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X. R. Wang, P. Yan and J. Lu[a]

Physics Department, The Hong Kong University of Science and Technology - Clear Water Bay,
Hong Kong SAR, China

Abstract – A theory of field-induced domain wall (DW) propagation is developed. The theory not only explains why a DW in a defect-free nanowire must propagate at a finite velocity, but also provides a proper definition of DW propagation velocity. This definition, valid for an arbitrary DW structure, allows one to compute the instantaneous DW velocity in a meaningful way even when the DW is not moving as a rigid body. A new velocity-field formula beyond the Walker breakdown field, which is in excellent agreement with both experiments and numerical simulations, is derived.

It is a textbook knowledge [1] that a magnetic field can drive a magnetic domain wall (DW) to move. However, our understanding of the field-induced DW motion is far from complete although it has been intensively studied for more than fifty years and many interesting phenomena of magnetization dynamics have been found. Recent development in nanomagnetism [2] demands a deep understanding of DW motion in nanowires, especially how a field affects DW propagation velocity. DW dynamics is governed by the Landau-Lifshtiz-Gilbert (LLG) equation that can only be solved analytically for some special problems [3,4]. A number of theories have been widely accepted and written in books [1], such as the kinetic potential approach that assumes zero damping, the Thiele dynamic force equilibrium formulation that is correct for rigid DW propagation, the Schryer and Walker analytical solution that is valid only for 1D and exact only for field smaller than a so-called Walker breakdown field H_W [3], Slonczewski formulation that simplifies a DW by its center and the cant angle of DW plane. None of these orthodox theories works beyond H_W although they have greatly enriched our current understanding of DW dynamics. For example, the kinetic potential approach cannot be a correct description of DW propagation because it violates the principle of "no damping, no propagation" that will be explained in this paper. The Thiele approach is a good way to describe a rigid DW propagation for small field $H < H_W$, but its assumptions are not valid for $H > H_W$. Schryer and Walker's approach is for 1D and $H < H_W$, and its predictions for $H > H_W$

are incorrect. For instance, its prediction that the $v - H$ line for $H \gg H_W$ passing through the origin differs from both experiments and micromagnetic simulations [5–8]. Its generalization predicts a saturated velocity [9] (bounded by the velocity at H_W) that does not agree either with experiments or with simulations [5–8]. Slonczewski formulation is a great simplification of LLG equation that not only replaces partial differential equations by ordinary differential ones, but also is based on Thiele rigid DW approximation although the Slonczewski equations have also been applied to the case of $H > H_W$ where it is known that DW deformation cannot be neglected. The problems with both Thiele and Slonczewski formulations can also be seen from their $v - H$ formula [8,10] that do not capture the trend for $H > H_W$. Even more surprising, none of the existing theories provides a proper definition of DW propagation velocity when a DW does not propagate like a rigid body.

In this paper, we develop a general formulation of DW propagation for both $H < H_W$ and $H > H_W$ that does not have all the problems with the existing theories. The theory reveals the origin of DW propagation. Firstly, we show that no static tail-to-tail (TT) or head-to-head (HH) DW is allowed in a homogeneous nanowire in the presence of an external magnetic field. Thus, a DW must propagate in order to release the Zeeman energy to compensate the energy dissipation due to the moving DW. This energy consideration can clearly explain DW velocity oscillation for $H > H_W$. Secondly, the energy conservation provides a proper definition for DW propagation velocity. This definition leads to a general relationship between DW
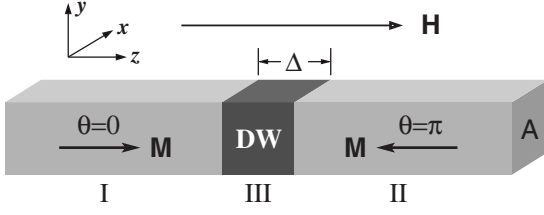
Fig. 1: Schematic diagram of a HH DW of width Δ in a magnetic nanowire of cross-section A. The wire consists of three phases: two domains and one DW. The magnetization in domains I and II is along $+z$-direction ($\theta = 0$) and $-z$-direction ($\theta = \pi$), respectively. III is the DW region whose magnetization structure could be very complicate. $\vec{H}$ is an external field along $+z$-direction.

propagation velocity and the DW structure. Finally, a new velocity-field formula beyond the Walker breakdown field is derived and is compared with both experiments and numerical simulations.

