In[ ]:=
```
Quit[]
```

In[891]:=
```
(*Definição dos parâmetros*)

par = {
g → 9.81, (*m^2/s*)
L₁ → 0.5, (*m*)
L₂ → 0.75, (*m*)
m₁ → 0.5, (*kg*)
m₂ → 0.75, (*kg*)
m₀ → 1.5, (*kg*)
l₁ → 0.25,
l₂ → 0.375
};

(*Simplificação para visualização*)
simp = {
L₁ → L,
L₂ → L,
l₁ → L/2,
l₂ → L/2,
m₀ → m,
m₁ → m,
m₂ → m
};
```

Cinemática

In[893]:=
```
(*Velocidades para o Lagrange*)

V₀ = θ₀'[t];
ω₁ = θ₁'[t];
ω₂ = θ₂'[t];


(*Momentos de Inércia das barras*)
I₁ = m₁L₁^2/12;
I₂ = m₂L₂^2/12;
```

Método de Lagrange

In[898]:=

```
(*Energia Cinética*)
T₀ = 1/2 m₀V₀^2;
T₁ = 1/2 m₁V₀^2 + 1/2 (m₁l₁^2+I₁)ω₁^2 + m₁l₁V₀ω₁Cos[θ₁[t]];
T₂ = 1/2 m₂V₀^2 + 1/2 m₂L₁^2ω₁^2 + 1/2 (m₂l₂^2+I₂)ω₂^2 + m₂L₁V₀ω₁Cos[θ₁[t]] + m₂l₂V₀ω₂Cos[θ₂[t]] + m₂L₁l
T = T₀+T₁+T₂;


(*Energia Potencial*)
P₀ = 0;
P₁ = m₁*g*l₁Cos[θ₁[t]];
P₂ = m₂*g*(L₁Cos[θ₁[t]]-l₂Cos[θ₂[t]]);
P = P₀+P₁+P₂;

(*Lagrangiano*)
ℒ = T - P;
```

In[∘]:= ℒ // FullSimplify

Out[∘]=

$$\frac{1}{24}\Big(12\,(m_0+m_1)\,\theta_0'[t]^2+12\,m_2\,\theta_0'[t]^2+12\,l_1^2\,m_1\,\theta_1'[t]^2+ L_1^2\,m_1\,\theta_1'[t]^2+12\,L_1^2\,m_2\,\theta_1'[t]^2-24\,\text{Cos}[\theta_1[t]]\,l_1\,m_1\,(g-\theta_0'[t]\,\theta_1'[t])- 24\,\text{Cos}[\theta_1[t]]\,L_1\,m_2\,(g-\theta_0'[t]\,\theta_1'[t])+12\,l_2^2\,m_2\,\theta_2'[t]^2+L_2^2\,m_2\,\theta_2'[t]^2+ 24\,l_2\,m_2\,(g\,\text{Cos}[\theta_2[t]]+(\text{Cos}[\theta_2[t]]\,\theta_0'[t]+\text{Cos}[\theta_1[t]-\theta_2[t]]\,L_1\,\theta_1'[t])\,\theta_2'[t])\Big)$$

In[∘]:= ℒ /. par // FullSimplify

Out[∘]=

$$-4.905\,\text{Cos}[\theta_1[t]]+2.75906\,\text{Cos}[\theta_2[t]]+1.375\,\theta_0'[t]^2+ 0.114583\,\theta_1'[t]^2+0.140625\,\text{Cos}[\theta_1[t]-\theta_2[t]]\,\theta_1'[t]\,\theta_2'[t]+ 0.0703125\,\theta_2'[t]^2+\theta_0'[t]\,(0.5\,\text{Cos}[\theta_1[t]]\,\theta_1'[t]+0.28125\,\text{Cos}[\theta_2[t]]\,\theta_2'[t])$$

In[907]:=

```
(*Equações de Lagrange*)

Lag₀ = D[D[ℒ,θ₀'[t]],t] - D[ℒ,θ₀[t]] == u;
Lag₁ = D[D[ℒ,θ₁'[t]],t] - D[ℒ,θ₁[t]] == 0;
Lag₂ = D[D[ℒ,θ₂'[ t]],t] - D[ℒ,θ₂[t]] == 0;

Lag = {Lag₀, Lag₁, Lag₂};
```

In[∘]:= MatrixForm[Lag] // FullSimplify // TraditionalForm

Out[∘]//TraditionalForm=

$$\begin{pmatrix} (l_1\,m_1+L_1\,m_2)\,\theta_1'(t)^2\sin(\theta_1(t))+l_2\,m_2\left(\theta_2'(t)^2\sin(\theta_2(t))-\theta_2''(t)\cos(\theta_2(t))\right)=(l_1\,m_1+L_1\,m_2)\,\theta_1''(t)\cos(\theta_1(\\ L_1\,m_2\left(-g\sin(\theta_1(t))+l_2\left(\theta_2'(t)^2\sin(\theta_1(t)-\theta_2(t))+\theta_2''(t)\cos(\theta_1(t)-\theta_2(t))\right)+\theta_0''(t)\cos(\theta_1(t))\right)+l_1^2\,m_1\,\theta_1''(t)+\frac{1}{12}\,L_1^2\,(m_1+1 \\ m_2\left(12\,l_2\left(g\sin(\theta_2(t))+L_1\left(\theta_1''(t)\cos(\theta_1(t)-\theta_2(t))-\theta_1'(t)^2\sin(\theta_1(t)-\theta_2(t))\right)+\theta_0''(t)\cos(\theta_2(t))\right)\right. \end{pmatrix}$$

In[ ]:=
```
MatrixForm[Lag] /. par // FullSimplify // TraditionalForm
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0.5625\, \theta_2'(t)^2 \sin(\theta_2(t)) + \theta_1'(t)\,(1.\, \theta_1'(t) \sin(\theta_1(t)) - 20.) = 5.5\, \theta_0''(t) + 1.\, \theta_1''(t) \cos(\theta_1(t)) + 0.5625\, \theta_2''(t) \cos(\theta_2(t \\ 1.\, \sin(\theta_1(t)) = 0.0286697\, \theta_2'(t)^2 \sin(\theta_1(t) - \theta_2(t)) + 0.046721\, \theta_1''(t) + 0.101937\, \theta_0''(t) \cos(\theta_1(t)) + 0.0286697\, \theta_2''(t) \cos(\theta_1 \\ 0.0509684\, \theta_2''(t) + 0.101937\, \theta_0''(t) \cos(\theta_2(t)) + 0.0509684\, \theta_1''(t) \cos(\theta_1(t) - \theta_2(t)) + 1.\, \sin(\theta_2(t)) = 0.0509684\, \theta_1'(t)^2 \sin(\theta \end{pmatrix}$$

Espaço de Estados

In[911]:=
```
(*Substituição para o Vetor de Estados*)

