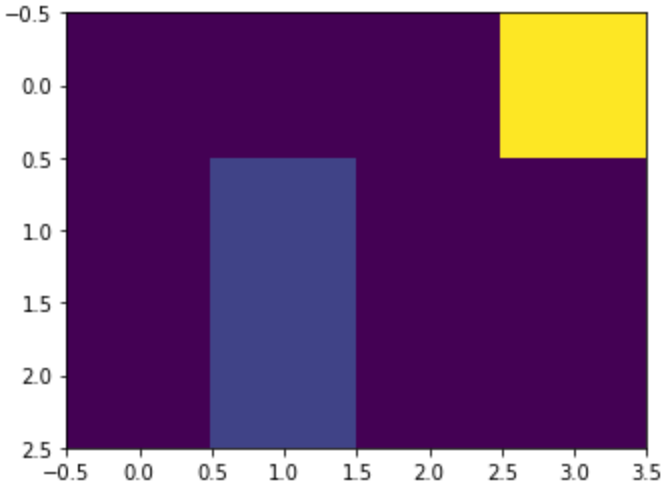
In [1]:
```
from itertools import product

import z3
import numpy as np
from matplotlib import pyplot as plt

%matplotlib inline
z3.set_option(html_mode=False)
```

In [2]:
```
layout = np.array([
    [0,0,0,5],
    [0,1,0,0],
    [0,1,0,0],
])
# layout = np.array([
#     [0,5],
#     [0,0],
# ])
plt.imshow(layout)
```

Out[2]:
```
<matplotlib.image.AxesImage at 0x7fdc09ea5a30>
```



Layout encoding (facts):

1. for each empty cell(i,j): $cell(i, j)$.
2. $goal(0, 3)$.
3. $block(1, 1). block(2, 1)$.

In [3]:
```
axiom_layout = []
cell = z3.Function('Cell', z3.IntSort(), z3.IntSort(), z3.BoolSort())
block = z3.Function('Block', z3.IntSort(), z3.IntSort(), z3.BoolSort())
goal = z3.Function('Goal', z3.IntSort(), z3.IntSort(), z3.BoolSort())
for i in range(layout.shape[0]):
    for j in range(layout.shape[1]):
        if layout[i, j] == 1:
            axiom_layout.append(block(i, j))
        else:
            axiom_layout.append(cell(i,j))
        if layout[i, j] > 1:
            axiom_layout.append(goal(i, j))
        else:
            axiom_layout.append(z3.Not(goal(i, j)))

i, j = z3.Ints('i j')
ij_range = z3.And(i >= 0, i < layout.shape[0], j >= 0, j < layout.shape[1])
```

```
# axiom_layout.append(z3.ForAll([i, j], cell(i, j) == z3.Not(block(i,j))))
axiom_layout.append(z3.ForAll([i, j], z3.Xor(cell(i, j), block(i, j)))) # Wang
axiom_layout.append(z3.ForAll([i, j], z3.Implies(z3.Or(i>=layout.shape[0], i<0, j>=layou
axiom_layout
```

Out[3]:

```
[Cell(0, 0),
 Not(Goal(0, 0)),
 Cell(0, 1),
 Not(Goal(0, 1)),
 Cell(0, 2),
 Not(Goal(0, 2)),
 Cell(0, 3),
 Goal(0, 3),
 Cell(1, 0),
 Not(Goal(1, 0)),
 Block(1, 1),
 Not(Goal(1, 1)),
 Cell(1, 2),
 Not(Goal(1, 2)),
 Cell(1, 3),
 Not(Goal(1, 3)),
 Cell(2, 0),
 Not(Goal(2, 0)),
 Block(2, 1),
 Not(Goal(2, 1)),
 Cell(2, 2),
 Not(Goal(2, 2)),
 Cell(2, 3),
 Not(Goal(2, 3)),
 ForAll([i, j], Xor(Cell(i, j), Block(i, j))),
 ForAll([i, j],
        Implies(Or(i >= 3, i < 0, j >= 4, j < 0),
                Block(i, j)))]
```

