



# Chapter 4

## SIMPLIFICATION of LOGIC CIRCUITS



# Don't Care conditions

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- The unspecified minterms (maxterms) of an incompletely specified function
- An **X** inside a map represents a don't care condition



# Don't Care conditions

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- There are 2 cases when this occurs.
  - The input combination never occurs
    - E.g. The BCD code does not use the 6 remaining codes.
  - The input combinations are expected to occur, but we do not care what the outputs are

# Representation of Don't Cares

- e.g:

$$F(W,X,Y,Z) = \sum m(0,1,2,4,6,7,8,10)$$

$$d(W,X,Y,Z) = \sum d(12,13,14,15)$$

- It could also be represented as:

$$F(W,X,Y,Z) = \sum m(0,1,2,4,6,7,8,10) \\ + \sum d(12,13,14,15)$$

# Example

- Simplify  $F = \sum m(1, 3, 7, 14, 15) + d(8)$

AB \ CD	CD			
	00	01	11	10
00		1	1	
01			1	
11			1	1
10	X			

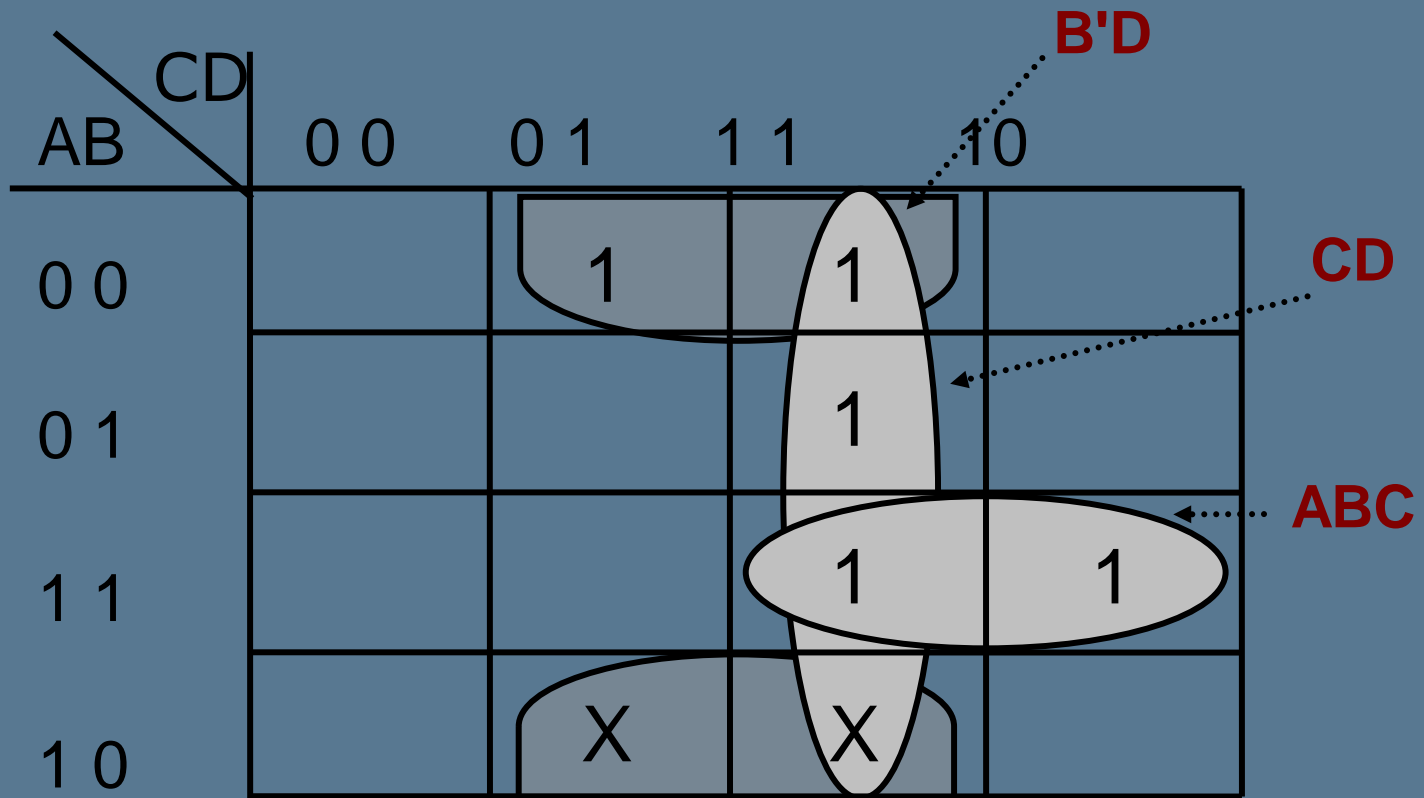
**A'B'D** (points to the group of 1s in row 00, columns 01 and 11)

