

# Thermochemica User Manual

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September 27, 2019

## 1 Introduction

### 1.1 Overview

The software library Thermochemica [1] has been developed to determine a unique combination of phases and their compositions at thermochemical equilibrium for given temperature, pressure, and chemical composition. Thermochemica has been designed for direct integration in multi-physics codes and to address several concerns with using commercially available software, including: licensing entanglements associated with code distribution, access to the source code, convenient incorporation into other codes with quality assurance considerations, and computational performance. In particular, significant research efforts have been dedicated to enhance computational performance through advanced algorithm development, such as improved estimation techniques and non-linear solvers. The overall goal of this undertaking is to provide an open source computational tool to enhance the predictive capability of multi-physics codes without significantly impeding computational performance.

The purpose of this document is to describe the design of Thermochemica and the basic use of the software library. A brief background to computational thermodynamics and the conditions for thermodynamic equilibrium is given in §2, details of the code development are discussed in §3, details of building and compiling Thermochemica are given in §4, and specific programming details for each subroutine in the library are given in the Appendix. The open license for Thermochemica is provided with the source code, which is maintained on an open GitHub repository <https://github.com/ORNL-CEES/thermochemica>.

As is always the case, Software Quality Assurance (SQA) is an extremely important subject in software development. The importance of this warrants an independent document dedicated to SQA, which has been developed in parallel to this document. The subject of thermodynamic model validation is also of extreme importance in scientific computing and must be given careful attention. To be clear, this document is concerned with the development and use of a *general thermodynamic solver* and the validation of any models should be included in the discussion of the development of a *thermodynamic model* that is specific to a particular system.

### 1.2 Purpose

Thermochemica can be used for stand-alone calculations or it can be directly coupled to other codes. The initial release of the software does not have a graphical user interface (GUI) and it can be executed from the command line or from an Application Programming Interface (API). Also, it is not intended at this time for thermodynamic model development or for constructing phase diagrams. The main purpose of the software is to be directly coupled to a multi-physics code to provide material properties and boundary conditions for various physical phenomena.

Temperature, hydrostatic pressure, the mass of each chemical element, and the units of the foregoing variables are provided as input in addition to a thermodynamic database (in the form of a ChemSage data-file). ChemSage data-files can be generated by the commercial software package FactSage [2] or through the auxiliary code CSFAP.

Various useful parameters can be provided as output from Thermochemica, such as:

- determination of which phases are stable at equilibrium,
- the mass of solution species and phases at equilibrium,
- mole fractions of solution phase species,
- site fractions of constituents in multi-sublattice phases,
- pair fractions of nearest neighbours in phase represented by the modified quasi-chemical model,
- thermochemical activities (which are related to partial pressures for gaseous species),
- chemical potentials of solution species, phases, and system components, and
- integral Gibbs energy (referenced relative to standard state).

Additional variables that are not listed above (e.g., heat capacity, enthalpy) could be made available with a modest level of programming effort. All input/output operations are performed via Fortran modules, specifically ModuleThermo and ModuleThermoIO. An example is given in §3.6 of how one can make use of the aforementioned variables for the carbon-oxygen system. Details for all variables used by all modules are documented in a dOxygen report, which is provided in the Appendix. dOxygen [3] is an automatic documenting tool that generates an HTML web site and L<sup>A</sup>T<sub>E</sub>X report from comments embedded in the source code. Thus, all documentation is consolidated directly in the source code in a convenient manner.

### 1.3 Capabilities, Limitations, and Assumptions

All calculations performed by Thermochemica apply at thermodynamic equilibrium and do not take chemical kinetic information into consideration. Although chemical equilibrium is not achieved instantaneously, chemical kinetics may not have a large part to play in some situations, depending on temperature, pressure and the particular system under consideration. Generally, thermodynamic calculations are more appropriate at high temperatures, over a sufficiently long period of time and when atoms of the various chemical elements are randomly mixed within the thermodynamic system. For instance, chemical equilibrium is achieved in relatively short time periods in nuclear fuel under normal operating conditions due to the high temperatures experienced in a reactor and the relatively long time periods between refueling<sup>1</sup>. Also, due to the nature of fission, the atoms of the various elements representing the fission and activation products are randomly mixed in irradiated nuclear fuel<sup>2</sup>. Many engineering fields have relied on the same assumption of local thermodynamic equilibrium in multi-physics codes, such as combustion, geology and metallurgy. The onus is on the user for judging whether equilibrium calculations are appropriate for a particular simulation.

The current capabilities of Thermochemica and the data-file parser are summarized below<sup>3</sup>:

- Systems comprised of various combinations of pure stoichiometric phases, and ideal and non-ideal solution phases,
- Non-ideal regular solution phases based on the “QKTO”, “RKMP”, “SUBL”, “SUBG”, and “SUBQ” identification tags,

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<sup>1</sup>The simulation of transient behaviour in a nuclear reactor may prove less appropriate for thermochemical equilibrium calculations. However, this approximation may still be significantly better than neglecting thermochemistry entirely, as is the case in many traditional nuclear fuel performance codes.

<sup>2</sup>For the case of simulating nuclear fuel behaviour in a multi-physics code, a thermodynamic system is represented by a finite element, which is sufficiently discretized to capture spatial variations in the local fission product inventory (resulting from neutron flux depression and mass diffusion across the fuel). Thus, the statement of atoms of the different elements being randomly mixed in nuclear fuel is valid in this context.

<sup>3</sup>Thermochemica is limited by the ChemSage data-files that are used as input, which is a function of the FactSage software. An earlier version of the software has been tested with a fictive system representing 118 elements, 1500 species and 20 solution phases [4].

- Between 2 and 4 constituents per mixing term in a QKTO phase,
- Systems with a minimum number of 2 elements and a maximum of 48 elements,
- Systems with up to 42 solution phases,
- Systems with up to 1500 species,
- Systems with fewer chemical elements than what is represented by the data-file,
- Species with magnetic contributions to the standard molar Gibbs energy terms, and
- Species with multiple particles per mole.

Several limitations in the current version include:

- Only ChemSage data-files can be parsed, although the CSFAP code can convert ThermoCalc (\*.tdb) data files to the ChemSage (\*.dat) format.
- Some non-ideal models available in FactSage are not currently available in Thermochemica,
- Real gases cannot be considered, and
- Aqueous solution phases cannot be used in the current version<sup>4</sup>.

