One-Dimensional Heat Flow Analysis of Various Aluminum Composite Panels via Leapfrog Technique

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Abstract

In this experiment, a numerical solution of the parabolic heat equation was found using finite-difference methods to solve for the temperature distribution between Aluminum Composite Panels with varying cores, namely Polyurethane, Polyethylene, and pure Aluminum. Space and time was discretized on a lattice using the leapfrog method and the solutions were solved on the lattice sites. von Neumann stability assessment was implemented after the simulation to check the stability of the approximation. With the boundary and initial conditions set at $T(x = 0, t) = 100°C$ and $T(x \neq 0, t = 0) = 0°C$, respectively, the temperature distribution over a one-hour time window was obtained. It was found that the ACP with a polyurethane core had a higher resistance to temperature variations than the other two materials, whose temperature variations exhibited a similar trend.

Keywords: Heat Flow, Aluminum Composite Panel, Time-Stepping techniques, Leapfrog method, Finite-Difference, von Neumann Stability Analysis,

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1. Introduction

As the laws of thermodynamics dictate, two systems are said to be at thermal equilibrium if and only if they have the same temperature. It is common knowledge by now that the temperature of a metallic object changes when it is not in thermal equilibrium with its environment. The change in temperature is explained by a transfer of energy from one substance to another. Heat flow or heat transfer is a specific type of energy transfer when it is caused solely by a difference in temperature between a system and its surroundings, and the energy transferred in this case is called heat.[**?**] It should be noted, however, that heat and temperature are two distinct entities. Temperature is a measure of the average kinetic energy of the atoms or molecules in the system[**?**] and is a quantitative measure of the physical state of a material's hotness or coldness[**?**], whereas heat refers to the amount of energy transferred between two substances of unequal temperature when in close proximity to one another.[**?**]

This experiment explores the heat flow of sandwich panels, specifically, Aluminum Composite Panels (ACPs). ACPs are composed of three layers, with two Aluminum outer layers covering a core layer. They

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have become highly popular due to their versatility, from being used as a cladding material, for home interior use in the kitchen, and for commercial spaces such as signages and display panels. Sandwich panels are known to have different fire behaviors. In 2000, Gordon Cooke, a leading fire safety consultant at the time, reported that sandwich panels can contribute to the severity and speed of fire development.[**?**] A study done by the Association of British Insurers[**?**] highlighted that "sandwich panels do not start a fire on their own, and where these systems have been implicated in fire spread, the fire has often started in high risk areas such as cooking areas, subsequently spreading as a result of poor fire risk management, prevention and containment measures." The article aims to subject various commonly used cores in ACPs through a numerical simulation in Python in order to determine which among the cores would exhibit the smoothest temperature distribution.

2. Theoretical Background

2.1 The Parabolic Heat Equation and its Analytic Solution

The heat equation is given by

$$\frac{\partial T(x,t)}{\partial t} = \frac{K}{C\rho}\nabla^2 T(x,t) \tag{1}$$

where C is the specific heat of the material and ρ is its density. The heat equation (**??**) is a parabolic partial differential equation that has space and time as independent variables. In the case of this experiment, there is no temperature variations in directions perpendicular to the material (i.e., y and z), and thus the Laplacian will only have one spatial coordinate:

$$\frac{\partial T(x,t)}{\partial t} = \frac{K}{C\rho}\frac{\partial^2 T(x,t)}{\partial x^2} \tag{2}$$

2.2 Time-Stepping using Finite-Difference Methods

The differential equation is converted to a finite-difference equation in order to obtain the numerical solution.[**?**] Space and time is discretized on a lattice and the solutions are solved on the lattice sites using the leapfrog technique[**?**], which is a robust method known for its stability when solving partial differential equations with oscillatory solutions. On the left hand side of equation (**??**), forward-difference approximation is used for the time derivative of the temperature:

$$\frac{\partial T(x,t)}{\partial t} \simeq \frac{T(x,t+\Delta t) - T(x,t)}{\Delta t} \tag{3}$$

