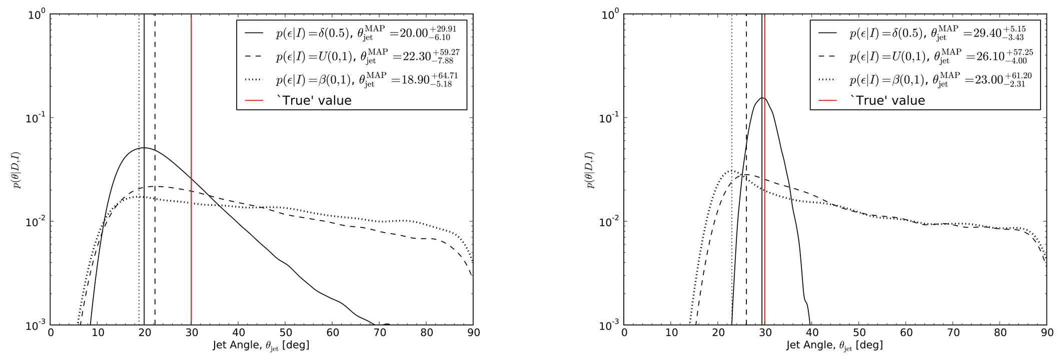


Figure 3: Figure caption

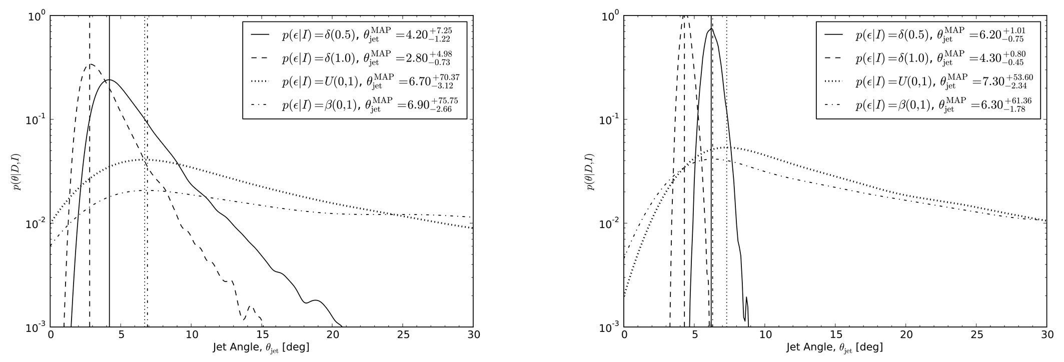
Validation



(a) Rate Posterior (b) Jet Posterior

Figure 4: Example results for $\theta_{\rm jet} = 30°$ and binary coalescence rates in [**?**] to derive a 'simulated' GRB rate.

Predictions



(a) Rate Posterior (b) Jet Posterior

Figure 5: Results in ADE assuming $\mathcal{R}_{\rm grb} = 3 \times 10^{-9}\,{\rm Mpc}^{-3}{\rm yr}^{-1}$ and binary coalescence rates in [**?**].

Conclusions

- Pellentesque eget orci eros. Fusce ultricies, tellus et pellentesque fringilla, ante massa luctus libero, quis tristique purus urna nec nibh. Phasellus fermentum rutrum elementum. Nam quis justo lectus.
- Vestibulum sem ante, hendrerit a gravida ac, blandit quis magna.
- Donec sem metus, facilisis at condimentum eget, vehicula ut massa. Morbi consequat, diam sed convallis tincidunt, arcu nunc.
- Nunc at convallis urna. isus ante. Pellentesque condimentum dui. Etiam sagittis purus non tellus tempor volutpat. Donec et dui non massa tristique adipiscing.

References