

# Chapter 1

## Data Representation (Part 1)

# Data Representation

- There are two basic ways of representing information, **analog** and **digital**.
- They are distinguished by the nature of the values that they allow information variables to assume.

# Analog vs. Digital

## Analog quantities

- can vary over a **continuous** range of values.
- Examples: voltage, room thermostat



## Digital quantities

- represented by symbols called **digits**.
- Examples: digital watch



# Analog and Digital Systems

## Analog System

- contains devices that manipulate physical quantities that are represented in **continuous** range of values.
- Examples: audio amplifiers, automobile odometer

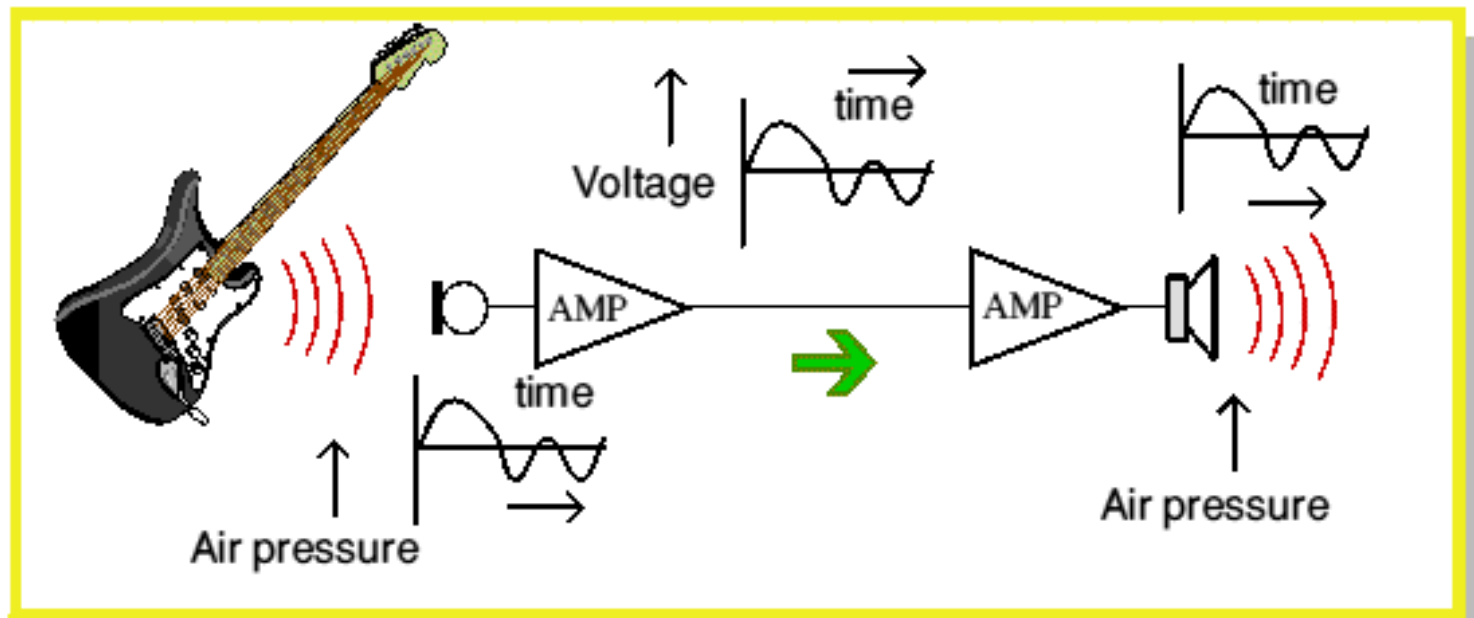
## Digital quantities

- is a combination of devices designed to manipulate physical quantities that are represented in **discrete** values.
- Examples: digital computers, digital audio/video equipment



# Example – Analog System

- *A simple guitar system*



# Example – Digital Systems

- Wireless Devices



- Smart Watch



- Game Consoles



# Advantages of Digital Techniques

- Digital system design is easier
- Information storage is easy
- Greater precision and accuracy
- Programmability
- Less susceptible to noise
- Digital circuitry can be fabricated on IC chips



# Limitation of Digital Techniques

- There is one major drawback when using digital techniques:

