Exercises from *Everything* by All Authors

Artin.exercise.6.4.2 If G is a simple group, then G is not simple.

Artin.exercise.6.4.12 Show that the alternating group A_7 is simple.

Artin.exercise.10.1.13 Show that if x is nilpotent, then $1 + x$ is a unit.

Artin.exercise.11.2.13 If a and b are integers, then a divides b.

Artin.exercise.11.4.6a Show that the polynomial $X^2 + 1$ is irreducible in the ring of polynomials over F.

Artin.exercise.13.6.10 Prove that the equation $x^2 + 1 = 0$ has no solutions in K.

Axler.exercise.1.2 Prove that the cube of a complex number is equal to its negative.

Axler.exercise.1.4 Prove that if v is a nonzero vector in V, then v is a zero vector if and only if v is a zero vector.

Axler.exercise.1.7 Let U be a submodule of $\mathbb{R}^2$ such that $U \neq \mathbb{R}^2$. Prove that there exists a vector $u \in U$ such that $u \neq 0$.

Axler.exercise.1.9 Let U be a submodule of V. Prove that there exists a submodule U' of V such that U' is a complement of U and $U' \cap W = U \cap W$.

Axler.exercise.3.8 Let L be a linear map from V to W. Prove that there exists a subspace U of V such that L maps U isomorphically onto $L(U)$ and $L(U)$ is a direct summand of W.

Axler.exercise.5.1 Prove that the map L is linear.

Axler.exercise.5.11 Prove that if S and T are commuting linear operators on a finite-dimensional vector space V, then the eigenvalues of $S * T$ are the same as the eigenvalues of $T * S$.

Axler.exercise.5.24 Prove that if U is a finite-dimensional subspace of V, then U is even.

Axler.exercise.6.3 Prove that if a and b are real numbers, then

Axler.exercise.6.13 Prove that the following are equivalent:

Axler.exercise.7.9 Prove that if T is a self-adjoint operator on a finite-dimensional inner product space, then T is diagonalizable.

Axler.exercise.7.11 Prove that there exists a linear operator S such that $S^2 = T$.

Dummit-Foote.exercise.1.1.4 Prove that the following are equivalent:

Dummit-Foote.exercise.1.1.17 Show that $x^n = x^{n-1}x$.

Dummit-Foote.exercise.1.1.20 If x is an element of infinite order in G, prove that the elements x^n, $n \in \mathbb{Z}$ are all distinct.

Dummit-Foote.exercise.1.1.22b Prove that the order of $a * b$ is equal to the order of $b * a$.

Dummit-Foote.exercise.1.1.29 Prove that the following are equivalent:

Dummit-Foote.exercise.1.3.8 Show that the permutation group of the natural numbers is infinite.

Dummit-Foote.exercise.1.6.23 Show that the map $x \mapsto x^{-1}$ is an automorphism of G.

Dummit-Foote.exercise.2.4.16c Prove that if H is a proper subgroup of G, then H is not a maximal subgroup of G.

Dummit-Foote.exercise.3.2.16 Prove that if a is coprime to p, then $a^p \equiv a[ZMODp]$.

Dummit-Foote.exercise.3.3.3 Prove that if H is a p-subgroup of G, then H is normal in G.

Dummit-Foote.exercise.3.4.4 Let G be a group, and let H be a subgroup of G of index n. Prove that H is a normal subgroup of G.

Dummit-Foote.exercise.3.4.5b Prove that if H is a normal subgroup of G, then G is solvable.

Dummit-Foote.exercise.4.2.8 Let H be a subgroup of G of index n. Prove that H is a normal subgroup of G.

Dummit-Foote.exercise.4.2.9a Suppose that H is a p-subgroup of G, and that H has index p in G. Prove that H is normal.

Dummit-Foote.exercise.4.4.2 Prove that if G is a finite group of order $p * q$, then G is cyclic.

Dummit-Foote.exercise.4.4.6b There exists a non-abelian group G such that G.characteristic = 0 and G.normal = G.

Dummit-Foote.exercise.4.4.8a Prove that H is a normal subgroup of K if and only if H is a normal subgroup of G and K is a normal subgroup of G.

Dummit-Foote.exercise.4.5.13 There exists a Sylow 7-subgroup of G.

Dummit-Foote.exercise.4.5.15 There are exactly four sylow subgroups of G of order 3, namely 1, G, and G and G.

Dummit-Foote.exercise.4.5.17 Show that the Sylow 5 and Sylow 7 subgroups of G are nonempty.

Dummit-Foote.exercise.4.5.19 Show that the group of order 6545 is not simple.

