ساختمان داده و الگوریتم ها (CE203)

جلسه هجدهم:
درهم سازی

سجاد شیرعلی شهرضا

پاییز 1400

شنبه، 13 آذر 1400

Based on Stanford CS161 Slides from Summer 2020 by Karey Shi

اطلاع رسانی

- بخش مرتبط کتاب برای این جلسه: 11

جدول درهم سازی

چه عملگرهایی برای ما مهم هستند؟

THE TASK

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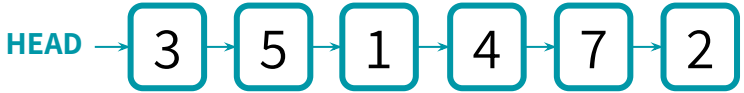
Sorted Arrays

O(n) INSERT/DELETE: first, find the relevant element (via SEARCH) and move a bunch of elements in the array

O(log n) SEARCH: use binary search to see if an element is in A

Linked Lists



O(1) INSERT: just insert the element at the head of the linked list

O(n) SEARCH/DELETE: since the list is not necessarily sorted, you need to scan the list (delete by manipulating pointers)

HASH TABLE MOTIVATION

OPERATION	SORTED ARRAY	UNSORTED LINKED LIST	HASH TABLES (HOPEFULLY)
SEARCH	O(log(n))	O(n)	O(1)
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What is a *naive* way to achieve these runtimes?

ATTEMPT 1: DIRECT ADDRESSING

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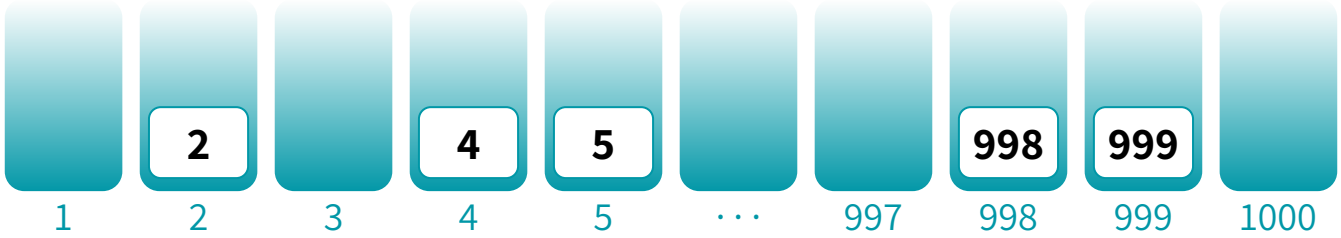

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Reasonable Attempt: *Direct Addressing!*

Not bad!

But what's the issue with this approach?

1 2 3 4 5 · · · 997 998 999 1000

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Suppose you're storing numbers from 1 - 10^{10}:

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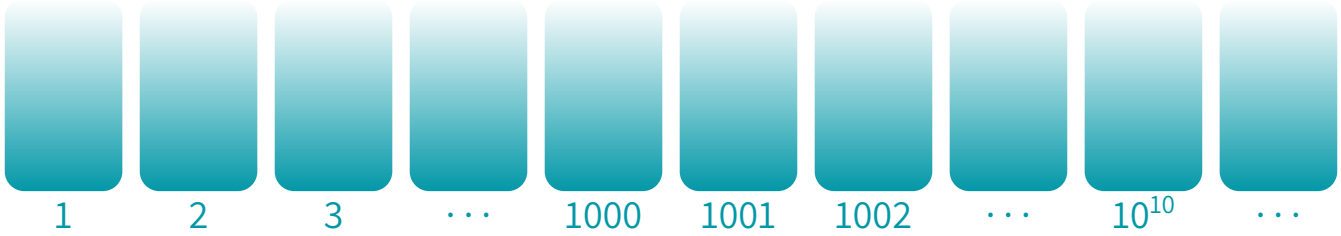

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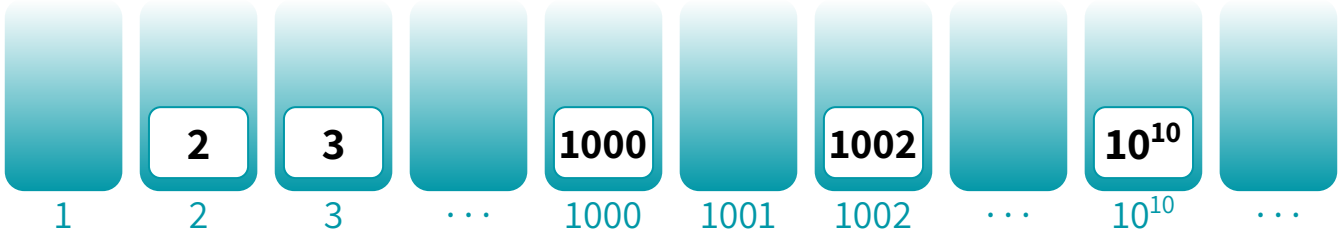

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But the space requirement is HUGE...

O(1) INSERT/DELETE/SEARCH: Just index into the bucket!

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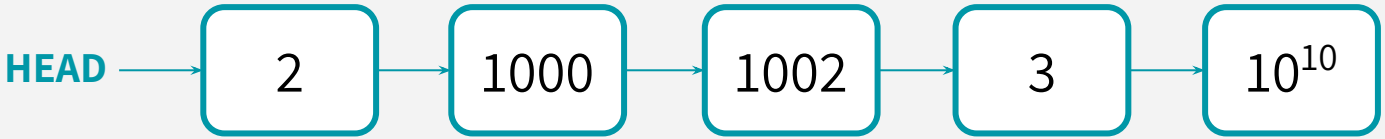
On the other extreme, we could save a lot of space by using linked lists!



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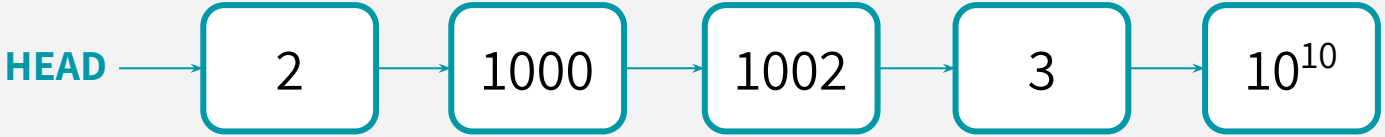

Good news: Space is now proportional to the number of objects you deal with

Bad news: Searching for an object is now going to scale with the number of inputs you deal with… not close to our desired O(1)!

The direct-addressing approach still has merit because of it's fast object search/access

HOW DO WE IMPROVE THIS?

We like the functionality of a direct-addressable array for constant time access (super fast INSERT/DELETE/SEARCH)

But reserving an bucket/array slot for each possible key leads to unreasonable space requirements… (kind of like CountingSort)

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Let's try bucketing **by the least-significant digit**…

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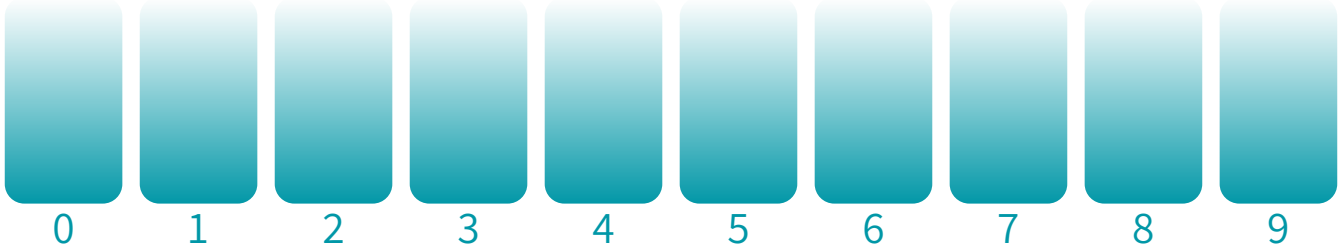
Bucket by last digit?



