FBRTL32

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1.

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\begin{aligned}
h_{w_H}(\delta) &= \min_{z\in(0,1]} \log_q \frac{f_{w_H}}{z^\delta} \\
&= \min_{z\in(0,1]} \left(\log_q f_{w_H}(z) - \log_q z^\delta\right) \\
&= \min_{z\in(0,1]} \left(\log_q \left(1 + (q-1)\,z\right) - \delta \log_q z\right) \\
&= \log_q \left(1 + (q-1)\frac{\delta}{(q-1)(1-\delta)}\right) - \delta \log_q \left(\frac{\delta}{(q-1)(1-\delta)}\right) \\
&= \log_q \left(\frac{1}{1-\delta}\right) - \delta \log_q \delta + \delta \log_q (q-1) + \delta \log_q (q-1) \\
&= \delta \log_q \frac{1}{\delta} + (1-\delta) \log_q \frac{1}{1-\delta} + \delta \log_q (q-1)\,.
\end{aligned}
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2.

$$
\begin{aligned}
ab &= [x_1, x_2]qx_2[x_1, x_2][x_1, x_2]x_1 + q^{-1}qx_2[x_1, x_2]\left[[x_1, x_2] + q^{-1}x_1 x_2\right][x_1, x_2]x_1 \\
&= [x_1, x_2]qx_2[x_1, x_2][x_1, x_2]x_1 + x_2[x_1, x_2][x_1, x_2][x_1, x_2]x_1 \\
&\quad + x_2[x_1, x_2]
\end{aligned}
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3.

$$
\begin{aligned}
[x_i, x_j] = 0, &\qquad \text{si } |i-j| > 1; &\qquad (1) \\
[[x_i, x_{i+1}], x_{i+1}] = 0, &\qquad \text{si } 1 \le i < n; &\qquad (2) \\
[x_i, [x_i, x_{i+1}]] = 0, &\qquad \text{si } 1 \le i < n. &\qquad (3)
\end{aligned}
$$

4.

$$
\begin{aligned}
[x_i, x_j] = 0, &\qquad \text{si } |i-j| > 1; & \\
[[x_i, x_{i+1}], x_{i+1}] = 0, &\qquad \text{si } 1 \le i < n; & \\
[x_i, [x_i, x_{i+1}]] = 0, &\qquad \text{si } 1 \le i < n. &\qquad (1)
\end{aligned}
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5.

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\begin{aligned}
e^{i\theta_1} e^{i\theta_2} &= (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) \\
&= (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + i(\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2) \\
&= \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2) \\
&= e^{i(\theta_1 + \theta_2)}
\end{aligned}
$$