Exercises from
Everything
by All Authors

Artin.exercise.3.7.2 Let V be a vector space over a field K, and let S be a finite subset of V. Prove that there exists a vector $v \in V$ such that $v \notin \sum_{s \in S} Ks$.

Artin.exercise.6.4.2 If G is a finite group, then G is simple.

Artin.exercise.6.4.12 If G is a simple group, then G is abelian.

Artin.exercise.10.1.13 If x is nilpotent, then $1 + x$ is invertible.

Artin.exercise.10.4.7a If I and J are ideals of R, then $I * J$ is an ideal of R.

Artin.exercise.10.6.7 Let I be an ideal of $gaussian_int$, and let z be a nonzero element of I. Prove that z is a nonzero element of $gaussian_int$.

Artin.exercise.11.2.13 If a and b are integers, then a divides b if and only if a divides b.

Artin.exercise.11.4.6a Let F be a field of characteristic 7, and let X be a polynomial in $F[X]$ of degree 2. Prove that X is irreducible.

Artin.exercise.11.4.6c Show that the polynomial $X^3 - 9$ is irreducible in $\mathbb{Z}[X]$.

Artin.exercise.11.13.3 Let p be a prime number greater than N. Prove that $p+1$ is not a square modulo 4.

Axler.exercise.1.2 Show that the cube root of -1 is equal to the cube root of 1.

Axler.exercise.1.4 If v is a nonzero vector in V, then v is a nonzero vector in F^n for some n.

Axler.exercise.1.9 Let U be a submodule of V, and let W be a submodule of V. Prove that U is a submodule of W if and only if U is a submodule of V and W is a submodule of V.

Axler.exercise.3.8 Let L be a linear map from V to W, and let U be the submodule of V generated by the kernel of L. Prove that L is injective if and only if U is a direct summand of V.

Axler.exercise.5.11 If S and T are endomorphisms of a finite-dimensional vector space V over a field F, then the eigenvalues of $S * T$ are the same as the eigenvalues of $T * S$.

Axler.exercise.5.13 Let T be a linear transformation of a finite-dimensional vector space V over a field F. Then there exists a basis B of V such that T is represented by a matrix with respect to B.

Axler.exercise.5.24 If U is a submodule of V, then V is even-dimensional.

Axler.exercise.6.3 Let a_i be a sequence of real numbers. Prove that $\sum_{i=1}^n a_i^2 \leq \sum_{i=1}^n a_i \sum_{i=1}^n a_i/i$.

Axler.exercise.6.13 Show that the orthonormal basis e is an orthonormal basis if and only if the orthogonal projection P onto the span of e is the identity.

Axler.exercise.7.5 Let T be a linear operator on V such that $T^2 = T$. Prove that T is a scalar multiple of the identity operator.

Axler.exercise.7.9 If T is a self-adjoint linear operator on a finite-dimensional complex vector space, then T is diagonalizable.

Axler.exercise.7.11 Let S be the unique linear map such that $S^2 = T$. Prove that S is self-adjoint.

Dummit-Foote.exercise.1.1.2a Let a, b be integers such that $a - b \neq b - a$. Prove that a and b are not relatively prime.

Dummit-Foote.exercise.1.1.4 Let n be a positive integer. Prove that the map $a \mapsto a^n$ is a group homomorphism from the additive group of integers modulo n to the multiplicative group of integers modulo n.

Dummit-Foote.exercise.1.1.15 Let G be a group, and let a be a list of elements of G. Prove that a is a group element if and only if a is a group element and a is a group element.

Dummit-Foote.exercise.1.1.17 Show that x^n is the only element of G of order n.

Dummit-Foote.exercise.1.1.20 If x is an element of finite order in G, prove that the elements x^n, $n \in \mathbb{Z}$ are all distinct.

Dummit-Foote.exercise.1.1.22b If a and b are elements of a group G such that $a^n = b^n$ for some $n \in \mathbb{N}$, then $a = b$.

Dummit-Foote.exercise.1.3.8 Show that the permutation group of the natural numbers is infinite.

Dummit-Foote.exercise.1.6.23 If x and y are elements of a group G such that $x * y = y * x$, then $x = y$.

Dummit-Foote.exercise.2.1.13 If H is a subgroup of $\mathbb{Q}$, then H is either $\mathbb{Q}$ or $\mathbb{Z}$.

