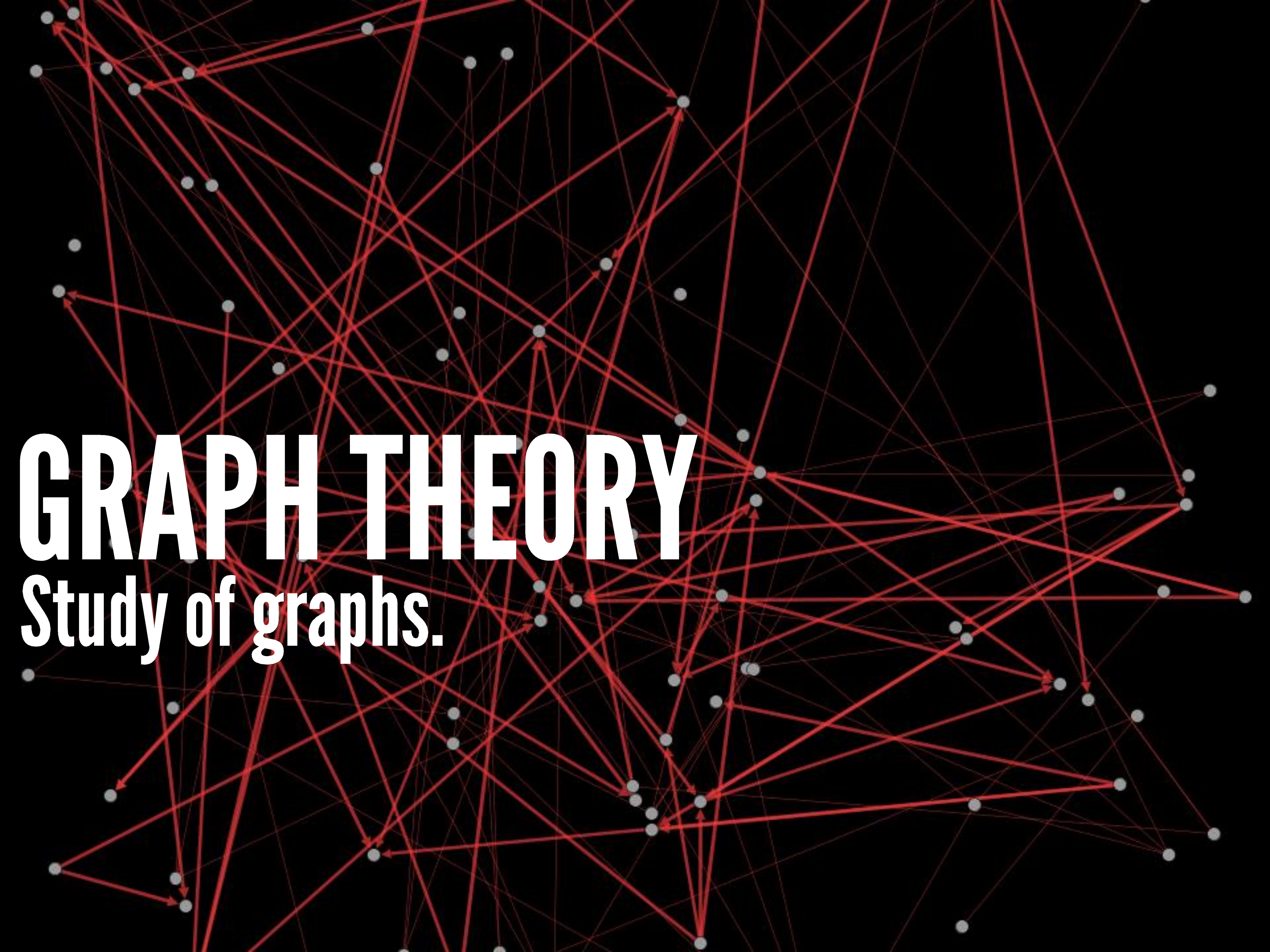
DISCRETE PROBABILITY

GRAPH THEORY

ALGEBRAIC STRUCTURES

COMBINATORICS



GRAPH THEORY

Study of graphs.

GRAPHS are used to represent **OBJECTS** and **RELATIONSHIP** among these objects.

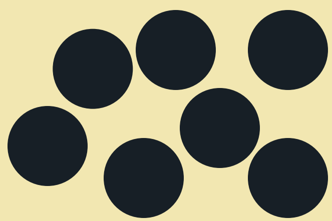
GRAPH

$$G = \{ \underset{\text{VERTICES}}{\text{Set of}} \; , \; \underset{\text{EDGES}}{\text{Set of}} \}$$

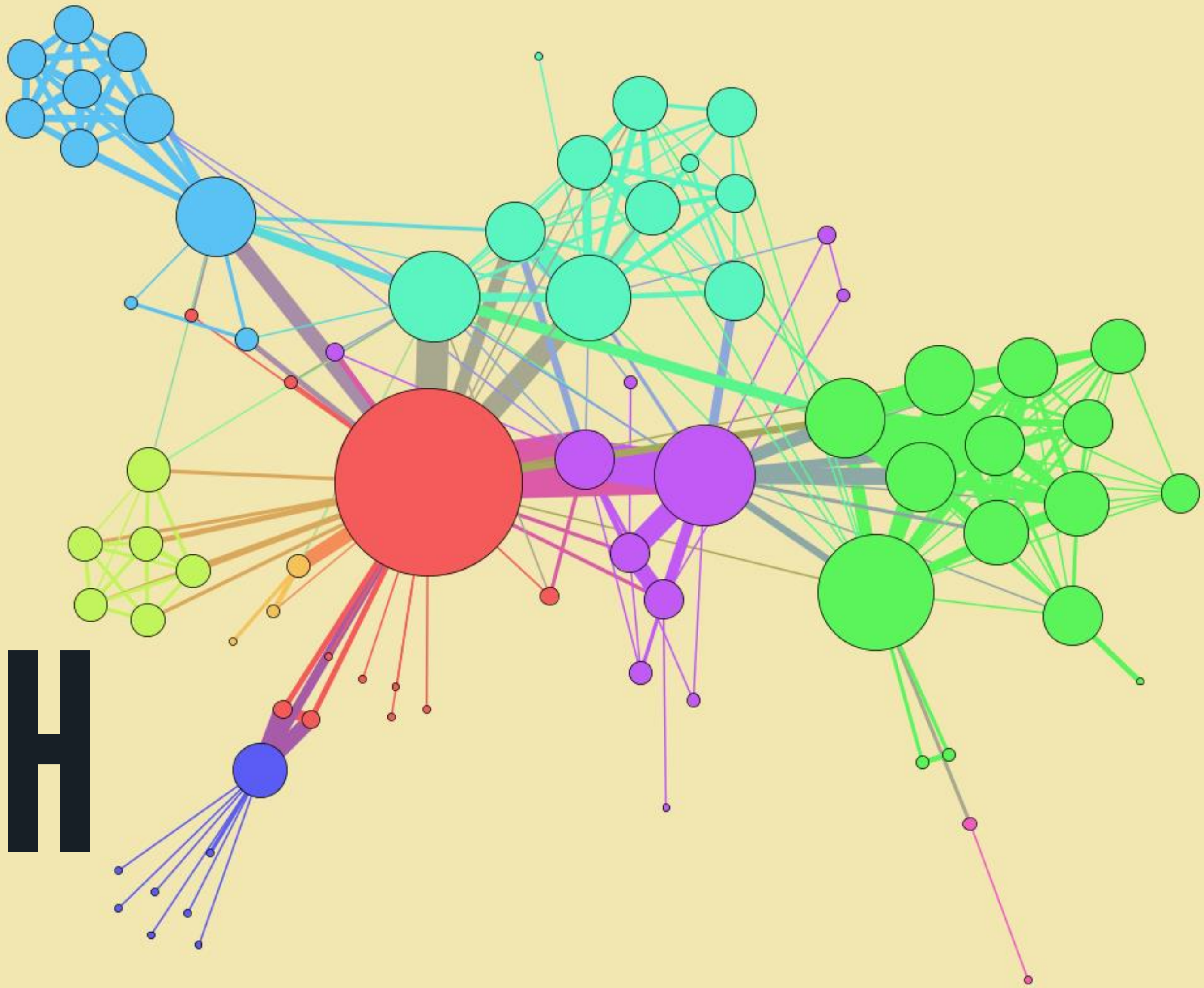
GRAPH

$G = \{ V(G), E(G) \}$

GRAPH

G = {  ,  }

GRAPH
example



gephi.org

KÖNIGSBERG
Bridge Problem.

Kaliningrad
(Калининград)
Frunze ulitsa
Shevchenko ulitsa
9 Aprelya ulitsa
Proletarskaya
Zhitomirskaya ulitsa
Kopernika ulitsa
Moskovskiy prospekt
Московский просп.
Leninskiy prospekt
Дом советов
Generala Butkova ulitsa
Krasnoyarskaya ulitsa
Маршала Баграмяна наб.
Витязь
Кнайпхоф
Kanta ulitsa
Генерала Карбышева наб.
Крестовоздвиженский собор
Портовая ул.
Portovaya ulitsa
Polotskaya ulitsa
Ленинский просп.
Серпуховская ул.
Novyy Val ulitsa
Solnechnyy bul'var
Семь мостов Кёнигсберга
Oktyabr'skaya ulitsa
Юбилейный мост
Октябрьская ул.
Краснооктябрьская ул.
Krasnooktyabr'skaya ulitsa
Skladskaya ulitsa
El'blongskaya ulitsa
Serpukhovskaya ulitsa
Generala Pavlova ulitsa
Bagrationa ulitsa
Багратиона ул.
Bol'shaya Pesochnaya
Pesochnaya
1000 ft
200 m
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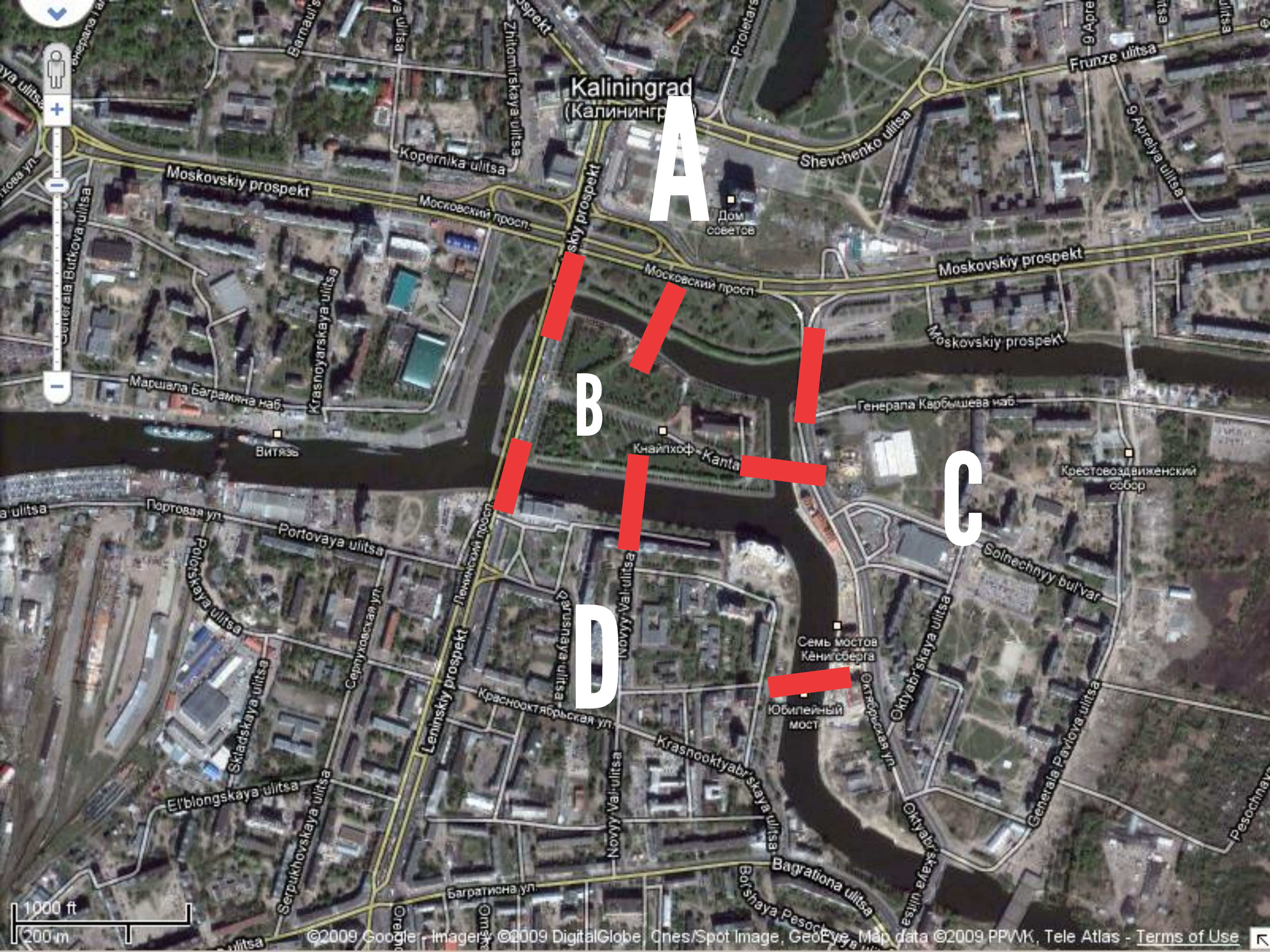

