Finite impulse response filtering

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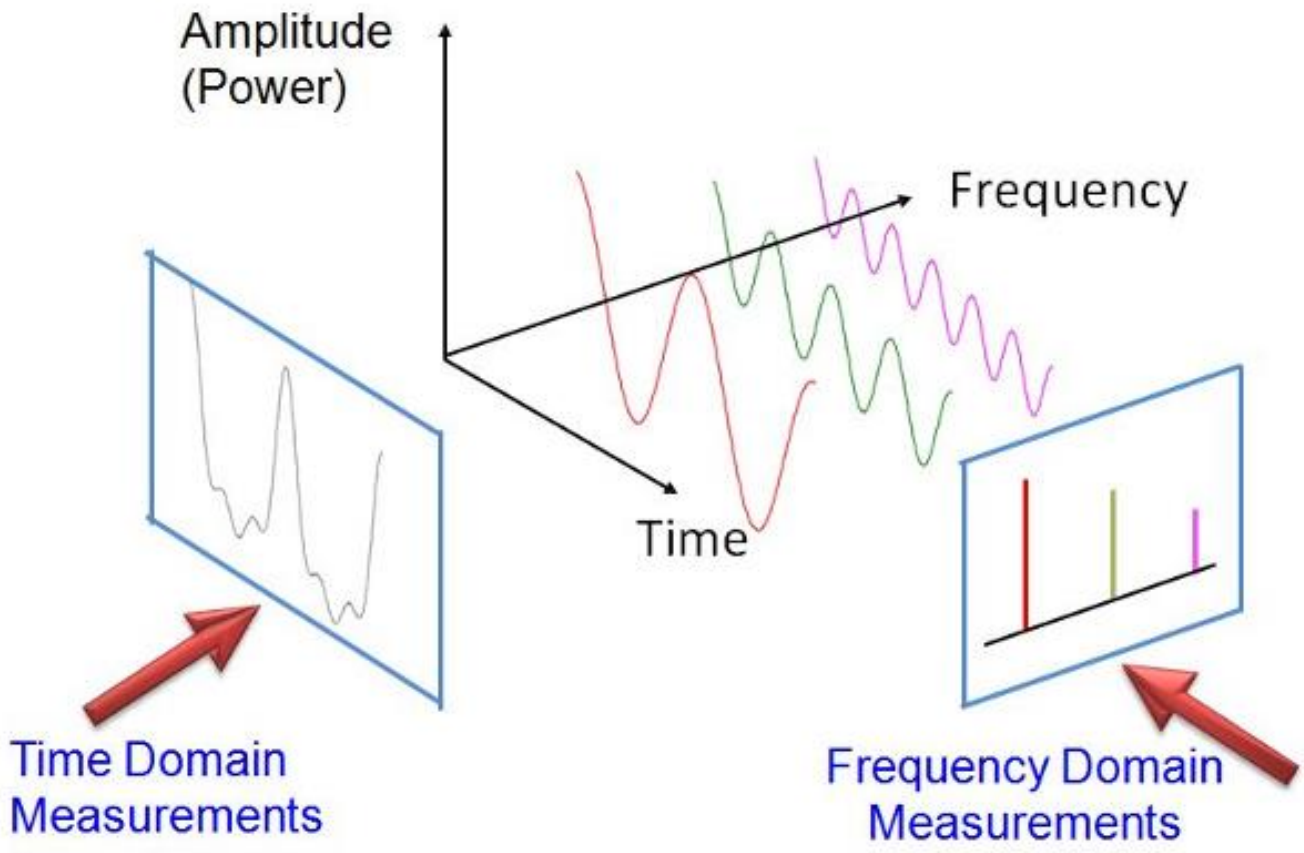
Summary

1. Introduction to Discrete Filters
   - Classification of discrete filters
2. FIR filtering in time domain
   - Time domain parameters
   - Moving average filter
   - Noise Reduction vs. Step Response
   - Frequency Response
   - MA cutoff frequency
3. Filtering in frequency domain
   - Frequency domain parameters
   - Filters by windowing
   - Kaiser window filter
   - FIR filter design
4. FIR structures

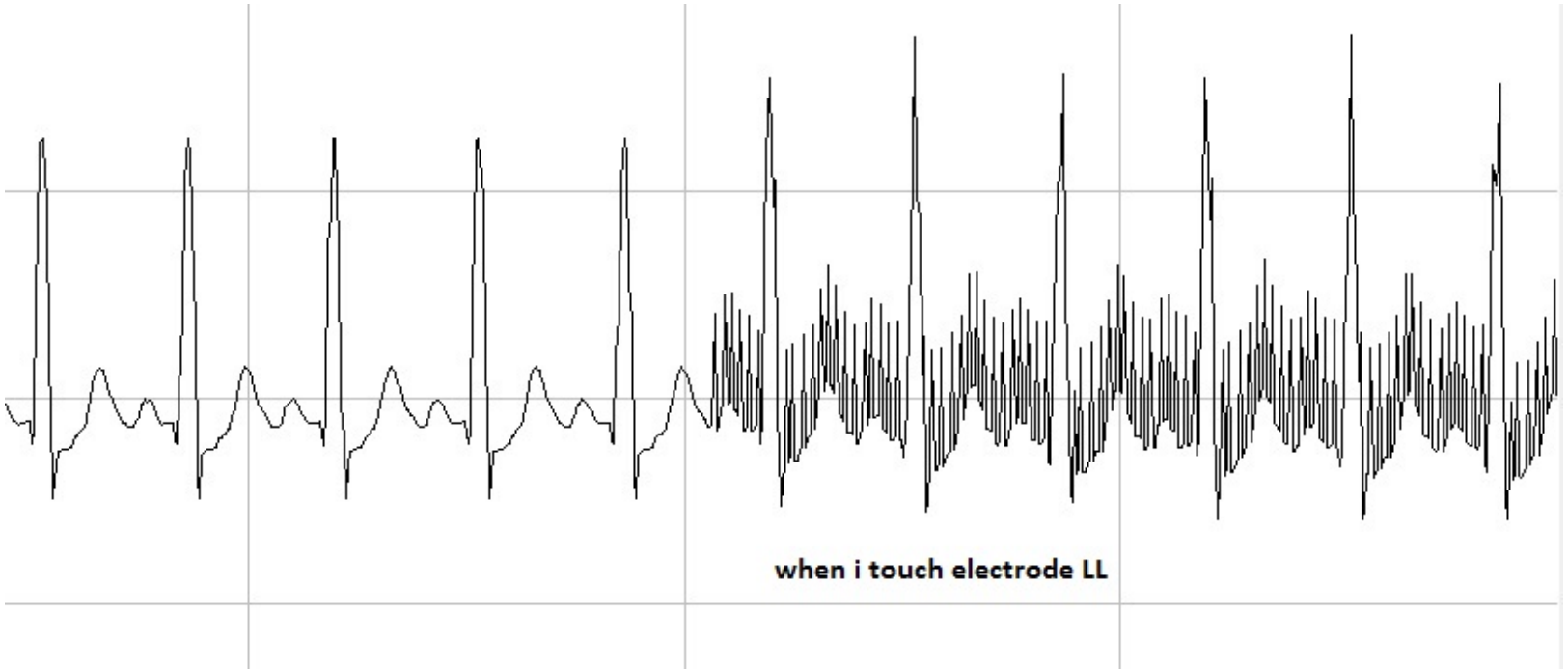
Filtering in different domains

- Filtering in **time domain** (signal restoration, smoothing, denoising).
- Filtering in **frequency domain** (signal separation).
- Choosing filtering in time or frequency domain depends on **where the information is**.