Ee = {
θ0[t] → x1[t],
θ1[t] → x2[t],
θ2[t] → x3[t],
θ0'[t] → x4[t],
θ1'[t] → x5[t],
θ2'[t] → x6[t],
θ0''[t] → x4'[t],
θ1''[t] → x5'[t],
θ2''[t] → x6'[t]
};
```

Linearização

In[912]:=
```
(*Ponto de equilíbrio escolhido*)

equi = {
u → 0,
x2[t] → 0,
x3[t] → 0,
x4[t] → 0,
x5[t] → 0,
x6[t] → 0,
x4'[t] → 0,
x5'[t] → 0,
x6'[t] → 0
};
```

In[913]:=

```
(*Matrizes de Linearização, de acordo com a literatura*)

d1 = m0 + m1 + m2;
d2 = m1l1 + m2L1;
d3 = m2l2;
d4 = (1/3 m1 + m2) L1^2;
d5 = m2L1l2;
d6 = 1/3 m2L2^2;
f1 = (1/2 m1 + m2) L1*g;
f2 = 1/2 m2L2*g;

D0 = {{d1, d2, d3},{d2, d4, d5},{d3, d5, d6}};
G0 = {{0},{-f1Sin[θ1[t]]},{-f2Sin[θ2[t]]}};

D-1 = Inverse[D0];
dG0 = {{0,0,0},{0,-f1,0},{0,0,-f2}};
A3 = -D-1.dG0;
B2 = D-1.{{1},{0},{0}};
A1 = Join[ConstantArray[0,{3,3}],IdentityMatrix[3],2];
A2 = Join[A3,ConstantArray[0,{3,3}],2];
Af = Join[A1,A2];
Bf = Join[ConstantArray[0,{3,1}],B2];
Cf = Join[IdentityMatrix[3], ConstantArray[0,{3,3}],2];
Df = ConstantArray[0,{3,1}];
```

In[ ]:=
```
Af // TraditionalForm // FullSimplify
Bf // TraditionalForm // FullSimplify
Cf // TraditionalForm // FullSimplify
Df // TraditionalForm // FullSimplify
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{3\, g\, L_1\, (m_1+2\, m_2)\left(l_1\, L_2^2\, m_1+L_1\, m_2\left(L_2^2-3\, l_2^2\right)\right)}{2\, L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)+2\, L_1\, m_2\left(L_2^2-3\, l_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)} & \frac{3\, g\, l_2\, L_1\, L_2\, m_1\, m_2\, (3\, l_1-L_1)}{2\, L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)+2\, L_1\, m_2\left(L_2^2-3\, l_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)} & 0 & 0 \\ 0 & \frac{3\, g\, L_1\, (m_1+2\, m_2)\left(L_2^2\, (m_0+m_1+m_2)-3\, l_2^2\, m_2\right)}{2\, L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)+2\, L_1\, m_2\left(L_2^2-3\, l_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)} & -\frac{9\, g\, l_2\, L_2\, m_2\, (L_1\, (m_0+m_1)-l_1\, m_1)}{2\, L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)+2\, L_1\, m_2\left(L_2^2-3\, l_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)} & 0 & 0 \\ 0 & -\frac{9\, g\, l_2\, L_1\, (m_1+2\, m_2)\, (L_1\, (m_0+m_1)-l_1\, m_1)}{2\, L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)+2\, L_1\, m_2\left(L_2^2-3\, l_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)} & \frac{3\, g\, L_2\left(-6\, l_1\, L_1\, m_1\, m_2-3\, l_1^2\, m_1^2+L_1^2\, (m_0\, (m_1+3\, m_2)+m_1\, (m_1+4\, m_2))\right)}{2\, L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)+2\, L_1\, m_2\left(L_2^2-3\, l_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)} & 0 & 0 \end{pmatrix}$$

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{L_1^2\left(L_2^2\, (m_1+3\, m_2)-9\, l_2^2\, m_2\right)}{L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)+L_1\, m_2\left(L_2^2-3\, l_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)} \\ \frac{3\, l_1\, L_2^2\, m_1+3\, L_1\, m_2\left(L_2^2-3\, l_2^2\right)}{L_1\, m_2\left(3\, l_2^2-L_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)-L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)} \\ \frac{3\, l_2\, L_1\, m_1\, (L_1-3\, l_1)}{L_1\, m_2\left(3\, l_2^2-L_2^2\right)(L_1\, (3\, m_0+4\, m_1)-6\, l_1\, m_1)-L_2^2\, m_1\left(L_1^2\, (m_0+m_1)-3\, l_1^2\, m_1\right)} \end{pmatrix}$$

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

In[ ]:=
```
Af /. par // TraditionalForm
Bf /. par // TraditionalForm
Cf /. par // TraditionalForm
Df /. par // TraditionalForm
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -7.3575 & 0.788304 & 0 & 0 & 0 \\ 0 & 73.575 & -33.1088 & 0 & 0 & 0 \\ 0 & -58.86 & 51.1521 & 0 & 0 & 0 \end{pmatrix}$$

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.607143 \\ -1.5 \\ 0.285714 \end{pmatrix}$$

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Função de Tranferência, Polos e Zeros

In[933]:=
```
(*Função de Transferência*)

s = Symbol["s"];
Gs = Cf.(Inverse[s*IdentityMatrix[6]-Af]).Bf + Df;
```

In[ ]:=
```
Gs // TraditionalForm // FullSimplify
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \dfrac{L_1 (L_2 m_1 (3 g-2 L_1 s^2)(3 g-2 L_2 s^2)-6 m_2 (L_2 (g-L_1 s^2)(2 L_2 s^2-3 g)+6 l_2^2 L_1 s^4))}{s^2 (6 g L_1 m_2^2 (L_2 (3 g-2 L_2 s^2)+6 l_2^2 s^2)+L_1 m_2 (6 l_2^2 s^2 (3 m_1 (g+4 l_1 s^2)-2 L_1 (3 m_0+4 m_1) s^2)+L_2 (2 L_2 s^2-3 g)(m_1 (4 s^2 (2 L_1-3 l_1)-9 g)-6 m_0 (g-L_1 s^2)))+L_2 m_1 (3 g-2 L_2 \cdots} \\ \dfrac{6 l_1 L_2 m_1 (2 L_2 s^2-3 g)-6 L_1 m_2 (L_2 (3 g-2 L_2 s^2)+6 l_2^2 s^2)}{-6 g L_1 m_2^2 (L_2 (3 g-2 L_2 s^2)+6 l_2^2 s^2)+L_2 m_1 (2 L_2 s^2-3 g)(3 g L_1 (m_0+m_1)+6 l_1^2 m_1 s^2-2 L_1^2 (m_0+m_1) s^2)+L_1 m_2 (L_2 (2 L_2 s^2-3 g)(m_1 (9 g+4 s^2 (3 l_1-2 L_1))+6 m_0 (g-L_1 s \cdots} \\ -\dfrac{6 l_2 L_1 (m_1 (3 g+6 l_1 s^2-2 L_1 s^2)+6 g m_2)}{-6 g L_1 m_2^2 (L_2 (3 g-2 L_2 s^2)+6 l_2^2 s^2)+L_2 m_1 (2 L_2 s^2-3 g)(3 g L_1 (m_0+m_1)+6 l_1^2 m_1 s^2-2 L_1^2 (m_0+m_1) s^2)+L_1 m_2 (L_2 (2 L_2 s^2-3 g)(m_1 (9 g+4 s^2 (3 l_1-2 L_1))+6 m_0 (g-L_1 \cdots} \end{pmatrix}$$

