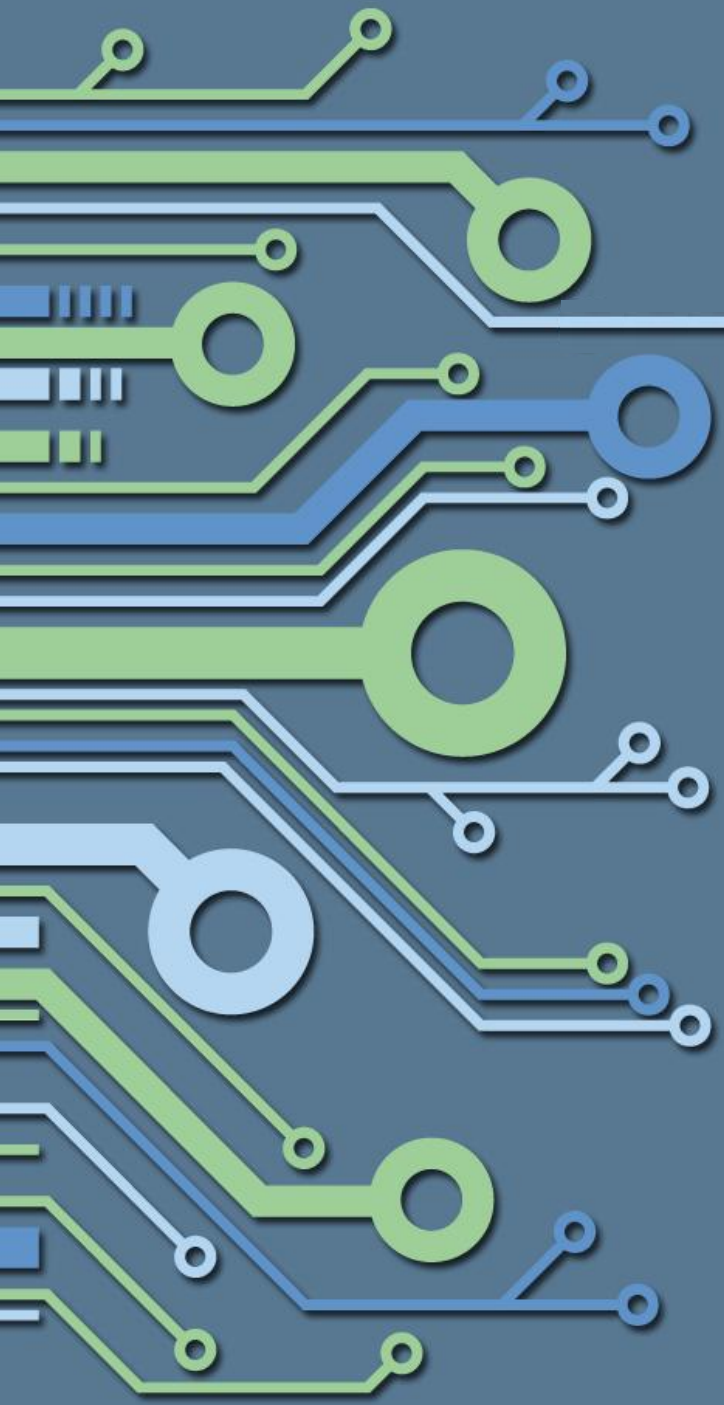


Chapter 4

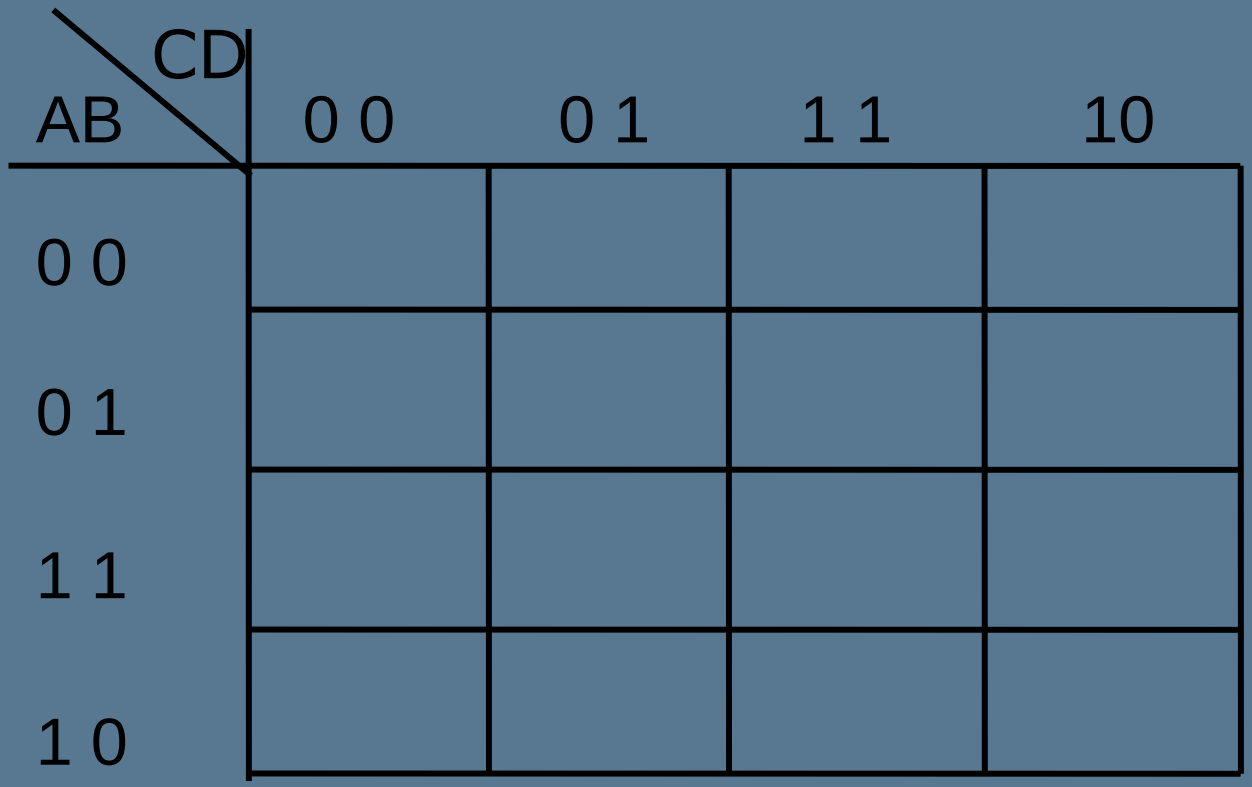
SIMPLIFICATION of LOGIC CIRCUITS

Simplification Example

- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'

