CMSC 141 Automata and Language Theory
Context-Free Languages

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Non-Regular Languages

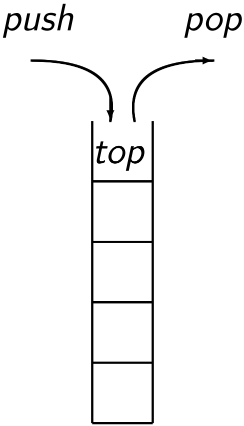
- Not all languages are regular
  - e.g. $\{a^n b^n : n > 0\}$
- There are many other non-regular languages that can be very useful
- We need something more powerful than finite automata that can express non-regular languages

What's Wrong With FAs?

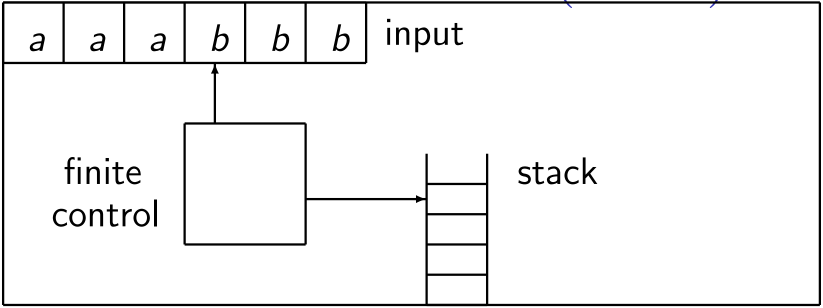
- We can only have finite number of states where we can store information, hence, finite memory.
- A more powerful machine needs more (theoretically infinite) memory
- One simple storage we can use is a *stack*

Stack

- A stack is a data structure with some basic operations
  - **PUSH**, store data to the top of the stack,
  - **POP**, read and remove data from the top of the stack,
  - PEEK/TOP, to just read data from the top of the stack, and
  - NOP, or no operation



Pushdown Automata (PDA)



- Addition of stack for storage increases the power of the automaton
- We can assume that the stack size is unbounded, giving us infinite memory

The class of languages PDAs recognize are called Context-Free Languages (CFL)

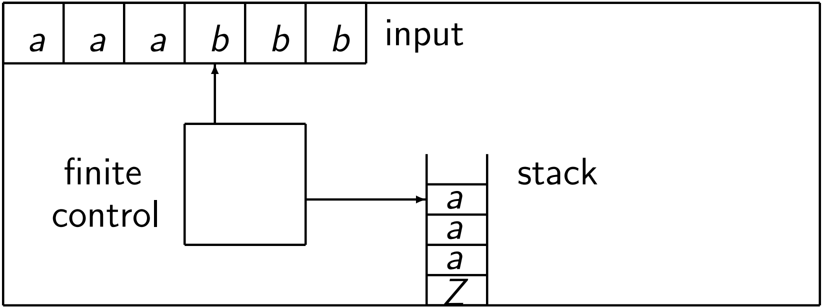
PDA for $\{a^n b^n : n > 0\}$

- Idea is to **push** a's while we are reading them, and **pop** them one by one for every matching b.
- We accept the string if we ended up at the bottom of the stack after reading the whole input

Bottom of the stack

Often times, a special marker Z is placed at the bottom of the stack

PDA FOR $\{a^n b^n : n > 0\}$



- If (a, Z) or (a, a), then push a
- If (b, a), then pop
- If (ε, Z), then accept the string

