Measuring Relative Stellar Elemental Abundance from Solar Spectroscopy

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1 Motivation

Determining the elemental composition of exoplanets can provide insight into their structure, formation history, and possible habitability. However, it is not possible to directly measure the composition of most exoplanets. Instead, we assume their composition to be correlated with the composition of the protoplanetary disk that formed them, which we can infer from the composition of the system's host star. To test this technique, we can start with the best-known star to us and the closest star to Earth: the Sun. In this paper, we measure the solar relative abundance of several elements in the Sun (namely Na, Mg, and Fe) by analyzing the solar spectrum according to the curve-of-growth method and ensuring that ground-state, excited-state, neutral, and ionized atoms of each element are all counted.

2 Methods

2.1 Curve-of-Growth

To find the number of atoms of a given element in an absorbing state, we first find the equivalent width of the sodium [Figure 1], magnesium [Figure 2], and iron [Figure 3] absorption lines. To do this, we defined a function that finds the indices of all of the wavelengths within a range that approximates the dip in the spectrum, finds the median step between each wavelength, and evaluates the following equation: $e.w. = \sum_{n=n_1}^{n_2}[1 - f(n)] * \Delta\lambda$, where $f(n)$

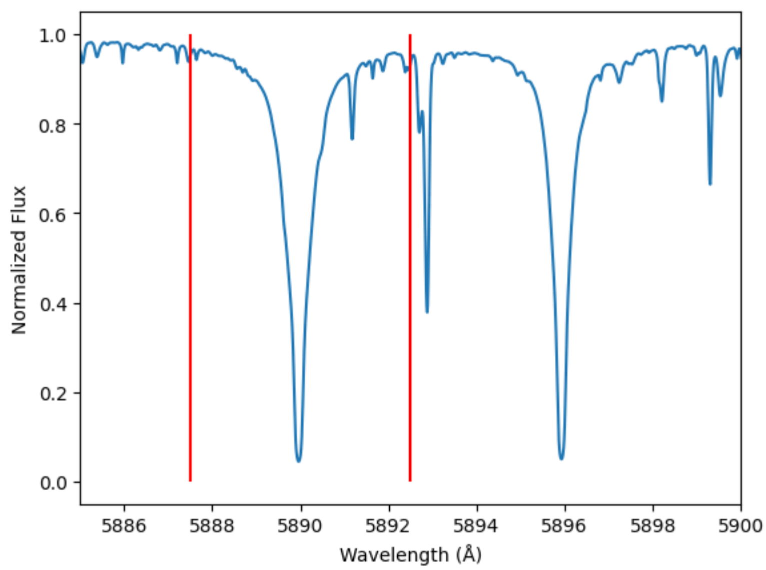

Figure 1: Solar spectrum in blue, with the bounds around the sodium absorption line in red.

is the flux at a given index, n_1 and n_2 are the indices of the lower and upper bounds respectively, and $\Delta\lambda$ is the median step between each wavelength in the range of indices. Table 1 lists the ranges of wavelengths considered for each element, as well as the equivalent width.

Element Symbol	$\lambda(n_1)$ [Å]	$\lambda(n_2)$ [Å]	e.w. [Å]
Na	5887.5	5892.5	0.83467
Mg	5181.4	5185.8	1.5257
Fe	4382.5	4384.9	1.3800

Table 1: Wavelength ranges considered and resulting equivalent widths for sodium, magnesium, and iron absorption lines, all in Angstroms.

Now that we have the equivalent width for each element's solar spectrum absorption line, we can find the number density of each element in the Sun by analyzing the solar curve of the growth plot [Figure 4]. We defined a function that outputs the $\log[Nf(\frac{\lambda}{5000\text{A}})]$ for an inputted $\log[\frac{\text{e.w.}}{\lambda_{\text{line}}}]$, where λ_{line} is the wavelength at the absorption line. By rearranging the output of this function to solve for N, the number of atoms in the Sun of a given element in the

