

# PID controllers and modified PID controllers

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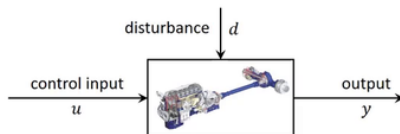
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## Feedback principles – model based design

Control objectives (specification)

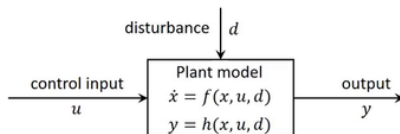
- Qualitative – minimize energy
- Quantitative – response time



## Feedback principles – model based design

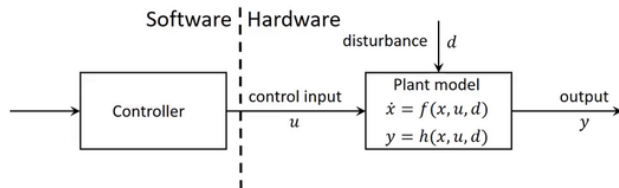
Description of the system/plant

- Level of abstraction
- Modeling – physical modeling or from measured data



## Feedback principles – model based design

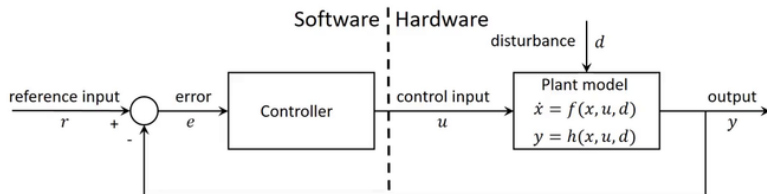
Design controller



## Feedback principles – model based design

Design controller

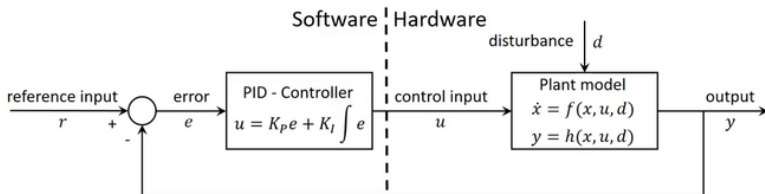
- Select technique – Open loop or closed loop



## Feedback principles – model based design

### Design controller

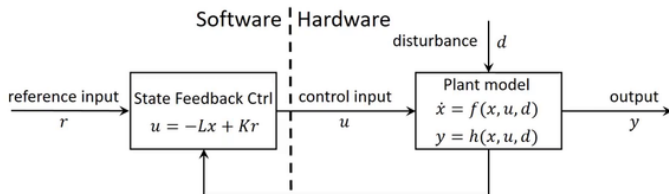
- Select technique – Open loop or closed loop
- Classical methods or state-space methods



## Feedback principles – model based design

### Design controller

- Select technique – Open loop or closed loop
- Classical methods or state-space methods
- Choose parameters (trial-and-error, design method, optimization)

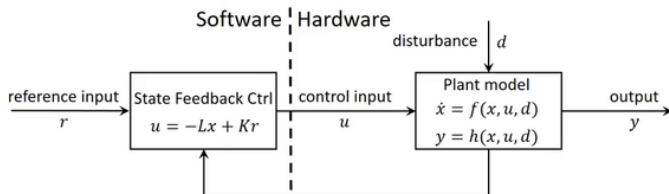




## Feedback principles – model based design

Analyze the performance

- Analysis
- Simulation
- Experiments



## Control design methods

### Classical control methods

- works well for simple systems,
- can be tuned based on trial-and-error or engineering intuition,
- do not require a mathematical model of the system

## Control design methods

### Classical control methods

- works well for simple systems,
  - can be tuned based on trial-and-error or engineering intuition,
  - do not require a mathematical model of the system
- but
- are typical iterative,
  - are difficult to use for larger-scale systems (complex systems) with multiple inputs and outputs (MIMO),