PDA FOR $\{a^n b^n : n > 0\}$

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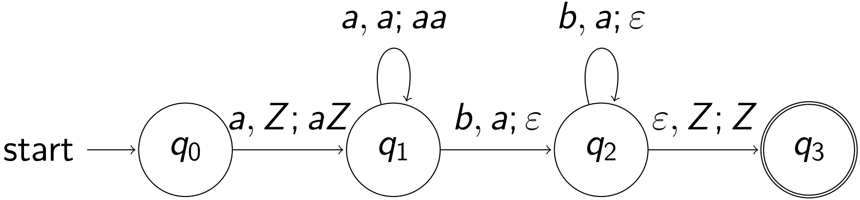
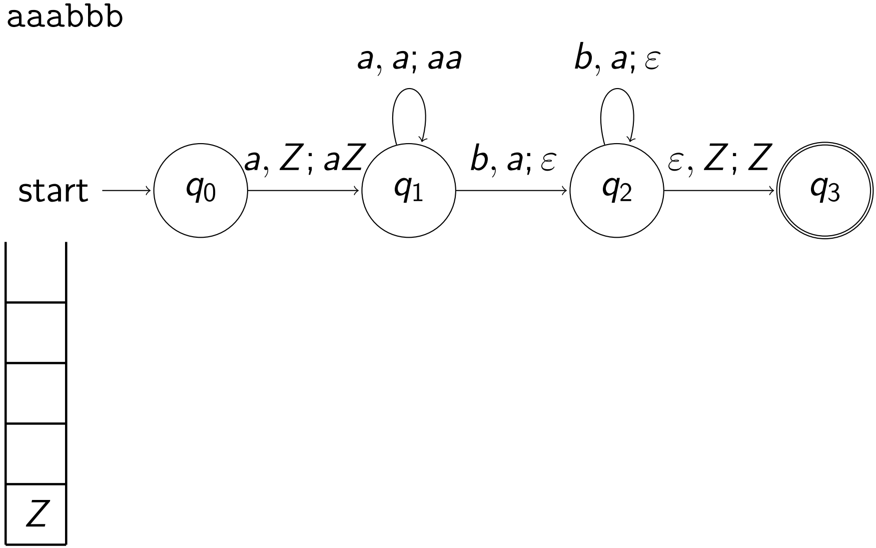


SYNTAX

(current symbol on tape,
symbol on top of the stack;
replacement symbols for the top)

PDA for $\{a^n b^n : n > 0\}$



PDA for $\{a^n b^n : n > 0\}$

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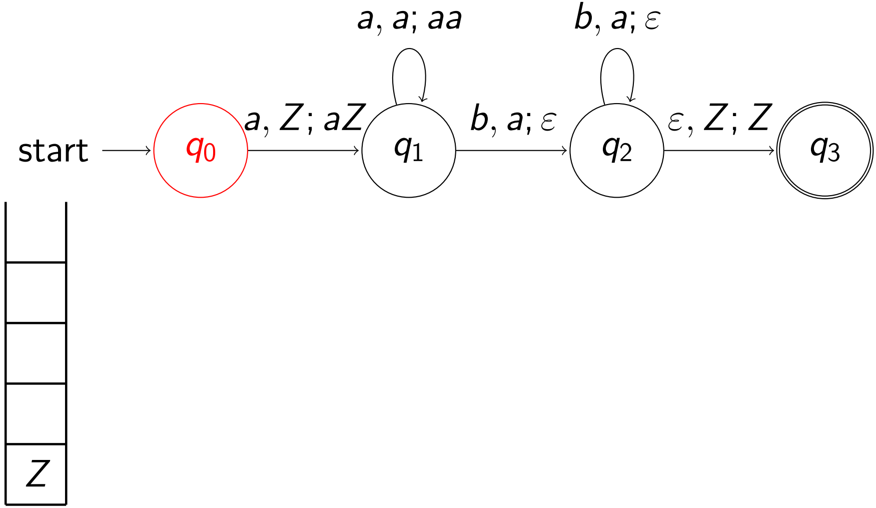