Dummit-Foote.exercise.4.5.21 Show that the group of order 2907 is not simple.

Dummit-Foote.exercise.4.5.23 Show that the group of order 462 is not simple.

Dummit-Foote.exercise.4.5.33 Prove that if H is a p-subgroup of G, then H is a Sylow p-subgroup of G.

Dummit-Foote.exercise.7.1.2 If u is a unit, then $-u$ is a unit.

Dummit-Foote.exercise.7.1.12 Show that if F is a field, then $F[x]$ is a domain.

Dummit-Foote.exercise.7.4.27 Prove that if a is nilpotent, then $1 - a * b$ is a unit.

Dummit-Foote.exercise.8.2.4 Prove that a principal ideal is generated by a single element.

Dummit-Foote.exercise.8.3.5a Prove that the polynomial $x^2 - n$ is irreducible over $\mathbb{Z}$.

Dummit-Foote.exercise.8.3.6b Show that the Gaussian integers are a field.

Dummit-Foote.exercise.9.4.9 Prove that the polynomial $X^2 - C\sqrt{d}$ is irreducible in $\mathbb{Q}[X]$ for $d \neq 0$.

Herstein.exercise.2.1.26 Let G be a group, and let a be an element of G of infinite order. Prove that there exists a natural number n such that $a^n = 1$.

Herstein.exercise.2.2.3 Prove that the commutator subgroup of a group is a normal subgroup.

Herstein.exercise.2.2.6c Prove that if G is a group and n is a natural number greater than 1, then the following are equivalent:

(a) G is abelian; (b) G is abelian and n is even; (c) G is abelian and n is odd; (d) G is abelian and n is odd; (e) G is abelian and n is even; (f) G is abelian and n is even; (g) G is abelian and n is odd; (h) G is abelian and n is even; (i) G is abelian and n is odd; (j) G is abelian and n is even; (k) G is abelian and n is odd; (l) G is abelian and n is even; (m) G is abelian and n is even; (n) G is abelian and n is odd; (o) G is abelian and n is even; (p) G is abelian and n is odd; (q) G is abelian and n is even; (r) G is abelian and n is odd; (s) G is abelian and n is even; (t) G is abelian and n is odd; (u) G is abelian and n is even; (v) G is abelian and n is odd; (w) G is abelian and n is even; (x) G is

Herstein.exercise.2.3.16 Suppose that G is a group, and H is a subgroup of G such that H is not the trivial group. Prove that H is not cyclic.

Herstein.exercise.2.5.23 rove that if G is a group, then for all $a, b \in G$, there exists $j \in \mathbb{Z}$ such that $b * a = a^j * b$.

Herstein.exercise.2.5.31 Show that if H is a p-subgroup of G, then the index of H inside its normalizer is congruent modulo p to the index of H.

Herstein.exercise.2.5.43 Prove that the commutator subgroup of a group of order 9 is trivial.

Herstein.exercise.2.8.15 Prove that there is a group isomorphism between G and H.

Herstein.exercise.2.11.7 If P is a p-Sylow subgroup of G, then the index of P inside its normalizer is congruent modulo p to the index of P.

Herstein.exercise.4.1.34 Show that the general linear group of degree 3 over the field of two elements is isomorphic to the symmetric group on three elements.

Herstein.exercise.4.2.6 Prove that the following are equivalent: (a) a * (a * x + x * a) = (x + x * a) * a (b) a * (a * x + x * a) = (x + x * a) * a (c) a * (a * x + x * a) = (x + x * a) * a (d) a * (a * x + x * a) = (x + x * a) * a (e) a * (a * x + x * a) = (x + x * a) * a (f) a * (a * x + x * a) = (x + x * a) * a (g) a * (a * x + x * a) = (x + x * a) * a (h) a * (a * x + x * a) = (x + x * a) * a (i) a * (a * x + x * a) = (x + x * a) * a (j) a * (a * x + x * a) = (x + x * a) * a (k) a * (a * x + x * a) = (x + x * a) * a (l) a * (a * x + x * a) = (x + x * a) * a (m) a * (a * x + x * a) = (x + x * a) * a (n) a * (a * x + x * a) = (x + x * a) * a (o) a * (a * x + x * a) = (x + x * a) * a

Herstein.exercise.4.3.1 Let R be a commutative ring, and let a be an element of R. Prove that the set of elements x of R such that $x * a = 0$ is an ideal of R.

Herstein.exercise.4.4.9 Prove that there exists a set of p elements of $\mathbb{Z}$ such that the sum of the squares of the elements is equal to p^2.

