PID controllers and modified PID controllers

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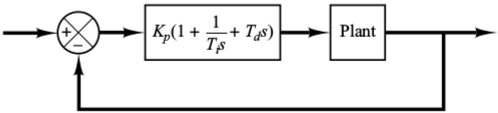
Summary

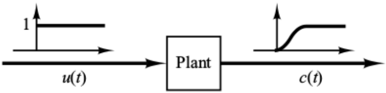
- If the plant mathematical model cannot be easily obtained, then an analytical or computational approach to the design of a PID controller is not possible.
- Then we must resort to experimental approaches to the tuning of PID controllers.
- Ziegler and Nichols suggested rules for tuning PID controllers (values K_p , T_i, and T_d) based on:
  - Experimental step responses (Method 1) or
  - Based on the value of K_p that results in marginal stability when only proportional control action is used (Method 2).

Figure 8–1
PID control of a plant.



- We obtain experimentally the response of the plant to a unit-step input.
- This method applies if the response to a step input exhibits an S-shaped curve.

Figure 8–2
Unit-step response of a plant.



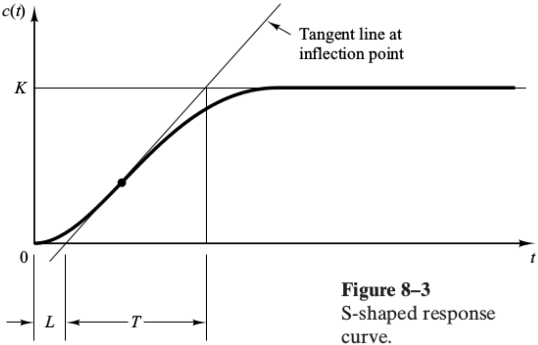


Figure 8–3
S-shaped response curve.

Table 8–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

Ziegler–Nichols Rules for Tuning PID Controllers
Second Method

- We first set $T_i = \infty$, and $T_d = 0$
- IncreaseK_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations.

Figure 8–4
Closed-loop system with a proportional controller.

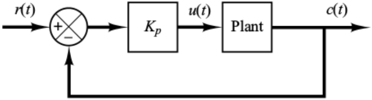


Figure 8–5
Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

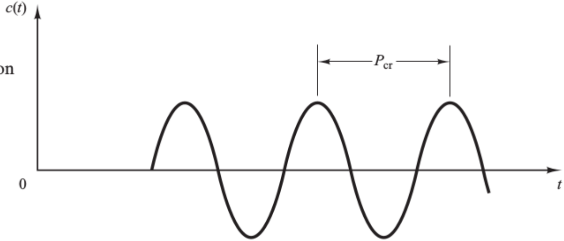


Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$
\begin{aligned}
G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\
&= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\
&= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}
\end{aligned}
$$

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

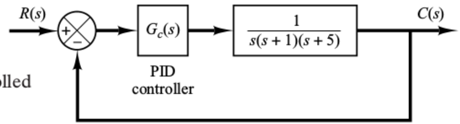


Figure 8–6 PID-controlled system.

EXAMPLE 8–2 Consider the PID-controlled system shown in Figure 8–19. The PID controller is given by

$$G_c(s) = K\frac{(s+a)^2}{s}$$

It is desired to find a combination of K and a such that the closed-loop system will have 10% (or less) maximum overshoot in the unit-step response. (We will not include any other condition in this problem. But other conditions can easily be included, such as that the settling time be less than a specified value. See, for example, Example 8–3.)

There may be more than one set of parameters that satisfy the specifications. In this example, we shall obtain all sets of parameters that satisfy the given specifications.

To solve this problem with MATLAB, we first specify the region to search for appropriate K and a. We then write a MATLAB program that, in the unit-step response, will find a combination of K and a which will satisfy the criterion that the maximum overshoot is 10% or less.

Note that the gain K should not be too large, so as to avoid the possibility that the system require an unnecessarily large power unit.

Assume that the region to search for K and a is

$$2 \le K \le 3 \qquad \text{and} \qquad 0.5 \le a \le 1.5$$

- If $u(t)$ change abruptly, the derivative term in the control action will involve an impulse function.
- Such a phenomenon is called *set-point kick*.

