

Modeling of physical systems

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Control y Sistemas

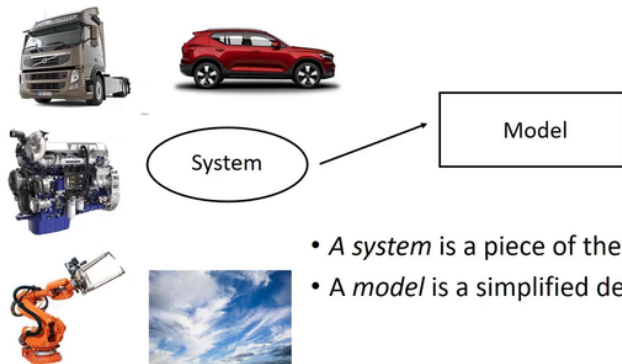
Ingeniería Mecatrónica,
Facultad de Ingeniería,
Universidad Nacional de Cuyo

May 2020



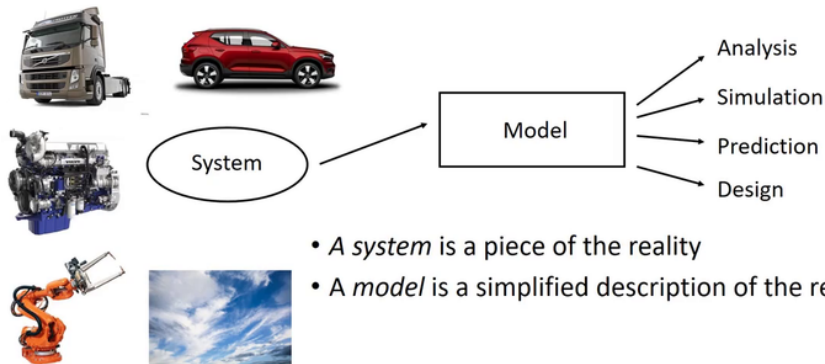
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Modeling of physical systems



- A *system* is a piece of the reality
- A *model* is a simplified description of the reality

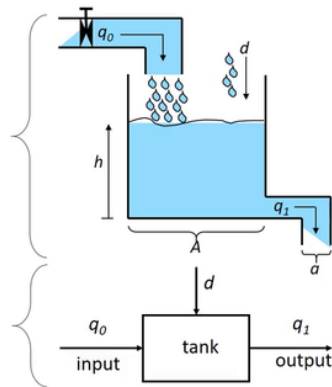
Modeling of physical systems



Modeling of physical systems

Models can be of different kinds:

- Drawings, schematics
- Block scheme, flow chart

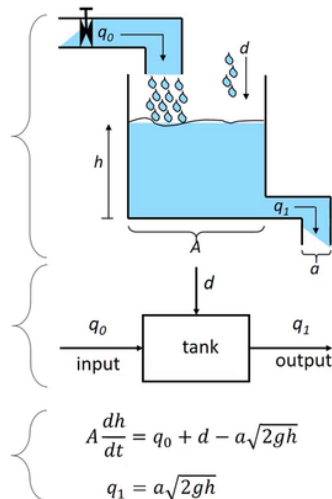


Modeling of physical systems

Models can be of different kinds:

- Drawings, schematics
- Block scheme, flow chart
- Mathematical models

We are interested in mathematical models of dynamical systems



Modeling of dynamic systems

Our focus: mathematical models of physical (dynamic) systems, in particular linear, time invariant models (LTI) in continuous or discrete time:

- Differential- and difference equations

$$\frac{d^2y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_2 y(t) = b_1 u(t)$$

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 u(k)$$

- Transfer functions

$$Y(s) = \frac{b_1}{s^2 + a_1 s + a_2} U(s)$$

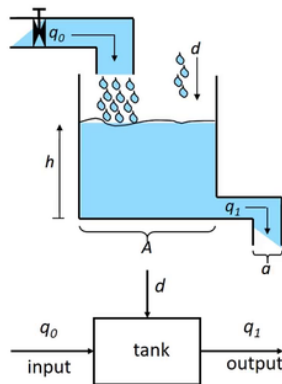
- State-space models

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

How do we build models?