Acceptable input units include [character strings expected by software are in square brackets]:

- Temperature units in Kelvin [K], Celsius [C], Fahrenheit [F] or Rankine [R],
- Hydrostatic pressure units can be in atmospheres [atm], pounds per square inch [psi], bar [bar], Pascals [Pa] and kilopascals [kPa], and
- Mass units can be in moles [mol], mole fraction [mole fraction], atoms [atoms], gram-atoms [gram-atoms], atom fraction [atom fraction], kilograms [kilograms], grams [grams], pounds [pounds], and mass fraction [mass fraction].

The units for temperature, hydrostatic pressure and mass are internally converted to Kelvin, atmospheres and atom fractions, respectively. Acceptable input quantities are as follows:

- Temperature,  $295 \text{ K} \leq T \leq 6,000 \text{ K}$ ,
- Hydrostatic pressure,  $10^{-6} \text{ atm} \leq P \leq 10^6 \text{ atm}$ , and
- Atom fraction,  $10^{-10} \leq \chi_j \leq 1$ . The motivation for applying a minimum atom fraction of  $10^{-10}$  is for computational reasons and is discussed in §3.2.

As is the case in any scientific or engineering software, the user is ultimately responsible for judging whether the results produced by the software are reasonable for a particular application. Significant efforts have been dedicated to ensure that the results produced by Thermochemica are numerically correct; however, the results produced by the software depend on the thermodynamic data and models specified by the user. The user should be aware of limitations in using certain thermodynamic datasets and models in addition to the limitation of performing *equilibrium* calculations.

## 2 Background

A brief background in classical thermodynamics and the conditions for thermodynamic equilibrium is provided for sake of completion. A thorough discussion of the conditions for thermodynamic equilibrium is provided by Piro [4] and details of the numerical methods employed by Thermochemica are given by Piro *et al.* [1, 5, 6].

### 2.1 Glossary

Table 1 summarizes many thermodynamic terms that are used in this document and in comments located in the source code.

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<sup>4</sup>An earlier version of the software was capable of handling aqueous solution phases [4].

Table 1: A glossary of thermodynamic terms relevant to this document.

Term	Symbol	Definition
Activity	$a_i$	A dimensionless quantity related to the chemical potential of a substance. The activity is equivalent to mole fraction, $x_i$ , for an ideal solution phase and unity for a stoichiometric phase that is stable.
Activity Coefficient	$\gamma_i$	The activity coefficient accounts for the departure of a substance from ideal behaviour and is defined by $\gamma_i = a_i/x_i$ . This is related to the partial molar excess Gibbs energy of mixing.
Atomic Number	$Z$	The number of protons in the nucleus of an atom, and also its positive charge. Each chemical element has its characteristic atomic number.
Chemical Potential	$\mu_i$	A measure of the effect on the change of the Gibbs energy of the system by the introduction of substance $i$ . The chemical potential is formally defined as $\mu_i = (\partial G/\partial n_i)_{T,P,n_{k \neq i}}$ .
Closed System		A system that permits the exchange of heat and work with its surroundings at constant mass.
Component		A phase component refers to a species of a solution phase that can be varied independently. A system component refers to the “minimum number of independent species necessary to define the composition in all phases of a system” [7].
Constituent		A constituent of a solution phase refers to a particular species in a particular phase and/or a particular sublattice. For example, “H <sub>2</sub> O” is a species, whereas “H <sub>2</sub> O <sub>(g)</sub> ” refers to H <sub>2</sub> O in the gaseous phase and “H <sub>2</sub> O <sub>(l)</sub> ” refers to H <sub>2</sub> O in a liquid phase.
Element		A chemical element as it appears on the Periodic Table of the Elements.
Gibbs Energy	$G$	A thermodynamic potential measuring the maximum amount of useful work obtainable from an isothermal-isobaric closed system. The Gibbs energy is defined as the difference between enthalpy and the product of temperature and entropy [7].
Ideal Solution		A solution phase that is comprised of constituents that mix ideally, i.e., the activity coefficient is equal to unity.
Ion		An atom or molecule where the number of electrons does not equal the number of protons, and thus has a positive or negative charge.
Isobaric		A system at constant hydrostatic pressure.
Isothermal		A system at constant temperature.
Isotope		One of several nuclides with the same atomic number (i.e., the same chemical element), but with different atomic weights and thus different nuclear properties.
Molality		Molality denotes the number of moles of a solute $i$ per kilogram of solvent (not solution).
Mole	$n_i$	A quantity of mass measured as $6.02214179 \times 10^{23}$ atoms. Equivalent to one gram-atom for a pure element.
Mole Fraction	$x_i$	The fraction of moles of a particular species in a particular solution phase.
Non-Ideal Solution		A solution phase that is comprised of constituents that do not mix ideally and the activity coefficient is not equal to unity. The departure from ideal mixing behaviour is represented by the partial molar excess Gibbs energy of mixing, which is typically provided by a thermodynamic model.
Partial Molar Excess Gibbs Energy of Mixing	$g_{i(\lambda)}^{ex}$	The partial molar excess Gibbs energy of mixing represents the contribution to the chemical potential term due to non-ideal behaviour. This is the difference between the real chemical potential of a substance and that from assuming ideal mixing behaviour.

Phase		A body of matter that is uniform in chemical composition and physical state. Phases are separated from one another by a physical discontinuity. A phase is not to be confused with a state of matter. For example, there are three different phases of pure uranium in the solid state (i.e., orthogonal, tetragonal and body centred cubic). Thermochemica considers two different types of phases: stoichiometric phases and solution phases. “Mixtures” are numerically treated as solution phases.
Phase Assemblage		A unique combination of phases predicted to be stable at thermodynamic equilibrium for a particular system.
Pure Condensed Phase	$\omega, \Omega$	A condensed phase with invariant stoichiometry and may be interpreted mathematically as containing a single species with unit concentration. A pure condensed phase may be in either liquid or solid states. The symbol $\omega$ represents the phase index and $\Omega$ represents the number of stable pure condensed phases.
Solution Phase	$\lambda, \Lambda$	A phase containing a mixture of multiple species. Traditionally, the term “solution” refers to a condensed phase. Since the mathematical representation of a gaseous mixture is identical to that of a solution phase, the programming is simplified in the current work by calling a gaseous “mixture” as a “solution”. The symbol $\lambda$ represents the phase index and $\Lambda$ represents the number of stable solution phases.
Species	$i$	A chemically distinct molecular entity. For example, $\text{H}_2\text{O}$ has a distinct chemical composition, but can be in gaseous, liquid or solid phases. This differs from the term constituent, which refers to a particular species in a particular phase and/or sublattice. The phase index is represented by the variable $i$ .
Standard Molar Gibbs Energy	$g_{i(\lambda)}^\circ$	The standard molar Gibbs energy of a pure constituent is the Gibbs energy of that constituent with unit activity. This quantity is computed using values from the ChemSage data-file.
State		A state of matter distinguishes the distinct form that matter may take on. This includes solid, liquid, gas, plasma, and the Bose-Einstein condensate. Thermochemica only considers the solid, liquid and gaseous states. The term “state” is not to be confused with the term “phase”.
Stoichiometry		This refers to the relative amounts of atoms per formula mass of a substance.
Surroundings		The space remaining in the Universe outside of a system.
System		A portion of the Universe with a perimeter defined by real or imaginary boundaries.

