

Three Body Isospin Eigenstates

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The three body isospin combinations $I_A \otimes I_B \otimes I_C$ that have definite total I , I_3 and I_{AB} , the isospin of the first two particles subsystem, are presented in the following pages. The operators I^2 and I_{AB}^2 have been simultaneously diagonalized to obtain the desired results. The notation is selfexplanatory and coincides with the one used in the *Hadron Spectroscopy in Lattice QCD* notes by Colin Morningstar, with the addition of a third particle, C . As in such reference, conventional phases have been ignored.

The results have been obtained thanks to Mathematica's symbolic diagonalization. Find the code at github.com/FernandoAlvarado1996/Three_Isospin_combination/tree/main .

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$|I \ I_3 \ I_{AB}\rangle$ states from $1 \otimes 1 \otimes 1$:

I	I_3	I_{AB}	
3	3	2	$\bar{A}_1 \bar{B}_1 \bar{C}_1$
3	2	2	$\frac{\bar{A}_1 \bar{B}_1 \bar{C}_0}{\sqrt{3}} + \frac{\bar{A}_1 \bar{B}_0 \bar{C}_1}{\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_1}{\sqrt{3}}$
3	1	2	$\frac{\bar{A}_1 \bar{B}_1 \bar{C}_{-1}}{\sqrt{15}} + \frac{2\bar{A}_1 \bar{B}_0 \bar{C}_0}{\sqrt{15}} + \frac{2\bar{A}_0 \bar{B}_1 \bar{C}_0}{\sqrt{15}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_1}{\sqrt{15}} + \frac{2\bar{A}_0 \bar{B}_0 \bar{C}_1}{\sqrt{15}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_1}{\sqrt{15}}$
3	0	2	$\frac{\bar{A}_1 \bar{B}_0 \bar{C}_{-1}}{\sqrt{10}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{-1}}{\sqrt{10}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_0}{\sqrt{10}} + \sqrt{\frac{2}{5}} \bar{A}_0 \bar{B}_0 \bar{C}_0 + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_0}{\sqrt{10}} + \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_1}{\sqrt{10}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_1}{\sqrt{10}}$
3	-1	2	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-1}}{\sqrt{15}} + \frac{2\bar{A}_0 \bar{B}_0 \bar{C}_{-1}}{\sqrt{15}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-1}}{\sqrt{15}} + \frac{2\bar{A}_0 \bar{B}_{-1} \bar{C}_0}{\sqrt{15}} + \frac{2\bar{A}_{-1} \bar{B}_0 \bar{C}_0}{\sqrt{15}} + \frac{\bar{A}_{-1} \bar{B}_{-1} \bar{C}_1}{\sqrt{15}}$
3	-2	2	$\frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{-1}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{-1}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_{-1} \bar{C}_0}{\sqrt{3}}$
3	-3	2	$\bar{A}_{-1} \bar{B}_{-1} \bar{C}_{-1}$
2	2	2	$-\sqrt{\frac{2}{3}} \bar{A}_1 \bar{B}_1 \bar{C}_0 + \frac{\bar{A}_1 \bar{B}_0 \bar{C}_1}{\sqrt{6}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_1}{\sqrt{6}}$
2	2	1	$\frac{\bar{A}_0 \bar{B}_1 \bar{C}_1}{\sqrt{2}} - \frac{\bar{A}_1 \bar{B}_0 \bar{C}_1}{\sqrt{2}}$
2	1	2	$-\frac{\bar{A}_1 \bar{B}_1 \bar{C}_{-1}}{\sqrt{3}} - \frac{\bar{A}_1 \bar{B}_0 \bar{C}_0}{2\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_1 \bar{C}_0}{2\sqrt{3}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_1}{2\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_0 \bar{C}_1}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_1}{2\sqrt{3}}$
2	1	1	$-\frac{1}{2} \bar{A}_1 \bar{B}_0 \bar{C}_0 + \frac{1}{2} \bar{A}_0 \bar{B}_1 \bar{C}_0 - \frac{1}{2} \bar{A}_1 \bar{B}_{-1} \bar{C}_1 + \frac{1}{2} \bar{A}_{-1} \bar{B}_1 \bar{C}_1$
2	0	2	$-\frac{1}{2} \bar{A}_1 \bar{B}_0 \bar{C}_{-1} - \frac{1}{2} \bar{A}_0 \bar{B}_1 \bar{C}_{-1} + \frac{1}{2} \bar{A}_0 \bar{B}_{-1} \bar{C}_1 + \frac{1}{2} \bar{A}_{-1} \bar{B}_0 \bar{C}_1$
2	0	1	$-\frac{\bar{A}_1 \bar{B}_0 \bar{C}_{-1}}{2\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{-1}}{2\sqrt{3}} - \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_0}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_0}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_1}{2\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_1}{2\sqrt{3}}$
2	-1	2	$-\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-1}}{2\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_0 \bar{C}_{-1}}{\sqrt{3}} - \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-1}}{2\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_0}{2\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_0}{2\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_{-1} \bar{C}_1}{\sqrt{3}}$
2	-1	1	$-\frac{1}{2} \bar{A}_1 \bar{B}_{-1} \bar{C}_{-1} + \frac{1}{2} \bar{A}_{-1} \bar{B}_1 \bar{C}_{-1} - \frac{1}{2} \bar{A}_0 \bar{B}_{-1} \bar{C}_0 + \frac{1}{2} \bar{A}_{-1} \bar{B}_0 \bar{C}_0$
2	-2	2	$-\frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{-1}}{\sqrt{6}} - \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{-1}}{\sqrt{6}} + \sqrt{\frac{2}{3}} \bar{A}_{-1} \bar{B}_{-1} \bar{C}_0$
2	-2	1	$\frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{-1}}{\sqrt{2}} - \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{-1}}{\sqrt{2}}$
1	1	2	$\sqrt{\frac{3}{5}} \bar{A}_1 \bar{B}_1 \bar{C}_{-1} - \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_1 \bar{B}_0 \bar{C}_0 - \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_0 \bar{B}_1 \bar{C}_0 + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_1}{2\sqrt{15}} + \frac{\bar{A}_0 \bar{B}_0 \bar{C}_1}{\sqrt{15}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_1}{2\sqrt{15}}$
1	1	1	$\frac{1}{2} \bar{A}_1 \bar{B}_0 \bar{C}_0 - \frac{1}{2} \bar{A}_0 \bar{B}_1 \bar{C}_0 - \frac{1}{2} \bar{A}_1 \bar{B}_{-1} \bar{C}_1 + \frac{1}{2} \bar{A}_{-1} \bar{B}_1 \bar{C}_1$
1	1	0	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_1}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_0 \bar{C}_1}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_1}{\sqrt{3}}$
1	0	2	$\frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_1 \bar{B}_0 \bar{C}_{-1} + \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_0 \bar{B}_1 \bar{C}_{-1} - \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_0}{\sqrt{15}} - \frac{2\bar{A}_0 \bar{B}_0 \bar{C}_0}{\sqrt{15}} - \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_0}{\sqrt{15}} + \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_0 \bar{B}_{-1} \bar{C}_1 + \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_{-1} \bar{B}_0 \bar{C}_1$
1	0	1	$\frac{1}{2} \bar{A}_1 \bar{B}_0 \bar{C}_{-1} - \frac{1}{2} \bar{A}_0 \bar{B}_1 \bar{C}_{-1} - \frac{1}{2} \bar{A}_0 \bar{B}_{-1} \bar{C}_1 + \frac{1}{2} \bar{A}_{-1} \bar{B}_0 \bar{C}_1$
1	0	0	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_0}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_0 \bar{C}_0}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_0}{\sqrt{3}}$
1	-1	2	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-1}}{2\sqrt{15}} + \frac{\bar{A}_0 \bar{B}_0 \bar{C}_{-1}}{\sqrt{15}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-1}}{2\sqrt{15}} - \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_0 \bar{B}_{-1} \bar{C}_0 - \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_{-1} \bar{B}_0 \bar{C}_0 + \sqrt{\frac{3}{5}} \bar{A}_{-1} \bar{B}_{-1} \bar{C}_1$
1	-1	1	$\frac{1}{2} \bar{A}_1 \bar{B}_{-1} \bar{C}_{-1} - \frac{1}{2} \bar{A}_{-1} \bar{B}_1 \bar{C}_{-1} - \frac{1}{2} \bar{A}_0 \bar{B}_{-1} \bar{C}_0 + \frac{1}{2} \bar{A}_{-1} \bar{B}_0 \bar{C}_0$
1	-1	0	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-1}}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_0 \bar{C}_{-1}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-1}}{\sqrt{3}}$
0	0	1	$-\frac{\bar{A}_1 \bar{B}_0 \bar{C}_{-1}}{\sqrt{6}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{-1}}{\sqrt{6}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_0}{\sqrt{6}} - \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_0}{\sqrt{6}} - \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_1}{\sqrt{6}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_1}{\sqrt{6}}$

$|I \ I_3 \ I_{AB}\rangle$ states from $1 \otimes 1 \otimes \frac{1}{2}$:

