# Estadística Descriptiva

| Métrica     | Individuales   | Agrupados   |
|-------------|--|---|
| Media       | $\overline{x} = \frac{1}{n} \sum_{k=0}^{n} x_k$  | $\bar{x} = \frac{1}{n} \sum_{j=1}^{k} f_j \cdot x_j$                  |
| Mediana     | $\tilde{x} = \begin{cases} x_{\frac{n+1}{2}} & n \text{ es impar} \\ \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2} & n \text{ es par} \end{cases}$ | $\tilde{x} = L_{i-1} + \frac{\frac{n}{2} - F_{i-1}}{f_i} A$           |
| Varianza    | $s^{2} = \frac{1}{n-1} \sum_{k=1}^{n} (x_{k} - \bar{x})^{2}$   | $s^{2} = \frac{1}{n-1} \sum_{j=1}^{k} f_{j} (x_{j} - \bar{x})^{2}$    |
| Desviación  | $s = \sqrt{s^2}$   | $s = \sqrt{s^2}$  |
| estándar    |  |   |
| Percentiles | $P_k = \begin{cases} \frac{x_j + x_{j+1}}{2} & r = 0\\ x_{\lceil j+1 \rceil} & r > 0 \end{cases}$  | $\frac{P_k}{\frac{nk/100 - F_{k-1}}{f_k}A} = L_{k-1} + \frac{1}{f_k}$ |

| Rango       | $x_{\text{máx}} - x_{\text{mín}}$  |
|-------------|--|
| Rango in-   | $RIQ = Q_3 - Q_1$  |
| tercuartil  |  |
| Coeficiente | $CV = \frac{s}{\bar{r}}$   |
| de varia-   | , and the second |
| ción        |  |
| Covariaza   | $Cov(x,y) = \frac{1}{n-1} \sum_{k=1}^{n} (x_k - \bar{x})(y_k - \bar{y})$   |
| Coeficiente | $r = \frac{\operatorname{Cov}(x, y)}{s_x s_y}$   |
| de relación | $S_X S_Y$  |
| Asimetría   | $A_s = \frac{1}{ns^3} \sum_{k=1}^{n} (x_k - \bar{x})^3$  |
| Curtosis    | $A_c = \frac{1}{ns^4} \sum_{k=1}^{n} (x_k - \bar{x})^4$  |

## Probabilidad de eventos

| $P(A) = \frac{\text{Casos Favorables}}{\text{Casos Totales}}$    | $P(A \cap B) = P(A)P(B)$ , independien-                                   |
|--|---|
| $\Gamma(\Lambda) = \frac{\Gamma(\Lambda)}{\text{Casos Totales}}$ | tes   |
| Probabilidad de la   | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$                                 |
| unión eventos  | $P\left(\bigcup_{k=1}^{n} A_k\right) = \sum_{k=1}^{n} P(A_k)$ , disjuntos |
| Probabilidad Condi-  | $P(B A) = \frac{P(A \cap B)}{P(A)}$                                       |
| cional   | 1 (11)  |
| Probabilidad Total   | $P(A) = \sum_{k=1}^{n} P(A B_k)P(B_k)$                                    |
| Teorema de Bayes   | $P(B_k A) = \frac{P(A H_k)P(B_k)}{n}$                                     |
|  | $\sum_{k=1} P(A B_k)P(B_k)$   |

## Análisis Combinatorio

| conjunto ti-   | Sin repetición                       | Con Repetición                                       |  |  |  |  |  |  |
|--|--------------------------------------|--|--|--|--|--|--|--|
| po   |                                      |  |  |  |  |  |  |  |
| $\{a,b,c,d\}$  |                                      |  |  |  |  |  |  |  |
| Con  | $n\mathbb{C}k = \frac{n!}{(n-k)!k!}$ | $n\mathbb{C}\mathbb{R}k = \frac{(n+k-1)!}{(n-1)!k!}$ |  |  |  |  |  |  |
| orden  | ( )                                  | (** **)****  |  |  |  |  |  |  |
| Sin orden  | $n\mathbb{P}k = \frac{n!}{(n-k)!}$   | $n\mathbb{P}\mathbb{R}k=n^k$                         |  |  |  |  |  |  |
| conjunto tip   | $oo \{a,a,a,b,b,b,c,c,c,$            | d, d}  |  |  |  |  |  |  |
| $n\mathbb{P}n_1, n_2, \dots, n_k = \frac{n!}{\prod\limits_{j=1}^k n_j!}$ , donde $\sum\limits_{j=1}^k n_j = n$ |                                      |  |  |  |  |  |  |  |

|                                       | Cor                         | ntinua   | Discreta                  |                           |  |  |  |  |  |
|---------------------------------------|-----------------------------|--|---------------------------|---------------------------|--|--|--|--|--|
| Propiedades                           | $P(X \le x)$                | Media y varianza   | Propiedades               | $P(X \le x)$              | Media y varianza                                     |  |  |  |  |
| $f(x) \ge 0$                          | $\int_{-\infty}^{x} f(t)dt$ | $\mu = E(X) = \int_{-\infty}^{+\infty} x f(x) dx$                | $p(x) \ge 0$              | $\sum_{k=1}^{\infty} p_k$ | $\mu = E(X) = \sum_{k=0}^{n} x_k p_k$                |  |  |  |  |
| $\int_{-\infty}^{+\infty} f(x)dx = 1$ | $J-\infty$                  | $\sigma^2 = V(X) = \int_{-\infty}^{+\infty} (x - \mu)^2 f(x) dx$ | $\sum_{k=0}^{n} p(k) = 1$ | k=0                       | $\sigma^2 = V(X) = \sum_{k=0}^{n} (x_k - \mu)^2 p_k$ |  |  |  |  |

