

DISEÑO DIGITAL - IST 7072 - UNIVERSIDAD DEL NORTE - EVALUACIÓN

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Punto 1

0.1. Represente los números decimales en binario usando 16 bits y realice las operaciones en complemento a 2:

- $7365 - 4192$ (7365 menos 4192)
 - Iniciamos haciendo la conversión del sistema decimal a binario de 7365

Número	Divisor	Restante	Residuo
7365	2	3682	1
3682	2	1841	0
1841	2	920	1
920	2	460	0
460	2	230	0
230	2	115	0
115	2	57	1
57	2	28	1
28	2	14	0
14	2	7	0
7	2	3	1
3	2	1	1
1	2	0	1

Número	Divisor	Restante	Residuo
4192	2	2096	0
2096	2	1048	0
1048	2	524	0
524	2	262	0
262	2	131	0
131	2	65	1
65	2	32	1
32	2	16	0
16	2	8	0
8	2	4	0
4	2	2	0
2	2	1	1
1	2	0	0

- Ahora, llevamos a 16 bits cada número

Número	Representación binaria en 16 bits
7365	0001110011000101
4192	0001000001100000

- El resultado en sistema decimal es: 3173. Ahora debemos de tomar el segundo número (4192) para llevarlo a complemento 2

4192	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0
complemento	1	1	1	0	1	1	1	1	1	0	0	1	1	1	1	1

- Ahora le sumamos +1

Complemento	1	1	1	0	1	1	1	1	1	0	0	1	1	1	1	1
suma																+1
Resultado	1	1	1	0	1	1	1	1	1	0	1	0	0	0	0	0

- A Resultado lo vamos a sumar con 7365

7365	0	0	0	1	1	1	0	0	1	1	0	0	0	1	0	1
Resultado	1	1	1	0	1	1	1	1	1	0	1	0	0	0	0	0
Suma	0	0	0	0	1	1	0	0	0	1	1	0	0	1	0	1

- El resultado binario debe concordar con 3173 en decimal

$$3173 = 0000110001100101$$

- $9274 - 5888$ (9274 menos 5888)

- Iniciamos haciendo la conversión a binario de ambos números

Número	Divisor	Restante	Residuo
9274	2	4637	0
4637	2	2318	1
2318	2	1159	0
1159	2	579	1
579	2	289	1
289	2	144	1
144	2	72	0
72	2	36	0
36	2	18	0
18	2	9	0
9	2	4	1
4	2	2	0
2	2	1	0
1	2	0	1

Número	Divisor	Restante	Residuo
5888	2	2944	0
2944	2	1472	0
1472	2	736	0
736	2	368	0
368	2	184	0
184	2	92	0
92	2	46	0
46	2	23	0
23	2	11	1
11	2	5	1
5	2	2	1
2	2	1	0
1	2	0	1

Cuadro 1: Caption

- Ahora, llevamos a 16 bits cada número

Número	Representación binaria en 16 bits
9274	0010010000111010
5888	0001011100000000

- El resultado en decimal es de 3386. Ahora procedemos a hallar el complemento de 5888

5888	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
Complemento	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1

- Ahora le sumamos un +1

Complemento	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1
suma																	+1
Resultado	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0

- A Resultado lo vamos a sumar con 9274

9274	0	0	1	0	0	1	0	0	0	0	1	1	1	0	1	0
+																
Resultado	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0
Suma	0	0	0	0	1	1	0	1	0	0	1	1	1	0	1	0

- el resultado tiene que ser acorde a 3386 en binario

$$3386 = 0000110100111010$$

Punto 2

$$F(A,B,C,D) = \sum m(0, 1, 2, 3, 6, 9, 11, 13, 14, 15) = \prod M(4, 5, 7, 8, 10, 12)$$

