practisingTrading

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```r
library(PortfolioAnalytics)
```

```
## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: xts

## Loading required package: foreach

## Loading required package: PerformanceAnalytics

##
## Attaching package: 'PerformanceAnalytics'

## The following object is masked from 'package:graphics':
##
##     legend
```

```r
library(foreach)
library(iterators)
library(ROI)
```

```
## ROI: R Optimization Infrastructure

## Registered solver plugins: nlminb, glpk, quadprog.

## Default solver: auto.

##
## Attaching package: 'ROI'

## The following objects are masked from 'package:PortfolioAnalytics':
##
##     is.constraint, objective
```

```r
library(ROI.plugin.quadprog)
library(ROI.plugin.glpk)
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------------- tidyverse 1.3.0 --

## v ggplot2 3.3.2     v purrr   0.3.4
## v tibble  3.0.1     v dplyr   1.0.0
## v tidyr   1.1.0     v stringr 1.4.0
```

```
## v readr   1.3.1      v forcats 0.5.0

## -- Conflicts --------------------------------------------------- tidyverse_conflicts() --
## x purrr::accumulate() masks foreach::accumulate()
## x dplyr::filter()     masks stats::filter()
## x dplyr::first()      masks xts::first()
## x dplyr::lag()        masks stats::lag()
## x dplyr::last()       masks xts::last()
## x purrr::when()       masks foreach::when()
```

```r
library(tidyquant)
```

```
## Loading required package: lubridate

##
## Attaching package: 'lubridate'

## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union

## Loading required package: quantmod

## Loading required package: TTR

## Version 0.4-0 included new data defaults. See ?getSymbols.

## == Need to Learn tidyquant? ============================================================
## Business Science offers a 1-hour course - Learning Lab #9: Performance Analysis & Portfolio Optimiza
## </> Learn more at: https://university.business-science.io/p/learning-labs-pro </>
```

```r
library(PerformanceAnalytics)
library(PortfolioAnalytics)
library(dplyr)
library(xts)
library(zoo)
library(tibble)

#Maximizing Mean Return
#The objective to maximize mean return is a linear problem of the form:

#maximize    wμ^(l)
#w

#Where μ^ is the estimated mean asset returns and w is the set of weights. Because this
#is a linear problem, it is well suited to be solved using a linear programming solver. For
#these types of problems, PortfolioAnalytics uses the ROI package with the glpk plugin




data(edhec)
data(edhec)
returns <- edhec[, 1:4]
colnames(returns) <- c("CA", "CTAG", "DS", "EM")
print(head(returns, 5))
```

```
##                 CA    CTAG      DS      EM
```

```
## 1997-01-31 0.0119  0.0393  0.0178  0.0791
## 1997-02-28 0.0123  0.0298  0.0122  0.0525
## 1997-03-31 0.0078 -0.0021 -0.0012 -0.0120
## 1997-04-30 0.0086 -0.0170  0.0030  0.0119
## 1997-05-31 0.0156 -0.0015  0.0233  0.0315
```

```
tail(returns,5)
```

```
##                 CA    CTAG     DS     EM
## 2019-07-31 0.0032  0.0206 -0.0032  0.0044
## 2019-08-31 0.0019  0.0300 -0.0089 -0.0348
## 2019-09-30 0.0023 -0.0273 -0.0022  0.0076
## 2019-10-31 0.0032 -0.0204 -0.0033  0.0204
## 2019-11-30 0.0060  0.0058 -0.0043 -0.0008
```

```r
#library(zoo)
#z <- read.zoo(df)
#returns <- as.data.frame(returns)
#class(returns)

#rownames_to_column()

#returns <- rownames_to_column(returns, var="Fecha")




#MyReturns <- returns %>%
#  filter(EM >= "0.0060" & EM <= "0.0065")


# Get a character vector of the fund names
funds <- colnames(returns)

# Create portfolio object

portf_maxret <- portfolio.spec(assets=funds)

# Add constraints to the portfolio object
portf_maxret <- add.constraint(portfolio=portf_maxret, type="full_investment")

portf_maxret <- add.constraint(portfolio=portf_maxret, type="box",
                           min=c(0.02, 0.05, 0.03, 0.02),
                           max=c(0.55, 0.6, 0.65, 0.5))



# Add objective to the portfolio object
portf_maxret <- add.objective(portfolio=portf_maxret, type="return", name="mean")


summary(portf_maxret)
```

```
## $assets
##   CA CTAG   DS   EM
```

```
## 0.25 0.25 0.25 0.25
##
## $enabled_constraints
## $enabled_constraints[[1]]
## An object containing 6 nonlinear constraints.
##
## $enabled_constraints[[2]]
## An object containing 5 nonlinear constraints.
##
##
## $disabled_constraints
## list()
##
## $enabled_objectives
## $enabled_objectives[[1]]
## $name
## [1] "mean"
##
## $target
## NULL
##
## $arguments
## list()
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] -1
##
## $call
## add.objective(portfolio = portf_maxret, type = "return", name = "mean")
##
## attr(,"class")
## [1] "return_objective" "objective"
##
##
## $disabled_objectives
## list()
##
## attr(,"class")
## [1] "summary.portfolio"
```

```
# Run the optimization

opt_maxret <- optimize.portfolio(R=returns, portfolio=portf_maxret,
                                 optimize_method="ROI", trace=TRUE)


print(opt_maxret)
```

```
## *********************************
## PortfolioAnalytics Optimization
## *********************************
##
```

```
## Call:
## optimize.portfolio(R = returns, portfolio = portf_maxret, optimize_method = "ROI",
##     trace = TRUE)
##
## Optimal Weights:
##   CA CTAG   DS   EM
## 0.02 0.05 0.65 0.28
##
## Objective Measure:
##     mean
## 0.006371
```

```
opt_maxret
```

```
## **********************************
## PortfolioAnalytics Optimization
## **********************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_maxret, optimize_method = "ROI",
##     trace = TRUE)
##
## Optimal Weights:
##   CA CTAG   DS   EM
## 0.02 0.05 0.65 0.28
##
## Objective Measure:
##     mean
## 0.006371
```

```
summary(opt_maxret)
```

```
## **************************************************
## PortfolioAnalytics Optimization Summary
## **************************************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_maxret, optimize_method = "ROI",
##     trace = TRUE)
##
## Optimal Weights:
##   CA CTAG   DS   EM
## 0.02 0.05 0.65 0.28
##
## Objective Measures:
##     mean
## 0.006371
##
##
## Portfolio Assets and Initial Weights:
##   CA CTAG   DS   EM
## 0.25 0.25 0.25 0.25
##
## **************************************************
## PortfolioAnalytics Portfolio Specification
```

```
## **************************************************
##
## Call:
## portfolio.spec(assets = funds)
##
## Number of assets: 4
## Asset Names
## [1] "CA"   "CTAG" "DS"   "EM"
##
## Constraints
## Enabled constraint types
##       - full_investment
##       - box
##
## Objectives:
## Enabled objective names
##       - mean
##
## ****************************************
## Constraints
## ****************************************
## Leverage Constraint:
## min_sum = 1
## max_sum = 1
## actual_leverage = 1
##
## Box Constraints:
## min:
##    CA CTAG   DS   EM
## 0.02 0.05 0.03 0.02
## max:
##    CA CTAG   DS   EM
## 0.55 0.60 0.65 0.50
##
## Position Limit Constraints:
## Maximum number of non-zero weights, max_pos:
## [1] "Unconstrained"
## Realized number of non-zero weights (i.e. positions):
## [1] 4
##
## Maximum number of long positions, max_pos_long:
## [1] "Unconstrained"
## Realized number of long positions:
## [1] 4
##
## Maximum number of short positions, max_pos_short:
## [1] "Unconstrained"
## Realized number of short positions:
## [1] 0
##
##
## Diversification Target Constraint:
## [1] "Unconstrained"
##
```

```
## Realized diversification:
## [1] 0.4962
##
## Turnover Target Constraint:
## [1] "Unconstrained"
##
## Realized turnover from initial weights:
## [1] 0.215
##
## ***************************************
## Objectives
## ***************************************
##
## Objective: return_objective
## $name
## [1] "mean"
##
## $target
## NULL
##
## $arguments
## list()
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] -1
##
## $call
## add.objective(portfolio = portf_maxret, type = "return", name = "mean")
##
## attr(,"class")
## [1] "return_objective" "objective"
##
## ***************************************
##
## Elapsed Time:
## Time difference of 0.01216578 secs
```

```r
names(opt_maxret)
```

```
##  [1] "weights"           "objective_measures" "opt_values"
##  [4] "out"               "call"               "portfolio"
##  [7] "R"                 "data_summary"       "elapsed_time"
## [10] "end_t"
```

```r
extractStats(opt_maxret)
```

```
##        mean          out          w.CA         w.CTAG         w.DS          w.EM
##  0.006370927 -0.006370927  0.020000000  0.050000000  0.650000000  0.280000000
```
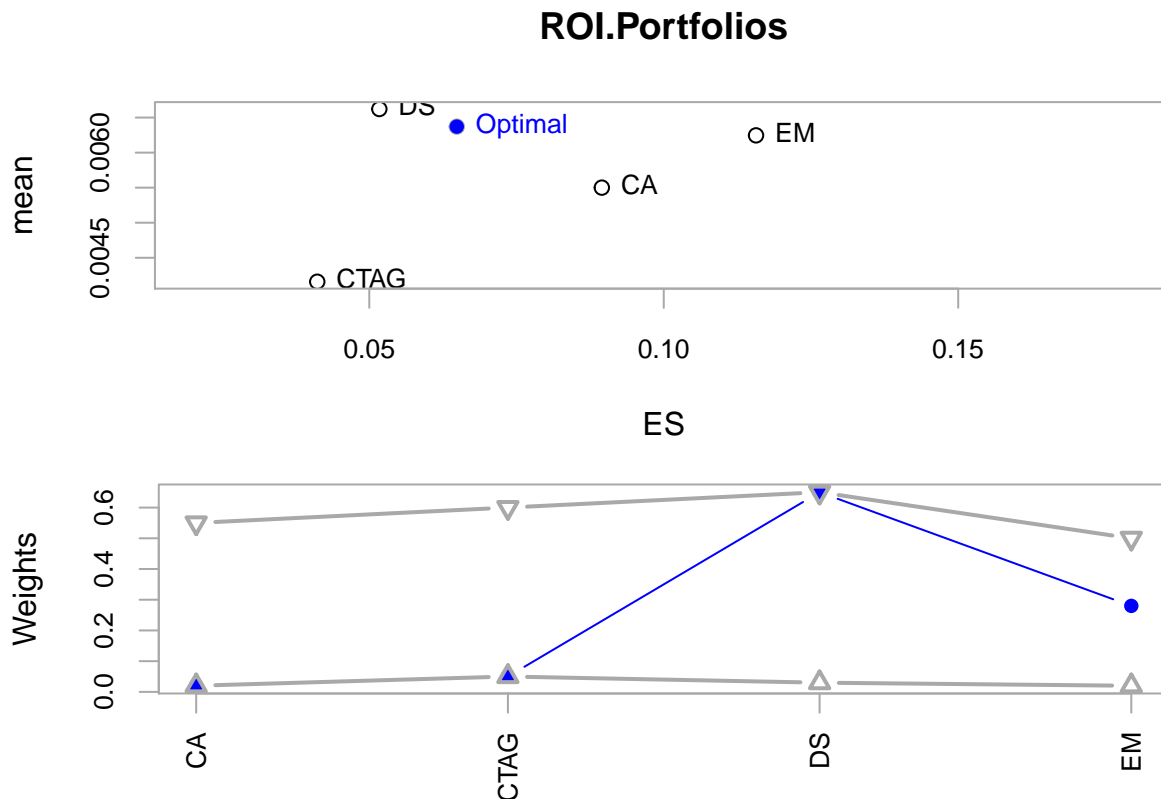
```r
extractWeights(opt_maxret)
```

```
##   CA CTAG   DS   EM
## 0.02 0.05 0.65 0.28
```

```r
#The plot method charts of the optimal weights with the box constraints along with the
#optimal portfolio in risk-return space. The blue dots are the optimal weights and the gray
#triangles are the min and max of the box constraints.
library(graphics)
plot.new()
plot(opt_maxret, chart.assets=TRUE, xlim=c(0.02, 0.18))
```
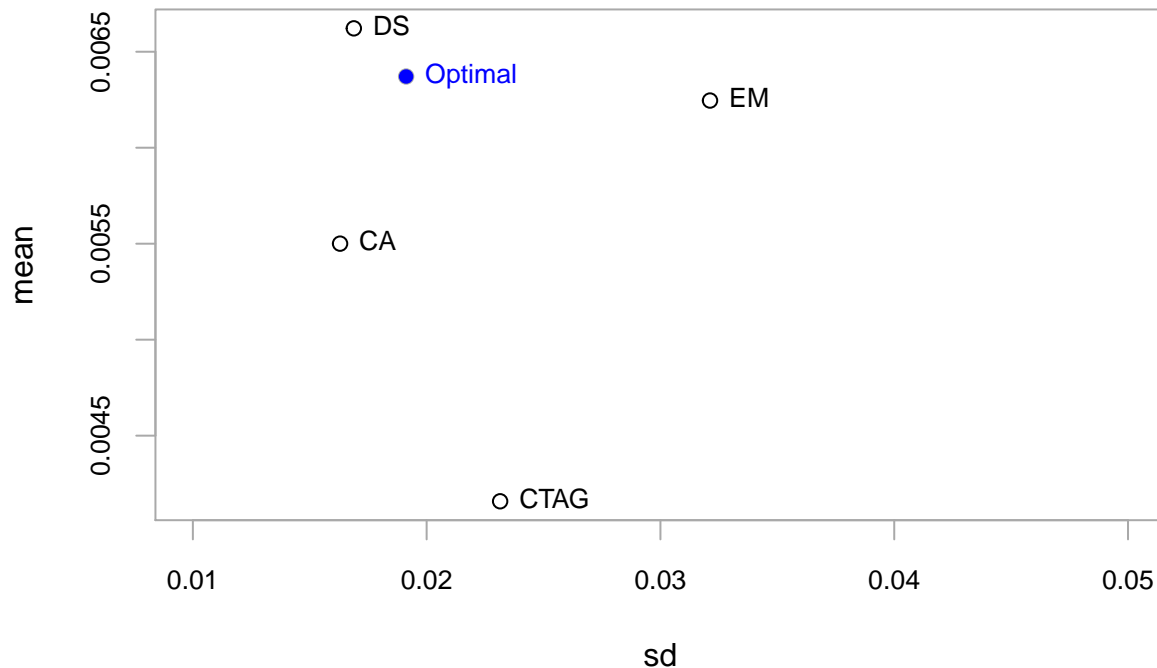
ROI.Portfolios


