Typical stages in digital signal processing

Aliasing prefiltering

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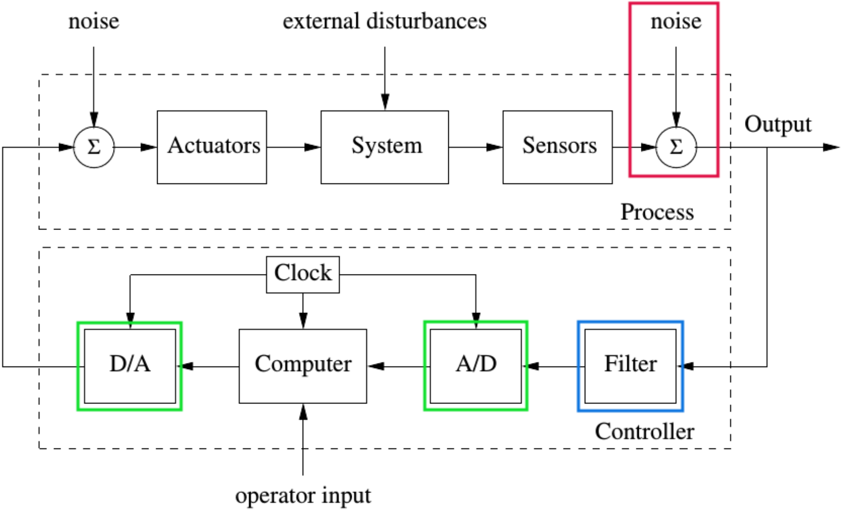
DSP in a the context of control systems



Digital processing of analog signals

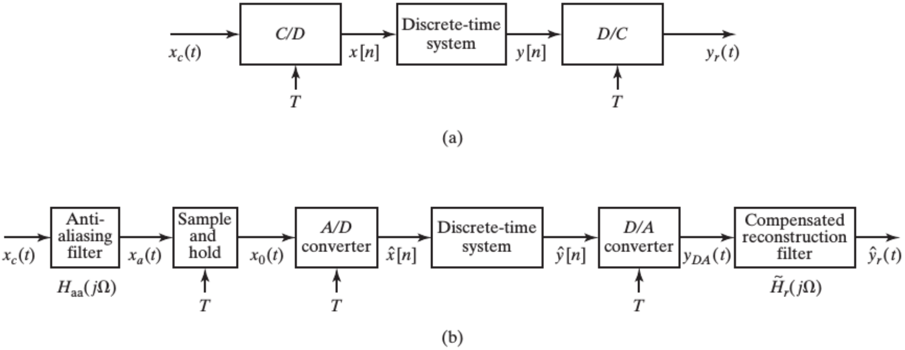


Figure 4.47 (a) Discrete-time filtering of continuous-time signals. (b) Digital processing of analog signals.

Periodic sampling

The discrete-time representation of a continuous-time signal is obtained through periodic sampling from a continuous-time signal $x_c(t)$ according to,

$$x[n] = x_c(nT), \qquad -\infty < n < \infty\,, \tag{1}$$

where T is the sampling period, and $f_s = 1/T$ is the sampling frequency, or $\Omega_s = 2\pi/T$ in radians/s

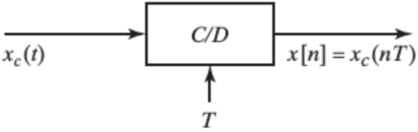


Figure 4.1 Block diagram representation of an ideal continuous-to-discrete-time (C/D) converter.

Sampling process

It is convenient to represent the sampling process mathematically in the two stages.

1. An impulse train $s(t)$ is multiplied by a continuous-time signal $x_c(t)$.
2. The continuous-time signal $x_s(t)$ is transformed to a discrete-time sequence $x[n]$.

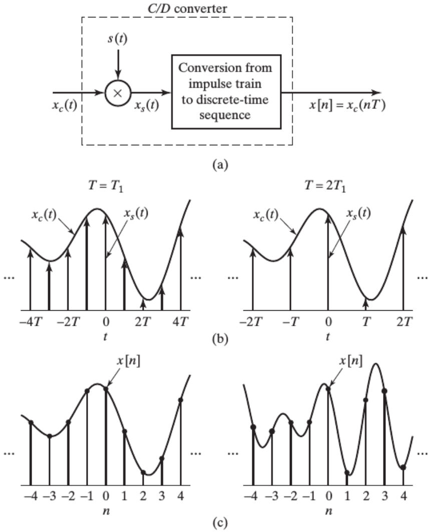


Figure 4.2 Sampling with a periodic impulse train, followed by conversion to a discrete-time sequence. (a) Overall system. (b) $x_s(t)$ for two sampling rates. (c) The output sequence for the two different sampling rates.

