

Lab4

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Problema 1

1.

$$\begin{aligned} T(n) &= T(n-1) + n: \\ &< c(n-1)^2 + n \\ &< c(n^2 - 2n + 1) + n \\ &< cn^2 - 2cn + c + n \\ &< cn^2 - n(2c - 1) + c \\ &< cn^2 \end{aligned}$$

2.

$$\begin{aligned} T(n) &= T(n/2) + 1: \\ T(n/2) &= [T(n/2^2) + 1] + 1 \\ T(n/2) &= [T(n/2^2) + 2] \\ T(n/2) &= [T(n/2^3)] + 2 \\ T(n/2) &= [T(n/2^3) + 3] \end{aligned}$$

$$T(n/2^i) + i$$

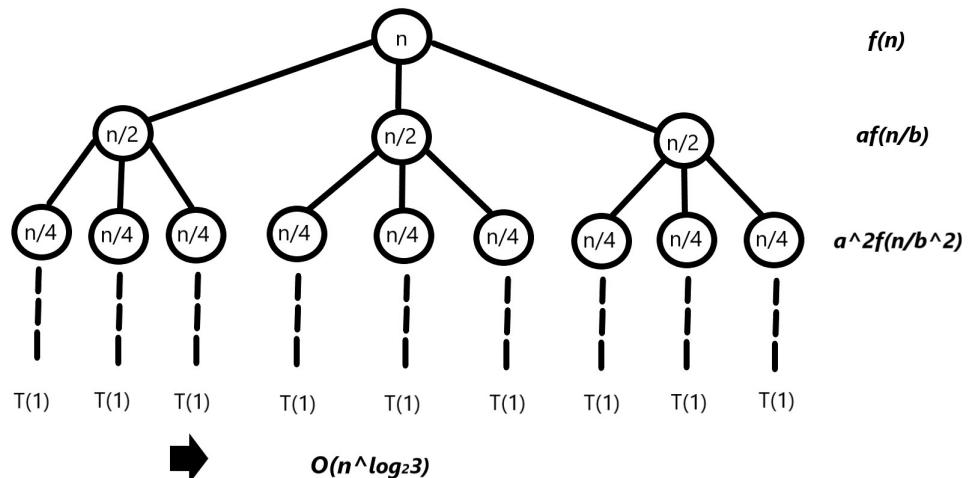
$$\begin{aligned} T(n) &= T(n/2^k) + k \\ n/2^k &= 1 \\ n &= 2^k \\ k &= \log n \end{aligned}$$

$$T(n) = T(1) + \log n$$

$$O(\log n)$$

Problema 2

$$T(n) = 3T(n/2) + n;$$



Problema 3

Master Method:

$$a < b^d \longrightarrow T(n) \in O(n^d)$$

$$a = b^d \longrightarrow T(n) \in O(n^d \log n)$$

$$a > b^d \longrightarrow T(n) \in O(n^{\log_b a})$$

1.

$$T(n) = 2T(n/4) + 1$$

$$a=2$$

$$b=4$$

$$d=0$$

$$2 > 1 \longrightarrow O(n^{\log_4 2})$$

2.

$$T(n) = 2T(n/4) + \sqrt{n}$$

$$a=2$$

$$b=4$$

$$d=1/2$$

$$2 = 1 \longrightarrow O(\sqrt{n} \log n)$$

3.

$$T(n) = 2T(n/4) + n$$

a=2
b=4
d=1

$$2 < 4 \longrightarrow O(n)$$

4.

$$T(n) = 2T(n/4) + n^2$$

a=2
b=4
d=2

$$2 < 16 \longrightarrow O(n^2)$$