```r
#The optimal portfolio can be plotted in risk-return space along with other feasible
#portfolios. The return metric is defined in the return.col argument and the risk metric
#is defined in the risk.col argument. The scatter chart includes the optimal portfolio (blue
#dot) and other feasible portfolios (gray circles) to show the overall feasible space given the
#constraints. By default, if rp is not passed in, the feasible portfolios are generated with
#random_portfolios to satisfy the constraints of the portfolio object.
#Volatility as the risk metric
library(PerformanceAnalytics)
library(PortfolioAnalytics)

chart.RiskReward(opt_maxret,return.col="mean", risk.col="sd",
                 chart.assets=TRUE, xlim=c(0.01, 0.05), main="Maximum Return")
```

Maximum Return


```
#Backtesting
#An out of sample backtest is run with optimize.portfolio.rebalancing. In this example, an initial trai
#quarterly


bt_maxret <- optimize.portfolio.rebalancing(R=returns, portfolio=portf_maxret,
                                             optimize_method="ROI",
                                             rebalance_on="quarters",
                                             training_period=36)
```

```
## Warning: executing %dopar% sequentially: no parallel backend registered
```

```
bt_maxret
```

```
## ***************************************************
## PortfolioAnalytics Optimization with Rebalancing
## ***************************************************
##
## Call:
## optimize.portfolio.rebalancing(R = returns, portfolio = portf_maxret,
##     optimize_method = "ROI", rebalance_on = "quarters", training_period = 36)
##
## Number of rebalancing dates:  81
## First rebalance date:
## [1] "1999-12-31"
## Last rebalance date:
## [1] "2019-11-30"
##
## Annualized Portfolio Rebalancing Return:
## [1] 0.06755494
##
```

```
## Annualized Portfolio Standard Deviation:
## [1] 0.06116238
```

```
summary(bt_maxret)
```

```
## *****************************************************
## PortfolioAnalytics Optimization with Rebalancing
## *****************************************************
##
## Call:
## optimize.portfolio.rebalancing(R = returns, portfolio = portf_maxret,
##     optimize_method = "ROI", rebalance_on = "quarters", training_period = 36)
##
## First rebalance date:
## [1] "1999-12-31"
##
## Last rebalance date:
## [1] "2019-11-30"
##
## Annualized Portfolio Rebalancing Return:
## [1] 0.06755494
##
## Annualized Portfolio Standard Deviation:
## [1] 0.06116238
##
## Downside Risk Measures:
##                             portfolio.returns
## Semi Deviation                       0.0140
## Gain Deviation                       0.0099
## Loss Deviation                       0.0162
## Downside Deviation (MAR=10%)         0.0152
## Downside Deviation (Rf=0%)           0.0117
## Downside Deviation (0%)              0.0117
## Maximum Drawdown                     0.2773
## Historical VaR (95%)                -0.0212
## Historical ES (95%)                 -0.0410
## Modified VaR (95%)                  -0.0274
## Modified ES (95%)                   -0.0622
```

```
names(bt_maxret)
```

```
## [1] "portfolio"      "R"              "call"           "elapsed_time"
## [5] "opt_rebalancing"
```

```
extractStats(bt_maxret)
```

```
## $`1999-12-31`
##       mean           out        w.CA         w.CTAG         w.DS          w.EM
##  0.008867083 -0.008867083  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2000-03-31`
##       mean           out        w.CA         w.CTAG         w.DS         w.EM
##  0.0101719  -0.0101719   0.4200000   0.0500000   0.0300000   0.5000000
##
## $`2000-06-30`
##       mean           out        w.CA         w.CTAG         w.DS          w.EM
```

```
##  0.009793381 -0.009793381  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2000-09-30`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009760978 -0.009760978  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2000-12-31`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.008994188 -0.008994188  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2001-03-31`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009534784 -0.009534784  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2001-06-30`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009650593 -0.009650593  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2001-09-30`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009560351 -0.009560351  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2001-12-31`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009340433 -0.009340433  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2002-03-31`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009155889 -0.009155889  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2002-06-30`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.008973348 -0.008973348  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2002-09-30`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.008659029 -0.008659029  0.550000000  0.400000000  0.030000000  0.020000000
##
## $`2002-12-31`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.008841972 -0.008841972  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2003-03-31`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009115213 -0.009115213  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2003-06-30`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009407872 -0.009407872  0.550000000  0.050000000  0.380000000  0.020000000
##
## $`2003-09-30`
##         mean          out         w.CA       w.CTAG         w.DS         w.EM
##  0.009311679 -0.009311679  0.550000000  0.050000000  0.380000000  0.020000000
##
```

```
## $`2003-12-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009517286 -0.009517286  0.280000000  0.050000000  0.650000000  0.020000000
##
## $`2004-03-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009622241 -0.009622241  0.280000000  0.050000000  0.650000000  0.020000000
##
## $`2004-06-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009377911 -0.009377911  0.280000000  0.050000000  0.650000000  0.020000000
##
## $`2004-09-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009258581 -0.009258581  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2004-12-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009727927 -0.009727927  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2005-03-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009620899 -0.009620899  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2005-06-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009443667 -0.009443667  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2005-09-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009658295 -0.009658295  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2005-12-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009598398 -0.009598398  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2006-03-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009873559 -0.009873559  0.020000000  0.050000000  0.430000000  0.500000000
##
## $`2006-06-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009719614 -0.009719614  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2006-09-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.009592889 -0.009592889  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2006-12-31`
##       mean        out       w.CA      w.CTAG       w.DS       w.EM
##  0.0098964 -0.0098964  0.0200000  0.0500000  0.4300000  0.5000000
##
## $`2007-03-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
```

```
##   0.009941878 -0.009941878   0.020000000   0.050000000   0.430000000   0.500000000
##
## $`2007-06-30`
##        mean          out        w.CA       w.CTAG         w.DS         w.EM
##   0.01016387 -0.01016387   0.02000000   0.05000000   0.43000000   0.50000000
##
## $`2007-09-30`
##        mean          out        w.CA       w.CTAG         w.DS         w.EM
##   0.01006947 -0.01006947   0.02000000   0.05000000   0.43000000   0.50000000
##
## $`2007-12-31`
##        mean          out        w.CA       w.CTAG         w.DS         w.EM
##   0.01000037 -0.01000037   0.02000000   0.05000000   0.43000000   0.50000000
##
## $`2008-03-31`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.009469563 -0.009469563   0.020000000   0.050000000   0.430000000   0.500000000
##
## $`2008-06-30`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.009369254 -0.009369254   0.020000000   0.050000000   0.430000000   0.500000000
##
## $`2008-09-30`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.008316411 -0.008316411   0.020000000   0.050000000   0.430000000   0.500000000
##
## $`2008-12-31`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.007159569 -0.007159569   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2009-03-31`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.007029884 -0.007029884   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2009-06-30`
##         mean          out        w.CA       w.CTAG         w.DS         w.EM
##   0.00772368 -0.00772368   0.02000000   0.05000000   0.43000000   0.50000000
##
## $`2009-09-30`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.008243464 -0.008243464   0.020000000   0.050000000   0.430000000   0.500000000
##
## $`2009-12-31`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.008427776 -0.008427776   0.020000000   0.050000000   0.430000000   0.500000000
##
## $`2010-03-31`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.008527208 -0.008527208   0.020000000   0.050000000   0.430000000   0.500000000
##
## $`2010-06-30`
##          mean           out         w.CA        w.CTAG          w.DS          w.EM
##   0.008234006 -0.008234006   0.020000000   0.050000000   0.650000000   0.280000000
##
```

```
## $`2010-09-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.008399364 -0.008399364  0.020000000  0.050000000  0.430000000  0.500000000
##
## $`2010-12-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.008539762 -0.008539762  0.020000000  0.050000000  0.430000000  0.500000000
##
## $`2011-03-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.008526725 -0.008526725  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2011-06-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.008365897 -0.008365897  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2011-09-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007776785 -0.007776785  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2011-12-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007691361 -0.007691361  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2012-03-31`
##        mean         out        w.CA       w.CTAG        w.DS        w.EM
##  0.00788053 -0.00788053  0.02000000  0.05000000  0.65000000  0.28000000
##
## $`2012-06-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007600333 -0.007600333  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2012-09-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007693063 -0.007693063  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2012-12-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007794495 -0.007794495  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2013-03-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007873144 -0.007873144  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2013-06-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007810646 -0.007810646  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2013-09-30`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
##  0.007802289 -0.007802289  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2013-12-31`
##         mean          out         w.CA        w.CTAG         w.DS         w.EM
```

```
##   0.007875696 -0.007875696   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2014-03-31`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.007871391 -0.007871391   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2014-06-30`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.00789241  -0.00789241    0.02000000    0.05000000    0.65000000    0.28000000
##
## $`2014-09-30`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.007701629 -0.007701629   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2014-12-31`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.007478778 -0.007478778   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2015-03-31`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.007426973 -0.007426973   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2015-06-30`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.007323072 -0.007323072   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2015-09-30`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.007017498 -0.007017498   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2015-12-31`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.006870886 -0.006870886   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2016-03-31`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.006736377 -0.006736377   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2016-06-30`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.00680644  -0.00680644    0.02000000    0.05000000    0.65000000    0.28000000
##
## $`2016-09-30`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.006913844 -0.006913844   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2016-12-31`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.006956275 -0.006956275   0.020000000   0.050000000   0.650000000   0.280000000
##
## $`2017-03-31`
##          mean          out         w.CA        w.CTAG          w.DS          w.EM
##   0.006978091 -0.006978091   0.020000000   0.050000000   0.650000000   0.280000000
##
```

```
## $`2017-06-30`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006918744 -0.006918744  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2017-09-30`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006935631 -0.006935631  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2017-12-31`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006950647 -0.006950647  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2018-03-31`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006899435 -0.006899435  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2018-06-30`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006824446 -0.006824446  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2018-09-30`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006746529 -0.006746529  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2018-12-31`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006481466 -0.006481466  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2019-03-31`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006560869 -0.006560869  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2019-06-30`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006542107 -0.006542107  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2019-09-30`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006417593 -0.006417593  0.020000000  0.050000000  0.650000000  0.280000000
##
## $`2019-11-30`
##          mean           out          w.CA        w.CTAG          w.DS          w.EM
##  0.006370927 -0.006370927  0.020000000  0.050000000  0.650000000  0.280000000
```

```r
extractWeights(bt_maxret)
```

```
##              CA CTAG   DS   EM
## 1999-12-31 0.55 0.05 0.38 0.02
## 2000-03-31 0.42 0.05 0.03 0.50
## 2000-06-30 0.55 0.05 0.38 0.02
## 2000-09-30 0.55 0.05 0.38 0.02
## 2000-12-31 0.55 0.05 0.38 0.02
## 2001-03-31 0.55 0.05 0.38 0.02
## 2001-06-30 0.55 0.05 0.38 0.02
## 2001-09-30 0.55 0.05 0.38 0.02
```

```
## 2001-12-31 0.55 0.05 0.38 0.02
## 2002-03-31 0.55 0.05 0.38 0.02
## 2002-06-30 0.55 0.05 0.38 0.02
## 2002-09-30 0.55 0.40 0.03 0.02
## 2002-12-31 0.55 0.05 0.38 0.02
## 2003-03-31 0.55 0.05 0.38 0.02
## 2003-06-30 0.55 0.05 0.38 0.02
## 2003-09-30 0.55 0.05 0.38 0.02
## 2003-12-31 0.28 0.05 0.65 0.02
## 2004-03-31 0.28 0.05 0.65 0.02
## 2004-06-30 0.28 0.05 0.65 0.02
## 2004-09-30 0.02 0.05 0.65 0.28
## 2004-12-31 0.02 0.05 0.65 0.28
## 2005-03-31 0.02 0.05 0.65 0.28
## 2005-06-30 0.02 0.05 0.65 0.28
## 2005-09-30 0.02 0.05 0.65 0.28
## 2005-12-31 0.02 0.05 0.65 0.28
## 2006-03-31 0.02 0.05 0.43 0.50
## 2006-06-30 0.02 0.05 0.65 0.28
## 2006-09-30 0.02 0.05 0.65 0.28
## 2006-12-31 0.02 0.05 0.43 0.50
## 2007-03-31 0.02 0.05 0.43 0.50
## 2007-06-30 0.02 0.05 0.43 0.50
## 2007-09-30 0.02 0.05 0.43 0.50
## 2007-12-31 0.02 0.05 0.43 0.50
## 2008-03-31 0.02 0.05 0.43 0.50
## 2008-06-30 0.02 0.05 0.43 0.50
## 2008-09-30 0.02 0.05 0.43 0.50
## 2008-12-31 0.02 0.05 0.65 0.28
## 2009-03-31 0.02 0.05 0.65 0.28
## 2009-06-30 0.02 0.05 0.43 0.50
## 2009-09-30 0.02 0.05 0.43 0.50
## 2009-12-31 0.02 0.05 0.43 0.50
## 2010-03-31 0.02 0.05 0.43 0.50
## 2010-06-30 0.02 0.05 0.65 0.28
## 2010-09-30 0.02 0.05 0.43 0.50
## 2010-12-31 0.02 0.05 0.43 0.50
## 2011-03-31 0.02 0.05 0.65 0.28
## 2011-06-30 0.02 0.05 0.65 0.28
## 2011-09-30 0.02 0.05 0.65 0.28
## 2011-12-31 0.02 0.05 0.65 0.28
## 2012-03-31 0.02 0.05 0.65 0.28
## 2012-06-30 0.02 0.05 0.65 0.28
## 2012-09-30 0.02 0.05 0.65 0.28
## 2012-12-31 0.02 0.05 0.65 0.28
## 2013-03-31 0.02 0.05 0.65 0.28
## 2013-06-30 0.02 0.05 0.65 0.28
## 2013-09-30 0.02 0.05 0.65 0.28
## 2013-12-31 0.02 0.05 0.65 0.28
## 2014-03-31 0.02 0.05 0.65 0.28
## 2014-06-30 0.02 0.05 0.65 0.28
## 2014-09-30 0.02 0.05 0.65 0.28
## 2014-12-31 0.02 0.05 0.65 0.28
## 2015-03-31 0.02 0.05 0.65 0.28
```