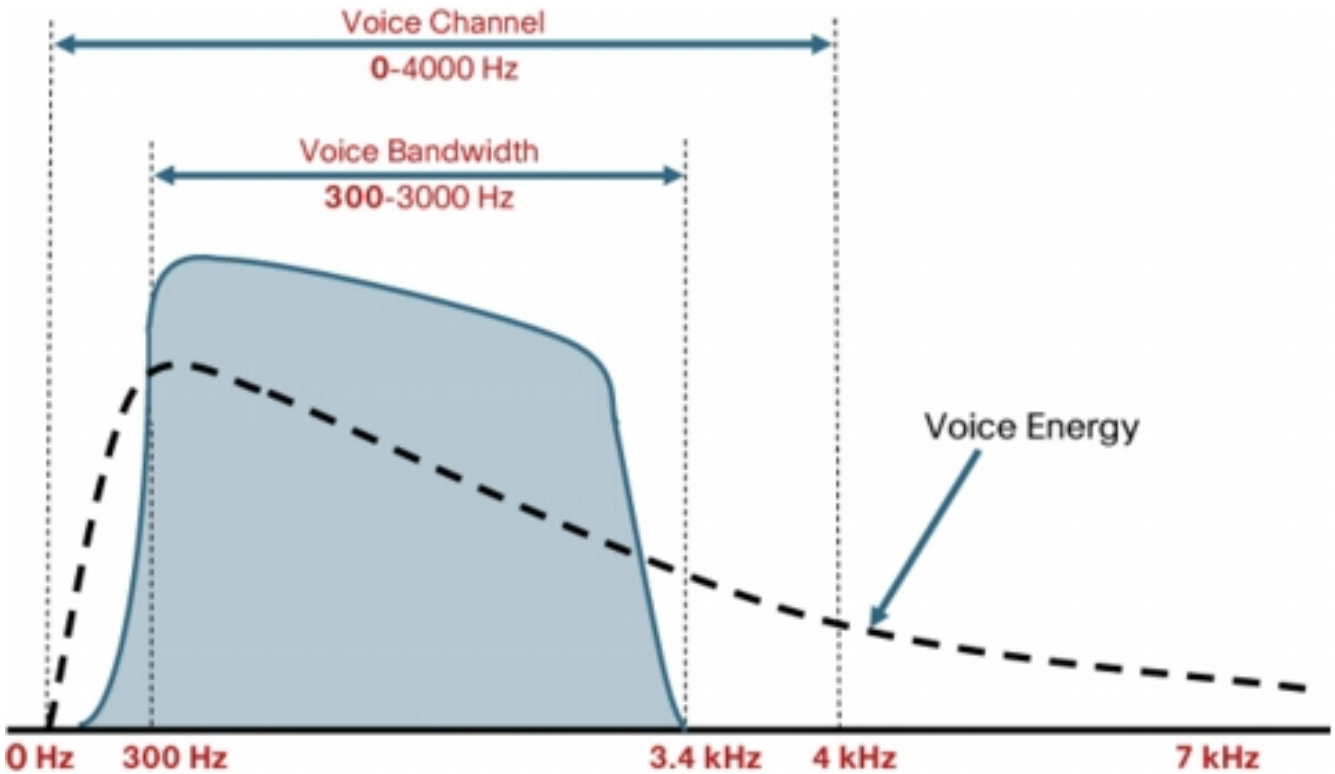
Information in time domain

- Information is contained in amplitude and time of the signal.
- Each sample contains information that is interpretable without reference to any other sample.
- The **step response** describes how information represented in the time domain is being modified by the system.
- Examples: electrocardiography (ECG) signal, accelerometer, gyroscope...



Information in frequency domain

- The information is contained in the relationship between many points in the signal.
- The frequency response shows how information represented in the frequency domain is being changed.
- Example: telephone voice channel, equalizer...



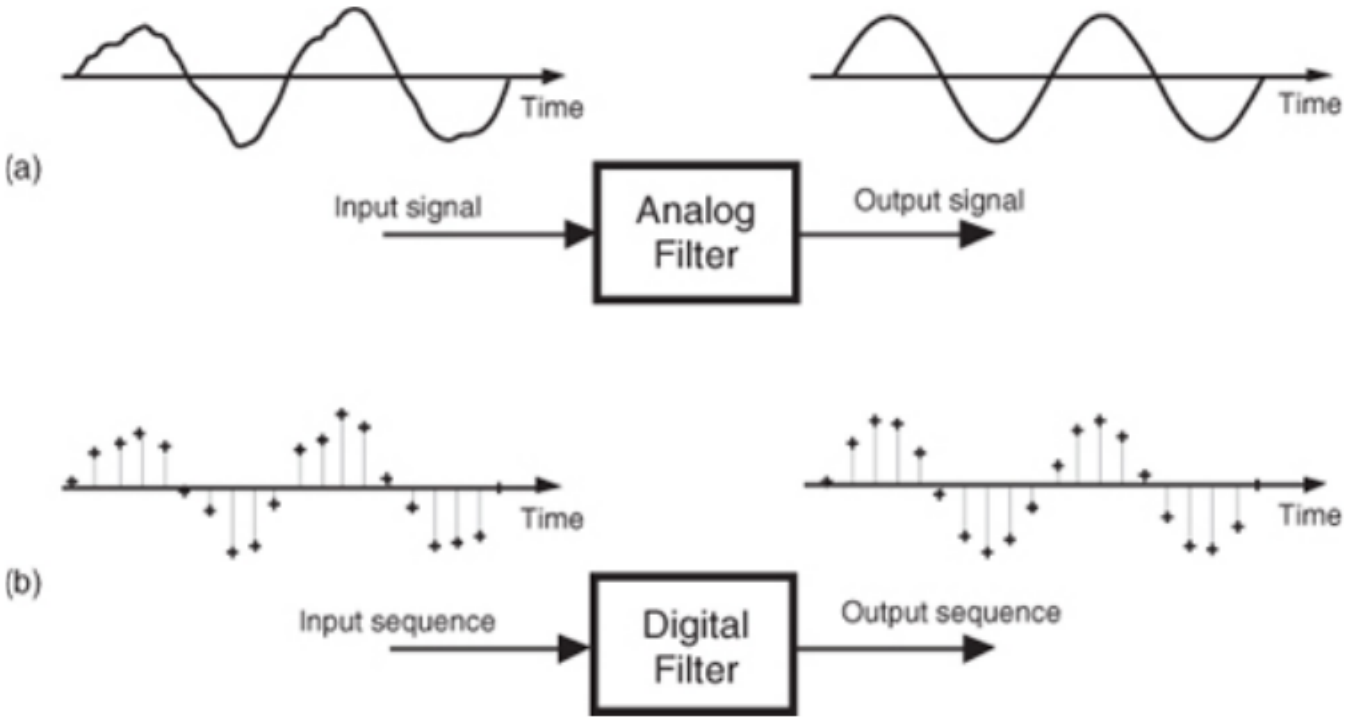
Classification of discrete filters

Table: Classification of discrete filters

	Finite impulse response (FIR)	Infinite impulse response (IIR)
Filtering in time domain	Moving average	Leaky Integrator
Filtering in frequency domain	Windowed Filters Equiripple Minimax	Bilinear z-transform

FIR filtering structure

Figure 5-1 Filters: (a) an analog filter with a noisy tone input and a reduced-noise tone output; (b) the digital equivalent of the analog filter.

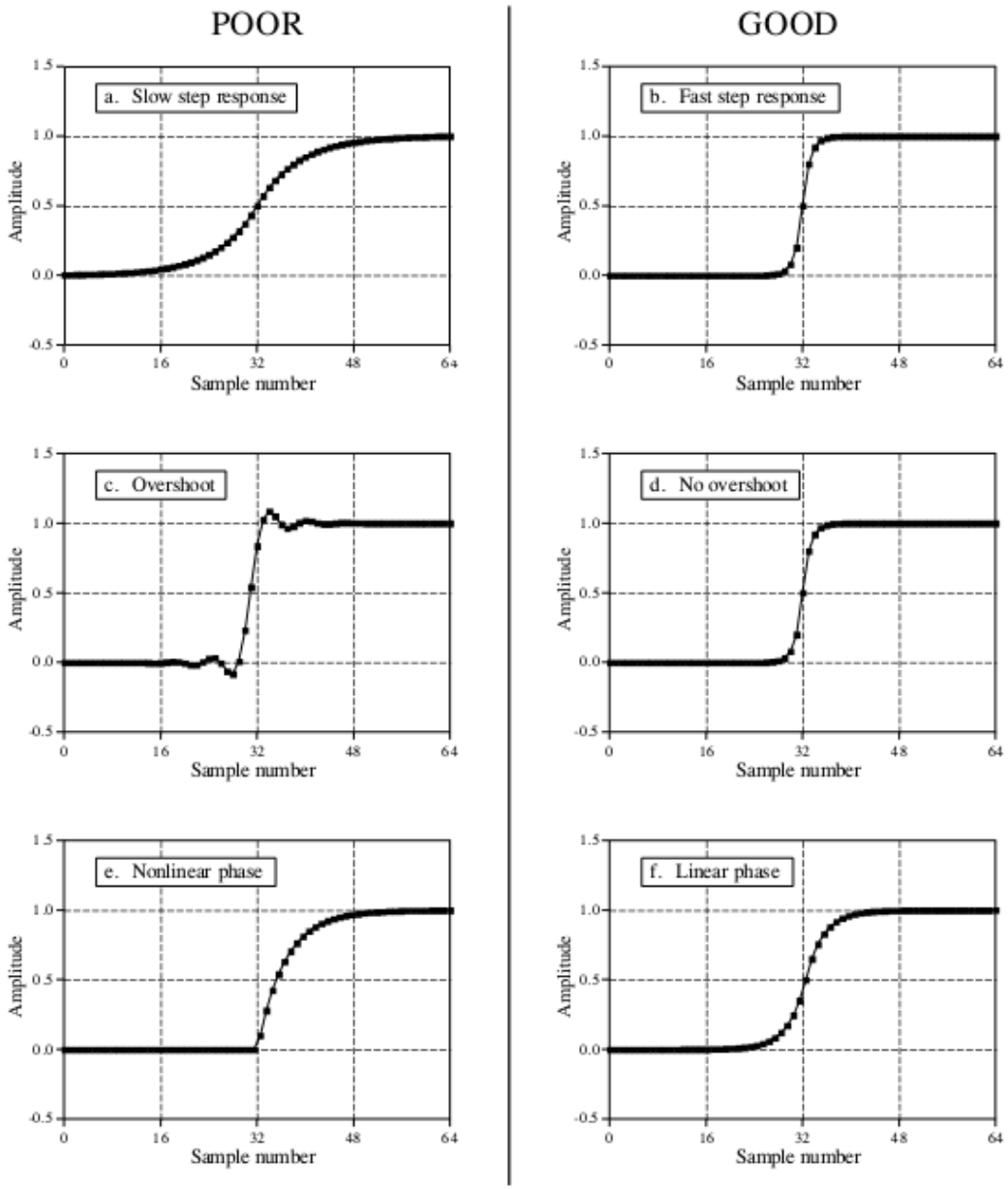


Time domain parameters, step response

- Risetime (between 10%∼90% amplitude).
- Overshoot.
- Linear phase.

It is not possible to optimize a filter for both domains.

Good performance in the time domain results in poor performance in the frequency domain, and vice versa.