Simplification Example

- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



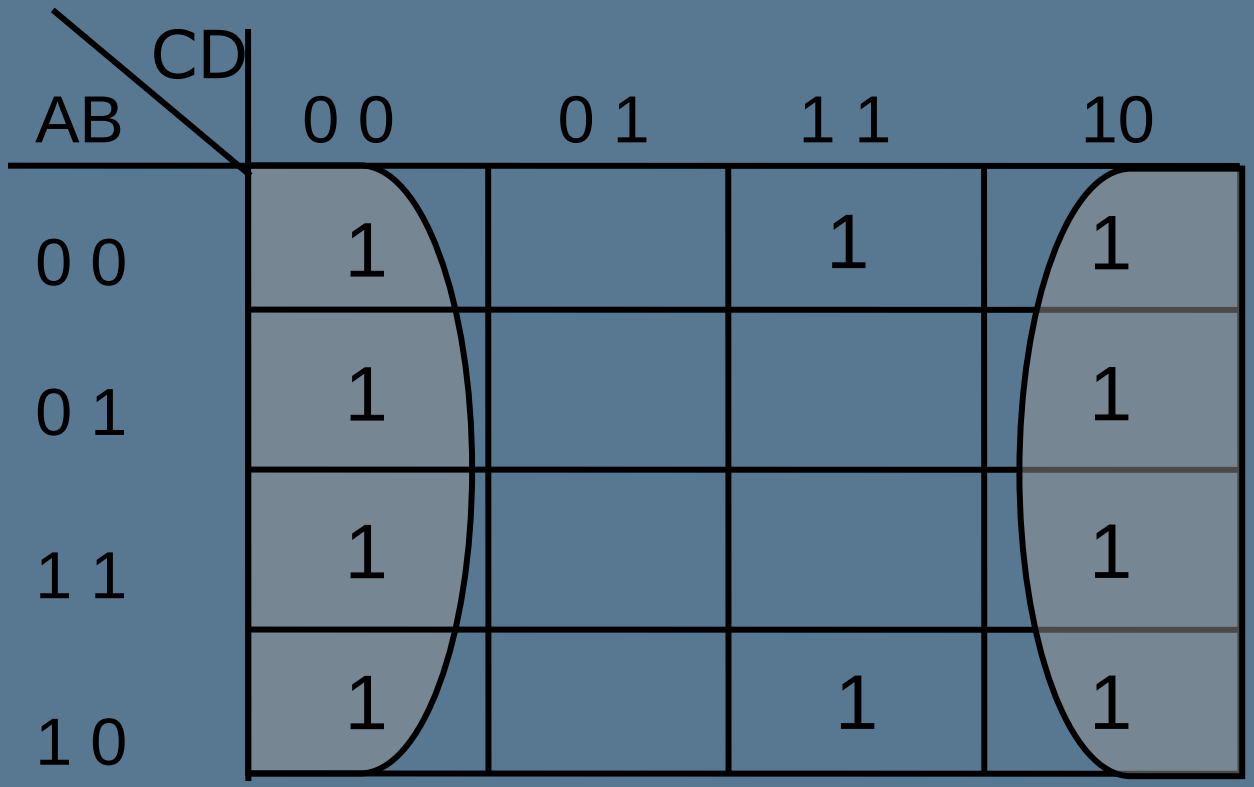
Simplification Example

- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'

