ASSIGMENT 1

Ejemplo 6.1: Assignment: Weakly interacting and confined bosons at low density (April 2021)

The theoretical description of the recent experiments on Bose Einstein condensation (BEC) is based on the Gross-Pitaevskii (GP) equation. The atoms are confined by a magnetic field, which effects are very well described by an harmonic oscillator potential. Here we assume a condensate of 87Rb. We made the assumption that all the atoms are in the condensate, $\Psi(\vec{r})$ normalized such that:

$$\int d\vec{r} |\Psi(\vec{r})|^2 = N \,. \tag{6.1}$$

In harmonic oscilator units, the GP equation reads:

$$\left[-\frac{1}{2}\nabla^2 + \frac{1}{2} r_1^2 + 4\pi \bar{a}_s N |\bar{\Psi}(\vec{r}_1)|^2 \right] \bar{\Psi}(\vec{r}_1) = \bar{\mu} \bar{\Psi}(\vec{r}_1) \tag{6.2}$$

where μ is the chemical potential, N the number of particles and $\bar{a}_s$ the $s-$wave scattering length in harmonic oscilator units. In this case, $\bar{\Psi}(\vec{r}_1)$ is normalized to 1.

a0) Using the GP program check that if you put the interaction equal to zero (in the program, cequ=0), then you recover the expected results for the harmonic oscillator. Why the energy per particle is equal to the chemical potential?

a) Take $\bar{a}_s = 0.00433$, appropriate for ^{87}Rb and, using the program, solve the Gross Pitaevskii equation. Study the dependence on the number of particles ($N = 100, 1000, 10000, 100000, 1000000$) of the chemical potential, and the following energies per particle: total, kinetic, harmonic oscillator and interaction energy. Construct a table with the results and comment their behavior.

b) Do the same using the Thomas-Fermi approximation. Notice that the kinetic energy is this approach is taken to zero.

c) Make a plot of the density profile $\rho(r_1)$ normalized such that

$$\int dr_1 r_1^2 \rho(r_1) = 1 \tag{6.3}$$

for $N = 1000$ and $N = 100000$ and compare the GP and the TF results.

d) Check numerically that the solutions of the GP equation fulfil the virial theorem for different values of N.