SGN – Assignment #1

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1 Periodic orbit

Exercise 1

Consider the 3D Earth–Moon Circular Restricted Three-Body Problem with $\mu = 0.012150$.

1) Find the x-coordinate of the Lagrange point L_1 in the rotating, adimensional reference frame with at least 10-digit accuracy.

Solutions to the 3D CRTBP satisfy the symmetry

$$\mathcal{S} : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \to (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t).$$

Thus, a trajectory that crosses perpendicularly the $y = 0$ plane twice is a periodic orbit.

2) Given the initial guess $\mathbf{x}_0 = (x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})$, with

$$\begin{aligned} x_0 &= 1.08892819445324 \\ y_0 &= 0 \\ z_0 &= 0.0591799623455459 \\ v_{x0} &= 0 \\ v_{y0} &= 0.257888699435051 \\ v_{z0} &= 0 \end{aligned}$$

Find the periodic halo orbit that passes through z_0; that is, develop the theoretical framework and implement a differential correction scheme that uses the STM either approximated through finite differences or achieved by integrating the variational equation.

The periodic orbits in the CRTBP exist in families. These can be computed by continuing the orbits along one coordinate, e.g., z_0. This is an iterative process in which one component of the state is varied, while the other components are taken from the solution of the previous iteration.

3) By gradually decreasing z_0 and using numerical continuation, compute the families of halo orbits until $z_0 = 0.034$.

(8 points)

Point 1

L_1 is a Collinear Lagrange point in the X-axis of the Circular Restricted Three Body Problem (CRTBP). To compute its location, the scalar potential function U is considered and Equation 1 must be solved:

$$\frac{\partial U}{\partial x} := x - \frac{1-\mu}{r_1^3}(\mu + x) + \frac{\mu}{r_2^3}(1 - \mu - x) = 0 \tag{1}$$

With $r_1 = \sqrt{(x+\mu)^2 + y^2 + z^2}$ and $r_2 = \sqrt{(x+\mu-1)^2 + y^2 + z^2}$.

As L_1 is on the X-axis, components y and z are set to zero. The solution is found using the *fsolve* algorithm with an initial guess of $x = 0.5$, as it is known that L_1 is located in between the two bodies of the CRTBP. The resulting coordinates for L_1 are:

$$L_1 = (0.8369180073, 0, 0)\,[-]$$

Point 2

A halo orbit passing through z_0 is the one which crosses perpendicular to the $y = 0$ plane twice. This means that when it crosses $y = 0$, both v_x and v_z must equal zero. To meet all the requirements imposed by the problem, a correction of the initial states x_0 and v_{y0} must be applied using the State Transition Matrix (STM). Therefore, the framework used is:

1. Propagate initial state $\underline{x_0}$
2. Calculate the error between the desired final conditions and the obtained through the propagation
3. Compute the STM through the finite differences method
4. Compute the corrections Δx_0 and Δv_{y0} by solving the system shown in Equation 2

$$\begin{pmatrix} \Phi_{41} & \Phi_{45} \\ \Phi_{61} & \Phi_{65} \end{pmatrix} \begin{pmatrix} \Delta x_0 \\ \Delta v_{y0} \end{pmatrix} = \begin{pmatrix} v_{xf} \\ \Delta v_{zf} \end{pmatrix} \tag{2}$$

5. Apply the calculated corrections to the initial state
6. Repeat until the error is less than the desired value (in this case 10^{-12})

Following this framework, the obtained Δx_0 and Δv_{y0} are:

$$\Delta x_0 = 0.0013498602$$
$$\Delta v_{y0} = 0.0024606856$$

Applying these two corrections to the given initial state, the halo orbit passing through z_0 shown in Figure 1 is retrieved.

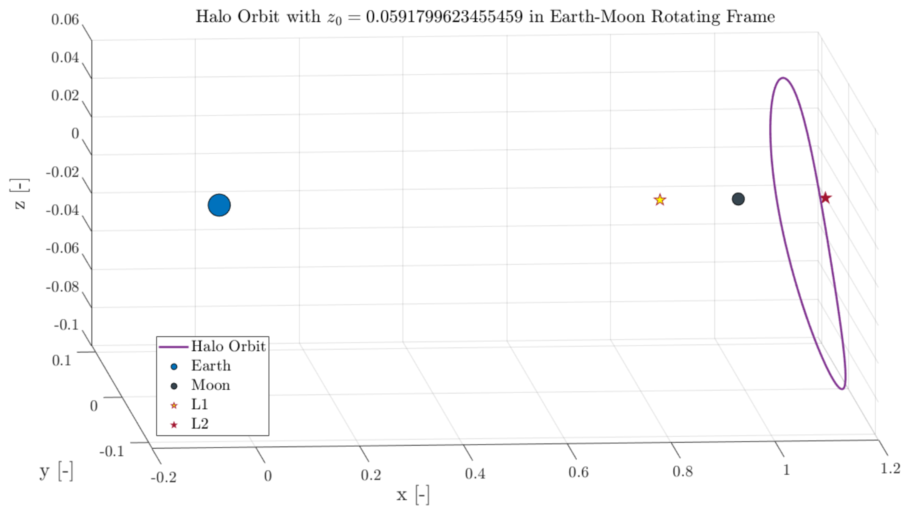

Figure 1: Halo orbit with $z_0 = 0.0591799623455459$ in Earth-Moon rotating frame

Point 3

To retrieve the families, the initial z_0 is gradually decreased until it reaches a value of 0.034. For each family, the framework described above is used. In order to hasten the computation of the families, a numerical continuation approach is adopted, in which the correction values found for the previous step are used as initial guesses for the current halo computation.

