Constraints On Short, Hard Gamma-Ray Burst Beaming Angles From Gravitational Wave Observations

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Apologies in advance for inconsistent conditioning statements in probabilities . . .

I. INTRODUCTION

It is common in the literature to draw inferences on the rate of binary coalescence $\mathcal{R}$, given some estimate for the beaming angle θ and the observed rate of sGRBs $\mathcal{R}_{\mathrm{grb}}$. In this work, we investigate what statements can *currently* be made on the beaming angle itself using the upper limits placed on $\mathcal{R}$ from all-sky, all-time gravitational wave searches and explore the potential for direct inference of sGRB beaming angles in the advanced detector era.

II. LIMITS ON SGRB BEAMING ANGLES FROM PAST GRAVITATIONAL WAVE SEARCHES

A. Complete Sky-Coverage & Known Observed GRB Rate

Assuming all-sky Gamma-ray coverage and that all compact binary coalescence events result in a short-hard gamma-ray burst, the rate of binary coalescences is,

$$\mathcal{R} = \frac{\mathcal{R}_{\mathrm{grb}}}{1 - \cos\theta}, \qquad (1)$$

where θ is the beaming angle of the outflow from the GRB and $\mathcal{R}_{\mathrm{grb}}$ is the *observed* sGRB rate. We take $\mathcal{R}_{\mathrm{grb}} = 10\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ [1, 2]. Inferences of the GRB beaming angle may then be drawn from the posterior probability density on the beaming angle, related to that on the rate posterior via,

$$
\begin{aligned}
p(\theta|D, I) &= p(\mathcal{R}|D, I) \left| \frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\theta} \right| & (2) \\
&= p(\mathcal{R}|D, I) \times \left| \frac{\mathcal{R}_{\mathrm{grb}} \sin\theta}{(1 - \cos\theta)^2} \right|, & (3)
\end{aligned}
$$

where the Jacobian is computed from equation 1, D represents our gravitational wave observations and we explicitly include the conditioning information I to remind us of the assumptions in the analysis.

Following [3, 4], the posterior on the binary coalescence rate may be determined from the loudest event in the gravitational wave analysis. Specifically, for a foreground event rate due to binary coalescence $\mathcal{R}$, the probability of obtaning no events with ranking statistic ρ greater than the observed loudest event ρ^* is,

$$P_F(\rho^*|\mathcal{R}, C_L, T) = e^{-\mathcal{R}C_L(\rho^*)T}, \qquad (4)$$

where $C_L(\rho^*)$ is the total luminosity to which the search is sensitive and T is the duration of the search. The overall probability of obtaining no events with ranking statistic $\rho > \rho^*$ is the product of obtaining no such events from foreground *and* the probability of obtaining no such events from the background in the detector, denoted $P_B(\rho^*)$,

$$P(\rho^*|\mathcal{R}, I) = P_B(\rho^*|I)e^{-\mathcal{R}C_L(\rho^*)T} \qquad (5)$$

Using a uniform prior on $\mathcal{R}$ and inverting the overall probability with Bayes' theorem, we arrive at,

$$p(\mathcal{R}|C_L(\rho^*, T, \Lambda) \propto p(\mathcal{R}) \left[\frac{1 + \Lambda C_L(\rho^*)T}{1 + \Lambda} \right] e^{-\mathcal{R}C_L(\rho^*)T} \qquad (6)$$

where $p(\mathcal{R})$ is the prior probability distribution on the rate. The quantity $\hat{\Lambda}$ measures the relative probability of detecting an event with ranking statistic x due to gravitational waves versus the probability of an equally loud event arising in the background distribution. The reader is directed to section 3 of [4] for a full discussion of this quanitity.

Now, recall that our objective is to derive a posterior on the jet angle θ_j from the posterior on the rate of binary coalescence. It is straightforward to reconstruct the rate posterior from the published rate upper limits: we simply require the value of Λ and the product $C_L T$. Assuming a uniform prior on the rate $\mathcal{R}$ and the posterior given by equation 6, the upper limit is given by equation 21 from [4]:

$$1 - \alpha = e^{-\mathcal{R}_\alpha C_L(\rho^*)T)} \left[1 + \left(\frac{\Lambda}{1 + \Lambda} \right) \mathcal{R}_\alpha T C_L(\rho^*) \right] \quad (7)$$

For now, let us focus attention on the interpretation of upper limits from past gravitational wave analyses. No gravitational wave signal has been observed and the loudest event is umabiguously due to background noise fluctuations. That is, the limit in which $\Lambda \to 0$. In this case, we simply have,