AB \ CD	0 0	0 1	1 1	10
0 0	1		1	1
0 1	1			1
1 1	1			1
1 0	1		1	1

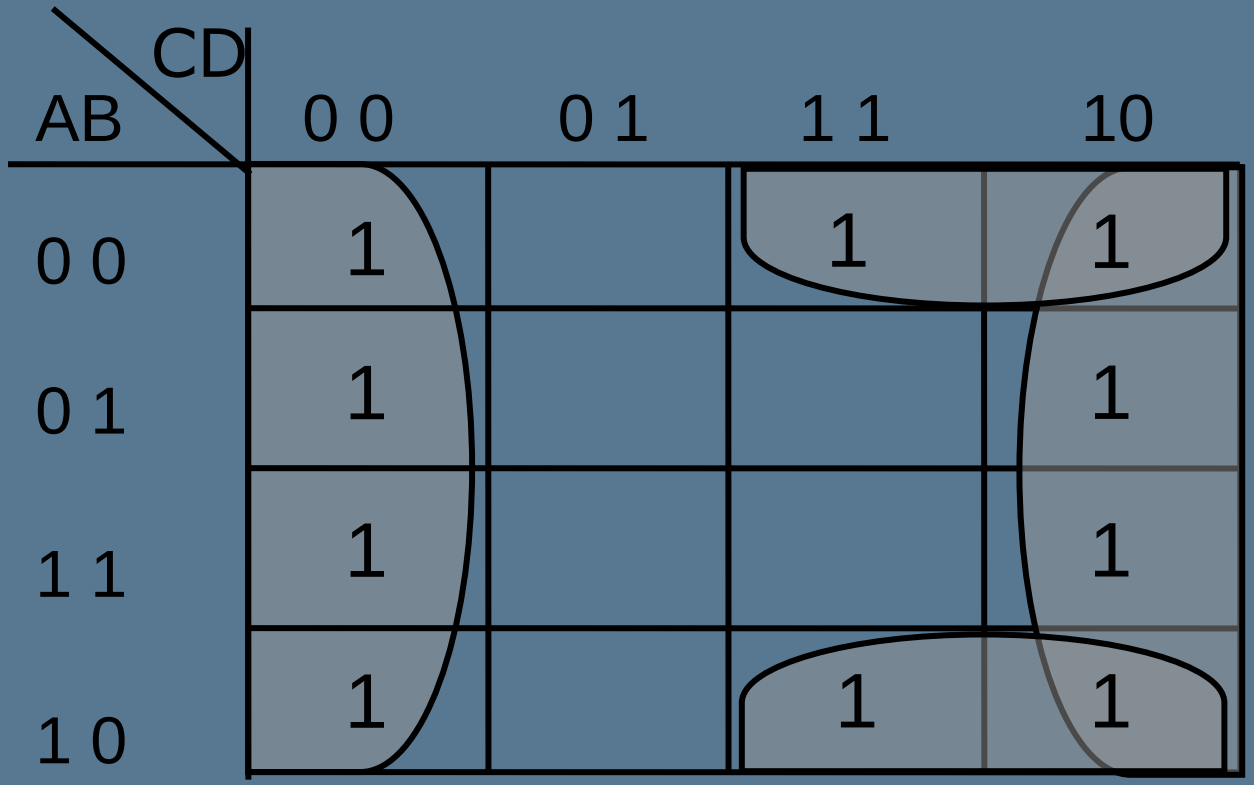
Simplification Example

- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



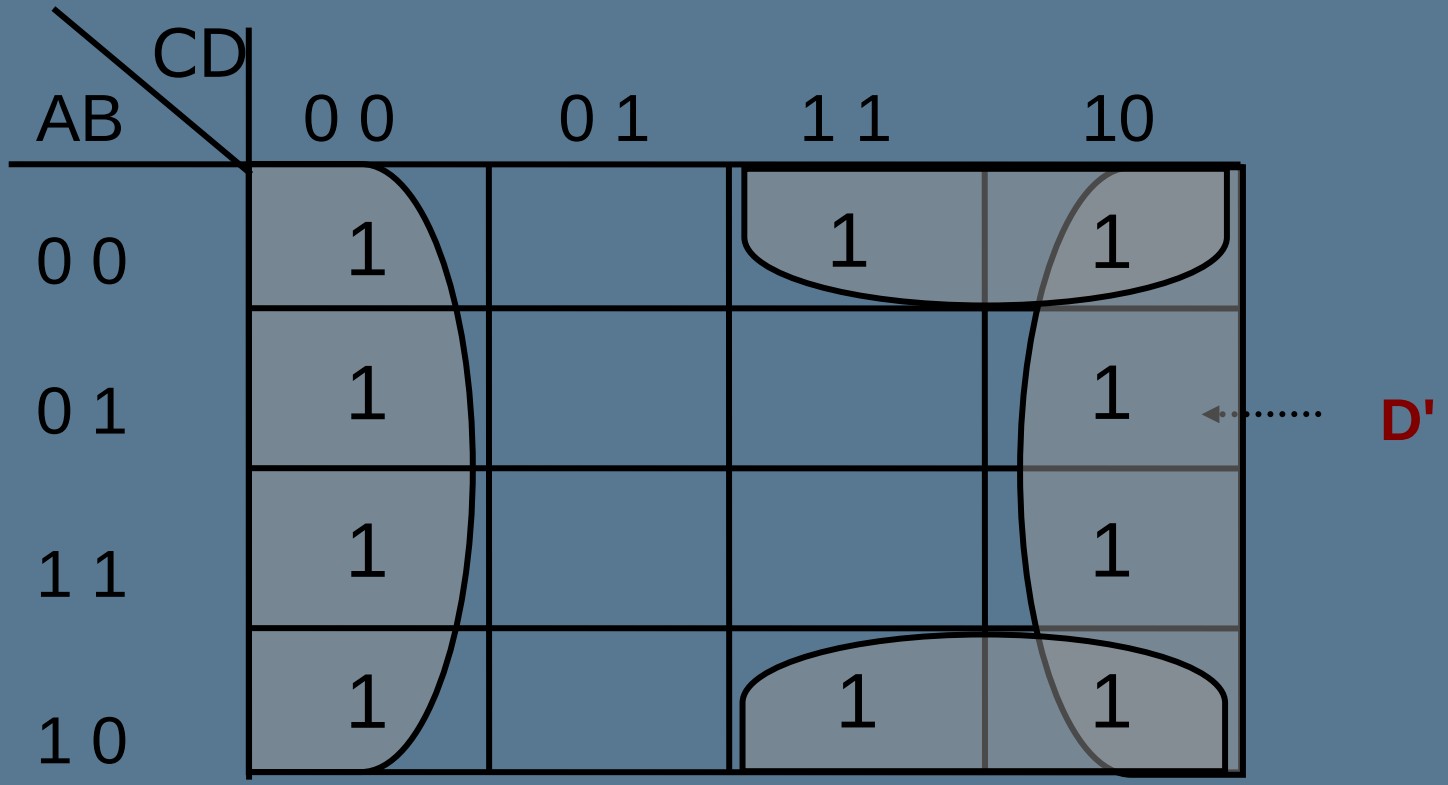
Simplification Example

- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



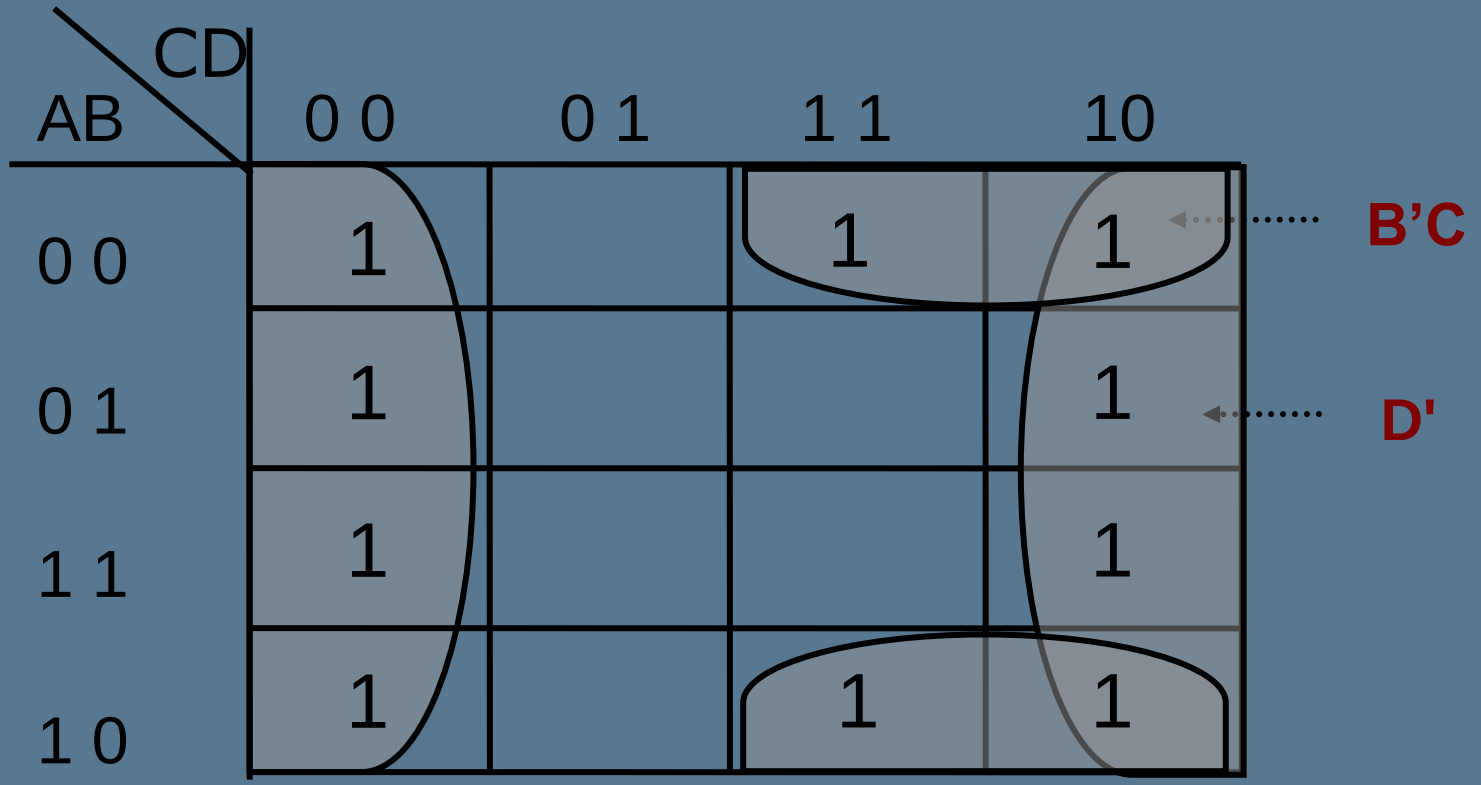
Simplification Example

- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



Simplification Example

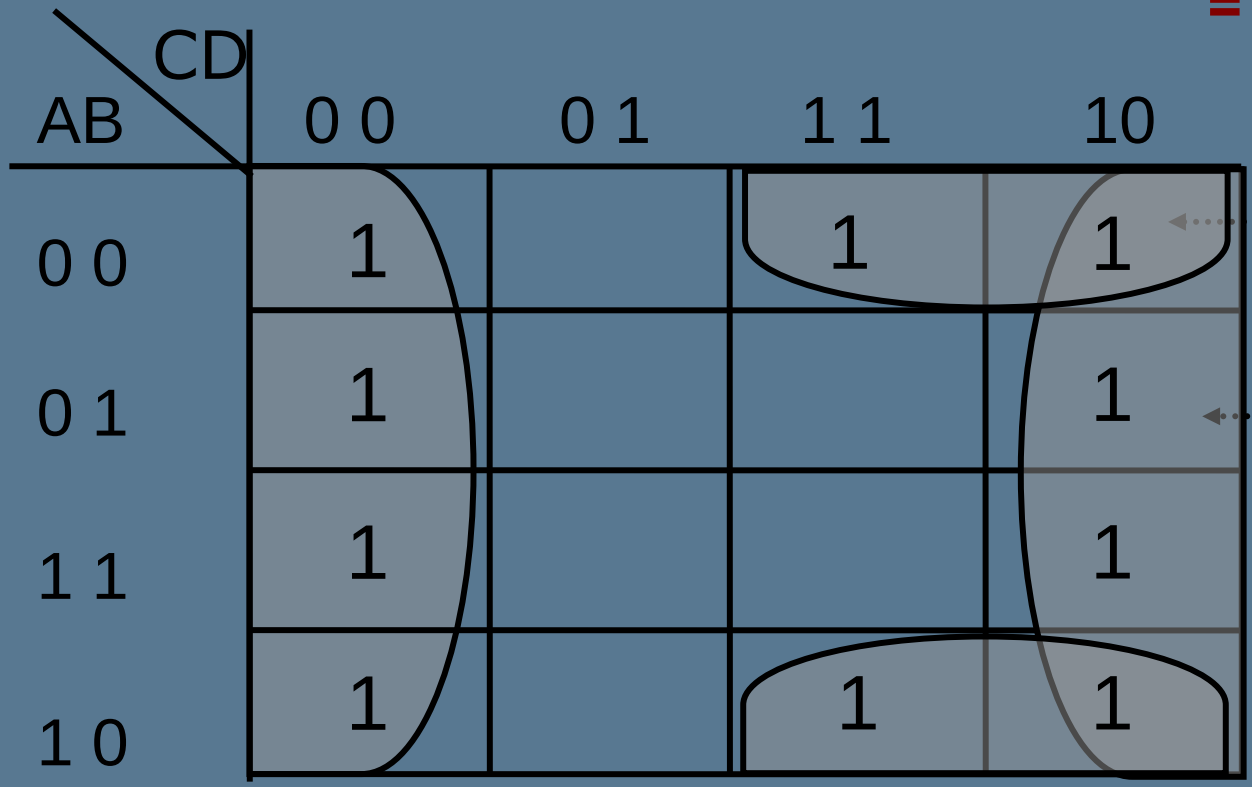
- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