$r_0 \cdot 10^{-2}$ [-]	5.520	4.990	4.460	3.930	3.400
$\Delta x_0 \cdot 10^{-2}$ [-]	0.609	1.142	1.587	1.961	2.274
$\Delta v_{y0} \cdot 10^{-2}$ [-])	-1.054	-2.523	-3.759	-4.805	-5.687

Table 1: Corrections applied to initial guess state

A total number of 20 halo orbit families are obtained and presented in Figures 2 and 3. For 5 equally spaced families, the corrections applied with respect to the initial guess state are reported in Table 1. The smaller the z_0 value, the closer the halo orbit is to L2 and the larger the correction found.

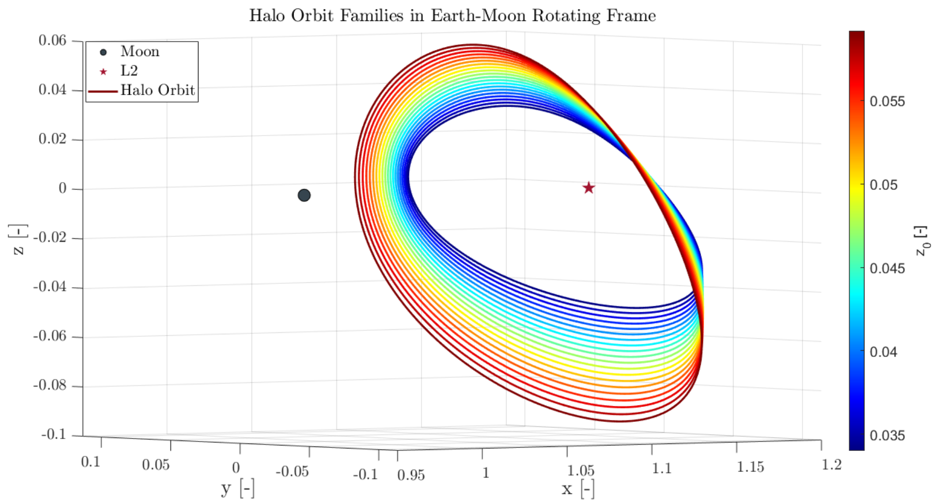

Figure 2: Halo orbit with $z_0 \in [0.059, 0.034]$ in Earth-Moon rotating frame, 3D View

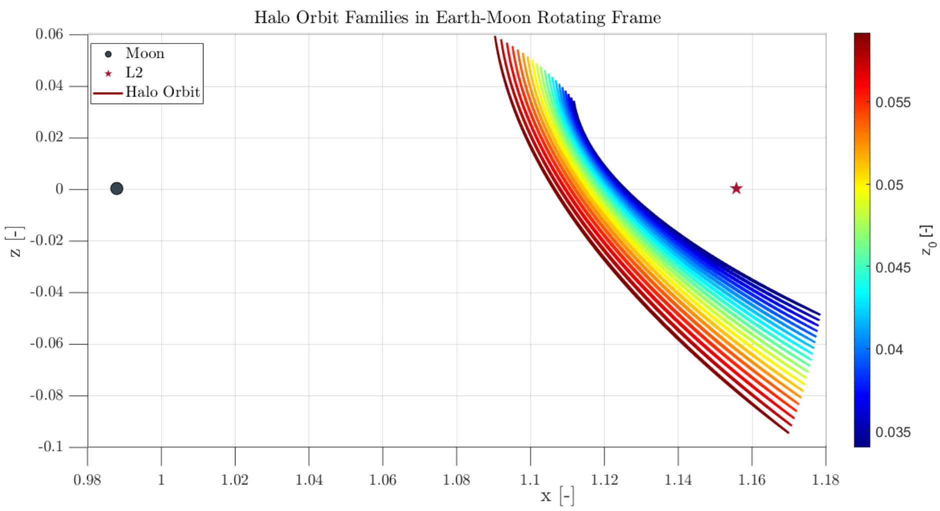

Figure 3: Halo orbit with $z_0 \in [0.059, 0.034]$ in Earth-Moon rotating frame, lateral view

2 Impulsive guidance

Exercise 2

The Apophis close encounter with Earth will occur in April 2029. You shall design a planetary protection guidance solution aimed at reducing the risk of impact with the Earth.

The mission shall be performed with an impactor spacecraft, capable of imparting a $\Delta\mathbf{v} = 0.00005\,\mathbf{v}(t_{\text{imp}})$, where $\mathbf{v}$ is the spacecraft velocity and t_{imp} is the impact time. The spacecraft is equipped with a chemical propulsion system that can perform impulsive manoeuvres up to a total Δv of 5 km/s.

The objective of the mission is to maximize the distance from the Earth at the time of the closest approach. The launch shall be performed between 2024-10-01 (LWO, Launch Window Open) and 2025-02-01 (LWC, Launch Window Close), while the impact with Apophis shall occur between 2028-08-01 and 2029-02-28. An additional Deep-Space Manoeuvre (DSM) can be performed between LWO+6 and LWC+18 months.

1) Analyse the close encounter conditions reading the SPK kernel and plotting in the time window [2029-01-01; 2029-07-31] the following quantities:
   a) The distance between Apophis and the Sun, the Moon and the Earth respectively.
   b) The evolution of the angle Earth-Apophis-Sun
   c) The ground track of Apophis for a time window of 12 hours centered around the time of closest approach (TCA).
2) Formalize an unambiguous statement of the problem specifying the considered optimization variables, objective function, the linear and non-linear equality and inequality constraints, starting from the description provided above. Consider a multiple-shooting problem with $N = 3$ points (or equivalently 2 segments) from t_0 to t_{imp}.
3) Solve the problem with multiple shooting. Propagate the dynamics of the spacecraft considering only the gravitational attraction of the Sun; propagate the post-impact orbit of Apophis using a full n-body integrator. Use an event function to stop the integration at TCA to compute the objective function; read the position of the Earth at t_0 and that of Apophis at t_{imp} from the SPK kernels. Provide the optimization solution, that is, the optimized departure date, DSM execution epoch and the corresponding $\Delta\mathbf{v}$'s, the spacecraft impact epoch, and time and Distance of Closest Approach (DCA) in Earth radii. Suggestion: try different initial conditions.

