

m231 - quiz #1

Prove or give a counter example:

① interior of ∂ is always in ∂ of interior.

FALSE. Here is a counterexample:

Let $X = \mathbb{Q}$. Then $\partial\mathbb{Q} = \mathbb{R}$ and $\text{int}(\partial\mathbb{Q}) = \mathbb{R}$.

Meanwhile $\text{int}\mathbb{Q} = \emptyset$ because $\forall q \in \mathbb{Q}, \forall r > 0$, there are irrational #'s in $B(r, q)$. Therefore $B(r, q) \not\subset \mathbb{Q}$ so that no q is an interior pt of \mathbb{Q} .

Thus $\partial(\text{int}\mathbb{Q}) = \emptyset$. $\longrightarrow \mathbb{R} \not\subset \emptyset$.

② ∂ of interior is always in interior of ∂

FALSE. Here is a counterexample:

Let $X = (a, b)$. Then $\text{int}X = X$ and $\partial(\text{int}X) = \{a, b\}$.

Meanwhile $\partial X = \{a, b\}$ and $\text{int}(\partial X) = \emptyset$.

Obviously, $\{a, b\} \not\subset \emptyset$.