	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

$\backslash CD$ AB	00	01	11	10
00	1	1	1	1
01	0	0	0	1
11	0	1	1	1
10	0	1	1	0

$\backslash CD$ AB	00	01	11	10
00	1	1	1	1
01	0	0	0	1
11	0	1	1	1
10	0	1	1	0

Suma de productos

$$F(A,B,C,D) = \overline{A}\overline{B} + BC\overline{D} + AD$$

Producto de sumas

$$F(A,B,C,D) = (\overline{B} + C + D)(A + \overline{B} + \overline{D})(\overline{A} + B + \overline{D})$$

$$\rightarrow \underbrace{\overline{A}\overline{B}}_{\textcircled{1}} + \underbrace{BC\overline{D}}_{\textcircled{2}} + \underbrace{AD}_{\textcircled{3}} = \underbrace{(\overline{B} + C + D)}_{\textcircled{4}} \underbrace{(A + \overline{B} + \overline{D})}_{\textcircled{5}} \underbrace{(\overline{A} + B + \overline{D})}_{\textcircled{6}}$$

$$\textcircled{2}Y\textcircled{3} : BC\overline{D} + AD$$

$$= \underline{(B + A)} \underline{(B + D)} \underline{(C + A)} \underline{(C + D)} \underline{(\overline{D} + A)} \underline{(\overline{D} + A)}$$

$$= \underbrace{(B + D)(C + D)(\overline{D} + A)}_{\textcircled{7}} \text{ Por consenso y } \overline{X} + X = 1$$

$$\begin{aligned}
& \textcircled{1}Y\textcircled{7} : (\overline{AB}) + (B + D)(C + D)(\overline{D} + A) \\
& = (\overline{A} + B + D)(\overline{A} + C + D)\cancel{(\overline{A} + \overline{D} + A)}\cancel{(\overline{B} + B + D)}(\overline{B} + C + D)(\overline{B} + \overline{D} + A) \\
& = (\overline{A} + B + D)(\overline{B} + C + D)(A + \overline{B} + \overline{D})(\overline{A} + C + D) \text{ Por } \overline{X} + X = 1 \\
& = (B + (\overline{A} + D))(\overline{B} + (C + D))(\underline{(\overline{A} + D) + (C + D)})(A + \overline{B} + \overline{D}) \\
& = (B + (\overline{A} + D))(\overline{B} + (C + D))(A + \overline{B} + \overline{D}) \\
& \text{Por consenso } (x + y)(\overline{x} + z)(y + z) = (x + y)(\overline{x} + z) \\
& = \underbrace{(\overline{B} + C + D)}_{\textcircled{4}} \underbrace{(A + \overline{B} + \overline{D})}_{\textcircled{5}} \underbrace{(\overline{A} + B + D)}_{\textcircled{6}}
\end{aligned}$$

1. Punto 3

$$F(A,B,C,D) = \prod M(0,1,2,3,6,7,11,13,14) = \sum m(4,5,8,9,10,12,15)$$

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1

$\backslash CD$ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	1	0	1	0
10	1	1	0	1

$\backslash CD$ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	1	0	1	0
10	1	1	0	1

Producto de sumas

$$F = (A + B)(A + \overline{C})(\overline{B} + \overline{C} + D)(B + \overline{C} + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})$$

Suma de productos

$$F = \overline{A}\overline{B}\overline{C} + ABCD + \overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}\overline{D}$$

$$\rightarrow \underbrace{(A + B)}_{\textcircled{1}} \underbrace{(A + \overline{C})}_{\textcircled{2}} \underbrace{(\overline{B} + \overline{C} + D)}_{\textcircled{3}} \underbrace{(B + \overline{C} + \overline{D})}_{\textcircled{4}} \underbrace{(\overline{A} + \overline{B} + C + \overline{D})}_{\textcircled{5}} = \underbrace{(\overline{A}\overline{B}\overline{C})}_{\textcircled{6}} \underbrace{(ABCD)}_{\textcircled{7}} \underbrace{(\overline{B}\overline{C}\overline{D})}_{\textcircled{8}} \underbrace{(\overline{A}\overline{B}\overline{C})}_{\textcircled{9}} \underbrace{(\overline{A}\overline{B}\overline{D})}_{\textcircled{10}}$$

$$\textcircled{1} \text{ Y } \textcircled{2}: (A + B)(A + \overline{C})$$

$$= AA + \underline{A\overline{C}} + \underline{BA} + B\overline{C}$$

$$= A + B\overline{C} \rightarrow \textcircled{11} \text{ por absorción.}$$

$$\textcircled{3} \text{ y } \textcircled{4}: (\overline{B} + \overline{C} + D)(B + \overline{C} + \overline{D})$$

$$= \overline{B}\overline{B} + \overline{B}\overline{C} + \overline{B}D + \overline{C}B + \overline{C}\overline{C} + \overline{C}\overline{D} + DB + \overline{D}\overline{C} + D\overline{D}$$

