

× × × ×

# ALGORITMO GENÉTICO

Técnicas de otimização

202106840041 - Gabriel Ferreira Vieira

× × × ×

# 1. QUESTÃO

## • ORIENTAÇÕES

Implementar algoritmo genético (AG), com representação binária, para encontrar o máximo da função Schaffer's , conhecida como F6.

$$f_6(x, y) = 0.5 - \frac{(\sin \sqrt{x^2 + y^2})^2 - 0.5}{(1 + 0.001 \times (x^2 + y^2))^2}$$

Parâmetros definidos:

- Domínio das duas variáveis [-100, +100].
- Precisão de 5 casas decimais para as duas variáveis.
- Representação Binária para as duas variáveis da F6.
- Número de bits necessários para representar cada uma das variáveis: 25 bits.
- Taxa de Cruzamento igual a 0,85.
- Taxa de Mutação igual a 0,01.
- Tamanho da população igual a 200 indivíduos.
- Método de Seleção por meio da Roleta proporcional a medida de aptidão do indivíduo.
- Método de Cruzamento binário com um ponto de corte.
- Parar o AG em 500 gerações.

## • IMPLEMENTAÇÃO

- Python
- Representação dos bits usando strings
- Conversão: string → bit → decimal → real
- Precisão de 5 casas decimais
- Primeira geração gerada aleatoriamente a partir de bit string.
- VA uniforme [0, 1] → Cruzamento ou repete os progenitores ou realiza cruzamento por ponto de corte com posição aleatória [1, 24] uniforme.
- Mesmo número de indivíduos ao longo das gerações.
- Mutação implementada logo após cada cruzamento, em cada bit → VA uniforme [0, 1] (limiar de taxa de mutação) → bit flip ou não.
- Taxa de mutação fixa ao longo das gerações
- Roleta:

$$Fatia = \frac{z_i}{\sum_{n=1}^{Populacao} z_n}$$

$$Real = \frac{int \times (max - min)}{2^k - 1} + min$$

$$Real = \frac{int \times (100 - (-100))}{2^{25} - 1} + (-100)$$

- Informações da população da geração armazenadas em dataframes pandas, de 10 em 10 salvas em .csv
- Foram plotados os gráficos das gerações no espaço de busca das gerações de 10 em 10
- Foi plotado o gráfico do melhor individuo ao longo das épocas

# 1. QUESTÃO

## • GERAÇÃO 1

```
1_csv > geracao1.csv > data
1 xbin,ybin,xreal,yreal,z,particao
2 11010110111110101011000,11000101100111101110110,67.96470188989346,54.39295036771745,0.4982704678890573,0.005011551125215734
3 1001100001011101110011100,011110111100011100101000,19.036272735484616,-1.6492337479959076,0.7333717139361425,0.007376174337099984
4 11011110000110110010101,0000101110000000110010,74.23947972772956,-91.5036079735639,0.49774539442983146,0.005006269992467585
5 001111001110001001110110,10001111101001001010001,-52.433882726248584,12.360582124012183,0.5196825449120236,0.00522691150800596
6 110001011000011000111000,00110101110111011001,54.30636865813639,-57.91649096955332,0.49870348912485896,0.005015906406536575
7 0111110010111010101101,110101010010101001011,-1.2776524209276516,70.76918991712301,0.4864054099390208,0.004892213640150304
8 01010011110101010010011,11001000110000111010110,-34.43973465084238,56.84713592669772,0.5094243598425409,0.005123735778676796
9 100110000010111000011110,10101100111001000001011,18.893614974427663,35.07392212968833,0.4686867122360296,0.004714000871918263
10 10100011011111000011010,11101111111001111001100,26.36429448021336,87.48138509635285,0.5049564075709708,0.005078797592135541
11 00100000110011110100000,0100110010000110011110001,-74.36637802023822,-40.21462619944293,0.5063841815392495,0.005093157990149083
12 100110111110001100000111,11001110001111010000111,21.830794865810716,61.814163977329855,0.5119563368311355,0.005149202132683225
13 11100000000110101110010,10000010000111111100100,87.52052150727872,0.829914236960235,0.5042459844565963,0.0050716522323607455
14 010111011101010000001,00111111100100100011000,-26.628104645851394,-50.083778801076974,0.526429079902278,0.005294767436062377
15 01011001101010011110,100100100000101001010110,-29.94805365646046,14.07827478880509,0.3886066537864674,0.003908564200685316
16 10111010001001100001011,010000011010110010110111,47.71430336577603,-49.345831553513754,0.5089534702080512,0.005118999621048235
17 011100100110100110110100,000010100010101101101011,-10.614893156733899,-92.05487346812706,0.49456127458676963,0.004974244455313192
18 11101010111010110010011,10000001101110011011111,83.38253448553485,0.6740540466920777,0.4923733270407169,0.00495223831267988
19 110001101100001010010001,10010011010000110100001,55.27735815278763,20.07084846707727,0.4951761216853963,0.00498042852173401
20 10111010010011101011011,100100110000100011100000,45.567868517871744,14.870837774003675,0.4978202270996779,0.005007022651464937
21 011111001010010101111010,0001110111010111101010011,-1.309999862611292,-76.68560077803137,0.49097051650446594,0.004938128994198299
22 10110001001110101000001000,000011010010001101001011,38.45830376321082,90.72618066611226,0.5030060610101174,0.005060139520422046
```

# 1. QUESTÃO

## • GERAÇÃO 10

```
1_csv > geracao10.csv > data
 1 xbin,ybin,xreal,yreal,z,particao
 2 1010110011010100010000000,000110111100100001111000,35.022739619694335,-78.20973331361215,0.4987851498614103,0.0044320393839570715
 3 1011100111010001010000110,11100011110101001111001,45.16987041145177,78.05935079036209,0.4984079119531914,0.004428687373042527
 4 000111101000100101011011,110010000010011001111011,-75.57166145955507,56.30872119393115,0.5051201516661091,0.004488330108537916
 5 000111000010110001000110,0011010101110010011011,-77.99030476779654,-58.24225420481724,0.5045329098849354,0.004483112072871076
 6 01111000000100110001100,0111000010010001001,-3.1104297372826863,-12.051144601438779,0.8640706705864302,0.007677845348092912
 7 110111011110000110010011,1001011100110010111011,73.39107910964128,18.125507775709266,0.5102314089213047,0.004533747044994139
 8 0111110010001001100001110,10010011110110010110001,-2.705296954670459,15.56607233184792,0.8148541076092097,0.007240523295664822
 9 11111110100110100001110,011000011100001000100101,99.72695707461111,-23.62629543621229,0.49729089140131255,0.004418761898958161
10 0111010001010011000100111,100000101000010010011101,-9.121468935056598,1.9672245373494803,0.9157210645579525,0.00813679361538253
11 101111001001111100010111,0111010100011111011001,47.36189685350348,-8.374253164954581,0.4833384092443263,0.004294784770848185
12 1000100000100101010000101,1111011100000011110001011,6.363710950723615,92.98026540816622,0.4973093818833826,0.004418926199248146
13 011110100111100010010,100010111101011001110101,-3.3949703989914184,9.31161371802132,0.7333710029201534,0.006516491457087956
14 101111010001111001010000,000000110100000111101010,47.751335732678655,-97.458016796649,0.4970617537525054,0.004416725857820723
15 110011001001000011011,0001010000110010111001111,59.81767057829114,-84.21965194403089,0.5027031216661121,0.004466853181738534
16 01111100111111101010010,10010111100011000011110,-2.3447842104668695,18.573371129434435,0.7651174989456746,0.006798580289778426
17 100111100010110101000100,100100101110000100010111,24.287823566431513,14.749429069442428,0.6469877540636428,0.005748918562921806
18 0001111100010010101011010,10001000010011001110000,-75.72425531519221,6.367305110910692,0.5040840738268987,0.0044791238645478905
19 0011101110000110011110011,000100011100000000100111,-53.49586467432572,-86.13257962860405,0.49939672892738035,0.004437473672663413
20 11110111010011110111000,10001111010000100101010,96.60559286491849,12.210336691449186,0.5045488706578594,0.004483253895004749
21 0001110010011111111001,10010011011001001001011,-77.66117387000244,15.155908917066725,0.5036754988396713,0.004475493402744457
22 110111010010011101001011,0111101101010111010111,72.77724661759277,-1.8221408671778647,0.5058617095452227,0.004494919345861097
```

