Experiment-2
Cramer's Rule

(Duration: 105 mins)

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Purpose: Any NxN square matrix $[A]$ is called invertible if there exist an NxN square matrix such that $AA^{-1} = I$ where I is identity matrix. In this lab, inverse of a 3x3 $[A]$ matrix will be calculated by using the Cramer's Rule.

Introduction

In linear algebra, Cramer's Rule is a way to solve linear equation systems by using matrix determinant. For a given system,

$$a1x + b1y + c1z = d1$$
$$a2x + b2y + c2z = d2$$
$$a3x + b3y + c3z = d3$$

Cramer's Rule can be applied as following,

$$\begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix} \tag{1}$$

$$D = \begin{vmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} d1 & b1 & c1 \\ d2 & b2 & c2 \\ d3 & b3 & c3 \end{vmatrix} \qquad D_y = \begin{vmatrix} a1 & d1 & c1 \\ a2 & d2 & c2 \\ a3 & d3 & c3 \end{vmatrix} \qquad D_z = \begin{vmatrix} a1 & b1 & d1 \\ a2 & b2 & d2 \\ a3 & b3 & d3 \end{vmatrix}$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

Cramer's Rule can be used to find the inverse of 3x3 A matrix by considering the $AA^{-1} = I$ equation where

$$A = \begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} p1 & q1 & r1 \\ p2 & q2 & r2 \\ p3 & q3 & r3 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AA^{-1} = I$$

$$\begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix} * \begin{bmatrix} p1 & q1 & r1 \\ p2 & q2 & r2 \\ p3 & q3 & r3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Equation (2) can be written separately.

$$\begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix} * \begin{bmatrix} p1 \\ p2 \\ p3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix} * \begin{bmatrix} q1 \\ q2 \\ q3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix} * \begin{bmatrix} r1 \\ r2 \\ r3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

Equations (3), (4) and (5) are in the same form as Equation (1). So, $\begin{bmatrix} p1 \\ p2 \\ p3 \end{bmatrix}$, $\begin{bmatrix} q1 \\ q2 \\ q3 \end{bmatrix}$ and $\begin{bmatrix} r1 \\ r2 \\ r3 \end{bmatrix}$ vectors can easily be found by applying the Cramer's Rule explained above and finally, inverse of the A matrix can be obtained by combining these vectors.

Problem Statement

You are asked to write a C program that calculates the inverse of a 3x3 matrix by using Cramer's Rule. You need to verify your functions by comparing the results with the examples in the laboratory procedure given in the last section.

The running program in bash shell should look in Figure 1 (on the last page). An ideal program should:

- Your program should ask the elements of the matrix A. (10p)
- Write a function that **creates** the 3x3 identity matrix.(10p) (Do not write the values manually)
- Write a function that calculates determinant of a 3x3 matrix, **manually**. (20p)
- Write a function that finds inverse of a 3x3 matrix by using Cramer's Rule (60p).
    - Use **at least one** for loop (20p)
    - Call "CreateDxyz_mat" function (given in Lab Procedure) in this for loop. (15p)
    - Other necessary lines. (25p)
- Demonstrate working functions (use prepared example in the last section of the lab procedure).

Lab Procedure

1. Write the main function as written below. Do not change it.

```
int main(){
        double A[N*N];                  // 3x3 A matrix , N=3
        CramersRuleForInverse(A);       // Call function
}
```

2. Write a function that has the following prototype to create the 3x3 identity matrix. (10p)

```
void CreateIdentityMat(double *I)       // I is 3x3 identity matrix
```

3. Write a function that has the following prototype to calculate the determinant of a 3x3 matrix **manually**. (20p)

```
void det3x3(double *Dxyz, double *det)      // Dxyz is the 3x3 matrix, det is determinant value
```

