Modeling of Hydraulic Systems

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Summary

- For incompressible fluids, conservation of mass is equivalent to conservation of volume, because the fluid density is constant.
- If we know the mass density ρ and the volume flow rate, we can compute **the mass flow rate**.
- That is, $q_m = \rho q_v$, where q_m and q_v are the mass and volume flow rates.
- The FPS and SI units for mass flow rate are slug/sec and kg/s, respectively.
- The units for volume rates are ft3/sec and m3/s, respectively.

- The units for **mass density** are slug/ft3 and kg/m3 .
- The units for **weight density**, λ, are lb/ft3 or N/m3.
- Weight density is related to the mass density as $\lambda = \rho\, g$.
- The mass density of fresh water near room temperature is 1.94 slug/ft3 , or 1000 kg/m3.

- Pressure is the force per unit area that is exerted by the fluid.
- The FPS and SI units of pressure are lb/ft2 and the Pascal (1 Pa = 1 N/m2), respectively.
- At sea level near room temperature, atmospheric pressure, usually abbreviated pa, is 14.7 psi (2117 lb/ft2) or 1.0133x105 pa.
- Gage pressure is the pressure difference between the absolute pressure and atmospheric pressure, and is often abbreviated as psig.
- Hydrostatic pressure is the pressure that exists in a fluid at rest. It is caused by the weight of the fluid.
- For example, the hydrostatic pressure at the bottom of a column of fluid of height h is $\rho g h$. If the atmospheric pressure above the column of liquid is pa, then the total pressure at the bottom of the column is $\rho g h + pa$.

Conservation of mass

Conservation of mass

For a container holding a mass of fluid m, the time rate of change $\dot{m}$ of mass in the container must equal the total mass inflow rate minus the total mass outflow rate. That is,

$$\dot{m} = q_{mi} - q_{mo} \tag{1}$$

where q_{mi} is the mass inflow rate and q_{mo} is the mass outflow rate.
The fluid mass m is related to the container volume V by

$$m = \rho V \tag{2}$$

For an incompressible fluid, ρ is constant, and thus $\dot{m} = \rho \dot{V}$.

Let q_{vi} and q_{vo} be the total volume inflow and outflow rates. Thus, $q_{mi} = \rho q_{vi}$, and $q_{mo} = \rho q_{vo}$. Substituting these relationships into Eq. 1 gives

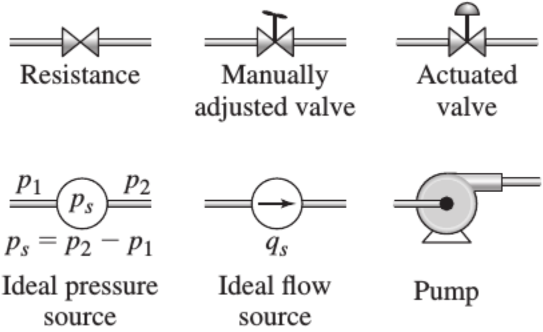
$$\rho V = \rho q_{vi} - \rho q_{vo} \tag{3}$$

$$V = q_{vi} - q_{vo} \tag{4}$$

This is a statement of conservation of volume for the fluid, and it is equivalent to conservation of mass, Eq. 1, when the fluid is incompressible.

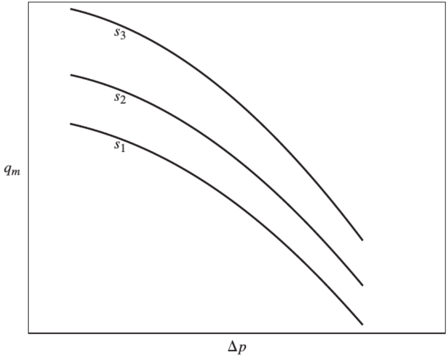
Table 7.2.1 Analogous fluid and electrical quantities.