For the right hand side of (**??**), central-difference is used for the space derivative of the temperature:

$$\frac{\partial^2 T(x,t)}{\partial x^2} \simeq \frac{T(x+\Delta x,t) + T(x-\Delta x,t) - -2T(x,t)}{(\Delta x)^2} \tag{4}$$

Combining (**??**) and (**??**) to the heat equation (**??**):

$$\frac{T(x,t+\Delta t) - T(x,t)}{\Delta t} = \frac{\kappa}{C\rho}\frac{T(x+\Delta x,t) - 2T(x,t) + T(x-\Delta x,t)}{(\Delta x)^2} \tag{5}$$

$$T(x,t+\Delta t) = T(x,t) + \eta[T(x+\Delta,t) - 2T(x,t) + T(x-\Delta x,t)] \tag{6}$$

where

$$\eta = \frac{2\kappa \Delta t}{C\rho(\Delta x)^2} \tag{7}$$

2.3 von Neumann Stability Analysis

The stability of numerical schemes is closely associated with numerical error. A finite difference scheme is stable if the errors made at one time step of calculation would not create a domino effect causing errors to magnify as the calculations are continued. If the errors grow with time the numerical scheme is said to be unstable. But on the contrary if the errors remain constant as the computations are carried forward, or if better yet decay and eventually dampen out, then it is said to be neutrally stable or stable.[**?**] Also known as Fourier stability analysis, von Neumann stability analysis is a procedure that is used to check the stability of finite difference schemes when it is applied to partial differential equations. The application of stability analysis is given by:

$$\mathcal{E}(k) = 1 + 2\eta \left[\cos(k\Delta x) - 1\right] \tag{8}$$

In order for $|\mathcal{E}(k)| < 1$ for all possible k values, it is crucial that the condition

$$\eta < \frac{1}{2} \tag{9}$$

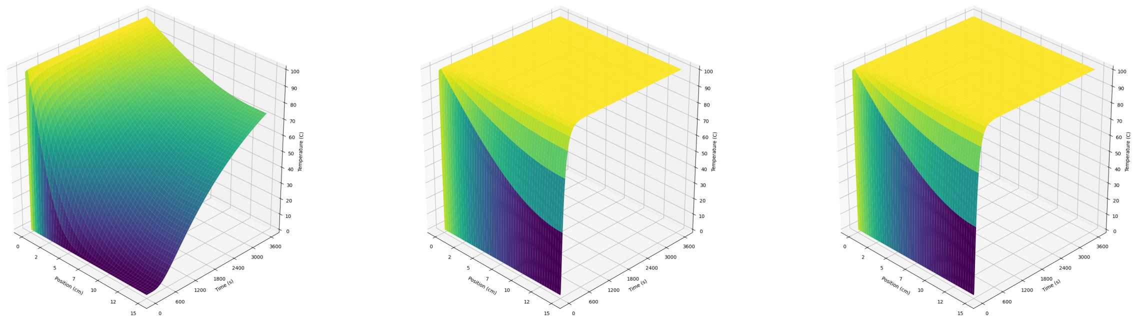
must be satisfied.

3. Experimental Procedure

The heat flow within an aluminum composite panel (ACP) was studied in this experiment. The temperature distribution was subject to the boundary and initial conditions $T(x = 0, t) = 100°C$, $T(x \neq 0, t = 0) = 0°C$. The thermal conductivity, specific heat, and density were obtained for APCs with a polyurethane core and polyethylene core. A 2D array `T`, of shape (500, 2), for the temperature as a function of space and time was then defined, wherein the first index, `500`, is for the space divisions of the bar, and the second index, `2`, is for present and past times. Therefore, `T[...,0]` represents the temperature values for all points along the bar at time t, and `T[...,1]` is for time $t + \Delta t$. At $t = 0$, all points on the bar except one end, that is `T[1:,0]`, were set to $0°C$, whereas one endpoint, `T[0,0]`, was set to $100°C$. Afterwards, (**??**) was applied to obtain the temperature at the next time step. In this experiment, Δx and Δt were set to 0.03 cm and 0.0004 s, respectively. The length of the panel, L, and time window were set to 15 cm and 3600 s, respectively. The present values of the temperature were then assigned to the past time values, `T[i,0]=T[i,1]` for `i=1,2,...,499`.

