GRB-beams & aLIGO Scenarios

James A. Clark

Recap

Key equation:

$$\mathcal{R}_{\mathrm{grb}} = \epsilon \mathcal{R}(1 - \cos\theta), \tag{1}$$

where θ is the *mean* of the distribution of GRB beaming angles.

Explicitly: the distribution of GRB beaming angles can be broad, but if we only look at the relative GRB and BNS rates ($\mathcal{R}_{\mathrm{grb}}$, $\mathcal{R}$, respectively), we only probe the mean value of that distribution

Beaming angle & rates

`monte-carlo demonstration of individual thetas on rates`

Posterior measurement

Our goal is to construct the following posterior:

$$p(\theta) = \frac{2\mathcal{R}_{\mathrm{grb}} \sin\theta \ p(\mathcal{R})}{(\cos\theta - 1)^2} \int_{\epsilon} \frac{p(\epsilon)}{\epsilon} \, \mathrm{d}\epsilon, \tag{2}$$

given some choice of prior on the 'efficiency' ϵ[1] and measurement of the BNS rate posterior $p(\mathcal{R})$

The rate posterior $p(\mathcal{R})$ is taken either from published results in the case of iLIGO or, in the case of aLIGO observing scenarios[2] is computed according to the Gregory formalism.

[1] probability that a BNS results in a GRB

[2] i.e., expected numbers of detections, observation times and sensitivities

Rate Posterior

This yields the following posterior on the signal detection rate (for known background rate b):

$$p(s|n, b, I) = p(r|n, b, I), \tag{3}$$

where n is the number of gravitational wave (GW) detections. From eq 14.8 of Gregory, we get to:

$$p(s|n, b, I) = C\,\frac{T\left[(s+b)T\right]^n e^{-(s+b)T}}{n!}, \tag{4}$$

where,

$$C^{-1} \quad = \quad \frac{e^{-bT}}{n!}\int_0^\infty \mathrm{d}(sT)(s+b)^n T^n e^{-sT} \tag{5}$$

and, finally, the posterior on the binary coalescence rate is,

$$p(\mathcal{R}|N_{\mathrm{det}}, I) \quad = \quad p(s|N_{\mathrm{det}}, I)\left|\frac{\mathrm{d}s}{\mathrm{d}\mathcal{R}}\right| \tag{6}$$

GRB 'Injections'

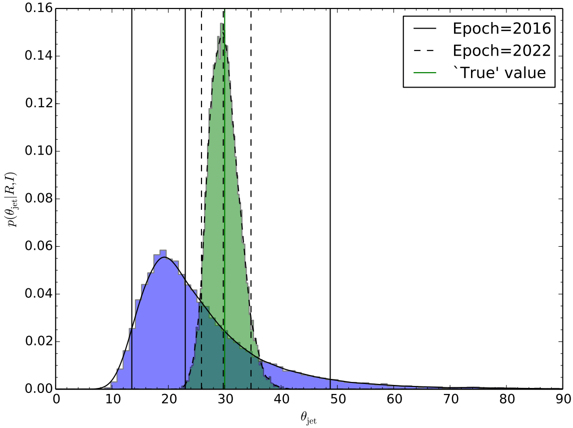
Test the posterior measurement / algorithm by constructing a detection scenario based on a known value for θ:

- pick values for θ and ϵ to compute a $\mathcal{R}$ via equation **??**
- Generate expected numbers for foreground/background triggers according to that rate
- Measure the rate posterior from those numbers using equation 6
- Obtain the jet posterior from equation 2
- For simplicity, we'll consider only the 2016 and 2022+ scenarios
- New: posteriors derived from MCMC sampling of equation 2. Following posteriors have KDE & histograms. Propose KDE for publication?

Example 1: known efficiency

$\epsilon = 0.5,\ p(\epsilon) = \delta(\epsilon - 0.1),\ \theta = 30^\circ$

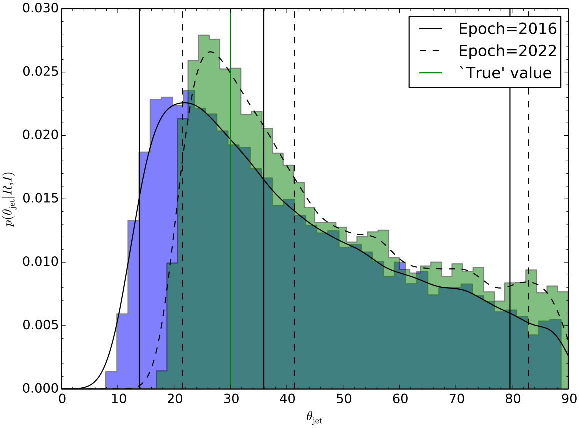
Lines: 95% confidence interval about median



Example 2: unknown efficiency, uniform prior

$\epsilon = 0.5,\ p(\epsilon) = U(0,1),\ \theta = 30^\circ$

Lines: 95% confidence interval about median



Example 3: unknown efficiency, Jeffreys prior

$\epsilon = 0.5$, $p(\epsilon) = \beta(0.5, 0.5)$, $\theta = 30^\circ$

Lines: 95% confidence interval about median