Dummit-Foote.exercise.2.4.16c If H is a proper subgroup of $zmodn$, then H is not a maximal subgroup of $zmodn$.

Dummit-Foote.exercise.3.2.16 If a is a natural number, then a^p is a natural number.

Dummit-Foote.exercise.3.3.3 If H is a p-subgroup of G, then the index of H inside its normalizer is congruent modulo p to the index of H inside G.

Dummit-Foote.exercise.3.4.4 Let H be a subgroup of G of finite index. Prove that H is normal in G.

Dummit-Foote.exercise.4.2.8 Let H be a subgroup of G of index n. Prove that there exists a subgroup K of G of index at most $n!$ such that K is normal in G and K is a subgroup of H.

Dummit-Foote.exercise.4.2.9a If H is a p-subgroup of G, then the index of H inside its normalizer is congruent modulo p to the index of H inside G.

Dummit-Foote.exercise.4.4.2 If G is a finite group, then G is cyclic.

Dummit-Foote.exercise.4.4.6b Let G be a group, and let H be a subgroup of G. Prove that H is characteristic in G if and only if H is normal in G.

Dummit-Foote.exercise.4.4.8a If H is a p-subgroup of G, then the index of H inside its normalizer is congruent modulo p to the index of H inside G.

Dummit-Foote.exercise.4.5.13 Let G be a finite group of order 56. Show that G is cyclic.

Dummit-Foote.exercise.4.5.15 Let G be a finite group of order 351. Then G is cyclic.

Dummit-Foote.exercise.4.5.17 Show that the Sylow 5-subgroup of G is nonempty and the Sylow 7-subgroup of G is nonempty.

Dummit-Foote.exercise.4.5.19 If G is a simple group, then G is not isomorphic to A_5.

Dummit-Foote.exercise.4.5.21 If G is a simple group, then G is not isomorphic to A_5.

Dummit-Foote.exercise.4.5.23 If G is a simple group, then G is not isomorphic to A_5.

Dummit-Foote.exercise.4.5.33 Let R be a Sylow p-subgroup of H. Then R is a Sylow p-subgroup of G.

Dummit-Foote.exercise.7.1.2 If u is a unit in R, then $-u$ is a unit in R.

Dummit-Foote.exercise.7.1.12 Let K be a subring of a field F. Prove that K is a domain if and only if K is a field.

Dummit-Foote.exercise.7.2.2 If p is a nonzero polynomial, then p divides 0.

Dummit-Foote.exercise.7.4.27 If a is a unit in R, then $1 - a$ is a unit in R.

Dummit-Foote.exercise.8.2.4 Let R be a ring. Prove that if R is a principal ideal domain, then R is a field.

Dummit-Foote.exercise.8.3.5a Show that the polynomial $x^2 + x + 1$ is irreducible in $\mathbb{Q}[x]$.

Dummit-Foote.exercise.8.3.6b Let R be a ring of Gaussian integers. Show that R is a field if and only if R is a finite field.

Dummit-Foote.exercise.9.1.10 Show that the minimal primes of the ideal generated by the coefficients of f are exactly the minimal primes of the ideal generated by the coefficients of f.

Dummit-Foote.exercise.9.4.2a Show that the polynomial $X^4 - 4 * X^3 + 6$ is irreducible over the integers.

Dummit-Foote.exercise.9.4.2c Show that the polynomial $X^4 + 4 * X^3 + 6 * X^2 + 2 * X + 1$ is irreducible over the integers.

Dummit-Foote.exercise.9.4.9 Show that the polynomial $z^2 - C$ is irreducible in $\mathbb{Q}[z]$.
continuous.

Herstein.exercise.2.1.18 If G is a finite group, then G is cyclic.

Herstein.exercise.2.1.26 Let n be the order of a in G. Prove that n is a power of p.

Herstein.exercise.2.2.3 Let G be a group, and let P be a predicate on G such that $P(n)$ is true if and only if n is a power of 2.

Herstein.exercise.2.3.16 If G is a cyclic group, then G is finite.

Herstein.exercise.2.5.23 et G be a group, and let $a, b \in G$ be elements such that a has finite order and b is not a root of unity. Prove that a and b generate a finite cyclic group.