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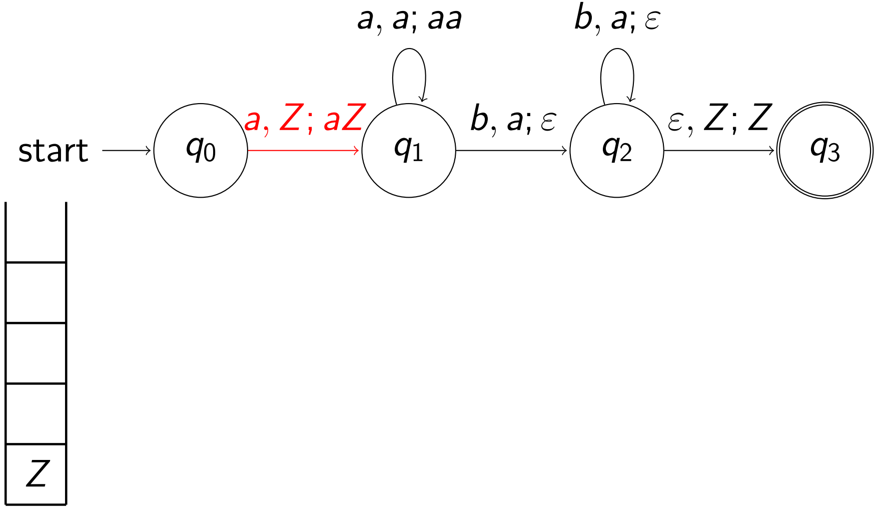

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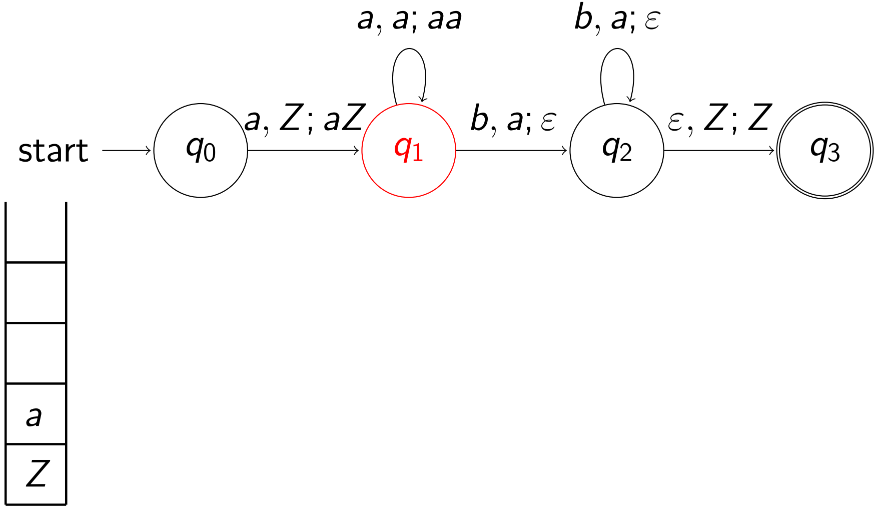

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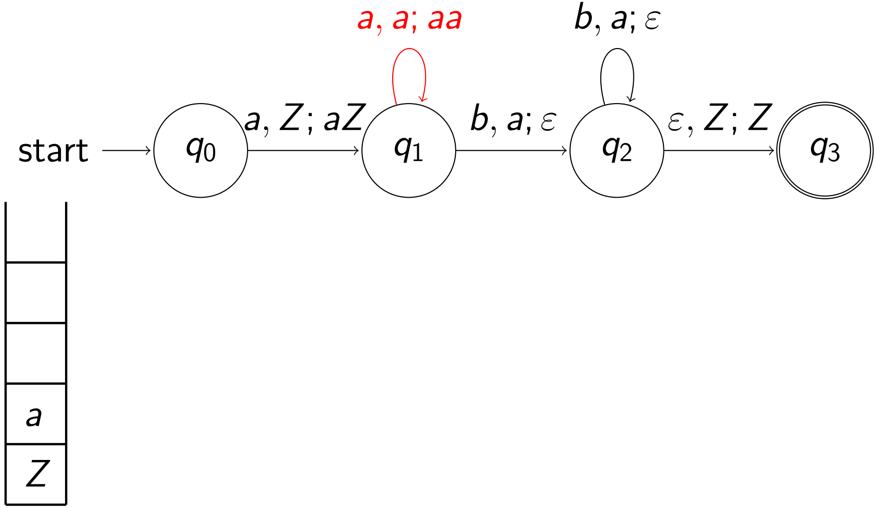