$$= \overline{B}D + \overline{C} + DB \textcircled{12} \text{ por } X \cdot \overline{X} = 0 \text{ y absorción.}$$

$$\rightarrow \underbrace{(A + B\overline{C})}_{\textcircled{11}} \underbrace{(\overline{B}D + \overline{C} + DB)}_{\textcircled{12}} \underbrace{(\overline{A} + \overline{B} + C + \overline{D})}_{\textcircled{5}}$$

$$\textcircled{11} \text{ y } \textcircled{5}: (A + B\overline{C})(\overline{A} + \overline{B} + C + \overline{D})$$

$$= \overline{A}\overline{A} + \overline{A}\overline{B} + \overline{A}C + \overline{A}\overline{D} + B\overline{C}\overline{A} + B\overline{C}\overline{B} + B\overline{C}C + B\overline{C}\overline{D}$$

$$= \overline{A}\overline{B} + \overline{A}C + \overline{A}\overline{D} + B\overline{C}\overline{A} + B\overline{C}\overline{D} \textcircled{13}$$

$$\textcircled{13} \text{ Y } \textcircled{12}: (\overline{B}D + \overline{C} + DB)(\overline{A}\overline{B} + \overline{A}C + \overline{A}\overline{D} + B\overline{C}\overline{A} + B\overline{C}\overline{D})$$

$$= \overline{A}\overline{B}D\overline{D} + \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}D\overline{B} + \overline{A}C\overline{B}D + \overline{A}C\overline{C} + \overline{A}CDB + \overline{A}\overline{D}B\overline{D} + \overline{A}\overline{D}\overline{C} + \overline{A}\overline{D}D\overline{B} + \overline{B}\overline{C}\overline{A}B\overline{D} +$$

$$\overline{B}\overline{C}\overline{A}C + \overline{B}\overline{C}\overline{A}DB + \overline{B}\overline{C}\overline{D}B\overline{D} + \overline{B}\overline{C}D\overline{C} + \overline{B}\overline{C}D\overline{D}\overline{B}$$

Por la ley $X \cdot \overline{X} = 0$ y $X \cdot X = X$

$$= \overline{A}\overline{B}D + \overline{A}\overline{B}\overline{C} + \overline{A}C\overline{B}D + \overline{A}CDB + \overline{A}\overline{D}\overline{B} + \overline{A}\overline{D}\overline{C} + \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C}D + B\overline{C}\overline{D}$$

Aplicando ley de absorción tenemos que:

$$= \overline{A}\overline{B}D + \overline{A}\overline{B}\overline{C} + \overline{A}CDB + \overline{A}\overline{C}D + \overline{A}\overline{B}\overline{C} + B\overline{C}\overline{D}$$

Reorganizando el siguiente termino:

$$= \overline{A}\overline{B}D + \overline{A}\overline{B}\overline{C} + \overline{A}CDB + \overline{A}\overline{C}D + \overline{A}\overline{B}\overline{C} + B\overline{C}\overline{D}$$

Por ley del consenso y $X \cdot X = X$: $B\overline{C}\overline{D} + \overline{A}\overline{B}D + \overline{A}\overline{C}D = B\overline{C}\overline{D} + \overline{A}\overline{B}D$

Obtenemos al final:

$$= \overline{A}\overline{B}D + \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}\overline{C} + \overline{A}CDB + B\overline{C}\overline{D}$$

Punto 4

$$F(A,B,C,D,E) = \sum m(0, 6, 8, 9, 15, 16, 26, 27, 28) \\ = \prod M(1, 2, 3, 4, 5, 7, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31)$$

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>Y</i>
0	0	0	0	0	0	1
1	0	0	0	0	1	0
2	0	0	0	1	0	0
3	0	0	0	1	1	0
5	0	0	1	0	1	0
6	0	0	1	1	0	1
7	0	0	1	1	1	0
8	0	1	0	0	0	1
9	0	1	0	0	1	1
10	0	1	0	1	0	0
11	0	1	0	1	1	0
12	0	1	1	0	0	0
13	0	1	1	0	1	0
14	0	1	1	1	0	0
15	0	1	1	1	1	1
16	1	0	0	0	0	1
17	1	0	0	0	1	0
18	1	0	0	1	0	0
19	1	0	0	1	1	0
20	1	0	1	0	0	0
21	1	0	1	0	1	0
22	1	0	1	1	0	0
23	1	0	1	1	1	0
24	1	1	0	0	0	0
25	1	1	0	0	1	0
26	1	1	0	1	0	1
27	1	1	0	1	1	1
28	1	1	1	0	0	1
29	1	1	1	0	1	0
30	1	1	1	1	0	0
31	1	1	1	1	1	0