## 2.2 Conditions for Thermodynamic Equilibrium

The foundation of computing thermodynamic equilibria is based on minimizing the integral Gibbs energy of a closed system at constant temperature and hydrostatic pressure. This equilibrium condition is a manifestation of the first and second laws of thermodynamics, which state that the energy of the Universe is constant and that the entropy of the Universe tends to a maximum [8]. Alternatively, one could minimize the Helmholtz energy at constant temperature and volume. Traditionally, the Gibbs energy function is used instead of the Helmholtz energy because hydrostatic pressure is more easily controlled in experiments than volume, particularly for systems involving condensed phases [4].

From a numerical point of view, the objective of computing thermodynamic equilibria is to determine a unique combination of phases and their composition that yields a global minimum in the integral Gibbs energy of an isothermal, isobaric system, which is generally given by

$$\frac{G}{RT} = \sum_{\lambda=1}^{\Lambda} n_{\lambda} \sum_{i=1}^{N_{\lambda}} x_{i(\lambda)} \tilde{\mu}_i + \sum_{\omega=1}^{\Omega} n_{\omega} \tilde{\mu}_{\omega} \quad (1)$$

where  $R$  [ $\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$ ] is the ideal gas constant,  $T$  [K] is the absolute temperature,  $N_\lambda$  denotes the number of constituents in solution phase  $\lambda$ , and  $\Lambda$ , and  $\Omega$  represent the total number of solution phases and pure condensed phases in the system, respectively. The mole fraction of constituent  $i$  in solution phase  $\lambda$  is  $x_{i(\lambda)}$  [unitless] and the number of moles of solution phase  $\lambda$  and pure condensed phase  $\omega$  are denoted by  $n_\lambda$  and  $n_\omega$  [mol] respectively. Finally,  $\tilde{\mu}_i$  and  $\tilde{\mu}_\omega$  [unitless] represent the dimensionless chemical potential of constituent  $i$  in solution phase  $\lambda$  and pure condensed phase  $\omega$ , respectively. The diacritic symbol  $\sim$  is used to emphasize that chemical potential terms used in this discussion are dimensionless.

The chemical potential of a constituent in an ideal solution phase incorporates the standard molar Gibbs energy of the pure species,  $\tilde{g}_{i(\lambda)}^o$  [unitless], and the entropic contribution due to mixing as a function of its mole fraction, such that [8]

$$\tilde{\mu}_i = \tilde{g}_{i(\lambda)}^o + \ln(x_{i(\lambda)}) \quad (2)$$

The chemical potential of a non-ideal solution phase constituent adds a partial molar excess Gibbs energy of mixing term,  $\tilde{g}_{i(\lambda)}^{ex}$  [unitless], to more closely represent experimentally observed behaviour

$$\tilde{\mu}_i = \tilde{g}_{i(\lambda)}^o + \tilde{g}_{i(\lambda)}^{ex} + \ln(x_{i(\lambda)}) \quad (3)$$

Finally, the chemical potential of a pure condensed phase does not include a composition dependent term, since the stoichiometry is by definition invariant. The equilibrium condition is subject to several constraints, including conservation of mass and the Gibbs phase rule. The total mass (in gram-atoms) of element  $j$  is

$$b_j = \sum_{\lambda=1}^{\Lambda} n_\lambda \sum_{i=1}^{N_\lambda} x_{i(\lambda)} \nu_{i,j} + \sum_{\omega=1}^{\Omega} n_\omega \nu_{\omega,j} \quad (4)$$

where  $\nu_{i,j}$  and  $\nu_{\omega,j}$  [g-at/mol] are the stoichiometric coefficients of element  $j$  in constituent  $i$  and pure condensed phase  $\omega$  respectively. The Gibbs phase rule is generally given as

$$F = C - \Phi + 2 + \Xi \quad (5)$$

where  $F$  represents the number of degrees of freedom,  $C$  is the number of system components<sup>5</sup>,  $\Phi$  (i.e.,  $\Phi = \Lambda + \Omega$ ) represents the number of phases currently predicted to be stable in the system, and  $\Xi$  represents any additional constraints (e.g., ionic charge). Maintaining constant temperature and pressure removes two degrees of freedom. Since the number of degrees of freedom must be non-negative, the phase rule dictates that the maximum number of coexisting phases at equilibrium cannot exceed the number of chemical elements in a closed isothermal-isobaric system.

Finally, Gibbs' criteria for equilibrium require that the Gibbs energy of a closed system be at a global minimum at constant temperature and pressure. An equivalent statement defines the chemical potential of any constituent in a stable phase by a linear function of the element potentials as [8, 9]

$$\tilde{\mu}_i = \sum_{j=1}^C \nu_{i,j} \tilde{\Gamma}_j \quad (6)$$

The above linear equality corresponds to species and phases that are stable at thermodynamic equilibrium. Non-stable phases are defined by the “driving force” or “tangent plane distance function”, which is defined as [5]:

$$\pi_\lambda = \min_x \sum_{i=1}^{N_\lambda} x_{i(\lambda)} \left( \mu_{i(\lambda)} - \sum_{j=1}^E \nu_{i,j} \Gamma_j \right) \quad (7)$$

which is positive for meta-stable phases and a negative value is used to indicate that the system is not at thermodynamic equilibrium.

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<sup>5</sup>Note that a system component may represent a pure chemical element, a fixed combination of chemical elements, or an electron (i.e., for ionic phases).

In summary, the necessary conditions for thermodynamic equilibrium require that the chemical potentials of all stable solution phase constituents and pure condensed phases abide by Gibbs’ criteria, and that the conservation of mass and the Gibbs phase rule are satisfied. The sufficient condition for equilibrium – which ensures that the system is at a global minimum yielding a unique solution – is that the driving force is greater than zero for all phases believed to not be stable.

The interested reader is referred to the literature on specific details on determining initial estimates for Thermochemica [10], a general overview of numerical techniques used by the solver [4, 1], and numerical methods in ensuring that the necessary [11] and sufficient conditions [5] have been satisfied.

