

AI Assisted Analysis of Fermat's Last Theorem

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AI Assistance

If the analyses generated by *deepai* (<https://deepai.org/chat/mathematics>) are correct, they confirm Fermat's conjecture.

Introduction

Pierre de Fermat (1601-1665) was a lawyer, a member of the Parliament in Toulouse, and a mathematician. His conjecture is easy to explain: he said that there are no solutions to the following equation when X , Y , and Z are positive integers, and n is a positive integer greater than two:

$$X^n + Y^n = Z^n$$

Fermat created his conjecture in 1637 when he wrote in a copy of *Arithmetica* that he had a proof that was too large to fit in the margin. In 1995, professors Taylorⁱ and Wilesⁱⁱ proved the conjecture.

Limiting the Scope of the Problem to Prime Exponents

Professor Van der Poortenⁱⁱⁱ shows that it is not necessary to prove Fermat's conjecture for any positive exponent n ; rather, it is sufficient to prove it when the exponent p is prime and the greatest common divisor of X and Y equals 1:

$$X^p + Y^p = Z^p \tag{1}$$

Variables

All variables represent integers.

Change of Variables

These new variables represent the differences among the original variables:

$$i = Z - Y$$

$$k = Z - X$$

$$r = X - i$$

$$h = i + k$$

These definitions imply:

$$X = r + i$$

$$Y = r + k$$

$$Z = r + i + k$$

If $p > 2$, then (1) can be written as:

$$(X + Y) \sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = Z^p$$

This implies that:

$$(2r + i + k) \sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = (r + i + k)^p$$

$$\sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = \frac{(r + i + k)^p}{(2r + i + k)}$$

$$\sum_{m=0}^{p-1} (-1)^m X^m Y^{p-m-1} = \frac{(r + h)^p}{(2r + h)}$$

All the terms on the left-hand side of this equation are integers. This implies that the fraction on the right hand-side is an integer. Using this, the following are question to, and responses from, *deepai*:

<u>Question</u>	<u>Response from <i>deepai</i></u>
Does $(2r + h)$ divide $-r^p$ (implying that $(2r + h)$ divides r^p)	Yes
Does $(2r + h)$ divide h^p	Yes
What is greatest common divisor of $(r^p$ and $h^p)$	1

Conclusion

Since $X + Y == (2r + h)$ divides both r^p and h^p , it also divides their greatest common divisor. This leads to a contradiction where the sum of two positive integers divides 1; therefore, Fermat's conjecture is correct.

deepai: Questions and Responses

Does $(2r + h)$ divide $-r^p$

PROMPT: given integers p,r,h where p is prime and greater than 2, and $(r+h)^p / ((2r+h))$ is an

integer; does $(2r + h)$ divide $-r^p$

deepai short-form response

Final answer:

Yes, given the initial conditions, $2r + h$ divides $-r^p$.

Summary:

- The key step was recognizing that $(r + h)^p$ divisible by $2r + h$ implies certain congruences.
- These lead to the conclusion that r and h are divisible by $2r + h$.
- Therefore, $2r + h$ divides r^p (and hence $-r^p$), fulfilling the question.

Does $(2r + h)$ divide h^p

PROMPT: given integers p, r, h where p is prime and greater than 2, and $(r+h)^p / ((2r+h))$ is an integer; does $(2r + h)$ divide h^p

deepai short-form response

Summary:

- The key step was expressing $r + h$ in terms of $d = 2r + h$.
- Using the binomial theorem, we showed that the divisibility of $\frac{(r+h)^p}{2r+h}$ implies $2r + h$ divides h^p .

What is greatest common divisor of (r^p) and h^p

PROMPT: given integers p, r, h , where $(h^p)/(2r+h)$ is an integer, and $(r^p)/(2r+h)$ is an integer, and p is prime and greater than 2, what is the greatest common divisor of r^p and h^p

deepai short-form response

Final answer:

$$\gcd(r^p, h^p) = 1$$

Raw output from *deepai*

Full text generated by prompts is available at:

<https://github.com/FermatAndAI/documents/tree/main/raw%20output%20from%20deepai>

ⁱ Richard Taylor and Andrew Wiles, "Ring-theoretic properties of certain Hecke algebras", *Annals of Mathematics*, Vol. 141 (1995) pp 553-572.

ⁱⁱ Andrew Wiles, "Modular elliptic curves and Fermat's Last Theorem", *Annals of Mathematics*, Vol. 141 (1995) pp 443-551

ⁱⁱⁱ Alf van der Poorten, "Notes on Fermat's Last Theorem", *Canadian Mathematical Society Series of Monographs and Advanced Texts*, Wiley Interscience, 1996, Page 8, note 1.9