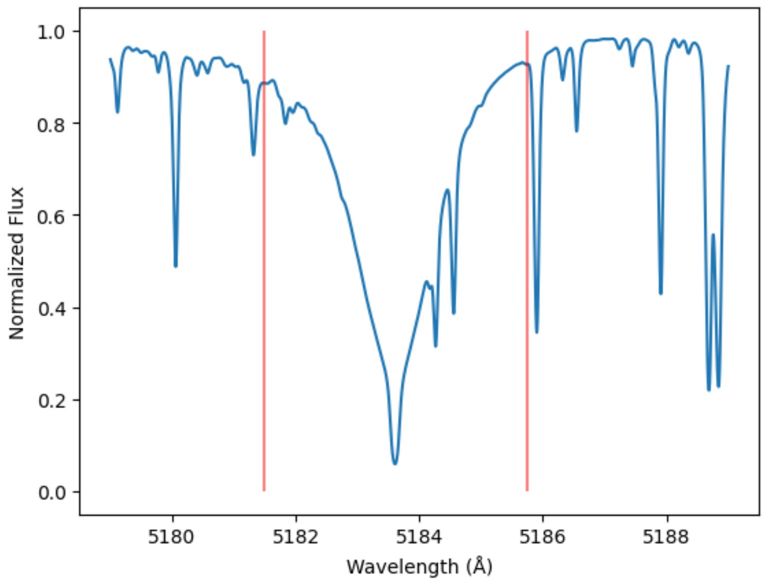

Figure 2: Solar spectrum in blue, with the bounds around the magnesium absorption line in red.

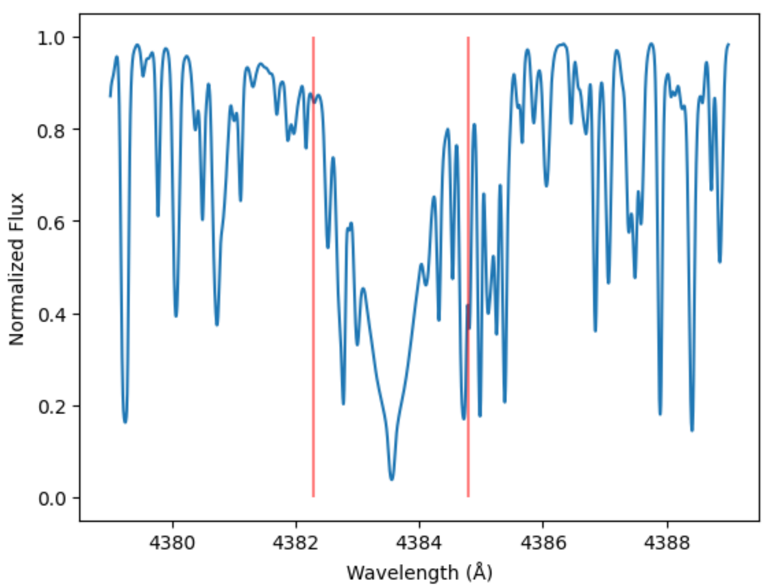

Figure 3: Solar spectrum in blue, with the bounds around the iron absorption line in red.

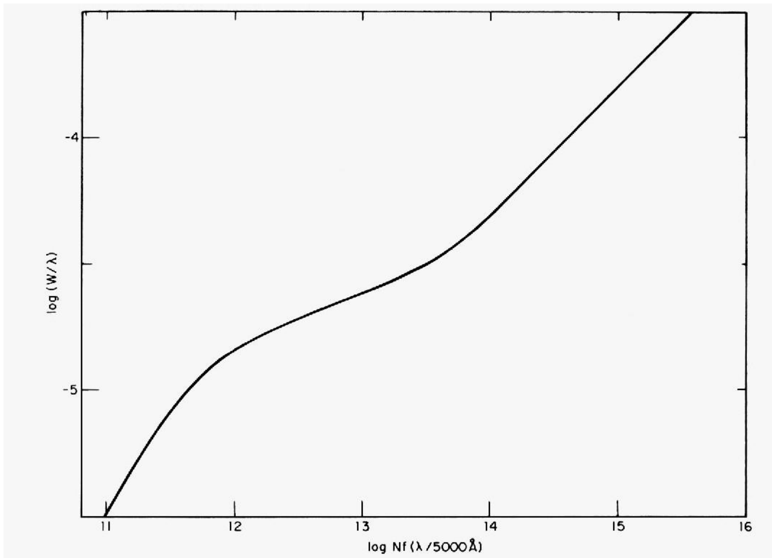

Figure 4: A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971)

absorbing state. The results are reported in Table 2.

