Notes about GRAVITY+ metrology demodulation

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1 The metrology table

The metrology data is stored as a FITS Table in the `METROLOGY` HDU (10^{th}) as in table 1. The voltages `VOLT` are composed of 80 columns:

- 2 directions (x and y) per diodes
- 4 diodes per telescope (one on each spiders)
- 1 fiber coupler diode (labeled `FC`) per telescope
- 4 telescopes
- 2 sides: `FT` and `SC`

The signals (the table column) are sample at 500 Hz leading to a very large number of rows.

2 The modulation model

When the pupil modulation is on (`ESO INS PMC1 MODULATE` keyword is `true`), the metrology signal is modulated at a frequency of $f = 1\,\text{Hz}$. This modulation does not affect the fiber coupler diode.

Name	Size	Type	TFORM
TIME		Int32	1J
VOLT	(80,)	Float32	80E
POWER_LASER		Float32	1E
LAMBDA_LASER		Float32	1E

Table 1: Metrology table

For each diode in {D1, D2, D3, D4}, we define the complex measurement $\boldsymbol{v} \in \mathbb{C}^N$ with $v_i = x_i + \jmath y_i\,, \quad \forall i \in [\![1, N]\!]$ and N is the number of row in the metrology data. The modulation as a function of time is:

$$\boldsymbol{v} = \exp\left(\jmath\, b \sin\left(\boldsymbol{\omega} + \phi\right)\right)\,. \tag{1}$$

where $\omega = 2\,\pi\,\boldsymbol{t}$ is the modulation pulsation built from the TIME column, the amplitude b and the phase ϕ are the modulation parameters that need to be estimated to demodulate the metrology signal.

3 The overall model

For a diode, the measured modulated voltage data $\boldsymbol{d}$ can be modeled as follows:

$$\boldsymbol{d} = \left(c + \boldsymbol{s} \times \exp\left(\jmath\, b \sin\left(\boldsymbol{\omega} + \phi\right)\right)\right) \times \exp\left(\jmath\, \Phi_{\mathrm{FC}}\right) + \boldsymbol{e}\,, \tag{2}$$

where $\times$ is the element-wise multiplication, $\boldsymbol{s}$ is the sought-after demodulated metrology signal, Φ_{FC} is the phase of the fiber coupler measurement accounting for FDDL movements. $c = x_0 + \jmath\, y_0$ is the center of the pupil and $\boldsymbol{e}$ is a vector representing the measurement errors, which are assumed to be Gaussian centered, independent, and identically distributed.

4 How it is actually coded in the pipeline

4.1 Processing in chunks

The demodulation signal is split in chunks of `MAX_SECONDS_PER_CHUNK = 100` seconds. In each of these chunks, the phase of the fiber coupler Φ_{FC} is assumed to be constant. As a consequence, it is averaged out, and each chunk is demodulated independently.

4.2 Centering

Before processing, the metrology signal is centered (such that $\boldsymbol{c} = \boldsymbol{0}$). If the keyword `gravity.metrology.use-dark-offsets=TRUE` it is centered using the dark file if it exists; otherwise, it use hard-coded diode centers computed by S. Gillessen and set in the array `diode_zeros` in the file `gravi_demodulate.c`.

4.3 Averaging

Within each chunk the metrology data is averaged on a single second such that:

$$\bar{d}_k = \langle d_{k+i\,T}, \forall i \in \mathbb{N}\rangle\,, \tag{3}$$

where T is the number of steps in 1 second given by the hard-coded constant `STEPS_PER_SECOND=500`. The model of the metrology described equation 2 is rewritten as:

$$\bar{\boldsymbol{d}} = a \times \exp\left(\jmath\, b \sin\left(\bar{\boldsymbol{\omega}} + \phi'\right) + \phi\right) + \boldsymbol{e}\,, \tag{4}$$

where $\bar{\omega}_k = k\,\frac{2\pi}{500}$ is the modulation pulsation at the k^{th} step within one second.

The pipeline code use the variables `a, b, pha1, pha2` where `pha1`$=\phi'$, `pha2`$=\phi$ and $s = a\exp(\jmath\,\phi)$.

