1

$L = \{ a^i b^j c^k \mid i \neq j \text{ o } j \neq k \}$

$S \to X \mid Y \mid \lambda$
$X \to HC$
$Y \to AN$
$H \to aHb \mid \lambda$
$C \to Cc \mid \lambda$
$A \to aA \mid \lambda$
$N \to bNc \mid Nc \mid \lambda$

(1) Remove λ

$S \to X \mid Y$
$X \to H \mid C \mid HC$
$Y \to A \mid N \mid AN$
$H \to aHb \mid ab$
$C \to Cc \mid c$
$A \to aA \mid a$
$N \to bNc \mid Nc \mid bc \mid c$

(2) Unitary unitarios

$S \to aHb \mid ab \mid Cc \mid c \mid HC \mid aA \mid a \mid Nc \mid Nc \mid bc \mid c \mid AN$

eliminamos X e Y

(3) Símbolos inútiles

Debido a que reemplazamos

terminales $\{a, b, c\}$
generadores $\{H, C, A, N, a, b, c\}$
$\Downarrow$
$\{S, H, C, A, N, a, b, c\}$

ya no hay símbolos inútiles

Chomsky

terminales $\to$ variables

$T_1 \to a$
$T_2 \to b$
$T_3 \to c$

$S \to T_1 H T_2 \mid T_1 T_2 \mid C T_3 \mid T_3 \mid HC \mid T_1 A \mid T_1 \mid T_2 N T_3 \mid N T_3 \mid T_2 T_3 \mid T_3 \mid \lambda$

$H \to T_1 H T_2 \mid T_1 T_2$

$C \to C T_3 \mid T_3$

$A \to T_1 A \mid T_1$

$N \to T_2 N T_3 \mid N T_3 \mid T_2 T_3 \mid T_3$

A longitud 3

X. casi está, cambiando:

$X_1 \to H T_2$

$X_2 \to N T_3$

2

$a^i b^j a^k \quad j = i+k \mid i,j,k \geq 0$

$S \to XY$
$X \to aXb \mid \lambda$
$Y \to bYa \mid \lambda$

todos son generadores, no hay inútiles

eliminando λ

$S \to X \mid Y \mid XY$
$X \to aXb \mid ab$
$Y \to bYa \mid ba$

longitud 2

$H \to Xb$
$N \to Ya$

unitarios

$S \to aXb \mid ab \mid bYa \mid ba \mid XY$
$X \to aXb \mid ab$
$Y \to bYa \mid ba$

$S \to aH \mid ab \mid bN \mid ba \mid XY$
$X \to aH \mid ab$
$Y \to bN \mid ba$

quedó sin reemplazar los terminales por variables

3

$(a+b)^* \quad N(a) \neq N(b)$

$S \to X_a X \mid Y_b Y$
$X \to X_a X X_b X \mid X_b X \mid aX \mid aX \mid \lambda$
$Y \to Y_a Y b Y \mid Y_b Y \mid aY \mid bY \mid \lambda$

eliminando λ

$S \to X_a X \mid a X \mid X_a \mid a \mid Y_b Y \mid bY \mid Y_b \mid b$
$X_a \to ab \mid aX_b X \mid X_a b X \mid X_a b \mid aX_b \mid ab X \mid X_a X_b X \mid$
$ba \mid bX_a X \mid X_b a X \mid X_b X_a \mid X_b a \mid bX_a \mid ba X \mid X_b X_a X \mid aX \mid a$
$Y \to ab \mid aY_b Y \mid Y_a b Y \mid Y_a Y_b \mid Y_a b \mid aY_b \mid abY \mid Y_a Y_b Y \mid$
$ba \mid bY_a Y \mid Y_b a Y \mid Y_b Y_a \mid Y_b a \mid bY_a \mid ba Y \mid Y_b Y_a Y \mid bY \mid b$

No hay unitarios

No hay inútiles

Pasar a longitud 2 $\to$ 26 variables auxiliares ... :/

4

$a^i b^j c^k \quad k \neq i+j$

eliminar λ $\to$ no hay

transiciones unitarias

$S \to M \mid N$
$M \to A \mid B \mid AB \mid W$
$W \to AW_c \mid X \mid a$
$X \to BX_c \mid b$
$N \to C \mid R$
$R \to ARC \mid Y \mid c$
$Y \to bYC \mid c$
$A \to A_a \mid a$
$B \to B_b \mid b$
$C \to C_c \mid c$

$S \to M \mid N$
$M \to A \mid B \mid W$
$W \to X$
$N \to C \mid R$
$R \to Y$

$S \to A_a \mid a \mid B_b \mid b \mid AB \mid AW_c \mid BX_c \mid b \mid a$
$M \to A_a \mid a \mid B_b \mid b \mid AB \mid AW_c \mid BX_c \mid b \mid a$
$W \to AW_c \mid BX_c \mid b \mid a$
$X \to BX_c \mid b$
$N \to C_c \mid a \mid ARC \mid bYC \mid c \mid c$
$R \to ARC \mid bYC \mid c \mid c$
$Y \to bYC \mid c$
$A \to A_a \mid a$
$B \to B_b \mid b$
$C \to C_c \mid c$

transiciones inútiles

$S \to A_a \mid a \mid B_b \mid b \mid C_c \mid c \mid AW_c \mid BX_c \mid ARC \mid bYC$
$M \to A_a \mid a \mid B_b \mid b \mid AW_c \mid BX_c$
$W \to AW_c \mid BX_c \mid b \mid a$
$X \to BX_c \mid b$
$N \to C_c \mid c \mid ARC \mid bYC$
$R \to ARC \mid bYC \mid c$
$Y \to bYC \mid c$
$A \to A_a \mid a$
$B \to B_b \mid b$
$C \to C_c \mid c$