I	I_3	I_{AB}	
$\frac{5}{2}$	$\frac{5}{2}$	2	$\bar{A}_1 \bar{B}_1 \bar{C}_{\frac{1}{2}}$
$\frac{5}{2}$	$\frac{3}{2}$	2	$\frac{\bar{A}_1 \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{5}} + \sqrt{\frac{2}{5}} \bar{A}_1 \bar{B}_0 \bar{C}_{\frac{1}{2}} + \sqrt{\frac{2}{5}} \bar{A}_0 \bar{B}_1 \bar{C}_{\frac{1}{2}}$
$\frac{5}{2}$	$\frac{1}{2}$	2	$\frac{\bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{5}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{5}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{10}} + \sqrt{\frac{2}{5}} \bar{A}_0 \bar{B}_0 \bar{C}_{\frac{1}{2}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{10}}$
$\frac{5}{2}$	$-\frac{1}{2}$	2	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{10}} + \sqrt{\frac{2}{5}} \bar{A}_0 \bar{B}_0 \bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{10}} + \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{5}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{5}}$
$\frac{5}{2}$	$-\frac{3}{2}$	2	$\sqrt{\frac{2}{5}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}} + \sqrt{\frac{2}{5}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_{-1} \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{5}}$
$\frac{5}{2}$	$-\frac{5}{2}$	2	$\bar{A}_{-1} \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}$
$\frac{3}{2}$	$\frac{3}{2}$	2	$-\frac{2\bar{A}_1 \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{5}} + \frac{\bar{A}_1 \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{10}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{10}}$
$\frac{3}{2}$	$\frac{3}{2}$	1	$\frac{\bar{A}_0 \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{2}} - \frac{\bar{A}_1 \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	2	$-\sqrt{\frac{3}{10}} \bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{1}{2}} - \sqrt{\frac{3}{10}} \bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{15}} + \frac{2\bar{A}_0 \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{15}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{15}}$
$\frac{3}{2}$	$\frac{1}{2}$	1	$-\frac{\bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{3}}$
$\frac{3}{2}$	$-\frac{1}{2}$	2	$-\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{15}} - \frac{2\bar{A}_0 \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{15}} - \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{15}} + \sqrt{\frac{3}{10}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{1}{2}} + \sqrt{\frac{3}{10}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{1}{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	1	$-\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{6}}$
$\frac{3}{2}$	$-\frac{3}{2}$	2	$-\frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{10}} - \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{10}} + \frac{2\bar{A}_{-1} \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{5}}$
$\frac{3}{2}$	$-\frac{3}{2}$	1	$\frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{2}} - \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{2}}$
$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{\bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} - \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{6}}$
$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{3}}$
$\frac{1}{2}$	$-\frac{1}{2}$	1	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{3}}$
$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{3}}$

$|I\ I_3\ I_{AB}\rangle$ states from $1 \otimes 1 \otimes \frac{3}{2}$:

I	I_3	I_{AB}	
$\frac{7}{2}$	$\frac{7}{2}$	2	$\bar{A}_1 \bar{B}_1 \bar{C}_{\frac{3}{2}}$
$\frac{7}{2}$	$\frac{5}{2}$	2	$\sqrt{\frac{3}{2}} \bar{A}_1 \bar{B}_1 \bar{C}_{\frac{1}{2}} + \sqrt{\frac{2}{7}} \bar{A}_1 \bar{B}_0 \bar{C}_{\frac{3}{2}} + \sqrt{\frac{2}{7}} \bar{A}_0 \bar{B}_1 \bar{C}_{\frac{3}{2}}$
$\frac{7}{2}$	$\frac{3}{2}$	2	$\frac{\bar{A}_1 \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{7}} + \sqrt{\frac{2}{7}} \bar{A}_1 \bar{B}_0 \bar{C}_{\frac{1}{2}} + \sqrt{\frac{2}{7}} \bar{A}_0 \bar{B}_1 \bar{C}_{\frac{1}{2}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{3}{2}}}{\sqrt{21}} + \frac{2 \bar{A}_0 \bar{B}_0 \bar{C}_{\frac{3}{2}}}{\sqrt{21}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{3}{2}}}{\sqrt{21}}$
$\frac{7}{2}$	$\frac{1}{2}$	2	$\frac{\bar{A}_1 \bar{B}_1 \bar{C}_{-\frac{3}{2}}}{\sqrt{35}} + \sqrt{\frac{6}{35}} \bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{1}{2}} + \sqrt{\frac{6}{35}} \bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{1}{2}} + \sqrt{\frac{3}{35}} \bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}} + 2 \sqrt{\frac{3}{35}} \bar{A}_0 \bar{B}_0 \bar{C}_{\frac{1}{2}} + \sqrt{\frac{3}{35}} \bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}} + \sqrt{\frac{2}{35}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{3}{2}} + \sqrt{\frac{2}{35}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{3}{2}}$
$\frac{7}{2}$	$-\frac{1}{2}$	2	$\sqrt{\frac{2}{35}} \bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{3}{2}} + \sqrt{\frac{2}{35}} \bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{3}{2}} + \sqrt{\frac{3}{35}} \bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}} + 2 \sqrt{\frac{3}{35}} \bar{A}_0 \bar{B}_0 \bar{C}_{-\frac{1}{2}} + \sqrt{\frac{3}{35}} \bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}} + \sqrt{\frac{3}{35}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{1}{2}} + \sqrt{\frac{6}{35}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{1}{2}} + \frac{\bar{A}_{-1} \bar{B}_{-1} \bar{C}_{\frac{3}{2}}}{\sqrt{35}}$
$\frac{7}{2}$	$-\frac{3}{2}$	2	$\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{3}{2}}}{\sqrt{21}} + \frac{2 \bar{A}_0 \bar{B}_0 \bar{C}_{-\frac{3}{2}}}{\sqrt{21}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{3}{2}}}{\sqrt{21}} + \sqrt{\frac{2}{7}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}} + \sqrt{\frac{2}{7}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_{-1} \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{7}}$
$\frac{7}{2}$	$-\frac{5}{2}$	2	$\sqrt{\frac{2}{7}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{-\frac{3}{2}} + \sqrt{\frac{2}{7}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{-\frac{3}{2}} + \sqrt{\frac{3}{7}} \bar{A}_{-1} \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}$
$\frac{7}{2}$	$-\frac{7}{2}$	2	$\bar{A}_{-1} \bar{B}_{-1} \bar{C}_{-\frac{3}{2}}$
$\frac{5}{2}$	$\frac{5}{2}$	2	$-\frac{2 \bar{A}_1 \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{7}} + \sqrt{\frac{3}{14}} \bar{A}_1 \bar{B}_0 \bar{C}_{\frac{3}{2}} + \sqrt{\frac{3}{14}} \bar{A}_0 \bar{B}_1 \bar{C}_{\frac{3}{2}}$
$\frac{5}{2}$	$\frac{5}{2}$	1	$\frac{\bar{A}_0 \bar{B}_1 \bar{C}_{\frac{3}{2}}}{\sqrt{2}} - \frac{\bar{A}_1 \bar{B}_0 \bar{C}_{\frac{3}{2}}}{\sqrt{2}}$