(11 points)

Point 1

To calculate the distances, the position vectors of each body are retrieved through the function *cspice_spkpos*, using the same frame (ECLIPJ2000) and center (SSB or Solar System Barycenter). Then, the position vectors are subtracted and the norm is calculated to obtain the distances. Figure 4 shows the distance values while Figure 5 provides a more detailed view of the closest approach time. It can be noticed that the distance from the Sun follows a sinusoidal-like curve. Moreover, the Earth and Moon distances have similar trends. They decrease until the closest approach is reached and then increase again. The closest approaches for both Earth and Moon are 1 day apart from each other.

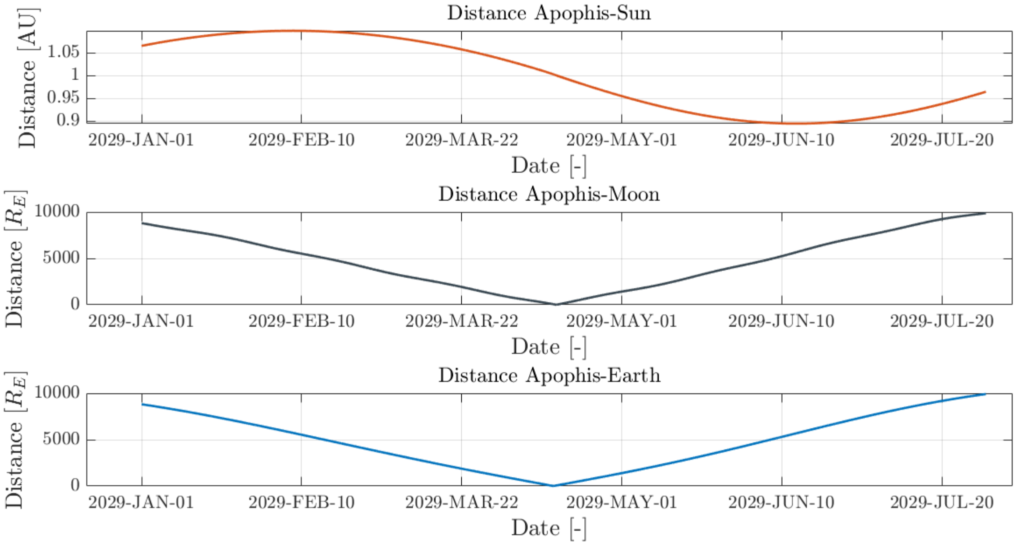

Figure 4: Distances with Apophis

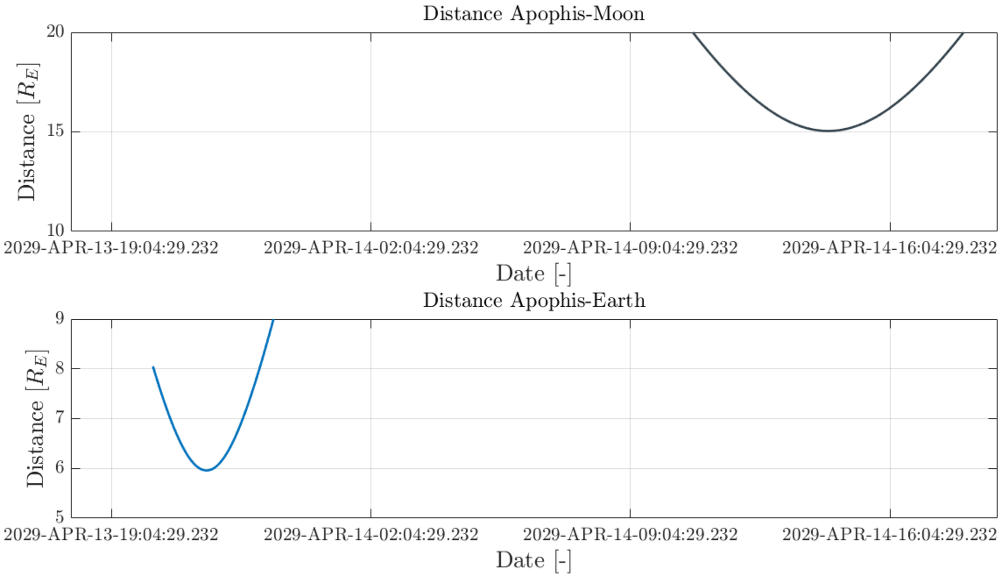

Figure 5: Distances with Apophis, close view

To retrieve the angle Earth-Apophis-Sun (θ_{EAS}), with Apophis as vertex, Equation 3 is used, where $\underline{rr}_{Ap-Sun}$ is the position of the Sun with respect to Apophis and $\underline{rr}_{Ap-Earth}$ Earth's position with respect to Apophis:

$$\theta_{EAS} = \arccos \frac{\underline{rr}_{Ap-Sun} \cdot \underline{rr}_{Ap-Earth}}{\|\underline{rr}_{Ap-Sun}\| \|\underline{rr}_{Ap-Earth}\|} \tag{3}$$

After solving Equation 3, Figure 6 is obtained.

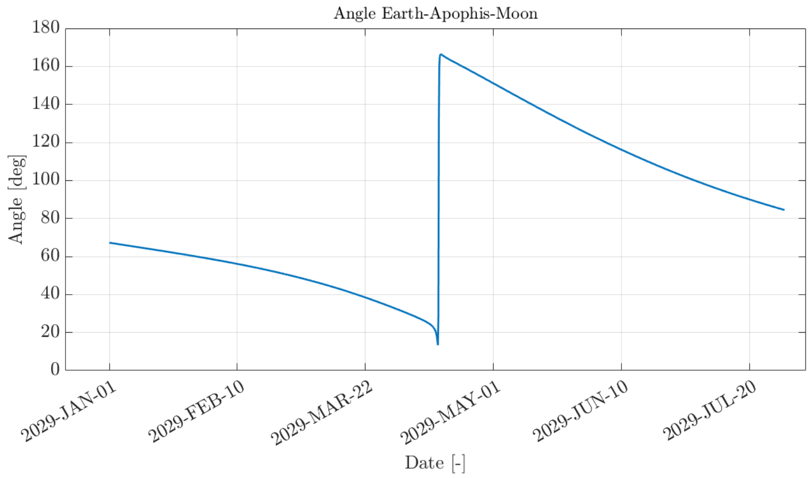

Figure 6: Angle Earth-Apophis-Sun evolution

Finally, to obtain the ground track of Apophis on Earth's surface, the position of the asteroid with respect to Earth in a body-fixed frame is retrieved. Afterwards, the function *cspice_recgeo* is applied to transform the coordinates into longitude and latitude. Figure 7 shows the results.

