



## Aulas 4 e 5 - Ordenação

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Módulo 1 - PrepTech Google

Ordenação

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# Ordenação

## Conceito

Dado um vetor **v** com **N** elementos, a ordenação consiste em organizar todos esses **N** elementos em uma ordem (crescente, não-decrescente etc)

### **Exemplo**

$v = [1, 9, 8, 5, 3, 7, 4]$

- Depois de ordenado de forma não-decrescente, fica:

$v = [1, 3, 4, 5, 7, 8, 9]$



Exemplos de **algoritmos de ordenação** por comparação:

- Selection Sort
- Bubble Sort
- Mergesort
- Quicksort
- Heapsort

Exemplos de **algoritmos de ordenação** sem comparação:

- Count Sort
- Radix Sort

## Selection Sort

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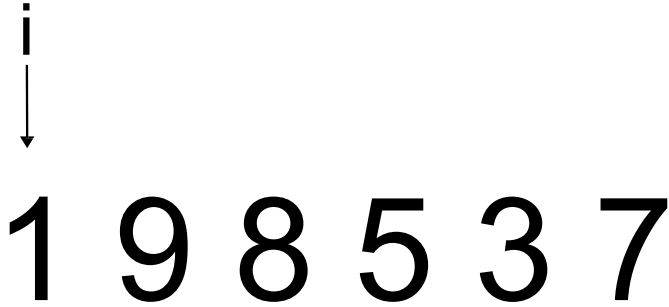
# Selection Sort

## Conceito

- Consiste em posicionar o menor ou o maior elemento em seu lugar correto **N** vezes consecutivas (onde N é o tamanho do vetor)

# Selection Sort

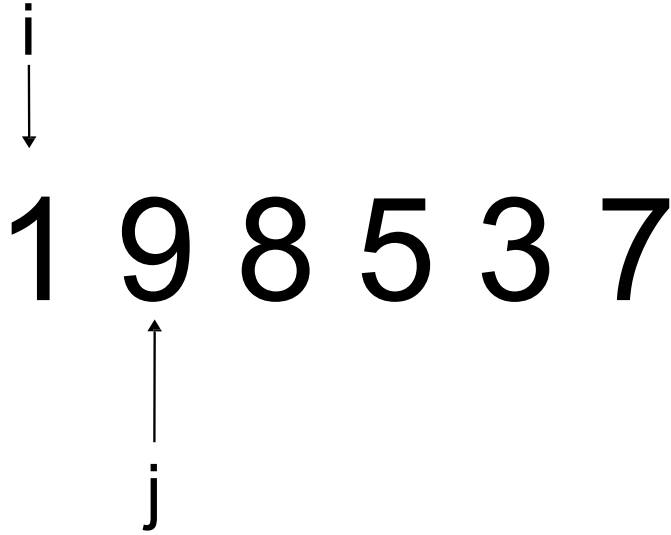
Funcionamento



O ponteiro **i** denota onde queremos inserir o menor elemento.

# Selection Sort

Funcionamento

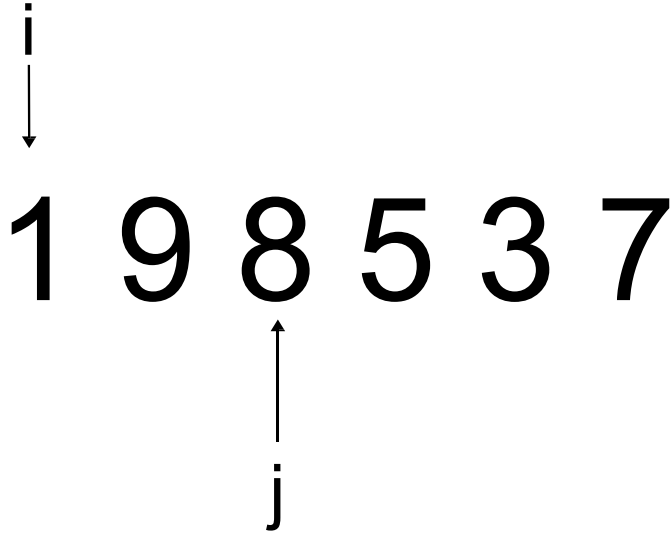


```
if v[j] < v[i]:  
    v[i], v[j] = v[j], v[i] #swap
```



# Selection Sort

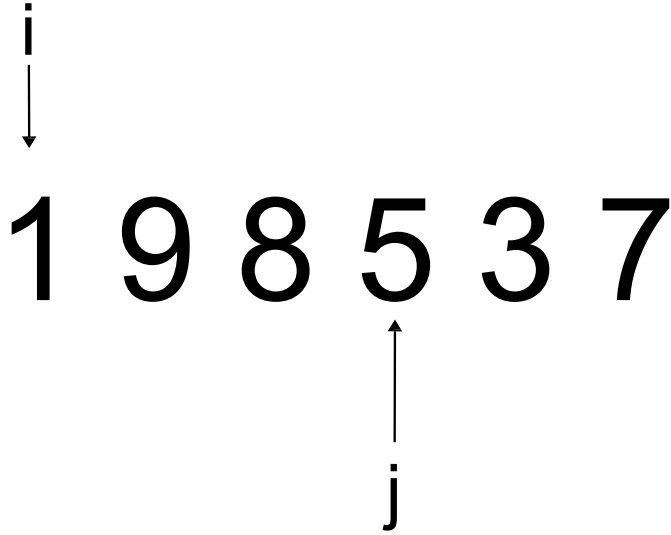
Funcionamento



```
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```

# Selection Sort

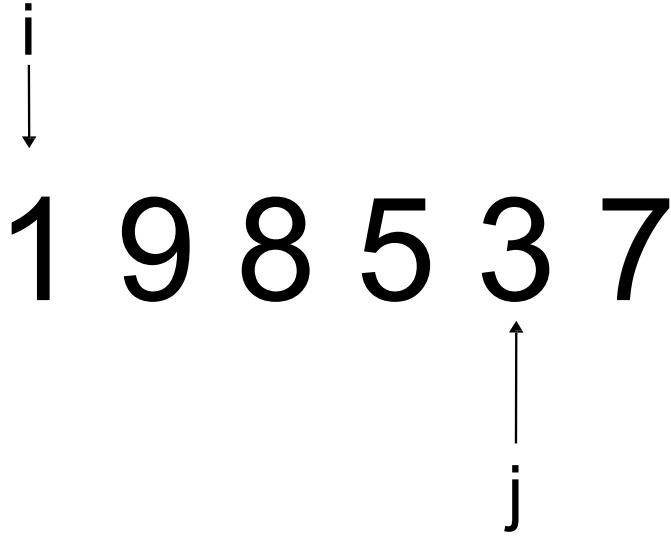
Funcionamento



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```

# Selection Sort

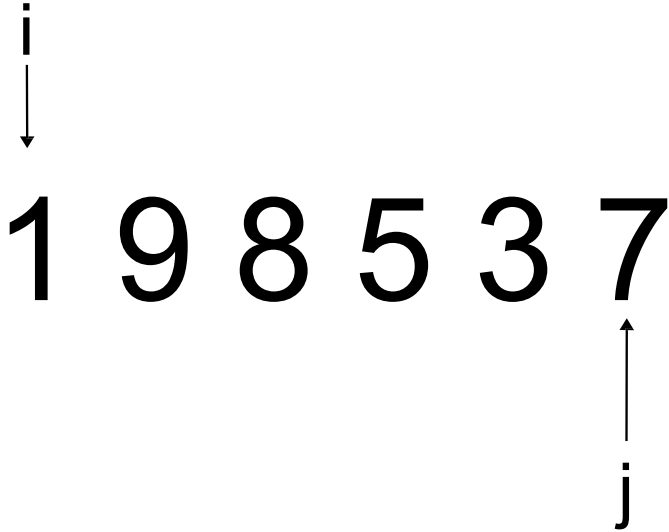
Funcionamento



```
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```

# Selection Sort

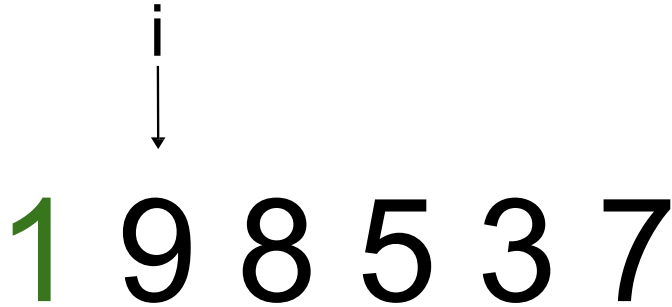
Funcionamento



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```

# Selection Sort

Funcionamento

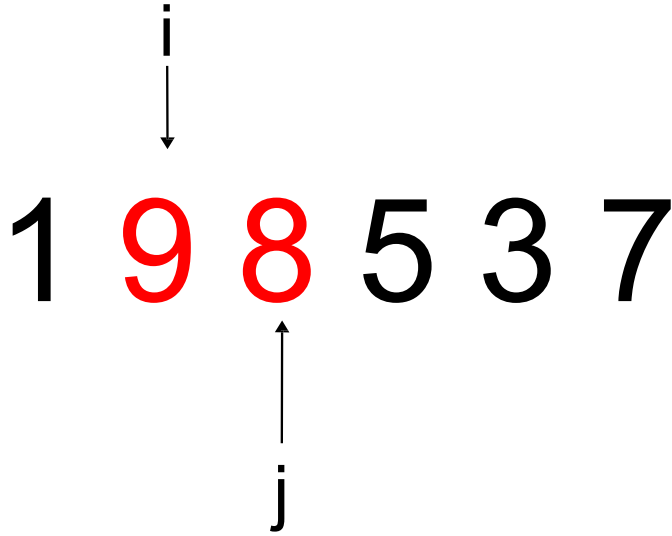


Elementos na esquerda de **i** já **ordenados**. O menor elemento na posição zero.



# Selection Sort

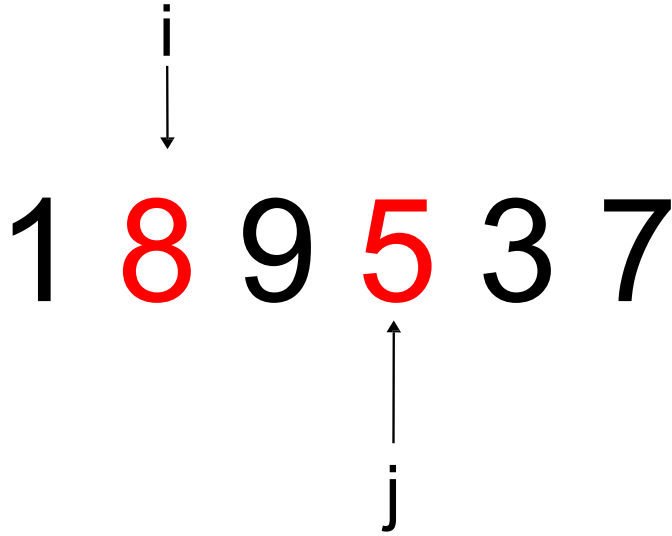
Funcionamento



```
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```

# Selection Sort

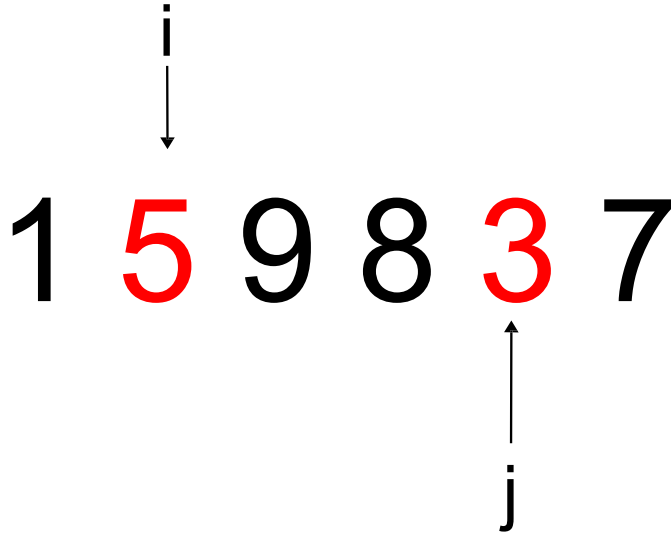
Funcionamento



```
if v[j] < v[i]:  
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```

# Selection Sort

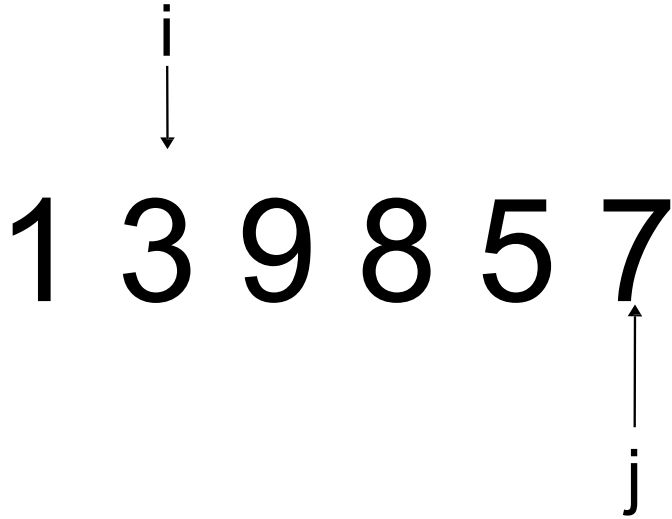
Funcionamento



```
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```

# Selection Sort

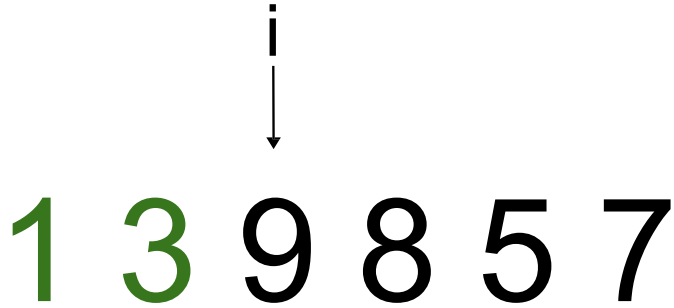
Funcionamento



```
if v[j] < v[i]:  
    v[i], v[j] = v[j], v[i] #swap
```

# Selection Sort

Funcionamento

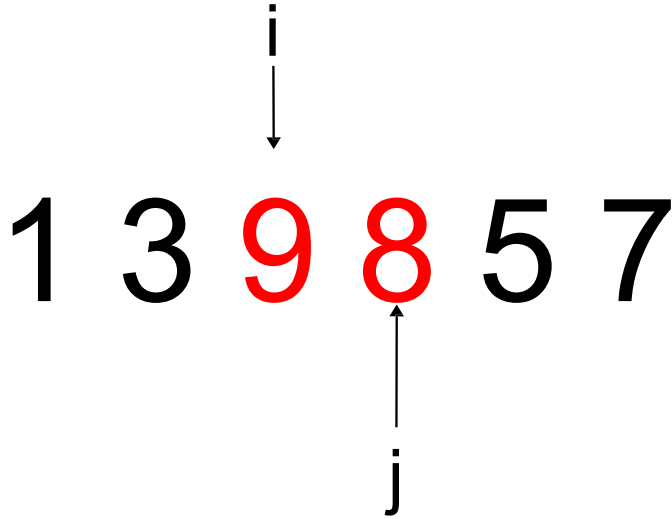


Elementos na esquerda de **i** já **ordenados**, em seu lugar definitivo.



# Selection Sort

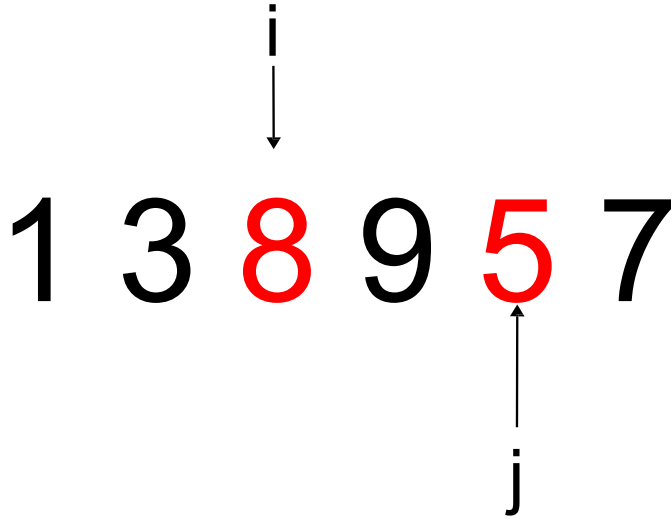
Funcionamento



```
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```

# Selection Sort

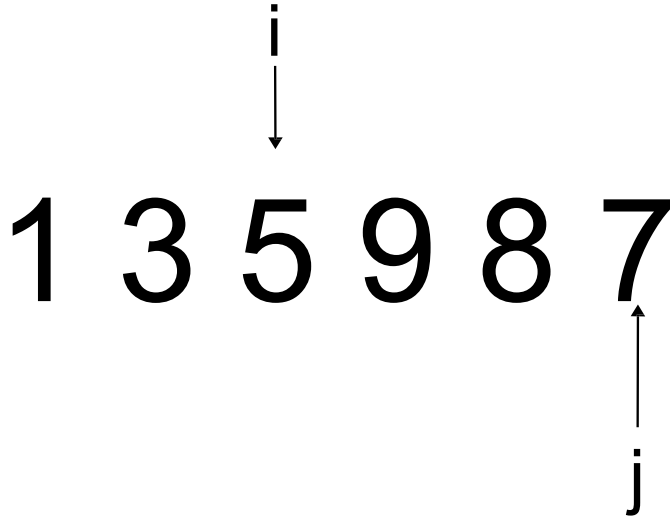
Funcionamento



```
if v[j] < v[i]:  
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```

# Selection Sort

Funcionamento



```
if v[j] < v[i]:  
    v[i], v[j] = v[j], v[i] #swap
```

# Selection Sort

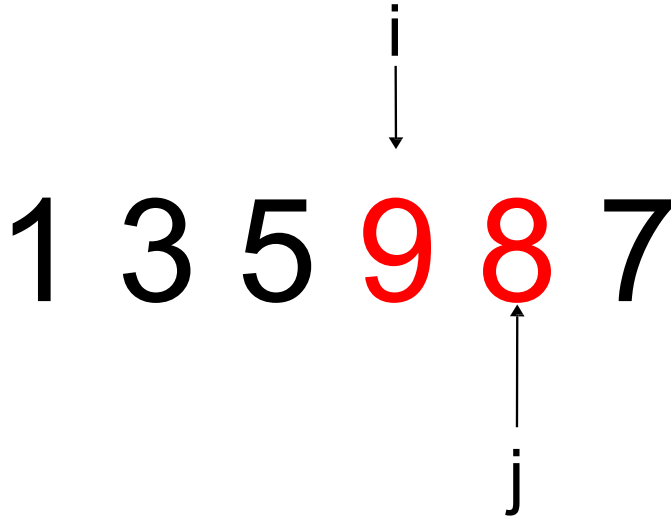
Funcionamento

i  
↓  
1 3 5 9 8 7

Elementos na esquerda de **i** já **ordenados**, em seu lugar definitivo.

# Selection Sort

Funcionamento

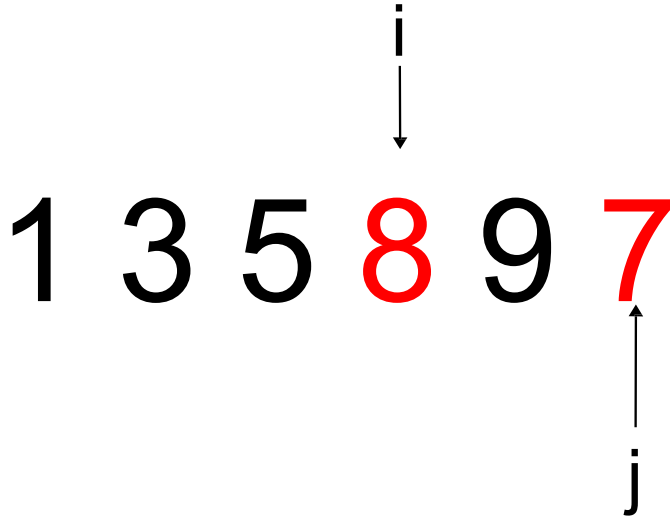


```
if v[j] < v[i]:  
    v[i], v[j] = v[j], v[i] #swap
```



# Selection Sort

Funcionamento



```
if v[j] < v[i]:  
    v[i], v[j] = v[j], v[i] #swap
```

# Selection Sort

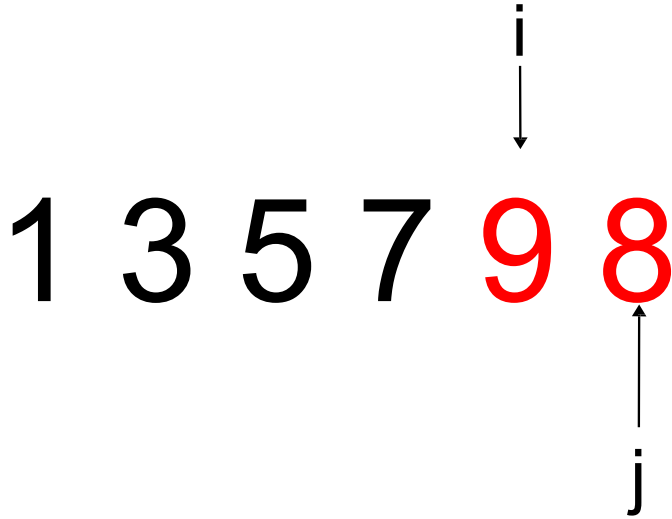
Funcionamento

$i$   
↓  
1 3 5 7 9 8

Elementos na esquerda de  **$i$**  já **ordenados**, em seu lugar definitivo.

# Selection Sort

Funcionamento



```
if v[j] < v[i]:  
    v[i], v[j] = v[j], v[i] #swap
```

# Selection Sort

Funcionamento

$i$   
↓  
1 3 5 7 8 9

Elementos na esquerda de  **$i$**  já **ordenados**, em seu lugar definitivo.

# Selection Sort

Funcionamento

1 3 5 7 8 9



The diagram illustrates the final state of an array after the Selection Sort algorithm. The array contains the numbers 1, 3, 5, 7, 8, and 9. The numbers 1, 3, 5, 7, and 8 are colored green, while the number 9 is black. An arrow points from the letter 'i' to the number 9, indicating its final position in the sorted array.

Agora todos os elementos na posição final.

# Selection Sort

Funcionamento

1 3 5 7 8 9

Vetor ordenado!

## Selection Sort

Código em Lua

```
for i=1,N do  
    for j=i+1,N do  
        if v[i]>v[j] then  
            v[i],v[j] = v[j],v[i]  
        end  
    end  
end
```

# Selection Sort

Código em Python

```
for i in range( len(arr) ):
    for j in range(i+1, len(arr)):
        if v[j]<v[i]:
            v[i], v[j] = v[j], v[i]
```



# Selection Sort

Código

1 3 5 7 8 9

**Número de operações:**

**$(N-1) + (N-2) + (N-3) + \dots + 3 + 2 + 1$**

$$\frac{N*(N-1)}{2}$$

## Bubble Sort

---

# Bubble Sort

Código

- Ideia:
  - Se o vetor já estiver ordenado em alguma interação, seria interessante detectar isso e parar o algoritmo.
- Como detectar se um vetor está ordenado?

```
sorted = true
```

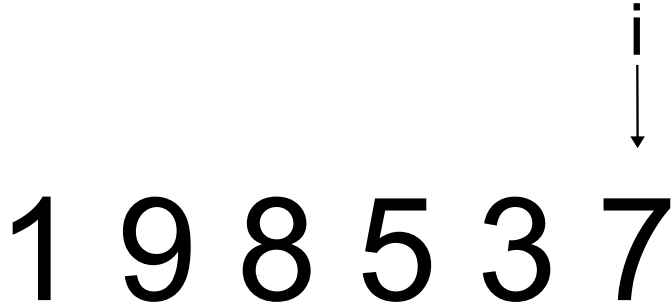
```
for i=1,N do
```

```
    if v[i]>v[i+1] then sorted = false end
```

```
end
```

# Bubble Sort

Funcionamento

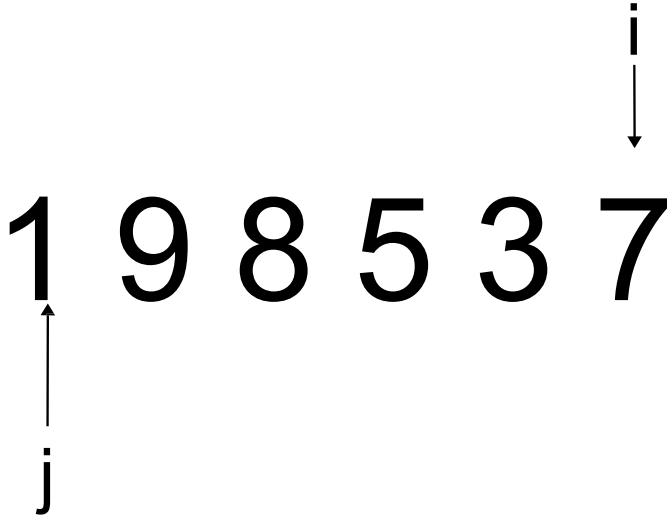
  
1 9 8 5 3 7

A ideia agora vai ser colocar o **maior elemento** no final do vetor, comparando os **consecutivos**.

O ponteiro **i** indica essa posição.

# Bubble Sort

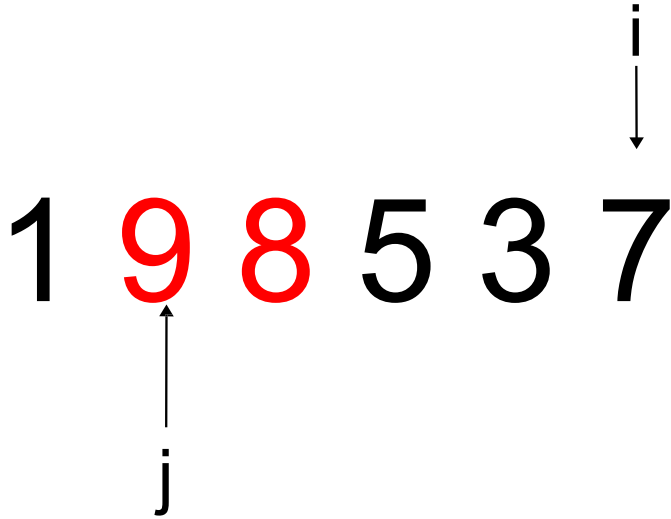
Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

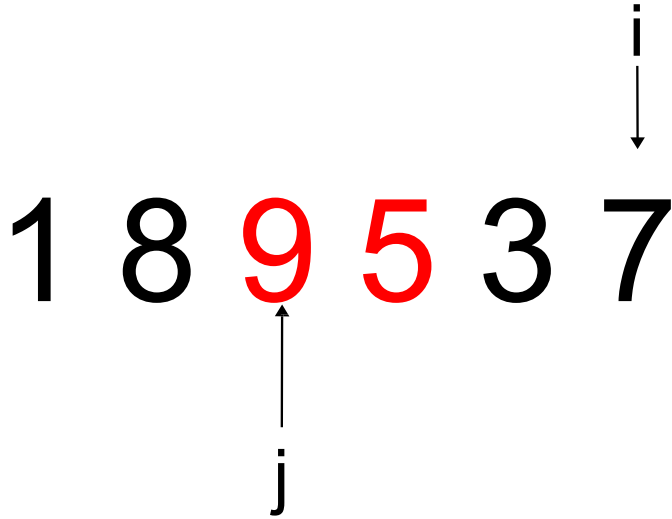
Funcionamento



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    v[j], v[j+1] = v[j+1],v[j] #swap
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# Bubble Sort

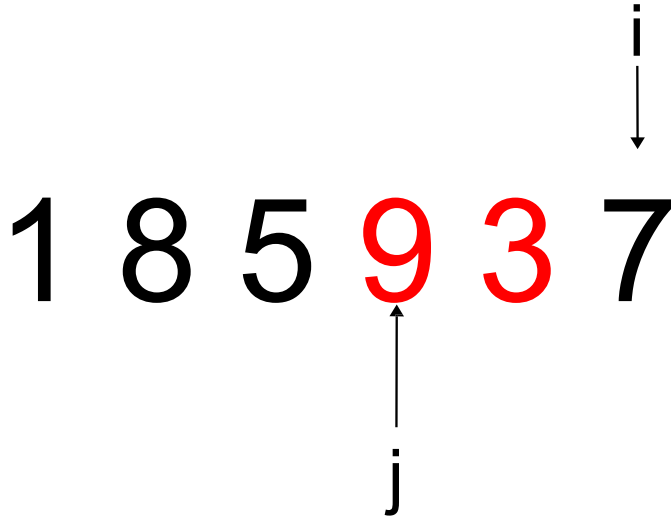
Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

Funcionamento

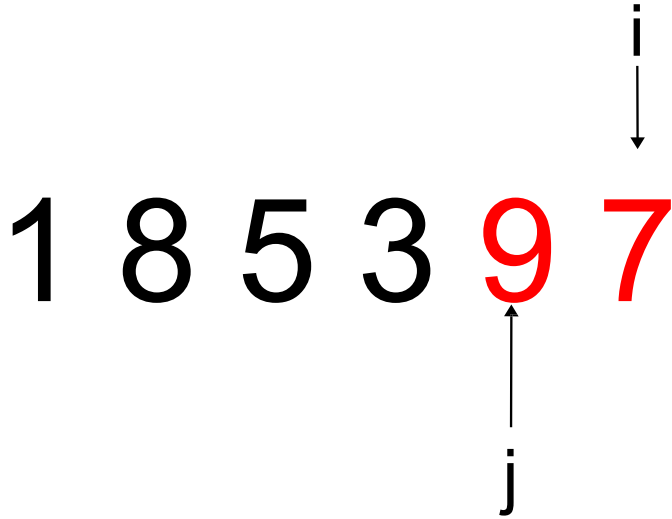


```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```



# Bubble Sort

Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

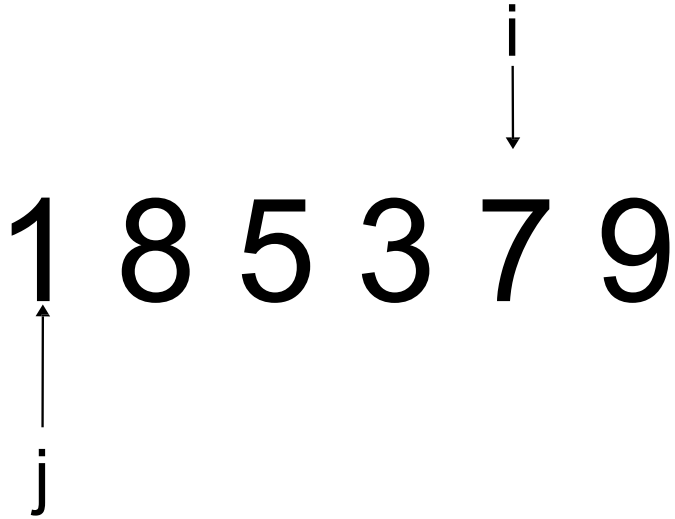
Funcionamento

$i$   
↓  
1 8 5 3 7 9

O vetor na direita de ***i*** está com os **elementos ordenados**, na posição definitiva

