Infinite impulse response filters

Leakey Integrator filter

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Classification of discrete filters

Table: Classification of discrete filters

	Finite impulse response (FIR)	Infinite impulse response (IIR)
Filtering in time domain	Moving average	Leaky Integrator
Filtering in frequency domain	Windowed Filters Equiripple Minimax	ZOH method Bilinear z-transform

Leaky integrator filter

The MA filter equation,

$$y[n] = x[n] * h[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k] \,, \tag{1}$$

$$y[n] = \frac{1}{M} \left[\sum_{k=1}^{M-1} x[n-k] + x[n] \right] = \frac{1}{M} \left[\sum_{k=1}^{M-1} x[n-k] \right] + \frac{1}{M} x[n] \,. \tag{2}$$

Since,

$$y[n-1] = \frac{1}{M-1} \left[\sum_{k=1}^{M-1} x[n-k] \right] \implies y[n-1](M-1) = \left[\sum_{k=1}^{M-1} x[n-k] \right] \,. \tag{3}$$

Then,

$$y[n] = \frac{M-1}{M} y[n-1] + \frac{1}{M} x[n] \,. \tag{4}$$

Defining $\lambda = \frac{M-1}{M}$,

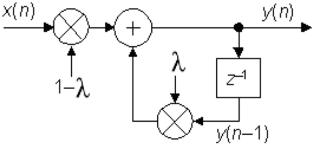
$$y[n] = \lambda \, y[n-1] + (1-\lambda) \, x[n] \,. \tag{5}$$

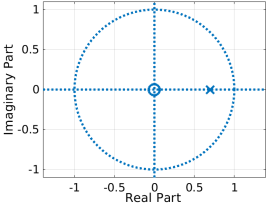
It can be seen that the leaky integrator filter is an IIR filter. Why?

$$y[n] = \lambda\, y[n-1] + (1-\lambda)\, x[n]\,.$$

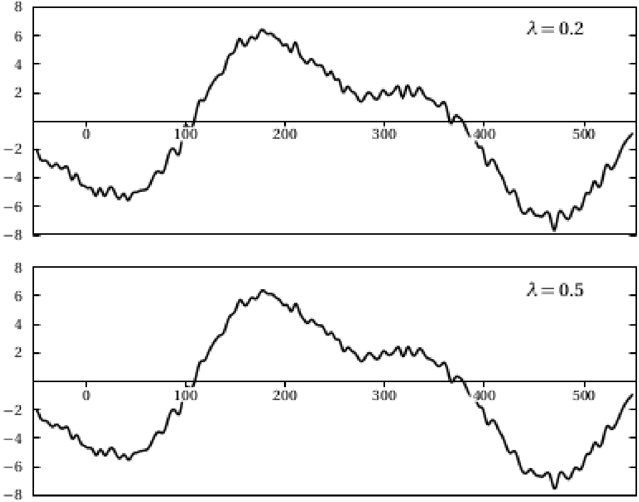
- No longer a convolution.
- Instead, a **constant coefficient difference equation**. Initial conditions must be set.
- LI is also known as **Single Pole Recursive filter** [2].
- The new system is LTI.
- LI is stable for $|\lambda| < 1$. Since $\lambda = \frac{M-1}{M}$, the filter is stable.
- The value of λ (which is the pole of the system) determines the smoothing power of the filter.

$$\frac{Y(z)}{X(z)} = \frac{1-\lambda}{1-\lambda z^{-1}}$$



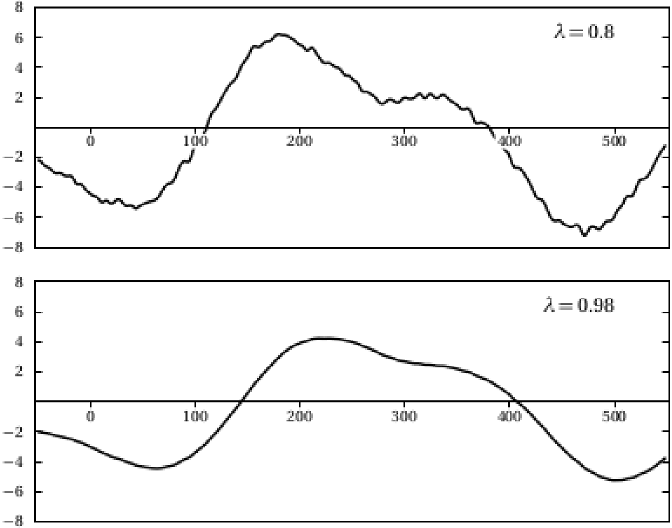


Time domain response



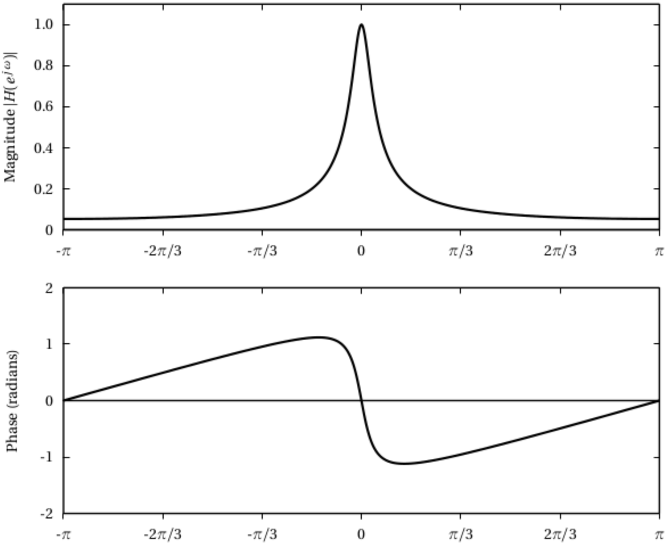
Time domain response, 2

Note how the signal is delayed as λ grows.

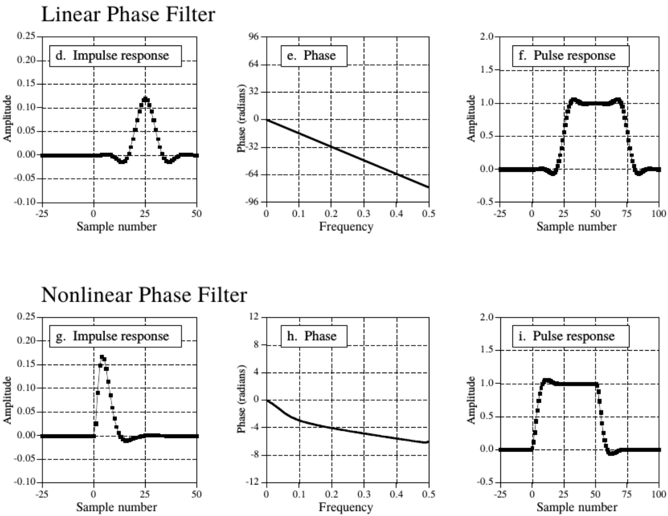


Frequency domain response

Magnitude and phase response of the leaky integrator for $\lambda = 0.9$. Phase response is **nonlinear**.



Nonlinear phase response



Bibliography

1 Paolo Prandoni and Martin Vetterli. Signal processing for communications. Taylor and Francis Group, LLC. 2008. Section 5.3.2.

2 Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing. Chapter 19. www.dspguide.com