Gravitational Wave Event Rate Constraints On GRB Jet Angles

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Outline

1. Most short gamma-ray bursts (GRBs) are probably associated with compact binary coalescence.
2. The observed rate of GRBs in the local Universe is a function of the opening angle of the beamed jet emmitted by GRBs, and the rate of compact binary coalescence; searches for gravitational waves (GWs) yield direct constraints on the binary coalescence rate.
3. It has previously been shown (e.g., [1, 2]) how coalescence rate limits may be used to constrain the jet opening angle.
4. **We demonstrate a how to transform the posterior measurement of the binary coalescence rate measured from GW observations to a direct measurement of the GRB beaming angle**, *while accounting for the uncertainty in the rate measurement and our ignorance in the details of the GRB progenitor model*.

GRB Beaming Angles From Coalescence Rate Measurements

Assuming at least some fraction of sGRBs are due to compact binary coalescence, the observed rate of sGRBs may be written,

$$\mathcal{R}_{\rm grb} = \epsilon \mathcal{R}(1-\cos\theta), \tag{1}$$

- θ is the *mean* GRB jet opening angle, given a population of angles.
- $\mathcal{R}$ is the rate of GRB progenitor events (binary neutron star coalescence).
- ϵ is the (unknown) probability that any given coalescence will sucessfully generate a GRB, a.k.a 'efficiency'.

OBJECTIVE: infer the value of and uncertainty in the jet angle θ, given GW constraints on the binary coalescence rate $\mathcal{R}$ and a clearly stated level of ignorance on the GRB efficiency ϵ.

For the purposes of this study, we assume that the sGRB rate $\mathcal{R}_{\rm grb}$ in the local Universe is known to arbitrary accuracy and adopt a fiducial value of $\mathcal{R}_{\rm grb} = 3\times10^{-9}\,{\rm Mpc}^{-3}{\rm yr}^{-1}$

Robustness of Mean Angle Inference To Distribution Width

How robust is our inference on the *mean* GRB beaming angle to the *width* of the distribution of angles?

We conduct a simple Monte-Carlo experiment: simulate a population of binary inclinations and count how many of these events would be observed, given a certain jet angle distribution.

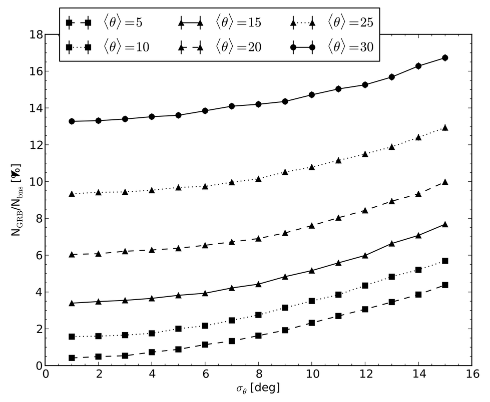


Figure 1: Expected relative numbers of observed GRBs and binary coalescences for different distributions on the GRB beaming angle. Lines in the figure correspond to jet angle population means, while the x-axis shows the width of the distribution. All distributions are Gaussian, truncated at $(0, 90]$ degrees.

Figure 1 shows the fraction of mergers observed as GRBs is quite insensitive to the the distribution width - the ratio only changes by a few %, for a given distribution mean.

Notice that the ratio is degenerate across a range of distribution means (e.g., the result for $p(\theta) = N(5, 10)$ is approximately the same as the result for $p(\theta) = N(10, 5)$).

Key finding: event rate-based inferences on θ are really *upper bounds* on the *mean* of the GRB jet angle population.

