PID controllers and modified PID controllers

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Summary

- If the plant mathematical model cannot be easily obtained, then an analytical or computational approach to the design of a PID controller is not possible.
- Then we must resort to experimental approaches to the tuning of PID controllers.
- Ziegler and Nichols suggested rules for tuning PID controllers (values K_p , T_i, and T_d) based on:
    - Experimental step responses (Method 1).
    - Based on the value of K_p that results in marginal stability when only proportional control action is used (Method 2).

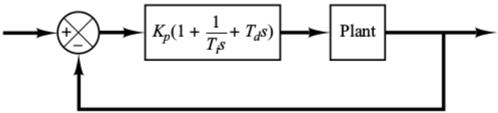
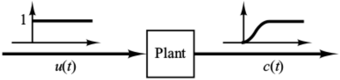


Figure 8–1
PID control of a plant.

Ziegler–Nichols Rules for Tuning PID Controllers
Method 1

- We obtain experimentally the response of the plant to a unit-step input.
- This method applies if the response to a step input exhibits an S-shaped curve.
- If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped.

Figure 8–2
Unit-step response of a plant.



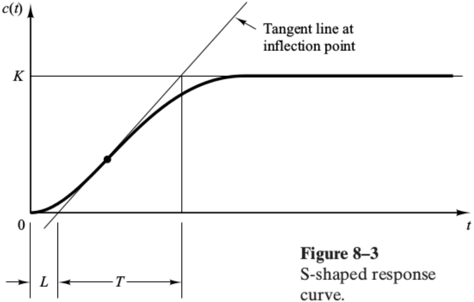


Figure 8–3
S-shaped response curve.

- The S-shaped curve may be characterized by two constants, delay time L and time constant T.

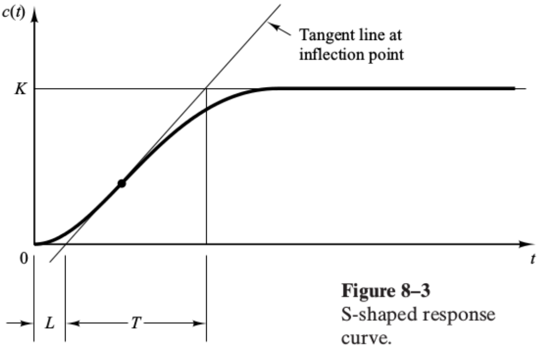


Figure 8–3
S-shaped response curve.

Table 8–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

- We first set $T_i = \infty$, and $T_d = 0$.
- Increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations.
- Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined.
- If the output does not exhibit sustained oscillations, then this method does not apply.

Figure 8–4 Closed-loop system with a proportional controller.

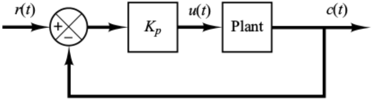


Figure 8–5 Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

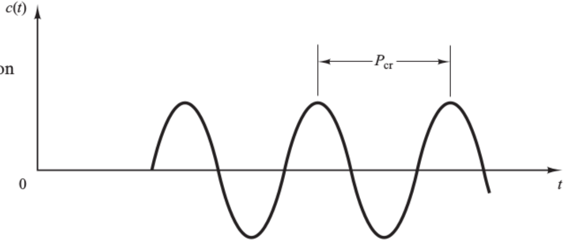


Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\ &= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

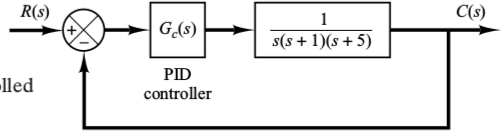
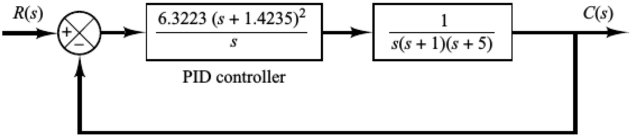


Figure 8–6 PID-controlled system.

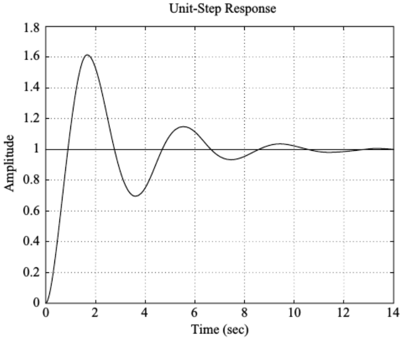
$$
\begin{aligned}
K_{cr} &= 30 \\
P_{cr} &= 2.8099 \\
K_p &= 0.6 \cdot K_{cr} = 18 \\
T_i &= 0.5 \cdot P_{cr} = 1.405 \\
T_d &= 0.125 \cdot P_{cr} = 0.35124
\end{aligned}
$$



Maximum overshoot is close to 62%.

