WANT User's Guide

Version 2.0.0

AN INTEGRATED APPROACH TO AB INITIO ELECTRONIC TRANSPORT FROM MAXIMALLY-LOCALIZED WANNIER FUNCTIONS

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(http://www.wannier-transport.org)

This *User's Guide* describes how to run and use the various features of the integrated WANT approach. This guide includes a description of the capabilities of the program, how to use these capabilities, the necessary input files and formats, and how to run the program on both serial and parallel machines.

WANT Version 2.0.0

CREDITS. The development and maintenance of the WANT code is promoted by the National Research Center on nanoStructures and bioSystems at Surfaces (S3) of the Italian INFM-CNR (http://www.s3.infm.it) and the Physics Department North Carolina State University (NCSU) (http://ermes.physics.ncsu.edu) under the coordination of Arrigo Calzolari, Andrea Ferretti and Marco Buongiorno Nardelli.

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The routines for the calculation of the maximally-localized Wannier functions were originally written by Nicola Marzari and David Vanderbilt (©1997); Ivo Souza, Nicola Marzari and David Vanderbilt (©2002); Arrigo Calzolari, Nicola Marzari, and Marco Buongiorno Nardelli (©2003).

The routines for the calculation of the quantum conductance were originally written by Marco Buongiorno Nardelli (©1998); Arrigo Calzolari, Nicola Marzari, and Marco Buongiorno Nardelli (©2003).

GENERAL DESCRIPTION. WANT is an open-source, GNU General Public License suite of codes that provides an integrated approach for the study of coherent electronic transport in nanostructures. The core methodology combines state-of-the-art Density Functional Theory (DFT), plane-waves, pseudopotential calculations with a Green's functions method based on the Landauer formalism to describe quantum conductance. The essential connection between the two, and a crucial step in the calculation, is the use of the maximally-localized Wannier function representation to introduce naturally the ground-state electronic structure into the lattice Green's function approach at the basis of the evaluation of the quantum conductance. Moreover, the knowledge of Wannier functions allows for a direct link between the electronic transport properties of the device with the nature of the chemical bonds, providing insight into the mechanisms that govern electron flow at the nanoscale.

The WanT package operates, in principles, as a simple post-processing of any standard electronic structure code. In its present version 2.0.0 the user will find a wrapper to run WanT from the results of a self-consistent calculation done using the PWscf package (http://www.pwscf.org).

WanT capabilities include: - Quantum conductance spectrum for a bulk (infinite, periodic) system and for a lead-conductor-lead geometry - Density of states spectrum projected on the conductor region - Centers and spreads of the maximally-localized Wannier functions of the system.

TERMS OF USE. Although users are not under any obligation in the spirit of the GNU General Public Licence, the developers of WanT would appreciate the acknowledgment of the effort to produce such codes in the form of the following reference:

In the text: "The results of this work have been obtained using the WANT package.[ref]"

In references: "[ref] WanT code by A. Calzolari, A. Ferretti, C. Cavazzoni, N. Marzari and M. Buongiorno Nardelli, (www.wannier-transport.org). See also: A. Calzolari, N. Marzari, I. Souza and M. Buongiorno Nardelli, Phys. Rev. B 69, 035108 (2004)."

DISCLAIMER. While the developers of WanT make every effort to deliver a high quality scientific software, we do not guarantee that our codes are free from defects. Our software is provided "as is". Users are solely responsible for determining the appropriateness of using this package and assume all risks associated with the use of it, including but not limited to the risks of program errors, damage to or loss of data, programs or equipment, and unavailability or interruption of operations. Due to the limited human resources involved in the development of this software package, no support will be given to individual users for either installation or execution of the codes. Finally, in the spirit of every open source project, any contribution from external users is welcome, encouraged and, if appropriate, will be included in future releases.

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1 Theoretical Background

WanT is an open-source, GNU General Public License suite of codes that provides an integrated approach for the study of coherent electronic transport in low-dimensional, extended nanostructures. The core methodology combines state-of-the-art Density Functional Theory (DFT), plane-waves, pseudopotential calculations with a Green's functions method based on the Landauer formalism to describe quantum conductance. The essential connection between the two, and a crucial step in the calculation, is the use of the maximally-localized Wannier function representation to introduce naturally the ground-state electronic structure into the lattice Green's function approach at the basis of the evaluation of the quantum conductance. Moreover, the knowledge of Wannier functions allows for a direct link between the electronic transport properties of the device with the nature of the chemical bonds, providing insight into the mechanisms that govern electron flow at the nanoscale.

WanT scheme was originally described in Ref. [1]: A. Calzolari, N. Marzari, I. Souza, and M. Buongiorno Nardelli, *Phys. Rev. B* **69**, 035108 (2004).

In the following we review the theoretical background that holds the WANT method.

1.1 Quantum transport

Calculations of the quantum conductance are based on a recently developed efficient method for evaluating quantum transport in extended systems [2, 3, 4]. This method is applicable to any Hamiltonian that can be expanded within a localized-orbital basis and can be used as a general theoretical scheme for the computation and analysis of the electrical properties of nanostructures.

1.1.1 Electron transmission and Green's functions

Let us consider a system composed of a conductor, C, connected to two semi-infinite leads, R and L, as in Fig. 1. A fundamental result in the theory of electronic transport is that the zero-temperature conductance through a region of non-interacting electrons (the C region in Fig. 1) is related to the scattering properties of the region itself via the Landauer formula [5]:

$$C = \frac{2e^2}{h} \mathcal{T}(E_f),\tag{1}$$

where \mathcal{T} is the transmission function, \mathcal{C} is the conductance and E_f the Fermi energy. The former represents the probability that an electron injected at one end of the conductor will transmit to the other end. In principle, we can compute the transmission function for a coherent conductor starting from the knowledge of the scattering matrix, S. The latter is the mathematical quantity that describes the response at one lead due an excitation at another. In principle, the scattering matrix can be uniquely computed from the solution of the Schroedinger equation and would suffice to describe the transport processes we are interested in this work. However, it is a general result of conductance theory that the elements of the S-matrix can be expressed in terms of the Green's function of the conductor [6, 7, 8] which, in practice, can be sometimes simpler to compute.

¹A conductor is said to be coherent if it can be characterized by a transmission matrix that relates each of the outgoing wave amplitudes to the incoming wave amplitudes at a given energy.

Let us consider a physical system represented by an Hamiltonian H. The Green's functions of the system can be defined as:

$$(\omega \pm i\eta - H) G(\omega) = I \tag{2}$$

where I is ht eidentity operator and $i\eta > 0$ is an infinitesimal imaginary part added to the energy to incorporate the boundary conditions into the equation. The solution with + sign is the retarded Green's function G^r , while the solution with - sign is called advanced Green's function G^a . The transmission function can then be expressed in terms of the Green's functions of the conductor and the couplings of the conductor to the leads in a simple manner using the Fisher and Lee formula [7]:

$$\mathcal{T}(\omega) = \text{Tr}\left[\Gamma_L G_C^r \Gamma_R G_C^a\right]. \tag{3}$$

Here $G_C^{\{r,a\}}$ are the retarded and advanced Green's functions of the conductor, and $\Gamma_{\{L,R\}}$ are functions that describe the coupling of the conductor to the leads.