4.4 Parameters fitting

For each chunk, the parameters `a, b, pha1, pha2` are estimated using the Nelder-Mead Simplex algorithm minimizing the chi-square χ^2 :

$$\chi^2\left(a, b, \phi, \phi'\right) = \left\|\bar{\boldsymbol{d}} - a \times \exp\left(\jmath\, b \sin\left(\bar{\boldsymbol{\omega}} + \phi'\right) + \phi\right)\right\|^2 \tag{5}$$

To ensure a global minimum, this minimization is performed multiple times with different initializations:

- $a_0 = \mathrm{std}(\boldsymbol{d}')$
- $b_0 = 0.25$
- $\phi_0 = \{-\pi/2, 0\}$
- $\phi'_0 = \{-\pi/2, 0\}$

The phases are then unwrapped if needed.

4.5 Computation of the modulation

Once the parameters a, b, , ϕ, ϕ' have been estimated for each chunk, the modulation ψ is computed over one second:

$$\psi_k = -\arctan(\bar{d_k}) - \phi \tag{6}$$

4.6 Demodulation

The metrology signal is finally demodulated, chunk-wise, with each demodulated sample p_i being:

$$p_i = d_i \exp\left(-\jmath\,\psi_{\mathrm{mod}(i,T)}\right) \tag{7}$$

where $\mathrm{mod}(i, T)$ is the modulo of i by T.

5 Estimating the modulation parameters

To estimate the modulation parameters we need to disentangle it from the signal $\boldsymbol{s}$. This is done using the expansion:

$$\boldsymbol{s} = a + \boldsymbol{\delta v} \tag{8}$$

$|a|$ and $\arg(a)$ being the mean amplitude and phase respectively of the metrology signal. $\boldsymbol{\delta v}$ is centered ($\mathrm{E}(\delta v) = 0$) and supposed to be Gaussian independent and identically distributed.

Under this assumption, the modulation parameters (b, ϕ), the center c and the mean of the metrology signal a can be estimated using least square:

$$(c^+, a^+, b^+, \phi^+) = \underset{c,a,b,\phi}{\arg\min} \|\boldsymbol{d} - (c + a \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi))) \times \exp(\jmath\, \Phi_{\text{FC}})\|_2^2 \tag{9}$$

that is equivalent with :

$$(c^+, a^+, b^+, \phi^+) = \underset{c,a,b,\phi}{\arg\min} \|\boldsymbol{r} - c - a \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi))\|_2^2 \tag{10}$$

with $\boldsymbol{r} = \boldsymbol{d} \times \exp(-\jmath\, \Phi_{\text{FC}})$.

5.1 Linear estimates

From the equation 10, we can see that the parameters c and a depends linearly of the $\boldsymbol{d}$ and the modulation phasor $\boldsymbol{m} = \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi))$. For a given values of b and ϕ, the optimal values c^+ and a^+ have a closed-form solution that is:

$$\begin{bmatrix} c^+ \\ a^+ \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \sum_i d_i \\ \boldsymbol{m}^{\mathrm{T}} \boldsymbol{d} \end{bmatrix} \tag{11}$$

where $\boldsymbol{m}^{\mathrm{T}}$ is the conjugate transpose of $\boldsymbol{m}$ and $\mathbf{A}$ is the 2×2 matrix:

$$\mathbf{A} = \begin{bmatrix} N & \sum_i m_i \\ \sum_i m_i^* & N \end{bmatrix}, \tag{12}$$

where m_i^* is the complex conjugate of m_i and N the number of measurements. Its inverse is:

$$\mathbf{A}^{-1} = \frac{1}{N^2 \left(1 - |\sum_i d_i|^2\right)} \begin{bmatrix} N & -\sum_i m_i \\ -\sum_i m_i^* & N \end{bmatrix} \tag{13}$$

For a couple (b, ϕ) the optimal $c^+(b, \phi)$ and $a^+(b, \phi)$ are given by:

$$c^+(b, \phi) = \frac{N \sum_i d_i - \boldsymbol{m}^{\mathrm{T}} \boldsymbol{d} \sum_i m_i}{N^2 \left(1 - |\sum_i d_i|^2\right)} \tag{14}$$

$$a^+(b, \phi) = \frac{-N \left(\sum_i d_i\right) \left(\sum_i m_i^*\right) + \boldsymbol{m}^{\mathrm{T}} \boldsymbol{d}}{N^2 \left(1 - |\sum_i d_i|^2\right)} \tag{15}$$