$\frac{5}{2}$	$\frac{3}{2}$	2	$-\frac{4 \bar{A}_1 \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{35}} - \frac{\bar{A}_1 \bar{B}_0 \bar{C}_{\frac{1}{2}}}{\sqrt{70}} - \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{70}} + \sqrt{\frac{3}{35}} \bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{3}{2}} + 2 \sqrt{\frac{3}{35}} \bar{A}_0 \bar{B}_0 \bar{C}_{\frac{3}{2}} + \sqrt{\frac{3}{35}} \bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{3}{2}}$
$\frac{5}{2}$	$\frac{3}{2}$	1	$-\sqrt{\frac{3}{10}} \bar{A}_1 \bar{B}_0 \bar{C}_{\frac{1}{2}} + \sqrt{\frac{3}{10}} \bar{A}_0 \bar{B}_1 \bar{C}_{\frac{1}{2}} - \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{3}{2}}}{\sqrt{5}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{3}{2}}}{\sqrt{5}}$
$\frac{5}{2}$	$\frac{1}{2}$	2	$-\sqrt{\frac{6}{35}} \bar{A}_1 \bar{B}_1 \bar{C}_{-\frac{3}{2}} - \frac{1}{2} \sqrt{\frac{5}{7}} \bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{1}{2}} - \frac{1}{2} \sqrt{\frac{5}{7}} \bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{70}} + \sqrt{\frac{2}{35}} \bar{A}_0 \bar{B}_0 \bar{C}_{\frac{1}{2}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}}}{\sqrt{70}} + \frac{3}{2} \sqrt{\frac{3}{35}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{3}{2}} + \frac{3}{2} \sqrt{\frac{3}{35}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{3}{2}}$
$\frac{5}{2}$	$\frac{1}{2}$	1	$-\frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{1}{2}} + \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{1}{2}} - \sqrt{\frac{3}{10}} \bar{A}_1 \bar{B}_{-1} \bar{C}_{\frac{1}{2}} + \sqrt{\frac{3}{10}} \bar{A}_{-1} \bar{B}_1 \bar{C}_{\frac{1}{2}} - \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{3}{2}}}{2\sqrt{5}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{3}{2}}}{2\sqrt{5}}$
$\frac{5}{2}$	$-\frac{1}{2}$	2	$-\frac{3}{2} \sqrt{\frac{3}{35}} \bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{3}{2}} - \frac{3}{2} \sqrt{\frac{3}{35}} \bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{3}{2}} - \frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{70}} - \sqrt{\frac{2}{35}} \bar{A}_0 \bar{B}_0 \bar{C}_{-\frac{1}{2}} - \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}}}{\sqrt{70}} + \frac{1}{2} \sqrt{\frac{5}{7}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{1}{2}} + \frac{1}{2} \sqrt{\frac{5}{7}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{1}{2}} + \sqrt{\frac{6}{35}} \bar{A}_{-1} \bar{B}_{-1} \bar{C}_{\frac{3}{2}}$
$\frac{5}{2}$	$-\frac{1}{2}$	1	$-\frac{\bar{A}_1 \bar{B}_0 \bar{C}_{-\frac{3}{2}}}{2\sqrt{5}} + \frac{\bar{A}_0 \bar{B}_1 \bar{C}_{-\frac{3}{2}}}{2\sqrt{5}} - \sqrt{\frac{3}{10}} \bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}} + \sqrt{\frac{3}{10}} \bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{1}{2}} - \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_0 \bar{B}_{-1} \bar{C}_{\frac{1}{2}} + \frac{1}{2} \sqrt{\frac{3}{5}} \bar{A}_{-1} \bar{B}_0 \bar{C}_{\frac{1}{2}}$
$\frac{5}{2}$	$-\frac{3}{2}$	2	$-\sqrt{\frac{3}{35}} \bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{3}{2}} - 2 \sqrt{\frac{3}{35}} \bar{A}_0 \bar{B}_0 \bar{C}_{-\frac{3}{2}} - \sqrt{\frac{3}{35}} \bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{3}{2}} + \frac{\bar{A}_0 \bar{B}_{-1} \bar{C}_{-\frac{1}{2}}}{\sqrt{70}} + \frac{\bar{A}_{-1} \bar{B}_0 \bar{C}_{-\frac{1}{2}}}{\sqrt{70}} + \frac{4 \bar{A}_{-1} \bar{B}_{-1} \bar{C}_{\frac{1}{2}}}{\sqrt{35}}$
$\frac{5}{2}$	$-\frac{3}{2}$	1	$-\frac{\bar{A}_1 \bar{B}_{-1} \bar{C}_{-\frac{3}{2}}}{\sqrt{5}} + \frac{\bar{A}_{-1} \bar{B}_1 \bar{C}_{-\frac{3}{2}}}{\sqrt{5}} - \sqrt{\frac{3}{10}} \bar{A}_0 \bar{B}_{-1$

