

# AI Assisted Analysis of Fermat's Last Theorem

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## AI Assistance

If the analyses generated by *deepai* (<https://deepai.org/chat/mathematics>) are correct, they confirm Fermat's conjecture.

## Introduction

Pierre de Fermat (1601-1665) was a lawyer, a member of the Parliament in Toulouse, and a mathematician. His conjecture is easy to explain: he said that there are no solutions to the following equation when  $X$ ,  $Y$ , and  $Z$  are positive integers, and  $n$  is a positive integer greater than two:

$$X^n + Y^n = Z^n$$

Fermat created his conjecture in 1637 when he wrote in a copy of *Arithmetica* that he had a proof that was too large to fit in the margin. In 1995, professors Taylor<sup>i</sup> and Wiles<sup>ii</sup> proved the conjecture.

## Limiting the Scope of the Problem to Prime Exponents

Professor Van der Poorten<sup>iii</sup> shows that it is not necessary to prove Fermat's conjecture for any positive exponent  $n$ ; rather, it is sufficient to prove it when the exponent  $p$  is prime and the greatest common divisor of  $X$  and  $Y$  equals 1:

$$X^p + Y^p = Z^p \tag{1}$$

## Variables

All variables represent integers.

## Change of Variables

These new variables represent the differences among the original variables:

$$i = Z - Y$$

$$k = Z - X$$

$$r = X - i$$

$$h = i + k$$

These definitions imply:

$$X = r + i$$

$$Y = r + k$$

$$Z = r + i + k$$

If  $p > 2$ , then (1) can be written as:

$$(X + Y) \sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = Z^p$$

This implies that:

$$(2r + i + k) \sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = (r + i + k)^p$$

$$\sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = \frac{(r + i + k)^p}{(2r + i + k)}$$

$$\sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = \frac{(r + h)^p}{(2r + h)}$$

All the terms on the left-hand of this equation are integers. This implies that the fraction on the right hand-side is an integer. Using this, the following are question to, and responses from, *deepai*:

<u>Question</u>	<u>Response from <i>deepai</i></u>
Does $(2r + h)$ divide $r^p$	Yes
Does $(2r + h)$ divide $h^p$	Yes
What is greatest common divisor of $(r^p$ and $h^p)$	1

## Conclusion

Since  $X + Y = (2r + h)$  divides both  $r^p$  and  $h^p$ , it also divides their greatest common divisor. This leads to a contradiction where the sum of two positive integers divide 1; therefore, Fermat's conjecture is correct.

## *deepai*: Questions and Responses

### Does $(2r + h)$ divide $r^p$

PROMPT: given integers p,r,h where p is prime and greater than 2, and  $\llbracket (r+h) \rrbracket^{p/(2r+h)}$  is

an integer; does  $(2r + h)$  divide  $r^p$

*deepai short-form response*

Summary:

- The key step was expressing  $r + h$  modulo  $d = 2r + h$ , which revealed that  $d$  divides  $r^p$ .
- Therefore, if  $(r + h)^p / (2r + h)$  is an integer, then  $2r + h$  divides  $r^p$ .

### Does $(2r + h)$ divide $h^p$

PROMPT: given integers  $p, r, h$  where  $p$  is prime and greater than 2, and  $\lfloor (r+h) \rfloor^p / (2r+h)$  is an integer; does  $(2r + h)$  divide  $h^p$

*deepai short-form response*

Summary:

- The key step was expressing  $r + h$  in terms of  $d = 2r + h$ .
- Using the binomial theorem, we showed that the divisibility of  $\frac{(r+h)^p}{2r+h}$  implies  $2r + h$  divides  $h^p$ .

### What is greatest common divisor of $(r^p)$ and $h^p$

PROMPT: given integers  $p, r, h$ , where  $(h^p)/(2r+h)$  is an integer, and  $(r^p)/(2r+h)$  is an integer, and  $p$  is prime and greater than 2, what is the greatest common divisor of  $r^p$  and  $h^p$

*deepai short-form response*

Final answer:

$$\gcd(r^p, h^p) = 1$$

<sup>i</sup> Richard Taylor and Andrew Wiles, "Ring-theoretic properties of certain Hecke algebras", Annals of Mathematics, Vol. 141 (1995) pp 553-572.

<sup>ii</sup> Andrew Wiles, "Modular elliptic curves and Fermat's Last Theorem", Annals of Mathematics, Vol. 141 (1995) pp 443-551

<sup>iii</sup> Alf van der Poorten, "Notes on Fermat's Last Theorem", Canadian Mathematical Society Series of Monographs and Advanced Texts", Wiley Interscience, 1996, Page 8, note 1.9