# ProofNet NL Statements

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### chapter 1

#### section 1

15: Prove that $(a_1a_2\dots a_n)^{-1} = a_n^{-1}a_{n-1}^{-1}\dots a_1^{-1}$ for all $a_1, a_2, \dots, a_n \in G$.
16: Let $x$ be an element of $G$. Prove that $x^2 = 1$ if and only if $x|x|$ is either 1 or 2.
17: Let $x$ be an element of $G$. Prove that if $|x| = n$ for some positive integer $n$ then $x^{-1} = x^{n-1}$.
18: Let $x$ and $y$ be elements of $G$. Prove that $xy = yx$ if and only if $y^{-1}xy = x$ if and only if $x^{-1}y^{-1}xy = 1$.
20: For $x$ an element in $G$ show that $x$ and $x^{-1}$ have the same order.
22a: If $x$ and $g$ are elements of the group $G$, prove that $|x| = \left|g^{-1}xg\right|$.
22b: Deduce that $|ab| = |ba|$ for all $a, b \in G$.
25: Prove that if $x^2 = 1$ for all $x \in G$ then $G$ is abelian.
29: Prove that $A\times B$ is an abelian group if and only if both $A$ and $B$ are abelian.
34: If $x$ is an element of infinite order in $G$, prove that the elements $x^n, n \in \mathbb{Z}$ are all distinct.

#### section 3

8: Prove that if $\Omega = \{1, 2, 3, \dots\}$ then $S_\Omega$ is an infinite group

#### section 6

4: Prove that the multiplicative groups $\mathbb{R}-\{0\}$ and $\mathbb{C}-\{0\}$ are not isomorphic.
11: Let $A$ and $B$ be groups. Prove that $A \times B \cong B \times A$.
12: Let $A$ and $B$ be groups. Prove that $A \times B \cong B \times A$.
17: Let $G$ be any group. Prove that the map from $G$ to itself defined by $g \mapsto g^{-1}$ is a homomorphism if and only if $G$ is abelian.
23: Let $G$ be a finite group which possesses an automorphism $\sigma$ such that $\sigma(g) = g$ if and only if $g = 1$. If $\sigma^2$ is the identity map from $G$ to $G$, prove that $G$ is abelian

**section 7**

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