In [4]:

```
# actions = ['stop', 'up', 'down', 'left', 'right']
# Action = z3.Datatype('Action')
# for a in actions:
#     Action.declare(a)
# Action = Action.create()
Action, (stop, up, down, left, right) = z3.EnumSort('Action', ('stop', 'up', 'down', 'le
```

Available action encoding:

- $\forall i, j.\, cell(i, j) \wedge cell(i + 1, j) \equiv avai\_action(i, j, down)$

similarly for stop, up, right, left

In [5]:

```
axiom_avai_actions = []
avai_action = z3.Function('Avai_Action', z3.IntSort(), z3.IntSort(), Action, z3.BoolSort
axiom_avai_actions.append(
    z3.ForAll([i, j], z3.Implies(ij_range, z3.And(cell(i, j), cell(i, j)) == avai_action
)
axiom_avai_actions.append(
    z3.ForAll([i, j], z3.Implies(ij_range, z3.And(cell(i, j), cell(i + 1, j)) == avai_ac
)
axiom_avai_actions.append(
    z3.ForAll([i, j], z3.Implies(ij_range, z3.And(cell(i, j), cell(i - 1, j)) == avai_ac
)
axiom_avai_actions.append(
    z3.ForAll([i, j], z3.Implies(ij_range, z3.And(cell(i, j), cell(i, j + 1)) == avai_ac
)
axiom_avai_actions.append(
    z3.ForAll([i, j], z3.Implies(ij_range, z3.And(cell(i, j), cell(i, j - 1)) == avai_ac
```

```
)
axiom_avai_actions
```

Out[5]:

```
[ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                And(Cell(i, j), Cell(i, j)) ==
                Avai_Action(i, j, stop))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                And(Cell(i, j), Cell(i + 1, j)) ==
                Avai_Action(i, j, down))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                And(Cell(i, j), Cell(i - 1, j)) ==
                Avai_Action(i, j, up))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                And(Cell(i, j), Cell(i, j + 1)) ==
                Avai_Action(i, j, right))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                And(Cell(i, j), Cell(i, j - 1)) ==
                Avai_Action(i, j, left)))]
```

Movement encoding:

- $\forall i, j, i', j' . move(i, j, down, i'j') \equiv cell(i, j) \wedge i' == i + 1 \wedge j' == j \wedge cell(i + 1, j)$

In [6]:

```
axiom_move = []
move = z3.Function('Move', z3.IntSort(), z3.IntSort(), Action, z3.IntSort(), z3.IntSort(
i_, j_ = z3.Ints("i' j'")
ij__range = z3.And(i_ >= 0, i_ < layout.shape[0], j_ >= 0, j_ < layout.shape[1])
all_range = z3.And(ij_range, ij__range)
axiom_move.append(
    z3.ForAll([i, j, i_, j_], z3.Implies(all_range, move(i, j, stop, i_, j_) ==
              z3.And(cell(i, j), i_ == i, j_ == j, cell(i, j))))
)
axiom_move.append(
    z3.ForAll([i, j, i_, j_], z3.Implies(all_range, move(i, j, down, i_, j_) ==
              z3.And(cell(i, j), i_ == i + 1, j_ == j, cell(i + 1, j))))
)
axiom_move.append(
    z3.ForAll([i, j, i_, j_], z3.Implies(all_range, move(i, j, up, i_, j_) ==
              z3.And(cell(i, j), i_ == i - 1, j_ == j, cell(i - 1, j))))
)
axiom_move.append(
    z3.ForAll([i, j, i_, j_], z3.Implies(all_range, move(i, j, right, i_, j_) ==
              z3.And(cell(i, j), i_ == i, j_ == j + 1, cell(i, j + 1))))
)
axiom_move.append(
    z3.ForAll([i, j, i_, j_], z3.Implies(all_range, move(i, j, left, i_, j_) ==
              z3.And(cell(i, j), i_ == i, j_ == j - 1, cell(i, j - 1))))
)
axiom_move
```