5.2 Non-linear estimates

The modulation parameters estimation amounts to estimate only b and ϕ, optimizing the function:

$$f(b, \phi) = \left\| \boldsymbol{r} - c^+(b, \phi) - a^+(b, \phi) \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi)) \right\|_2^2 \tag{16}$$

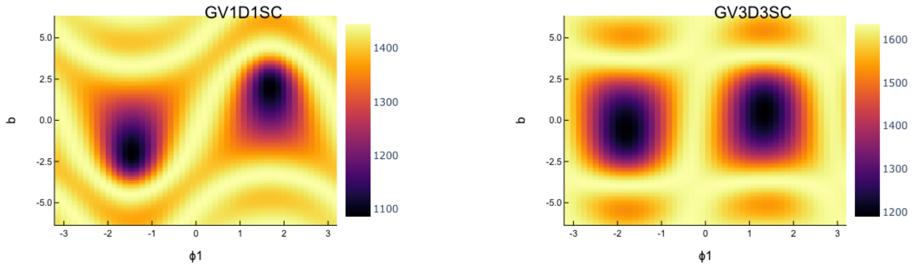

Figure 1: $f(b, \phi)$ for two different diodes

Name		keyword
center	$\mathrm{Re}(c^+)$	`DEMODULATION CENTER X0`
center	$\mathrm{Im}(c^+)$	`DEMODULATION CENTER Y0`
metrology mean amplitude	$\lvert a^+ \rvert$	`DEMODULATION AMPLITUDE ABS`
metrology mean phase	$\arg(a^+)$	`DEMODULATION AMPLITUDE ARG`
modulation amplitude	b^+	`DEMODULATION SIN AMPLITUDE`
modulation phase	ϕ^+	`DEMODULATION SIN PHASE`

Table 2: New keywords in the metrology table

This function is non-linear and non-convex as it can be seen on figure 1. The two main optima are equivalent as $f(b, \phi) = f(-b, \phi+\pi)$. An initialization with b sufficiently small seems to ensure to end in the global optimum. Note that the case $b = 0$ is singular as $f(b, \phi)$ is equal to the variance of $\boldsymbol{r}$ in this case whatever is ϕ and c.

In the code, this function is minimized by the means of the derivative free NEWOA method of Powell that seems faster than VMLMB (with derivative) and the Simplex method.

6 Demodulation

Once the modulation parameters are estimated the demodulated signal is given by:

$$\boldsymbol{s} = \left(\boldsymbol{r} - c^+\right) \times \exp\left(-\jmath\, b^+ \sin\left(\boldsymbol{\omega} + \phi_1^+\right)\right) \times \exp\left(\jmath\, \Phi_{\mathrm{FC}}\right) , \tag{17}$$

All the $6 \times 4 \times 4 \times 2$ parameters are stored in keywords of the METROLOGY table header as shown on the table 2. These keywords are suffixed with the side, the telescope number and the diode (*e.g.* `DEMODULATION CENTER X0 SC T4 D1`). In addition, the keyword `PROCSOFT` of the METROLOGY table header is set to `GPPupilDemodulation.jl`

Keywords	unit	
`ESO INS ANLO3 RATE1`	second	Rate of repetition of the first state
`ESO INS ANLO3 RATE2`	second	Rate of repetition of the second state
`ESO INS ANLO3 REPEAT1`	Int	Number of repetition of the first state
`ESO INS ANLO3 REPEAT2`	Int	Number of repetition of the second state
`ESO INS ANLO3 TIMER1`	second (unix time)	Starting time of the first state
`ESO INS ANLO3 TIMER2`	second (unix time)	Starting time of the second state
`ESO INS ANLO3 VOLTAGE1`	V	Voltage of the first state
`ESO INS ANLO3 VOLTAGE2`	V	Voltage of the second state

Table 3: Keywords of the FAINT mode

7 Faint model

The FAINT mode of the metrology is active when the keyword `ESO INS MET MODE` is set to `FAINT`. In this case metrology laser is dimmed (the LOW state) during observations and maximized between the frame (the HIGH state) according to the keyword shown on table 3. This alternating HIGH/LOW state is shown on figure 2. In the firsts and the lasts second the metrology power is in NORMAL state.

