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PROMPT:

given integers p, r, i, k , where p is prime and greater than 2, none of $(i+k), r, i, k$ equal to zero, using polynomial division with numerator equal to $(r+i)^p + (r+k)^p$, with denominator equal to $(r+i+k)$, what is the remainder

RESPONSE:

We view the given “numerator”

$$f(r) = (r + i)^p + (r + k)^p$$

as a polynomial in r , and we are dividing by the linear factor

$$d(r) = r + (i + k).$$

By the Remainder-Theorem, the remainder of $f(r)$ upon division by $r - a$ is $f(a)$. Here our divisor is $r - [-(i+k)]$, so we plug in

$$r = -(i + k).$$

Compute:

$$\begin{aligned} f(-(i + k)) &= (-(i+k) + i)^p + (-(i+k) + k)^p \\ &= (-k)^p + (-i)^p. \end{aligned}$$

Since $p > 2$ is an odd prime, $(-k)^p = -k^p$ and $(-i)^p = -i^p$. Hence

$$\text{remainder} = -k^p - i^p = -(i^p + k^p).$$

Answer: The remainder is $-(i^p + k^p)$.