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**PROBLEM 1** *Proving Complexity*

Given the following functions,  $f(n) = 5n^3 + 8n^2 - 3n + 14$  and  $g(n) = n^3$ .

1. Show that  $f(n) = \Theta(g(n))$  using a direct proof, as shown in lecture. Choose integer values for  $n_0$  and  $c$  that are low, or about as low, as possible.

**Solution:** Choose values for  $n_0$  and  $c$ . Possible legal values for  $n_0$  and  $c$  are:  $c = 5, n_0 = 2$ . Other valid combinations exist. Result should be that  $n^3 = n^3$ .

2. Using the limit definitions, show which of the complexity classes  $f(n)$  is in relative to  $g(n)$ . In other words, show whether  $f(n) = \Omega(g(n))$ , and similarly for the other complexity classes ( $O, \Theta, o, \omega$ ).

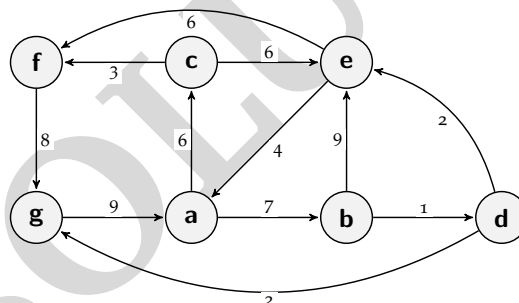
**Solution:**

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 5.$$

- As this is  $< \infty$ ,  $f(n) \in O(g(n))$
- As this is  $> 0$ ,  $f(n) \in \Omega(g(n))$
- As this is a constant  $c$  and  $0 < c < \infty$ ,  $f \in \Theta(g)$
- As this is  $\neq 0$ ,  $f(n) \notin o(g(n))$
- As this is  $\neq \infty$ ,  $f(n) \notin \omega(g(n))$

**PROBLEM 2** *Graph Representations*

Consider the following weighted digraph:



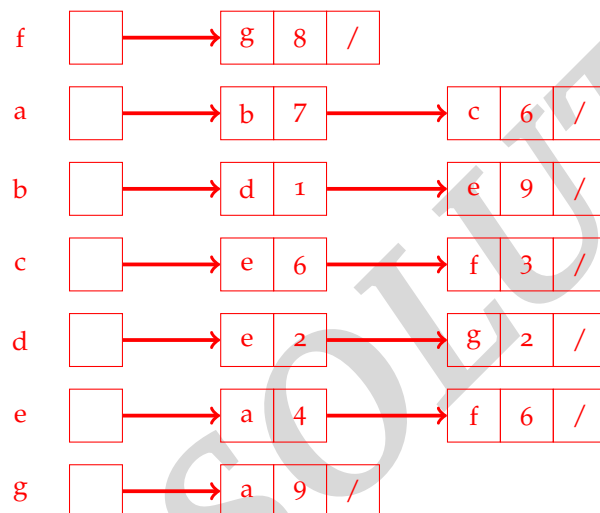
## Graph G

1. Show a representation of this graph using an adjacency matrix. Your representation should look similar in format to the adjacency matrix shown in lecture. The format of a graph is given to you as a starting point. If no edge exists, leave the cell blank. The number "2025" has been placed in the first cell just to show you where to modify cell values. Remove it and either leave it blank or replace it with a value if appropriate.

**Solution:**

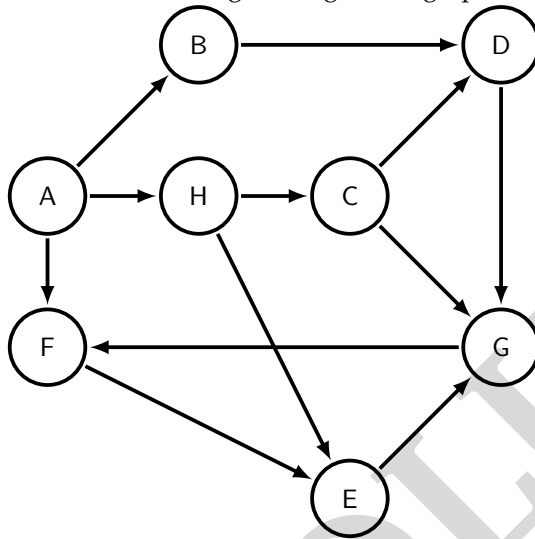
	a	b	c	d	e	f	g
a		7	6				
b				1	9		
c					6	3	
d					2		2
e	4					6	
f							8
g	9						

2. Show a representation of this graph using an adjacency list. Your representation should look similar in format to the adjacency list shown in lecture. Example formatting in given to you below. Within each list node, the node letter is first, followed by the edge cost.

**Solution:**

PROBLEM 3 *BFS vs. DFS*

Given the following unweighted digraph:



1. Perform a breadth first search starting with node A. List the nodes visited in order of when they are first visited. Note: if multiple nodes are enqueued onto the queue in a single step, enqueue them alphabetically (i.e. If both nodes F and H follow node C, enqueue F before H).

**Solution:** A B F H D E C G. If they have A B F H D C G E, then they copied the answer from last semester.

2. Perform a depth first search starting with node A. List the nodes visited in order of when they are first visited. Note: if multiple nodes are enqueued onto the queue in a single step, enqueue them alphabetically (i.e. If both nodes F and H follow node C, enqueue F before H).

**Solution:** A B D G F E H C or A H E G F C D B (depending on how they interpreted the question). If they have A B D E H G F C, then they copied the answer from last semester.