<i>DE</i> \ <i>BC</i>	00	01	11	10
00	1	0	0	0
01	0	0	0	1
11	0	0	1	0
10	1	1	0	0

<i>DE</i> \ <i>BC</i>	00	01	11	10
00	1	0	0	0
01	0	0	0	1
11	0	0	1	0
10	1	1	0	0

⏟
 \bar{A}

DE \ BC	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	1	0	0	0
10	0	0	1	1

DE \ BC	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	1	0	0	0
10	0	0	1	1

A

Suma de productos

$$F(A,B,C,D,E) = \underbrace{\bar{A}}_X (\underbrace{C\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E} + BC\bar{D}\bar{E} + \bar{B}C\bar{D}}_Z) + \underbrace{A}_X (\underbrace{\bar{B}C\bar{D}\bar{E} + BC\bar{D}\bar{E} + \bar{B}C\bar{D}}_Y)$$

Producto de sumas

$$F(A,B,C,D,E) = \underbrace{(A + (C + \bar{D})(B + \bar{B})(\bar{C} + D)(\bar{B} + \bar{D} + E))}_X \underbrace{(\bar{A} + (D + \bar{E})(B + \bar{C})(\bar{B} + C + D)(B + \bar{D}) + \bar{C} + \bar{D}))}_{\bar{X}} \underbrace{(\bar{B} + C + D)(B + \bar{D}) + \bar{C} + \bar{D})}_W$$

$$XY + \bar{X}Z = (X + V)(\bar{X} + W) \rightarrow (X + Z)(\bar{X} + Y) = (X + V)(\bar{X} + W)$$

$$Z = V \rightarrow \bar{Z}V = 0, Y = W$$

$$\underbrace{(\bar{C}\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E} + BC\bar{D}\bar{E} + \bar{B}C\bar{D})}_Z = \underbrace{(C + \bar{D})(B + \bar{B})(\bar{C} + D)(\bar{B} + \bar{D} + E))}_V$$

Ley de Morgan en Z

$$\bar{Z} = \overline{(\bar{C}\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E} + BC\bar{D}\bar{E} + \bar{B}C\bar{D})} = (C + D + E)(B + \bar{C} + \bar{D} + E)(\bar{B}C\bar{D}\bar{E})(\bar{B} + C + D)$$

$$\bar{Z}V = \underbrace{(C + D + E)}_{\textcircled{1}} \underbrace{(B + \bar{C} + \bar{D} + E)}_{\textcircled{2}} \underbrace{(\bar{B} + \bar{C} + \bar{D} + \bar{E})}_{\textcircled{3}} \underbrace{(\bar{B} + C + D)}_{\textcircled{4}} \underbrace{(C + \bar{D})}_{\textcircled{5}} \underbrace{(B + \bar{E})}_{\textcircled{6}} \underbrace{(\bar{C} + D)}_{\textcircled{7}} \underbrace{(\bar{B} + \bar{D} + E)}_{\textcircled{8}}$$

$$\textcircled{1}Y\textcircled{4} : (C + D + E)(\bar{B} + C + D)$$

$$= \underline{C\bar{B}} + \underline{CC} + \underline{CD} + \underline{D\bar{B}} + \underline{DC} + \underline{DD} + \underline{E\bar{B}} + \underline{EC} + \underline{ED}$$

$$= \underbrace{C + D + E\bar{B}}_{\textcircled{9}} \text{ Por absorcion}$$

$$\textcircled{2}Y\textcircled{3} : (B + \bar{C} + \bar{D} + E)(\bar{B} + \bar{C} + \bar{D} + \bar{E})$$

$$= \underline{B\bar{B}} + \underline{B\bar{C}} + \underline{B\bar{D}} + \underline{B\bar{E}} + \underline{\bar{C}B} + \underline{\bar{C}\bar{C}} + \underline{\bar{C}\bar{D}} + \underline{\bar{C}\bar{E}} + \underline{\bar{D}B} + \underline{\bar{D}\bar{C}} + \underline{\bar{D}\bar{D}} + \underline{\bar{D}\bar{E}} + \underline{EB} + \underline{E\bar{C}} + \underline{E\bar{D}} + \underline{E\bar{E}}$$

