Chapter 2

COMPUTER ARITHMETIC

Arithmetic in various Number Systems

- Addition of numbers in any number system
  - Add numbers starting at the least significant digit.
  - Perform addition on numbers of the same number base.
- Subtraction of numbers
  - Must use complements

Binary Addition

- To add binary numbers: (X + Y)
  - Get the SCR of the negative numbers
  - Add the two numbers
  - If the SCR used is:
    - 2's C: Discard end carry
    - 1's C: Add the end carry to the sum

Examples: Addition

- $(999.5 + 281.6)_{10}$

$$
\begin{array}{r}
{\scriptstyle 1\ \ 1\ 1} \\
999.5 \\
+ \quad 281.6 \\
\hline
1281.1
\end{array}
$$

Examples: Addition

- $(999.5 + 281.6)_{10}$

$$
\begin{array}{lr}
 & {}^{1\ \ 1\,1} \\
 & 999.5 \\
+ & 281.6 \\
\hline
 & 1281.1
\end{array}
$$

- $(110.11 + 101010.11)_2$

$$
\begin{array}{lr}
 & {}^{1\,1\ \,1\ \ \ 1\ \ \ 1} \\
 & 000110.11 \\
+ & 101010.11 \\
\hline
 & 110001.10
\end{array}
$$

Examples: Addition

- $(355.45 + 240.664)_8$

$$
\begin{array}{r}
355.45 \\
+\quad 240.664 \\
\hline
=\qquad\qquad
\end{array}
$$

Examples: Addition

- $(355.45 + 240.664)_8$

$$\begin{array}{rr} & \overset{1}{}355.45 \\ + & 240.664 \\ \hline = & 4 \end{array}$$

Examples: Addition

- $(355.45 + 240.664)_8$

$$
\begin{array}{rr}
 & \overset{}{355.\overset{1}{4}5} \\
+ & 240.664 \\
\hline
= & 34
\end{array}
$$

Examples: Addition

- $(355.45 + 240.664)_8$

$$
\begin{array}{rl}
 & \overset{}{3}\overset{}{5}\overset{1}{5}.\overset{1}{4}5 \\
+ & 240.664 \\
\hline
= & 6.334
\end{array}
$$

Examples: Addition

- $(355.45 + 240.664)_8$

$$
\begin{array}{rl}
 & \overset{1}{3}5\overset{1}{5}.\overset{1}{4}5 \\
+ & 240.664 \\
\hline
= & 616.334
\end{array}
$$

Examples: Addition

- $(355.45 + 240.664)_8$
- $(A0C.D + E72.9)_{16}$

$$
\begin{array}{rl}
 & \overset{1}{3}55.\overset{1}{4}\overset{1}{5} \\
+ & 240.664 \\
\hline
= & 616.334
\end{array}
\qquad
\begin{array}{rl}
 & A0C.D \\
+ & E72.9 \\
\hline
= &
\end{array}
$$

Examples: Addition

- $(355.45 + 240.664)_8$
- $(A0C.D + E72.9)_{16}$

$$
\begin{array}{lr}
 & \overset{1}{3}55.\overset{1}{4}\overset{1}{5} \\
+ & 240.664 \\
\hline
= & 616.334
\end{array}
\qquad
\begin{array}{lr}
 & A0\overset{1}{C}.D \\
+ & E72.9 \\
\hline
= & .6
\end{array}
$$

Examples: Addition

- $(355.45 + 240.664)_8$
- $(A0C.D + E72.9)_{16}$

$$\begin{array}{rr} & \overset{1}{3}5\overset{1}{5}.\overset{1}{4}5 \\ + & 240.664 \\ \hline = & 616.334 \end{array}$$

$$\begin{array}{rr} & \mathrm{A0}\overset{1}{\mathrm{C}}.\mathrm{D} \\ + & \mathrm{E72.9} \\ \hline = & \mathrm{187F.6} \end{array}$$

Binary Addition

- To add binary numbers: (X + Y)
  - Get the SCR of the negative numbers
  - Add the two numbers
  - If the SCR used is:
    - 2's C: Discard end carry
    - 1's C: Add the end carry to the sum

Binary Subtraction

- To subtract binary numbers: (X – Y)
  - Take the complement of the subtrahend.
  - Then, add the two numbers.
  - If the complement used is:
    - 1's C: Add the end carry to the sum
    - 2's C: Discard end carry

(X – Y) >>> X + (complement of Y)

Example: Binary Subtraction

- Subtract the following numbers. Use 8 bits to represent each number.