# 1. QUESTÃO

## • GERAÇÃO 50

```
1_csv > geracao50.csv > data
 1 xbin,ybin,xreal,yreal,z,particao
 2 011110001000100110011000,1000011110000010100011010,-2.915045705886058,5.867162521694979,0.8951145380675456,0.005771905797139757
 3 0111010001000001101011010,100001110010001111101111,-9.174570714669542,5.578515099838825,0.15042161468861376,0.0009699534002773804
 4 011110011001010010010001,100001100110111011110001,-2.4256617553729285,5.026158840243781,0.5777802854327578,0.003725661060272877
 5 011110111001011010010101,10000111010010101010001,-1.7233521259829985,5.696591904657836,0.8675035121622777,0.005593863509019121
 6 01111000110101111011010,100001101101110010011001,-2.7956337569842873,5.360719721338739,0.9139299240604688,0.005893231762554677
 7 011110001000101110011110,1000001101000110111110011,-2.9119581851946776,2.5603503751859193,0.5479710574414516,0.0035334442561276423
 8 011110001011001110101100,1000001110101010111001010,-2.850839580620516,5.990281879612269,0.8503403836171625,0.005483191682192943
 9 011110111001001111100011,1000001110010001010001110,-1.727464846595069,5.574217604822451,0.7925484180179794,0.005110535706802166
10 01111100001011010111011,100000101010101011110011,-0.746563099222714,2.084276142247802,0.36102977028893707,0.0023280035520029775
11 0101110011010001101011001,100101111101100111110101,-27.460710032603444,18.691954573749143,0.3989383129882892,0.002572446612708801
12 01111010000001101100011,1000001100001010111000110,-2.3385793667688255,2.376976680069461,0.9531092534218094,0.0061458691498961675
13 0111100110011010101101,100000111001011110010001,-2.4983078985901983,5.931958732961377,0.939481837079472,0.0060579964140157015
14 01111001100001111100011,1000001100001100111000110,-2.435472680195346,2.38308019587636,0.921142002500827,0.005939736914229678
15 0111100110011010011000,1000001110101011110001,-2.4984807520652055,5.9905143377338135,0.9213211661047591,0.005940892202631546
16 111110001100111000010001,100000110100101111001101,97.18943825928682,2.6547283725359563,0.5043331361105527,0.0032520568354197717
17 011110010000011010101010,1000001101001000110101100,-5.458578630047398,2.5660306980022938,0.9078308157767964,0.005853903300148165
18 01111000100010111011100,1000001101011100011000111,-2.911588636386057,5.750701002797513,0.936753029282892,0.006040400429512574
19 01111001000101110011010,1000001101011100100110011,-2.521357015411766,5.751344732980272,0.9627696648719153,0.0062081617197074
20 01111010000111010010001,1000001100100010110101,-2.3215205169177153,5.579695271840549,0.9128784973543718,0.005886451919705413
21 01111001100001101000011,10000010100000011001011,-2.4314672479470687,2.0519942656753756,0.988459449087837,0.006373815396568985
22 01111001100001101000011,10000010100000011001011,-2.4314672479470687,2.0519942656753756,0.988459449087837,0.006373815396568985
```

# 1. QUESTÃO

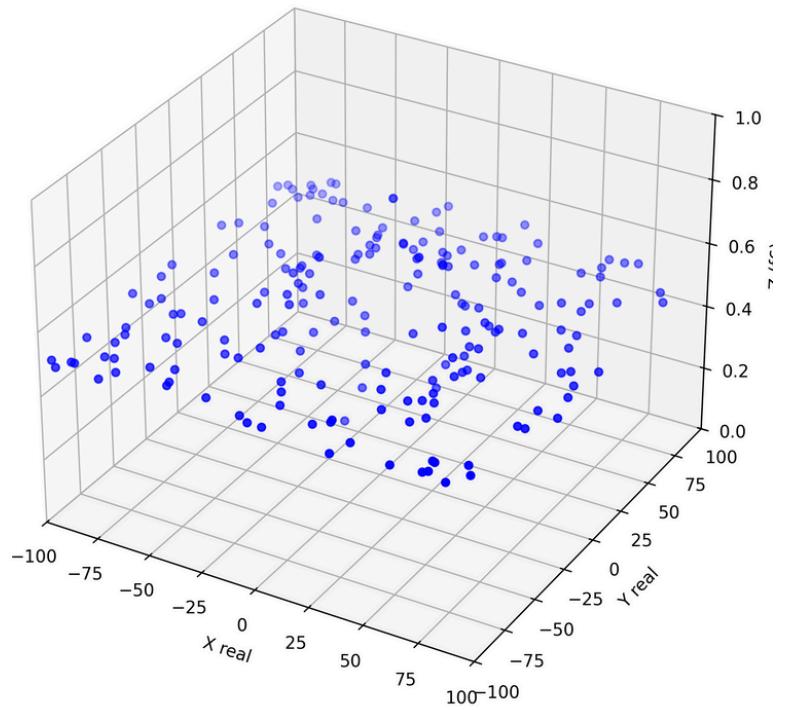
## • GERAÇÃO 500

```
1_csv > geracao500.csv > data
1 xbin,ybin,xreal,yreal,z,particao
2 011111010001111110010000,1000001010010110010011101,-2.2467584087478656,2.021202505266743,0.9770394662488562,0.006231103032183213
3 011101111011011100010111,1100011110100110011110010,-6.47112563345216,55.9767888777491,0.5264378084053817,0.0033573753543494583
4 011110110001100101010000,1000011110000100110011010,-1.9144982670098045,5.8740289769777405,0.9537109288113732,0.006082324476777268
5 0101110100100011100110101,1000011100101000100111001,-27.23509452447577,5.59268908478883,0.5935644291327296,0.003785477702722614
6 01111001110110000110111,1000001011011000111101100,-2.4044544221298167,2.2246152825538843,0.9719361965885671,0.0061985567531916775
7 01111010001010000110101,100011110001100011111001,-1.9222885943141108,12.147483591660375,0.8244450810289252,0.005257927055895818
8 0111100110011111101011,1000011110001111110000,-2.490356638740195,6.078913989034717,0.8863118414432214,0.005652484462966502
9 011110011110010110100111,100000101110101101101100,-2.3839504237160156,2.2810728037676995,0.9650989204748,0.006154951787992128
10 011110011100111100011111,100001111000110111110000,-2.4183303838470636,5.901912030634648,0.953370153452119,0.0060801511701221184
11 01111011011011111101011,100000111000000000111000,-1.7823488051399181,2.734711847743739,0.9748537533298027,0.006217163572347793
12 01111010001111100010011,10010111001011010010001,-2.2475034668297553,18.42890734758697,0.7330730337869065,0.00467519866027386
13 0111100111010100010111,100000101010011001110000,-2.4092227938539565,2.0705253502883068,0.988851697120787,0.006306435943642277
14 01111011101100000110,10000111000011110000110,-1.6799539828286782,5.492440029753453,0.7212095702833726,0.004599539010932493
15 01110100001010101111011,1000001110100100011101,-8.560970680742585,2.8947682051291537,0.8052231502583496,0.005135338526725347
16 01111001111110110010110,1000011100100001001000,-2.347430656773767,5.909064588220858,0.956701328475275,0.006101395854195335
17 011110011101100011111,1000001101000011101001,-2.3969680785229173,2.857837166125691,0.6867281822137355,0.004379632764381879
18 01111010001111100000110,1000001010000001110100,-2.2475809528702797,2.0521909610089892,0.9814547126392108,0.006259261418943576
19 010111110010100010111,100000111000011001111000,-25.163129721973235,2.754119120660988,0.6737350115456311,0.004296768368475196
20 011110111010110100110,100001110000111110011100,-1.6881555821941987,5.516985223203463,0.7421372623303342,0.004733006091715906
21 0111101011001101011011,10000111001000111101001,-1.0151416365844455,5.914172706430335,0.8934382004865998,0.005697933064560976
22 011110111010101111011,100000111010100011101,-1.6762048505605662,2.8947682051291537,0.9490681805673267,0.0060527151890662914
23 01111011110100010111111,1000001101001001001001,-1.633456398053653,2.6508898332980237,0.9896832314424819,0.00631173908257603
24 0111101110111101111111,100011110001101110100011,-1.6616851586605605,12.151548628555204,0.8110663219163193,0.005172603556330885
25 0111100110010100110111,100000101100000011111101,-2.506688311895374,2.1499485418185174,0.9641871495942412,0.006149136937625521
26 01111001011001101011011,100001110010001111101001,-2.577641683150574,5.914172706430335,0.9349785913449649,0.005962858345859394
```

