

Definition of convexity

2.1 Let $C \subseteq \mathbf{R}^n$ be a convex set, with $x_1, \dots, x_k \in C$, and let $\theta_1, \dots, \theta_k \in \mathbf{R}$ satisfy $\theta_i \geq 0$, $\theta_1 + \dots + \theta_k = 1$. Show that $\theta_1 x_1 + \dots + \theta_k x_k \in C$. (The definition of convexity is that this holds for $k = 2$; you must show it for arbitrary k .) *Hint.* Use induction on k .