Moving average filter

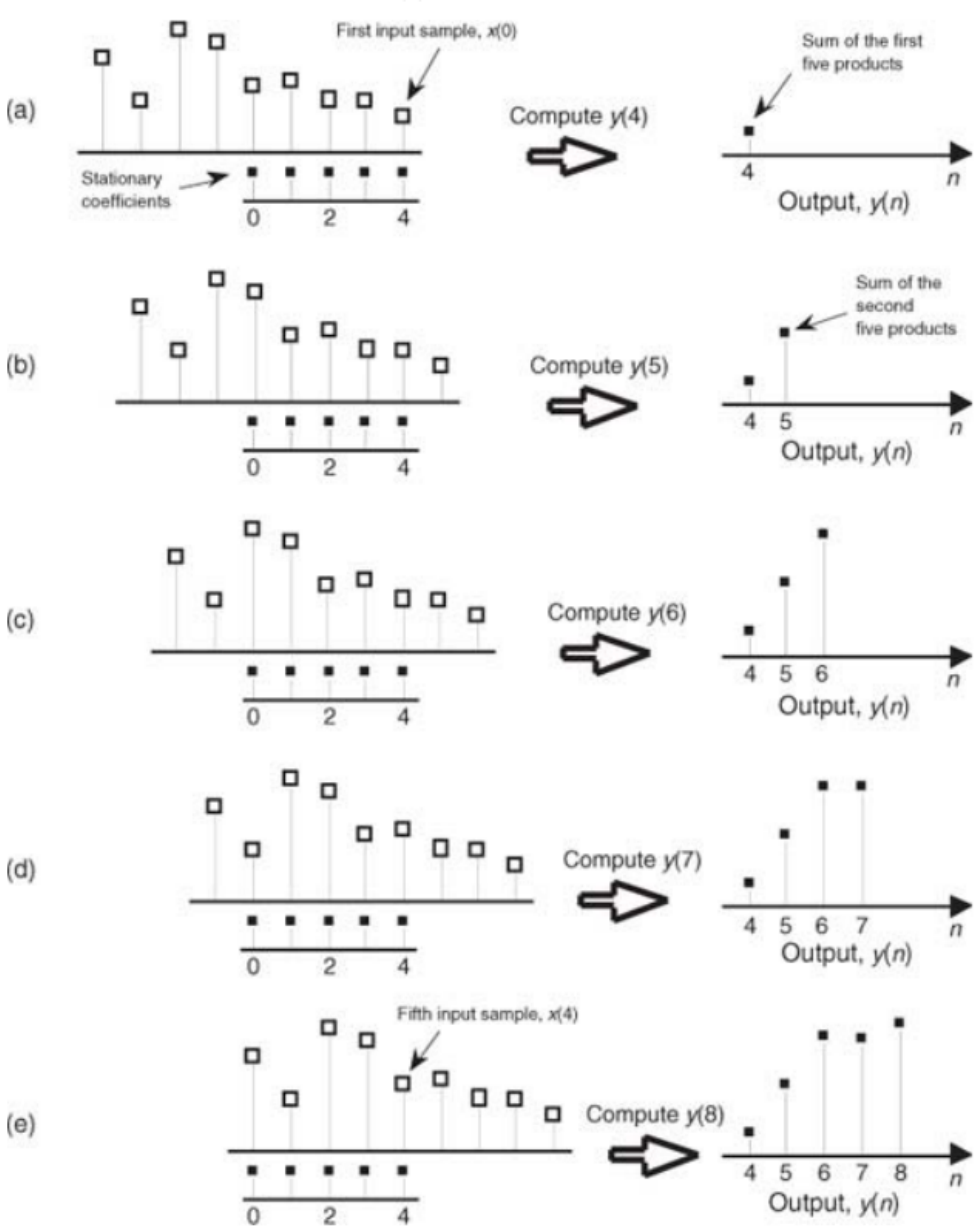
- The moving average filter is a convolution of the input signal with a rectangular pulse having an area of one.
- *Local average*.
- There is a delay of N/2 samples between input and output.

$$h[n] = \frac{1}{N} \sum_{k=0}^{N-1} \delta[n-k]\,, \tag{1}$$

$$y[n] = x[n] * h[n] = \frac{1}{N} \sum_{k=0}^{N-1} x[n-k] \tag{2}$$

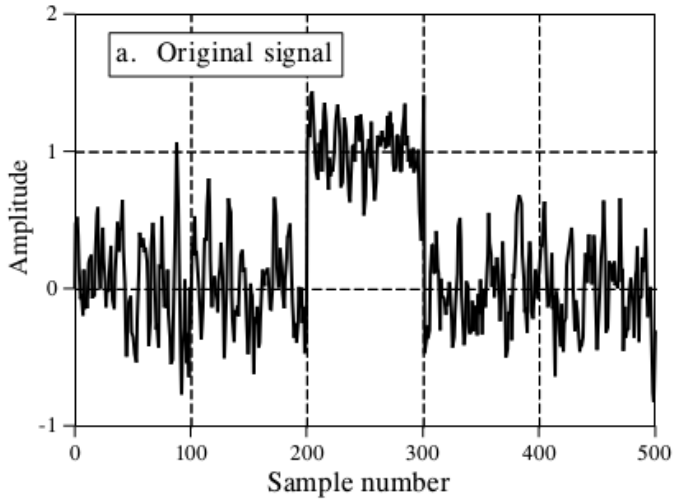
- It can be seen that the moving average filter is a FIR filter. Why?

Moving average filter, example



Noise Reduction vs. Step Response

- MA reduces random white noise while trying to keep the sharpest step response.



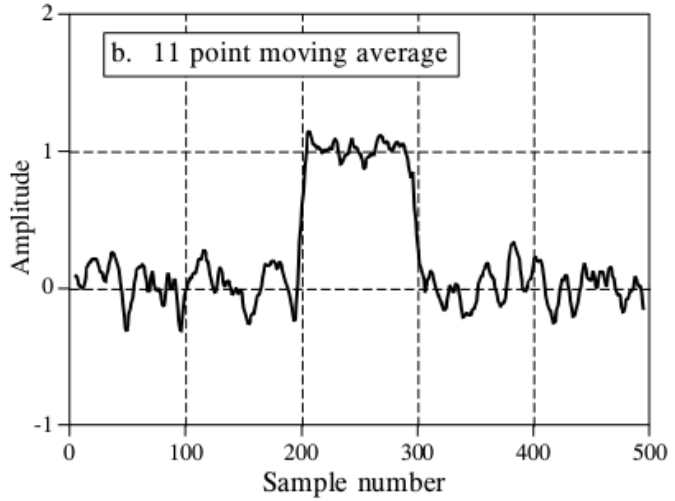
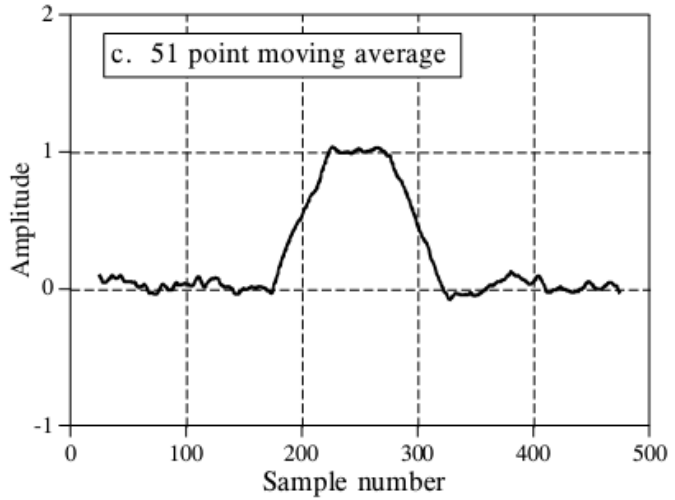
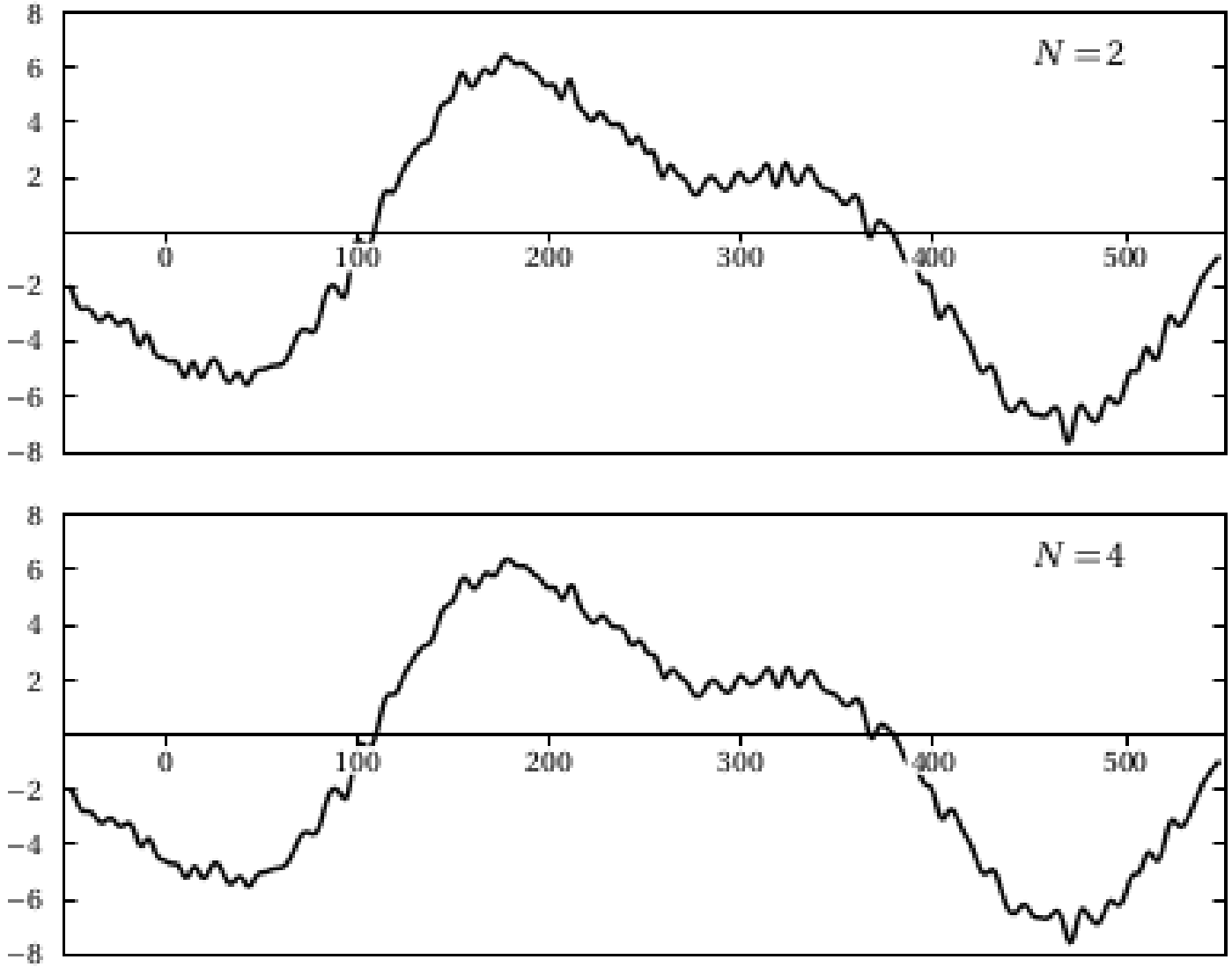


