CHAPTER 2

FUNCTIONS OF MORE THAN ONE VARIABLE

2.1 Functions of more than one variable

The set of all ordered *n*- tuples of real numbers is called the *n-dimensional number space* and is denoted by R^n.

Each ordered *n*-tuple

$$(x_1, x_2, \ldots, x_n)$$

is called a *point* in R^n.

Example 2.1.1

1. **The set of all *ordered pairs* of real numbers is the 2-dimensional number space denoted by R^2.**

   Each *ordered pair* is called a *point* in R^2.

2. **The set of all *ordered triples* of real numbers is the 3-dimensional number space denoted by R^3.**

   Each *ordered triple* is called a *point* in R^3.

Fill in the blanks.

1. $(3,2,-4)$ **is a point in** $\underline{\;R^3\;}$.
2. $(3,2,5,4)$ **is a point in** $\underline{\;R^4\;}$.
3. $(0,0,0,1,2)$ **is a point in** $\underline{\;R^5\;}$.
4. **A point in R^7 has** $\underline{\;7\;}$ **coordinates.**
5. **A point in R^{101} has** $\underline{\;101\;}$ **coordinates.**

If $P(x_1, x_2, \ldots, x_n)$ and $A(a_1, a_2, \ldots, a_n)$ are two points in R^n, the *distance between P and A*, denoted by

$$\|P-A\|$$

is given by

$$\|P-A\| = \sqrt{(x_1-a_1)^2 + (x_2-a_2)^2 + \ldots + (x_n-a_n)^2}.$$

Example 2.1.2

1. **The distance between $P(1,3)$ and $A(-2,7)$ is**

$$\|P-A\| = \sqrt{(1-(-2))^2 + (3-7)^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5.$$

2. **The distance between $P(1,2,3)$ and $A(7-2,5)$ is**

$$\|P-A\| = \sqrt{(1-7)^2 + (2-(-2))^2 + (3-5)^2}$$
$$= \sqrt{36+16+4} = \sqrt{56} = 2\sqrt{14}.$$

Recall ….

A *function* is a set of ordered pairs, such that no two distinct ordered pairs have the same first element.

The following are examples of functions:

1. $f = \{(1,2),(2,3),(3,4)\}$
2. $g = \{(1,1),(2,4),(3,9),(4,16),...\}$
3. $h = \{(1,2),(2,2),(3,2),(4,2),...\}$

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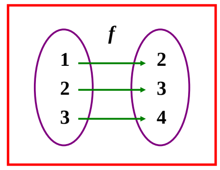
1. $f = \{(1,2),(2,3),(3,4)\}$

$D_f = \{1,2,3\}$
$R_f = \{2,3,4\}$



$$f(1) = 2$$
$$f(2) = 3$$
$$f(3) = 4$$

$$f(x) = x+1, \qquad x = 1,2,3$$

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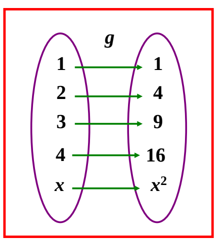
2. $g = \{(1,1),(2,4),(3,9),(4,16),...\}$

$D_g = \mathbf{N} = \{1,2,3,...\}$
$R_g = \{1,4,9,16,...\} = \{y : y = x^2, x \in \mathbf{N}\}$



$$g(1) = 1$$
$$g(2) = 4$$
$$g(3) = 9$$
$$\vdots$$

$$g(x) = x^2, x \in \mathbf{N}$$

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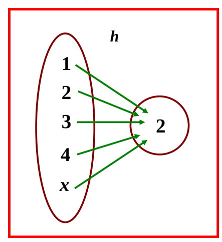
3. $h = \{(1,2),(2,2),(3,2),(4,2),...\}$

$D_h = \mathrm{N}$
$R_h = \{2\}$



$$h(x) = 2, \quad x \in \mathbf{N}$$

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A *function of n variables* is a set of ordered pairs, such that no two distinct ordered pairs have the same first element.

If f is a function of n variables and

$$(P,w) \in f$$

we write

$$f(P) = w.$$

In (P,w) P is a point in R^n and w is a real number.

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Let M be the amount of money you spend per day.

Then we can view M as a function of different variables:

- ***f*: food expenses**
- ***p*: transpo. fare**
- ***l*: communication expenses**
- ***g*: gimmick (social life) expenses**
- ***s*: school supplies**

$M(f, p, l, g, s) = f + p + l + g + s$

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If $f(P) = w$, the set of all admissible points P is called the ***domain*** of the function, and the set of all resulting values of w is called the ***range*** of the function.

Example 2.1.3 **Determine the domain of the indicated function and sketch/describe the domain.**

a. $f(x,y) = \sqrt{4 - x^2 - y^2}$ b. $g(x,y) = \dfrac{1}{xy}$ c. $h(x,y) = ln(xy)$

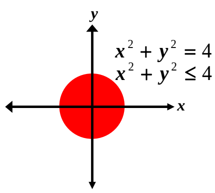
solution:

a. $D_f = \{(x,y) \in R^2 : 4 - x^2 - y^2 \geq 0\}$
$\quad = \{(x,y) \in R^2 : x^2 + y^2 \leq 4\}$

b. $D_g = \{(x,y) \in R^2 : xy \neq 0\}$

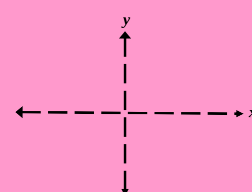
c. $D_h = \{(x,y) \in R^2 : xy > 0\}$

a. $D_f = \{(x,y) \in R^2 : x^2 + y^2 \leq 4\}$



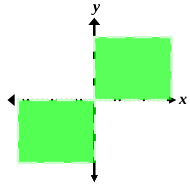
The domain of *f* consists of all points inside or on the circle given by $x^2 + y^2 = 4$.

b. $D_g = \{(x,y) \in R^2 : xy \neq 0\}$



The domain of *g* consists of all points in the plane except those on the *y*-axis or *x*-axis.

c. $D_h = \{(x,y) \in R^2 : xy > 0\}$



The domain of *h* consists of all points in the first or third quadrant.

Example 2.1.4 **Determine the domain of the indicated function and sketch the domain.**

a. $f(x,y,z) = \sqrt{9 - x^2 - y^2 - z^2}$

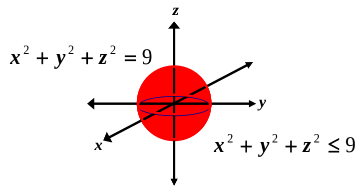
b. $g(x,y,z) = ln\,x + ln\,y + ln\,z$

solution:

a. $D_f = \{(x,y,z) \in R^3 : 9 - x^2 - y^2 - z^2 \geq 0\}$
$\quad = \{(x,y,z) \in R^3 : x^2 + y^2 + z^2 \leq 9\}$

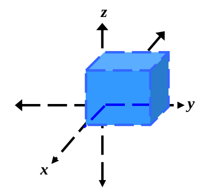
b. $D_g = \{(x,y,z) \in R^3 : x > 0, y > 0, z > 0\}$

$$D_f = \{(x,y,z) \in R^3 : x^2 + y^2 + z^2 \le 9\}$$



The domain of *f* consists of all points inside or on the sphere given by $x^2 + y^2 + z^2 = 9$.

$$D_g = \{(x,y,z) \in R^3 : x > 0, y > 0, z > 0\}$$



The domain of *g* consists of all points in the first octant.