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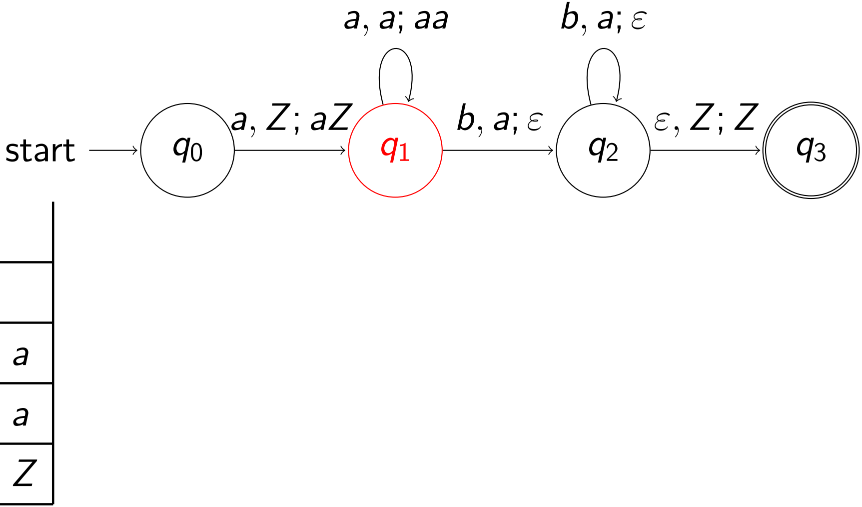

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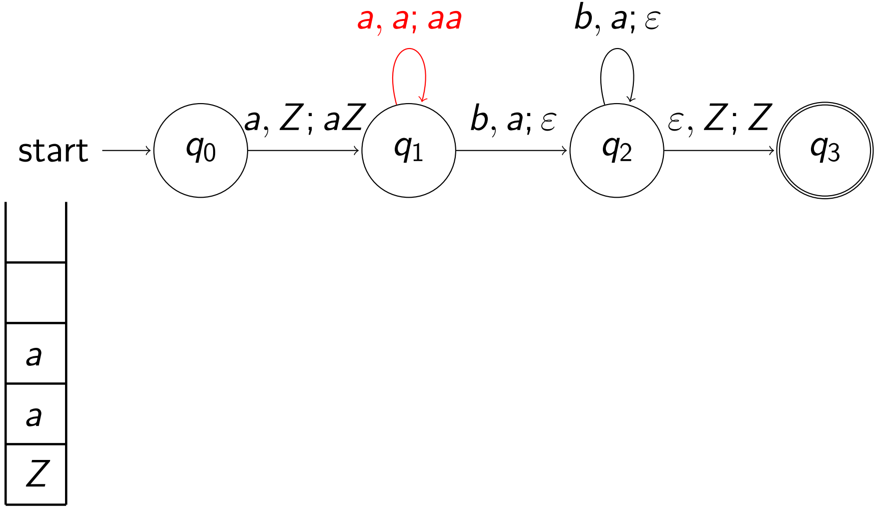