Posterior Inferences On The Jet Angle

To build the jet angle posterior:

1. Construct the joint-PDF on the jet angle θ and efficiency ϵ from the joint-PDF on the rate and the efficiency,

$$p(\theta,\epsilon) = p(\mathcal{R},\epsilon)\left|\left|\frac{\partial(\mathcal{R},\epsilon)}{\partial(\theta,\epsilon)}\right|\right|, \tag{2}$$

   where the second term is the Jacobian determinant for the transformation and we assume: $p(\mathcal{R},\epsilon) = p(\epsilon|\mathcal{R})p(\mathcal{R}) = p(\epsilon)p(\mathcal{R})$

2. Get the 1-D jet angle posterior by marginalising the joint posterior $p(\theta,\epsilon)$ over the unknown efficency:

$$p(\theta) = \int_\epsilon p(\theta,\epsilon)\,{\rm d}\epsilon = \int_\epsilon {\rm d}\epsilon p(\mathcal{R})(\epsilon)\left|\left|\frac{\partial(\mathcal{R},\epsilon)}{\partial(\theta,\epsilon)}\right|\right|, \tag{3}$$

Priors On GRB 'Efficiency' ϵ

- $p(\epsilon|I) = \delta(\epsilon-\epsilon_0)$: Efficiency known to be ϵ_0
- $p(\epsilon|I) = U(0,1]$: Unknown efficiency, uniform prior.
- $p(\epsilon|I) = \beta(0,1]$: Unknown efficiency, Jeffrey's prior for Bernoulli trial (success=GRB!)

Rate Posteriors

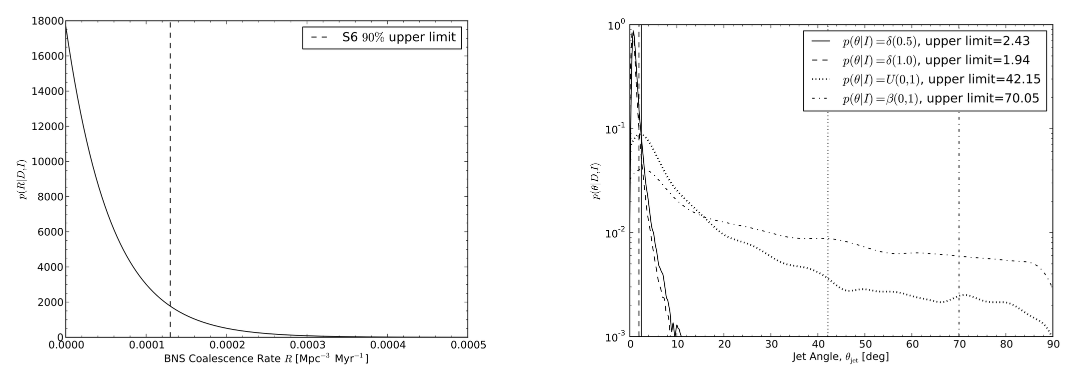
We consider two forms for the rate posterior in this study:

GW Null-detection / initial detector era results
The final binary neutron star coalescence rate posterior from observing runs in the initial-detector era: the **S6/VSR2,3 loudest event rate posterior**, constructed via the formalism of [3], and using the results from [4].

Multiple GW detections / advanced detector era results
We consider two scenarios based on those described in [5]: a 2016 observing run and a 2022 observing run. The rate posteriors are constructed via a Bayesian analysis of a Poission signal rate in the presence of known background, described in [6].

The results presented here are concerned with the hypothesis that BNS are GRB progenitors; the result is quite general and trivially extended to rate posteriors for other sources.

Constraints From The Initial Detector Era



(a) S6/VSR2,3 BNS Rate Posterior (b) S6/VSR2,3 Jet Posterior

Figure 2: *Left*: The binary neutron star merger rate posterior constructed from the 90% rate upper limit (dashed line) reported in [4] and the formalism described in [3]. *Right*: the jet angle posteriors obatined from equation 3. Different lines for the jet posterior correspond to the different priors on the GRB efficiency.