## 3 Code Development

### 3.1 Overview

Thermochemica is written in Fortran and complies with the Fortran 90/2003 standard. A description of each subroutine is provided in the dOxygen documentation located in the Appendix. The documentation for each subroutine includes a description of the purpose of each subroutine and a description of all input/output parameters. A description of each local variable is provided for each subroutine and a description of each global variable is provided in each module. All of the programming in Thermochemica is original unless otherwise specified in the source code. These subroutines have been modified from Numerical Recipes [12] and have been referenced appropriately in the source code.

### 3.2 Checking Input Values

Thermochemica will test to ensure that the input variables are appropriate for consideration. For instance, the units for temperature, pressure and mass are compared with the list of acceptable units highlighted in §1.3. Temperature, pressure and mass are internally converted to units of Kelvin, atmospheres and atom fractions, respectively. The values of temperature, pressure and mass are tested to be real and within an acceptable range. The acceptable range for temperature is  $295\text{ K} \leq T \leq 6000\text{ K}$  and the acceptable range for hydrostatic pressure is  $10^{-6}\text{ atm} \leq P \leq 10^6\text{ atm}$ . An error is recorded (via the variable `INFOThermo`) and execution halts if either temperature or pressure is out of range or not real.

An element that has an atom fraction that is out of range will not be considered part of the system. An error will be recorded and execution will halt if the number of elements in the system is less than two or if any of the atom fractions are non-real or negative. The minimum allowable atom fraction of any element is defined as the division of machine precision ( $10^{-15}$  for double precision variables) by the relative error tolerance for the mass balance equations. By default, the tolerance for the relative errors for the mass balance equations is  $10^{-5}$ , which yields a minimum atom fraction tolerance of  $10^{-10}$ . The motivation for applying this tolerance is in accordance with numerical errors associated with solving a system of linear equations representing the mass balance constraints. This tolerance is generally exceedingly small in comparison to experimental errors and uncertainties, and it is considered more than sufficient for investigating a wide array of engineering problems.

### 3.3 Example Fortran Application Programming Interface

Listing 1 provides an example Fortran wrapper to call the data-file parser for ChemSage-style .dat files (`ParseCSDDataFile.f90`), Thermochemica (`Thermochemica.f90`), the destructor (`ResetThermoAll.f90`, optional) and the debugger (`ThermoDebug.f90`, optional). This example wrapper is provided as a Fortran program with the software in the *test* directory and is labeled `Thermo.F90`. The destructor deallocates all allocatable arrays used by both the parser and the solver. The debugger prints an error message to the screen in the event that an error has been encountered. The integer scalar variable `INFOThermo` is used to identify a successful exit or to record a specific error encountered in execution. One should verify a null value for `INFOThermo` after calling the parser and before calling Thermochemica. An error may be returned in the event that there

is an error opening, reading or closing the data-file. This particular example considers the carbon-oxygen system at 1000 K and 1 atm with 2 mol C and 1 mol O.



```

!-----
!
! Purpose:
! =====
!
! The purpose of this program is to provide an example Application
! Programming Interface (API) for Thermochemica to simulate the interaction
! with another code. The module "ModuleThermoIO" is intended to be used by
! another code that calls Thermochemica for input/output operations.
!-----

program thermo

  USE ModuleThermoIO

  implicit none

  ! Specify units:
  cInputUnitTemperature = 'K'
  cInputUnitPressure    = 'atm'
  cInputUnitMass        = 'moles'
  cThermoFileName       = DATA_DIRECTORY // 'C-O.dat'

  ! Specify values:
  dPressure              = 1D0
  dTemperature           = 1000D0
  dElementMass(6)        = 2D0          ! C
  dElementMass(8)        = 1D0          ! O

  ! Specify output and debug modes:
  iPrintResultsMode      = 2
  lDebugMode             = .FALSE.

  ! Parse the ChemSage data-file:
  call ParseCSDDataFile(cThermoFileName)

  ! Call Thermochemica:
  if (INFOThermo == 0)    call Thermochemica

  ! Perform post-processing of results:
  if (iPrintResultsMode > 0) call PrintResults

  ! Destruct everything:
  if (INFOThermo == 0)    call ResetThermoAll

  ! Call the debugger:
  call ThermoDebug

end program thermo

```

Listing 1: A sample Fortran wrapper is provided that calls the ChemSage data-file parser, Thermochemica, the destructor (optional) and the debugger (optional).

As described in the Appendix, the variable `dElementMass` is a double real vector indexed from (0:118), where each coefficient corresponds to an element on the periodic table. The zeroth coefficient corresponds to the electron, which is used in ionic phases, such as aqueous, plasma or the solid solution phases based on the compound energy formalism (yet to be implemented). The system shown in Listing 1 represents carbon and oxygen, which have atomic numbers 6 and 8 respectively. Therefore, the example system above has 2 mole of carbon and 1 moles of oxygen.

### 3.4 Example C++ Application Programming Interface

Using Thermochemica with C++ requires declaration of its functions using the C convention and keyword `extern` keyword. Keywords `extern "C"` specify that the linkage conventions of C language are being used for the Thermochemica function declarations. This avoids C++ name mangling, and makes the function symbol name to be the same as the function name. FORTRAN compilers usually append an underline to the function symbol which is done using `FORTRAN_ALL` macro. An example header file, `Thermochemica.h`, for the C++ API for Thermochemica is given in the Listing 2 below.

```
/* FORTRAN function wrapper */
#ifdef __bg__ // On Blue Gene Architectures there is no underscore
#define FORTRAN_CALL(name) name
#else // One underscore everywhere else
#define FORTRAN_CALL(name) name##_
#endif

namespace Thermochemica
{
    extern "C"
    {
        void FORTRAN_CALL(setthermofilename)(const char*);
        void FORTRAN_CALL(ssparsecsdatafile)();
        void FORTRAN_CALL(setunittemperature)(const char*);
        void FORTRAN_CALL(setunitpressure)(const char*);
        void FORTRAN_CALL(setunitmass)(const char*);
        void FORTRAN_CALL(setelementmass)(int*, double*);
        void FORTRAN_CALL(setstandardunits)();
        void FORTRAN_CALL(checkinfothermo)(int*);
        void FORTRAN_CALL(thermochemica)();
        void FORTRAN_CALL(getmolfraction)(int*, double*, int*);
        void FORTRAN_CALL(getchemicalpotential)(int*, double*, int*);
        void FORTRAN_CALL(getelementpotential)(int*, double*, int*);
        void FORTRAN_CALL(getoutputchempot)(char*, double*, int*);
        void FORTRAN_CALL(thermodebug)();
        void FORTRAN_CALL(resetinfothermo)();
        void FORTRAN_CALL(resetthermo)();
        void FORTRAN_CALL(resetthermoall)();
    }
    void ConvertToFortran(char*, std::size_t, const char*);
}
```

Listing 2: A sample C++ header for application programming interface definition.