Podemos pasar a longitud 2 con:

$Z_1 \to X_c$
$Z_2 \to RC$
$Z_3 \to YC$
$Z_4 \to WC$

Y cuentan pasando terminales a variables

5

$S \to aSD \mid AT$
$T \to BTC \mid B$
$A \to aA \mid a$
$B \to bB \mid b$
$C \to c \mid \lambda$
$D \to d \mid \lambda$

elimina λ

$S \to aSD \mid aSD \mid AT$
$T \to BTC \mid BT \mid B$
$A \to aA \mid a$
$B \to bB \mid b$
$C \to c$
$D \to d$

unitarios

$S \to aS \mid aSD \mid AT$
$T \to BTC \mid BT \mid B$
$A \to aA \mid a$
$B \to bB \mid b$

No hay inútiles

terminales = abc
generadores = $A\ BT\ abc$
$\Downarrow$
$S\ ABT\ abc$

pasar los terminales a variables

$S \to aS \mid aX \mid AT$ longitud 2
$T \to BX \mid BT \mid B$
$A \to aA \mid a$
$B \to bB \mid b$

$X_1 \to SD$
$X_2 \to TC$

6

$a^i (bc)^k a^j \quad k \neq i+j$

$S \to AXY \mid XYA \mid AXYA \mid XBY$
$X \to aXZ \mid \lambda$
$Y \to ZYa \mid \lambda$
$Z \to bc$
$B \to bB \mid b \mid cB \mid c$
$A \to Aa \mid a$

λ

$S \to AX \mid AY \mid A \mid XA \mid YA \mid B \mid XB \mid BY \mid AX \mid AXY \mid XYA \mid AXYA \mid XBY \mid AXA \mid AYA$
$X \to aXZ \mid aZ$
$Y \to ZYa \mid Za$
$Z \to bc$
$B \to bB \mid b \mid cB \mid c$
$A \to Aa \mid a$

Símbolos inútiles

terminales = a b c
generadores = $Z\ B\ A\ a\ bc$
$\Downarrow$
$X\ Y\ Z\ B\ A\ a\ bc$

$S\ X\ Y\ Z\ B\ A\ a\ bc$

No hay

transiciones unitarias

Combinando en S

$S \to A \to S \to Aa \mid a$
$S \to B \to S \to bB \mid b \mid cB \mid c$

7

$a^i b^j c^k a^m b^p \quad i = j+k \text{ o } m+p$

$S \to MN$
$M \to aMc \mid O$
$O \to aOb \mid \lambda$
$N \to aNb \mid \lambda$

elimina λ

$S \to MN \mid M \mid N$
$M \to aMc \mid O \mid ac$
$O \to aOb \mid ab$
$N \to aNb \mid ab$

elimina unitarios

$S \to MN \mid aMc \mid aOb \mid ab \mid ac \mid aNb$
$M \to aMc \mid aOb \mid ab \mid ac$
$O \to aOb \mid ab$
$N \to aNb \mid ab$

terminales a variables

$S \to MN \mid aX_c \mid aX_b \mid ab \mid ac \mid aX_b$
$M \to aX_c \mid aX_c \mid ab \mid ac$
$O \to aX_b \mid ab$
$N \to aX_b \mid ab$

longitud 2

$X_c \to Mc$
$X_b \to Ob$
$X_b \to Nb$

todos son generadores, no hay inútiles

8

$a^i b^j c^k d^n \quad i+k = j+n \text{ o } i+j+k = n$

$S \to X \mid Y$
$X \to aXd \mid M$
$M \to bMd \mid N$
$N \to cNd \mid \lambda$
$Y \to aYd \mid PQR$
$P \to aPb \mid \lambda$
$Q \to bQc \mid \lambda$
$R \to cRd \mid \lambda$

sin λ

No hay inútiles

Quitando transiciones unitarias

$S \to X \mid Y$
$X \to aXd \mid M$
$M \to bMd \mid N$
$N \to cNd \mid cd$
$Y \to aYd \mid PQR \mid QR \mid PR \mid PQ \mid P \mid Q \mid R$
$P \to aPb \mid ab$
$Q \to bQc \mid bc$
$R \to cRd \mid cd$

todos son generadores, no hay inútiles

$S \to aXd \mid bMd \mid cNd \mid cd \mid aYd \mid PQR \mid QR \mid PR \mid PQ \mid aPb \mid ab \mid bQc \mid bc \mid cRd \mid cd$

$X \to aXd \mid bMd \mid cNd \mid cd$
$M \to bMd \mid cNd \mid cd$
$N \to cNd \mid cd$
$Y \to aYd \mid PQR \mid QR \mid PR \mid PQ \mid aPb \mid ab \mid bQc \mid bc \mid cRd \mid cd$
$P \to aPb \mid ab$
$Q \to bQc \mid bc$
$R \to cRd \mid cd$