In[ ]:=
```
Gs /. par // TraditionalForm // FullSimplify
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \dfrac{0.607143 s^4-64.4657 s^2+659.905}{s^6-124.727 s^4+1814.74 s^2} \\ \dfrac{67.2686-1.5 s^2}{s^4-124.727 s^2+1814.74} \\ \dfrac{0.285714 s^2+67.2686}{s^4-124.727 s^2+1814.74} \end{pmatrix}$$

In[935]:=

```
(*Polos do sistema → Denominadores == 0*)

Polo1 = Solve[Gs[[1,1]]^(-1) == 0, s];
Polo2 = Solve[Gs[[2,1]]^(-1) == 0, s];
Polo3 = Solve[Gs[[3,1]]^(-1) == 0, s];

PValor1 = s/.Polo1;
PValor2 = s/.Polo2;
PValor3 = s/.Polo3;

PoloLista = {{PValor1}, {PValor2}, {PValor3}};
```

In[∘]:= **PoloLista // TraditionalForm // FullSimplify**

Out[∘]//TraditionalForm=

$$\left(\begin{array}{l} \left\{0,\, 0,\, -\frac{1}{2}\sqrt{3}\sqrt{\frac{g\left(-L_2\left(m_0\left(m_1+3m_2\right)+m_1\left(m_1+4m_2\right)\right)L_1^2+\left(3m_2\left(m_1+2m_2\right)l_2^2+6l_1L_2m_1m_2-L_2^2\left(m_0+m_1+m_2\right)\left(m_1+2m_2\right)\right)L_1+3l_1^2L_2m_1^2\right)+\sqrt{g^2\left(\left(L_2\left(m_0\left(m_1+3m\right.\right.\right.\right.}}{L_1}}\right. \\ \left\{-\frac{1}{2}\sqrt{3}\sqrt{\frac{g\left(-L_2\left(m_0\left(m_1+3m_2\right)+m_1\left(m_1+4m_2\right)\right)L_1^2+\left(3m_2\left(m_1+2m_2\right)l_2^2+6l_1L_2m_1m_2-L_2^2\left(m_0+m_1+m_2\right)\left(m_1+2m_2\right)\right)L_1+3l_1^2L_2m_1^2\right)+\sqrt{g^2\left(\left(L_2\left(m_0\left(m_1+3m_2\right)+\right.\right.\right.\right.}}{L_1\left(3l\right.}}\right. \\ \left\{-\frac{1}{2}\sqrt{3}\sqrt{\frac{g\left(-L_2\left(m_0\left(m_1+3m_2\right)+m_1\left(m_1+4m_2\right)\right)L_1^2+\left(3m_2\left(m_1+2m_2\right)l_2^2+6l_1L_2m_1m_2-L_2^2\left(m_0+m_1+m_2\right)\left(m_1+2m_2\right)\right)L_1+3l_1^2L_2m_1^2\right)+\sqrt{g^2\left(\left(L_2\left(m_0\left(m_1+3m_2\right)+\right.\right.\right.\right.}}{L_1\left(3l\right.}}\right. \end{array}\right.$$

In[∘]:= **PoloLista /. par // TraditionalForm // FullSimplify**

Out[∘]//TraditionalForm=

$$\left(\begin{array}{l} \{0,\, 0,\, -4.10087,\, 4.10087,\, -10.388,\, 10.388\} \\ \{-4.10087,\, 4.10087,\, -10.388,\, 10.388\} \\ \{-4.10087,\, 4.10087,\, -10.388,\, 10.388\} \end{array}\right)$$

In[942]:=

```
(*Zeros do sistema → Numeradores == 0*)

Zero1 = Solve[Gs[[1,1]] == 0, s];
Zero2 = Solve[Gs[[2,1]] == 0, s];
Zero3 = Solve[Gs[[3,1]] == 0, s];

ZValor1 = s/.Zero1;
ZValor2 = s/.Zero2;
ZValor3 = s/.Zero3;

ZeroMatriz = {{ZValor1}, {ZValor2}, {ZValor3}};
```

In[∘]:= **ZeroMatriz // FullSimplify // TraditionalForm**

Out[∘]//TraditionalForm=

$$\left(\left\{-\frac{1}{2}\sqrt{-\frac{3\sqrt{g^2L_2\left(-2L_1\left(m_1+2m_2\right)\left(L_2^2\left(m_1+3m_2\right)-18l_2^2m_2\right)+L_2^3\left(m_1+2m_2\right)^2+L_1^2L_2\left(m_1+3m_2\right)^2\right)}-3gL_2\left(L_2\left(m_1+2m_2\right)+L_1\left(m_1+3m_2\right)\right)}{L_1\left(L_2^2\left(m_1+3m_2\right)-9l_2^2m_2\right)}},\, \frac{1}{2}\sqrt{-\frac{3\sqrt{g^2L_2\left(-2L\right.}}{}}\right.\right.$$

In[•]:= ZeroMatriz /. par // TraditionalForm // FullSimplify

Out[•]//TraditionalForm=

$$\begin{pmatrix} \{-3.38779, 3.38779, -9.73148, 9.73148\} \\ \{-6.69669, 6.69669\} \\ \{0. - 15.3441\, i, 0. + 15.3441\, i\} \end{pmatrix}$$

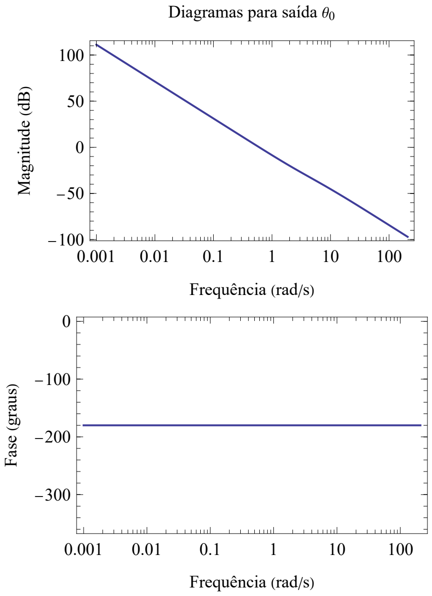
Resposta em frequência

In[949]:=

```
(*Função de Transferência com parâmetros substituídos*)

Gpar = Simplify[Gs /.par];
Gpar1 = Gs[[1,1]]/.par;
Gpar2 = Gs[[2,1]]/.par;
Gpar3 = Gs[[3,1]]/.par;

(*Diagramas de Bode*)

BodeG1 = BodePlot[Gpar1,PlotTheme→"Classic",
FrameLabel→
{{"Frequência (rad/s)","Magnitude (dB)","Diagramas para saída θ0"},
{"Frequência (rad/s)","Fase (graus)"}},
GridLines → Automatic];