We define the laser power p_k as the expected laser power at row k for the state $\mathrm{ST}(k)$. This vector is filled with mean of the modulus of the data $\boldsymbol{d}$ for each state (NORMAL, LOW, HIGH):

$$p_k = \langle |d_m| \rangle_{m \in \mathrm{ST}(k)} \tag{18}$$

In addition we consider that the variance of the noise $\boldsymbol{e}$ varies with the power of the metrology laser. We define the precision (*i.e.* inverse variance) of a measurements at row k as:

$$w_k = 1/\operatorname{Var}\left(|d_m|\right)_{m \in \mathrm{ST}(k)} \tag{19}$$

The noise $\boldsymbol{e}$ is probably Gaussian with identical variance for both real and imaginary part. As consequence, the variance of its modulus follows a Rayleigh distribution but as we treat equally all states it should not make much difference to assume Normal distribution.

7.1 Lag on voltage switching

As seen on figure 3, there is a lag of about 10ms between the switching of voltage and the actual dimming/brightening of the laser. This lag can be automatically estimated and corrected in the Julia code (not by default). In addition, there is also a 10ms transient before the laser power stabilizes. These lags and transients are handle by pre/post-switching delays.

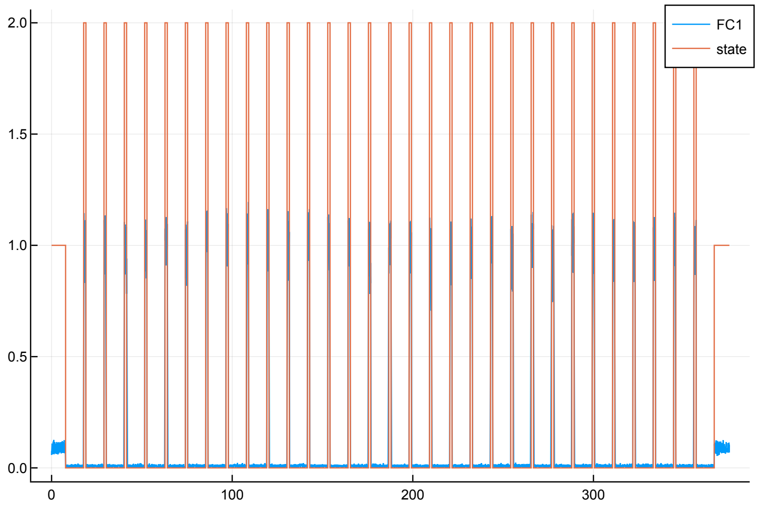

Figure 2: Modulus of the T1 FT fiber coupler diode and the state of the metrology laser during a whole observation in FAINT mode.

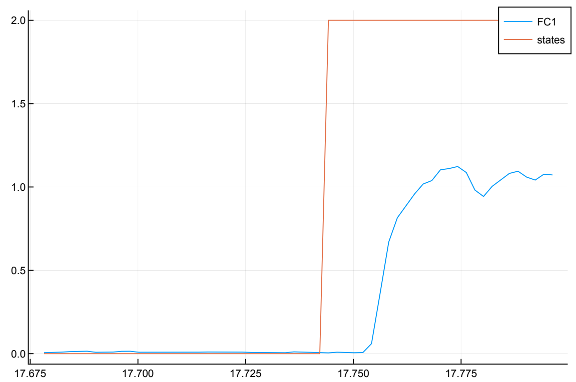

Figure 3: Zoom on the modulus of the T1 FT fiber coupler diode and the state of the metrology laser when the laser is switching to HIGH.

7.2 Faint model (guess 1)

In this mode we consider that the laser power affects both c and the modulated metrology the equation 2 then rewritten as:

$$\boldsymbol{d} = \boldsymbol{p} \times (c + \boldsymbol{s} \times \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi))) \times \exp(\jmath\, \Phi_{\mathrm{FC}}) + \boldsymbol{e}\,, \tag{20}$$

In that case, the equation 10 solved to retrieve the modulation parameters can be updated as:

$$(c^+, a^+, b^+, \phi^+) = \underset{c,a,b,\phi}{\arg\min} \|\boldsymbol{r} - \boldsymbol{p} \times (c + a \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi)))\|_2^2 \tag{21}$$