| E · · · · · D/V · · · · ·   | Г                            | X7   | A  | C 1   |
|---|------------------------------|--|--|---|
| Function $P(X = x)$   | Esperanza                    | Varianza   | Asimetria  | Curtosis  |
|   |                              |  |  | 4 (4  |
|   | пр                           | np(1-p)  | $\frac{1-2p}{\sqrt{np(1-p)}}$  | $3+\frac{1-6p(1-p)}{np(1-p)}$   |
| $\overline{\hspace{1cm}}$   | $\frac{an}{N}$               | $np(1-p)\frac{N-n}{N-1}$   | $\frac{(N-2a)(N-2n)\sqrt{N-1}}{(n-2)\sqrt{na(N-a)(N-n)}}$  |   |
| $\frac{e^{-\lambda}\lambda^x}{x!}$  | λ                            | λ  | $\frac{1}{\sqrt{\lambda}}$   | $3+\frac{1}{\lambda}$   |
| $ \begin{pmatrix} x-1 \\ k-1 \end{pmatrix} p^k q^{x-k} \qquad \begin{pmatrix} x+k-1 \\ k-1 \end{pmatrix} p^k q^x $                                  | $\frac{k}{p}$ $\frac{qk}{p}$ | $\frac{k(1-p)}{p^2}$   | $\frac{2-p}{\sqrt{k(1-p)}}$  | $3 + \frac{p^2 - 6p + 6}{k(1 - p)}$   |
| $p(1-p)^{x-1} \qquad p(1-p)^x$  | $\frac{1}{p}$ $\frac{q}{p}$  | $\frac{1-p}{p^2}$  | $\frac{2-p}{\sqrt{1-p}}$   | $3 + \frac{p^2 - 6p + 6}{1 - p}$  |
| 0 otro caso   | $\frac{a+b}{2}$              | $\frac{(b-a)^2}{12}$   | 0  | 9<br>5  |
| $\begin{cases} \frac{1}{\beta^{\alpha} \Gamma(\alpha)} x^{\alpha - 1} \exp\left(-\frac{x}{\beta}\right) & x > 0\\ 0 & \text{otro caso} \end{cases}$ | αβ                           | $\alpha \beta^2$   | $\frac{2}{\sqrt{\alpha}}$  | $3\left(1+\frac{2}{\alpha}\right)$  |
| $\begin{cases} \frac{1}{\beta} \exp\left(-\frac{x}{\beta}\right) & x > 0\\ 0 & \text{otro caso} \end{cases}$  | β                            | $\beta^2$  | 2  | 9   |
| $\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$  | μ                            | $\sigma^2$   | 0  | 3   |
| $\begin{cases} \frac{1}{2^{\frac{v}{2}}}\Gamma(\frac{v}{2})}x^{\frac{v}{2}-1}e^{-\frac{x}{2}} & x > 0\\ 0 & \text{otro caso} \end{cases}$           | v                            | 2v   | $\sqrt{\frac{8}{v}}$   | $\frac{12}{v}$  |
|   |                              | $ \begin{pmatrix} n \\ x \end{pmatrix} p^{x} (1-p)^{n-x} & np \\ \frac{\binom{a}{x} \binom{N-a}{n-x}}{\binom{N}{n}} & \frac{an}{N} \\ \frac{e^{-\lambda}\lambda^{x}}{x!} & \lambda \\ \frac{e^{-\lambda}\lambda^{x}}{x!} & \lambda \\ \begin{pmatrix} x-1 \\ k-1 \end{pmatrix} p^{k}q^{x-k} & \begin{pmatrix} x+k-1 \\ k-1 \end{pmatrix} p^{k}q^{x} & \frac{k}{p} & \frac{qk}{p} \\ \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otro caso} \\ \frac{1}{\beta^{\alpha}} \frac{1}{\Gamma(\alpha)} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right) & x > 0 \\ 0 & \text{otro caso} \\ \frac{1}{\beta} \exp\left(-\frac{x}{\beta}\right) & x > 0 \\ 0 & \text{otro caso} \\ \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right) & \mu \\ \frac{1}{2^{\frac{v}{2}}} \frac{1}{\Gamma(\frac{v}{2})} x^{\frac{v}{2}-1} e^{-\frac{v}{2}} & x > 0 \\ 0 & \text{otro caso} \\ \end{pmatrix} $ | $ \begin{pmatrix} \binom{n}{x} p^x (1-p)^{n-x} & np & np(1-p) \\ \frac{\binom{a}{x} \binom{N-a}{n-x}}{\binom{N}{n}} & \frac{an}{N} & np(1-p) \frac{N-n}{N-1} \\ \frac{e^{-\lambda} \lambda^x}{x!} & \lambda & \lambda \\ \frac{x-1}{k-1} p^k q^{x-k} & \binom{x+k-1}{k-1} p^k q^x & \frac{k}{p} & \frac{qk}{p} & \frac{k(1-p)}{p^2} \\ p(1-p)^{x-1} & p(1-p)^x & \frac{1}{p} & \frac{q}{p} & \frac{1-p}{p^2} \\ \frac{1}{b-a} & a \leq x \leq b & \frac{a+b}{2} & \frac{(b-a)^2}{12} \\ \frac{1}{\beta^\alpha \Gamma(a)} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right) & x > 0 & \alpha\beta & \alpha\beta^2 \\ 0 & \text{otro caso} & \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) & \mu & \sigma^2 \\ \frac{1}{2^{\frac{\nu}{2}} \Gamma(\frac{\nu}{2})} x^{\frac{\nu}{2}-1} e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{otro caso} & v & 2v \\ \end{pmatrix} $ | $ \begin{pmatrix} \binom{n}{x} p^x (1-p)^{n-x} & np & np(1-p) & \frac{1-2p}{\sqrt{np(1-p)}} \\ \frac{\binom{a}{x} \binom{N-a}{n-x}}{\binom{N}{n}} & \frac{an}{N} & np(1-p) \frac{N-n}{N-1} & \frac{(N-2a)(N-2n)\sqrt{N-1}}{(n-2)\sqrt{na(N-a)(N-n)}} \\ \frac{e^{-\lambda}\lambda^x}{x!} & \lambda & \lambda & \frac{1}{\sqrt{\lambda}} \\ \binom{x-1}{k-1} p^k q^{x-k} & \binom{x+k-1}{k-1} p^k q^x & \frac{k}{p} & \frac{qk}{p} & \frac{k(1-p)}{p^2} & \frac{2-p}{\sqrt{k(1-p)}} \\ p(1-p)^{x-1} & p(1-p)^x & \frac{1}{p} & \frac{q}{p} & \frac{1-p}{p^2} & \frac{2-p}{\sqrt{1-p}} \\ \frac{1}{b-a} & a \le x \le b & \frac{a+b}{2} & \frac{(b-a)^2}{12} & 0 \\ 0 & \text{otro caso} & \alpha\beta & \alpha\beta^2 & \frac{2}{\sqrt{\alpha}} \\ \frac{1}{\beta} \exp\left(-\frac{x}{\beta}\right) & x > 0 & \alpha\beta & \alpha\beta^2 & \frac{2}{\sqrt{\alpha}} \\ \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) & \mu & \sigma^2 & 0 \\ \frac{1}{2^{\frac{1}{2}} \Gamma(\frac{p}{2})} x^{\frac{p}{2}-1} e^{-\frac{p}{2}} & x > 0 \\ 0 & \text{otro caso} & v & 2v & \sqrt{\frac{8}{p}} \end{aligned} $ |

