Constraints On Short, Hard Gamma-Ray Burst Beaming Angles From Gravitational Wave Observations

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Apologies in advance for inconsistent conditioning statements in probabilities ...

I. INTRODUCTION

It is common in the literature to draw inferences on the rate of binary coalescence $\mathcal{R}_{\rm cbc}$, given some estimate for the beaming angle θ and the observed rate of sGRBs $\mathcal{R}_{\rm grb}$. In this work, we investigate what statements can *currently* be made on the beaming angle itself using the upper limits placed on $\mathcal{R}_{\rm cbc}$ from all-sky, all-time gravitational wave searches and explore the potential for direct inference of sGRB beaming angles in the advanced detector era.

II. LIMITS ON SGRB BEAMING ANGLES FROM PAST GRAVITATIONAL WAVE SEARCHES

A. Complete Sky-Coverage & Known Observed GRB Rate

Assuming all-sky Gamma-ray coverage and that all compact binary coalescence events result in a short-hard gamma-ray burst, the rate of binary coalescences is,

$$\mathcal{R}_{\rm cbc} = \frac{\mathcal{R}_{\rm grb}}{1-\cos\theta}, \tag{1}$$

where θ is the beaming angle of the outflow from the GRB and $\mathcal{R}_{\rm grb}$ is the *observed* sGRB rate. We take $\mathcal{R}_{\rm grb} = 10\,{\rm Gpc}^{-3}\,{\rm yr}^{-1}$ [1, 2]. Inferences of the GRB beaming angle may then be drawn from the posterior probability density on the beaming angle, related to that on the rate posterior via,

$$\begin{aligned} p(\theta|D,I) &= p(\mathcal{R}_{\rm cbc}|D,I)\left|\frac{{\rm d}\mathcal{R}_{\rm cbc}}{{\rm d}\theta}\right| & (2)\\ &= p(\mathcal{R}_{\rm cbc}|D,I)\times\left|\frac{\mathcal{R}_{\rm grb}\sin\theta}{(1-\cos\theta)^2}\right|, & (3)\end{aligned}$$

where the Jacobian is computed from equation 1, D represents our gravitational wave observations and we explicitly include the conditioning information I to remind us of the assumptions in the analysis.

Following [3, 4], the posterior on the binary coalescence rate may be determined from the loudest event in the gravitational wave analysis. Specifically, for a foreground event rate due to binary coalescence $\mathcal{R}_{\rm cbc}$, the probability of obtaning no events with ranking statistic ρ greater than the observed loudested event ρ^* is,

$$P_F(\rho^*|\mathcal{R}_{\rm cbc}, C_L, T) = e^{-\mathcal{R}_{\rm cbc}C_L(\rho^*)T}, \tag{4}$$

where $C_L(\rho^*)$ is the total luminosity to which the search is sensitive and T is the duration of the search. The overall probability of obtaining zero events with ranking statistic $\rho > \rho^*$ is the product of obtaining no events from foreground *and* the probability of obtaining no events from the background in the detector, denoted $P_B(\rho^*)$,

$$P(\rho^*|\mathcal{R}_{\rm cbc}, I) = P_B(\rho^*|I)e^{-\mathcal{R}_{\rm cbc}C_L(\rho^*)T} \tag{5}$$

Using a uniform prior on $\mathcal{R}_{\rm cbc}$ and inverting the overall probability with Bayes' theorem, we arrive at,

$$p(\mathcal{R}_{\rm cbc}|\hat{\epsilon},\hat{\Lambda}) = \frac{\hat{\epsilon}}{1+\hat{\Lambda}}\left(1+\mathcal{R}_{\rm cbc}\hat{\epsilon}\hat{\Lambda}\right)e^{-\mathcal{R}_{\rm cbc}\hat{\epsilon}}, \tag{6}$$

where, for notational and computational convenience and for consistency with [3], we set,

$$\epsilon = C_L(\rho^*)T \tag{7}$$

The quantity $\hat{\Lambda}$ measures the relative probability of detecting an event with ranking statistic x due to gravitational waves versus the probability of an equally loud event arising in the background distribution. For now, let us focus attention on the interpretation of existing upper limits from past gravitational wave searches where no gravitational wave signal has been observed and the loudest event is umabiguously due to background noise fluctuations. That is, the limit in which $\hat{\Lambda}\to 0$. In this case, the posterior on the rate goes to,

$$p(\mathcal{R}_{\rm cbc}|\hat{\epsilon},\hat{\Lambda}\to 0) = \hat{\epsilon}e^{-\mathcal{R}_{\rm cbc}\hat{\epsilon}} \tag{8}$$

Now, our objective is to compute a posterior on the GRB beaming angle via equation 3, for which we require the (unpublished) numerical value of $\hat{\epsilon}$. To proceed, we note that the reported 90% confidence rate upper limits in [5] are found by solving for ${\mathcal{R}_{\rm cbc}}^{90\%}$ in,

$$\begin{aligned} 0.9 &= \int_0^{{\mathcal{R}_{\rm cbc}}^{90\%}} p(\mathcal{R}_{\rm cbc}|\hat{\epsilon},\hat{\Lambda}\to 0)\,{\rm d}\mathcal{R}_{\rm cbc} \\ &= 1-\left[1+\frac{{\mathcal{R}_{\rm cbc}}^{90\%}\hat{\epsilon}\hat{\Lambda}}{1+\hat{\Lambda}}\right]e^{-{\mathcal{R}_{\rm cbc}}^{90\%}\hat{\epsilon}}. & (9)\end{aligned}$$

So, *given* ${\mathcal{R}_{\rm cbc}}^{90\%}$, we simply solve equation 9 for $\hat{\epsilon}$ to reconstruct the rate posterior obtained in the search NOTE: I only just noticed that [3] actually has the expression for this. Existing results do the calculation for $\hat{epsilon}$ iteratively but this is an unnecessary (but fun) brute force approach. I still need to make that change.