MATLAB Program 8–1

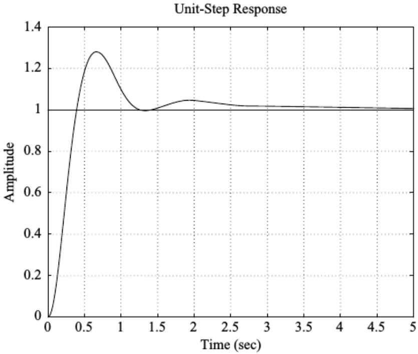
```
% ---------- Unit-step response ----------

num = [6.3223  18  12.811];
den = [1  6  11.3223  18  12.811];
step(num,den)
grid
title('Unit-Step Response')
```



The Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

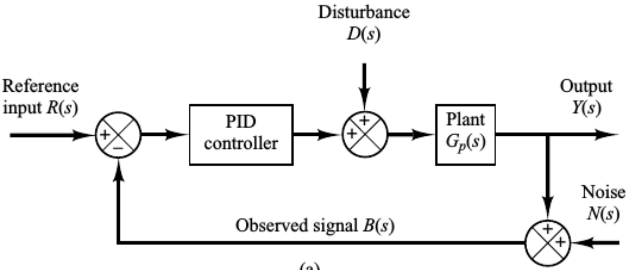
$$K_p = 39.42$$
$$T_i = 3.077$$
$$T_d = 0.7692$$

Maximum overshoot is fairly close to 25%.



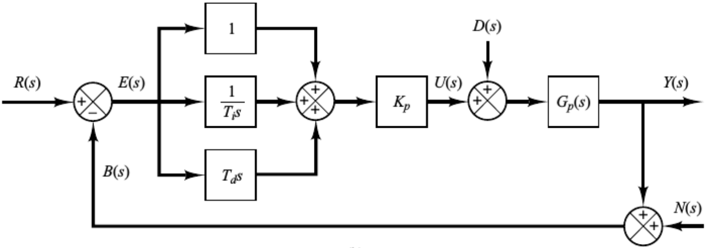
- The value of Kp increases the speed of response.
- However, varying the location of the double zero has a significant effect on the maximum overshoot.

- In practical cases, there may be one requirement on the *response to disturbance input* and another requirement on the *response to reference input*.
- Often these two requirements conflict with each other and cannot be satisfied in the single-degree-of-freedom case.
- By increasing the degrees of freedom, we are able to satisfy both, response to disturbance and response to reference input.



(a)

- If reference input is a step function, the derivative term in the control action will produce that $u(t)$ will involve an impulse function.
- If T_d is multiplied by 0.1, $u(t)$ will involve an pulse function.
- Such a phenomenon is called *set-point kick*.

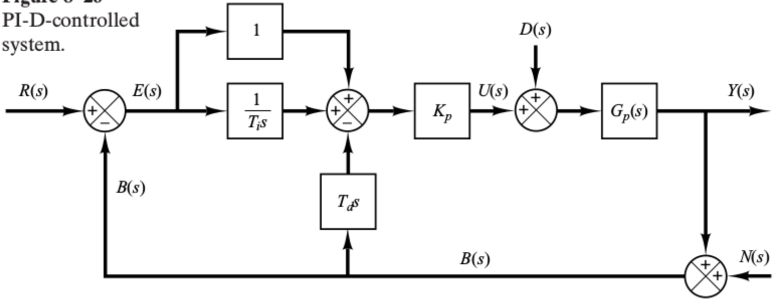


(b)

To avoid the set-point kick phenomenon (a pulse input), we may wish to operate the derivative action only in the feedback path so that differentiation occurs only on the feedback signal and not on the reference signal.

$$U(s) = K_p\left(1 + \frac{1}{T_i s}\right)R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$

Figure 8–26
PI-D-controlled system.



