Chapter 9

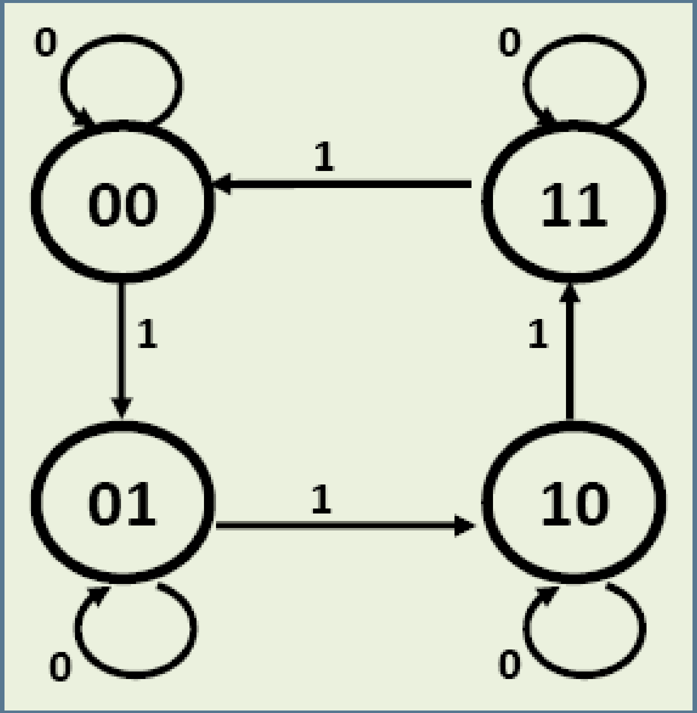
DESIGN OF SEQUENTIAL CIRCUITS

Design Procedure

Given the problem (a word description of the circuit behavior):

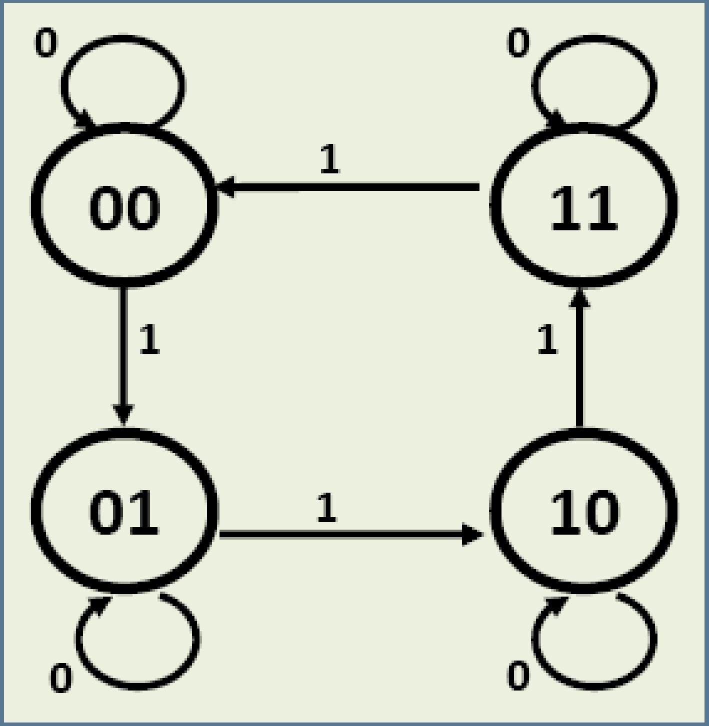
- Derive the state diagram
- Obtain the state table
- Reduce the number of states by state reduction method
- Determine the number of flip-flops needed
- Choose the type of flip-flops to be used
- Derive excitation equations
- Derive the output functions and the flip-flop input functions
- Draw the logic diagram

Example 1



Design a circuit whose state diagram is shown above. Use T flip-flop(s).