Kaliningrad
A
B
C
D
Moskovskiy prospekt
Московский просп.
Leninskiy prospekt
Shevchenko ulitsa
Frunze ulitsa
9 Aprelya ulitsa
Kopernika ulitsa
Zhitomirskaya ulitsa
Дом советов
Knaiphof - Kanta
Генерала Карбышева наб.
Маршала Баграмяна наб.
Витязь
Крестовоздвиженский собор
Портовая ул.
Portovaya ulitsa
Novyy Val ulitsa
Solnechnyy bul'var
Oktyabr'skaya ulitsa
Семь мостов Кёнигсберга
Юбилейный мост
Краснооктябрьская ул.
Krasnooktyabr'skaya ulitsa
Generala Pavlova ulitsa
Bagrationa ulitsa
Багратиона ул.
El'blongskaya ulitsa
Serpukhovskaya ulitsa
Skladskaya ulitsa
Polotskaya ulitsa
Krasnoyarskaya ulitsa
Generala Butkova ulitsa
1000 ft
200 m
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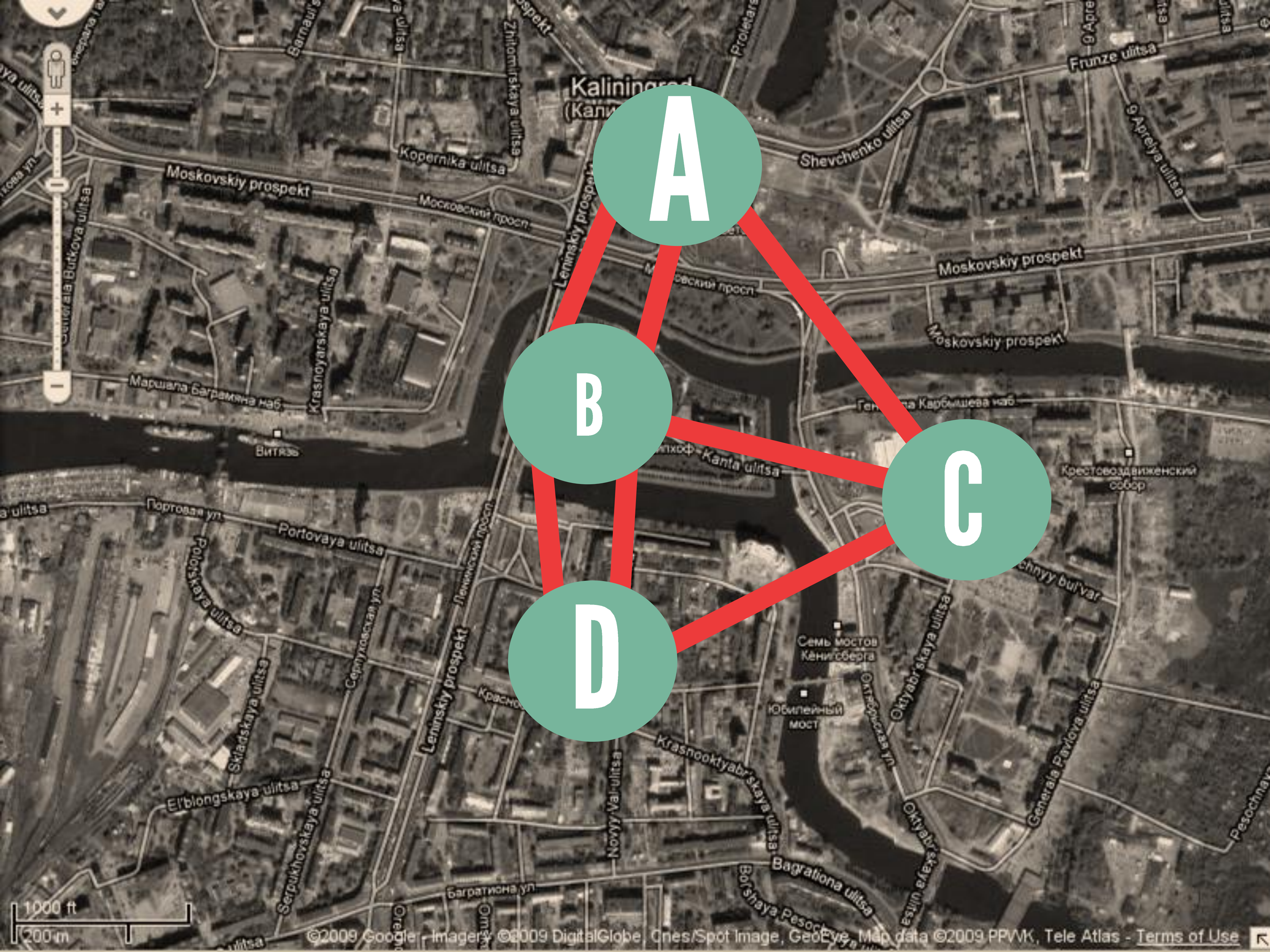

