Modeling of Pneumatic Systems

Dr. Ing. Rodrigo Gonzalez

rodralez@fing.uncu.edu.ar

Control y Sistemas

Facultad de Ingeniería,
Universidad Nacional de Cuyo

UNCUYO
UNIVERSIDAD
NACIONAL DE CUYO

Symmary

Resistance of Pressure Systems

The gas flow resistance R may be defined as follows:

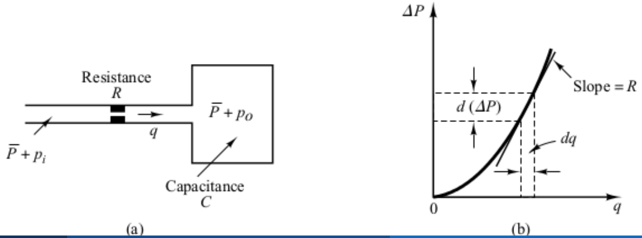
$$R = \frac{\text{change in gas pressure difference, lb}_f/\text{ft}^2}{\text{change in gas flow rate, lb/sec}}$$

or

$$R = \frac{d(\Delta P)}{dq} \qquad (4\text{–}8)$$

- where $d(\Delta P)$ is a small change in the gas pressure difference, and dq is a small change in the gas flow rate.
- Computation of the value of the gas flow resistance R may be quite time consuming.
- Experimentally, it can be determined from a plot of the pressure difference versus flow rate.

Figure 4–4
(a) Schematic diagram of a pressure system; (b) pressure-difference-versus-flow-rate curve.



Capacitance of Pressure Systems

The capacitance of the pressure vessel may be defined by

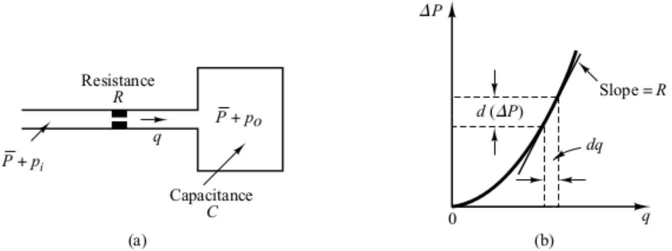
$$C = \frac{\text{change in gas stored, lb}}{\text{change in gas pressure, lb}_f/\text{ft}^2}$$

or

$$C = \frac{dm}{dp} = V\,\frac{d\rho}{dp} \qquad (4\text{–}9)$$

$$C = \frac{V}{nR_{\text{gas}}T} \qquad (4\text{–}12)$$

Figure 4–4
(a) Schematic diagram of a pressure system; (b) pressure-difference-versus-flow-rate curve.



Let us define

$\bar{P}$ = gas pressure in the vessel at steady state (before changes in pressure have occurred), $\mathrm{lb_f/ft^2}$

p_i = small change in inflow gas pressure, $\mathrm{lb_f/ft^2}$

p_o = small change in gas pressure in the vessel, $\mathrm{lb_f/ft^2}$

V = volume of the vessel, $\mathrm{ft^3}$

m = mass of gas in the vessel, lb

q = gas flow rate, lb/sec

ρ = density of gas, $\mathrm{lb/ft^3}$

For small values of p_i and p_o, the resistance R given by Equation (4–8) becomes constant and may be written as

$$R = \frac{p_i - p_o}{q}$$

The capacitance C is given by Equation (4–9), or

$$C = \frac{dm}{dp}$$

Since the pressure change dp_o times the capacitance C is equal to the gas added to the vessel during dt seconds, we obtain

$$C\, dp_o = q\, dt$$

or

$$C \frac{dp_o}{dt} = \frac{p_i - p_o}{R}$$

which can be written as

$$RC \frac{dp_o}{dt} + p_o = p_i$$

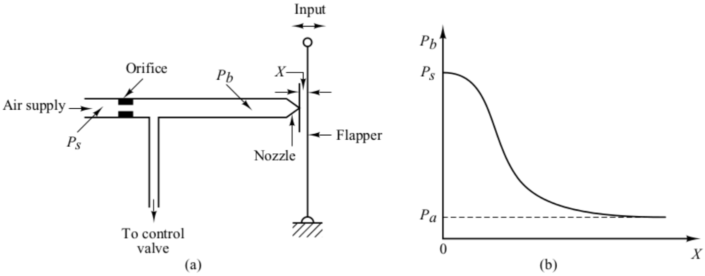
If p_i and p_o are considered the input and output, respectively, then the transfer function of the system is

$$\frac{P_o(s)}{P_i(s)} = \frac{1}{RCs + 1}$$

Pneumatic Nozzle–Flapper Amplifiers

- The nozzle–flapper amplifier converts displacement into a pressure signal.
- The power source for this amplifier is a supply of air at constant pressure.
- The nozzle–flapper amplifier converts small changes in the position of the flapper into large changes in the back pressure in the nozzle.
- Thus a large power output can be controlled by the very little power that is needed to position the flapper.
- To ensure proper functioning of the amplifier, the nozzle diameter must be larger than the orifice diameter (0.4 mmm vs 0.25 mm).
- The lowest possible pressure will be the ambient pressure P_a.