# Bubble Sort

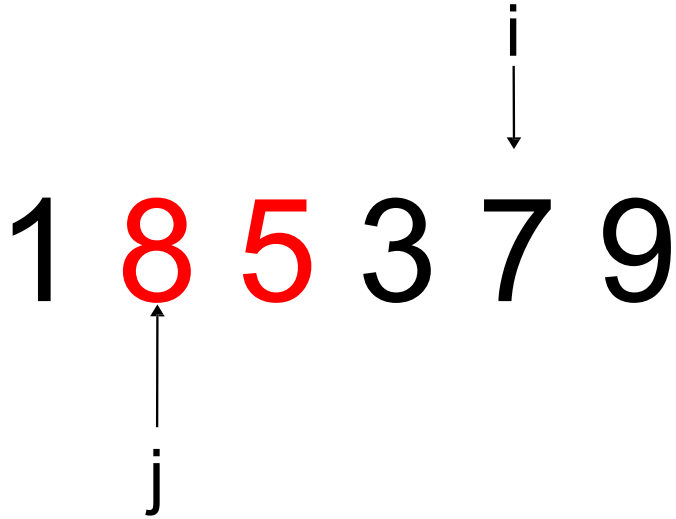
Funcionamento



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if v[j]>v[j+1]:  
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# Bubble Sort

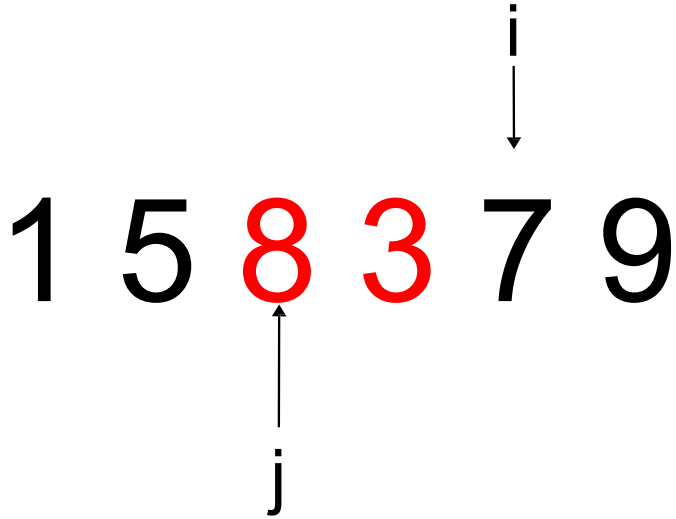
Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

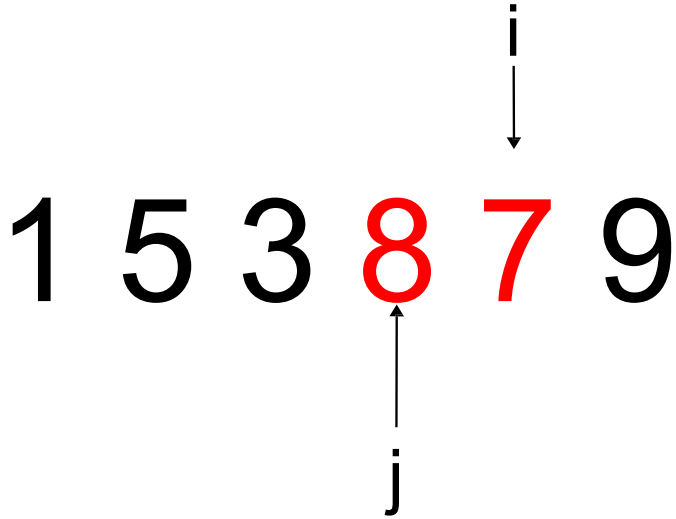
Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

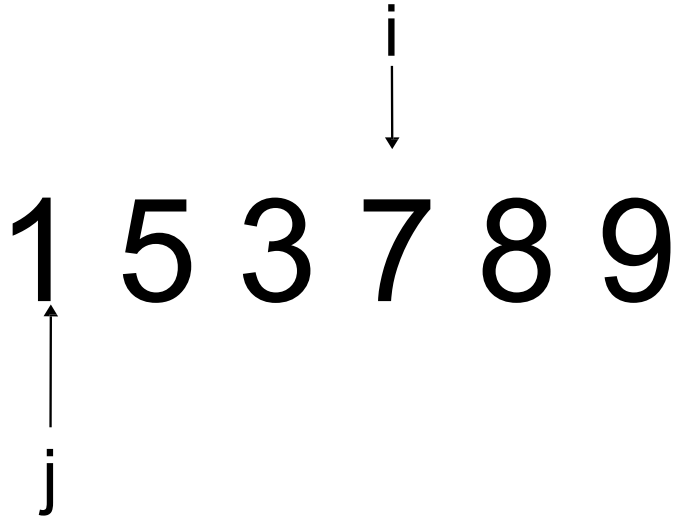
Funcionamento

$i$   
↓  
1 5 3 7 8 9

O vetor na direita de ***i*** está com os **elementos ordenados**, na posição definitiva

# Bubble Sort

Funcionamento

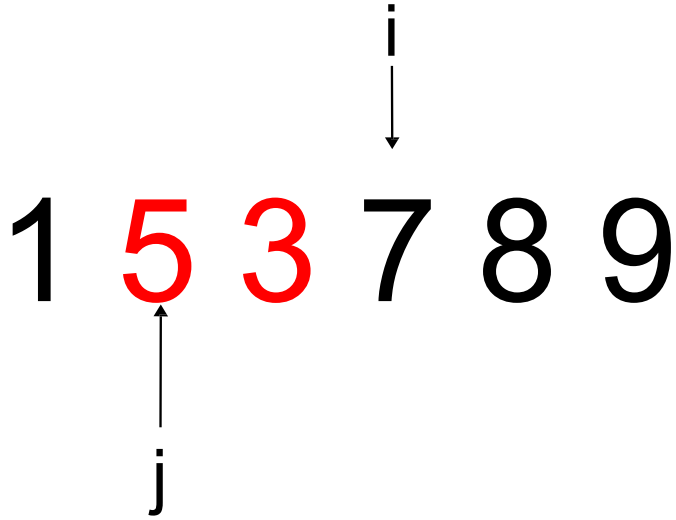


```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```



# Bubble Sort

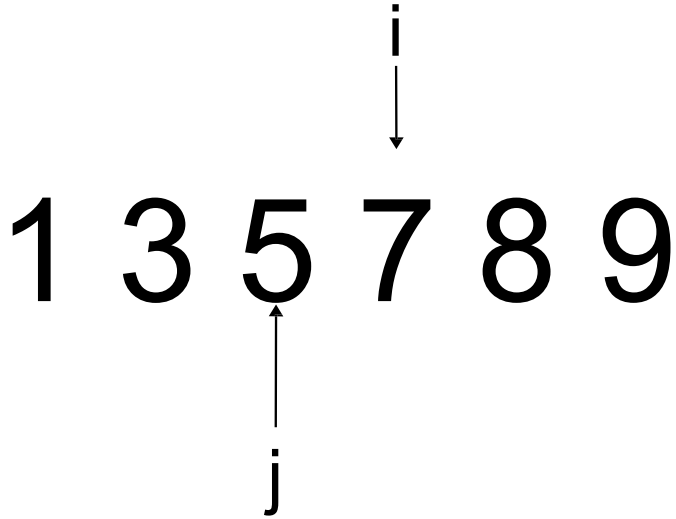
Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

Funcionamento



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if v[j]>v[j+1]:  
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# Bubble Sort

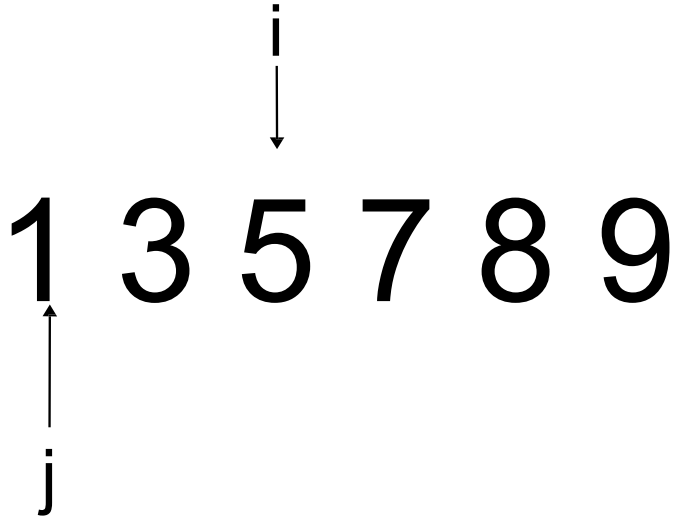
Funcionamento

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↓  
1 3 5 7 8 9

O vetor na direita de ***i*** está com os **elementos ordenados**, na posição definitiva

# Bubble Sort

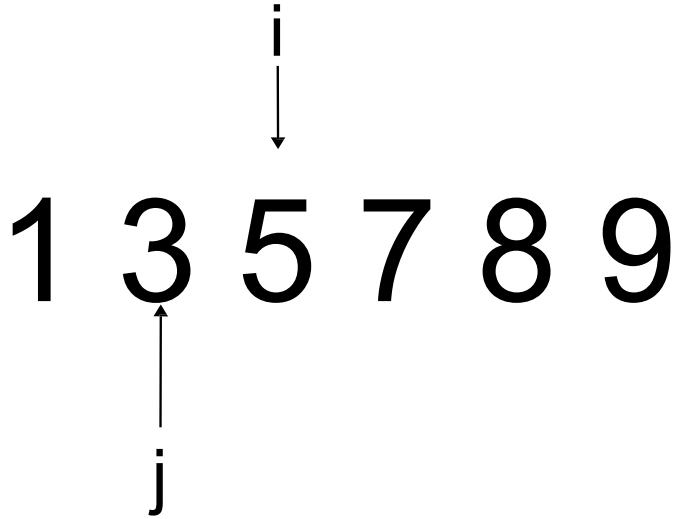
Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

Funcionamento



```
if v[j]>v[j+1]:  
    v[j], v[j+1] = v[j+1],v[j] #swap
```

# Bubble Sort

Funcionamento

i  
↓  
1 3 5 7 8 9

Não houve trocas! O vetor está  
ordenado!

# Bubble Sort

Código em Lua

```
for i=N,2,-1 do  
    anySwap = false  
    for j=1,i-1 do  
        if v[j]>v[j+1] then  
            v[j],v[j+1] = v[j+1],v[j]  
            anySwap = true  
        end  
    end  
    if not anySwap then break end  
end
```

# Bubble Sort

Código em Python

```
for i in range( len(arr)-1, 0, -1 ):
    anySwap = False
    for j in range(i):
        if v[j]>v[j+1]:
            v[j], v[j+1] = v[j+1], v[j]
            anySwap = True
    if not anySwap:
        break
```



## Selection and Bubble Sort

Prática no LeetCode - <https://leetcode.com/problems/sort-an-array/description/>

Implementar os dois algoritmos (*Insert* e *Bubble Sort*) e comparar o resultado de cada um deles em termos de tempo de processamento.

## Mergesort

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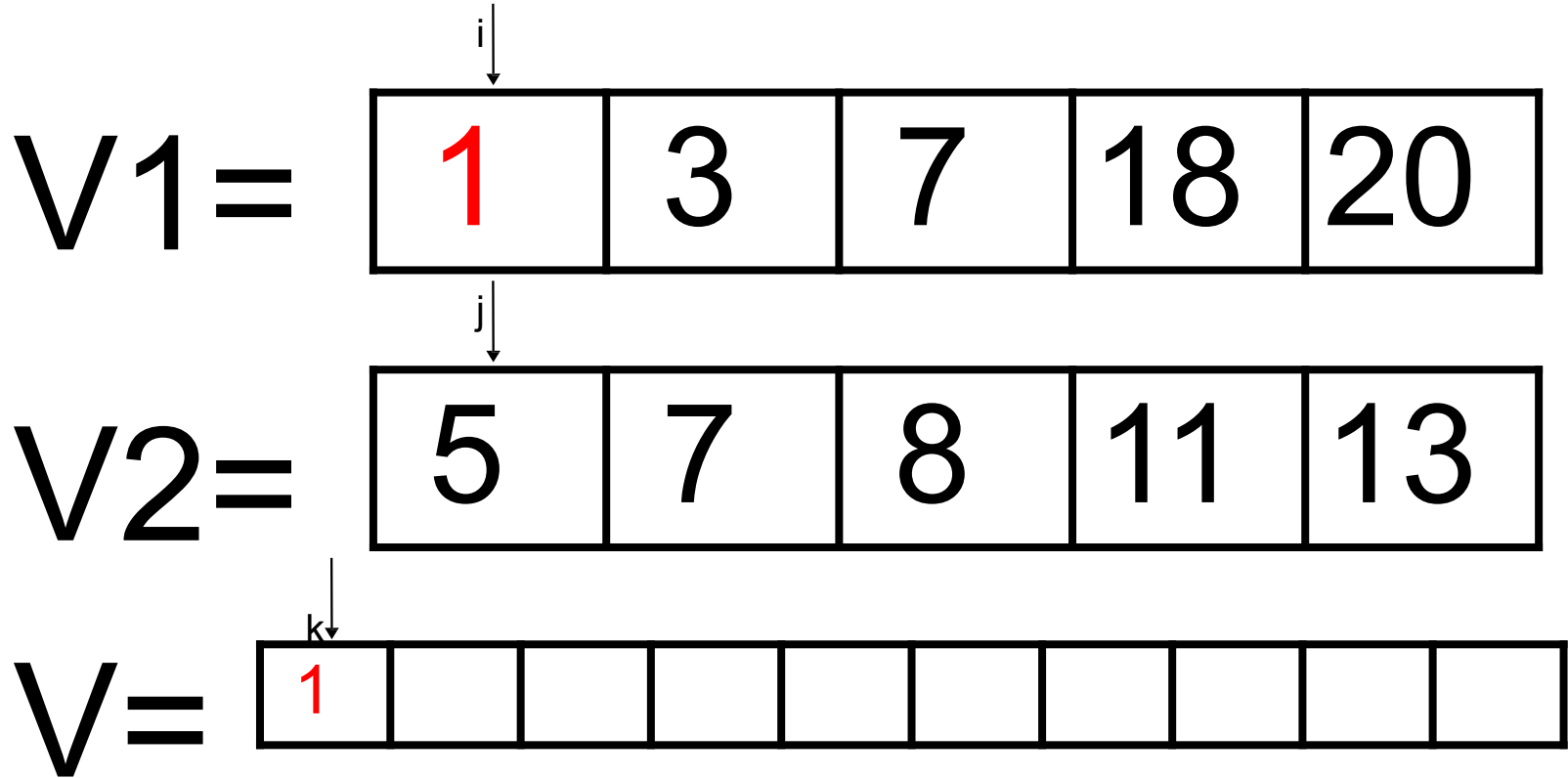
# Mergesort

## Conceito

- Ordena inicialmente sub-vetores de tamanho 1 ou 2 e recursivamente dobra esse tamanho até ordenar o vetor inteiro
- Baseia-se no merge de dois subvetores ordenados (veja animação disso nos próximos slides)

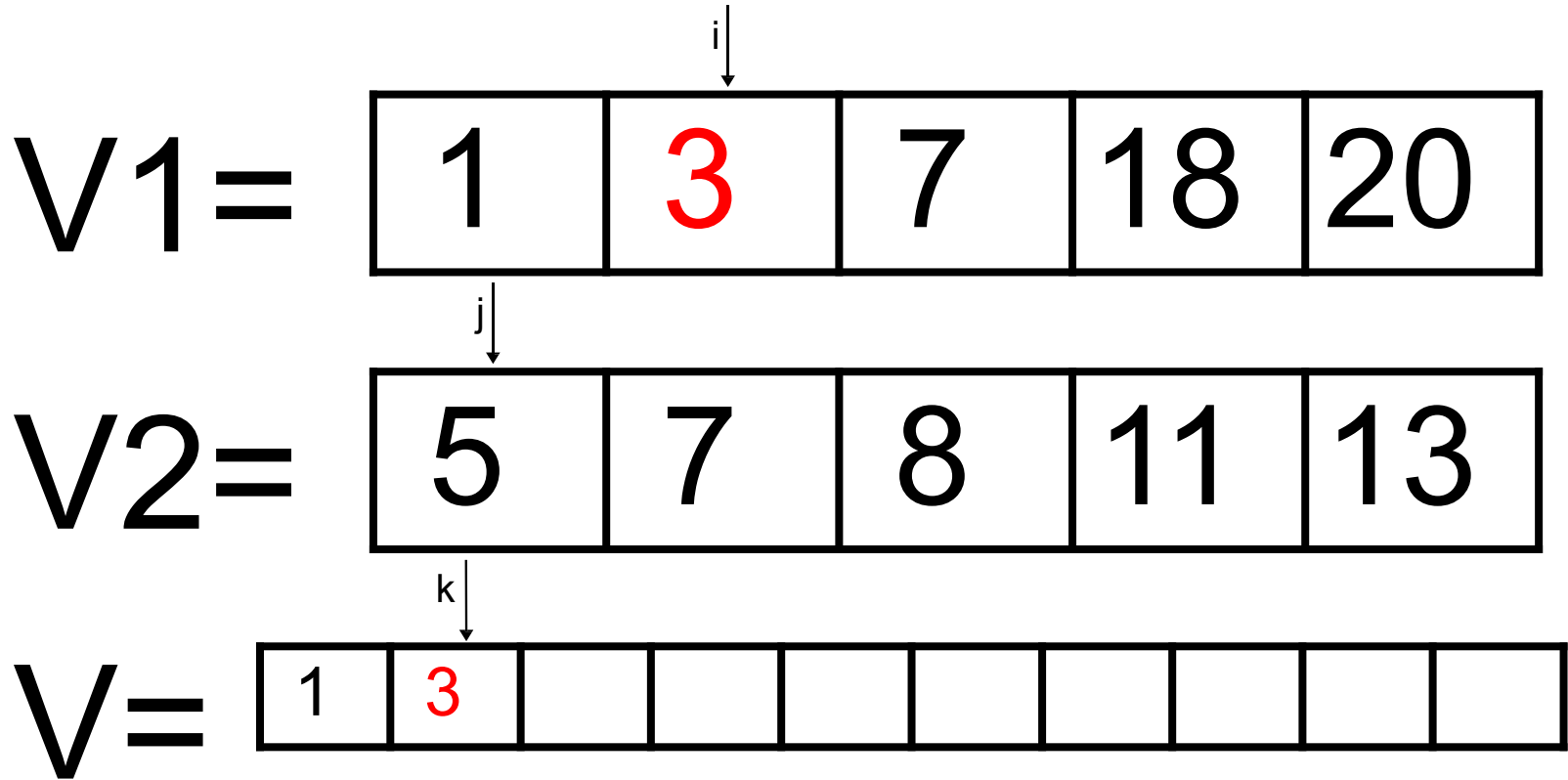
## Merge de dois vetores ordenados

Código em Python



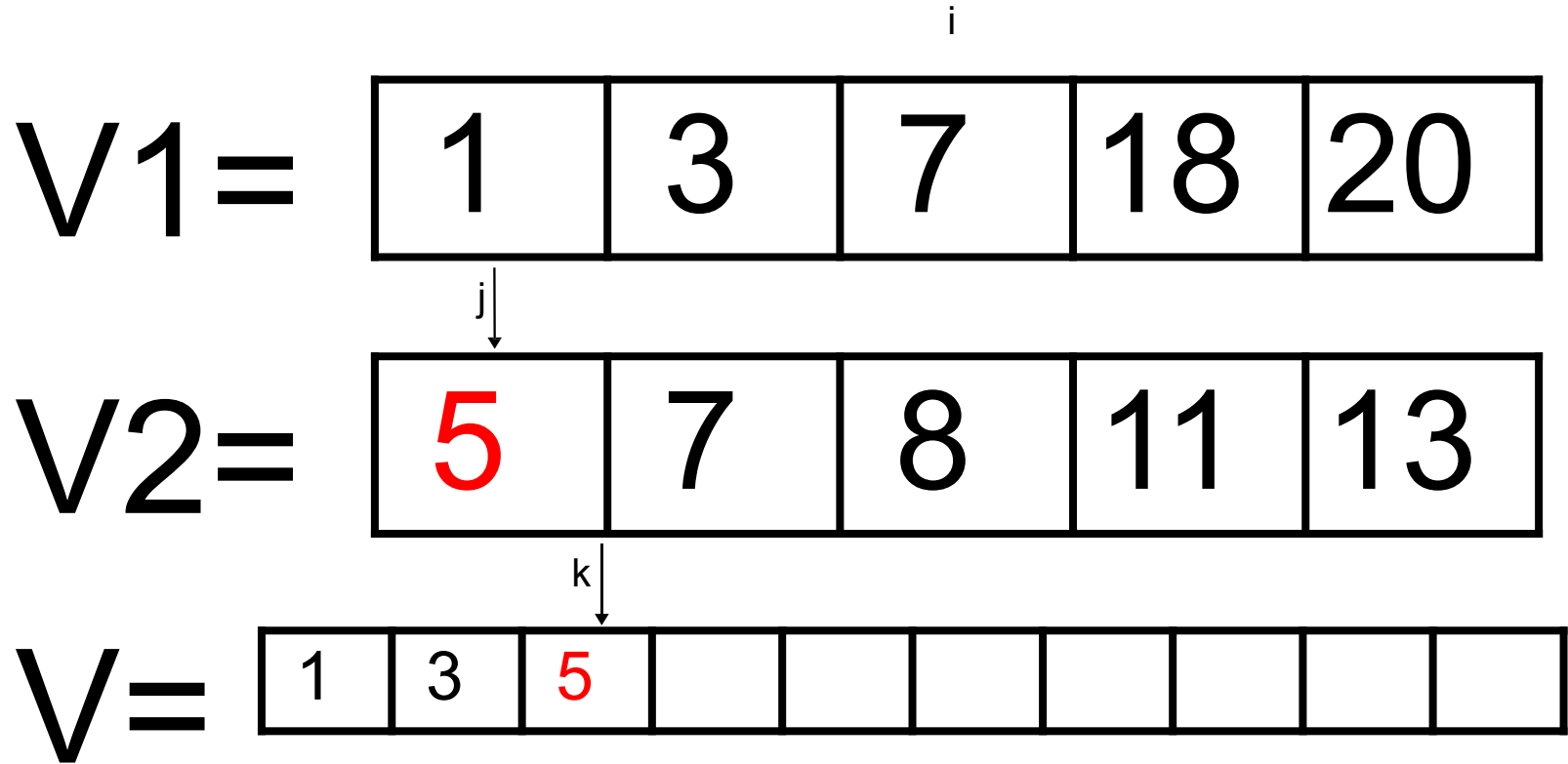
## Merge de dois vetores ordenados

Código em Python



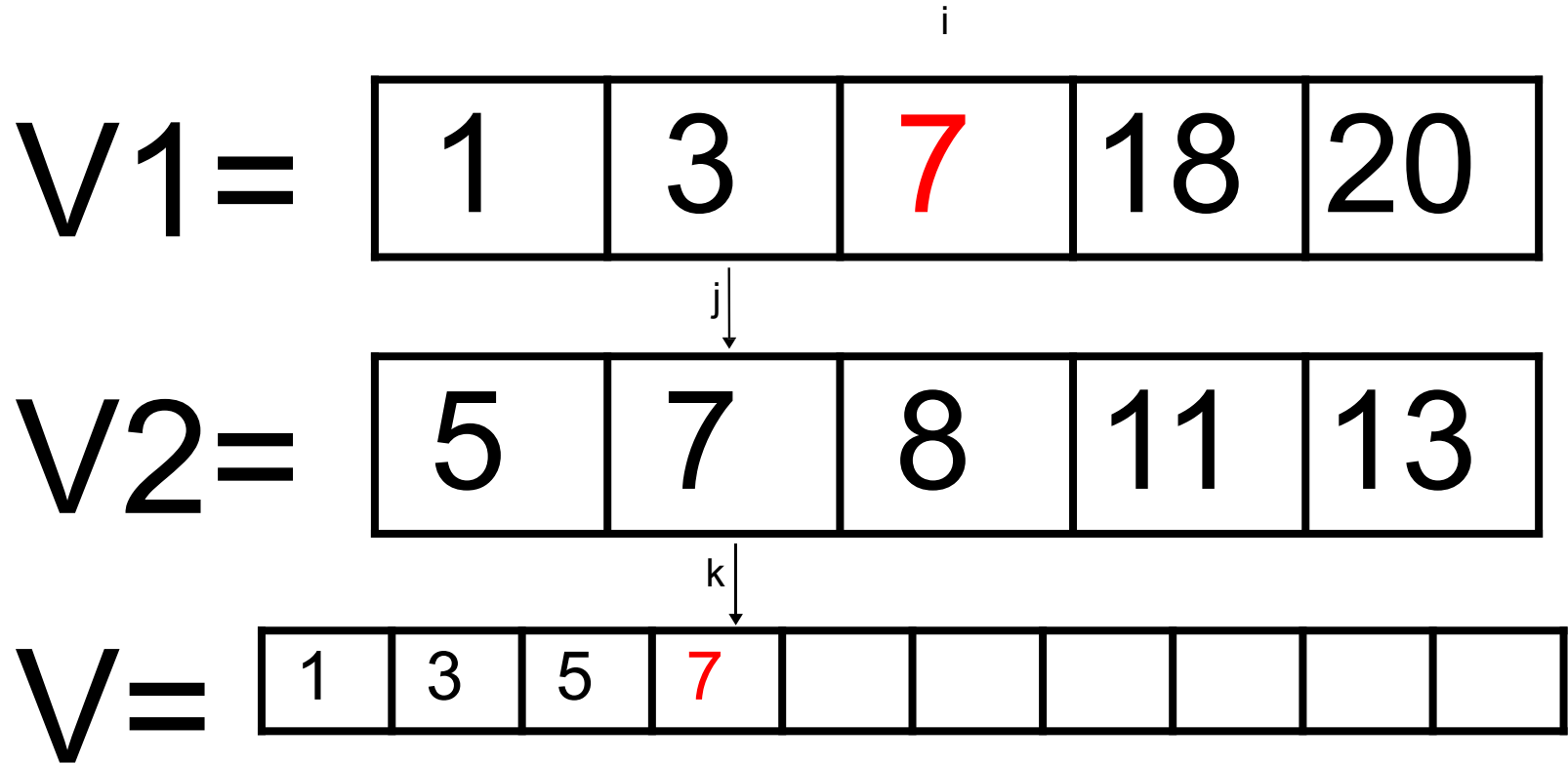
## Merge de dois vetores ordenados

Código em Python



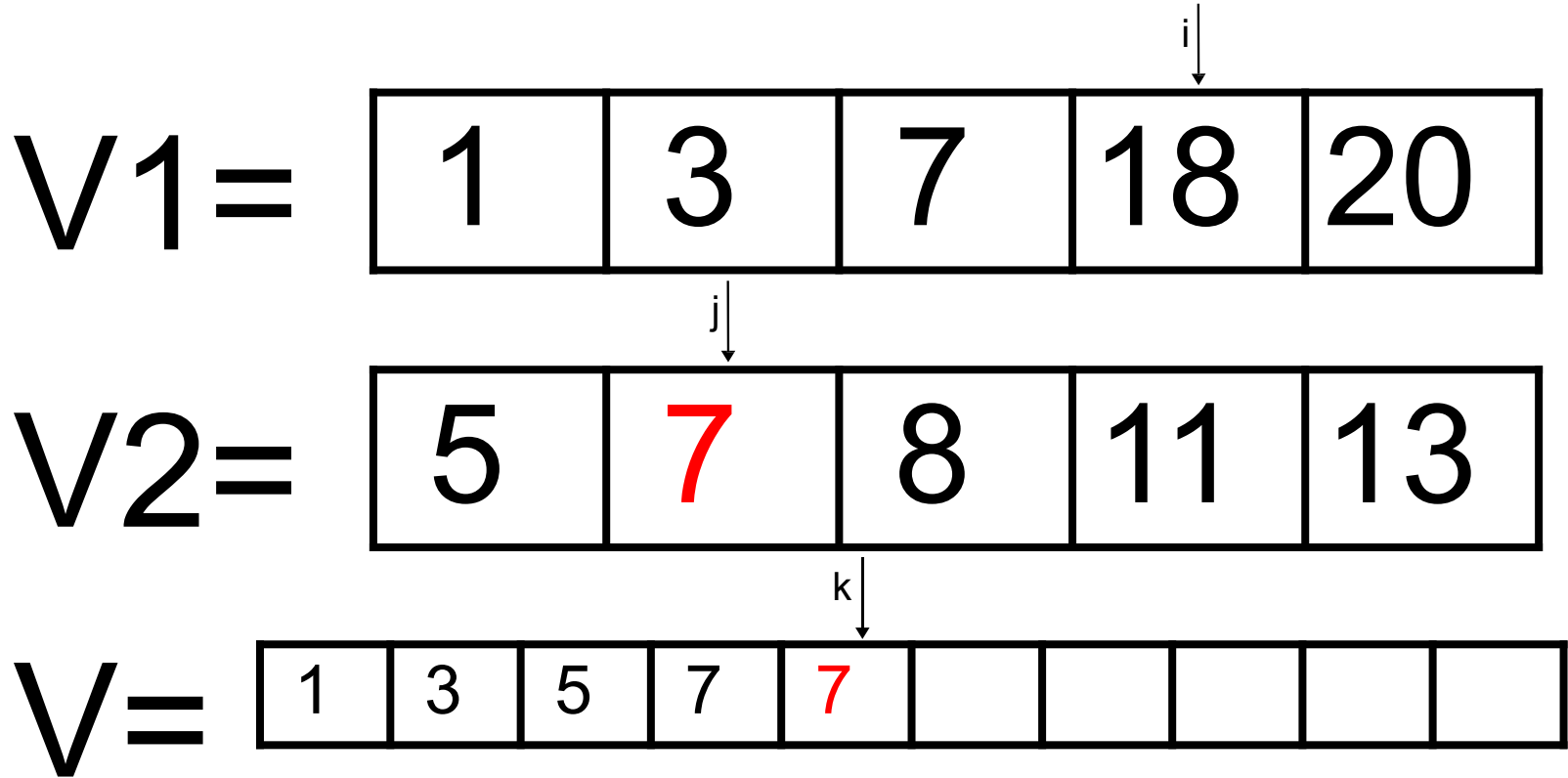
## Merge de dois vetores ordenados

Código em Python



## Merge de dois vetores ordenados

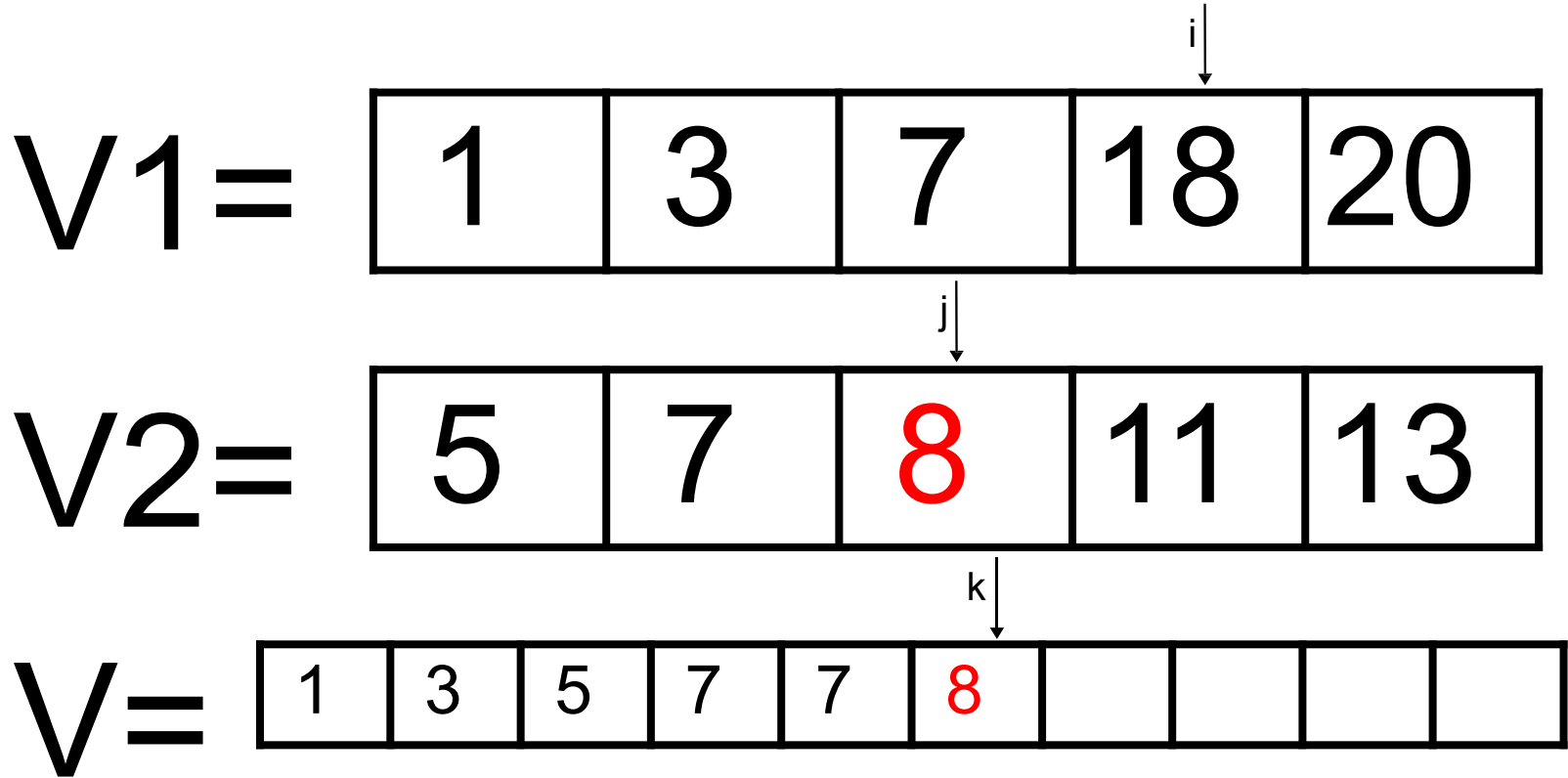
Código em Python





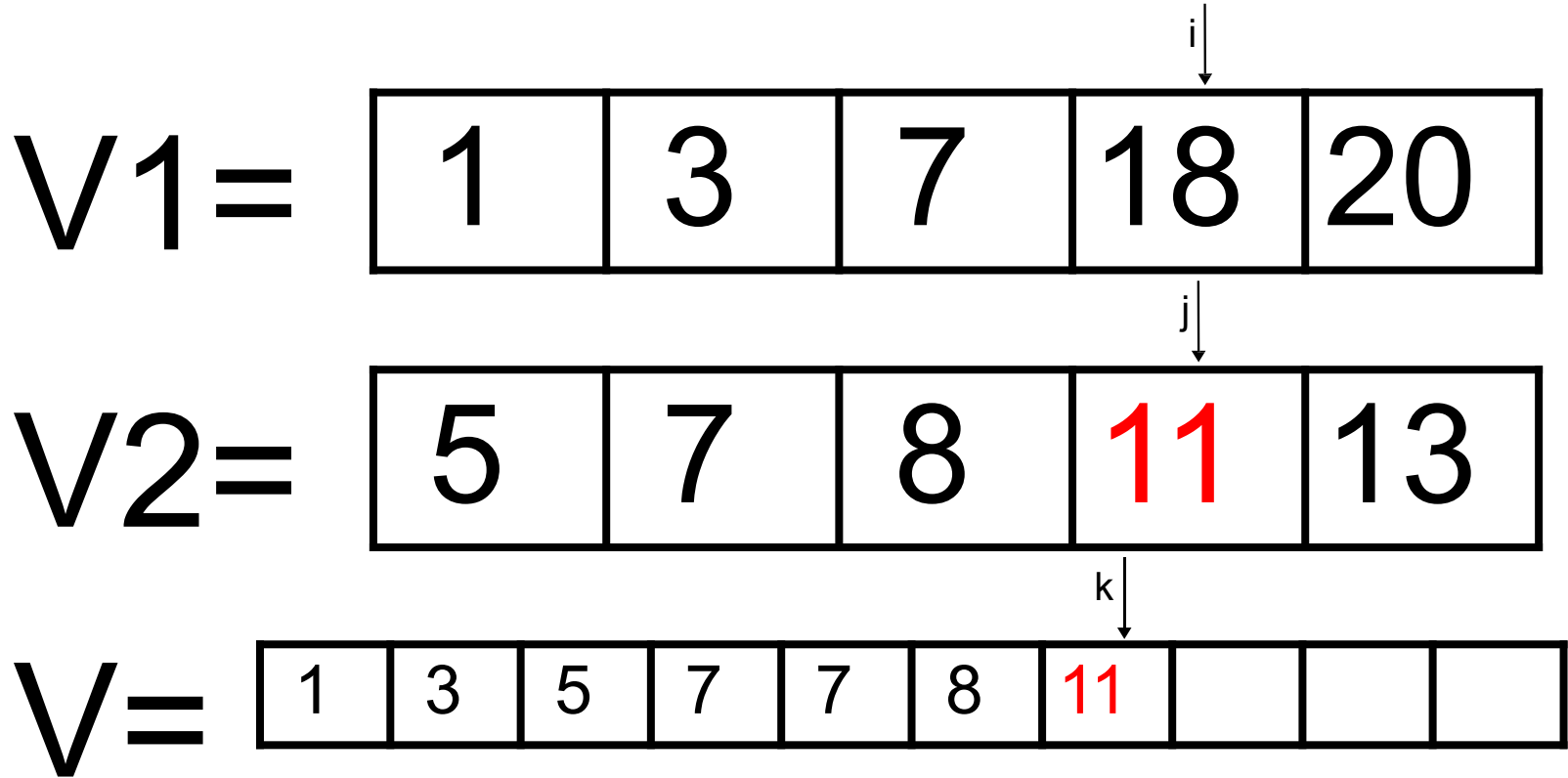
## Merge de dois vetores ordenados

Código em Python



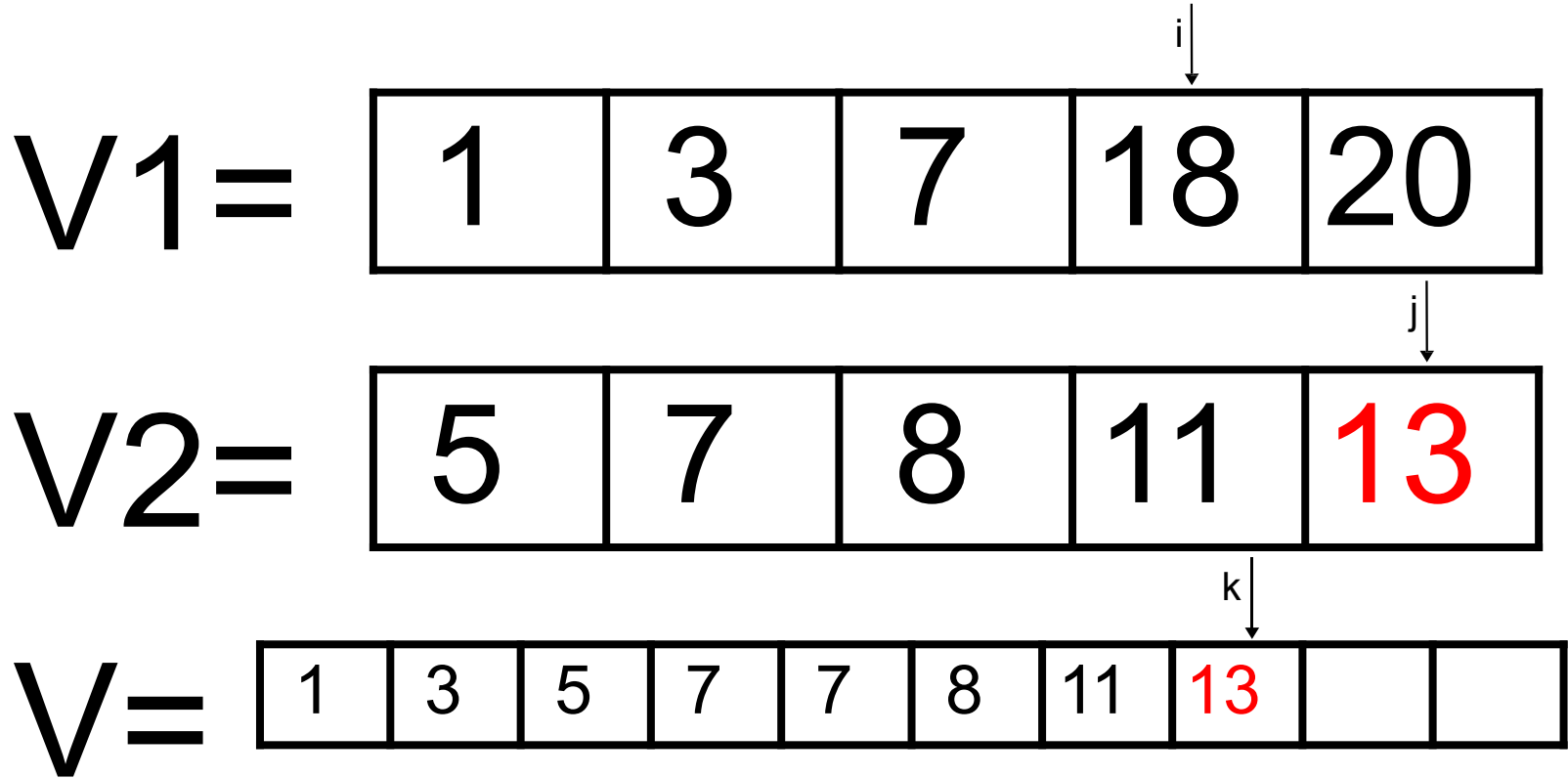
## Merge de dois vetores ordenados

Código em Python



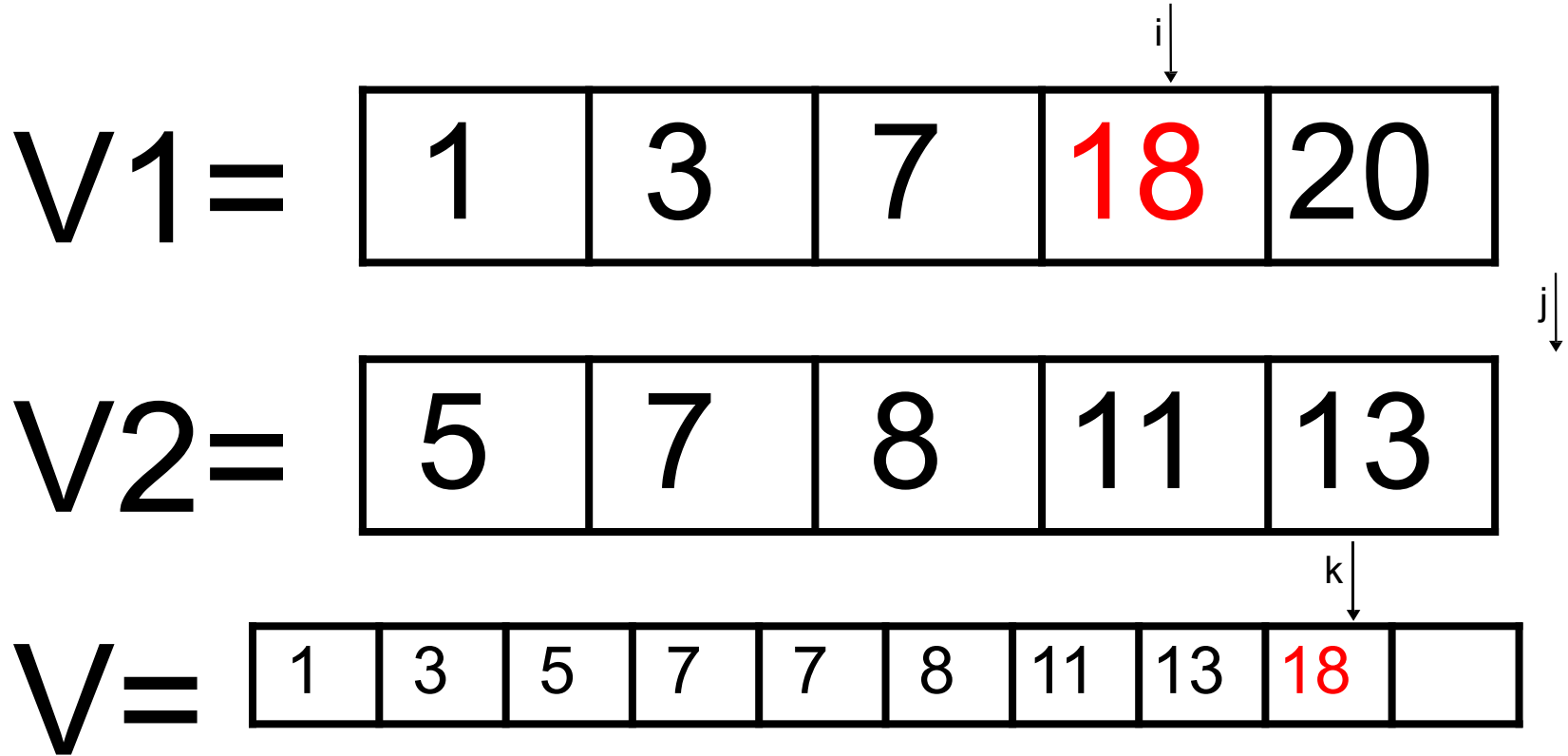
## Merge de dois vetores ordenados

Código em Python



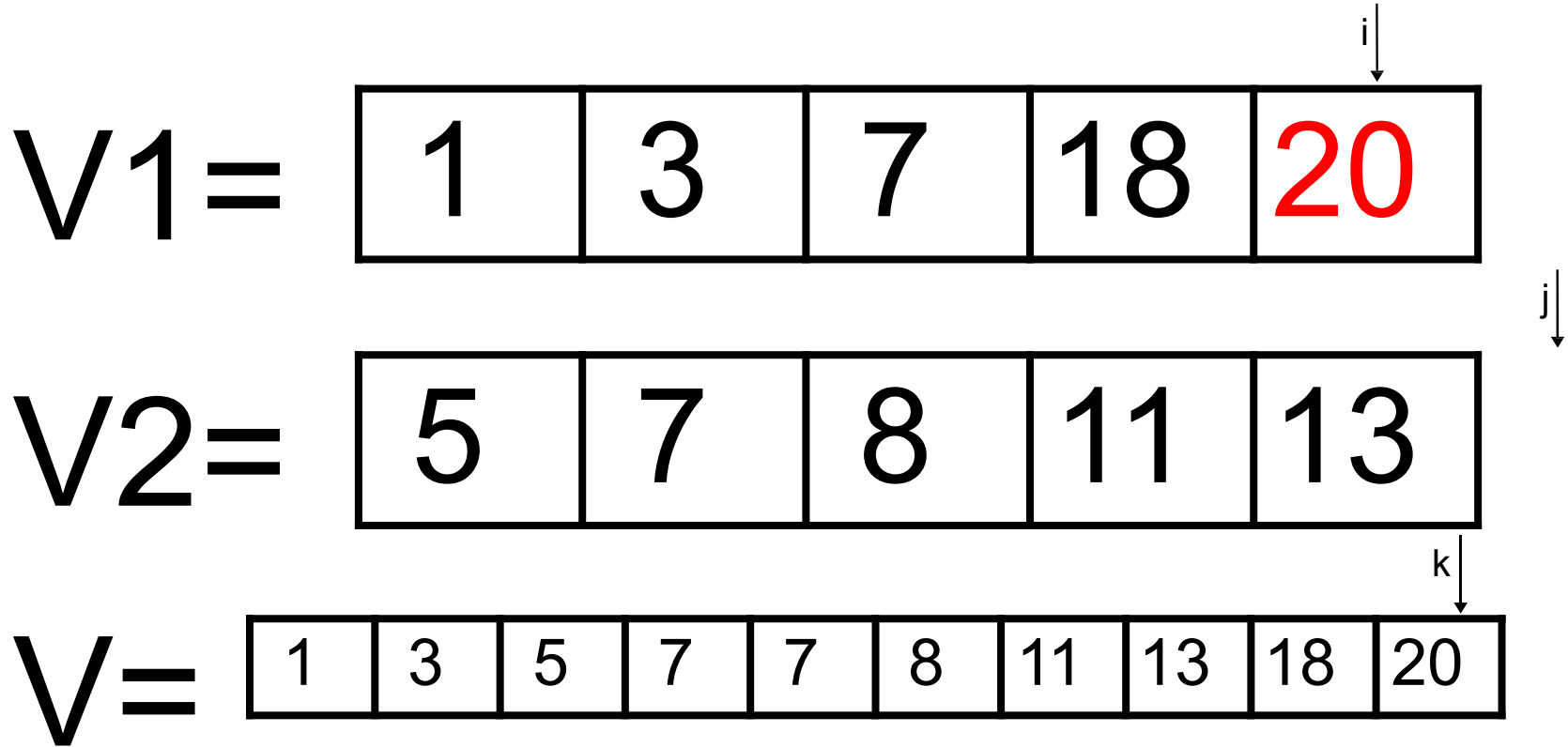
## Merge de dois vetores ordenados

Código em Python



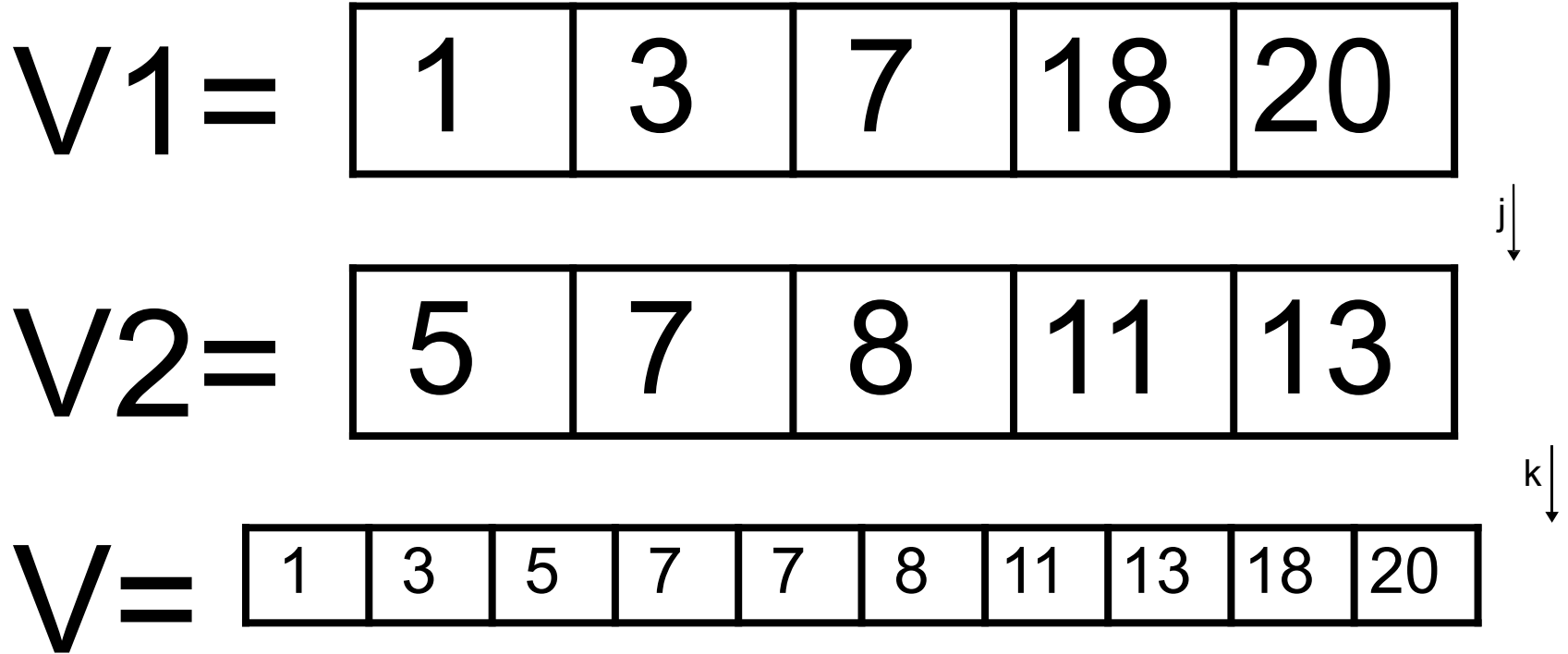
## Merge de dois vetores ordenados

Código em Python



## Merge de dois vetores ordenados

Código em Python



# Merge de dois vetores ordenados

Código em Python

- Dados dois vetores *nums1* e *nums2*, já ordenados, o **merge** cria um outro vetor *mergedArr* ordenado com os elementos de *nums1* e *nums2*