Out[6]:

```
[ForAll([i, j, i', j'],
        Implies(And(And(i >= 0, i < 3, j >= 0, j < 4),
                    And(i' >= 0, i' < 3, j' >= 0, j' < 4)),
                Move(i, j, stop, i', j') ==
                And(Cell(i, j), i' == i, j' == j, Cell(i, j)))),
 ForAll([i, j, i', j'],
        Implies(And(And(i >= 0, i < 3, j >= 0, j < 4),
                    And(i' >= 0, i' < 3, j' >= 0, j' < 4)),
                Move(i, j, down, i', j') ==
                And(Cell(i, j),
```

```
                    i' == i + 1,
                    j' == j,
                    Cell(i + 1, j)))),
 ForAll([i, j, i', j'],
        Implies(And(And(i >= 0, i < 3, j >= 0, j < 4),
                    And(i' >= 0, i' < 3, j' >= 0, j' < 4)),
                Move(i, j, up, i', j') ==
                And(Cell(i, j),
                    i' == i - 1,
                    j' == j,
                    Cell(i - 1, j)))),
 ForAll([i, j, i', j'],
        Implies(And(And(i >= 0, i < 3, j >= 0, j < 4),
                    And(i' >= 0, i' < 3, j' >= 0, j' < 4)),
                Move(i, j, right, i', j') ==
                And(Cell(i, j),
                    i' == i,
                    j' == j + 1,
                    Cell(i, j + 1)))),
 ForAll([i, j, i', j'],
        Implies(And(And(i >= 0, i < 3, j >= 0, j < 4),
                    And(i' >= 0, i' < 3, j' >= 0, j' < 4)),
                Move(i, j, left, i', j') ==
                And(Cell(i, j),
                    i' == i,
                    j' == j - 1,
                    Cell(i, j - 1))))]
```

Policy encoding:

1. $\forall i, j, a.\, policy(i, j, a) \implies cell(i, j) \wedge avai\_action(i, j, a)$
2. $\forall i, j.\, cell(i, j) \implies one\_true\{policy(i, j, a) \text{ for a in actions}\}$

In [7]:
```
axiom_policy = []
policy = z3.Function('Policy', z3.IntSort(), z3.IntSort(), Action, z3.BoolSort())
a = z3.Const('a', Action)
axiom_policy.append(
    z3.ForAll([i, j, a], z3.Implies(policy(i, j, a), z3.And(cell(i, j), avai_action(i, j
)
actions = (stop, up, down, left, right)
axiom_policy.append(
    z3.ForAll([i, j],
              z3.Implies(z3.And(ij_range, cell(i, j)),
                  z3.PbEq([(policy(i, j, action), 1) for action in actions], 1)
              )
    )
)
axiom_policy.append(
    z3.ForAll([i, j, a],
              z3.Implies(z3.Or(i>=layout.shape[0], i<0, j>=layout.shape[1], j<0),
                         z3.Implies(policy(i, j, a), False)
              )
    )
)
axiom_policy
```

Out[7]:
```
[ForAll([i, j, a],
        Implies(Policy(i, j, a),
                And(Cell(i, j), Avai_Action(i, j, a)))),
 ForAll([i, j],
        Implies(And(And(i >= 0, i < 3, j >= 0, j < 4),
                    Cell(i, j)),
                PbEq(((Policy(i, j, stop), 1),
                      (Policy(i, j, up), 1),
```

```
                    (Policy(i, j, down), 1),
                    (Policy(i, j, left), 1),
                    (Policy(i, j, right), 1)),
                   1))),
 ForAll([i, j, a],
        Implies(Or(i >= 3, i < 0, j >= 4, j < 0),
                Implies(Policy(i, j, a), False)))]
```