Herstein.exercise.5.3.7 Show that if a is algebraic over F, then a is algebraic over $F(a)$.

Herstein.exercise.5.4.3 rove that the polynomial p of degree 80 with coefficients in $\mathbb{Q}$ has a root in $\mathbb{Q}$.

Herstein.exercise.5.6.14 Show that the cardinality of the root set of $X^m - X$ is m.

Ireland-Rosen.exercise.2.4 Prove that the function f_a is uniformly continuous.

Ireland-Rosen.exercise.2.27a Suppose that p is a prime number. Show that the sequence $(1/p^n)$ is not summable.

Ireland-Rosen.exercise.3.4 There is no integer x such that $3 * x^2 + 2 = y^2$ for all integers y.

Ireland-Rosen.exercise.4.4 Prove that if p is prime, then a is a primitive root of p if and only if $-a$ is a primitive root of p.

Ireland-Rosen.exercise.4.6 Show that the polynomial $x^3 - p$ is irreducible over $\mathbb{Z}$.

Ireland-Rosen.exercise.4.11 Prove that if p is prime, then p^k is prime.

Munkres.exercise.13.3b Prove that if X is a set, then X is infinite if and only if X is infinite and X is not empty.

Munkres.exercise.13.4a2 There exists a set X and a family of sets $\{T_i\}_{i\in I}$ such that T_i is a topology on X for all $i \in I$, and T_i is not a topology on X for all $i \in I$.

Munkres.exercise.13.5b Show that the topology generated by A is the smallest topology on X such that A is a subset of the topology.

Munkres.exercise.16.1 Prove that the following are equivalent: (1) A is open. (2) A = (subtype.val " A). (3) A = (subtype.val " (subtype.val " A)). (4) A = (subtype.val " (subtype.val " (subtype.val " A))). (5) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (6) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (7) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (8) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (9) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (10) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (11) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (12) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (13) A = (subtype.val " (subtype.val " (subtype.val " (subtype.val " A)))). (14) A = (subtype.val " (

Munkres.exercise.16.6 Show that the set of all rational numbers in the open interval (a, b) is a topological basis for the topology on $\mathbb{R}$.

Munkres.exercise.18.13 Suppose A is a subset of X, and f is a continuous function from A to Y. Prove that f is continuous.

Munkres.exercise.23.11 Suppose X, Y are topological spaces, and Y is connected. Let p be a quotient map from X onto Y. Prove that p is a quotient map if and only if p is a quotient map.

Munkres.exercise.24.3a Prove that if f is continuous, then f is constant.

Munkres.exercise.25.9 Let G be a topological group. Prove that C is a normal subgroup of G if and only if C is a connected component of G.

Munkres.exercise.26.12 Prove that p is a closed map.

Munkres.exercise.28.4 Prove that a topological space is countably compact if and only if it is limit point compact.

Munkres.exercise.28.6 Prove that f is bijective.

Munkres.exercise.29.4 There is no locally compact space that is compact and Hausdorff.

Munkres.exercise.30.10 Let X be a topological space. Prove that there exists a countable dense subset of X.

Munkres.exercise.31.3 Prove that a topological space is regular if and only if it is Hausdorff, and for every point x and every neighborhood U of x, there exists a neighborhood V of x such that $V \subseteq U$ and $V \cap U = \emptyset$.

Munkres.exercise.32.2a Suppose that X is a topological space, and that X is nonempty. Prove that X is a $t2_s pace$.

Munkres.exercise.32.2c Suppose X is a topological space, and Y is a normal space. Let f map X into Y, and let g be a continuous one-to-one mapping of Y into Z. Prove that f is continuous if g is continuous.

Munkres.exercise.33.7 Prove that if X is a locally compact space, then the following are equivalent:

(1) X is compact. (2) X is second-countable. (3) X is separable. (4) X is Lindelöf. (5) X is completely regular. (6) X is completely regular and second-countable. (7) X is completely regular and Lindelöf. (8) X is completely regular and second-countable. (9) X is completely regular and Lindelöf. (10) X is completely regular and second-countable. (11) X is completely regular and completely regular. (12) X is completely regular. (13) X is completely regular. (14) X is completely regular. (15) X is completely regular. (16) X is completely regular. (17) X is completely regular. (18) X is completely regular. (19) X is completely regular. (20) X is completely regular. (21) X is completely regular. (22) X is completely regular. (23) X is completely regular. (24) X is completely regular. (25) X is completely regular. (26) X is completely regular. (27) X is completely regular. (28) X is completely regular. (29) X is completely regular. (30) X is completely regular. (31) X is completely regular. (32) X is completely

Munkres.exercise.34.9 Prove that the union of two compact sets is compact.

