Notes on a general form for the Quantum optimal control Hamiltonian

March 29, 2018

Abstract

Notes on the form of Hamiltonian and rules for constructing the general matrix form of H. Some additional comments on notation.

A fairly general form of the system Hamiltonian we are interested in looks like this:

$$H0 = \sum_i \omega_i a_i^\dagger a_i - \chi_{ii} a_i^\dagger a_i^\dagger a_i a_i - \sum_{i \neq j} \chi_{ij} a_i^\dagger a_i a_j^\dagger a_j$$

where a_i are "annihilation operators" that are operating on a subspace of the full hilbert space (In a way I'll expand on below) $a|n\rangle = \sqrt{n}|n-1\rangle$

$$a = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \sqrt{3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ z0 & 0 & 0 & 0 & \cdots & 0 & \sqrt{n_a - 1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{1}$$

a is a matrix of size $n_a \times n_a$. $|n\rangle$ is a column vector where only the nth entry is nonzero

$$\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

More detail on this flaky notation

$$\langle A|B\rangle \doteq A_1^* B_1 + A_2^* B_2 + \cdots + A_N^* B_N = \begin{pmatrix} A_1^* & A_2^* & \cdots & A_N^* \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix}$$

$$
|\phi\rangle\langle\psi| \doteq \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix} \begin{pmatrix} \psi_1^* & \psi_2^* & \cdots & \psi_N^* \end{pmatrix} = \begin{pmatrix} \phi_1\psi_1^* & \phi_1\psi_2^* & \cdots & \phi_1\psi_N^* \\ \phi_2\psi_1^* & \phi_2\psi_2^* & \cdots & \phi_2\psi_N^* \\ \vdots & \vdots & \ddots & \vdots \\ \phi_N\psi_1^* & \phi_N\psi_2^* & \cdots & \phi_N\psi_N^* \end{pmatrix}
$$

The full Hamiltonian is built up from tensor products of a and $a^\dagger$ operators acting on tensored subspaces. For example, in the case of qubits, each a acts on a two level subsystem.

$$
a = \begin{pmatrix} 0.0 & 1.0 \\ 0.0 & 0.0 \end{pmatrix}
$$

the full Hamiltonian matrix H for a two qubit system would then be made up of products of opperators $a_0 \otimes I_2$ and $I_2 \otimes a_1$ that look like

$$
a_0 \otimes I_2 = \begin{pmatrix} 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}
$$

$$
I_2 \otimes a_1 = \begin{pmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}
$$

So the full Hamiltonian for a two qubit system would look like

$$
\begin{aligned}
H = \quad & \omega_0(a_0 \otimes I_2)^\dagger(a_0 \otimes I_2) + \omega_1(I_2 \otimes a_1)^\dagger(I_2 \otimes a_1) + \\
& \chi_{00}(a_0 \otimes I_2)^\dagger(a_0 \otimes I_2)^\dagger(a_0 \otimes I_2)(a_0 \otimes I_2) + \\
& \chi_{11}(I_2 \otimes a_1)^\dagger(I_2 \otimes a_1)^\dagger(I_2 \otimes a_1)(I_2 \otimes a_1) + \\
& \chi_{01}(a_0 \otimes I_2)^\dagger(a_0 \otimes I_2)(I_2 \otimes a_1)^\dagger(I_2 \otimes a_1)
\end{aligned}
$$