```
## 2015-06-30 0.02 0.05 0.65 0.28
## 2015-09-30 0.02 0.05 0.65 0.28
## 2015-12-31 0.02 0.05 0.65 0.28
## 2016-03-31 0.02 0.05 0.65 0.28
## 2016-06-30 0.02 0.05 0.65 0.28
## 2016-09-30 0.02 0.05 0.65 0.28
## 2016-12-31 0.02 0.05 0.65 0.28
## 2017-03-31 0.02 0.05 0.65 0.28
## 2017-06-30 0.02 0.05 0.65 0.28
## 2017-09-30 0.02 0.05 0.65 0.28
## 2017-12-31 0.02 0.05 0.65 0.28
## 2018-03-31 0.02 0.05 0.65 0.28
## 2018-06-30 0.02 0.05 0.65 0.28
## 2018-09-30 0.02 0.05 0.65 0.28
## 2018-12-31 0.02 0.05 0.65 0.28
## 2019-03-31 0.02 0.05 0.65 0.28
## 2019-06-30 0.02 0.05 0.65 0.28
## 2019-09-30 0.02 0.05 0.65 0.28
## 2019-11-30 0.02 0.05 0.65 0.28
```

```r
#Maximizing Mean Return
#The objective to maximize mean return is a linear problem of the form:

#maximize    wμ^(l)
#w

#Where μ^ is the estimated mean asset returns and w is the set of weights. Because this
#is a linear problem, it is well suited to be solved using a linear programming solver. For
#these types of problems, PortfolioAnalytics uses the ROI package with the glpk plugin


#Minimizing Portfolio Variance
#The objective to minimize portfolio variance is a quadratic problem of the form:

#minimize    w^(0) *Σw
#w

#Where Σ is the estimated covariance matrix of asset returns and w is the set of weights.
#Because this is a quadratic problem, it is well suited to be solved using a quadratic programming solv
#with the quadprog plugin



# Create portfolio object
portf_minvar <- portfolio.spec(assets=funds)

# Add full investment constraint to the portfolio object
portf_minvar <- add.constraint(portfolio=portf_minvar, type="full_investment")

# Add objective to minimize variance
#The only constraint specified is the full investment constraint, therefore the #optimization problem i


portf_minvar <- add.objective(portfolio=portf_minvar, type="risk", name="var")
```

```r
# Run the optimization

opt_gmv <- optimize.portfolio(R=returns, portfolio=portf_minvar,
 optimize_method="ROI", trace=TRUE)

opt_gmv
```

```
## ************************************
## PortfolioAnalytics Optimization
## ************************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_minvar, optimize_method = "ROI",
##     trace = TRUE)
##
## Optimal Weights:
##      CA    CTAG      DS      EM
##  0.3637  0.2887  0.5581 -0.2105
##
## Objective Measure:
##  StdDev
## 0.01187
```

```r
summary(opt_gmv)
```

```
## ***************************************************
## PortfolioAnalytics Optimization Summary
## ***************************************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_minvar, optimize_method = "ROI",
##     trace = TRUE)
##
## Optimal Weights:
##      CA    CTAG      DS      EM
##  0.3637  0.2887  0.5581 -0.2105
##
## Objective Measures:
##  StdDev
## 0.01187
##
##
## Portfolio Assets and Initial Weights:
##   CA CTAG   DS   EM
## 0.25 0.25 0.25 0.25
##
## ***************************************************
## PortfolioAnalytics Portfolio Specification
## ***************************************************
##
## Call:
## portfolio.spec(assets = funds)
##
## Number of assets: 4
```

```
## Asset Names
## [1] "CA"   "CTAG" "DS"   "EM"
##
## Constraints
## Enabled constraint types
##      - full_investment
##
## Objectives:
## Enabled objective names
##      - var
##
## ****************************************
## Constraints
## ****************************************
## Leverage Constraint:
## min_sum = 1
## max_sum = 1
## actual_leverage = 1
##
## Box Constraints:
## min:
## [1] -Inf -Inf -Inf -Inf
## max:
## [1] Inf Inf Inf Inf
##
## Position Limit Constraints:
## Maximum number of non-zero weights, max_pos:
## [1] "Unconstrained"
## Realized number of non-zero weights (i.e. positions):
## [1] 4
##
## Maximum number of long positions, max_pos_long:
## [1] "Unconstrained"
## Realized number of long positions:
## [1] 3
##
## Maximum number of short positions, max_pos_short:
## [1] "Unconstrained"
## Realized number of short positions:
## [1] 1
##
##
## Diversification Target Constraint:
## [1] "Unconstrained"
##
## Realized diversification:
## [1] 0.4285863
##
## Turnover Target Constraint:
## [1] "Unconstrained"
##
## Realized turnover from initial weights:
## [1] 0.2302493
##
```

```
## ***************************************
## Objectives
## ***************************************
##
## Objective: portfolio_risk_objective
## $name
## [1] "var"
##
## $target
## NULL
##
## $arguments
## $arguments$portfolio_method
## [1] "single"
##
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] 1
##
## $call
## add.objective(portfolio = portf_minvar, type = "risk", name = "var")
##
## attr(,"class")
## [1] "portfolio_risk_objective" "objective"
##
## ***************************************
##
## Elapsed Time:
## Time difference of 0.01085997 secs
```

```r
names(opt_gmv)
```

```
##  [1] "weights"            "objective_measures" "opt_values"
##  [4] "out"                "call"               "portfolio"
##  [7] "R"                  "data_summary"       "elapsed_time"
## [10] "end_t"
```

```r
extractStats(opt_gmv)
```

```
##       StdDev          out         w.CA        w.CTAG         w.DS
##  0.0118713641  0.0001409293  0.3637452496  0.2886587657  0.5580945505
##        w.EM
## -0.2104985658
```

```r
extractWeights(opt_gmv)
```

```
##        CA       CTAG         DS         EM
##  0.3637452  0.2886588  0.5580946 -0.2104986
```

```r
#Constrained Minimum Variance Portfolio


# Add long only constraints
portf_minvar <- add.constraint(portfolio=portf_minvar, type="box",
```

```
  min=0, max=1)

  # Add group constraints
portf_minvar <- add.constraint(portfolio=portf_minvar,
 type="group",
 groups=list(groupA=1,
 groupB=c(2, 3),
 groupC=4),
 group_min=c(0, 0.25, 0.10),
 group_max=c(0.45, 0.6, 0.5))



# Run the optimization
opt_minvar <- optimize.portfolio(R=returns, portfolio=portf_minvar,
 optimize_method="ROI", trace=TRUE)

opt_minvar
```

```
## ***********************************
## PortfolioAnalytics Optimization
## ***********************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_minvar, optimize_method = "ROI",
##      trace = TRUE)
##
## Optimal Weights:
##      CA    CTAG      DS      EM
## 0.4008  0.3217  0.1774  0.1000
##
## Objective Measure:
## StdDev
## 0.0136
```

```
summary(opt_minvar)
```

```
## ***************************************************
## PortfolioAnalytics Optimization Summary
## ***************************************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_minvar, optimize_method = "ROI",
##      trace = TRUE)
##
## Optimal Weights:
##      CA    CTAG      DS      EM
## 0.4008  0.3217  0.1774  0.1000
##
## Objective Measures:
## StdDev
## 0.0136
##
##
```

```
## Portfolio Assets and Initial Weights:
##   CA CTAG   DS   EM
## 0.25 0.25 0.25 0.25
##
## **************************************************
## PortfolioAnalytics Portfolio Specification
## **************************************************
##
## Call:
## portfolio.spec(assets = funds)
##
## Number of assets: 4
## Asset Names
## [1] "CA"   "CTAG" "DS"   "EM"
##
## Constraints
## Enabled constraint types
##       - full_investment
##       - box (long only)
##       - group
##
## Objectives:
## Enabled objective names
##       - var
##
## ***************************************
## Constraints
## ***************************************
## Leverage Constraint:
## min_sum = 1
## max_sum = 1
## actual_leverage = 1
##
## Box Constraints:
## min:
##   CA CTAG   DS   EM
##    0    0    0    0
## max:
##   CA CTAG   DS   EM
##    1    1    1    1
##
## Group Constraints:
## Groups:
## $groupA
## [1] "CA"
##
## $groupB
## [1] "CTAG" "DS"
##
## $groupC
## [1] "EM"
##
##
## Lower bound on group weights, group_min:
```

```
## [1] 0.00 0.25 0.10
##
## Upper bound on group weights, group_max:
## [1] 0.45 0.60 0.50
##
## Group Weights:
##     groupA     groupB     groupC
## 0.4008296 0.4991704 0.1000000
##
## Position Limit Constraints:
## Maximum number of non-zero weights, max_pos:
## [1] "Unconstrained"
## Realized number of non-zero weights (i.e. positions):
## [1] 4
##
## Maximum number of long positions, max_pos_long:
## [1] "Unconstrained"
## Realized number of long positions:
## [1] 4
##
## Maximum number of short positions, max_pos_short:
## [1] "Unconstrained"
## Realized number of short positions:
## [1] 0
##
##
## Diversification Target Constraint:
## [1] "Unconstrained"
##
## Realized diversification:
## [1] 0.6943398
##
## Turnover Target Constraint:
## [1] "Unconstrained"
##
## Realized turnover from initial weights:
## [1] 0.1112807
##
## ****************************************
## Objectives
## ****************************************
##
## Objective: portfolio_risk_objective
## $name
## [1] "var"
##
## $target
## NULL
##
## $arguments
## $arguments$portfolio_method
## [1] "single"
##
##
```

```
## $enabled
## [1] TRUE
##
## $multiplier
## [1] 1
##
## $call
## add.objective(portfolio = portf_minvar, type = "risk", name = "var")
##
## attr(,"class")
## [1] "portfolio_risk_objective" "objective"
##
## *****************************************
##
## Elapsed Time:
## Time difference of 0.008088589 secs
```

```r
names(opt_minvar)
```

```
##  [1] "weights"           "objective_measures" "opt_values"
##  [4] "out"               "call"               "portfolio"
##  [7] "R"                 "data_summary"       "elapsed_time"
## [10] "end_t"
```

```r
extractStats(opt_minvar)
```

```
##       StdDev          out         w.CA        w.CTAG         w.DS         w.EM
## 0.0136033595 0.0001850514 0.4008295616 0.3217317910 0.1774386474 0.1000000000
```

```r
extractWeights(opt_minvar)
```

```
##        CA       CTAG         DS         EM
## 0.4008296 0.3217318 0.1774386 0.1000000
```

```r
# Backtesting

bt_gmv <- optimize.portfolio.rebalancing(R=returns, portfolio=portf_minvar,
 optimize_method="ROI",
 rebalance_on="quarters",
 training_period=36)

bt_gmv
```

```
## ***************************************************
## PortfolioAnalytics Optimization with Rebalancing
## ***************************************************
##
## Call:
## optimize.portfolio.rebalancing(R = returns, portfolio = portf_minvar,
##     optimize_method = "ROI", rebalance_on = "quarters", training_period = 36)
##
## Number of rebalancing dates:  81
## First rebalance date:
## [1] "1999-12-31"
## Last rebalance date:
## [1] "2019-11-30"
##
```

```
## Annualized Portfolio Rebalancing Return:
## [1] 0.05608581
##
## Annualized Portfolio Standard Deviation:
## [1] 0.04840955
```

```
summary(bt_gmv)
```

```
## **************************************************
## PortfolioAnalytics Optimization with Rebalancing
## **************************************************
##
## Call:
## optimize.portfolio.rebalancing(R = returns, portfolio = portf_minvar,
##     optimize_method = "ROI", rebalance_on = "quarters", training_period = 36)
##
## First rebalance date:
## [1] "1999-12-31"
##
## Last rebalance date:
## [1] "2019-11-30"
##
## Annualized Portfolio Rebalancing Return:
## [1] 0.05608581
##
## Annualized Portfolio Standard Deviation:
## [1] 0.04840955
##
## Downside Risk Measures:
##                              portfolio.returns
## Semi Deviation                          0.0108
## Gain Deviation                          0.0082
## Loss Deviation                          0.0123
## Downside Deviation (MAR=10%)            0.0127
## Downside Deviation (Rf=0%)              0.0089
## Downside Deviation (0%)                 0.0089
## Maximum Drawdown                        0.1899
## Historical VaR (95%)                   -0.0182
## Historical ES (95%)                    -0.0302
## Modified VaR (95%)                     -0.0213
## Modified ES (95%)                      -0.0496
```

```
names(bt_gmv)
```

```
## [1] "portfolio"        "R"                "call"             "elapsed_time"
## [5] "opt_rebalancing"
```

```
tail(extractStats(bt_gmv))
```

```
## $`2018-09-30`
##       StdDev          out         w.CA        w.CTAG          w.DS          w.EM
## 0.0137260884 0.0001884055 0.3836741205 0.3252923385 0.1910335409 0.1000000000
##
## $`2018-12-31`
##       StdDev          out         w.CA        w.CTAG          w.DS          w.EM
## 0.0137812383 0.0001899225 0.3987372221 0.3252963000 0.1759664779 0.1000000000
```

```
## 
## $`2019-03-31`
##      StdDev         out        w.CA      w.CTAG        w.DS        w.EM
## 0.013727601 0.000188447 0.394917980 0.325538799 0.179543221 0.100000000
## 
## $`2019-06-30`
##       StdDev         out         w.CA       w.CTAG         w.DS         w.EM
## 0.0136936813 0.0001875169 0.3957064089 0.3242861094 0.1800074817 0.1000000000
## 
## $`2019-09-30`
##       StdDev         out         w.CA       w.CTAG         w.DS         w.EM
## 0.0136413590 0.0001860867 0.3981167198 0.3224871099 0.1793961703 0.1000000000
## 
## $`2019-11-30`
##       StdDev         out         w.CA       w.CTAG         w.DS         w.EM
## 0.0136033595 0.0001850514 0.4008295616 0.3217317910 0.1774386474 0.1000000000
```

```r
tail(extractWeights(bt_gmv))
```

```
##                   CA      CTAG        DS  EM
## 2018-09-30 0.3836741 0.3252923 0.1910335 0.1
## 2018-12-31 0.3987372 0.3252963 0.1759665 0.1
## 2019-03-31 0.3949180 0.3255388 0.1795432 0.1
## 2019-06-30 0.3957064 0.3242861 0.1800075 0.1
## 2019-09-30 0.3981167 0.3224871 0.1793962 0.1
## 2019-11-30 0.4008296 0.3217318 0.1774386 0.1
```