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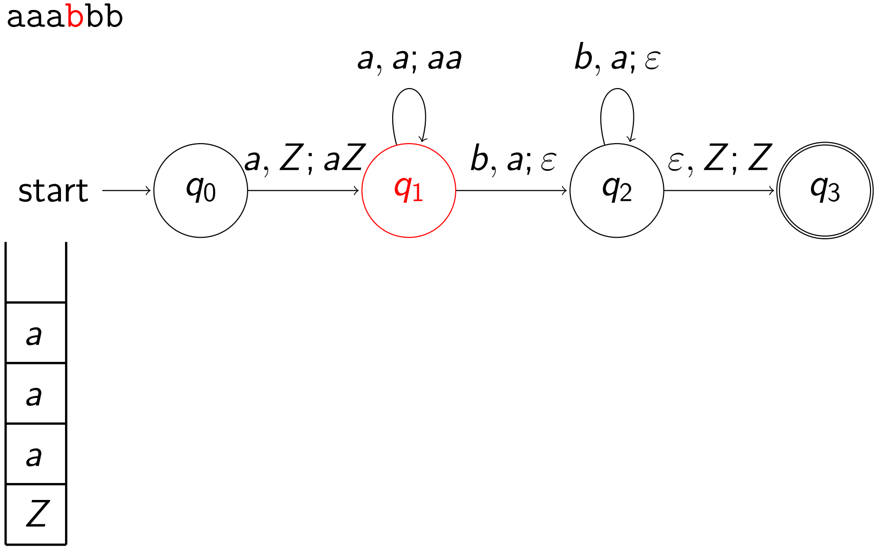

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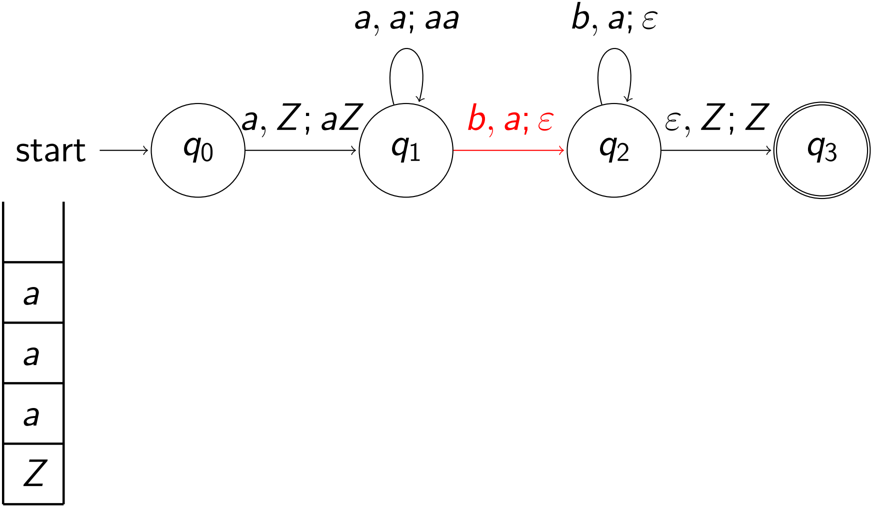

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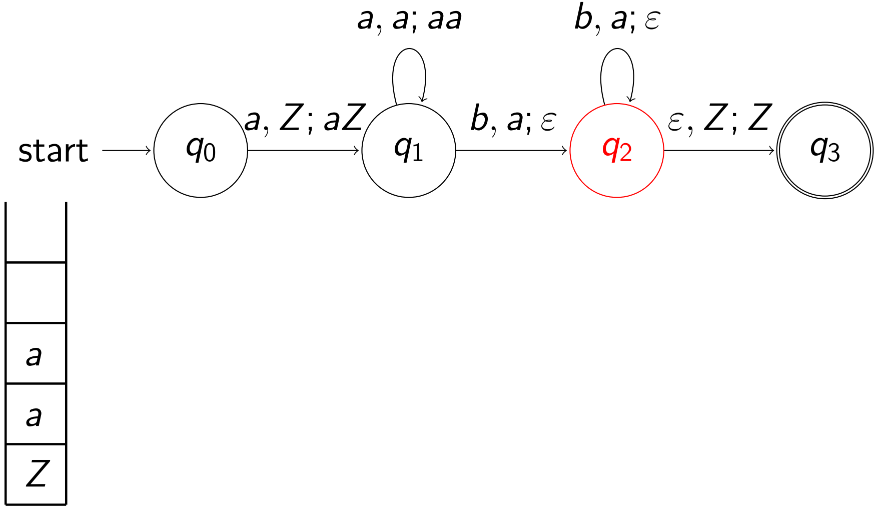

