Capítulo 1

Lista de reglas

Reglas 1.1. (Reglas básicas de universos)

$$\frac{\Gamma \text{ ctx}}{\Gamma \vdash \mathcal{U}_i : \mathcal{U}_{i+1}}\ \mathcal{U}_i\text{-INTRO} \qquad\qquad \frac{\Gamma \vdash A : \mathcal{U}_i}{\Gamma \vdash A : \mathcal{U}_{i+1}}\ \mathcal{U}_i\text{-CUMUL}$$

Reglas 1.2. (Reglas básicas de contextos y variables)

$$\frac{}{\cdot \text{ ctx}}\ \text{ctx-EMP} \qquad\qquad \frac{x_1{:}A_1, \ldots, x_{n-1}{:}A_{n-1} \vdash A_n : \mathcal{U}_i}{(x_1{:}A_1, \ldots, x_n{:}A_n) \text{ ctx}}\ \text{ctx-EXT}$$

$$\frac{(x_1{:}A_1, \ldots, x_n{:}A_n) \text{ ctx}}{x_1{:}A_1, \ldots, x_n{:}A_n \vdash x_i : A_i}\ \text{Vble}$$

donde la regla ctx-EXT tiene la condición adicional de que x_n debe ser distinta a las demás variables $x_1, \ldots, x_{n-1}$, y la regla Vble requiere que $1 \leq i \leq n$. La regla ctx-EMP tiene 0 hipótesis, por lo que siempre es posible aplicarla.

Reglas 1.3. (Reglas básicas de igualdades por definición)

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash a \equiv a : A} \qquad \frac{\Gamma \vdash a \equiv b : A}{\Gamma \vdash b \equiv a : A} \qquad \frac{\Gamma \vdash a \equiv b : A \qquad \Gamma \vdash b \equiv c : A}{\Gamma \vdash a \equiv c : A}$$

$$\frac{\Gamma \vdash a : A \qquad \Gamma \vdash A \equiv B : \mathcal{U}_i}{\Gamma \vdash a : B} \qquad \frac{\Gamma \vdash a \equiv b : A \qquad \Gamma \vdash A \equiv B : \mathcal{U}_i}{\Gamma \vdash a \equiv b : B}$$

Reglas 1.4. (Reglas de funciones dependientes)

$$\frac{\Gamma \vdash A : \mathcal{U}_i \qquad \Gamma, x : A \vdash B : \mathcal{U}_i}{\Gamma \vdash \prod_{(x:A)} B : \mathcal{U}_i}\ \Pi\text{-FORM}$$

$$\frac{\Gamma, x : A \vdash b : B}{\Gamma \vdash \lambda(x : A).\, b : \prod_{(x:A)} B}\ \Pi\text{-INTRO}$$

donde la expresión $\prod_{(x:A)} B$ liga a x hasta el final de esta.

$$\frac{\Gamma \vdash f : \prod_{(x:A)} B \qquad \Gamma \vdash a : A}{\Gamma \vdash f(a) : B[a/x]}\ \Pi\text{-ELIM}$$

$$\frac{\Gamma, x : A \vdash b : B \qquad \Gamma \vdash a : A}{\Gamma \vdash (\lambda(x : A).\, b)(a) \equiv b[a/x] : B[a/x]}\ \Pi\text{-COMP}$$

$$\frac{\Gamma \vdash f : \prod_{(x:A)} B}{\Gamma \vdash f \equiv (\lambda(x:A).\, f(x)) : \prod_{(x:A)} B}\ \Pi\text{-UNIQ}$$

Reglas 1.5. (Reglas de pares dependientes)

$$\frac{\Gamma \vdash A : \mathcal{U}_i \qquad \Gamma \vdash B : A \to \mathcal{U}_i}{\Gamma \vdash \sum_{(x:A)} B : \mathcal{U}_i}\ \Sigma\text{-FORM}$$

$$\frac{\Gamma \vdash B : A \to \mathcal{U}_i \qquad \Gamma \vdash a : A \qquad \Gamma \vdash b : B(a)}{\Gamma \vdash (a,b) : \sum_{(x:A)} B}\ \Sigma\text{-INTRO}$$

donde la expresión $\sum_{(x:A)} B$ liga a x hasta el final de esta.