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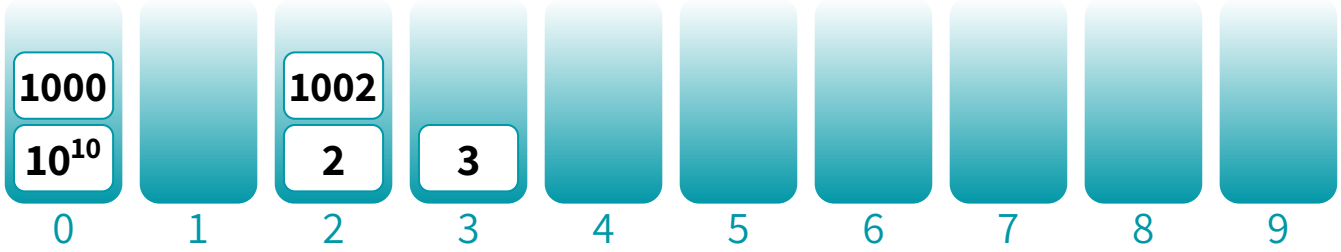

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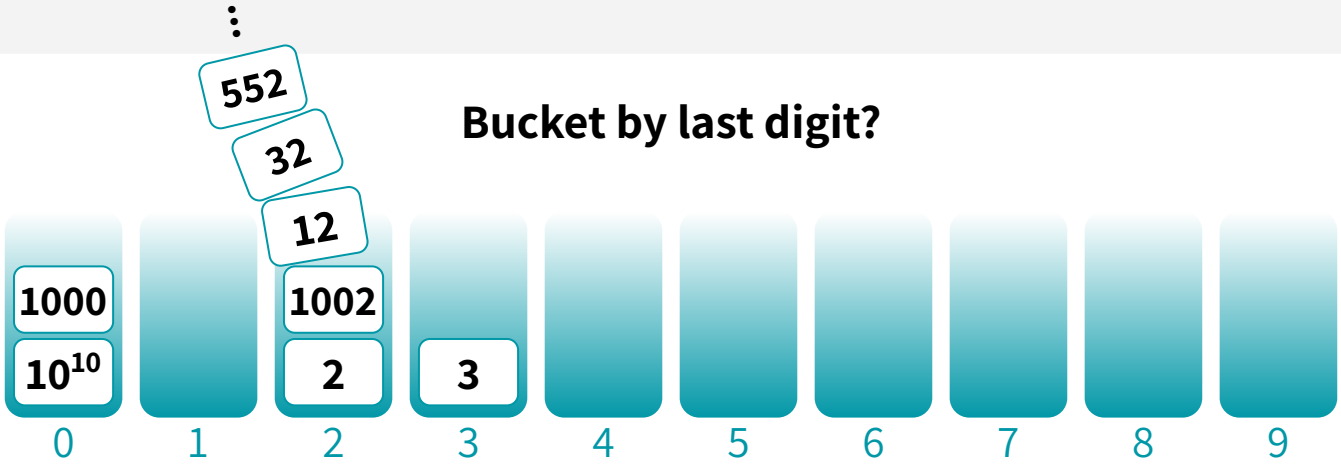
Under this scheme, a bad guy could give us inputs that yields quite ugly worst-case runtimes...



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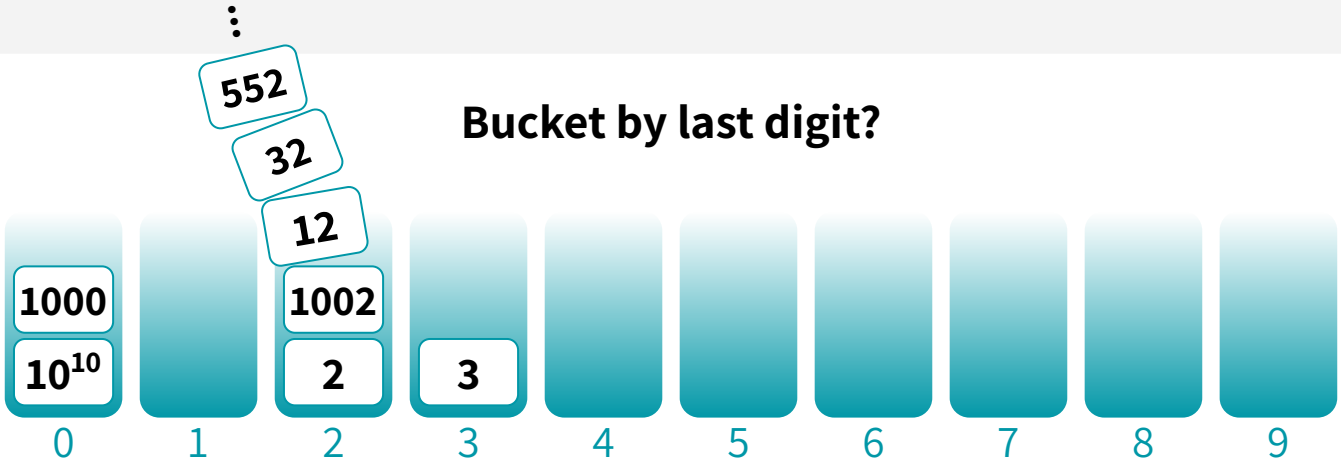

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Maybe another bucketing scheme?

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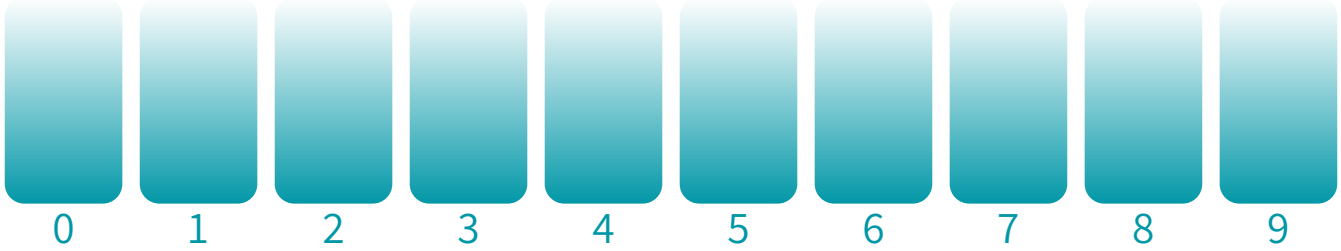
Suppose you're storing numbers from 1 - 10^{10}: **2** **3** **1000** **1002** **10^{10}**

Bucket by last digit of (number * 7) mod 3



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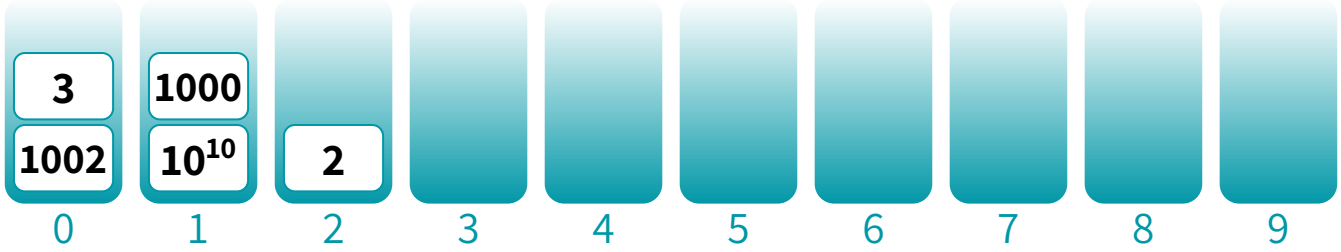