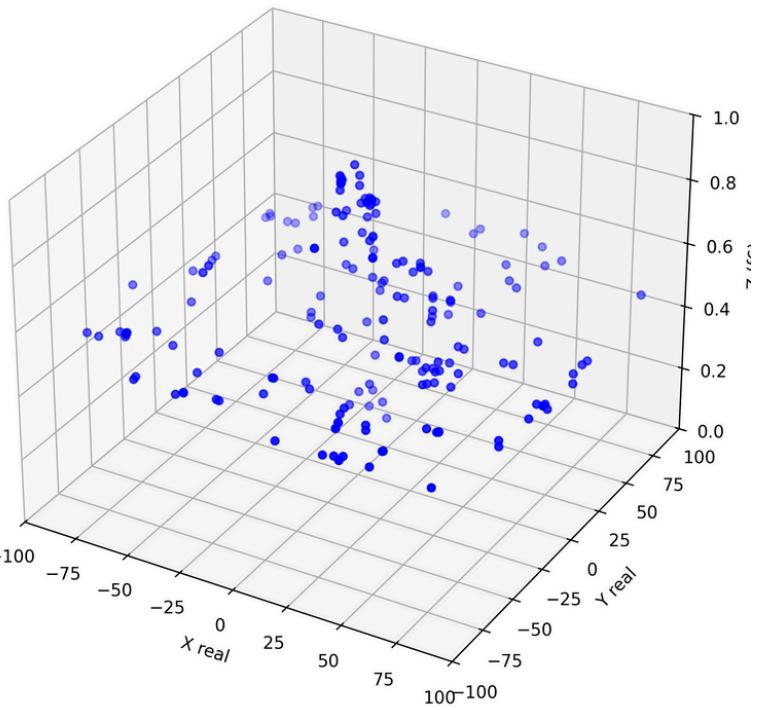
# 1. QUESTÃO

- GERAÇÕES NO ESPAÇO DE BUSCA

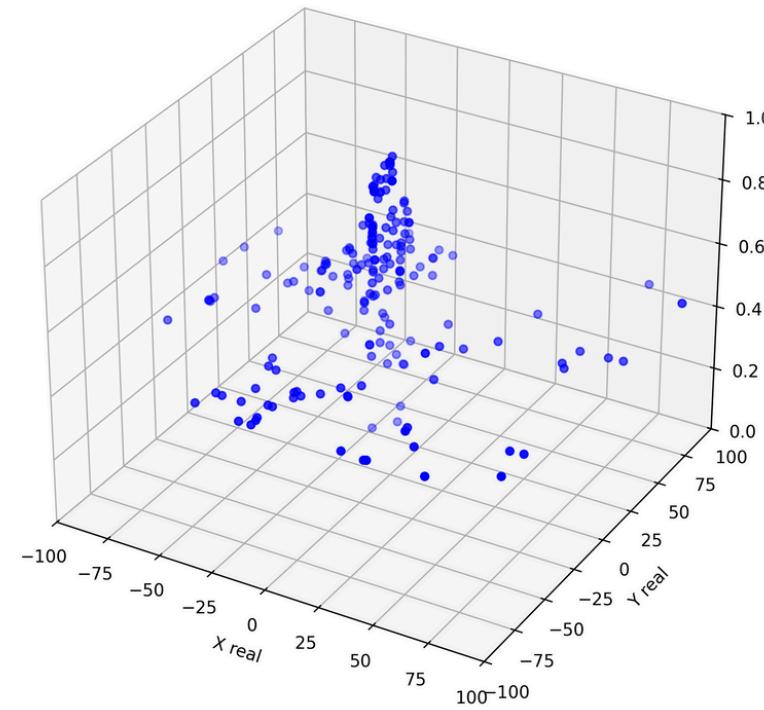
Geração 1



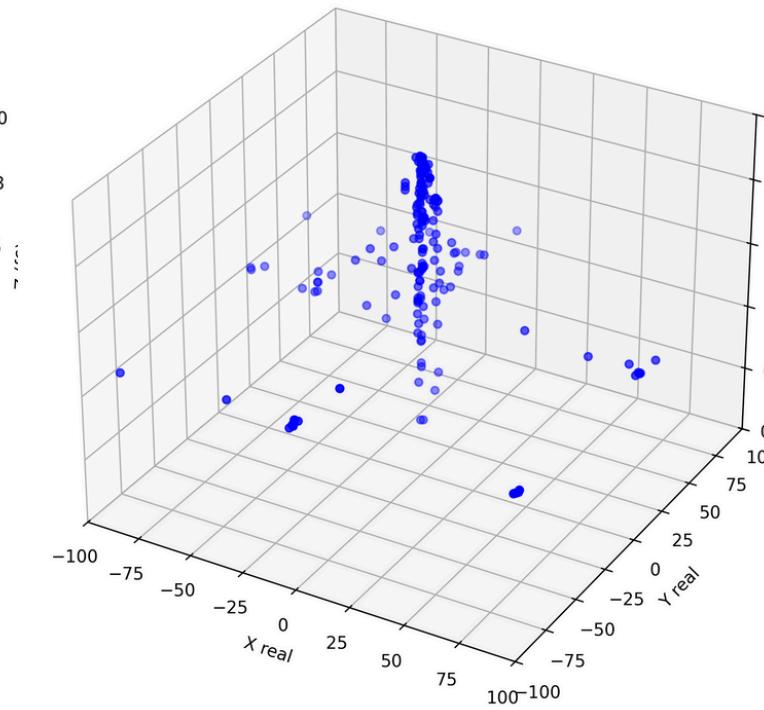
Geração 10



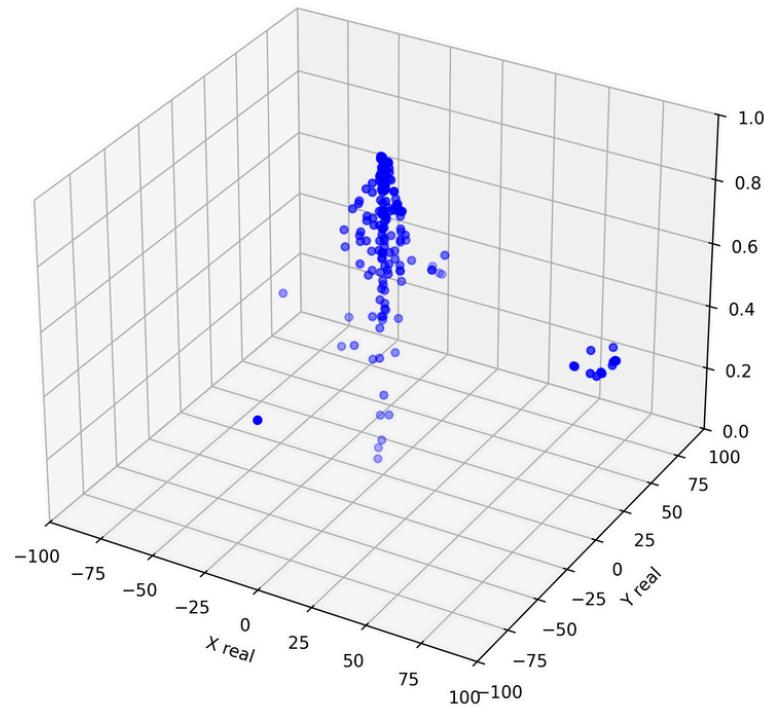
Geração 20



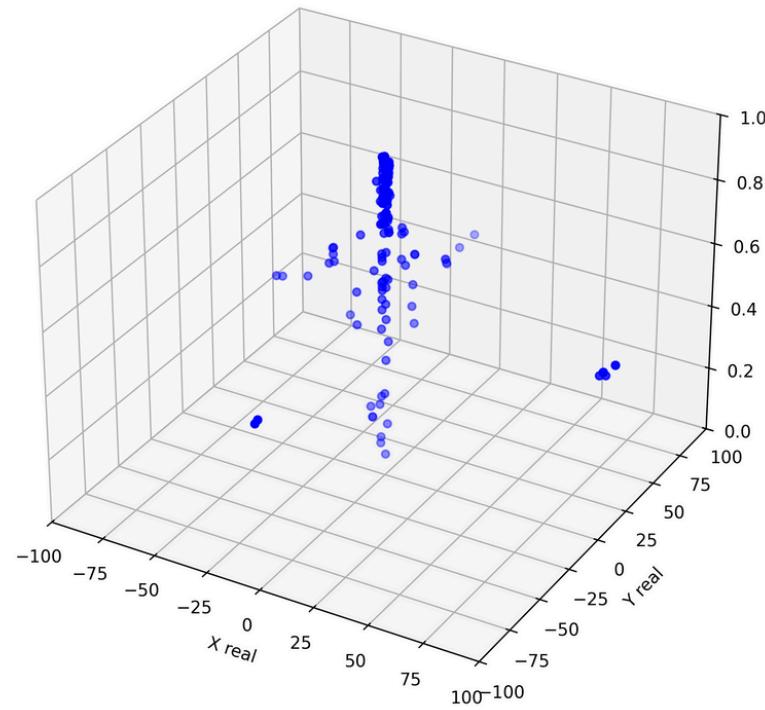
Geração 30



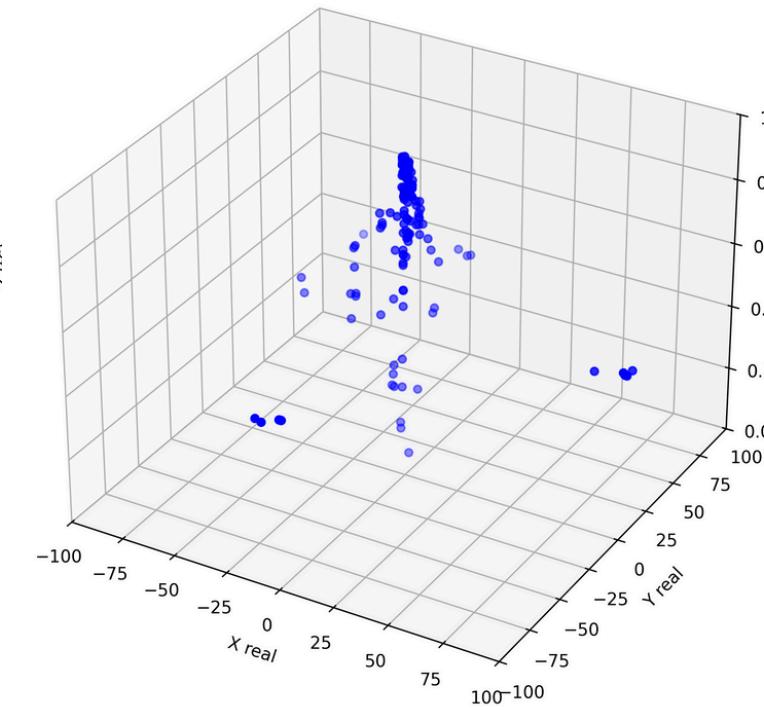
Geração 40



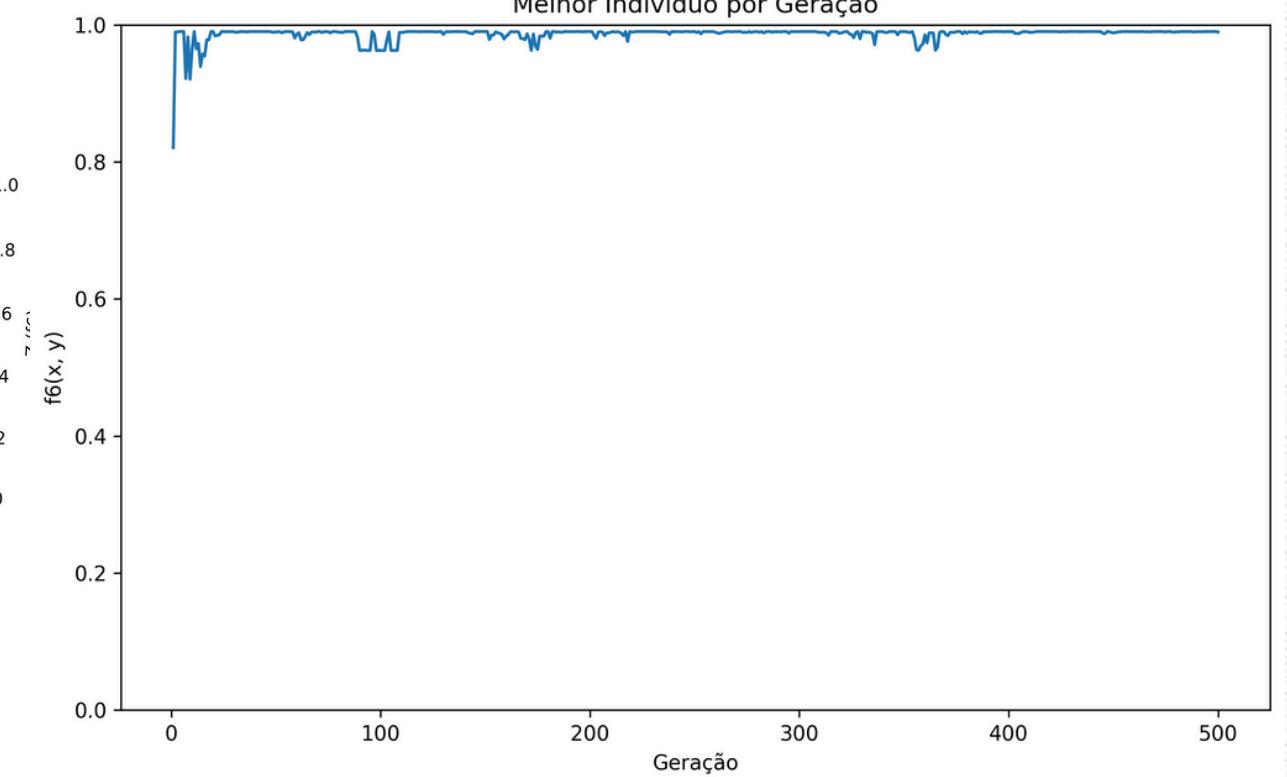
Geração 50



Geração 500



Melhor Indivíduo por Geração



## 2. QUESTÃO

### • ORIENTAÇÕES

Utilizar o Algoritmo Genético (AG), com representação real, para encontrar o máximo da função Schaffer's , conhecida como F6, a seguir:

$$f_6(x, y) = 0.5 - \frac{(\sin \sqrt{x^2 + y^2})^2 - 0.5}{(1 + 0.001 \times (x^2 + y^2))^2}$$

Parâmetros definidos:

- Domínio das duas variáveis [-100, +100].
- Taxa de Cruzamento igual a 0,85.
- Taxa de Mutação igual a 0,01.
- Tamanho da população igual a 200 indivíduos.
- Método de Seleção por meio da Roleta proporcional a medida de aptidão do indivíduo.
- Método de Cruzamento Aritmético com  $a = 0,35$ .
- Método de Mutação Randômica Não Uniforme, com  $rand =$  normal de média zero e variância a ser definida pelo aluno, optou-se por utilizar variância = 1.
- Parar o AG em 500 gerações.