FIGURE 15-1
Example of a moving average filter. In (a), a rectangular pulse is buried in random noise. In (b) and (c), this signal is filtered with 11 and 51 point moving average filters, respectively. As the number of points in the filter increases, the noise becomes lower; however, the edges becoming less sharp. The moving average filter is the *optimal* solution for this problem, providing the lowest noise possible for a given edge sharpness.

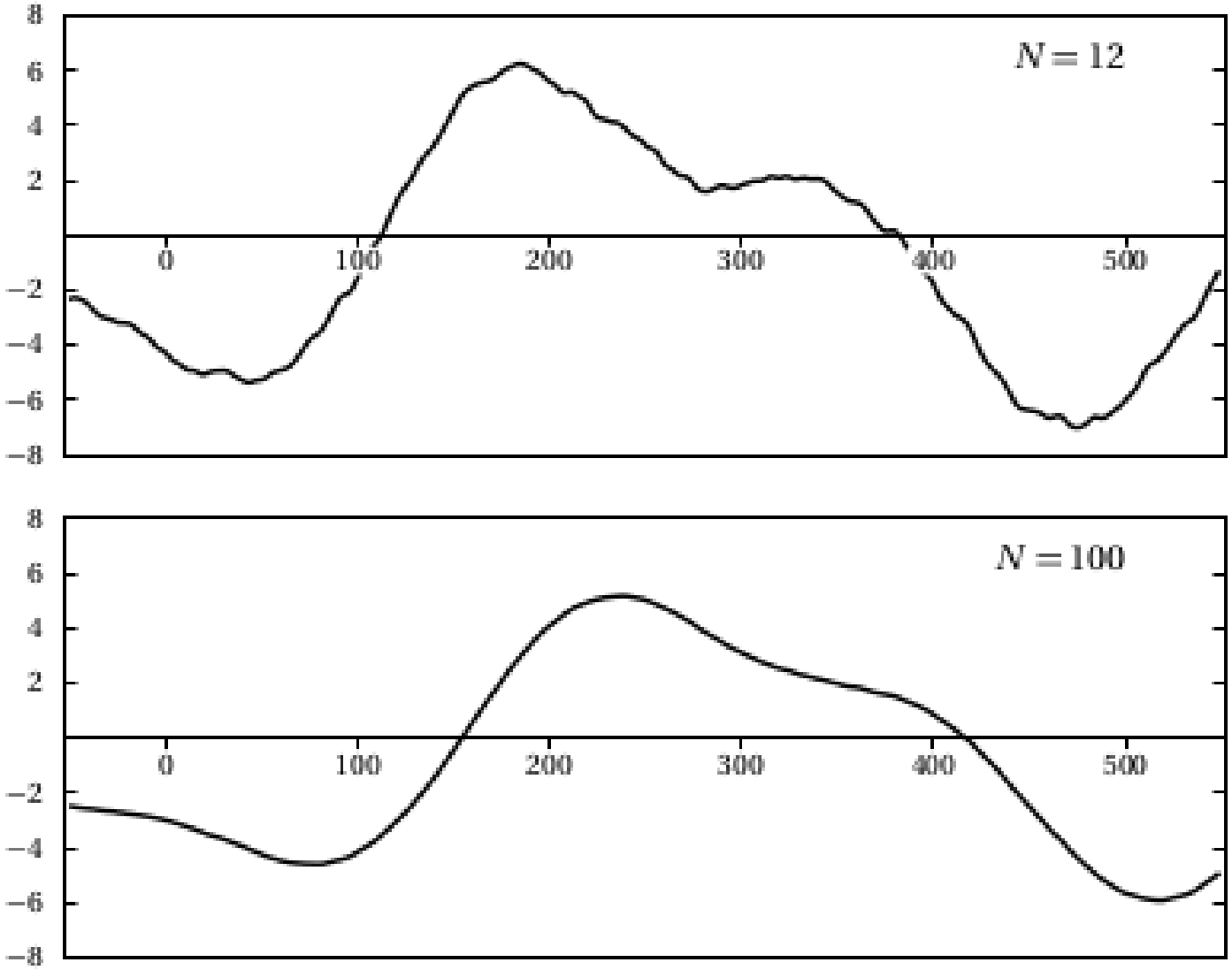


Noise Reduction

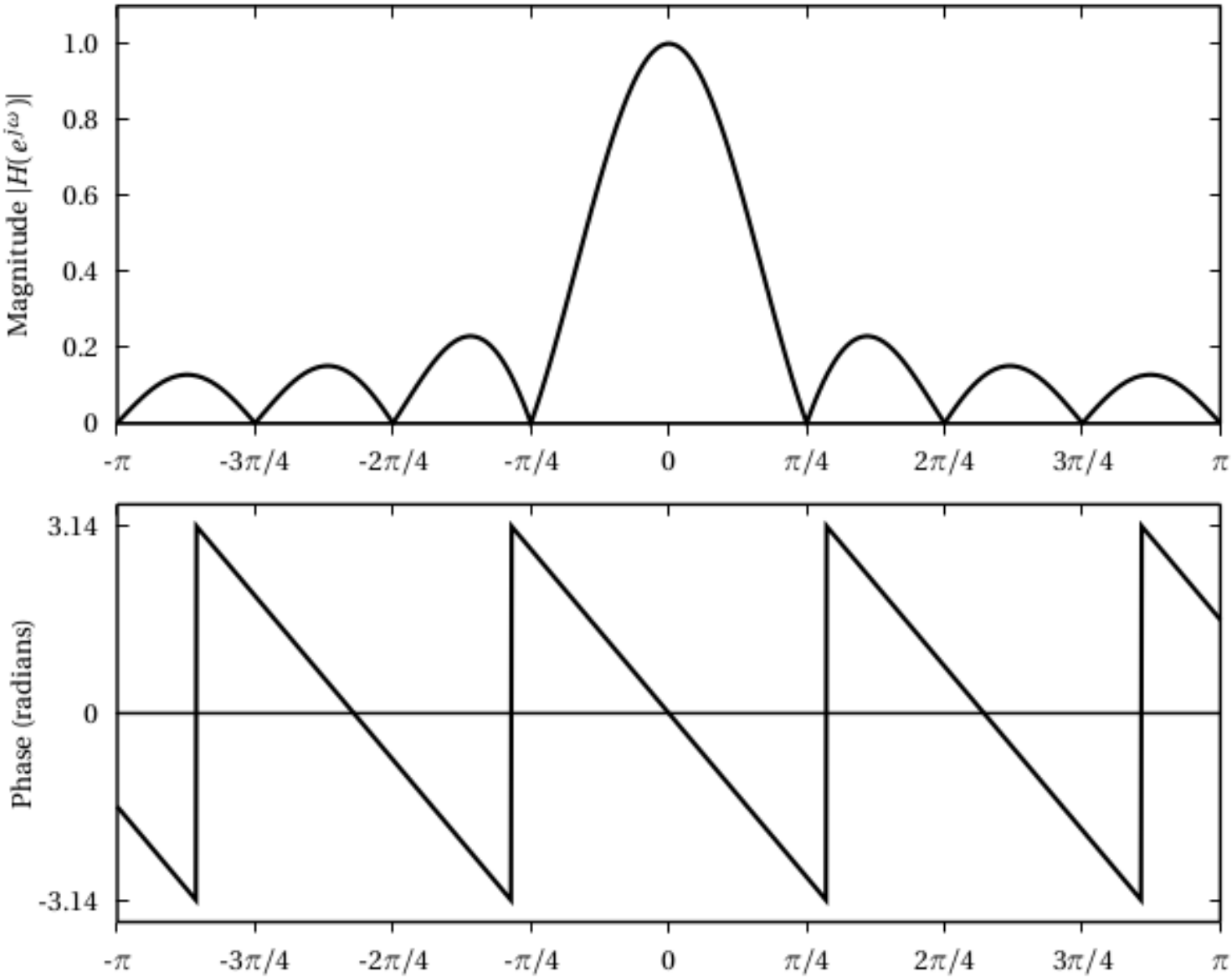


Noise Reduction

- Note how the signal is delayed as N grows.