$$\frac{\begin{array}{c}\Gamma \vdash C : (\sum_{(x:A)} B) \to \mathcal{U}_i \qquad \Gamma \vdash g : \prod_{(a:A)} \prod_{(b:B(a))} C((a,b)) \\ \Gamma \vdash p : \sum_{(x:A)} B\end{array}}{\Gamma \vdash \mathsf{ind}^{C,g,p}_{\sum_{(x:A)} B} : C(p)}\ \Sigma\text{-ELIM}$$

$$\frac{\begin{array}{c}\Gamma \vdash C : (\sum_{(x:A)} B) \to \mathcal{U}_i \qquad \Gamma \vdash g : \prod_{(a:A)} \prod_{(b:B(a))} C(a,b) \\ \Gamma \vdash a : A \qquad \Gamma \vdash b : B[a/x]\end{array}}{\Gamma \vdash \mathsf{ind}^{C,g,(a,b)}_{\sum_{(x:A)} B} \equiv g(a)(b) : C((a,b))}\ \Sigma\text{-COMP}$$

Reglas 1.6. (Reglas del tipo **0**.)

$$\frac{\Gamma \text{ ctx}}{\Gamma \vdash \mathbf{0} : \mathcal{U}_0}\ \mathbf{0}\text{-FORM} \qquad \frac{\Gamma \vdash C : \mathbf{0} \to \mathcal{U}_0 \qquad \Gamma \vdash z : \mathbf{0}}{\Gamma \vdash \mathsf{ind}^{C,z}_{\mathbf{0}} : C(z)}\ \mathbf{0}\text{-ELIM}$$

Reglas 1.7. (Reglas del tipo **1**.)

$$\frac{\Gamma \text{ ctx}}{\Gamma \vdash \mathbf{1} : \mathcal{U}_i}\ \mathbf{1}\text{-FORM} \qquad \frac{\Gamma \text{ ctx}}{\Gamma \vdash \star : \mathbf{1}}\ \mathbf{1}\text{-INTRO}$$

$$\frac{\Gamma \vdash C : \mathbf{1} \to \mathcal{U}_i \qquad \Gamma \vdash c : C(\star) \qquad \Gamma \vdash a : \mathbf{1}}{\Gamma \vdash \mathsf{ind}^{C,c,a}_{\mathbf{1}} : C(a)}\ \mathbf{1}\text{-ELIM}$$

$$\frac{\Gamma \vdash C : \mathbf{1} \to \mathcal{U}_i \qquad \Gamma \vdash c : C(\star)}{\Gamma \vdash \mathsf{ind}^{C,c,\star}_{\mathbf{1}} \equiv c : C(\star)}\ \mathbf{1}\text{-COMP}$$

Reglas 1.8. (Reglas del coproducto.)

$$\frac{\Gamma \vdash A : \mathcal{U}_i \qquad \Gamma \vdash B : \mathcal{U}_i}{\Gamma \vdash A + B : \mathcal{U}_i}\ +\text{-FORM}$$

$$\frac{\Gamma \vdash A : \mathcal{U}_i \qquad \Gamma \vdash B : \mathcal{U}_i \qquad \Gamma \vdash a : A}{\Gamma \vdash \mathsf{inl}(a) : A + B}\ +\text{-INTRO}_1$$

$$\frac{\Gamma \vdash A : \mathcal{U}_i \qquad \Gamma \vdash B : \mathcal{U}_i \qquad \Gamma \vdash b : B}{\Gamma \vdash \mathsf{inr}(b) : A + B}\ +\text{-INTRO}_2$$

$$\frac{\begin{array}{c}\Gamma \vdash C : A + B \to \mathcal{U}_i \qquad \Gamma \vdash c : \prod_{(x:A)} C(\mathsf{inl}(x)) \\ \Gamma \vdash d : \prod_{(y:B)} C(\mathsf{inr}(y)) \qquad \Gamma \vdash e : A + B\end{array}}{\Gamma \vdash \mathsf{ind}^{C,c,d,e}_{A+B} : C(e)}\ +\text{-ELIM}$$

$$\frac{\begin{array}{c}\Gamma \vdash C : A + B \to \mathcal{U}_i \qquad \Gamma \vdash c : \prod_{(x:A)} C(\mathsf{inl}(x)) \\ \Gamma \vdash d : \prod_{(y:B)} C(\mathsf{inr}(y)) \qquad \Gamma \vdash a : A\end{array}}{\Gamma \vdash \mathsf{ind}_{A+B}^{C,c,d,\mathsf{inl}(a)} \equiv c(a) : C(\mathsf{inl}(a))} \text{ +-COMP}_1$$

$$\frac{\begin{array}{c}\Gamma \vdash C : A + B \to \mathcal{U}_i \qquad \Gamma \vdash c : \prod_{(x:A)} C(\mathsf{inl}(x)) \\ \Gamma \vdash d : \prod_{(y:B)} C(\mathsf{inr}(y)) \qquad \Gamma \vdash b : B\end{array}}{\Gamma \vdash \mathsf{ind}_{A+B}^{C,c,d,\mathsf{inr}(b)} \equiv d(b) : C(\mathsf{inr}(b))} \text{ +-COMP}_2$$