```
mergedArr = []
```

```
i, j = 0, 0
```

```
while i<len(nums1) and j<len(nums2):
```

```
    if nums1[i]<nums2[j]:
```

```
        mergedArr.append(nums1[i])
```

```
        i+=1
```

```
    else:
```

```
        mergedArr.append(nums2[j])
```

```
        j+=1
```

```
while i<len(nums1):
```

```
    mergedArr.append(nums1[i])
```

```
    i+=1
```

```
while j<len(nums2):
```

```
    mergedArr.append(nums2[j])
```

```
    j+=1
```

# Mergesort

Prática no LeetCode - <https://leetcode.com/problems/merge-sorted-array/>

## 88. Merge Sorted Array

Solved 

Easy  Topics  Companies  Hint

You are given two integer arrays `nums1` and `nums2`, sorted in **non-decreasing order**, and two integers `m` and `n`, representing the number of elements in `nums1` and `nums2` respectively.

**Merge** `nums1` and `nums2` into a single array sorted in **non-decreasing order**.

The final sorted array should not be returned by the function, but instead be *stored inside the array* `nums1`. To accommodate this, `nums1` has a length of `m + n`, where the first `m` elements denote the elements that should be merged, and the last `n` elements are set to `0` and should be ignored. `nums2` has a length of `n`.

### Example 1:

Input: `nums1 = [1,2,3,0,0,0]`, `m = 3`, `nums2 = [2,5,6]`, `n = 3`

Output: `[1,2,2,3,5,6]`

Explanation: The arrays we are merging are `[1,2,3]` and `[2,5,6]`.

The result of the merge is `[1,2,2,3,5,6]` with the underlined elements coming from `nums1`.

### Example 2:

Input: `nums1 = [1]`, `m = 1`, `nums2 = []`, `n = 0`

Output: `[1]`

Explanation: The arrays we are merging are `[1]` and `[]`.

The result of the merge is `[1]`.





## Mergesort

Prática no LeetCode - <https://leetcode.com/problems/merge-sorted-array/>

# 88 - Merge Sorted Array

```
def merge(self, nums1: List[int], m: int, nums2: List[int], n: int) -> None:  
    mergedArr = []  
    i = 0  
    j = 0  
    while i<m and j<n:  
        if nums1[i]<nums2[j]:  
            mergedArr.append(nums1[i])  
            i+=1  
        else:  
            mergedArr.append(nums2[j])  
            j+=1  
    while i<m:  
        mergedArr.append(nums1[i])  
        i+=1  
    while j<n:  
        mergedArr.append(nums2[j])  
        j+=1  
    for i in range(len(mergedArr)):  
        nums1[i] = mergedArr[i]
```

## 88 - Merge Sorted Array

O enunciado pede para resolver *in loco*, sem usar espaço extra de memória.

**Dica:** pense em uma solução que preencha o vetor *nums1* de trás para frente!

# 88 - Merge Sorted Array