**Teorema Central del Límite:** Sean  $X_1, X_2, ..., X_n$ ; n variables aleatorias independientes con media  $\mu$  y varianza  $\sigma^2$ , (con cualquier distribución de probabilidad) entonces, la **variable promedio**  $\bar{X} = \frac{1}{n} \sum_{k=1}^{n} X_k$  tiene media  $\mu$  y desviación estándar  $\frac{\sigma}{\sqrt{n}}$  y tiende a una ley normal de probabilidades conforme n tiende al infinito. La variable estandarizada:  $Z = \frac{(\bar{X} - \mu)\sqrt{n}}{\sigma}$  converge a una ley normal estándar.

**Resultado:** Siendo X variable aleatoria Binomial. La variable  $Y = \frac{X - np}{\sqrt{npq}}$  converge a la ley normal estandarizada.

#### Distribuciones de muestreo de las variables media, total y proporción

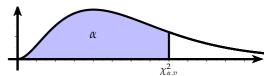
|            |   |   | , , , , ,                         |   |   |  |  |  |  |  |
|------------|---|---|-----------------------------------|---|---|--|--|--|--|--|
| Variable   |   | Varianza de la poblac                       | ión conocida                      | Varianza de la población desconocida (Estimada)   |   |  |  |  |  |  |
|            |   | Tamaño de la Población N                    | Población infinita                | Tamaño de la Población N                          | Población infinita                      |  |  |  |  |  |
| Media      | $\bar{X} = \frac{\sum_{k=1}^{n} X_k}{n}$ $E(\bar{X}) = \mu$ | $V(\bar{X}) = \frac{\sigma^2(N-n)}{n(N-1)}$ | $V(\bar{X}) = \frac{\sigma^2}{n}$ | $\hat{V}(\bar{X}) = \frac{s^2(N-n)}{nN}$          | $\hat{V}(\bar{X}) = \frac{s^2}{n}$      |  |  |  |  |  |
| Total      | $T = n\bar{X}$ $E(T) = n\mu$                                | $V(T) = n\sigma^2 \frac{(N-n)}{(N-1)}$      | $V(T) = n\sigma^2$                | $\hat{V}(T) = ns^2 \frac{(N-n)}{N}$               | $\hat{V}(T) = ns^2$                     |  |  |  |  |  |
| Proporción | $P = \frac{X}{n}$ $E(P) = p$                                | $V(P) = \frac{pq(N-n)}{n(N-1)}$             | $V(P) = \frac{pq}{n}$             | $\hat{V}(P) = \frac{\hat{p}\hat{q}(N-n)}{N(n-1)}$ | $\hat{V}(P) = \frac{\hat{p}\hat{q}}{n}$ |  |  |  |  |  |