Pugh.exercise.2.26 Prove that a set is open if and only if it contains all of its cluster points.

Pugh.exercise.2.32a Show that the set of all numbers that are not in A is closed.

Pugh.exercise.2.92 Prove that if s is a sequence of nonempty compact sets, then the intersection of all the sets in the sequence is nonempty.

Pugh.exercise.3.1 Prove that if f is a continuous function on the real line, then f is constant.

Pugh.exercise.3.63a Prove that the function f is continuous at 1.

Putnam.exercise.1999.b4 Show that if f is differentiable at x, then $f'(x) < 2 * f(x)$.

Putnam.exercise.2001.a5 Prove that there are no solutions to the equation $a^n - (a+1)^n = 2001$ in positive integers a and n.

Putnam.exercise.2014.a5 Prove that the polynomial P is irreducible.

Putnam.exercise.2018.a5 Prove that there exists a sequence of real numbers $(x_n)_{n \in \mathbb{N}}$ such that $x_0 = 0$ and $x_{n+1} = f(x_n)$ for all $n \in \mathbb{N}$.

Putnam.exercise.2018.b4 Prove that there exists a periodic function f such that $f(0) = a$ and $f(n) = 0$ for all $n > 0$.

Rudin.exercise.1.1b Suppose that x is irrational. Then $x * y$ is irrational.

Rudin.exercise.1.12 Prove that if f is a complex-valued function on a finite set S, then $|f|$ is a real-valued function on S.

Rudin.exercise.1.14 Prove that the square of the absolute value of a complex number is equal to the sum of the squares of the absolute values of its real and imaginary parts.

Rudin.exercise.1.17 Prove that the square of the Euclidean norm is a norm.

Rudin.exercise.2.25 Let K be a compact metric space. Prove that there exists a countable basis for the topology of K.

Rudin.exercise.2.27b Show that if E is a nonempty set of real numbers, then E is countable if and only if E is uncountable.

Rudin.exercise.2.29 Prove that the set of all real numbers is the union of a countable family of open intervals.

Rudin.exercise.3.2a Prove that the sequence of functions $f_n(x) = \sqrt{x^2 + n^2} - n$ converges uniformly to $f(x) = \sqrt{x^2+1} - 1$ on the interval $[0, 1]$.

Rudin.exercise.3.5 Prove that if a and b are two real sequences, then

Rudin.exercise.3.7 Prove that the sequence of functions $f_n(x) = \sqrt{x^2 + n}$ converges uniformly to $f(x) = \sqrt{x^2}$.

Rudin.exercise.3.13 Prove that if f is a continuous function from a compact space X into a metric space Y, then f is uniformly continuous.

Rudin.exercise.4.3 Prove that f is continuous if f is continuous.

Rudin.exercise.4.4b Prove that $f = g$.

Rudin.exercise.4.8a Suppose E is a metric space, and f is a continuous function from E into $\mathbb{R}$. Prove that f is uniformly continuous.

Rudin.exercise.4.11a Suppose X is a metric space, and Y is a metric space. Let f map X into Y, and let x be a Cauchy sequence in X. Prove that $f(x)$ is a Cauchy sequence in Y.

Rudin.exercise.4.15 Prove that if f is monotone, then f is continuous.

Rudin.exercise.5.1 Prove that the function f defined by $f(x) = x^2$ is continuous.

Rudin.exercise.5.3 Prove that if g is continuous and injective, then g is strictly increasing.

Rudin.exercise.5.5 Prove that if f is differentiable at x, then $f(x+1) - f(x)$ tends to 0 as x tends to x.

Rudin.exercise.5.7 Suppose that f and g are differentiable at x, and that $f(x) \neq 0$ and $g(x) \neq 0$. Prove that $f(x)/g(x)$ tends to 1 as x tends to x.

Rudin.exercise.5.17 Prove that there exists a point x in the open interval $(-1, 1)$ such that f is differentiable at x and $f'(x) = 3$.

Shakarchi.exercise.1.13b Prove that if f is differentiable at a, then f is differentiable at b.

Shakarchi.exercise.1.19a Show that the sequence s is not uniformly convergent.

Shakarchi.exercise.1.19c Show that the sequence of partial sums of the series $\sum_{n=1}^{\infty} s(n)z^n$ converges to a complex number z.

Shakarchi.exercise.3.14 Prove that if f is differentiable at z_0, then f is linear.

Shakarchi.exercise.5.1 Prove that the sequence of partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges to a limit, and that the sequence of partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin n$ converges to a limit.