## Control design methods

### State-space methods

- can easily handle larger-scale systems (complex systems) with multiple inputs and outputs (MIMO),
- tuning can be formed as an optimization problem,
- are easy to implement

## Control design methods

### State-space methods

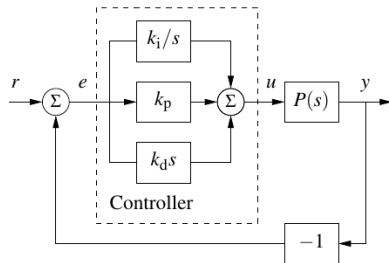
- can easily handle larger-scale systems (complex systems) with multiple inputs and outputs (MIMO),
- tuning can be formed as an optimization problem,
- are easy to implement

but

- require a mathematical model of the system,

A survey of controllers for more than 100 boiler-turbine units in the Guangdong Province in China [1]:

- 94.4% of all controllers were PI,
- 3.7% PID,
- 1.9% used advanced control.



$$u(s) = k_p \cdot e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt} = k_p \left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right), \quad (1)$$

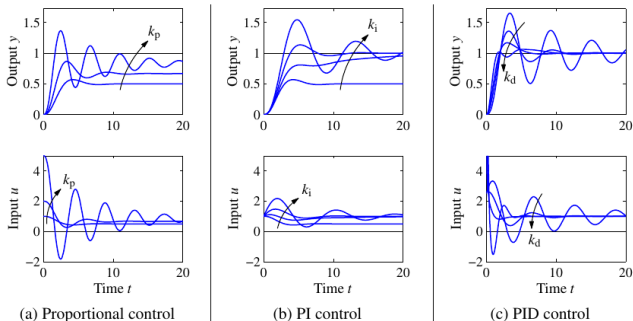
$$k_i = \frac{k_p}{T_i}, \quad (2)$$

$$k_d = k_p \cdot T_d. \quad (3)$$

[1] Li Sun, Donghai Li, and Kwang Y. Lee. Optimal disturbance rejection for pi controller with constraints on relative delay margin. ISA Transactions, 2016.

# PID Control

## PID parameters



**Figure 11.2:** Responses to step changes in the reference value for a system with a proportional controller (a), PI controller (b) and PID controller (c). The process has the transfer function  $P(s) = 1/(s+1)^3$ , the proportional controller has parameters  $k_p = 1, 2$  and  $5$ , the PI controller has parameters  $k_p = 1, k_i = 0, 0.2, 0.5$ , and  $1$ , and the PID controller has parameters  $k_p = 2.5, k_i = 1.5$  and  $k_d = 0, 1, 2$ , and  $4$ .

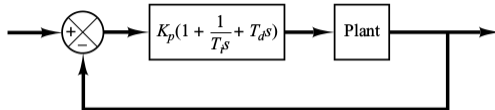
$$u = k_p \cdot e + k_i \int_0^t e(\tau) d\tau + k_d \frac{de}{dt} = k_p \left( e + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de}{dt} \right) \quad (4)$$

# PID Control Tuning

## Ziegler–Nichols Rules

- If the plant mathematical model cannot be obtained at all, then an analytical or computational approach to the design of a PID controller is not possible.
- Then we must resort to experimental approaches to the tuning of PID controllers.
- Ziegler and Nichols suggested rules for tuning PID controllers (values  $K_p$ ,  $T_i$ , and  $T_d$ ) based on:
  - Experimental step responses (Method 1).
  - Based on the value of  $K_p$  that results in marginal stability when only proportional control action is used (Method 2).