Figure 4–5
(a) Schematic diagram of a pneumatic nozzle–flapper amplifier; (b) characteristic curve relating nozzle back pressure and nozzle–flapper distance.



- As the nozzle back pressure P_b increases, the diaphragm valve moves downward.
- The atmosphere decreases and the opening to the pneumatic valve increases, thereby increasing the control pressure P_c.
- P_c can vary from 0 psi (pound-force per square inch) to full supply pressure, usually 20 psi.
- Since P_c changes almost instantaneously with changes in P_b, the time constant of the pneumatic relay is negligible.

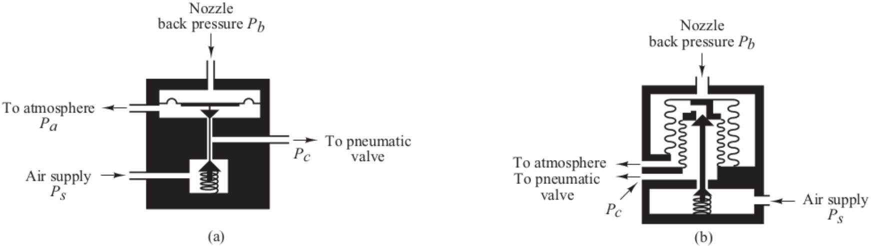
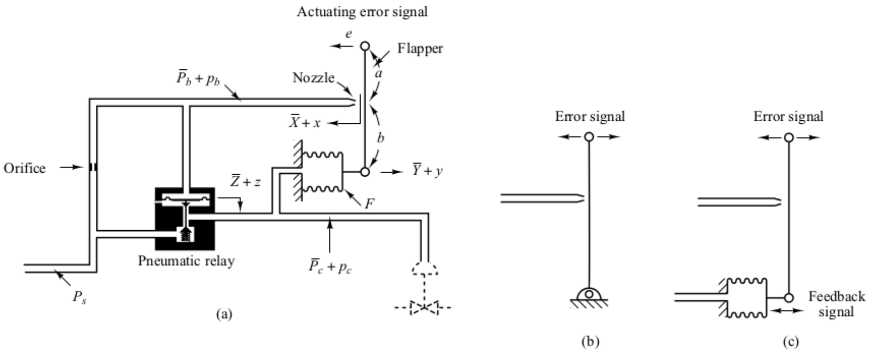


Figure 4–6
(a) Schematic diagram of a bleed-type relay; (b) schematic diagram of a nonbleed-type relay.

- The nozzle-flapper amplifier constitutes the first-stage amplifier, and the relay-type amplifier constitutes the second-stage amplifier.
- Feedback of the pneumatic output reduces the amount of actual movement of the flapper.
- Error signal and bellows F now move the flapper.
- If these two movements were equal, no control action would result.
- The effect of the feedback bellows is thus to reduce the sensitivity of the controller.



Assuming that the relationship between the variation in the nozzle back pressure and the variation in the nozzle–flapper distance is linear, we have

$$p_b = K_1 x \tag{4–13}$$

where K_1 is a positive constant. For the diaphragm valve,

$$p_b = K_2 z \tag{4–14}$$

where K_2 is a positive constant. The position of the diaphragm valve determines the control pressure. If the diaphragm valve is such that the relationship between p_c and z is linear, then

$$p_c = K_3 z \tag{4–15}$$

where K_3 is a positive constant. From Equations (4–13), (4–14), and (4–15), we obtain

$$p_c = \frac{K_3}{K_2} p_b = \frac{K_1 K_3}{K_2} x = K x \tag{4–16}$$

where $K = K_1 K_3 / K_2$ is a positive constant. For the flapper, since there are two small movements (e and y) in opposite directions, we can consider such movements separately and add up the results of two movements into one displacement x. See Figure 4–8(d). Thus, for the flapper movement, we have

$$x = \frac{b}{a + b} e - \frac{a}{a + b} y \tag{4–17}$$

The bellows acts like a spring, and the following equation holds true:

$$A p_c = k_s y \tag{4–18}$$

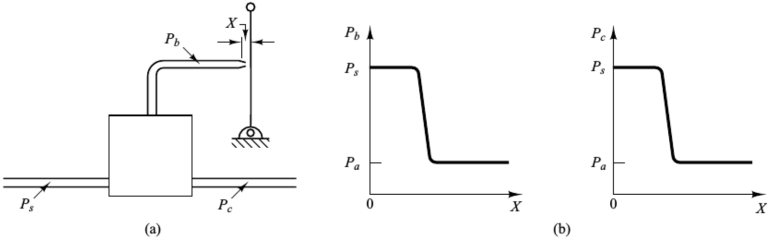
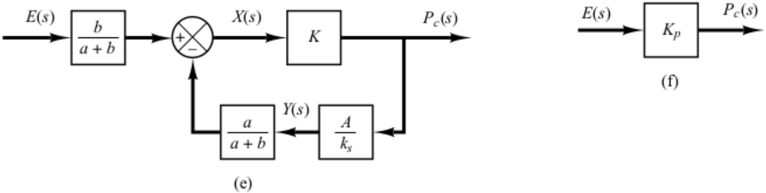
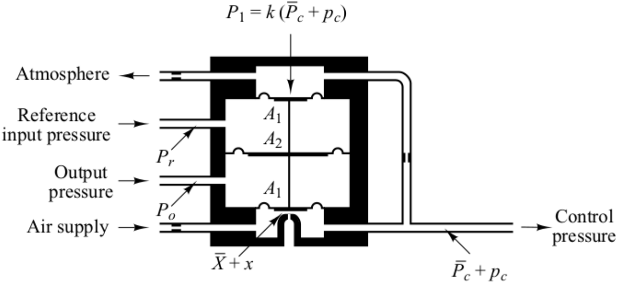


Figure 4–9
(a) Pneumatic controller without a feedback mechanism; (b) curves P_b versus X and P_c versus X.

$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b}K}{1 + K\frac{a}{a+b}\frac{A}{k_s}} = K_p \qquad (4\text{–}19)$$



- Such controllers are called stack controllers.
- The main advantage of the force-balance type controller is that it eliminates many mechanical linkages and pivot joints, thereby reducing the effects of friction.



$$p_e = P_r - P_o \tag{4–20}$$

If $p_e = 0$, there is an equilibrium state with the nozzle–flapper distance equal to $\bar{X}$ and the control pressure equal to $\bar{P}_c$. At this equilibrium state, $P_1 = \bar{P}_c k$ (where $k < 1$) and

$$\bar{X} = \alpha\left(\bar{P}_c A_1 - \bar{P}_c k A_1\right) \tag{4–21}$$

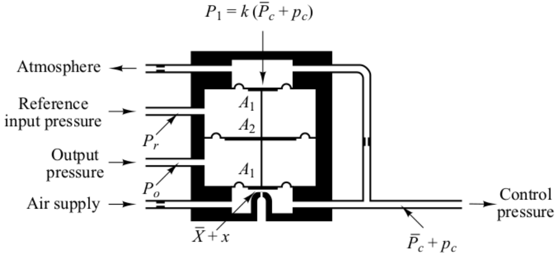
where α is a constant.

Let us assume that $p_e \neq 0$ and define small variations in the nozzle–flapper distance and control pressure as x and p_c, respectively. Then we obtain the following equation:

$$\bar{X} + x = \alpha\left[\left(\bar{P}_c + p_c\right)A_1 - \left(\bar{P}_c + p_c\right)kA_1 - p_e\left(A_2 - A_1\right)\right] \tag{4–22}$$

From Equations (4–21) and (4–22), we obtain

$$x = \alpha\left[p_c(1 - k)A_1 - p_e\left(A_2 - A_1\right)\right] \tag{4–23}$$



At this point, we must examine the quantity x. In the design of pneumatic controllers, the nozzle–flapper distance is made quite small. In view of the fact that x/α is very much smaller than $p_c(1-k)A_1$ or $p_e(A_2 - A_1)$—that is, for $p_e \neq 0$

$$\frac{x}{\alpha} \ll p_c(1-k)A_1$$

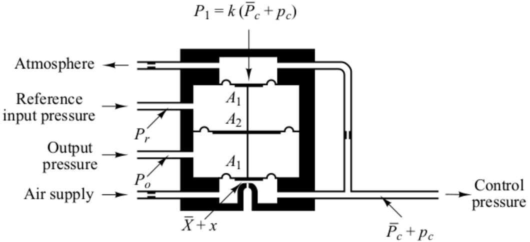
$$\frac{x}{\alpha} \ll p_e(A_2 - A_1)$$

we may neglect the term x in our analysis. Equation (4–23) can then be rewritten to reflect this assumption as follows:

$$p_c(1-k)A_1 = p_e(A_2 - A_1)$$

and the transfer function between p_c and p_e becomes

$$\frac{P_c(s)}{P_e(s)} = \frac{A_2 - A_1}{A_1}\frac{1}{1-k} = K_p$$



$$Ap_c = m\ddot{x} + b\dot{x} + kx$$

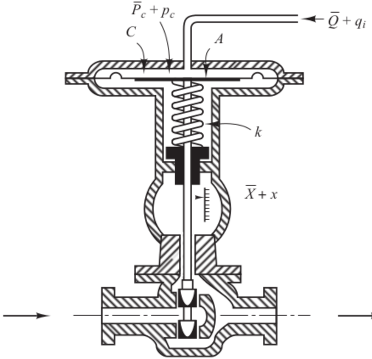
where m = mass of the valve and valve stem
b = viscous-friction coefficient
k = spring constant

If the force due to the mass and viscous friction are negligibly small, then this last equation can be simplified to

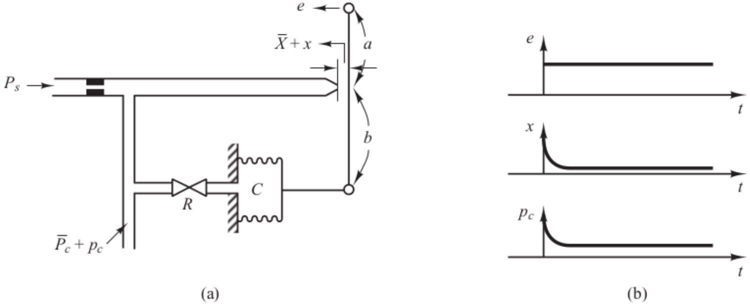
$$Ap_c = kx$$

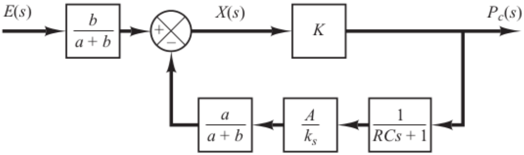
The transfer function between x and p_c thus becomes

$$\frac{X(s)}{P_c(s)} = \frac{A}{k} = K_c$$



Basic Principle for Obtaining Derivative Control Action





Proportional-Plus-Derivative Control Action (PD)

$$\frac{P_c(s)}{E(s)} = \frac{\dfrac{b}{a + b}K}{1 + \dfrac{Ka}{a + b}\dfrac{A}{k_s}\dfrac{1}{RCs + 1}}$$

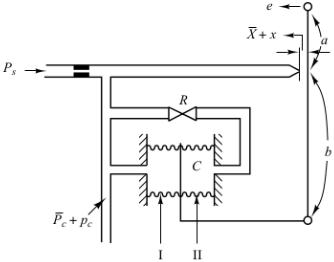
In such a controller the loop gain $\left|KaA/\left[(a + b)k_s(RCs + 1)\right]\right|$ is made much greater than unity. Thus the transfer function $P_c(s)/E(s)$ can be simplified to give

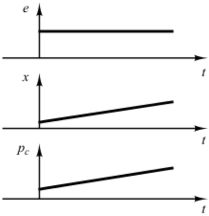
$$\frac{P_c(s)}{E(s)} = K_p(1 + T_d s)$$

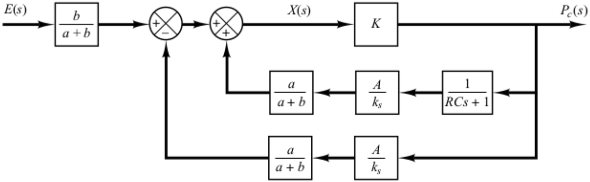
where

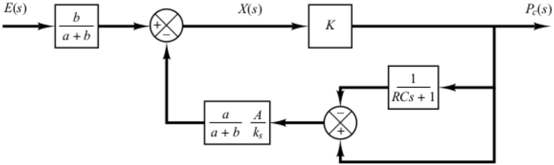
$$K_p = \frac{bk_s}{aA}, \qquad T_d = RC$$