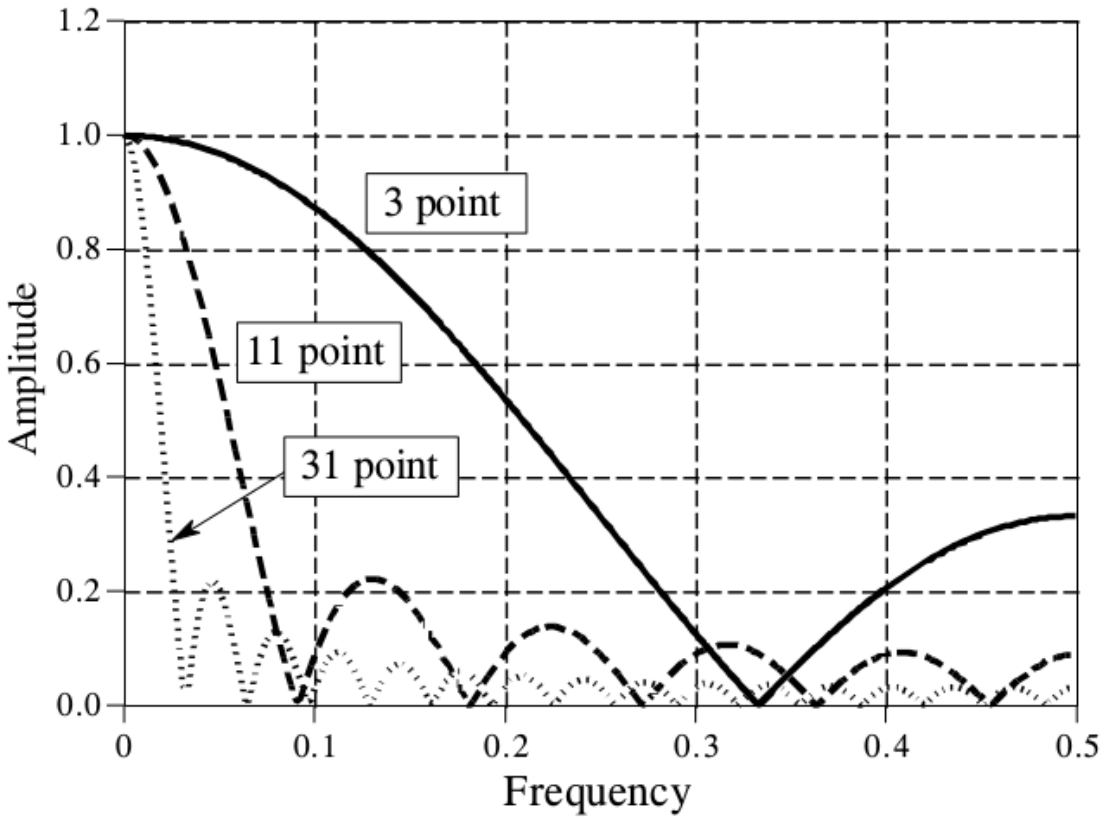
Frequency Response



Frequency Response II

- The moving average filter is a bad low-pass filter.
- In short, the moving average is a good *smoothing filter* (the action in the time domain), but a bad low-pass filter (the action in the frequency domain).

$$H[\omega] = \frac{\sin(\omega \cdot T_w/2)}{(\omega \cdot T_w/2)} \tag{3}$$



MA cutoff frequency

The cutoff frequency, *fco*, can be defined as the frequency at which the magnitude ratio is 0.707 (-3 dB).

$$|H[\omega]| = \left| \frac{\sin(\omega \cdot T_w/2)}{(\omega \cdot T_w/2)} \right| = \frac{|\sin(\pi f \cdot T_w)|}{(\pi f \cdot T_w)} \tag{4}$$

$$0.707 = \frac{|\sin(\pi f_{co} \cdot T_w)|}{(\pi f_{co} \cdot T_w)} \tag{5}$$

$$f_{co} = \frac{0.443}{T_w} \tag{6}$$

$$\implies T_w = \frac{0.443}{f_{co}} \tag{7}$$

Since,

$$T_w = N \cdot dt = \frac{N}{f_s} \tag{8}$$

$$\implies \frac{N}{f_s} = \frac{0.443}{f_{co}} \tag{9}$$

$$N = 0.443 \cdot \frac{f_s}{f_{co}} \tag{10}$$

Maximum filter order

The following equation imposes a limit to the value of N from the expected frequency response of the signal that it is going to be smoothed.

$$N = 0.443 \cdot \frac{f_s}{f_{co}} \tag{11}$$

Example:

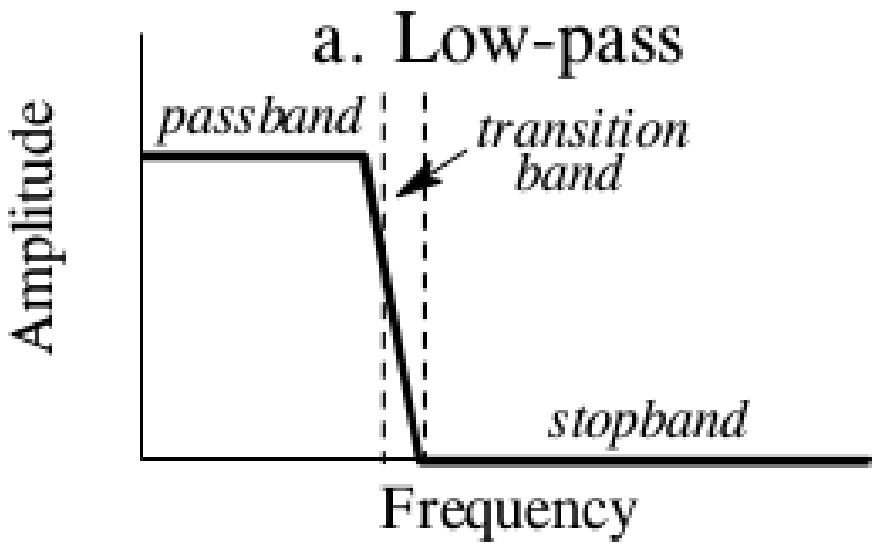
- The dynamics of a robots will be studied using an accelerometer.
- The maximum operation frequency of the robot it is assumed to be around 10 Hz.
- If the accelerometer is sampled at 100 Hz, the maximum order of an MA filter is:

$$N_{max} = round\left(0.443 \cdot \frac{100}{10}\right) = 4 \tag{12}$$

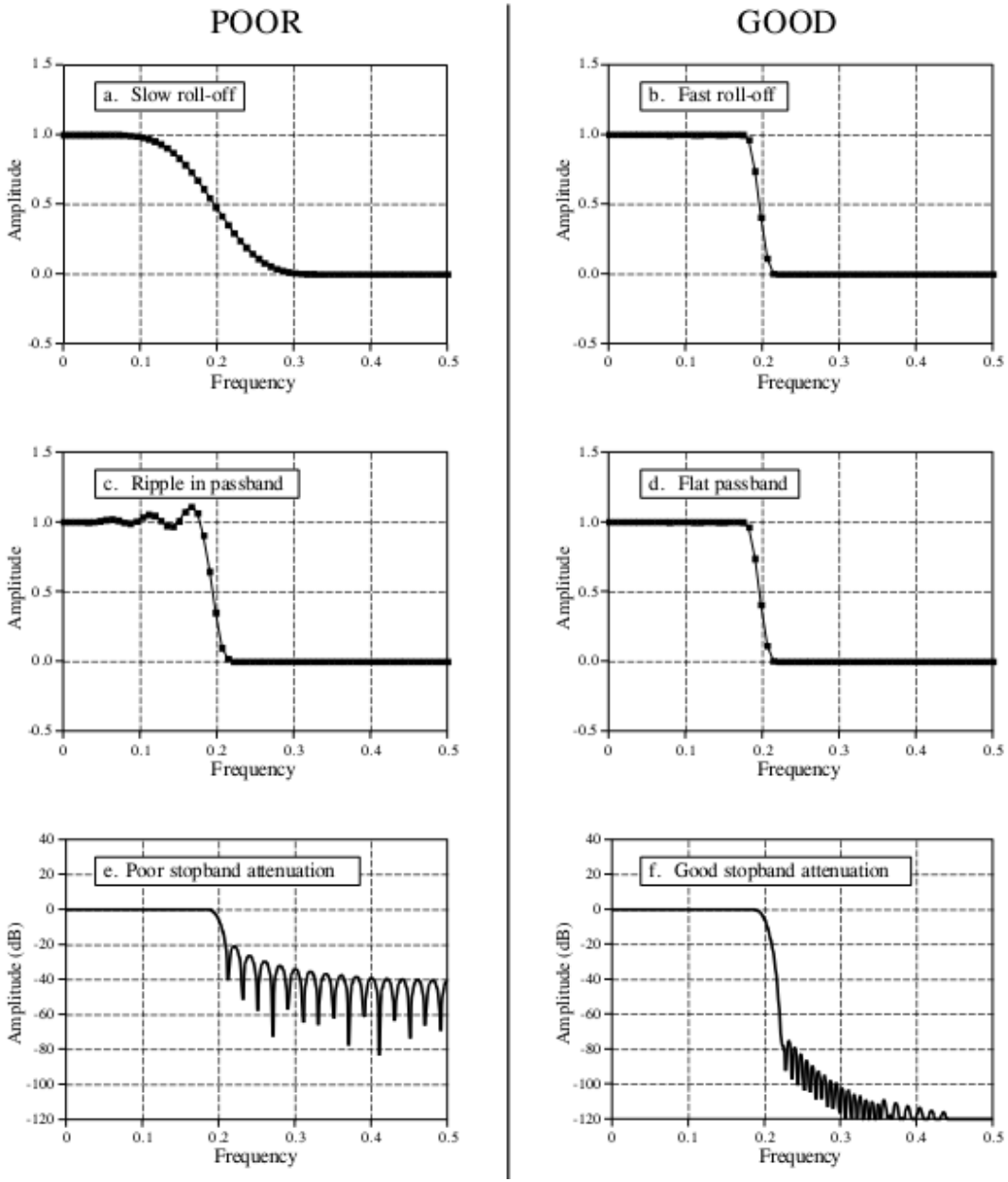
Suppose thatthe noise attenuation is not enough, what can be done?