```r
chart.Weights(opt_minvar)
```

Weights
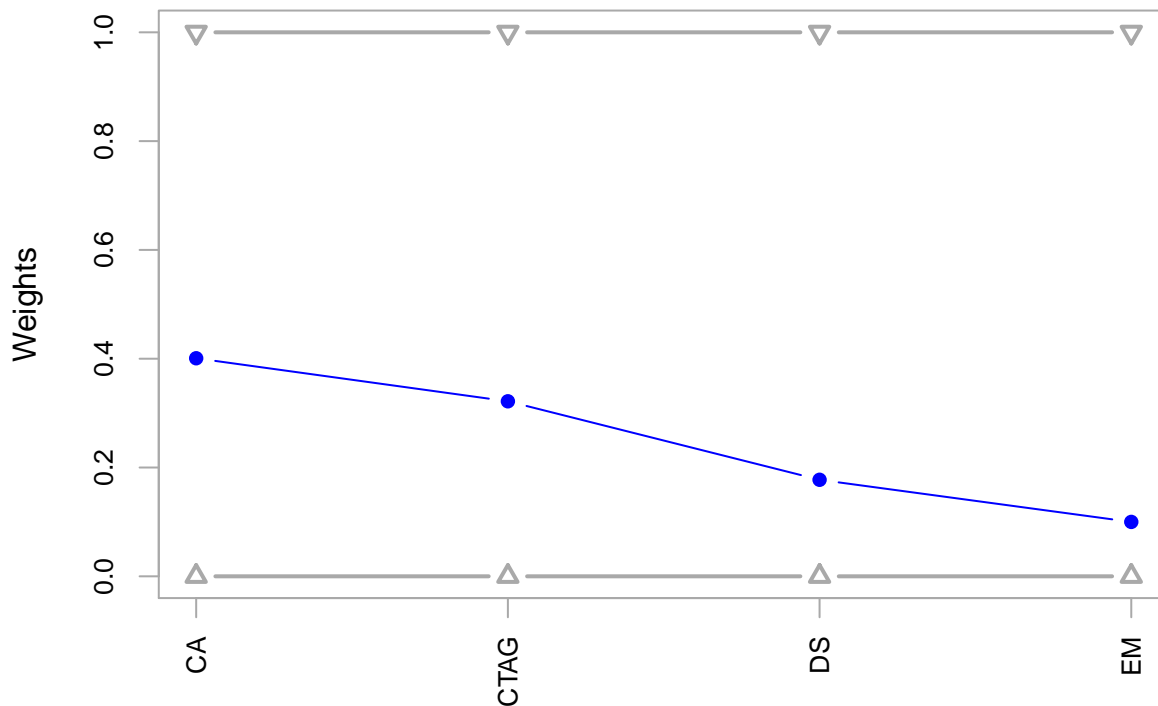
```
chart.Weights(bt_gmv)
```

Weights

| | CA | | CTAG | | DS | | EM |
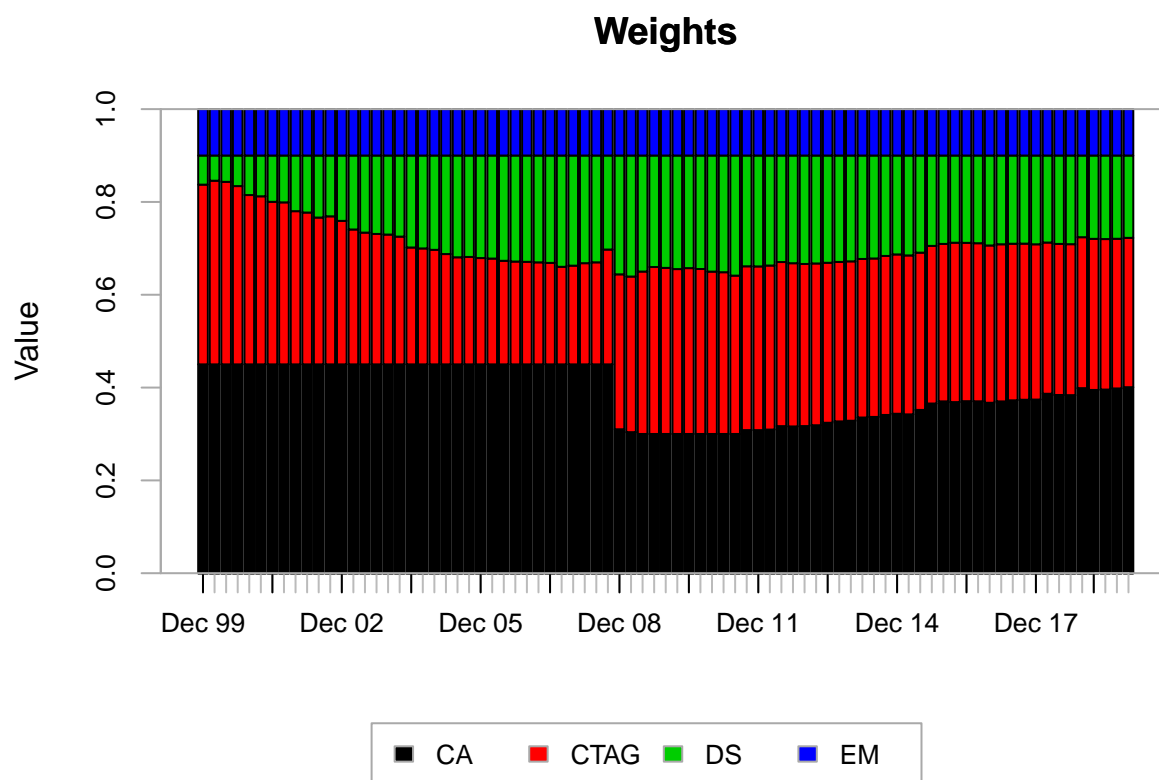
```
extractObjectiveMeasures(bt_gmv)
```

```
##                  StdDev
## 1999-12-31 0.01237827
## 2000-03-31 0.01206091
## 2000-06-30 0.01182820
## 2000-09-30 0.01182062
## 2000-12-31 0.01177995
## 2001-03-31 0.01178284
## 2001-06-30 0.01161512
## 2001-09-30 0.01136747
## 2001-12-31 0.01142580
## 2002-03-31 0.01132767
## 2002-06-30 0.01113168
## 2002-09-30 0.01093345
## 2002-12-31 0.01103885
## 2003-03-31 0.01132628
## 2003-06-30 0.01144957
## 2003-09-30 0.01136533
## 2003-12-31 0.01126266
## 2004-03-31 0.01118000
## 2004-06-30 0.01160160
## 2004-09-30 0.01148188
## 2004-12-31 0.01143848
## 2005-03-31 0.01158838
## 2005-06-30 0.01190338
## 2005-09-30 0.01176193
```

```
## 2005-12-31 0.01168768
## 2006-03-31 0.01168751
## 2006-06-30 0.01167024
## 2006-09-30 0.01155538
## 2006-12-31 0.01146036
## 2007-03-31 0.01133322
## 2007-06-30 0.01123049
## 2007-09-30 0.01146900
## 2007-12-31 0.01157293
## 2008-03-31 0.01179729
## 2008-06-30 0.01169594
## 2008-09-30 0.01349727
## 2008-12-31 0.01488515
## 2009-03-31 0.01476215
## 2009-06-30 0.01506476
## 2009-09-30 0.01519816
## 2009-12-31 0.01509369
## 2010-03-31 0.01508059
## 2010-06-30 0.01521782
## 2010-09-30 0.01515158
## 2010-12-31 0.01521594
## 2011-03-31 0.01510879
## 2011-06-30 0.01517437
## 2011-09-30 0.01535731
## 2011-12-31 0.01527571
## 2012-03-31 0.01522972
## 2012-06-30 0.01516837
## 2012-09-30 0.01506692
## 2012-12-31 0.01499358
## 2013-03-31 0.01492127
## 2013-06-30 0.01492090
## 2013-09-30 0.01483380
## 2013-12-31 0.01473384
## 2014-03-31 0.01465801
## 2014-06-30 0.01455811
## 2014-09-30 0.01448512
## 2014-12-31 0.01443381
## 2015-03-31 0.01433813
## 2015-06-30 0.01435485
## 2015-09-30 0.01435663
## 2015-12-31 0.01432939
## 2016-03-31 0.01427183
## 2016-06-30 0.01420150
## 2016-09-30 0.01412442
## 2016-12-31 0.01405362
## 2017-03-31 0.01398880
## 2017-06-30 0.01392534
## 2017-09-30 0.01385365
## 2017-12-31 0.01379087
## 2018-03-31 0.01386201
## 2018-06-30 0.01379728
## 2018-09-30 0.01372609
## 2018-12-31 0.01378124
## 2019-03-31 0.01372760
```

```
## 2019-06-30 0.01369368
## 2019-09-30 0.01364136
## 2019-11-30 0.01360336
```

```
  rr <- Return.portfolio(returns, weights = extractWeights(bt_gmv))
charts.PerformanceSummary(rr)
```
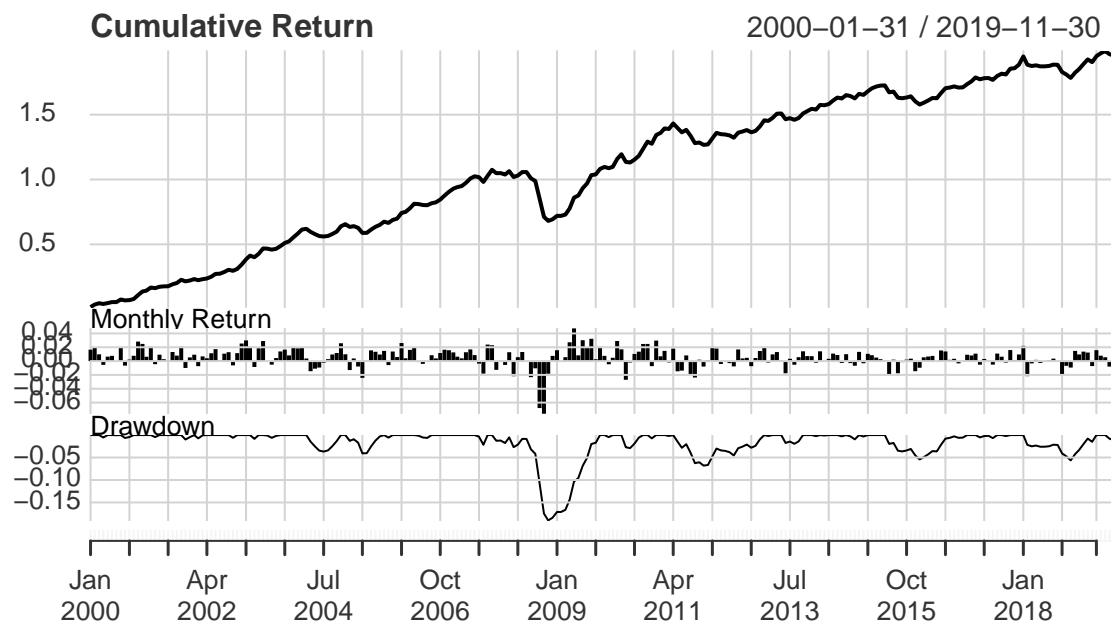
portfolio.returns Performance


```
#Constrained Minimum Variance Portfolio

# Add long only constraints
portf_minvar <- add.constraint(portfolio=portf_minvar, type="box",
 min=0, max=1)
 # Add group constraints
 portf_minvar <- add.constraint(portfolio=portf_minvar,
 type="group",
 groups=list(groupA=1,
 groupB=c(2, 3),
 groupC=4),
 group_min=c(0, 0.25, 0.10),
 group_max=c(0.45, 0.6, 0.5))

# Run the optimization
 opt_minvar <- optimize.portfolio(R=returns, portfolio=portf_minvar,
 optimize_method="ROI", trace=TRUE)


# Backtesting
 bt_minvar <- optimize.portfolio.rebalancing(R=returns, portfolio=portf_minvar,
 optimize_method="ROI",
 rebalance_on="quarters",
 training_period=36)
```

```
opt_minvar
```

```
## **********************************
## PortfolioAnalytics Optimization
## **********************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_minvar, optimize_method = "ROI",
##     trace = TRUE)
##
## Optimal Weights:
##     CA   CTAG     DS     EM
## 0.4008 0.3217 0.1774 0.1000
##
## Objective Measure:
## StdDev
## 0.0136
```

```
summary(opt_minvar)
```

```
## **************************************************
## PortfolioAnalytics Optimization Summary
## **************************************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = portf_minvar, optimize_method = "ROI",
##     trace = TRUE)
##
## Optimal Weights:
##     CA   CTAG     DS     EM
## 0.4008 0.3217 0.1774 0.1000
##
## Objective Measures:
## StdDev
## 0.0136
##
##
## Portfolio Assets and Initial Weights:
##   CA CTAG   DS   EM
## 0.25 0.25 0.25 0.25
##
## **************************************************
## PortfolioAnalytics Portfolio Specification
## **************************************************
##
## Call:
## portfolio.spec(assets = funds)
##
## Number of assets: 4
## Asset Names
## [1] "CA"   "CTAG" "DS"   "EM"
##
## Constraints
## Enabled constraint types
```

```
##        - full_investment
##        - box (long only)
##        - group
##        - box (long only)
##        - group
##
## Objectives:
## Enabled objective names
##        - var
##
## ****************************************
## Constraints
## ****************************************
## Leverage Constraint:
## min_sum = 1
## max_sum = 1
## actual_leverage = 1
##
## Box Constraints:
## min:
##    CA CTAG   DS   EM
##     0    0    0    0
## max:
##    CA CTAG   DS   EM
##     1    1    1    1
##
## Group Constraints:
## Groups:
## $groupA
## [1] "CA"
##
## $groupB
## [1] "CTAG" "DS"
##
## $groupC
## [1] "EM"
##
##
## Lower bound on group weights, group_min:
## [1] 0.00 0.25 0.10
##
## Upper bound on group weights, group_max:
## [1] 0.45 0.60 0.50
##
## Group Weights:
##     groupA     groupB     groupC
## 0.4008296 0.4991704 0.1000000
##
## Position Limit Constraints:
## Maximum number of non-zero weights, max_pos:
## [1] "Unconstrained"
## Realized number of non-zero weights (i.e. positions):
## [1] 4
##
```

```
## Maximum number of long positions, max_pos_long:
## [1] "Unconstrained"
## Realized number of long positions:
## [1] 4
##
## Maximum number of short positions, max_pos_short:
## [1] "Unconstrained"
## Realized number of short positions:
## [1] 0
##
##
## Diversification Target Constraint:
## [1] "Unconstrained"
##
## Realized diversification:
## [1] 0.6943398
##
## Turnover Target Constraint:
## [1] "Unconstrained"
##
## Realized turnover from initial weights:
## [1] 0.1112807
##
## ****************************************
## Objectives
## ****************************************
##
## Objective: portfolio_risk_objective
## $name
## [1] "var"
##
## $target
## NULL
##
## $arguments
## $arguments$portfolio_method
## [1] "single"
##
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] 1
##
## $call
## add.objective(portfolio = portf_minvar, type = "risk", name = "var")
##
## attr(,"class")
## [1] "portfolio_risk_objective" "objective"
##
## ****************************************
##
## Elapsed Time:
```