- Physical relations
- Empirical knowledge
- Data

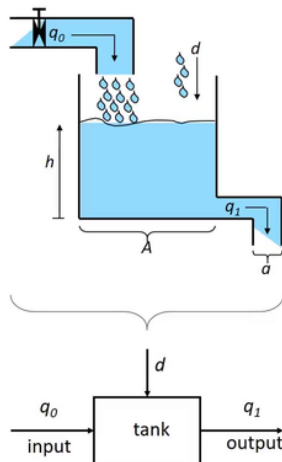


$$A \frac{dh}{dt} = q_0 + d - a\sqrt{2gh}$$

$$q_1 = a\sqrt{2gh}$$

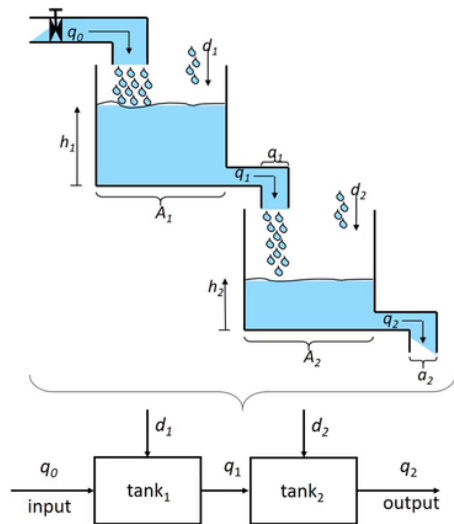
Three phase method

- Structuring
 - Divide into subsystems
 - Inputs, outputs, internal variables?
- Basic equations
 - Conservation laws
 - Constitutive relations
- Form state-space model
 - Choose state variables
 - Form $\dot{x} = \dots$



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Conservation laws (balance equation):

- Mass balance [kg]
- Force balance [$\text{kgm}/\text{s}^2 = \text{N}$] (Newton's law)
- Torque balance [$\text{kgm}^2/\text{s}^2 = \text{Nm}$] (Newton)
- Voltage balance [V] (Kirchhoff's voltage law)
- Current flows [A] (Kirchhoff's current law)
- Volume flows [m^3/s]
- Energy flows [$\text{J}/\text{s} = \text{W}$]
- ...

Three phase method

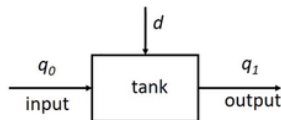
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Constitutive relations (relate variables of different kind):

- Ohm's law: $U=RI$ (voltage and current)
- Ideal gas law: $pV=nRT$ (pres, vol and temp)
- Hooke's law: $F=kx$ (force and distance)
- Air resistance: $F=bv^2$ (force and velocity)
- ...

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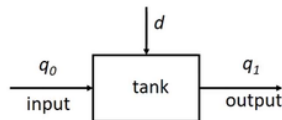
Conservation laws (balance equation):

What is it that changes?

↓
[change of volume
per time instant,
"accumulation"]

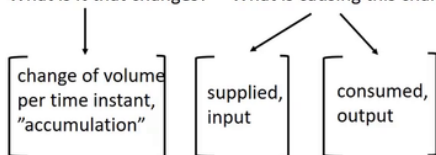
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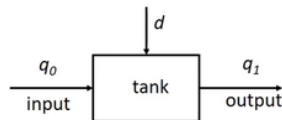
Conservation laws (balance equation):

What is it that changes? What is causing this change?



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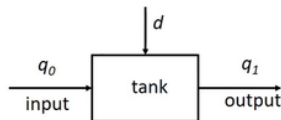
Conservation laws (balance equation):

What is it that changes? What is causing this change?

$$\left[\begin{array}{c} \text{change of volume} \\ \text{per time instant,} \\ \text{"accumulation"} \end{array} \right] = \left[\begin{array}{c} \text{supplied,} \\ \text{input} \end{array} \right] - \left[\begin{array}{c} \text{consumed,} \\ \text{output} \end{array} \right]$$

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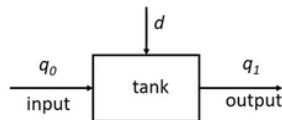
Conservation laws (balance equation):

- Volume flows [m³/s]

$$\frac{dV}{dt} = q_0 + d - q_1$$

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Conservation laws (balance equation):