Frequency domain parameters

- Passband.
- Stopband.
- Transition band (fast roll-off).
- Passband ripple.
- Stopband ripple.



Frequency response parameters II



Strategy of filtering by windowing

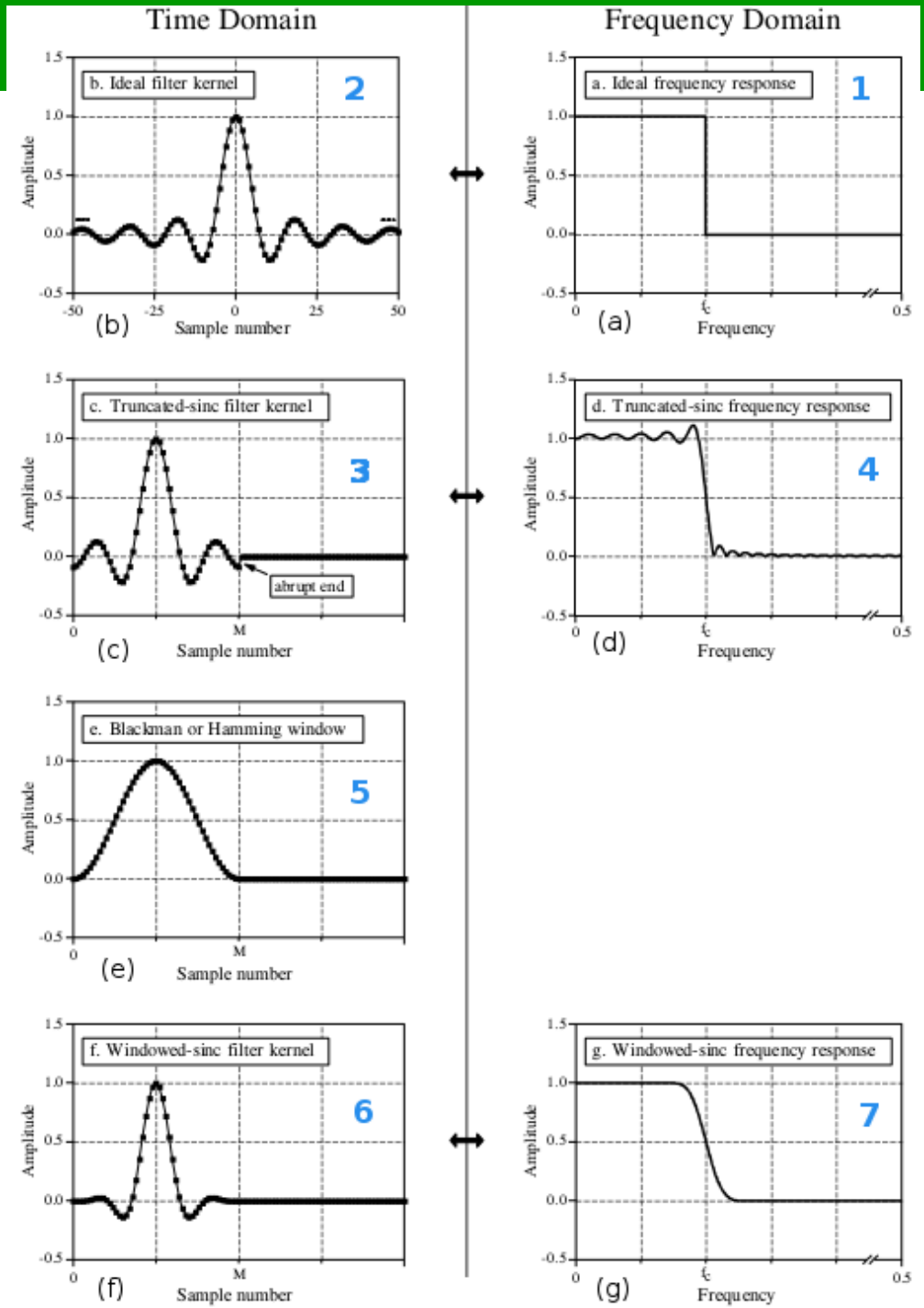
- Taking the Inverse Fourier Transform of an ideal frequency response (1) produces an ideal sinc filter kernel (2, impulse response).

$$hs[i] = \frac{\sin(2\pi f_C i)}{i\pi}$$

- Truncated-sinc (3) produces the Gibbs phenomenon in frequency response (4), no matter how long M is made.

$$h[i] = hs[i] \cdot w[i]$$

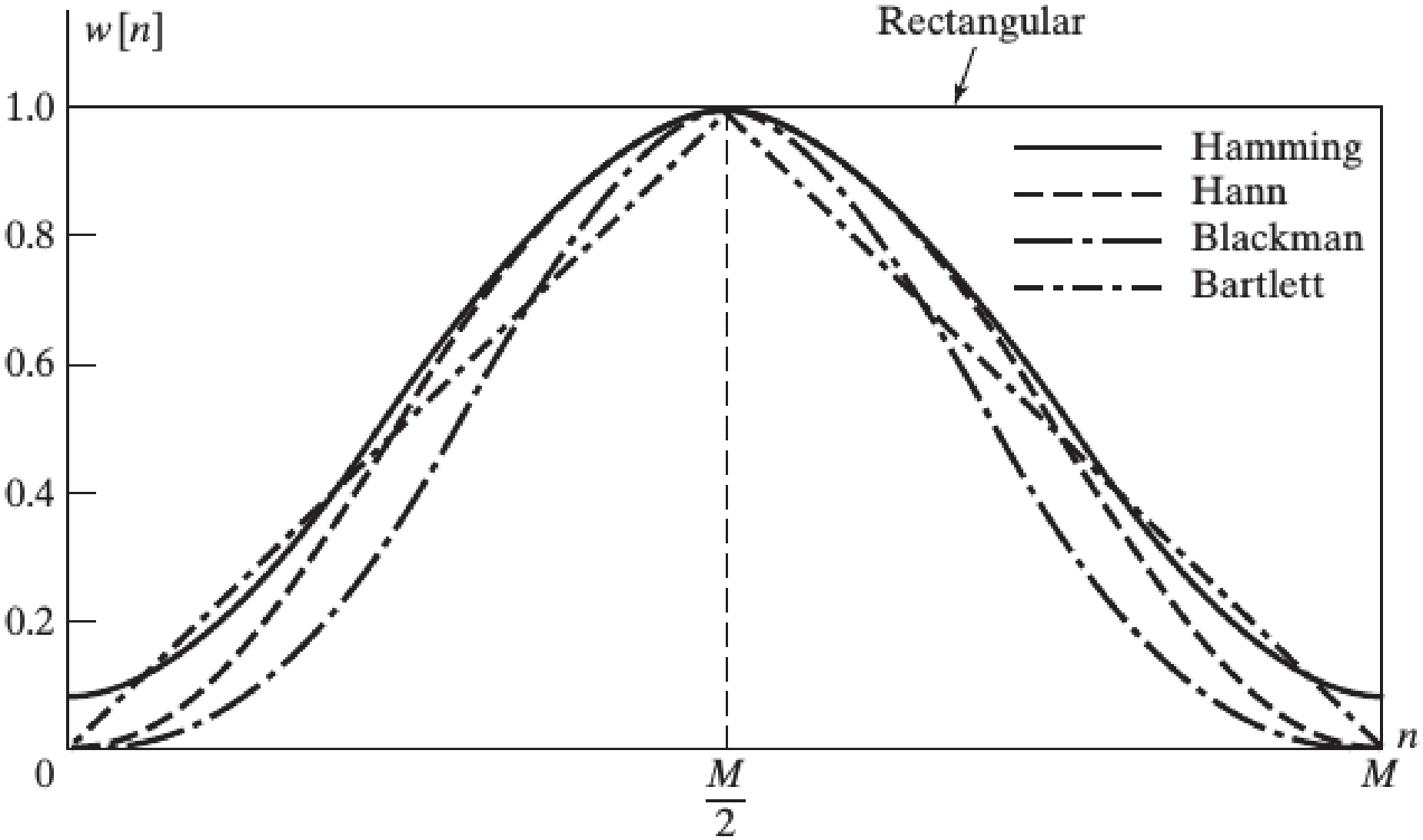
- Multiplying the truncated-sinc (3) by the Blackman window (5) results in the windowed-sinc filter kernel (6) with frequency response (7).



