



LIMIT COMPARISON TEST

Let $\sum_{n=1}^{+\infty} u_n$ and $\sum_{n=1}^{+\infty} v_n$ be two series of positive terms and $L = \lim_{n \to \infty} \frac{u_n}{v_n}$ a. If L>0, then the two series either both converge or both diverge.

b. If L=0 and $\sum_{n=1}^{+\infty} v_n$ is convergent, then $\sum_{n=1}^{+\infty} u_n$ is also convergent

c. If L= ∞ and $\sum_{n=1}^{+\infty} v_n$ is divergent, then $\sum_{n=1}^{+\infty} u_n$ is also divergent

Solution
$$let \ u_n = \frac{n}{4n^4 - 3} \quad v_n = \frac{1}{n^3}$$

$$\lim_{x \to \infty} \frac{u_n}{v_n} = \lim_{x \to \infty} \left(\frac{n}{4n^4 - 3}\right) \left(\frac{n^3}{1}\right) = \frac{1}{4} > 0$$
Since
$$\sum_{n=1}^{+\infty} \frac{1}{n^3} \text{ is convergent, then } \sum_{n=1}^{+\infty} \frac{n}{4n^4 - 3}$$
is also convergent.

ILLUSTRATION
$$\sum_{n=1}^{+\infty} \frac{5n^3}{3n^4 + n}$$
Solution
$$let \ u_n = \frac{5n^3}{3n^4 + n} \quad v_n = \frac{1}{n}$$

$$\lim_{x \to \infty} \frac{u_n}{v_n} = \lim_{x \to \infty} \left(\frac{5n^3}{3n^4 + n}\right) \left(\frac{n}{1}\right) = \frac{5}{3} > 0$$
Since
$$\sum_{n=1}^{+\infty} \frac{1}{n}$$
 is divergent, then
$$\sum_{n=1}^{+\infty} \frac{5n^3}{3n^4 + n}$$
 is also divergent.

Alternating series

If
$$u_n > 0$$
 for each natural number n , then the following are alternating series

$$\sum_{n=1}^{\infty} (-1)^n u_n = -u_1 + u_2 - u_3 + \dots$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - \dots$$

1.
$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n}$$
 alternating harmonic series
2.
$$\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2}{3^{n-1}}$$

Alternating Series Test

Given
$$\sum_{n=1}^{\infty} (-1)^n u_n$$
 or $\sum_{n=1}^{\infty} (-1)^{n+1} u_n$

If i. $u_n > u_{n+1}$ for each n

ii. $\lim_{n \to +\infty} u_n = 0$

then the series is convergent.

Illustration: Show convergence. Conclude properly.

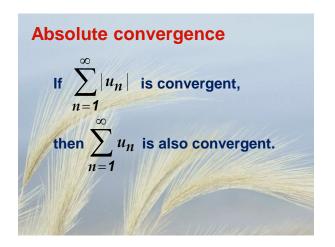
$$\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2}{3^{n-1}}$$

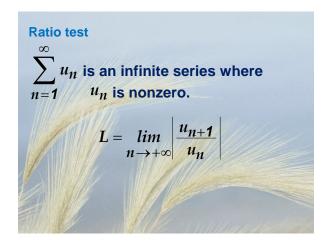
$$\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2}{3^{n-1}}$$
Solution
$$u_n = \frac{2}{3^{n-1}} \qquad since \ u_n = \frac{2}{3^{n-1}} > \frac{2}{3^n} = u_{n+1}$$
and $\lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{2}{3^{n-1}} = 0$
By AST, $\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2}{3^{n-1}}$ is convergent.

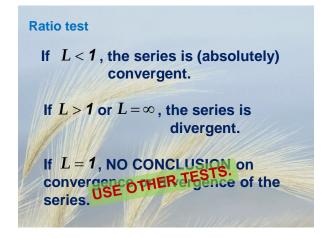
ASIDE ... (other approach)
$$\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2}{3^{n-1}} = \sum_{n=1}^{+\infty} (-1)^{n-1} \frac{2}{3^{n-1}}$$

$$= \sum_{n=1}^{+\infty} 2 \cdot \left(-\frac{1}{3}\right)^{n-1}$$

$$= \frac{a}{1-r} = \frac{3}{2}$$







Example. Test for convergence. $\sum_{n=1}^{+\infty} \frac{1}{n!}$ Example. Test for convergence. $\sum_{n=1}^{+\infty} (-1)^n \frac{2^n}{n!}$