Herstein.exercise.2.5.31 If H is a p-subgroup of G, then the index of H inside its normalizer is congruent modulo p to the index of H inside G.

Herstein.exercise.2.5.43 Show that the commutator subgroup of G is of order 9.

Herstein.exercise.2.8.15 Let G be a finite group, and let H be a subgroup of G of index p. Then H is isomorphic to a subgroup of G of index p.

Herstein.exercise.2.11.7 If P is a Sylow p-subgroup of G, then P is cyclic.

Herstein.exercise.4.1.34 The general linear group of the vector space of 3×3 matrices over the field of two elements is isomorphic to the general linear group of the vector space of 2×2 matrices over the field of two elements.

Herstein.exercise.4.2.6 Let R be a ring, and let a be an element of R such that $a^2 = 0$. Prove that a is a zero-divisor.

Herstein.exercise.4.4.9 Let S be the set of all elements of $zmodp$ that are squares modulo p. Show that S is a subgroup of $zmodp$ and that S is cyclic of order $p - 1$. Show that S is the unique subgroup of $zmodp$ of order $p - 1$. Show that S is the unique subgroup of $zmodp$ of order $p - 1$.

Herstein.exercise.4.5.23 Show that the polynomial $X^3 - 2$ is irreducible in $\mathbb{Z}[X]$ and the polynomial $X^3 + 2$ is irreducible in $\mathbb{Z}[X]$. Show that the ideal generated by $X^3 - 2$ in $\mathbb{Z}[X]$ is equal to the ideal generated by $X^3 + 2$ in $\mathbb{Z}[X]$. Show that the ideal generated by $X^3 - 2$ in $\mathbb{Z}[X]$ is not equal to the ideal generated by $X^3 + 2$ in $\mathbb{Z}[X]$. Conclude that the ideal generated by $X^3 - 2$ in $\mathbb{Z}[X]$ is not equal to the ideal generated by $X^3 + 2$ in $\mathbb{Z}[X]$. Conclude that the polynomial $X^3 - 2$ is irreducible in $\mathbb{Z}[X]$ and the polynomial $X^3 + 2$ is irreducible in $\mathbb{Z}[X]$. Conclude that the ideal generated by $X^3 - 2$ in $\mathbb{Z}[X]$ is not equal to the ideal generated by $X^3 + 2$ in $\mathbb{Z}[X]$. Conclude that the polynomial $X^3 - 2$ is irreducible in $\mathbb{Z}[X]$ and the polynomial $X^3 + 2$ is irreducible in $\mathbb{Z}[X]$. Conclude that the polynomial $X^3 - 2$ is irreducible in $\mathbb{Z}[X]$ and the polynomial $X^3 + 2$ is irreducible in $\mathbb{Z}[X]$. Conclude that the polynomial $X^3 - 2$ is irreducible in $\mathbb{Z}[X]$ and the polynomial $X^3 + 2$ is irreducible in $\mathbb{Z}[X]$. Conclude that the polynomial $X^3 - 2$ is irreducible

Herstein.exercise.4.6.2 Show that the polynomial $X^3 + 3 * X + 2$ is irreducible over the rationals.

Herstein.exercise.5.1.8 Let F be a field of characteristic p, and let $a, b \in F$ be such that $a + b \neq 0$. Prove that $a + b$ is a root of $x^m - x$.

Herstein.exercise.5.3.7 Let F be a subfield of K, and let a be an element of K such that $a^2 \in F$. Prove that $a \in F$.

Herstein.exercise.5.4.3 et p be the polynomial $p(x) = x^5 + x^3 + x^2 + x + 11$. Let a be the root of p in the complex numbers. Show that a is a root of p in the algebraic numbers. Show that a is a root of p in the algebraic numbers. Show that a is a root of p in the algebraic numbers.

Herstein.exercise.5.6.14 Show that the roots of $X^m - X$ are the m-th roots of unity.

Ireland-Rosen.exercise.1.30 Show that there is no integer a such that $a^2 + a + 1 = 0$.

Ireland-Rosen.exercise.2.4 Let f_a be the function defined by $f_a(n) = a^n$ for $n \in \mathbb{Z}$.

Ireland-Rosen.exercise.2.27a Show that the series $\sum_{i=1}^{\infty} 1/i$ diverges.