$$= \underbrace{\bar{B}\bar{E} + \bar{C} + \bar{D} + E\bar{B}}_{\textcircled{10}} \text{ Por } X\bar{X} = 0 \text{ y absorcion}$$

$$\begin{aligned}
& \textcircled{5}Y\textcircled{7} : (C + \overline{D})(\overline{C} + D) \\
& = \cancel{C}\overline{C} + CD + \overline{C}\overline{D} + = \cancel{D}\overline{D} \\
& = \underbrace{CD + \overline{C}\overline{D}}_{\textcircled{11}} \text{ Por } X\overline{X} = 0 \\
& \textcircled{6}Y\textcircled{8} : (B + \overline{E})(\overline{B} + \overline{D} + E) \\
& = \cancel{B}\overline{B} + B\overline{D} + BE + \overline{E}\overline{B} + \overline{E}\overline{D} + \cancel{E}\overline{E} \\
& = \underline{B\overline{D}} + BE + \overline{E}\overline{B} + \overline{E}\overline{D} \text{ Por } X\overline{X} = 0 \\
& = \underbrace{BE + \overline{E}\overline{B} + \overline{E}\overline{D}}_{\textcircled{12}} \text{ Por consenso}
\end{aligned}$$

$$\rightarrow (C + D + E\overline{B})(\overline{B}\overline{E} + \overline{C} + \overline{D} + E\overline{B})(CD + \overline{C}\overline{D})(BE + \overline{B}\overline{E} + \overline{D}\overline{E})$$

$$\begin{aligned}
& \textcircled{9}Y\textcircled{11} : (C + D + E\overline{B})(CD + \overline{C}\overline{D}) \\
& = CCD + \cancel{C}\overline{C}\overline{D} + \underline{DCD} + \cancel{D}\overline{D}\overline{C} + \underline{E\overline{B}C\overline{D}} + E\overline{B}\overline{C}\overline{D} \\
& = \underbrace{CD + E\overline{B}\overline{C}\overline{D}}_{\textcircled{13}} \text{ Por } X\overline{X} = 0, \text{ Por absorcion y } X\overline{X} = X
\end{aligned}$$

$$\begin{aligned}
& \textcircled{10}Y\textcircled{12} : (\overline{B}\overline{E} + \overline{C} + \overline{D} + E\overline{B})(BE + \overline{B}\overline{E} + \overline{D}\overline{E}) \\
& = \underline{\overline{B}\overline{E}\overline{B}\overline{E}} + \underline{\overline{B}\overline{E}\overline{E}\overline{B}} + \underline{\overline{B}\overline{E}\overline{E}\overline{D}} + \underline{\overline{C}BE} + \underline{\overline{B}\overline{C}\overline{E}} + \underline{\overline{C}\overline{E}\overline{D}} + \underline{\overline{D}BE} + \underline{\overline{B}\overline{D}\overline{E}} + \underline{\overline{E}\overline{B}\overline{B}\overline{E}} + \underline{\overline{E}\overline{B}\overline{E}\overline{B}} + \underline{\overline{E}\overline{B}\overline{E}\overline{D}} \\
& = \underline{\overline{C}BE + \overline{C}\overline{E}\overline{B} + \overline{D}BE + \overline{D}\overline{E}} \text{ Por absorcion y } X\overline{X} = 0 \\
& \textcircled{14}
\end{aligned}$$

$$\textcircled{13} \text{ y } \textcircled{14} : (CD + E\overline{B}\overline{D}\overline{C})(\overline{C}BE + \overline{C}\overline{E}\overline{B} + \overline{D}BE + \overline{D}\overline{E})$$

$$= CD\overline{C}BE + CD\overline{C}\overline{E}\overline{B} + CD\overline{D}BE + CD\overline{D}\overline{E} + E\overline{B}\overline{D}\overline{C}\overline{C}BE + E\overline{B}\overline{D}\overline{C}\overline{C}\overline{E}\overline{B} + E\overline{B}\overline{D}\overline{C}\overline{D}BE + E\overline{B}\overline{D}\overline{C}\overline{D}\overline{E}$$

$$= 0 \text{ por } X * \overline{X} = 0 \text{ por lo tanto } \overline{Z} * V = 0$$

$$\rightarrow y = w \rightarrow \overline{y} \cdot w = 0$$

$$\rightarrow (\overline{B}\overline{C}\overline{D}\overline{E} + B\overline{C}\overline{D}\overline{E} + B\overline{C}\overline{D}) = (D + \overline{E})(B + \overline{C})(\overline{B} + C + D)(B + \overline{D})(\overline{C} + \overline{D})$$