In the following we are going to restrict the discussion to discrete systems that we can describe by ordinary matrix algebra. More precisely, we are going to work with matrices representing a physical system in the basis of localized electronic orbitals centered on the atoms constituting the system. It includes in particular the tight-binding model. For a discrete media, the Green's function defined in Eq. (2) is then the inverse of the $(\omega - H)$ matrix. To simplify the notation, we drop the exponentis $\{a, r\}$ referring to advanced and retarded functions and include the $\pm i\eta$ factor in ω . For an open system, consisting of a conductor and two semi-infinite leads (see Fig. 1), the above Green's function can be partitioned into sub-matrices that correspond to the individual subsystems:

$$\begin{pmatrix} g_{L} & g_{LC} & g_{LCR} \\ g_{CL} & G_{C} & g_{CR} \\ g_{LRC} & g_{RC} & g_{R} \end{pmatrix} = \begin{pmatrix} \omega - h_{L} & -h_{LC} & 0 \\ -h_{LC}^{\dagger} & \omega - H_{C} & -h_{CR} \\ 0 & -h_{CR}^{\dagger} & \omega - h_{R} \end{pmatrix}^{-1}, \tag{4}$$

where the matrix $(\omega - H_C)$ represents the finite "isolated" conductor (with no coupling elements to the leads), $(\omega - h_{\{R,L\}})$ represent the semi-infinite leads, and h_{CR} and h_{LC} are the coupling matrices between the conductor and the leads. As a convention, we use lower case letters for (semi-)infinite matrices and upper case for finite dimension matrices. In Eq. (4) we have made the assumption that there is no direct interaction between the left and right leads. From this equation it is straightforward to obtain an explicit expression for $G_C[6]$:

$$G_C(\omega) = (\omega - H_C - \Sigma_L(\omega) - \Sigma_R(\omega))^{-1}$$
(5)

where the finite dimension matrices

$$\Sigma_L(\omega) = h_{LC}^{\dagger}(\omega - h_L)^{-1} h_{LC}, \quad \Sigma_R(\omega) = h_{RC}(\omega - h_R)^{-1} h_{RC}^{\dagger}$$
 (6)

are defined as the self-energies due to the semi-infinite leads. These terms can be viewed as effective Hamiltonians that arise from the coupling of the conductor with the leads. The coupling functions $\Gamma_{\{L,R\}}(\omega)$ can then be obtained as [6]:

$$\Gamma_{\{L,R\}} = i \left[\Sigma_{\{L,R\}}^r(\omega) - \Sigma_{\{L,R\}}^a(\omega) \right], \tag{7}$$

where the advanced self-energy $\Sigma^a_{\{L,R\}}$ is the Hermitian conjugate of the retarded self-energy $\Sigma^r_{\{L,R\}}$. The core of the problem lies in the calculation of the self-energies of the semi-infinite leads.



Figure 1: A conductor described by the Hamiltonian H_C , connected to two semi-infinite leads L and R, through the coupling matrices h_{LC} and h_{CR} .

It is well known that any solid (or surface) can be viewed as an infinite (semi-infinite in the case of surfaces) stack of principal layers with nearest-neighbor interactions [9, 10]. This corresponds to transforming the original system into a linear chain of principal layers. For a lead-conductor-lead system, the conductor can be considered as one principal layer sandwiched between two semi-infinite stacks of principal layers. The next sections are devoted to the computation of the self-energies using the principal layers approach.

1.1.2 Transmission through a bulk system.

Within the principal layer approach, the matrix elements of Eq. (2) between layer orbitals yield a series of matrix equations for the Green's functions:

$$(\omega - H_{00})G_{00} = I + H_{01}G_{10}$$

$$(\omega - H_{00})G_{10} = H_{01}^{\dagger}G_{00} + H_{01}G_{20}$$

$$\cdots$$

$$(\omega - H_{00})G_{n0} = H_{01}^{\dagger}G_{n-1,0} + H_{01}G_{n+1,0}$$
(8)

where the finite dimension matrices H_{nm} and G_{nm} are formed by the matrix elements of the Hamiltonian and Green's function between the layer orbitals. We assume that in a bulk system $H_{00} = H_{11} = \ldots$ and $H_{01} = H_{12} = \ldots$ Following Refs. [11, 12], this chain can be transformed in order to express the Green's function of an individual layer in terms of the Green's function of the preceding (or following) one. This is done via the introduction of the transfer matrices T and \overline{T} , defined such that $G_{10} = TG_{00}$ and $G_{00} = \overline{T}G_{10}$. Using these definitions, we can write the bulk Green's function as [13]:

$$G(\omega) = (\omega - H_{00} - H_{01}T - H_{01}^{\dagger}\overline{T})^{-1}.$$
 (9)

The transfer matrix can be easily computed from the Hamiltonian matrix elements via an iterative procedure, as outlined in [11, 12]. In particular T and \overline{T} can be written as:

$$T = t_0 + \tilde{t}_0 t_1 + \tilde{t}_0 \tilde{t}_1 t_2 + \ldots + \tilde{t}_0 \tilde{t}_1 \tilde{t}_2 \cdots t_n,$$

$$\overline{T} = \tilde{t}_0 + t_0 \tilde{t}_1 + t_0 t_1 \tilde{t}_2 + \ldots + t_0 t_1 t_2 \cdots \tilde{t}_n,$$
(10)

where t_i and \tilde{t}_i are defined via the recursion formulas:

$$t_{i} = (I - t_{i-1}\tilde{t}_{i-1} - \tilde{t}_{i-1}t_{i-1})^{-1}t_{i-1}^{2},$$

$$\tilde{t}_{i} = (I - t_{i-1}\tilde{t}_{i-1} - \tilde{t}_{i-1}t_{i-1})^{-1}\tilde{t}_{i-1}^{2},$$
(11)

and

$$t_0 = (\omega - H_{00})^{-1} H_{01}^{\dagger},$$

$$\tilde{t}_0 = (\omega - H_{00})^{-1} H_{01}.$$
(12)

The process is repeated until $t_n, \tilde{t}_n \leq \delta$ with δ arbitrarily small. Usually, no more than 5 or 6 iterations are required to converge the above sum.

If we compare Eq. (9) with Eq. (5), in the hypothesis of leads and conductors being of the same material (bulk conductivity), we can identify one principal layer of the bulk system with the conductor C, so that $H_{00} \equiv H_C$. In particular, by comparing with Eq.(5), we obtain the expression of the self-energies of the conductor-leads system:

$$\Sigma_L = H_{01}^{\dagger} \overline{T}, \quad \Sigma_R = H_{01} T. \tag{13}$$

The coupling functions are then obtained [2] from the sole knowledge of the transfer matrices and the coupling Hamiltonian matrix elements: $\Gamma_L = -\text{Im}(H_{01}^{\dagger}\overline{T})$ and $\Gamma_R = -\text{Im}(H_{01}\overline{T})$.

1.1.3 Transmission through a left lead-conductor-right lead (LCR) system.

The procedure outlined above can also be applied to the case of electron transmission through one or more interfaces, between different media. For the calculation of conductances in realistic experimental geometry, the method can be expanded to the general configuration of a Left-lead-Conductor-Right-lead (LCR) systems — as displayed in Fig .1. To study this case we make use of the Surface Green's Function Matching (SGFM) theory, pioneered by [14, 13].

We have to compute the Green's function G_I , where the subscript I refers to the interface region composed of two principal layers — one in each media — (L, C, R in our case). Using the SGFM method, G_I is calculated from the bulk Green's function of the isolated systems, and the coupling between the two principal layers at the two sides of the interface. Via the calculation of the transmitted and reflected amplitudes of an elementary excitation that propagates from one medium to another, it can be shown that the interface Green's function obeys the following secular equation [13]:

$$G_{LCR} = \begin{pmatrix} G_L & G_{LC} & G_{LR} \\ G_{CL} & G_C & G_{CR} \\ G_{RL} & G_{RC} & G_R \end{pmatrix}$$

$$= \begin{pmatrix} \omega - H_{00}^L - (H_{01}^L)^{\dagger} \overline{T} & -H_{LC} & 0 \\ -H_{CL} & \omega - H_C & -H_{CR} \\ 0 & -H_{RC} & \omega - H_{00}^R - H_{01}^R T \end{pmatrix}^{-1} . \tag{14}$$

where $H_{nm}^{\{L,R\}}$ are the block matrices of the Hamiltonian between the layer orbitals in the left and right leads respectively, and $T_{\{L,R\}}$ and $\overline{T}_{\{L,R\}}$ are the appropriate transfer matrices. The latter are easily computed from the Hamiltonian matrix elements via the iterative procedure already described in the bulk case (Sec.1.1.2). Correspondingly, H_{LC} and H_{CR} are the coupling matrices between the conductor and the leads principal layers in contact with the conductor. It is straightforward to obtain in the form of Eq.(5), $G_C = (\omega - H_C - \Sigma_L - \Sigma_R)^{-1}$, where Σ_L and Σ_R are the self-energy terms due to the semi-infinite leads, and identify [2]:

$$\Sigma_{L}(\omega) = H_{LC}^{\dagger} (\omega - H_{00}^{L} - (H_{01}^{L})^{\dagger} \overline{T}_{L})^{-1} H_{LC},$$

$$\Sigma_{R}(\omega) = H_{CR} (\omega - H_{00}^{R} - H_{01}^{R} T_{R})^{-1} H_{CR}^{\dagger}.$$
(15)