A
B
C
D
Kaliningrad
Moskovskiy prospekt
Московский просп.
Kopernika ulitsa
Shevchenko ulitsa
Frunze ulitsa
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Leninskiy prospekt
Krasnoyarskaya ulitsa
Маршала Баграмяна наб.
Витязь
Портовая ул.
Portovaya ulitsa
Kanta ulitsa
Крестовоздвиженский собор
Семь мостов Кенигсберга
Юбилейный мост
Krasnooktyabr'skaya ulitsa
Oktyabr'skaya ulitsa
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Багратиона ул.
El'blongskaya ulitsa
Serpukhovskaya ulitsa
Skladskaya ulitsa
Polotskaya ulitsa
Novyy Val ulitsa
Zhitomirskaya ulitsa
Ленинский просп.
1000 ft
200 m
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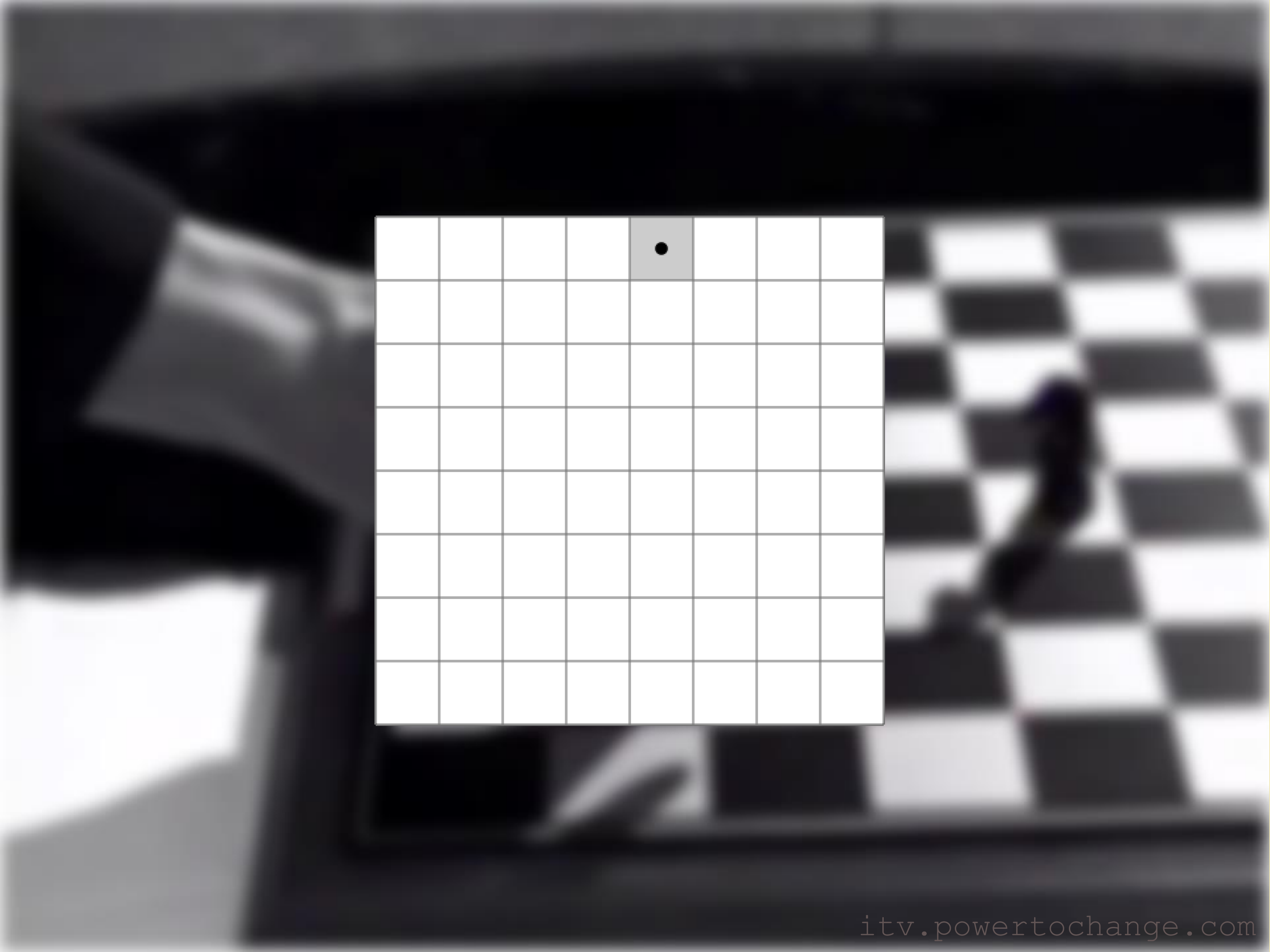
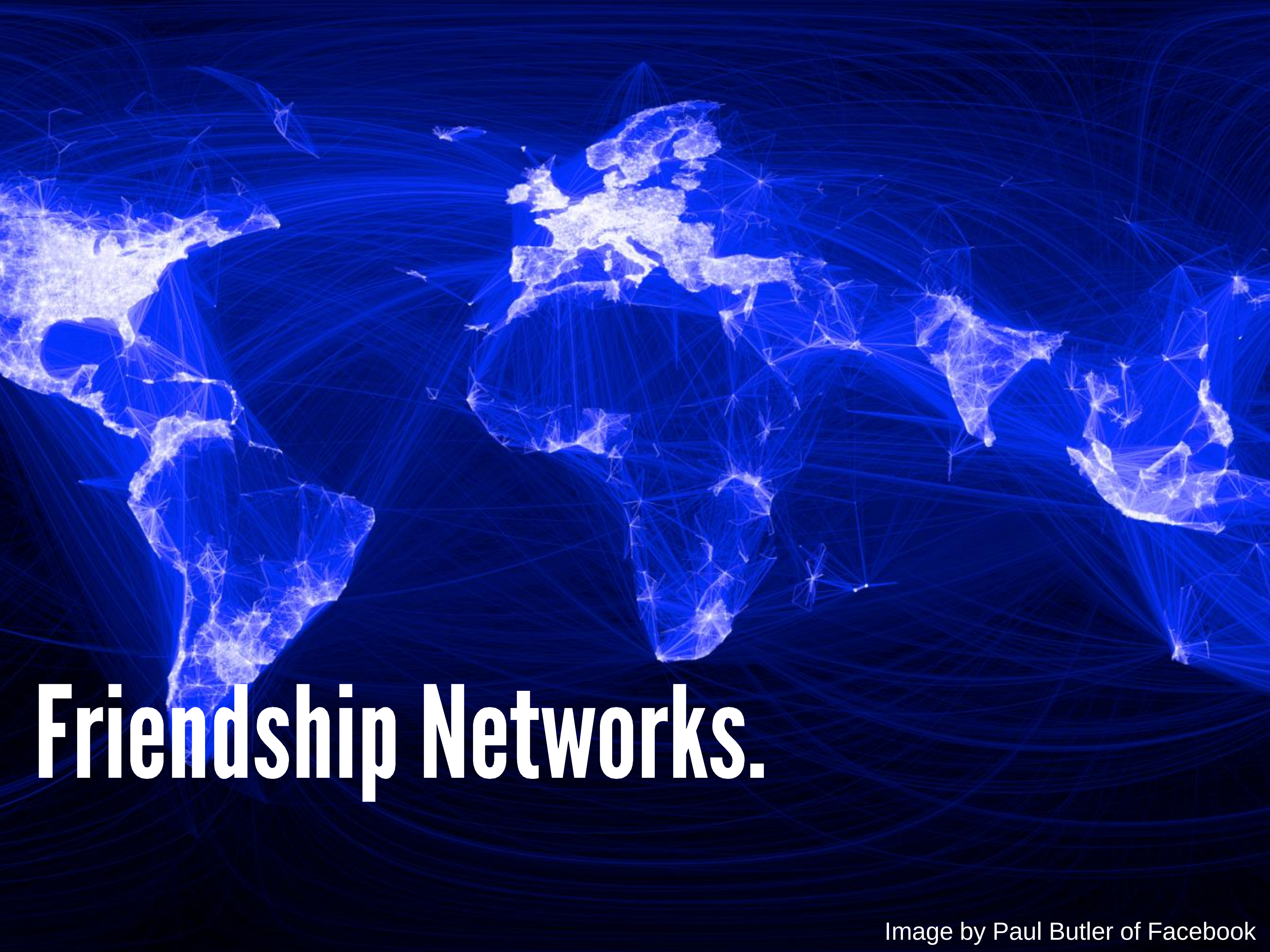
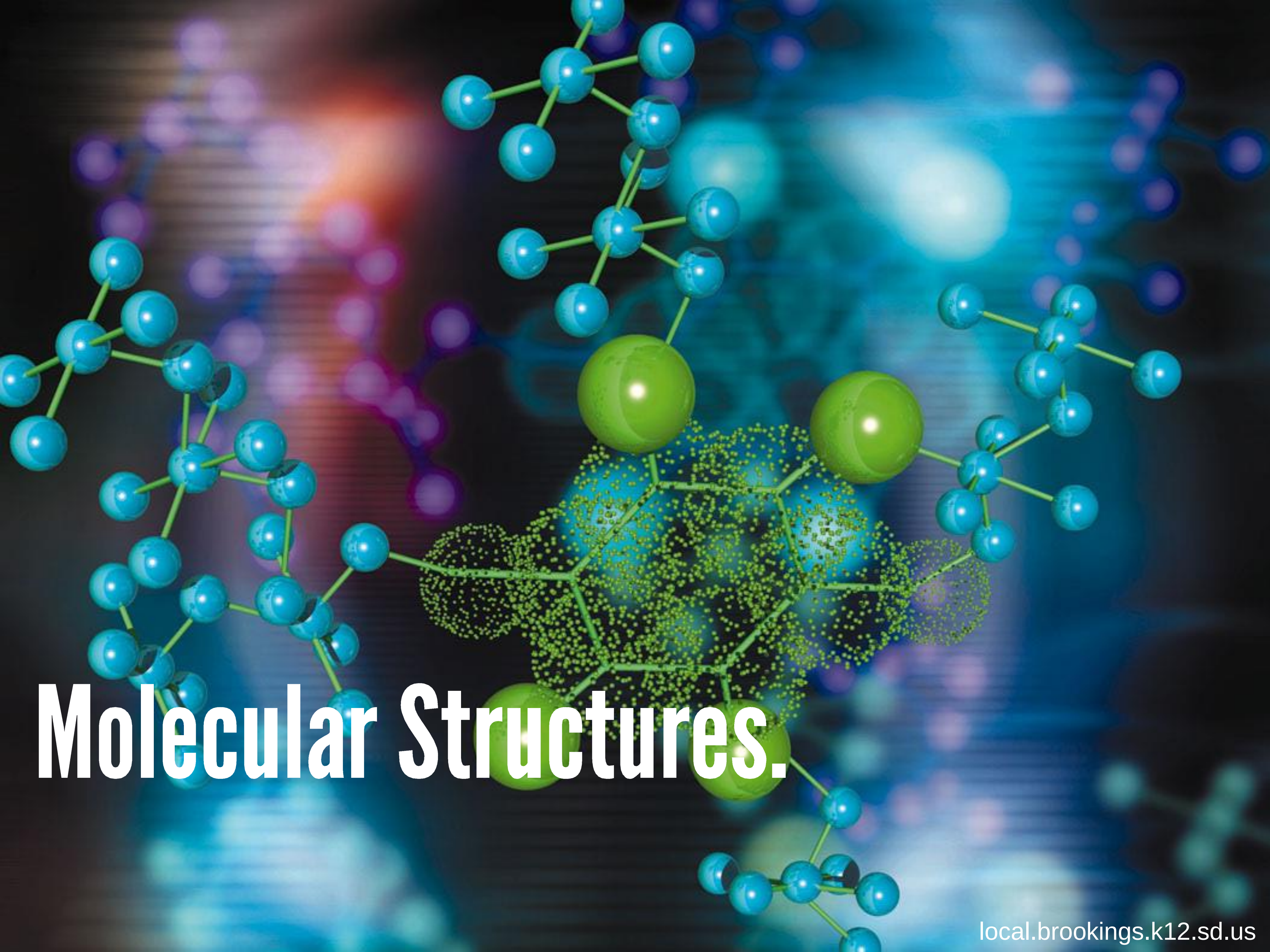
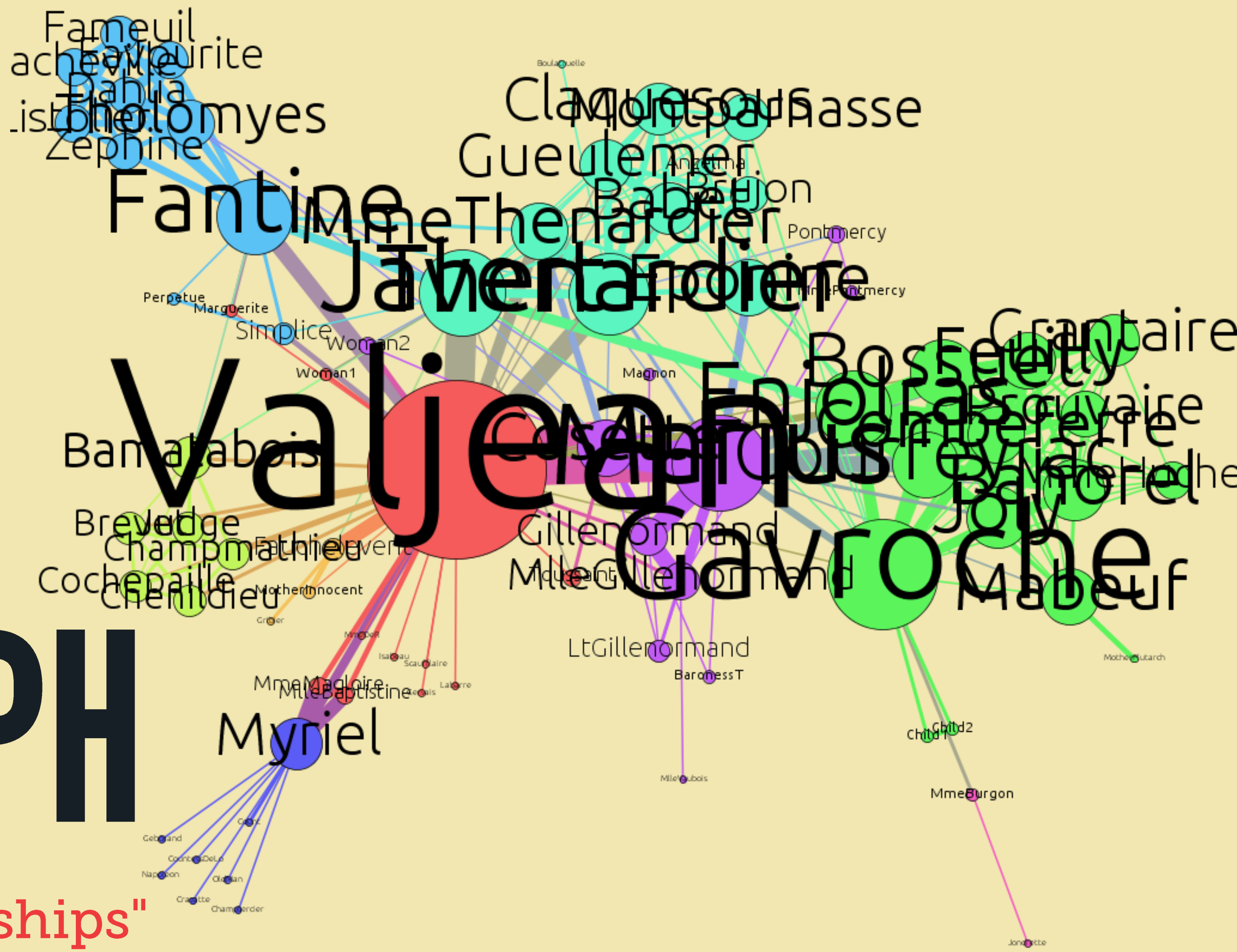
KNIGHT’S
Tour.
itv.powertochange.com




Graph Theory
Applications


Friendship Networks.
Image by Paul Butler of Facebook


Molecular Structures.
local.brookings.k12.sd.us



GRAPH

that shows the"relationships" of characters in Les Miserables

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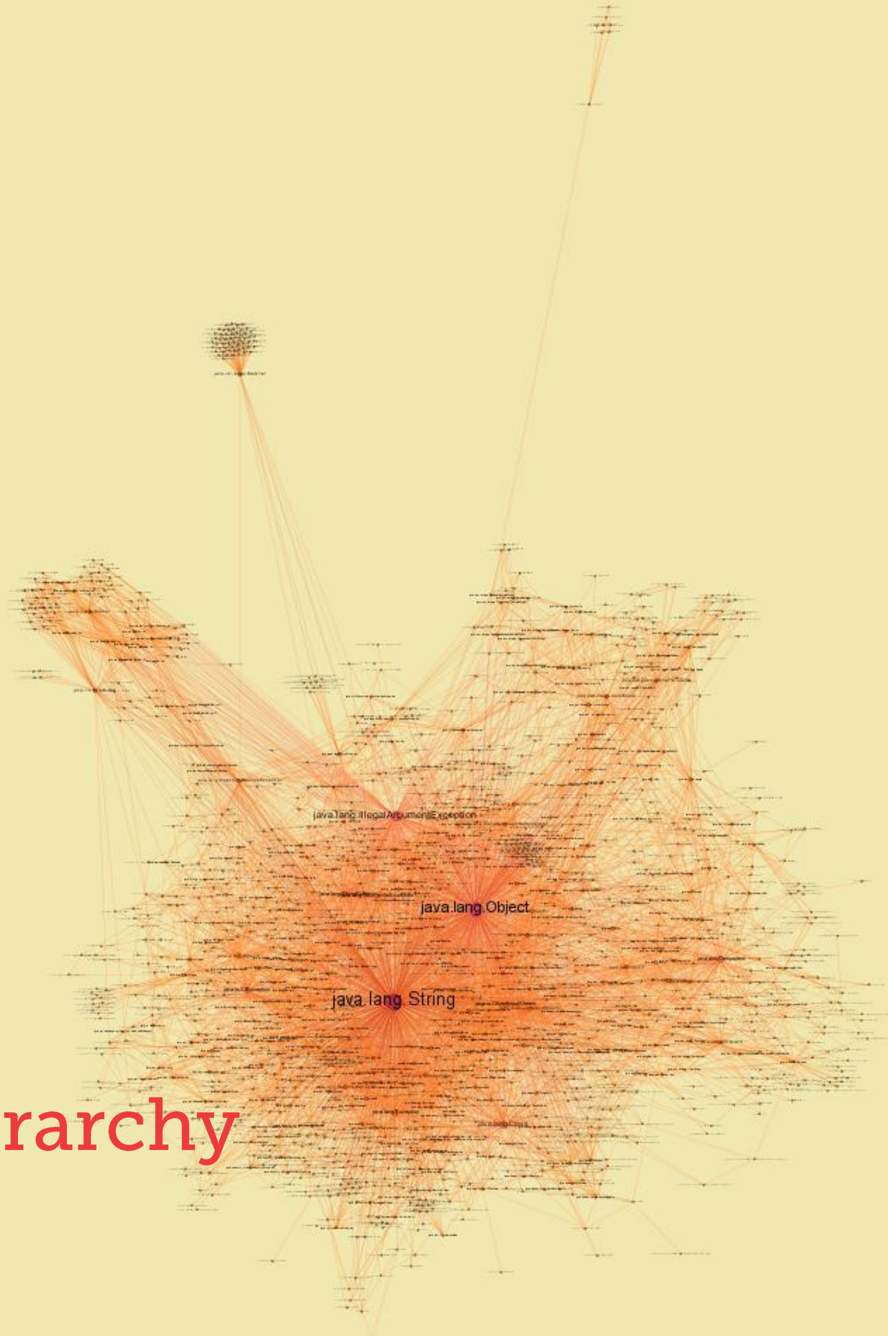
GRAPH

that shows the hierarchy
of classes in Java



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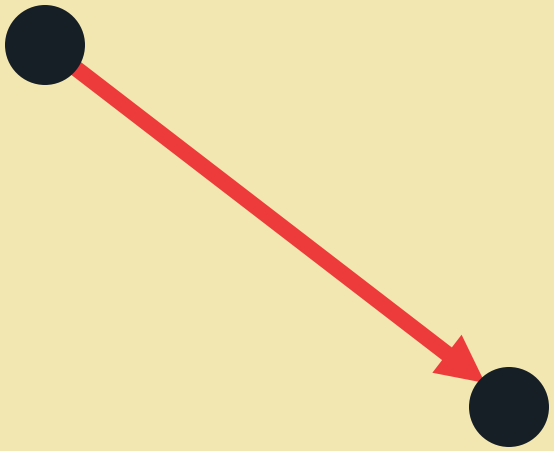
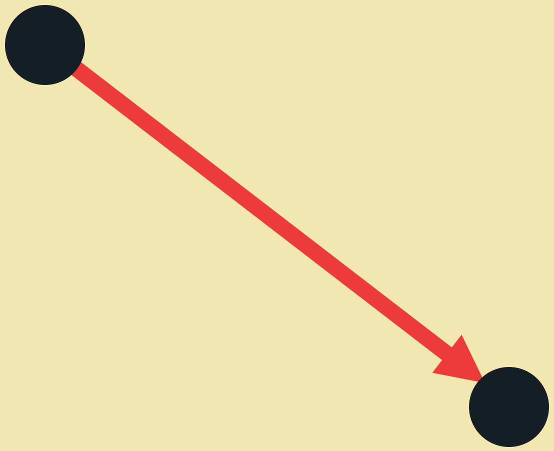
types of