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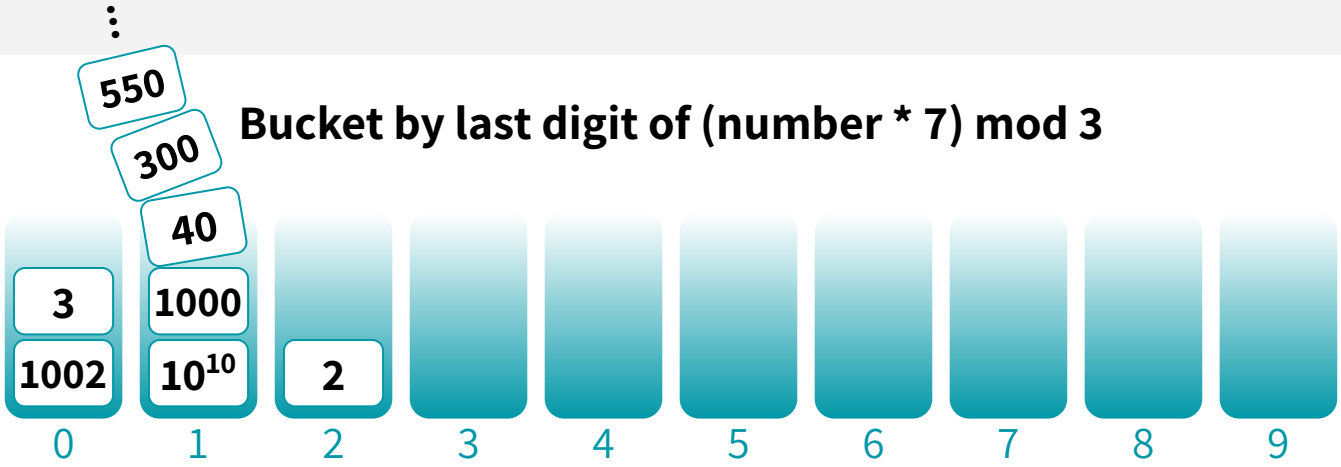

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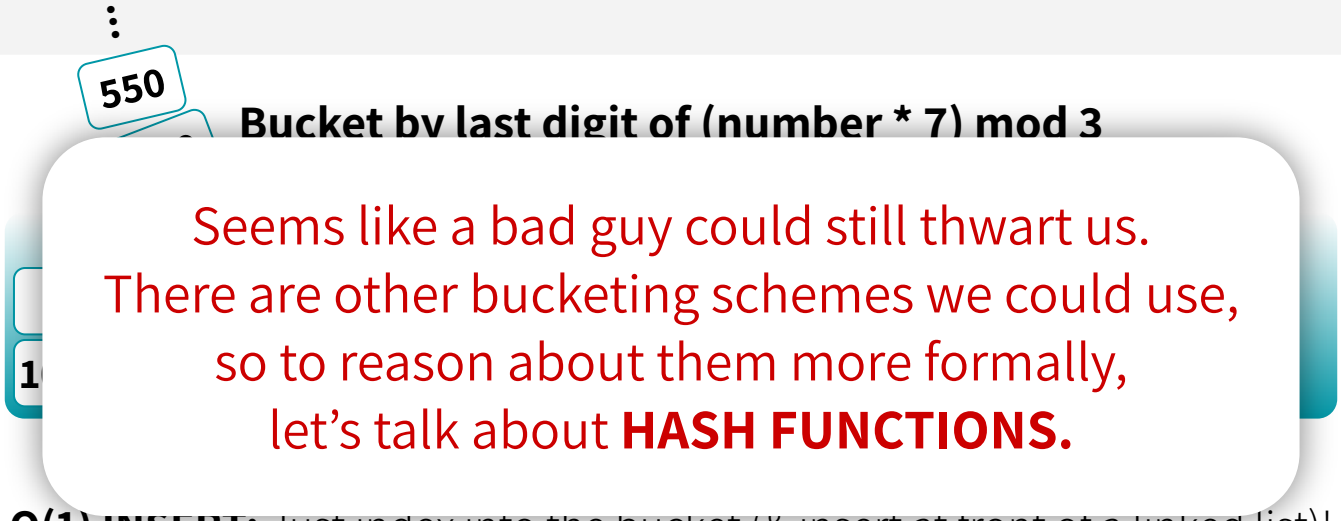

Seems like a bad guy could still thwart us.
There are other bucketing schemes we could use,
so to reason about them more formally,
let's talk about **HASH FUNCTIONS.**

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تابع درهم ساز

چه تابع درهم سازی خوب است؟

SOME TERMINOLOGY

There exists a universe **U** of keys, with size M.

Generally, M is *really big*. Examples:

- U = the set of all ASCII strings of length 20. M = 26^{20}
- U = the set of all IPv4 addresses. M = 2^{32}
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NOTE:

For this lecture, I'm assuming that the # of elements I receive is the same as the # of buckets (both are **n**). This doesn't have to be the case, but we usually aim for **#buckets = O(# elements that show up)** (otherwise, we're using "too much" space)

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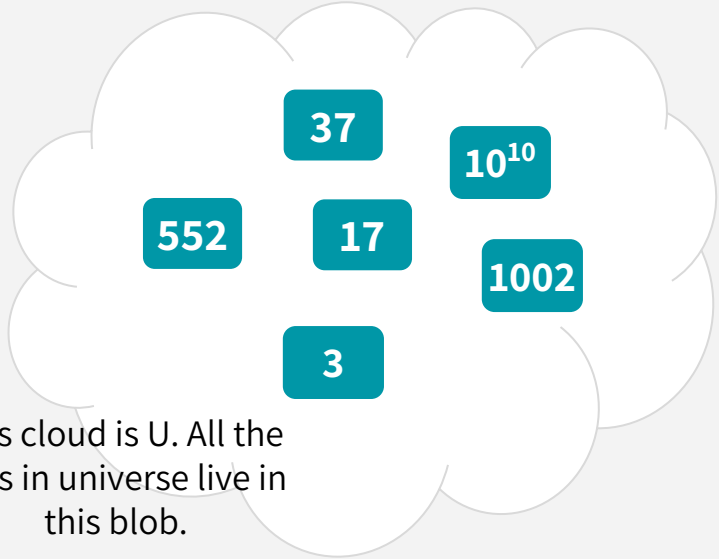
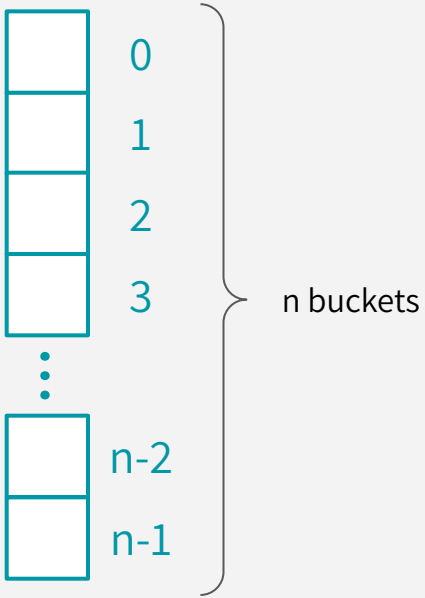


This cloud is U. All the keys in universe live in this blob.