BodeG2 = BodePlot[Gpar2, PlotTheme→"Classic",
FrameLabel→
{{"Frequência (rad/s)","Magnitude (dB)","Diagramas para saída θ1"},
{"Frequência (rad/s)","Fase (graus)"}},
GridLines → Automatic];

BodeG3 = BodePlot[Gpar3, PlotTheme→"Classic",
FrameLabel→
{{"Frequência (rad/s)","Magnitude (dB)","Diagramas para saída θ2"},
{"Frequência (rad/s)","Fase (graus)"}},
GridLines → Automatic];
```

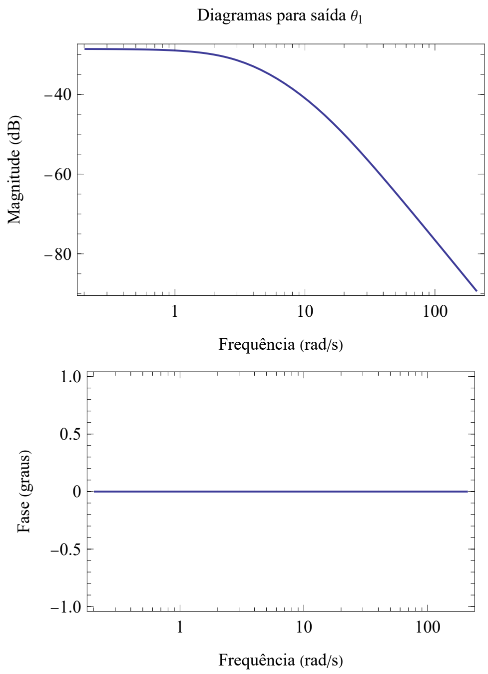
In[ ]:= **BodeG$_1$**

Out[ ]=



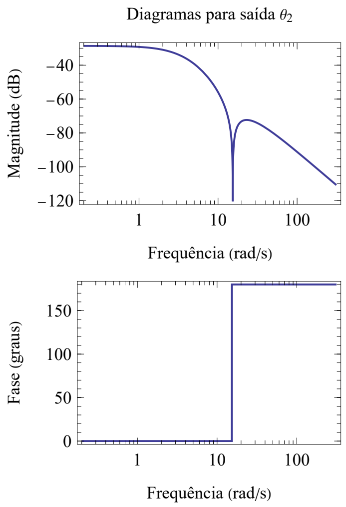
In[ ]:= **BodeG$_2$**

Out[ ]=



In[ ]:= BodeG$_3$

Out[ ]=



Resposta no Tempo

In[956]:=

```
(*Preparação para o OutputResponse*)

Gtfm = TransferFunctionModel[Gpar, s];
Gee = StateSpaceModel[Gtfm];
```

In[ ]:= G_{tfm}

$$\begin{pmatrix} \dfrac{659.905 - 64.4657\, s^2 + 0.607143\, s^4}{1814.74\, s^2 - 124.727\, s^4 + s^6} \\ \dfrac{67.2686 - 1.5\, s^2}{1814.74 - 124.727\, s^2 + s^4} \\ \dfrac{67.2686 + 0.285714\, s^2}{1814.74 - 124.727\, s^2 + s^4} \end{pmatrix}^{\mathcal{T}}$$

In[ ]:= G_{ee}

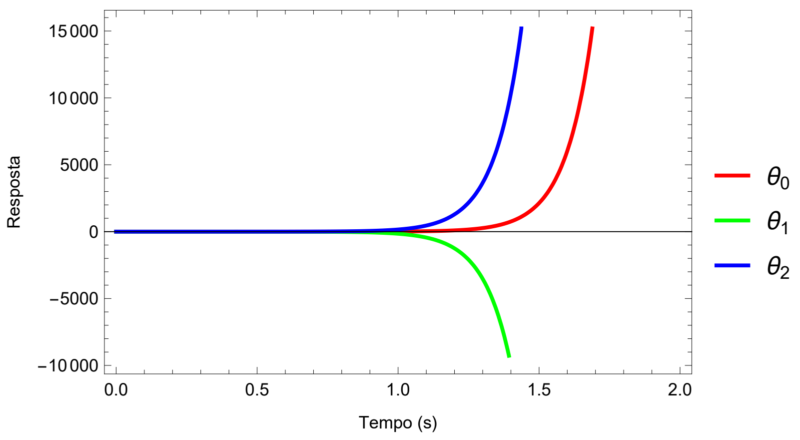
Out[ ]=

$$\left(\begin{array}{cccccc|c} 0 & 1. & 0. & 0. & 0. & 0. & 0 \\ 0 & 0. & 1. & 0. & 0. & 0. & 0 \\ 0 & 0. & 0. & 1. & 0. & 0. & 0 \\ 0 & 0. & 0. & 0. & 1. & 0. & 0 \\ 0 & 0. & 0. & 0. & 0. & 1. & 0 \\ 0. & 0. & -1814.74 & 0. & 124.727 & 0. & 1. \\ \hline 659.905 & 0. & -64.4657 & 0. & 0.607143 & 0 & 0 \\ -1.7807\times 10^{-12} & 0. & 67.2686 & 0. & -1.5 & 0 & 0 \\ -1.7807\times 10^{-12} & 0. & 67.2686 & 0. & 0.285714 & 0 & 0 \end{array}\right)^{\mathcal{S}}$$

In[958]:=

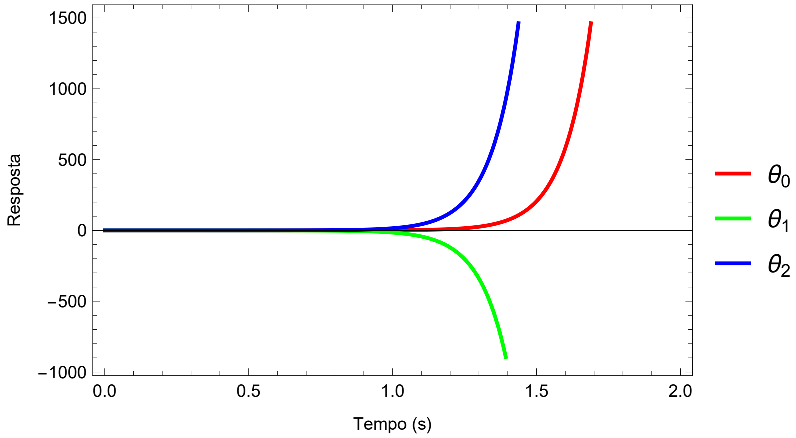
```
(*Resposta a degrau unitário*)

Resp_unit = OutputResponse[G_ee, UnitStep[t] ,{t,0,2}];

Pl_unit = Plot[Evaluate[Resp_unit], {t, 0, 2},
 PlotStyle → {Red,Green,Blue,Thick, Dashed},
 Frame → True, FrameLabel → {"Tempo (s)", "Resposta"},
 PlotLegends→{"θ0","θ1","θ2"},
 GridLines → Automatic];

