Chapter 2

Computer Arithmetic (Part 2)

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Recall: Binary Addition

- To add binary numbers: (X + Y)
  - Get the SCR of the negative numbers
  - Add the two numbers
  - If the SCR used is:
    - 2's C: Discard end carry
    - 1's C: Add the end carry to the sum

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Binary Subtraction

- To subtract binary numbers: (X – Y)
  - Take the complement of the subtrahend.
  - Then, add the two numbers.
  - If the complement used is:
    - 2's C: Discard end carry
    - 1's C: Add the end carry to the sum

(X – Y) >>> X + (complement of Y)

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Example: Binary Subtraction

- Subtract the following numbers. Use 8 bits to represent each number.
  - 6 -13
  - 6 – (-13)
  - (-6) -13
  - (-6) – (-13)

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Example: Binary Subtraction

- 6 – 13 = –7

0 0000110

\+ 1 1110010 (1's)

---

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Example: Binary Subtraction

- 6 – 13 = –7

$$
\begin{array}{rl}
 & 0\ 0000110 \\
+ & 1\ 1110010 \quad (1\text{'s}) \\
\hline
 & 1\ 1111000
\end{array}
$$

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Example: Binary Subtraction

- $6 - 13 = -7$

```
  0 0000110
+ 1 1110010   (1's)
-------------
  1 1111000
```

- $6 - (-13) = 19$

```
  0 0000110
+ 0 0001101
-----------
```

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Example: Binary Subtraction

- $6 - 13 = -7$

$$
\begin{array}{rr}
 & 0\ 0000110 \\
+ & 1\ 1110010 \quad (1\text{'s}) \\
\hline
 & 1\ 1111000
\end{array}
$$

- $6 - (-13) = 19$

$$
\begin{array}{rr}
 & 0\ 0000110 \\
+ & 0\ 0001101 \\
\hline
 & 0\ 0010011
\end{array}
$$

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Example: Binary Subtraction

- (–6) – 13 = –19

$$\begin{array}{rl} & 1\ 1111010 \quad (2\text{'s}) \\ + & 1\ 1110011 \quad (2\text{'s}) \\ \hline \end{array}$$

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Example: Binary Subtraction

- (–6) – 13 = –19

$$
\begin{array}{ll}
 & 1\ 1111010 \quad (2\text{'s}) \\
+ & 1\ 1110011 \quad (2\text{'s}) \\
\hline
 & 11\ 1101101
\end{array}
$$

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Example: Binary Subtraction

- (–6) –13 = –19

```
  1 1111010   (2's)
+ 1 1110011   (2's)
-------------------
 11 1101101
```

- (–6) – (–13) = 7

```
  1 1111001   (1's)
+ 0 0001101
-----------
```

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Example: Binary Subtraction

- (–6) –13 = –19

```
  1 1111010   (2's)
+ 1 1110011   (2's)
-------------------
 11 1101101
```

- (–6) – (–13) = 7

```
  1 1111001   (1's)
+ 0 0001101
-------------------
 10 0000110
```

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Example: Binary Subtraction

- $(-6) - 13 = -19$

$$
\begin{array}{rl}
 & 1\ 1111010 \quad (2\text{'s}) \\
+ & 1\ 1110011 \quad (2\text{'s}) \\
\hline
 & 1\ 1101101
\end{array}
$$

- $(-6) - (-13) = 7$

$$
\begin{array}{rl}
 & 1\ 1111001 \quad (1\text{'s}) \\
+ & 0\ 0001101 \\
\hline
 & 10\ 0000110 \\
+ & \longrightarrow\ 1 \\
\hline
\end{array}
$$

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Example: Binary Subtraction

- (–6) –13 = –19

```
  1 1111010   (2's)
+ 1 1110011   (2's)
-------------------
 11 1101101
```

- (–6) – (–13) = 7

```
  1 1111001   (1's)
+ 0 0001101
-------------------
 10 0000110
+           1
-------------------
  0 0000111
```

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Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$\begin{array}{r} 999.5 \\ + \\ \hline \end{array}$$

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Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$\begin{array}{rr} & 999.5 \\ + & 718.4 \\ \hline \end{array}$$

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Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$\begin{array}{r} 999.5 \\ +\ 718.4 \\ \hline 1717.9 \end{array}$$

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Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$
\begin{array}{r}
999.5 \\
+\ 718.4 \\
\hline
1717.9
\end{array}
$$

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Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$\begin{array}{r} 999.5 \\ +\ 718.4 \\ \hline 1717.9 \end{array}$$

- $(00000110.11 - 00101010.11)_2$

$$\begin{array}{r} 00000110.11 \\ + \\ \hline \end{array}$$

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Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$\begin{array}{r} 999.5 \\ +\ 718.4 \\ \hline 1717.9 \end{array}$$

- $(00000110.11 - 00101010.11)_2$

$$\begin{array}{r} 00000110.11 \\ +\ 11010101.00 \\ \hline \end{array}$$

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Examples: Subtraction

- $(999.5 - 281.6)_{10}$

$$\begin{array}{r} 999.5 \\ +\ 718.4 \\ \hline 1717.9 \end{array}$$

- $(00000110.11 - 00101010.11)_2$

$$\begin{array}{r} 00000110.11 \\ +\ 11010101.00 \\ \hline 11011011.11 \end{array}$$