Fluid quantity	Electrical quantity
Fluid mass, m	Charge, Q
Mass flow rate, q_m	Current, i
Pressure, p	Voltage, v
Fluid linear resistance, R	Electrical resistance, R
$R = p/q_m$	$R = v/i$
Fluid capacitance, C	Electrical capacitance, C
$C = m/p$	$C = Q/v$
Fluid inertance, I	Electrical inductance, L
$I = p/(dq_m/dt)$	$L = v/(di/dt)$


Resistance
Manually adjusted valve
Actuated valve
p_1
p_2
p_s
$p_s = p_2 - p_1$
Ideal pressure source
q_s
Ideal flow source
Pump

Steady-state flow-pressure relation for a centrifugal pump

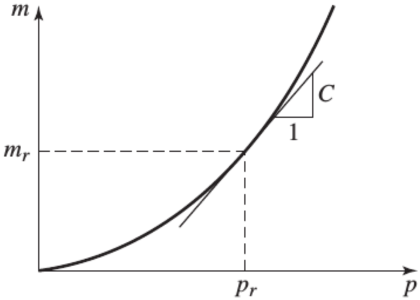
- q_m is the mass flow rate produced by the pump when the pressure difference across the pump is Δp.
- When the outlet pressure is greater than the inlet pressure, $\Delta p > 0$.
- Curves depend on the pump speed, labeled s1 , s2, and so on.
- To determine the operating condition of the pump for a given speed, we need the load connected to the pump outlet.

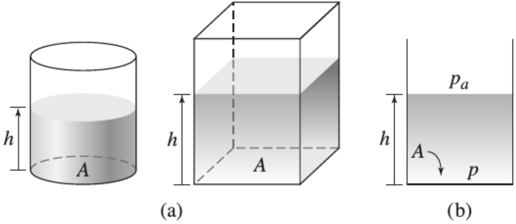


Fluid capacitance is the relation between stored fluid mass and the resulting pressure caused by the stored mass. At a particular reference point (pr, mr) the slope is C, where

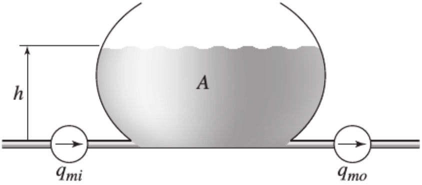
$$C = \left.\frac{dm}{dp}\right|_{p=pr} \tag{5}$$

Thus, fluid capacitance C is the ratio of the change in stored mass to the change in pressure.





h
A
h
A
h
p_a
A
p
(a)
(b)

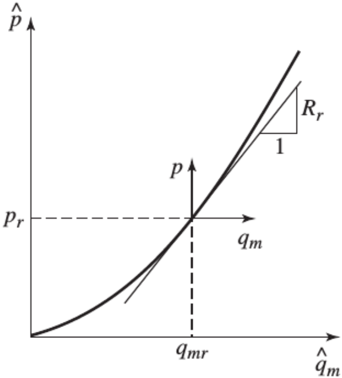


h
A
q_{mi}
q_{mo}

The mass flow rate $\hat{q}_m$ through a resistance is related to the pressure difference $\hat{p}$ across the resistance.

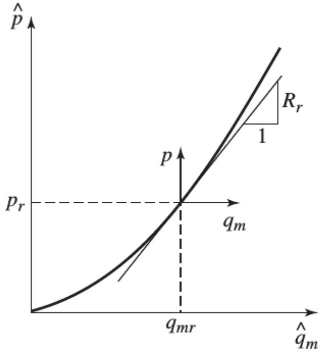
The fluid resistance R is define as,

$$R_r = \left.\frac{d\hat{p}}{d\hat{q}_m}\right|_{\hat{q}_m=\hat{q}_{mr}} \tag{6}$$



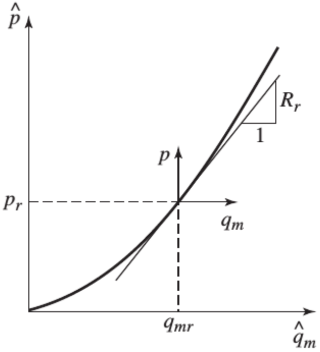
An approximate linear model of the pressure-flow rate relation using Taylor series expansion to linearize the expression,

$$\hat{p} = p_r + \left(\frac{d\hat{p}}{d\hat{q}_m}\right)(q_m - q_{mr}) = p_r + R_r(q_m - q_{mr}) \tag{7}$$



$$R_r = \frac{\text{small pressure change}}{\text{small change in mass flow rate}} \tag{8}$$

In a limited number of cases, such as pipe flow under certain conditions, the relation of $\hat{p}$ versus $\hat{q}_m$ is linear so that $\hat{p} = R\hat{q}_m$.



Bibliography

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