• P Probability Dying

$P_1 = \frac{1}{3} + \frac{2}{3} P_2$

$P_2 = \frac{1}{3} P_1 + \frac{2}{3} P_3 = \frac{1}{9} + \frac{2}{9} P_2 + \frac{2}{3} P_3 \rightarrow P_2 = \frac{1}{7} + \frac{6}{7} P_3$

$P_3 = \frac{1}{3} P_2 + \frac{2}{3} P_4 = \frac{1}{21} + \frac{2}{7} P_3 + \frac{2}{3} P_4 \rightarrow P_3 = \frac{1}{15} + \frac{14}{15} P_4$

$\vdots$

$P_{n-1} = \frac{1}{2^n - 1} + \frac{(2^n - 2)}{2^n - 1} \cdot P_n = \frac{1 + (2^n - 2) P_n}{2^n - 1}$

$\rightarrow P_1 = \frac{1}{3} + \frac{2}{3}\left[\frac{1}{7} + \frac{6}{7}\left[\frac{1}{15} + \frac{14}{15}\left[\cdots \right]\right]\right]$

$P_{1000} = \frac{1}{3} P_{999}$

$P_{999} = \frac{1}{3} P_{998} + \frac{2}{3} P_{1000} = \frac{1}{3} P_{998} + \frac{2}{9} P_{999} \rightarrow P_{999} = \frac{3}{7} P_{998}$

$P_{998} = \frac{1}{3} P_{997} + \frac{2}{3} P_{999} = \frac{1}{3} P_{997} + \frac{2}{7} P_{998} \rightarrow P_{998} = \frac{7}{15} P_{997}$

$P_n = \frac{(2^{1000-n+1} - 1)}{(2^{1000-n+2} - 1)} P_{n-1}$

$P_2 = \frac{(2^{999} - 1)}{(2^{1000} - 1)} P_1 \approx \frac{P_1}{2}$

$P_1 = \frac{1}{3} + \frac{\cancel{2}}{3} \frac{P_1}{\cancel{2}} \rightarrow P_1 = \underline{1/2}$

• P Probability Surviving

$P_1 = \frac{2}{3} P_2$

$P_2 = \frac{2}{3} P_3 + \frac{1}{3} P_1 = \frac{2}{3} P_3 + \frac{2}{9} P_2 \rightarrow P_2 = \frac{6}{7} P_3$

$P_3 = \frac{2}{3} P_4 + \frac{1}{3} P_2 = \frac{2}{3} P_4 + \frac{2}{7} P_3 \rightarrow P_3 = \frac{14}{15} P_4$

$\vdots$

$P_{n-1} = \frac{(2^n - 2)}{(2^n - 1)} P_n \rightarrow P_1 = \overbrace{\frac{2}{\cancel{3}} \frac{\cancel{6}}{\cancel{7}} \cdot \frac{\cancel{14}}{\cancel{15}} \cdots \frac{(\cancel{2^{999}} - 2)}{2^{1000} - 1}}^{2^{999}} P_{1000}$

$P_1 = \frac{P_{1000}}{2}$

$P_{1000} = \frac{2}{3} + \frac{1}{3} P_{999}$

$= \frac{2}{3} + \frac{1}{3} \frac{(\cancel{2^{1000}} - 2)}{(\cancel{2^{1000}} - 1)} P_{1000} \rightarrow P_{1000} = 1$

$\rightarrow P_1 = \underline{1/2}$