$|I \ I_3 \ I_{AB}\rangle$ states from $1 \otimes \frac{1}{2} \otimes \frac{1}{2}$:

I	I_3	I_{AB}	
2	2	$\frac{3}{2}$	$\bar{A}_1 \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}$
2	1	$\frac{3}{2}$	$\frac{1}{2} \bar{A}_1 \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}} + \frac{1}{2} \bar{A}_1 \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}} + \frac{\bar{A}_0 \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{2}}$
2	0	$\frac{3}{2}$	$\frac{\bar{A}_1 \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_0 \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{6}}$
2	-1	$\frac{3}{2}$	$\frac{\bar{A}_0 \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{2}} + \frac{1}{2} \bar{A}_{-1} \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}} + \frac{1}{2} \bar{A}_{-1} \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}$
2	-2	$\frac{3}{2}$	$\bar{A}_{-1} \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}$
1	1	$\frac{3}{2}$	$-\frac{1}{2} \sqrt{3} \bar{A}_1 \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_1 \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{2\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{6}}$
1	1	$\frac{1}{2}$	$\frac{\bar{A}_0 \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{3}} - \sqrt{\frac{2}{3}} \bar{A}_1 \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}$
1	0	$\frac{3}{2}$	$-\frac{\bar{A}_1 \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_0 \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-1} \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{6}}$
1	0	$\frac{1}{2}$	$-\frac{\bar{A}_1 \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_0 \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_0 \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_{-1} \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{3}}$
1	-1	$\frac{3}{2}$	$-\frac{\bar{A}_0 \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_{-1} \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{2\sqrt{3}} + \frac{1}{2} \sqrt{3} \bar{A}_{-1} \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}$
1	-1	$\frac{1}{2}$	$\sqrt{\frac{2}{3}} \bar{A}_{-1} \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}} - \frac{\bar{A}_0 \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{3}}$
0	0	$\frac{1}{2}$	$\frac{\bar{A}_1 \bar{B}_{-\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{3}} - \frac{\bar{A}_0 \bar{B}_{\frac{1}{2}} \bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_0 \bar{B}_{-\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_{-1} \bar{B}_{\frac{1}{2}} \bar{C}_{\frac{1}{2}}}{\sqrt{3}}$