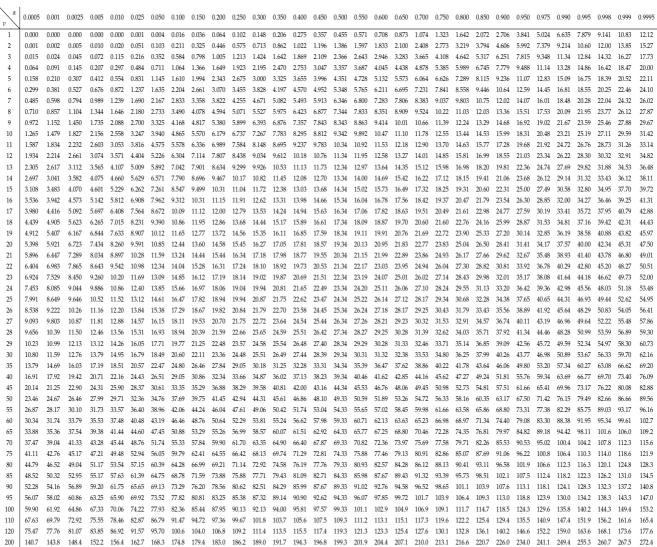
Intervalos de confianza

| Media   | Varianza poblacional conocida $(\sigma^2)$   | $\bar{X} - z_{1-\frac{\alpha}{2}} \sqrt{V(\bar{X})} < \mu < \bar{X} + z_{1-\frac{\alpha}{2}} \sqrt{V(\bar{X})}$                       |  |  |  |  |  |  |
|---|--|---|--|--|--|--|--|--|
| μ   | Varianza muestral conocida (s <sup>2</sup> ) | $\bar{X} - t_{1-\frac{\alpha}{2}, n-1} \sqrt{\hat{V}(\bar{X})} < \mu < \bar{X} + t_{1-\frac{\alpha}{2}, n-1} \sqrt{\hat{V}(\bar{X})}$ |  |  |  |  |  |  |
| Proporción  | Varianza poblacional conocida $(\sigma^2)$   | $P - z_{1 - \frac{\alpha}{2}} \sqrt{V(P)}$  |  |  |  |  |  |  |
| р   | Varianza muestral conocida (s <sup>2</sup> ) | $P - z_{1 - \frac{\alpha}{2}} \sqrt{\hat{V}(P)}$  |  |  |  |  |  |  |
|   |  |   |  |  |  |  |  |  |
| $z_{1-\frac{\alpha}{4}}=1.64$ al 90 % confianza, $z_{1-\frac{\alpha}{4}}=1.96$ al 95 % confianza, $z_{1-\frac{\alpha}{4}}=2.58$ al 99 % confianza |  |   |  |  |  |  |  |  |

#### Pruebas de Hipótesis

| Media   | Proporción  | Bondad de Ajuste  | Varianza                               |
|---|---|---|--|
| $t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ | $z = \frac{ P - P_0  - \frac{1}{2n}}{\sqrt{\frac{P_0 Q_0}{n}}}$ | $\chi^{2} = \sum_{j=1}^{k} \frac{(O_{j} - e_{j})^{2}}{e_{j}}$ | $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$ |