 (*Resposta a rampa*)

 Resp_rampa = OutputResponse[G_ee, Ramp[t] ,{t,0,2}];

 Pl_rampa = Plot[Evaluate[Resp_rampa], {t, 0, 2},
 PlotStyle → {Red,Green,Blue,Thick, Dashed},
 Frame → True, FrameLabel → {"Tempo (s)", "Resposta"},
 PlotLegends→{"θ0","θ1","θ2"},
 GridLines → Automatic];

 (*Resposta a impulso*)
  Resp_Dirac = OutputResponse[G_ee, DiracDelta[t] ,{t,0,2}];

 Pl_Dirac = Plot[Evaluate[Resp_Dirac], {t, 0, 2},
 PlotStyle → {Red,Green,Blue,Thick, Dashed},
 Frame → True, FrameLabel → {"Tempo (s)", "Resposta"},
 PlotLegends→{"θ0","θ1","θ2"},
 GridLines → Automatic];

  (*Resposta a senoide*)
  Resp_seno = OutputResponse[G_ee, Sin[10t] ,{t,0,2}];

 Pl_seno = Plot[Evaluate[Resp_seno], {t, 0, 2},
 PlotStyle → {Red,Green,Blue,Thick, Dashed},
 Frame → True, FrameLabel → {"Tempo (s)", "Resposta"},
 PlotLegends→{"θ0","θ1","θ2"},
 GridLines → Automatic];
```

In[ ]:= `Pl_unit`



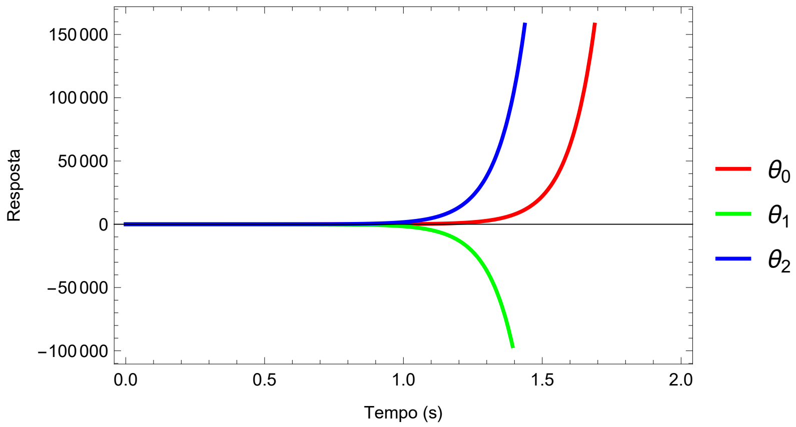
In[∘]:= Pl_{rampa}

Out[∘]=



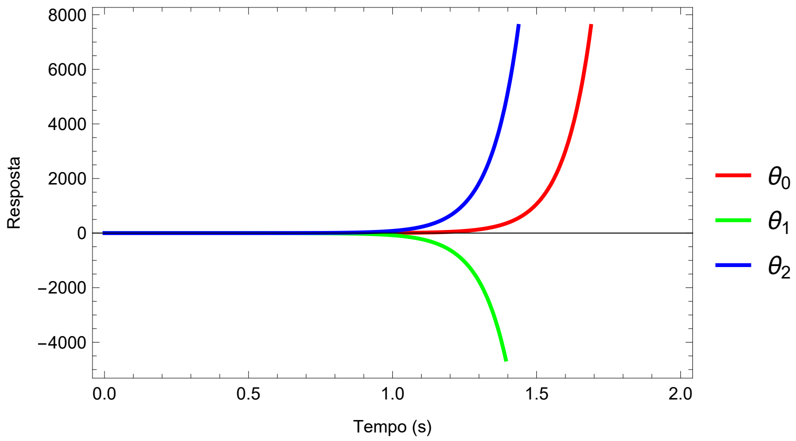
In[∘]:= Pl_{Dirac}

Out[∘]=



In[∘]:= Pl_{seno}

Out[∘]=



Simulação Numérica

In[966]:=

```
ClearAll[u]

(*Força de controle*)
u = 0;

(*Equações Diferenciais*)
ordem₁ = { Lag⟦1⟧, Lag⟦2⟧, Lag⟦3⟧} /. par;

(*Condições Iniciais*)
Ini₁ = {θ₀[0] == 7, θ₁[0] == 0, θ₂[0] == 0,
        θ₀'[0] == 0, θ₁'[0] == 0, θ₂'[0] == 0};

(*Integração Numérica - método stiff ou BDF*)
sol₁ = NDSolve[{ordem₁,Ini₁},{θ₀[t],θ₁[t],θ₂[t],θ₀'[t],θ₁'[t],θ₂'[t]},{t,0,10},Method → "BDF"

Pl₁ = Plot[
   Evaluate[{θ₀[t], θ₁[t], θ₂[t]} /. sol₁ /. t → t],
   {t, 0, 10},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Equilíbrio (sistema não-linear)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
```

In[972]:=

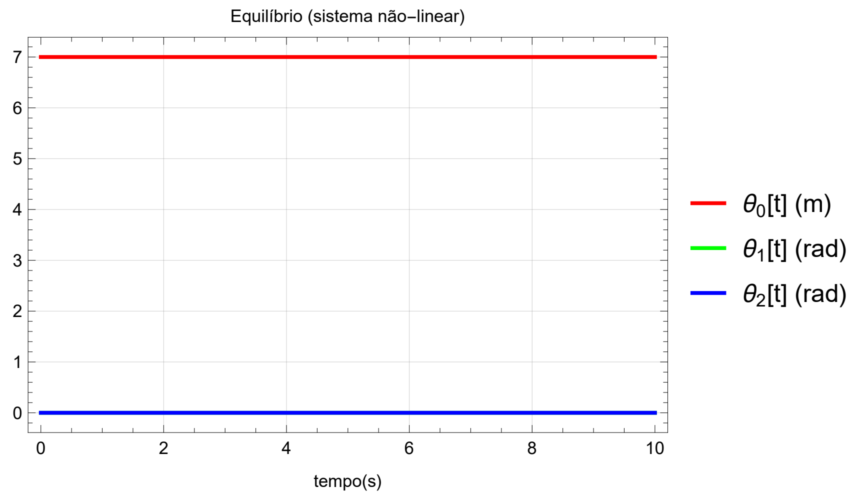
```
ClearAll[u]

x[t_] = {{x1[t]}, {x2[t]},{x3[t]},{x4[t]},{x5[t]},{x6[t]}};
u[t_] = 0;

EspaEstados = x'[t] == Af.x[t] + Bf*u[t] /.par ;

EspaEstadosmod = {
EspaEstados[[1,1]] == EspaEstados[[2,1]],
EspaEstados[[1,2]] == EspaEstados[[2,2]],
EspaEstados[[1,3]] == EspaEstados[[2,3]],
EspaEstados[[1,4]] == EspaEstados[[2,4]],
EspaEstados[[1,5]] == EspaEstados[[2,5]],
EspaEstados[[1,6]] == EspaEstados[[2,6]]
};