**BCD** (points to the group of 1s in column 11, rows 01 and 11)

**ABC** (points to the group of 1s in row 11, columns 11 and 10)

# Example

- Simplify  $F = \sum m(1, 3, 7, 14, 15) + d(9, 11)$



# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0				
0 1				
1 1				
1 0				

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1			1
0 1	1			
1 1	1			
1 0	1	1		



# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1			1
0 1	1			X
1 1	1	X	X	
1 0	1	1		

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1				X
1 1	1	X	X		
1 0	1	1			

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1				X
1 1	1	X	X		
1 0	1	1			

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1				X
1 1	1	X	X		
1 0	1	1			

Diagram illustrating the Karnaugh map for the function  $F(A,B,C,D)$ . The map shows minterms (1) and don't care terms (X) for the function  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$ .

The map is a 4x4 grid with rows labeled AB and columns labeled CD. The cells contain values 1 or X.

Groupings shown:

- A group of four cells (1s) is circled, corresponding to the term  $A'D'$ .
- A group of four cells (1s and Xs) is circled, corresponding to the term  $AB'$ .
- A group of four cells (1s and Xs) is circled, corresponding to the term  $BC$ .
- A group of two cells (1s) is circled, corresponding to the term  $CD$ .

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1				X
1 1	1	X	X		
1 0	1	1			

Groupings and Simplifications:

- A'D'**: Grouped cells (0,0), (0,1), (4,0), (4,1) are shaded gray. A dotted arrow points from the label to the cell (4,0).
- AC'**: Grouped cells (0,0), (0,1), (4,0), (4,1) are shaded light gray. A dotted arrow points from the label to the cell (4,1).

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15) = A'D' + AC'$

AB \ CD		CD			
		00	01	11	10
00	1				1
01	1				X
11	1	X	X		
10	1	1			

Diagram illustrating the Karnaugh map for the function  $F(A,B,C,D)$ . The map shows minterms (1s) and don't care terms (Xs) for the function  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$ . The map is a 4x4 grid with rows labeled AB and columns labeled CD. The minterms are circled, and the don't care terms are marked with X. The function is simplified to  $A'D' + AC'$ .

Annotations:

- $A'D'$  is indicated by a dotted line pointing to the minterms 00 and 01.
- $AC'$  is indicated by a dotted line pointing to the minterms 11 and 10.

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

0    0000

2    0010

4    0100

8    1000

9    1001

12   1100

6    0110

13   1101

15   1111

# Example – Quine McCluskey

•  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1
		0 0000
0	0000	
2	0010	2 0010
4	0100	4 0100
8	1000	8 1000
9	1001	6 0110
12	1100	9 1001
6	0110	12 1100
13	1101	
15	1111	13 1101
		15 1111



# Example – Quine McCluskey

•  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2
		0 0000	✓	0,2 00-0
0	0000			
2	0010	2 0010	✓	
4	0100	4 0100		
8	1000	8 1000		
9	1001	6 0110		
12	1100	9 1001		
6	0110	12 1100		
13	1101	13 1101		
15	1111	15 1111		