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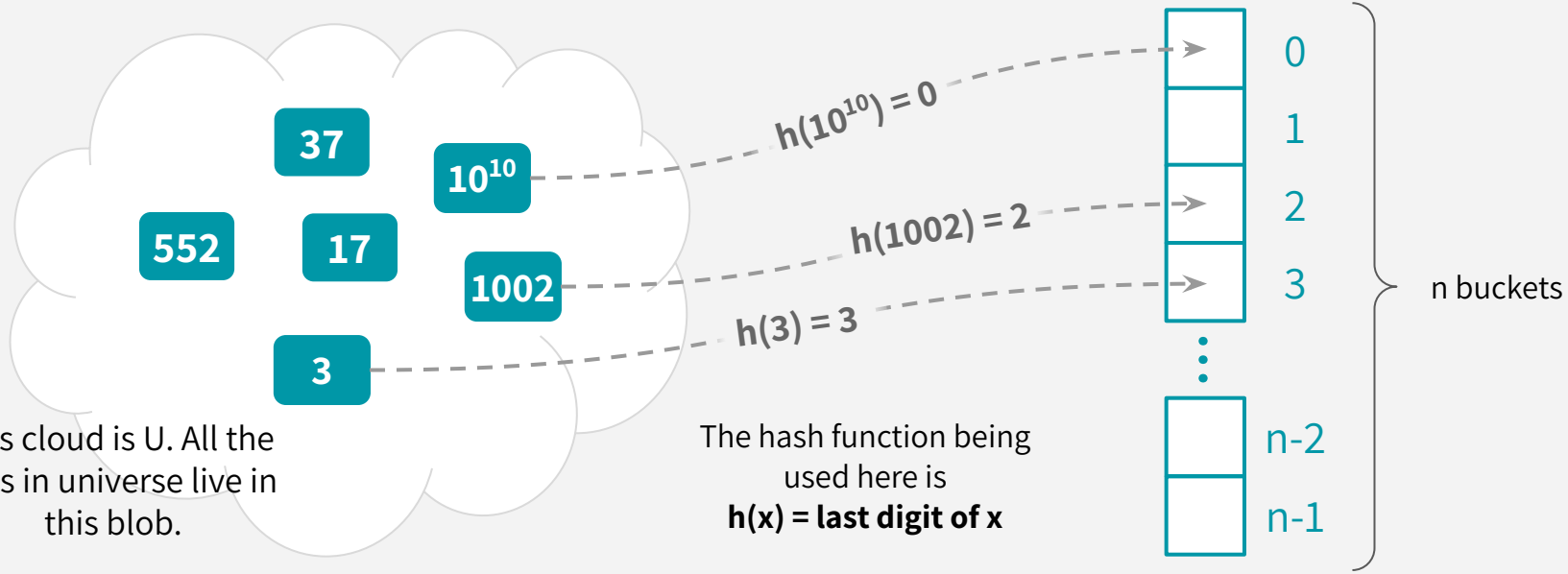

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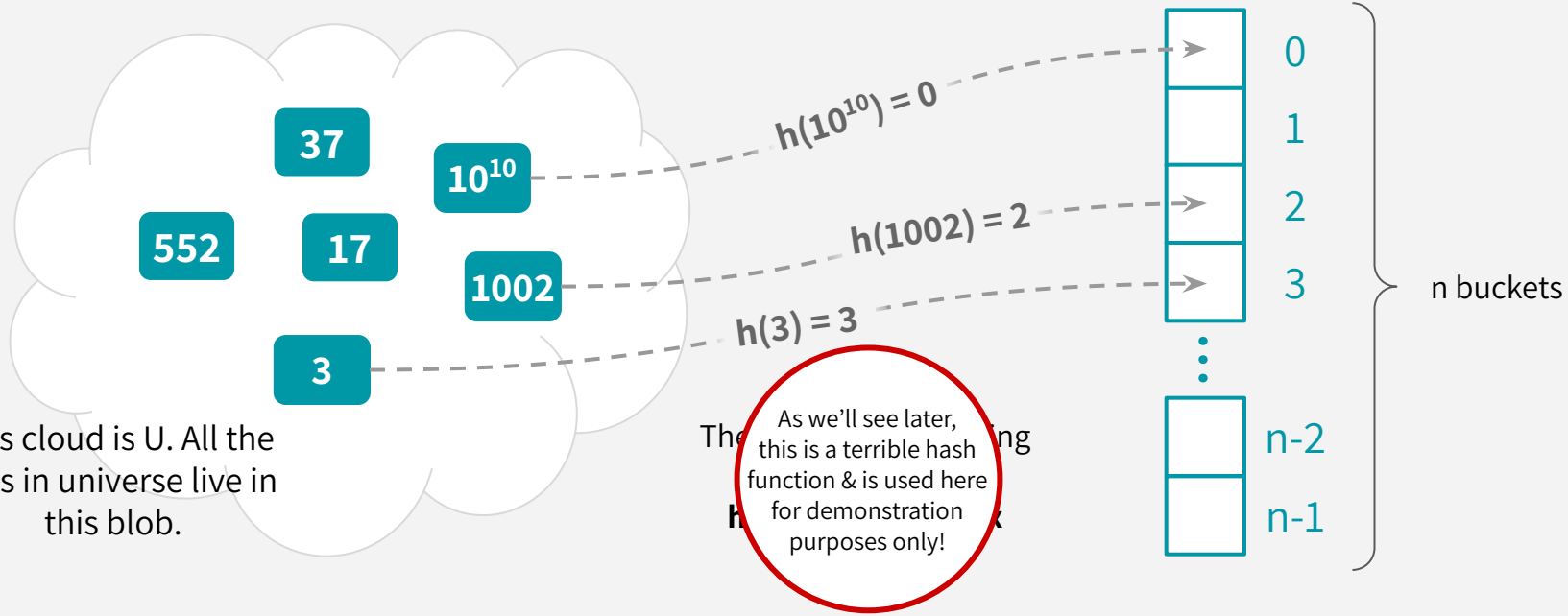

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A hash function tells you where to start looking for an object.

For example, if a particular hash function **h** has **h(1002) = 2** , then we say "1002 *hashes* to 2", and we go to bucket 2 to search for 1002, or insert 1002, or delete 1002.

n buckets

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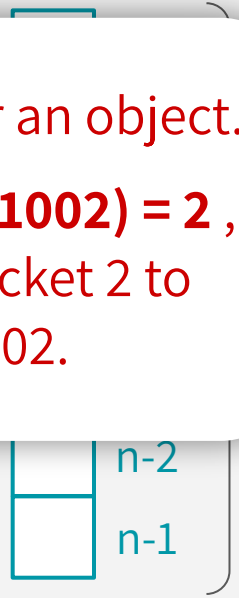


سوال؟

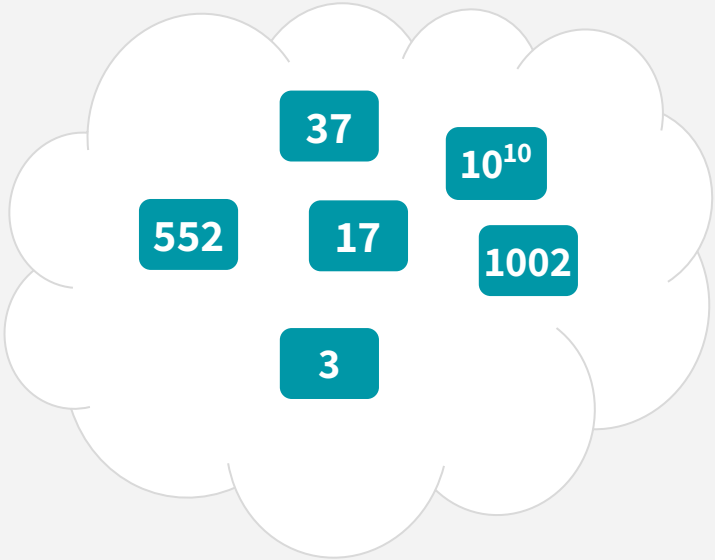
برخورد در درهم ساز!

چه مشکلی ممکن است پیش بیاید؟

COLLISIONS

Collisions (when a hash function would map 2 different keys to the same bucket) **are inevitable!**

This is because of the *Pigeonhole Principle*. Since the size of universe U > # of buckets, every hash function (no matter how clever), suffers from at least one collision.