|                        |                              |                        |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    | f <sub>0.05</sub> ( | $v_n, v_d$ )       |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |
|------------------------|------------------------------|------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|---------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $v_d$                  | 1                            | 2                      | 3                  | 4                  | 5                  | 6                  | 7                  | 8                  | 9                  | 10                 | 11                 | 12                 | 13                 | 14                  | 15                 | 16                 | 17                 | 18                 | 19                 | 20                 | 21                 | 22                 | 23                 | 24                 | 25                 | 26                 | 27                 | 28                 | 29                 | 30                 |
| 1 v <sub>n</sub>       | 161.448                      | 18.513                 | 10.128             | 7.709              | 6.608              | 5.987              | 5.591              | 5.318              | 5.117              | 4.965              | 4.844              | 4.747              | 4.667              | 4.600               | 4.543              | 4.494              | 4.451              | 4.414              | 4.381              | 4.351              | 4.325              | 4.301              | 4.279              | 4.260              | 4.242              | 4.225              | 4.210              | 4.196              | 4.183              | 4.171              |
| 2                      | 199.500                      | 19.000                 | 9.552              | 6.944              | 5.786              | 5.143              | 4.737              | 4.459              | 4.256              | 4.103              | 3.982              | 3.885              |                    | 3.739               | 3.682              | 3.634              | 3.592              | 3.555              | 3.522              | 3.493              | 3.467              | 3.443              | 3.422              | 3.403              | 3.385              | 3.369              | 3.354              | 3.340              | 3.328              | 3.316              |
| 3<br>4                 | 215.707<br>224.583           | 19.164<br>19.247       | 9.277<br>9.117     | 6.591              | 5.409<br>5.192     | 4.757<br>4.534     | 4.347<br>4.120     | 4.066<br>3.838     | 3.863              | 3.708<br>3.478     | 3.587              | 3.490              |                    | 3.344               | 3.287              | 3.239              | 3.197<br>2.965     | 3.160<br>2.928     | 3.127<br>2.895     | 3.098<br>2.866     | 3.072<br>2.840     | 3.049<br>2.817     | 3.028<br>2.796     | 3.009<br>2.776     | 2.991              | 2.975<br>2.743     | 2.960              | 2.947              | 2.934              | 2.922              |
| 5                      | 230.162                      | 19.296                 | 9.013              | 6.256              | 5.050              | 4.387              | 3.972              | 3.687              | 3.482              | 3.326              | 3.204              | 3.106              |                    | 2.958               | 2.901              | 2.852              | 2.810              | 2.773              | 2.740              | 2.711              | 2.685              | 2.661              | 2.640              | 2.621              | 2.603              | 2.587              | 2.572              | 2.558              | 2.545              | 2.534              |
| 6                      | 233.986                      | 19.330                 | 8.941              | 6.163              | 4.950              | 4.284              | 3.866              | 3.581              | 3.374              | 3.217              | 3.095              | 2.996              |                    | 2.848               | 2.790              | 2.741              | 2.699              | 2.661              | 2.628              | 2.599              | 2.573              | 2.549              | 2.528              | 2.508              | 2.490              | 2.474              | 2.459              | 2.445              | 2.432              | 2.421              |
| 7<br>8                 | 236.768                      | 19.353<br>19.371       | 8.887<br>8.845     | 6.094              | 4.876<br>4.818     | 4.207              | 3.787              | 3.500              | 3.293              | 3.135              | 3.012<br>2.948     | 2.913              |                    | 2.764               | 2.707              | 2.657              | 2.614              | 2.577              | 2.544              | 2.514              | 2.488              | 2.464              | 2.442              | 2.423              | 2.405              | 2.388              | 2.373              | 2.359              | 2.346              | 2.334              |
| 9                      | 240.543                      | 19.385                 | 8.812              | 5.999              | 4.772              | 4.099              | 3.677              | 3.388              | 3.179              | 3.020              | 2.896              | 2.796              |                    | 2.646               | 2.588              | 2.538              | 2.494              | 2.456              | 2.423              | 2.393              | 2.366              | 2.342              | 2.320              | 2.300              | 2.282              | 2.265              | 2.250              | 2.236              | 2.223              | 2.211              |
| 10                     | 241.882                      | 19.396                 | 8.786              | 5.964              | 4.735              | 4.060              | 3.637              | 3.347              | 3.137              | 2.978              | 2.854              | 2.753              |                    | 2.602               | 2.544              | 2.494              | 2.450              | 2.412              | 2.378              | 2.348              | 2.321              | 2.297              | 2.275              | 2.255              | 2.236              | 2.220              | 2.204              | 2.190              | 2.177              | 2.165              |
| 11<br>12               | 242.983                      | 19.405<br>19.413       | 8.763<br>8.745     | 5.936<br>5.912     | 4.704<br>4.678     | 4.027              | 3.603              | 3.313              | 3.102              | 2.943              | 2.818              | 2.717              |                    | 2.565               | 2.507              | 2.456              | 2.413              | 2.374              | 2.340              | 2.310              | 2.283              | 2.259              | 2.236              | 2.216              | 2.198              | 2.181              | 2.166              | 2.151              | 2.138              | 2.126              |
| 13                     | 243.906                      | 19.415                 | 8.729              | 5.891              | 4.655              | 3.976              | 3.550              | 3.259              | 3.048              | 2.887              | 2.761              | 2.660              | 2.577              | 2.507               | 2.448              | 2.425              | 2.353              | 2.342              | 2.280              | 2.276              | 2.222              | 2.198              | 2.204              | 2.155              | 2.136              | 2.119              | 2.103              | 2.118              | 2.104              | 2.092              |
| 14                     | 245.364                      | 19.424                 | 8.715              | 5.873              | 4.636              | 3.956              | 3.529              | 3.237              | 3.025              | 2.865              | 2.739              | 2.637              | 2.554              | 2.484               | 2.424              | 2.373              | 2.329              | 2.290              | 2.256              | 2.225              | 2.197              | 2.173              | 2.150              | 2.130              | 2.111              | 2.094              | 2.078              | 2.064              | 2.050              | 2.037              |