In a magnetic material, magnetic domains are formed in order to minimize the stray field energy. We consider a HH DW in a magnetic nanowire with its easy axis along the wire axis which is chosen as the z-axis as illustrated in fig. 1. Since the magnitude of the magnetization $\vec{M}$ does not change according to the LLG equation [4], the magnetic state of the wire can be conveniently described by the polar angle $\theta(\vec{x}, t)$ (angle between $\vec{M}$ and the z-axis) and the azimuthal angle $\phi(\vec{x}, t)$. The magnetic energy of the wire can be written in general as

$$
\begin{aligned}
E &= \int F(\theta, \phi, \vec{\nabla}\theta, \vec{\nabla}\phi) \mathrm{d}^3\vec{x}, \\
F &= f(\theta, \phi) + \frac{J}{2}\left[(\vec{\nabla}\theta)^2 + \sin^2\theta(\vec{\nabla}\phi)^2\right] - MH\cos\theta,
\end{aligned}
\tag{1}
$$

where f is the energy density due to *all kinds* of magnetic anisotropies which has two equal minima at $\theta = 0$ and π ($f(\theta = 0, \phi) = f(\theta = \pi, \phi)$). J describes the exchange energy. M is the magnitude of magnetization, and H is the external magnetic field along z-axis. However, the non-local anisotropy due to the magnetostatic interaction is neglected. The small wire diameter of a magnetic nanowire generates a small magnetostatic energy that does not alter too much of the total magnetic energy. Furthermore, part of the magnetostatic energy (that can be modeled by a local anisotropy) may also be absorbed into f. For the homogeneous nanowires, our numerical simulations show later that this is the case indeed. Thus, the theory presented below should still work as long as the long-range magnetostatic interaction, which cannot be modeled by a local energy function of magnetization, is not too big. In the absence of H, a static HH DW that separates $\theta = 0$ and $\theta = \pi$ domains can exist [3] in the wire.

To show that no intrinsic static HH DW is allowed in the presence of an external field ($H \neq 0$), one only needs to show that following equations have no solution with $\theta = 0$

at the far left and $\theta = \pi$ at the far right,

$$
\begin{aligned}
\frac{\delta E}{\delta\theta} &= J\nabla^2\theta - \frac{\partial f}{\partial\theta} - HM\sin\theta - J\sin\theta\cos\theta(\vec{\nabla}\phi)^2 = 0, \\
\frac{\delta E}{\delta\phi} &= J\vec{\nabla}\cdot(\sin^2\theta\vec{\nabla}\phi) - \frac{\partial f}{\partial\phi} = 0.
\end{aligned}
\tag{2}
$$

Let us multiply the first equation by $\nabla\theta$ and the second equation by $\nabla\phi$, then add up the two equations. One can show a tensor $\mathbf{T}$ satisfying $\nabla\cdot\mathbf{T} = 0$ with

$$
\begin{aligned}
\mathbf{T} &= \left[-f + HM\cos\theta - \frac{J}{2}(|\nabla\theta|^2 + \sin^2\theta|\nabla\phi|^2)\right]\mathbf{1} \\
&\quad + J(\nabla\theta\otimes\nabla\theta + \sin^2\theta\nabla\phi\otimes\nabla\phi),
\end{aligned}
\tag{3}
$$

where $\mathbf{1}$ is 3×3 unit matrix. $\nabla\theta\otimes\nabla\theta$ and $\nabla\phi\otimes\nabla\phi$ are the usual dyadic products. The diagonal terms of $\mathbf{T}$ are just magentic Lagrangian density. If a HH DW exists with $\theta = 0$ at the far left and $\theta = \pi$ at the far right, it requires $-f(0, \phi) + HM = -f(\pi, \phi) - HM$ that holds only for $H = 0$ since $f(0, \phi) = f(\pi, \phi)$. In other words, a static DW can only exist between two equal-energy-density domains. A HH DW in a nanowire under an external field must vary with time because two domains separated by the DW have different magnetic energy density. It should be clear that the above argument is only true for a HH DW in a homogeneous wire, but not valid with defect pinning that changes eq. (2). Static DWs do exist in the presence of a weak field in reality because of pinning.