Reachability encoding:

1. $\forall i, j.\, goal(i, j) \implies policy(i, j, stop)$
2. $\forall i, j.\, goal(i, j) \equiv reachable(0, i, j)$
3. For each k <= MAX, $\forall i,j.
   reachable(k, i,j) \equiv reachable(k-1, i,j) \lor
   \{\exists i',j',a. reachable(k-1, i', j') \land policy(i,j,a) \land
   move(i,j,a,i',j')\}$
4. $\forall i, j.\, cell(i, j) \implies reachable(MAX, i, j)$

In [8]:
```
MAX = 7
axiom_reachable = []
reachable = z3.Function('Reachable', z3.IntSort(), z3.IntSort(), z3.IntSort(), z3.BoolSo
axiom_reachable.append(
    z3.ForAll([i, j], reachable(0, i, j) == goal(i, j))
)
axiom_reachable.append(
    z3.ForAll([i, j], z3.Implies(goal(i, j), policy(i, j, stop)))
)

for k in range(1, MAX):
    axiom_reachable.append(
        z3.ForAll([i, j],z3.Implies(ij_range,
                  reachable(k, i, j) ==
                  z3.Or(reachable(k - 1, i, j),
                        z3.Exists(
                            [i_, j_, a],
                            z3.And(cell(i_, j_), reachable(k - 1, i_, j_), policy(i, j,
                        )
                  )
        ))

    )
axiom_reachable.append(
    z3.ForAll([i, j], z3.Implies(cell(i, j), reachable(k, i, j)))
)
axiom_reachable
```

Out[8]:
```
[ForAll([i, j], Reachable(0, i, j) == Goal(i, j)),
 ForAll([i, j], Implies(Goal(i, j), Policy(i, j, stop))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                Reachable(1, i, j) ==
                Or(Reachable(0, i, j),
                   Exists([i', j', a],
                          And(Cell(i', j'),
                              Reachable(0, i', j'),
                              Policy(i, j, a),
                              Move(i, j, a, i', j')))))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                Reachable(2, i, j) ==
                Or(Reachable(1, i, j),
                   Exists([i', j', a],
                          And(Cell(i', j'),
```

```
                                Reachable(1, i', j'),
                                Policy(i, j, a),
                                Move(i, j, a, i', j'))))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                Reachable(3, i, j) ==
                Or(Reachable(2, i, j),
                   Exists([i', j', a],
                          And(Cell(i', j'),
                                Reachable(2, i', j'),
                                Policy(i, j, a),
                                Move(i, j, a, i', j')))))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                Reachable(4, i, j) ==
                Or(Reachable(3, i, j),
                   Exists([i', j', a],
                          And(Cell(i', j'),
                                Reachable(3, i', j'),
                                Policy(i, j, a),
                                Move(i, j, a, i', j')))))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                Reachable(5, i, j) ==
                Or(Reachable(4, i, j),
                   Exists([i', j', a],
                          And(Cell(i', j'),
                                Reachable(4, i', j'),
                                Policy(i, j, a),
                                Move(i, j, a, i', j')))))),
 ForAll([i, j],
        Implies(And(i >= 0, i < 3, j >= 0, j < 4),
                Reachable(6, i, j) ==
                Or(Reachable(5, i, j),
                   Exists([i', j', a],
                          And(Cell(i', j'),
                                Reachable(5, i', j'),
                                Policy(i, j, a),
                                Move(i, j, a, i', j')))))),
 ForAll([i, j], Implies(Cell(i, j), Reachable(6, i, j)))]
```