**Figure 8–1**  
PID control  
of a plant.



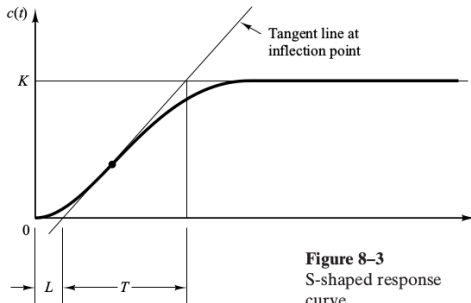
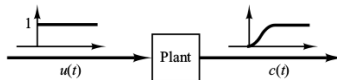


# PID Control Tuning

## Method 1

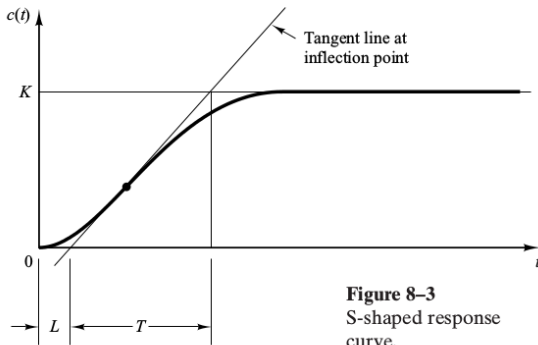
- We obtain experimentally the response of the plant to a unit-step input.
- This method applies if the response to a step input exhibits an S-shaped curve.
- If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped.

**Figure 8-2**  
Unit-step response  
of a plant.



**Figure 8-3**  
S-shaped response  
curve.

- The S-shaped curve may be characterized by two constants, delay time  $L$  and time constant  $T$ .



**Table 8-1** Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	$K_p$	$T_i$	$T_d$
P	$\frac{T}{L}$	$\infty$	0
PI	$0.9 \frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2 \frac{T}{L}$	$2L$	$0.5L$

function  $C(s)/U(s)$  may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of  $K_p$ ,  $T_i$ , and  $T_d$  according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \\ &= 1.2 \frac{T}{L} \left( 1 + \frac{1}{2Ls} + 0.5Ls \right) \\ &= 0.6T \frac{\left( s + \frac{1}{L} \right)^2}{s} \end{aligned}$$

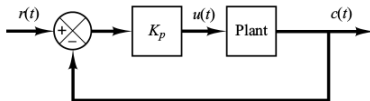
Thus, the PID controller has a pole at the origin and double zeros at  $s = -1/L$ .

# PID Control Tuning

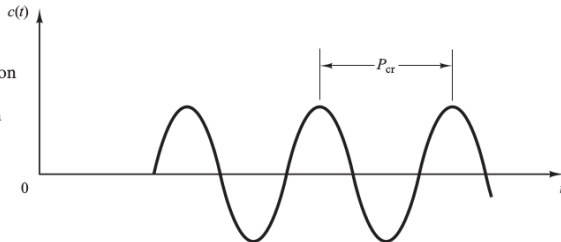
## Method 2

- We first set  $T_i = \infty$ , and  $T_d = 0$ .
- Increase  $K_p$  from 0 to a critical value  $K_{cr}$  at which the output first exhibits sustained oscillations.
- Thus, the critical gain  $K_{cr}$  and the corresponding period  $P_{cr}$  are experimentally determined.
- If the output does not exhibit sustained oscillations, then this method does not apply.

**Figure 8-4**  
Closed-loop system with a proportional controller.



**Figure 8-5**  
Sustained oscillation with period  $P_{cr}$ .  
( $P_{cr}$  is measured in sec.)



**Table 8-2** Ziegler–Nichols Tuning Rule Based on Critical Gain  $K_{cr}$  and Critical Period  $P_{cr}$  (Second Method)

Type of Controller	$K_p$	$T_i$	$T_d$
P	$0.5K_{cr}$	$\infty$	0
PI	$0.45K_{cr}$	$\frac{1}{1.2} P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

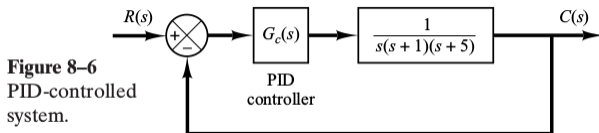
$$\begin{aligned}
 G_c(s) &= K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \\
 &= 0.6K_{cr} \left( 1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s \right) \\
 &= 0.075K_{cr}P_{cr} \frac{\left( s + \frac{4}{P_{cr}} \right)^2}{s}
 \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at  $s = -4/P_{cr}$ .