```
## Time difference of 0.008313179 secs
```

```r
names(opt_minvar)
```

```
##  [1] "weights"            "objective_measures" "opt_values"
##  [4] "out"                "call"               "portfolio"
##  [7] "R"                  "data_summary"       "elapsed_time"
## [10] "end_t"
```

```r
extractStats(opt_minvar)
```

```
##      StdDev           out          w.CA        w.CTAG          w.DS          w.EM
## 0.0136033595 0.0001850514 0.4008295616 0.3217317910 0.1774386474 0.1000000000
```
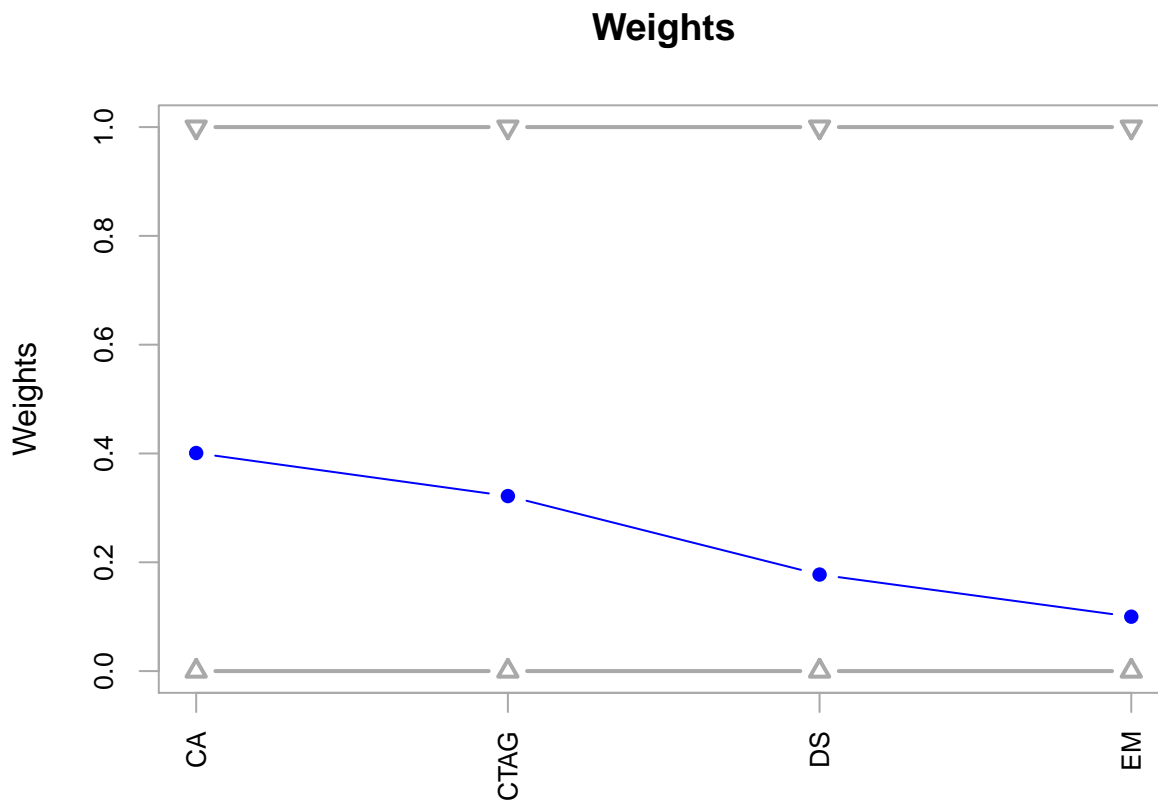
```r
extractWeights(opt_minvar)
```

```
##        CA      CTAG        DS        EM
## 0.4008296 0.3217318 0.1774386 0.1000000
```

```r
extractWeights(opt_minvar)
```

```
##        CA      CTAG        DS        EM
## 0.4008296 0.3217318 0.1774386 0.1000000
```

```r
 chart.Weights(opt_minvar)
```



```r
 chart.Weights(bt_minvar)
```

Weights

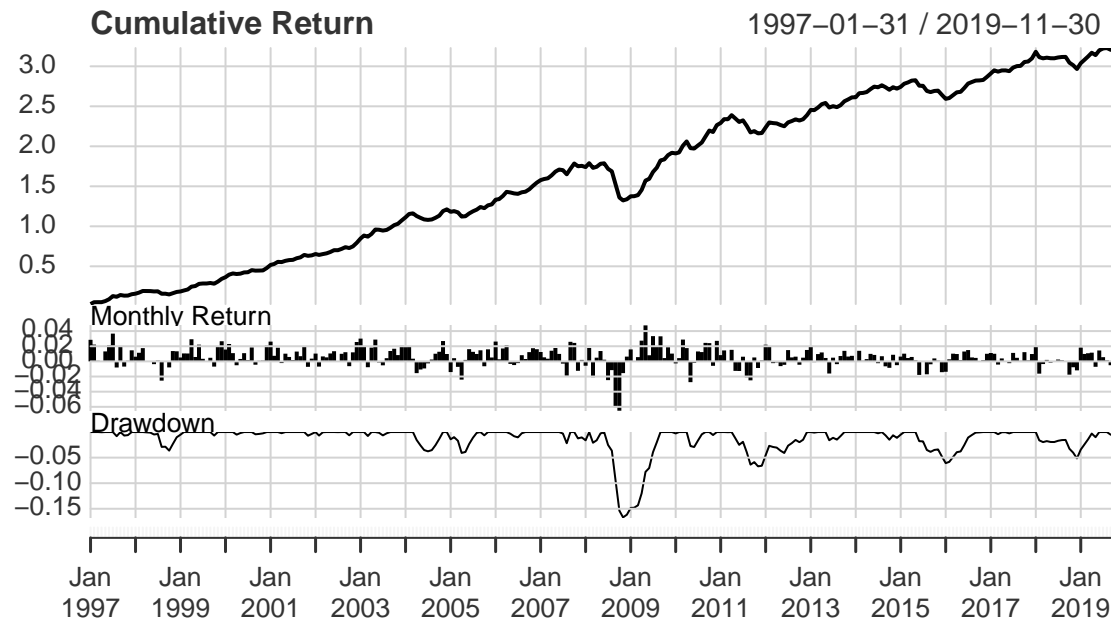| ■ CA | ■ CTAG | ■ DS | ■ EM |
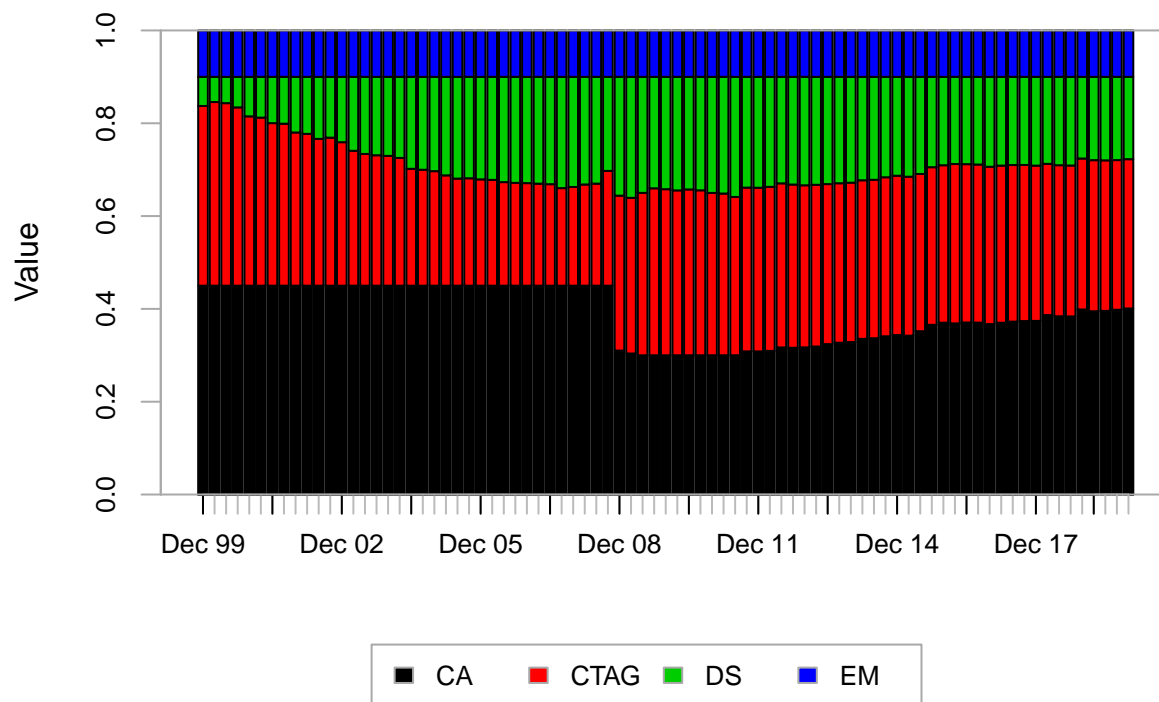
```
extractObjectiveMeasures(opt_minvar)
```

```
## $StdDev
##      StdDev
## 0.01360336
```
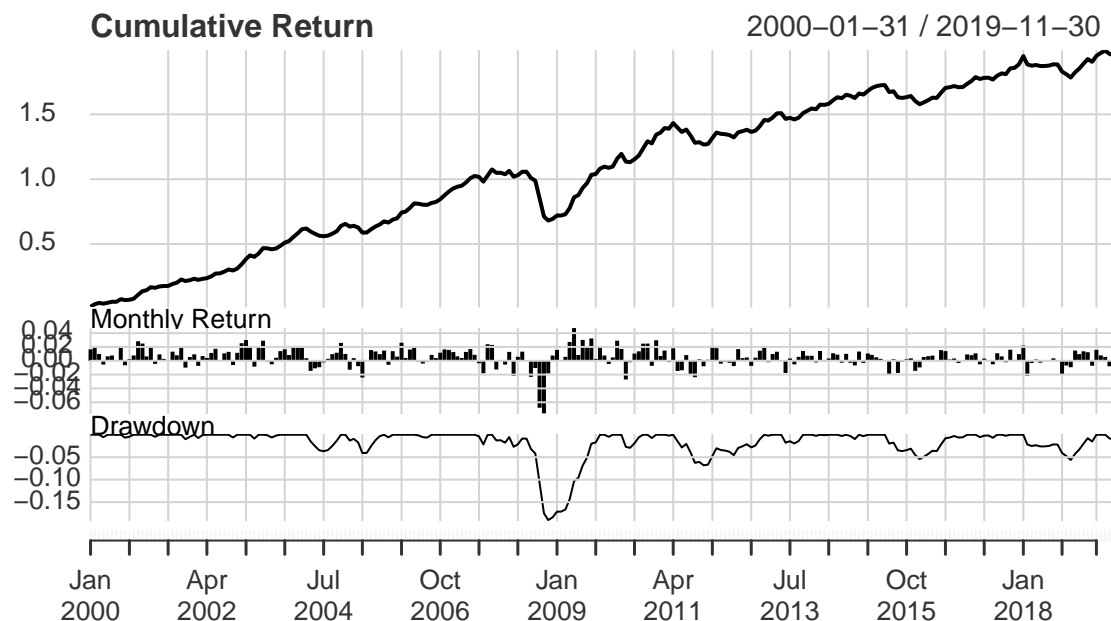
```
  rr <- Return.portfolio(returns, weights = extractWeights(opt_minvar))
charts.PerformanceSummary(rr)
```

portfolio.returns Performance

Cumulative Return   1997−01−31 / 2019−11−30


```
  rr <- Return.portfolio(returns, weights = extractWeights(bt_minvar))
charts.PerformanceSummary(rr)
```

portfolio.returns Performance

Cumulative Return   2000−01−31 / 2019−11−30


```
 # Maximizing Quadratic Utility


# The objective to maximize quadratic utility is a quadratic problem of the form:
#maximize w^(0)*μ  -  /2 * w^(0) *Σw
```

```
#w

#Where μ is the estimated mean asset returns,   is the risk aversion parameter, Σ is
#the estimated covariance matrix of asset returns and w is the set of weights. Quadratic
#utility maximizes return while penalizing variance. The   risk aversion parameter controls
#how much portfolio variance is penalized. Because this is a quadratic problem, it is well
#suited to be solved using a quadratic programming solver. For these types of problems,
#PortfolioAnalytics uses the ROI package with the quadprog plugin




 #Portfolio Object
# Create initial portfolio object
init_portf <- portfolio.spec(assets=funds)

# Create full investment constraint

fi_constr <- weight_sum_constraint(type="full_investment")

 # Create long only constraint

lo_constr <- box_constraint(type="long_only", assets=init_portf$assets)

# Combine the constraints in a list
qu_constr <- list(fi_constr, lo_constr)



# Create return objective
ret_obj <- return_objective(name="mean")


# Create variance objective specifying a risk_aversion parameter which controls
#how much the variance is penalized
var_obj <- portfolio_risk_objective(name="var", risk_aversion=0.25)

 # Combine the objectives into a list

qu_obj <- list(ret_obj, var_obj)

# Run the optimization
opt_qu <- optimize.portfolio(R=returns, portfolio=init_portf,
 constraints=qu_constr,
 objectives=qu_obj,
 optimize_method="ROI",
 trace=TRUE)

 opt_qu
```

```
## ***********************************
## PortfolioAnalytics Optimization
## ***********************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = init_portf, constraints = qu_constr,
```

```
##       objectives = qu_obj, optimize_method = "ROI", trace = TRUE)
##
## Optimal Weights:
##   CA CTAG   DS   EM
##    0    0    1    0
##
## Objective Measure:
##      mean
## 0.006622
##
##
##  StdDev
## 0.01689
```

```
summary(opt_qu)
```

```
## ************************************************
## PortfolioAnalytics Optimization Summary
## ************************************************
##
## Call:
## optimize.portfolio(R = returns, portfolio = init_portf, constraints = qu_constr,
##       objectives = qu_obj, optimize_method = "ROI", trace = TRUE)
##
## Optimal Weights:
##   CA CTAG   DS   EM
##    0    0    1    0
##
## Objective Measures:
##      mean
## 0.006622
##
##
##  StdDev
## 0.01689
##
##
## Portfolio Assets and Initial Weights:
##   CA CTAG   DS   EM
## 0.25 0.25 0.25 0.25
##
## ************************************************
## PortfolioAnalytics Portfolio Specification
## ************************************************
##
## Call:
## portfolio.spec(assets = funds)
##
## Number of assets: 4
## Asset Names
## [1] "CA"   "CTAG" "DS"    "EM"
##
## Constraints
## Enabled constraint types
##       - full_investment
```