The transmission function in the LCR geometry can then be derived from Eq.(3) and (7). The knowledge of the conductor's Green's function G_C gives also direct information on the electronic

spectrum of the system via the spectral density of states:

$$N(\omega) = -\frac{1}{\pi} \text{Im}[\text{Tr}(G_C(\omega))]. \tag{16}$$

We have assumed a truly one-dimensional chain of principal layers, which is physical only for systems like nanotubes or quantum wires that have a definite quasi-one-dimensional character. The extension to a truly three-dimensional case is straightforward using Bloch functions in the directions perpendicular to the transport axis. The introduction of the principal layer concept implies that along the direction of the layer expansion the system is described by an infinite set of k_{\perp} while k_{\parallel} are still good quantum numbers for the problem. The above procedure effectively reduces the three-dimensional system to a set of non-interacting linear-chains, one for each k_{\parallel} [9, 10]. We can then use the usual k-point summation techniques to evaluate the quantum conductance:

$$T(\omega) = \sum_{k_{\parallel}} w_{k_{\parallel}} T_{k_{\parallel}}(\omega) \tag{17}$$

where $w_{k_{\parallel}}$ are the relative weights of the different k_{\parallel} in the irreducible wedge of the surface Brillouin zone [15].

1.2 Maximally localized Wannier functions

1.2.1 Definition of the problem

Bloch orbitals cannot be used directly to evaluate electronic transport with the method outlined in Sec. 1.1. As we have pointed out, quantum conductance is computed starting from the knowledge of the lattice Green's function, whose calculation relies on a localized orbital representation. Bloch orbitals, that are intrinsically delocalized, have to be transformed into *localized* functions in order to construct the sparse, short-ranged matrix elements of the Hamiltonian. The core of our proposed methodology is to use maximally-localized Wannier functions (WFs) for the system considered. These are the most natural choice for a set of localized orbitals that still span the same Hilbert space of the Hamiltonian eigenfunctions: they allow to bridge plane-wave electronic structure and lattice Green's function calculations in a coherent fashion.

A Wannier function $w_{n\mathbf{R}}(\mathbf{r})$, labeled by the Bravais lattice vector \mathbf{R} , is usually defined via a unitary transformation of the Bloch functions $\psi_{n\mathbf{k}}(\mathbf{r})$ of the nth band:

$$w_{n\mathbf{R}}(\mathbf{r}) = \frac{V}{(2\pi)^3} \int_{BZ} \psi_{n\mathbf{k}}(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{R}} d^3k, \qquad (18)$$

where V is the volume of the unit cell and the integration is performed over the entire Brillouin Zone. It is easy to show that the WFs defined as above form an orthonormal basis set, and that any two of them, for a given index n and different \mathbf{R} and \mathbf{R}' , are just translational images of each other. Note that, as WFs are (continuous) linear combinations of Bloch functions with different energies, they do not represent stationary states, but still span the original Hilbert space.

The *ab-initio* eigenstates are well-defined, modulus an arbitrary **k**-dependent phase factor; thus, the definition above does not lead to a unique set of Wannier functions [16, 17], since the electronic structure problem is invariant for the transformation $\psi_{n\mathbf{k}} \to e^{\phi_n(\mathbf{k})}\psi_{n\mathbf{k}}$. Besides this freedom in the choice of phases $\phi_n(\mathbf{k})$ for the Bloch functions, there is a more comprehensive gauge freedom stemming from the fact that the many-body wavefunction is actually a Slater determinant: a unitary transformation between orbitals will not change the manifold, and will not change the total

energy and the charge density of the system. In all generality, starting with a set of \mathcal{N} Bloch functions with periodic parts $u_{n\mathbf{k}}$, we can constructs infinite sets of \mathcal{N} WFs displaying different spatial characteristics:

$$w_{n\mathbf{R}}(\mathbf{r}) = \frac{V}{(2\pi)^3} \int_{BZ} \left[\sum_{m} U_{mn}^{(\mathbf{k})} \psi_{m\mathbf{k}}(\mathbf{r}) \right] e^{-i\mathbf{k}\cdot\mathbf{R}} d^3k.$$
 (19)

The unitary matrices $U^{(\mathbf{k})}$ include also the gauge freedom on phase factors afore mentioned [18].

The present WanT method is based on a localization algorithm that allows to transform a set of Bloch functions – calculated by means of *ab initio* approaches – into a unique set of Maximally localized Wannier functions, as proposed by Marzari and Vanderbilt in 1997 [18]. The formulation of this minimum-spread criterion extends the concept of *localized molecular orbitals*, proposed by Boys [19] for molecules, to the solid-state case. However, its generality allows to deal with both "extended" (periodic and disordered) systems as well as with "isolated" clusters and molecules, in the limit of large supercells.

1.2.2 Localization procedure

For our purposes, we need to transform the Bloch eigenstates in WFs with the narrowest spatial distribution. Following the procedure proposed by Marzari and Vanderbilt [18], we search the particular unitary matrix $U_{mn}^{(\mathbf{k})}$ that transform the Bloch eigenstates in the WFs with the narrowest spatial distribution.

A measure of the spatial delocalization of WFs is given by a *Spread Operator* Ω , defined as the sum of the second moments of all the Wannier functions in a reference cell:

$$\Omega = \sum_{n} [\langle r^2 \rangle_n - \langle \mathbf{r} \rangle_n^2], \tag{20}$$

where the sum is over a selected group of bands, and

$$\langle \mathbf{r} \rangle_n = \langle \mathbf{0} n | \mathbf{r} | \mathbf{0} n \rangle,$$
 (21)
 $\langle r^2 \rangle_n = \langle \mathbf{0} n | r^2 | \mathbf{0} n \rangle.$

The value of the spread Ω depends on the choice of unitary matrices $U^{(\mathbf{k})}$; thus it is possible to evolve any arbitrary set of $U^{(\mathbf{k})}$ until we reach the stationarity condition:

$$\frac{\delta\Omega_{\mathbf{k}}}{\delta U^{(\mathbf{k})}} = 0 \tag{22}$$

At the minimum, we obtain the matrices $U^{(\mathbf{k}),ML}$ that transform the first-principles $\psi_{n\mathbf{k}}^{FP}(\mathbf{r})$ into the maximally-localized WFs, according to Eq. (19). If we restrict to the case of **k**-point mesh calculations, we can use finite differences in reciprocal space to evaluate the derivatives of Eq. (22). For this purpose we rewrite the expectation values $\langle \mathbf{r} \rangle$ and $\langle r^2 \rangle$ as proposed by Blount [20]:

$$\langle \mathbf{0}n|\mathbf{r}|\mathbf{0}n\rangle = i\frac{1}{N}\sum_{\mathbf{k}} e^{+i\mathbf{k}\cdot\mathbf{R}}\langle u_{\mathbf{k}n}|\nabla_{\mathbf{k}}|u_{\mathbf{k}n},\rangle$$

$$\langle \mathbf{0}n|r^{2}|\mathbf{0}n\rangle = \frac{1}{N}\sum_{\mathbf{k}} e^{+i\mathbf{k}\cdot\mathbf{R}}\langle u_{\mathbf{k}n}|\nabla_{\mathbf{k}}^{2}|u_{\mathbf{k}n}\rangle,$$
(23)

where $|u_{n\mathbf{k}}\rangle = e^{-i\mathbf{k}\cdot\mathbf{r}}|\psi_{n\mathbf{k}}\rangle$ is the periodic part of the Bloch function. Making the assumption that the BZ has been discretized into a uniform **k**-point mesh, and letting **b** being the vectors that connect a mesh point to its near neighbors, we can define the overlap matrix between Bloch orbitals as:

$$M_{mn}^{(\mathbf{k},\mathbf{b})} = \langle u_{m\mathbf{k}} | u_{n\mathbf{k}+\mathbf{b}} \rangle = \langle \psi_{m\mathbf{k}} | e^{-i\mathbf{k}\mathbf{b}} | \psi_{n\mathbf{k}+\mathbf{b}} \rangle. \tag{24}$$

Using the expression of the gradient in terms of finite differences and substituting $M_{mn}^{(\mathbf{k},\mathbf{b})}$ in Eq. (23) we obtain the expressions for $\langle \mathbf{r} \rangle$ and $\langle r^2 \rangle$ to be used in the localization procedure:

$$\langle \mathbf{r} \rangle_{n} = -\frac{1}{N} \sum_{\mathbf{k}, \mathbf{b}} w_{b} \operatorname{Im} \operatorname{Ln} M_{nn}^{(\mathbf{k}, \mathbf{b})}$$

$$\langle r^{2} \rangle_{n} = \frac{1}{N} \sum_{\mathbf{k}, \mathbf{b}} w_{b} \left[\left(1 - |M_{nn}^{(\mathbf{k}, \mathbf{b})}|^{2} \right) + \left(\operatorname{Im} \operatorname{Ln} M_{nn}^{(\mathbf{k}, \mathbf{b})} \right)^{2} \right]$$

$$(25)$$

Here, w_b are the weights of the **b**-vectors, and must satisfy the completeness condition $\sum_{\mathbf{b}} w_b b_{\alpha} b_{\beta} = \delta_{\alpha\beta}$. Substituting the above expression into Eq. (20), we obtain the expression for the spread operator as a function of the overlap matrix $M_{mn}^{(\mathbf{k},\mathbf{b})}$.