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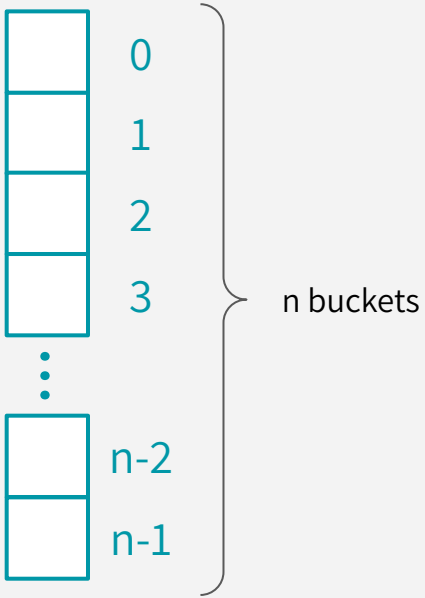

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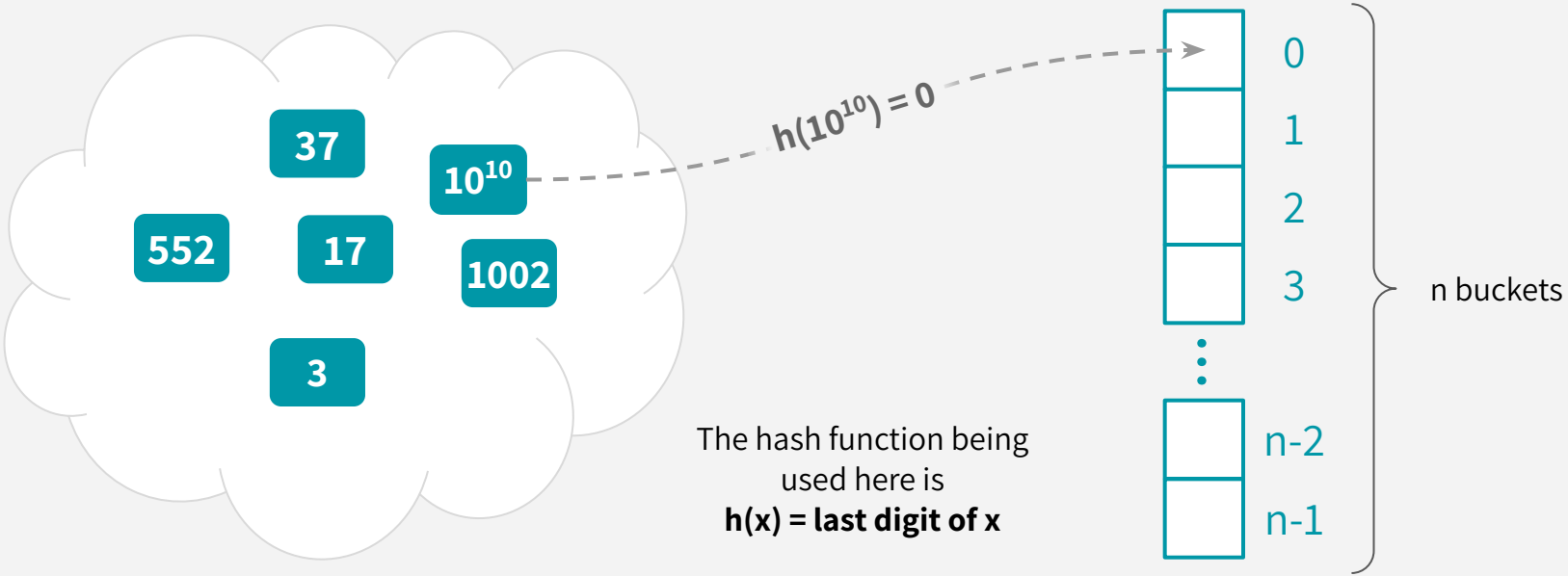

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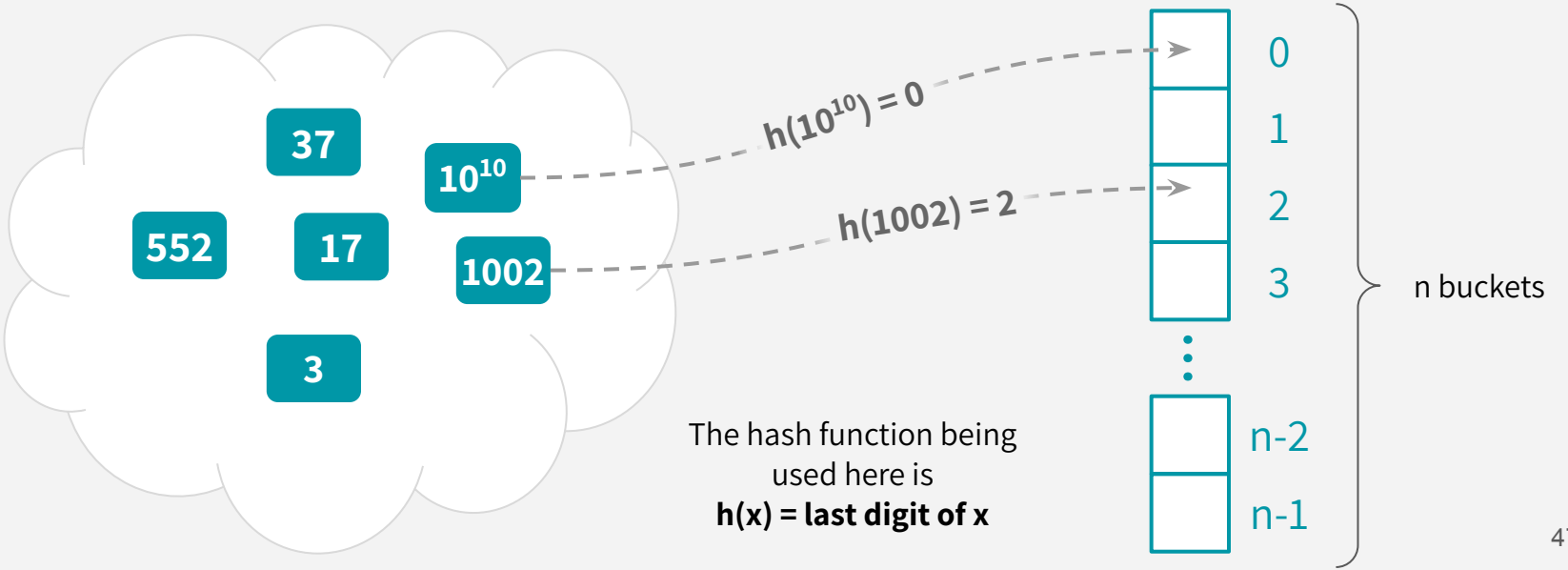