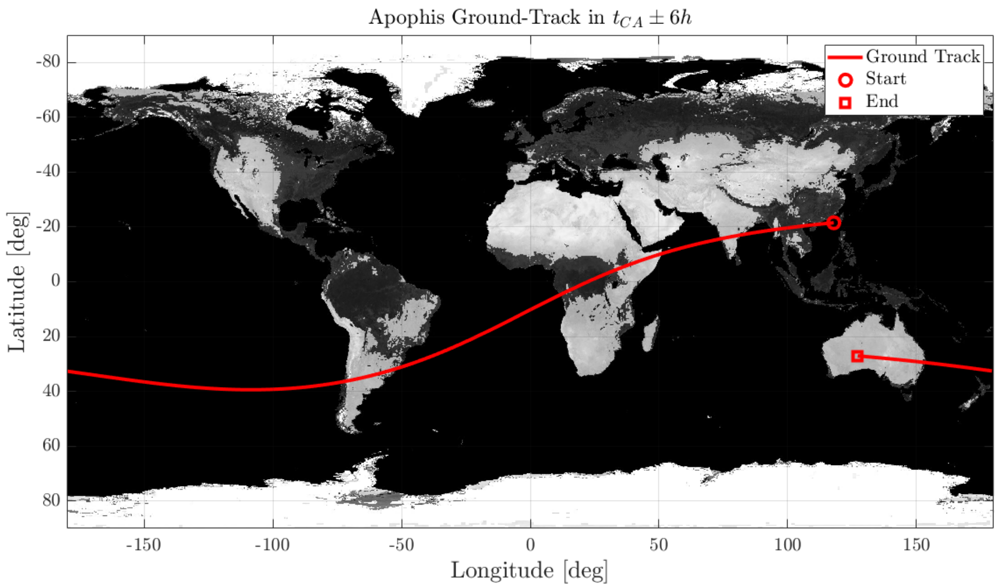

Figure 7: Apophis ground track in $t_{CA} \pm 6$ hours

Point 2

The problem statement is defined as follows:

Variables ($\underline{var}$):

$$\underline{var} = \{\underline{x}_{DEP}, \underline{x}_{DSM}, \underline{x}_{IMP}, t_{DEP}, t_{DSM}, t_{IMP}\}$$

Where $\underline{x}_{DEP}$, $\underline{x}_{DSM}$, $\underline{x}_{IMP}$ are respectively the spacecraft's states at departure time, deep space manoeuvre time and impact time; and t_{DEP} , t_{DSM} , t_{IMP} are respectively the times at departure, deep space manoeuvre and impact.

Objective Function ($\rho(\underline{var})$):

$$\rho(\underline{var}) = -\|\underline{rr}_{Ap}(t_{CA}) - \underline{rr}_{Earth}(t_{CA})\|$$

With $\underline{rr}_{Ap}(t_{CA})$, $\underline{rr}_{Earth}(t_{CA})$ the position vectors of Apophis and Earth at closest approach time respectively, using ECLIPJ2000 frame and SSB as the center.

Regarding the constraints, they must be divided into equality and inequality constraints. No linear constraints have been found as the time variables are bounded using upper and lower bounds.

Non-Linear Equality Constraints ($\underline{c}_{eq}(\underline{var})$):

$$\underline{c}_{eq}(\underline{var}) = \begin{cases} \underline{\Phi}_i := \underline{x}_{DEP}(1:3) - \underline{rr}_{Earth}(t_{DEP}) & \text{Initial Boundary Condition} \\ \underline{\Phi}_f := \underline{x}_{IMP}(1:3) - \underline{rr}_{Ap}(t_{IMP}) & \text{Final Boundary Condition} \\ \underline{z}_1 := \underline{\phi}(\underline{x}_{DEP}, t_{DEP}, t_{DSM})(1:3) - \underline{x}_{DSM}(1:3) & \text{Position Defect in DSM Time} \\ \underline{z}_2 := \underline{\phi}(\underline{x}_{DSM}, t_{DSM}, t_{IMP}) - \underline{x}_{IMP} & \text{Full Defect in Impact Time} \end{cases}$$

Where $\underline{rr}_{Earth}(t_{DEP})$ is the Earth's position at departure time, $\underline{rr}_{Ap}(t_{IMP})$ is the Apophis' position at impact time and $\underline{\Phi}$ is the computed flow between desired times.

Non-Linear Inequality Constraints ($c(\underline{var})$):

$$c(\underline{var}) = \left\{ \; \underline{g}_1 := \|\underline{\Delta v}_1\| + \|\underline{\Delta v}_2\| - 5 \quad \text{Maximum Total Cost} \right.$$

With

$$\underline{\Delta v}_1 = \underline{x}_{DEP}(4:6) - \underline{vv}_{Earth}(t_{DEP})$$
$$\underline{\Delta v}_2 = \underline{x}_{DSM}(4:6) - \underline{\phi}(\underline{x}_{DEP}, t_{DEP}, t_{DSM})(4:6)$$

Where $\underline{vv}_{Earth}(t_{DEP})$ is Earth's velocity at departure time.