Types of windows

Name of window function $w(n)$	Mathematical definition
Rectangular	1
Hanning	$0.5 - 0.5\cos\left[\frac{2\pi n}{N-1}\right]$
Hamming	$0.54 - 0.46\cos\left[\frac{2\pi n}{N-1}\right]$
Blackman	$0.42 - 0.5\cos\left[\frac{2\pi n}{N-1}\right] + 0.08\cos\left[\frac{2\pi n}{N-1}\right]$
Kaiser	$\frac{I_0\left[\beta\sqrt{1-\left(\frac{\lvert 2n-N+1\rvert}{N-1}\right)^2}\right]}{-I_o(\beta)}$ Where, $I_0(x) = \sum_{k=0}^{\infty}\left(\frac{x^k}{2^k\, k!}\right)^2$

Windows in time domain



Windows in frequency domain

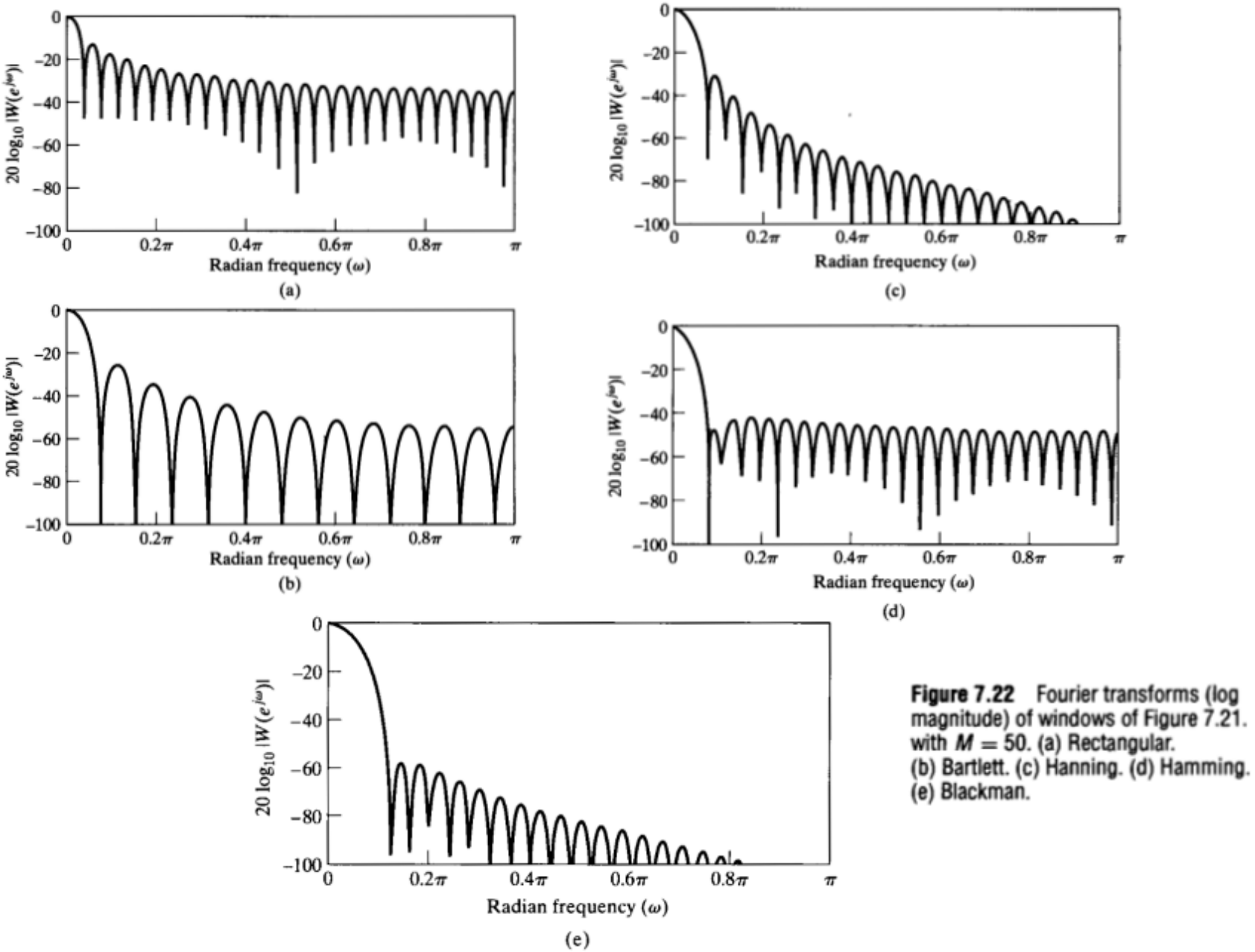
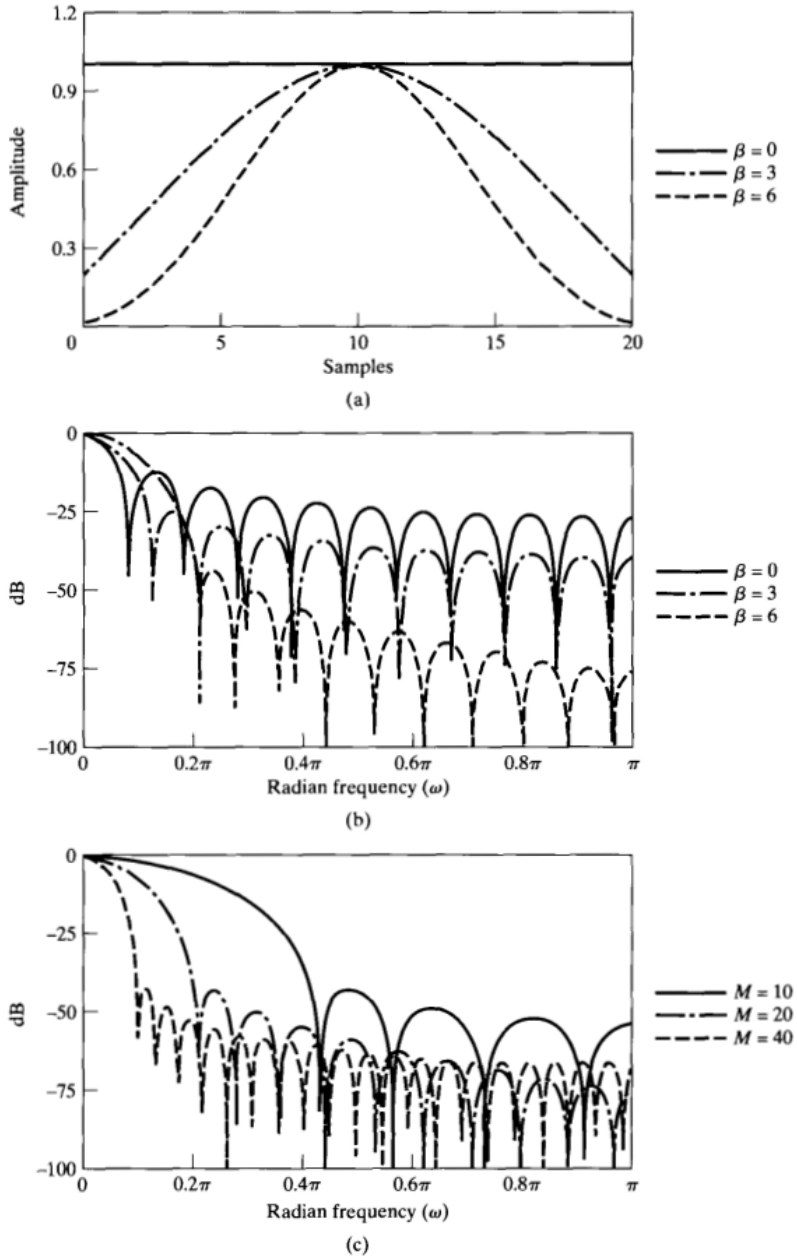