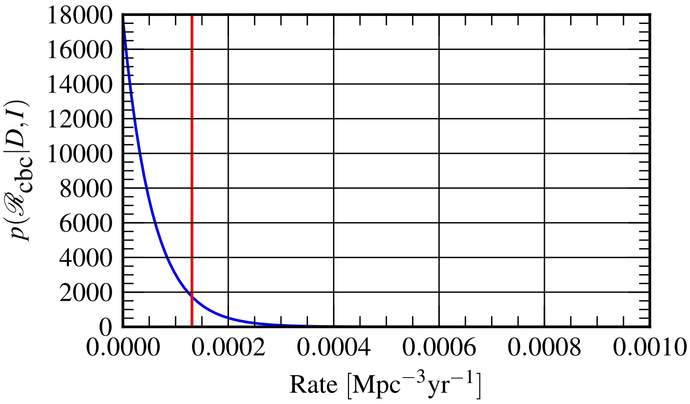

FIG. 1: Rate posterior for S6/VSR2,3 Low-mass Search For Compact Binary Coalescence.

The reconstructed rate posterior, $p(\mathcal{R}_{\mathrm{cbc}}|\hat{\epsilon},\hat{\Lambda}\rightarrow 0)$ and the **upper limit for BNS** is shown in figure 1. Note that this procedure necessarily confines our jet angle inferences based on progenitor systems for which the rate upper limits are available; we are not free to choose our own mass configurations.

Finally, figure 2 shows the resulting posterior on the GRB beaming angle, using the transformation in equation 3 and our results so far. Informal: To interpret this posterior and hence obtain a limit on the range of possible beaming angles, we need to consider the question we're asking. I (James) think this should probably be, 'what is the smallest jet angle that produces a result consistent with our observations?'. That is, we want a *lower* limit. Why not the *largest* jet angle? I think that would involve allowing infinitely small angles which doesn't really make sense. I can't quite put my finger on it, though.

Now, the quantity of interest is the inferred *lower* limit on the jet angle $\theta^{90\%}$:

$$0.9 = \int_{\theta^{90\%}}^{\infty} p(\theta|\mathcal{R}_{\mathrm{cbc}}{}^{90\%})\ \mathrm{d}\theta \qquad (10)$$

This is indicated with the vertical red line in figure 2. We find $\theta^{90\%} = 3.3°$.

B. Incomplete Sky-Coverage & Unknown GRB Rate

Marginalise over some stuff for the case of an 'impure' GRB sample (unknown fraction of CBCs).

C. Astrophysical Interpretation & Comparison With Other Limits

The comparison with other limits is straightforward. I think it looks pretty consistent with Dietz, Holtz etc (but should double check).

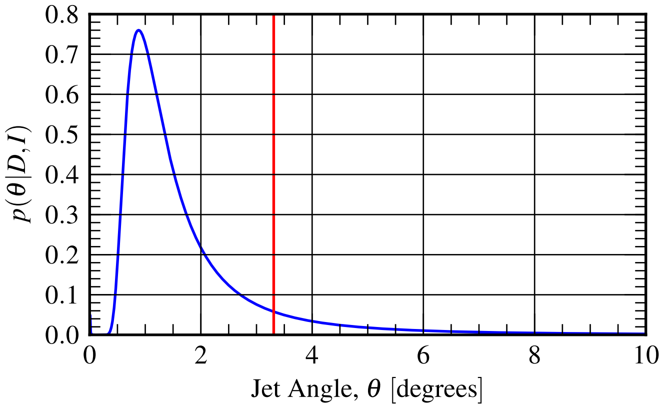

FIG. 2: Jet angle posterior derived from S6/VSR2,3 Low-mass Search For Compact Binary Coalescence posterior / upper limit on compact binary coalescence rate.

More importantly, however, we've demonstrated how to get a limit on the beaming angle. That's all well and good but we need some astrophysical interpretation, really. The most obvious thing (to James) is that the beaming angle is really a proxy for the Lorentz factor of the outflow (I think):

$$\theta \sim \frac{1}{\Gamma}. \qquad (11)$$

So, what *physics* of the progenitor can we constrain with this?

III. INFERENCES ON BEAMING ANGLES EXPECTED FROM FUTURE DETECTIONS

I'm less sure how to address this but I could imagine having a) a mock result based on the Big Dog or b) a collection of mock results for e.g., a measured non-zero rate after x years of observation (say). Anything here is going to be pretty speculative since we don't know about satellites. In fact, maybe this is an opportunity to devise a simple figure of merit to state what sort of sky-coverage is required in each of the timelines in the observing scenarios document to measure the jet angle to some accuracy?

IV. CONCLUSION

[1] E. Nakar, Physics Reports **442**, 166 (2007).
[2] A. Dietz, Astron. Astrophys. **529**, A97 (2011), 0802.0874.
[3] R. Biswas, P. R. Brady, J. D. E. Creighton, and S. Fairhurst, Class. Quantum Grav. **26**, 175009 (2009), gr-qc/0308069).
[4] P. R. Brady and S. Fairhurst, Classical and Quantum Gravity **25**, 105002 (2008), 0707.2410.
[5] (????).