Lower Bounds ($\underline{lb}$):

$$\underline{lb} = [-\infty\, ones(18,1)\ ;\ 2024/Oct/01\ ;\ 2025/Apr/01\ ;\ 2028/Aug/01]$$

Upper Bounds ($\underline{ub}$):

$$\underline{ub} = [+\infty\, ones(18,1)\ ;\ 2025/Feb/01\ ;\ 2026/Aug/01\ ;\ 2029/Feb/28]$$

Statement of the Problem:

$$\begin{aligned} \min_{\underline{var}\,\in\,\underline{lb}\ \&\ \underline{ub}} \quad & \rho(\underline{var}) := -\|\underline{rr}_{Ap}(t_{CA}) - \underline{rr}_{Earth}(t_{CA})\| \\ \text{s.t.} \quad & \begin{cases} \underline{c}_{eq}(\underline{var}) = 0 \\ c(\underline{var}) \le 0 \end{cases} \end{aligned} \tag{4}$$

After the problem statement has been properly presented, it is necessary to understand how to retrieve the objective function. In order to do so, the following framework is followed:

1. Retrieve Apophis state at t_{IMP}: $\underline{rv}_{Ap}(t_{IMP})$
2. Compute Apophis new state after the spacecraft impact:
$$\underline{rv}_{Ap}(t_{IMP}) = \underline{rv}_{Ap}(t_{IMP})(4:6) + 0.00005\, \underline{x}_{IMP}(4:6)$$
3. Propagate altered Apophis state until the closest approach to Earth. To stop the propagation at the right time, Equation 5 must be equal to zero. From a computational point of view, it is much better to set the derivative of the squared distance to zero. In this way, the expression to be evaluated is not a fraction, facilitating its evaluation and avoiding the need to calculate a denominator related to the quantity of interest. Therefore, a faster computation is obtained.
$$\frac{\partial \rho^2}{\partial t} = 2\left(\underline{rr}_{Ap}(t) - \underline{rr}_{Earth}(t)\right) \cdot \left(\underline{vv}_{Ap}(t) - \underline{vv}_{Earth}(t)\right) \tag{5}$$
4. Compute $\rho(\underline{var})$

Point 3

To solve the optimisation problem, an initial guess must be given. After trying several combinations, the following guess is used:

$$\underline{x}_{DEP} = \{1.149 \cdot 10^8,\ 9.216 \cdot 10^7,\ 2.218 \cdot 10^4,\ -18.000,\ 23.000,\ 0.735\}\ [km, km/s]$$
$$\underline{x}_{DSM} = \{-1.280 \cdot 10^8,\ -4.179 \cdot 10^7,\ 1.169 \cdot 10^6,\ 11.900,\ -27.851,\ -3.400\}\ [km, km/s]$$
$$\underline{x}_{IMP} = \{1.097 \cdot 10^8,\ 4.646 \cdot 10^7,\ 9.842 \cdot 10^4,\ -10.700,\ 32.523,\ 3.820\}\ [km, km/s]$$
$$t_{DEP} = 9070.0\ [day]$$
$$t_{DSM} = 9221.5\ [day]$$
$$t_{IMP} = 10500.0\ [day]$$

Table 2 provides the results after performing the optimisation problem.

Launch	2024-11-01-00:11:30.469 UTC		
DSM	2025-04-01-07:04:51.962 UTC		
Impact	2028-10-01-23:10:19.236 UTC		
TCA	2029-04-13-23:04:12.028 UTC		
$\Delta \underline{v}_L$ [km/s]	1.0722	−0.0810	0.7327
$\Delta \underline{v}_{DSM}$ [km/s]	1.1602	2.3232	−2.6335
DCA [Re]	22.3946		

Table 2: Guidance solution for the impactor mission

Figure 8 shows the interplanetary trajectory followed by the impactor spacecraft, highlighting the departure manoeuvre, the deep space manoeuvre, its impact with Apophis and the closest approach point.

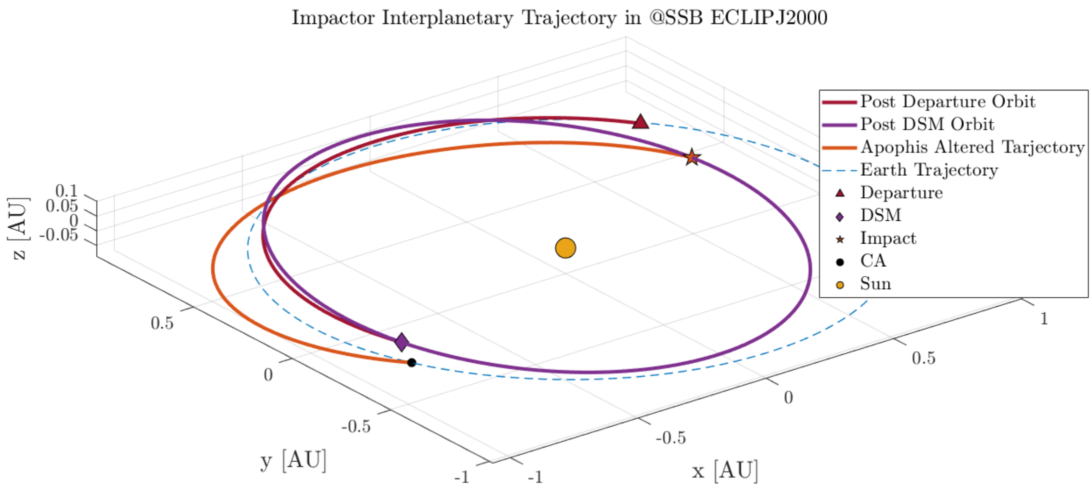

Figure 8: Impactor mission interplanetary trajectory @SSB ECLIPJ2000

The impactor departures from Earth, performs a deep space manoeuvre and, after two orbits, it hits Apophis, increasing the closest approach distance to 22.3946 R_e

3 Continuous guidance

Exercise 3

A low-thrust option is being considered for an Earth-Venus transfer*. Provide a *time-optimal* solution under the following assumptions: the spacecraft moves in the heliocentric two-body problem, Venus instantaneous acceleration is determined only by the Sun gravitational attraction, the departure date is fixed, and the spacecraft initial and final states are coincident with those of the Earth and Venus, respectively.