Reglas 1.9. (Reglas del tipo $\mathbb{N}$.)

$$\frac{\Gamma \text{ ctx}}{\Gamma \vdash \mathbb{N} : \mathcal{U}_i} \text{ } \mathbb{N}\text{-FORM}$$

$$\frac{\Gamma \text{ ctx}}{\Gamma \vdash 0 : \mathbb{N}} \text{ } \mathbb{N}\text{-INTRO}_1 \qquad \frac{\Gamma \text{ ctx}}{\Gamma \vdash \mathsf{succ} : \mathbb{N} \to \mathbb{N}} \text{ } \mathbb{N}\text{-INTRO}_2$$

$$\frac{\begin{array}{c}\Gamma \vdash C : \mathbb{N} \to \mathcal{U}_i \qquad \Gamma \vdash c_0 : C(0) \\ \Gamma \vdash c_s : \prod_{(n:\mathbb{N})} C(n) \to C(\mathsf{succ}(n)) \qquad \Gamma \vdash n : \mathbb{N}\end{array}}{\Gamma \vdash \mathsf{ind}_{\mathbb{N}}^{C,c_0,c_s,n} : C(n)} \text{ } \mathbb{N}\text{-ELIM}$$

$$\frac{\begin{array}{c}\Gamma \vdash C : \mathbb{N} \to \mathcal{U}_i \qquad \Gamma \vdash c_0 : C(0) \\ \Gamma \vdash c_s : \prod_{(n:\mathbb{N})} C(n) \to C(\mathsf{succ}(n))\end{array}}{\Gamma \vdash \mathsf{ind}_{\mathbb{N}}^{C,c_0,c_s,0} \equiv c_0 : C(n)} \text{ } \mathbb{N}\text{-COMP}_1$$

$$\frac{\begin{array}{c}\Gamma \vdash C : \mathbb{N} \to \mathcal{U}_i \qquad \Gamma \vdash c_0 : C(0) \\ \Gamma \vdash c_s : \prod_{(n:\mathbb{N})} C(n) \to C(\mathsf{succ}(n)) \qquad \Gamma \vdash n : \mathbb{N}\end{array}}{\Gamma \vdash \mathsf{ind}_{\mathbb{N}}^{C,c_0,c_s,\mathsf{succ}(n)} \equiv c_s(n, \mathsf{ind}_{\mathbb{N}}^{C,c_0,c_s,n}) : C(\mathsf{succ}(n))} \text{ } \mathbb{N}\text{-COMP}_2$$

Reglas 1.10. (Reglas del tipo de identidades.)

$$\frac{\Gamma \vdash A : \mathcal{U}_i \qquad \Gamma \vdash a : A \qquad \Gamma \vdash b : A}{\Gamma \vdash a =_A b : \mathcal{U}_i} \text{ =-FORM}$$

$$\frac{\Gamma \vdash A : \mathcal{U}_i \qquad \Gamma \vdash a : A}{\Gamma \vdash \mathsf{refl}_a : a =_A a} \text{ =-INTRO}$$

$$\frac{\begin{array}{c}\Gamma \vdash C : \prod_{(x,y:A)} (x =_A y) \to \mathcal{U}_i \qquad \Gamma \vdash c : \prod_{(z:A)} C(z, z, \mathsf{refl}_z) \\ \Gamma \vdash a : A \qquad \Gamma \vdash b : A \qquad \Gamma \vdash p : a =_A b\end{array}}{\Gamma \vdash \mathsf{ind}_{=_A}^{C,c,a,b,p} : C(a, b, p)} \text{ =-ELIM}$$

$$\frac{\begin{array}{c}\Gamma \vdash C : \prod_{(x,y:A)} (x =_A y) \to \mathcal{U}_i \qquad \Gamma \vdash c : \prod_{(z:A)} C(z, z, \mathsf{refl}_z) \\ \Gamma \vdash a : A\end{array}}{\Gamma \vdash \mathsf{ind}_{=_A}^{C,c,a,a,\mathsf{refl}_a} : C(a, a, \mathsf{refl}_a)} \text{ =-COMP}$$