In order to calculate the gradient in Eq. (22), we consider the first order change in Ω arising from an infinitesimal transformation $U_{mn}^{(\mathbf{k})} = \delta_{mn} + dW_{mn}^{(\mathbf{k})}$, where dW is an infinitesimal antiunitary matrix $(dW^{\dagger} = -dW)$. The gauge transformation rotates the wave functions according to Eq. (19) into $|u_{\mathbf{k}n}\rangle \to |u_{\mathbf{k}n}\rangle + \sum_{m} dW_{mn}^{(\mathbf{k})}|u_{\mathbf{k}m}\rangle$. Following the elegant description of Ref. [18], we obtain the final expression for the gradient of the spread functional:

$$G^{(\mathbf{k})} = \frac{\delta\Omega}{dW^{(\mathbf{k})}} = 4\sum_{\mathbf{b}} w_b \left(\frac{R^{\mathbf{k},\mathbf{b})} - R^{(\mathbf{k},\mathbf{b})\dagger}}{2} - \frac{T^{(\mathbf{k},\mathbf{b})} + T^{(\mathbf{k},\mathbf{b})\dagger}}{2i} \right)$$
(26)

where

$$R_{mn}^{(\mathbf{k},\mathbf{b})} = M_{mn}^{(\mathbf{k},\mathbf{b})} M_{nn}^{(\mathbf{k},\mathbf{b})*}; \qquad T_{mn}^{(\mathbf{k},\mathbf{b})} = \frac{M_{mn}^{(\mathbf{k},\mathbf{b})}}{M_{nn}^{(\mathbf{k},\mathbf{b})}} \left[\operatorname{Im} \operatorname{Ln} M_{nn}^{(\mathbf{k},\mathbf{b})} + \mathbf{b} \cdot \langle \mathbf{r} \rangle_{n} \right].$$
 (27)

Note that the entire expression $G^{(\mathbf{k})}$ is a function of the overlap matrices $M^{(\mathbf{k},\mathbf{b})}$. The minimization of the spread functional Ω is obtained via steepest descent or conjugate gradient schemes. The procedure does not require the updating of wavefunctions, but only of the overlap and unitary matrixes. This is the most demanding task (scaling as N^3) for each iteration in Wannier localization.

Wannier functions obtained with the above procedure should be almost real, except for an overall phase factor. This conjecture can also be used as a check of the convergence of the localization procedure. It is important to notice that whenever a Born-von Karman discretization of the Brillouin Zone is introduced, even the above-mentioned WFs are not truly localized, but will be periodic in real-space, with a *superperiodicity* determined by the BZ discretization. The truly isolated limit is recovered only in the case of continuous BZ integrations. This is easily seen remembering that $\psi_{n\mathbf{k}}(\mathbf{r}) = u_{n\mathbf{k}}(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}$, and $u_{n\mathbf{k}}(\mathbf{r})$ has the periodicity of the direct lattice; thus the phase factors $e^{i\mathbf{k}\cdot\mathbf{r}}$ determine the *superperiodicity* of the $\psi_{n\mathbf{k}}$ themselves.

In the standard language of electronic-structure calculations, if the $\psi_{n\mathbf{k}}$ have \mathbf{k} 's that are restricted to a uniform Monkhorst-Pack mesh, they will all be periodic with a wavelength inversely proportional to the spacing of the mesh; this periodicity is consequently inherited by the WFs. For \mathcal{N} \mathbf{k} -points along a direction of the BZ, the WFs will repeat along the corresponding direction

every \mathcal{N} cells. A dense mesh of **k**-points guarantees that the adjacent replicas of a WF are sufficiently far and do not interact. However, even the case of Γ -sampling is encompassed by the above formulation. In this case the neighboring **k**-points for Γ are given by the homologous Γ -points of the neighboring cells. In this case the algebra becomes simpler and an equivalent real-space formulation is preferred [21, 22].

The method described above works properly in the case of isolated groups of bands. A Bloch band is called isolated if it does not become degenerate with any other band anywhere in the BZ. Conversely, a group of bands is said to form a composite group if bands are inter-connected by degeneracy, but are isolated from all the other bands [18]. On the other hand to study quantum conductance in extended systems we often need to compute WFs for a subset of energy bands that are entangled or mixed with other bands. Most often we are interested in the states that lie in the vicinity of the Fermi level of a conductor in a restricted energy range. Since the unitary transformations $U^{(\mathbf{k})}$ mix energy bands at each **k**-point, any arbitrary choice of states inside a prescribed window will affect the localization properties of WFs unless energy gaps effectively separate the manifold of interest from higher and lower bands. This problem has been solved by Souza, Marzari, and Vanderbilt [23], introducing an additional disentanglement procedure that automatically extracts the best possible manifold of a given dimension from the states falling in a predefined energy window. This is the generalization to entangled or metallic cases of the maximally-localized WF formulation. The procedure relies on minimizing the subspace dispersion across the Brillouin Zone, and effectively extracts the bands of interest from the overall band structure.

In practice, first we select a desired number of bands in an energy window; then we determine the optimally-connected subspace that can be extracted from that band structure; and finally proceed with a standard localization procedure inside the selected subspace. The resulting orbitals have the same good localization properties, and allow to apply our formalism to arbitrary systems, independently of the insulating or metallic nature of the band manifold. It should be stressed that the WFs obtained in the later case are not the WFs of the occupied subspace (that would exhibit poor localization properties), but are those of a well connected, continuous subspace that in general will contain both occupied and unoccupied Bloch functions.

1.2.3 Real space hamiltonians

In order to calculate the conductance according to the prescriptions outlined in Sec. 1.1, we need as an input the matrix elements of the Hamiltonian calculated on a localized basis: in our case, it is the minimal basis of the maximally-localized WFs. Assuming a BZ sampling fine enough to eliminate the interaction with the WF periodic images, we can simply compute the WF Hamiltonians $H_{ii}(\mathbf{R}) = \langle w_{i0}|H|w_{i\mathbf{R}}\rangle$, from the unitary rotations $U^{(\mathbf{k})}$ obtained in the localization procedure.

In the Bloch representation we have by definition $H_{mn}(\mathbf{k}) = \epsilon_{m\mathbf{k}}\delta_{m,n}$. Moving to the Wannier basis we have:

$$H^{(rot)}(\mathbf{k}) = U^{(\mathbf{k})\dagger} H(\mathbf{k}) U^{(\mathbf{k})}. \tag{28}$$

Next we Fourier transform $H^{(rot)}(\mathbf{k})$ into the corresponding set of Bravais lattice vectors $\{\mathbf{R}\}$:

$$H_{ij}(\mathbf{R}) = \frac{1}{N_{kp}} \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}} H_{ij}^{(rot)}(\mathbf{k})$$
(29)

2 Installation procedure

<u>NOTES</u>: (i) The present version of the code adopts the installation procedure of the PWscF package (for more details see also http://www.pwscf.org). (ii) This installation procedure is still experimental, and only a limited number of architectures are currently supported. Details are also reported in the \$TOPDIR/docs/README.install file, where \$TOPDIR is the top directory of the WanT source tree.

