



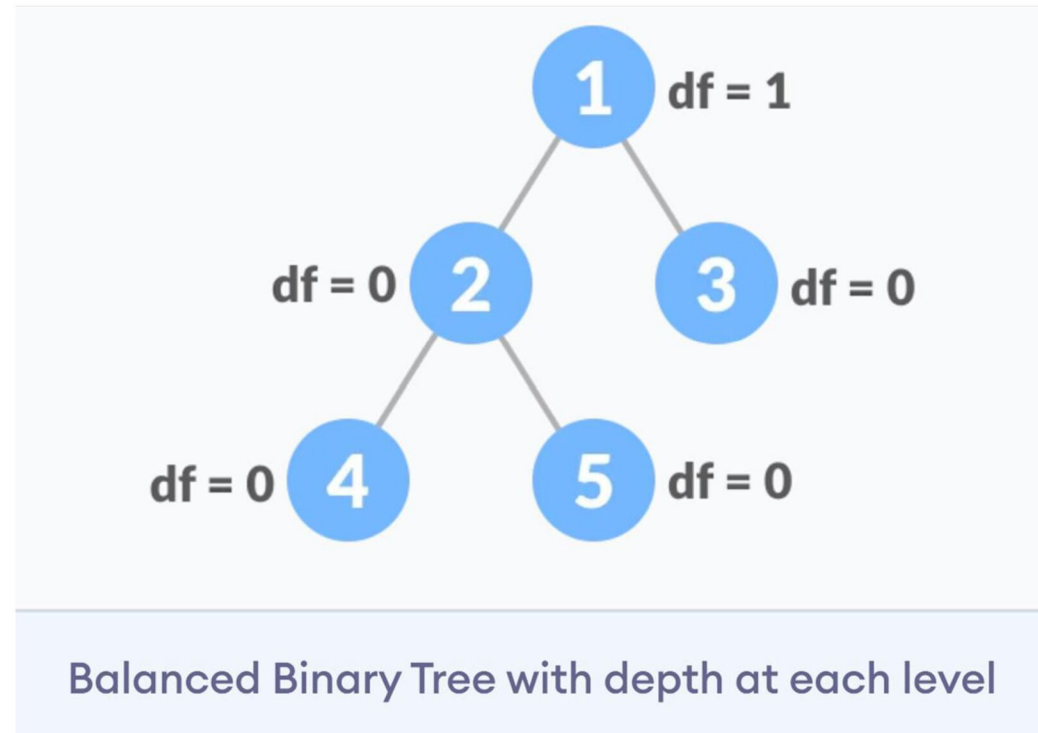
Data Structure & Algorithms

Balanced Binary Tree

Definition

- A **balanced binary tree** is a binary tree in which the difference between the height of the left subtree and right subtree of each node is not more than 1.
- A balanced binary tree is also known as a *height balanced tree*.
- In a balanced binary tree, the tree height is **$O(\log N)$** , where N is the number of nodes.

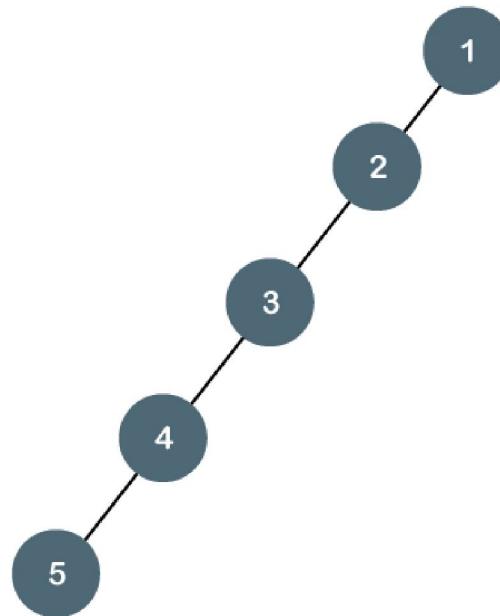
Example



$df = \text{depth} = |\text{height of left child} - \text{height of right child}|$

Example

An example of a *non-balanced* binary tree is a **degenerate tree** where every parent node has only one child node:



Why is it important that a binary tree be balanced?

- We have seen that the efficiency of many important operations on trees is related to the Height of the tree - for example searching, inserting, and deleting in a BST are all $O(\text{Height})$.
- In general, the relation between Height (H) and the number of nodes (N) in a tree can vary from $H = N$ (degenerate tree) to $H = \log(N)$ (balanced binary tree).
- **For efficiency's sake, we would like to guarantee that H equals to $O(\log N)$.**

Self-balancing binary search tree

- **Self-Balancing Binary Search Trees** are *height-balanced* binary search trees that automatically keeps height as small as possible when insertion and deletion operations are performed on tree. The height is typically maintained in order of $\log n$ so that all operations take $O(\log n)$ time on average.
- A **red-black** tree is a kind of self-balancing binary search tree.