Simplification Example

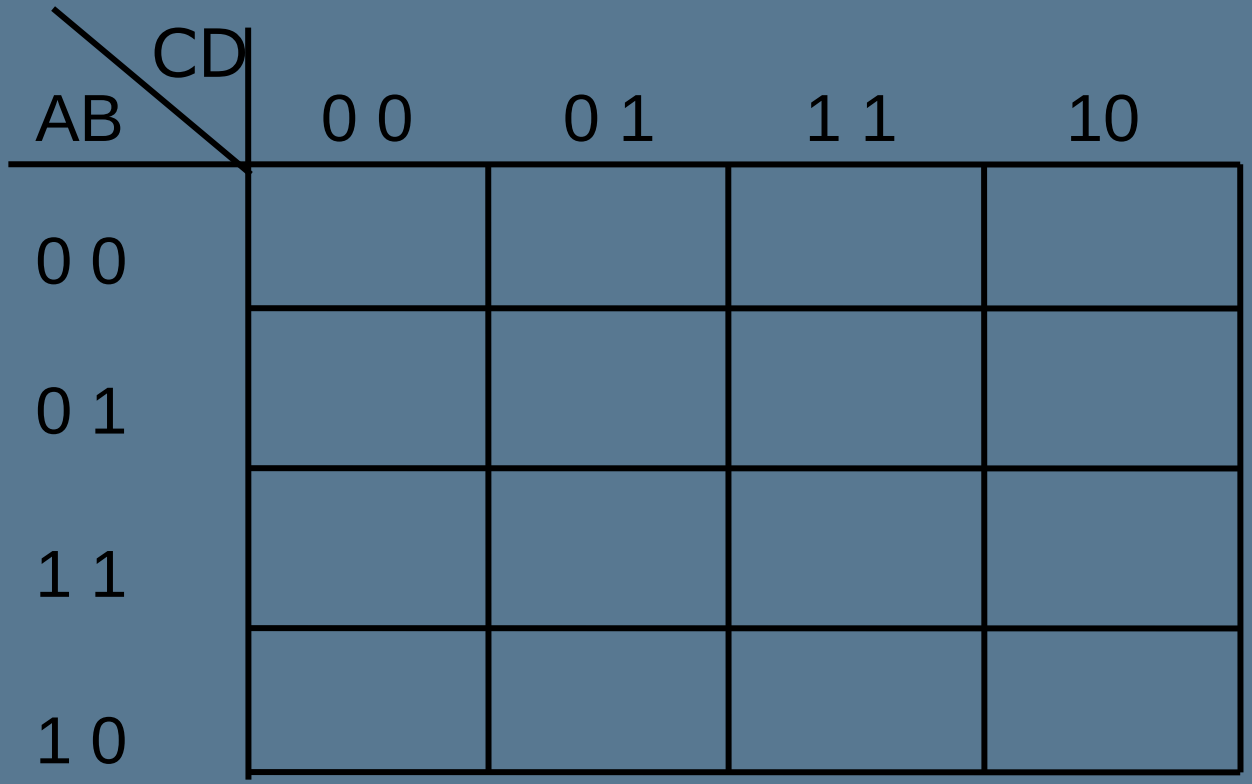
- Simplify B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'

= B'C + D'



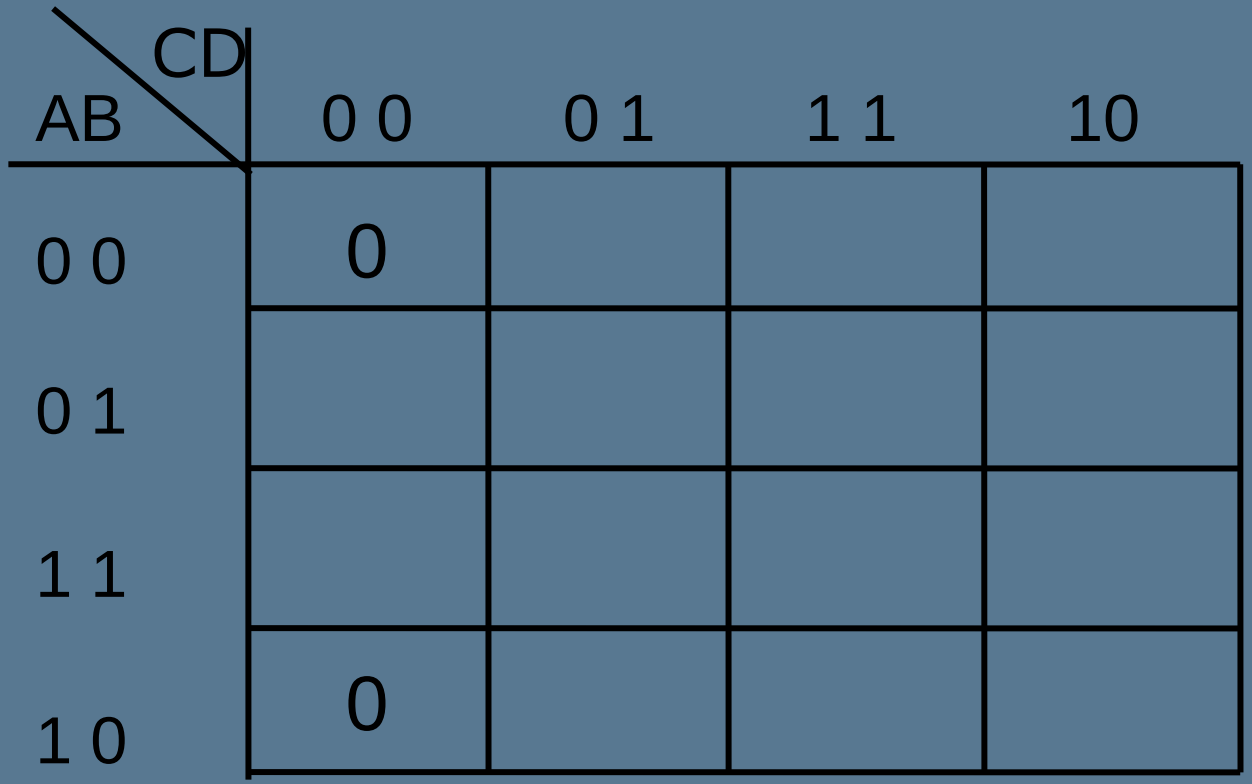
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	0 0	0 1	1 1	10
0 0	0			0
0 1				
1 1				
1 0	0			

Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	0 0	0 1	1 1	10
0 0	0			0
0 1				
1 1				
1 0	0	0		

Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	0 0	0 1	1 1	10
0 0	0			0
0 1	0			
1 1				
1 0	0	0		

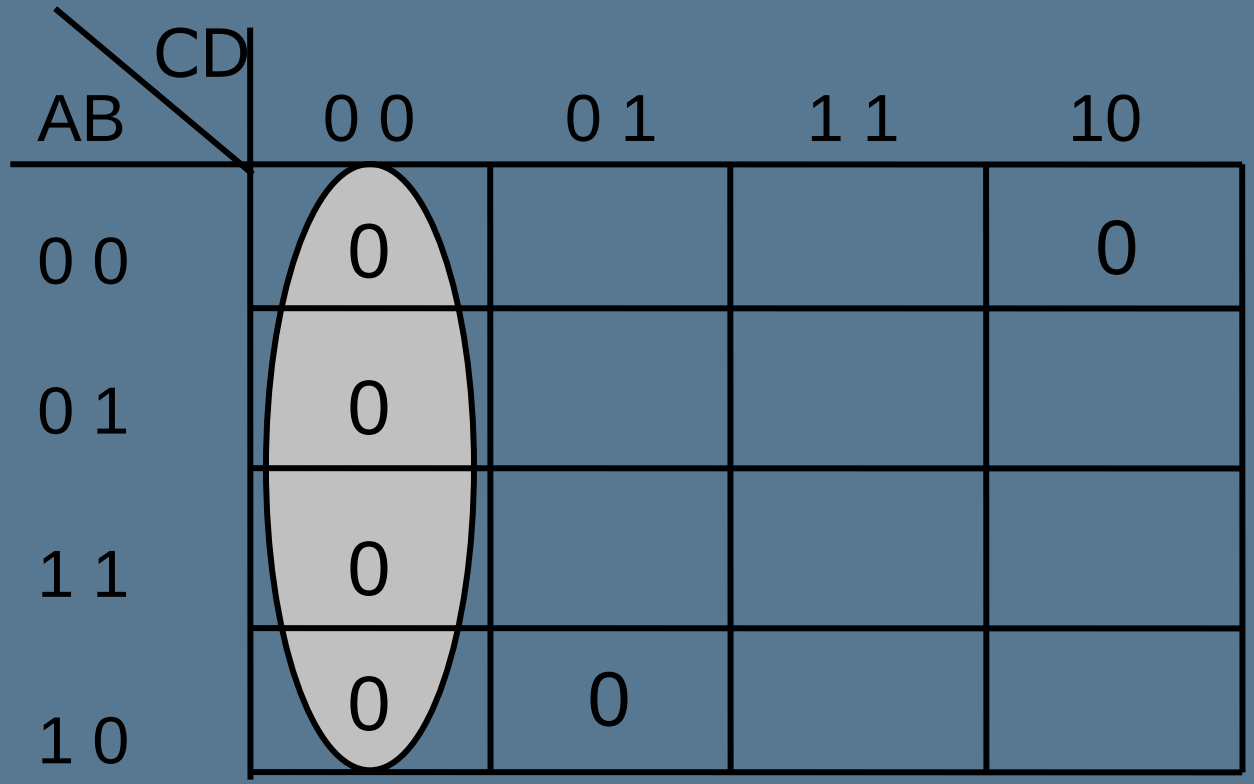
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	0 0	0 1	1 1	10
0 0	0			0
0 1	0			
1 1	0			
1 0	0	0		

