the polar axis $\mathbf{p}$) are introduced, as shown in Figure 6-12, the phonon contribution becomes

$$\Sigma^{\mathrm{ph}}(p) = \frac{i}{(2\pi)^3|\mathbf{p}|}\int_{-\infty}^{\infty} dq_0 \int p'\,dp' \frac{1}{(p_0+q_0)(1+i\delta)-\epsilon_{p'}} \times \int q\,dq\{\bar{g}_{ql}\}^2 D_l(-q) \quad (6\text{-}30)$$

For convenience we measure all energies with respect to the Fermi energy E_F so that $\epsilon_{p_F} = 0$. Since D decreases as $1/q_0{}^2$ for large q_0, the dominant part of the integral comes from $|q_0| \lesssim \omega_{\mathrm{av}}$ [a typical phonon energy, i.e., $\simeq (m/M)^{1/2}E_F \simeq 10^{-2}E_F$]. We shall be interested in electron energies $|p_0| \lesssim \omega_{\mathrm{av}}$ so that the most important values of $|\epsilon_{p'}|$ are also of order ω_{av} or less. For this reason the p'-integral can be replaced by an integral over $\epsilon_{p'}$ with the limits extending from $-\infty$ to ∞. Thus,

$$\Sigma^{\mathrm{ph}}(p) \cong \frac{im}{(2\pi)^3 p}\int_{-\infty}^{\infty} dq_0 \int_{-\infty}^{\infty} d\epsilon_{p'} \frac{1}{(p_0+q_0)(1+i\delta)-\epsilon_{p'}} \times \int_0^{2k_F} q\,dq\{\bar{g}_{ql}\}^2 D_l(q) \quad (6\text{-}31)$$

The limits on the q integral have been simplified by using the fact that only states with $|p'| \simeq k_F$ contribute strongly to the

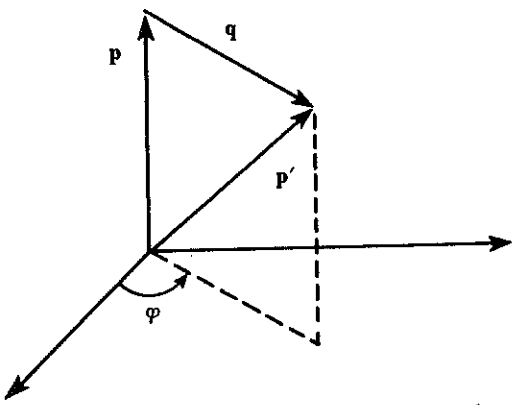

FIGURE 6-12 Coordinate system for carrying out the momentum integral in the expression for Σ^{ph}.

If we use the relation $2|g_{ql}|^2/\Omega_{q,l} = V(q)$, which holds for jellium, we find the simple result

$$\Sigma(p) = i\int G_0(p+q)\mathscr{V}_c(q)\left[\frac{{q_0}^2}{{q_0}^2 - \dfrac{{\Omega_{ql}}^2}{\kappa(q)} + i\delta}\right]\frac{d^4q}{(2\pi)^4} \quad \text{(6-29a)}$$

or

$$\Sigma(p) = i\int G_0(p+q)\frac{V(q)}{1 + V(q)P(q) - \dfrac{{\Omega_{ql}}^2}{{q_0}^2} + i\delta}\frac{d^4q}{(2\pi)^4} \quad \text{(6-29b)}$$

The denominator in (6-29b) is just the total dynamical dielectric constant of the system including electronic and ionic polarizabilities, since the ionic polarizability is given by the high-frequency

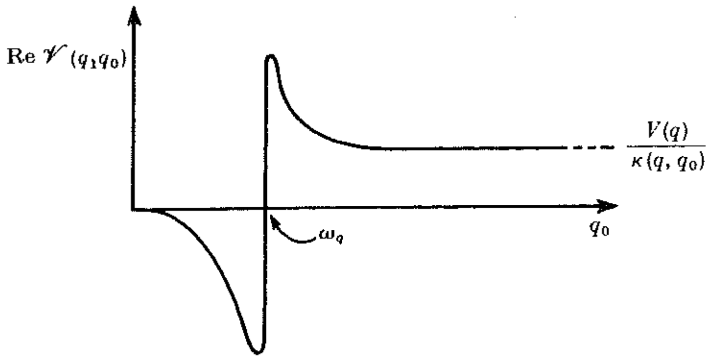

FIGURE 6-11 The real part of the effective interaction between electrons due to the screened Coulomb interaction and the exchange of a dressed phonon, plotted as a function of the energy transfer q_0 for a fixed momentum transfer **q**. The plot is shown for the RPA treatment of the "jellium" model of a metal. The resonance occurs at the dressed phonon frequency ω_q, illustrating the effect of ionic overscreening of the bare Coulomb interaction for $q_0 < \omega_q$ and underscreening for $q_0 > \omega_q$. For high-frequency $q_0 \gg \omega_q$, the ions do not respond and $\mathscr{V}(\mathbf{q}, q_0)$ approaches the bare Coulomb interaction reduced by the electronic dielectric function $\kappa(\mathbf{q}, q_0)$.

2. $d\kappa_0/dq \to \infty$ as $q \to 2k_F$. This fact leads to the result, first discussed by Kohn, Langer, and Vosko,[109a, b] that the asymptotic form of the screened Coulomb potential for large distance is not a Yukawa potential but rather the oscillatory function

$$\mathscr{V}(r) \propto \frac{\cos(2k_F r + \phi)}{r^3} \tag{6-9}$$

There is good experimental evidence to support this result.[110]

3. $\kappa_0 \to 1$ as $q \to \infty$. Thus, screening is ineffective for very large momentum transfers.

For $q_0 \neq 0$, the imaginary part of κ is nonzero only when q_0 and q are related so that the argument of the delta function in (6-5) can vanish for some $|\mathbf{p}| < p_F$ and $|\mathbf{p} + \mathbf{q}| > p_F$, that is to say,

$$q^2 - 2qk_F \leqslant 2m|q_0| \leqslant q^2 + 2qk_F \tag{6-10}$$

The situation is illustrated in Figure 6-3. For large frequency, i.e., $|q_0| \gg q^2 + (2qk_F/2m)$, we have the familiar limiting form

$$\operatorname{Re} \kappa_0(\mathbf{q}, q_0) = 1 - \frac{\omega_p^2}{q_0^2} \tag{6-11}$$

where $\omega_p^2 = 4\pi ne^2/m$. The two expressions (6-8) and (6-11) are useful limiting forms for making physical arguments.

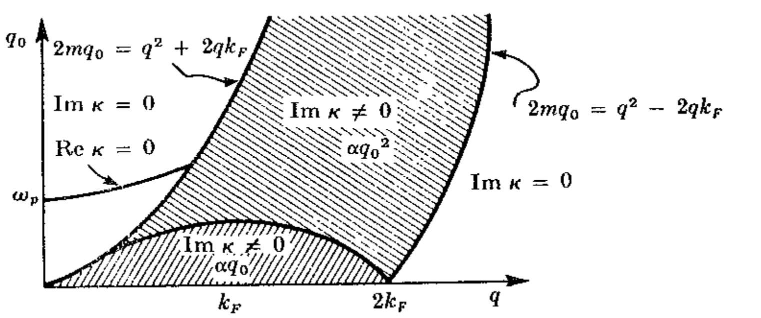

FIGURE 6-3 A plot showing the behavior of the imaginary part of the RPA dielectric function for general wave number q and frequency q_0.