### • IMPLEMENTAÇÃO

- Python
- VA uniforme [0, 1] → Cruzamento ou repete os progenitores ou realiza cruzamento aritmético, dando origem a 2 indivíduos.
- Mesmo número de indivíduos ao longo das gerações.
- Mutação implementada logo após cada cruzamento, em cada individuo → VA uniforme [0, 1] (limiar taxa de mutação) → Soma VA distribuição normal média 0 e variância 1 ou não.
- Taxa de mutação fixa.
- Se o individuo ultrapassar 100 em uma variável, é ajustado para 100, se for menor que -100 é ajustado para -100.
- Roleta:

$$Fatia = \frac{z_i}{\sum_{n=1}^{Populacao} z_n}$$

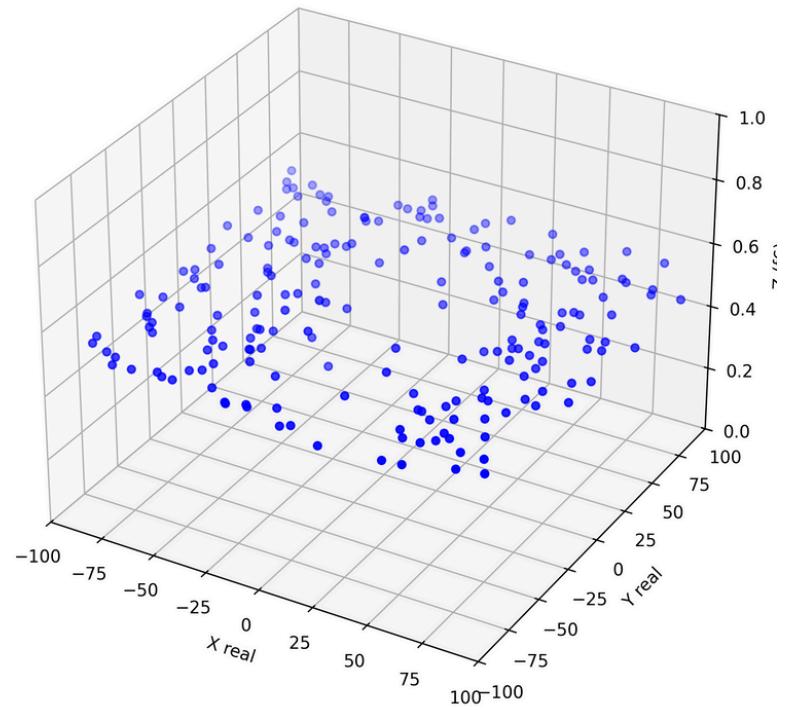
- Informações da população da geração armazenadas em dataframes pandas
- Foram plotados os gráficos das gerações no espaço de busca das gerações de 10 em 10
- Foi plotado o gráfico do melhor individuo ao longo das épocas