```
def merge(self, nums1: List[int], m: int, nums2: List[int], n: int) ->
None:
    if n == 0: return
    end_idx = len(nums1)-1
    while n > 0 and m > 0 :
        if nums2[n-1] >= nums1[m-1]:
            nums1[end_idx] = nums2[n-1]
            n-=1
        else:
            nums1[end_idx] = nums1[m-1]
            m-=1
        end_idx-=1
    while n > 0:
        nums1[end_idx] = nums2[n-1]
        n-=1
        end_idx-=1
```

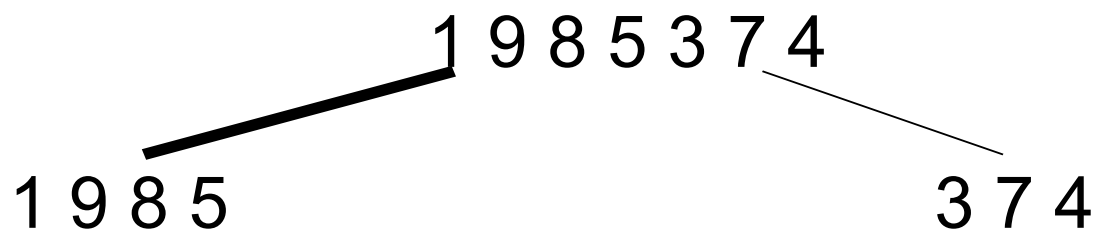
# Mergesort

Funcionamento

1 9 8 5 3 7 4

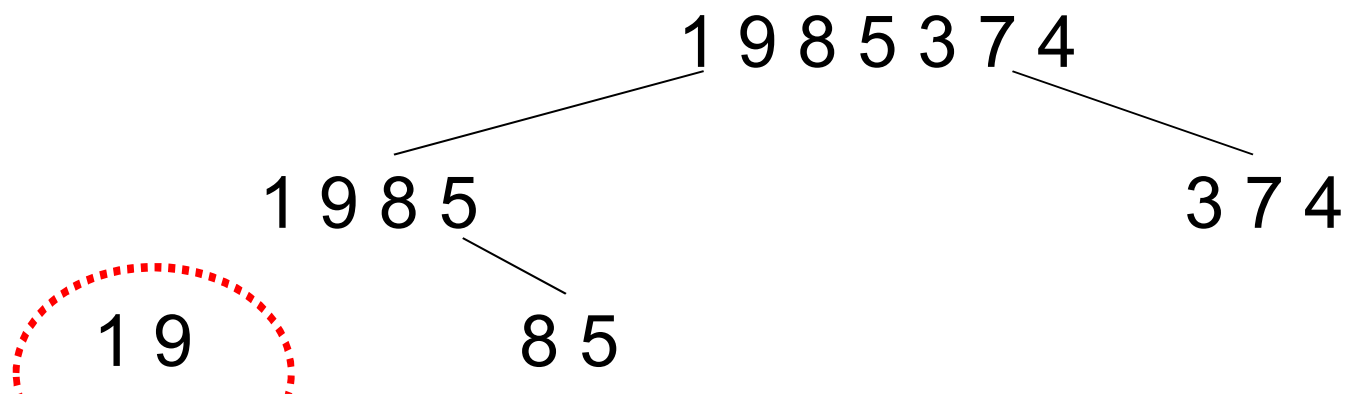
# Mergesort

Funcionamento



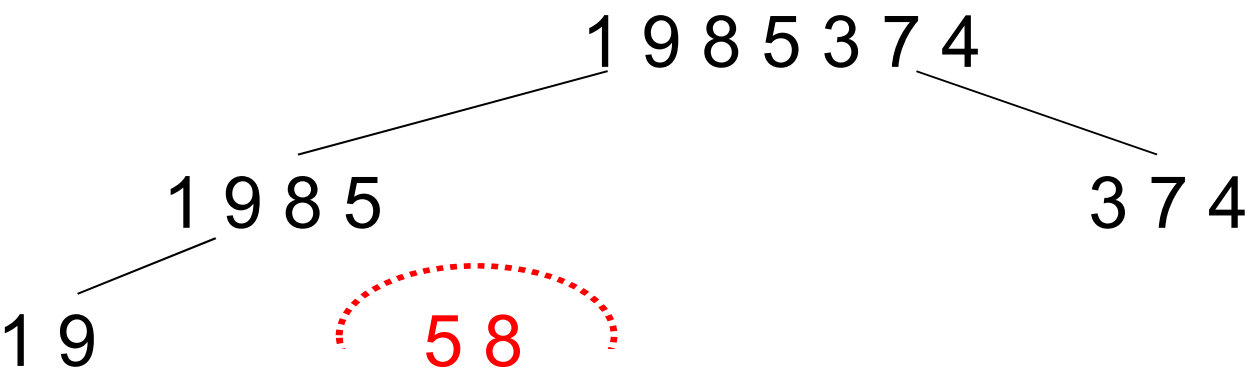
# Mergesort

Funcionamento



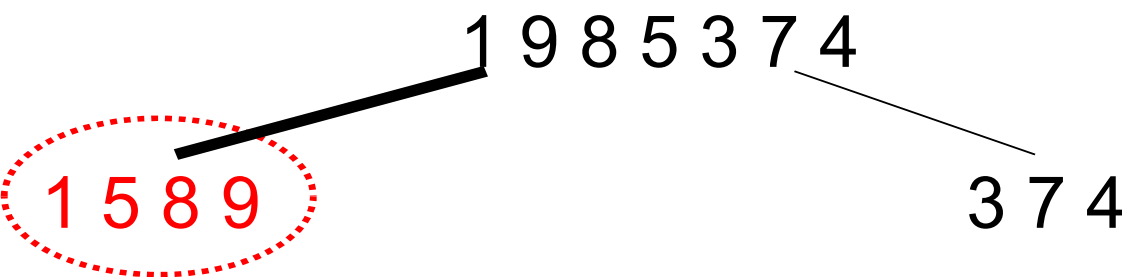
# Mergesort

Funcionamento



# Mergesort

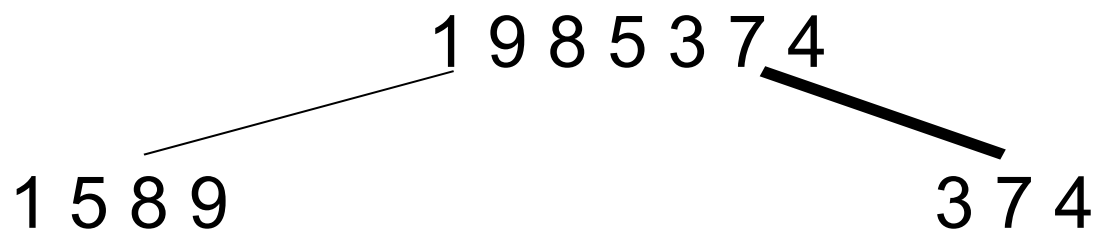
Funcionamento





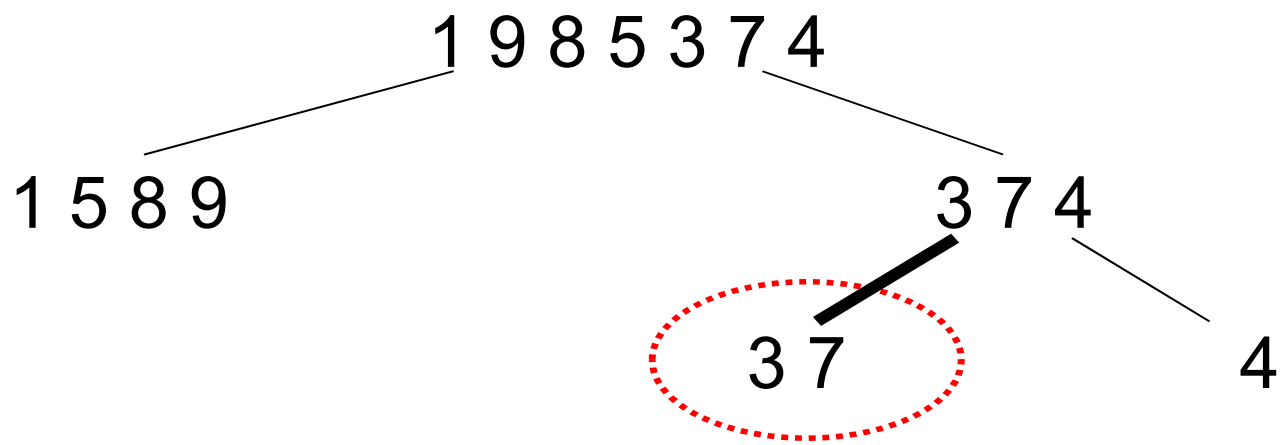
# Mergesort

Funcionamento



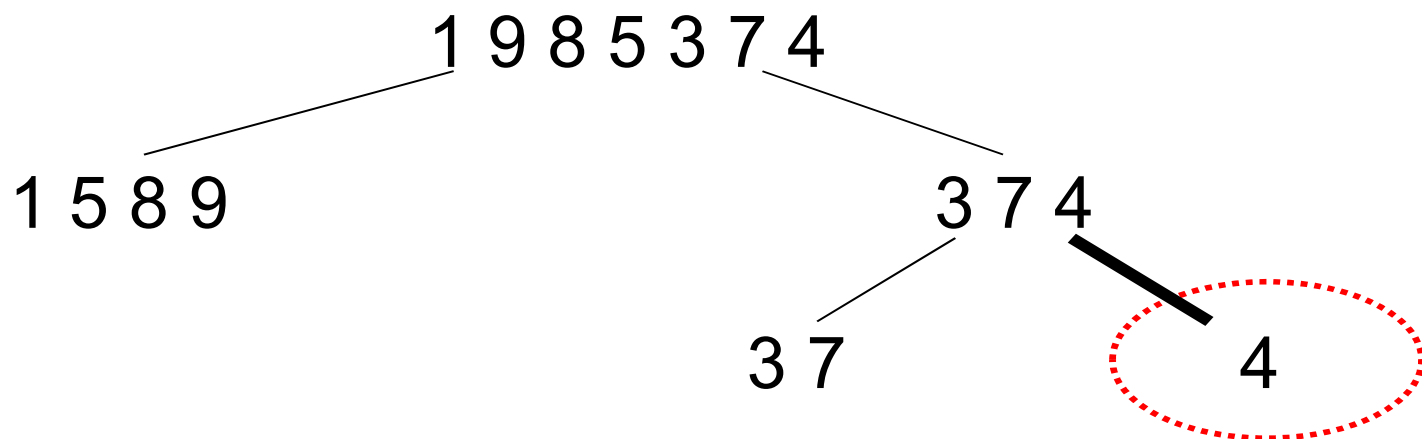
# Mergesort

Funcionamento



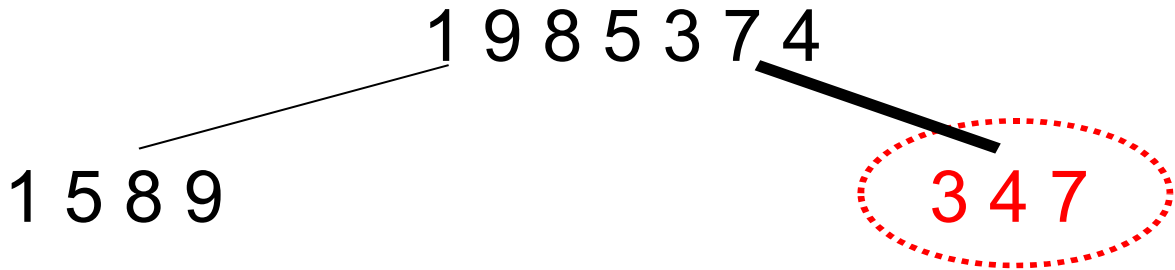
# Mergesort

Funcionamento



# Mergesort

Funcionamento



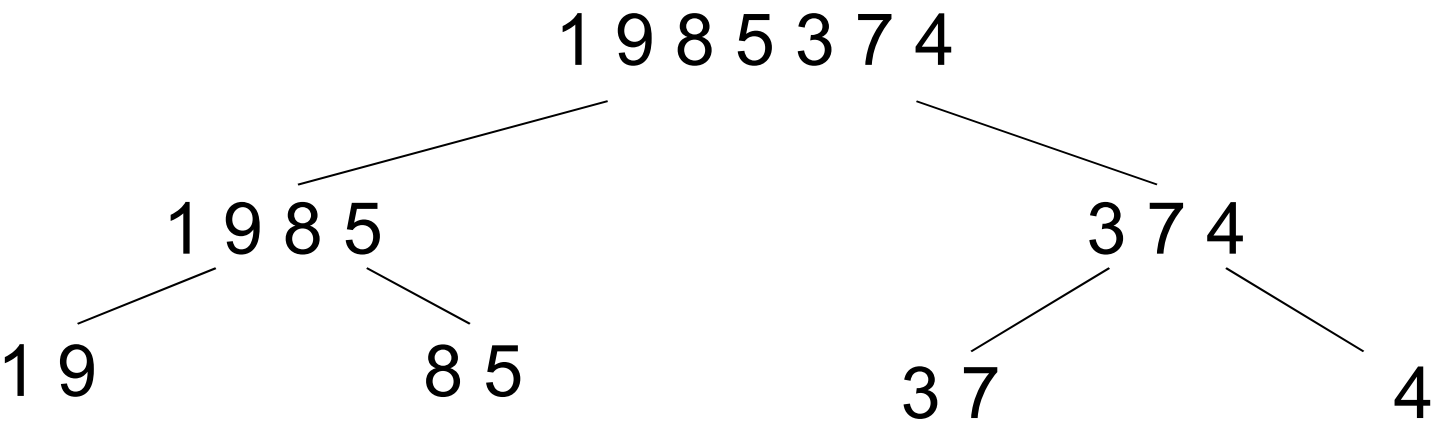
# Mergesort

Funcionamento



# Mergesort

Quantidade de Operações



### Quantidade de Operações



## Quicksort

---



# Quicksort

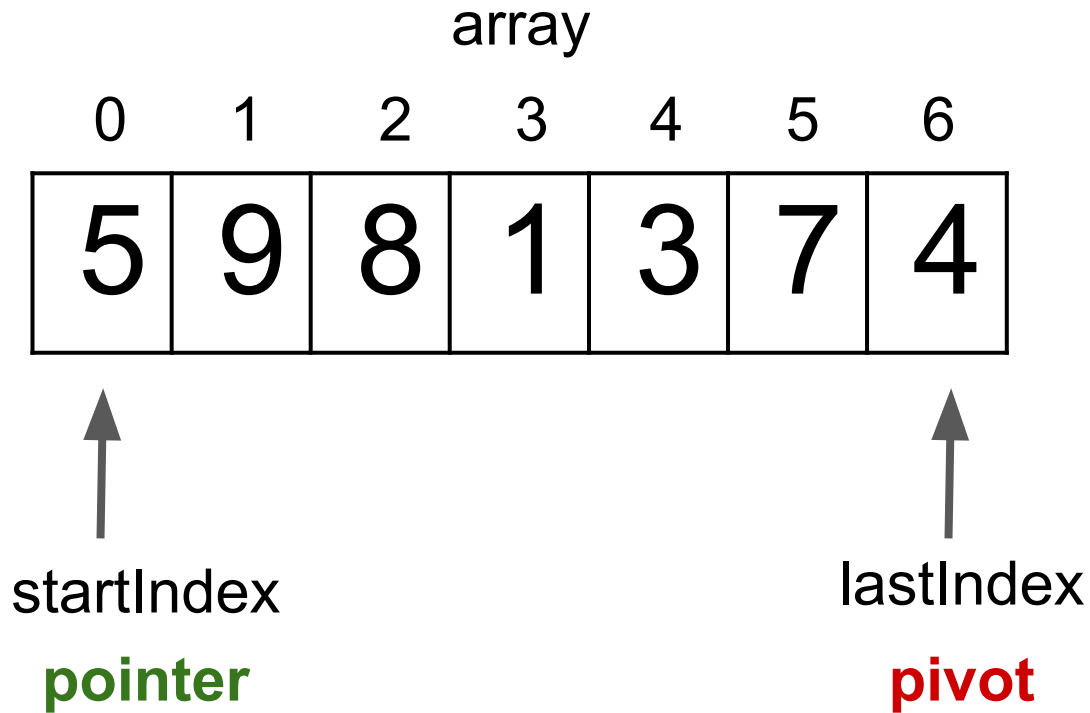
## Conceito

- Ordena o vetor particionando recursivamente.
- Usa a técnica "***dividir para conquistar***":
  - A cada iteração, localiza a posição final de um elemento aleatório (pivô) e subdivide o vetor em duas partes para prosseguir a ordenação.
- O *Quick Sort* pode ser implementado com duas funções:
  - `partition(Array, startIndex, lastIndex)`
  - `quick_sort(Array, startIndex, lastIndex)`

# Quicksort

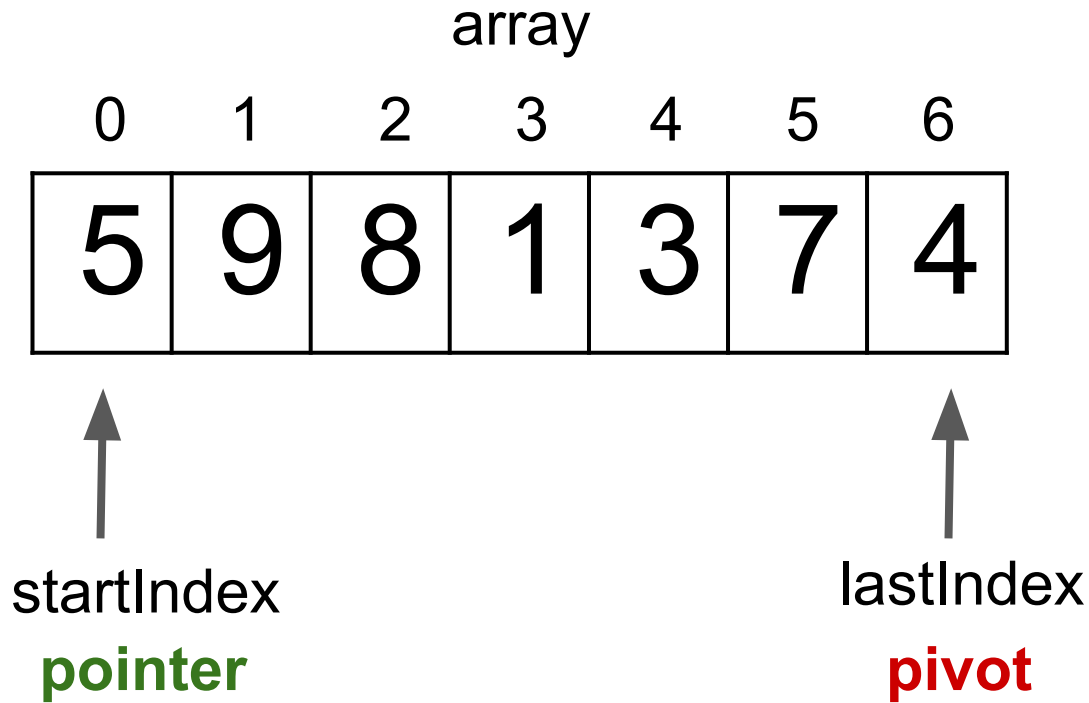
Funcionamento

```
int pivot = Partition(array, startIndex, lastIndex);
```



# Quicksort

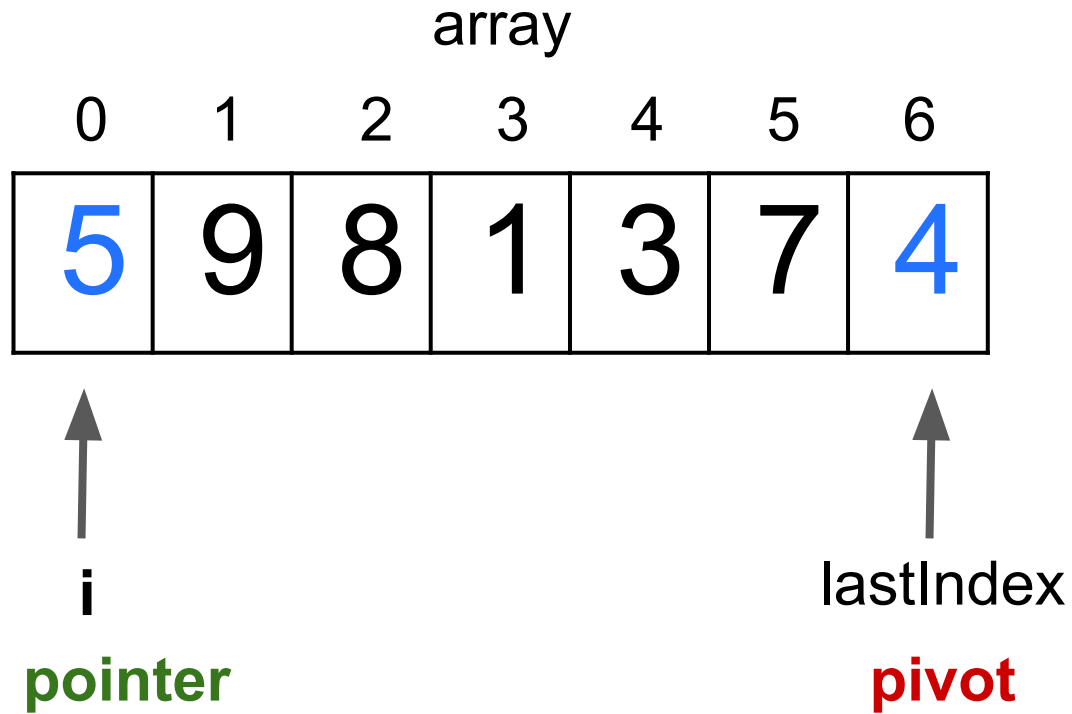
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++;  
    }
```

# Quicksort

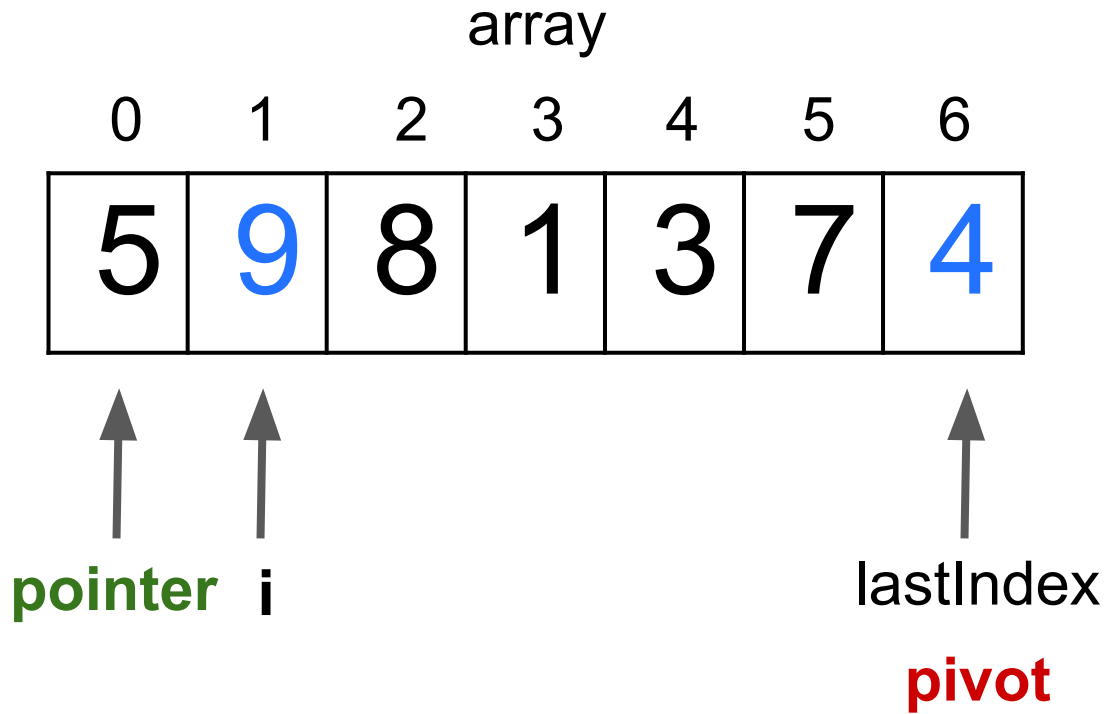
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```

# Quicksort

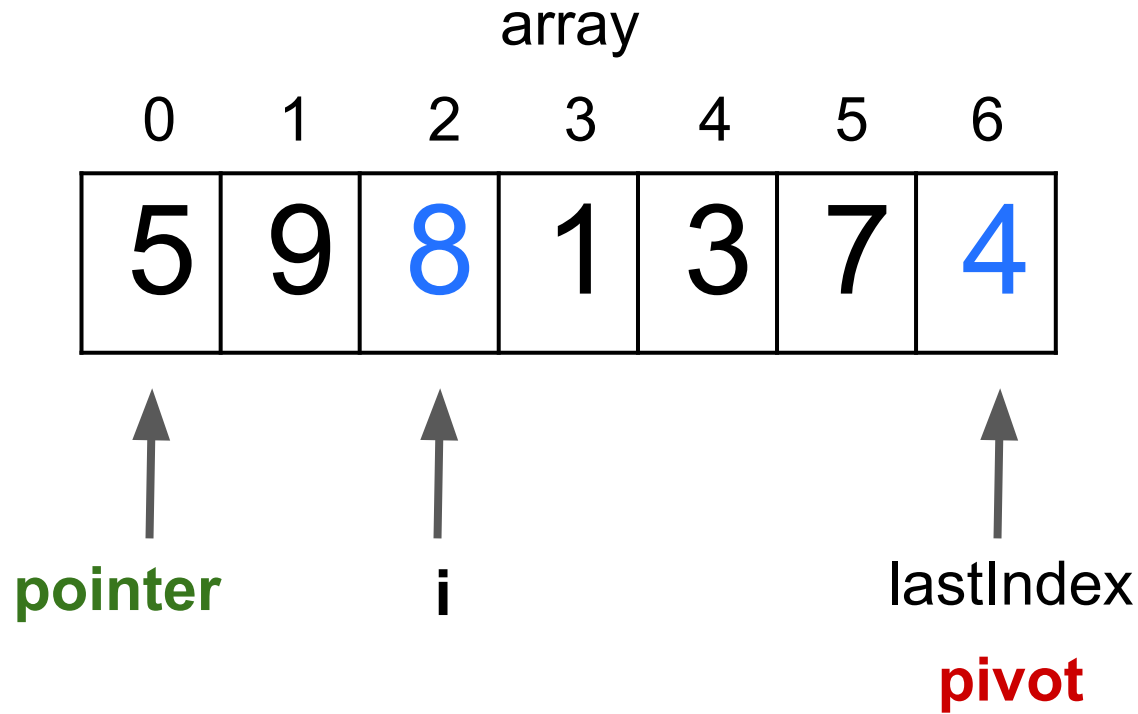
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```

# Quicksort

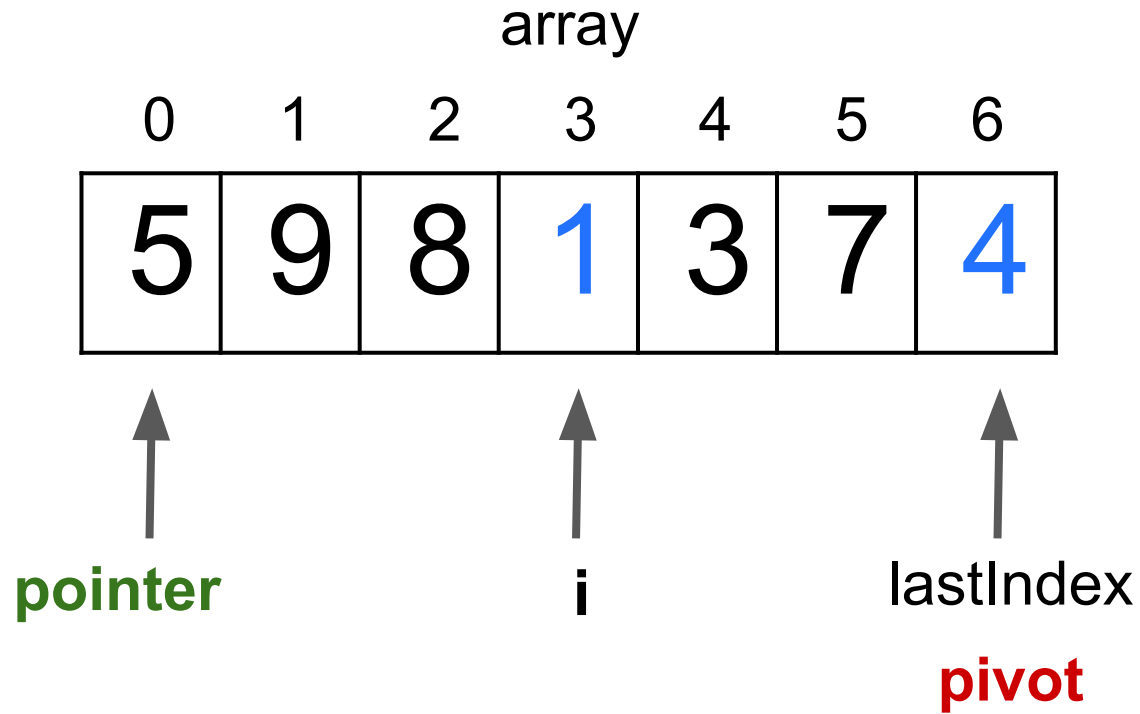
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```

# Quicksort

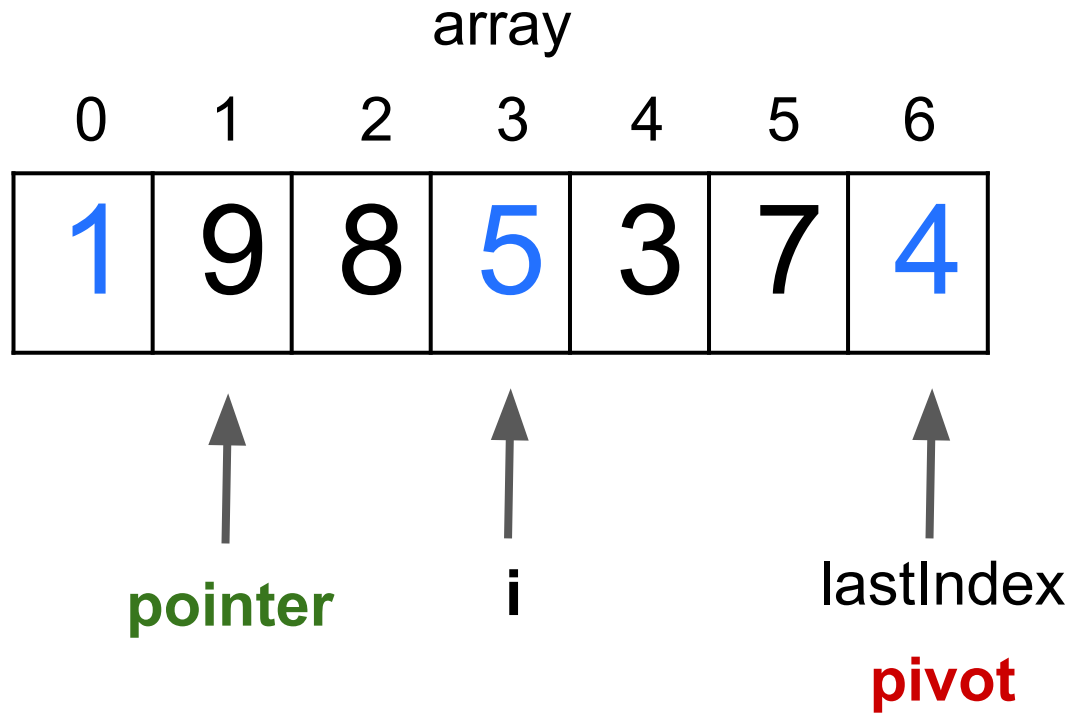
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```

# Quicksort

Funcionamento

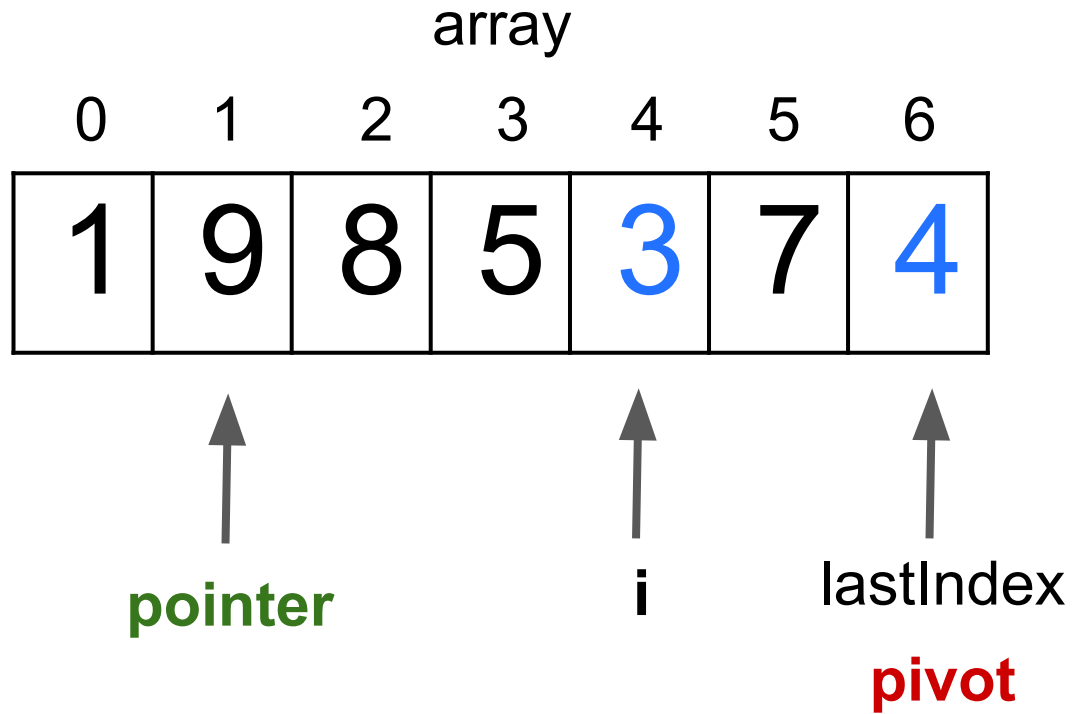


```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```



# Quicksort

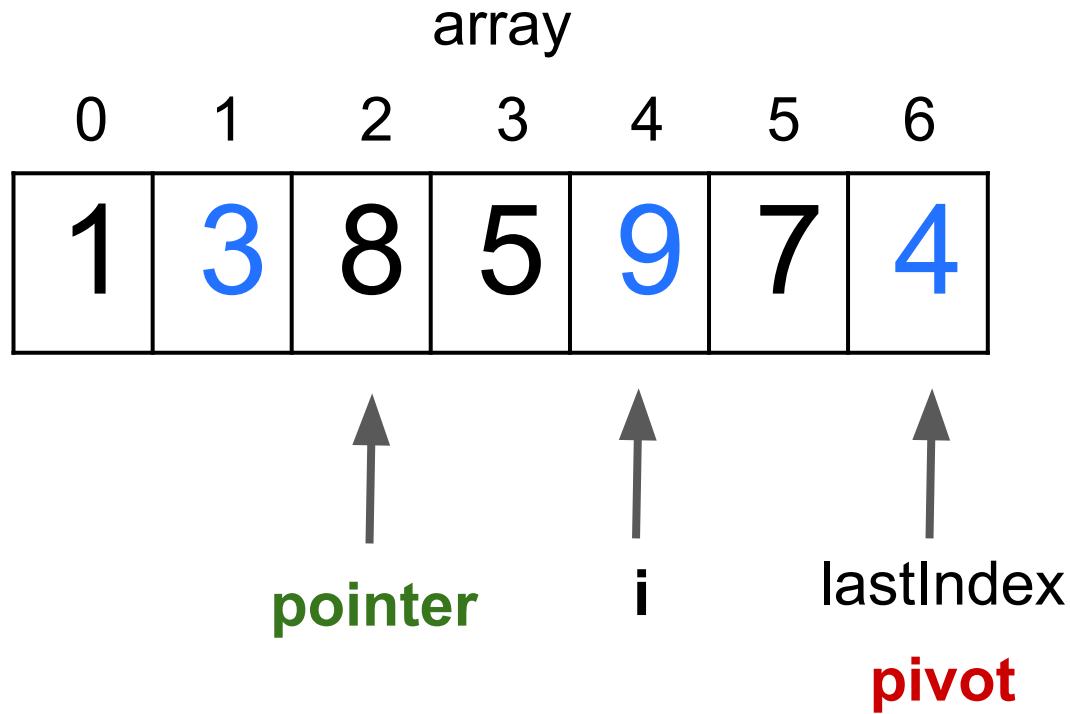
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```

# Quicksort

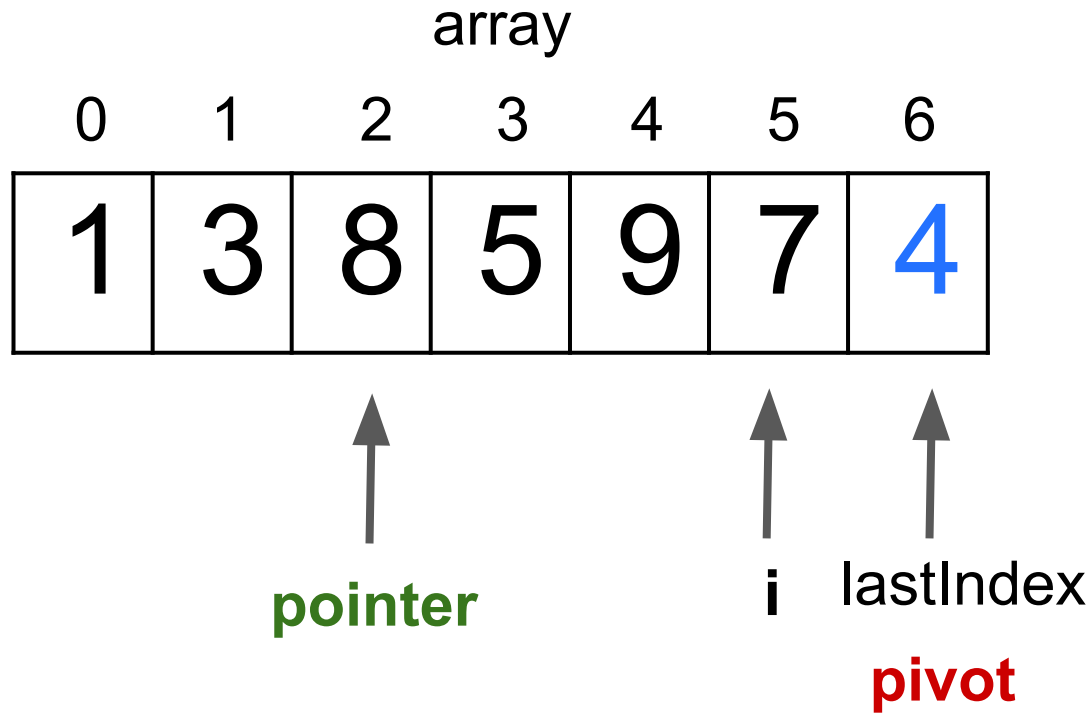
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```

# Quicksort

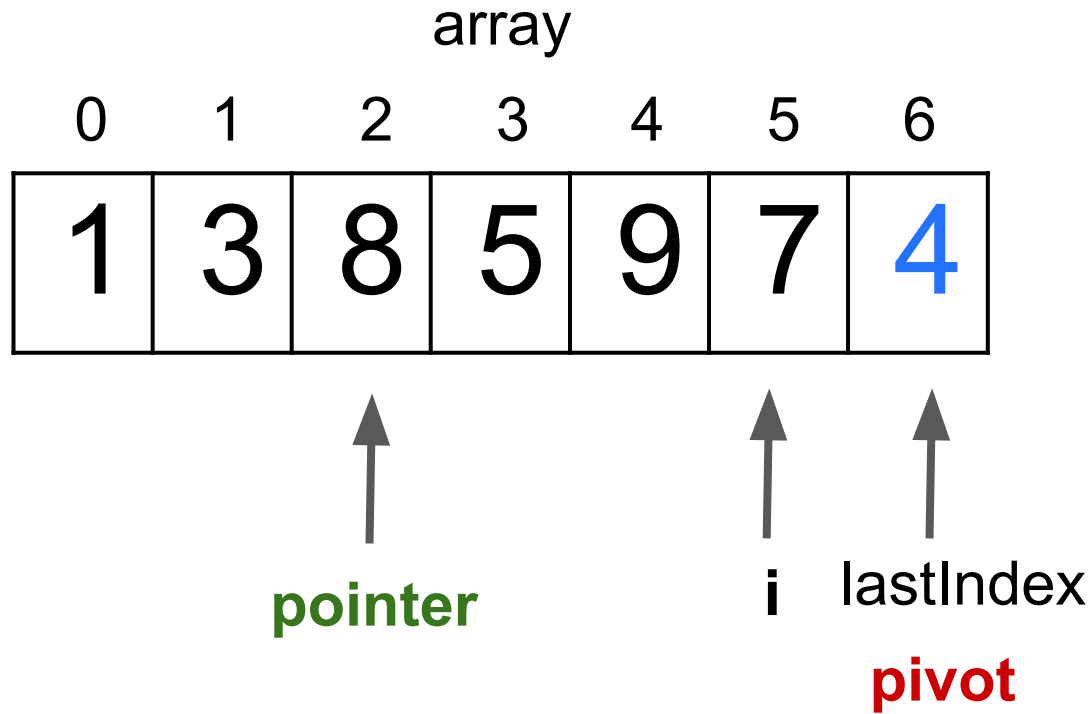
Funcionamento