Nyquist-Shannon Sampling Theorem

Let $x_c(t)$ be a bandlimited signal with,

$$X_c(j\Omega) = 0 \text{ para } |\Omega| \geq \Omega_N \,. \tag{2}$$

Then $x_c(t)$ is uniquely determined by its samples $x[n] = x_c(nT)$, $n = 0, \pm 1, \pm 2, \ldots$ if,

$$\Omega_s = \frac{2\pi}{T} \geq 2\Omega_N \,. \tag{3}$$

The frequency $2\Omega_N$ is commonly referred to as the **Nyquist frequency**.

The Nyquist frequency is the minimum sampling frequency in order to be able of reconstructing $x_c(t)$.

Frequency-domain representation of sampling

$x_s(t)$ is obtained multiplying $x_c(t)$ by a periodic impulse train $s(t)$,

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)\,, \tag{4}$$

$$x_s(t) = x_c(t)\, s(t)\,, \tag{5}$$

$$= x_c(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)\,, \tag{6}$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT)\, \delta(t - nT) \qquad \text{by sifting property.} \tag{7}$$

The Fourier transform of the periodic impulse train $s(t)$ is the periodic impulse train,

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s), \qquad \text{where } \Omega_s = \frac{2\pi}{T}\,. \tag{8}$$

The Fourier transform of $x_s(t)$ is the continuous-variable convolution of $X_c(j\Omega)$ and $S(j\Omega)$,

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)\,, \tag{9}$$

$$X_s(j\Omega) = \frac{1}{T} X_c[j(\Omega - k\Omega_s)]\,. \tag{10}$$

Frequency-domain representation of sampling, 2

- Fourier transform of $x_s(t)$ consists of periodically repeated copies of $X_c(j\Omega)$
- These copies are shifted by integer multiples of the sampling frequency.
- It is evident that

$\Omega_s - \Omega_N \geq \Omega_N$, or,

$\Omega_s \geq 2\Omega_N$

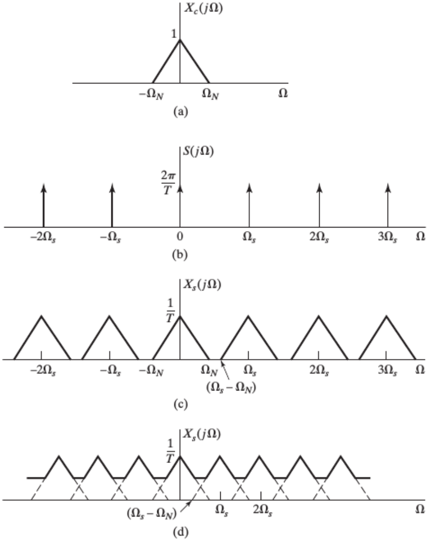


Figure 4.3 Frequency-domain representation of sampling in the time domain. (a) Spectrum of the original signal. (b) Fourier transform of the sampling function. (c) Fourier transform of the sampled signal with $\Omega_S > 2\Omega_N$. (d) Fourier transform of the sampled signal with $\Omega_S < 2\Omega_N$.

The aliasing filter is an **analog** low-pass or band-pass filter.

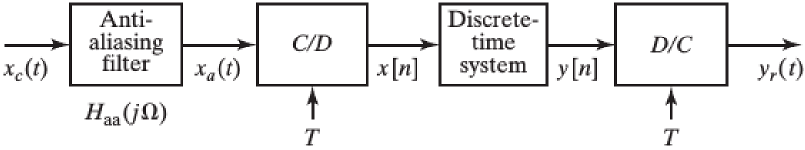
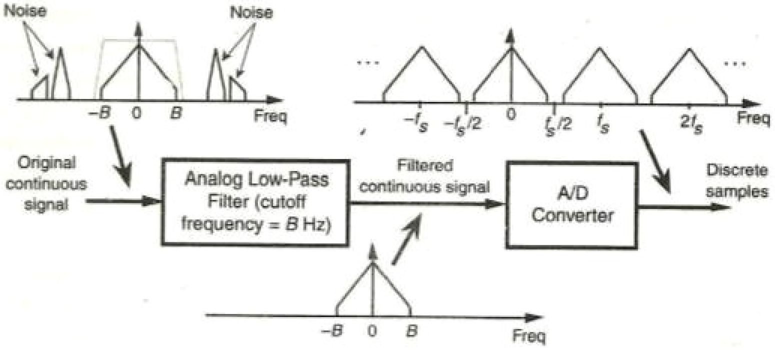


Figure 4.48 Use of prefiltering to avoid aliasing.