Constraints In The Advanced Detector Era

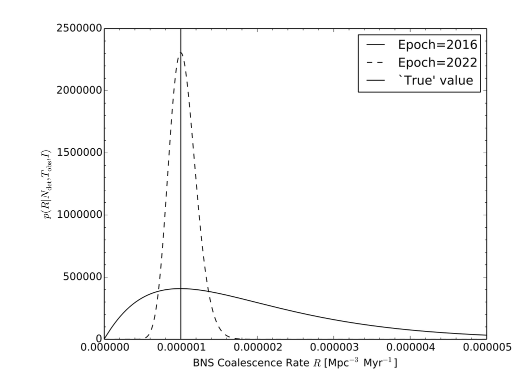
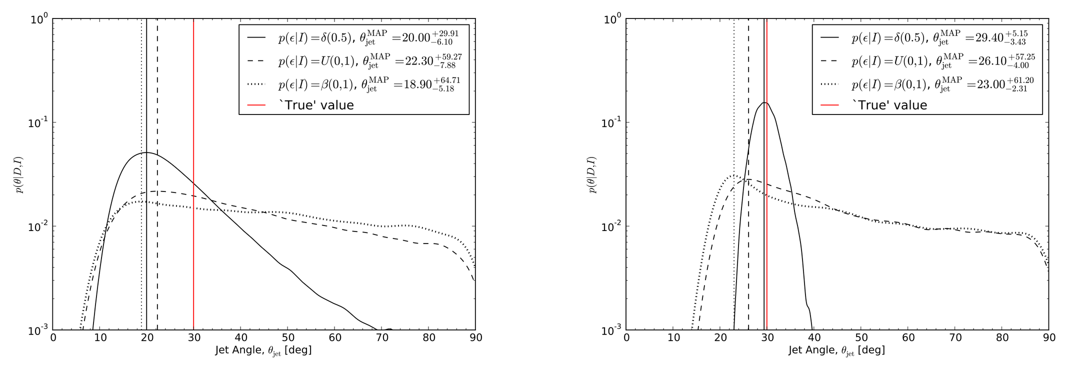


Figure 3: Rate posterior PDFs expected following the 2016 and 2022 observing scenarios described in [5], assuming the 'realistic' rate of BNS coalescence from [5] (solid vertical line) and using the Bayesian Poisson rate determination described in [6].

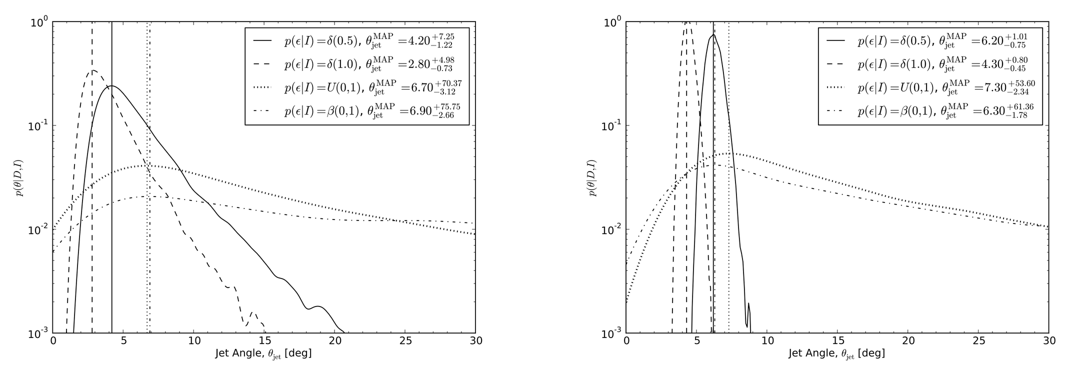
Validation: Posteriors For 'Known' Jet Angle



(a) Simulated 2016 jet posteriors (b) Simulated 2022 jet posteriors

Figure 4: Simulated jet angle posterior PDFs generated by setting $\theta = 30°$ and computing the corresponding $\mathcal{R}_{\rm grb}$ using $\epsilon = 1.0$ and $\mathcal{R}\sim10^{-6}\,{\rm Mpc}^{-3}\,{\rm yr}^{-1}$.

Predictions: Jet Angle Posteriors In ADE Observing Scenarios



(a) Expected 2016 jet posteriors (b) Expected 2022 jet posteriors

Figure 5: Results Expected in ADE, given [5].

Conclusions

- General framework described for astrophysical inferences (θ measurement) from GW observations
- Interpretation of GRB beaming angles in terms of GW rates contingent on prior knowledge of progenitor physics (ϵ)
- Conversely, independent measurements of jet angle could be used to infer mean probability (ϵ) that BNS $\rightarrow$ GRBs

References

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