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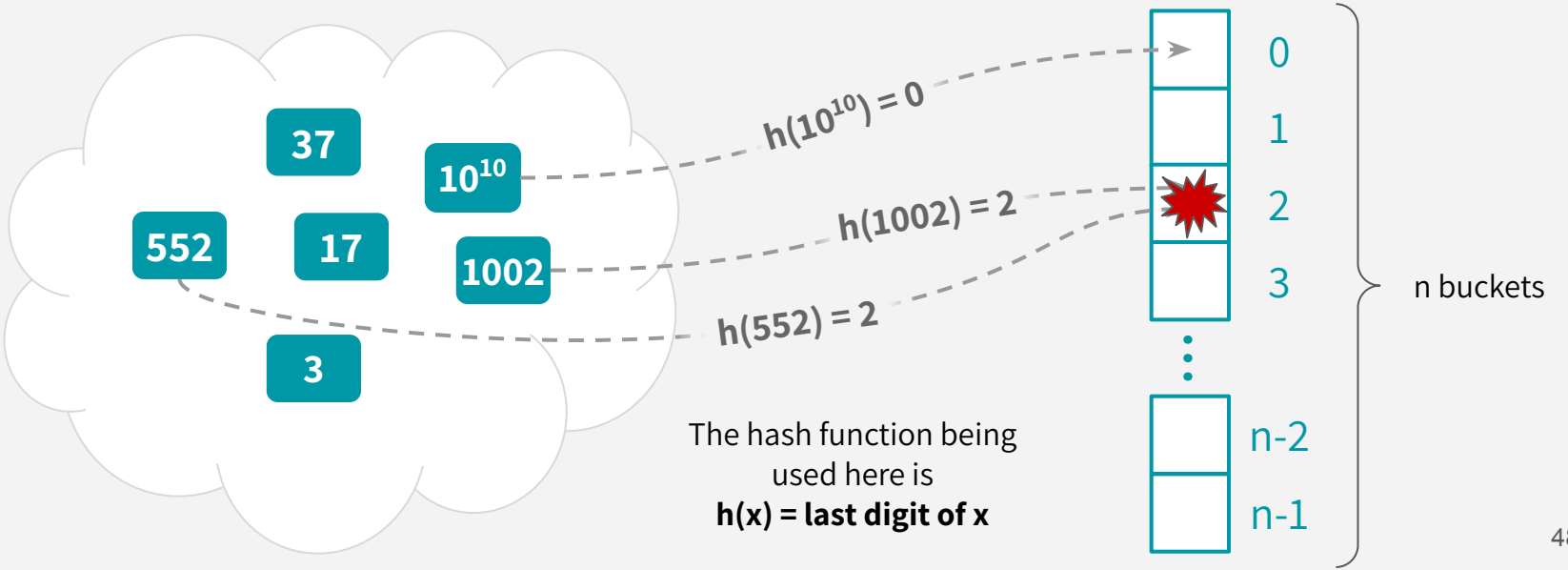

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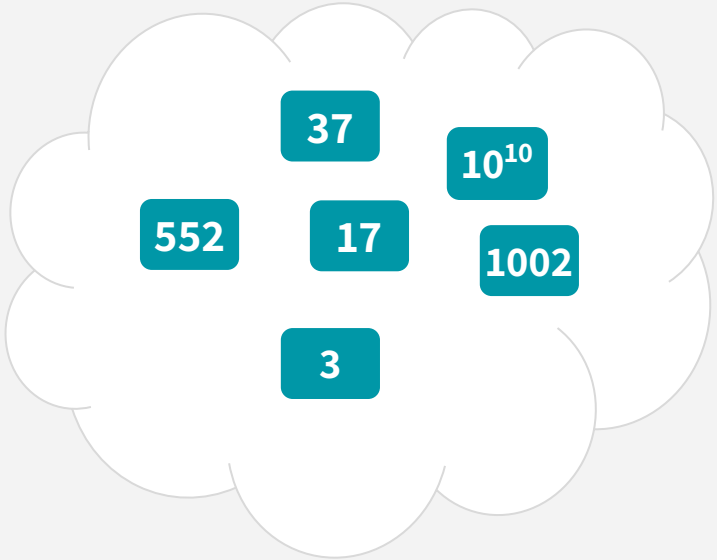

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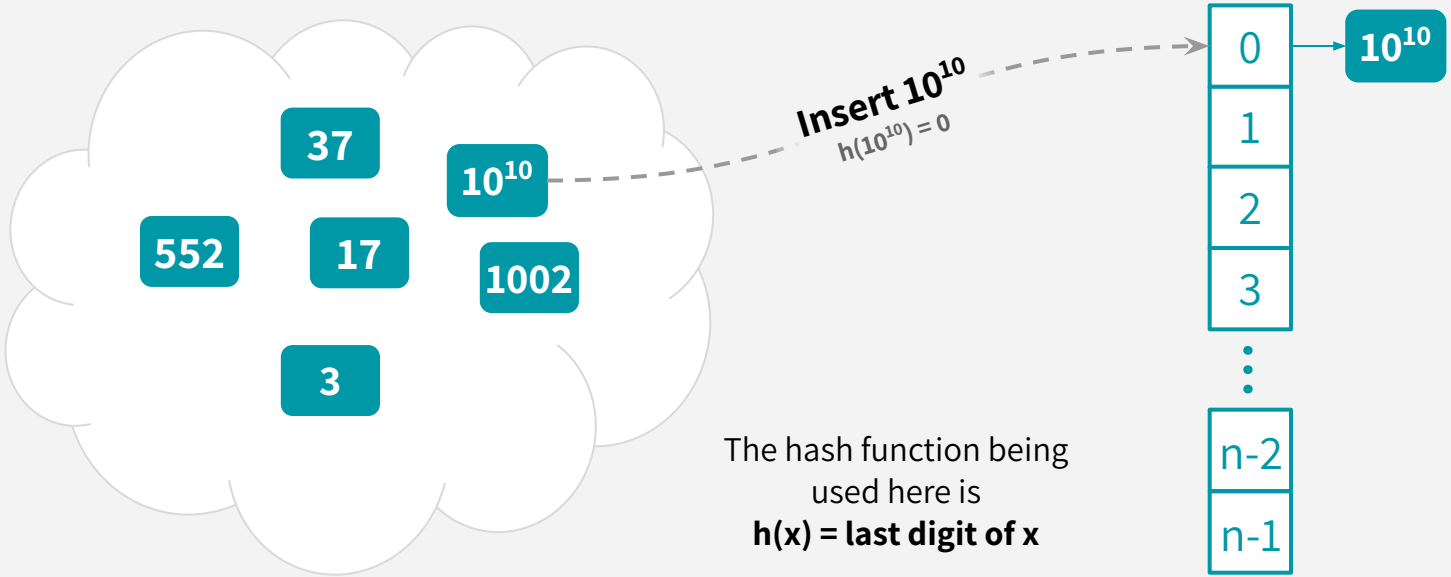

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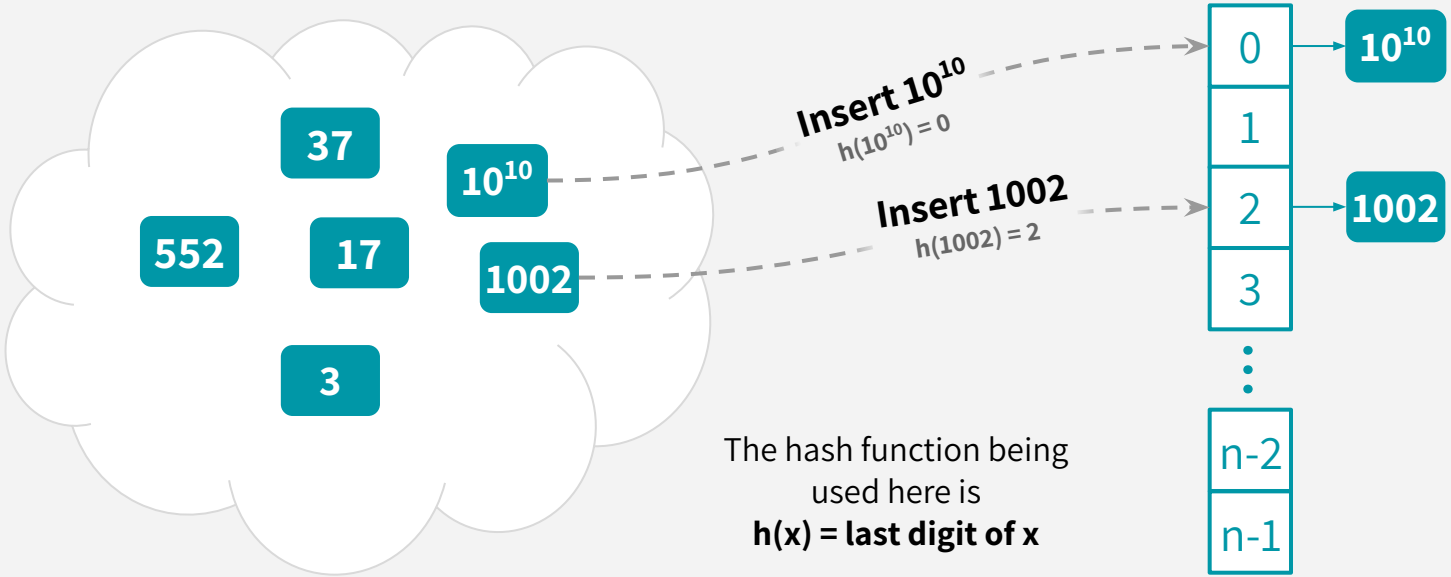