The consequence of the non-existence of a static DW is that the DW has to move when an external magnetic field is applied along the nanowire as shown in fig. 1. It is well known [11] that a moving magnetization must dissipate its energy to its environments with a rate, $\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{\alpha M}{\gamma}\int_{-\infty}^{+\infty}(\mathrm{d}\vec{m}/\mathrm{d}t)^2\mathrm{d}^3\vec{x}$, where $\vec{m}$ is the unit vector of $\vec{M}$, α and γ are the Gilbert damping constant and gyromagnetic ratio, respectively. Following the similar method in ref. [12] for a Stoner particle, one can also show that the energy dissipation rate of a DW is related to the DW structure as

$$
\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{\alpha\gamma}{(1+\alpha^2)M}\int_{-\infty}^{+\infty}(\vec{M}\times\vec{H}_{eff})^2\mathrm{d}^3\vec{x},
\tag{4}
$$

where $\vec{H}_{eff} = -\frac{\delta F}{\delta\vec{M}}$ is the effective field. In regions I and II or inside a static DW (fig. 1), $\vec{M}$ is parallel to $\vec{H}_{eff}$, and no energy dissipation is possible there. The energy dissipation can only occur in the DW region when $\vec{M}$ is not parallel to $\vec{H}_{eff}$.

For a magnetic nanowire in a static magnetic field, the dissipated energy must come from the magnetic energy released from the DW propagation. The total energy of the wire equals the sum of the energies of regions I, II, and III (fig. 1), $E = E_I + E_{II} + E_{III}$. E_I increases while E_{II} decreases when the DW propagates to the right along the wire. The net energy change of region I plus II due to the DW propagation is $\frac{\mathrm{d}(E_I+E_{II})}{\mathrm{d}t} = -2HMvA$, where v is the DW propagating speed, and A is the area of wire

cross-section. This is the released Zeeman energy stored in the wire. The energy of region III should not change much because the DW width Δ is finite, typically of the order of 10–100 nm. A DW cannot absorb or release too much energy, and can at most adjust temporarily energy dissipation rate. In other words, $\frac{\mathrm{d}E_{III}}{\mathrm{d}t}$ is either zero or fluctuates between positive and negative values with zero time average. Since the energy release from the magnetic wire should be equal to the energy dissipated (to the environment), one has

$$v = \frac{\alpha\gamma}{2(1+\alpha^2)HA} \int_{III} \left(\vec{m} \times \vec{H}_{eff}\right)^2 \mathrm{d}^3\vec{x} + \frac{1}{2HMA}\frac{\mathrm{d}E_{III}}{\mathrm{d}t}. \tag{5}$$

Equation (5) can serve as a proper definition of DW propagation velocity that is completely defined by the *instantaneous* DW structure.

A DW can have two possible types of motion under an external magnetic field. One is that a DW behaves like a *rigid body* propagating along the wire. This case occurs often at low field, and it is the basic assumption in the Slonczewski model [13] and Walker's solution for $H < H_W$. Obviously, both energy dissipation and DW energy are time independent, $\frac{\mathrm{d}E_{III}}{\mathrm{d}t} = 0$. Thus, the DW velocity should be time independent. The other case is that a DW structure varies with time which occurs at large field $H > H_W$. In this case, the DW structure deforms and the DW precesses around the wire axis, experiencing different transverse magnetic anisotropy energy. As a result, the DW width breathes periodically since it is defined by the balance between the magnetic anisotropy energy and the exchange energy. Thus, one should expect both $\frac{\mathrm{d}E_{III}}{\mathrm{d}t}$ and energy dissipation rate oscillate with time. According to eq. (5), the DW velocity will oscillate. The DW velocity may oscillate periodically or irregularly, depending on whether the ratio of precession period and breathe period is rational or irrational. Indeed, this oscillation was observed in a recent experiment [7]. How can one understand the wire width dependence of DW velocity? According to eq. (5), the velocity is a functional of the DW structure which is very sensitive to the wire width. For a very narrow wire, only transverse DW is possible while a vortex DW is preferred for a wide wire (larger than DW width). Different vortexes yield different values of $|\vec{m} \times \vec{H}_{eff}|$, which in turn results in different DW propagation speed.

The time-averaged velocity is

$$\bar{v} = \frac{\alpha\gamma}{2(1+\alpha^2)HA} \int_{III} \overline{\left(\vec{m} \times \vec{H}_{eff}\right)^2} \mathrm{d}^3\vec{x}, \tag{6}$$

where the bar denotes the time average. It says that the averaged velocity is proportional to the energy dissipation rate. The right-hand side of eq. (6) is positive and non-zero since a time-dependent DW requires $\vec{m} \times \vec{H}_{eff} \neq 0$,

implying a zero intrinsic critical field for DW propagation. It is straightforward to show

$$\left(\vec{m} \times \vec{H}_{eff}\right)^2 = \left\{ \frac{1}{M\sin\theta} \left[J\frac{\partial}{\partial z}\left(\sin^2\theta \frac{\partial\phi}{\partial z}\right) - \frac{\partial f}{\partial\phi} \right] \right\}^2$$
$$+ \left\{ H\sin\theta - \frac{1}{M}\left[J\frac{\partial^2\theta}{\partial z^2} - \frac{\partial f}{\partial\theta} - J\sin\theta\cos\theta\left(\frac{\partial\phi}{\partial z}\right)^2 \right] \right\}^2. \tag{7}$$