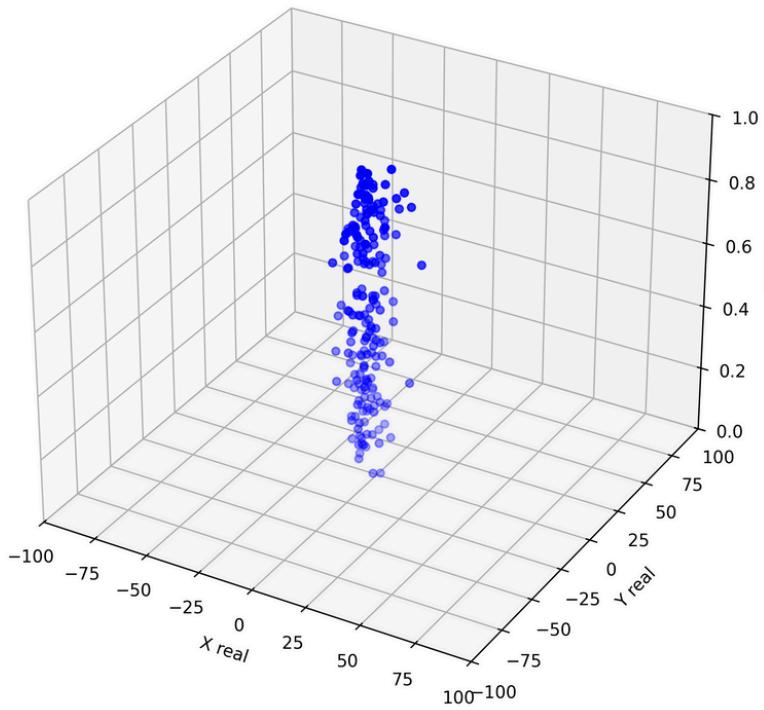
## 2. QUESTÃO

- GERAÇÕES NO ESPAÇO DE BUSCA

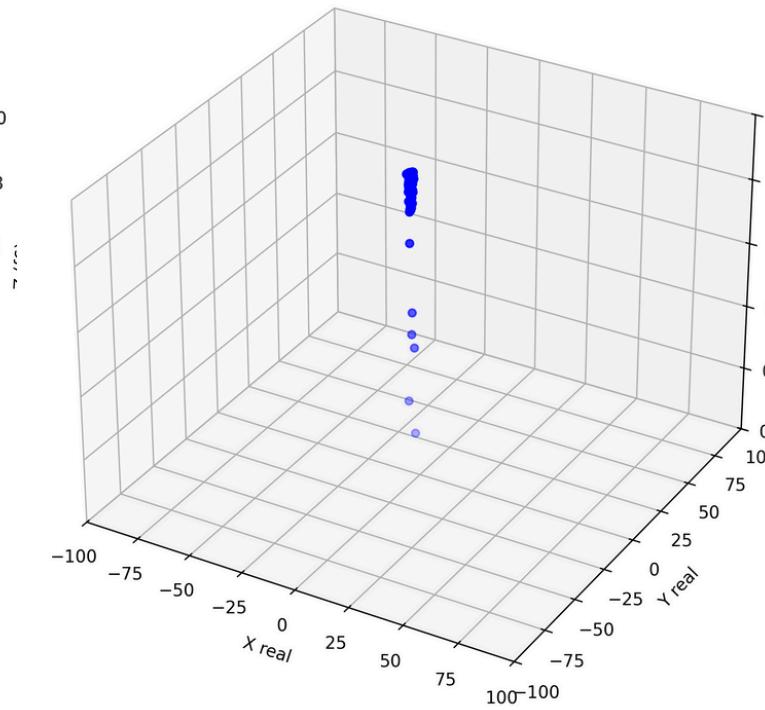
Geração 1



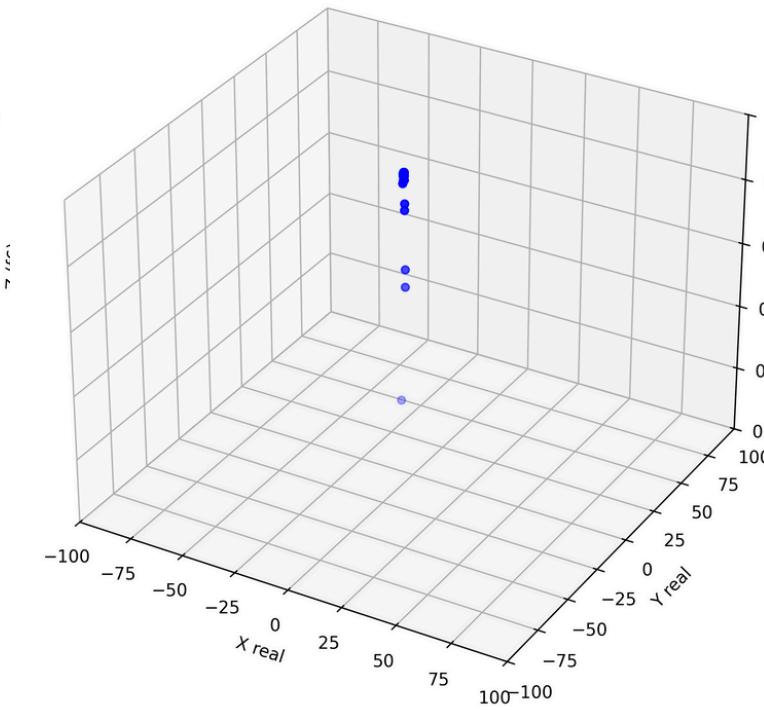
Geração 10



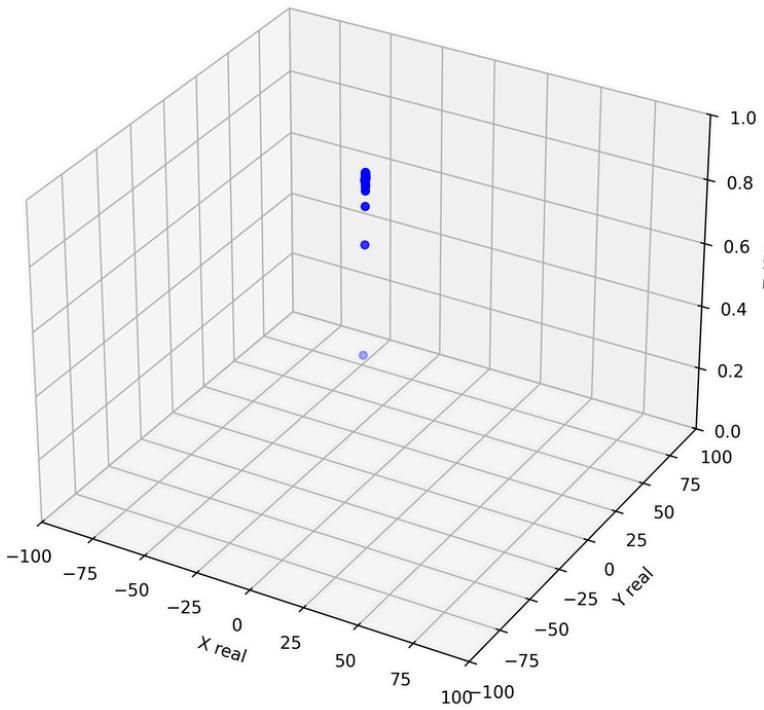
Geração 20



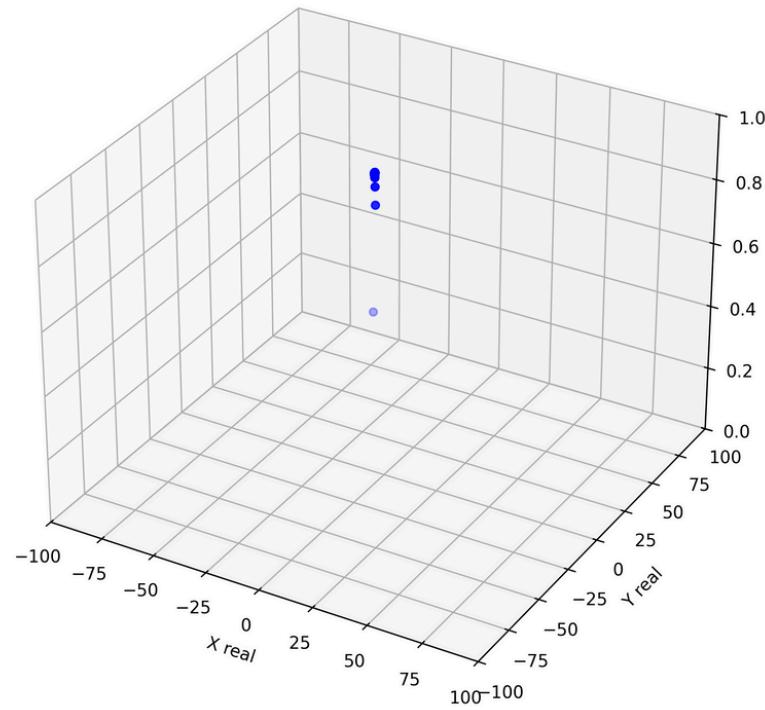
Geração 30



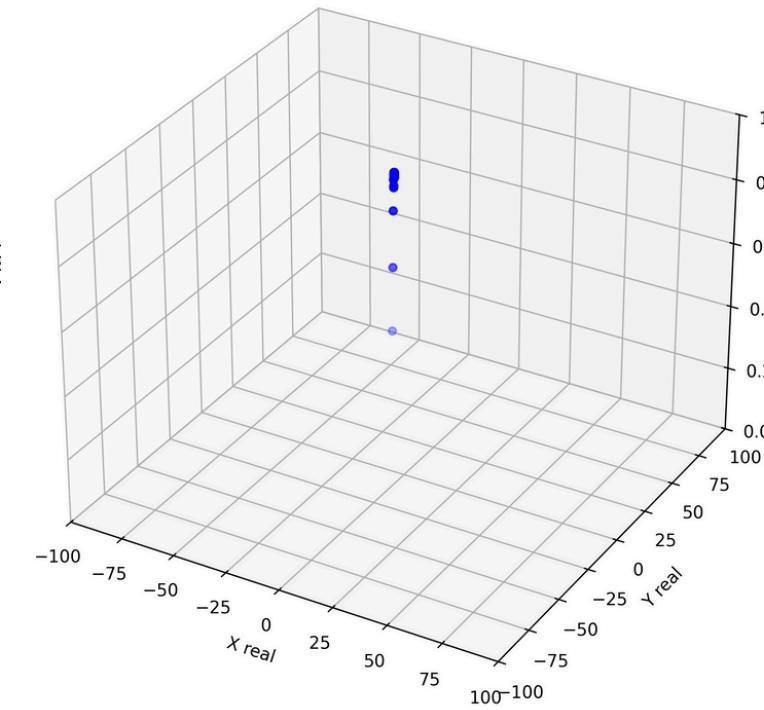
Geração 40



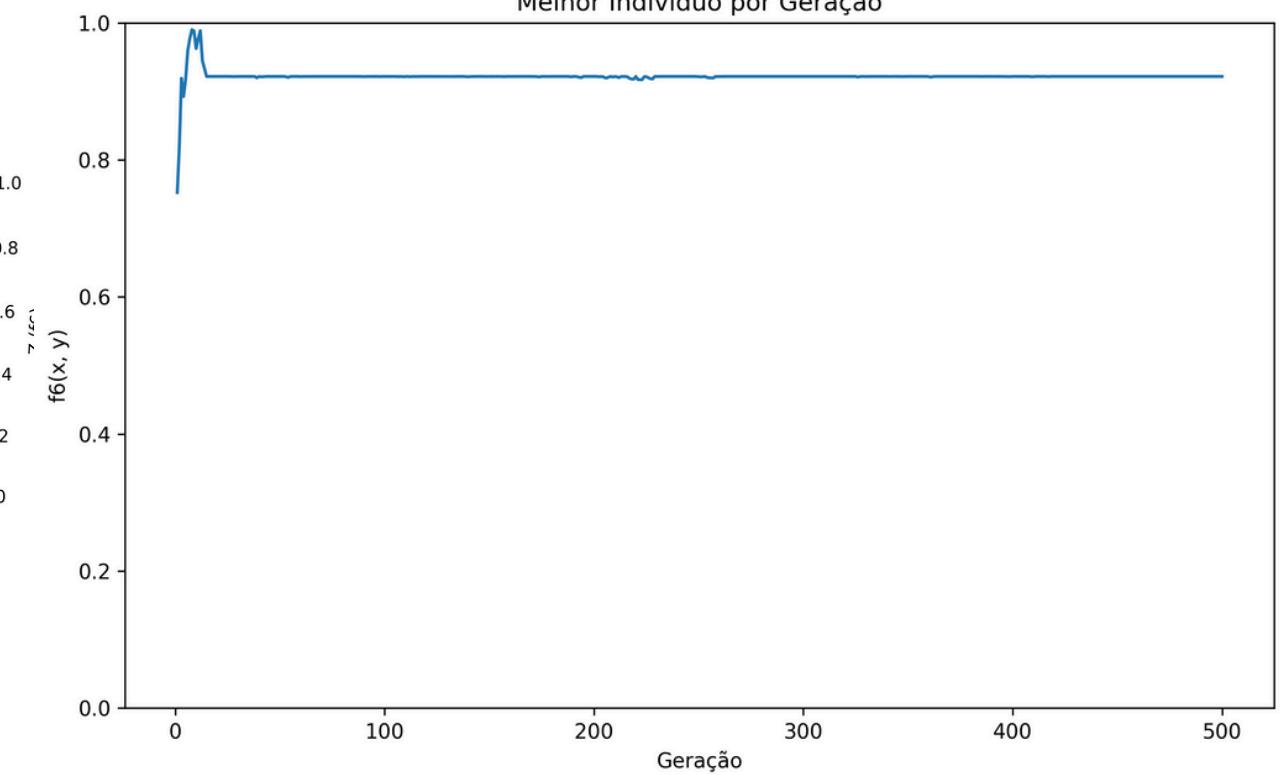
Geração 50



Geração 500



Melhor Indivíduo por Geração



Q1

### 3. QUESTÃO: COMPARAÇÃO DOS RESULTADOS

Mudanças no cruzamento e mutação.