$$C_L(\rho^*)T = -\frac{\log(1 - \alpha)}{\mathcal{R}_\alpha}, \qquad (8)$$

and the value of $\mathcal{R}$ can be taken straight from the literature. Note that this procedure necessarily confines our jet angle inferences based on progenitor systems for which the rate upper limits are available. Given that the binary coalesence rate limits are quoted for canonical binary neutron star and neutron star-black hole systems,
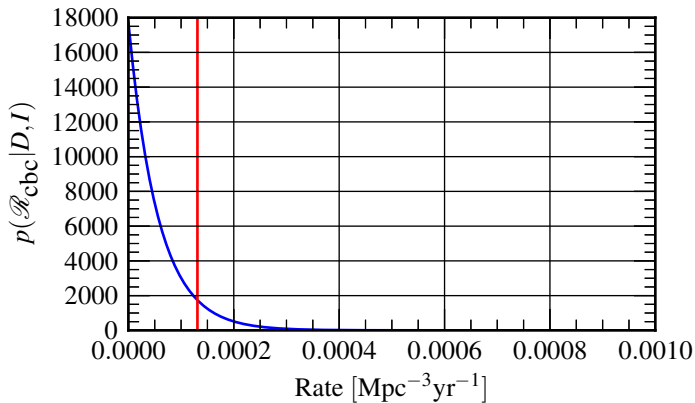
FIG. 1: Rate posterior for S6/VSR2,3 Low-mass Search For Compact Binary Coalescence.
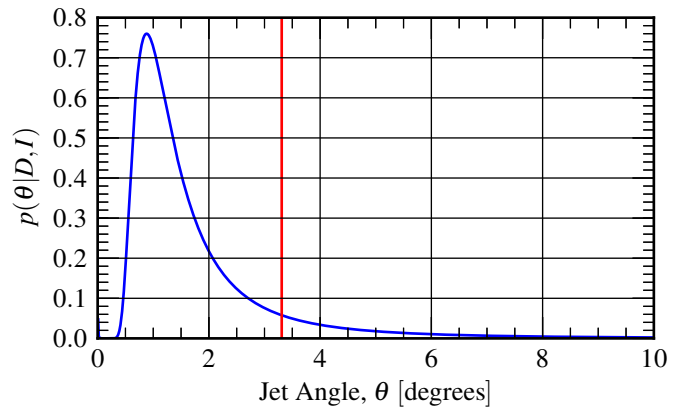

FIG. 2: Jet angle posterior derived from S6/VSR2,3 Low-mass Search For Compact Binary Coalescence posterior / upper limit on compact binary coalescence rate.

both plausible sGRB progenitors, this simply means our inferences on the jet angle are specific to each system and treated separately.

Using the most stringent 90% confidence upper limit from gravitational wave observations on the rate of binary neutron star coalescences to date, $\mathcal{R}_{\text{bns}}^{90\%} = 1.3 \times 10^{-4} \, \text{Mpc}^{-3}\text{yr}^{-1}$ [5], gives $C_L(\rho^*)T = 17712$. Similarly, for NS-BH systems, $\mathcal{R}_{\text{nsbh}}^{90\%} = 3.1 \times 10^{-5} \, \text{Mpc}^{-3}\text{yr}^{-1}$ gives $C_L(\rho^*)T = 74277$. The posteriors on the rates, assuming these values and $\Lambda = 0$, are shown in figure 1.

Now that we have our, figure 2 shows the resulting posterior on the GRB beaming angle, using the transformation in equation 3 and our results so far. Informal: To interpret this posterior and hence obtain a limit on the range of possible beaming angles, we need to consider the question we're asking. I (James) think this should probably be, 'what is the smallest jet angle that produces a result consistent with our observations?'. That is, we want a *lower* limit. Why not the *largest* jet angle? I think that would involve allowing infinitely small angles which doesn't really make sense. I can't quite put my finger on it, though.

Now, the quantity of interest is the inferred *lower* limit on the jet angle $\theta^{90\%}$:

$$0.9 = \int_{\theta^{90\%}}^{\infty} p(\theta | \mathcal{R}^{90\%}) \, \text{d}\theta \qquad (9)$$

This is indicated with the vertical red line in figure 2. We find $\theta^{90\%} = 3.3°$.

B. Incomplete Sky-Coverage & Unknown GRB Rate

Marginalise over some stuff for the case of an 'impure' GRB sample (unknown fraction of CBCs).

C. Astrophysical Interpretation & Comparison With Other Limits

The comparison with other limits is straightforward. I think it looks pretty consistent with Dietz, Holtz etc (but should double check).

More importantly, however, we've demonstrated how to get a limit on the beaming angle. That's all well and good but we need some astrophysical interpretation, really. The most obvious thing (to James) is that the beaming angle is really a proxy for the Lorentz factor of the outflow (I think):

$$\theta \sim \frac{1}{\Gamma}. \qquad (10)$$

So, what *physics* of the progenitor can we constrain with this?

III. INFERENCES ON BEAMING ANGLES EXPECTED FROM FUTURE DETECTIONS

I'm less sure how to address this but I could imagine having a) a mock result based on the Big Dog or b) a collection of mock results for e.g., a measured non-zero rate after x years of observation (say). Anything here is going to be pretty speculative since we don't know about satellites. In fact, maybe this is an opportunity to devise a simple figure of merit to state what sort of sky-coverage is required in each of the timelines in the observing scenarios document to measure the jet angle to some accuracy?

IV. CONCLUSION

[1] E. Nakar, Physics Reports **442**, 166 (2007).
[2] A. Dietz, Astron. Astrophys. **529**, A97 (2011), 0802.0874.
[3] R. Biswas, P. R. Brady, J. D. E. Creighton, and S. Fairhurst, Class. Quantum Grav. **26**, 175009 (2009), gr-qc/0308069).
[4] P. R. Brady and S. Fairhurst, Classical and Quantum Gravity **25**, 105002 (2008), 0707.2410.
[5] (????).