```
for (i = startIndex; i < lastIndex; i++)  
    if (array[i] <= pivot) {  
        temp = array[i];  
        array[i] = array[pointer];  
        array[pointer] = temp;  
        pointer++; }  
}
```

# Quicksort

Funcionamento

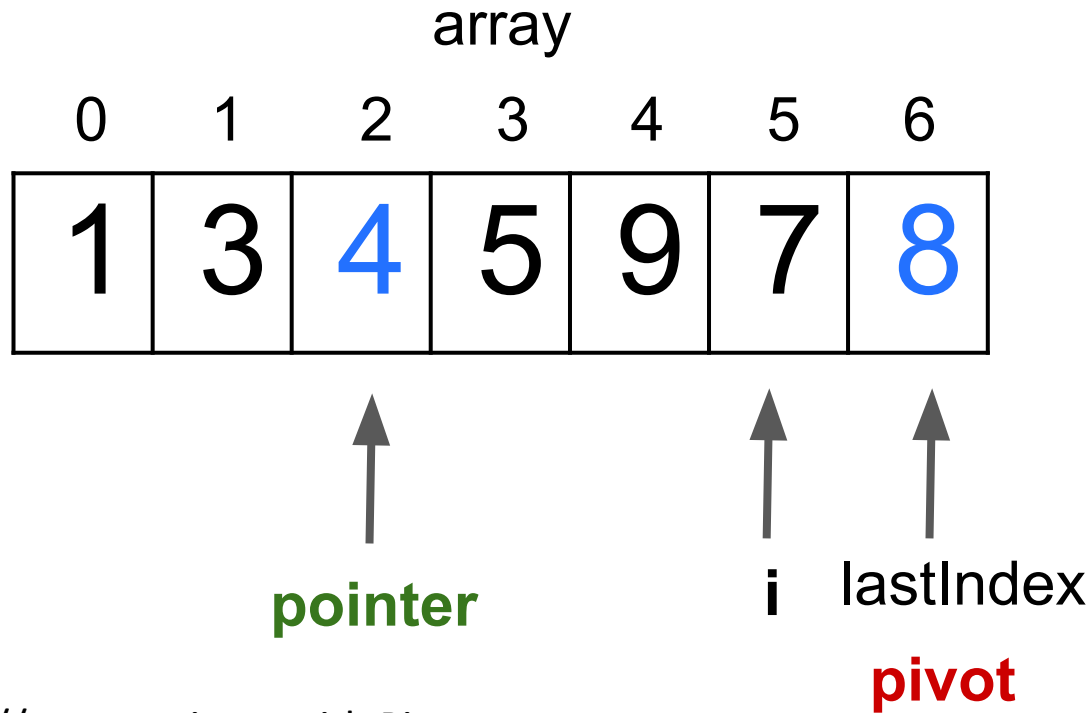


```
// swap pointer with Pivot
temp = array[pointer];
array[pointer] = array[lastIndex];
array[lastIndex] = temp;
```

```
// returning the index of the pivot
return pointer;
```

# Quicksort

Funcionamento

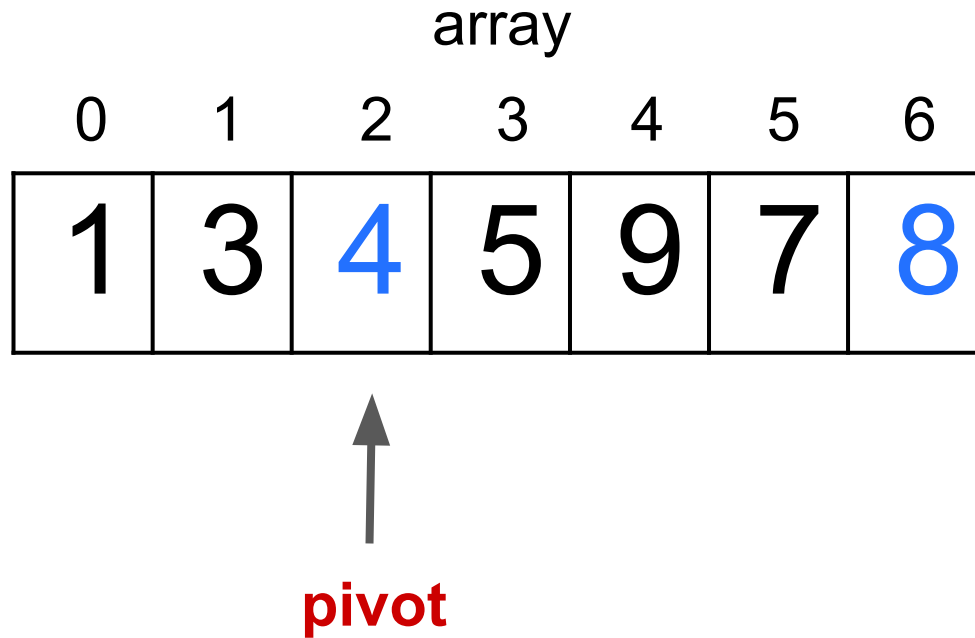


```
// swap pointer with Pivot
temp = array[pointer];
array[pointer] = array[lastIndex];
array[lastIndex] = temp;
```

```
// returning the index of the pivot
return pointer; // índice 2 será o próximo pivô
```

# Quicksort

Funcionamento



*// this will contain the elements that are less than pivot*

```
QuickSort(array, startIndex, pivot - 1);
```

*// this will contain the elements that are greater than pivot*

```
QuickSort(array, pivot + 1, lastIndex);
```

# Quicksort

Funcionamento

**Vamos implementar o  
Quick Sort ?**





Heapsort

---



# Heapsort

## Conceito

- Ordenação baseada na estrutura de dados *heap*
- Heap é a fila de prioridades
- O heap tem o vetor como a representação de uma árvore binária completa, onde cada nó é um elemento do vetor, com a raiz na primeira posição

# Heapsort

Primitivas

- `length(V)`
  - número de elementos do vetor `V`
- `heap-size(V)`
  - número de elementos do heap armazenados no vetor `V`

# Heapsort

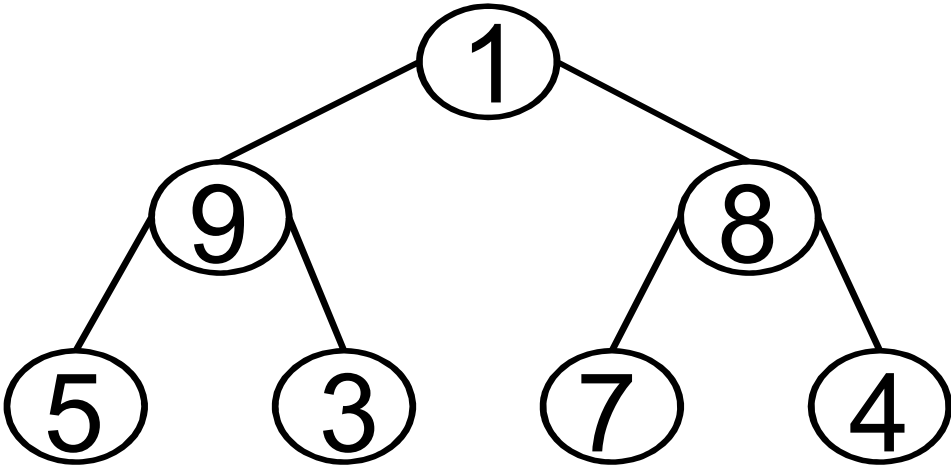
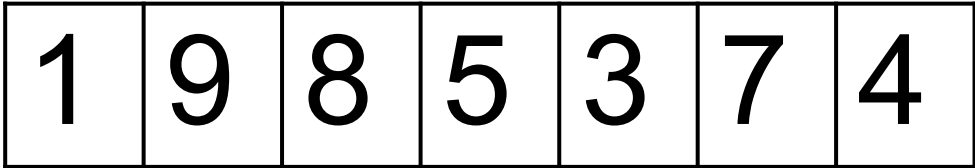
Primitivas

- `parent(i)`
  - `return i/2`
- `left(i)`
  - `return 2i`
- `right(i)`
  - `return 2i+1`

**LEMBRETE:** Em Lua, os vetores são *indexados a partir de 1*. Isso exige que esse código fique diferente em sua linguagem de programação se os *índices começarem de 0*.

# Heapsort

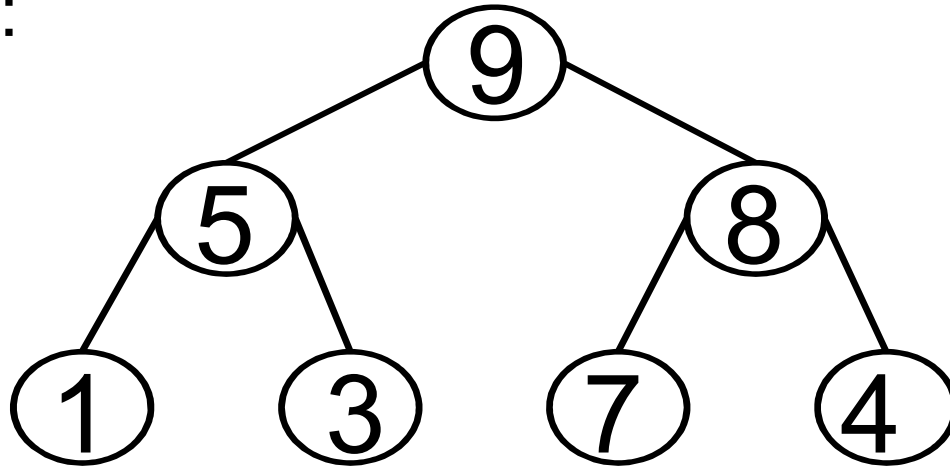
Funcionamento



# Heapsort

Propriedade do Heap

- Para cada nó  $i$  que não seja a raiz, então:
  - $V[\text{parent}(i)] \geq V[i]$
- Exemplo:



# Heapsort

Primitiva Heapify

## • Função para manter a propriedade da heap

```
function heapify(V, i)
    l = left(i)
    r = right(i)
    largest = i
    if l <= heap-size(V) and V[l] > V[i] then
        largest = l
    end
    if r <= heap-size(V) and V[r] > V[largest] then
        largest = r
    end
    if largest != i:
        V[i], V[largest] = V[largest], V[i]
        heapify(V, largest)
    end
```

# Heapsort

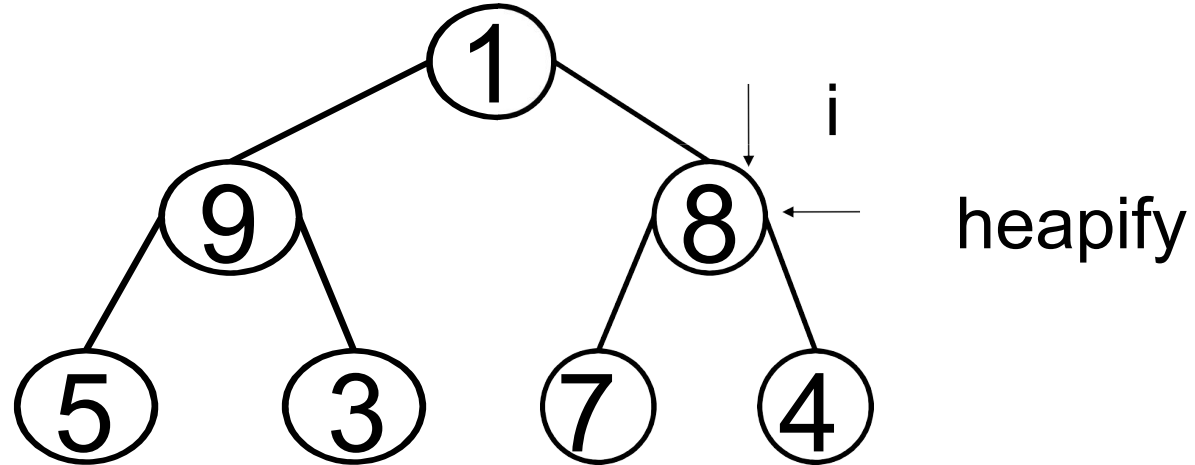
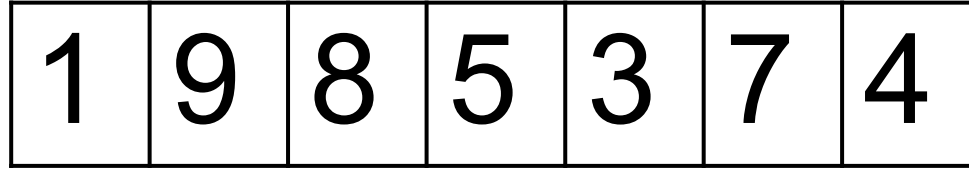
Construindo uma heap

- Primitiva que constrói uma *heap* a partir de um vetor de N elementos

```
function build_heap(V)
    heap-size(V) = length(V) -- #V
    for i=length(V)/2,1,-1 do
        heapify(V,i)
    end
end
```

# Heapsort

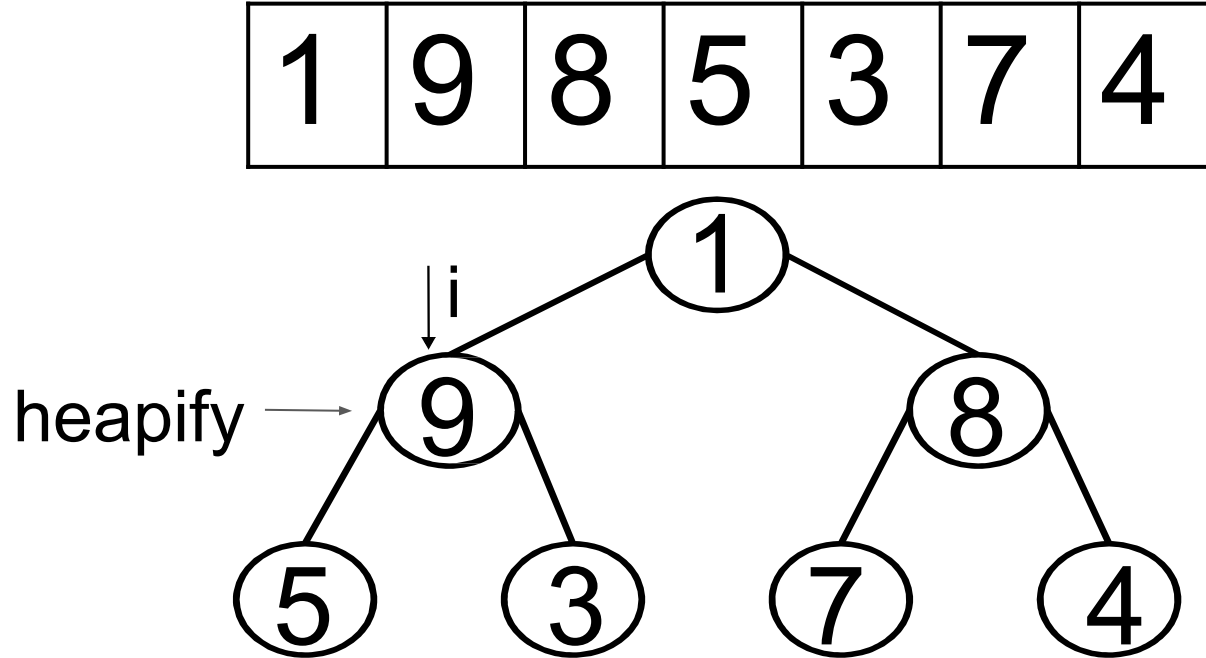
Construindo uma Heap





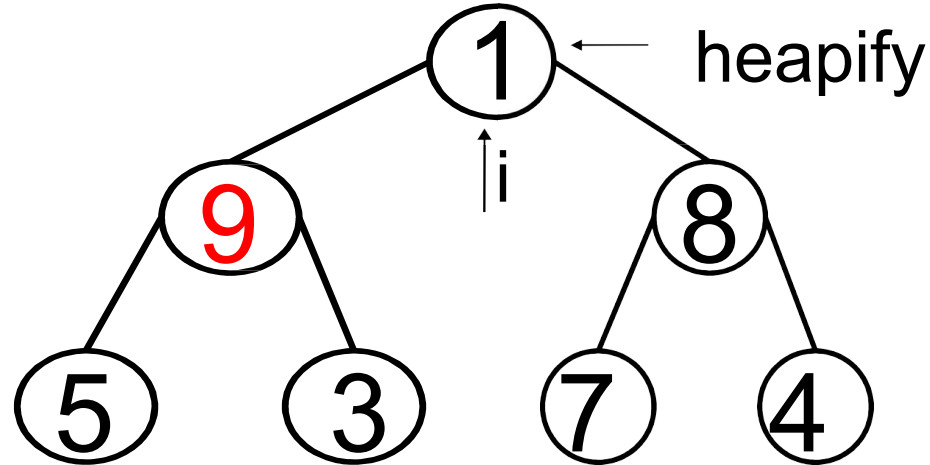
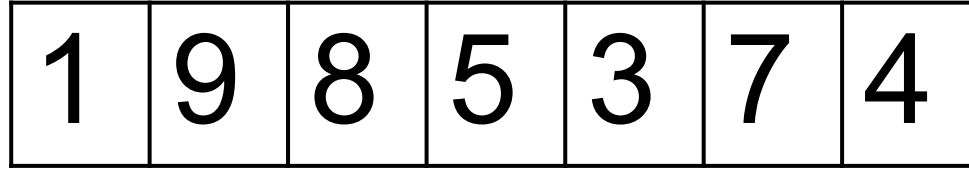
# Heapsort

Construindo uma Heap



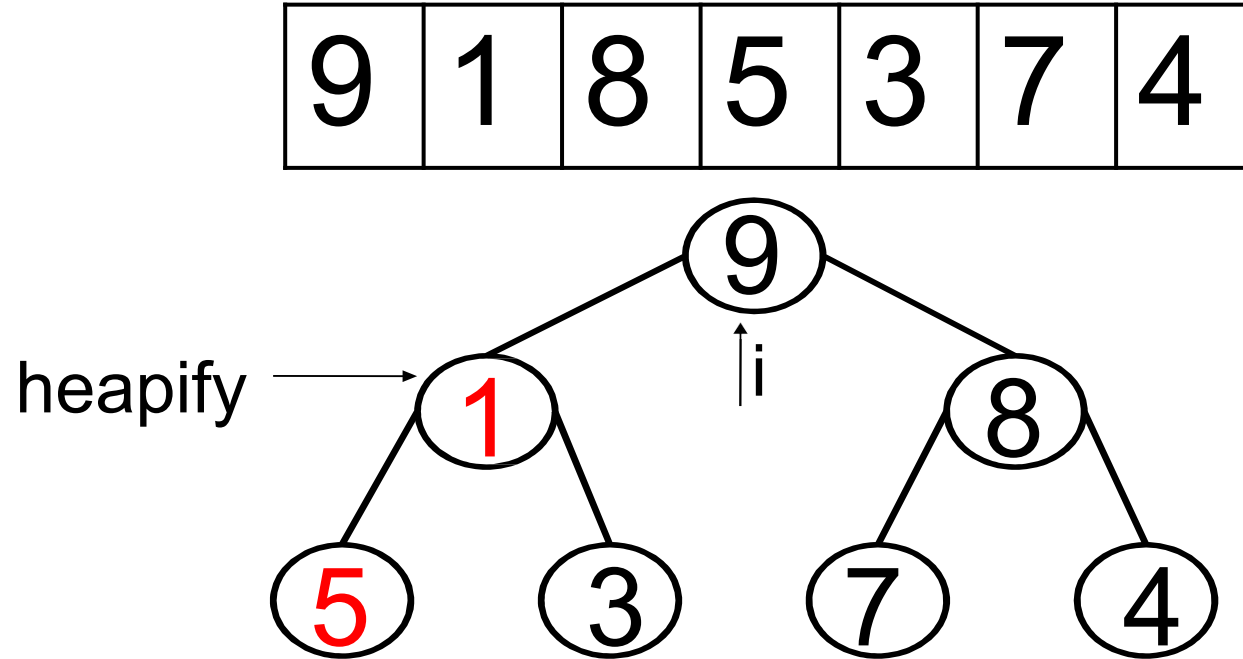
# Heapsort

Construindo uma Heap



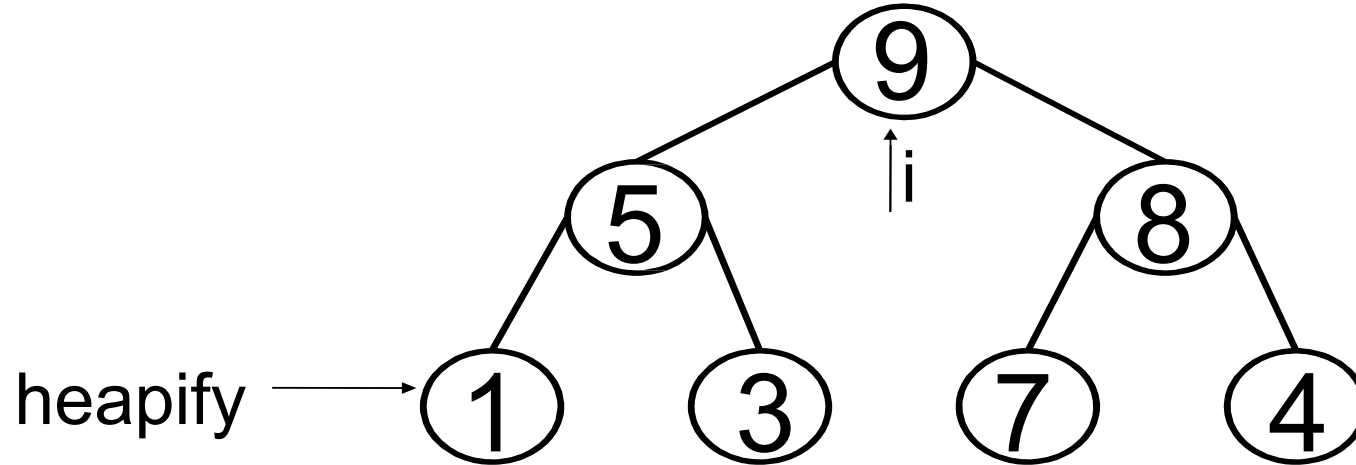
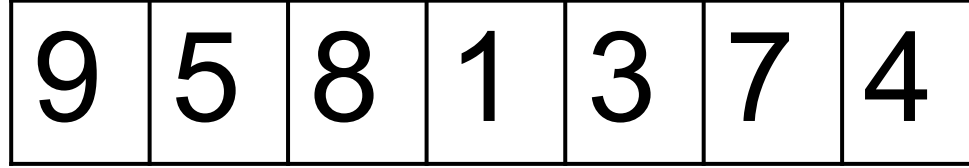
# Heapsort

Construindo uma Heap



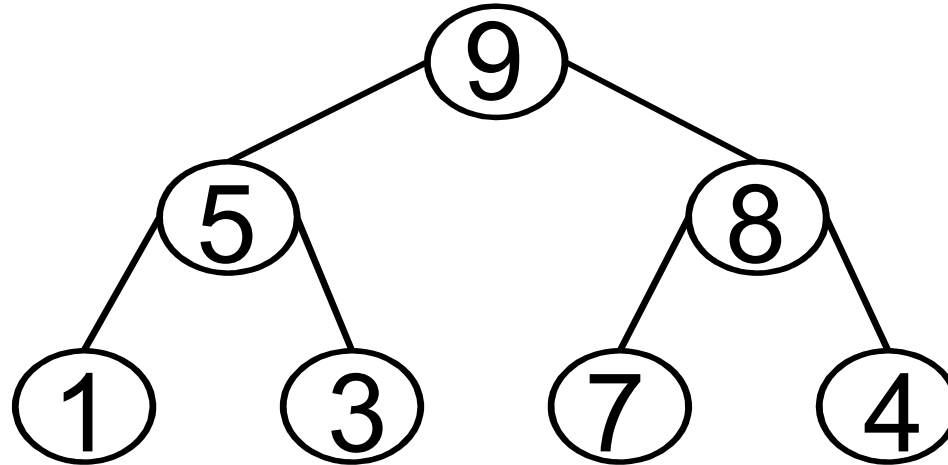
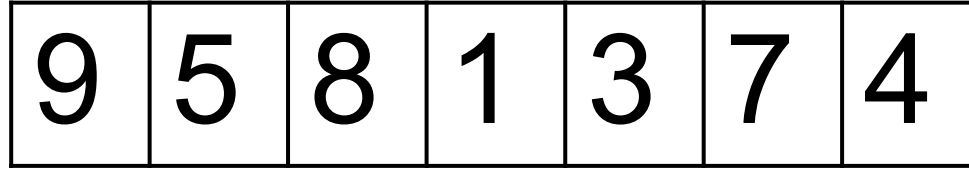
# Heapsort

Construindo uma Heap



# Heapsort

Construindo uma Heap



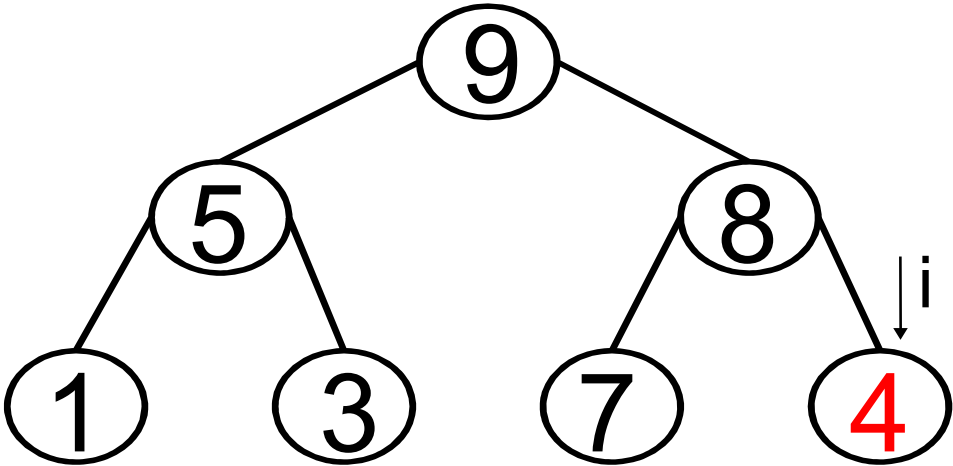
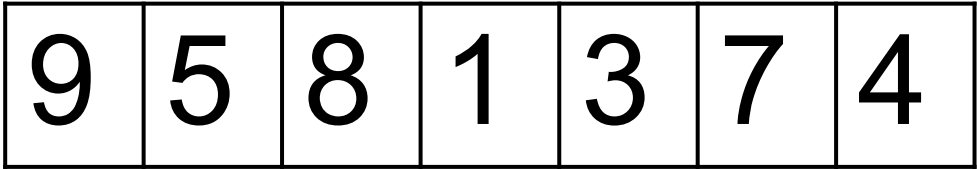
# Heapsort

Código