GRAPHS

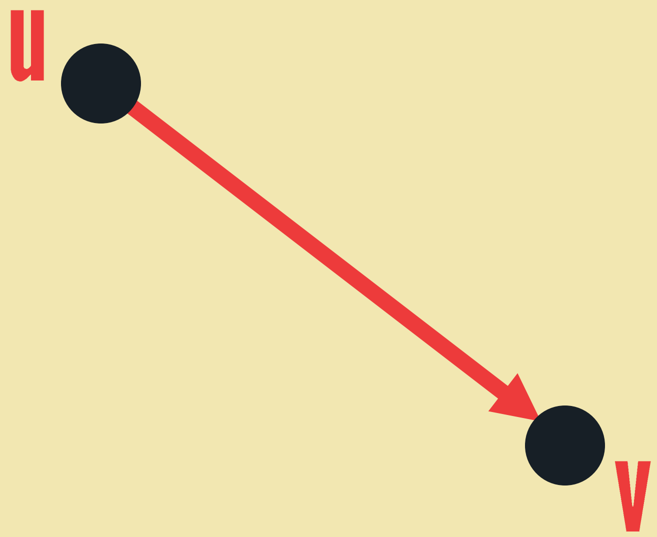
directed

GRAPHS



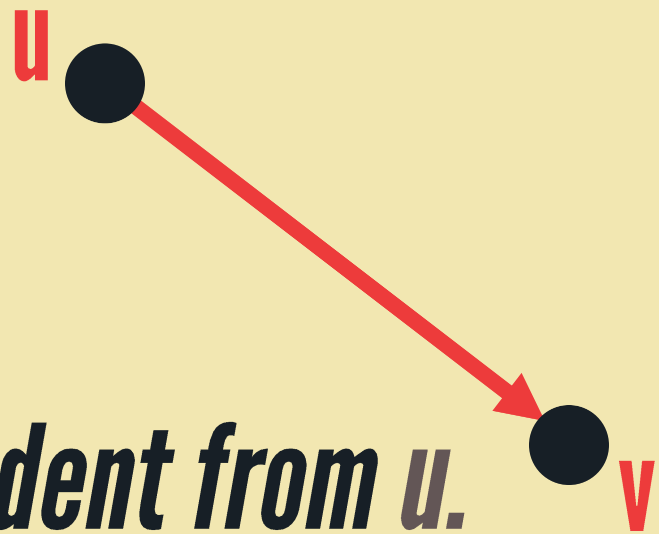


Directed Graphs
have edges that are
***directed** (ordered pairs).*



***Vertices:** u, v*

***Directed** edge: (u,v)*



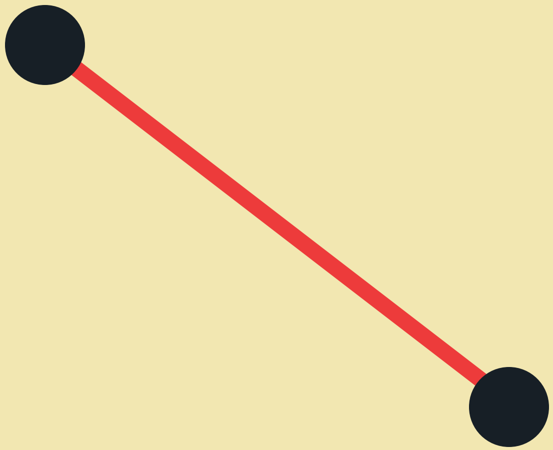
Edge (u,v) is ***incident from*** *u.*

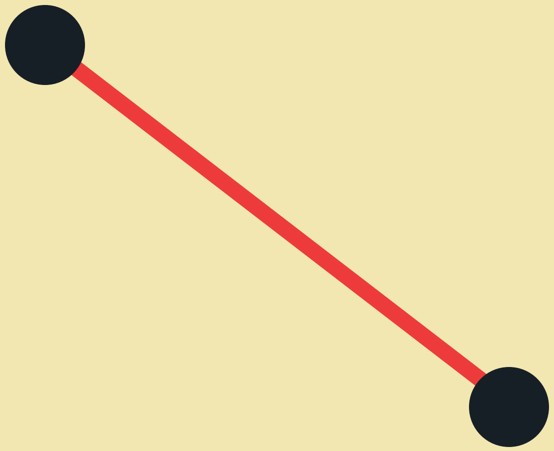
Edge (u,v) is ***incident to*** *u.*

Vertex v is ***adjacent to*** *vertex u.*

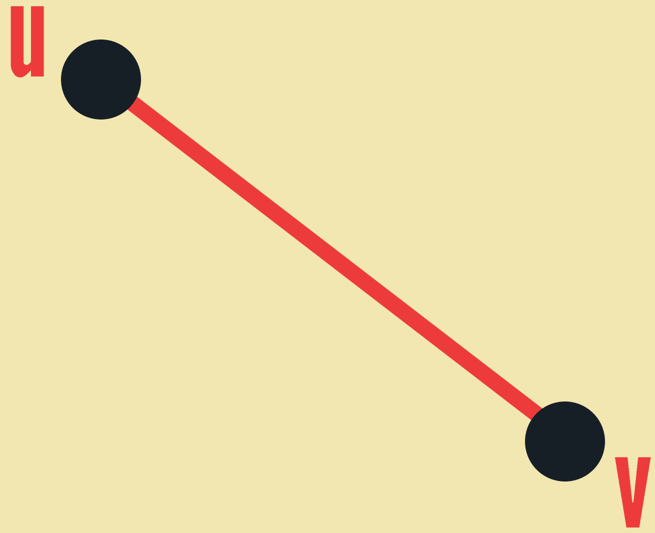
undirected

GRAPHS



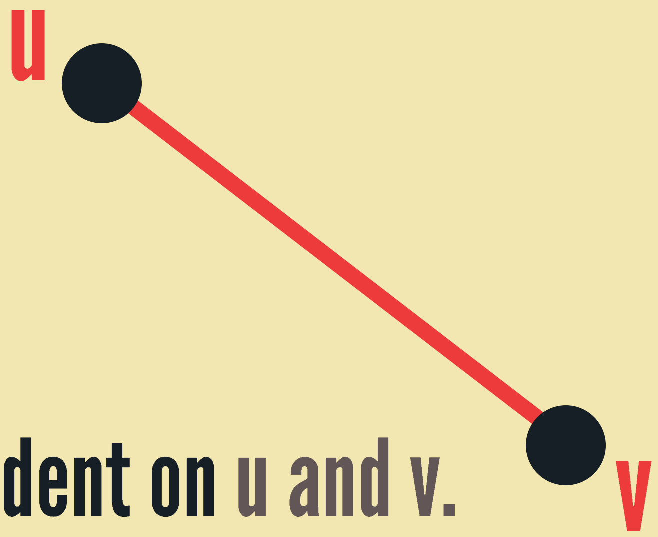


Undirected Graphs have edges that are **undirected**.



Vertices: u, v

Undirected edge: (u,v) or (v,u)



Edge (u,v) or (v,u) is incident on u and v.

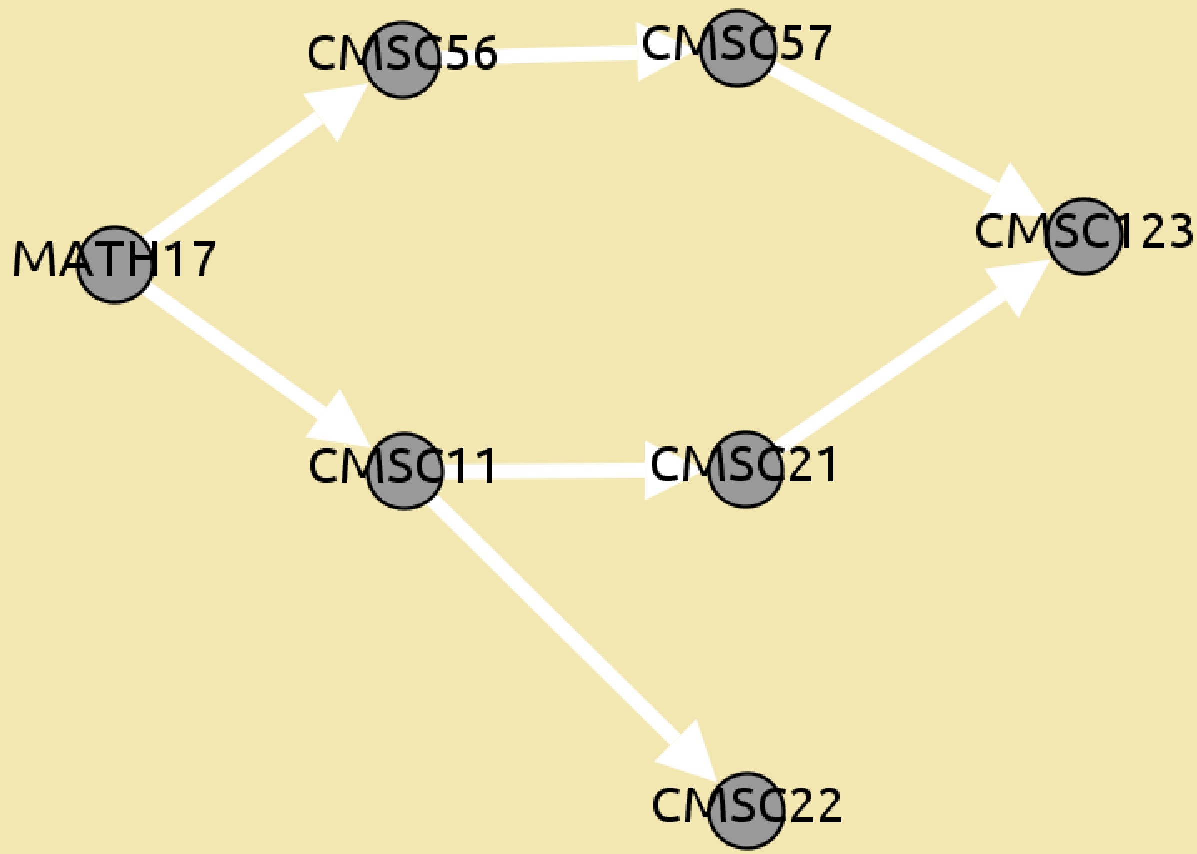
Vertex v is adjacent to vertex u.

Vertex u is adjacent to vertex v.

Graph

that shows the hierarchy
of courses in BSCS.

Directed or undirected?