4. Results and Discussion

As illustrated in Fig. **??**, the temperature distribution over a one-hour time window varies smoothly with time and eventually reaches equilibrium. Both the ACP with polyethylene core and pure aluminum



(a) ACP with polyurethane core

(b) ACP with polyethylene core

(c) ACP with aluminum core

Figure 1: Temperature distribution through the entire length of various ACPs as time evolves.

material exhibit a very similar trend in heat flow. On the other hand, the ACP with polyurethane core appears to be far more resistant to temperature variations than the other two materials studied. It was also observed during experimentation that, as expected, increasing time step or decreasing the width of the material would allow heat to transfer from one end of the material to the other at a much faster rate. Other observations noted from the experiment include the increased resistance to temperature variations by decreasing the thermal conductivity or decreasing either the specific heat capacity or density of the material. Errors during experimentation were generated when $\eta \geq 1/2$. These errors were caused by increasing Δt by a substantial amount or decreasing Δx without a simultaneous quadratic increase in the time step.

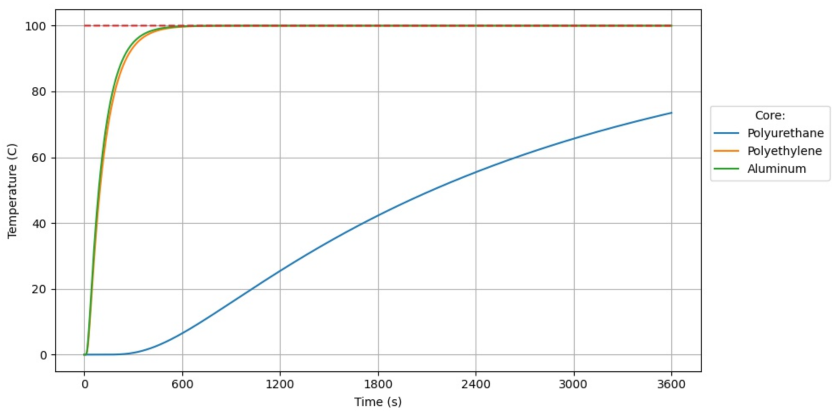


Figure 2: Comparison of amount of heat at $x = L$ for various ACPs.

The temperature at the end of the material was recorded and is shown in Fig. **??**. As illustrated, it is evident that the ACP with polyethylene core exhibits minimal resistance to heat, performing only slightly better than the ACP with aluminum core (pure aluminum material). This is exhibited by the leveling-off of the graph in Fig. **??** within the 600-second mark. This means that the entire material is at thermal equilibrium (i.e., end-to-end) well within 10 minutes. It is also quite evident that, in the ACP with polyurethane core, heat only begins to reach the end of material at around the 300-second mark.

5. Conclusion and Recommendations

The temperature distribution along a aluminum composite panel (ACP) at later times was studied. The cores used in the ACP for this study were polyurethane and polyethylene. The temperature distribution along a pure aluminum material was also studied for comparison with the ACPs. The main feature revealed by the analysis is that the ACP with polyurethane core exhibits higher durability (i.e., more resistant to temperature variations) in comparison to the ACP with polyethylene core and pure aluminum material. It was further revealed that the ACP with polyethylene core exhibits very low resistance to heat, performing only slightly better than the pure aluminum material. Thus, it is highly suggested to use polyurethane as the core of an aluminum composite panel. An open question remains as to how the temperature distribution at later times would be affected if the material was in contact with an environment at a temperature T_e, rather than being insulated. This would lead one to modify (**??**) and consider Newton's law of cooling (radiation). Finally, an open question remains as to how the heat flow would be affected due to a change in temperature that would, in turn, affect the thermal properties of the material.