An example C++ program using using Thermochemica is given in the Listing 3 below. Note that conversion between null terminated strings in C++ and FORTRAN strings is required.

```

#include <cstdlib>
#include <iostream>
#include <string>
#include "Thermochimica.h"

int main()
{
    double Pressure      = 1D0;
    double Temperature    = 1000D0;
    double ElementMass6   = 2D0;
    double ElementMass8   = 1D0;
    int     idbg           = 0;
    int     iElement       = 0;
    double  dMol           = 0.0;

    char cThermoFileName[120];
    std::string thermofile ("file.dat");

    // Set the thermodynamic model file name
    Thermochimica::ConvertToFortran(cThermoFileName,
                                     sizeof cThermoFileName, thermofile.c_str());
    FORTRAN_CALL(Thermochimica::setthermofilename)(cThermoFileName);
    // Read the thermodynamic model from file
    FORTRAN_CALL(Thermochimica::ssparssecsdatafile)();
    FORTRAN_CALL(Thermochimica::checkinfothermo>(&idbg);
    if( idbg != 0 ){
        cout<<"Thermochimica_data_file_cannot_be_parsed.";
        exit(1)
    }
    // Set units, temperature and pressure
    FORTRAN_CALL(Thermochimica::setstandardunits)();
    FORTRAN_CALL(Thermochimica::settemperaturepressure>(&Temperature, &Pressure);

    // Reset all elemental amounts
    FORTRAN_CALL(Thermochimica::setelementmass>(&iElement, &dMol);
    // Set element amounts
    iElement=6; dMol=ElementMass6;
    FORTRAN_CALL(Thermochimica::setelementmass>(&iElement, &dMol);
    iElement=8; dMol=ElementMass8;
    FORTRAN_CALL(Thermochimica::setelementmass>(&iElement, &dMol);

    // Calculate thermochemical equilibrium
    FORTRAN_CALL(Thermochimica::thermochimica)();
    // Check for error status
    FORTRAN_CALL(Thermochimica::checkinfothermo>(&idbg);
    if( idbg != 0 ){
        FORTRAN_CALL(Thermochimica::thermodebug)();
    }
}

```

Listing 3: A sample C++ program.

### 3.5 Useful Output Variables

There are several useful variables that are provided as output that may be used by another code. Table [2](#) summarizes many of these variables organized by their corresponding module. The Hungarian variable naming convention is adopted by Thermochemica, which precedes each variable name by a single lower case character that identifies the type of variable. This includes the prefixes **n-** to identify the number of something (e.g., the number of elements in the system), **i-** to identify an integer, **d-** to identify a double real, **c-** to identify a character and **l-** to identify a logical variable. There are a few exceptions to this naming convention, such as the coefficients **i** and **j** (that typically represent array coefficients) and the error code **INFOThermo**.

Table 2: Summary of useful variables provided as output from Thermochemica.

Module Name	Variable Name (Array Dimension)	Description
ModuleThermo	nElements	An integer scalar representing the number of elements in the system.
	nSpecies	An integer scalar representing the number of species in the system.
	nConPhases	An integer scalar representing the number of coexisting pure condensed phases.
	nSolnPhases	An integer scalar representing the number of coexisting solution phases.
	nSolnPhasesSys	An integer scalar representing the total number of solution phases in system data-file.
	iAssemblage (1:nElements)	An integer vector representing the indices of the phases predicted to coexist at equilibrium.
	iPhase (1:nSpecies)	An integer vector representing the phase index of all species in the system.
	dMolesPhase (1:nElements)	A double real vector representing the number of moles of coexisting phases at equilibrium (corresponds directly to iAssemblage).
	dMolesSpecies (1:nSpecies)	A double real vector representing the number of moles of all species in the system.
	dMolFraction (1:nSpecies)	A double real vector representing the mole fraction of each species in the system data-file.
	dStdGibbsEnergy (1:nSpecies)	A double real vector representing the standard Gibbs energies of all species in the system.
	dChemicalPotential (1:nSpecies)	A double real vector representing the chemical potentials of all species in the system.
	dElementPotential (1:nElements)	A double real vector representing the element potentials.
	cElementName (1:nElements)	A character vector representing the name of each chemical element in the system.
	cSolnPhaseName (1:nSolnPhasesSys)	A character vector representing the name of all solution phases in the system data-file.
	cSpeciesName (1:nSpecies)	A character vector representing the name of all species in the system data-file.
ModuleThermoIO	INFOThermo	An integer scalar representing a successful exit or an error (used in a similar style as LAPACK).
ModuleGEMSolver	dPartialExcessGibbs (1:nSpecies)	A double real vector representing the partial molar excess Gibbs energy of mixing of each species in the system data-file.

The integer vector **iAssemblage** stores the indices of all phases predicted to be stable at equilibrium. The dimension of this vector is defined as the number of elements in the system, thus the Gibbs phase rule is necessarily satisfied. The coefficients 1:nConPhases in **iAssemblage** represent the pure condensed phases, the coefficients nElements-nSolnPhases+1:nElements represent solution phases and all other entries are zero. The values of the coefficients for pure condensed phases (which are positive) correspond directly to coefficients in **dStdGibbsEnergy** and **cSpeciesName**. The values of the coefficients for solution phases are stored as negative integers in **iAssemblage**. The absolute values of coefficients in **iAssemblage** corresponding

to solution phases correspond directly to coefficients in `cSolnPhaseName` (see next section for an example). All phases in `iAssemblage` correspond directly to `dMolesPhase`. Also, the vector `iPhase` represents the phase index corresponding to each species in the system. Furthermore, the chemical elements represented in the character vector `cElementName` correspond directly to the element potentials in `dElementPotential`. Finally, the variable `INFOThermo` indicates a successful exit or an error. An example is provided in the next section.

### 3.6 Example System

Consider the C-O system from Listing 1, which contains 1 solution phase containing a total of 16 species and pure condensed phases. Table 3 summarizes pertinent variables that are provided by Thermochemica at thermodynamic equilibrium.

Table 3: Sample calculated data from the example given in Listing 1 in the format of Table 2. All double real variables are cropped to three significant figures.

