REVIEW ITEMS FOR MIDTERM EXAM

PART 1. FILL THE BLANKS WITH CORRECT EXPRESSIONS OR WORDS.

1. If $\vec{A} = \langle 2,-1,-2 \rangle$, then $\|\vec{A}\|$ =_______.
2. The unit vector in the same direction as $\vec{A} = \langle 2,-1,-2 \rangle$ is $\vec{u}_{\vec{A}}$ =______.
3. If $\vec{C} = \langle 3,-4,1 \rangle$ and $\vec{D} = \langle -8,6,3 \rangle$, then $3\vec{C} + 2\vec{D} =$ ______.
4. The direction angles of $\langle 0,0,-3 \rangle$ are $\alpha = \frac{\pi}{2}, \beta = \frac{\pi}{2}$ and $\gamma =$ ______.
5. If $\vec{A} = \left\langle \frac{-\sqrt{3}}{2}, \frac{1}{2} \right\rangle$ and $\vec{B} = \langle 0,2 \rangle$, then $\vec{A} \cdot \vec{B} =$ ______.
6. In problem no.5, the radian measure of the angle between $\vec{A}$ and $\vec{B}$ is ______.
7. In problem no.5, the scalar projection of $\vec{A}$ onto $\vec{B}$ is ______.
8. In problem no.5, the vector projection of $\vec{A}$ onto $\vec{B}$ is ______.
9. If the direction angle of a vector $\vec{G}$ is $\frac{5\pi}{4}$ and its magnitude is 4, then $\vec{G} =$ __________________
10. Consider the points $C(4,-5)$ and $D(-3,2)$. If $\overrightarrow{DC}$ is a representation of $\vec{E}$, then $\vec{E} =$ ____________________
11. An equation of a plane that is parallel to the xz-plane and which passes through the point $(1,2,3)$ is
12. The distance between $A(1,2,3)$ and $B(-2,3,-4)$ is ______.
13. The midpoint of the segment whose endpoints are $A(1,2,3)$ and $B(-2,3,-4)$ is ______.
14. The standard equation of the sphere with $A(1,2,3)$ and $B(-2,3,-4)$ as endpoints of a diameter is
15. The point $(1,2,3)$ lies ______ (on, inside, outside) the sphere given by $x^2 + y^2 + (z-1)^2 = 5$.
16. The graph of $x^2 + 4x + y^2 - 6y + z^2 - 2z - 10 = 0$ is a/an ______.
17. A standard equation of the plane passing through $(1,2,3)$ and having $\langle -2,3,-4 \rangle$ as a normal vector is given by ______.
18. The distance between the parallel planes given by $2x - 2y + z + 5 = 0$ and $4x - 4y + 2z + 6 = 0$ is ______.
19. The distance from the point $(1,2,3)$ to the plane given by $2x - 2y + z + 5 = 0$ is ______.
20. The parametric equations of the line passing through $(1,2,3)$ and is parallel to $\langle 4,5,6 \rangle$ are given by ______.
21. If $\vec{A} = \langle 1,2,3 \rangle$ and $\vec{B}(-2,3,-4)$, then $\vec{A} \times \vec{B} =$ ______.
22. In R^3, the graph of $x^2 - 4y = 1$ is called a/an ______ cylinder.
23. The trace of $\frac{x^2}{2} - \frac{y^2}{9} - z^2 = 1$ on the **xz-plane** is called a/an ______.
24. The limit of the sequence 1,-1,1,-1,1,-1,... is ______.
25. The limit of the sequence $\left\{ \frac{\sin n}{n} \right\}$ as $n \to \infty$ is ______.
26. $\lim\limits_{n \to \infty} \frac{2n+1}{1-3n^2}$ is equal to ______________.
27. The *k*-th partial sum of the geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ is ______.
28. The series $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^n$ is ______ (absolutely convergent, conditionally convergent, divergent).
29. The sum $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} + \ldots$ is equal to ______.
30. The sequence $\left\{ \frac{(-1)^n}{n} \right\}$ is ______ (convergent, divergent)

31. If $f'(x) < 0$ for all $x \geq 1$, then $\{f(n)\}$ is ______ (decreasing, increasing, neither)

32. The sum of the infinite series $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{n+1}$ is _______.

33. The sum of the infinite series $\sum_{n=1}^{\infty}\frac{2}{n(n+1)}$ is _______.

34. The *p*-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if _______.

35. The series $\sum_{n=1}^{\infty}\left(\frac{1}{n}+\frac{1}{2^n}\right)$ is _______ (convergent, divergent)

36. If $\sum_{n=1}^{\infty}\frac{k}{n}$ converges, then *k* = _______.

37. The series $\sum_{n=1}^{\infty}\frac{n^2}{2^{n^2}}$ is absolutely convergent. Using the *ratio* test, the value of $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|$ is ______.

38. In its interval of convergence, the sum of the power series $\sum_{n=0}^{\infty}(x-1)^n$ is expressed by ______.

39. The interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ is ______.

40. The Maclaurin series expansion of the function $f(x) = \sin x$ is ______.

PART 2. PROBLEM SOLVING. WRITE YOUR SOLUTIONS NEATLY, COMPLETELY AND LOGICALLY.

1. Determine the general equation of the plane through the point $P(0,2,-1)$ and parallel to the plane $2x - y + 3z + 8 = 0$.
2. Find an equation of a plane containing the point $(3,1,-1)$ and parallel to the lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$.
3. Identify and sketch the graph of the following surfaces:
   a. $4x^2 - 9z^2 = 36$
   b. $\frac{x^2}{4}+\frac{y^2}{16}-\frac{z^2}{2}=1$
   c. $4y^2 + z^2 = 4x$
4. Given the series $\sum_{n=0}^{\infty}u_n = \frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots$
   a. Find u_n.
   b. Let $S_n = u_1 + u_2 + \ldots + u_n$. Find a formula for S_n.
   c. Find $\lim_{n\to\infty} S_n$, if it exists.
   d. Is the series $\sum_{n=0}^{\infty}u_n$ convergent? Why?
5. Use Ratio Test to determine whether the series $\sum_{n=1}^{+\infty}\frac{3^n}{n^2}$ is convergent or divergent.
6. Consider the power series $\sum_{n=0}^{\infty}\frac{(-1)^n(x-3)^n}{2^{2n+1}}$. Find its radius of convergence and determine its interval of convergence.

End of items