```
function heapsort(V)
    for i=length(V),2,-1 do
        V[1], V[i] = V[i], V[1]
        heap-size(V) = heap-size(V) - 1
        heapify(V,1)
    end
end
```

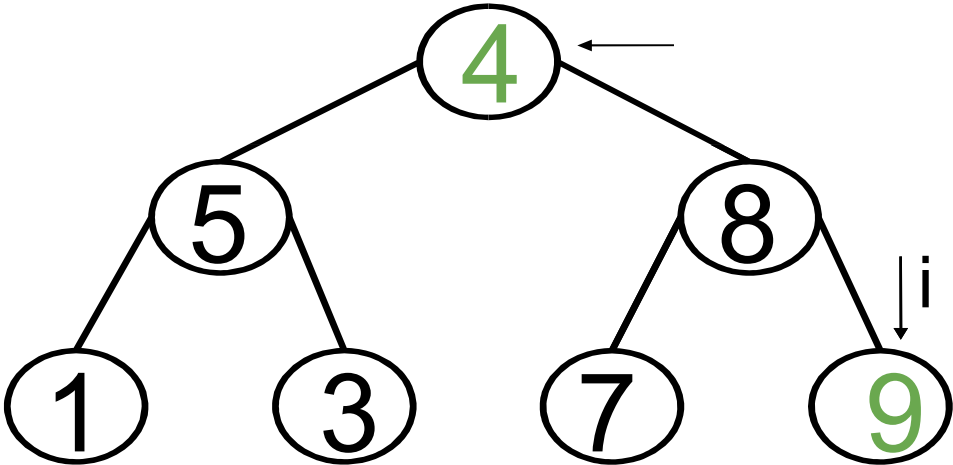
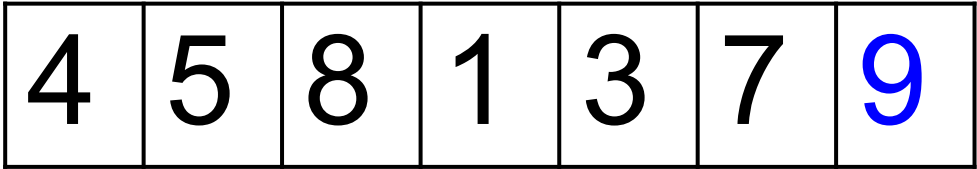
# Heapsort

Código



# Heapsort

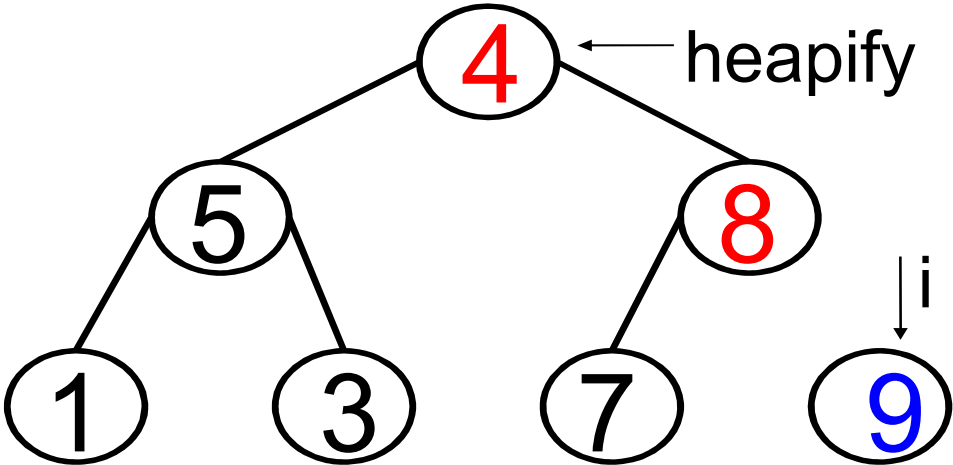
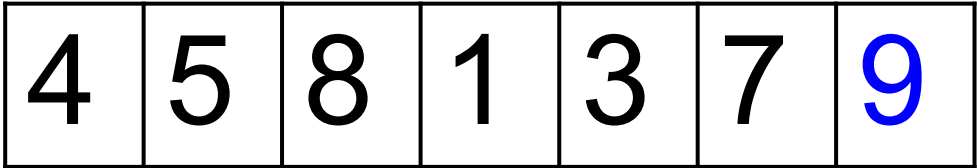
Código





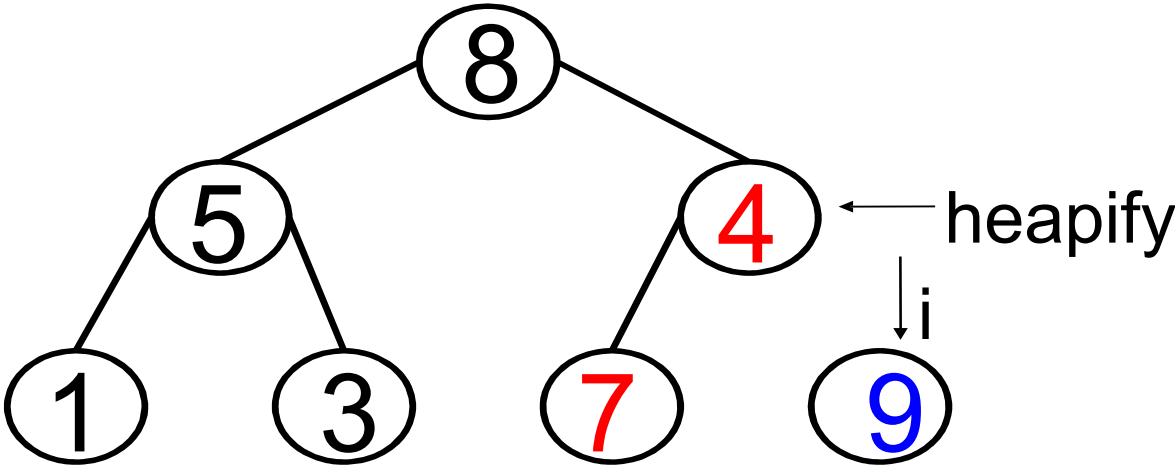
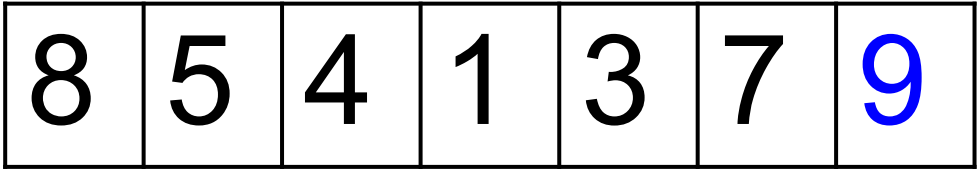
# Heapsort

Código



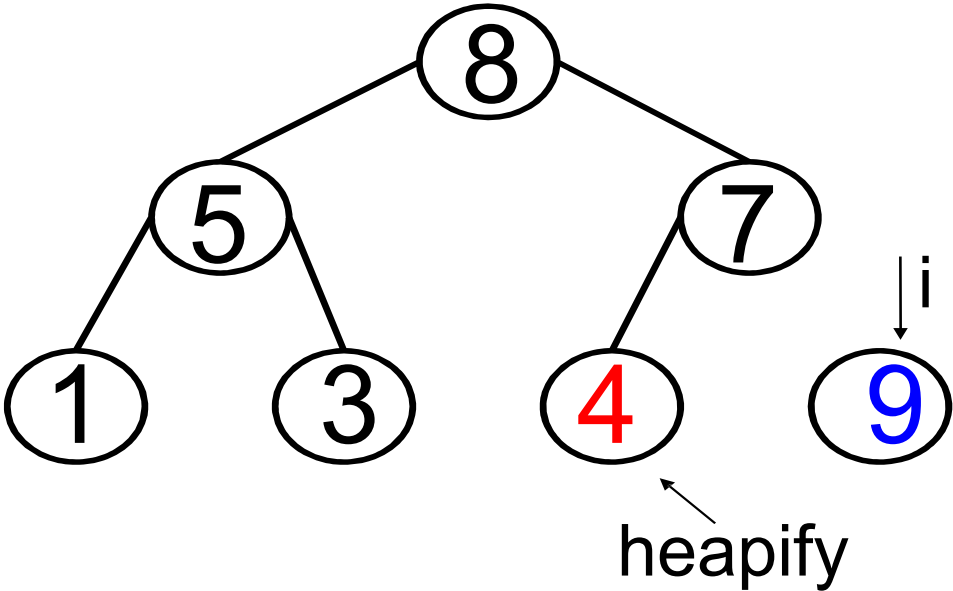
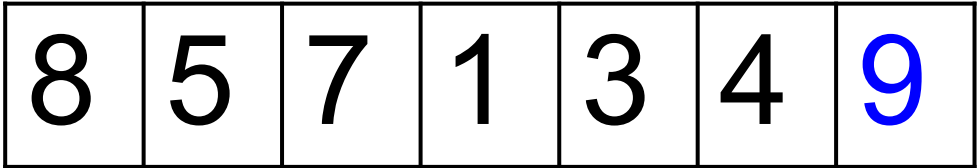
# Heapsort

Código



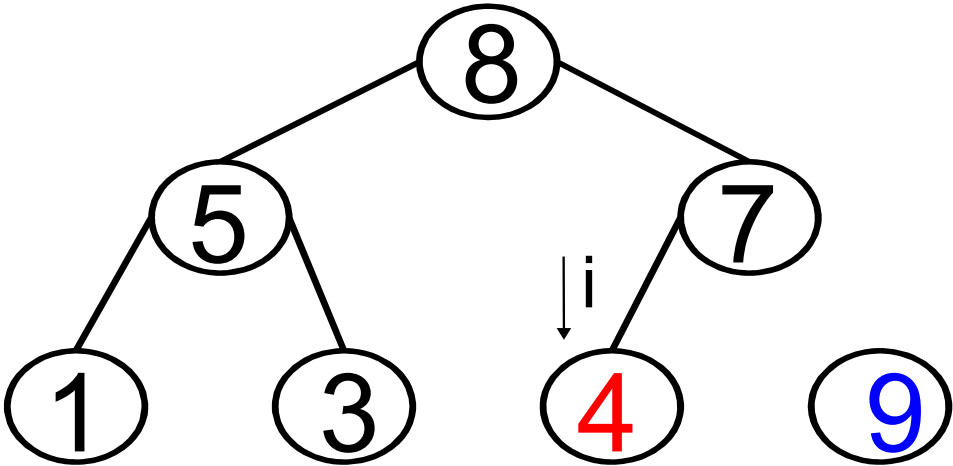
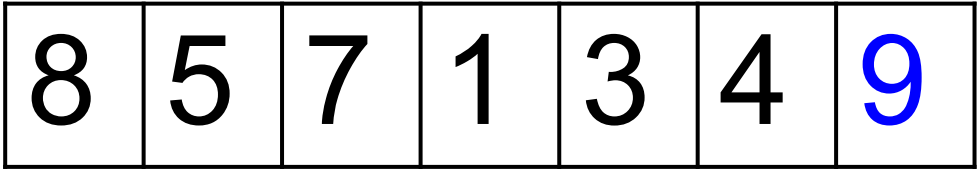
# Heapsort

Código



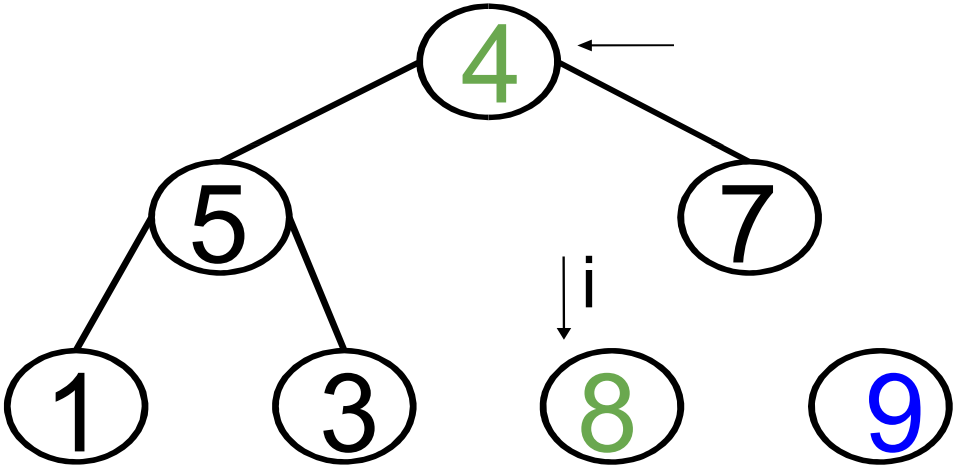
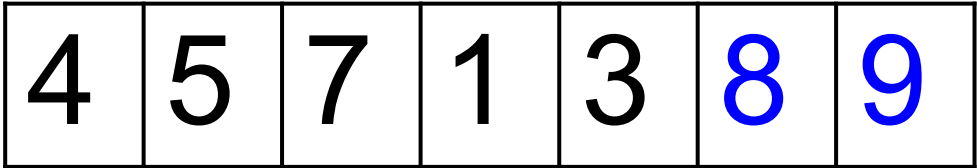
# Heapsort

Código



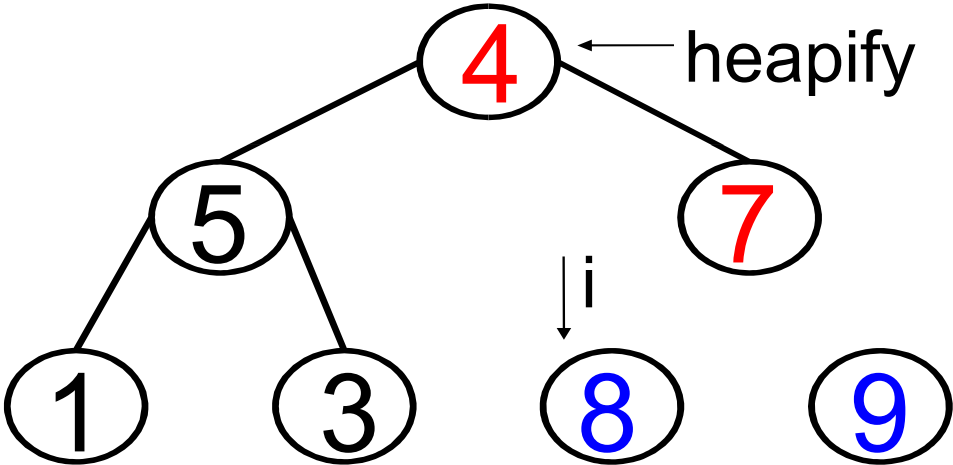
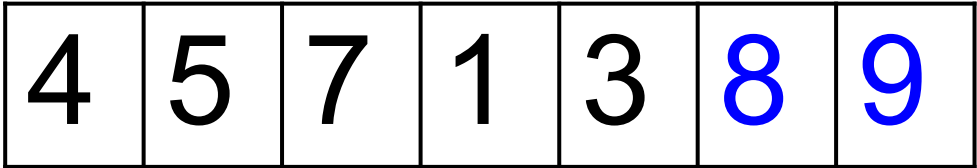
# Heapsort

Código



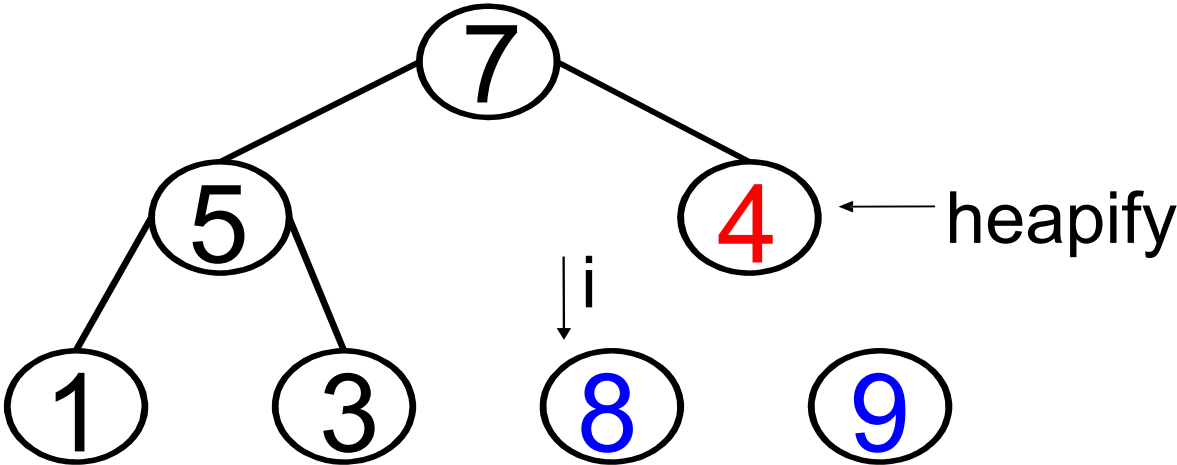
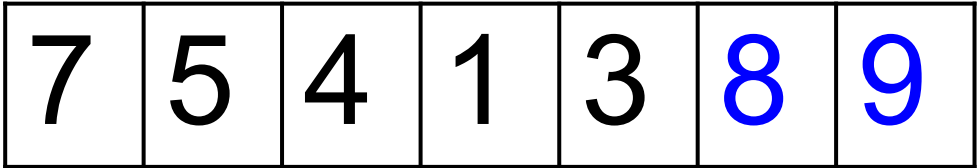
# Heapsort

Código



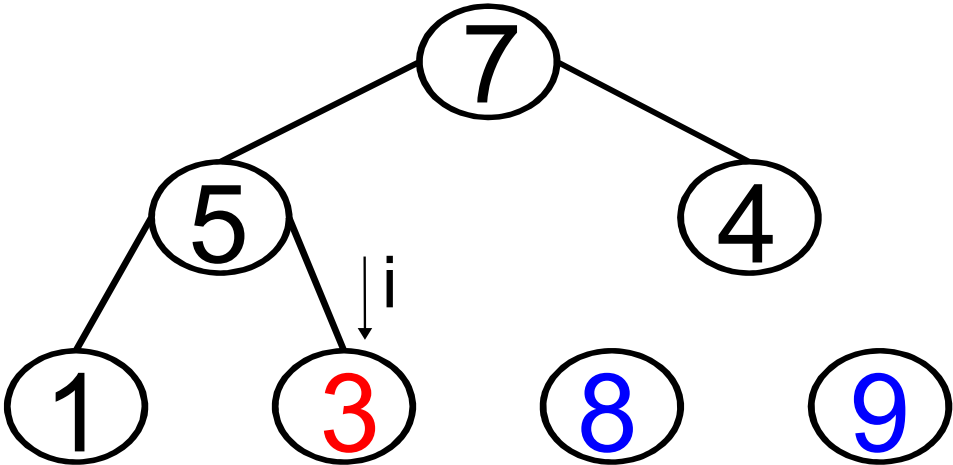
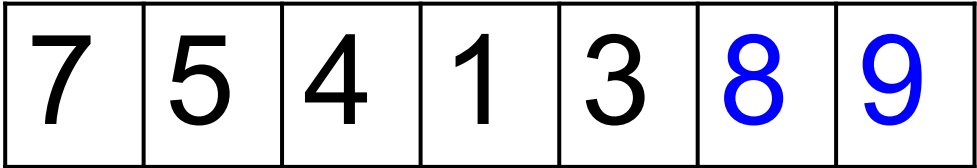
# Heapsort

Código



# Heapsort

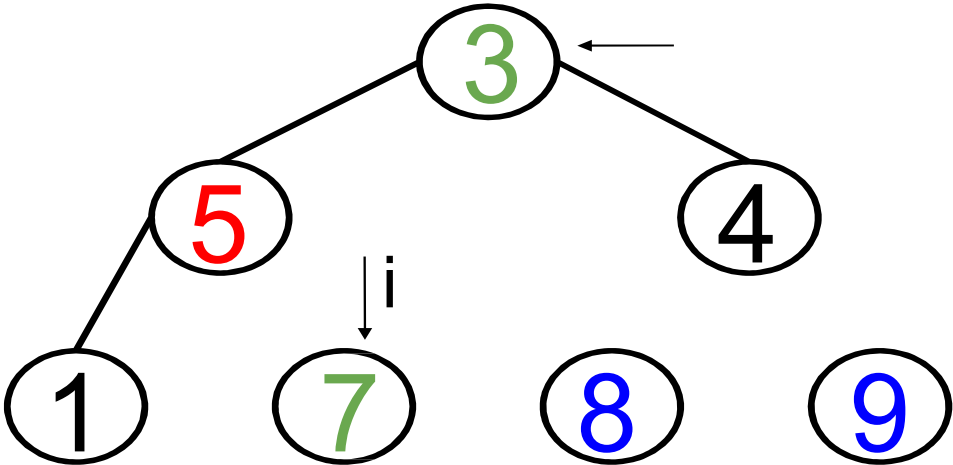
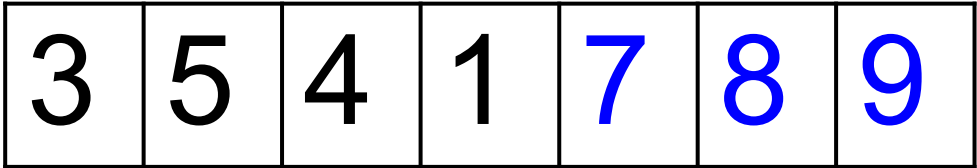
Código





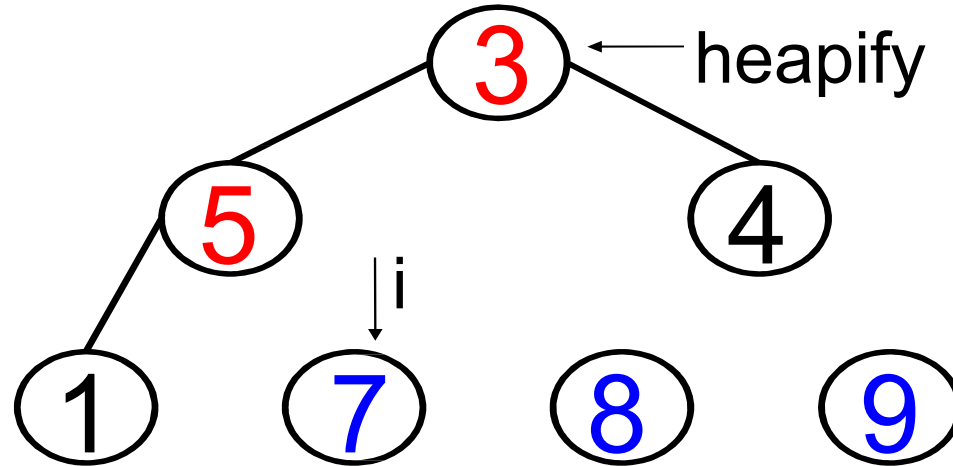
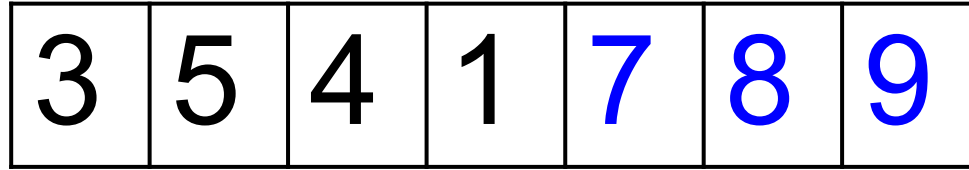
# Heapsort

Código



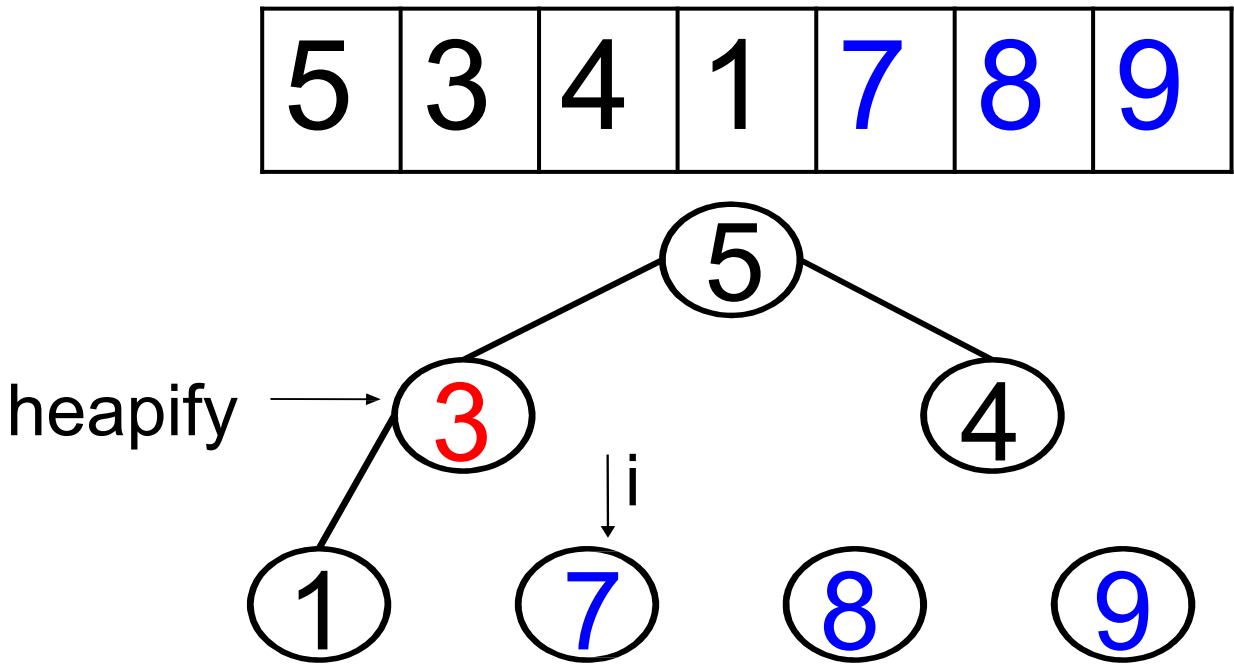
# Heapsort

Código



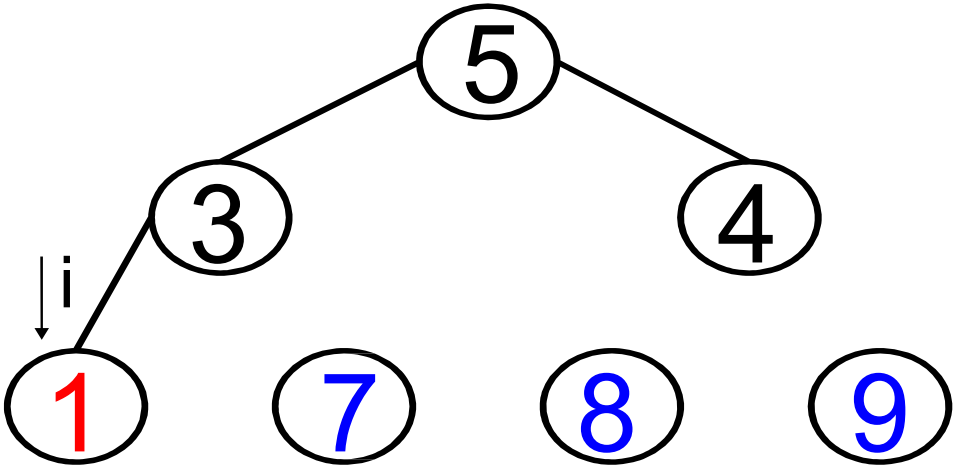
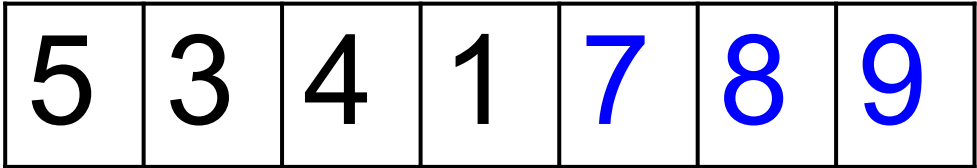