1) Using the PMP, write down the spacecraft equations of motion, the costate dynamics, and the zero-finding problem for the unknowns $\{\boldsymbol{\lambda}_0, t_f\}$ with the appropriate transversality condition.

2) Adimensionalize the problem using as reference length $LU = 1$ AU† and reference mass $MU = m_0$, imposing that $\mu = 1$. Report all the adimensionalized parameters.

3) Solve the problem considering the following data:

   – Launch date: `2023-05-28-14:13:09.000 UTC`
   – Spacecraft mass: $m_0 = 1000$ kg
   – Electric propulsion properties: $T_{\max} = 800$ mN, $I_{sp} = 3120$ s

   To obtain an initial guess for the costate, generate random numbers such that $\lambda_{0,i} \in [-20; +20]$, while $t_f < 2\pi$. Report the obtained solution in terms of $\{\boldsymbol{\lambda}_0, t_f\}$ and the error with respect to the target. Assess your results exploiting the properties of the Hamiltonian in problems that are not time-dependent and time-optimal solutions.

4) Solve the problem for a lower thrust level $T_{\max} = [500]$ mN. Tip: exploit numerical continuation.

(11 points)

Point 1

The entire exercise is solved considering the frame ECLIPJ2000 centered at the Sun to be consistent with the Keplerian motion imposed in the interplanetary transfer.

The Keplerian PMP for this problem can be stated as follows.

Dynamics:

$$
\begin{cases}
\dot{\underline{r}} = \underline{v} \\
\dot{\underline{v}} = -\dfrac{\mu}{\|\underline{r}\|^3}\,\underline{r} - u\,\dfrac{T_{max}}{m}\,\hat{\underline{\alpha}} \\
\dot{m} = -u\,\dfrac{T_{max}}{I_{sp}\,g_0} \\
\dot{\underline{\lambda_r}} = -\dfrac{3\,\mu}{\|\underline{r}\|^5}\,(\underline{r}\cdot\underline{\lambda_v})\,\underline{r} + \dfrac{\mu}{\|\underline{r}\|^3}\,\underline{\lambda_v} \\
\dot{\underline{\lambda_v}} = -\underline{\lambda_r} \\
\dot{\lambda}_m = -u\,\dfrac{\|\underline{\lambda_v}\|\,T_{max}}{m}
\end{cases}
$$

*Read the necessary gravitational constants and planets positions from SPICE. Use the kernels provided on WeBeep for this assignment.

†Read the value from SPICE

Where $\underline{r}$ is the position, $\underline{v}$ the velocity, $u \in [0, 1]$ the thrust throttle factor, T_{max} the maximum thrust, m the spacecraft's mass, $\underline{\hat{\alpha}}$ the thrust pointing unit vector, I_{sp} the specific impulse, g_0 the standard gravitational acceleration, $\underline{\lambda_r}$ the position's costate, $\underline{\lambda_v}$ the velocity's costate and λ_m the mass' costate.

Due to the problem's nature, time-independent and time-optimal solution, some simplifications can be applied to both u and $\underline{\hat{\alpha}}$, shown in Equations 6 and 7. The thrust is always exploited at its maximum capability, to reduce the transfer time, while the thrust pointing only depends on the velocity's costate.

$$u = 1 \tag{6}$$

$$\underline{\hat{\alpha}} = \frac{\underline{\lambda_v}}{\|\underline{\lambda_v}\|} \tag{7}$$

Boundary Conditions:

$$\begin{cases} \underline{r}(t_0) = \underline{r_{Earth}}(t_0) & \underline{r}(t_f) = \underline{r_{Venus}}(t_f) \\ \underline{v}(t_0) = \underline{v_{Earth}}(t_0) & \underline{v}(t_f) = \underline{v_{Venus}}(t_f) \\ m(t_0) = m_0 & \lambda_m(t_f) = 0 \end{cases}$$

With $\underline{r_{Earth}}$, $\underline{r_{Venus}}$ as Earth and Venus position vectors respectively and $\underline{v_{Earth}}$, $\underline{v_{Venus}}$ as their velocity vectors. Therefore, the spacecraft states at initial and final times are bounded by Earth and Venus states, the initial mass is known and, to complete the set of boundary conditions, the mass costate at final time is enforced to zero.

Zero-Finding System:

$$\begin{cases} \underline{r}(t_f) - \underline{r_{Venus}}(t_f) = 0 & \text{Match with Venus Position} \\ \underline{v}(t_f) - \underline{v_{Venus}}(t_f) = 0 & \text{Match with Venus Velocity} \\ \lambda_m(t_f) = 0 & \text{Costate Boundary Condition} \\ H(t_f) - [\underline{\lambda}_r(t_f)\,;\ \underline{\lambda}v(t_f)] \cdot [\underline{v_{Venus}}(t_f)\,;\ \underline{a_{Venus}}(t_f)] = 0 & \text{Transversality Condition} \end{cases}$$

$$\text{With } H(t_f) := 1 + \underline{\lambda_r}(t_f) \cdot \underline{v}(t_f) - \frac{\mu}{\|\underline{r}(t_f)\|^3}\,\underline{r}(t_f) \cdot \underline{\lambda_v}(t_f) + \frac{T_{max}}{I_{sp}\,g_0}\,S(t_f)$$

Where $S(t_f) := \left(-\frac{\|\underline{\lambda_v}(t_f)\|}{m(t_f)}\, I_{sp}\, g_0 - \lambda_m(t_f) \right)$ is the so-called switching function and $\underline{a_{Venus}}(t_f)$ is Venus' acceleration at final time, estimated with the Keplerian dynamics around the Sun.