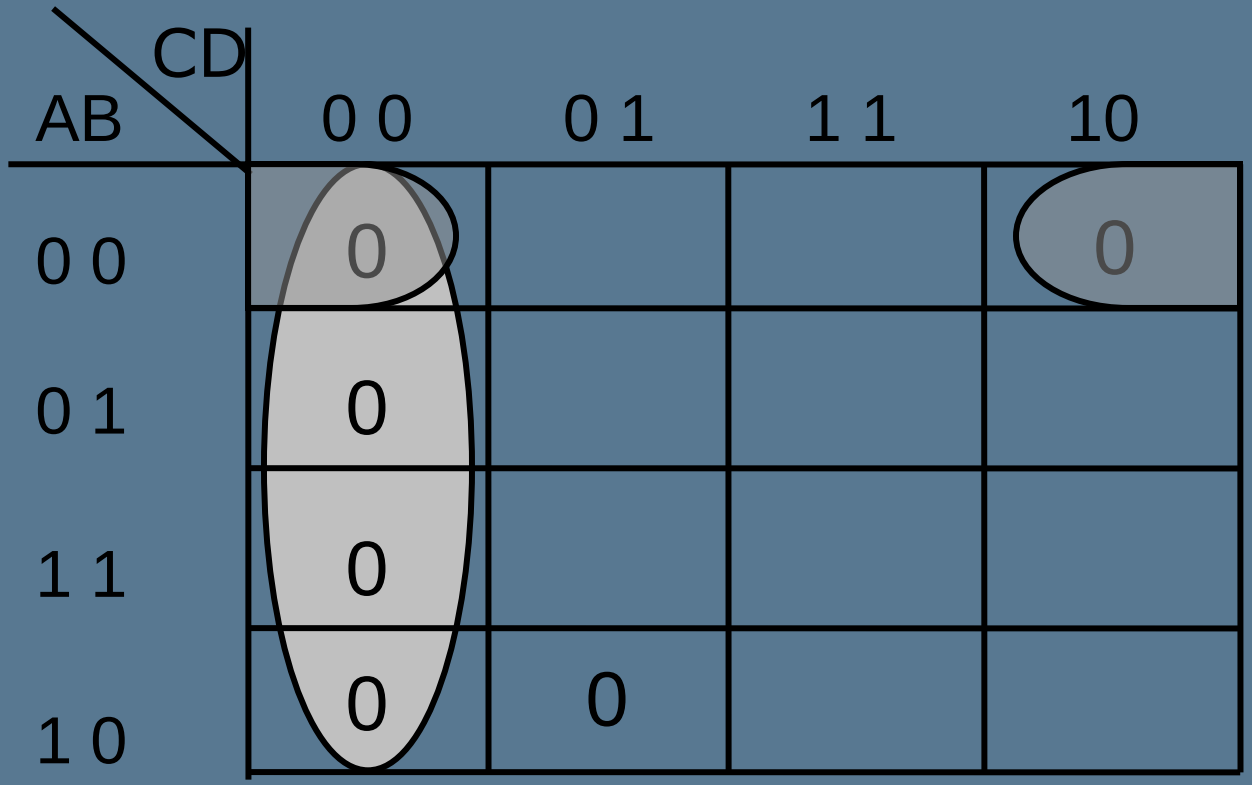
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



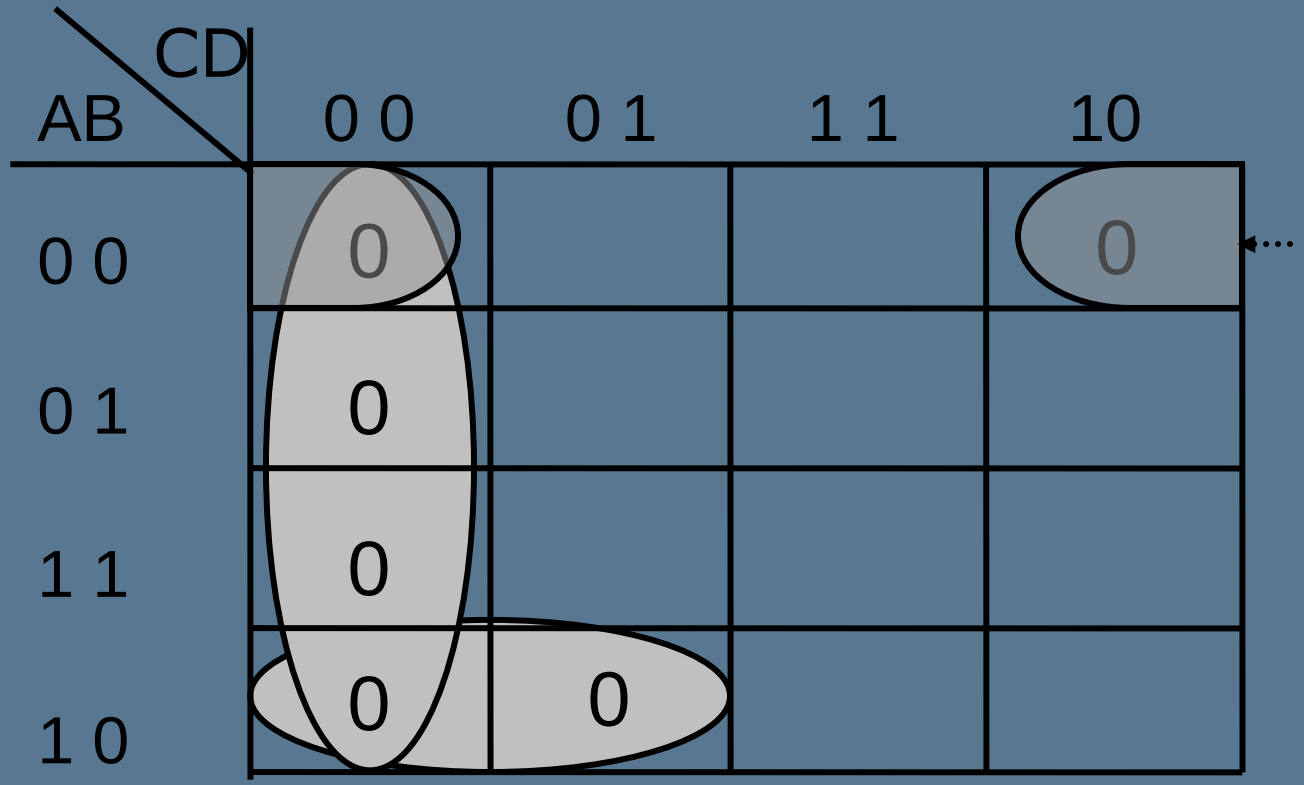
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