Proportional-Plus-Integral Control Action









Proportional-Plus-Integral Control Action, II

$$\frac{P_c(s)}{E(s)} = \frac{\dfrac{b}{a+b}K}{1 + \dfrac{Ka}{a+b}\dfrac{A}{k_s}\left(1 - \dfrac{1}{RCs+1}\right)}$$

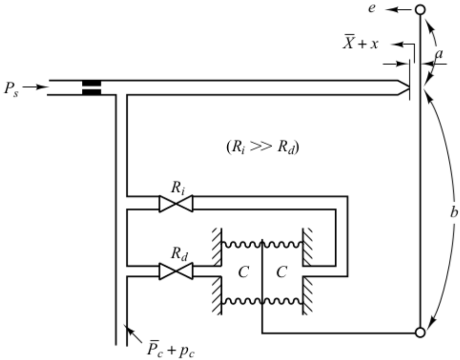
where K is a constant, A is the area of the bellows, and k_s is the equivalent spring constant of the combined bellows. If $\left|KaARCs/\left[(a+b)k_s(RCs+1)\right]\right| \gg 1$, which is usually the case, the transfer function can be simplified to

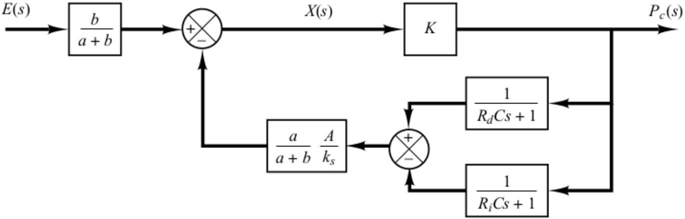
$$\frac{P_c(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$$

where

$$K_p = \frac{bk_s}{aA}, \qquad T_i = RC$$

Proportional-Plus-Integral-Plus-Derivative Control Action





The transfer function of this controller is

$$\frac{P_c(s)}{E(s)} = \frac{\dfrac{bK}{a+b}}{1 + \dfrac{Ka}{a+b}\dfrac{A}{k_s}\dfrac{(R_iC - R_dC)s}{(R_dCs+1)(R_iCs+1)}}$$

By defining

$$T_i = R_iC, \qquad T_d = R_dC$$

and noting that under normal operation $|KaA(T_i - T_d)s/[(a+b)k_s(T_ds+1)(T_is+1)]| \gg 1$ and $T_i \gg T_d$, we obtain

$$\begin{aligned}\frac{P_c(s)}{E(s)} &\doteq \frac{bk_s}{aA}\frac{(T_ds+1)(T_is+1)}{(T_i - T_d)s} \\ &\doteq \frac{bk_s}{aA}\frac{T_dT_is^2 + T_is + 1}{T_is} \\ &= K_p\left(1 + \frac{1}{T_is} + T_ds\right) \qquad (4\text{–}24)\end{aligned}$$

where

$$K_p = \frac{bk_s}{aA}$$

Comparison Between Pneumatic Systems and Hydraulic Systems. The fluid generally found in pneumatic systems is air; in hydraulic systems it is oil. And it is primarily the different properties of the fluids involved that characterize the differences between the two systems. These differences can be listed as follows:

1. Air and gases are compressible, whereas oil is incompressible (except at high pressure).
2. Air lacks lubricating property and always contains water vapor. Oil functions as a hydraulic fluid as well as a lubricator.
3. The normal operating pressure of pneumatic systems is very much lower than that of hydraulic systems.
4. Output powers of pneumatic systems are considerably less than those of hydraulic systems.
5. Accuracy of pneumatic actuators is poor at low velocities, whereas accuracy of hydraulic actuators may be made satisfactory at all velocities.
6. In pneumatic systems, external leakage is permissible to a certain extent, but internal leakage must be avoided because the effective pressure difference is rather small. In hydraulic systems internal leakage is permissible to a certain extent, but external leakage must be avoided.
7. No return pipes are required in pneumatic systems when air is used, whereas they are always needed in hydraulic systems.
8. Normal operating temperature for pneumatic systems is 5° to 60°C (41° to 140°F). The pneumatic system, however, can be operated in the 0° to 200°C (32° to 392°F) range. Pneumatic systems are insensitive to temperature changes, in contrast to hydraulic systems, in which fluid friction due to viscosity depends greatly on temperature. Normal operating temperature for hydraulic systems is 20° to 70°C (68° to 158°F).
9. Pneumatic systems are fire- and explosion-proof, whereas hydraulic systems are not, unless nonflammable liquid is used.

Bibliography

- Ogata, Katsuhiko. Modern Control Engineering. Fifth Edition. Prentice Hall. 2009. Chapter 4.