Ireland-Rosen.exercise.3.4 There exists a non-zero integer x such that $3x^2 + 2$ is a square.

Ireland-Rosen.exercise.3.10 If n is a prime number, then $n - 1$ is a prime number.

Ireland-Rosen.exercise.4.4 Show that if p is prime, then $zmodp$ is a field if and only if -1 is a square in $zmodp$.

Ireland-Rosen.exercise.4.6 Show that the primitive root of 3 is $2^n + 1$.

Ireland-Rosen.exercise.4.11 Let k be a positive integer, and let s be a positive integer. Prove that there exists a positive integer n such that n^2 is a multiple of p and $n^2 + 1$ is a multiple of s.

Ireland-Rosen.exercise.12.12 Show that the algebraic numbers are exactly the numbers of the form $a + b\sqrt{2}$ where $a, b \in \mathbb{Q}$.

Munkres.exercise.13.3b If X is a set, then the set of all subsets of X is infinite if and only if X is infinite.

Munkres.exercise.13.4b2 Let T be a set of subsets of X such that T is a topology on X and T is closed under finite intersections and arbitrary unions. Then T is a topology on X.

Munkres.exercise.13.5b Let A be a set of subsets of X. Then A is a topology on X if and only if A is a topology on X and A is closed under finite intersections.

Munkres.exercise.13.8a Show that the set of all open intervals in the real line is a basis for the topology of the real line.

Munkres.exercise.16.1 If U is an open subset of A, then U is open in Y.

Munkres.exercise.18.8a If f is continuous, then the set of points x such that $f(x) \leq g(x)$ is closed.

Munkres.exercise.18.13 If g is a continuous one-to-one mapping of A into Y, then g is uniformly continuous.

Munkres.exercise.22.5 Let A be a subset of X, and let f map A into Y. Prove that f is uniformly continuous if f is uniformly continuous when restricted to A.

Munkres.exercise.23.11 If X is a topological space and Y is a connected space, then X is connected if and only if Y is connected.

Munkres.exercise.24.3a Let I be a set, and let f be a function from I into I. Prove that f is a constant function if and only if f is injective.

Munkres.exercise.25.9 If C is a normal subgroup of G, then G is a normal subgroup of G.

Munkres.exercise.28.4 If X is a countable limit point compact space, then X is compact.

Munkres.exercise.28.6 Show that f is a bijection.

Munkres.exercise.31.1 Let U and V be open sets in X such that $x \in U$ and $y \in V$. Prove that there exists a point $z \in U \cap V$ such that $x = z$ and $y = z$.

Munkres.exercise.31.3 A regular space is a space in which every point has a neighborhood basis consisting of regular open sets.

Munkres.exercise.32.2c Let X be a topological space. Prove that X is normal if and only if for every i, the space Xi is normal.

Munkres.exercise.33.7 Let X be a topological space, and let A be a subset of X. Prove that A is closed if and only if A is the inverse image of $\{0\}$ under a continuous function.

Munkres.exercise.34.9 Show that the union of two compact sets is compact.

Munkres.exercise.43.2 Let g be the function defined by $g(x) = f(x)$ for $x \in A$ and $g(x) = x$ for $x \notin A$. Prove that g is uniformly continuous.

Pugh.exercise.2.26 If U is an open set, then U is a cluster point of U.

Pugh.exercise.2.32a Show that the set of all clopen subsets of A is a topology on A.

Pugh.exercise.3.1 Let f be a continuous function from $\mathbb{R}$ to $\mathbb{R}$. Prove that f is uniformly continuous.

Putnam.exercise.1998.b6 Let n be the smallest positive integer such that $n^3 + a * n^2 + b * n + c$ is a perfect square. Show that n is the smallest positive integer such that $n^3 + a * n^2 + b * n + c$ is a perfect square.

Putnam.exercise.1999.b4 If f is a real-valued function on the real line, then the function $x \mapsto f(x) - 2 * f(x+1)$ is strictly increasing on the interval $[0, 1]$.

Putnam.exercise.2001.a5 Find the smallest positive integer a such that $a^2 - (a+1)^2 = 2001$.

Putnam.exercise.2014.a5 Let P be a polynomial with integer coefficients. Prove that P is irreducible if and only if P is not a product of two non-constant polynomials.