Installation is a two-step procedure:

- 1. cd to the top directory of the WanT tree, and issue this command at the shell prompt: ./configure [options]
- 2. Now run: make $\langle \texttt{target} \rangle$

where $\langle \text{target} \rangle$ is one (or more) of the following: wannier, transport, libwant, libiotk, all, clean, wash. Running make without arguments prints a short manual. Cross-compilation is not currently supported.

2.1 Step one: configuring

configure is a GNU-style configuration script, automatically generated by GNU Autoconf. (If you want to play with it, its source file is \$TOPDIR/conf/configure.ac; you may also want to edit \$TOPDIR/conf/make.sys.in) It generates the following files:

```
$TOPDIR/make.sys compilation settings and flags
$TOPDIR/*/make.depend dependencies, in each source dir
```

Files make.depend are actually generated by the makedeps.sh shell script. If you modify the program sources, you might have to rerun it. Note that you must run it from the directory it is in.

To force using a particular compiler, or compilation flags, or libraries, you may set the appropriate environment variables when running the configuration script. For example:

```
./configure CC=gcc CFLAGS=-03 LIBS="-llapack -lblas -lfftw"
```

Some of those environment variables are:

```
TOPDIR : top directory of the WanT tree (defaults to 'pwd')
F90, F77, CC : Fortran 90, Fortran 77, and C compilers
```

: source file preprocessor (defaults to "\$CC -E")

: linker (defaults to \$F90)

CFLAGS, FFLAGS,

F90FLAGS, CPPFLAGS, LDFLAGS : compilation flags

LIBDIRS : extra directories to search for libraries (see below)

You should always be able to compile the WANT suite of programs without having to edit any of the generated files. If you ever have to, that should be considered a bug in the configuration script and you are encouraged to submit a bug report. IMPORTANT: Wan't can take advantage of several optimized numerical libraries:

- ESSL on AIX systems (shipped by IBM)
- MKL together with Intel compilers (shipped by Intel, free for non-commercial use)
- ATLAS (freely downloadable from http://math-atlas.sourceforge.net)
- FFTW (freely downloadable from http://www.fftw.org)

The configuration script attempts to find those libraries, but may fail if they have been installed in non-standard locations. You should look at the LIBS environment variable (either in the output of the configuration script, or in the generated make.sys) to check whether all available libraries were found. If any libraries weren't found, you can rerun the configuration script and pass it a list of directories to search, by setting the environment variable LIBDIRS; directories in the list must be separated by spaces. For example:

```
./configure LIBDIRS="/opt/intel/mkl/mkl61/lib/32 /usr/local/lib/fftw-2.1.5"
```

If this still fails, you may set the environment variable LIBS manually and retry. For example:

```
./configure LIBS="-L/cineca/prod/intel/lib -lfftw -llapack -lblas"
```

Beware that in this case, you must specify **all** the libraries that you want to link to. The configuration script will blindly accept the specified value, and will **not** search for any extra library.

If you want to use the FFTW library, the fftw.h include file is also required. If the configuration script wasn't able to find it, you can specify the correct directory in the INCLUDEFFTW environment variable. For example:

```
./configure INCLUDEFFTW="/cineca/lib/fftw-2.1.3/fftw"
```

2.2 Step two: compiling

Here is a list of available compilation targets:

${\tt make}$	wannier	compile	disentangle.x	(step 1)	
			wannier.x	(step 2)	
			bands.x	(post pr	oc)
			plot.x	(post pr	oc)
			blc2wan.x	(post pr	oc)
${\tt make}$	transport	compile	conductor.x	(step 3)	
${\tt make}$	all		make wannier + transport		
make	libwant	compile	WanT basic libs		
make	libiotk	compile	Input-Output toolkit lib (iotk)		
${\tt make}$	clean	remove	Object files, libs and executables		
${\tt make}$	${\tt clean_test}$	remove	test output files		
make	wash	remove	Configuration files too		

<u>IMPORTANT</u>: If you change any compilation or precompilation option after a previous (successful or failed) compilation, you must run make clean before recompiling, unless you know exactly which routines are affected by the changed options and how to force their recompilation.

2.3 List of directories

Within the top directory of the WanT tree there are the following directories:

conf configuration files

docs documentation files and manuals

iotk source files for the iotk library (input-output toolkit, by G. Bussi)

wannier source files for the Wannier functions suite /par

transport source files and modules for transport code

bin links to all the executables

include include files *.h

libs source files for basic common libraries

tests tutorial examples for the use of WanT suite

utility general utility and tools.

3 How to run WanT: a step by step description

<u>NOTE</u>: At present the WANT code is implemented to work as post processing of DFT calculations done using the PWSCF package (http://www.pwscf.org); we will refer to that code in the following.

Following the theoretical description of Section 1, the evaluation of transport properties requires three separate steps:

- 1. Calculation of DFT electronic structure
- 2. Calculation of maximally-localized Wannier functions
- 3. Calculation of quantum conductance

<u>IMPORTANT</u>: For a correct results, the following steps **MUST** be done in the reported order.

3.1 Preliminary Steps: DFT Calculations

- Self-consistent calculation using the PWscf code.
 For the description of the input and for further details see the PWscf manual.
- ii) Bandstucture calculation.
 - Starting from the self-consistent charge calculated in point (1), we calculate the Bloch functions for a REGULAR k-point grid in the COMPLETE Brillouin Zone; Gamma point must be included. Reduction of k-points due to time-reversal symmetry is not (yet) allowed. The complete list of k-points should be specified in the K_POINTS card. The simple program kgrid.f90 in \$TOPDIR/utility can be used to generate non-symmetrized Monkhorst-Pack grids.
- iii) From PWscf to the Wannier code.

 Use the post processing pw_export.x (distributed in the PWscf package ²), to extract the input data necessary for the following Wannier calculations from PWscf output datafile.

 Data will be stored in the newly-created directory \$prefix.export/.

<u>NOTE</u>: Steeps (i-iii) should be be run, using the parallel version of the code, paying attention to use the same number of processors. From this point to the end, instead, the code is scalar.

3.2 Calculation of maximally-localized Wannier functions

Following the list of input parameters as in Section 4 (also reported in the README.input file in \$TOPDIR/docs/) and the examples in directory \$TOPDIR/tests/, create your own input file, that will be used for the following two steps (a-b).

a) disentangle.x: Starting from the data stored at level (iii), the code selects the working energy window. From there it will extract a selected number (N) of WFs to setup the Hilbert subspace for Wannier localization. For each k-point, the energy window **must** contain a

²The export utility is already distributed in the Qantum-Espresso v3.0, but a patch to include it also in versions v2.1.x is available at http://www.wannier-transport.org or http://www.pwscf.org.

number of bands not lower than N. It is possible to use an inner window (inside the working one) to treat frozen-states, for details see Ref. [23]: these frozen Bloch-states will be kept as they are in the Wannier subspace. Trial Wannier centers are not mandatory in this step (depending on the subspace_init flag). The code produces a standard output with the main results, and two internal files: $prefix_postfix_openion and projection integrals (to be re-used in further or restarted calculations) while the latter describe the wannier optimal subspace.$

b) wannier.x: the code reads the same input of disentangle.x and performs the localization procedure leading to the maximally localized WFs. The optimal unitary matrices $U(\mathbf{k})$ governing the transformation between Bloch and Wannier states are obtained. In this step different trial centers can be used, but it is a standard procedure to keep those used in the disentangle.x run. wannier.x produces a standard output with describing the iterative path to the optimally localized WFs. The code also writes two internal data-files: $prefix_postfix.wan$ containing the $U(\mathbf{k})$ and $prefix_postfix.ham$ with the hamiltonian matrix elements on the Wannier basis.

Further physical information may be obtained as external post-processing of the Wfs calculation. They are not necessary for the transport calculation, but they may be useful for a better understanding of the intrinsic electronic properties of the system. These codes require a separate input, to be generated following Sec. 4 or the file \$TOPDIR/docs/README.input.