Inilin1 = x[0] == {7,0,0,0,0,0};

sollin1 = NDSolve[{EspaEstadosmod,Inilin1},{x1[t],x2[t],x3[t],x4[t],x5[t],x6[t]},{t,0,10},Method

Pllin1 = Plot[
   Evaluate[{x1[t], x2[t], x3[t]} /. sollin1 /. t → t],
   {t, 0, 10},
   PlotLegends → {"θ0[t] (m)", "θ1[t] (rad)", "θ2[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Equilíbrio (sistema não-linear)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
```

In[461]:=

In[462]:=

```
Pl1
```

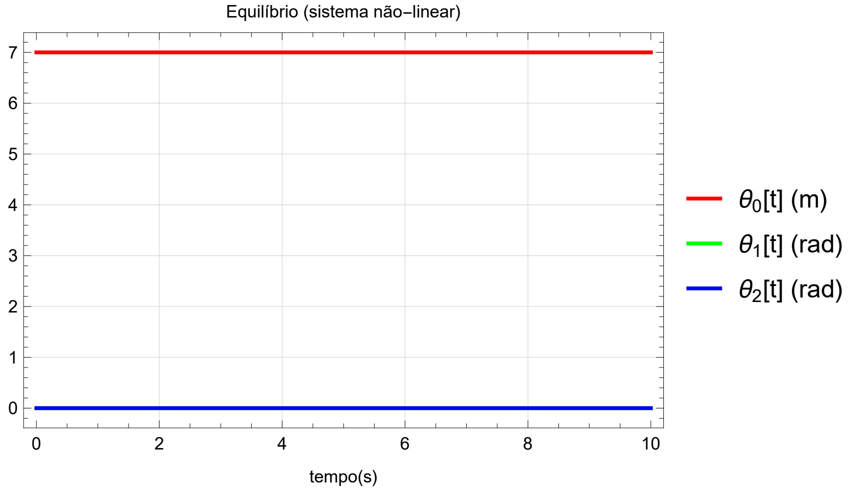
Out[462]=



In[463]:=

```
Pl_lin1
```

Out[463]=



In[980]:=

```
ClearAll[u]

u = 0;

ordem₂ = { Lag⟦1⟧, Lag⟦2⟧, Lag⟦3⟧} /. par;

Ini₂ = {θ₀[0] == 0, θ₁[0] == π/180, θ₂[0] == 0,
        θ₀'[0] == 0, θ₁'[0] == 0, θ₂'[0] == 0};

sol₂ = NDSolve[{ordem₂,Ini₂},{θ₀[t],θ₁[t],θ₂[t],θ₀'[t],θ₁'[t],θ₂'[t]},{t,0,10},Method → "BDF"

Pl₂ = Plot[
   Evaluate[{θ₀[t], θ₁[t], θ₂[t]} /. sol₂ /. t → t],
   {t, 0, 0.5},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Pequena perturbação em θ₁ (sistema não-linear)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];

Pl₂ex = Plot[
   Evaluate[{θ₀[t], θ₁[t], θ₂[t]} /. sol₂ /. t → t],
   {t, 0, 5},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Pequena perturbação em θ₁ (sistema não-linear)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
```

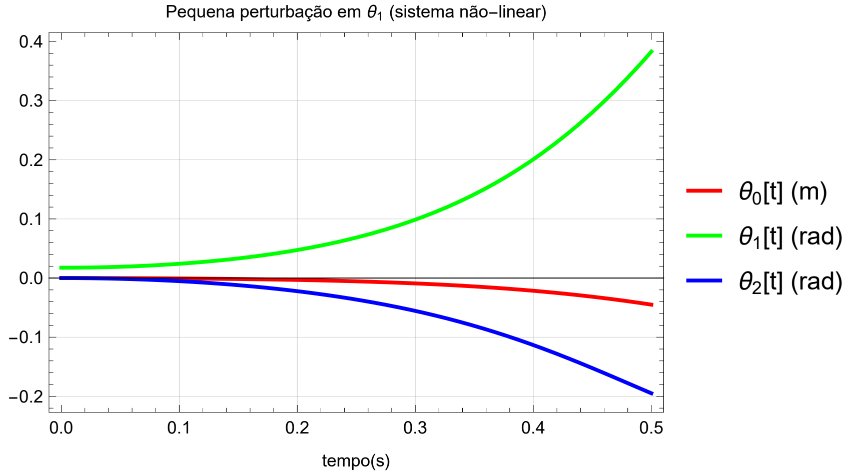
In[987]:=

```
ClearAll[u]

x[t_] = {{x₁[t]}, {x₂[t]},{x₃[t]},{x₄[t]},{x₅[t]},{x₆[t]}};
u[t_] = 0;

EspaEstados = x'[t] == Af.x[t] + Bf*u[t] /.par ;

EspaEstadosmod = {
EspaEstados[[1,1]] == EspaEstados[[2,1]],
EspaEstados[[1,2]] == EspaEstados[[2,2]],
EspaEstados[[1,3]] == EspaEstados[[2,3]],
EspaEstados[[1,4]] == EspaEstados[[2,4]],
EspaEstados[[1,5]] == EspaEstados[[2,5]],
EspaEstados[[1,6]] == EspaEstados[[2,6]]
};

Inilin2 = x[0] == {0,π/180,0,0,0,0};

sollin2 = NDSolve[{EspaEstadosmod,Inilin2},{x₁[t],x₂[t],x₃[t],x₄[t],x₅[t],x₆[t]},{t,0,10},Method

Pllin2 = Plot[
   Evaluate[{x₁[t], x₂[t], x₃[t]} /. sollin2 /. t → t],
   {t, 0, 0.5},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Pequena perturbação em θ₁ (sistema linearizado)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
Pllin2ex = Plot[
   Evaluate[{x₁[t], x₂[t], x₃[t]} /. sollin2 /. t → t],
   {t, 0, 3},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Pequena perturbação em θ₁ (sistema linearizado)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
```

In[873]:=