Putnam.exercise.2018.a5 Show that the iterated derivative of f is identically zero.

Putnam.exercise.2018.b4 Let x be a periodic function with period p. Prove that x is a constant function.

Rudin.exercise.1.1b If x is irrational, then $x * y$ is irrational.

Rudin.exercise.1.12 Let f be a function from the set of natural numbers to the complex numbers. Prove that f is uniformly continuous if and only if f is uniformly continuous as a function from the set of natural numbers to the complex numbers.

Rudin.exercise.1.14 Show that z is a root of the polynomial $x^2 + x + 1$.

Rudin.exercise.1.17 Show that the square of the Euclidean norm is a norm.

Rudin.exercise.1.18b If x is a real number, prove that the set of real numbers y such that $x * y = 0$ is a closed set.

Rudin.exercise.2.19a Let A be a closed set, and let B be a closed set disjoint from A. Prove that A and B are separated by a continuous function.

Rudin.exercise.2.25 Let B be a set of closed balls in K such that B is a countable basis for the topology on K. Prove that B is a countable basis for the topology on K.

Rudin.exercise.2.27b Show that if E is a nonempty open set, then E P is nonempty.

Rudin.exercise.3.2a Show that the sequence $n \mapsto (n^2 + n) - n$ converges to $1/2$.

Rudin.exercise.3.7 Show that if a_n is a sequence of positive real numbers such that $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n/n^2$ converges.

Rudin.exercise.3.13 Let a_n be a sequence of real numbers such that $a_n \to 0$ as $n \to \infty$. Prove that there exists a sequence b_n of real numbers such that $b_n \to 0$ as $n \to \infty$ and $b_n \sum_{i=1}^{n} a_i \to 0$ as $n \to \infty$.

Rudin.exercise.4.1a There exists a function f such that f is continuous, f is not uniformly continuous, and f is not differentiable at any point.

Rudin.exercise.4.3 If z is a closed set, then f is continuous.

Rudin.exercise.4.4b If f and g are continuous, then $f = g$.

Rudin.exercise.4.5b Let f be a continuous function from $\mathbb{R}$ to $\mathbb{R}$. Prove that f is uniformly continuous.

Rudin.exercise.4.8a Show that if f is uniformly continuous on E, then f is uniformly continuous on $f''E$.

Rudin.exercise.4.11a Show that if x is a Cauchy sequence in X, then $f(x)$ is a Cauchy sequence in Y.

Rudin.exercise.4.15 Show that if f is monotone, then f is continuous.

Rudin.exercise.5.1 Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ such that f is differentiable and f' is bounded. Prove that f is differentiable and $f' = f$.

Rudin.exercise.5.3 Let g be a continuous one-to-one mapping of $\mathbb{R}$ into $\mathbb{R}$. Prove that g is uniformly continuous.

Rudin.exercise.5.5 Show that if f is differentiable at 0, then f is differentiable at 1.

Rudin.exercise.5.7 Show that if f and g are differentiable at x, then f/g is differentiable at x.

Rudin.exercise.5.17 Show that the derivative of the function $f(x) = x^3$ is not bounded on the interval $(-1, 1)$.

Shakarchi.exercise.1.13b Let f be a function from $\mathbb{C}$ to $\mathbb{C}$ that is differentiable on $\mathbb{C}$ and let a, b be points in $\mathbb{C}$ such that $f(a) = f(b)$. Prove that $a = b$.

Shakarchi.exercise.1.19a Show that the series $\sum_{n=0}^{\infty} z^n$ is not absolutely convergent.

Shakarchi.exercise.1.19c Show that the series $\sum_{n=1}^{\infty} z^n/n$ converges to z.

Shakarchi.exercise.2.2 Show that the function f defined by $f(x) = \int_0^x \sin t/t \, dt$ is continuous at 0.

Shakarchi.exercise.2.13 Let f be a function from the complex numbers to the complex numbers. Prove that f is holomorphic if and only if f is continuous and f is holomorphic at 0.

Shakarchi.exercise.3.4 Show that the function f defined by $f(x) = x * real.sinx/(x^2 + a^2)$ is continuous at 0.

Shakarchi.exercise.3.14 Let f be a function from the complex numbers to the complex numbers. Prove that f is differentiable if and only if f is differentiable at every point of the complex numbers.