Aliasing prefiltering, 2

Even if the signal is naturally bandlimited (as music), wideband additive noise may fill in the higher frequency range, and as a result of sampling, these noise components would be aliased into the low-frequency band.



Aliasing prefiltering, example

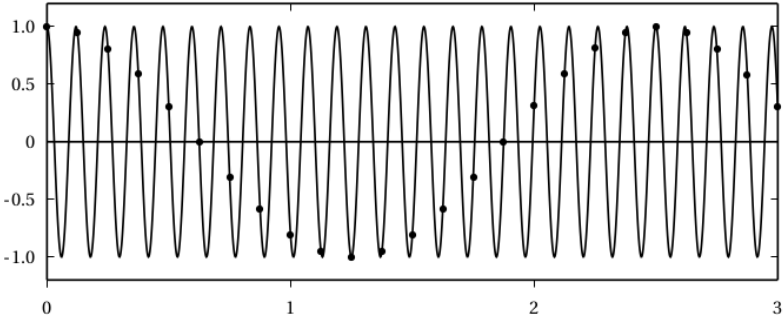


Figure 9.8 Example of aliasing: a sinusoid at 8400 Hz, $x(t) = \cos(2\pi \cdot 8400t)$ (solid line) is sampled at $F_s = 8000$ Hz. The sampled values (dots) are indistinguishable from those of at 400 Hz sinusoid sampled at F_s.

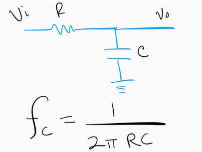


Figure: RC low-pass filter.

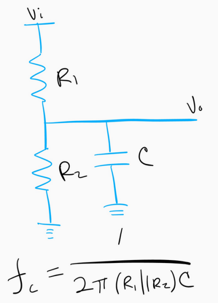


Figure: RC low-pass filter with voltage divider.

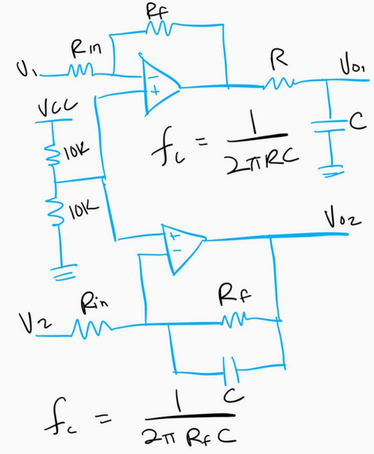


Figure: The top circuit is filtered using a passive filter while the bottom part uses an active filter.

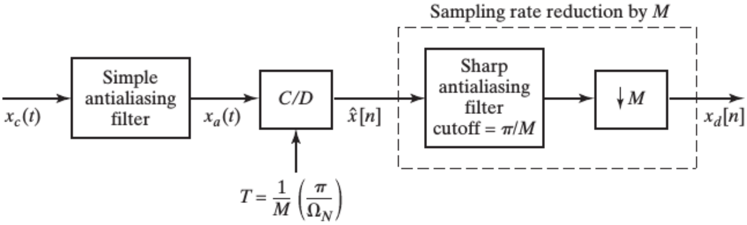


Figure 4.49 Using oversampled A/D conversion to simplify a continuous-time antialiasing filter.

Oversampling frequency response

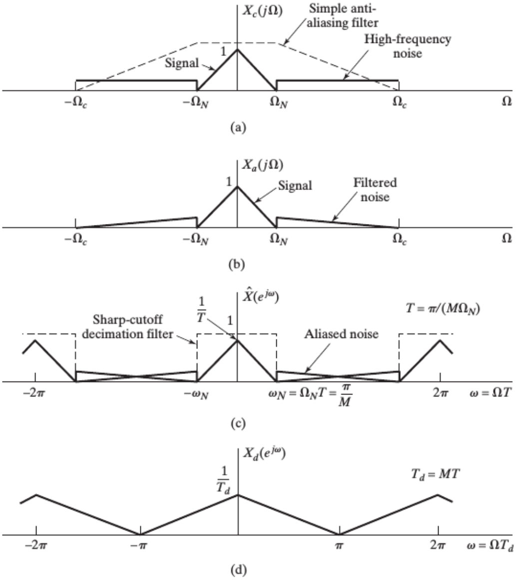


Figure 4.50 Use of oversampling followed by decimation in C/D conversion.

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