# Heapsort

Código



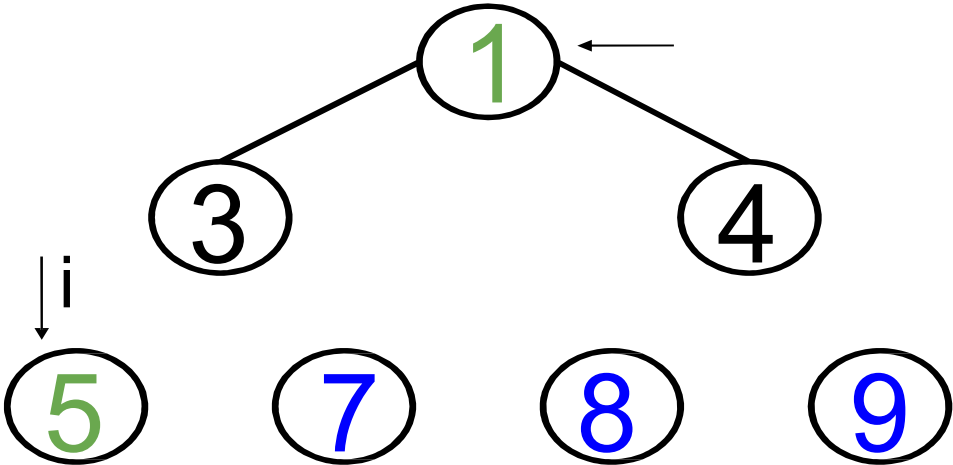
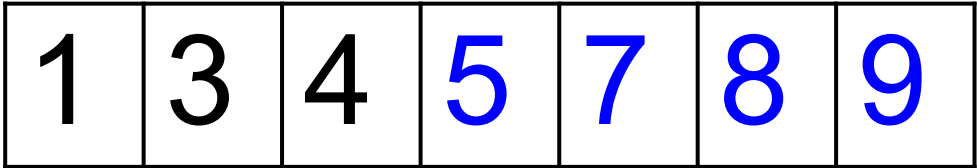
# Heapsort

Código



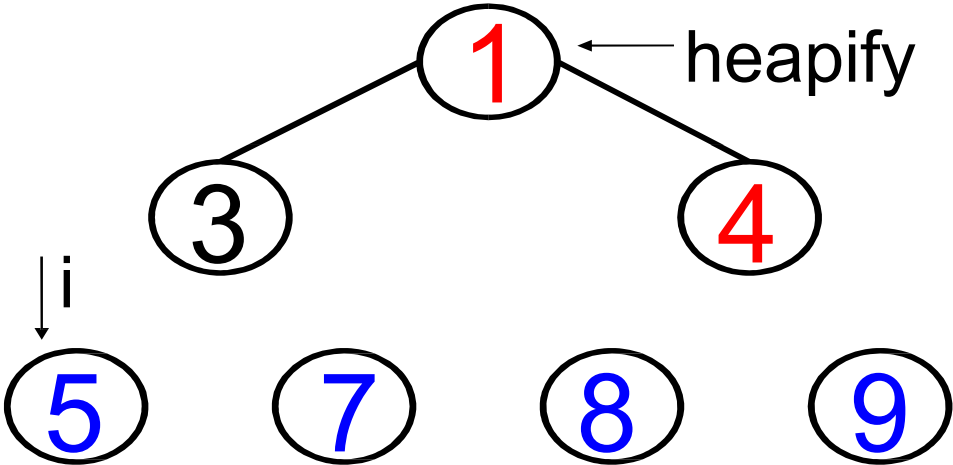
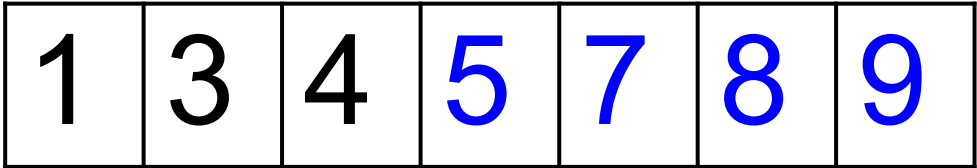
# Heapsort

Código



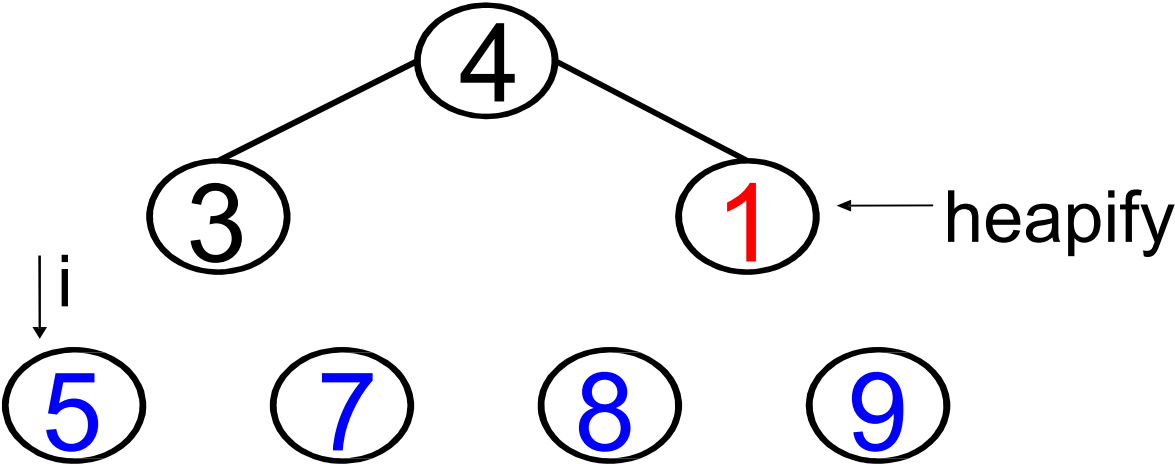
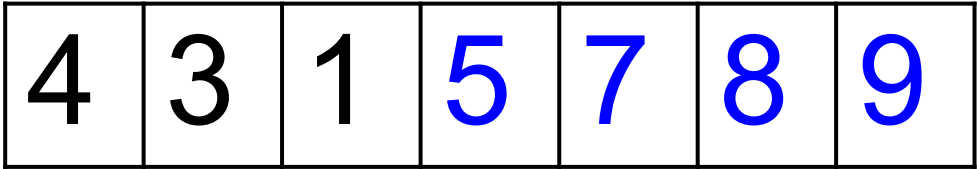
# Heapsort

Código



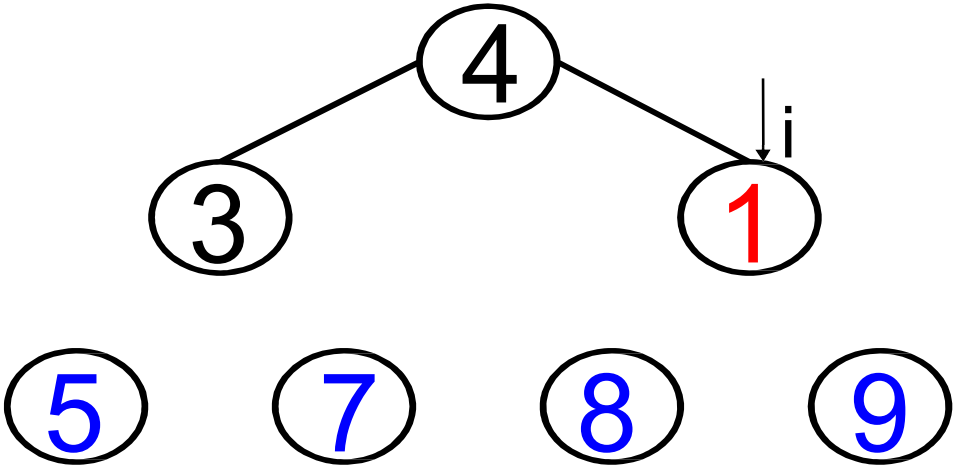
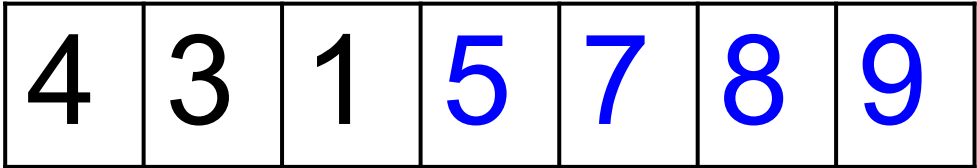
# Heapsort

Código



# Heapsort

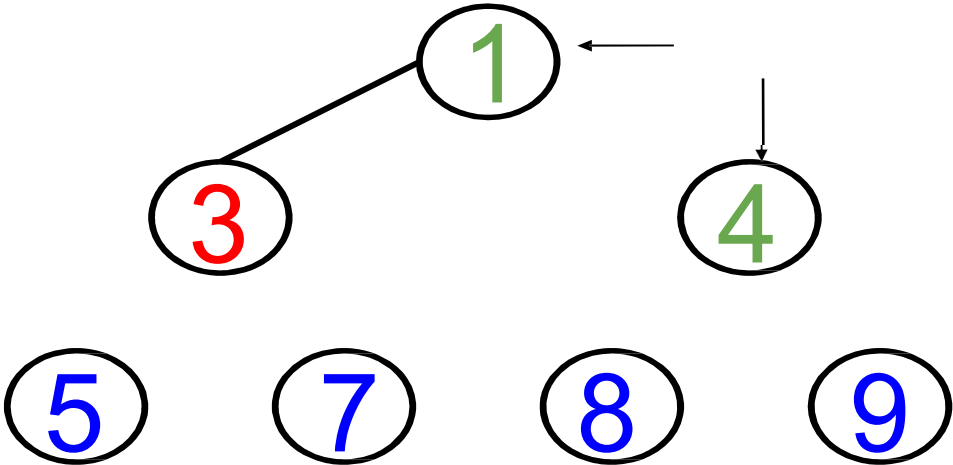
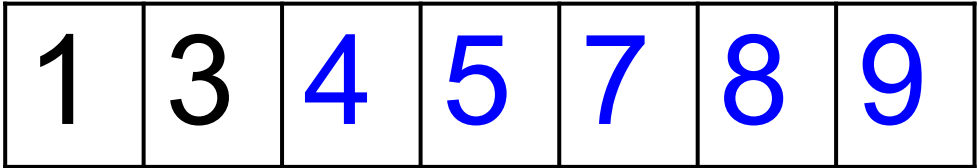
Código





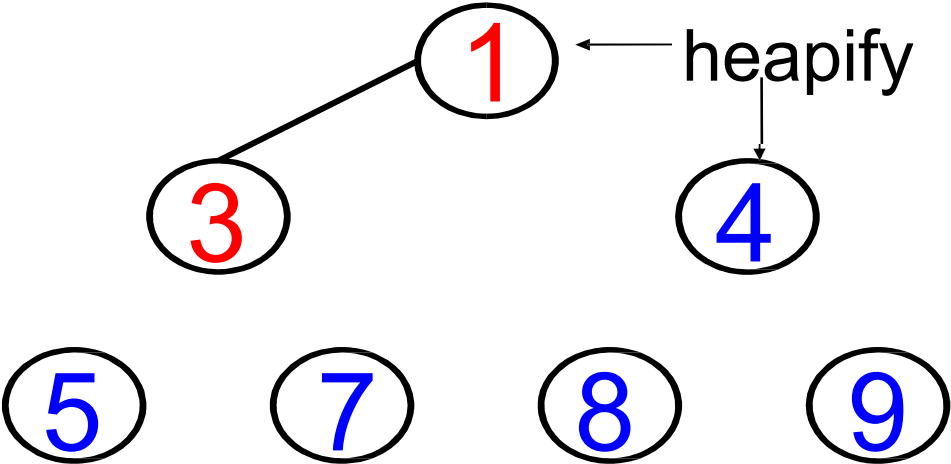
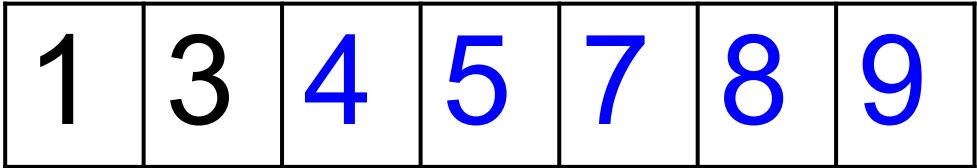
# Heapsort

Código



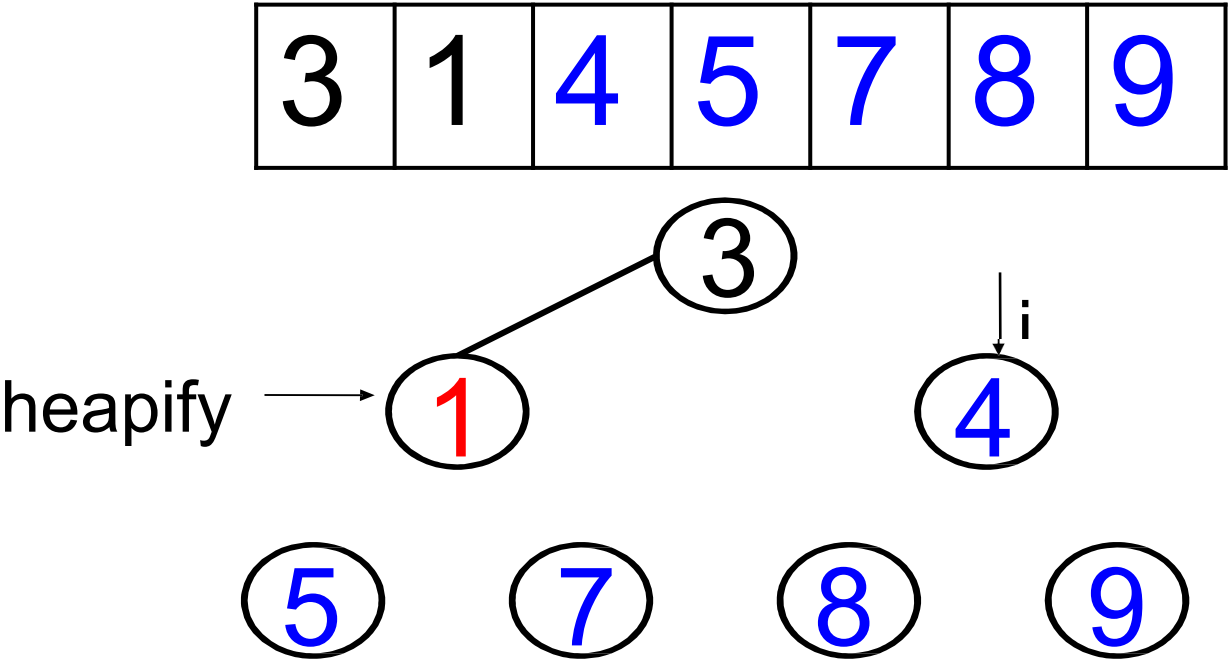
# Heapsort

Código



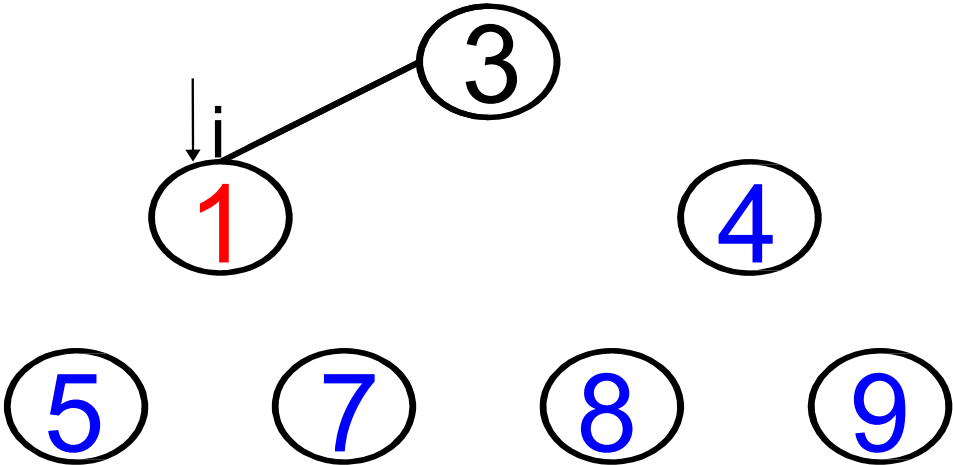
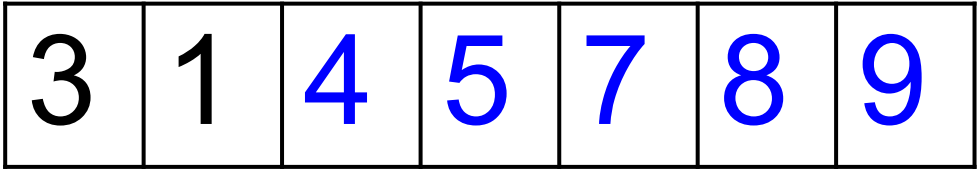
# Heapsort

Código



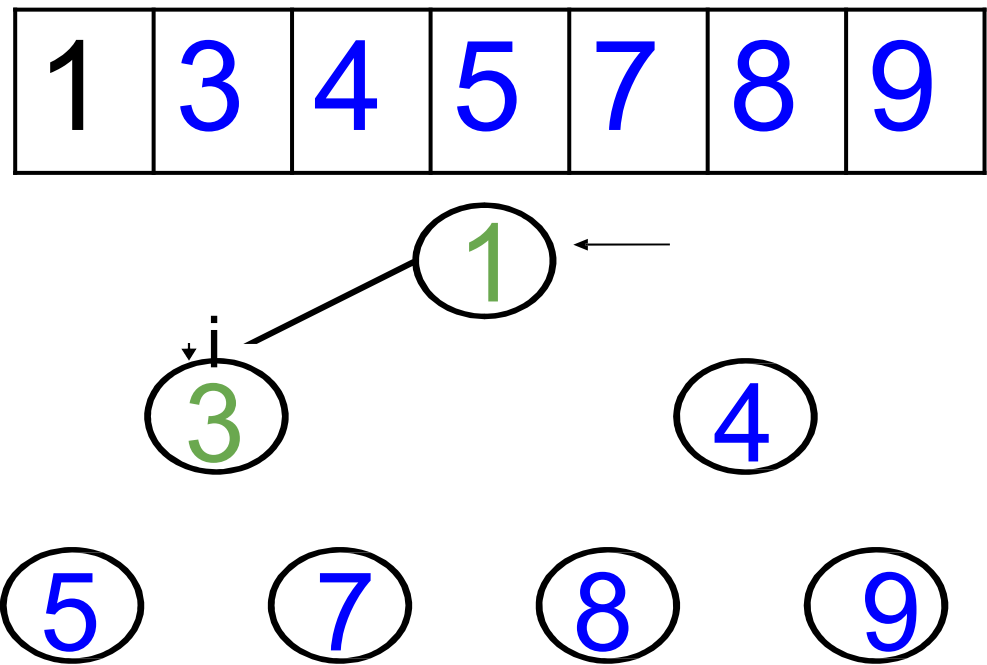
# Heapsort

Código



# Heapsort

Código



# Heapsort

Código

1	3	4	5	7	8	9
---	---	---	---	---	---	---

1

3

4

5

7

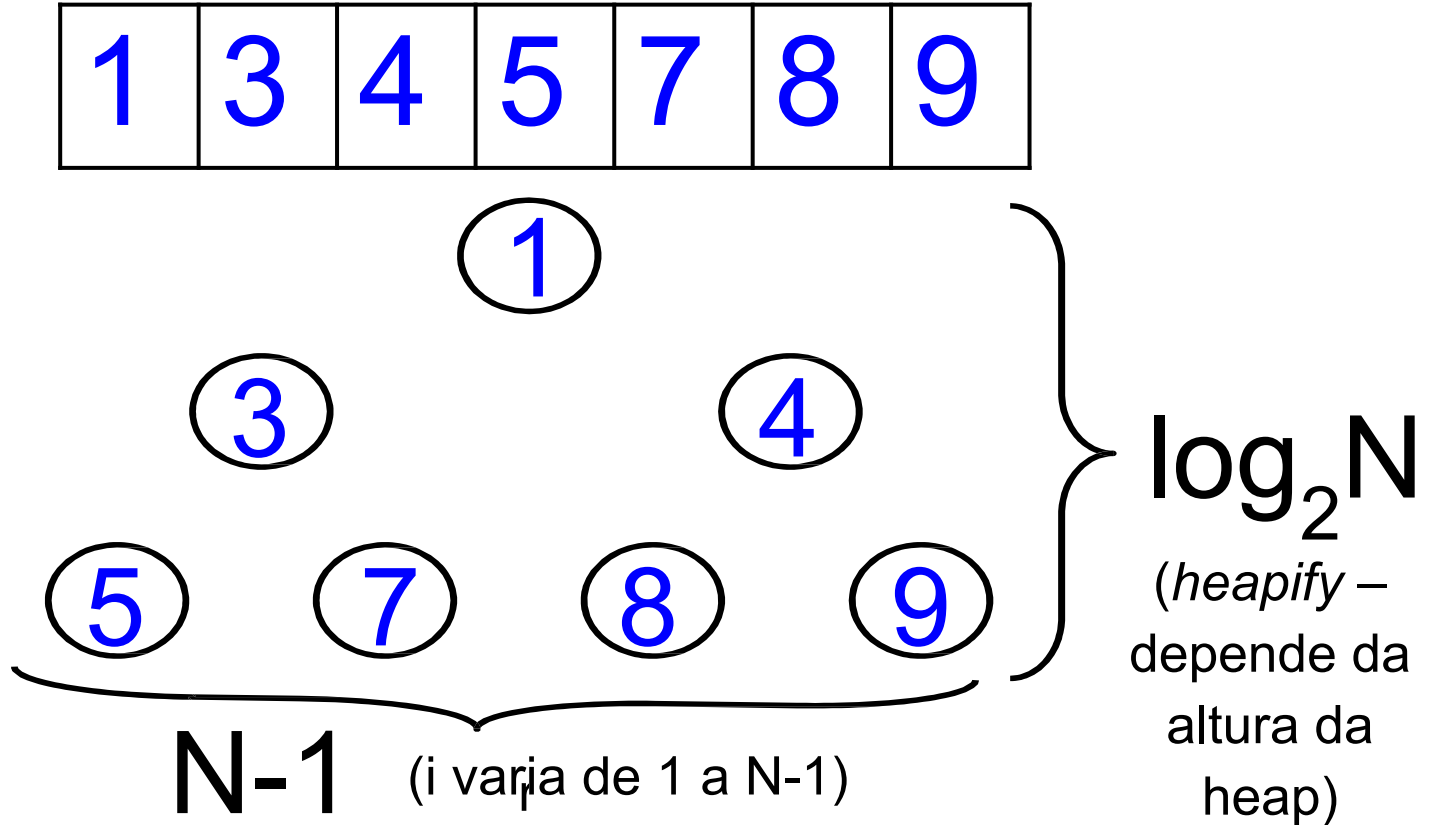
8

9

# Heapsort

Complexidade

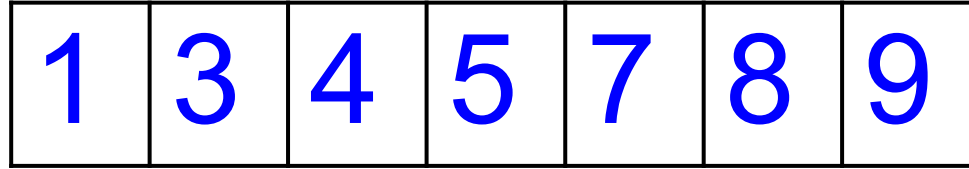
## Operações do Heapsort



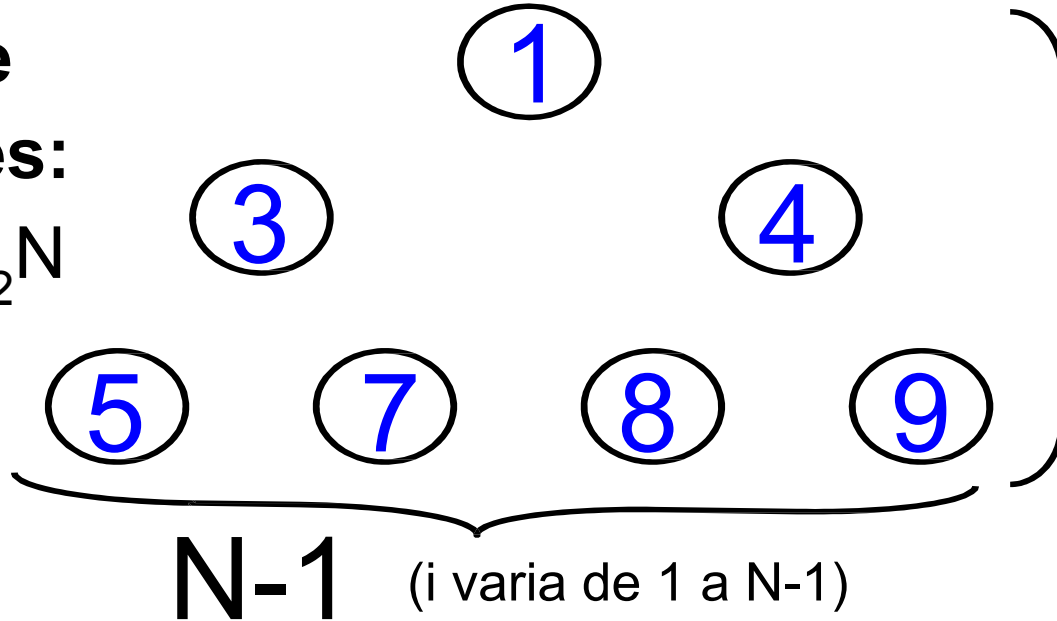
# Heapsort

Complexidade

## Operações do Heapsort



**Total de  
operações:**  
 $(N-1) * \log_2 N$



$\log_2 N$   
(*heapify* –  
depende da altura  
da heap)



## Countsort

---

# Countsort

Apenas quando os valores do vetor **v** variarem em um intervalo pequeno (Ex.: 0 a 100).