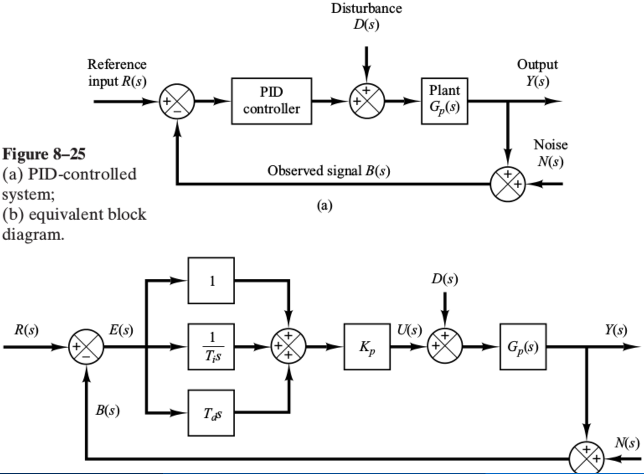
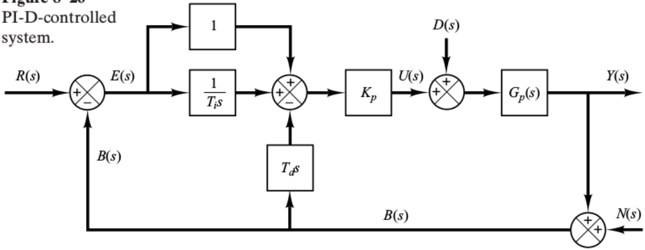


Figure 8–25
(a) PID-controlled system;
(b) equivalent block diagram.

$$U(s) = K_p\left(1 + \frac{1}{T_i s}\right)R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$

Figure 8–26
PI-D-controlled system.



Notice that in the absence of the disturbances and noises, the closed-loop transfer function of the basic PID control system [shown in Figure 8–25(b)] and the PI-D control system (shown in Figure 8–26) are given, respectively, by

$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s} + T_d s\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

and

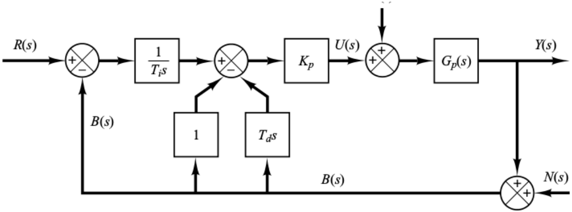
$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

It is important to point out that in the absence of the reference input and noises, the closed-loop transfer function between the disturbance $D(s)$ and the output $Y(s)$ in either case is the same and is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

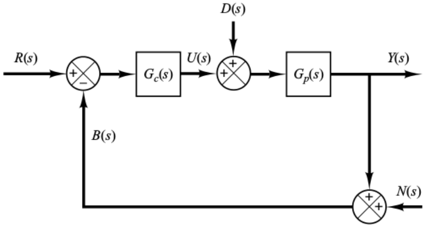
- The proportional action and derivative action are moved to the feedback path so that these actions affect the feedback signal only.

$$U(s) = K_p \frac{1}{T_i s} R(s) - K_p \left(1 + \frac{1}{T_i s} + T_d s\right) B(s)$$



The closed-loop transfer function $Y(s)/R(s)$ in the absence of the disturbance input and noise input is given by

$$\frac{Y(s)}{R(s)} = \left(\frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + K_p G_p(s)\left(1 + \frac{1}{T_i s} + T_d s\right)}$$



For this system, three closed-loop transfer functions $Y(s)/R(s) = G_{yr}$, $Y(s)/D(s) = G_{yd}$, and $Y(s)/N(s) = G_{yn}$ may be derived. They are

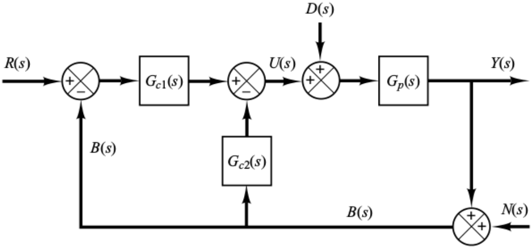
$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_c G_p}{1 + G_c G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_c G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_c G_p}{1 + G_c G_p}$$

$$G_{yr} = \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$



$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{(G_{c1} + G_{c2})G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yr} = G_{c1}G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