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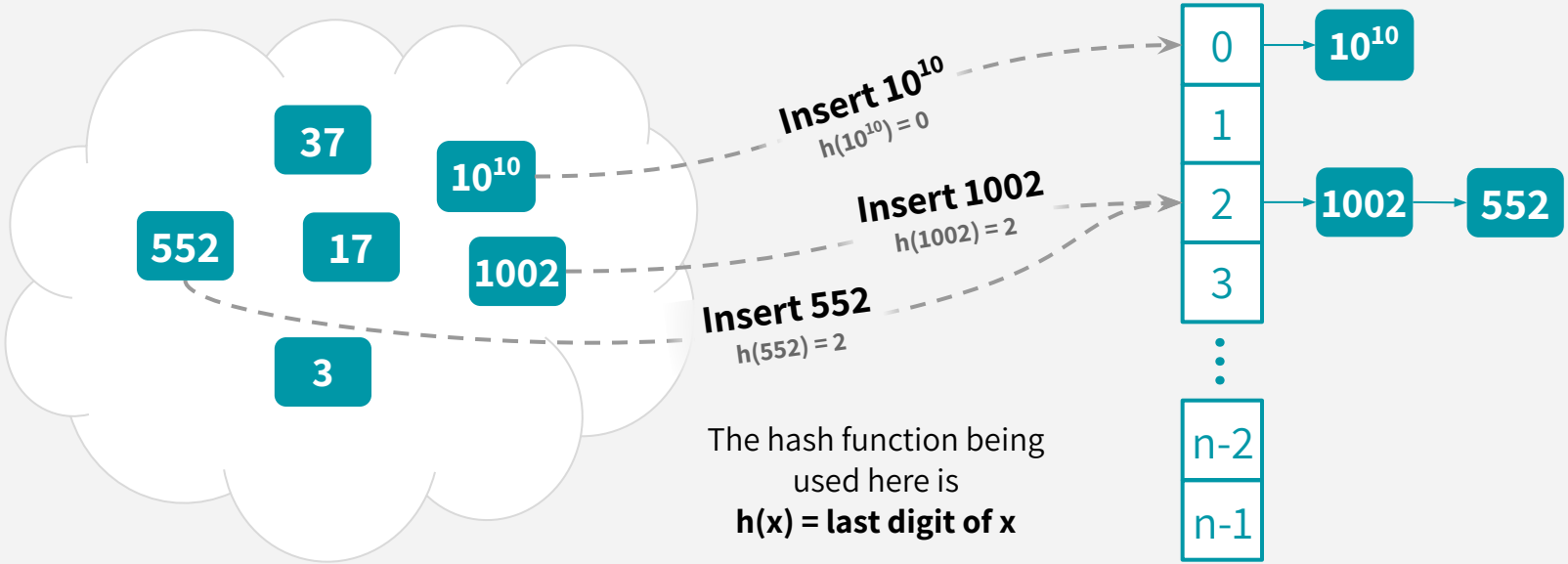

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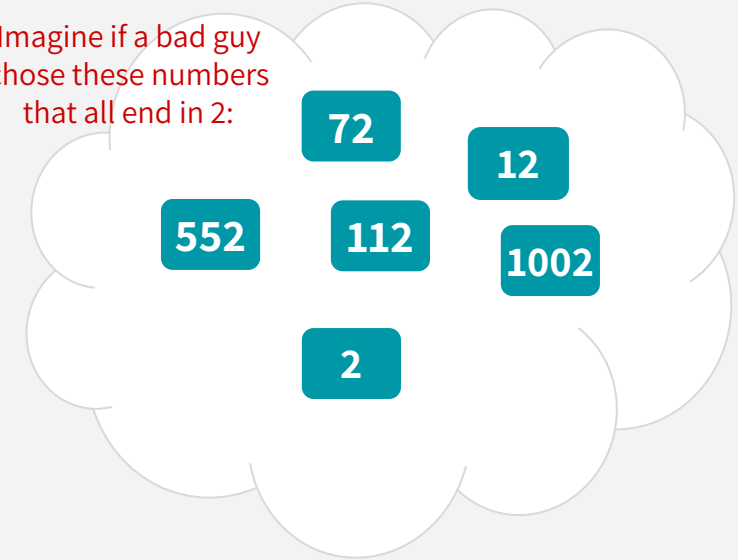

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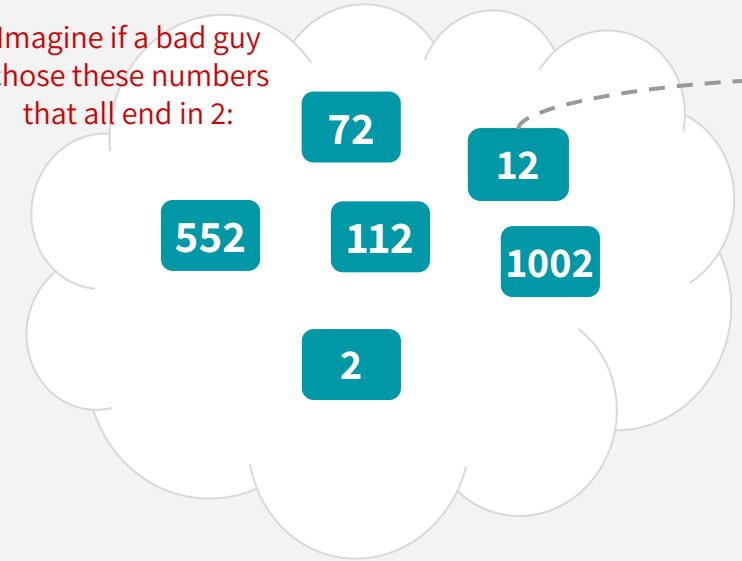
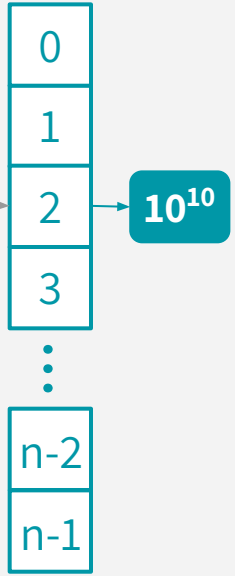