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Examples: Subtraction

- $(355.45 - 240.664)_8$

  355.45

  +

Examples: Subtraction

- $(355.45 - 240.664)_8$

$$\begin{array}{rr} & 355.45 \\ + & 537.114 \\ \hline \end{array}$$

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Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rr}
 & 355.45 \\
+ & 537.114 \\
\hline
 & .564
\end{array}
$}

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Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rr}
 & 355.45 \\
+ & 537.114 \\
\hline
 & 1114.564
\end{array}
$$

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Examples: Subtraction

- $(355.45 - 240.664)_8$

$$
\begin{array}{rr}
 & 355.45 \\
+ & 537.114 \\
\hline
 & 1114.564
\end{array}
$$

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Examples: Subtraction

- $(355.45 – 240.664)_8$

$$\begin{array}{rr} & 355.45 \\ + & 537.114 \\ \hline & 1114.564 \end{array}$$

- $(A0C.D – E72.9)_{16}$

$$\begin{array}{rr} & A0C.D \\ + & \\ \hline & \end{array}$$

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Examples: Subtraction

- $(355.45 – 240.664)_8$

$$\begin{array}{r} 355.45 \\ +\ 537.114 \\ \hline 1114.564 \end{array}$$

- $(A0C.D – E72.9)_{16}$

$$\begin{array}{r} A0C.D \\ +\ 18D.6 \\ \hline \end{array}$$

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Examples: Subtraction

- $(355.45 – 240.664)_8$

$$\begin{array}{rr} & 355.45 \\ + & 537.114 \\ \hline & 1114.564 \end{array}$$

- $(A0C.D – E72.9)_{16}$

$$\begin{array}{rr} & A0C.D \\ + & 18D.6 \\ \hline & .3 \end{array}$$

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Examples: Subtraction

- $(355.45 – 240.664)_8$

$$\begin{array}{rr} & 355.45 \\ + & 537.114 \\ \hline & 1114.564 \end{array}$$

- $(A0C.D – E72.9)_{16}$

$$\begin{array}{rr} & A0C.D \\ + & 18D.6 \\ \hline & B9A.3 \end{array}$$

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Error Detection

- Overflow
  - occurs when an arithmetic operation yields a result that is **greater than the range's positive limit**.
- Example:

+5	0 101	
+6	0 110	
−5	1 011	(2's)

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Error Detection

- Underflow
  - occurs when an arithmetic operation yields a result that is **lesser than the range's negative limit**.
- Example:

-3	1 101
-6	1 010
+7	0 111

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Error Detection

- *Simple Rule:* An addition operation produces an error if the signs of the addends are the same and the sign of the sum is different from the addend's sign.
- *OR:* An error occurs when the last carry-in is not equal to the carry-in is not equal to the carry-out (end-carry).

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Error Detection

- Example: Use 1's C for negative values

$$
\begin{array}{rr}
 & 9 + 5 \\
 & 0\ 1001 \\
+ & 0\ 0101 \\
\hline
 & 0\ 1110
\end{array}
$$

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Error Detection

- Example: Use 1's C for negative values

```
  9 + 5
  0 1001
+ 0 0101
-------
  0 1110
```

No error!

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Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7
0 1001	0 1101
+ 0 0101	+ 0 0111
0 1110	1 0100

No error!

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Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7
0 1001	0 1101
+ 0 0101	+ 0 0111
0 1110	1 0100
No error!	**Overflow!**

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Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7	−9 + −9
0 1001	0 1101	1 0110
+ 0 0101	+ 0 0111	+ 1 0110
0 1110	1 0100	0 1100
No error!	**Overflow!**	

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Error Detection

- Example: Use 1's C for negative values

9 + 5	13 + 7	−9 + −9
0 1001	0 1101	1 0110
+ 0 0101	+ 0 0111	+ 1 0110
0 1110	1 0100	0 1100
No error!	**Overflow!**	**Underflow!**

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BCD Addition

- Sum less than or equal to 9
  - Normal binary addition
- Sum greater than 9
  - Add the codes
  - Add a correction value of 0110 to any sum

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Example: BCD Addition

45 + 55 = 100

45 = 0100 0101

55 = 0101 0101

100 = 1001 1010

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Example: BCD Addition

45 + 55 = 100

45 = 0100 0101

55 = 0101 0101

100 = 1001 1010

\+ 0110

1010 0000

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Example: BCD Addition

45 + 55 = 100

45 = 0100 0101

55 = 0101 0101

100 = 1001 1010

\+ 0110

1010 0000

\+ 0110

0001 0000 0000

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Example: BCD Addition

19 + 65 = 84

19 = 0001 1001

65 = 0110 0101

84 = 0111 1110

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Example: BCD Addition

19 + 65 = 84

19 = 0001 1001

65 = 0110 0101

84 = 0111 1110

\+ 0110

1000 0100

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