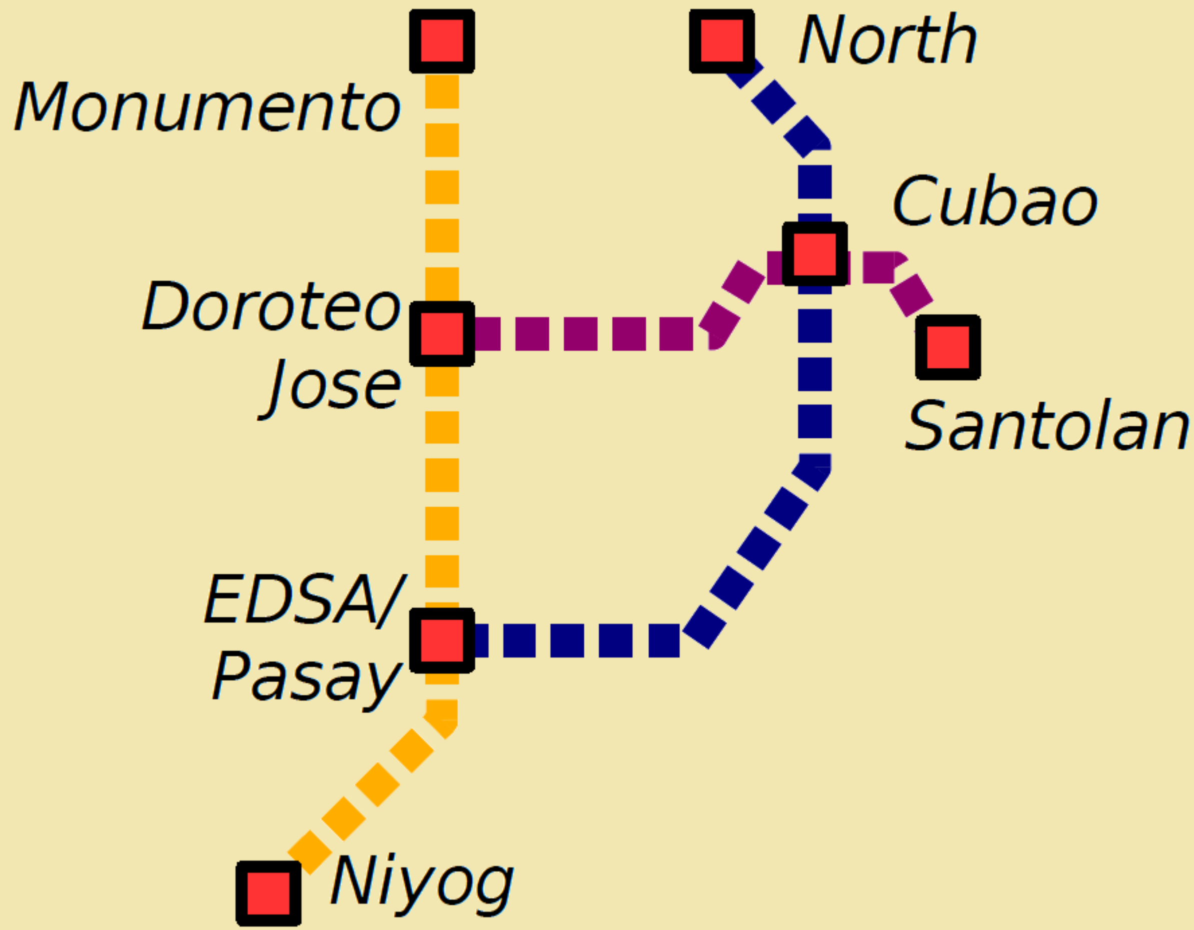


Directed or undirected?

Graph

that shows the
Manila MRT System.

Directed or undirected?

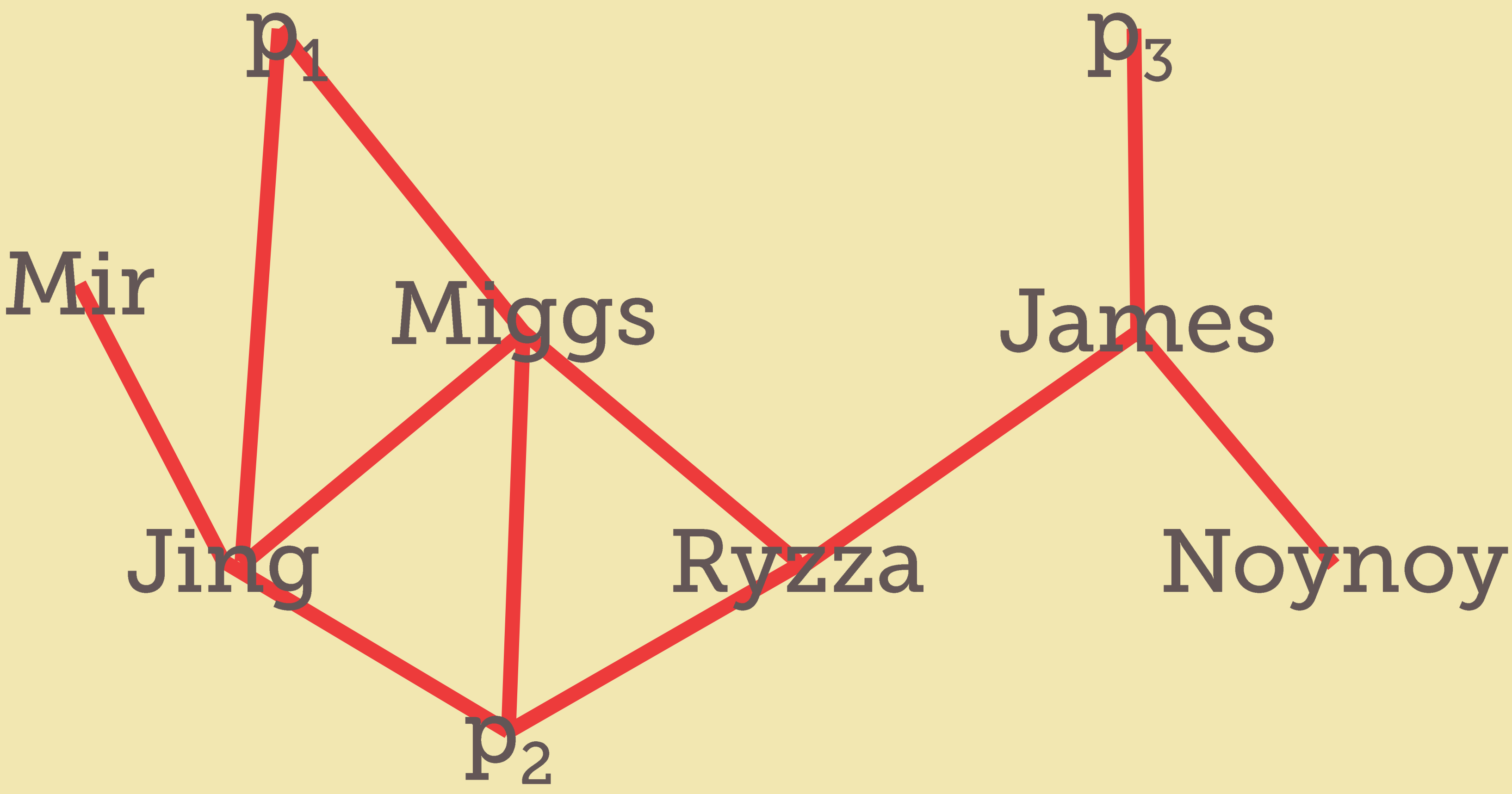


Directed or undirected?

Graph

that shows friendship connections on Facebook.

Directed or undirected?



Directed or undirected?

Graph

that shows what
users follow on Twitter.

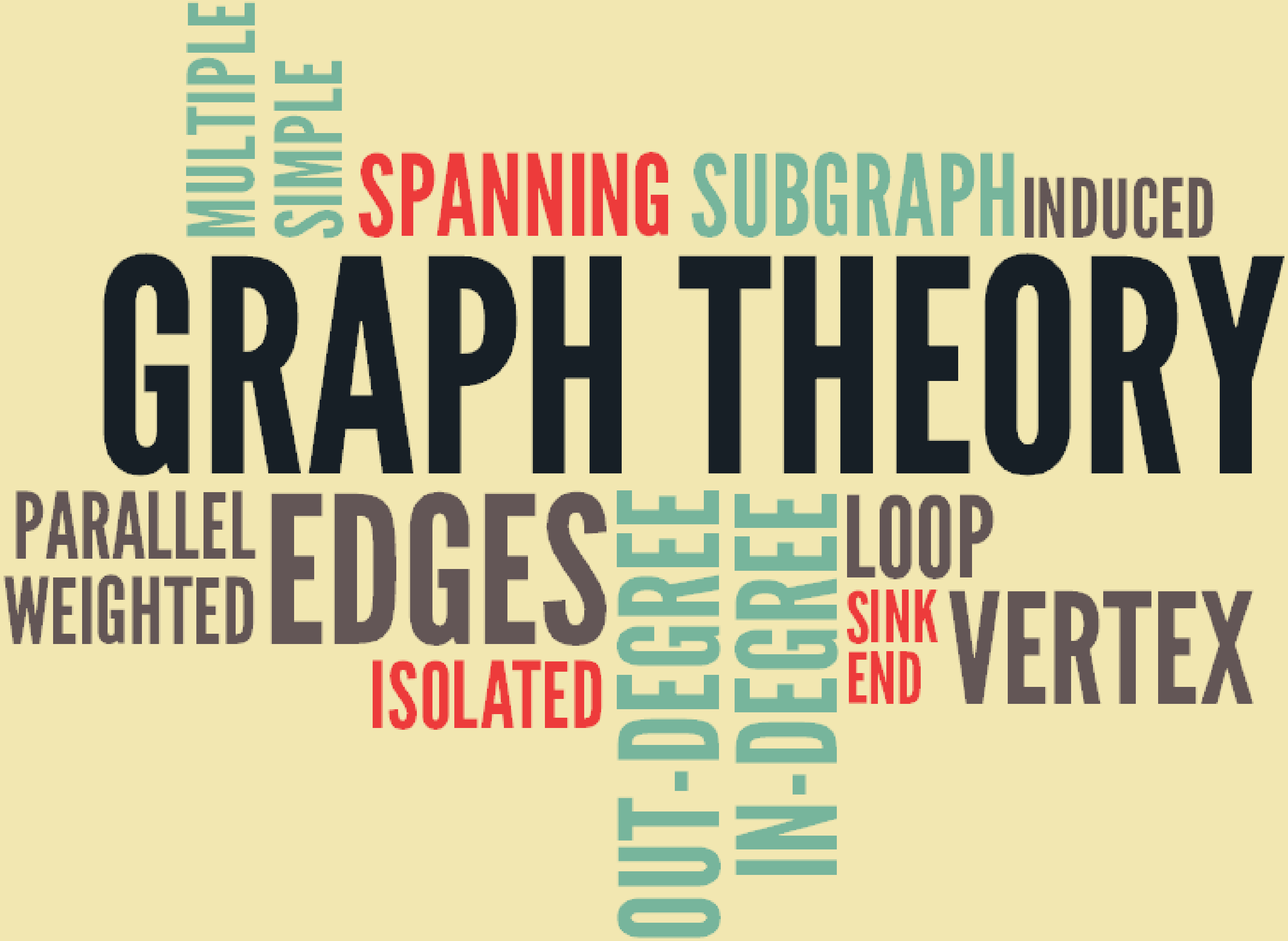
Directed or undirected?