Aplicamos Ley de Morgan en y :

$$\bar{Y} = \overline{(BCDE + BC\bar{D}E + B\bar{C}D)} = (B + C + D + E)(\bar{B} + \bar{C} + D + E)(\bar{B} + C + \bar{D})$$

$$\bar{Y} \cdot W = \underbrace{(B + C + D + E)}_{\textcircled{1}} \underbrace{(\bar{B} + \bar{C} + D + E)}_{\textcircled{2}} \underbrace{(\bar{B} + C + \bar{D})}_{\textcircled{3}} \underbrace{(D + \bar{E})}_{\textcircled{4}} \underbrace{(B + \bar{C})}_{\textcircled{5}} \underbrace{(\bar{B} + C + D)}_{\textcircled{6}} \underbrace{(B + \bar{D})}_{\textcircled{7}} \underbrace{(\bar{C} + \bar{D})}_{\textcircled{8}}$$

$$\textcircled{1} \text{ y } \textcircled{5}: (B + C + D + E)(B + \bar{C})$$

$$= BB + \underline{B\bar{C}} + \underline{CB} + \cancel{CC} + \underline{DB} + \underline{D\bar{C}} + \underline{EB} + \underline{E\bar{C}}$$

$$= B + D\bar{C} + E\bar{C} \textcircled{9} \rightarrow \text{por ley de absorción y } X \cdot \bar{X} = 0$$

$$\textcircled{2} \text{ Y } \textcircled{6}: (\bar{B} + \bar{C} + D + E)(\bar{B} + C + D)$$

$$= \bar{B}\bar{B} + \underline{\bar{B}C} + \underline{\bar{B}D} + \underline{\bar{C}\bar{B}} + \cancel{\bar{C}C} + \underline{\bar{C}D} + \underline{D\bar{B}} + \underline{DC} + DD + \underline{E\bar{B}} + EC + ED$$

$$= \bar{B} + D + EC + ED \textcircled{10} \rightarrow \text{por } X \cdot \bar{X} = 0 \text{ y absorción.}$$

$$\textcircled{3} \text{ Y } \textcircled{8}: (\bar{B} + C + \bar{D})(\bar{C} + \bar{D})$$

$$= \bar{B}\bar{C} + \underline{\bar{B}\bar{D}} + \cancel{C\bar{C}} + \underline{C\bar{D}} + \underline{\bar{D}\bar{C}} + \bar{D}\bar{D}$$

$$= \bar{B}\bar{C} + \bar{D} \rightarrow \textcircled{11} \text{ por } X \cdot \bar{X} = 0 \text{ y absorción.}$$

$$\textcircled{4} \text{ Y } \textcircled{9}: (D + \bar{E})(B + \bar{D})$$

$$= DB + \cancel{D\bar{D}} + \underline{\bar{E}B} + \underline{\bar{E}\bar{D}}$$

$$= DB + \bar{E}\bar{D} \rightarrow \textcircled{12} \text{ por } X \cdot \bar{X} = 0 \text{ y consenso.}$$

$$\rightarrow \underbrace{(B + D\bar{C} + E\bar{C})}_{\textcircled{9}} \underbrace{(\bar{B} + D + EC + ED)}_{\textcircled{10}} \underbrace{(\bar{B}\bar{C} + \bar{D})}_{\textcircled{11}} \underbrace{(DB + \bar{E}\bar{D})}_{\textcircled{12}}$$

$$\textcircled{9} \text{ Y } \textcircled{12}: (B + D\bar{C} + E\bar{C})(DB + \bar{E}\bar{D})$$

$$= BDB + \underline{B\bar{E}\bar{D}} + \underline{D\bar{C}DB} + \cancel{D\bar{C}\bar{E}\bar{D}} + \underline{E\bar{C}DB} + \cancel{E\bar{C}\bar{E}\bar{D}}$$

