CMSC 57 Discrete Mathematical Structures in Computer Science II
Exercise 8. Random Variables and Probability Distribution

A. Identify whether the described random variable is valid or not. If it is invalid, explain why it is invalid.

For numbers 1-3, consider a random experiment of playing two songs from a playlist.
1. Let random variable X be the average length of the two songs in minutes rounded up to the nearest whole number.
   Valid
2. Let random variable Y=0 if the songs played are from the same artist/band, Y=1 if otherwise.
   Valid
3. Let random variable Z=0 if at least one song is rock, Z=1 if at least one song is not rock.
   Invalid - we can assign both zero and one if one is rock and the other is not rock
4. Consider the random experiment of counting the number of people wearing red everyday. Let C=0 if the number of people wearing red is between 0 and 50, C=1 if the count is between 51 and 100.
   Invalid - no assignment to values exceeding to one hundred

B. Compute the probability of each specific value and show the probability distribution for the random variable.
5. In a random experiment or answering a ten-question multiple choice exam with four choices per question. We define a random variable C for counting the number of correct answers. Fill up the table below.
   a. Which c has the greatest chance of occuring? **P(C = 2)**
   b. What is the probability of getting at least 8 incorrect answers? **P (C = 8,9,10) 0.0004**
   c. What is the probability of getting 1,2, or 3 correct answers? **P (C = 1,2,3) 0.719**

C	0	1	2	3	4	5
P(C=c)	$3^{10}/4^{10}$	$\dfrac{C(10,1)3^9}{4^{10}}$	$\dfrac{C(10,2)3^8}{4^{10}}$	$\dfrac{C(10,3)3^7}{4^{10}}$	$\dfrac{C(10,4)3^6}{4^{10}}$	$\dfrac{C(10,5)3^5}{4^{10}}$
C	6	7	8	9	10	
P(C=c)	$\dfrac{C(10,6)3^4}{4^{10}}$	$\dfrac{C(10,7)3^3}{4^{10}}$	$\dfrac{C(10,8)3^2}{4^{10}}$	$\dfrac{C(10,9)3^1}{4^{10}}$	$\dfrac{C(10,10)3^0}{4^{10}}$	

C. Use some known probability distributions to answer the following problems:

6. In a certain experiment, it was found that out of 160 six-year olds, 32 have mastered how to do the "cups" which was popularized by Anna Kendrick in the movie *Pitch Perfect*. 16 can sing the cup song perfectly, and it is known that there is 25% probability that a kid can sing the cup song perfectly given that he/she has mastered the "cups". If you were to train a kid named Jhai on how to perform the "cup song", what is the probability that she can both do the "cups" and sing the cup song on the 15th try?
   p = P(mastered_cups&sing) = p(mastered_cups) * p(sing|mastered_cups)

p = 32/160 * 0.25

p = 0.2 * 0.25 = 0.05;   1-p = 0.95; k = 15 $G(X=15;0.05) = (0.95)^{14}(0.05)$

7. In a ballet class of 60, 12 are known as exceptional ballerinas. The school will be conducting a recital at the end of the month. The principal randomly chose eight students from the class to perform. What is the probability that this group will include 2 exceptional ballerinas?

$$\frac{\underline{C(12,2)\ C(48,6)}}{C(60,8)}$$

8. Kate is listening to a radio program wherein songs from a single artist are shuffled and played. Tonight's artist is Taylor Swift, and she has a total of 125 songs. Kate, being broken-hearted and emotional, hopes that the songs "Back to December" or "Breathe" will be played. If all songs can be replayed,

   A. what is the probability that "Back to December" or "Breathe" will be played as the fifth song?

   $2*((124/125)^4(1/125))$

   B. and if there are a total of 50 songs played in the program, what is the probability that "All Too Well" will be played thrice?

   $C(50,3)(1/125)^3(124/125)^{47}$

9. Khris is practicing for a target archery competition. Based on her previous stats, she has 89% chance that she will hit bull's eye. In a normal outdoor match, an archer has to shoot six times in four minutes. What is the probability that Khris will hit the bull's eye exactly four times in the actual competition?

   $C(6,4)\ (0.89)^4(0.11)^2$

10. Kim has 12 pairs of shoes: two Keds, three Aldo, two Converse, four Mendrez, and one Primadonna. She has to choose four pairs for the whole week to be worn on Tuesday, Wednesday, Thursday and Friday. What is the probability the she randomly chooses one primadonna and three Aldo shoes? Note: she can wear all pairs more than once a week.

   $4!/(1!3!)\ (1/12)^1(3/12)^3$