Draw the directed graph

$$G = \{ V(G), E(G) \}$$

$$V(G) = \{a,b,c,d,e,f\}$$

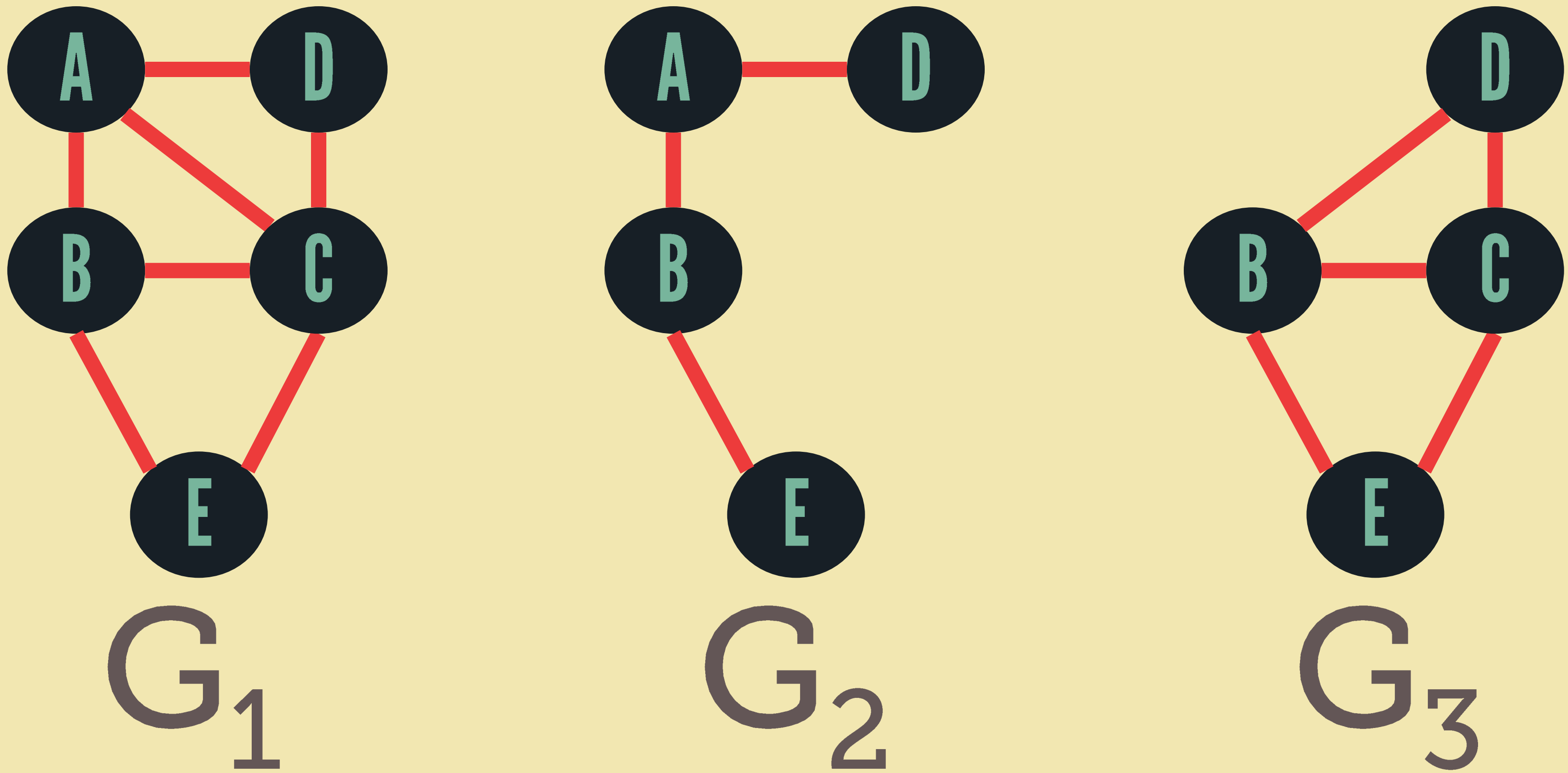
$$E(G) = \{(a,d),(b,a),(b,e),(d,c),(f,e)\}$$

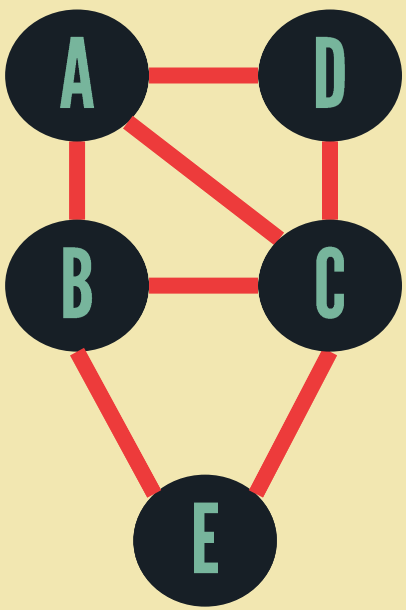

MULTIPLE
SIMPLE
SPANNING
SUBGRAPH
INDUCED
GRAPH THEORY
PARALLEL
WEIGHTED
EDGES
ISOLATED
OUT-DEGREE
IN-DEGREE
LOOP
SINK
END
VERTEX

SUBGRAPH

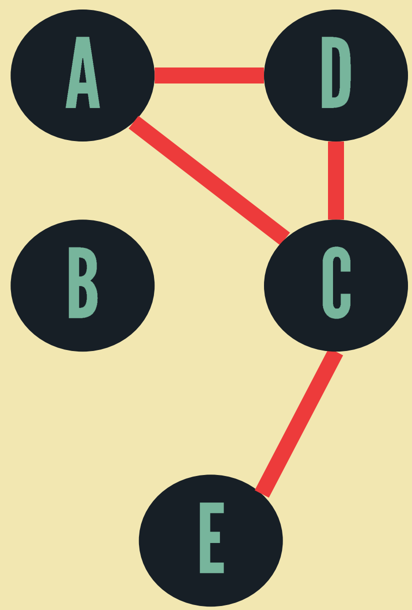
$$G_S = \{ V(G_S), E(G_S) \}$$

where $V(G_S) \subseteq V(G)$ and $E(G_S) \subseteq E(G)$

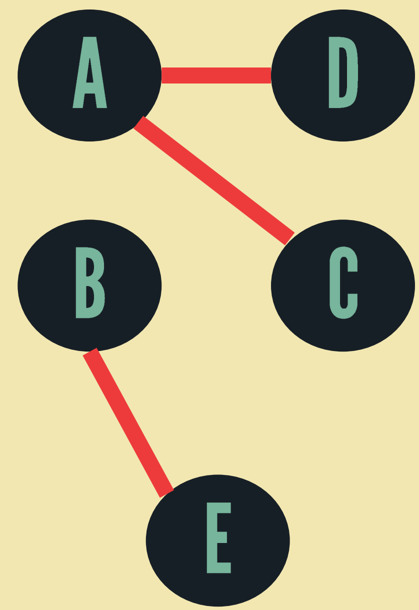

A
D
B
C
E
G1
A
D
B
E
G2
D
B
C
E
G3



G_1



G_4



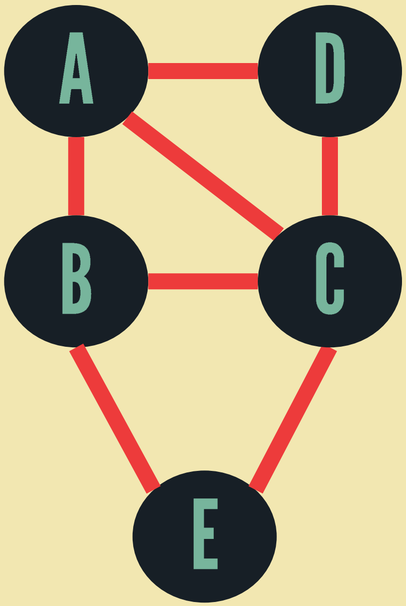
G_5

SPANNING SUBGRAPH

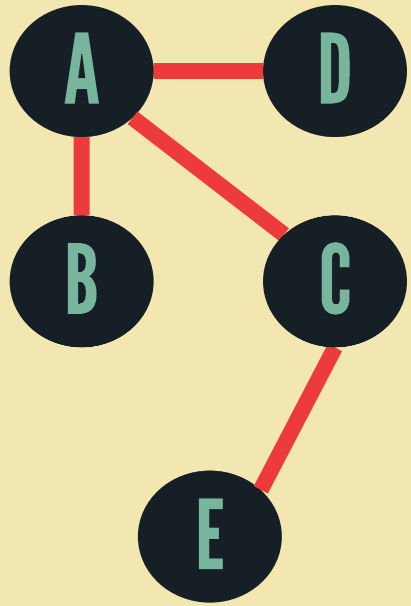
A subgraph

$$G_S = \{ V(G_S), E(G_S) \}$$

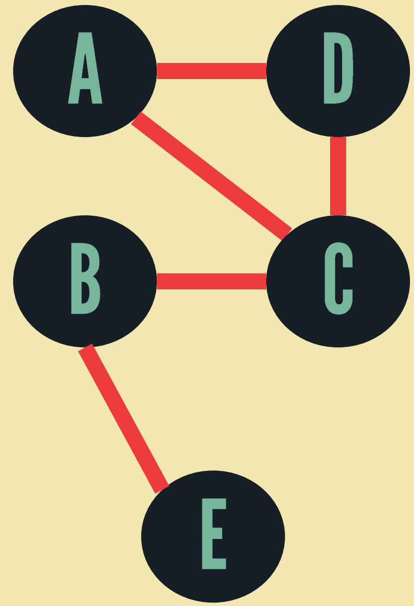
Where $V(G_S) = V(G)$



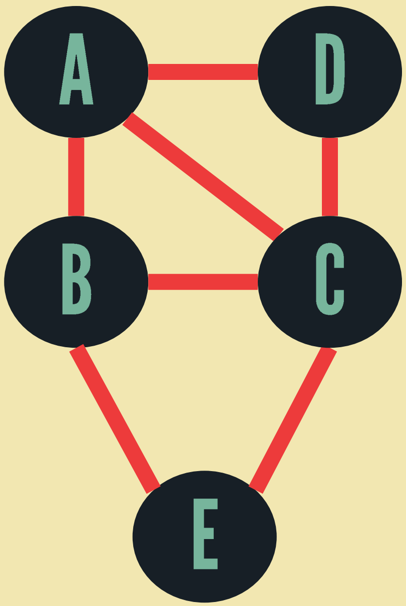
G_1



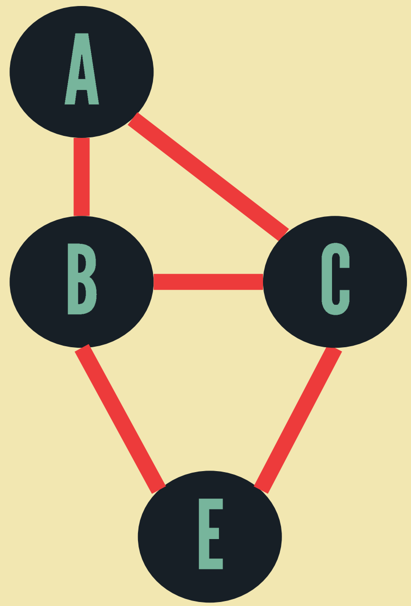
G_6



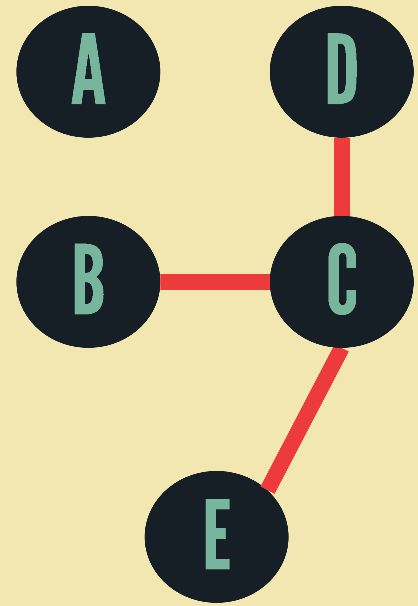
G_7



G_1



G_8



G_9

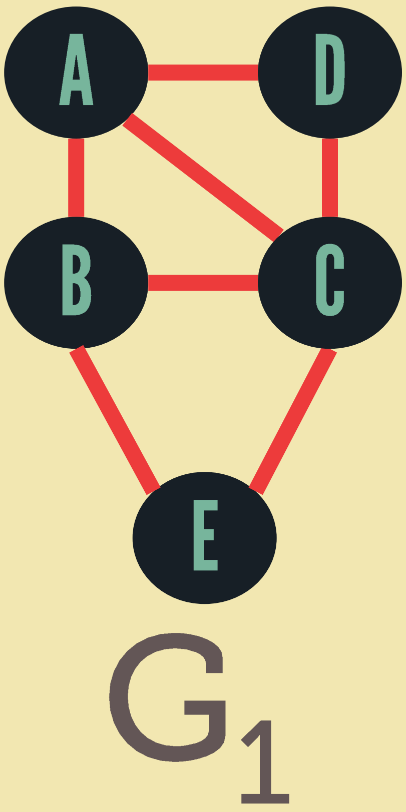
SUBGRAPH INDUCED

by a set of vertices W

A subgraph

$$G_S = \{ V(G_S), E(G_S) \}$$

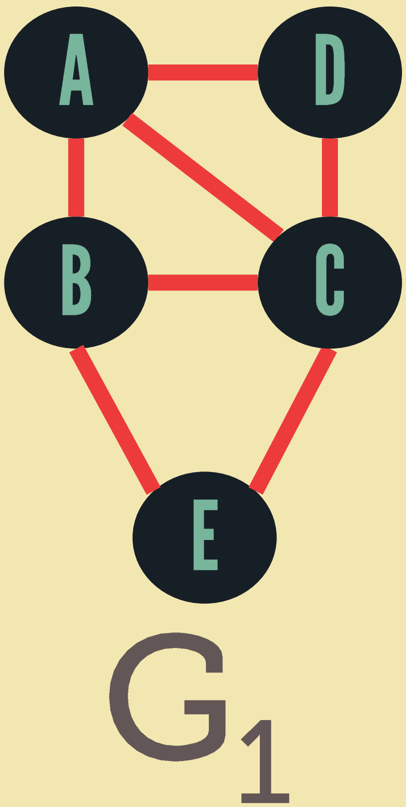
where $V(G_S) = W$ and
$E(G_S)$ are edges of G
that join pairs of vertices in W.