| 15                     | 245.950                      | 19.429                 | 8.703              | 5.858              | 4.619              | 3.938              | 3.511              | 3.218              | 3.006<br>2.989     | 2.845              | 2.719              | 2.617              |                    | 2.463               | 2.403              |                    | 2.308              | 2.269              | 2.234              | 2.203              |                    | 2.151              | 2.128              | 2.108              | 2.089              |                    | 2.056              | 2.041              | 2.027              | 2.015              |
| 16<br>17               | 246.464<br>246.918           | 19.433                 | 8.692<br>8.683     | 5.844<br>5.832     | 4.604              | 3.922              | 3.494              | 3.202              | 2.989              | 2.828              | 2.701              | 2.599              |                    | 2.445<br>2.428      | 2.368              | 2.333              | 2.289              | 2.250              | 2.215              | 2.184              | 2.156<br>2.139     | 2.131              | 2.109              | 2.088              | 2.069              | 2.052              | 2.036              | 2.021              | 2.007<br>1.989     | 1.995              |
| 18                     | 247.323                      | 19.440                 | 8.675              | 5.821              | 4.579              | 3.896              | 3.467              | 3.173              | 2.960              | 2.798              | 2.671              | 2.568              |                    | 2.413               | 2.353              | 2.302              | 2.257              | 2.217              | 2.182              | 2.151              | 2.123              | 2.098              | 2.075              | 2.054              | 2.035              | 2.018              | 2.002              | 1.987              | 1.973              | 1.960              |
| 19                     | 247.686                      | 19.443                 | 8.667              | 5.811              | 4.568              | 3.884              | 3.455              | 3.161              | 2.948              | 2.785              | 2.658              | 2.555              |                    | 2.400               | 2.340              | 2.288              | 2.243              | 2.203              | 2.168              | 2.137              | 2.109              | 2.084              | 2.061              | 2.040              | 2.021              | 2.003              | 1.987              | 1.972              | 1.958              | 1.945              |
| 20<br>21               | 248.013                      | 19.446<br>19.448       | 8.660<br>8.654     | 5.803<br>5.795     | 4.558              | 3.874              | 3.445              | 3.150              | 2.936              | 2.774              | 2.646              | 2.544              |                    | 2.388               | 2.328              | 2.276              | 2.230              | 2.191              | 2.155              | 2.124              | 2.096              | 2.071              | 2.048              | 2.027              | 2.007<br>1.995     | 1.990              | 1.974              | 1.959              | 1.945              | 1.932              |
| 22                     | 248.579                      | 19.450                 | 8.648              | 5.787              | 4.541              | 3.856              | 3.426              | 3.131              | 2.917              | 2.754              | 2.626              | 2.523              |                    | 2.367               | 2.306              | 2.254              | 2.208              | 2.168              | 2.133              | 2.102              | 2.073              | 2.048              | 2.025              | 2.003              | 1.984              | 1.966              | 1.950              | 1.935              | 1.921              | 1.908              |
| 23                     | 248.826                      | 19.452                 | 8.643              | 5.781              | 4.534              | 3.849              | 3.418              | 3.123              | 2.908              | 2.745              | 2.617              | 2.514              | 2.429              | 2.357               | 2.297              | 2.244              | 2.199              | 2.159              | 2.123              | 2.092              | 2.063              | 2.038              | 2.014              | 1.993              | 1.974              | 1.956              | 1.940              | 1.924              | 1.910              | 1.897              |
| 24                     | 249.052                      | 19.454<br>19.456       | 8.639              | 5.774              | 4.527              | 3.841              | 3.410              | 3.115              | 2.900              | 2.737              | 2.609              | 2.505              |                    | 2.349               | 2.288              | 2.235              | 2.190              | 2.150              | 2.114              | 2.082              | 2.054              | 2.028              | 2.005<br>1.996     | 1.984              | 1.964              | 1.946              | 1.930              | 1.915              | 1.901              | 1.887              |
| 25<br>26               | 249.260<br>249.453           | 19.456                 | 8.634<br>8.630     | 5.769<br>5.763     | 4.521<br>4.515     | 3.835              | 3.404              | 3.108              | 2.893              | 2.730              | 2.594              | 2.498              |                    | 2.341               | 2.280              | 2.227              | 2.174              | 2.141              | 2.106              | 2.074              | 2.045              | 2.020              | 1.988              | 1.967              | 1.955              | 1.938              | 1.921              | 1.906<br>1.897     | 1.891              | 1.878<br>1.870     |
| 27                     | 249.631                      | 19.459                 | 8.626              | 5.759              | 4.510              | 3.823              | 3.391              | 3.095              | 2.880              | 2.716              | 2.588              | 2.484              | 2.398              | 2.326               | 2.265              | 2.212              | 2.167              | 2.126              | 2.090              | 2.059              | 2.030              | 2.004              | 1.981              | 1.959              | 1.939              | 1.921              | 1.905              | 1.889              | 1.875              | 1.862              |
| 28                     | 249.797                      | 19.460                 | 8.623              | 5.754              | 4.505              | 3.818              | 3.386              | 3.090              | 2.874              | 2.710              | 2.582              | 2.478              |                    | 2.320               | 2.259              | 2.206              | 2.160              | 2.119              | 2.084              | 2.052              | 2.023              | 1.997              | 1.973              | 1.952              | 1.932              | 1.914              | 1.898              | 1.882              | 1.868              | 1.854              |
| 29<br>30               | 249.951<br>250.095           | 19.461<br>19.462       | 8.620<br>8.617     | 5.750<br>5.746     | 4.500              | 3.813              | 3.381              | 3.084              | 2.869<br>2.864     | 2.705              | 2.576              | 2.472              |                    | 2.314               | 2.253              | 2.200              | 2.154              | 2.113              | 2.077              | 2.045              | 2.016              | 1.990<br>1.984     | 1.967<br>1.961     | 1.945              | 1.926              | 1.907<br>1.901     | 1.891<br>1.884     | 1.875              | 1.861              | 1.847              |