Notice that in the absence of the disturbances and noises, the closed-loop transfer function of the basic PID control system [shown in Figure 8–25(b)] and the PI-D control system (shown in Figure 8–26) are given, respectively, by

$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s} + T_d s\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

and

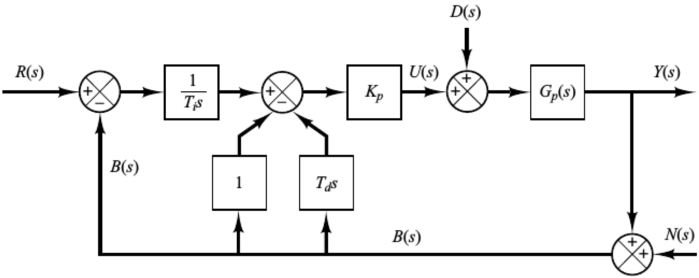
$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

It is important to point out that in the absence of the reference input and noises, the closed-loop transfer function between the disturbance $D(s)$ and the output $Y(s)$ in either case is the same and is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

- If reference input is a step function, both PID control and PI-D control involve a step function in the manipulated signal.
- The proportional action and derivative action are moved to the feedback path so that these actions affect the feedback signal only.

$$U(s) = K_p \frac{1}{T_i s} R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$



The closed-loop transfer function $Y(s)/R(s)$ in the absence of the disturbance input and noise input is given by

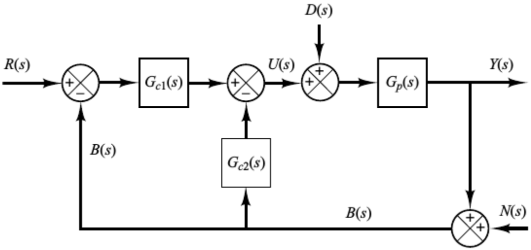
$$\frac{Y(s)}{R(s)} = \left(\frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

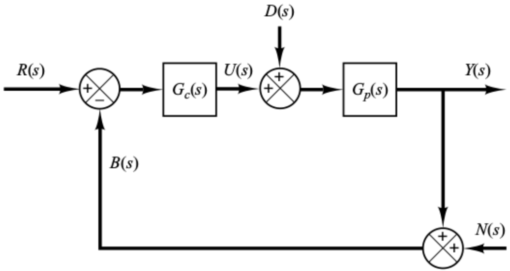
It is noted that in the absence of the reference input and noise signals, the closed-loop transfer function between the disturbance input and the output is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

This expression is the same as that for PID control or PI-D control.

- Instead of moving the entire derivative control action or proportional control action to the feedback path, it is possible to move only portions of these control actions to the feedback path.
- The characteristics of PI-PD control scheme lie between PID control and I-PD control.
- Similarly, PID-PD control can be considered.
- We will have one controller in the feedforward path and another controller in the feedback path. Such control schemes lead us to a two-degrees-of-freedom control scheme.





For this system, three closed-loop transfer functions $Y(s)/R(s) = G_{yr}$, $Y(s)/D(s) = G_{yd}$, and $Y(s)/N(s) = G_{yn}$ may be derived. They are

$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_c G_p}{1 + G_c G_p}$$

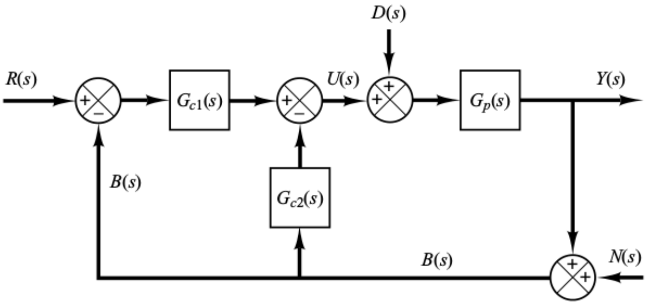
$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_c G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_c G_p}{1 + G_c G_p}$$

$$G_{yr} = \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

How many of these closed-loop transfer functions are independent?



$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{(G_{c1} + G_{c2})G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yr} = G_{c1}G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

How many of these closed-loop transfer functions are independent?

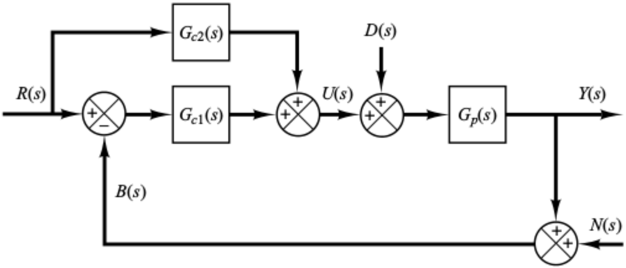
$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + G_{c1}G_p} + \frac{G_{c2}G_p}{1 + G_{c1}G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_{c1}G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_{c1}G_p}{1 + G_{c1}G_p}$$

$$G_{yr} = G_{c2}G_{yd} + \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$



How many of these closed-loop transfer functions are independent?

Bibliography

- Ogata, Katsuhiko. Modern Control Engineering. Fifth Edition. Prentice Hall. 2009. Chapter 8.