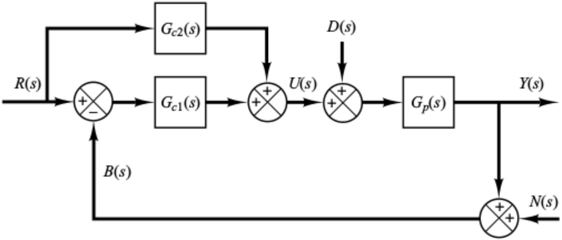
$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + G_{c1}G_p} + \frac{G_{c2}G_p}{1 + G_{c1}G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_{c1}G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_{c1}G_p}{1 + G_{c1}G_p}$$

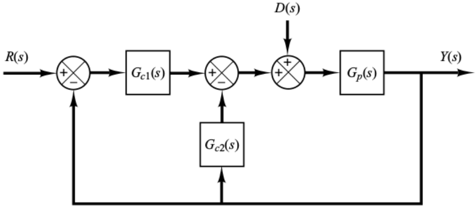
$$G_{yr} = G_{c2}G_{yd} + \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$



Consider the two-degrees-of-freedom control system shown in Figure 8–31. Assume that the plant transfer function $G_p(s)$ is a minimum-phase transfer function and is given by

$$G_p(s) = K\frac{A(s)}{B(s)}$$



where

$$A(s) = (s + z_1)(s + z_2)\cdots(s + z_m)$$
$$B(s) = s^N(s + p_{N+1})(s + p_{N+2})\cdots(s + p_n)$$

where N may be 0, 1, 2 and $n \geq m$. Assume also that G_{c1} is a PID controller followed by a filter $1/A(s)$, or

$$G_{c1}(s) = \frac{\alpha_1 s + \beta_1 + \gamma_1 s^2}{s}\frac{1}{A(s)}$$

and G_{c2} is a PID, PI, PD, I, D, or P controller followed by a filter $1/A(s)$. That is

$$G_{c2}(s) = \frac{\alpha_2 s + \beta_2 + \gamma_2 s^2}{s}\frac{1}{A(s)}$$

where some of α_2, β_2, and γ_2 may be zero. Then it is possible to write $G_{c1} + G_{c2}$ as

$$G_{c1} + G_{c2} = \frac{\alpha s + \beta + \gamma s^2}{s}\frac{1}{A(s)} \tag{8–3}$$

where α, β, and γ are constants. Then

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p} = \frac{K\dfrac{A(s)}{B(s)}}{1 + \dfrac{\alpha s + \beta + \gamma s^2}{s}\dfrac{K}{B(s)}}$$

$$= \frac{sKA(s)}{sB(s) + (\alpha s + \beta + \gamma s^2)K}$$

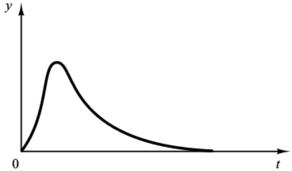
if the disturbance input is a step function of magnitude d, or

$$D(s) = \frac{d}{s}$$

and assuming the system is stable, then

$$y(\infty) = \lim_{s\to 0} s\left[\frac{sKA(s)}{sB(s) + (\alpha s + \beta + \gamma s^2)K}\right]\frac{d}{s}$$
$$= \lim_{s\to 0} \frac{sKA(0)d}{sB(0) + \beta K}$$

Figure 8–32 Typical response curve to a step disturbance input.



The response $y(t)$ to a step disturbance input will have the general form shown in Figure 8–32.