In [9]:
```
solver = z3.Solver()
solver.add(axiom_layout)
solver
```

Out[9]: [Cell(0, 0), ¬Goal(0, 0), Cell(0, 1), ¬Goal(0, 1), Cell(0, 2), ¬Goal(0, 2), Cell(0, 3), Goal(0, 3), Cell(1, 0), ¬Goal(1, 0), Block(1, 1), ¬Goal(1, 1), Cell(1, 2), ¬Goal(1, 2), Cell(1, 3), ¬Goal(1, 3), Cell(2, 0), ¬Goal(2, 0), Block(2, 1), ¬Goal(2, 1), Cell(2, 2), ¬Goal(2, 2), Cell(2, 3), ¬Goal(2, 3), ∀i, j : Xor(Cell(i, j), Block(i, j)), ∀i, j : i ≥ 3 ∨ i < 0 ∨ j ≥ 4 ∨ j < 0 ⇒ Block(i, j)]

In [10]:
```
if solver.check() == z3.sat:
    m = solver.model()
    Cell = np.full(layout.shape, True)
    Goal = np.full(layout.shape, True)
    Block = np.full(layout.shape, True)
    for i in range(layout.shape[0]):
        for j in range(layout.shape[1]):
            Cell[i, j] = m.evaluate(cell(i, j))
            Goal[i, j] = m.evaluate(goal(i, j))
            Block[i, j] = m.evaluate(block(i, j))
    print('Cell\n', Cell)
    print('Goal\n', Goal)
    print('Block\n', Block)
```

```
Cell
```

```
 [[ True  True  True  True]
 [ True False  True  True]
 [ True False  True  True]]
Goal
 [[False False False  True]
 [False False False False]
 [False False False False]]
Block
 [[False False False False]
 [False  True False False]
 [False  True False False]]
```

In [11]:
```
solver.add(axiom_avai_actions)
solver
res = solver.check()
```

In [12]:
```
if res == z3.sat:
    m = solver.model()
    for i in range(layout.shape[0]):
        for j in range(layout.shape[1]):
            for a in actions:
                print(f'cell({i},{j}): action {a} {m.evaluate(avai_action(i, j, a))}')
```

```
cell(0,0): action stop True
cell(0,0): action up False
cell(0,0): action down True
cell(0,0): action left False
cell(0,0): action right True
cell(0,1): action stop True
cell(0,1): action up False
cell(0,1): action down False
cell(0,1): action left True
cell(0,1): action right True
cell(0,2): action stop True
cell(0,2): action up False
cell(0,2): action down True
cell(0,2): action left True
cell(0,2): action right True
cell(0,3): action stop True
cell(0,3): action up False
cell(0,3): action down True
cell(0,3): action left True
cell(0,3): action right False
cell(1,0): action stop True
cell(1,0): action up True
cell(1,0): action down True
cell(1,0): action left False
cell(1,0): action right False
cell(1,1): action stop False
cell(1,1): action up False
cell(1,1): action down False
cell(1,1): action left False
cell(1,1): action right False
cell(1,2): action stop True
cell(1,2): action up True
cell(1,2): action down True
cell(1,2): action left False
cell(1,2): action right True
cell(1,3): action stop True
cell(1,3): action up True
cell(1,3): action down True
cell(1,3): action left True
cell(1,3): action right False
cell(2,0): action stop True
cell(2,0): action up True
```

```
cell(2,0): action down False
cell(2,0): action left False
cell(2,0): action right False
cell(2,1): action stop False
cell(2,1): action up False
cell(2,1): action down False
cell(2,1): action left False
cell(2,1): action right False
cell(2,2): action stop True
cell(2,2): action up True
cell(2,2): action down False
cell(2,2): action left False
cell(2,2): action right True
cell(2,3): action stop True
cell(2,3): action up True
cell(2,3): action down False
cell(2,3): action left True
cell(2,3): action right False
```