It is interesting to notice that two terms on the right-hand side of eq. (7) are just $\left[\frac{1}{M\sin\theta}\frac{\delta E}{\delta\phi}\right]^2$ and $\left[\frac{1}{M}\frac{\delta E}{\delta\theta}\right]^2$. In the case that there is no distortion on the DW plane, *i.e.* $\partial\phi/\partial z = 0$ and $\partial^2\phi/\partial z^2 = 0$, then

$$\left(\vec{m} \times \vec{H}_{eff}\right)^2 = (H - H_0')^2 \sin^2\theta + \left(\frac{1}{M\sin\theta}\frac{\partial f}{\partial\phi}\right)^2, \tag{8}$$

where $H_0' \equiv \frac{1}{M\sin\theta}\left(J\frac{\partial^2\theta}{\partial z^2} - \frac{\partial f}{\partial\theta}\right)$. To reproduce the result of Schryer and Walker [3] for $H < H_W$, one sets $\frac{\partial\phi}{\partial z} = 0$, $\frac{\partial\phi}{\partial t} = 0$, and $J\frac{\partial^2\theta}{\partial z^2} = \frac{\partial f}{\partial\theta}$ (similar to the original analysis of Schryer and Walker [3]) so that $H_0' = 0$. Under this assumption, the LLG equation gives $\alpha\frac{\partial\theta}{\partial t} = \gamma H\sin\theta$ and $\frac{\partial\theta}{\partial t} = -\frac{\gamma}{M\sin\theta}\frac{\partial f}{\partial\phi}$. Thus, $\frac{1}{M\sin\theta}\frac{\partial f}{\partial\phi} = -\frac{H}{\alpha}\sin\theta$, and the right-hand side of eq. (8) is just $\frac{1+\alpha^2}{\alpha^2}H^2\sin^2\theta$. Using the definition of DW width Δ, $\int\sin^2\theta\,\mathrm{d}^3\vec{x} = 2\Delta A$, then eq. (6) reproduces the famous Walker's mobility coefficient $\mu = \frac{\gamma\Delta}{\alpha}$ for $H < H_W$. For $H > H_W$, ϕ varies periodically with time between 0 and 2π. Substitute expression in eq. (7) into eq. (6) and take the time average, the field-dependent averaged velocity takes a form of $\bar{v} = aH - a_0 + a_{-1}/H$ that can be rewritten as

$$\bar{v} = a(H - H_0)^2/H + b/H, \tag{9}$$

where a is proportional to the averaged DW width, H_0 and b (and a_0, a_{-1}) depend on the DW structure and magnetic anisotropy. From eqs. (6), (7) and (9), one has $H_0 = \overline{\int_{-\infty}^{+\infty} g\sin\theta\,\mathrm{d}^3\vec{x}}/\overline{\int_{-\infty}^{+\infty}\sin^2\theta\,\mathrm{d}^3\vec{x}}$, where $g = \frac{1}{M}[J\frac{\partial^2\theta}{\partial z^2} - \frac{\partial f}{\partial\theta} - J\sin\theta\cos\theta(\frac{\partial\phi}{\partial z})^2]$ is the $\hat{e}_\phi$-component of transverse torque exerted by the internal field. When the external field is larger than H_W, but not too close to it, the angular velocity of the DW plane does not vary too much during its precession, and the DW plane experiences all transverse state with nearly the same possibility. This would lead to a nearly H-independent H_0. H_0 may have a H-dependence very close to H_W. This is also why the new high-field formula (9) cannot fit well near H_W.