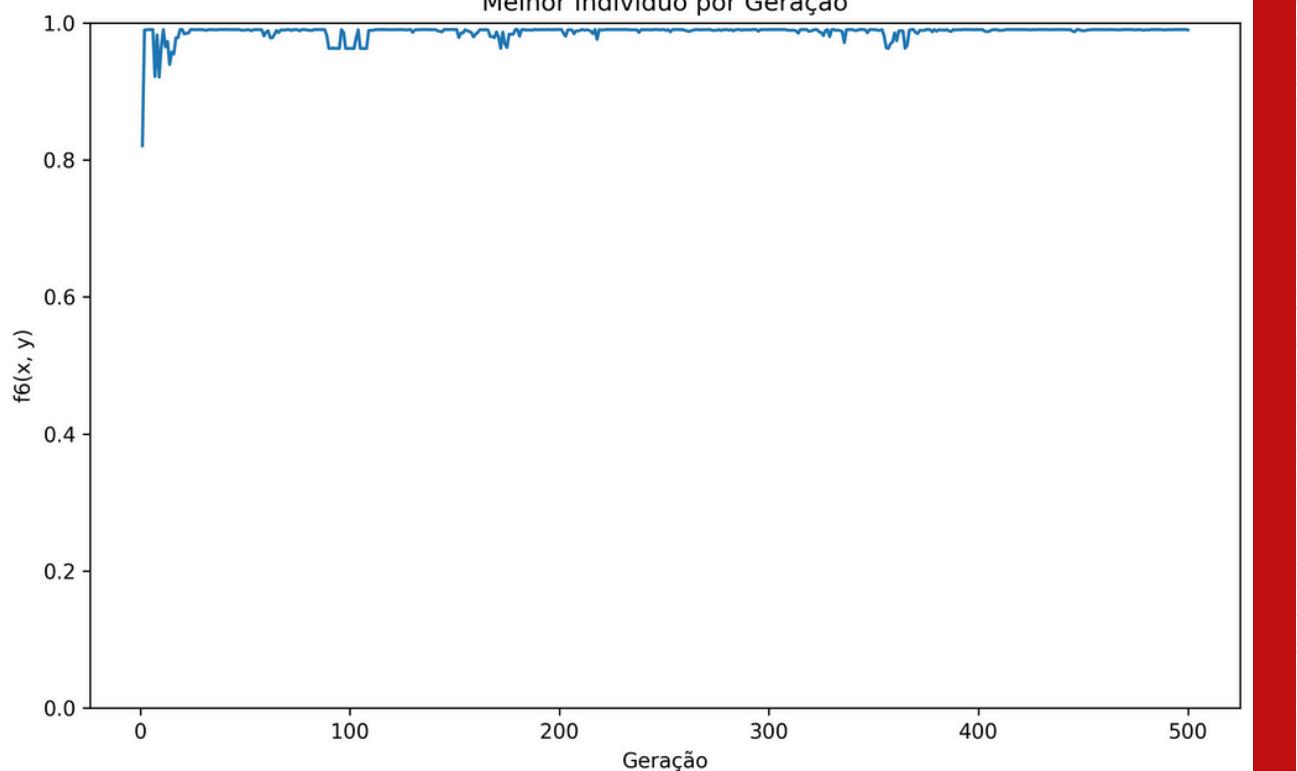
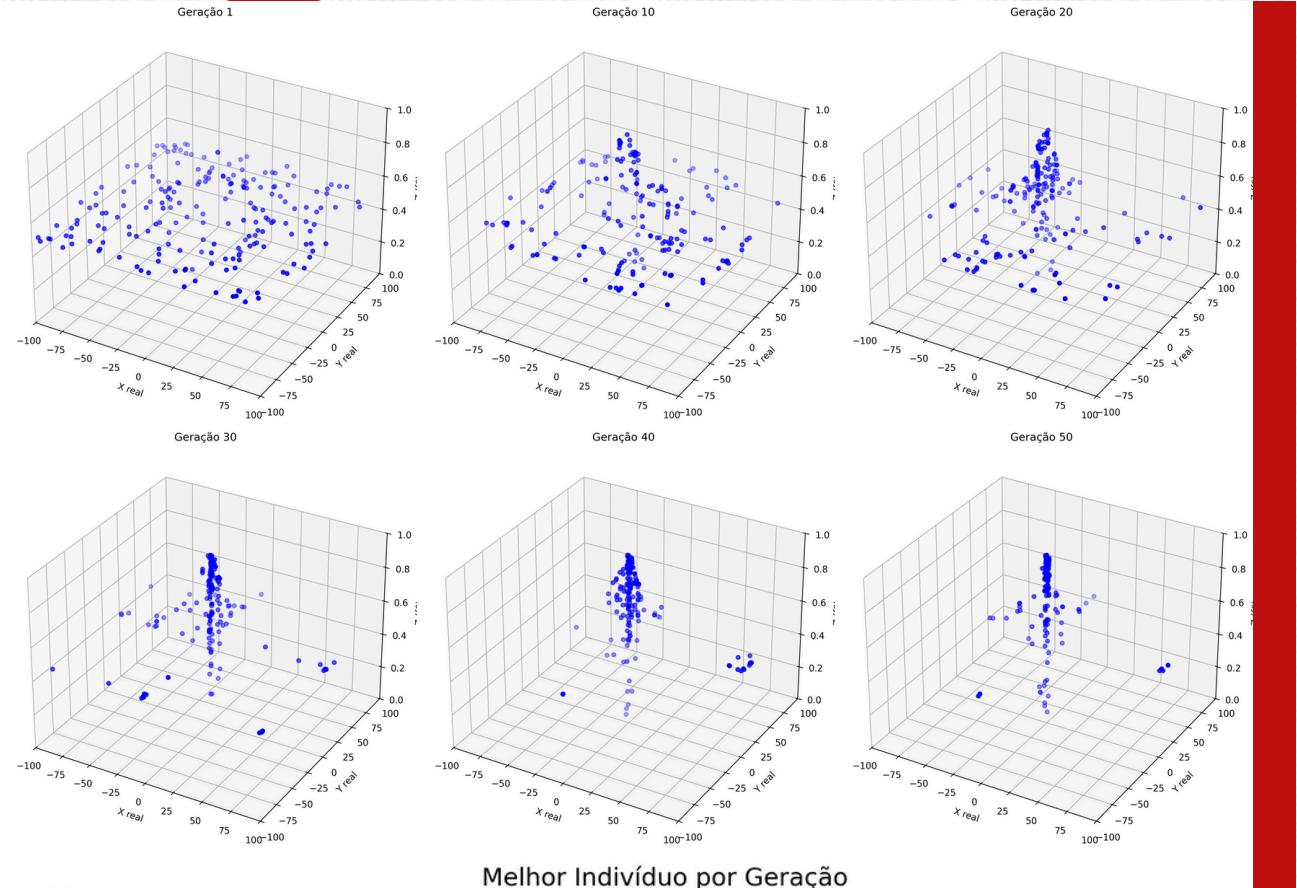
Convergência mais lenta no 1 que no 2:

- O cruzamento por ponto de corte convergem mais lento que cruzamento aritmético.
- Mutação por bit flip da primeira questão tem mais chance de dispersar a população que a normal implementada. (Q1 sempre apresentou alguns indivíduos mais dispersos no espaço de busca)
- Q1 tem taxa de mutação avaliada bit a bit, enquanto Q2 é avaliado por individuo. Probabilidade de mutação de individuo em Q1 é maior que em Q2.