In [13]:
```
solver.add(axiom_move)
solver
```

Out[13]: [Cell(0, 0), ¬Goal(0, 0), Cell(0, 1), ¬Goal(0, 1), Cell(0, 2), ¬Goal(0, 2), Cell(0, 3), Goal(0, 3), Cell(1, 0), ¬Goal(1, 0), Block(1, 1), ¬Goal(1, 1), Cell(1, 2), ¬Goal(1, 2), Cell(1, 3), ¬Goal(1, 3), Cell(2, 0), ¬Goal(2, 0), Block(2, 1), ¬Goal(2, 1), Cell(2, 2), ¬Goal(2, 2), Cell(2, 3), ¬Goal(2, 3), ∀i, j : Xor(Cell(i, j), Block(i, j)), ∀i, j : i ≥ 3 ∨ i < 0 ∨ j ≥ 4 ∨ j < 0 ⇒ Block(i, j), ∀i, j : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ⇒ (Cell(i, j) ∧ Cell(i, j)) = Avai_Action(i, j, stop), ∀i, j : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ⇒ (Cell(i, j) ∧ Cell(i + 1, j)) = Avai_Action(i, j, down), ∀i, j : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ⇒ (Cell(i, j) ∧ Cell(i - 1, j)) = Avai_Action(i, j, up), ∀i, j : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ⇒ (Cell(i, j) ∧ Cell(i, j + 1)) = Avai_Action(i, j, right), ∀i, j : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ⇒ (Cell(i, j) ∧ Cell(i, j - 1)) = Avai_Action(i, j, left), ∀i, j, i', j' : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ∧ i' ≥ 0 ∧ i' < 3 ∧ j' ≥ 0 ∧ j' < 4 ⇒ Move(i, j, stop, i', j') = (Cell(i, j) ∧ i' = i ∧ j' = j ∧ Cell(i, j)), ∀i, j, i', j' : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ∧ i' ≥ 0 ∧ i' < 3 ∧ j' ≥ 0 ∧ j' < 4 ⇒ Move(i, j, down, i', j') = (Cell(i, j) ∧ i' = i + 1 ∧ j' = j ∧ Cell(i + 1, j)), ∀i, j, i', j' : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ∧ i' ≥ 0 ∧ i' < 3 ∧ j' ≥ 0 ∧ j' < 4 ⇒ Move(i, j, up, i', j') = (Cell(i, j) ∧ i' = i - 1 ∧ j' = j ∧ Cell(i - 1, j)), ∀i, j, i', j' : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ∧ i' ≥ 0 ∧ i' < 3 ∧ j' ≥ 0 ∧ j' < 4 ⇒ Move(i, j, right, i', j') = (Cell(i, j) ∧ i' = i ∧ j' = j + 1 ∧ Cell(i, j + 1)), ∀i, j, i', j' : i ≥ 0 ∧ i < 3 ∧ j ≥ 0 ∧ j < 4 ∧ i' ≥ 0 ∧ i' < 3 ∧ j' ≥ 0 ∧ j' < 4 ⇒ Move(i, j, left, i', j') = (Cell(i, j) ∧ i' = i ∧ j' = j - 1 ∧ Cell(i, j - 1))]

In [14]:
```
if solver.check() == z3.sat:
    m = solver.model()
    for i in range(layout.shape[0]):
        for j in range(layout.shape[1]):
            print(f'cell({i},{j}):')
            for a in actions:
                for i_ in range(layout.shape[0]):
                    for j_ in range(layout.shape[1]):
                        if m.evaluate(move(i,j,a,i_,j_)):
                            print(f'\t{a}\t to ({i_,j_})')
```

```
cell(0,0):
        stop     to ((0, 0))
        down     to ((1, 0))
        right    to ((0, 1))
cell(0,1):
        stop     to ((0, 1))
        left     to ((0, 0))
        right    to ((0, 2))
cell(0,2):
        stop     to ((0, 2))
        down     to ((1, 2))
        left     to ((0, 1))
        right    to ((0, 3))
```

```
cell(0,3):
        stop     to ((0, 3))
        down     to ((1, 3))
        left     to ((0, 2))
cell(1,0):
        stop     to ((1, 0))
        up       to ((0, 0))
        down     to ((2, 0))
cell(1,1):
cell(1,2):
        stop     to ((1, 2))
        up       to ((0, 2))
        down     to ((2, 2))
        right    to ((1, 3))
cell(1,3):
        stop     to ((1, 3))
        up       to ((0, 3))
        down     to ((2, 3))
        left     to ((1, 2))
cell(2,0):
        stop     to ((2, 0))
        up       to ((1, 0))
cell(2,1):
cell(2,2):
        stop     to ((2, 2))
        up       to ((1, 2))
        right    to ((2, 3))
cell(2,3):
        stop     to ((2, 3))
        up       to ((1, 3))
        left     to ((2, 2))
```