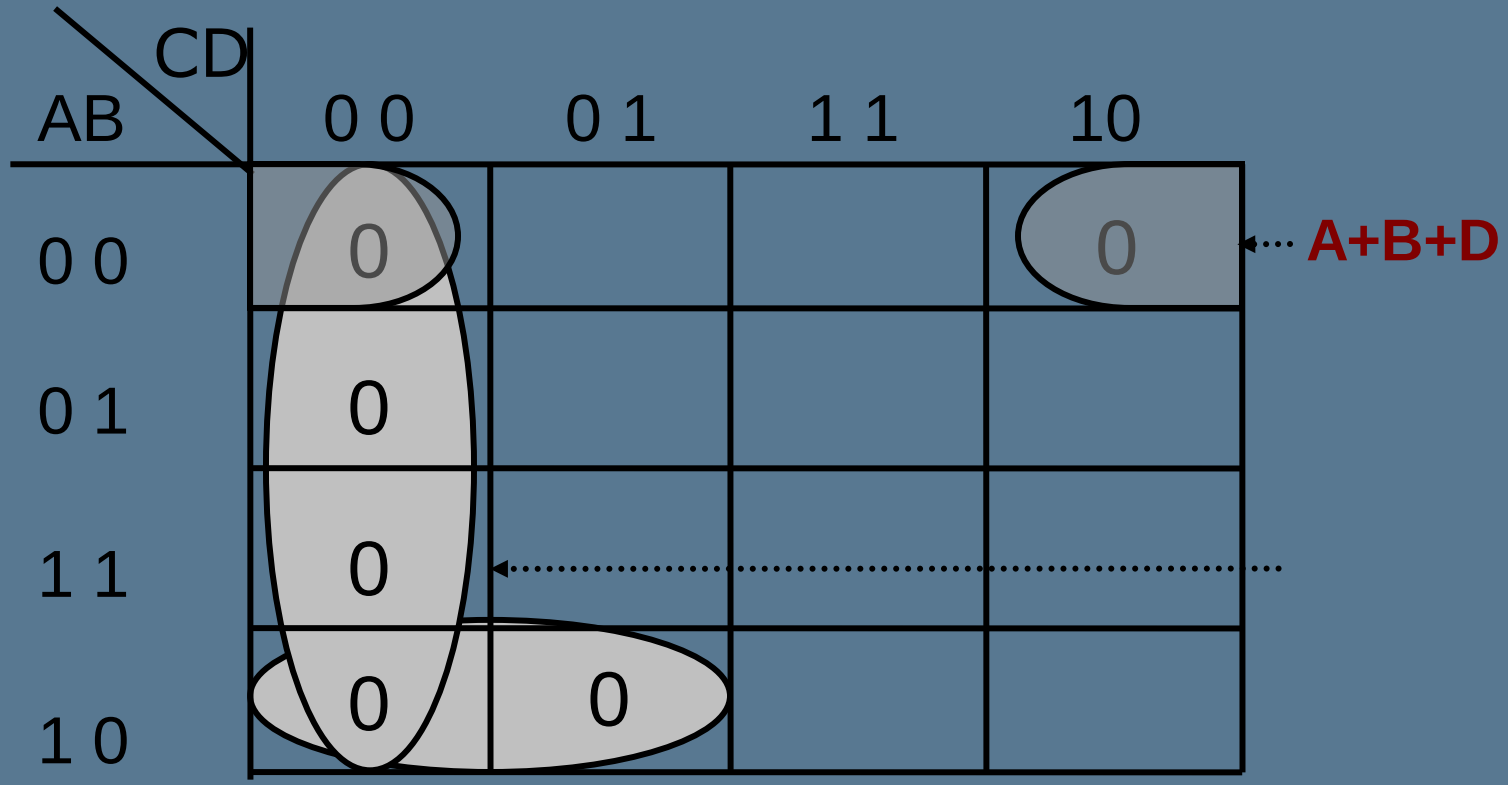
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



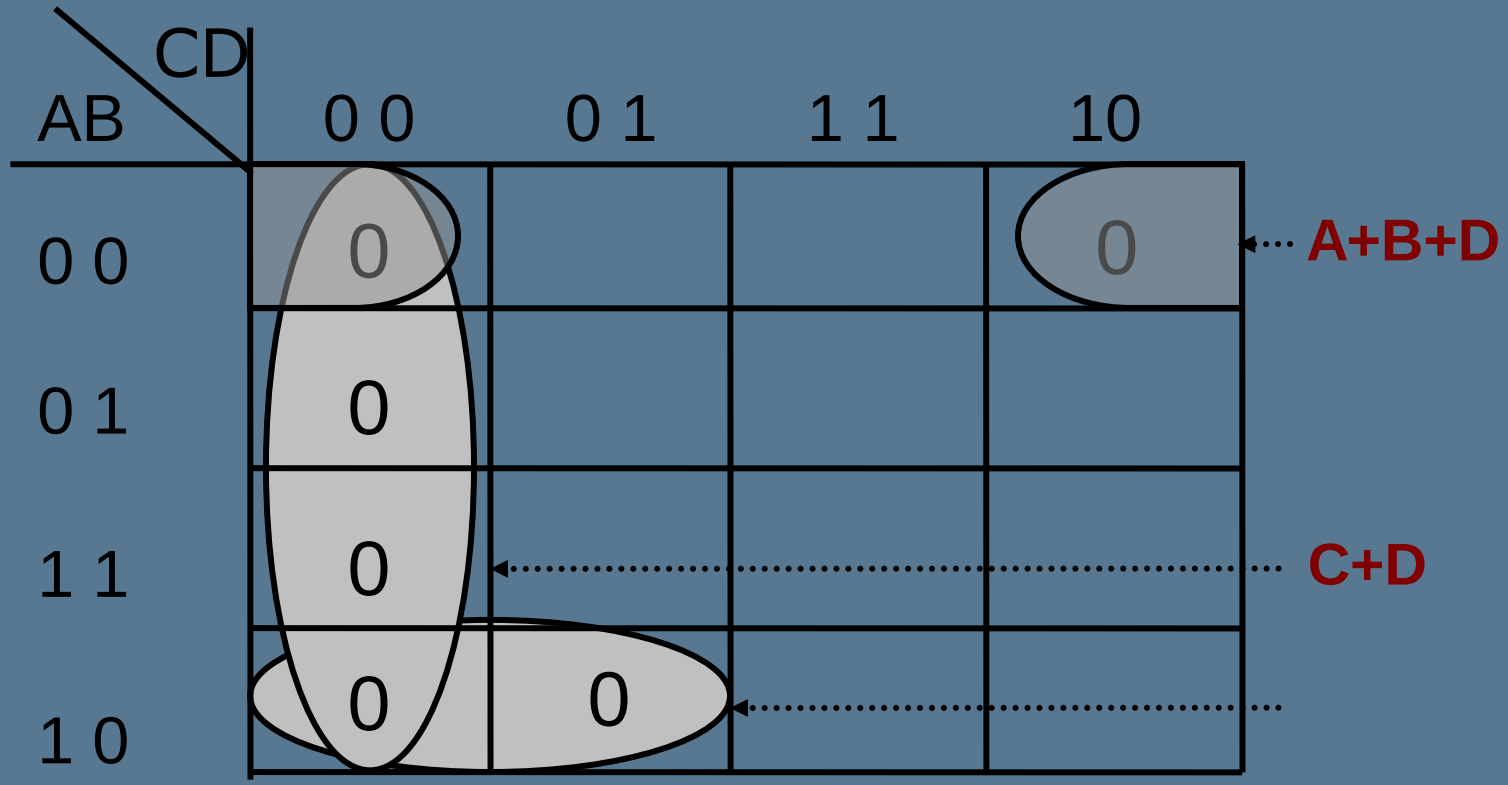
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