```
##       - long_only
##
## Objectives:
## Enabled objective names
##       - mean
##       - var
##
## ****************************************
## Constraints
## ****************************************
## Leverage Constraint:
## min_sum = 1
## max_sum = 1
## actual_leverage = 1
##
## Box Constraints:
## min:
##   CA CTAG   DS   EM
##    0    0    0    0
## max:
##   CA CTAG   DS   EM
##    1    1    1    1
##
## Position Limit Constraints:
## Maximum number of non-zero weights, max_pos:
## [1] "Unconstrained"
## Realized number of non-zero weights (i.e. positions):
## [1] 1
##
## Maximum number of long positions, max_pos_long:
## [1] "Unconstrained"
## Realized number of long positions:
## [1] 1
##
## Maximum number of short positions, max_pos_short:
## [1] "Unconstrained"
## Realized number of short positions:
## [1] 0
##
##
## Diversification Target Constraint:
## [1] "Unconstrained"
##
## Realized diversification:
## [1] 1.110223e-15
##
## Turnover Target Constraint:
## [1] "Unconstrained"
##
## Realized turnover from initial weights:
## [1] 0.375
##
## ****************************************
## Objectives
```

```
## ****************************************
##
## Objective: return_objective
## $name
## [1] "mean"
##
## $target
## NULL
##
## $arguments
## list()
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] -1
##
## attr(,"class")
## [1] "return_objective" "objective"
##
## ****************************************
## Objective: portfolio_risk_objective
## $name
## [1] "var"
##
## $target
## NULL
##
## $arguments
## $arguments$portfolio_method
## [1] "single"
##
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] 1
##
## $risk_aversion
## [1] 0.25
##
## attr(,"class")
## [1] "portfolio_risk_objective" "objective"
##
## ****************************************
##
## Elapsed Time:
## Time difference of 0.0086236 secs
# Backtesting

bt_qu <- optimize.portfolio.rebalancing(R=returns, portfolio=init_portf,
```

```
constraints=qu_constr,
objectives=qu_obj,
optimize_method="ROI",
rebalance_on="quarters",
training_period=36)
```

```
bt_qu
```

```
## **************************************************
## PortfolioAnalytics Optimization with Rebalancing
## **************************************************
##
## Call:
## optimize.portfolio.rebalancing(R = returns, portfolio = init_portf,
##     constraints = qu_constr, objectives = qu_obj, optimize_method = "ROI",
##     rebalance_on = "quarters", training_period = 36)

## Warning in Return.portfolio.geometric(R = R, weights = weights, wealth.index =
## wealth.index, : The weights for one or more periods do not sum up to 1: assuming
## a return of 0 for the residual weights

## Number of rebalancing dates:  81
## First rebalance date:
## [1] "1999-12-31"
## Last rebalance date:
## [1] "2019-11-30"
##
## Annualized Portfolio Rebalancing Return:
## [1] 0.1261984
##
## Annualized Portfolio Standard Deviation:
## [1] 0.03543838
```

```
summary(bt_qu)
```

```
## Warning in Return.portfolio.geometric(R = R, weights = weights, wealth.index =
## wealth.index, : The weights for one or more periods do not sum up to 1: assuming
## a return of 0 for the residual weights

## **************************************************
## PortfolioAnalytics Optimization with Rebalancing
## **************************************************
##
## Call:
## optimize.portfolio.rebalancing(R = returns, portfolio = init_portf,
##     constraints = qu_constr, objectives = qu_obj, optimize_method = "ROI",
##     rebalance_on = "quarters", training_period = 36)
##
## First rebalance date:
## [1] "1999-12-31"
##
## Last rebalance date:
## [1] "2019-11-30"
```

```
##
## Annualized Portfolio Rebalancing Return:
## [1] 0.1261984
##
## Annualized Portfolio Standard Deviation:
## [1] 0.03543838
##
## Downside Risk Measures:
##                                portfolio.returns
## Semi Deviation                            0.0072
## Gain Deviation                            0.0081
## Loss Deviation                            0.0044
## Downside Deviation (MAR=10%)              0.0063
## Downside Deviation (Rf=0%)                0.0028
## Downside Deviation (0%)                   0.0028
## Maximum Drawdown                          0.0215
## Historical VaR (95%)                     -0.0073
## Historical ES (95%)                      -0.0105
## Modified VaR (95%)                       -0.0067
## Modified ES (95%)                        -0.0107
```

```r
names(bt_qu)
```

```
## [1] "portfolio"        "R"                "call"             "elapsed_time"
## [5] "opt_rebalancing"
```

```r
extractStats(bt_qu)
```

```
## $`1999-12-31`
##          mean         StdDev           out           w.CA         w.CTAG
##  8.936111e-03   1.189271e-02  -8.900752e-03   1.000000e+00   2.182070e-17
##          w.DS           w.EM
##  2.287059e-14  0.000000e+00
##
## $`2000-03-31`
##          mean         StdDev           out           w.CA         w.CTAG
##  1.023495e-02   2.009404e-02  -1.013401e-02   7.556488e-01  -5.504568e-17
##          w.DS           w.EM
##  0.000000e+00  2.443512e-01
##
## $`2000-06-30`
##          mean         StdDev           out           w.CA         w.CTAG
##  1.072619e-02   1.194628e-02  -1.069051e-02   1.000000e+00   0.000000e+00
##          w.DS           w.EM
##  2.153833e-14  0.000000e+00
##
## $`2000-09-30`
##          mean         StdDev           out           w.CA         w.CTAG
##  1.089111e-02   1.157330e-02  -1.085763e-02   1.000000e+00  -9.369392e-17
##          w.DS           w.EM
##  0.000000e+00  0.000000e+00
##
## $`2000-12-31`
##          mean         StdDev           out           w.CA         w.CTAG
##  1.014583e-02   1.165353e-02  -1.011188e-02   1.000000e+00   0.000000e+00
```

```
##          w.DS          w.EM
## -2.220446e-16 -5.286908e-17
##
## $`2001-03-31`
##          mean        StdDev           out          w.CA        w.CTAG
##  1.089804e-02  1.186908e-02 -1.086282e-02  1.000000e+00  0.000000e+00
##          w.DS          w.EM
##  1.325329e-14  0.000000e+00
##
## $`2001-06-30`
##          mean        StdDev           out          w.CA        w.CTAG
##  1.066667e-02  1.166813e-02 -1.063263e-02  1.000000e+00  1.203510e-30
##          w.DS          w.EM
##  0.000000e+00  1.784556e-16
##
## $`2001-09-30`
##          mean        StdDev           out          w.CA        w.CTAG
##  1.065088e-02  1.136948e-02 -1.061856e-02  1.000000e+00  0.000000e+00
##          w.DS          w.EM
##  2.198242e-14  1.747802e-16
##
## $`2001-12-31`
##          mean        StdDev           out          w.CA        w.CTAG
##  1.029000e-02  1.138032e-02 -1.025762e-02  1.000000e+00 -8.709944e-17
##          w.DS          w.EM
##  1.398881e-14  0.000000e+00
##
## $`2002-03-31`
##          mean        StdDev           out          w.CA        w.CTAG
##  1.004127e-02  1.129750e-02 -1.000936e-02  1.000000e+00 -5.693326e-31
##          w.DS          w.EM
##  2.220446e-16  0.000000e+00
##
## $`2002-06-30`
##          mean        StdDev           out          w.CA        w.CTAG
##  9.786364e-03  1.112694e-02 -9.755411e-03  1.000000e+00  8.402358e-17
##          w.DS          w.EM
##  2.220446e-15  0.000000e+00
##
## $`2002-09-30`
##          mean        StdDev           out          w.CA        w.CTAG
##  9.414493e-03  1.133961e-02 -9.382346e-03  1.000000e+00  0.000000e+00
##          w.DS          w.EM
##  4.163779e-30  5.162658e-17
##
## $`2002-12-31`
##          mean        StdDev           out          w.CA        w.CTAG
##  9.733333e-03  1.127324e-02 -9.701562e-03  1.000000e+00 -3.041291e-31
##          w.DS          w.EM
##  6.661338e-16 -4.508381e-17
##
## $`2003-03-31`
##          mean        StdDev           out          w.CA        w.CTAG
##  1.001733e-02  1.125574e-02 -9.985660e-03  1.000000e+00  0.000000e+00
```