|                        | 0.00 0.10                    | 0.20                   | 0.30               | 0.40               | 0.50               | 0.60               | 0.70               | 0.80               | 0.90               | 1.00               | 1.10               | 1.20               | 1.30               | 1.40                | 1.50               | 1.60               | 1.70               | 1.80               | 1.90               | 2.00               | 2.10               | 2.20               | 2.30               | 2.40               | 2.50               | 2.60               | 2.70               | 2.80               | 2.90               | 3.00               |
|                        | 50000 0.4601<br>49900 0.4591 |                        |                    | 0.34458            | 0.30854            | 0.27425            | 0.24196            | 0.21186            | 0.18406<br>0.18340 | 0.15866<br>0.15805 | 0.13567<br>0.13512 | 0.11507<br>0.11458 |                    | 0.08076             | 0.06681            | 0.05480            | 0.04457            | 0.03593            | 0.02872            | 0.02275            | 0.01786            | 0.01390            | 0.01072            | 0.00820            | 0.00621            | 0.00466            | 0.00347            | 0.00256            | 0.00187            | 0.00135            |
|                        | 49801 0.4581<br>49701 0.4573 |                        |                    |                    | 0.30678            | 0.27259            | 0.24041            | 0.21041            | 0.18273            | 0.15745            |                    | 0.11410            |                    | 0.08001             | 0.06616            | 0.05425            | 0.04410            | 0.03554            | 0.02839            | 0.02248            | 0.01765            | 0.01373            | 0.01058            | 0.00809            | 0.00612            |                    | 0.00342            |                    | 0.00184            | 0.00133            |
| 0.0100 0.              | 49601 0.4562                 | 0 0.41683              | 0.37828            | 0.34090            | 0.30503            | 0.27093            | 0.23885            | 0.20897            | 0.18141            | 0.15625            | 0.13350            | 0.11314            | 0.09510            | 0.07927             | 0.06552            | 0.05370            | 0.04363            | 0.03515            | 0.02807            | 0.02222            | 0.01743            | 0.01355            | 0.01044            | 0.00798            | 0.00604            | 0.00453            | 0.00336            | 0.00248            | 0.00181            | 0.00131            |
|                        | 49501 0.4552<br>49402 0.4542 |                        | 0.37733<br>0.37638 | 0.33999            | 0.30415<br>0.30328 | 0.27010<br>0.26928 | 0.23808<br>0.23730 | 0.20825<br>0.20754 | 0.18075<br>0.18010 | 0.15565<br>0.15505 | 0.13296<br>0.13243 | 0.11266<br>0.11218 | 0.09468<br>0.09425 | 0.07890<br>0.07853  | 0.06520<br>0.06489 | 0.05343<br>0.05316 | 0.04340<br>0.04317 | 0.03495<br>0.03476 | 0.02791<br>0.02775 | 0.02208<br>0.02195 | 0.01732<br>0.01721 | 0.01347<br>0.01338 | 0.01038<br>0.01031 | 0.00792<br>0.00787 | 0.00599            | 0.00449<br>0.00446 | 0.00334            | 0.00246<br>0.00244 | 0.00179<br>0.00178 | 0.00130<br>0.00128 |
|                        | 49302 0.4532<br>49202 0.4522 |                        |                    | 0.33816            | 0.30240            |                    | 0.23653            | 0.20682            | 0.17944            | 0.15446<br>0.15386 | 0.13189<br>0.13136 | 0.11171<br>0.11123 |                    | 0.07817<br>0.07780  | 0.06457            | 0.05289            |                    | 0.03457            | 0.02759            |                    | 0.01711            | 0.01329            | 0.01024<br>0.01017 | 0.00781            | 0.00591            | 0.00443            | 0.00329            | 0.00242            | 0.00176<br>0.00175 | 0.00127<br>0.00126 |
| 0.0225 0.              | 49102 0.4512                 | 5 0.41196              | 0.37354            | 0.33633            | 0.30066            | 0.26681            | 0.23499            | 0.20540            | 0.17813            | 0.15327            | 0.13082            | 0.11076            | 0.09300            | 0.07744             | 0.06394            | 0.05235            | 0.04249            | 0.03419            | 0.02727            | 0.02156            | 0.01690            | 0.01312            | 0.01010            | 0.00771            | 0.00583            | 0.00436            | 0.00324            | 0.00238            | 0.00174            | 0.00125            |
| 0.0200                 | 49003 0.4502<br>48903 0.4492 |                        | 0.001              | 0.33542<br>0.33451 | 0.29979 0.29892    | 0.26599<br>0.26517 | 0.23423<br>0.23346 | 0.20469 0.20398    | 0.17748<br>0.17683 | 0.15268<br>0.15209 | 0.13029<br>0.12977 | 0.11029<br>0.10982 |                    | 0.07708<br>0.07672  | 0.06363            | 0.05208<br>0.05182 | 0.04226<br>0.04204 | 0.03400            | 0.02711<br>0.02696 | 0.02143<br>0.02131 | 0.01679<br>0.01669 | 0.01304<br>0.01296 | 0.01004<br>0.00997 | 0.00765<br>0.00760 | 0.00578<br>0.00574 | 0100.00            | 0.00322            | 0.00236<br>0.00235 | 0.00172<br>0.00171 | 0.00124<br>0.00123 |
| 010000                 | 48803 0.4482<br>48704 0.4473 |                        | 0.37070            | 0.33360            | 0.29806            | 0.26435            | 0.23270            | 0.20327            | 0.17619            | 0.15151            | 0.12924            | 0.10935            |                    | 0.07636             | 0.06301            | 0.05155            | 0.04182            | 0.03362            | 0.02680            | 0.02118            | 0.01659            | 0.01287            | 0.00990            | 0.00755            | 0.00570            | 0.00427            | 0.00317            | 0.00233            | 0.00169            | 0.00122            |
| 0.0350 0.              | 48604 0.4463                 | 1 0.40710              | 0.36881            | 0.33178            | 0.29632            | 0.26271            | 0.23117            | 0.20186            | 0.17489            | 0.15033            | 0.12819            | 0.10842            | 0.09094            | 0.07564             | 0.06239            | 0.05102            | 0.04137            | 0.03325            | 0.02650            | 0.02093            | 0.01638            | 0.01271            | 0.00977            | 0.00745            | 0.00562            | 0.00421            | 0.00312            | 0.00229            | 0.00167            | 0.00120            |
|                        | 48504 0.4453<br>48405 0.4443 |                        |                    |                    | 0.29546<br>0.29460 |                    | 0.23041<br>0.22965 |                    |                    | 0.14975 $0.14917$  | 0.12766<br>0.12714 |                    |                    | 0.07529<br>0.07493  | 0.06209<br>0.06178 | 0.05076<br>0.05050 | 0.04115<br>0.04093 | 0.03307<br>0.03288 |                    | 0.02080<br>0.02068 | 0.01628<br>0.01618 | 0.01263<br>0.01255 | 0.00971<br>0.00964 | 0.00739<br>0.00734 | 0.00558<br>0.00554 | 0.00418<br>0.00415 |                    |                    | 0.00165<br>0.00164 | 0.00119<br>0.00118 |
|                        | 48305 0.4433<br>48205 0.4423 | 4 0.40420<br>6 0.40323 | 0.36599            | 0.32906            | 0.29374            | 0.26027            | 0.22889            | 0.19975            | 0.17297            | 0.14859            | 0.12662<br>0.12610 | 0.10703            | 0.08972            | 0.07458             | 0.06148            | 0.05024            | 0.04071            | 0.03270            | 0.02604            | 0.02055            | 0.01608            | 0.01246<br>0.01238 | 0.00958            | 0.00729            | 0.00550            | 0.00411            | 0.00305            | 0.00224            | 0.00163            | 0.00117            |
