Great question! Let's carefully analyze the problem and see what we can deduce about the greatest common divisor (GCD) of r^p and h^p.

Given:

- p, r, h are integers.
- p is **prime** and **greater than 2**.
- The fractions $\frac{h^p}{2r + h}$ and $\frac{r^p}{2r + h}$ are **integers**.

Goal:

Find the **greatest common divisor** $\gcd(r^p, h^p)$.

Step 1: Understand the divisibility conditions

Since both $\frac{h^p}{2r + h}$ and $\frac{r^p}{2r + h}$ are integers, this implies:

$$
2r + h \mid h^p \quad \text{and} \quad 2r + h \mid r^p
$$

In other words, **$2r + h$ divides both h^p and r^p**.

Step 2: Use properties of divisibility and GCD

Because $2r + h$ divides both h^p and r^p, it **divides any linear combination** of these two.

Step 3: Explore the divisibility of $\gcd(r^p, h^p)$

Recall a key property:

$$
\gcd(r^p, h^p) = (\gcd(r, h))^p
$$

- The GCD of the powers is the power of the GCD of the bases.

Our goal reduces to understanding $\gcd(r, h)$.

Step 4: How does $2r + h$ relate to r and h?

Since $2r + h$ divides both h^p and r^p, it must **also divide any linear combination**:

$$
A r + B h
$$

for integers A, B. Let's see if we can relate $2r + h$ to r and h.

Step 5: Consider the divisibility of r and h

Suppose $d = \gcd(r, h)$. Then:

$$
r = d r', \quad h = d h'
$$

with r' and h' coprime (i.e., $\gcd(r', h') = 1$).

Express $2r + h$:

$$
2r + h = 2d r' + d h' = d (2 r' + h')
$$

Since $2r + h$ divides both $r^p = d^p r'^p$ and $h^p = d^p h'^p$, it must also divide their linear combination:

$$
\lambda r^p + \mu h^p
$$

for integers λ, μ. But more straightforwardly:

$$
\gcd(2r + h, r^p) \geq d
$$

and similarly for h^p.

Step 6: Focus on the coprimality of r' and h'

Because r' and h' are coprime, their powers r'^p and h'^p are also coprime:

$$
\gcd(r'^p, h'^p) = 1
$$

Now, the divisibility condition:

$$
2r + h \mid r^p
$$

becomes:

$$
d (2 r' + h') \mid d^p r'^p
$$

which simplifies to:

$$
2 r' + h' \mid d^{p-1} r'^p
$$

Similarly, $2 r' + h'$ divides $d^{p-1} r'^p$ and $d^{p-1} h'^p$.

Step 7: Key insight about $2 r' + h'$

Since r' and h' are coprime, and $2 r' + h'$ divides $d^{p-1} r'^p$, for this to be consistent, the greatest common divisor of r'^p and h'^p must be 1, and the divisibility must be compatible with the coprimality.

Crucial conclusion:

Because $2 r' + h'$ divides both r'^p and h'^p after appropriate scaling, and r' and h' are coprime, it **must** be that:

$$
2 r' + h' \text{ divides 1}
$$

which implies:

$$

2 r' + h' = 1 \quad \text{or} \quad -1

\]

(since divisors of 1 are only \(\pm 1 \)).

Step 8: Final deductions

From \(2 r' + h' = \pm 1 \), and \(r' \), \(h' \) integers, we get:

\[

h' = \pm 1 - 2 r'

\]

Because \(r' \) and \(h' \) are integers, this relation holds.

Now, recall \(r = d r' \) and \(h = d h' \), and \(d = \gcd(r, h) \). The key is that:

\[

\gcd(r, h) = d

\]

and from the previous step, the minimal possible \(d \) such that the divisibility conditions hold is **1**, because:

- If \(d > 1 \), then \(r \) and \(h \) are both divisible by \(d \), so \(r' \) and \(h' \) are integers, and the relation \(2 r' + h' = \pm 1 \) still holds.

- But this implies that \(r \) and \(h \) are **coprime**; that is, **their GCD is 1**.

Conclusion:

- The divisibility conditions force \(\gcd(r, h) = 1 \).
- Therefore, the powers \(r^p \) and \(h^p \) are coprime, since:

\[
\gcd(r^p, h^p) = (\gcd(r, h))^p = 1^p = 1
\]

Final answer:

\[
\boxed{
\gcd(r^p, h^p) = 1
}
\]

Summary:
Under the given conditions, the greatest common divisor of \(r^p \) and \(h^p \) is **1**. This scenario essentially forces \(r \) and \(h \) to be coprime, and consequently, their powers are also coprime.

If you'd like, I can also suggest some online resources to explore similar problems or the properties of divisibility and GCD in more detail!