- Volume flows [m^3/s]

$$\frac{dV}{dt} = q_0 + d - q_1$$

Constitutive relations:

$$V = Ah$$

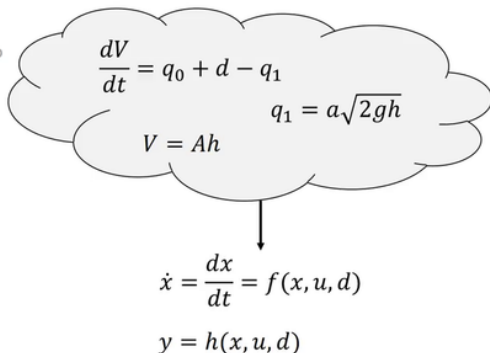
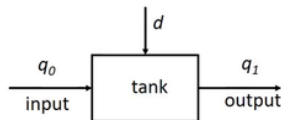
Volume and level

$$q_1 = a\sqrt{2gh}$$

Bernoulli equation

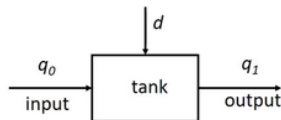
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A cloud-shaped box containing the following equations:

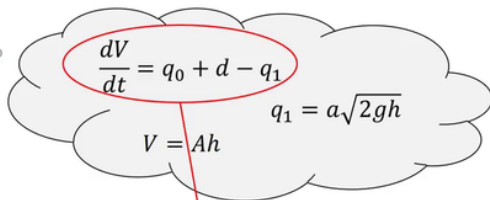
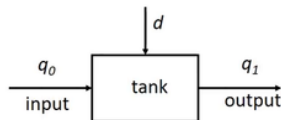
$$\frac{dV}{dt} = q_0 + d - q_1$$
$$q_1 = a\sqrt{2gh}$$
$$V = Ah$$

Choose state variables:

- What is changing? V or h

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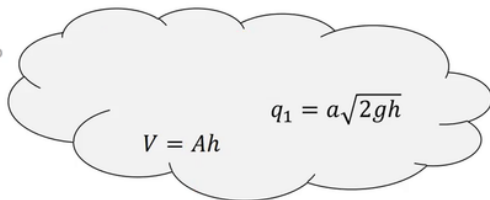
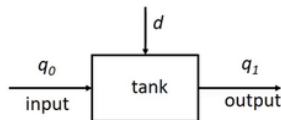


Choose state variables: V

$$\frac{dV}{dt} =$$

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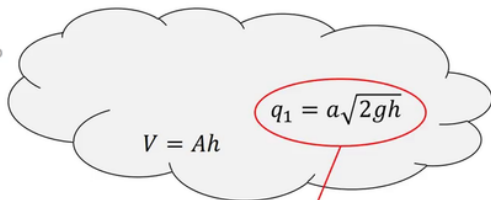
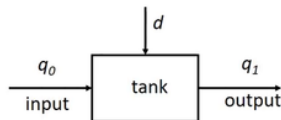


Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - q_1$$

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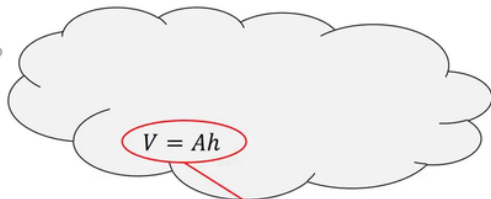
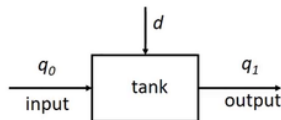


Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - q_1$$

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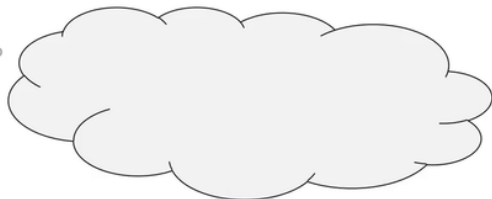
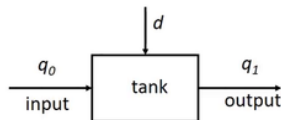


Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2gh}$$

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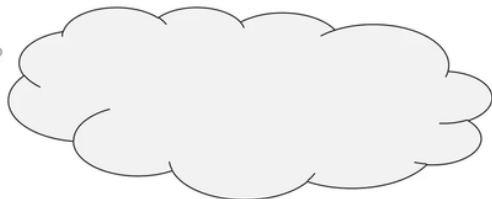
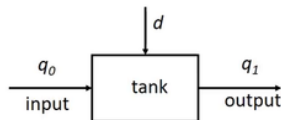


Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2gV/A}$$

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Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2gV/A}$$

$$q_1 = a\sqrt{2gh}$$

Three phase method

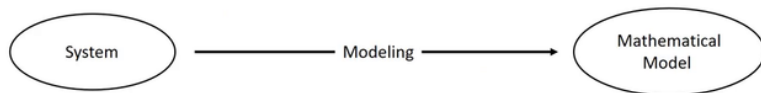
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$$\left\{ \begin{array}{l} \frac{dV}{dt} = q_0 + d - a\sqrt{2gV/A} \\ q_1 = a\sqrt{2gV/A} \end{array} \right.$$

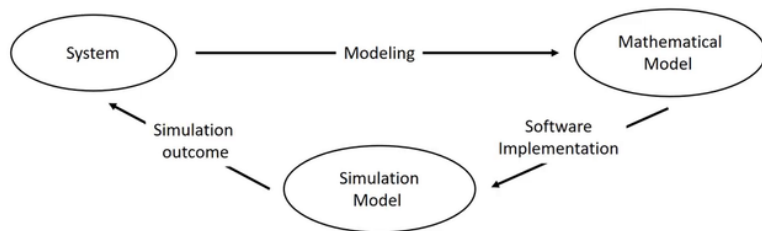
Variable change: $x = V, u = q_0$ and $y = q_1$

$$\left\{ \begin{array}{l} \dot{x} = \frac{dx}{dt} = u + d - a\sqrt{2gx/A} \\ y = a\sqrt{2gx/A} \end{array} \right.$$

Model verification and validation

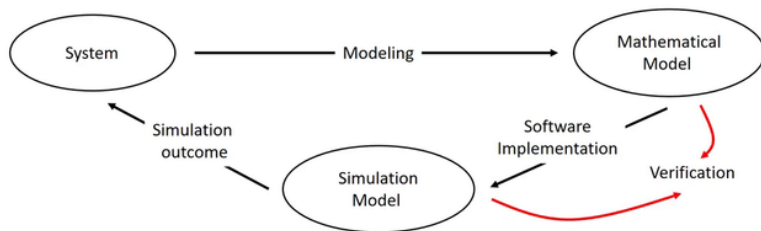


Model verification and validation



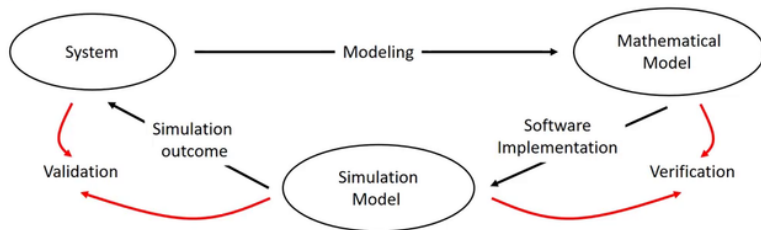
Model verification and validation

- Verification is the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.



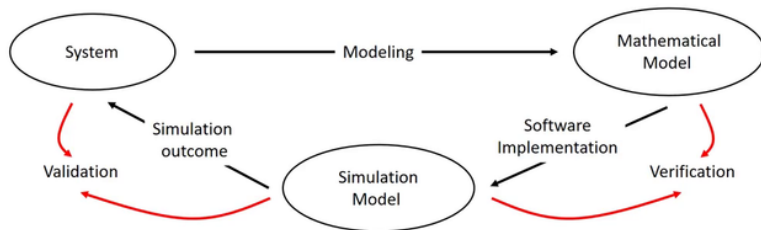
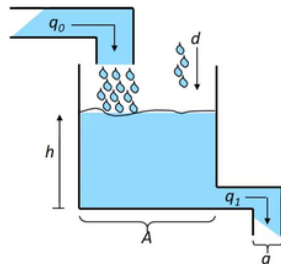
Model verification and validation

- Verification is the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.
- Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.



Model verification and validation

- All models have a limited domain of validity.
- Be aware of the model's (lack of) accuracy.



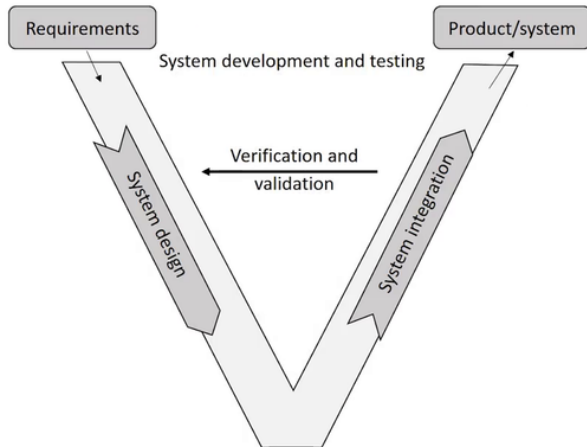
"All models are wrong but some are useful."