**The real world is mainly analog.**



# Limitation of Digital Techniques

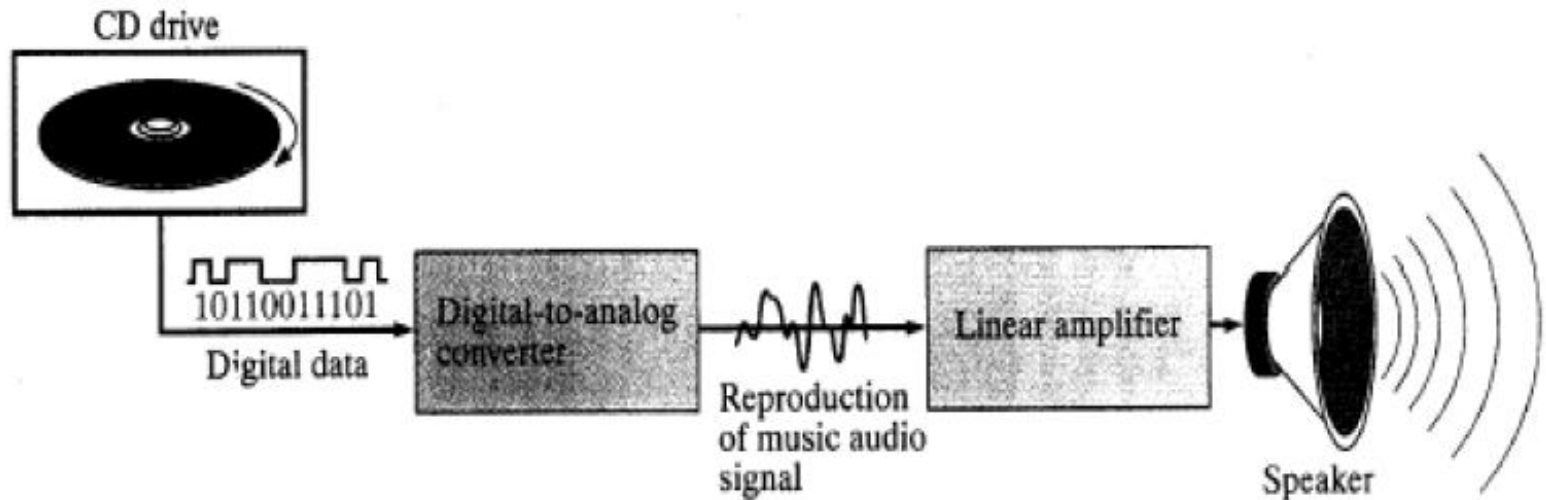
- There is one major drawback when using digital techniques:

**The real world is mainly analog.**

- *Hybrid systems* – both digital and analog techniques employed within the same system.

# Example – Hybrid System

- CD Player



# Digital Number Systems

- Many number systems are in use in digital technology.
- The most common are the *decimal*, *binary*, *octal*, and *hexadecimal* systems.

# Number Systems

Number System	Base	Coefficients
Decimal	10	0 – 9
Binary	2	0 , 1
Octal	8	0 – 7
Hexadecimal	16	0 – 9, A – F

# Base Conversion

- From any *base- $r$*  to Decimal
- From Decimal to any *base- $r$*
- From Binary to either Octal or Hexadecimal
- From either Octal or Hexadecimal to Binary



# Conversion: *Base- $r$* to *Decimal*

## Procedure

- **Step 1:**

Multiply each coefficient with the corresponding power of  $r$ .

- **Step 2:**

Get the sum.

# Conversion: *Base- $r$* to *Decimal*

Examples:

1.  $(0111.101)_2 = \underline{\hspace{2cm}}_{10}$

# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + \end{aligned}$$

# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + \end{aligned}$$

# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + \end{aligned}$$



# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + \end{aligned}$$

# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + \\ &\quad (1 \times 2^{-1}) + \end{aligned}$$

# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + \\ &\quad (1 \times 2^{-1}) + (0 \times 2^{-2}) + \end{aligned}$$

# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + \\ &\quad (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) \end{aligned}$$

# Conversion: *Base- $r$* to *Decimal*

Examples:

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + \\ &\quad (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) \\ &= 0 + 4 + 2 + 1 + 0.5 + 0 + 0.125 \end{aligned}$$



# Conversion: *Base- $r$* to *Decimal*

Example (Binary-to-Decimal) :

$$\begin{aligned} 1. \quad (0111.101)_2 &= \underline{\hspace{2cm}}_{10} \\ &= (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + \\ &\quad (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) \\ &= 0 + 4 + 2 + 1 + 0.5 + 0 + 0.125 \\ &= (7.625)_{10} \end{aligned}$$