SUBGRAPH INDUCED

by a set of vertices

W = {A, B, C, E}

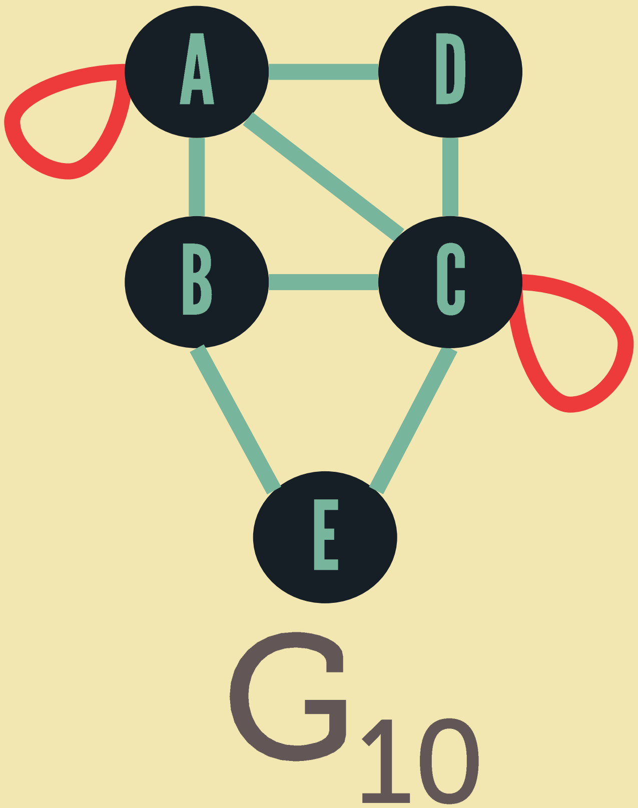


SUBGRAPH INDUCED

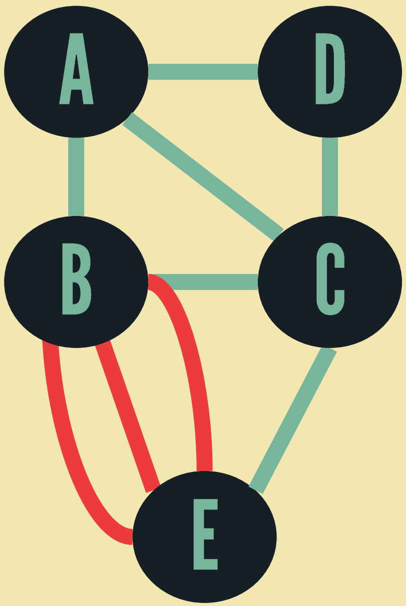
by a set of vertices

$W = \{B, C, D\}$

LOOPS (EDGES)

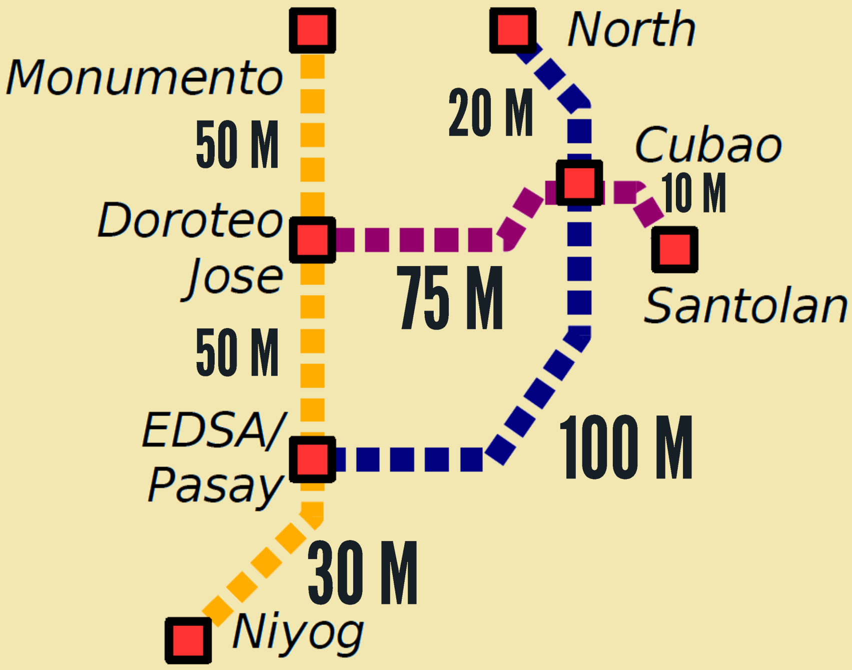


PARALLEL EDGES



G_{11}

WEIGHTED EDGES / LABELED EDGES



SIMPLE GRAPH

NO LOOPS
PARALLEL EDGES

MULTIGRAPH

HAVE LOOPS

PARALLEL EDGES

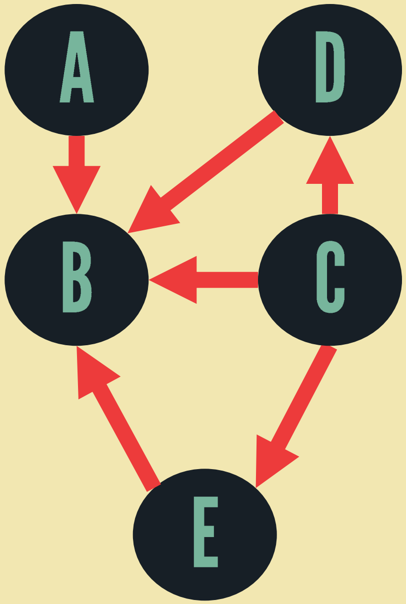
DEGREE of a vertex
(for DIRECTED GRAPHS)

in-degree, $\rho^{+}(v)$

# of edges incident to v

out-degree, $\rho^{-}(v)$

# of edges incident from v



G_{12}

Degree of vertex v, $\rho(v)$

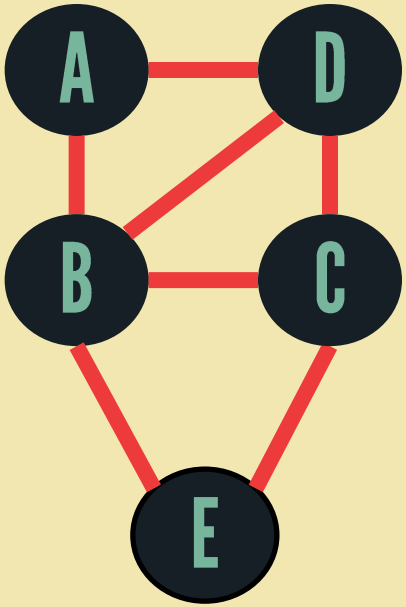
of edges incident on v

$= \rho^-(v) + \rho^+(v)$

Degree of vertex v, $\rho(v)$

of edges incident on v.

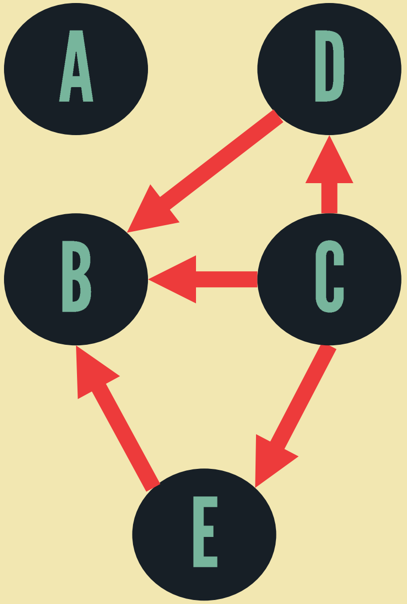
(also applicable for undirected graphs)



G_1

ISOLATED VERTEX

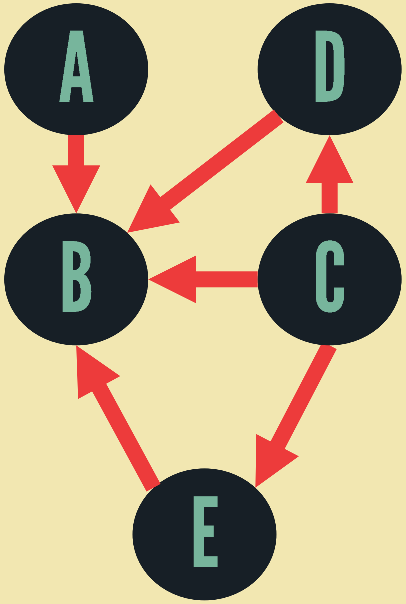
a vertex v with $\rho(v) = 0$



G_{13}

END VERTEX

a vertex v with $\rho(v) = 1$



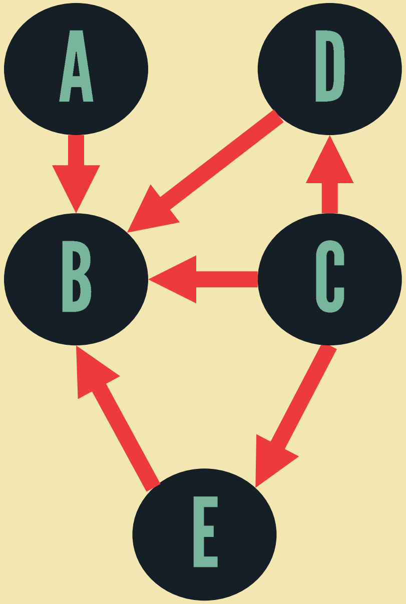
G_{12}

SINK VERTEX

a vertex v with

$\rho^{+}(\mathbf{v}) = |V(G)\text{-}1|$ and

$\rho^{-}(\mathbf{v}) = 0$



G_{12}

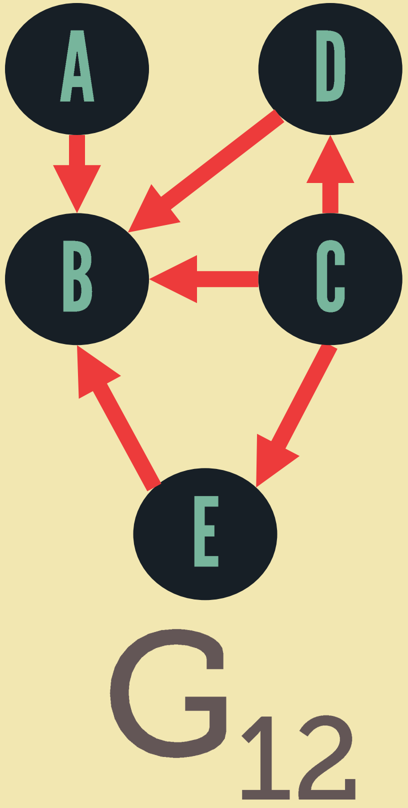
Theorems on

GRAPH THEORY

$$\sum \rho(v) = 2|E(G)|$$

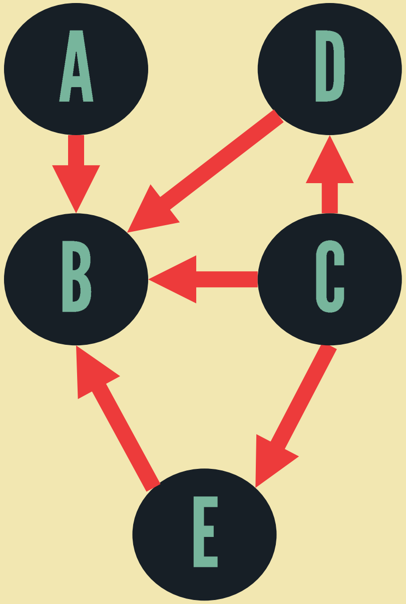
$$\sum \rho^{+}(v) = |E(G)|$$

$$\sum \rho^{-}(v) = |E(G)|$$



The HANDSHAKING lemma

Every graph has an even number of vertices with odd degree.



