AI Assisted Analysis of Fermat's Last Theorem

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AI Assistance

If the analyses generated by *deepai* (https://deepai.org/chat/mathematics) are correct, they confirm Fermat's conjecture.

Introduction

Pierre de Fermat (1601-1665) was a lawyer, a member of the Parliament in Toulouse, and a mathematician. His conjecture is easy to explain: he said that there are no solutions to the following equation when X, Y, and Z are positive integers, and n is a positive integer greater than two:

$$X^n + Y^n = Z^n$$

Fermat created his conjecture in 1637 when he wrote in a copy of *Arithmetica* that he had a proof that was too large to fit in the margin. In 1995, professors Taylor[i] and Wiles[ii] proved the conjecture.

Limiting the Scope of the Problem to Prime Exponents

Professor Van der Poorten[iii] shows that it is not necessary to prove Fermat's conjecture for any positive exponent n; rather, it is sufficient to prove it when the exponent p is prime and the greatest common divisor of X and Y equals 1:

$$X^p + Y^p = Z^p \tag{1}$$

Variables

All variables represent integers.

Change of Variables

These new variables represent the differences among the original variables:

$$i = Z - Y$$

$$k = Z - X$$

$$r = X - i$$

$$h = i + k$$

These definitions imply:

$$X = r + i$$

$$Y = r + k$$

$$Z = r + i + k$$

If $p > 2$, then (1) can be written as:

$$(X+Y)\sum_{m=0}^{p-1}(-1)^m X^m \; Y^{p-m-1} = Z^p$$

This implies that:

$$(2r+i+k)\sum_{m=0}^{p-1}(-1)^m X^m \; Y^{p-m-1} = (r+i+k)^p$$

$$\sum_{m=0}^{p-1}(-1)^m X^m \; Y^{p-m-1} = \frac{(r+i+k)^p}{(2r+i+k)}$$

$$\sum_{m=0}^{p-1}(-1)^m X^m \; Y^{p-m-1} = \frac{(r+h)^p}{(2r+h)}$$

All the terms on the left-hand of this equation are integers. This implies that the fraction on the right hand-side is an integer. Using this, the following are question to, and responses from, *deepai*:

Question	Response from *deepai*
Does $(2r+h)\ divide\ {-r^p}$ (implying that $(2r+h)\ divides\ r^p$)	Yes
Does $(2r+h)\ divide\ h^p$	Yes
What is greatest common divisor of (r^p and h^p)	1

Conclusion

Since $X + Y == (2r + h)$ divides both r^p and h^p, it also divides their greatest common divisor. This leads to a contradiction where the sum of two positive integers divide 1; therefore, Fermat's conjecture is correct.

deepai: Questions and Responses

Does $(2r+h)\ divide\ {-r^p}$

PROMPT: given integers p,r,h where p is prime and greater than 2, and 〖(r+h)〗^p/((2r+h)) is an integer; does (2r + h) divide -r^p

deepai short-form response

Final answer:

Yes, given the initial conditions, $2r + h$ **divides** $-r^p$.

Summary:

- The key step was recognizing that $(r + h)^p$ divisible by $2r + h$ implies certain congruences.
- These lead to the conclusion that r and h are divisible by $2r + h$.
- Therefore, $2r + h$ divides r^p (and hence $-r^p$), fulfilling the question.

Does $(2r + h)$ *divide* h^p

PROMPT: given integers p,r,h where p is prime and greater than 2, and 〖(r+h)〗^p/((2r+h)) is an integer; does (2r + h) divide h^p

deepai short-form response

Summary:

- The key step was expressing $r + h$ in terms of $d = 2r + h$.
- Using the binomial theorem, we showed that the divisibility of $\frac{(r+h)^p}{2r+h}$ implies $2r + h$ divides h^p.

What is greatest common divisor of (r^p and h^p)

PROMPT: given integers p,r,h, where (h^p)/(2r+h) is an integer, and (r^p)/(2r+h) is an integer, and p is prime and greater than 2, what is the greatest common divisor of r^p and h^p

deepai short-form response

Final answer:

$$\boxed{\gcd(r^p, h^p) = 1}$$

Raw output from *deepai*

Full text generated by prompts is available at:

https://github.com/FermatAndAI/documents/tree/main/raw%20output%20from%20deepai

[i] Richard Taylor and Andrew Wiles, "Ring-theoretic properties of certain Hecke algebras", Annals of Mathematics, Vol. 141 (1995) pp 553-572.

[ii] Andrew Wiles, "Modular elliptic curves and Fermat's Last Theorem", Annals of Mathematics, Vol. 141 (1995) pp 443-551

[iii] Alf van der Poorten, "Notes on Fermat's Last Theorem", Canadian Mathematical Society Series of Monographs and Advanced Texts", Wiley Interscience, 1996, Page 8, note 1.9