  6 - 13

  6 - (-13)

  (-6) - 13

  (-6) - (-13)

Example: Binary Subtraction

- 6 – 13 = -7

$$
\begin{array}{rl}
 & 0\ 0000110 \\
+ & \underline{1\ 1110010} \quad (1's) \\
= &
\end{array}
$$

Example: Binary Subtraction

- 6 – 13 = -7

$$
\begin{array}{rl}
 & 0\ 0000110 \\
+ & 1\ 1110010 \quad (1's) \\
\hline
= & 1\ 1111000
\end{array}
$$

Example: Binary Subtraction

- 6 – 13 = -7

```
   0 0000110
+  1 1110010  (1's)
= 1 1111000
```

- 6 - (-13) = 19

```
   0 0000110
+  0 0001101
=
```

Example: Binary Subtraction

- 6 – 13 = -7

```
   0 0000110
+  1 1110010  (1's)
  ___________
=  1 1111000
```

- 6 - (-13) = 19

```
   0 0000110
+  0 0001101
  ___________
=  0 0010011
```

Example: Binary Subtraction

- (-6) – 13 = -19

```
    1 1111010  (2's)
+   1 1110011  (2's)
  ____________
=
```

Example: Binary Subtraction

- (-6) – 13 = -19

```
     1 1111010  (2's)
+    1 1110011  (2's)
  ______________
= 11 1101101
```

Example: Binary Subtraction

- (-6) – 13 = -19

$$
\begin{array}{rl}
 1\ 1111010 & (2's) \\
+\quad 1\ 1110011 & (2's) \\
\hline
= \cancel{1}1\ 1101101 &
\end{array}
$$

Example: Binary Subtraction

- (-6) – 13 = -19

```
    1 1111010  (2's)
+   1 1110011  (2's)
= 11 1101101
```

- (-6) - (-13) = 7

```
    1 1111001  (1's)
+   0 0001101
```

Example: Binary Subtraction

- (-6) – 13 = -19

```
    1 1111010  (2's)
+   1 1110011  (2's)
---------------
= 11 1101101
```

- (-6) - (-13) = 7

```
    1 1111001  (1's)
+   0 0001101
---------------
   10 0000110
```

Example: Binary Subtraction

- (-6) – 13 = -19

$$
\begin{array}{rrl}
 & 1\ 1111010 & (2's) \\
+ & 1\ 1110011 & (2's) \\
\hline
= & \cancel{1}1\ 1101101 &
\end{array}
$$

- (-6) - (-13) = 7

$$
\begin{array}{rrl}
 & 1\ 1111001 & (1's) \\
+ & 0\ 0001101 & \\
\hline
 & 10\ 0000110 & \\
+ & 1 & \\
\hline
= & &
\end{array}
$$

Example: Binary Subtraction

- (-6) – 13 = -19

$$
\begin{array}{rrl}
 & 1\ 1111010 & (2\text{'s}) \\
+ & 1\ 1110011 & (2\text{'s}) \\
\hline
= & \cancel{1}1\ 1101101 &
\end{array}
$$

- (-6) - (-13) = 7

$$
\begin{array}{rrl}
 & 1\ 1111001 & (1\text{'s}) \\
+ & 0\ 0001101 & \\
\hline
 & 10\ 0000110 & \\
+ & 1 & \\
\hline
= & 0\ 0000111 &
\end{array}
$$

Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$
\begin{array}{r}
999.5 \\
+ \\
\hline
\end{array}
$$

Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$
\begin{array}{lr}
 & 999.5 \\
+ & 718.4 \\
\hline
\end{array}
$$

Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$
\begin{array}{rr}
 & {}^{1\,1} \\
 & 999.5 \\
+ & 718.4 \\
\hline
 & 1717.9
\end{array}
$$

Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$
\begin{array}{rr}
 & {\scriptstyle 1\;1} \\
 & 999.5 \\
+ & 718.4 \\
\hline
 & \cancel{1}717.9
\end{array}
$$

Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$
\begin{array}{rr}
 & {}^{1\ 1}\\
 & 999.5 \\
+ & 718.4 \\
\hline
 & \cancel{1}717.9
\end{array}
$$