4. The function to create the Dx, Dy and Dz matrix are as following. Note: Do not forget to define N=3, Dxyz(i,j), A(i,j) and I(i,j).

```
//Create Dx, Dy and Dz matrix for each column vector calculation of inverse A matrix
// A_colNum is for creating Dx-->A_colNum=0, Dy-->A_colNum=1 or Dz-->A_colNum=2
// invA_colNum shows which column of the inverse A matrix will be calculated
void CreateDxyz_mat(double *A,            // 3x3 A matrix
                    double *I,            // 3x3 Identity matrix
                    int A_colNum,         // to calculate Dx → A_colNum=0,  Dy → A_colNum=1,  Dz → A_colNum=2
                    double *Dxyz,         // Dx or Dy or Dz matrix (Depends on the A_colNum value)
                    int invA_colNum){     // for col 1→ invA_colNum=0, col 2→ invA_colNum=1, col 3→ invA_colNum=2
  for (int i=0;i<N;i++){
        for(int j=0;j<N;j++){
              if (j==A_colNum)
                    Dxyz(i,j)=I(i,invA_colNum);
              else
                    Dxyz(i,j)=A(i,j);
              }
        }
}
```

5. Write a function to find the Inverse of a 3x3 matrix by using the Cramer's Rule function that has the following prototype. Use **at least one** for loop(20p) and **call** CreateDxyz_mat in this for loop(15p). (60p)

```
void CramersRuleForInverse(double *A) {
  ................
  ................
  for(int i=0;i<N;i++){
        ................
        ................
        CreateDxyz_mat(................);
        ................
        ................
        }
}
```

6. Run the code to find the inverse A matrix.
   - For Column 1 of inverse A→ show Dx, Dy and Dz matrix and their determinant values, respectively.
   - For Column 2 of inverse A→ show Dx, Dy and Dz matrix and their determinant values, respectively.
   - For Column 3 of inverse A→ show Dx, Dy and Dz matrix and their determinant values, respectively as shown in **Figure 1** .

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & 3 \\ 0 & -1 & -3 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -1.2 & 1.1 & 1.9 \\ 0.6 & -0.3 & -0.7 \\ -0.2 & 0.1 & -0.1 \end{bmatrix}$$

```
Write the elements of the A matrix:
1 3 -2 2 5 3 0 -1 -3

A matrix
1.000000        3.000000        -2.000000
2.000000        5.000000        3.000000
0.000000        -1.000000       -3.000000
Determinant of A matrix: 10.000000

-------------------------------------------------
Calculation of the column #1 of the inverse A
1.000000        3.000000        -2.000000
0.000000        5.000000        3.000000
0.000000        -1.000000       -3.000000
Determinant: -12.000000
1.000000        1.000000        -2.000000
2.000000        0.000000        3.000000
0.000000        0.000000        -3.000000
Determinant: 6.000000
1.000000        3.000000        1.000000
2.000000        5.000000        0.000000
0.000000        -1.000000       0.000000
Determinant: -2.000000

-------------------------------------------------
Calculation of the column #2 of the inverse A
0.000000        3.000000        -2.000000
1.000000        5.000000        3.000000
0.000000        -1.000000       -3.000000
Determinant: 11.000000
1.000000        0.000000        -2.000000
2.000000        1.000000        3.000000
0.000000        0.000000        -3.000000
Determinant: -3.000000
1.000000        3.000000        0.000000
2.000000        5.000000        1.000000
0.000000        -1.000000       0.000000
Determinant: 1.000000

-------------------------------------------------
Calculation of the column #3 of the inverse A
0.000000        3.000000        -2.000000
0.000000        5.000000        3.000000
1.000000        -1.000000       -3.000000
Determinant: 19.000000
1.000000        0.000000        -2.000000
2.000000        0.000000        3.000000
0.000000        1.000000        -3.000000
Determinant: -7.000000
1.000000        3.000000        0.000000
2.000000        5.000000        0.000000
0.000000        -1.000000       1.000000
Determinant: -1.000000

-------------------------------------------------
Inverse A matrix
-1.200000       1.100000        1.900000
0.600000        -0.300000       -0.700000
-0.200000       0.100000        -0.100000
```

Figure 1