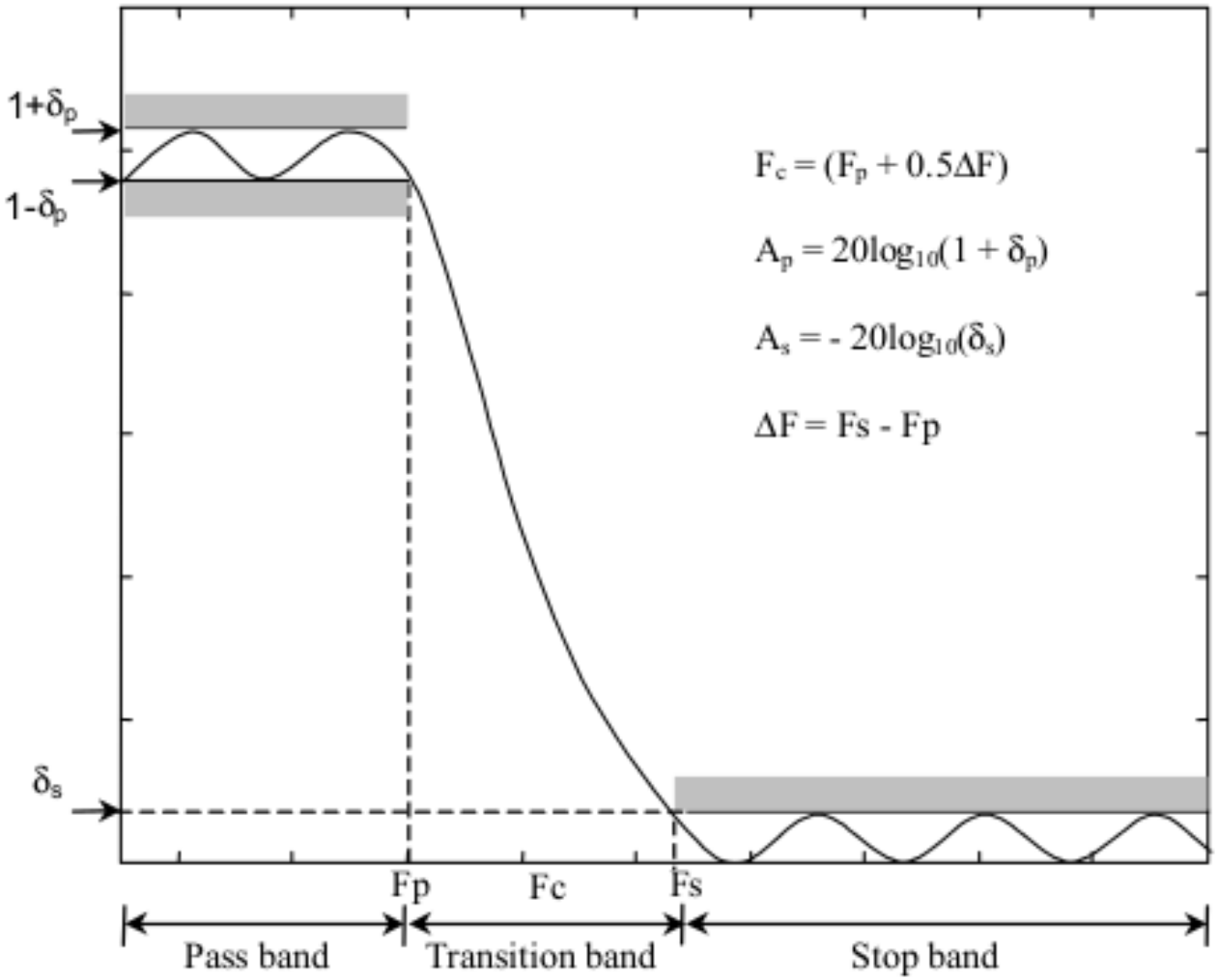
Figure 7.22 Fourier transforms (log magnitude) of windows of Figure 7.21. with $M = 50$. (a) Rectangular. (b) Bartlett. (c) Hanning. (d) Hamming. (e) Blackman.

Kaiser window filter

- The Kaiser window has two parameters:
  - Lenght, M+1.
  - Shape parameter, β.
- Trade-off between side-lobe amplitude and main-lobe width.



FIR filter design



FIR filter design II

Name of window function $w[n]$	Transition width ΔF in (Hz), (normalised)	Pass-band ripple A_p in (dB)	Ripple δ_p, δ_s	Side-lobe level in (dB)	Stop-band attenuation A_s in (dB)
Rectangular	0.9/N	0.741	0.089	-13	21
Hanning	3.1/N	0.0546	0.063	-31	44
Hamming	3.3/N	0.0194	0.0022	-41	53
Blackman	5.5/N	0.0017	0.000196	-57	74
Kaiser β=4.54	2.93/N	0.0274			50
β=5.65	3.63/N	0.00867			60
β=6.76	4.32/N	0.00275			70
β=8.96	5.71/N	0.000275			90

Example of FIR design III

A FIR low-pass filter is required to have the following specifications:

1. Pass-band edge frequency $f_p = 2$ kHz
2. Transition band $\Delta f = 200$ Hz
3. Pass-band ripple $A_p = 0.1$dB
4. Minimum stop-band attenuation $A_s = 50$dB
5. Sampling frequency of $f_s = 10$ kHz

We have to define:

- The type of window (Hanning, Blackman, ...)
- The order of the filter (N).

Example of FIR design IV

Pass-band ripple, $A_p = 20\log_{10}(1 + \delta_p)$

$$\delta_p = \log_{10}^{-1}\left[\frac{0.1}{20}\right] - 1 = 0.0116$$

Minimum stop-band attenuation $A_s = -20\log_{10}(\delta_s)$

$$\delta_s = \log_{10}^{-1}\left[\frac{-50}{20}\right] = 0.00316$$

Name of window function $w[n]$	Transition width ΔF in (Hz), (normalised)	Pass-band ripple A_p in (dB)	Ripple δ_p, δ_s	Side-lobe level in (dB)	Stop-band attenuation A_s in (dB)
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β=6.76	4.32/N	0.00275			70
β=8.96	5.71/N	0.000275			90

A Hamming window is selected according to the required level of δ_s.

FIR filter design V

Pass-band ripple, $A_p = 20\log_{10}(1 + \delta_p)$

$$\delta_p = \log_{10}^{-1}\left[\frac{0.1}{20}\right] - 1 = 0.0116$$

Minimum stop-band attenuation $A_s = -20\log_{10}(\delta_s)$

$$\delta_s = \log_{10}^{-1}\left[\frac{-50}{20}\right] = 0.00316$$

The normalised pass-band edge frequency

$$F_p = f_p / f_s = \frac{2x10^3}{10x10^3} = 0.2$$

The normalised transition width

$$\Delta F = \Delta f / f_s = \frac{200}{10x10^3} = 0.02$$

$$N = \frac{3.3}{0.02} = 165$$

FIR structures

$$H(z) = b_0 + b_1 z^{-1} + ... + b_{M-1} z^{M-1}$$

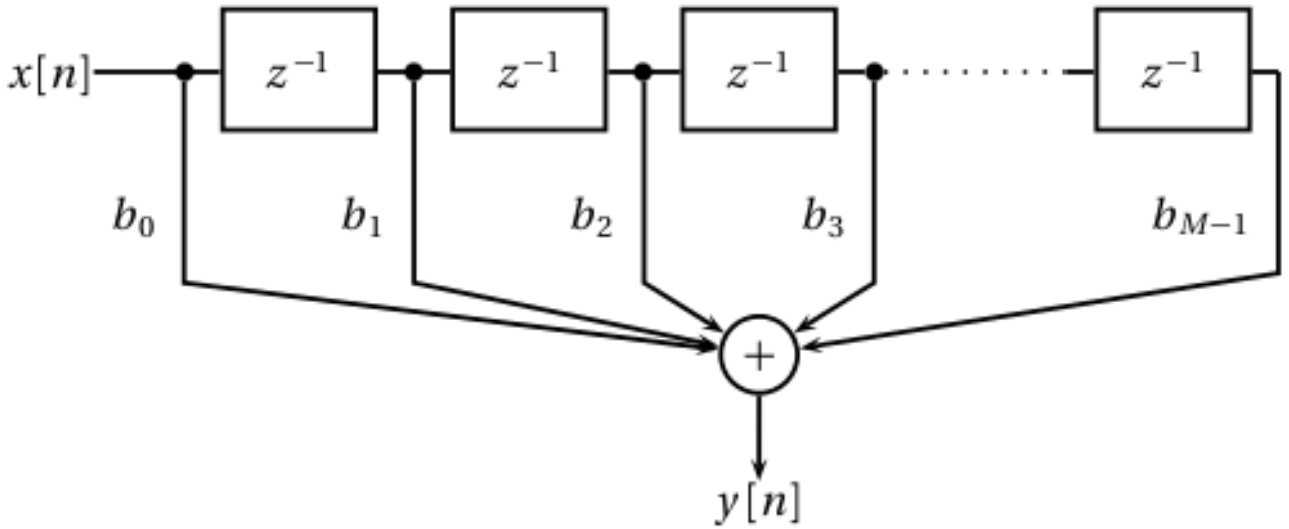

Figure 7.22 Direct FIR implementation.

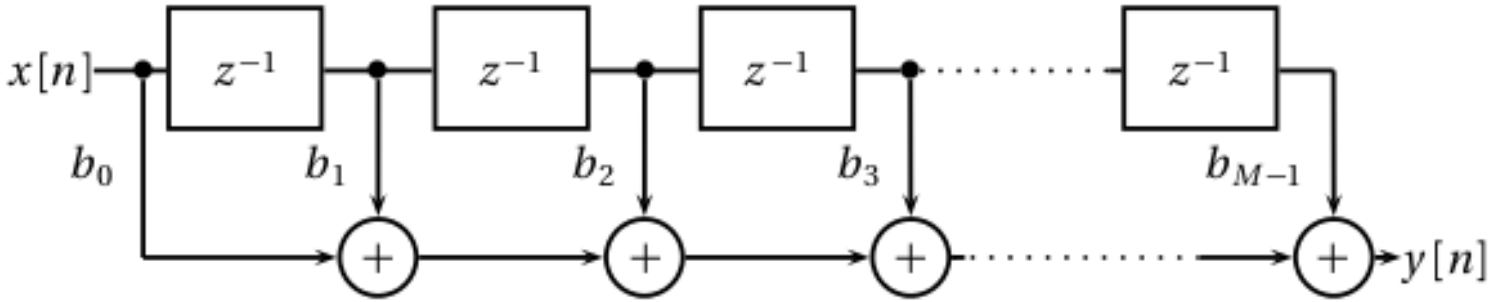

Figure 7.23 Transversal FIR implementation.

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