Insert 12

h(12) = 2



The hash function being used here is

h(x) = last digit of x

COLLISION RESOLUTION: CHAINING

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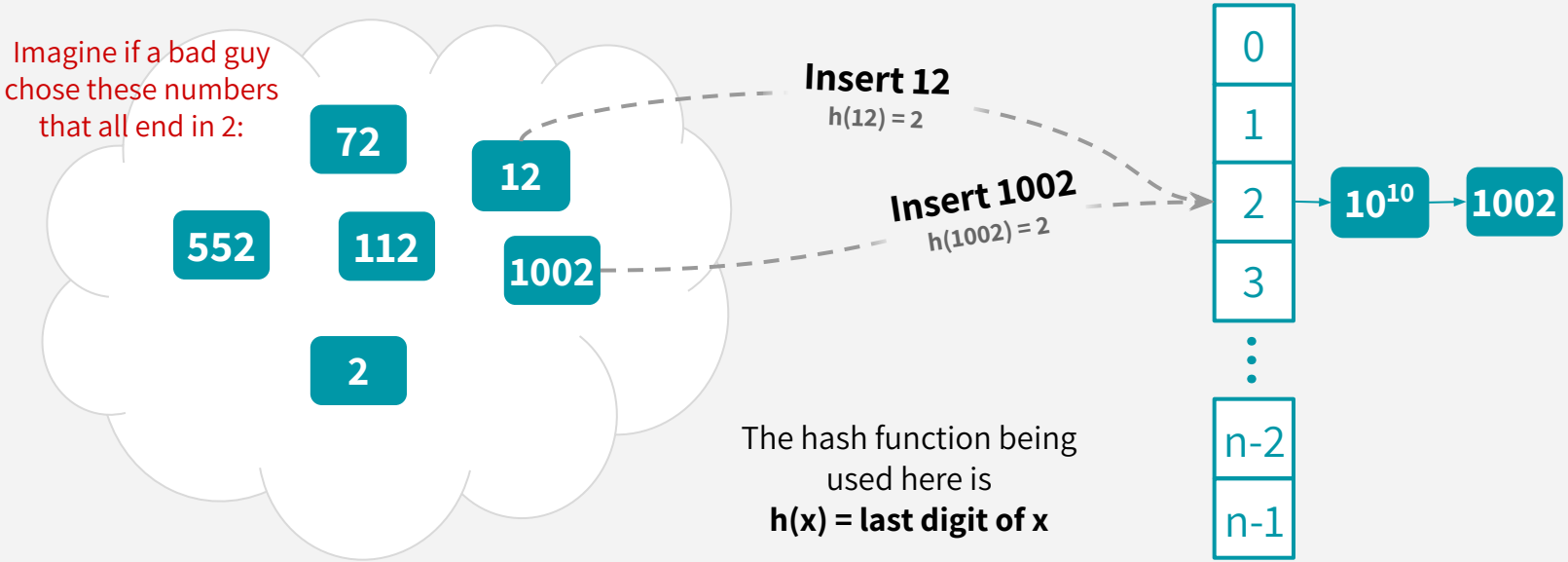

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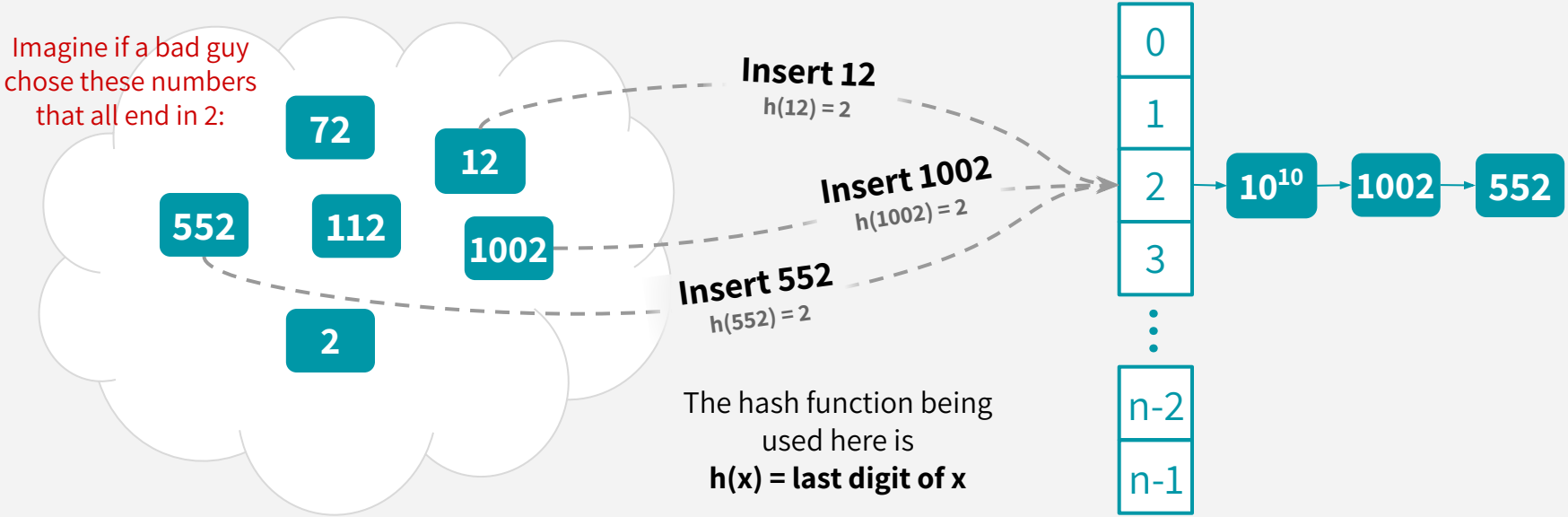

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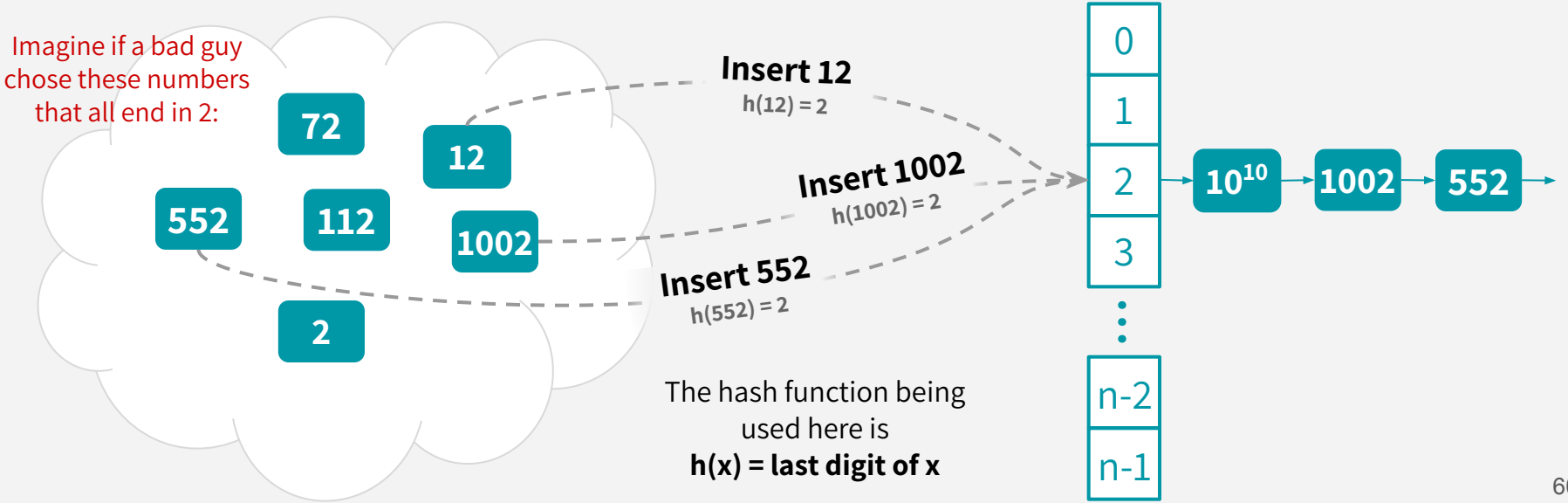

سوال؟

درهم سازی اعلی!

هدف جدول درهم سازی چیست؟

HASH TABLE GOALS

Remember worst-case analysis:

***OUR GOAL*:** Design a function **h: U → {1, …, n}** so that no matter what **n** items of **U** a bad guy chooses & the operations they choose to perform, the buckets will be balanced.

(Here, balanced means O(1) entries per bucket)

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No *deterministic* hash function can defeat worst-case input!

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In next class!

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