```
Pl2
```

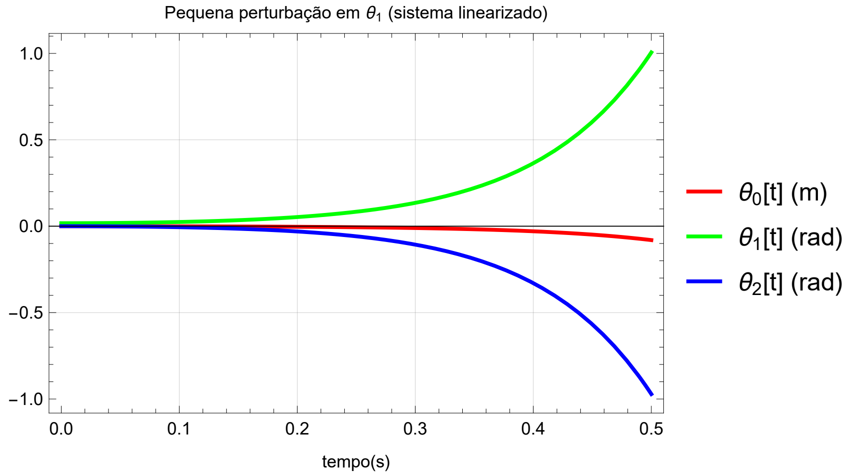
Out[873]=



In[1025]:=

```
Pllin2
```

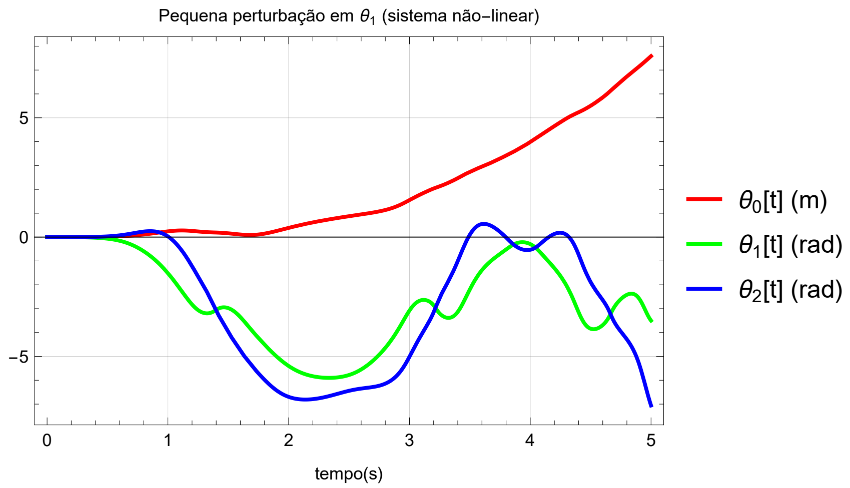
Out[1025]=



In[482]:=

```
Pl2 ex
```

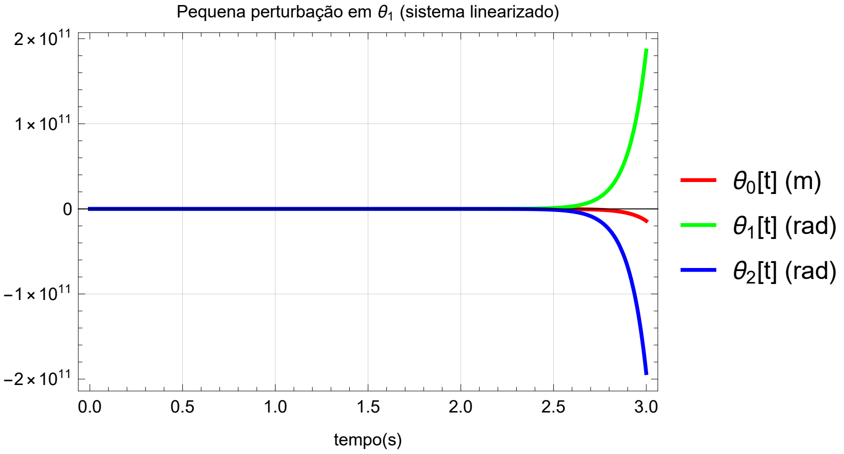
Out[482]=



In[483]:=

```
Pl_lin2ex
```

Out[483]=



In[996]:=

```
ClearAll[u]

u = -10θ₁[t];;

ordem₃ = { Lag⟦1⟧, Lag⟦2⟧, Lag⟦3⟧} /. par;

Ini₃ = {θ₀[0] == 0, θ₁[0] == π/180, θ₂[0] == 0,
        θ₀'[0] == 0, θ₁'[0] == 0, θ₂'[0] == 0};

sol₃ = NDSolve[{ordem₃,Ini₃},{θ₀[t],θ₁[t],θ₂[t],θ₀'[t],θ₁'[t],θ₂'[t]},{t,0,10},Method → "BDF"

Pl₃ = Plot[
   Evaluate[{θ₀[t], θ₁[t], θ₂[t]} /. sol₃ /. t → t],
   {t, 0, 0.5},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Força de entrada u = -10θ₁ (sistema não-linear)"},
   PlotStyle→{Red,Green,Blue},
      GridLines → Automatic
];

Pl₃ex = Plot[
   Evaluate[{θ₀[t], θ₁[t], θ₂[t]} /. sol₃ /. t → t],
   {t, 0, 3},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Força de entrada u = -10θ₁ (sistema não-linear)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
```

In[491]:=

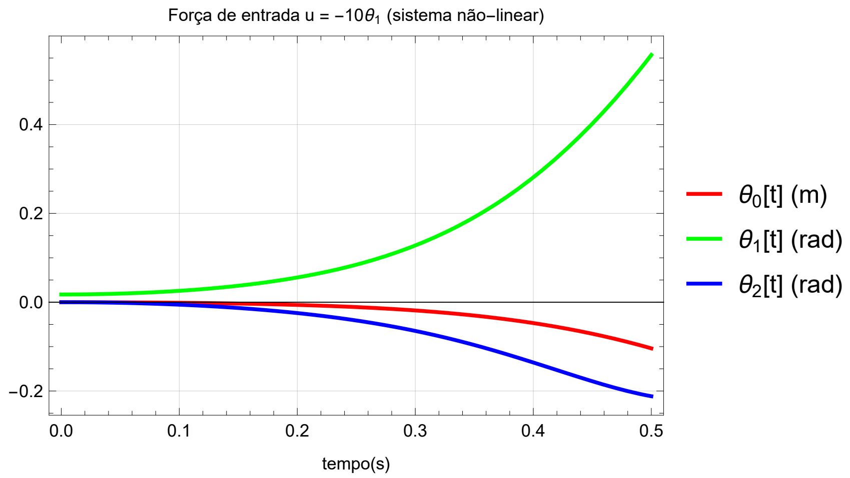
In[1003]:=

```
ClearAll[u]

x[t_] = {{x₁[t]}, {x₂[t]},{x₃[t]},{x₄[t]},{x₅[t]},{x₆[t]}};
u[t_] = -10x₂[t];

EspaEstados = x'[t] == Af.x[t] + Bf*u[t] /.par ;

EspaEstadosmod = {
EspaEstados[[1,1]] == EspaEstados[[2,1]],
EspaEstados[[1,2]] == EspaEstados[[2,2]],
EspaEstados[[1,3]] == EspaEstados[[2,3]],
EspaEstados[[1,4]] == EspaEstados[[2,4]],
EspaEstados[[1,5]] == EspaEstados[[2,5]],
EspaEstados[[1,6]] == EspaEstados[[2,6]]
};

Inilin3 = x[0] == {0,π/180,0,0,0,0};

sollin3 = NDSolve[{EspaEstadosmod,Inilin3},{x₁[t],x₂[t],x₃[t],x₄[t],x₅[t],x₆[t]},{t,0,10},Method