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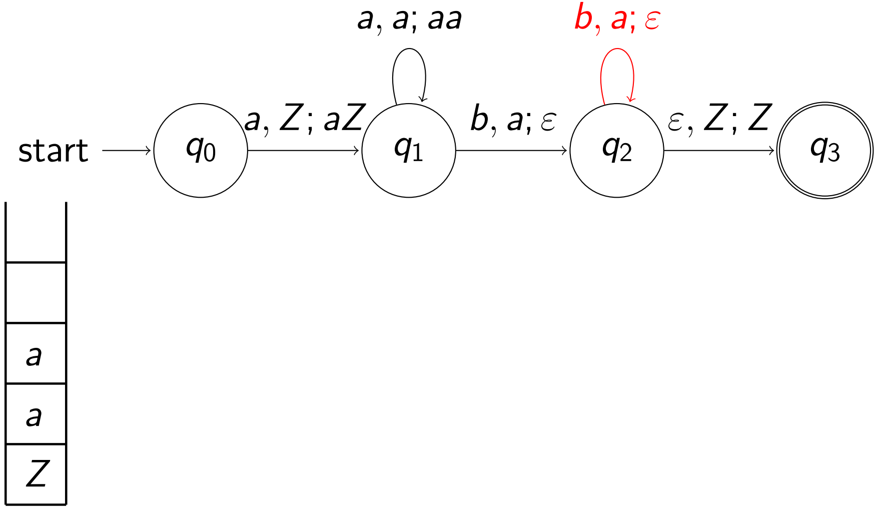

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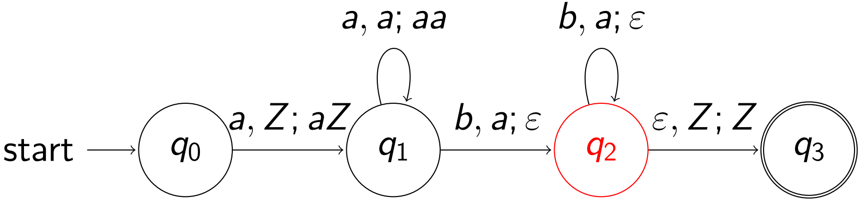

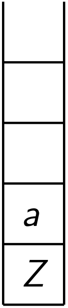

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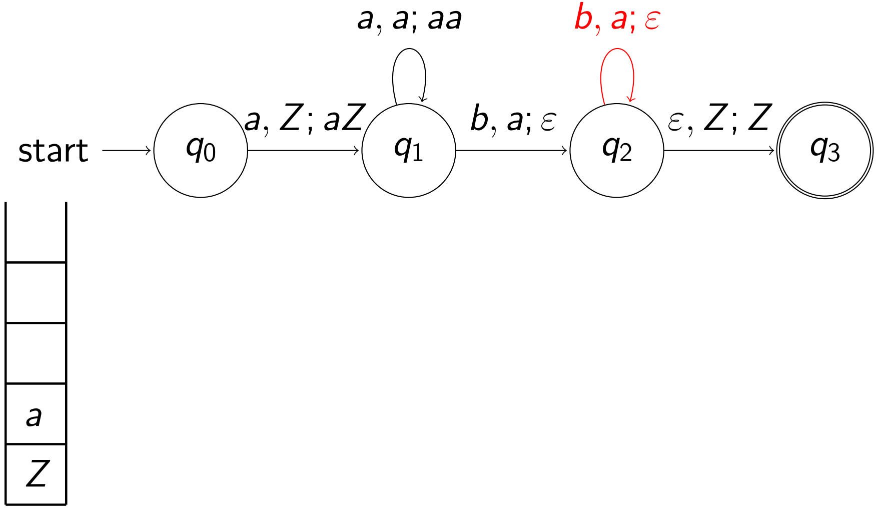