Module Name	Variable Name (Array Dimension)	Value
ModuleThermo	nElements	2
	nSpecies	16
	nConPhases	1
	nSolnPhases	1
	nSolnPhasesSys	1
	iAssemblage (1:nElements)	13, -1
	iPhase (1:nSpecies)	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, -1, -1
	dMolesPhase (1:nElements)	1.23, 0.77
	dMolesSpecies (1:nSpecies)	$4.01 \times 10^{-30}$ , $1.75 \times 10^{-34}$ , $3.04 \times 10^{-32}$ , $1.20 \times 10^{-40}$ , $1.27 \times 10^{-40}$ , $2.97 \times 10^{-21}$ , $4.69 \times 10^{-22}$ , $1.00 \times 10^{-43}$ , 0.55, $4.66 \times 10^{-20}$ , 0.23, $3.55 \times 10^{-14}$ , 0, 0, 0, 0
	dMolFraction (1:nSpecies)	$5.19 \times 10^{-30}$ , $2.26 \times 10^{-34}$ , $3.93 \times 10^{-32}$ , $1.55 \times 10^{-40}$ , $1.64 \times 10^{-40}$ , $3.85 \times 10^{-21}$ , $6.06 \times 10^{-22}$ , $1.29 \times 10^{-43}$ , 0.71, $6.02 \times 10^{-20}$ , 0.29, $4.60 \times 10^{-14}$ , 0.1, 0.1, 0.1, 0.1
	dStdGibbsEnergy (1:nSpecies)	65.91, 74.42, 67.75, 85.58, 84.00, 9.30, -26.55, -14.37, -38.88, 3.51, -75.70, -49.26, -1.52, -0.82, 100000, 100000
	dChemicalPotential (1:nSpecies)	-1.52, -3.05, -4.57, -6.09, -7.61, -37.70, -75.41, -113.12, -39.23, -40.75, -76.93, -79.98, 0, 0, 0, 0
	dElementPotential (1:nElements)	-37.70, -1.52
	cElementName (1:nElements)	“O”, “C”
	cSolnPhaseName (1:nSolnPhasesSys)	“gas_ideal”
	cSpeciesName (1:nSpecies)	“C”, “C2”, “C3”, “C4”, “C5”, “O”, “O2”, “O3”, “CO”, “C2O”, “CO2”, “C3O2”, “C_graphite(s)”, “C_Diamond(s2)”, “C”, “O”
ModuleThermoIO	INFOThermo	0
ModuleGEMSolver	dPartialExcessGibbs (1:nSpecies)	0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0

The first coefficient of the integer vector **iAssemblage** corresponds to the only pure condensed phase in the system and the second coefficient corresponds to the only solution phase in the system. Thus, the first coefficient in **iAssemblage** refers to the 13<sup>th</sup> species in the system, which corresponds to “C\_Graphite(s)” in the **cSpeciesName** vector. Similarly, the second coefficient in **iAssemblage** refers to the 1<sup>st</sup> solution phase in the system, which corresponds to “gas.ideal” in the **cSolnPhaseName** vector.

One can compare the results from Table 3 to the equations established in §2. For example, one could

compute the chemical potential of C in the graphite phase with equation (3) using values from Table 3:

$$\begin{aligned}\tilde{\mu}_{C_{graphite(s)}} &= \tilde{g}_{C_{graphite(s)}}^o + \tilde{g}_{C_{graphite(s)}}^{ex} + \ln(x_{C_{graphite(s)}}) \\ \tilde{\mu}_{C_{graphite(s)}} &= -1.52 - 0 + \ln(1) \\ \tilde{\mu}_{C_{graphite(s)}} &= -1.52\end{aligned}\tag{8}$$

which can also be compared to equation (6),

$$\begin{aligned}\tilde{\mu}_{C_{graphite(s)}} &= \sum_{j=1}^E a_{C_{graphite(s)},j} \tilde{\Gamma}_{,j} \\ \tilde{\mu}_{C_{graphite(s)}} &= (1)(-1.52) \\ \tilde{\mu}_{C_{graphite(s)}} &= -1.52\end{aligned}\tag{9}$$

Also, the chemical potential of C in the gaseous phase must be equivalent to C in the graphite phase. The chemical potential of C in the graphite phase is computed using Equation 2

$$\begin{aligned}\tilde{\mu}_{C_{gas-ideal}} &= \tilde{g}_{C_{gas-ideal}}^o + \ln(a_{C_{gas-ideal}}) \\ \tilde{\mu}_{C_{gas-ideal}} &= 65.91 + \ln(5.19 \times 10^{-30}) \\ \tilde{\mu}_{C_{gas-ideal}} &= -1.52\end{aligned}\tag{10}$$

To reduce memory requirements in a multi-physics code that makes use of Thermochemica, it is recommended that one stores `dElementPotential` instead of `dChemicalPotential` because the former requires significantly less memory than the latter, and one can compute `dChemicalPotential` for any solution phase constituent or pure condensed phase from `dElementPotential`.

### 3.7 Errors and Debugging

The variable `INFOThermo` is used to identify a successful exit or an error that has been encountered by either the parser or Thermochemica. The subroutine `ThermoDebug` is available for debugging purposes, which prints an error message to the screen that corresponds to a particular value of `INFOThermo`. Table 4 summarizes the errors that may be encountered by Thermochemica.



Table 4: Summary of error codes from INFOThermo.

Value of INFOThermo	Description of Error
0	Successful exit.
1	Temperature is out of range or a NAN.
2	Pressure is out of range or a NAN.
3	Mass is out of range or a NAN.
4	The input variable cThermoInputUnits is not recognizable.
5	The number of elements in the system is out of range.
6	The specified data-file was not found.
7	There is an unknown error in reading the data-file.
8	The number of solution phases in the system exceeds the maximum allowable number.
9	A pure chemical species is needed to be present in the database for each element.
10	The Leveling subroutine failed to determine an appropriate phase assemblage for further consideration.
11	The PostLeveling subroutine failed.
12	The non-linear solver failed to converge.
13	The non-linear solver detected a NAN.
14	The non-linear solver determined that there are no solution phases present, but the system cannot be represented by only pure condensed phases.
15	Failed to deallocate allocatable arrays from the ModuleThermo, ModuleThermoIO, and/or ModuleGEMSolver modules.
16	The LAPACK driver routines were not able to invert the Jacobian matrix in the non-linear solver.
17	The solution phase type is not supported.
18	Failed to deallocate allocatable arrays used in the ModuleParseCS module.
19	Failed to deallocate allocatable arrays used in the CheckSystem subroutine.
20	Failed to deallocate allocatable arrays used in the LevelingSolver subroutine.
21	Failed to deallocate allocatable arrays used in the InitGEM subroutine.
22	Failed to deallocate allocatable arrays used in the CompMolSolnPhase subroutine.
23	Failed to deallocate allocatable arrays used in the GEMBroyden subroutine.
24	A NAN was detected in the CompStoichSolnPhase subroutine.
25	A NAN was detected in the CompMolFraction subroutine.
26	Failed to deallocate allocatable arrays used in the CompMolAllSolnPhases subroutine.
27	The CheckQKTOSolnPhase subroutine failed to converge.
[100,1000)	Error in reading line $\text{int}(\text{INFOThermo} - 100)$ of the header section of the data-file.
[1000,10000)	Error in reading entry $\text{int}(\text{INFOThermo} - 1000)/100$ of solution phase $\text{int}(\text{INFOThermo} - 1000 - 100 * \text{int}(\text{INFOThermo} - 1000)/100)$ of the data-block section of the data-file.
Other	An unknown error has occurred.