Pllin3 = Plot[
   Evaluate[{x₁[t], x₂[t], x₃[t]} /. sollin3 /. t → t],
   {t, 0, 0.5},
   PlotLegends → {"θ0[t] (m)", "θ1[t] (rad)", "θ2[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","Força de entrada u = -10θ1 (sistema linearizado)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
```

In[889]:=

```
Pl3
```

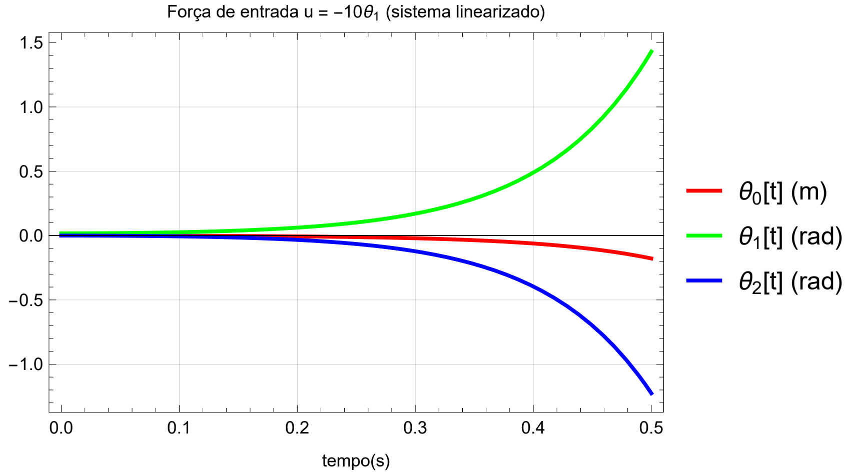
Out[889]=



In[890]:=

Pl_{lin3}

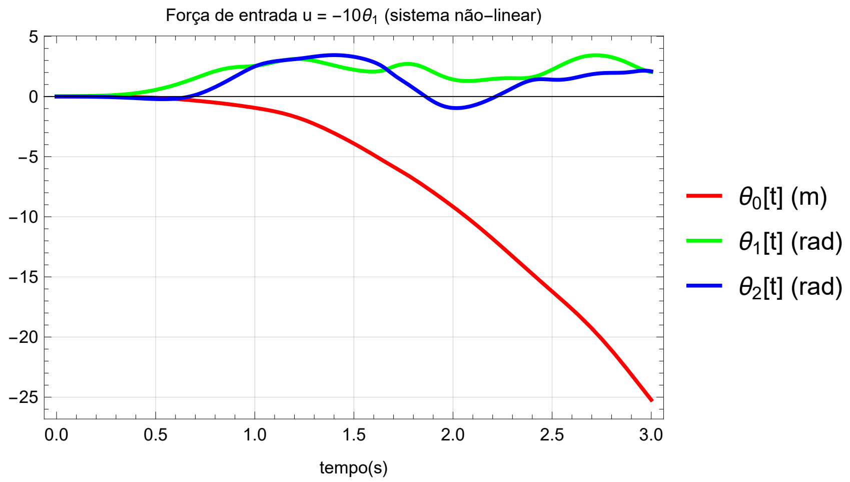
Out[890]=



In[802]:=

$Pl_{3\,ex}$

Out[802]=



In[1011]:=

```
ClearAll[u]

u = 0;

ordem4 = { Lag⟦1⟧, Lag⟦2⟧, Lag⟦3⟧} /. par;

Ini4 = {θ0[0] == 0, θ1[0] == 0, θ2[0] == 0,
        θ0'[0] == 0, θ1'[0] == 0, θ2'[0] == π/180};

sol4 = NDSolve[{ordem4,Ini4},{θ0[t],θ1[t],θ2[t],θ0'[t],θ1'[t],θ2'[t]},{t,0,10},Method → "BDF"

Pl4 = Plot[
   Evaluate[{θ0[t], θ1[t], θ2[t]} /. sol4 /. t → t],
   {t, 0, 0.5},
   PlotLegends → {"θ0[t] (m)", "θ1[t] (rad)", "θ2[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","θ2'[0] ≠ 0 (sistema não-linear)"},
   PlotStyle→{Red,Green,Blue},
      GridLines → Automatic
];
```

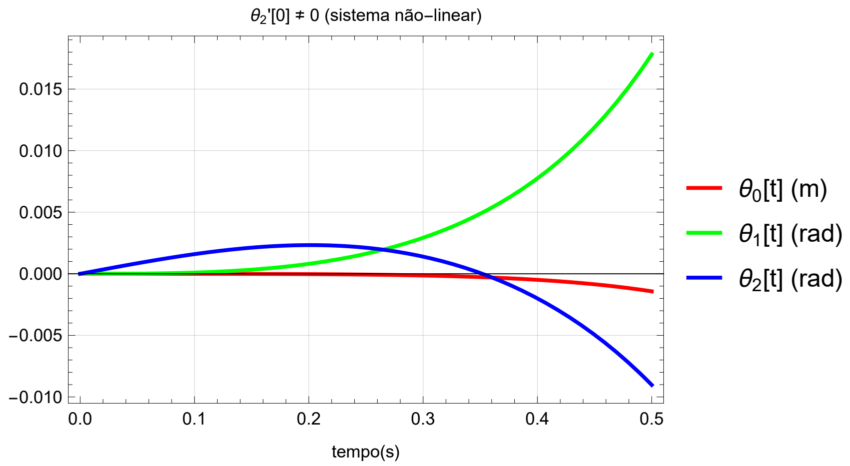
In[1017]:=

```
ClearAll[u]

x[t_] = {{x₁[t]}, {x₂[t]},{x₃[t]},{x₄[t]},{x₅[t]},{x₆[t]}};
u[t_] = 0;

EspaEstados = x'[t] == Af.x[t] + Bf*u[t] /.par ;

EspaEstadosmod = {
EspaEstados[[1,1]] == EspaEstados[[2,1]],
EspaEstados[[1,2]] == EspaEstados[[2,2]],
EspaEstados[[1,3]] == EspaEstados[[2,3]],
EspaEstados[[1,4]] == EspaEstados[[2,4]],
EspaEstados[[1,5]] == EspaEstados[[2,5]],
EspaEstados[[1,6]] == EspaEstados[[2,6]]
};

Inilin4 = x[0] == {0,0,0,0,0,π/180};

sollin4 = NDSolve[{EspaEstadosmod,Inilin4},{x₁[t],x₂[t],x₃[t],x₄[t],x₅[t],x₆[t]},{t,0,10},Method

Pllin4 = Plot[
   Evaluate[{x₁[t], x₂[t], x₃[t]} /. sollin4 /. t → t],
   {t, 0, 0.5},
   PlotLegends → {"θ₀[t] (m)", "θ₁[t] (rad)", "θ₂[t] (rad)"},
   PlotRange → All,
    Frame → True,
    FrameLabel → {"tempo(s)","","θ₂'[0] ≠ 0 (sistema linearizado)"},
   PlotStyle→{Red,Green,Blue},
   GridLines → Automatic
];
```

In[841]:=

```
Pl4
```

Out[841]=



In[842]:=

```
Pl_lin4
```

Out[842]=