- bands.x: the code computes the interpolated band-structure of the system along selected direction in the Brillouin Zone. The comparison with indipendently calculated DFT eigenvalues is a nice test to check the localization of the obtained WFs. When they are well behaved (i.e. localized), only few \mathbf{R} lattice vectors are required to described the Hamiltonian on the Wannier basis $[H_{ij}(\mathbf{R})]$ Starting with a small set of \mathbf{k} in the DFT calculation we obtain the Hamiltonian on the related (small) set of lattice vectors. When WFs are well localized the Hamiltonian is fully described on this set of \mathbf{R} and we can diagonalize it for an arbitrary large set of \mathbf{k} points (as a post-processing of the Wannier code). This is the interpolation of the band structure using WFs. If they are not localized we are essentially throuwing away some non-negligible matrix elements in the Hamiltonian representation, and the bands are not accurate. Typical unphysical oscillations appears in these cases. The code produces three files $prefix_postfix_dftband.dat$, $prefix_postfix_wanband.dat$ and $prefix_postfix_intband.dat$ which cab be used for a direct visualization.
- plot.x: this is an utility to plot WFs in real space. The plotting region can be tuned and a genereic number of WFs can be handled in a single run. The real or immaginary parts of the WFs as well as their squared moduli are allowed fields to be plotted. The code produces plot files in various formats (txt, gaussian cube, xsf, plt) allowing the use of standard open source visualization-packages such as gOpenMol or xCrysDen. Output files are labelled as \$prefix_\$postfix_WF\$type_\$num.\$fmt, where \$type can be R, I, M according to the plotted field (real or immaginary part, squared modulus), \$num is the index of the chosen WF and \$fmt is the output format. If needed, an auxiliary file with the atomic structure in the xyz format is also produced.
- blc2wan.x: transforms a generic (eventually dynamic) operator given on the Bloch eigenstates basis $A_{mn}(\mathbf{k}) = \langle \psi_{m\mathbf{k}} | \widehat{A}(\omega) | \psi_{n\mathbf{k}} \rangle$ to the WFs representation. Using the unitary matrices $U(\mathbf{k})$

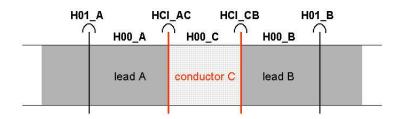


Figure 2: Schematic definitions for Hamiltonian block matrices used in transport calculations.

calculated during the Wannier localization (b) and the definition of the Wannier subspace in terms of the Bloch eigenstates (a), $\widehat{A}(\omega)$ is described in terms of the matrix elements $A_{ij}(\mathbf{R}) = \langle w_{i0} | \widehat{A}(\omega) | w_{j\mathbf{R}} \rangle$.

3.3 Calculation of electronic transport

Using the hamiltonian matrices calculated in step (b) we can calculate both the bulk and the two-terminal transmittance. Input file may be generated following Sec. 4 or the file README.input in \$TOPDIR/docs. The main result of the calculation is the quantum transmittance across the conductor region which is written on file \$SCRATCH/cond.dat. Conductance is given in $2e^2/h$ units. Calculations are performed using the Fisher-Lee formula according to Sec. 1.1. As an auxiliary piece of information, the density of states (DOS) projected on the conductor is also writte in \$SCRATCH/dos.dat. This DOS is computed as the trace of the conductor Green's function $N(E) = -(1/\pi) \operatorname{Im} \operatorname{Tr} G_C(E)$.

Except the special case of the bulk transmittance (see below), the systems one is interested in are generally not periodic along the transport direction. This does not in principle avoid a 2D periodicity in the orthogonal plane. The use of 2D mesh of k-points in this context is implemented in the present version of the code.

Before discussing in more ditail the actual calculations performed by the transport code, we precisely define the adopted convention for the names of the real-space (i.e. WF) Hamiltonian blocks. Given a conductor (C) connected to left (L) and right (R) leads, containing respectively N_C , N_L and N_R WFs, we define the following matrices [see Fig. 2]:

```
HOO_L = N_L \times N_L on-site hamiltonian of the leads L (from L-bulk calculation)
```

H01_L = $N_L \times N_L$ hopping hamiltonian of the leads L (from L-bulk calculation)

HOO_R = $N_R \times N_R$ on site hamiltonian of the leads R (from R-bulk calculation)

H01_R = $N_R \times N_R$ hopping hamiltonian of the leads R (from R-bulk calculation)

HOO_C = $N_C \times N_C$ on site hamiltonian of the conductor C (from C-supercell calculation)

H_LC = $N_L \times N_C$ coupling matrices between lead L and conductor C

H_CR = $N_C \times N_R$ coupling matrices between conductor C and lead R

In the general case we need to compute the electronic structure and WFs for three different regions L, C, R. This picture may be simplified as we will discuss below.

<u>NOTES</u>: (i) The definition of the coupling matrices HCL MUST BE DONE CAREFULLY case by case, depending on the particular system definition. (ii) In order to match the hamiltonian matrices at the boundary, it is necessary to check that the diagonal elements of the H00 matrices were aligned. If not, a rigid shift may be applied.

conductor.x calculates both the bulk and the two-terminal transmittance for the systems.

- bulk transmittance: the conductor and the leads are the same and the whole system is periodic. According to the We only need hamiltonian matrices are needed: that belonging to the reference cell (000 three last number in the first line, see step (b)) and that belonging to the first cell in the direction along which we calculate the transport (e.g. 100 in the first line, in order to calculate the transport along x-direction).
- In the case of **two-terminal** transmittance we need, in principle, three sets of calculations (two only if the leads are of the same material): bulk calculations for the two infinite leads and a supercell calculation for the conductor and the contacts (see PRB 69, 035108 (2004)). The labels for the hamiltonian matrices used in the input of conductor.x refer to the scheme below.

4 How to prepare an input file

According to the methodological scheme of Section 3, it is necessary to use separate input files at the different step of the WanT procedure.

Input files organized as several **NAMELIST**, followed by other fields introduced by **CARDS**. Namelist are defined from the flag "&NAMELIST" at the beginning to the "/" bar at the end. The order of variables within a namelist is arbitrary. Most variables have default value. If a variable is not explicitly defined in the input file, its default value is assumed.

In the following we report the list and the description of the parameters for each required input file.

4.1 Input for DFT-PW calculations

Step 1. i-ii: pw.x

WANT is currently interfaced with PWSCF code. For the description of the input for steps 1-2 (Sec. 3) and for further details see the PWSCF manual at http://www.pwscf.org.

Step 1. iii: pw_export.x

Namelists allowed : &inputpp

Input file layout

&inputpp
...
/

List of variables _____

prefix STRING

the first part of the name of all the file written by the code

DEFAULT = mandatory

outdir STRING

the scratch directory where the massive data file will be written

DEFAULT = "./"

pseudo_dir STRING

directory containing pseudopotential (PP) files

DEFAULT = "./"

psfile(i) STRING

files containing i-th PP, where i=1, ntype

PP numbering must follow the ordering defined in the input of pw.x

single_file LOGICAL

XXXXXXX aggiungere!!!
DEFAULT = ".FALSE."

ascii LOGICAL

XXXXXXX aggiungere!!!
DEFAULT = ".FALSE."

Namelists allowed: &CONTROL, &SUBSPACE, &LOCALIZATION

4.2 Input for Wannier function calculations

Step 2. a-b: disentangle.x wannier.x

Both codes for the WF calculation (disentangle.x and wannier.x) uses the same input file.

 $Cards allowed: WANNIER_CENTERS$ Input file layout &CONTROL &SUBSPACE &LOCALIZATION WANNIER_CENTERS ("crystal" | "angstrom" | "bohr") <type1><specific_fmt><typeN><specific_fmt>_____ NAMELIST &CONTROL _____

prefix STRING

the first part of the name of all the file written by the code

DEFAULT = mandatory

postfix STRING

the tail of the names of the above mentioned files (useful e.g. to distinguish

among different calculations having a common part)

DEFAULT = ""

work_dir STRING

the scratch directory where the massive data file will be written)

DEFAULT = "./"

title STRING

the title of the calculation)

DEFAULT = "Wannier Transport Calculation"

restart_mode STRING

("from_scratch" | "restart")

define whether to restart a previous calculation; at the moment the "restart" choice implies to overwrite the input variables OVERLAPS, PROJECTIONS, START_MODE_DIS and START_MODE_WAN, with the value "from_file" (see

below for thier meanings)
DEFAULT = "from_scratch"

verbosity STRING

("low" | "medium" | "high")

the level of detail of the code output

DEFAULT = "medium"

overlaps STRING

("from_scratch" | "from_file")

determine how to get overlap integrals:

"from_scratch": overlaps are calculated from wfcs

"from_file": overlaps are read from a previous data file. In this second case

dimensions should be consistent DEFAULT = "from_scratch"

projections STRING

("from_scratch" | "from_file")

determine how to get projections integrals:

Meaning as before

DEFAULT = "from_scratch"

assume_ncpp LOGICAL

if .TRUE. avoids the reading of pseudopotential files assuming that the DFT calculation has been performed within norm-conserving pseudopotentials (no

knowledge of them is required in the WanT calc)

DEFAULT = ".FALSE."

unitary_thr REAL

threshold for the check of matrix unitary

DEFAULT = 1.0d-8

__ NAMELIST &SUBSPACE _____

dimwann INTEGER

the number of wannier functions, i.e. the dimension of the wannier subspace

DEFAULT = mandatory

win_min REAL

the lower limit [eV] of the energy window containing the states forming the

starting subspace for Wannier functions)

DEFAULT = 0.0

win_max REAL

the upper limit [eV] of the energy window described above

DEFAULT = 0.0

froz_min REAL

the lower limit [eV] of the energy window containing 'frozen' states which will

not enter the calculation of WFs

DEFAULT = -2.0000

froz_max REAL

upper limit [eV] of the frozen window described above

DEFAULT = -1.0000

alpha_dis REAL

mixing parameter for the disentangle iterative procedure

DEFAULT = 0.5

maxiter_dis INTEGER

maximum number of iterations during the disentangle procedure

DEFAULT = 1000

nprint_dis INTEGER

every nprint_dis iterations in disentangle minimization write to stdout

DEFAULT = 10

nsave_dis INTEGER

every nsave_dis iterations save subspace data to disk

DEFAULT = 10

use_blimit LOGICAL

if .TRUE., b vectors are set to zero when calculation overlap augmentations. This essentially means we are doing a sort of thermodynamic limit even if

this is not consistent with the actual kpt grid. The .TRUE. value should be

considered for debug purposes

DEFAULT = ".FALSE."

disentangle_thr REAL

threshold for convergence of the iterative disentangle procedure

DEFAULT = 1.0d-8

subspace_init STRING

("randomized" | "lower_states" | "upper_states" |

"center_projections" | "from_file")

Determine how the trial subspace is chosen

"randomized": random starting point is chosen

"lower_states" : the lower DIMWANN bands from DFT calculation are used to define the subspace

"upper_states" : the upper DIMWANN bands from DFT calculation are used to define the subspace

"center_projections" : a subspace is extracted from the DFT bands by means of a projections on the given WANNIER_TRIAL_CENTERS (see the section WANNIER_CENTERS)

"from_file": subspace initialization is read from an existing data file; this is the choice used during restart

DEFAULT = "center_projections"

spin_component STRING

("up" | "down" | "none")

defines whether the calculation is spin polarized and if the case which spin

component is to be treated

DEFAULT = "none"

__NAMELIST &LOCALIZATION _____

wannier_thr REAL

threshold for convergence of the iterative wannier minimization

DEFAULT = 1.0d-6

aplha0_wan REAL

mixing parameter during the first CG part of the wannier minimization

DEFAULT = 0.5

aplha1_wan REAL

mixing parameter during the second part of the wannier minimization

DEFAULT = 0.5

maxiter0_wan INTEGER

maximum number of iterations for the first CG part of the wannier minimiza-

tion

DEFAULT = 0.5

maxiter1_wan INTEGER

maximum number of iterations for second part of the wannier minimization

DEFAULT = 0.5

nsave_wan INTEGER

every nsave_dis iterations save subspace data to disk

DEFAULT = 10

ncg INTEGER

every ncg iterations in the second minimization part, do a CG minimization

DEFAULT = 3

localization_init STRING

("no_guess" | "randomized" | "center_projections" | "from_file")

Determine how the wannier localization is started

"no_guess": disentangle states are used as starting point without any further localization guess

"randomized": a random rotation is applied to the states found by the disentangle procedure

"center_projections" : a subspace is extracted from the DFT bands by means of a projections on the given WANNIER_TRIAL_CENTERS (see the section WANNIER_CENTERS)

"from_file": subspace initialization is read from an existing data file; this is the choice used during restart

DEFAULT = "center_projections"

ordering_mode STRING

("none" | "spatial" | "spread" | "complete") specifies whether to order the computed Wannier functions and which ordering criterion adopt

"none": no ordering is performed

"spatial": ordering based on WF center positions

"spread": ordering based on WF increasing spreads

"complete": SPATIAL + SPREAD for WF with the same centers

DEFAULT = "none"

a_condmin REAL

the amplitude of the conditioned minimization functional. If set to zero ordinary minimization is performed

DEFAULT = 0.0

niter_condmin INTEGER

the number of steps for which minimization is conditioned.

dump_condmin REAL

the dumping factor for a_condmin during the conditioned minimization.

If the dumping factor is specified, after niter_condmin iterations a_condmin is dumped according to a_condmin = a_condmin * dump_condmin at each iteration

DEFAULT = 0.0

____ CARD WANNIER_CENTERS _____

WANNIER_CENTERS ("crystal" | "angstrom" | "bohr")

Aside the tag WANNIER_CENTERS, units for positions maybe specified:

"crystal": relative coordinates on the basis of a1,a2,a3 lattice vector (default)

"bohr" : cartesian coordinates in bohr "angstrom" : cartesian coordinates in angstrom

Next the card contains DIMWANN lines giving the trial centers for the WFs. Depending on the <TYPE> flag at the beginning of the line, formats are different.

<TYPE> may assume the values: "atomic", "1gauss", "2gauss"

TYPE == "1gauss"

The trial center is given by a single gaussian set at a given position with a given angular momentum. Standard positions are usually atomic sites or bond midpoints.

$\mathbf{x}, \mathbf{y}, \mathbf{z}$ REAL

define the position of the trial function. Units maybe specified aside the tag WANNIER_CENTERS: see above for more details.

l, m INTEGER

are the angular momentum quantum numbers for the spherical harmonics giving the angular part of the trial WF. l can be set equal to 0, 1, or 2, (and m values are then as usual) for standard spherical harmonics or l == -1 indicate the sp3 geometry. Here spherical harmonics are the real ones:

rloc REAL

specifies the spread of the gaussian used for the radial part of the trial WF. Units are bohr for both "bohr" and "crystal" and angstrom for "angstrom" specifier.

weight REAL

this value is required when conditioned minimization is performed. In case, it should be in the interval [0, 1] and weights the relative importance of each center in the penalty functional, weight = 0 is used to switch off the constrain for a given center

TYPE == "2gauss"

The trial function is given as the difference between gaussians with s-symmetry placed at positions selected by the user. This is useful to mimic a antibonding state.

 $\mathbf{x}, \mathbf{y}, \mathbf{z}$ REAL

as before for TYPE == "1gauss"

xx, yy, zz REAL

as before for x,y,z for the units, specify the center of a second gaussian used to build up the trial WF. This second case could be useful to describe anti-bonding

WF.

rloc REAL

as before for TYPE == "1gauss"

weight REAL

as before for TYPE == "1gauss"

TYPE == "atomic"

Atomic (pseudo)-orbitals from pseudopotential files are used as trial functions. They are specified by the atomic index and the required angular momentum quantum numbers.

iatom INTEGER

the index of the chosen atom, the same of PWSCF calculation

l, m REAL

as before for TYPE == "1gauss"

weight REAL

as before for TYPE == "1gauss"

4.3 Input for electronic transport calculations

Step 3: conductor.x

Both bulk and two-terminal calculations use similar input. Labels follows the scheme of Section 3.

Namelists allowed: &INPUT_CONDUCTOR

Cards allowed (tagged): <HAMILTONIAN_DATA>, <subcard> (xlm format) Input file layout

&INPUT_CONDUCTOR

```
...
/
<HAMILTONIAN_DATA>
<subcard>
&MATRIX_DATA
```

...