George Box, *Robustness in the strategy of scientific model building*, in Launer, R. L.; Wilkinson, G. N., *Robustness in Statistics*, Academic Press, pp. 201–236, 1979

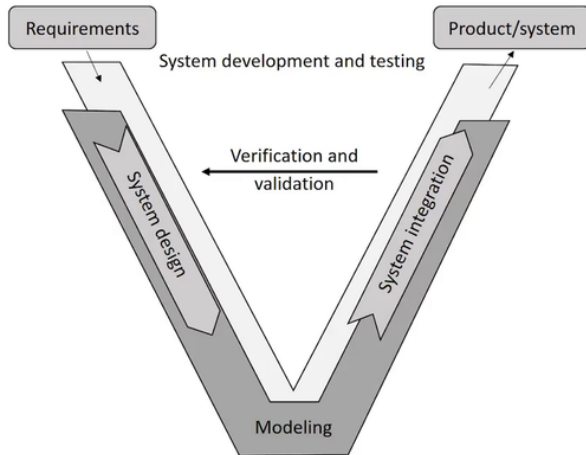
"Models and simulations can never replace observations and experiments – but they constitute an important and useful complement."

Lennart Ljung and Torkel Glad, *Modeling and Identification of Dynamic Systems*, Studentlitteratur, 2016

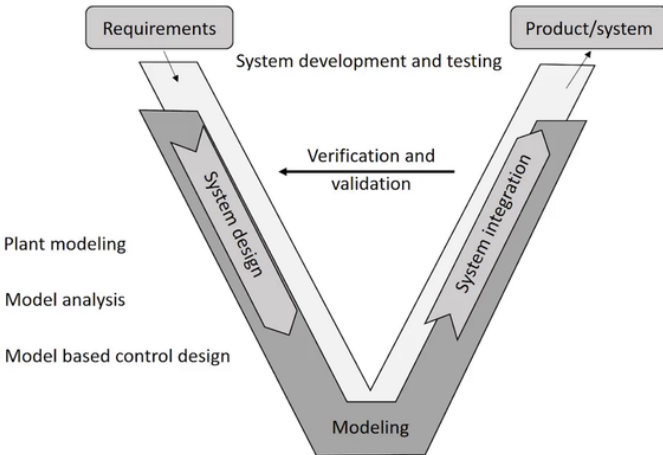
Model based design



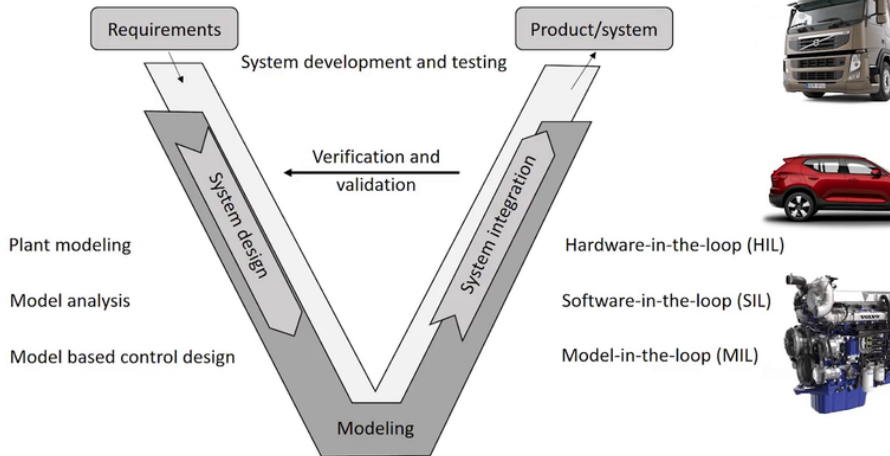
Model based design



Model based design



Model based design



- Karl J. Astrom and Richard M. Murray *Feedback Systems*. Version v3.0i. Princeton University Press. September 2018. Chapter 3.
- Karmopp, Dean et al. *Systems Dynamics: Modeling, Simulation, and Control of Mechatronic Systems*. Fifth Edition. John Wiley & Sons, Inc. 2012. Chapters 1 y 2.1.