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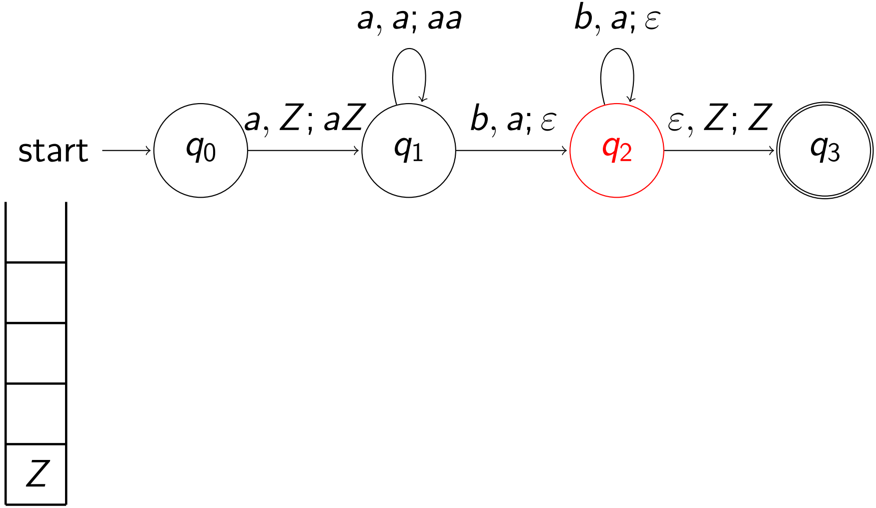

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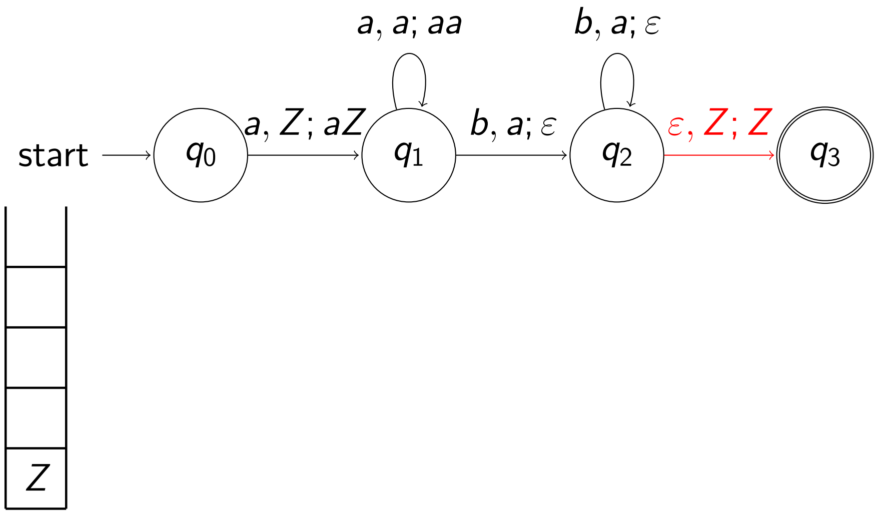

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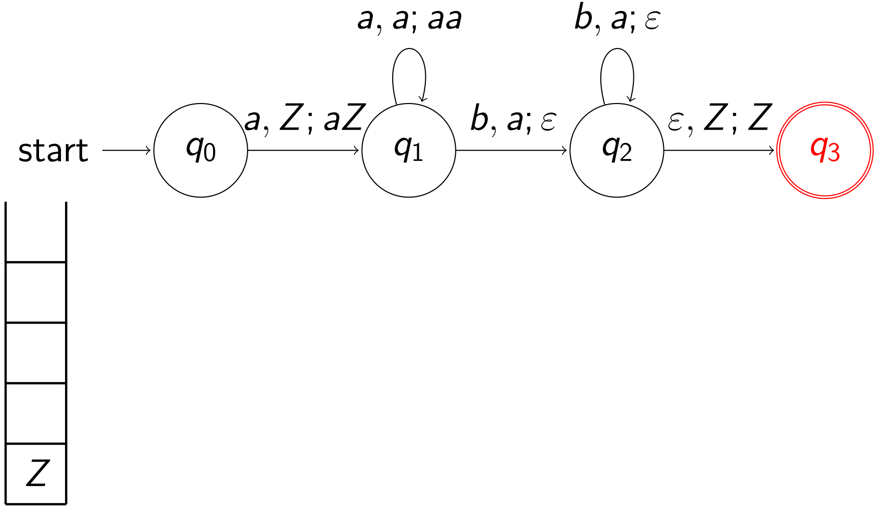