- $(00000110.11 - 00101010.11)_2$

$$
\begin{array}{rr}
 & 00000110.11 \\
+ & \\
\hline
\end{array}
$$

Examples: Subtraction

- $(999.5 - 281.6)_{10}$
- $(00000110.11 - 00101010.11)_2$

$$
\begin{array}{rr}
 & {}^{1\,1} \\
 & 999.5 \\
+ & 718.4 \\
\hline
 & \cancel{1}717.9
\end{array}
$$

$$
\begin{array}{rr}
 & 00000110.11 \\
+ & 11010101.00 \\
\hline
\end{array}
$$

Examples: Subtraction

- $(999.5 - 281.6)_{10}$
- $(00000110.11 - 00101010.11)_2$

$$\begin{array}{r} {}^{1\ 1} \\ 999.5 \\ +\quad 718.4 \\ \hline \cancel{1}717.9 \end{array}$$

$$\begin{array}{r} 00000110.11 \\ +\quad 11010101.00 \\ \hline 11011011.11 \end{array}$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{r}
355.45 \\
+\underline{\hphantom{0000000}}
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rl}
 & 355.45 \\
+ & 537.114 \\
\hline
= &
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{lr}
 & 355.45 \\
+ & 537.114 \\
\hline
= & .564
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rr}
 & \overset{1}{3}55.45 \\
+ & 537.114 \\
\hline
= & 4.564
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{lr}
 & \overset{1}{3}\overset{1}{5}5.45 \\
+ & 537.114 \\
\hline
= & 1114.564
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rl}
 & \overset{1}{3}\overset{1}{5}5.45 \\
+ & 537.114 \\
\hline
= & \cancel{1}114.564
\end{array}
$$

Examples: Subtraction

- $(355.45 – 240.664)_8$
- $(A0C.D - E72.9)_{16}$

$$\begin{array}{lr} & {}^{1}3{}^{1}55.45 \\ + & 537.114 \\ \hline = & \cancel{1}114.564 \end{array}$$

$$\begin{array}{lr} & A0C.D \\ + & \\ \hline = & \end{array}$$

Examples: Subtraction

- $(355.45 - 240.664)_8$
- $(A0C.D - E72.9)_{16}$

$$\begin{array}{lr} & {}^{1}\,{}^{1} \\ & 355.45 \\ + & 537.114 \\ \hline = & \cancel{1}114.564 \end{array}$$

$$\begin{array}{lr} & A0C.D \\ + & 18D.6 \\ \hline = & \end{array}$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{lr}
 & {\scriptstyle 1\ \ 1} \\
 & 355.45 \\
+ & 537.114 \\
\hline
= & \cancel{1}114.564
\end{array}
$$

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{lr}
 & {\scriptstyle 1} \\
 & A0C.D \\
+ & 18D.6 \\
\hline
= & .3
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rr}
 & {}^{1}\,{}^{1} \\
 & 355.45 \\
+ & 537.114 \\
\hline
= & \cancel{1}114.564
\end{array}
$$

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{rr}
 & {}^{1}\,{}^{1} \\
 & A0C.D \\
+ & 18D.6 \\
\hline
= & B9A.3
\end{array}
$$

Examples: Subtraction

- $(00101010.11 - 00000110.11)_2$

1 1 1 1

```
  00101010.11
+ 11111001.00 (1's)
-----------------
 100100011.11
          1
-----------------
  00100100.00
```

- $(00000110.11 - 00101010.11)_2$

1

```
  00000110.11
+ 11010101.00 (1's)
-----------------
  11011011.11

= - 0100100.00
```

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{ll}
 & 355.45 \\
+ & \underline{\hspace{5cm}} \text{ (8's)} \\
= &
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{lr}
 & 355.45 \\
+ & 537.114 \text{ (8's)} \\
\hline
= &
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$\begin{array}{rr} & \overset{1}{3}\overset{1}{5}5.45 \\ + & 537.114 \text{ (8's)} \\ \hline = & 1114.564 \end{array}$$

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rl}
 & \overset{1\,1}{355.45} \\
+ & 537.114 \quad (8\text{'s}) \\
\hline
= & \cancel{1}114.564
\end{array}
$$

Examples: Subtraction

- $(355.45 – 240.664)_8$
- $(240.664 - 355.45)_8$

$$
\begin{array}{rl}
 & \overset{1\ 1}{}355.45 \\
+ & 537.114 \quad (8\text{'s}) \\
\hline
= & \cancel{1}114.564
\end{array}
$$

$$
\begin{array}{rl}
 & 240.664 \\
+ & \quad (7\text{'s}) \\
\hline
= &
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$
- $(240.664 - 355.45)_8$

$$
\begin{array}{rrl}
 & \overset{1\ 1}{355.45} & \\
+ & 537.114 & (8\text{'s}) \\
\hline
= & \cancel{1}114.564 &
\end{array}
\qquad
\begin{array}{rrl}
 & 240.664 & \\
+ & 422.327 & (7\text{'s}) \\
\hline
= & &
\end{array}
$$