< /HAMILTONIAN_DATA>

_ NAMELIST &INPUT_CONDUCTOR _____

dimA INTEGER

number of sites in the lead A

DEFAULT = 0

dimB INTEGER

number of sites in the lead B

DEFAULT = 0

dimC INTEGER

number of sites in the conductor C

DEFAULT = 0

calculation_type STRING

("conductor" | "bulk") determines which kind of calculation should be per-

formed:

"conductor": ordinary transport calculation for a leads|conductor|lead inter-

face

"bulk": transport in a bulk system

DEFAULT: "conductor"

loverlap LOGICAL

If .TRUE. reads the overlap matrices from file, otherwise basis orthonormality

is assumed (which is by definition the case of Wannier functions)

DEFAULT: .FALSE.

ne INTEGER

dimension of the energy grid for transmittance and spectral function calculation

DEFAULT = 1000

niterx INTEGER

maximum number of iterations in the calculation of transfer matrices

DEFAULT = 200

emin REAL

lower limit [eV] of the energy grid dimensioned by NE

DEFAULT = -10.0

emax REAL

upper limit [eV] of the energy grid dimensioned by NE

DEFAULT = +10.0

bias REAL

bias voltage [eV] across the conductor region.

DEFAULT = 0.0

delta REAL

small imaginary part used to get off the real axix in the calculation of Green's

DEFAULT = 1.0E-5

__ CARD <HAMILTONIAN_DATA> _____

The tagged <HAMILTONIAN_DATA> is a mandatory card that specifies the hamiltonian matrices to be used in the transport calculation. It includes necessary "<subcards>" (xlm format) to be used the order shown below. The name and the number of the subcards depend on the "calculation_type" flag:

```
IF (calculation_type = "bulk")
                                      \rightarrow two subcards are necessary
                                          < H00_{-}C >, < HCI_{-}CB >
IF (calculation_type = "conductor") \rightarrow six subcards are necessary
                                          < H00_C >, < HCI_CB >, < HCI_AC >,
                                          < H00_A > < H01_A > 
                                          < H00_B > < H01_B >
```

Names of the subcards refers to Figure ?? of Sec. 3.3.

Each subcard contains the namelist "&MATRIX_DATA" and the following variables:

filename STRING "prefix_postfix.ham"

name of the file .ham, produced by wannier.x that includes the hamiltonian

DEFAULT = mandatory

INTEGER rows

It is a string of format e.g. "1-6" (analogous to the fmt used to specify pages to

very common print tools).

It defines the number of rows to be extracted from the original hamiltonian matrix, specified by "filename". The number of selected columns MUST be coherent case by case with the dimension of the hamiltonian matrices defined

by dimA, dimB, dimC (see Fig. ?? for details).

DEFAULT = mandatory

INTEGER cols

It is a string of format e.g. "1-6" (analogous to the fmt used to specify pages to

very common print tools).

It defines the number of columns to be extracted from the original hamiltonian matrix, specified by "filename". The number of selected columns MUST be coherent case by case with the dimension of the hamiltonian matrices defined by dimA, dimB, dimC (see Fig. ?? for details).

DEFAULT = mandatory

direction INTEGER.

> It defines the real space cell, over which we calculate the real space hamiltonian matrices. The spatial direction of the hopping matrices defines the direction of

electronic transport Allowed value are:

$$\mathbf{R} = \mathbf{0} \rightarrow \text{On site H00 hamiltonian}$$
 $\mathrm{H00}_{mn} = \langle w_{m\mathbf{0}} | \widehat{H} | w_{n\mathbf{0}} \rangle$
i.e. the hamiltonian matrix corresponding to WFs localized in the origin cell $(\mathbf{R} = \mathbf{0}, \mathbf{0}, \mathbf{0})$

in the origin cell $(\mathbf{R} = \mathbf{0} \ \mathbf{0} \ \mathbf{0})$ $\mathbf{R} = \mathbf{1} \rightarrow \text{Hopping H01 hamiltonian}$

$$\mathbf{R} = \mathbf{1} \rightarrow \text{Hopping H01 hamiltonian}$$
 $\mathrm{H01}_{mn} = \langle w_{m0} | \widehat{H} | w_{n1} \rangle$
i.e. the hamiltonian matrix

i.e. the hamiltonian matrix among to WFs localized in the origin cell and in the first neighbor cell along x direction $(\mathbf{R} = \mathbf{1} \ \mathbf{0} \ \mathbf{0})$

$$\mathbf{R} = \mathbf{2} \rightarrow \text{Hopping H01 hamiltonian}$$
 $\mathrm{H01}_{mn} = \langle w_{m0} | \widehat{H} | w_{n1} \rangle$
i.e. the hamiltonian matrix among to WFs localized in the origin cell and in the first neighbor cell along y direction $(\mathbf{R} = \mathbf{0} \ \mathbf{1} \ \mathbf{0})$

$${f R}={f 3} \ o \ ext{Hopping H01 hamiltonian}$$
 ${f H01}_{mn}=\langle w_{m{f 0}}|\widehat{H}|w_{n{f 1}}\rangle$ i.e. the hamiltonian matrix among to WFs localized in the origin cell and in the first neighbor cell along

z direction $(\mathbf{R} = \mathbf{0} \ \mathbf{0} \ \mathbf{1})$

4.4 Input for post-processing calculations

4.4.1 bands.x

Namelists allowed : &INPUT Input file layout

```
&INPUT
...
/
kpt_label_1
...
```

kpt_label_N

__ NAMELIST &INPUT _____

prefix

STRING

the first part of the name of all the file written by the code should be equal to the value given in the main calculations.

DEFAULT = mandatory

postfix STRING

the tail of the names of the above mentioned files (useful e.g. to distinguish among different calculations having a common part). should be equal to the

value given in the main calculations.

DEFAULT = ""

work_dir STRING

the scratch directory where the massive data file will be written

DEFAULT = "./"

verbosity STRING

("low" | "medium" | "high")

the level of detail of the code output

DEFAULT = "medium"

nkpts_in INTEGER

number of edge kpts defining the directions on which bands will be calculated

DEFAULT: mandatory

nkpts_max INTEGER

maximum number of interpolated kpoints

DEFAULT = 100

spin_component STRING

("up" | "down" | "none")

define whether the calculation is spin polarized and if the case which spin

component is to be treated.

DEFAULT = 100

After the INPUT namelist for each of the NSPTS kpts two lines with the following format must be provided:

kpt_label kx ky kz kpt_label CHARACTER(*)

it is a string with the name of the kpoint

kx, ky, kz REAL

component of the kpt vector in units of crystal reciprocal lattice vector

i.e. k = kx * b1 + ky * b2 + kz * b3

4.4.2 plot.x

Namelists allowed: &INPUT

Input file layout

&INPUT

..

prefix STRING

the first part of the name of all the file written by the code should be equal to the value given in the main calculations.

DEFAULT = mandatory

postfix STRING

the tail of the names of the above mentioned files (useful e.g. to distinguish among different calculations having a common part). should be equal to the value given in the main calculations.

DEFAULT = ""

work_dir STRING

the scratch directory where the massive data file will be written

DEFAULT = "./"

wann STRING

specifies the indexes of the Wannier functions to be plotted. It is a string of format e.g "1-3,5,7-9" (analogous to the fmt used to specify pages to very

common print tools)
DEFAULT = mandatory

data_type STRING

("modulus" | "real" | "imaginary") specifies the type of data plotted:

"modulus": plot the real space square modulus of the WFs. "real": plot the real part (in real space) of the WFs. "imaginary": plot the imaginary part (in real space) of the WFs this choice should be intended as a check because WFs

are expected to be more or less "real".

DEFAULT = "modulus"

output_fmt STRING

("plt" | "txt" | "cube")

Define the format of the output file. PLT is binary and smaller than CUBE and TXT. While CUBE deals also with non-orthorombic lattices, TXT is suitable to be converted to further format.

DEFAULT = "plt"

r1min, r1max REAL

the starting and ending points of the plotting cell along a1 dir, in units of a1

lattice vector (crystal coord).

DEFAULT = -0.5, 0.5

r2min, r2max REAL

as before but for a2 direction.

DEFAULT = -0.5, 0.5

r3min, r3max REAL

as before but for a3 direction.

DEFAULT = -0.5, 0.5

 $assume_ncpp$ LOGICAL

if using DFT pseudoptentials not readable in WanT set this value to .TRUE.

in order to avoid PP reading. $\label{eq:def-problem} \mbox{DEFAULT} = ".\mbox{FALSE}."$

locate_wf LOGICAL

if .TRUE. move the WFs in a unit cell centered around the midpoint of the

plotting cell. Useful to plot purposes.

DEFAULT = ".TRUE."

 $\mathbf{spin_component} \quad \mathrm{STRING}$

("up" | "down" | "none")

define whether the calculation is spin polarized and if the case which spin

component is to be treated.

DEFAULT = "none"

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