### 3.8 Re-Initializing Calculations

In many applications – particularly when coupling Thermochemica to other applications – numerous calls to Thermochemica must be made with similar input conditions (i.e. temperature, pressure, and concentrations of elements). For example, in a finite element simulation the conditions at one integration point are likely similar to those at neighboring integration points, as well as to the conditions at that integration point at the

next timestep. As the resulting phase assemblages and species concentrations are likely to be similar, making use of the results of the preceding calculations in place of the leveling solver can yield a better starting point for the Gibbs Energy Minimization (GEM) procedure. This results in cost savings both by bypassing leveling and by reducing the number of GEM iterations required to converge the system.

To accommodate calculation re-initialization it is necessary to store a number of variables. If Thermochimica is coupled to another code, it is likely that these variables will have to be passed out to that code, stored until needed, and then passed back into Thermochimica. For example, in a finite element code that is using data from previous timesteps to re-initialize Thermochimica calculations, after each call at each integration point the re-initialization data must be stored so that it can be re-used during the next timestep when Thermochimica is called for that point again. Between those calls, Thermochimica will be called at many other integration points and thus the re-initialization data cannot simply be expected to be held within Thermochimica.

`iAssemblage`, `dChemicalPotential`, `dMolesPhase`, `dElementPotential`, and `dMolFraction`, as well as an integer array created just for re-initialization, `iElementsUsed`, must be stored to enable re-initialization. `iElementsUsed` lists which elements were present in any non-zero amount in the calculation for which data was saved. If this differs from the current collection of elements, the re-initialization attempt is aborted. The suffix `_Old` is added to each when they are saved, and the variables with this prefix are held in the module `ModuleReinit`. Additional flags to track the status of the re-initialization procedure are also available. These include `iReinitRequested`, which the user must set to `.TRUE.` if re-initialization is desired, `iReinitAvailable`, which is set to `.TRUE.` after re-initialization data is successfully loaded, and `iReinitLoaded`, which is set to `.TRUE.` after a re-initialization is successfully loaded.

To make use of re-initialization, after a successful Thermochimica calculation `SaveReinitData` must be called. Then before the following calculation, `iReinitRequested` must be set to `.TRUE.`. If Thermochimica is coupled to another code, the developer must ensure that the `_Old` re-initialization variables are correctly supplied to Thermochimica. Useful subroutines for copying these variables to/from a coupled code are available in `CouplingUtils.f90`.

## 4 Building and Compiling Thermochimica

The first thing that one should do is pull all files from the online repository, which includes source files, a makefile, documentation, etc. The directory structure of Thermochimica is illustrated in Figure 1 below. The *bin* directory stores executables, *data* stores thermodynamic data-files, *doc* stores dOxygen documentation files (in both html and Latex format), *obj* stores Fortran object files and *src* stores all source code. The *src* directory contains a *shared* subdirectory containing all shared source f90 files, and the *test* directory contains all unit and application tests. Tests that are performed on a daily and weekly basis are stored in the *daily* and *weekly* subdirectories, respectively. The main directory contains the Makefile, Doxyfile and modules that are built by Fortran. Additionally, a script file (`"rundailytests"`) is provided that runs all daily tests.

Thermochimica is compiled on a regular basis with GNU compilers<sup>6</sup> on Mac OS-X and Linux operating systems. The only external library that Thermochimica is dependent on is LAPACK [13], which can be downloaded free of charge from the LAPACK web site<sup>7</sup>. The compiler can be specified with the *FC* variable in the Makefile, in addition to several compiler (i.e., FCFLAGS) and LAPACK (i.e., LDFLAGS) flags (if one chooses to change them).

To compile Thermochimica, go to the Thermochimica directory in the Terminal and type `make`. By default, a sample executable called "thermo" is created. Table 5 summarizes the commands that can be used from the provided Makefile.

<sup>6</sup>The most recent version of this compiler used is gfortran 7.4.0.

<sup>7</sup>The LAPACK linear algebra library can be downloaded from [netlib.org](http://netlib.org).