$$= BD + \bar{B}\bar{E}\bar{D} \textcircled{13} \rightarrow \text{por absorción y } X \cdot \bar{X} = 0$$

$$\textcircled{10} \text{ Y } \textcircled{11}: (\overline{B} + D + EC + ED)(\overline{BC} + \overline{D})$$

$$= \overline{B}\overline{BC} + \overline{B}\overline{D} + \underline{D}\overline{BC} + \cancel{D}\overline{D} + EC\overline{BC} + EC\overline{D} + \underline{ED}\overline{BC} + \cancel{ED}\overline{D}$$

$$= \overline{BC} + \overline{BD} + EC\overline{D} \rightarrow \textcircled{14} \text{ por absorción y } X \cdot \overline{X} = 0$$

$$\textcircled{13} \text{ Y } \textcircled{14}: (BD + B\overline{ED})(\overline{BC} + \overline{BD} + EC\overline{D})$$

$$= BD\overline{BD} + BD\overline{BC} + BDE\overline{CD} + \overline{B}ED\overline{BC} + \overline{B}ED\overline{BD} + \overline{B}E\overline{D}EC\overline{D}$$

$$= 0 \rightarrow \text{por } X \cdot \overline{X} = 0$$

$$\text{Por lo tanto, } \overline{Y} \cdot W = 0 \text{ o } \overline{Z} \cdot V = \overline{Y} \cdot W$$

Punto 5

$$F(A,B,C,D,E) = \prod M(0, 6, 8, 9, 15, 16, 26, 27, 28) \\ = \sum m(1, 2, 3, 4, 5, 7, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31)$$

	A	B	C	D	E	Y
0	0	0	0	0	0	0
1	0	0	0	0	1	1
2	0	0	0	1	0	1
3	0	0	0	1	1	1
5	0	0	1	0	1	1
6	0	0	1	1	0	0
7	0	0	1	1	1	1
8	0	1	0	0	0	0
9	0	1	0	0	1	0
10	0	1	0	1	0	1
11	0	1	0	1	1	1
12	0	1	1	0	0	1
13	0	1	1	0	1	1
14	0	1	1	1	0	1
15	0	1	1	1	1	0
16	1	0	0	0	0	0
17	1	0	0	0	1	1
18	1	0	0	1	0	1
19	1	0	0	1	1	1
20	1	0	1	0	0	1
21	1	0	1	0	1	1
22	1	0	1	1	0	1
23	1	0	1	1	1	1
24	1	1	0	0	0	1
25	1	1	0	0	1	1
26	1	1	0	1	0	0
27	1	1	0	1	1	0
28	1	1	1	0	0	0
29	1	1	1	0	1	1
30	1	1	1	1	0	1
31	1	1	1	1	1	1

DE \ BC	00	01	11	10
00	0	1	1	1
01	1	1	1	0
11	1	1	0	1
10	0	0	1	1

DE \ BC	00	01	11	10
00	0	1	1	1
01	1	1	1	0
11	1	1	0	1
10	0	0	1	1

⏟
 \bar{A}

BC \ DE	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	0	1	1	1
10	1	1	0	0

BC \ DE	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	0	1	1	1
10	1	1	0	0

A

Producto de sumas

$$F(A,B,C,D,E) = \underbrace{\left(\underbrace{A}_{X} + \underbrace{(C+D+E)(B+\bar{C}+\bar{D}+E)(\bar{B}+\bar{C}+\bar{D}+\bar{E})(\bar{B}+C+D)}_{Z} \right)}_{\bar{X}} \cdot \underbrace{\left(\underbrace{\bar{A}}_{\bar{X}} + \underbrace{(B+C+D+E)(\bar{B}+\bar{C}+D+E)(\bar{B}+C+\bar{D})}_{Y} \right)}_{Y}$$

Suma de productos

$$F(A,B,C,D,E) = \underbrace{\bar{A}}_{\bar{X}} \underbrace{(\bar{C}D + \bar{B}E + C\bar{D} + B\bar{D}\bar{E})}_{V} + \underbrace{A}_{X} \underbrace{(\bar{D}E + \bar{B}C + B\bar{C}\bar{D} + \bar{B}D + CD)}_{W}$$

$$XY + \bar{X}Z = (X+V)(\bar{X}+W) \rightarrow (X+Z)(\bar{X}+Y) = (X+V)(\bar{X}+W)$$

$$Z = V \rightarrow \bar{Z}V = 0, Y = W$$

$$\rightarrow \underbrace{(C+D+E)(B+\bar{C}+\bar{D}+E)(\bar{B}+\bar{C}+\bar{D}+\bar{E})(\bar{B}+C+D)}_{Z} = \underbrace{(\bar{C}D + \bar{B}E + C\bar{D} + B\bar{D}\bar{E})}_{V}$$

