CMSC 130 – Logic Design and Digital Computer Circuits
Handout # 3: LOGIC FUNCTIONS, DIAGRAMS AND BOOLEAN ALGEBRA

Theorems of Boolean Algebra

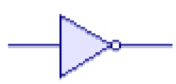
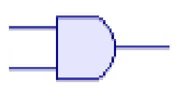
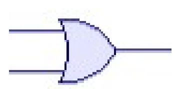
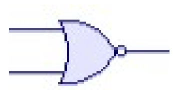
Identity	X + 0 = X	X • 1 = X
Contra-identity	X + 1 = 1	X • 0 = 0
Inverse	X + X'= 1	X • X'= 0
Idempotency	X + X = X	X • X = X
Involution	(X')' = X	
Commutativity	X + Y = Y + X	XY = YX
Associativity	X +(Y + Z)= (X + Y) + Z	X(YZ)= (XY)Z
Distributivity	X (Y + Z)= XY + XZ	X + YZ= (X + Y)(X + Z)
De Morgan's	(X + Y)'=X'Y'	(XY)'=X' + Y'
Absorption	X + XY= X	X(X + Y)= X

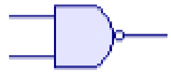
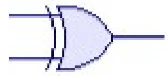
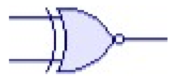
Operator Precedence: (), NOT, AND, OR (*from highest to lowest*)

Minterms and Maxterms for three binary variables:

			Minterms		Maxterms	
X	Y	Z	Term	Designation	Term	Designation
0	0	0	X'Y'Z'	m_0	X + Y + Z	M_0
0	0	1	X'Y'Z	m_1	X + Y + Z'	M_1
0	1	0	X'Y Z'	m_2	X + Y'+ Z	M_2
0	1	1	X'Y Z	m_3	X + Y'+ Z'	M_3
1	0	0	X Y'Z'	m_4	X'+ Y + Z	M_4
1	0	1	X Y'Z	m_5	X'+ Y + Z'	M_5
1	1	0	X Y Z'	m_6	X'+ Y'+ Z	M_6
1	1	1	X Y Z	m_7	X'+ Y'+ Z'	M_7

Operators and Logic Gates

Name	Algebraic Function	Symbol	Truth Table
NOT	F= x'		x F 0 1 1 0
AND	F= xy		x y F 0 0 0 0 1 0 1 0 0 1 1 1
OR	F= x + y		x y F 1 0 0 0 1 1 1 0 1 1 1 1
NOR	F= (x + y)'		x y F 0 0 1 0 1 0 1 0 0 1 1 0

Name	Algebraic Function	Symbol	Truth Table
NAND	F= (xy)’		x y F 0 0 1 0 1 1 1 0 1 1 1 0
XOR	F= xy’+ x’y = x ⊕ y		x y F 0 0 0 0 1 1 1 0 1 1 1 0
XNOR	F= xy + x’y’= x ⊙ y		x y F 0 0 1 0 1 0 1 0 0 1 1 1

Example1: Assuming w=0, x=0, y=1, and z=1, Find the resulting value of the following function: F= (wx)’ + y’z’ +(x+z) + (x’ + w’)’.

Solution:

F= (wx)’ + y’z’ +(x+z) + (x’ + w’)’
F= (0•0)’+(1’•1’)+(0+1)’+(0’+0’)’
F= (0)’ + (0•0)+ (1)’ +(1+1)’
F= 1 + 0 + 0 + 0
F= 1

Example2: Construct the truth table for the following function.
F= x’ + x’y + x’z.

Solution:

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Example3: Write the expressions (in Canonical form) for the following truth table. Then construct the corresponding logic circuits for them.

X	Y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Solutions:

a. Minterms: F = x’y’z + x’yz’+xy’z’+xy’z
= Σ(m1,m2,m4,m5)

b. Maxterms:
F = (x+y+z)(x+y’+z’)(x’+y’+z)(x’+y’+z’)
= Π(M0,M3,M6,M7)

Example4: Prove or disprove the following Boolean equation.

$$xy + (x'y') + x'yz = xyz' + (x'y') + yz$$

Solution:

$xy(1) + x'y' + x'yz$	identity
$xy(z + z') + x'y' + x'yz$	inverse
$xyz + xyz' + x'y' + x'yz$	distributivity
$xyz' + xyz + x'yz + x'y'$	commutativity
$xyz' + yz(x + x') + x'y'$	distributivity
$xyz' + yz(1) + x'y'$	inverse
$xyz' + yz + x'y'$	identity
$xyz' + x'y' + yz$	commutativity

Example5: Use Boolean Algebra to simplify the following expression. Indicate the theorem used for each step.

$$(x'y') + (x'y) + (yz') + (xz')$$

Solution:

$x'(y' + y) + (yz') + (xz')$	distributivity
$x'(1) + yz' + xz'$	inverse
$x' + yz' + xz'$	identity
$x' + xz' + yz'$	commutativity
$(x' + x)(x' + z') + yz'$	distributivity
$(1)(x' + z') + yz'$	inverse
$(x' + z') + yz'$	identity
$x' + (z' + yz')$	associativity
$x' + z'(1 + y)$	distributivity
$x' + z'(1)$	contra-identity
$x' + z'$	identity