In [15]:
```
solver.add(axiom_policy + axiom_reachable)
# solver
res = solver.check()
res
```

Out[15]: **sat**

In [16]:
```
%%time
if res == z3.sat:
    m = solver.model()
    for i in range(layout.shape[0]):
        for j in range(layout.shape[1]):
            for a in actions:
                if m.evaluate(policy(i, j, a)):
                    print(f'cell{i, j} do {a}')
```

```
cell(0, 0) do right
cell(0, 1) do right
cell(0, 2) do right
cell(0, 3) do stop
cell(1, 0) do up
cell(1, 2) do up
cell(1, 3) do down
cell(2, 0) do up
cell(2, 2) do up
cell(2, 3) do left
CPU times: user 2.24 s, sys: 29.1 ms, total: 2.27 s
Wall time: 2.28 s
```

In [17]:
```
m = solver.model()
for i in range(layout.shape[0]):
    for j in range(layout.shape[1]):
        for k in range(6):
```

```
            if m.evaluate(reachable(k, i, j)):
                print(f'cell{i, j} can reach goal within {k} steps')
```

```
cell(0, 0) can reach goal within 3 steps
cell(0, 0) can reach goal within 4 steps
cell(0, 0) can reach goal within 5 steps
cell(0, 1) can reach goal within 2 steps
cell(0, 1) can reach goal within 3 steps
cell(0, 1) can reach goal within 4 steps
cell(0, 1) can reach goal within 5 steps
cell(0, 2) can reach goal within 1 steps
cell(0, 2) can reach goal within 2 steps
cell(0, 2) can reach goal within 3 steps
cell(0, 2) can reach goal within 4 steps
cell(0, 2) can reach goal within 5 steps
cell(0, 3) can reach goal within 0 steps
cell(0, 3) can reach goal within 1 steps
cell(0, 3) can reach goal within 2 steps
cell(0, 3) can reach goal within 3 steps
cell(0, 3) can reach goal within 4 steps
cell(0, 3) can reach goal within 5 steps
cell(1, 0) can reach goal within 4 steps
cell(1, 0) can reach goal within 5 steps
cell(1, 2) can reach goal within 2 steps
cell(1, 2) can reach goal within 3 steps
cell(1, 2) can reach goal within 4 steps
cell(1, 2) can reach goal within 5 steps
cell(1, 3) can reach goal within 5 steps
cell(2, 0) can reach goal within 5 steps
cell(2, 2) can reach goal within 3 steps
cell(2, 2) can reach goal within 4 steps
cell(2, 2) can reach goal within 5 steps
cell(2, 3) can reach goal within 4 steps
cell(2, 3) can reach goal within 5 steps
```

In [18]:
```
m = solver.model()
for i in range(layout.shape[0]):
    for j in range(layout.shape[1]):
        for a in actions:
            if m.evaluate(policy(i, j, a)):
                print(f'cell{i, j} do {a}')
```

```
cell(0, 0) do right
cell(0, 1) do right
cell(0, 2) do right
cell(0, 3) do stop
cell(1, 0) do up
cell(1, 2) do up
cell(1, 3) do down
cell(2, 0) do up
cell(2, 2) do up
cell(2, 3) do left
```

In [ ]:
```

```