# Conversion: *Base- $r$* to *Decimal*

Example (Octal-to-Decimal) :

$$\begin{aligned} 1. \quad (515.4)_8 &= \underline{\hspace{2cm}}_{10} \\ &= (5 \times 8^2) + (1 \times 8^1) + (5 \times 8^0) + (4 \times 8^{-1}) \end{aligned}$$

# Conversion: *Base-r* to *Decimal*

Example (Octal-to-Decimal) :

$$\begin{aligned} 1. \quad (515.4)_8 &= \underline{\hspace{2cm}}_{10} \\ &= (5 \times 8^2) + (1 \times 8^1) + (5 \times 8^0) + (4 \times 8^{-1}) \\ &= 320 + 8 + 5 + 0.5 \end{aligned}$$

# Conversion: *Base-r* to *Decimal*

Example (Octal-to-Decimal) :

$$\begin{aligned} 1. \quad (515.4)_8 &= \underline{\hspace{2cm}}_{10} \\ &= (5 \times 8^2) + (1 \times 8^1) + (5 \times 8^0) + (4 \times 8^{-1}) \\ &= 320 + 8 + 5 + 0.5 \\ &= (333.5)_{10} \end{aligned}$$

# Binary to Decimal

## Examples

1.  $(11011100)_2 = \underline{\hspace{2cm}}_{10}$



# Binary to Decimal

## Examples

1.  $(11011100)_2 = \underline{\hspace{2cm}}_{10}$

128	64	32	16	8	4	2	1
1	1	0	1	1	1	0	0

# Binary to Decimal

## Examples

1.  $(11011100)_2 = (220)_{10}$

128	64	32	16	8	4	2	1
1	1	0	1	1	1	0	0

# Binary to Decimal

## Examples

1.  $(11011100)_2 = (220)_{10}$

128	64	32	16	8	4	2	1
1	1	0	1	1	1	0	0

2.  $(010101)_2 = \underline{\hspace{2cm}}_{10}$

# Binary to Decimal

## Examples

1.  $(11011100)_2 = (220)_{10}$

128	64	32	16	8	4	2	1
1	1	0	1	1	1	0	0

2.  $(010101)_2 = \underline{\hspace{2cm}}_{10}$

32	16	8	4	2	1
0	1	0	1	0	1

# Binary to Decimal

## Examples

1.  $(11011100)_2 = (220)_{10}$

128	64	32	16	8	4	2	1
1	1	0	1	1	1	0	0

2.  $(010101)_2 = (21)_{10}$

32	16	8	4	2	1
0	1	0	1	0	1

# Conversion: *Decimal to Base- $r$*

## Procedure

### Step 1:

Separate integer  
from fraction.

### *Integer to base- $r$*

- Divide integer by  $r$
- Accumulate remainders

### Step 2:

Convert integer to  
base- $r$

### *Fraction to base- $r$*

### Step 3:

Convert fraction to  
base- $r$

- Multiply fraction by  $r$
- Accumulate integers

# Decimal to Binary

## Examples

$$1. \quad (41.6875)_{10} = \underline{\hspace{2cm}}_2$$

Step 1:

Integer = 41

Fraction = 0.6875



# Decimal to Binary

Examples

$$1. \quad (41.6875)_{10} = \underline{\hspace{2cm}}_2$$

Step 2:

$$41 / 2 = 20 \text{ rem } 1$$

Step 1:

Integer = 41

Fraction = 0.6875

# Decimal to Binary

## Examples

$$1. \quad (41.6875)_{10} = \underline{\hspace{2cm}}_2$$

Step 2:

$$41 / 2 = 20 \text{ rem } 1$$

$$20 / 2 = 10 \text{ rem } 0$$

Step 1:

Integer = 41

Fraction = 0.6875

# Decimal to Binary

## Examples

$$1. \quad (41.6875)_{10} =$$

---

$$\qquad\qquad\qquad 2$$

Step 1:

Integer = 41

Fraction = 0.6875

Step 2:

$$41 / 2 = 20 \text{ rem } 1$$

$$20 / 2 = 10 \text{ rem } 0$$

$$10 / 2 = 5 \text{ rem } 0$$

# Decimal to Binary

## Examples

$$1. \quad (41.6875)_{10} =$$
$$\underline{\hspace{2cm}}_2$$

Step 1:

Integer = 41

Fraction = 0.6875

Step 2:

$$41 / 2 = 20 \text{ rem } 1$$

$$20 / 2 = 10 \text{ rem } 0$$

$$10 / 2 = 5 \text{ rem } 0$$

$$5 / 2 = 2 \text{ rem } 1$$

# Decimal to Binary

## Examples

1.  $(41.6875)_{10} =$   
 $\underline{\hspace{2cm}}_2$

Step 1:

Integer = 41

Fraction = 0.6875

Step 2:

$$41 / 2 = 20 \text{ rem } 1$$

$$20 / 2 = 10 \text{ rem } 0$$

$$10 / 2 = 5 \text{ rem } 0$$

$$5 / 2 = 2 \text{ rem } 1$$

$$2 / 2 = 1 \text{ rem } 0$$

# Decimal to Binary

## Examples

1.  $(41.6875)_{10} =$   
 $\underline{\hspace{2cm}}_2$

Step 1:

Integer = 41

Fraction = 0.6875

Step 2:

$$41 / 2 = 20 \text{ rem } 1$$

$$20 / 2 = 10 \text{ rem } 0$$

$$10 / 2 = 5 \text{ rem } 0$$

$$5 / 2 = 2 \text{ rem } 1$$

$$2 / 2 = 1 \text{ rem } 0$$

$$1 / 2 = 0 \text{ rem } 1$$

# Decimal to Binary

## Examples

1.  $(41.6875)_{10} =$   
\_\_\_\_\_2

Step 1:

Integer = 41

Fraction = 0.6875

Step 2:

$$41 / 2 = 20 \text{ rem}$$

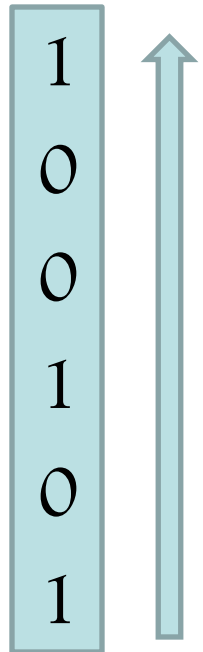
$$20 / 2 = 10 \text{ rem}$$

$$10 / 2 = 5 \text{ rem}$$

$$5 / 2 = 2 \text{ rem}$$

$$2 / 2 = 1 \text{ rem}$$

$$1 / 2 = 0 \text{ rem}$$





# Decimal to Binary

Step 3:

$$0.6875 \times 2 = 1.3750 \quad 1$$

# Decimal to Binary

Step 3:

$$0.6875 \times 2 = 1.3750 \quad 1$$

$$0.3750 \times 2 = 0.7500 \quad 0$$

# Decimal to Binary

Step 3:

$$0.6875 \times 2 = 1.3750 \quad 1$$

$$0.3750 \times 2 = 0.7500 \quad 0$$

$$0.7500 \times 2 = 1.5000 \quad 1$$

# Decimal to Binary

Step 3:

$$0.6875 \times 2 = 1.3750 \quad 1$$

$$0.3750 \times 2 = 0.7500 \quad 0$$

$$0.7500 \times 2 = 1.5000 \quad 1$$

$$0.5000 \times 2 = 1.0000 \quad 1$$

# Decimal to Binary

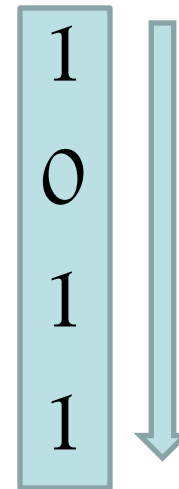
Step 3:

$$0.6875 \times 2 = 1.3750$$

$$0.3750 \times 2 = 0.7500$$

$$0.7500 \times 2 = 1.5000$$

$$0.5000 \times 2 = 1.0000$$



# Decimal to Binary

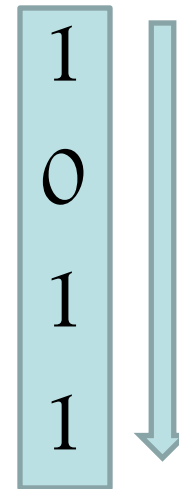
Step 3:

$$0.6875 \times 2 = 1.3750$$

$$0.3750 \times 2 = 0.7500$$

$$0.7500 \times 2 = 1.5000$$

$$0.5000 \times 2 = 1.0000$$



Thus:  $(41.6875)_{10} = (101001.1011)_2$

# Decimal to Octal

Examples

$$2. \quad (143.44)_{10} = \underline{\hspace{2cm}}_8$$

Step 1:

Integer = 143

Fraction = 0.44



# Decimal to Octal

Examples

$$2. \quad (143.44)_{10} = \underline{\hspace{2cm}}_8$$

Step 2:

$$143 / 8 = 17 \text{ rem } 7$$

Step 1:

Integer = 143

Fraction = 0.44

# Decimal to Octal

Examples

$$2. \quad (143.44)_{10} = \underline{\hspace{2cm}}_8$$

Step 2:

$$143 / 8 = 17 \text{ rem } 7$$

$$17 / 8 = 2 \text{ rem } 1$$

Step 1:

Integer = 143

Fraction = 0.44

# Decimal to Octal

Examples

$$2. \quad (143.44)_{10} = \underline{\hspace{2cm}}_8$$

Step 2:

$$143 / 8 = 17 \text{ rem } 7$$

$$17 / 8 = 2 \text{ rem } 1$$

$$2 / 8 = 0 \text{ rem } 2$$

Step 1:

Integer = 143

Fraction = 0.44

# Decimal to Octal

Examples

$$2. \quad (143.44)_{10} = \underline{\hspace{2cm}}_8$$

Step 1:

Integer = 143

Fraction = 0.44

Step 2:

$$143 / 8 = 17 \text{ rem}$$

$$17 / 8 = 2 \text{ rem}$$

$$2 / 8 = 0 \text{ rem}$$

7

1

2



# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52 \quad 3$$

# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52 \quad 3$$

$$0.52 \times 8 = 4.16 \quad 4$$

# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52 \quad 3$$

$$0.52 \times 8 = 4.16 \quad 4$$

$$0.16 \times 8 = 1.28 \quad 1$$



# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52 \quad 3$$

$$0.52 \times 8 = 4.16 \quad 4$$

$$0.16 \times 8 = 1.28 \quad 1$$

$$0.28 \times 8 = 2.24 \quad 2$$

# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52 \quad 3$$

$$0.52 \times 8 = 4.16 \quad 4$$

$$0.16 \times 8 = 1.28 \quad 1$$

$$0.28 \times 8 = 2.24 \quad 2$$

$$0.24 \times 8 = 1.92 \quad 1$$

# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52 \quad 3$$

$$0.52 \times 8 = 4.16 \quad 4$$

$$0.16 \times 8 = 1.28 \quad 1$$

$$0.28 \times 8 = 2.24 \quad 2$$

$$0.24 \times 8 = 1.92 \quad 1$$

$$0.92 \times 8 = 7.36 \quad 7$$

# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52$$

$$0.52 \times 8 = 4.16$$

$$0.16 \times 8 = 1.28$$

$$0.28 \times 8 = 2.24$$

$$0.24 \times 8 = 1.92$$

$$0.92 \times 8 = 7.36$$

3

4

1

2

1

7



# Decimal to Octal

Step 3:

$$0.44 \times 8 = 3.52$$

$$0.52 \times 8 = 4.16$$

$$0.16 \times 8 = 1.28$$

$$0.28 \times 8 = 2.24$$

$$0.24 \times 8 = 1.92$$

$$0.92 \times 8 = 7.36$$

3

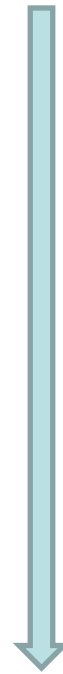
4

1

2

1

7



Thus:  $(143.44)_{10} = (217.341217)_8$

# Decimal to Binary

## Examples

1.  $(130)_{10} = \underline{\hspace{2cm}}_2$

128	64	32	16	8	4	2	1
1							

# Decimal to Binary

## Examples

1.  $(130)_{10} = \underline{\hspace{2cm}}_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	

# Decimal to Binary

## Examples

1.  $(130)_{10} = \underline{\hspace{2cm}}_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0



# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
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# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
1						

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
1	1					

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
1	1	1				

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
1	1	1	1			

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
1	1	1	1	0		



# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
1	1	1	1	0	1	

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = \underline{\hspace{2cm}}_2$

64	32	16	8	4	2	1
1	1	1	1	0	1	1

# Decimal to Binary

## Examples

1.  $(130)_{10} = (10000010)_2$

128	64	32	16	8	4	2	1
1	0	0	0	0	0	1	0

2.  $(123)_{10} = (1111011)_2$

64	32	16	8	4	2	1
1	1	1	1	0	1	1