

$\chi_S$  is integrable over  $S \subset \mathbb{R}^n$ .

$\iff$  Given  $\varepsilon > 0$ , there is a partition  $P$  over a rectangle  $R$  containing  $S$  such that upper sum minus lower sum  $< \varepsilon$ .

$$\iff \sum_{R_{ij} \cap S \neq \emptyset} - \sum_{R_{ij} \subset S} < \varepsilon$$

$$\iff \sum_{R_{ij} \cap \partial S \neq \emptyset} < \varepsilon$$

$\iff$  Given  $\varepsilon > 0$ ,  $S$  can be covered by finitely many boxes of total volume less than  $\varepsilon$ .

$\stackrel{\text{defn}}{\iff} \partial S$  has zero content.

$\stackrel{\text{defn}}{\iff} S$  is Jordan measurable.

The circled implication is your exercise to work out.