The zero-finding system is derived from the need to match the boundary conditions at the final time and the transversality condition, an additional equation helping to determine the final time. Therefore, a system of 8 equations is obtained, 7 used for the initial coestates and the other one to retrieve the final time. u has already been set to 1.

Point 2

Even though the reference length and mass are given, it is necessary to compute two more reference parameters: a reference time, TU, and a reference velocity, VU. As for the reference time, it can be computed by exploiting the fact that μ must be equal to 1, as shown in Equation 8.

$$TU = \sqrt{\frac{LU^3}{\mu}} \tag{8}$$

Once TU is calculated, Equation 9 is applied to retrieve VU.

$$VU = \frac{LU}{TU} \tag{9}$$

With all the reference parameters already obtained, the adimensional quantities of interest can be calculated, leading to a completely adimensionalized problem. The computation of these parameters is shown in Equation 10 and the results are retrieved in Table 3.

$$
\begin{aligned}
&\underline{r}_{0-ad} = \frac{\underline{r}_0}{LU} && \underline{v}_{0-ad} = \frac{\underline{v}_0}{VU} && \\
&m_{0-ad} = \frac{m_0}{MU} && I_{sp0-ad} = \frac{I_{sp0}}{TU} && \mu_{ad} = \frac{TU^2}{LU^3} \\
&T_{max-ad} = \frac{T_{max}\,TU}{MU\,VU} && g_{0-ad} = \frac{g_0\,TU^2}{LU} &&
\end{aligned}
\tag{10}
$$

$\underline{r}_0$	-0.40093	-0.93063	0.00005
$\underline{v}_0$	0.90233	-0.39917	0.00004
m_0	1.00000		
I_{sp}	0.00062		
$T_{\max}$	0.13491		
g_0	1653.77097		
GM	1.00000		

Table 3: Adimensionalized quantities ($T_{\max} = 800$ mN).

Point 3

To solve the time-optimal Keplerian PMP problem, a *while* loop is implemented to generate random initial guesses for the costate variables and the final time between the given ranges until the solution converges. The initial costate vectors and final time are reported in Table 4.

$\underline{\lambda}_{0,r}$	0.4978	-13.8154	0.1082
$\underline{\lambda}_{0,v}$	5.1824	-10.3977	1.4848
$\lambda_{0,m}$	1.4966		
t_f	`2023-10-17-03:10:44.040 UTC`		
TOF [days]	141.5733		

Table 4: Time-optimal Earth-Venus transfer solution ($T_{\max} = 800$ mN).

The position and velocity errors with respect to Venus' state are reported in Table 5. The errors are indeed small, meaning that an appropriate solution which matches Venus at the end of the transfer has been obtained.

$\lvert\lvert\underline{r}_f(t_f) - \underline{r}_V(t_f)\rvert\rvert$	[km]	$1.4281 \cdot 10^{-5}$
$\lvert\lvert\underline{v}_f(t_f) - \underline{v}_V(t_f)\rvert\rvert$	[m/s]	$7.7000 \cdot 10^{-9}$

Table 5: Final state error with respect to Venus' center ($T_{\max} = 800$ mN).

The thrust pointing vector can be retrieved using Equation 3 and it can be input in the transfer plot to indicate the thrust direction during the trajectory. This is done on the right side of Figure 10. The left plot shows the transfer placed in the interplanetary scenario along Earth and Venus orbits.

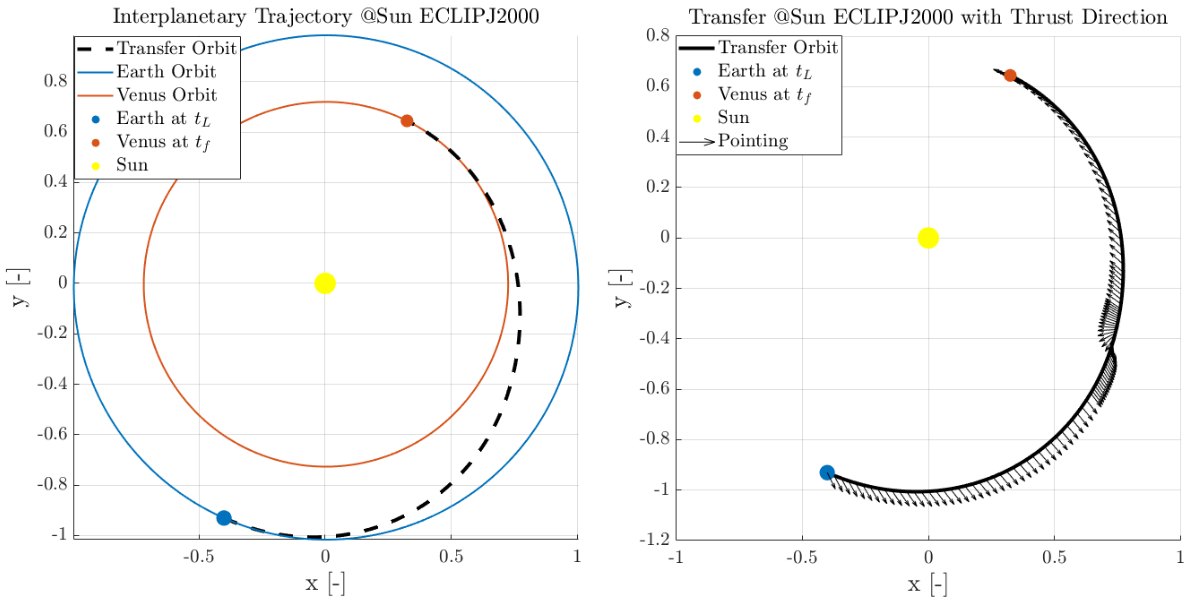

Figure 9: Interplanetary transfer @Sun ECLIPJ2000, $T_{max} = 800mN$

To assess the results, the properties of the Hamiltonian in problems that are not time-dependent and time-optimal solutions can be exploited. For this kind of problem, the Hamiltonian must be constant during the whole integration. Figure 11 shows the evolution of this parameter during the propagation. It can be noticed that it is indeed constant. Some fluctuations can be perceived due to the use of relative and absolute tolerances of 10^{-6} in the propagator to ease the optimal solution computation.