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2	
		0 0000	✓	0,2 00-0	
0	0000			0,4 0-00	
2	0010	2 0010	✓		
4	0100	4 0100	✓		
8	1000	8 1000			
9	1001	6 0110			
12	1100	9 1001			
6	0110	12 1100			
13	1101	13 1101			
15	1111	15 1111			

# Example – Quine McCluskey

$$F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$$

		Column 1	Column 2
		0 0000 ✓	0,2 00-0
0	0000		0,4 0-00
2	0010	2 0010 ✓	0,8 -000
4	0100	4 0100 ✓	
8	1000	8 1000 ✓	
9	1001	6 0110	
12	1100	9 1001	
6	0110	12 1100	
13	1101	13 1101	
15	1111	15 1111	

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2	
		0	0000 ✓	0,2	00-0
0	0000			0,4	0-00
2	0010	2	0010 ✓	0,8	-000
4	0100	4	0100 ✓		
8	1000	8	1000 ✓	2,6	0-10
9	1001	6	0110 ✓		
12	1100	9	1001		
6	0110	12	1100		
13	1101				
15	1111	13	1101		
		15	1111		

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2
		0 0000 ✓		0,2 00-0
0	0000			0,4 0-00
2	0010	2 0010 ✓		0,8 -000
4	0100	4 0100 ✓		
8	1000	8 1000 ✓		2,6 0-10
9	1001			4,6 01-0
12	1100	6 0110 ✓		
6	0110	9 1001		
13	1101	12 1100		
15	1111	13 1101		
		15 1111		

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2	
		0	0000 ✓	0,2	00-0
0	0000			0,4	0-00
2	0010	2	0010 ✓	0,8	-000
4	0100	4	0100 ✓		
8	1000	8	1000 ✓	2,6	0-10
9	1001			4,6	01-0
12	1100	6	0110 ✓	4,12	-100
6	0110	9	1001		
13	1101	12	1100 ✓		
15	1111				
		13	1101		
		15	1111		

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2
		0 0000 ✓		0,2 00-0
0	0000			0,4 0-00
2	0010	2 0010 ✓		0,8 -000
4	0100	4 0100 ✓		
8	1000	8 1000 ✓		2,6 0-10
9	1001			4,6 01-0
12	1100	6 0110 ✓		4,12 -100
6	0110	9 1001 ✓		8,9 100-
13	1101	12 1100 ✓		
15	1111	13 1101		
		15 1111		

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2	
		0	0000 ✓	0,2	00-0
0	0000			0,4	0-00
2	0010	2	0010 ✓	0,8	-000
4	0100	4	0100 ✓		
8	1000	8	1000 ✓	2,6	0-10
9	1001			4,6	01-0
12	1100	6	0110 ✓	4,12	-100
6	0110	9	1001 ✓	8,9	100-
13	1101	12	1100 ✓	8,12	1-00
15	1111				
		13	1101		
		15	1111		



# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2	
		0 0000	✓	0,2	00-0
0	0000			0,4	0-00
2	0010	2 0010	✓	0,8	-000
4	0100	4 0100	✓		
8	1000	8 1000	✓	2,6	0-10
9	1001			4,6	01-0
12	1100	6 0110	✓	4,12	-100
6	0110	9 1001	✓	8,9	100-
13	1101	12 1100	✓	8,12	1-00
15	1111				
		13 1101	✓	9,13	1-01
		15 1111			

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2	
		0	0000 ✓	0,2	00-0
0	0000			0,4	0-00
2	0010	2	0010 ✓	0,8	-000
4	0100	4	0100 ✓		
8	1000	8	1000 ✓	2,6	0-10
9	1001			4,6	01-0
12	1100	6	0110 ✓	4,12	-100
6	0110	9	1001 ✓	8,9	100-
13	1101	12	1100 ✓	8,12	1-00
15	1111				
		13	1101 ✓	9,13	1-01
				12,13	110-
		15	1111		