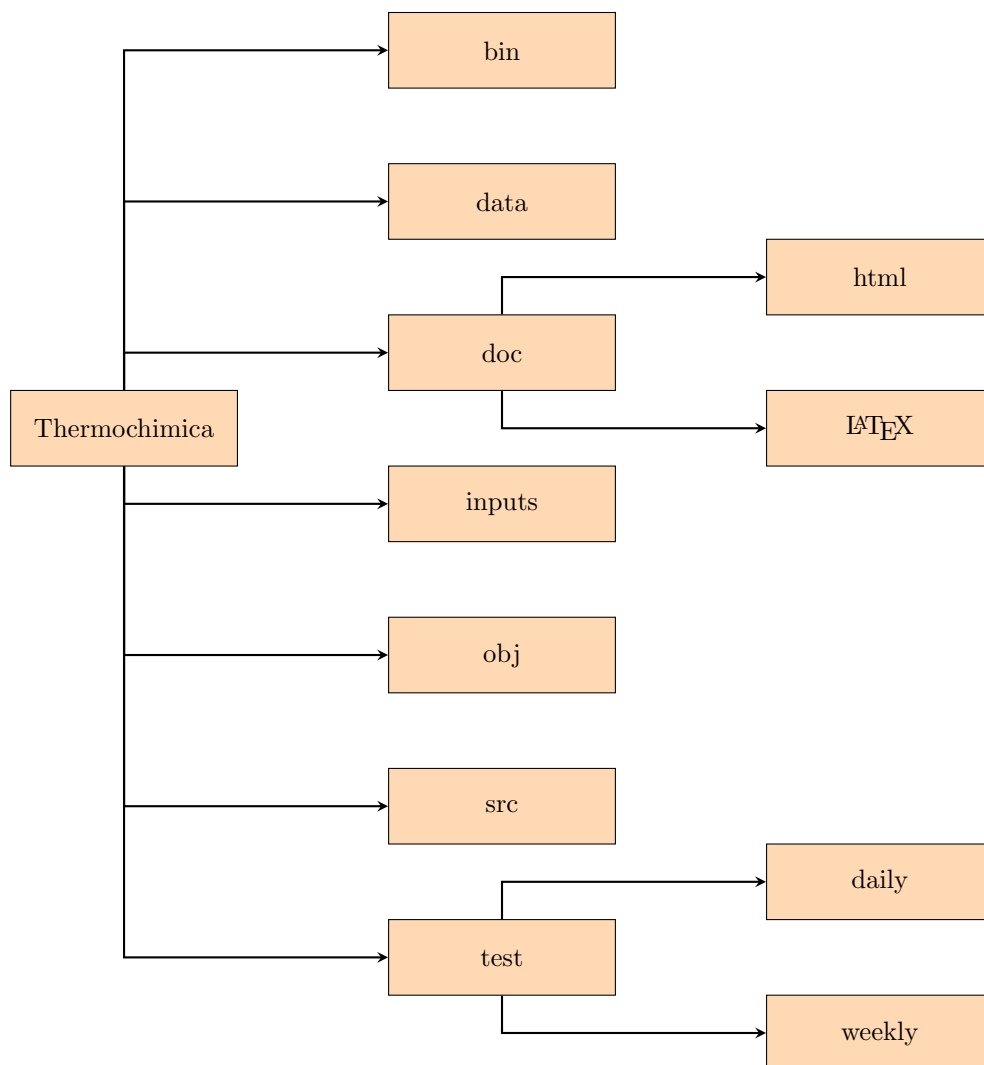


Figure 1: The data structure for Thermochemica.

To ensure that Thermochemica compiles without any issues and that it is producing correct results, the user is encouraged to run all daily and weekly unit and application tests. To run all daily tests, simply run the script file `./runtests` from the Terminal from the Thermochemica directory. Thermochemica will return the following for a successful test:

```
TestThermo01: PASS
```

and the following for a failure:

```
TestThermo01: FAIL <---
```

In the event that any tests fail, please report an issue on Thermochemica's [GitHub page](#) with the data-file that was used and a detailed description of the circumstances of the error. Thermochemica is automatically tested every time an update is made, and both the current status and past test results can be checked by visiting Thermochemica's [Travis-CI page](#). The Docker container used to build and run Thermochemica on Travis-CI is also available at [Docker Hub](#).

Table 5: Summary of all make commands.

Make Command	Description
<code>make</code> (or <code>make all</code> )	Compile Thermochemica with a default executable called “thermo”.
<code>make test</code>	Compile all unit tests (executables located in the <code>bin</code> directory).
<code>make dailytest</code>	Compile all daily unit tests (executables located in the <code>bin</code> directory).
<code>make weeklytest</code>	Compile all weekly unit tests (executables located in the <code>bin</code> directory).
<code>make doc</code>	Compile dOxygen HTML and L <sup>A</sup> T <sub>E</sub> X documents.
<code>make docHTML</code>	Compile dOxygen HTML documents only.
<code>make doclatex</code>	Compile dOxygen L <sup>A</sup> T <sub>E</sub> X documents only.
<code>make clean</code>	Clean object files.
<code>make cleandoc</code>	Clean all dOxygen files.
<code>make veryclean</code>	Clean all object files, dOxygen files, modules and executables.

## 5 Running Stand-Alone Thermochemica Calculations

Now that Thermochemica is installed and all the tests have passed, you may wish to use Thermochemica as a stand-alone application to perform some equilibrium calculations. The example Fortran API provided in §3.3 is a good starting point for custom stand-alone calculations. As in that example, you must set the units for temperature (`cUnitTemperature`: K, C, F, and R are accepted), pressure (`cUnitPressure`: atm, psi, bar, Pa, kPa accepted), and mass (`cUnitMass`: mass fraction, kilograms, grams, pounds, mole fraction, atom fraction, atoms, moles, and gram-atoms accepted). You must also supply a ChemSage style .dat file, and specify the path to this file in `cThermoFileName`. Next, specify the pressure (`dPressure`) and temperature (`dTemperature`), then the masses of all elements to be included in the system by using their atomic numbers to index into `dElementMass` (1 through 118 accepted). You may also optionally specify the output verbosity through `iPrintResultsMode`, where 0 is no printing, 1 is standard, and 2 adds details such as the functional norm and the numbers of stable pure and solution phases.

With that setup done, you can now run Thermochemica. Just call the parser, Thermochemica, print, reset, and debug in the same order as in the example API. Save your custom file in the `test` directory with any name ending with “.F90”, run `make`, and a corresponding executable will be placed in the `bin` directory.

Alternatively, a simple script input is available for stand-alone Thermochemica calculations. An example input script can be found in the `inputs` directory. To run this example, use the `ThermochemicaInputFileMode` executable in the `bin` directory, and supply the path to the example input script as a command-line argument (e.g. run `./bin/ThermochemicaInputFileMode inputs/trial-input.ti` from the root directory). To run your own calculations, make an input script similar to the example given, supplying the same inputs as required when running Thermochemica in the traditional mode. You may then execute the same executable but with the path to your input script as the argument. The input script is flexible with respect to order, spacing, and comments (comment lines start with any special character such as `!`, `#`, `%`, `/`, etc.). There is no need to recompile when using the input script mode!

## 6 Conclusions

A new equilibrium thermochemistry library called Thermochemica has been described that is intended for direct integration into multi-physics codes. Specifically, this solver is intended to provide input to thermodynamic material properties and boundary conditions for continuum-scale and meso-scale simulations. The development of this software is specifically aimed at addressing concerns with integrating commercial thermodynamic software into multi-physics codes. First of all, the development of new software facilitates the distribution of software that could otherwise be complicated by licensing issues. Secondly, advancements in the numerical approach in computing thermodynamic equilibria enhance computational performance and robustness, which is especially needed when integrated into large multi-physics codes. Finally, the software

has been intentionally designed from the beginning to handle extraordinarily large thermodynamic systems, thus eliminating any concern of software limitations regarding the number of chemical elements, species, pure condensed phases or solution phases.

The general application and use of Thermochemica have been reviewed in this document with an example Fortran application programming interface that demonstrates how it can be called from another code. Additionally, a sample problem is provided that demonstrates how one could make use of the data computed by Thermochemica. Finally, specific programming details of each variable and subroutine are described in a dOxygen-generated documentation.

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