AI Assisted Analysis of Fermat’s Last Theorem

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AI Assistance

If the analyses generated by *deepai* (https://deepai.org/chat/mathematics) are correct, they confirm Fermat's conjecture.

Introduction

Pierre de Fermat (1601-1665) was a lawyer, a member of the Parliament in Toulouse, and a mathematician. His conjecture is easy to explain: he said that there are no solutions to the following equation when X, Y, and Z are positive integers, and n is a positive integer greater than two:

$$X^n + Y^n = Z^n$$

Fermat created his conjecture in 1637 when he wrote in a copy of *Arithmetica* that he had a proof that was too large to fit in the margin. In 1995, professors Taylor[1] and Wiles[2] proved the conjecture.

Limiting the Scope of the Problem to Prime Exponents

Professor Van der Poorten[3] shows that it is not necessary to prove Fermat's conjecture for any positive exponent n; rather, it is sufficient to prove it when the exponent p is prime and the greatest common divisor of X and Y equals 1:

$$X^p + Y^p = Z^p \qquad (1)$$

Variables

All variables represent integers.

Change of Variables

These new variables represent the differences among the original variables:

$$i = Z - Y$$

$$k = Z - X$$

[1] Richard Taylor and Andrew Wiles, "Ring-theoretic properties of certain Hecke algebras", Annals of Mathematics, Vol. 141 (1995) pp 553-572.

[2] Andrew Wiles, "Modular elliptic curves and Fermat's Last Theorem", Annals of Mathematics, Vol. 141 (1995) pp 443-551

[3] Alf van der Poorten, "Notes on Fermat's Last Theorem", Canadian Mathematical Society Series of Monographs and Advanced Texts", Wiley Interscience, 1996, Page 8, note 1.9

$r = X - i$

These definitions imply:

$X = r + i$

$Y = r + k$

$Z = r + i + k$

If $p > 2$, then (1) can be written as:

$$(X + Y) \sum_{m=0}^{p-1} (-1)^m X^m \; Y^{p-m-1} = Z^p$$

This implies that:

$$(2r + i + k) \sum_{m=0}^{p-1} (-1)^m X^m \; Y^{p-m-1} = (r + i + k)^p$$

$$\sum_{m=0}^{p-1} (-1)^m X^m \; Y^{p-m-1} = \frac{(r + i + k)^p}{(2r + i + k)}$$

Similarly,

$$X^p + Y^p = Z^p$$

$$\frac{X^p + Y^p}{Z} = Z^{p-1}$$

$$\frac{(2r + i + k) \sum_{m=0}^{p-1} (-1)^m X^m \; Y^{p-m-1}}{r + i + k} = Z^{p-1}$$

Performing polynomial division on the left-hand side of this equation, using r as the reference variable, yields a polynomial remainder of $-(i^p + k^p)$, which implies that

$$\frac{i^p + k^p}{r + i + k}$$

is an integer (because all the variables in the polynomial quotient are integers).

Therefore, both of these expressions must (simultaneously) be integers:

$$\frac{(r+i+k)^p}{(2r+i+k)} \;\; and \;\; \frac{i^p + \; k^p}{r+i+k}$$

If we use the following prompt:

> given integers p, r, i, k, where p is prime and greater than 2, none of (i+k), r, i, k equal to zero, can the following expressions simultaneously be integers: (r+i+k)^p/(2r+i+k), (i^p + k^p)/(r+i+k)

deepai responds with:

> Conclusion. Except for the very special (and easily described) choice 2r + i + k = ±1, there is no way to make both (r+i+k)^p/(2r+i+k) and (i^p + k^p)/(r+i+k) simultaneously integers once p>2 and none of r,i,k or i+k vanishes. (full response on github[4])

Since the two fractions cannot simultaneously be integers, Fermat's conjecture is correct.

[4] documents/raw output from deepai/simultaneous integers.pdf at main · FermatAndAI/documents