```
c = {}  
for i=0, 100 do  
    c[i] = 0  
end  
for i=1,#v do  
    c[ v[i] ] = c[ v[i] ]+1 end  
j = 1  
for i=0,100 do while  
    c[i]>0 do  
        v[j] = i  
        j = j+1  
        c[i] = c[i]-1 end  
end
```

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$C =$

0	0	0	0
0	1	2	3

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow$   
 $i$

$C =$

0	0	0	1
0	1	2	3

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow$   
 $i$

$C =$

0	1	0	1
0	1	2	3

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow$   
 $i$

$C =$

0	1	1	1
0	1	2	3

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow$   
 $i$

$C =$

0	2	1	1
0	1	2	3



# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow_i$

$C =$

0	3	1	1
0	1	2	3

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow$   
 $i$

$C =$

0	3	1	2
0	1	2	3

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow j$

$C = \begin{array}{cccc} 0 & 3 & 1 & 2 \end{array}$

$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}$

$\uparrow i$

# Countsort

$V = 3 \ 1 \ 2 \ 1 \ 1 \ 3$

$\uparrow_j$

$C = \begin{array}{cccc} 0 & 3 & 1 & 2 \end{array}$

$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}$

$\uparrow_i$

# Countsort

$V =$  **1** 1 2 1 1 3

$\uparrow_j$

$C =$     0    **2**    1    2

         0    1    2    3

$\uparrow_i$

# Countsort

$V = 1 \text{ } \color{red}{1} \text{ } 2 \text{ } 1 \text{ } 1 \text{ } 3$

$\uparrow$   
j

$C = \begin{array}{cccc} 0 & \color{red}{1} & 1 & 2 \end{array}$

$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}$

$\uparrow$   
i

# Countsort

$V = 1 \ 1 \ 1 \ 1 \ 1 \ 3$

$\uparrow j$

$C = 0 \ 0 \ 1 \ 2$

$\uparrow i$

# Countsort

$V = 1\ 1\ 1\ 1\ 1\ 3$

$\uparrow$   
 $j$

$C = \begin{array}{cccc} 0 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \end{array}$

$\uparrow$   
 $i$



# Countsort

$V = 1 \ 1 \ 1 \ 2 \ 1 \ 3$

$\uparrow j$

$C = \begin{array}{cccc} 0 & 0 & 0 & 2 \end{array}$

$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}$

$\uparrow i$

# Countsort

$V = 1 \ 1 \ 1 \ 2 \ 1 \ 3$

$\uparrow j$

$C = \begin{array}{cccc} 0 & 0 & 0 & 2 \end{array}$

$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}$

$\uparrow i$

# Countsort

$V =$  1 1 1 2 **3** 3  
                                  ↑  
                                  j

$C =$     0       0       0       **1**  
         0       1       2       3  
                                  ↑  
                                  i

# Countsort

$V = 1 \ 1 \ 1 \ 2 \ 3 \ 3$

$\uparrow_j$

$C =$

0	0	0	0
0	1	2	3

$\uparrow_i$

# Countsort

$V = 1\ 1\ 1\ 2\ 3\ 3$

$C = \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{array}$

Radixsort

---

# Radixsort

- Ordena o vetor da casa decimal menos significativa para a mais significativa
- Não precisa necessariamente ser de dígito em dígito, pois pode ser de byte em byte

# Radixsort



- 133
- 252
- 411
- 323
- 510
- 523
- 101



# Radixsort



- 133
  - 252
  - 411
  - 323
  - 510
  - 523
  - 101
- 510
  - 411
  - 101
  - 252
  - 133
  - 323
  - 523

# Radixsort



- |       |       |
|-------|-------|
| • 133 | • 510 |
| • 252 | • 411 |
| • 411 | • 101 |
| • 323 | • 252 |
| • 510 | • 133 |
| • 523 | • 323 |
| 101   | • 523 |

# Radixsort



• 133	• 510	• 101
• 252	• 411	• 510
• 411	• 101	• 411
• 323	• 252	• 323
• 510	• 133	• 523
• 523	• 323	• 133
101	• 523	• 252

# Radixsort



• 133	• 510	• 101
• 252	• 411	• 510
• 411	• 101	• 411
• 323	• 252	• 323
• 510	• 133	• 523
• 523	• 323	• 133
101	• 523	• 252

# Radixsort

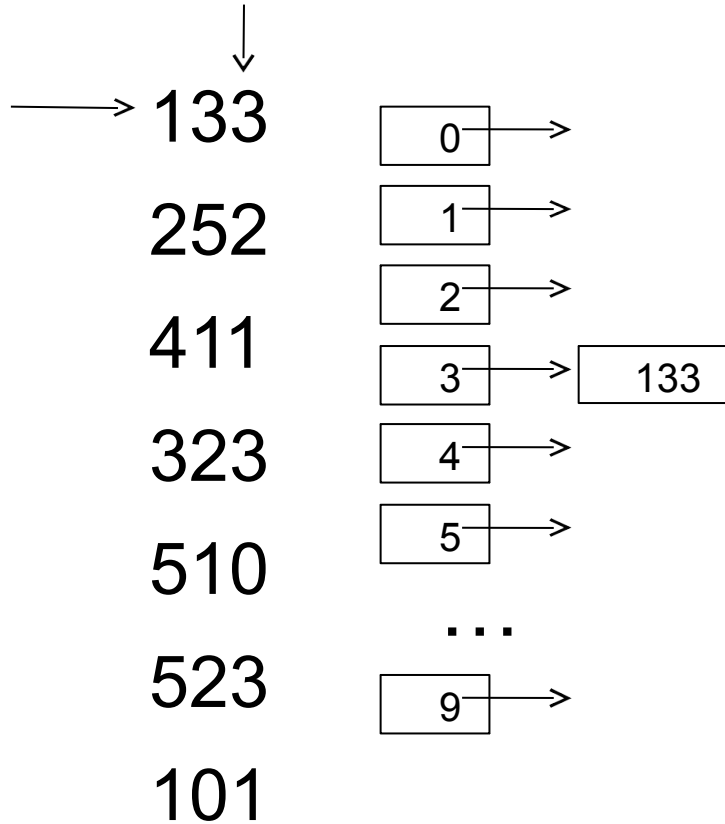


- |       |       |       |       |
|-------|-------|-------|-------|
| • 133 | • 510 | • 101 | • 101 |
| • 252 | • 411 | • 510 | • 133 |
| • 411 | • 101 | • 411 | • 252 |
| • 323 | • 252 | • 323 | • 323 |
| • 510 | • 133 | • 523 | • 411 |
| • 523 | • 323 | • 133 | • 510 |
| 101   | • 523 | • 252 | • 523 |

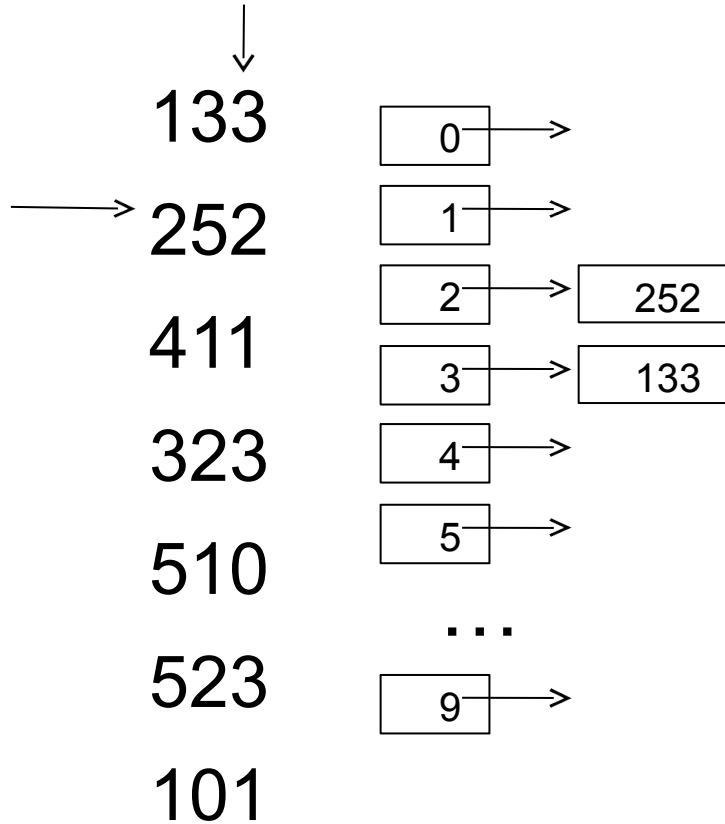
# Radixsort

- |       |       |       |       |
|-------|-------|-------|-------|
| • 133 | • 510 | • 101 | • 101 |
| • 252 | • 411 | • 510 | • 133 |
| • 411 | • 101 | • 411 | • 252 |
| • 323 | • 252 | • 323 | • 323 |
| • 510 | • 133 | • 523 | • 411 |
| • 523 | • 323 | • 133 | • 510 |
| 101   | • 523 | • 252 | • 523 |

# Radixsort – listas encadenadas

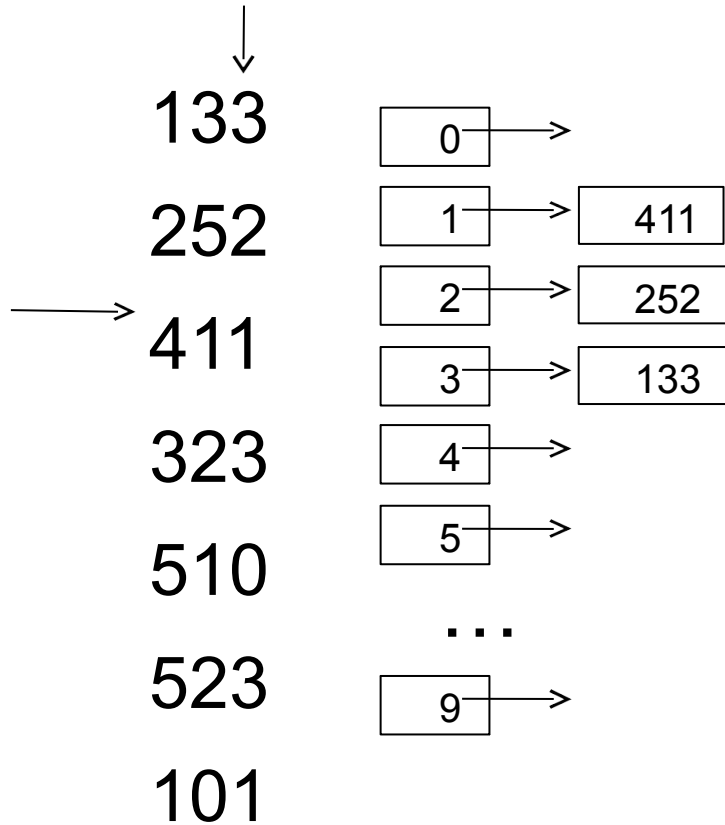


# Radixsort – listas encadenadas

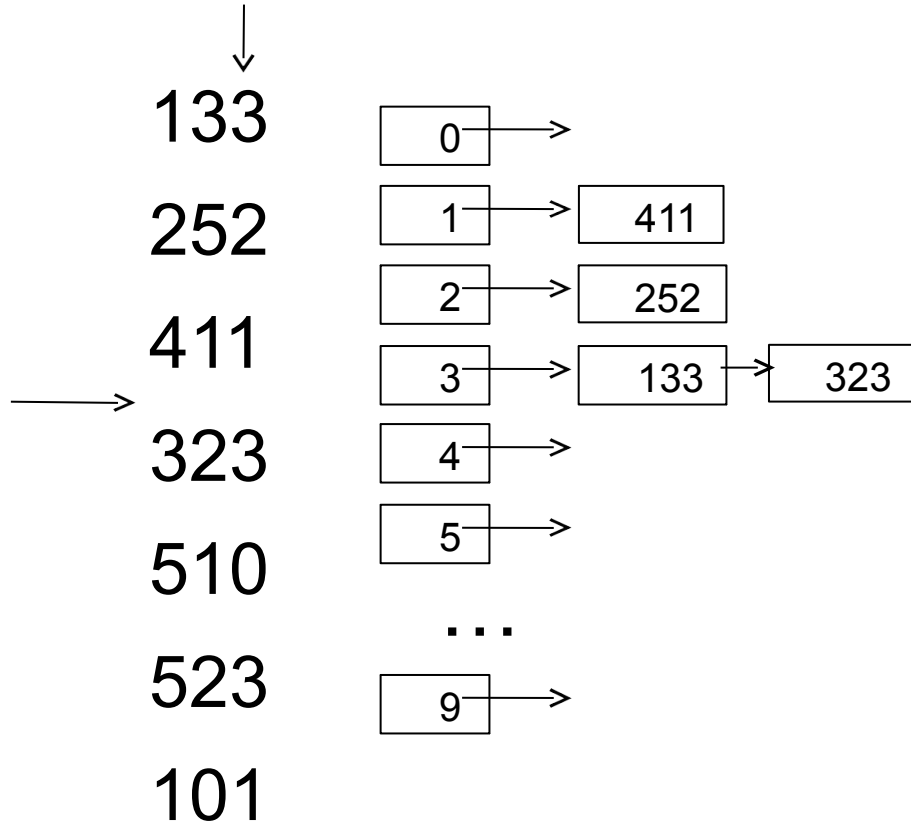




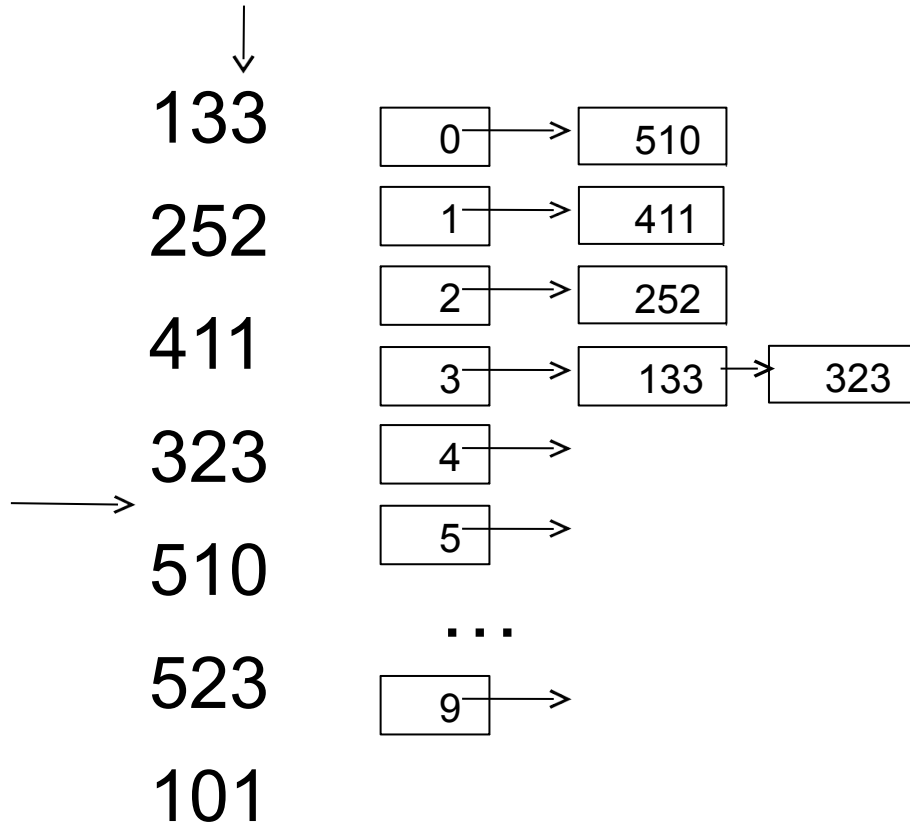
# Radixsort – listas encadenadas



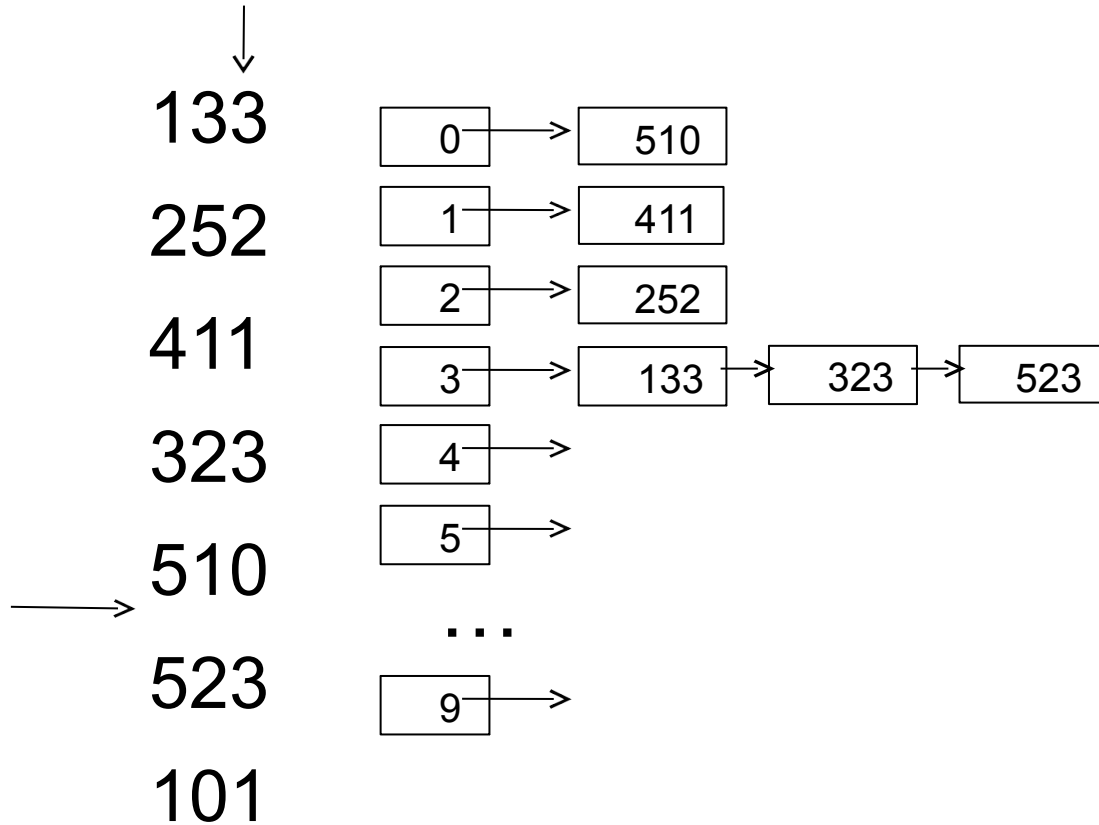
# Radixsort – listas encadenadas



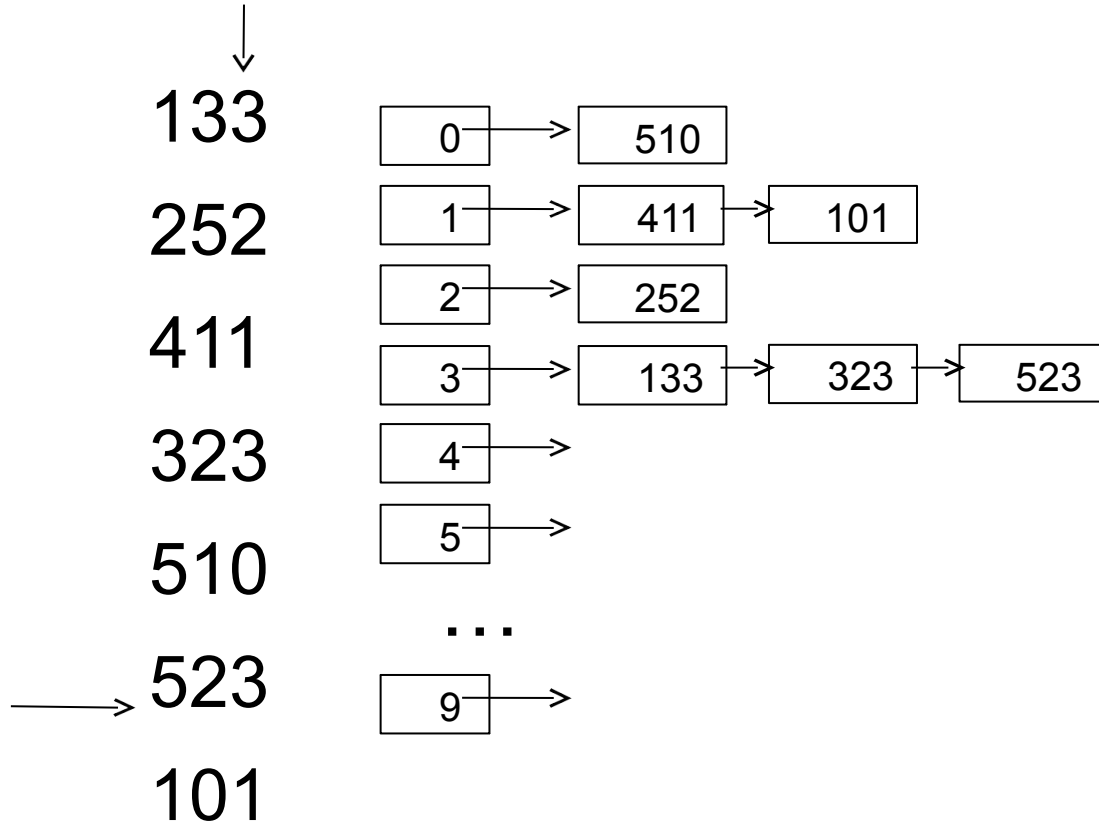
# Radixsort – listas encadenadas



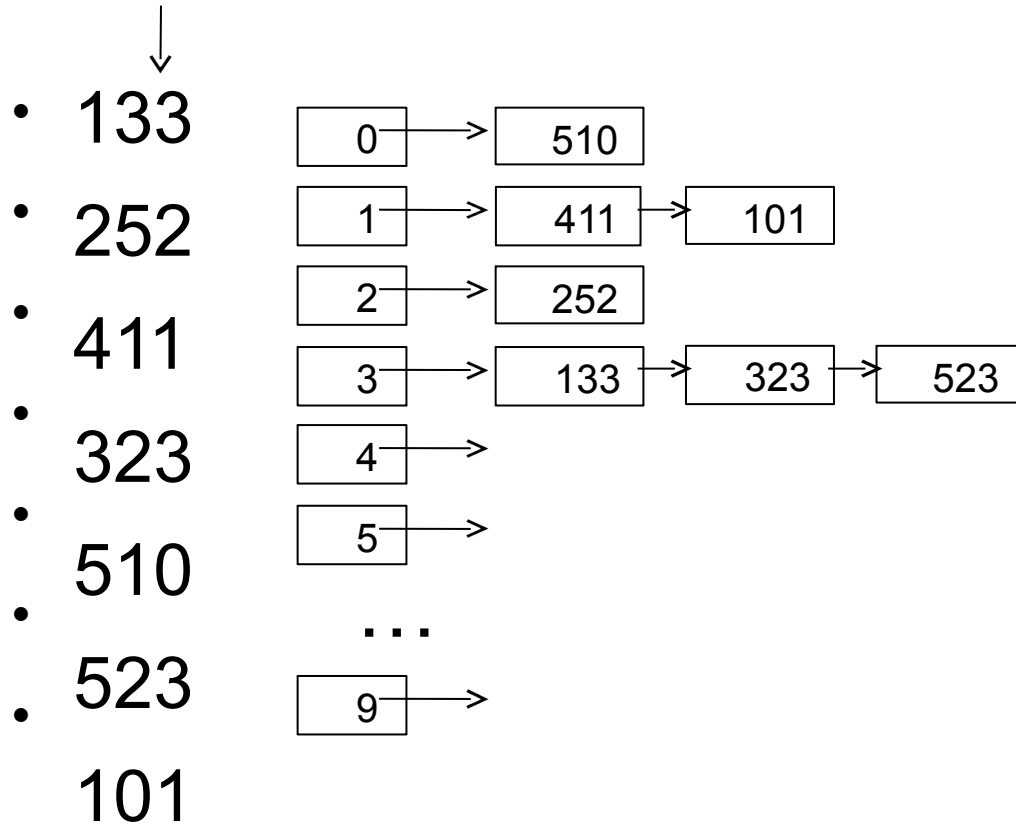
# Radixsort – listas encadenadas



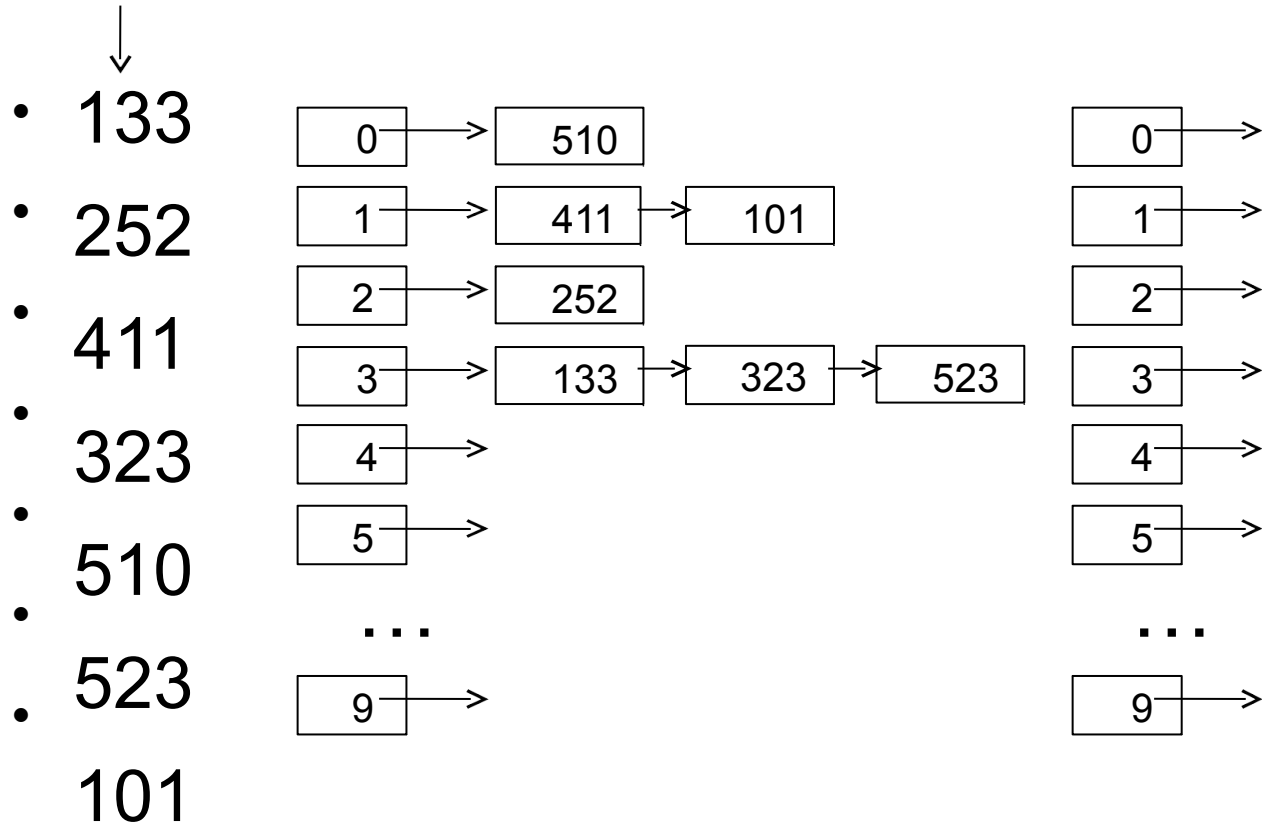
# Radixsort – listas encadenadas



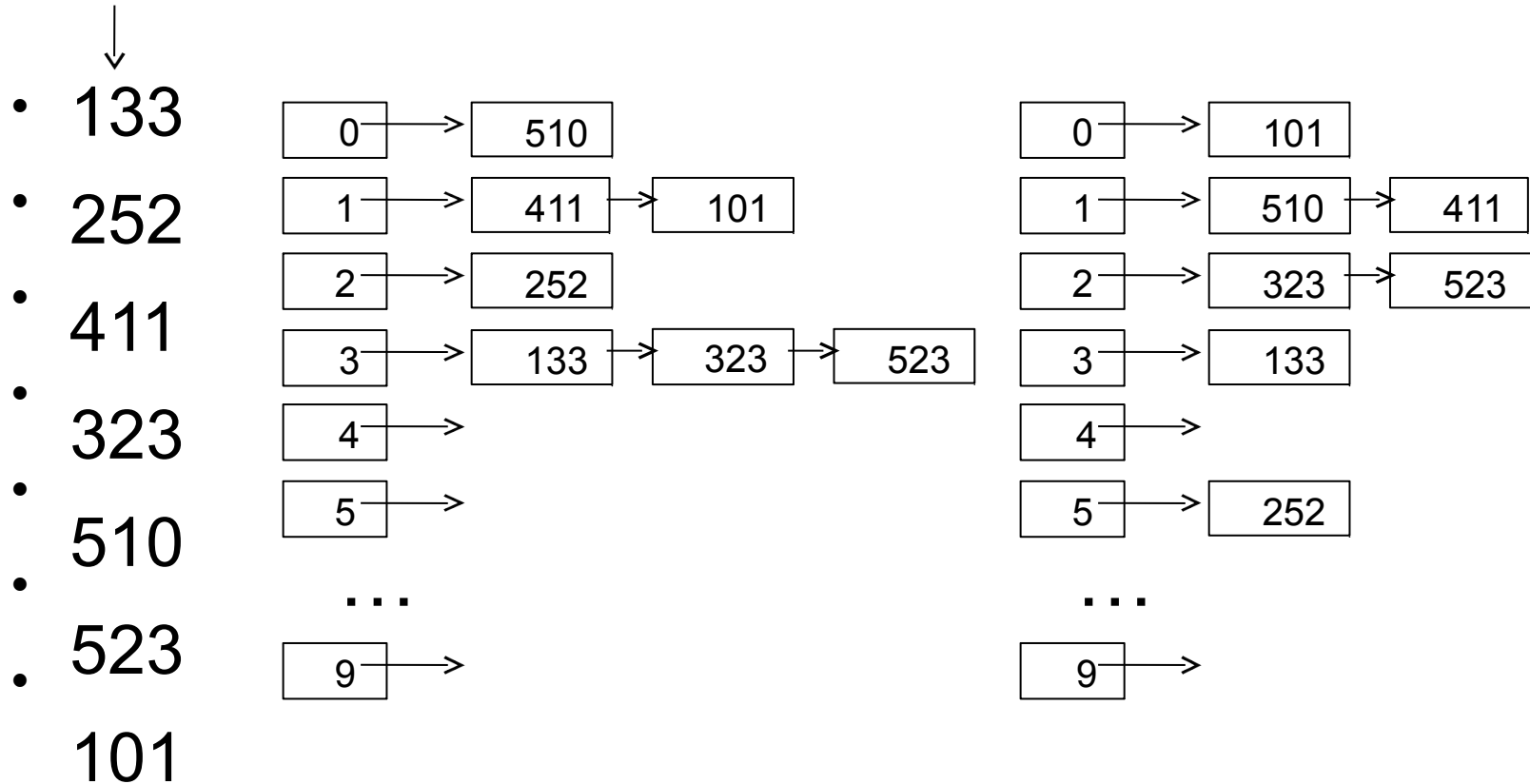
# Radixsort – listas encadenadas



# Radixsort – listas encadenadas

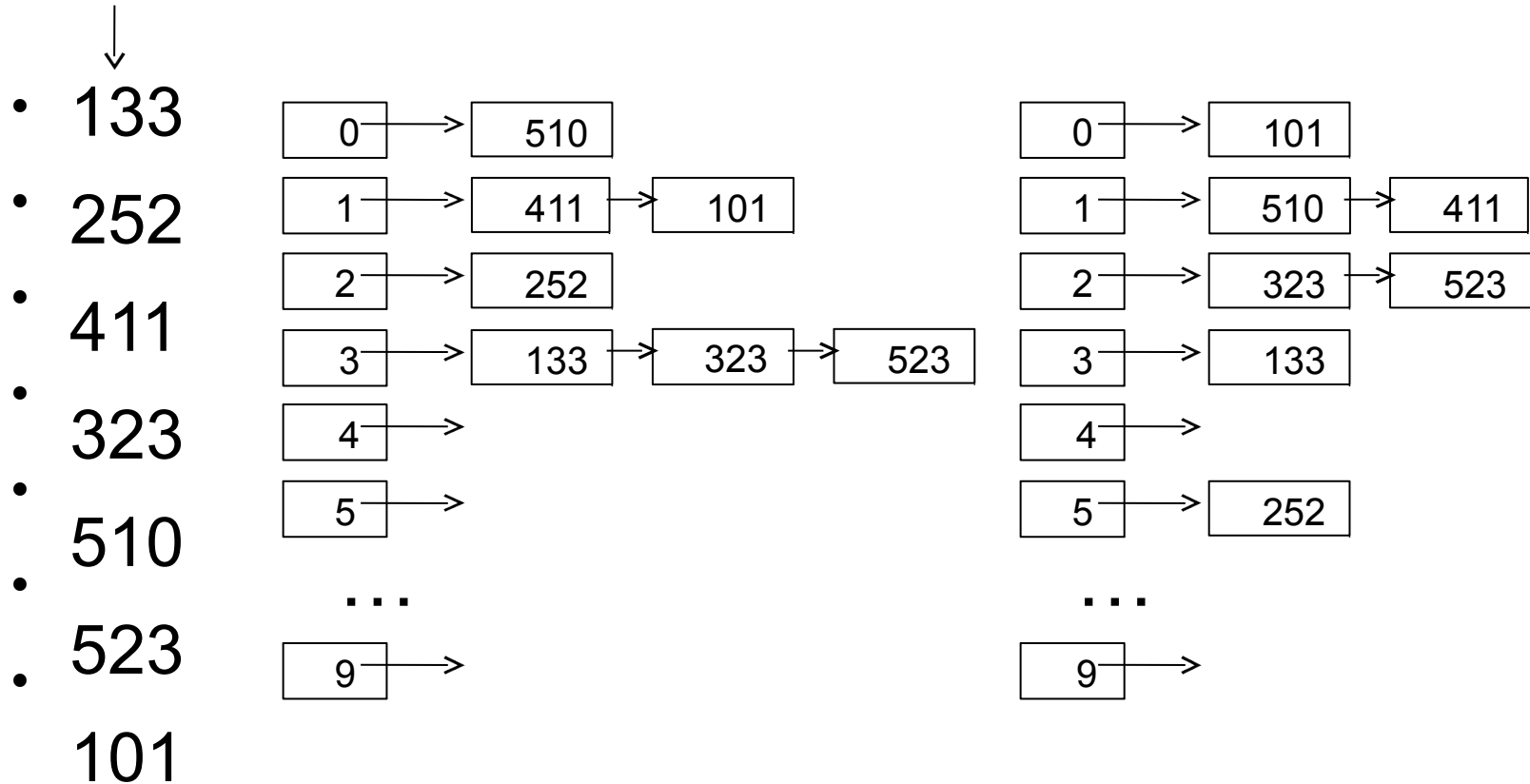


# Radixsort – listas encadenadas

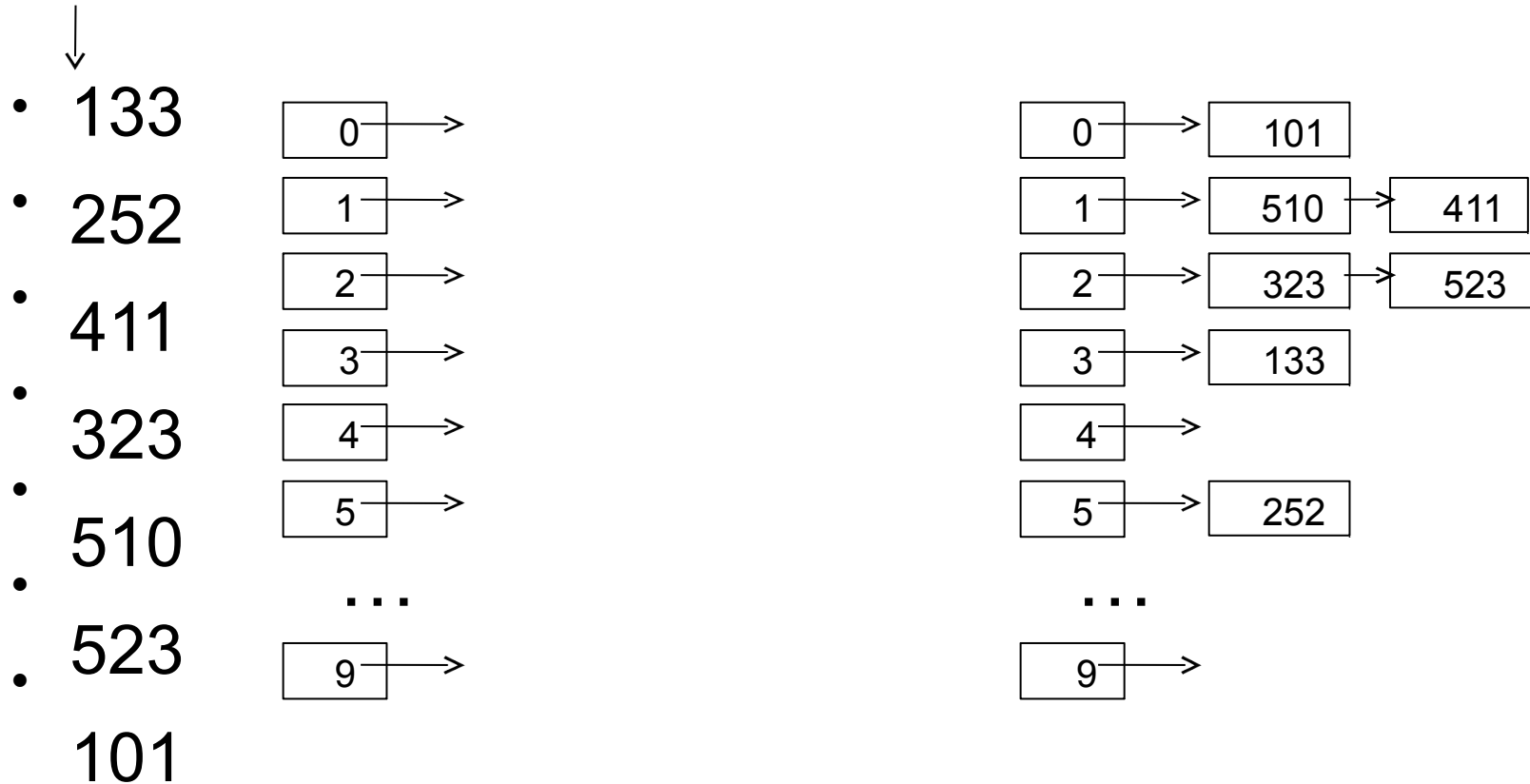




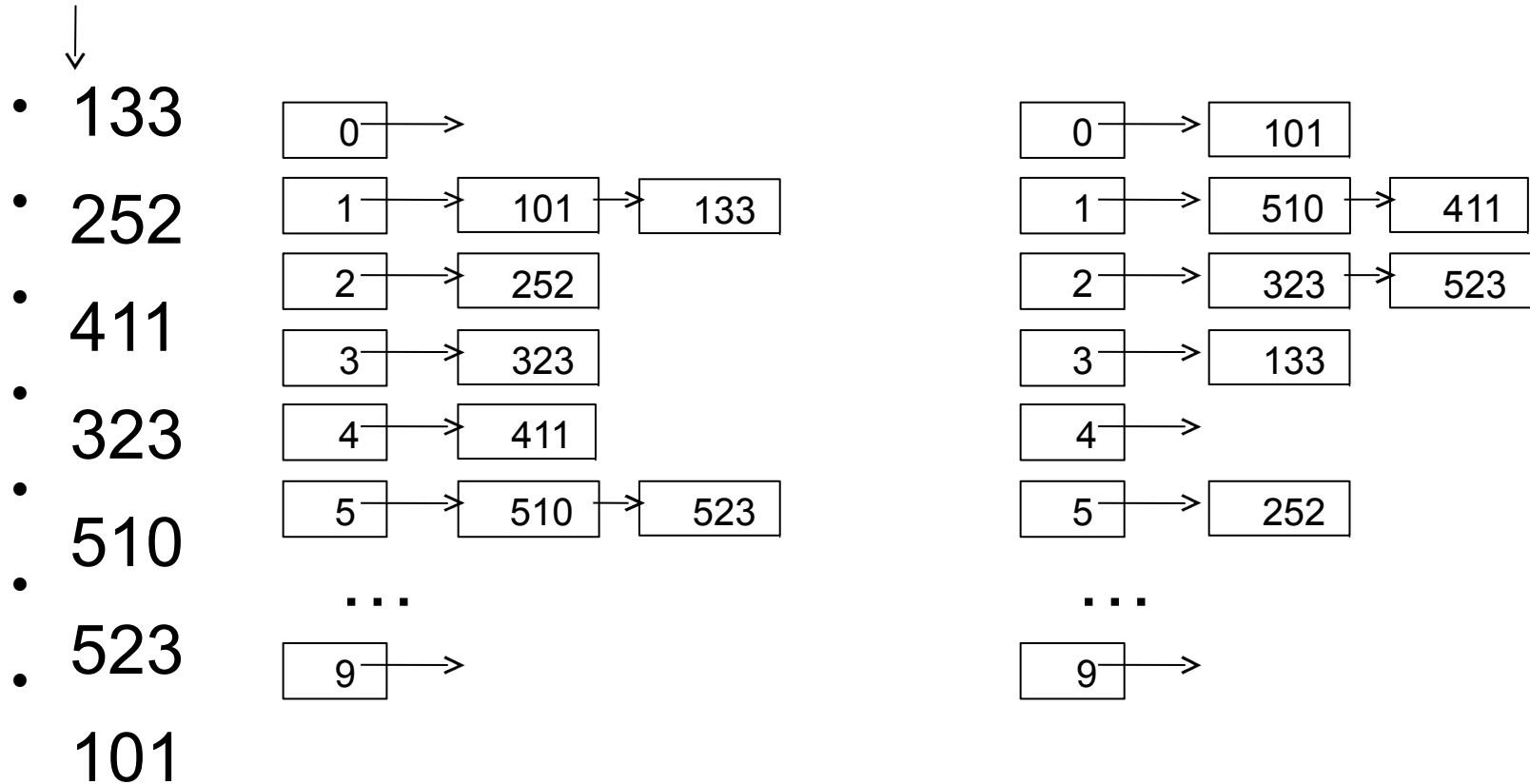
# Radixsort – listas encadenadas



# Radixsort – listas encadenadas



# Radixsort – listas encadenadas



# Radixsort - operações

- Número de operações:
  - Dígitos do maior número \* N (ordenação)
- Sem listas encadeadas (ordenando por heapsort):
  - Dígitos do maior número \* N logN
- Dígitos do maior número:
  - $\log_{10} K$ , onde K é o maior número
  -

Obrigado