

Let's carefully analyze the problem and see what we can determine.

Given:

- p is a prime number **greater than 2**.
 - r, h are integers.
 - The expression $\frac{(r+h)^p}{2r+h}$ is an **integer**.
- (In other words, $2r+h$ divides $(r+h)^p$.)

Question:

Does $2r+h$ **divide** r^p ?

Step 1: Understand what is given

- The key is that $\frac{(r+h)^p}{2r+h}$ is an integer.
- So, $2r+h \mid (r+h)^p$.

Step 2: Consider the divisibility

Since $2r+h$ divides $(r+h)^p$, and $r+h$ is involved, perhaps it's helpful to relate $2r+h$ to $r+h$.

Express $2r+h$ in terms of $r+h$:

$[$

$$2r+h = 2r+h$$

\]

But note that:

\[

$$r + h = (r) + h$$

\]

Step 3: Find a relation between $(2r + h)$ and $(r+h)$

Suppose we look at the difference:

\[

$$(2r + h) - 2(r + h) = 2r + h - 2r - 2h = -h$$

\]

So,

\[

$$(2r + h) = 2(r + h) - h$$

\]

But perhaps more directly, think about the divisibility:

\[

$$2r + h \mid (r+h)^p$$

\]

Step 4: Test with some concrete values

To understand whether $(2r + h)$ divides $(-r^p)$, try specific examples satisfying the initial condition.

Example 1:

Let $(r=1, h=1)$:

$$- (2r + h = 2(1) + 1 = 3)$$

$$- (r+h = 1+1=2)$$

Check:

$$\left[\frac{(r+h)^p}{2r+h} = \frac{2^p}{3} \right]$$

For this to be an integer, $3 \mid 2^p$. But 2^p is a power of 2, and 3 does not divide any power of 2, unless $p=0$, which isn't prime.

So, **not** valid for this example.

Example 2:

Let $(r=2, h=2)$:

$$- \ (\ 2r + h = 4 + 2 = 6 \)$$

$$- \ (\ r+h=4 \)$$

Check:

$$\left[\frac{4^p}{6} \right]$$

For this to be integer, $(\ 6 \mid 4^p \)$. But 4's prime factorization is $(\ 2^2 \)$, so $(\ 4^p=2^{\{2p\}} \)$. For $(\ 6=2 \times 3 \)$ to divide $(\ 2^{\{2p\}} \)$, $(\ 3 \)$ must divide $(\ 2^{\{2p\}} \)$, which it does not. So, again, not valid unless $(\ p=0 \)$.

Example 3:

Choose $(\ r=0 \)$:

$$- \ (\ r=0 \), \ (\ h=1 \)$$

Then:

$$- \ (\ 2r+h = 0+1=1 \)$$

$$- \ (\ r+h=1 \)$$

Check:

$$\left[\frac{1^p}{1} \right] = 1$$

which is integer, satisfying the condition.

Now, check whether $(2r+h+1)$ divides $(-r^p=0)$:

$$\begin{aligned} & \lfloor \\ & 1 \mid 0 \\ & \rfloor \end{aligned}$$

Yes, 1 divides 0.

Step 5: General reasoning

From the above, it seems that the divisibility condition depends heavily on the specific values of (r) and (h) . Let's analyze more systematically.

Key observations:

- Since $(r+h)^p$ is divisible by $(2r+h)$, and (p) is prime greater than 2, perhaps **Fermat's Little Theorem** or properties of prime exponents can help.

Step 6: Use properties of primes and divisibility

Recall **Fermat's Little Theorem**:

$$\begin{aligned} & \\ a^p &\equiv a \pmod{p} \\ & \end{aligned}$$

for prime (p) and integer (a) not divisible by (p) .

Suppose $(2r + h = d)$.

Given that $(d \mid (r+h)^p)$, then:

$$\begin{aligned} & \\ (r+h)^p &\equiv 0 \pmod{d} \\ & \end{aligned}$$

meaning (d) divides $((r+h)^p)$.

Step 7: Express (r) in terms of (d)

Suppose $(r \equiv x \pmod{d})$, and $(h \equiv y \pmod{d})$.