G_{12}

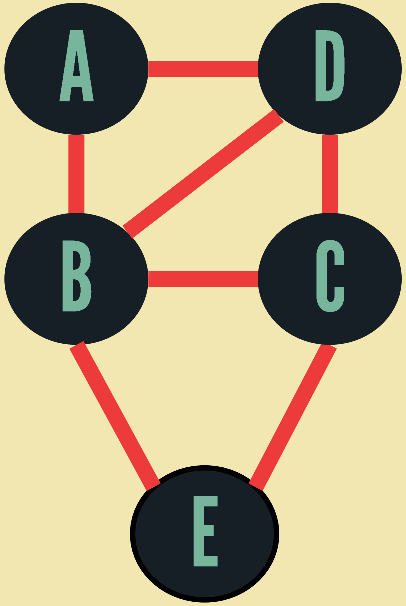
Graph
Representations

INCIDENCE
ADJACENCY

MATRIX

INCIDENCE MATRIX for undirected graphs

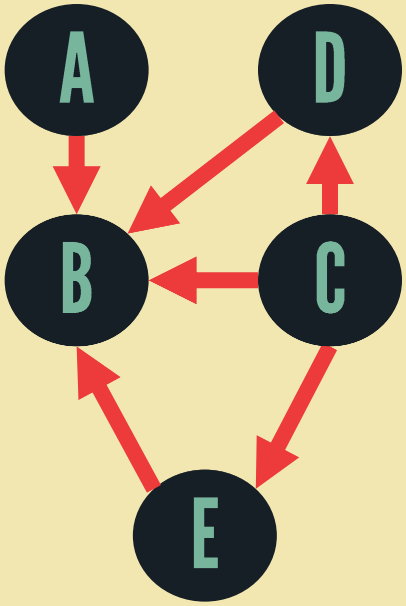
	EDGES
V E R T I C E S	Entry for row v, column e = 1 if e is incident on v = 0 otherwise



G_1

INCIDENCE MATRIX for directed graphs

	EDGES
V E R T I C E S	Entry for row v, column e = -1 if edge e leaves vertex v = 1 if edge e enters vertex v



G_{12}

ADJACENCY
MATRIX

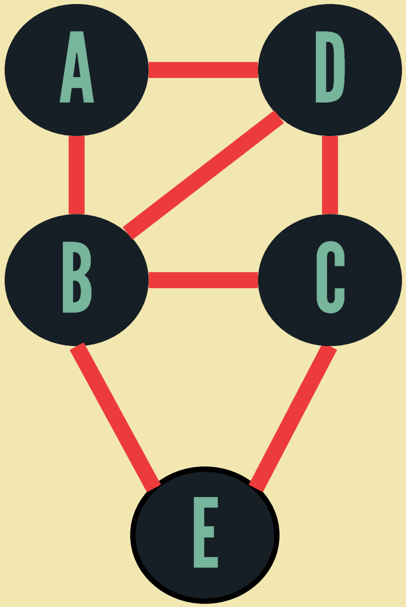
	EDGES
VERTICES	Entry for row i, column j = 1 if vertex i and j are adjacent

Graph
Operations

Removal of a vertex v

$V(G\text{-}v) = V(G) - \{v\}$

$E(G\text{-}v) =$ E(G) except those incident on v



G_1

Removal of an edge e

$$V(G-e) = V(G)$$
$$E(G-e) = E(G) - \{e\}$$

Addition of an edge e

$$V(G+e) = V(G)$$
$$E(G+e) = E(G) + \{e\}$$

Complement of a graph (simple only)

$V(G^C) = V(G)$

$E(G^C)$ have edges that are not in $E(G)$

DISCONNECTED
CONNECTED
CUT
EDGE
DIAMETER
EULERIAN
VERTEX
DISTANCE

GRAPH THEORY

HAMILTONIAN
CYCLES
WALKS
PATHS
CIRCUITS
TRAILS

walk

finite non-empty sequence of edges (v_1,v_2), (v_2,v_3), ..., (v_{n-1},v_n) such that (v_i,v_{i+1}) is an edge in G.

walk

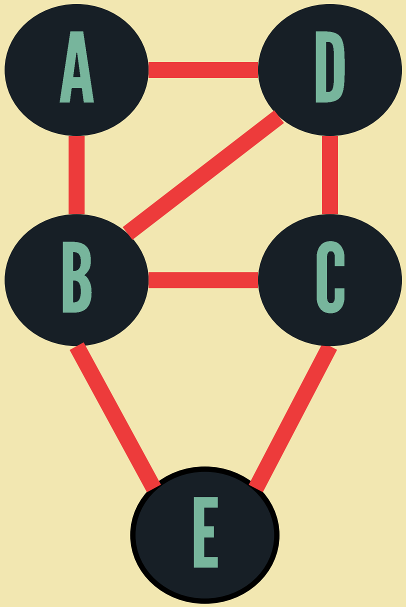
$$v_1\ v_2\ v_3 \cdots\ v_{n-1}\ v_n$$

trail

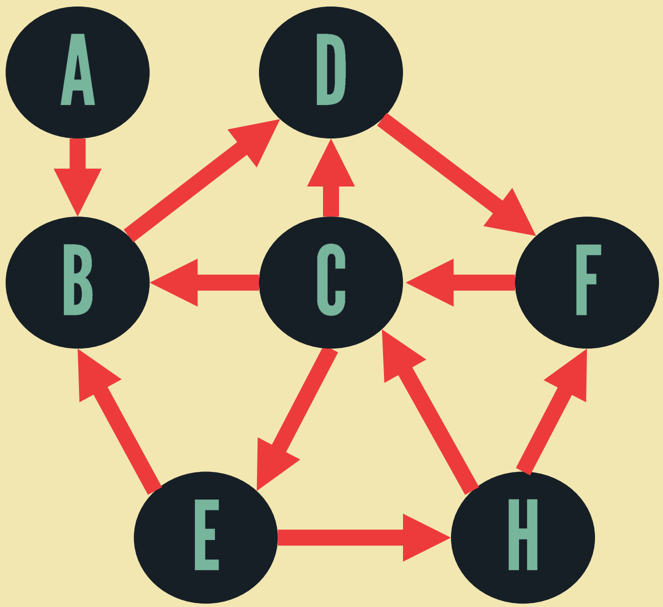
a walk with no repeated edges.

path

a walk with no repeated vertices.



G_1



G_{14}

closed walk

a walk that begins and ends at the same vertex.

closed trail / circuit

closed walk with no repeated edges.

closed path / cycle

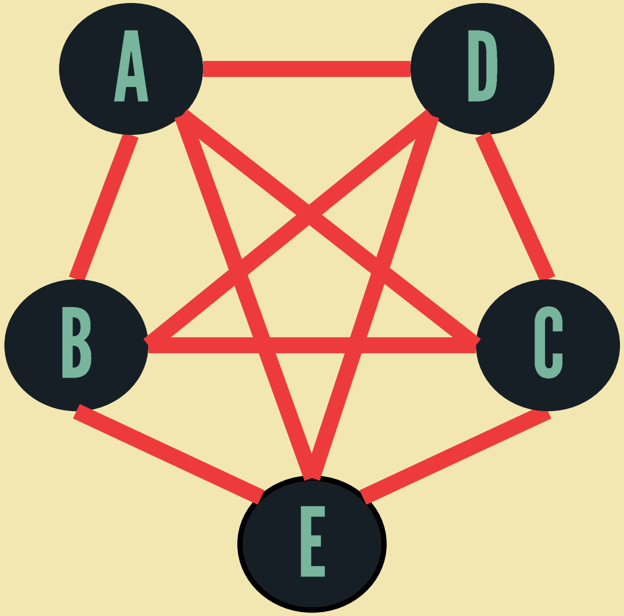
closed walk with no repeated vertices.

Eulerian circuit

a circuit which includes all vertices and all the edges of G.

Eulerian graph

a graph that contains an Eulerian circuit.



G_{15}

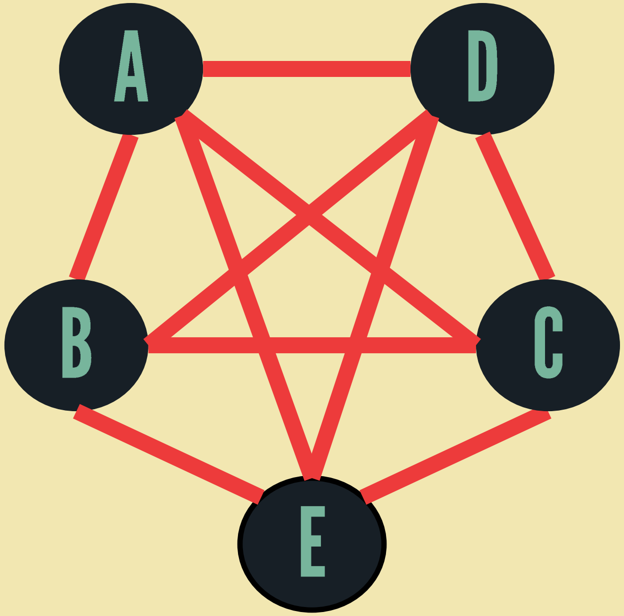
Eulerian circuit: **ACBDEBADCE**

Hamiltonian cycle

a cycle which includes every vertices of G exactly once (except for the initial and final vertices).

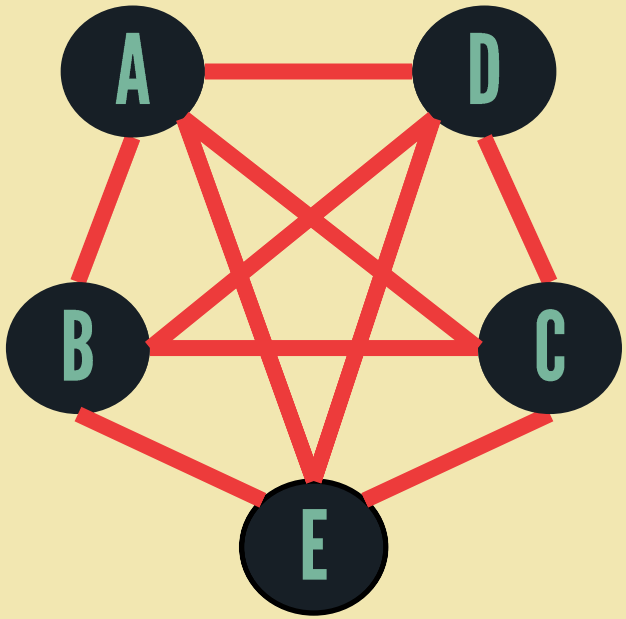
Hamiltonian graph

a graph that contains a Hamiltonian cycle.



G_{15}

Hamiltonian cycle: **ABECDA**



G_{15}

Hamiltonian cycle: **ACBDEA**

CONNECTED
DISCONNECTED
GRAPHS

connected graph

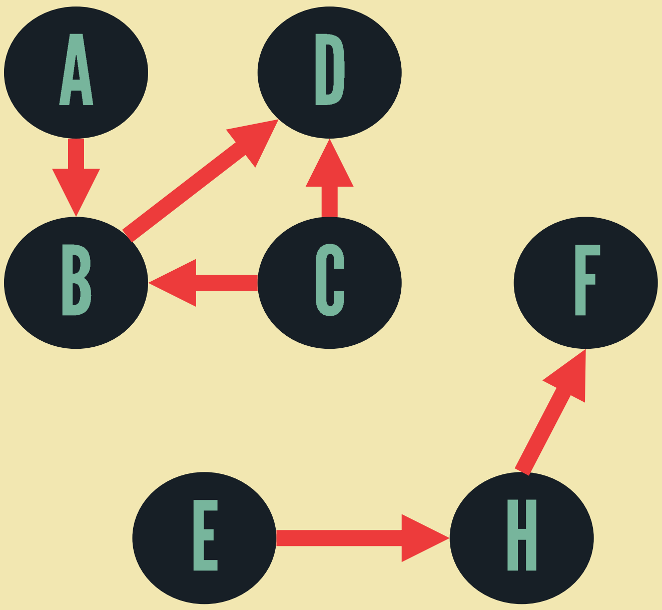
(undirected only)

there is a path between any two of its vertices.

disconnected graph

(undirected only)

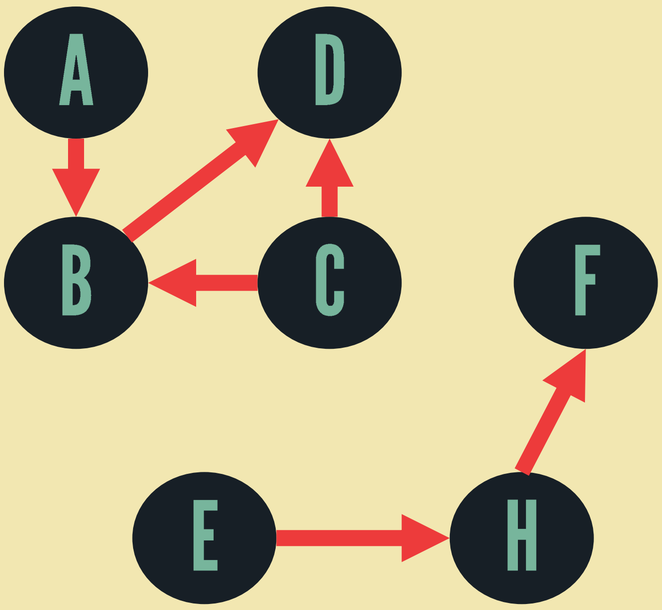
a graph that is not connected.



G_{15}

components

connected subgraphs
of a graph



G_{15}