**EXAMPLE 8-1** Consider the control system shown in Figure 8-6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters  $K_p$ ,  $T_i$ , and  $T_d$ . Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.



# PID Control Tuning

## Method 2, Example 8-1, II

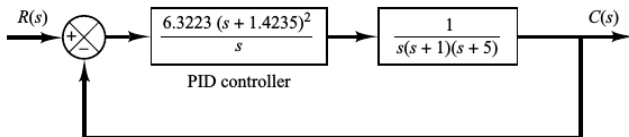
$$K_{cr} = 30$$

$$P_{cr} = 2.8099$$

$$K_p = 0.6 \cdot K_{cr} = 18$$

$$T_i = 0.5 \cdot P_{cr} = 1.405$$

$$T_d = 0.125 \cdot P_{cr} = 0.35124$$





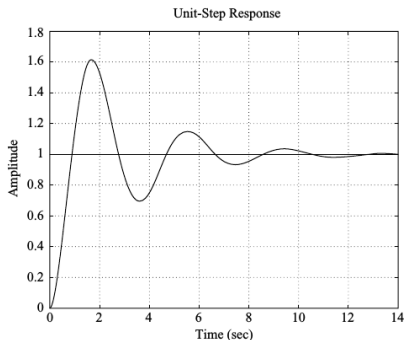
# PID Control Tuning

## Method 2, Example 8-1, III

Maximum overshoot is close to 62%.

### MATLAB Program 8-1

```
% ----- Unit-step response -----  
num = [6.3223 18 12.811];  
den = [1 6 11.3223 18 12.811];  
step(num,den)  
grid  
title('Unit-Step Response')
```



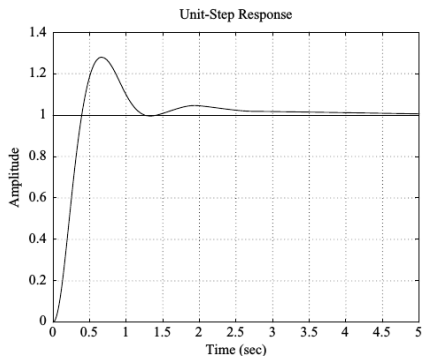
The Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

$$K_p = 39.42$$

$$T_i = 3.077$$

$$T_d = 0.7692$$

Maximum overshoot is fairly close to 25%.

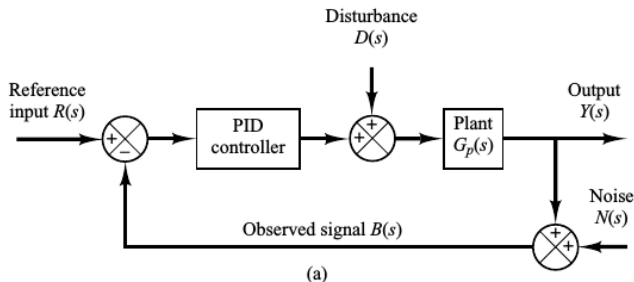


- The value of  $K_p$  increases the speed of response.
- However, varying the location of the double zero has a significant effect on the maximum overshoot.

# Modifications of PID Control Schemes

## Introduction

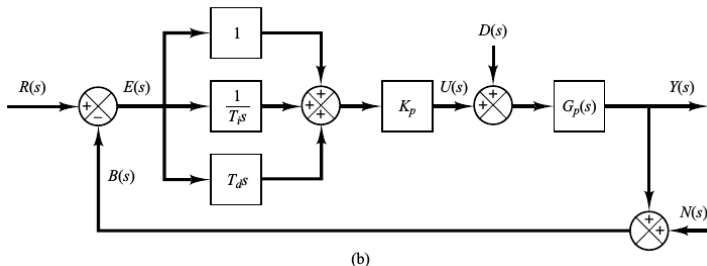
- In practical cases, there may be one requirement on the *response to disturbance input* and another requirement on the *response to reference input*.
- Often these two requirements conflict with each other and cannot be satisfied in the single-degree-of-freedom case.
- By increasing the degrees of freedom, we are able to satisfy both, response to disturbance and response to reference input.