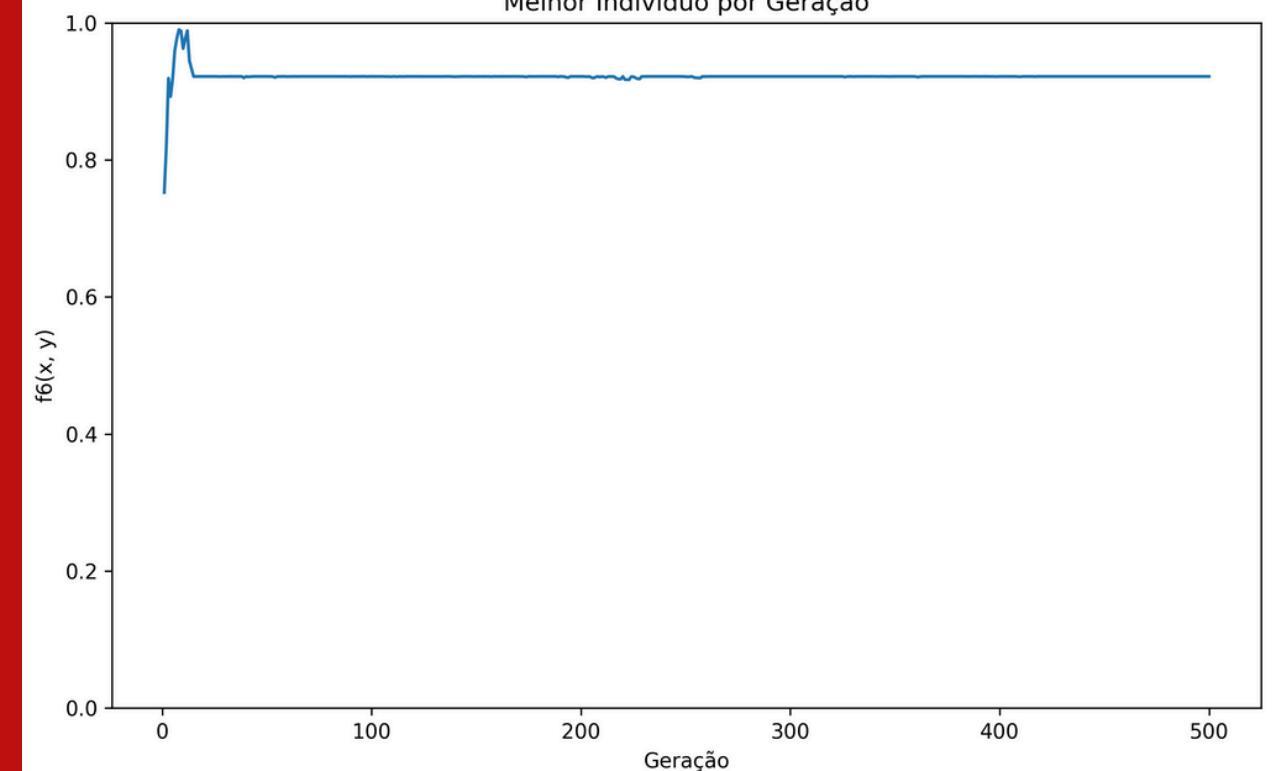
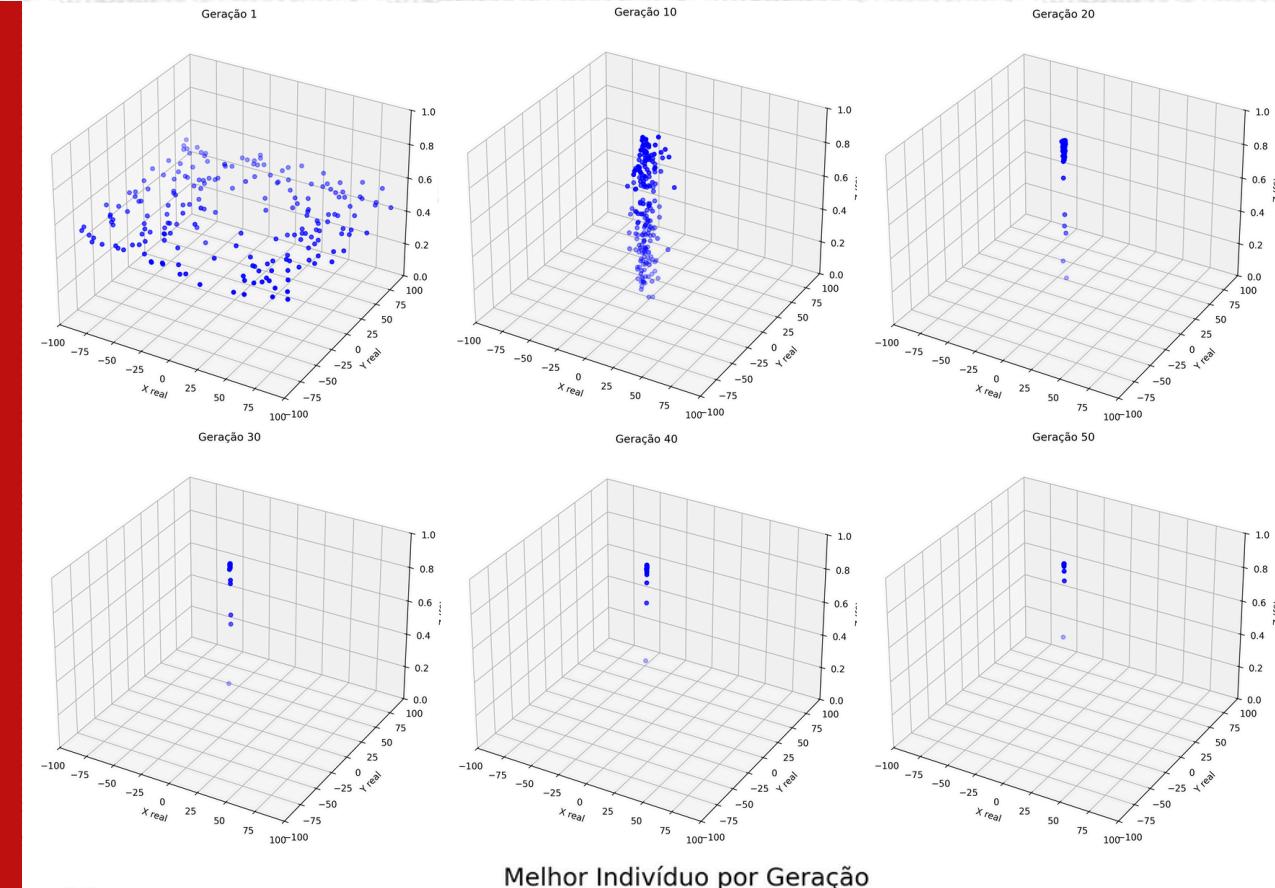
$$P_{mutacao} = 1 - (1 - Taxa_{mutacao})^{N_{bits}}$$

$$P_{mutacao} = 1 - (1 - 0.01)^{50} \approx 0.395$$

- A mutação seguindo a distribuição normal, de média 0 e variância 1, teve menor capacidade de dispersão no espaço de busca que a mutação por bit flip, mas permitiu alta dispersão nas proximidades do ponto encontrado de máximo.
- Ponto de convergência.



Q2



# REFERÊNCIAS

- [1] <https://github.com/FerrariusF/Algoritimo-Genetico>