Point 4

To solve this case, numerical continuation can be exploited. This is done by gradually decreasing the thrust until reaching 500 mN and using the previous step solution as the initial guess for the current iteration. A total number of 5 equally spaced thrust values is used, going from 800 mN to 500 mN. The solution found for the case of interest is reported in Table 6. It can be noticed that the time of flight is greater than in the previous point, something coherent as the thrust has been reduced.

$\underline{\lambda}_{0,r}$	-0.5636	-24.5369	-0.4752
$\underline{\lambda}_{0,v}$	14.1819	-18.9093	1.9963
$\lambda_{0,m}$	2.3833		
t_f	`2024-01-01-04:01:01.506 UTC`		
TOF [days]	217.5826		

Table 6: Time-optimal Earth-Venus transfer solution ($T_{\max} = 500$ mN).

As done in the previous point, the errors with respect to Venus are reported in Table 7. The values are also small, so an appropriate solution has been found.

$\|\|\underline{r}_f(t_f) - \underline{r}_V(t_f)\|\|$	[km]	$4.7415 \cdot 10^{-4}$
$\|\|\underline{v}_f(t_f) - \underline{v}_V(t_f)\|\|$	[m/s]	$6.6500 \cdot 10^{-8}$

Table 7: Final state error with respect to Venus' center ($T_{\max} = 500$ mN).

Figure 10 shows the interplanetary transfer with Earth and Venus orbits, left plot, and with the thrust pointing vector, right plot. It is clear that the transfer is indeed longer than the one presented in Figure 9, as previously mentioned.

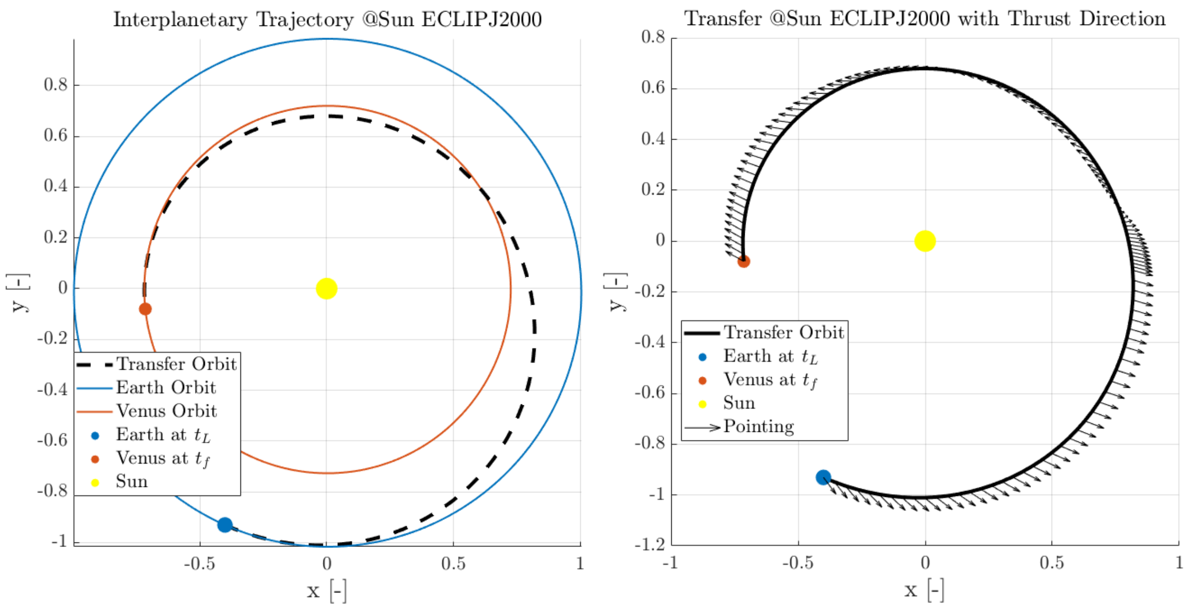

Figure 10: Interplanetary transfer @Sun ECLIPJ2000, $T_{max} = 500mN$

Finally, Figure 12 displays the Hamiltonian evolution for the solution of Point 4. As previously mentioned, it can be considered constant even though there are some fluctuations in the value caused by the use of tolerances less restrictive in the propagator.

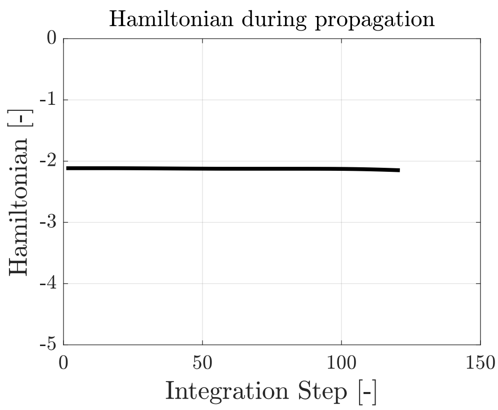

Figure 11: Hamiltonian evolution for Point 3

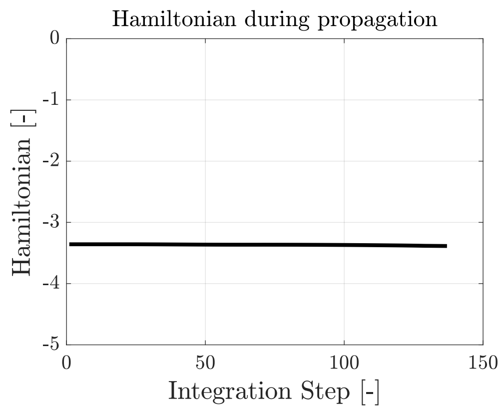

Figure 12: Hamiltonian evolution for Point 4