# Modifications of PID Control Schemes

## PI-D Control

- If reference input is a step function, the derivative term in the control action will produce that  $u(t)$  will involve an impulse function.
- If  $T_d$  is multiplied by 0.1,  $u(t)$  will involve an pulse function.
- Such a phenomenon is called *set-point kick*.

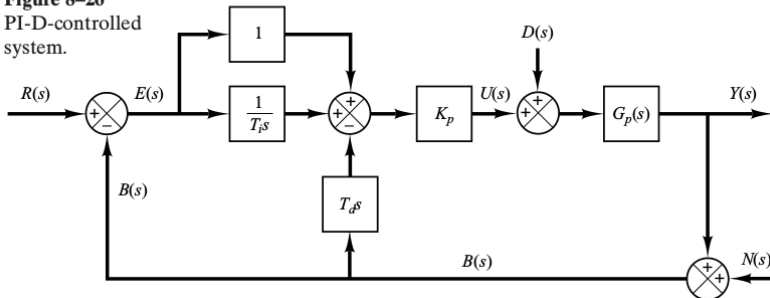


To avoid the set-point kick phenomenon (a pulse input), we may wish to operate the derivative action only in the feedback path so that differentiation occurs only on the feedback signal and not on the reference signal.

$$U(s) = K_p \left( 1 + \frac{1}{T_i s} \right) R(s) - K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) B(s)$$

**Figure 8–26**

PI-D-controlled system.



Notice that in the absence of the disturbances and noises, the closed-loop transfer function of the basic PID control system [shown in Figure 8–25(b)] and the PI-D control system (shown in Figure 8–26) are given, respectively, by

$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s} + T_d s\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

and

$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + \left(1 + \frac{1}{T_i s} + T_d s\right) K_p G_p(s)}$$

It is important to point out that in the absence of the reference input and noises, the closed-loop transfer function between the disturbance  $D(s)$  and the output  $Y(s)$  in either case is the same and is given by

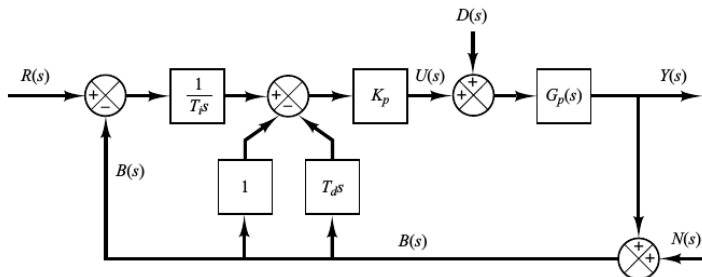
$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left(1 + \frac{1}{T_i s} + T_d s\right)}$$

# Modifications of PID Control Schemes

## I-PD Control

- If reference input is a step function, both PID control and PI-D control involve a step function in the manipulated signal.
- The proportional action and derivative action are moved to the feedback path so that these actions affect the feedback signal only.

$$U(s) = K_p \frac{1}{T_i s} R(s) - K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) B(s)$$



The closed-loop transfer function  $Y(s)/R(s)$  in the absence of the disturbance input and noise input is given by

$$\frac{Y(s)}{R(s)} = \left( \frac{1}{T_i s} \right) \frac{K_p G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

It is noted that in the absence of the reference input and noise signals, the closed-loop transfer function between the disturbance input and the output is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s) \left( 1 + \frac{1}{T_i s} + T_d s \right)}$$

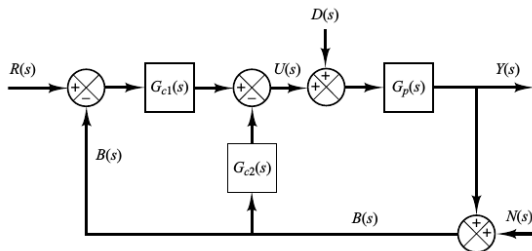
This expression is the same as that for PID control or PI-D control.