Note that $Y(s)/R(s)$ and $Y(s)/D(s)$ are given by

$$\frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + (G_{c1} + G_{c2})G_p}, \qquad \frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$\frac{Y(s)}{R(s)} = \frac{p(s)}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

If we choose $p(s)$ as

$$p(s) = a_2 s^2 + a_1 s + a_0 = a_2(s + s_1)(s + s_2)$$

that is, choose the zeros $s = -s_1$ and $s = -s_2$ such that, together with a_2, the numerator polynomial $p(s)$ is equal to the sum of the last three terms of the denominator polynomial—then the system will exhibit no steady-state errors in response to the step input, ramp input, and acceleration input.

Determination of G_{c2}. Now that the coefficients of the transfer function $Y(s)/R(s)$ are all known and $Y(s)/R(s)$ is given by

$$\frac{Y(s)}{R(s)} = \frac{a_2 s^2 + a_1 s + a_0}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0} \qquad (8\text{–}4)$$

we have

$$\begin{aligned}
\frac{Y(s)}{R(s)} &= G_{c1}\frac{Y(s)}{D(s)} \\
&= \frac{G_{c1} sKA(s)}{sB(s) + (\alpha s + \beta + \gamma s^2)K} \\
&= \frac{G_{c1} sKA(s)}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}
\end{aligned}$$

Since G_{c1} is a PID controller and is given by

$$G_{c1} = \frac{\alpha_1 s + \beta_1 + \gamma_1 s^2}{s} \frac{1}{A(s)}$$

$Y(s)/R(s)$ can be written as

$$\frac{Y(s)}{R(s)} = \frac{K(\alpha_1 s + \beta_1 + \gamma_1 s^2)}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

Therefore, we choose

$$K\gamma_1 = a_2, \qquad K\alpha_1 = a_1, \qquad K\beta_1 = a_0$$

so that

$$G_{c1} = \frac{a_1 s + a_0 + a_2 s^2}{Ks} \frac{1}{A(s)} \tag{8–5}$$

The response of this system to the unit-step reference input can be made to exhibit the maximum overshoot between the chosen upper and lower limits, such as

$$2\% < \text{maximum overshoot} < 10\%$$

The response of the system to the ramp reference input or acceleration reference input can be made to exhibit no steady-state error. The characteristic of the system of Equation (8–4) is that it generally exhibits a short settling time. If we wish to further shorten the settling time, then we need to allow a larger maximum overshoot—for example,

$$2\% < \text{maximum overshoot} < 20\%$$

The controller G_{c2} can now be determined from Equations (8–3) and (8–5). Since

$$G_{c1} + G_{c2} = \frac{\alpha s + \beta + \gamma s^2}{s} \frac{1}{A(s)}$$

we have

$$\begin{aligned} G_{c2} &= \left[\frac{\alpha s + \beta + \gamma s^2}{s} - \frac{a_1 s + a_0 + a_2 s^2}{Ks} \right] \frac{1}{A(s)} \\ &= \frac{(K\alpha - a_1)s + (K\beta - a_0) + (K\gamma - a_2)s^2}{Ks} \frac{1}{A(s)} \qquad (8\text{–}6) \end{aligned}$$

The two controllers G_{c1} and G_{c2} can be determined from Equations (8–5) and (8–6).

EXAMPLE 8–4 Consider the two-degrees-of-freedom control system shown in Figure 8–33. The plant transfer function $G_p(s)$ is given by

$$G_p(s) = \frac{10}{s(s+1)}$$

Design controllers $G_{c1}(s)$ and $G_{c2}(s)$ such that the maximum overshoot in the response to the unit-step reference input be less than 19%, but more than 2%, and the settling time be less than 1 sec. It is desired that the steady-state errors in following the ramp reference input and acceleration reference input be zero. The response to the unit-step disturbance input should have a small amplitude and settle to zero quickly.

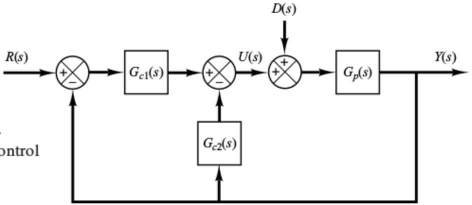


Figure 8–33 Two-degrees-of-freedom control system.

Bibliography

- Ogata, Katsuhiko. Modern Control Engineering. Fifth Edition. Prentice Hall. 2009. Chapter 8.