Zhuo Chen
TIN093 / DIT093 Algorithms
September 4, 2025

assignment

1. Combinatorics

1.1. Ordering 99 Distinct Elements. The number of ways to order a list of n distinct elements is the number of permutations, given by $n!$.

$$P(n) = n!$$

For $n = 99$, the number of different orderings is:

$$P(99) = 99! \approx 9.3326 \times 10^{155}$$

1.2. Edges in a Complete Graph. In a complete graph, every pair of distinct vertices is connected by an edge. The number of edges is the number of ways to choose 2 vertices from a set of n, given by the binomial coefficient $\binom{n}{2}$.

$$\text{Edges} = \binom{n}{2} = \frac{n(n-1)}{2}$$

- For $n = 4$ vertices:
$$\binom{4}{2} = \frac{4(4-1)}{2} = \frac{12}{2} = 6$$
- For $n = 40$ vertices:
$$\binom{40}{2} = \frac{40(40-1)}{2} = \frac{1560}{2} = 780$$
- For a general case of n vertices, the number of edges is $\frac{n(n-1)}{2}$.

1.3. Ordering Letters in ”engineering”. This is a problem of permutations of a multiset. The word ”engineering” has $n = 11$ letters with the following frequencies for repeated letters:

- ’e’: $n_1 = 3$
- ’n’: $n_2 = 3$
- ’g’: $n_3 = 2$
- ’i’: $n_4 = 2$
- ’r’: $n_5 = 1$

The number of distinct arrangements is given by the multinomial coefficient:

$$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1! n_2! \dots n_k!}$$

$$\text{Arrangements} = \frac{11!}{3! \cdot 3! \cdot 2! \cdot 2! \cdot 1!} = \frac{39,916,800}{(6)(6)(2)(2)(1)} = \frac{39,916,800}{144} = 277,200$$

2. Asymptotic Order

The functions are arranged in ascending order of growth, where $f(n) \ll g(n)$ denotes $f(n) = o(g(n))$.

Final Order.

$$f_6(n) \ll f_2(n) \ll f_4(n) \ll f_7(n) \ll f_1(n) \ll f_5(n) \ll f_3(n)$$

Proofs. **1.** $f_6(n) = o(f_2(n))$**:**

$$\lim_{n\to\infty} \frac{n(\log n)^2}{n^{1.5}} = \lim_{n\to\infty} \frac{(\log n)^2}{n^{0.5}} \xrightarrow{\text{L'H}} \lim_{n\to\infty} \frac{2(\log n)\cdot\frac{1}{n}}{0.5n^{-0.5}} = \lim_{n\to\infty} \frac{4\log n}{n^{0.5}} \xrightarrow{\text{L'H}} \lim_{n\to\infty} \frac{4/n}{0.5n^{-0.5}} = \lim_{n\to\infty} \frac{8}{n^{0.5}} = 0$$

2. $f_2(n) = o(f_4(n))$**:**

$$\lim_{n\to\infty} \frac{n^{1.5}}{n^{100}} = \lim_{n\to\infty} \frac{1}{n^{98.5}} = 0$$

3. $f_4(n) = o(f_7(n))$**:** Compare natural logarithms, $L(f) = \ln(f(n))$.

$$L(f_4) = \ln(n^{100}) = 100\ln n \quad \text{and} \quad L(f_7) = \ln(n^{\ln n}) = (\ln n)^2$$

$$\lim_{n\to\infty} \frac{L(f_4)}{L(f_7)} = \lim_{n\to\infty} \frac{100\ln n}{(\ln n)^2} = \lim_{n\to\infty} \frac{100}{\ln n} = 0 \implies f_4 = o(f_7)$$

4. $f_7(n) = o(f_1(n))$**:** Compare natural logarithms.

$$L(f_7) = (\ln n)^2 \quad \text{and} \quad L(f_1) = \ln(4^n) = n\ln 4$$

$$\lim_{n\to\infty} \frac{L(f_7)}{L(f_1)} = \lim_{n\to\infty} \frac{(\ln n)^2}{n\ln 4} \xrightarrow{\text{L'H}} \lim_{n\to\infty} \frac{2(\ln n)\cdot\frac{1}{n}}{\ln 4} = \lim_{n\to\infty} \frac{2\ln n}{n\ln 4} \xrightarrow{\text{L'H}} \lim_{n\to\infty} \frac{2/n}{\ln 4} = 0 \implies f_7 = o(f_1)$$

5. $f_1(n) = o(f_5(n))$**:** Compare exponents with a common base of 2.

$$f_1(n) = 4^n = 2^{2n} \quad \text{and} \quad f_5(n) = 2^{n^2}$$

We compare exponents: $\lim_{n\to\infty} \frac{2n}{n^2} = \lim_{n\to\infty} \frac{2}{n} = 0 \implies 2n = o(n^2) \implies f_1 = o(f_5)$.

6. $f_5(n) = o(f_3(n))$**:** Compare exponents.

$$f_5(n) = 2^{n^2} \quad \text{and} \quad f_3(n) = 2^{2^n}$$

We compare exponents: $\lim_{n\to\infty} \frac{n^2}{2^n} \xrightarrow{\text{L'H}} \lim_{n\to\infty} \frac{2n}{2^n \ln 2} \xrightarrow{\text{L'H}} \lim_{n\to\infty} \frac{2}{2^n(\ln 2)^2} = 0 \implies n^2 = o(2^n) \implies f_5 = o(f_3)$.

ByLatex