Ley de Morgan en Z

$$\bar{Z} = \overline{(C+D+E)(B+\bar{C}+\bar{D}+E)(\bar{B}+\bar{C}+\bar{D}+\bar{E})(\bar{B}+C+D)}$$

$$= \bar{C}\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E} + BCDE + B\bar{C}\bar{D}$$

$$\bar{Z}V = (\bar{C}\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E} + BCDE + B\bar{C}\bar{D}) \cdot (\bar{C}D + \bar{B}E + C\bar{D} + B\bar{D}\bar{E})$$

$$= \bar{C}\bar{D}\bar{E}C\bar{D} + \bar{C}\bar{D}\bar{E}B\bar{E} + \bar{C}\bar{D}\bar{E}C\bar{D} + \bar{C}\bar{D}\bar{E}B\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E}C\bar{D} + \bar{B}C\bar{D}\bar{E}B\bar{E} + \bar{B}C\bar{D}\bar{E}C\bar{D} + \bar{B}C\bar{D}\bar{E}B\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E}C\bar{D} + \bar{B}C\bar{D}\bar{E}B\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E}C\bar{D} + \bar{B}C\bar{D}\bar{E}B\bar{D}\bar{E} + \bar{B}C\bar{D}\bar{E}C\bar{D} + \bar{B}C\bar{D}\bar{E}B\bar{D}\bar{E} = 0$$

$$\text{Por } X\bar{X} = 0, \bar{Z}V = 0 \rightarrow Z = V$$

$$\blacksquare Y = W \rightarrow \bar{Y} \cdot W = 0$$

$$\rightarrow \underbrace{(B+C+D+E)(\bar{B}+\bar{C}+D+E)(\bar{B}+C+\bar{D})}_Y = \underbrace{(\bar{D}E + \bar{B}C + B\bar{C}\bar{D} + \bar{B}D + CD)}_W$$

Ley de Morgan en Y:

$$\overline{Y} = \overline{(B + C + D + E)(\overline{B} + \overline{C} + D + E)(\overline{B} + C + \overline{D})} = \overline{BCDE} + \overline{BCDE} + \overline{BCD}$$

$$\overline{Y} \cdot W = (\overline{BCDE} + \overline{BCDE} + \overline{BCD})(\overline{DE} + \overline{BC} + \overline{BCD} + \overline{BD} + CD)$$

$$= \overline{BCDEDE} + \overline{BCDEBC} + \overline{BCDEBCD} + \overline{BCDECD} + \overline{BCDEDE} + \overline{BCDEBC} + \overline{BCDEBCD} + \overline{BCDECD} + \overline{BCDDE} + \overline{BCD\overline{BC}} + \overline{BCD\overline{BCD}} + \overline{BCD\overline{BD}} + \overline{BCD\overline{CD}} = 0$$

por $X \cdot X = 0$ por lo tanto $\overline{Y} \cdot V = 0 \rightarrow Y = W$

Entonces $Y = W \wedge Z = W$

Punto 6

$$F(A,B,C,D) = \sum m(1, 2, 3, 7, 10, 11, 13), \quad d(A,B,C,D) = \sum m(4, 6, 15)$$

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	x
5	0	1	0	1	0
6	0	1	1	0	x
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	x

$\begin{array}{c} CD \\ \backslash AB \end{array}$	00	01	11	10
00	0	1	1	1
01	x	0	1	x
11	0	1	x	0
10	0	0	1	1

Grupos

$$\left| \begin{array}{l} (2, 3, 10, 11) \\ (2, 3, 6, 7) \\ (1, 3) \\ (13, 15) \end{array} \right| \begin{array}{l} \overline{B}C \\ \overline{A}C \\ \overline{A}\overline{B}D \\ ABD \end{array}$$

Suma de productos

$$F(A,B,C,D) = \overline{B}C + \overline{A}C + \overline{A}\overline{B}D + ABD$$