To demonstrate the goodness of eq. (9), we fit the experiment data (symbols in fig. 2a) from the first of ref. [7] by the expression (solid line in fig. 2a) with $a = 2.98\,\mathrm{m/(s\cdot Oe)}$, $b = 563\,\mathrm{Oe\cdot m/s}$, and $H_0 = 11\,\mathrm{Oe}$. The experimental mobility at large field is measured to be $2.5\,\mathrm{m/(s\cdot Oe)}$ that compares well with $a = 2.98\,\mathrm{m/(s\cdot Oe)}$. According to our theory, b should be proportional to $\alpha\gamma\bar{\Delta}K_2^2/(1+\alpha^2)$ (K_2, defined later, measures the transverse magnetic anisotropy (TMA)), where $\bar{\Delta}$ is the time-averaged DW width. Using material parameters [7]

$\Delta = 14\,\text{nm}$, and TMA constant to be $K_2 M = 34 \times 10^3\,\text{A/m}$ from $\Delta = \mu\alpha/\gamma$ and $H_W = \alpha K_2 M$, the 1D result from Walker's original paper. It is known that the DW width should vary as the DW precesses around the wire axis for $H > H_W$. Although the exact value of the averaged width $\bar{\Delta}$ is not known from the experiment, the theory requires its value to be about 16 nm in order to obtain the fitting value of $b = 563\,\text{Oe} \cdot \text{m/s}$. 16 nm is a fair value for DW width. The good agreement between eq. (9) and experimental results is not accidental. In fact, if one changes the fitting formula slightly to $a(H - H_0)^2 + b/H$ or $a(H - H_0)^2 + b$, the best fits (dashed line and dash-dotted line in fig. 2a) not only show poor agreement with experiment, but also give unreasonable fitting parameters. This proves that a good fitting is not due to the three fitting parameters introduced.

To further test the validity of eq. (9) and usefulness of both eqs. (5) and (6) in evaluating the DW propagation speed from a DW structure, we carry out micromagnetic simulations on a strap wire of $4\,\text{nm} \times 20\,\text{nm} \times 3\,\mu\text{m}$ whose magnetic energy density is $F = -\vec{M} \cdot \vec{H} + \frac{J}{M^2}(\nabla\vec{M})^2 - K_1 M_z^2 + K_2 M_x^2 + F_M$, where F_M is the magnetostatic energy density. We use the OOMMF package [14] to find the instantaneous DW structures and then use eq. (5) to obtain the instantaneous velocity. The average velocity is the time average of this instantaneous velocity within 4–5 periods. Another average velocity is obtained by the best linear fit to the time dependence of the position of the cell with $\theta = \pi/2$ in the OOMMF simulation over a long time (at least more than 5 periods of velocity oscillation). This velocity is referred as "simulated velocity". Figure 2b is the simulation results for system parameters of $K_1 = 10^{-7}\,\text{N/A}^2$, $K_2 = 0.8 \times 10^{-7}\,\text{N/A}^2$, $J = 4 \times 10^{-11}\,\text{J/m}$, $M = 10^6\,\text{A/m}$, and $\alpha = 0.1$. The calculated velocities are denoted by crosses and numerical simulations are the open circles with their error bars smaller than the symbol sizes. The good overlap between the crosses and open circles confirms the correctness of eq. (6). The $\bar{v} - H$ curve for $H > H_W$ can be fit well by eq. (9). The insets are instantaneous DW propagation velocities for both $H < H_W$ and $H > H_W$, by eq. (5) from the instantaneous DW structures obtained from OOMMF. It should be emphasized that both ϕ and θ are complicated functions of the time and the space. The left inset is the instantaneous DW speed at $H = 200\,\text{Oe} < H_W$, reaching its steady value in about 1 ns. The right inset is the instantaneous DW speed at $H = 1600\,\text{Oe} > H_W$, showing clearly an oscillation. They confirm that the theory is capable of capturing all the features of DW propagation.