Formal Definition of PDA

$$M = \{Q, \Sigma, \Gamma, \delta, q_0, Z, F\}$$

- $Q \rightarrow$ finite set of states
- $\Sigma \rightarrow$ input alphabet
- $\Gamma \rightarrow$ stack alphabet
- $\delta \rightarrow$ transition function
- $q_0 \rightarrow$ start/initial state
- $Z \rightarrow$ initial/bottom stack symbol
- $F \rightarrow$ set of final/accepting states

Transition Function

δ takes as argument a triple $\delta(q, a, X)$ where:

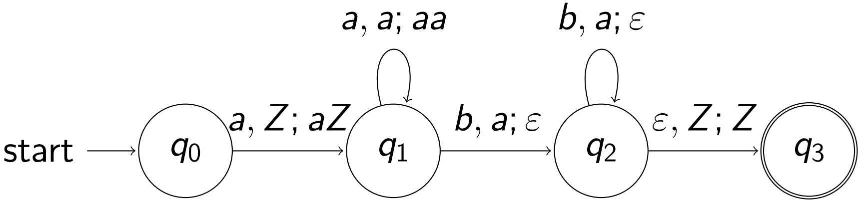
- q is a state in Q
- a is either an input symbol in Σ or $a = \varepsilon$
- X is a stack symbol that is a member of Γ

The output is a finite set of pairs (p, γ), where p is the new state, and γ is the string of stack symbols to replace X

Instantaneous Description for PDA

Syntax

(current state, remaining input, stack contents)



Execution of aabb

$(q_0, aabb, Z) \vdash (q_1, abb, aZ) \vdash (q_1, bb, aaZ) \vdash$
$(q_2, b, aZ) \vdash (q_2, \varepsilon, Z) \vdash (q_3, \varepsilon, Z)$

Acceptance in PDAs

- A PDA accepts a string x **by final state** if (q_0, x, Z) eventually leads to $(p, \varepsilon, ?)$ for some final state p
- A PDA accepts a string x **by empty stack** if (q_0, x, Z) eventually leads to $(p, \varepsilon, \varepsilon)$
- A PDA accepts a language L if every string in L is accepted and every other string is rejected
- The two forms of acceptance in PDAs are shown to be equivalent. That is one can be converted to the other

Examples/Exercises

Construct PDAs for the following languages:

- $\{a^n b^{2n} : n > 0\} =$
  $\{\texttt{abb,aabbbb,aaabbbbbb, ...}\}$
- *palindromes* $=$
  $\{\texttt{a, b, aa, bb, aaa, bbb, aba, bab, ...}\}$
- equal number of a's and b's (in any order) $=$
  $\{\texttt{ab,ba,aabb,abab,baba,bbaa,...}\}$
- balance pair of parentheses $=$
  $\{\texttt{(), (()), ()(), ((())), (())(), ...}\}$

References

- Previous slides on CMSC 141
- M. Sipser. Introduction to the Theory of Computation. Thomson, 2007.
- J.E. Hopcroft, R. Motwani and J.D. Ullman. Introduction to Automata Theory, Languages and Computation. 2nd ed, Addison-Wesley, 2001.
- E.A. Albacea. Automata, Formal Languages and Computations, UPLB Foundation, Inc. 2005
- JFLAP, www.jflap.org
- Various online LaTeX and Beamer tutorials