To take into an account the precision of the noise, we can rewrite this equation as:

$$(c^+, a^+, b^+, \phi^+) = \underset{c,a,b,\phi}{\arg\min} \|\boldsymbol{r} - \boldsymbol{p} \times (c + a \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi)))\|_{\boldsymbol{w}}^2 \tag{22}$$

with the weighted norm $\|\boldsymbol{x}\|_{\boldsymbol{w}}^2 = \boldsymbol{x}^{\mathrm{T}}\, \mathrm{diag}(\boldsymbol{w})\, \boldsymbol{x} = \sum_k w_k\, |x_k|^2$. This can be also rewritten as:

$$(c^+, a^+, b^+, \phi^+) = \underset{c,a,b,\phi}{\arg\min} \|\boldsymbol{r} \div \boldsymbol{p} - (c + a \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi)))\|_{\boldsymbol{w}'}^2\ , \tag{23}$$

where $\div$ is the element-wise division and $\boldsymbol{w}' = \boldsymbol{w} \times \boldsymbol{p} \times \boldsymbol{p}$

The closed-form solution for becomes:

$$\begin{bmatrix} c^+ \\ a^+ \end{bmatrix} = \begin{bmatrix} \sum_i w'_i & \sum_i w'_i\, m_i \\ \sum_i w'_i\, m_i^* & \sum_i w'_i\, |m_i|^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum_i w'_i\, d_i \\ \sum_i w'_i\, m_i^*\, d_i \end{bmatrix} \tag{24}$$

7.3 Faint model (guess 2)

In this mode we consider that the laser power affects both only the modulated metrology and not the c component. The equation 2 is then rewritten as:

$$\boldsymbol{d} = c + \boldsymbol{p} \times \boldsymbol{s} \times \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi)) \times \exp(\jmath\, \Phi_{\mathrm{FC}}) + \boldsymbol{e}\,, \tag{25}$$

In that case, the equation 10 solved to retrieve the modulation parameters can be updated as:

$$(c^+, a^+, b^+, \phi^+) = \underset{c,a,b,\phi}{\arg\min} \left\|\boldsymbol{d} - c - a\, \boldsymbol{p}' \times \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi))\right\|_{\boldsymbol{w}}^2\ , \tag{26}$$

where $\boldsymbol{p}' = \boldsymbol{p} \times \exp(\jmath\, \Phi_{\mathrm{FC}})$.

The closed-form solution for (c^+, a^+) becomes:

$$\begin{bmatrix} c^+ \\ a^+ \end{bmatrix} = \begin{bmatrix} \sum_i w_i & \sum_i w_i\ g_i \\ \sum_i w_i\, g_i^* & \sum_i w_i\, |g_i|^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum_i w_i\, d_i \\ \sum_i w' i\, g_i^*\, d_i \end{bmatrix} \tag{27}$$

with $\boldsymbol{g} = \boldsymbol{p}' \exp(\jmath\, b \sin(\boldsymbol{\omega} + \phi))$

An example on demodulated phase signal is shown on figure 4.

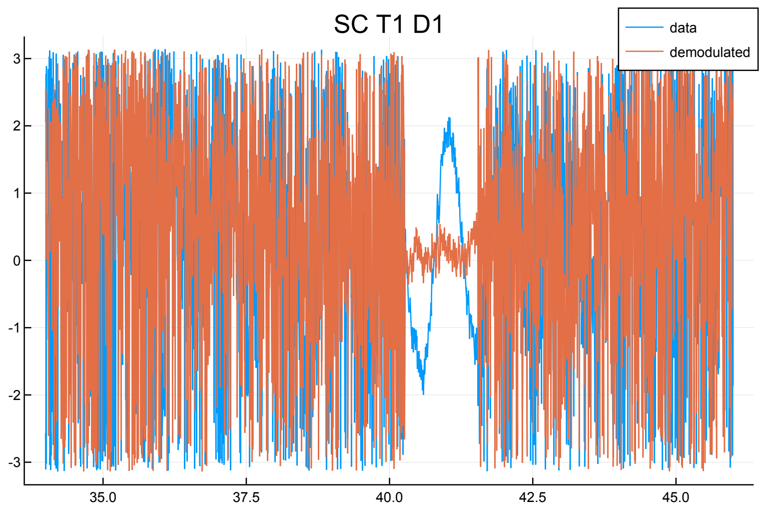

Figure 4: Zoom around a HIGH state of the phase of the diode D1 of T1 SC and its demodulation.