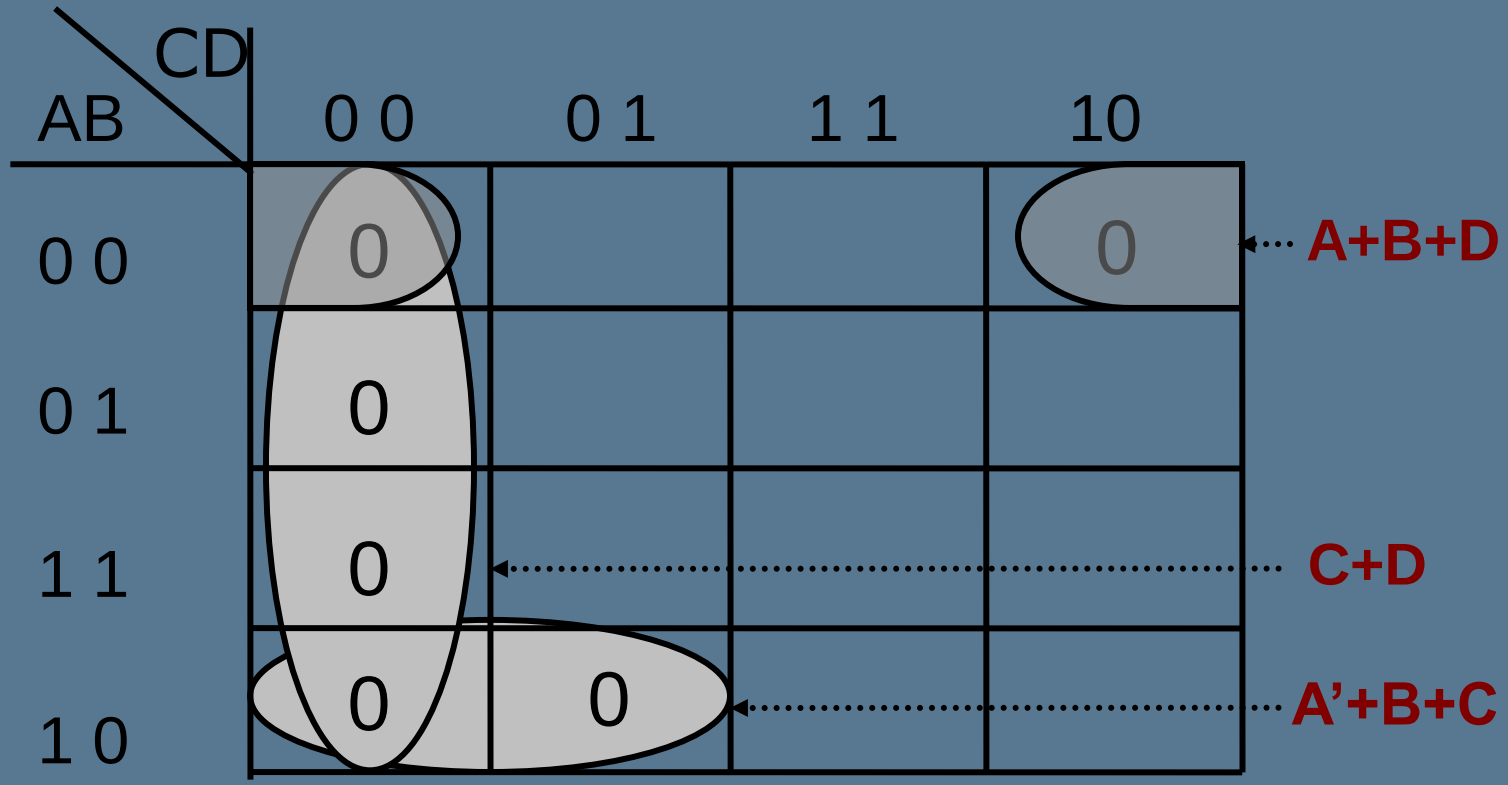
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



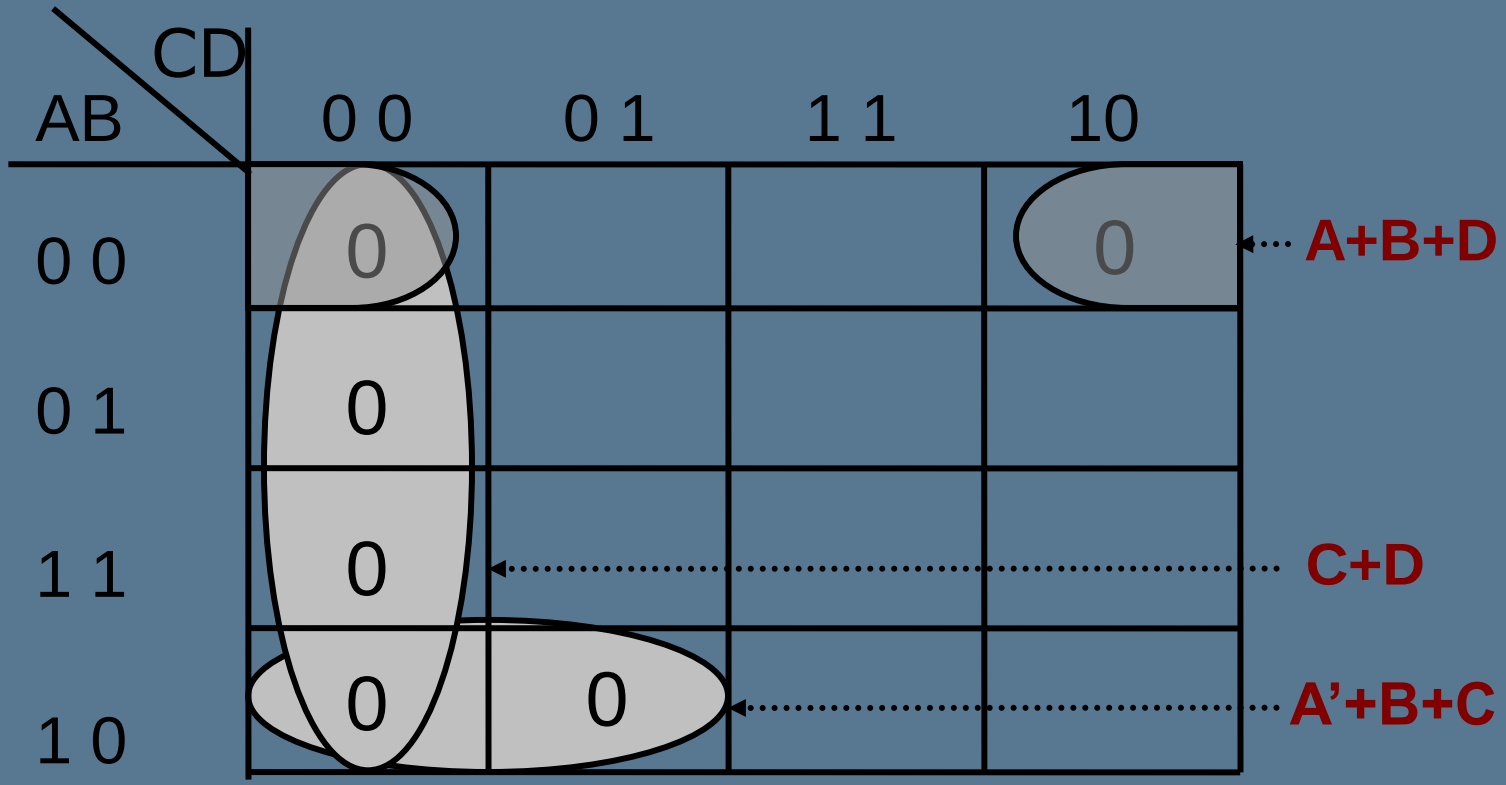
Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



Simplification Example

- Simplify (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D) **=(A+B+D)(C+D)(A'+B+C)**



Advantages of using K-map

- Can be done more systematically
- Much simpler to find minimum solutions
- Easier to see what is happening

Simplification: Quine-McCluskey

- Advantages
  - Specific step by step procedure
  - Can be applied to problems with many variables
  - Suitable for machine computation

Simplification: Quine-McCluskey

- Steps
  - Construct prime implicants table
  - Construct prime implicants chart
  - Select all essential prime implicants
  - Select a minimal cover from the remaining prime implicants

Constructing Prime Implicants Table

- List terms in a column using their binary representation
  - Group terms so that each group contains minterms with the same number of 1's
  - Place groupings which differ by only one literal adjacent to one another

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

1 0001
3 0011
7 0111
8 1000
14 1110
15 1111

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

1	0001
3	0011
7	0111
8	1000
14	1110
15	1111

Column 1

1	0001
8	1000
3	0011
7	0111
14	1110
15	1111

Constructing Prime Implicants Table

- Perform exhaustive search for logically adjacent terms between adjacent groups
  - Each term should be checked off
  - Combine each pair of terms into a single term replacing the differing literal with '-'
  - Repeat procedure until no further terms can be created
  - All unchecked terms are prime implicants

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

		Column 1		Column 2
1	0001	1	0001	
3	0011	8	1000	
7	0111			
8	1000	3	0011	
14	1110			
15	1111	7	0111	
		14	1110	
		15	1111	

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111					
8	1000	3	0011	√		
14	1110					
15	1111	7	0111			
		14	1110			
		15	1111			

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111				3,7	0-11
8	1000	3	0011	√		
14	1110					
15	1111	7	0111	√		
		14	1110			
		15	1111			

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111				3,7	0-11
8	1000	3	0011	√		
14	1110				7,15	-111
15	1111	7	0111	√		
		14	1110			
		15	1111	√		

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111				3,7	0-11
8	1000	3	0011	√		
14	1110				7,15	-111
15	1111	7	0111	√	14,15	111-
		14	1110	√		
		15	1111	√		

Example

- Simplify $F=\sum m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111				3,7	0-11
8	1000	3	0011	√		
14	1110				7,15	-111
15	1111	7	0111	√	14,15	111-
		14	1110	√		
		15	1111	√		

Prime implicants:

AB'C'D'

A'B'D

A'CD

BCD

ABC

Construct Prime Implicants Chart

- Terms are listed horizontally
- Prime implicants are listed vertically
- Place an X whenever a prime implicant covers a minterm

Example

		1	3	7	8	14	15
8	1000						
1,3	00-1						
3,7	0-11						
7,15	-111						
14,15	111-						

Example

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X

Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X

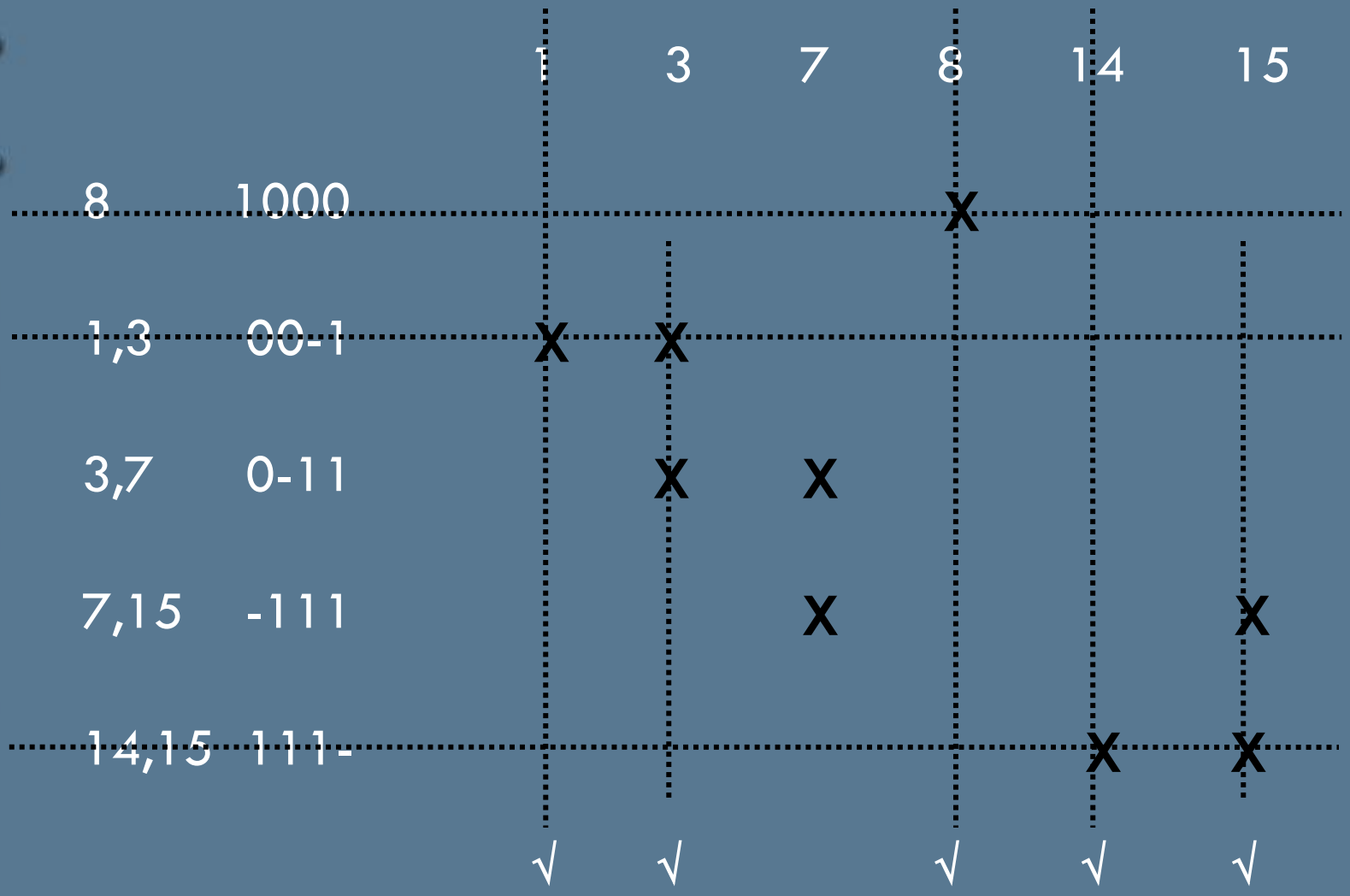
Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X

Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X
		√			√	√	

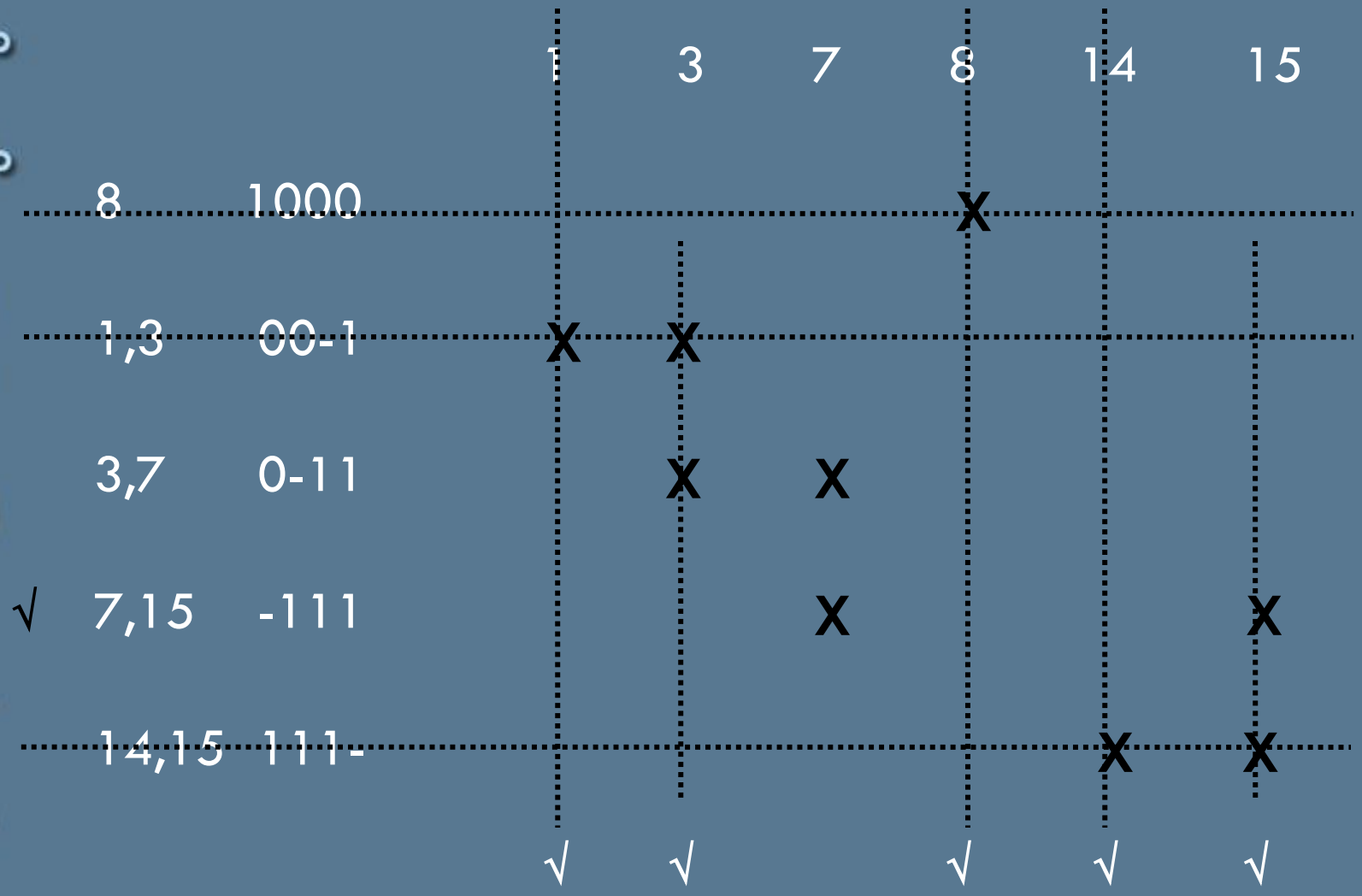
Select Minimum Cover



Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
  - F = AB'C'D' + A'B'D + ABC

Select Minimum Cover



Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
  - F = AB'C'D' + A'B'D + ABC +BCD