Figure 2c is another set of OOMMF simulations on a wire of $4\,\text{nm} \times 16\,\text{nm} \times 4\,\mu\text{m}$ with system parameters of $M = 500\,\text{kA/m}$ for the saturation magnetization, $J = 20 \times 10^{-12}\,\text{J/m}$, the axial crystalline anisotropy constant $K_1 = 8 \times 10^{-7}\,\text{N/A}^2$, and $\alpha = 0.1$. The symbols are simulation results, and the solid lines are the fits to eq. (9). The perfect agreement demonstrate the correctness of eq. (6). Figures 2d–f show how a, b and H_0 depend on the
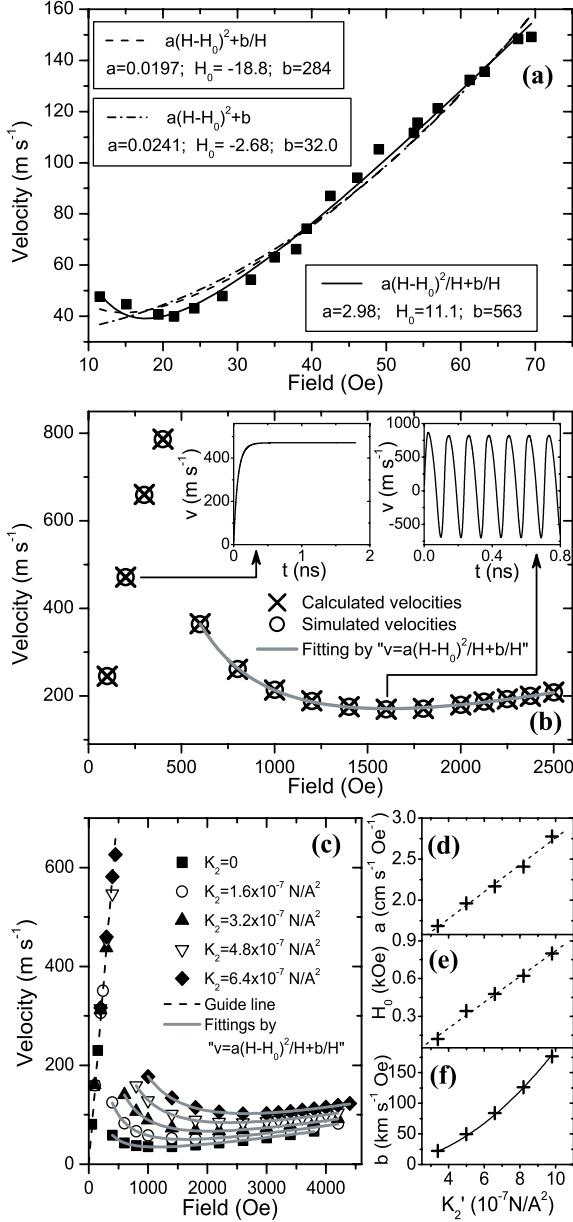
Fig. 2: a) Symbols are from the first of ref. [7]. The solid line is the best fit to eq. (8). The dashed and dash-dotted lines are fits to some modified expressions of eq. (8). b) The time-averaged DW propagation speed *vs.* the applied magnetic field for a biaxial magnetic nanowire. The crosses are the calculated velocities from eq. (6), and the open circles are simulated average velocities. The solid curve is the fit to eq. (8). Insets: the instantaneous DW speed calculated from eq. (5) for $H = 200\,\text{Oe} < H_W$ (left) and $H = 1600\,\text{Oe} > H_W$ (right). c) The symbols are numerical simulations of another set of parameters. The dashed line is the linear fit for low field and the solid lines are the fits to eq. (8). d)–f) The dependence of a, H_0, and b on K_2' defined in the text. The dashed lines are linear fits, and the solid line is the quadratic fit.

$M = 860 \times 10^3\,\text{A/m}$; $J = 13 \times 10^{-12}\,\text{J/m}$; $K_1 M^2 = 500\,\text{J/m}^3$ (defined later); $\alpha = 0.01$, and measured low-field mobility $\mu = 25\,\text{m/(s} \cdot \text{Oe)}$; and the Walker breakdown field $H_W = 4\,\text{Oe}$, the DW width for $H < H_W$ is about

normalized TMA constant $K_2' = K_2 + 3.4 \times 10^{-7} \, \text{N/A}^2$. This normalized parameter comes from the demagnetization effect that generates an extra magnetic anisotropic energy $(D_1 - D_2)M_x^2 + (D_3 - D_2)M_z^2 + D_2$, where D_i is the demagnetization factor along i-th axis. Using the theory in ref. [15] one can find $D_1 - D_2$ to be $3.4 \times 10^{-7} \, \text{N/A}^2$ for our geometry. a and H_0 are linear in K_2' while b is quadratic in K_2', in agreement with eq. (7). It should be pointed out that velocity expressions in both refs. [8] and [10] cannot fit either the experimental curve (fig. 2a) or simulations (figs. 2b and c). The correctness of result eq. (5) depends only on the LLG equation and the general energy expression of eq. (1). It does not depend on the details of a DW structure whether they are transverse or vortex like. In this sense, our result is very general and robust, and it is applicable to an arbitrary magnetic wire.

In conclusion, a proper definition of DW propagation velocity is obtained, and a velocity-field formula for high field (above the Walker breakdown field) is proposed. This new formula agrees well with both experiments and numerical simulations.

$$* * *$$

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