```
##         w.DS          w.EM
## 1.765255e-14 -5.308605e-17
##
## $`2003-06-30`
##         mean        StdDev          out          w.CA        w.CTAG
## 9.924359e-03 1.120202e-02 -9.892988e-03 1.000000e+00 -1.433757e-30
##         w.DS          w.EM
## 3.330669e-16 8.879249e-17
##
## $`2003-09-30`
##         mean        StdDev          out          w.CA        w.CTAG
## 9.571605e-03 1.137095e-02 -9.539280e-03 1.000000e+00 -1.273131e-30
##         w.DS          w.EM
## 6.661338e-16 0.000000e+00
##
## $`2003-12-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 9.633544e-03 1.526133e-02 -9.575317e-03 2.003848e-01 9.461819e-18
##         w.DS          w.EM
## 7.996152e-01 0.000000e+00
##
## $`2004-03-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 9.800000e-03 1.707024e-02 -9.727152e-03 5.551115e-17 4.290624e-17
##         w.DS          w.EM
## 1.000000e+00 -5.551115e-17
##
## $`2004-06-30`
##         mean        StdDev          out          w.CA        w.CTAG
## 9.790000e-03 1.685525e-02 -9.718975e-03 5.551115e-16 0.000000e+00
##         w.DS          w.EM
## 1.000000e+00 0.000000e+00
##
## $`2004-09-30`
##         mean        StdDev          out          w.CA        w.CTAG
## 9.701075e-03 1.659875e-02 -9.632196e-03 5.551115e-16 1.932332e-17
##         w.DS          w.EM
## 1.000000e+00 0.000000e+00
##
## $`2004-12-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 1.017500e-02 1.660971e-02 -1.010603e-02 -1.110223e-16 -1.383496e-16
##         w.DS          w.EM
## 1.000000e+00 3.330669e-16
##
## $`2005-03-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 1.007172e-02 1.638464e-02 -1.000460e-02 2.551128e-31 1.826609e-16
##         w.DS          w.EM
## 1.000000e+00 0.000000e+00
##
## $`2005-06-30`
##         mean        StdDev          out          w.CA        w.CTAG
## 9.860784e-03 1.624002e-02 -9.794850e-03 0.000000e+00 1.806270e-16
```

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```
##           w.DS           w.EM
##   1.000000e+00  0.000000e+00
##
## $`2005-09-30`
##           mean         StdDev            out           w.CA         w.CTAG
##   9.968571e-03  1.602280e-02 -9.904389e-03  0.000000e+00  1.784641e-16
##           w.DS           w.EM
##   1.000000e+00 -2.220446e-16
##
## $`2005-12-31`
##           mean         StdDev            out           w.CA         w.CTAG
##   9.867593e-03  1.584894e-02 -9.804795e-03  1.110223e-16  0.000000e+00
##           w.DS           w.EM
##   1.000000e+00  1.332268e-15
##
## $`2006-03-31`
##           mean         StdDev            out           w.CA         w.CTAG
##   1.004235e-02  1.571924e-02 -9.980576e-03  0.000000e+00  1.749255e-16
##           w.DS           w.EM
##   9.999277e-01  7.232907e-05
##
## $`2006-06-30`
##   mean StdDev    out   w.CA w.CTAG   w.DS   w.EM
##     NA     NA     NA     NA     NA     NA     NA
##
## $`2006-09-30`
##           mean         StdDev            out           w.CA         w.CTAG
##   9.873504e-03  1.540371e-02 -9.814186e-03  0.000000e+00 -1.726289e-16
##           w.DS           w.EM
##   1.000000e+00  0.000000e+00
##
## $`2006-12-31`
##           mean         StdDev            out           w.CA         w.CTAG
##   1.007894e-02  1.574027e-02 -1.001700e-02 -3.469447e-18  1.699204e-16
##           w.DS           w.EM
##   9.644896e-01  3.551035e-02
##
## $`2007-03-31`
##           mean         StdDev            out           w.CA         w.CTAG
##   1.015691e-02  1.508489e-02 -1.010002e-02  0.000000e+00  3.380425e-16
##           w.DS           w.EM
##   1.000000e+00  2.220446e-16
##
## $`2007-06-30`
##           mean         StdDev            out           w.CA         w.CTAG
##   1.053187e-02  3.200120e-02 -1.027586e-02  0.000000e+00 -1.461122e-17
##           w.DS           w.EM
##   1.608224e-01  8.391776e-01
##
## $`2007-09-30`
##           mean         StdDev            out           w.CA         w.CTAG
##   1.067984e-02  3.578437e-02 -1.035971e-02  0.000000e+00 -6.327720e-17
##           w.DS           w.EM
##   1.998401e-15  1.000000e+00
```

```
## 
## $`2007-12-31`
##         mean       StdDev         out        w.CA        w.CTAG
##  1.072348e-02  3.565357e-02 -1.040569e-02  0.000000e+00  6.266547e-17
##          w.DS         w.EM
##  0.000000e+00  1.000000e+00
## 
## $`2008-03-31`
##         mean       StdDev         out        w.CA        w.CTAG
##  1.003926e-02  3.591528e-02 -9.716782e-03  1.551358e-16  4.686309e-31
##          w.DS         w.EM
##  0.000000e+00  1.000000e+00
## 
## $`2008-06-30`
##         mean       StdDev         out        w.CA        w.CTAG
##  9.878261e-03  3.567546e-02 -9.560076e-03  1.544404e-16  0.000000e+00
##          w.DS         w.EM
##  1.554312e-15  1.000000e+00
## 
## $`2008-09-30`
##         mean       StdDev         out        w.CA        w.CTAG
##  8.418440e-03  1.602578e-02 -8.354233e-03  4.445725e-16  0.000000e+00
##          w.DS         w.EM
##  1.000000e+00 -4.440892e-16
## 
## $`2008-12-31`
##   mean StdDev   out   w.CA w.CTAG   w.DS   w.EM
##     NA     NA    NA     NA     NA     NA     NA
## 
## $`2009-03-31`
##         mean       StdDev         out        w.CA        w.CTAG
##  7.105442e-03  1.794859e-02 -7.024904e-03  5.161488e-18  0.000000e+00
##          w.DS         w.EM
##  1.000000e+00  1.110223e-15
## 
## $`2009-06-30`
##         mean       StdDev         out        w.CA        w.CTAG
##  7.791123e-03  2.514480e-02 -7.633058e-03 -2.658501e-16  0.000000e+00
##          w.DS         w.EM
##  6.013453e-01  3.986547e-01
## 
## $`2009-09-30`
##         mean       StdDev         out        w.CA        w.CTAG
##  8.449347e-03  3.280577e-02 -8.180292e-03 -6.678978e-17  0.000000e+00
##          w.DS         w.EM
##  2.495612e-01  7.504388e-01
## 
## $`2009-12-31`
##         mean       StdDev         out        w.CA        w.CTAG
##  8.543316e-03  2.536431e-02 -8.382479e-03 -2.284523e-16  0.000000e+00
##          w.DS         w.EM
##  5.848416e-01  4.151584e-01
## 
## $`2010-03-31`
```

```
##           mean          StdDev            out          w.CA         w.CTAG
##    8.627673e-03    1.838978e-02  -8.543127e-03    1.110223e-16    1.581894e-16
##           w.DS            w.EM
##    1.000000e+00    1.776357e-15
##
## $`2010-06-30`
##           mean          StdDev            out          w.CA         w.CTAG
##    8.403704e-03    1.851522e-02  -8.318000e-03    0.000000e+00    1.570929e-16
##           w.DS            w.EM
##    1.000000e+00    3.330669e-16
##
## $`2010-09-30`
##           mean          StdDev            out          w.CA         w.CTAG
##    8.492313e-03    2.243737e-02  -8.366455e-03    0.000000e+00   -1.565722e-16
##           w.DS            w.EM
##    7.305023e-01    2.694977e-01
##
## $`2010-12-31`
##           mean          StdDev            out          w.CA         w.CTAG
##    8.635119e-03    1.835120e-02  -8.550927e-03    0.000000e+00    0.000000e+00
##           w.DS            w.EM
##    1.000000e+00    5.551115e-16
##
## $`2011-03-31`
##           mean          StdDev            out          w.CA         w.CTAG
##    8.694152e-03    1.820900e-02  -8.611260e-03    0.000000e+00   -3.074847e-16
##           w.DS            w.EM
##    1.000000e+00    2.220446e-16
##
## $`2011-06-30`
##           mean          StdDev            out          w.CA         w.CTAG
##    8.571264e-03    1.810751e-02  -8.489294e-03    0.000000e+00    0.000000e+00
##           w.DS            w.EM
##    1.000000e+00    6.661338e-16
##
## $`2011-09-30`
##           mean          StdDev            out          w.CA         w.CTAG
##    7.981356e-03    1.865031e-02  -7.894397e-03    0.000000e+00    0.000000e+00
##           w.DS            w.EM
##    1.000000e+00    6.661338e-16
##
## $`2011-12-31`
##           mean          StdDev            out          w.CA         w.CTAG
##    7.947778e-03    1.858504e-02  -7.861427e-03    0.000000e+00    0.000000e+00
##           w.DS            w.EM
##    1.000000e+00    4.440892e-16
##
## $`2012-03-31`
##           mean          StdDev            out          w.CA         w.CTAG
##    8.154645e-03    1.854143e-02  -8.068699e-03    0.000000e+00    4.538389e-16
##           w.DS            w.EM
##    1.000000e+00    0.000000e+00
##
## $`2012-06-30`
```

```
##         mean        StdDev          out          w.CA        w.CTAG
## 7.911828e-03  1.851853e-02 -7.826094e-03  0.000000e+00 -1.510361e-16
##         w.DS          w.EM
## 1.000000e+00  4.440892e-16
##
## $`2012-09-30`
##         mean        StdDev          out          w.CA        w.CTAG         w.DS
## 0.008007937  0.018391458 -0.007923375  0.000000000  0.000000000  1.000000000
##         w.EM
## 0.000000000
##
## $`2012-12-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 8.121875e-03  1.829403e-02 -8.038207e-03  0.000000e+00  1.490868e-16
##         w.DS          w.EM
## 1.000000e+00 -2.220446e-16
##
## $`2013-03-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 8.212308e-03  1.818646e-02 -8.129621e-03  0.000000e+00  2.960303e-16
##         w.DS          w.EM
## 1.000000e+00  1.110223e-16
##
## $`2013-06-30`
##         mean        StdDev          out          w.CA        w.CTAG
## 8.235859e-03  1.813650e-02 -8.153625e-03  0.000000e+00 -4.416281e-16
##         w.DS          w.EM
## 1.000000e+00  7.771561e-16
##
## $`2013-09-30`
##         mean        StdDev          out          w.CA        w.CTAG         w.DS
## 0.008247264  0.018032607 -0.008165970  0.000000000  0.000000000  1.000000000
##         w.EM
## 0.000000000
##
## $`2013-12-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 8.323529e-03  1.791127e-02 -8.243326e-03  0.000000e+00  0.000000e+00
##         w.DS          w.EM
## 1.000000e+00 -6.661338e-16
##
## $`2014-03-31`
##         mean        StdDev          out          w.CA        w.CTAG
## 8.373430e-03  1.781746e-02 -8.294065e-03  0.000000e+00 -1.448811e-16
##         w.DS          w.EM
## 1.000000e+00  4.440892e-16
##
## $`2014-06-30`
##         mean        StdDev          out          w.CA        w.CTAG
## 8.378095e-03  1.769893e-02 -8.299782e-03  0.000000e+00 -1.439759e-16
##         w.DS          w.EM
## 1.000000e+00  4.440892e-16
##
## $`2014-09-30`
```

```
##         mean         StdDev          out           w.CA         w.CTAG
##  8.133803e-03  1.771769e-02 -8.055324e-03  0.000000e+00  1.429946e-16
##         w.DS           w.EM
##  1.000000e+00  8.881784e-16
##
## $`2014-12-31`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.886111e-03  1.772948e-02 -7.807527e-03  0.000000e+00 -1.421090e-16
##         w.DS           w.EM
##  1.000000e+00 -1.221245e-15
##
## $`2015-03-31`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.804566e-03  1.769594e-02 -7.726280e-03  0.000000e+00  2.220446e-16
##         w.DS           w.EM
##  1.000000e+00 -8.881784e-16
##
## $`2015-06-30`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.670721e-03  1.765077e-02 -7.592833e-03  1.110223e-16  1.404122e-16
##         w.DS           w.EM
##  1.000000e+00  1.110223e-16
##
## $`2015-09-30`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.387111e-03  1.772273e-02 -7.308587e-03 -3.560813e-31 -1.643489e-31
##         w.DS           w.EM
##  1.000000e+00  0.000000e+00
##
## $`2015-12-31`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.171053e-03  1.771902e-02 -7.092562e-03  0.000000e+00  2.785997e-16
##         w.DS           w.EM
##  1.000000e+00 -6.661338e-16
##
## $`2016-03-31`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.003030e-03  1.778380e-02 -6.923964e-03  0.000000e+00  0.000000e+00
##         w.DS           w.EM
##  1.000000e+00  4.440892e-16
##
## $`2016-06-30`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.094872e-03  1.769710e-02 -7.016575e-03  0.000000e+00 -1.379660e-16
##         w.DS           w.EM
##  1.000000e+00  1.221245e-15
##
## $`2016-09-30`
##         mean         StdDev          out           w.CA         w.CTAG
##  7.215190e-03  1.762523e-02 -7.137528e-03  0.000000e+00  0.000000e+00
##         w.DS           w.EM
##  1.000000e+00  9.992007e-16
##
## $`2016-12-31`
```

```
##          mean        StdDev          out          w.CA        w.CTAG
##  7.334167e-03  1.755141e-02 -7.257154e-03  0.000000e+00  0.000000e+00
##          w.DS          w.EM
##  1.000000e+00 -2.220446e-16
##
## $`2017-03-31`
##          mean        StdDev          out          w.CA        w.CTAG
##  7.316872e-03  1.748794e-02 -7.240415e-03  0.000000e+00  0.000000e+00
##          w.DS          w.EM
##  1.000000e+00  1.110223e-16
##
## $`2017-06-30`
##          mean        StdDev          out          w.CA        w.CTAG
##  7.240244e-03  1.740057e-02 -7.164549e-03  0.000000e+00 -1.348155e-16
##          w.DS          w.EM
##  1.000000e+00  1.332268e-15
##
## $`2017-09-30`
##          mean        StdDev          out          w.CA        w.CTAG
##  7.221687e-03  1.730436e-02 -7.146827e-03  0.000000e+00  0.000000e+00
##          w.DS          w.EM
##  1.000000e+00 -2.220446e-16
##
## $`2017-12-31`
##          mean        StdDev          out          w.CA        w.CTAG
##  7.205159e-03  1.722147e-02 -7.131014e-03  0.000000e+00  0.000000e+00
##          w.DS          w.EM
##  1.000000e+00  4.440892e-16
##
## $`2018-03-31`
##          mean        StdDev          out          w.CA        w.CTAG
##  7.157647e-03  1.716141e-02 -7.084019e-03  0.000000e+00  1.330476e-16
##          w.DS          w.EM
##  1.000000e+00  4.440892e-16
##
## $`2018-06-30`
##          mean        StdDev          out          w.CA        w.CTAG
##  7.160465e-03  1.706209e-02 -7.087686e-03  0.000000e+00 -2.654893e-16
##          w.DS          w.EM
##  1.000000e+00  4.440892e-16
##
## $`2018-09-30`
##          mean        StdDev          out          w.CA        w.CTAG
##  7.133716e-03  1.696989e-02 -7.061722e-03  0.000000e+00 -1.324144e-16
##          w.DS          w.EM
##  1.000000e+00  8.881784e-16
##
## $`2018-12-31`
##          mean        StdDev          out          w.CA        w.CTAG
##  6.831061e-03  1.711145e-02 -6.757860e-03  1.709127e-31  1.320061e-16
##          w.DS          w.EM
##  1.000000e+00  0.000000e+00
##
## $`2019-03-31`
```

```
##           mean          StdDev           out           w.CA          w.CTAG
##   6.853184e-03    1.704818e-02  -6.780523e-03   0.000000e+00   1.313924e-16
##            w.DS            w.EM
##   1.000000e+00    4.440892e-16
##
## $`2019-06-30`
##           mean          StdDev           out           w.CA          w.CTAG
##   6.825556e-03    1.697085e-02  -6.753553e-03  -1.110223e-16  -2.619477e-16
##            w.DS            w.EM
##   1.000000e+00    6.661338e-16
##
## $`2019-09-30`
##           mean          StdDev           out           w.CA          w.CTAG
##   6.698168e-03    1.692321e-02  -6.626570e-03   0.000000e+00   3.922686e-16
##            w.DS            w.EM
##   1.000000e+00   -8.881784e-16
##
## $`2019-11-30`
##           mean          StdDev           out           w.CA          w.CTAG
##   6.621818e-03    1.688505e-02  -6.550542e-03   0.000000e+00   2.607881e-16
##            w.DS            w.EM
##   1.000000e+00    0.000000e+00
```

extractWeights(opt_qu)

```
##            CA           CTAG             DS             EM
## 0.000000e+00 2.607881e-16   1.000000e+00  0.000000e+00
```