Then,

$$\begin{aligned} & \\ 2r + h &\equiv 0 \pmod{d} \Rightarrow 2x + y \equiv 0 \pmod{d} \\ & \end{aligned}$$

and

$$\begin{aligned} & \left[\right. \\ & r+h \equiv x+y \pmod{d} \\ & \left. \right] \end{aligned}$$

Since $(d \mid (r+h)^p)$, then

$$\begin{aligned} & \left[\right. \\ & (x+y)^p \equiv 0 \pmod{d} \\ & \left. \right] \end{aligned}$$

which implies

$$\begin{aligned} & \left[\right. \\ & x+y \equiv 0 \pmod{d} \\ & \left. \right] \end{aligned}$$

because (p) is prime and (d) divides $((x+y)^p)$, so $(d \mid x+y)$.

Now, from earlier:

$$\begin{aligned} & \left[\right. \\ & 2x+y \equiv 0 \pmod{d} \\ & \left. \right] \end{aligned}$$

and

$$\begin{aligned} & \left[\right. \\ & x+y \equiv 0 \pmod{d} \\ & \left. \right] \end{aligned}$$

Subtract these two:

$$\begin{aligned} & \backslash[\\ (2x + y) - (x + y) &= x \equiv 0 \pmod{d} \\ & \backslash] \end{aligned}$$

Thus, $\backslash(x \equiv 0 \pmod{d} \backslash)$.

Since $\backslash(x \equiv 0 \pmod{d} \backslash)$, then:

$$\begin{aligned} & \backslash[\\ x + y &\equiv y \equiv 0 \pmod{d} \\ & \backslash] \end{aligned}$$

and

$$\begin{aligned} & \backslash[\\ 2x + y &\equiv y \equiv 0 \pmod{d} \\ & \backslash] \end{aligned}$$

So, $\backslash(y \equiv 0 \pmod{d} \backslash)$.

****Conclusion:****

- Both $\backslash(x \equiv 0 \pmod{d} \backslash)$ and $\backslash(y \equiv 0 \pmod{d} \backslash)$.

In particular:

$$\begin{aligned} & \left[\begin{aligned} r &\equiv x \pmod{d} \end{aligned} \right] \\ & \left[\begin{aligned} h &\equiv y \pmod{d} \end{aligned} \right] \end{aligned}$$

which means:

$$\begin{aligned} & \left[\begin{aligned} r &\equiv 0 \pmod{2r+h} \end{aligned} \right] \\ & \left[\begin{aligned} h &\equiv 0 \pmod{2r+h} \end{aligned} \right] \end{aligned}$$

But since $(r \equiv 0 \pmod{d})$, then:

$$\begin{aligned} & \left[\begin{aligned} r &= k d \end{aligned} \right] \end{aligned}$$

for some integer (k) . Similarly for (h) .

Now, recall that the question is whether $(2r + h)$ divides $(-r^p)$.

But:

$$\begin{aligned} & \backslash[\\ & -r^p = -(kd)^p = -k^p d^p \\ & \backslash] \end{aligned}$$

and since $(d = 2r + h)$, and $(r \equiv 0 \pmod{d})$, then $(r = md)$, with (m) integer.

Thus,

$$\begin{aligned} & \backslash[\\ & r^p = (md)^p = m^p d^p \\ & \backslash] \end{aligned}$$

and:

$$\begin{aligned} & \backslash[\\ & -r^p = -m^p d^p \\ & \backslash] \end{aligned}$$

which is divisible by (d) because $(d \mid d^p)$.

Therefore, $*(2r + h = d)$ divides $(-r^p)$.

Final answer:

Yes, given the initial conditions, $*(2r + h)$ divides $(-r^p)$.

Summary:

- The key step was recognizing that $(r+h)^p$ divisible by $(2r+h)$ implies certain congruences.
- These lead to the conclusion that r and h are divisible by $(2r+h)$.
- Therefore, $(2r+h)$ divides r^p (and hence $-r^p$), fulfilling the question.

If you'd like, I can help you explore specific numerical examples or further properties related to this problem!