Risk Analytics 2024 – Practical 1

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Part 1: Financial returns and normality

In mathematical finance, many forecasting and inference methods critically rely on a setting under which the financial returns are independent and the logarithm of the returns are Normally distributed. In this exercise, we would like to check whether these assumptions are satisfied using the Bitcoin dataset.

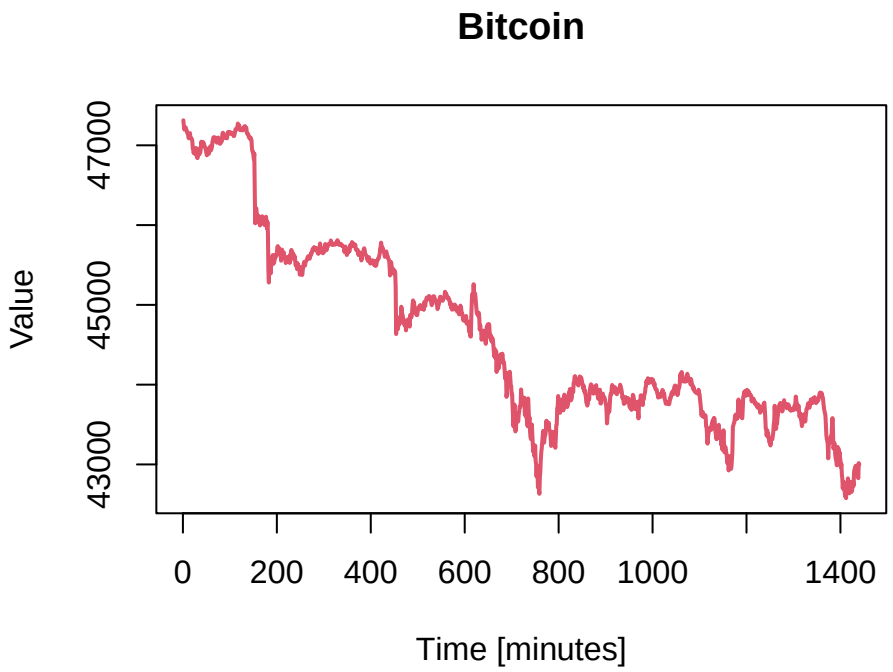

Figure 1: Tick chart of Bitcoin stock prices during 24 hours.

(a) Read in the Bitcoin data from file `Crypto_data.csv`. Then, assess the stationarity of the (raw) Bitcoin prices.

(b) Create a function to transform the Bitcoin prices into their negative log returns counterparts. Plot the latter series and assess their stationarity. To compare the series, also plot the negative log returns on a common scale.

(c) Are the negative log returns normally distributed? Draw histograms, check QQ-plots and use an Anderson-Darling testing procedure to answer this question.

(d) Fit a t-distribution to the negative log returns using `fitdistr()`. Using a QQ-plot, decide whether the fit is better than with a Normal distribution, based on your answer in (c).

(e) Compare the tails of the densities of the t-distribution and the normal distribution. Can we expect more extreme, unexpected events in t-distribution or in normal distribution? What can you conclude about the extreme events of our bitcoin data?

Part 2: Financial time series, heteroscedasticity and the random walk hypothesis

Another crucial hypothesis in asset pricing is the so-called homoscedasticity, i.e. constant variance of the residuals. We would also like to check this assumption.

We use the same Bitcoin data as in Part 1.

(a) Plot the ACF of the raw series as well as the negative log returns. Which one do you think are easier to model?

(b) Use a Ljung-Box procedure to formally test for (temporal) serial dependence in the raw series and in the negative log return series. What is your conclusion?

(c) Propose ARIMA models for the negative log returns series, based on visualization tools (e.g. ACF, PACF). Select an ARIMA model using `auto.arima()` (forecast package) for the negative log returns series. Comment on the difference. Assess the residuals of the resulting models.

(d) Fit GARCH models to the negative log returns with both normal and standardized t-distributions, with order $(1, 1)$, using the `garchFit()` function from the `fGarch` library. Assess the quality of the fit by evaluating the residuals.

(e) Residual serial correlation can be present when fitting a GARCH directly on the negative log returns. Hence, in order to circumvent this problem, it is possible to use the following two-step approach:

- Fit an ARIMA(p, d, q) on the negative log returns with the choices p, d and q from part (c);
- Fit a GARCH$(1, 1)$ on the residuals of the ARIMA(p, d, q) fit.

Proceed with the above recipe. Assess the quality of the above fit.

(f) Compare the three models from the previous parts. Which is more suitable? In which of these models is the homoscedasticity assumption violated?

Part 3: Dependence between time series

We would like to know the connection between the two most prominent financial crypto-stock prices — Bitcoin and Ethereum. Are they dependent? Are the extreme events between these time series connected?

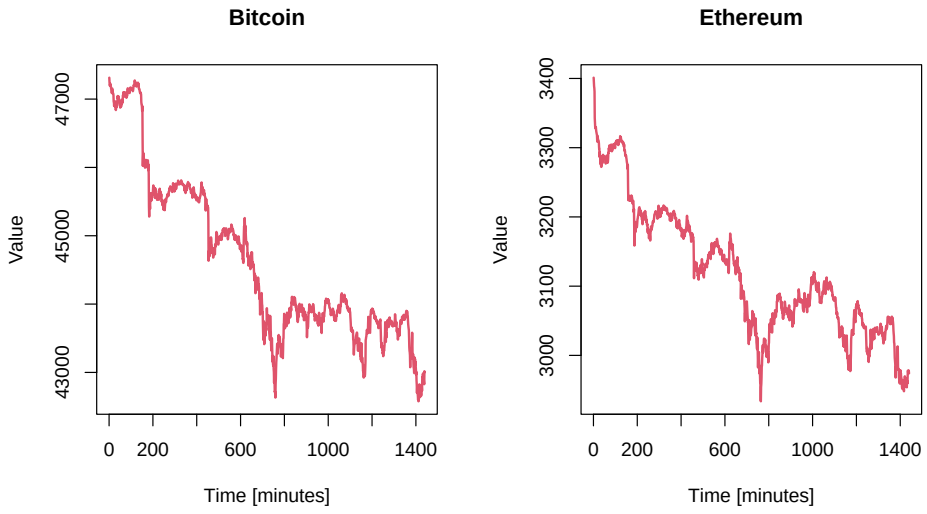

Figure 2: Bitcoin and Ethereum stocks - high frequency data measured each minute

For this part, use also the Ethereum data (ETH). Compute the negative log returns of ETH.

(a) Are the negative log returns of Bitcoin and ETH dependent? Compute the correlation using `cor.test()` function. Can we conclude that these series are independent?

(b) Calculate the cross-correlation function (CCF) between the negative log returns of Bitcoin and ETH. What do you observe?

(c) Is one of the time series good predictor of the second? Assess whether there is any predictive power between the negative log returns of Bitcoin and ETH. You can use `grangertest()` in the `lmtest` package with carefully chosen hyperparameter `order`. What is your conclusion?

(d) Based on your answer in (c), answer the following questions:

1) We observe an extreme sudden drop in Bitcoin stocks. What should we expect that will happen with ETH stocks?
2) We observe an extreme sudden drop in ETH stocks. What should we expect that will happen with Bitcoin stocks?