extractWeights(bt_qu)

```
##                       CA           CTAG             DS             EM
## 1999-12-31   1.000000e+00   2.182070e-17   2.287059e-14   0.000000e+00
## 2000-03-31   7.556488e-01  -5.504568e-17   0.000000e+00   2.443512e-01
## 2000-06-30   1.000000e+00   0.000000e+00   2.153833e-14   0.000000e+00
## 2000-09-30   1.000000e+00  -9.369392e-17   0.000000e+00   0.000000e+00
## 2000-12-31   1.000000e+00   0.000000e+00  -2.220446e-16  -5.286908e-17
## 2001-03-31   1.000000e+00   0.000000e+00   1.325329e-14   0.000000e+00
## 2001-06-30   1.000000e+00   1.203510e-30   0.000000e+00   1.784556e-16
## 2001-09-30   1.000000e+00   0.000000e+00   2.198242e-14   1.747802e-16
## 2001-12-31   1.000000e+00  -8.709944e-17   1.398881e-14   0.000000e+00
## 2002-03-31   1.000000e+00  -5.693326e-31   2.220446e-16   0.000000e+00
## 2002-06-30   1.000000e+00   8.402358e-17   2.220446e-15   0.000000e+00
## 2002-09-30   1.000000e+00   0.000000e+00   4.163779e-30   5.162658e-17
## 2002-12-31   1.000000e+00  -3.041291e-31   6.661338e-16  -4.508381e-17
## 2003-03-31   1.000000e+00   0.000000e+00   1.765255e-14  -5.308605e-17
## 2003-06-30   1.000000e+00  -1.433757e-30   3.330669e-16   8.879249e-17
## 2003-09-30   1.000000e+00  -1.273131e-30   6.661338e-16   0.000000e+00
## 2003-12-31   2.003848e-01   9.461819e-18   7.996152e-01   0.000000e+00
## 2004-03-31   5.551115e-17   4.290624e-17   1.000000e+00  -5.551115e-17
## 2004-06-30   5.551115e-16   0.000000e+00   1.000000e+00   0.000000e+00
## 2004-09-30   5.551115e-16   1.932332e-17   1.000000e+00   0.000000e+00
## 2004-12-31  -1.110223e-16  -1.383496e-16   1.000000e+00   3.330669e-16
## 2005-03-31   2.551128e-31   1.826609e-16   1.000000e+00   0.000000e+00
## 2005-06-30   0.000000e+00   1.806270e-16   1.000000e+00   0.000000e+00
## 2005-09-30   0.000000e+00   1.784641e-16   1.000000e+00  -2.220446e-16
```

```
## 2005-12-31  1.110223e-16  0.000000e+00  1.000000e+00  1.332268e-15
## 2006-03-31  0.000000e+00  1.749255e-16  9.999277e-01  7.232907e-05
## 2006-06-30            NA            NA            NA            NA
## 2006-09-30  0.000000e+00 -1.726289e-16  1.000000e+00  0.000000e+00
## 2006-12-31 -3.469447e-18  1.699204e-16  9.644896e-01  3.551035e-02
## 2007-03-31  0.000000e+00  3.380425e-16  1.000000e+00  2.220446e-16
## 2007-06-30  0.000000e+00 -1.461122e-17  1.608224e-01  8.391776e-01
## 2007-09-30  0.000000e+00 -6.327720e-17  1.998401e-15  1.000000e+00
## 2007-12-31  0.000000e+00  6.266547e-17  0.000000e+00  1.000000e+00
## 2008-03-31  1.551358e-16  4.686309e-31  0.000000e+00  1.000000e+00
## 2008-06-30  1.544404e-16  0.000000e+00  1.554312e-15  1.000000e+00
## 2008-09-30  4.445725e-16  0.000000e+00  1.000000e+00 -4.440892e-16
## 2008-12-31            NA            NA            NA            NA
## 2009-03-31  5.161488e-18  0.000000e+00  1.000000e+00  1.110223e-15
## 2009-06-30 -2.658501e-16  0.000000e+00  6.013453e-01  3.986547e-01
## 2009-09-30 -6.678978e-17  0.000000e+00  2.495612e-01  7.504388e-01
## 2009-12-31 -2.284523e-16  0.000000e+00  5.848416e-01  4.151584e-01
## 2010-03-31  1.110223e-16  1.581894e-16  1.000000e+00  1.776357e-15
## 2010-06-30  0.000000e+00  1.570929e-16  1.000000e+00  3.330669e-16
## 2010-09-30  0.000000e+00 -1.565722e-16  7.305023e-01  2.694977e-01
## 2010-12-31  0.000000e+00  0.000000e+00  1.000000e+00  5.551115e-16
## 2011-03-31  0.000000e+00 -3.074847e-16  1.000000e+00  2.220446e-16
## 2011-06-30  0.000000e+00  0.000000e+00  1.000000e+00  6.661338e-16
## 2011-09-30  0.000000e+00  0.000000e+00  1.000000e+00  6.661338e-16
## 2011-12-31  0.000000e+00  0.000000e+00  1.000000e+00  4.440892e-16
## 2012-03-31  0.000000e+00  4.538389e-16  1.000000e+00  0.000000e+00
## 2012-06-30  0.000000e+00 -1.510361e-16  1.000000e+00  4.440892e-16
## 2012-09-30  0.000000e+00  0.000000e+00  1.000000e+00  0.000000e+00
## 2012-12-31  0.000000e+00  1.490868e-16  1.000000e+00 -2.220446e-16
## 2013-03-31  0.000000e+00  2.960303e-16  1.000000e+00  1.110223e-16
## 2013-06-30  0.000000e+00 -4.416281e-16  1.000000e+00  7.771561e-16
## 2013-09-30  0.000000e+00  0.000000e+00  1.000000e+00  0.000000e+00
## 2013-12-31  0.000000e+00  0.000000e+00  1.000000e+00 -6.661338e-16
## 2014-03-31  0.000000e+00 -1.448811e-16  1.000000e+00  4.440892e-16
## 2014-06-30  0.000000e+00 -1.439759e-16  1.000000e+00  4.440892e-16
## 2014-09-30  0.000000e+00  1.429946e-16  1.000000e+00  8.881784e-16
## 2014-12-31  0.000000e+00 -1.421090e-16  1.000000e+00 -1.221245e-15
## 2015-03-31  0.000000e+00  2.220446e-16  1.000000e+00 -8.881784e-16
## 2015-06-30  1.110223e-16  1.404122e-16  1.000000e+00  1.110223e-16
## 2015-09-30 -3.560813e-31 -1.643489e-31  1.000000e+00  0.000000e+00
## 2015-12-31  0.000000e+00  2.785997e-16  1.000000e+00 -6.661338e-16
## 2016-03-31  0.000000e+00  0.000000e+00  1.000000e+00  4.440892e-16
## 2016-06-30  0.000000e+00 -1.379660e-16  1.000000e+00  1.221245e-15
## 2016-09-30  0.000000e+00  0.000000e+00  1.000000e+00  9.992007e-16
## 2016-12-31  0.000000e+00  0.000000e+00  1.000000e+00 -2.220446e-16
## 2017-03-31  0.000000e+00  0.000000e+00  1.000000e+00  1.110223e-16
## 2017-06-30  0.000000e+00 -1.348155e-16  1.000000e+00  1.332268e-15
## 2017-09-30  0.000000e+00  0.000000e+00  1.000000e+00 -2.220446e-16
## 2017-12-31  0.000000e+00  0.000000e+00  1.000000e+00  4.440892e-16
## 2018-03-31  0.000000e+00  1.330476e-16  1.000000e+00  4.440892e-16
## 2018-06-30  0.000000e+00 -2.654893e-16  1.000000e+00  4.440892e-16
## 2018-09-30  0.000000e+00 -1.324144e-16  1.000000e+00  8.881784e-16
## 2018-12-31  1.709127e-31  1.320061e-16  1.000000e+00  0.000000e+00
## 2019-03-31  0.000000e+00  1.313924e-16  1.000000e+00  4.440892e-16
```

```
## 2019-06-30 -1.110223e-16 -2.619477e-16  1.000000e+00  6.661338e-16
## 2019-09-30  0.000000e+00  3.922686e-16  1.000000e+00 -8.881784e-16
## 2019-11-30  0.000000e+00  2.607881e-16  1.000000e+00  0.000000e+00
```

```
extractObjectiveMeasures(bt_qu)
```

```
##                    mean     StdDev
## 1999-12-31 0.008936111 0.01189271
## 2000-03-31 0.010234949 0.02009404
## 2000-06-30 0.010726190 0.01194628
## 2000-09-30 0.010891111 0.01157330
## 2000-12-31 0.010145833 0.01165353
## 2001-03-31 0.010898039 0.01186908
## 2001-06-30 0.010666667 0.01166813
## 2001-09-30 0.010650877 0.01136948
## 2001-12-31 0.010290000 0.01138032
## 2002-03-31 0.010041270 0.01129750
## 2002-06-30 0.009786364 0.01112694
## 2002-09-30 0.009414493 0.01133961
## 2002-12-31 0.009733333 0.01127324
## 2003-03-31 0.010017333 0.01125574
## 2003-06-30 0.009924359 0.01120202
## 2003-09-30 0.009571605 0.01137095
## 2003-12-31 0.009633544 0.01526133
## 2004-03-31 0.009800000 0.01707024
## 2004-06-30 0.009790000 0.01685525
## 2004-09-30 0.009701075 0.01659875
## 2004-12-31 0.010175000 0.01660971
## 2005-03-31 0.010071717 0.01638464
## 2005-06-30 0.009860784 0.01624002
## 2005-09-30 0.009968571 0.01602280
## 2005-12-31 0.009867593 0.01584894
## 2006-03-31 0.010042350 0.01571924
## 2006-06-30          NA          NA
## 2006-09-30 0.009873504 0.01540371
## 2006-12-31 0.010078936 0.01574027
## 2007-03-31 0.010156911 0.01508489
## 2007-06-30 0.010531875 0.03200120
## 2007-09-30 0.010679845 0.03578437
## 2007-12-31 0.010723485 0.03565357
## 2008-03-31 0.010039259 0.03591528
## 2008-06-30 0.009878261 0.03567546
## 2008-09-30 0.008418440 0.01602578
## 2008-12-31          NA          NA
## 2009-03-31 0.007105442 0.01794859
## 2009-06-30 0.007791123 0.02514480
## 2009-09-30 0.008449347 0.03280577
## 2009-12-31 0.008543316 0.02536431
## 2010-03-31 0.008627673 0.01838978
## 2010-06-30 0.008403704 0.01851522
## 2010-09-30 0.008492313 0.02243737
## 2010-12-31 0.008635119 0.01835120
## 2011-03-31 0.008694152 0.01820900
## 2011-06-30 0.008571264 0.01810751
## 2011-09-30 0.007981356 0.01865031
```

```
## 2011-12-31 0.007947778 0.01858504
## 2012-03-31 0.008154645 0.01854143
## 2012-06-30 0.007911828 0.01851853
## 2012-09-30 0.008007937 0.01839146
## 2012-12-31 0.008121875 0.01829403
## 2013-03-31 0.008212308 0.01818646
## 2013-06-30 0.008235859 0.01813650
## 2013-09-30 0.008247264 0.01803261
## 2013-12-31 0.008323529 0.01791127
## 2014-03-31 0.008373430 0.01781746
## 2014-06-30 0.008378095 0.01769893
## 2014-09-30 0.008133803 0.01771769
## 2014-12-31 0.007886111 0.01772948
## 2015-03-31 0.007804566 0.01769594
## 2015-06-30 0.007670721 0.01765077
## 2015-09-30 0.007387111 0.01772273
## 2015-12-31 0.007171053 0.01771902
## 2016-03-31 0.007003030 0.01778380
## 2016-06-30 0.007094872 0.01769710
## 2016-09-30 0.007215190 0.01762523
## 2016-12-31 0.007334167 0.01755141
## 2017-03-31 0.007316872 0.01748794
## 2017-06-30 0.007240244 0.01740057
## 2017-09-30 0.007221687 0.01730436
## 2017-12-31 0.007205159 0.01722147
## 2018-03-31 0.007157647 0.01716141
## 2018-06-30 0.007160465 0.01706209
## 2018-09-30 0.007133716 0.01696989
## 2018-12-31 0.006831061 0.01711145
## 2019-03-31 0.006853184 0.01704818
## 2019-06-30 0.006825556 0.01697085
## 2019-09-30 0.006698168 0.01692321
## 2019-11-30 0.006621818 0.01688505
```
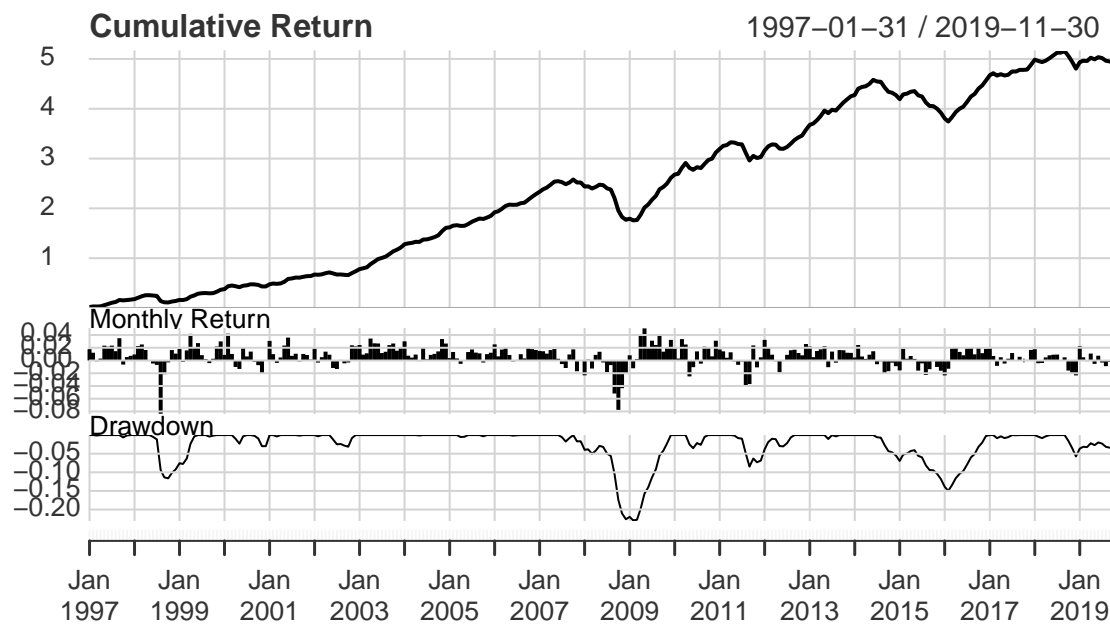
```
chart.Weights(opt_qu)
```

Weights
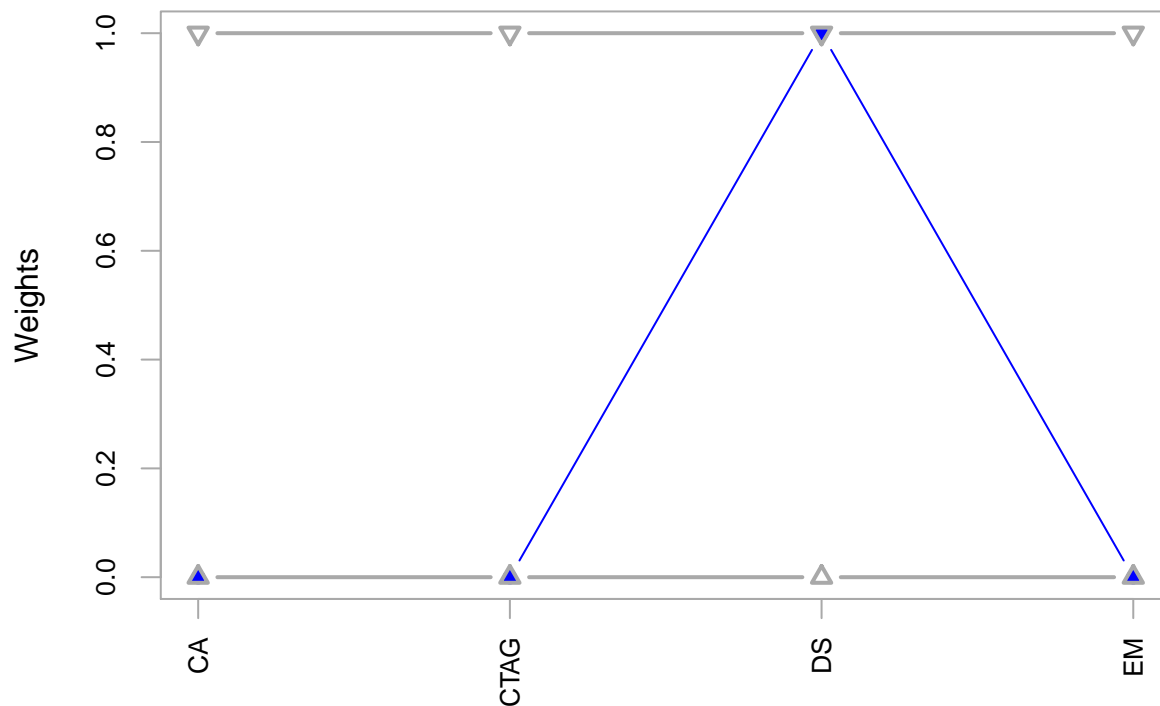

```
   rr <- Return.portfolio(returns, weights = extractWeights(opt_qu))
charts.PerformanceSummary(rr)
```

portfolio.returns Performance


```
weights <- extractWeights(bt_qu)

weights <- as.data.frame(weights)
```

```
weights <- weights %>%
  filter(CA != "NA" & CTAG != "NA" & DS != "NA" & EM != "NA")


rr <- Return.portfolio(returns, weights = weights)
charts.PerformanceSummary(rr)
```

portfolio.returns Performance