$|I\ I_3\ I_{AB}\rangle$ states from $1 \otimes \frac{3}{2} \otimes \frac{3}{2}$:

I	I_3	I_{AB}	
4	4	$\frac{5}{2}$	$\bar A_1 \bar B_{\frac{3}{2}} \bar C_{\frac{3}{2}}$
4	3	$\frac{5}{2}$	$\frac{1}{2} \sqrt{\frac{3}{2}} \bar A_1 \bar B_{\frac{3}{2}} \bar C_{\frac{3}{2}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_1 \bar B_{\frac{3}{2}} \bar C_{\frac{3}{2}} + \frac{1}{2} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{\frac{3}{2}}$
4	2	$\frac{5}{2}$	$\frac{1}{2} \sqrt{\frac{3}{7}} \bar A_1 \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}} + \frac{3 \bar A_1 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{7}} + \sqrt{\frac{3}{14}} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}} + \frac{1}{2} \sqrt{\frac{3}{7}} \bar A_1 \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}} + \sqrt{\frac{3}{14}} \bar A_0 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}} + \frac{\bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{7}}$
4	1	$\frac{5}{2}$	$\frac{\bar A_1 \bar B_{\frac{3}{2}} \bar C_{-\frac{3}{2}}}{2\sqrt{14}} + \frac{3 \bar A_1 \bar B_{\frac{1}{2}} \bar C_{-\frac{1}{2}}}{2\sqrt{14}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}} + \frac{3 \bar A_1 \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{14}} + \frac{3 \bar A_0 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{7}} + \frac{1}{2} \sqrt{\frac{3}{14}} \bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}} + \frac{\bar A_{1B_{-\frac{3}{2}}} \bar C_{\frac{1}{2}}}{2\sqrt{14}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_0 \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}} + \frac{1}{2} \sqrt{\frac{3}{14}} \bar A_{-1} \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}}$
4	0	$\frac{5}{2}$	$\sqrt{\frac{3}{70}} \bar A_1 \bar B_{\frac{1}{2}} \bar C_{-\frac{3}{2}} + \frac{\bar A_0 \bar B_{\frac{3}{2}} \bar C_{-\frac{3}{2}}}{\sqrt{35}} + \frac{3 \bar A_1 \bar B_{-\frac{1}{2}} \bar C_{-\frac{1}{2}}}{\sqrt{70}} + \frac{3 \bar A_0 \bar B_{\frac{1}{2}} \bar C_{-\frac{1}{2}}}{\sqrt{35}} + \sqrt{\frac{3}{70}} \bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}} + \frac{3 \bar A_0 \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}}}{\sqrt{35}} + \frac{3 \bar A_{-1} \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}}}{\sqrt{70}} + \frac{\bar A_0 \bar B_{-\frac{3}{2}} \bar C_{\frac{1}{2}}}{\sqrt{35}} + \sqrt{\frac{3}{70}} \bar A_{-1} \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}}$
4	-1	$\frac{5}{2}$	$\frac{1}{2} \sqrt{\frac{3}{14}} \bar A_1 \bar B_{-\frac{1}{2}} \bar C_{-\frac{3}{2}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_0 \bar B_{\frac{1}{2}} \bar C_{-\frac{3}{2}} + \frac{\bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{-\frac{3}{2}}}{2\sqrt{14}} + \frac{1}{2} \sqrt{\frac{3}{14}} \bar A_1 \bar B_{-\frac{3}{2}} \bar C_{-\frac{1}{2}} + \frac{3 \bar A_0 \bar B_{-\frac{1}{2}} \bar C_{-\frac{1}{2}}}{2\sqrt{7}} + \frac{3 \bar A_{-1} \bar B_{\frac{1}{2}} \bar C_{-\frac{1}{2}}}{2\sqrt{14}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_0 \bar B_{-\frac{3}{2}} \bar C_{\frac{1}{2}} + \frac{3 \bar A_{-1} \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{14}} + \frac{\bar A_{-1} \bar B_{-\frac{3}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{14}}$
4	-2	$\frac{5}{2}$	$\frac{\bar A_{1B_{-\frac{3}{2}}} \bar C_{-\frac{3}{2}}}{2\sqrt{7}} + \sqrt{\frac{3}{14}} \bar A_0 \bar B_{-\frac{1}{2}} \bar C_{-\frac{3}{2}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_{-1} \bar B_{\frac{1}{2}} \bar C_{-\frac{3}{2}} + \sqrt{\frac{3}{14}} \bar A_0 \bar B_{-\frac{3}{2}} \bar C_{-\frac{1}{2}} + \frac{3 \bar A_{-1} \bar B_{-\frac{1}{2}} \bar C_{-\frac{1}{2}}}{2\sqrt{7}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_{-1} \bar B_{-\frac{3}{2}} \bar C_{\frac{1}{2}}$