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2
		0 0000 ✓		0,2 00-0
0	0000			0,4 0-00
2	0010	2 0010 ✓		0,8 -000
4	0100	4 0100 ✓		
8	1000	8 1000 ✓		2,6 0-10
9	1001			4,6 01-0
12	1100	6 0110 ✓		4,12 -100
6	0110	9 1001 ✓		8,9 100-
13	1101	12 1100 ✓		8,12 1-00
15	1111			
		13 1101 ✓		9,13 1-01
				12,13 110-
		15 1111 ✓		13,15 11-1

# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2		Column 3	
		0	0000 ✓	0,2	00-0 ✓	0,2,4,6	0--0
0	0000			0,4	0-00		
2	0010	2	0010 ✓	0,8	-000		
4	0100	4	0100 ✓				
8	1000	8	1000 ✓	2,6	0-10		
9	1001			4,6	01-0 ✓		
12	1100	6	0110 ✓	4,12	-100		
6	0110	9	1001 ✓	8,9	100-		
13	1101	12	1100 ✓	8,12	1-00		
15	1111	13	1101 ✓	9,13	1-01		
				12,13	110-		
		15	1111 ✓	13,15	11-1		

# Example – Quine McCluskey

•  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2		Column 3
		0 0000 ✓		0,2 00-0 ✓		0,2,4,6 0--0
0	0000			0,4 0-00 ✓		0,4,2,6 0--0
2	0010	2 0010 ✓		0,8 -000		0,4,8,12 --00
4	0100	4 0100 ✓				
8	1000	8 1000 ✓		2,6 0-10 ✓		
9	1001			4,6 01-0 ✓		
12	1100	6 0110 ✓		4,12 -100		
6	0110	9 1001 ✓		8,9 100-		
13	1101	12 1100 ✓		8,12 1-00 ✓		
15	1111	13 1101 ✓		9,13 1-01		
				12,13 110-		
		15 1111 ✓		13,15 11-1		

# Example – Quine McCluskey

•  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2		Column 3	
		0 0000 ✓		0,2 00-0 ✓		0,2,4,6 0--0	
0	0000			0,4 0-00 ✓		0,4,2,6 0--0	
2	0010	2 0010 ✓		0,8 -000 ✓		0,4,8,12 --00	
4	0100	4 0100 ✓				0,8,4,12 --00	
8	1000	8 1000 ✓		2,6 0-10 ✓			
9	1001			4,6 01-0 ✓			
12	1100	6 0110 ✓		4,12 -100 ✓			
6	0110	9 1001 ✓		8,9 100-			
13	1101	12 1100 ✓		8,12 1-00 ✓			
15	1111	13 1101 ✓		9,13 1-01			
				12,13 110-			
		15 1111 ✓		13,15 11-1			

# Example – Quine McCluskey

•  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2		Column 3	
		0 0000 ✓		0,2 00-0 ✓		0,2,4,6 0--0	
0	0000			0,4 0-00 ✓		0,4,2,6 0--0	
2	0010	2 0010 ✓		0,8 -000 ✓		0,4,8,12 --00	
4	0100	4 0100 ✓				0,8,4,12 --00	
8	1000	8 1000 ✓		2,6 0-10 ✓			
9	1001			4,6 01-0 ✓		8,9,12,13 1-0-	
12	1100	6 0110 ✓		4,12 -100 ✓			
6	0110	9 1001 ✓		8,9 100- ✓			
13	1101	12 1100 ✓		8,12 1-00 ✓			
15	1111	13 1101 ✓		9,13 1-01			
				12,13 110- ✓			
		15 1111 ✓		13,15 11-1			