Element Symbol	N_1
Na	109×10^{13} atoms
Mg	2.49×10^{13} atoms
Fe	1470×10^{13} atoms

Table 2: The number of sodium, magnesium, and iron atoms in the ground state in the Sun.

2.2 Boltzmann Equation

To ensure we count all of the atoms of each element in an excited state, we use the Boltzmann equation to calculate the expected ratio between excited and ground state atoms for each element:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{E_2 - E_1}{kT}\right).$$

We defined a function to perform this calculation, using known values for each element's g (number of degenerate states) and E (energy of each state), and use the known values of $T = T_\odot \approx 5772$ K and the Boltzmann constant k. The results for each element are displayed in Table 3.

Element Symbol	$\frac{N_2}{N_1}$
Na	0.044
Mg	0.014
Fe	0.004

Table 3: The ratio between sodium, magnesium, and iron atoms in an excited state vs the ground state, as calculated using the Boltzmann equation.

2.3 Saha Equation

To ensure we account for ionized atoms of each element, we use the Saha equation to calculate the ratio between ionized atoms and neutral atoms for each element:

$$\frac{Na_{II}}{Na_I} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi}{kT}\right).$$

Again, we defined a function to perform this calculation, using known values for each element's Z (partition functions) and χ (ionization energy), along with known values for the mass of the electron (m_e) and the electron pressure ($P_e = n_e kT$). The element-specific known values and the final Saha ratios are displayed in Table 4.

Element Symbol	χ [eV]	Z_I	Z_{II}	$\frac{Na_{II}}{Na_I}$
Na	5.1	2.4	1	2490
Mg	7.6	421	252	21.4
Fe	7.9	1	.78	16.6

Table 4: The ratio between ionized sodium, magnesium, and iron atoms vs the neutral atoms, as calculated using the Saha equation. Also included are the partition functions, and ionization energy for each element.

2.4 Column Density

To finally calculate the total number of atoms of each element in the Sun, we connect the values calculated in Sections 2.1, 2.2, and 2.3 using the following relation:

$$N_{\text{total}} = N_1 \times (1 + \frac{N_2}{N_1}) \times (1 + \frac{Na_{II}}{Na_I}).$$

The results of this simple calculation are displayed in Table 5.

Element Symbol	N_{total}
Na	28,268 $\times 10^{14}$ atoms
Mg	5.6 $\times 10^{14}$ atoms
Fe	2609 $\times 10^{14}$ atoms

Table 5: The total number of sodium, magnesium, and iron atoms in the Sun, taking into account ionization and excitation.

3 Results

Using the column density of these elements and the known column density of hydrogen (6.6×10^{23}), we can calculate the solar abundance of each of these elements. In the astronomer's way, this is calculated by the following equation:

$$12 + log_{10}(N_{element}/N_H).$$

The solar abundance of each element is displayed in Table 6. The magne-

Element Symbol	Abundance
Na	6.632
Mg	4.827
Fe	6.913

Table 6: The solar abundance of sodium, magnesium, and iron atoms. Note: magnesium and iron abundances have been adjusted to include lower excitation states.

sium and iron abundances are different from the accepted solar values. The accepted magnesium solar abundance is 7.54, and the accepted iron solar abundance is 7.48.

4 Conclusion

Through analysis of the solar spectrum, we are able to reasonably calculate the relative solar abundances of sodium, magnesium, and iron. We applied the Boltzmann equation to account for excited states, and the Saha equation to account for ionized atoms. The number of ground state atoms was calculated using the curve of growth for the Sun. In the end, our abundances were slightly off from the accepted values. This is likely due to not considering other ionizations or excitations of the elements' atoms, which is a subject deserving of future research.

5 References

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