4	-3	$\frac{5}{2}$	$\frac{1}{2} \bar A_0 \bar B_{-\frac{3}{2}} \bar C_{-\frac{3}{2}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_{-1} \bar B_{-\frac{1}{2}} \bar C_{-\frac{3}{2}} + \frac{1}{2} \sqrt{\frac{3}{2}} \bar A_{-1} \bar B_{-\frac{3}{2}} \bar C_{-\frac{1}{2}}$
4	-4	$\frac{5}{2}$	$\bar A_{-1} \bar B_{-\frac{3}{2}} \bar C_{-\frac{3}{2}}$
3	3	$\frac{5}{2}$	$-\frac{1}{2} \sqrt{\frac{5}{2}} \bar A_1 \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}} + \frac{3 \bar A_1 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{10}} + \frac{1}{2} \sqrt{\frac{5}{2}} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}}$
3	3	$\frac{3}{2}$	$\sqrt{\frac{3}{5}} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{\frac{3}{2}} - \sqrt{\frac{5}{2}} \bar A_1 \bar B_{\frac{1}{2}} \bar C_{\frac{3}{2}}$
3	2	$\frac{5}{2}$	$-\frac{1}{2} \sqrt{\frac{5}{3}} \bar A_1 \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}} - \frac{\bar A_{1B_{\frac{3}{2}}} \bar C_{\frac{1}{2}}}{2\sqrt{5}} - \frac{\bar A_0 \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}}}{\sqrt{30}} + \frac{1}{2} \sqrt{\frac{5}{6}} \bar A_1 \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}} + \sqrt{\frac{3}{10}} \bar A_0 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}} + \frac{\bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}}}{2\sqrt{5}}$
3	2	$\frac{3}{2}$	$-\frac{\bar A_{1B_{\frac{1}{2}}} \bar C_{\frac{1}{2}}}{\sqrt{5}} + \sqrt{\frac{3}{10}} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}} - \frac{2 \bar A_{1B_{-\frac{1}{2}}} \bar C_{\frac{1}{2}}}{\sqrt{15}} + \frac{\bar A_0 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}}}{\sqrt{30}} + \frac{\bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}}}{\sqrt{5}}$
3	1	$\frac{5}{2}$	$-\frac{\bar A_{1B_{\frac{3}{2}}} \bar C_{-\frac{3}{2}}}{2\sqrt{2}} - \frac{7 \bar A_{1B_{\frac{1}{2}}} \bar C_{-\frac{1}{2}}}{10\sqrt{2}} - \frac{7 \bar A_0 \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}}}{10\sqrt{3}} + \frac{\bar A_{1B_{-\frac{3}{2}}} \bar C_{-\frac{1}{2}}}{10\sqrt{2}} + \frac{1}{10} \bar A_0 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}} + \frac{\bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{\frac{1}{2}}}{10\sqrt{6}} + \frac{3 \bar A_{1B_{-\frac{1}{2}}} \bar C_{\frac{1}{2}}}{10\sqrt{2}} + \frac{3}{10} \sqrt{3} \bar A_0 \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}} + \frac{3}{10} \sqrt{\frac{3}{2}} \bar A_{-1} \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}}$
3	1	$\frac{3}{2}$	$-\frac{1}{5} \sqrt{2} \bar A_1 \bar B_{\frac{1}{2}} \bar C_{-\frac{1}{2}} + \frac{1}{5} \sqrt{3} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}} - \frac{2}{5} \sqrt{2} \bar A_{1B_{-\frac{1}{2}}} \bar C_{-\frac{1}{2}} + \frac{1}{5} \bar A_0 \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}} + \frac{1}{5} \sqrt{6} \bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}} - \frac{1}{5} \sqrt{2} \bar A_{1B_{-\frac{3}{2}}} \bar C_{-\frac{1}{2}} - \frac{\bar A_0 \bar B_{-\frac{1}{2}} \bar C_{\frac{1}{2}}}{5\sqrt{3}} + \frac{2}{5} \sqrt{\frac{3}{2}} \bar A_{-1} \bar B_{\frac{1}{2}} \bar C_{\frac{1}{2}}$
3	0	$\frac{5}{2}$	$-\frac{3 \bar A_{1B_{\frac{3}{2}}} \bar C_{-\frac{3}{2}}}{5\sqrt{2}} - \frac{1}{5} \sqrt{3} \bar A_0 \bar B_{\frac{3}{2}} \bar C_{-\frac{3}{2}} - \frac{1}{5} \sqrt{\frac{3}{2}} \bar A_1 \bar B_{-\frac{1}{2}} \bar C_{-\frac{1}{2}} - \frac{1}{5} \sqrt{3} \bar A_0 \bar B_{\frac{1}{2}} \bar C_{-\frac{1}{2}} - \frac{\bar A_{-1} \bar B_{\frac{3}{2}} \bar C_{-\frac{1}{2}}}{5\sqrt{2}} + \frac{\bar A_{1B_{-\frac{3}{2}}} \bar C_{-\frac{1}{2}}}{5\sqrt{2}} + \frac{1}{5$