| 0.0475 0.              | 48106 0.4413                 | 7 0.40226              | 0.36411            | 0.32726            | 0.29202            | 0.25865            | 0.22738            | 0.19836            | 0.17169            | 0.14743            | 0.12559            | 0.10611            | 0.08891            | 0.07388             | 0.06087            | 0.04973            | 0.04028            | 0.03234            | 0.02574            | 0.02030            | 0.01588            | 0.01230            | 0.00945            | 0.00719            | 0.00542            | 0.00405            | 0.00300            | 0.00220            | 0.00160            | 0.00115            |
|                        | 48006 0.4403<br>47907 0.4394 |                        |                    | 0.32636<br>0.32545 | 0.29116<br>0.29030 | 0.25785<br>0.25704 | 0.22663<br>0.22588 | 0.19766<br>0.19697 |                    | 0.14686<br>0.14629 | 0.12507<br>0.12456 | 0.10565<br>0.10519 |                    | 0.07353<br>0.07318  | 0.06057<br>0.06027 | 0.04947<br>0.04922 | 0.04006<br>0.03984 | 0.03216<br>0.03198 | 0.02559<br>0.02544 | 0.02018<br>0.02006 | 0.01578<br>0.01568 | 0.01222<br>0.01215 | 0.00939            | 0.00714            | 0.00539<br>0.00535 |                    | 0.00298<br>0.00296 | 0.00219<br>0.00217 | 0.00159<br>0.00158 | 0.00114<br>0.00113 |
|                        | 47807 0.4384<br>47707 0.4374 | 1 0.39936<br>3 0.39840 |                    | 0.32455            | 0.28945            |                    |                    | 0.19628            | 0.16979            |                    |                    | 0.10474            |                    | 0.07283             | 0.05997            | 0.04896            | 0.03963            | 0.03180            | 0.02529            |                    |                    | 0.01207            | 0.00926            | 0.00704            |                    | 0.00397            | 0.00293            | 0.00215            | 0.00156            | 0.00113            |
| 0.0600 0.              | 47608 0.4364                 | 4 0.39743              | 0.35942            | 0.32276            | 0.28774            | 0.25463            | 0.22363            | 0.19489            | 0.16853            | 0.14457            | 0.12302            | 0.10383            | 0.08691            | 0.07215             | 0.05938            | 0.04846            | 0.03920            | 0.03144            | 0.02500            | 0.01970            | 0.01539            | 0.01191            | 0.00914            | 0.00695            | 0.00523            | 0.00391            | 0.00289            | 0.00212            | 0.00154            | 0.00111            |
|                        | 47508 0.4354<br>47409 0.4344 | 6 0.39647<br>7 0.39550 |                    | 0.32186<br>0.32097 | 0.28689<br>0.28604 | 0.25383<br>0.25303 | 0.22288<br>0.22214 | 0.19421<br>0.19352 | 0.16790<br>0.16727 |                    | 0.12252<br>0.12201 | 0.10338<br>0.10294 |                    | 0.07180<br>0.07146  | 0.05909            | 0.04821<br>0.04796 | 0.03899<br>0.03878 | 0.03127<br>0.03109 | 0.02485<br>0.02471 | 0.01958<br>0.01946 | 0.01529<br>0.01519 | 0.01183<br>0.01176 | 0.00908<br>0.00902 | 0.00690<br>0.00685 | 0.00520<br>0.00516 | 0.00388<br>0.00385 | 0.00287<br>0.00285 | 0.00210<br>0.00209 | 0.00153<br>0.00151 | 0.00110<br>0.00109 |
|                        | 47309 0.4334<br>47210 0.4325 | 0.07454                | 0.00002            | 0.32007            | 0.28519            | 0.25223            | 0.22139            | 0.19283            | 0.16665            | 0.14287<br>0.14231 | 0.12150<br>0.12100 | 0.10249            | 0.08573            | 0.07112             | 0.05850            | 0.04771            | 0.03857            | 0.03092            | 0.02456            | 0.01934            | 0.01510            | 0.01168            | 0.00895            | 0.00680            | 0.00512            | 0.00382            | 0.00282            | 0.00207            | 0.00150            | 0.00108<br>0.00107 |
| 0.0725 0.              | 47110 0.4315                 | 2 0.39262              | 0.35476            | 0.31828            | 0.28349            | 0.25063            | 0.21991            | 0.19147            | 0.16540            | 0.14175            | 0.12050            | 0.10160            | 0.08495            | 0.07044             | 0.05792            | 0.04721            | 0.03816            | 0.03057            | 0.02428            | 0.01911            | 0.01491            | 0.01153            | 0.00883            | 0.00671            | 0.00505            | 0.00376            | 0.00278            | 0.00204            | 0.00148            | 0.00106            |
| 0.0775 0.              | 47011 0.4305<br>46911 0.4295 |                        |                    | 0.51757            | 0.28265<br>0.28180 | 0.24904            | 0.21917<br>0.21843 | 0.19011            |                    | 0.14063            | 0.12000<br>0.11950 | 0.10071            | 0.08418            | 0.07011<br>0.06977  | 0.05763<br>0.05734 | 0.04697<br>0.04672 | 0.03795<br>0.03774 | 0.03040<br>0.03022 | 0.02399            | 0.01899<br>0.01888 | 0.01482<br>0.01472 | 0.01138            | 0.00877<br>0.00872 | 0.00666<br>0.00662 | 0.00501<br>0.00498 |                    | 0.00274            | 0.00202            | 0.00146<br>0.00145 | 0.00105<br>0.00104 |
|                        | 46812 0.4285<br>46712 0.4276 |                        | 0.000              |                    | 0.28096<br>0.28011 | 0.24825            | 0.21770            | 0.205.20           |                    | 0.14007<br>0.13952 | 0.11900<br>0.11850 | 0.10027            |                    | 0.06944             | 0.05705            | 0.04648            | 0.03754            | 0.03005            | 0.02385            | 0.01876<br>0.01865 | 0.01463            | 0.01130<br>0.01123 | 0.00866            | 0.00657            | 0.00494            |                    |                    | 0100255            | 0.00144            | 0.00104            |
| 0.0850 0.              | 46613 0.4266                 | 1 0.38782              | 0.35012            | 0.31384            | 0.27927            | 0.24667            | 0.21623            | 0.18808            | 0.16231            | 0.13896            | 0.11801            | 0.09940            | 0.08303            | 0.06877             | 0.05648            | 0.04599            | 0.03713            | 0.02971            | 0.02357            | 0.01853            | 0.01444            | 0.01116            | 0.00854            | 0.00648            | 0.00487            | 0.00363            | 0.00268            | 0.00196            | 0.00142            | 0.00102            |
| 0.0900 0.              | 46514 0.4256<br>46414 0.4246 | 5 0.38591              |                    |                    |                    |                    |                    | 0.18740<br>0.18673 |                    |                    |                    |                    |                    |                     | 0.05620<br>0.05592 |                    |                    |                    |                    |                    |                    |                    |                    | 0.00643<br>0.00639 |                    |                    |                    |                    |                    | 0.00101<br>0.00100 |
| 0.0925 0.<br>0.0950 0. | 46315 0.4236<br>46216 0.4227 |                        |                    | 0.31118            |                    |                    |                    | 0.18606            |                    |                    |                    |                    |                    | 0.06778             |                    |                    |                    |                    |                    |                    |                    |                    |                    | 0.00634            |                    |                    |                    |                    | 0.00138<br>0.00137 | 0.00099            |
|                        | 46116 0.4217                 |                        |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                     |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |
|                        |                              |                        |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                     |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |                    |