# Example – Quine McCluskey

•  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

		Column 1		Column 2		Column 3	
		0 0000 ✓		0,2 00-0 ✓		0,2,4,6 0--0	
0	0000			0,4 0-00 ✓		0,4,2,6 0--0	
2	0010	2 0010 ✓		0,8 -000 ✓		0,4,8,12 --00	
4	0100	4 0100 ✓				0,8,4,12 --00	
8	1000	8 1000 ✓		2,6 0-10 ✓			
9	1001			4,6 01-0 ✓		8,9,12,13 1-0-	
12	1100	6 0110 ✓		4,12 -100 ✓		8,12,9,13 1-0-	
6	0110	9 1001 ✓		8,9 100- ✓			
13	1101	12 1100 ✓		8,12 1-00 ✓			
15	1111	13 1101 ✓		9,13 1-01 ✓			
				12,13 110- ✓			
		15 1111 ✓		13,15 11-1			



# Example – Quine McCluskey

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

Column 1			Column 2			Column 3	
0	0000	✓	0,2	00-0	✓	0,2,4,6	0--0
			0,4	0-00	✓	0,4,2,6	0--0
2	0010	✓	0,8	-000	✓	0,4,8,12	--00
4	0100	✓				0,8,4,12	--00
8	1000	✓	2,6	0-10	✓		
			4,6	01-0	✓	8,9,12,13	1-0-
6	0110	✓	4,12	-100	✓	8,12,9,13	1-0-
9	1001	✓	8,9	100-	✓		
12	1100	✓	8,12	1-00	✓		
13	1101	✓	9,13	1-01	✓		
			12,13	110-	✓		
15	1111	✓	13,15	11-1			

# Example – Quine McCluskey

•  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

Column 1			Column 2			Column 3	
0	0000	✓	0,2	00-0	✓	0,2,4,6	0--0
			0,4	0-00	✓	0,4,2,6	0--0
2	0010	✓	0,8	-000	✓	0,4,8,12	--00
4	0100	✓				0,8,4,12	--00
8	1000	✓	2,6	0-10	✓		
			4,6	01-0	✓	8,9,12,13	1-0-
6	0110	✓	4,12	-100	✓	8,12,9,13	1-0-
9	1001	✓	8,9	100-	✓		
12	1100	✓	8,12	1-00	✓		
13	1101	✓	9,13	1-01	✓		
			12,13	110-	✓		
15	1111	✓	13,15	11-1			

Prime implicants:  
 $ABD$  ,  $A'D'$  ,  $C'D'$  ,  $AC'$

# Example – Quine McCluskey

	0	2	4	8	9	12
13,15						
11-1						
0,2,4,6						
0--0						
0,4,8,12						
--00						
8,9,12,13						
1-0-						

# Example – Quine McCluskey

		0	2	4	8	9	12
13,15	11-1						
0,2,4,6	0--0	X	X	X			
0,4,8,12	--00	X		X	X		X
8,9,12,13	1-0-				X	X	X

# Example – Quine McCluskey

		0	2	4	8	9	12
13,15	11-1						
0,2,4,6	0--0	X	X	X			
0,4,8,12	--00	X		X	X		X
8,9,12,13	1-0-				X	X	X

# Example – Quine McCluskey

		0	2	4	8	9	12
13,15	11-1						
0,2,4,6	0--0	X	X	X			
0,4,8,12	--00	X		X	X		X
8,9,12,13	1-0-				X	X	X

# Example – Quine McCluskey

		0	2	4	8	9	12
13,15	11-1						
0,2,4,6	0--0	X	X	X			
0,4,8,12	--00	X		X	X		X
8,9,12,13	1-0-				X	X	X

# Example – Quine McCluskey

		0	2	4	8	9	12
13,15	11-1						
0,2,4,6	0--0	X	X	X			
0,4,8,12	--00	X		X	X		X
8,9,12,13	1-0-				X	X	X
		✓	✓	✓	✓	✓	✓



# Example – Quine McCluskey

		0	2	4	8	9	12
13,15	11-1						
0,2,4,6	0--0	X	X	X			
0,4,8,12	--00	X		X	X		X
8,9,12,13	1-0-				X	X	X
<b><math>F = A'D' + AC'</math></b>		✓	✓	✓	✓	✓	✓



# Quiz

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- Simplify

$F(w,x,y,z) = \sum m(4,9,11,12,15) + \sum d(5,13)$   
using Karnaugh Map