Examples: Subtraction

- $(355.45 - 240.664)_8$
- $(240.664 - 355.45)_8$

$$
\begin{array}{rl}
 & \overset{1}{3}\overset{1}{5}5.45 \\
+ & 537.114 \quad (8\text{'s}) \\
\hline
= & \cancel{1}114.564
\end{array}
$$

$$
\begin{array}{rl}
 & 240.664 \\
+ & 422.327 \quad (7\text{'s}) \\
\hline
= & 663.213
\end{array}
$$

Examples: Subtraction

- $(355.45 – 240.664)_8$
- $(240.664 - 355.45)_8$

$$
\begin{array}{lr}
 & \overset{1}{3}\overset{1}{5}5.45 \\
+ & 537.114 \quad (8\text{'s}) \\
\hline
= & \cancel{1}114.564
\end{array}
$$

$$
\begin{array}{lr}
 & 240.664 \\
+ & 422.327 \quad (7\text{'s}) \\
\hline
= & 663.213 \\
= & -114.564
\end{array}
$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{rl}
 & A0C.D \\
+ & \qquad\qquad (15\text{'s}) \\
\hline
= &
\end{array}
$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{lll}
 & \text{A0C.D} & \\
+ & \text{18D.6} & (15\text{'s}) \\
\hline
= & &
\end{array}
$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{rll}
 & ^{1}\,^{1} & \\
 & A0C.D & \\
+ & 18D.6 & (15\text{'s}) \\
\hline
= & B9A.3 &
\end{array}
$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{rrl}
 & ^{1}\,^{1} & \\
 & A0C.D & \\
+ & 18D.6 & (15\text{'s}) \\
\hline
= & B9A.3 & \\
= & -\ 465.C &
\end{array}
$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{lr}
 & \overset{1}{}\ \overset{1}{} \\
 & A0C.D \\
+ & 18D.6\ (15\text{'s}) \\
\hline
= & B9A.3 \\
= & -\ 465.C
\end{array}
$$

- $(E72.9 - A0C.D)_{16}$

$$
\begin{array}{lr}
 & E72.9 \\
+ & \qquad (16\text{'s}) \\
\hline
= &
\end{array}
$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$\begin{array}{rrl} & \overset{1}{A}0\overset{1}{C}.D & \\ + & 18D.6 & (15\text{'s}) \\ \hline = & B9A.3 & \\ = & -\ 465.C & \end{array}$$

- $(E72.9 - A0C.D)_{16}$

$$\begin{array}{rrl} & E72.9 & \\ + & 5F3.3 & (16\text{'s}) \\ \hline = & & \end{array}$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{lrl}
 & {}^{1\ 1} & \\
 & A0C.D & \\
+ & 18D.6 & (15\text{'s}) \\
\hline
= & B9A.3 & \\
= & -\ 465.C &
\end{array}
$$

- $(E72.9 - A0C.D)_{16}$

$$
\begin{array}{lrl}
 & {}^{1} & \\
 & E72.9 & \\
+ & 5F3.3 & (16\text{'s}) \\
\hline
= & 1465.C &
\end{array}
$$

Examples: Subtraction

- $(A0C.D - E72.9)_{16}$

$$
\begin{array}{lll}
 & ^{1}\,^{1} & \\
 & A0C.D & \\
+ & 18D.6 & (15\text{'s}) \\
\hline
= & B9A.3 & \\
= & -\ 465.C &
\end{array}
$$

- $(E72.9 - A0C.D)_{16}$

$$
\begin{array}{lll}
 & ^{1} & \\
 & E72.9 & \\
+ & 5F3.3 & (16\text{'s}) \\
\hline
= & \cancel{1}\,465.C &
\end{array}
$$

Error Detection

- Overflow
  - occurs when an arithmetic operation yields a result that is greater than the range's positive limit
- Example:

```
+5    0  101
+6    0  110
-5    1  011  (2's)
```

Error Detection

- Underflow
  - occurs when an arithmetic operation yields a result that is lesser than the range's negative limit
- Example:

- 3	1	101
- 6	1	010
+7	0	111

Error Detection

- *Simple Rule*: An addition operation produces an error if the signs of the addends are the same and the sign of the sum is different from the addend's sign.
- OR: An error occurs when the last carry-in is not equal to the carry-out (end-carry).

Error Detection

- Example: Use 1's C for negative values

$$
\begin{array}{r}
9 + 5 \\
0\ 1001 \\
+\ \ 0\ 0101 \\
\hline
0\ 1110
\end{array}
$$

Error Detection

- Example: Use 1's C for negative values

$$\begin{array}{r} 9 + 5 \\ \\ 0\ 1001 \\ +\ \ 0\ 0101 \\ \hline 0\ 1110 \end{array}$$

No error!

Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7
0 1001	0 1101
+ 0 0101	+ 0 0111
0 1110	1 0100

No error!

Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7
0 1001	0 1101
+ 0 0101	+ 0 0111
0 1110	1 0100
No error!	**Overflow!**

Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7	-9 + -9
0 1001	0 1101	1 0110
+ 0 0101	+ 0 0111	+ 1 0110
0 1110	1 0100	0 1100

No error! **Overflow!**

Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7	-9 + -9
0 1001	0 1101	1 0110
+ 0 0101	+ 0 0111	+ 1 0110
0 1110	1 0100	0 1100
No error!	**Overflow!**	**Underflow!**

BCD Addition

- Sum less than or equal to 9
  - Normal binary addition
- Sum greater than 9
  - Add the codes
  - Add a correction value of 0110 to any sum

Example: BCD Addition

45 + 55 = 100

$$
\begin{array}{rcl}
45 & = & 0100\ \ 0101 \\
\underline{55} & \underline{=} & \underline{0101\ \ 0101} \\
100 & = & 1001\ \ 1010
\end{array}
$$

Example: BCD Addition

45 + 55 = 100

45	=	0100	0101
55	=	0101	0101
100	=	1001	1010
	+		0110
		1010	0000

Example: BCD Addition

45 + 55 = 100

$$
\begin{array}{rcrr}
45 & = & 0100 & 0101 \\
\underline{55} & \underline{=} & \underline{0101} & \underline{0101} \\
100 & = & 1001 & 1010 \\
 & + & & \underline{0110} \\
 & & 1010 & 0000 \\
 & + & \underline{0110} & \\
0001 & & 0000 & 0000
\end{array}
$$

Example: BCD Addition

19 + 65 = 84

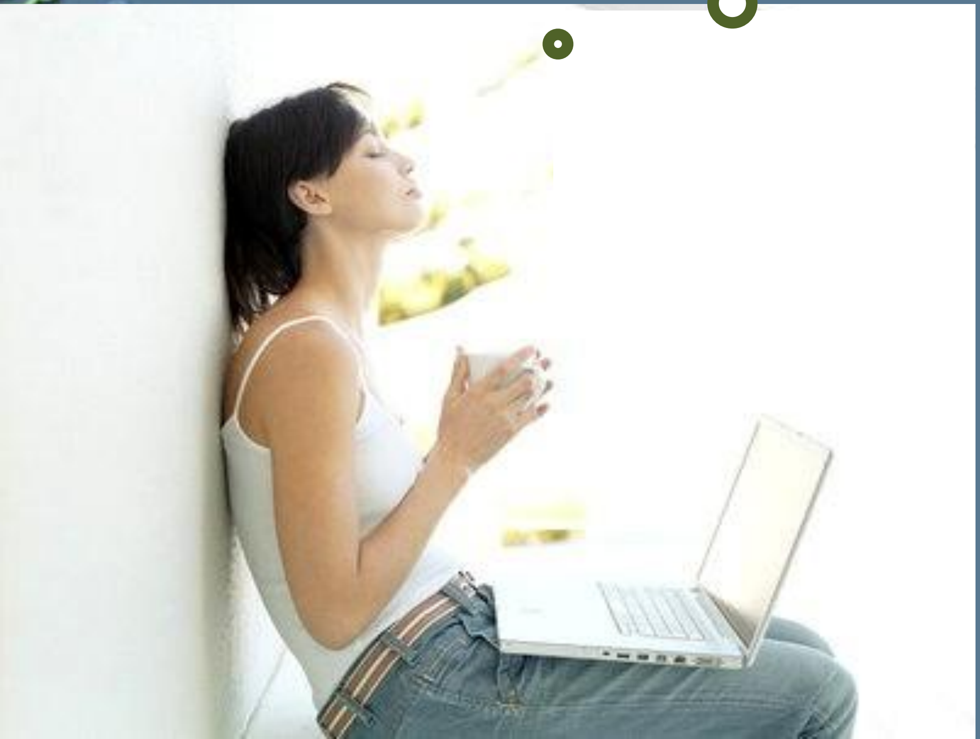
19	=	0001	1001
65	=	0110	0101
84	=	0111	1110

Example: BCD Addition

19 + 65 = 84

19	=	0001	1001
65	=	0110	0101
84	=	0111	1110
	+		0110
		1000	0100

Quiz



```
  91FB8C6D
+ 68D3239F
```