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$|I \ I_3 \ I_{AB}\rangle$ states from $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2}$:

I	I_3	I_{AB}	
$\frac{3}{2}$	$\frac{3}{2}$	1	$\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	1	$\frac{\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{3}}$
$\frac{3}{2}$	$-\frac{1}{2}$	1	$\frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{3}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{3}}$
$\frac{3}{2}$	$-\frac{3}{2}$	1	$\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}$
$\frac{1}{2}$	$\frac{1}{2}$	1	$-\sqrt{\frac{2}{3}}\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{6}}$
$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	1	$-\frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{6}} + \sqrt{\frac{2}{3}}\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{2}}$

$|I \ I_3 \ I_{AB}\rangle$ states from $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{3}{2}$:

I	I_3	I_{AB}	
$\frac{5}{2}$	$\frac{5}{2}$	1	$\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{3}{2}}$
$\frac{5}{2}$	$\frac{3}{2}$	1	$\sqrt{\frac{3}{5}}\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}} + \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{3}{2}}}{\sqrt{5}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{3}{2}}}{\sqrt{5}}$
$\frac{5}{2}$	$\frac{1}{2}$	1	$\sqrt{\frac{3}{10}}\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}} + \sqrt{\frac{3}{10}}\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}} + \sqrt{\frac{3}{10}}\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{3}{2}}}{\sqrt{10}}$
$\frac{5}{2}$	$-\frac{1}{2}$	1	$\frac{\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{3}{2}}}{\sqrt{10}} + \sqrt{\frac{3}{10}}\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}} + \sqrt{\frac{3}{10}}\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}} + \sqrt{\frac{3}{10}}\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}$
$\frac{5}{2}$	$-\frac{3}{2}$	1	$\frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{3}{2}}}{\sqrt{5}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{3}{2}}}{\sqrt{5}} + \sqrt{\frac{3}{5}}\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}$
$\frac{5}{2}$	$-\frac{5}{2}$	1	$\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{3}{2}}$
$\frac{3}{2}$	$\frac{3}{2}$	1	$-\sqrt{\frac{2}{5}}\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}} + \sqrt{\frac{3}{10}}\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{3}{2}} + \sqrt{\frac{3}{10}}\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{3}{2}}$
$\frac{3}{2}$	$\frac{3}{2}$	0	$\frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{3}{2}}}{\sqrt{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{3}{2}}}{\sqrt{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	1	$-2\sqrt{\frac{2}{15}}\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}} + \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{30}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{30}} + \sqrt{\frac{2}{5}}\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{3}{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	0	$\frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	1	$-\sqrt{\frac{2}{5}}\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{3}{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{30}} - \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{30}} + 2\sqrt{\frac{2}{15}}\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	0	$\frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{2}}$
$\frac{3}{2}$	$-\frac{3}{2}$	1	$-\sqrt{\frac{3}{10}}\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{3}{2}} - \sqrt{\frac{3}{10}}\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{3}{2}} + \sqrt{\frac{2}{5}}\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}$
$\frac{3}{2}$	$-\frac{3}{2}$	0	$\frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{3}{2}}}{\sqrt{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{3}{2}}}{\sqrt{2}}$
$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{3}{2}}}{\sqrt{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	1	$\frac{\bar{A}_{\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{3}{2}}}{\sqrt{2}} - \frac{\bar{A}_{\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{6}} - \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{\frac{1}{2}}\bar{C}_{-\frac{1}{2}}}{\sqrt{6}} + \frac{\bar{A}_{-\frac{1}{2}}\bar{B}_{-\frac{1}{2}}\bar{C}_{\frac{1}{2}}}{\sqrt{6}}$

$|I\ I_3\ I_{AB}\rangle$ states from $\frac{1}{2} \otimes \frac{3}{2} \otimes \frac{3}{2}$:[illegible]

$|I\ I_3\ I_{AB}\rangle$ states from $\frac{3}{2} \otimes \frac{3}{2} \otimes \frac{3}{2}$:[illegible]

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