# Two-Degrees-of-Freedom Control Systems

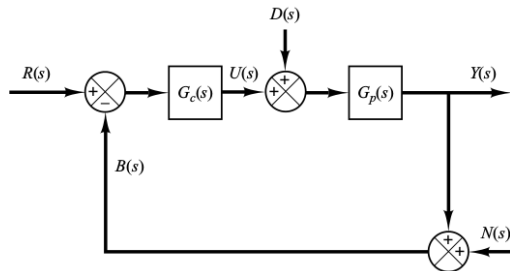
## Introduction

- Instead of moving the entire derivative control action or proportional control action to the feedback path, it is possible to move only portions of these control actions to the feedback path.
- The characteristics of PI-PD control scheme lie between PID control and I-PD control.
- Similarly, PID-PD control can be considered.
- We will have one controller in the feedforward path and another controller in the feedback path. Such control schemes lead us to a two-degrees-of-freedom control scheme.



# Two-Degrees-of-Freedom Control Systems

## One-degree-of-freedom Control System



For this system, three closed-loop transfer functions  $Y(s)/R(s) = G_{yr}$ ,  $Y(s)/D(s) = G_{yd}$ , and  $Y(s)/N(s) = G_{yn}$  may be derived. They are

$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_c G_p}{1 + G_c G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_c G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_c G_p}{1 + G_c G_p}$$

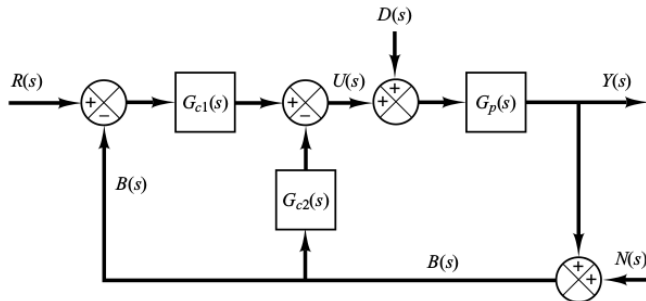
$$G_{yr} = \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

How many of these closed-loop transfer functions are independent?

# Two-Degrees-of-Freedom Control Systems

## Two-degree-of-freedom Control System



$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{(G_{c1} + G_{c2})G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yr} = G_{c1}G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

How many of these closed-loop transfer functions are independent?

# Two-Degrees-of-Freedom Control Systems

## Two-degree-of-freedom Control System, II

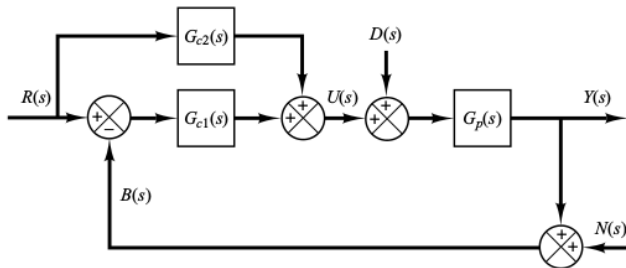
$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + G_{c1}G_p} + \frac{G_{c2}G_p}{1 + G_{c1}G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_{c1}G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_{c1}G_p}{1 + G_{c1}G_p}$$

$$G_{yr} = G_{c2}G_{yd} + \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$



How many of these closed-loop transfer functions are independent?

- Karl J. Astrom and Richard M. Murray *Feedback Systems*. Version v3.0i. Princeton University Press. September 2018. Chapter 11.
- Ogata, Katsuhiko. *Modern Control Engineering*. Fifth Edition. Prentice Hall. 2009. Chapter 8.