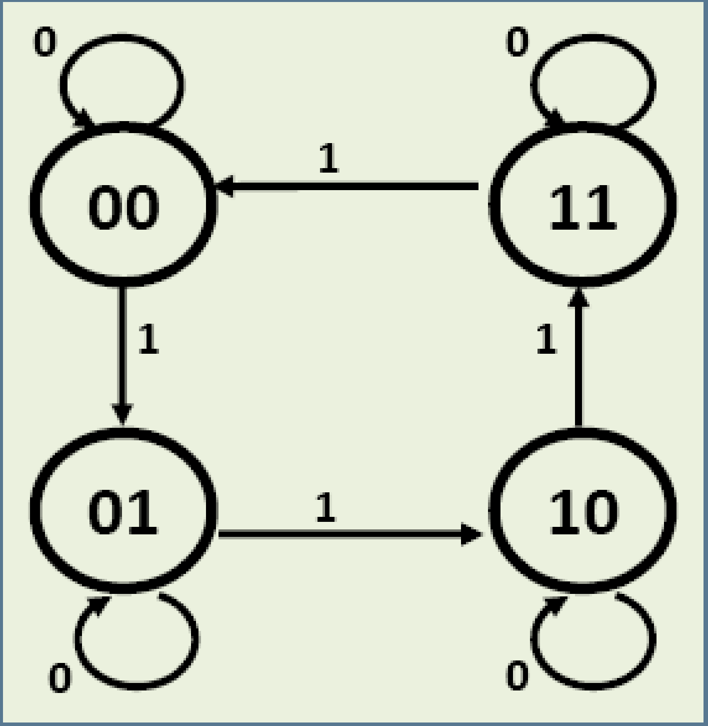
Step 2: State Table



Number of rows in the state table: 2^n

where n = no. of bits in a state + no. of inputs

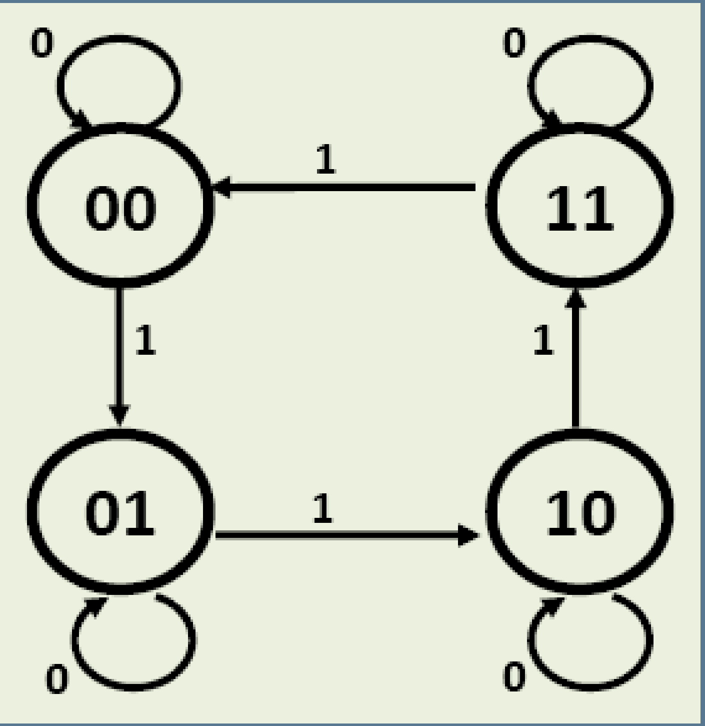
Step 2: State Table



State table will look like:

Present State		input	Next State	
A	B	x	A	B

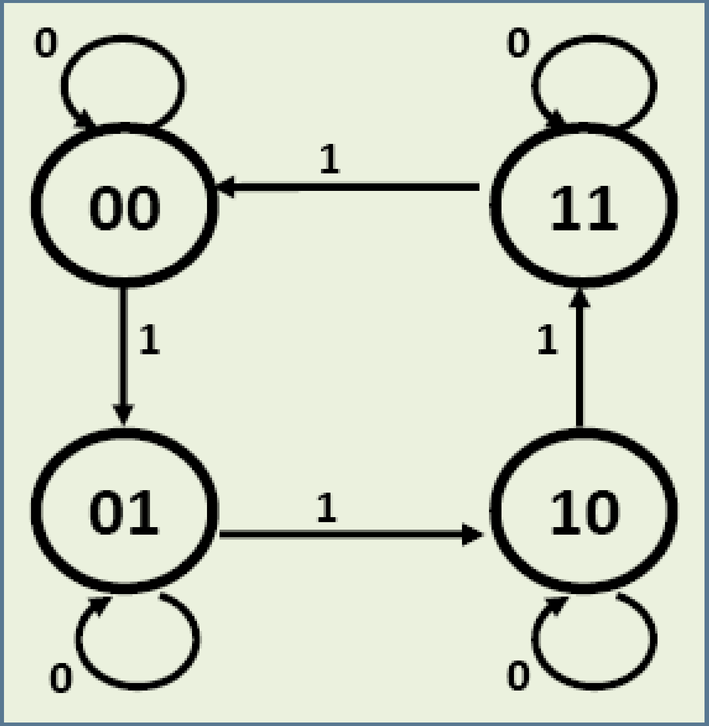
Step 2: State Table



State table will look like:

Present State		input	Next State	
A	B	x	A	B
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Step 2: State Table



State table will look like:

Present State		input	Next State	
A	B	x	A	B
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

Steps 3-5:

- Step 3: State reduction
  - Goal: to design a circuit with less number of flip-flops
- Step 4: Number of flip-flops
  - Binary numbers inside each circle identifies the state of the flip-flops
- Step 5: Type of flip-flop to be used in the design
  - In the example, use T flip-flops

Step 6: Excitation equation

Q(t)	Q(t+1)	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

Q(t)	Q(t+1)	D
0	0	0
0	1	1
1	0	0
1	1	1

Q(t)	Q(t+1)	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Q(t)	Q(t+1)	T
0	0	0
0	1	1
1	0	1
1	1	0

Flip-flop Excitation Tables

Step 6: Excitation equation for T flip-flops

Present State		input	Next State		FF input functions	
A	B	x	A	B	TA	TB
0	0	0	0	0		
0	0	1	0	1		
0	1	0	0	1		
0	1	1	1	0		
1	0	0	1	0		
1	0	1	1	1		
1	1	0	1	1		
1	1	1	0	0		

State table to get the excitation equations

Step 6: Excitation equation for T flip-flops

Present State		input	Next State		FF input functions	
A	B	x	A	B	TA	TB
0	0	0	0	0		
0	0	1	0	1		
0	1	0	0	1		
0	1	1	1	0		
1	0	0	1	0		
1	0	1	1	1		
1	1	0	1	1		
1	1	1	0	0		

State table to get the excitation equations

Step 6: Excitation equation for T flip-flops

Present State		input	Next State		FF input functions	
A	B	x	A	B	TA	TB
0	0	0	0	0	0	
0	0	1	0	1	0	
0	1	0	0	1	0	
0	1	1	1	0	1	
1	0	0	1	0	0	
1	0	1	1	1	0	
1	1	0	1	1	0	
1	1	1	0	0	1	

State table to get the excitation equations

Step 6: Excitation equation for T flip-flops

Present State		input	Next State		FF input functions	
A	B	x	A	B	TA	TB
0	0	0	0	0	0	
0	0	1	0	1	0	
0	1	0	0	1	0	
0	1	1	1	0	1	
1	0	0	1	0	0	
1	0	1	1	1	0	
1	1	0	1	1	0	
1	1	1	0	0	1	

State table to get the excitation equations

Step 6: Excitation equation for T flip-flops

Present State		input	Next State		FF input functions	
A	B	x	A	B	TA	TB
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	0	1	0	0
0	1	1	1	0	1	1
1	0	0	1	0	0	0
1	0	1	1	1	0	1
1	1	0	1	1	0	0
1	1	1	0	0	1	1

State table to get the excitation equations

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0				
1				

TA =

A \ Bx	00	01	11	10
0				
1				

TB =

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1			1	

A \ Bx	00	01	11	10
0				
1				

TA =

TB =

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1			1	

TA =

A \ Bx	00	01	11	10
0				
1				

TB =

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1			1	

TA = Bx

A \ Bx	00	01	11	10
0				
1				

TB =

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1			1	

TA = Bx

A \ Bx	00	01	11	10
0		1	1	
1		1	1	

TB =

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1			1	

TA = Bx

A \ Bx	00	01	11	10
0		1	1	
1		1	1	

TB =

Step 7: Simplification of Expressions

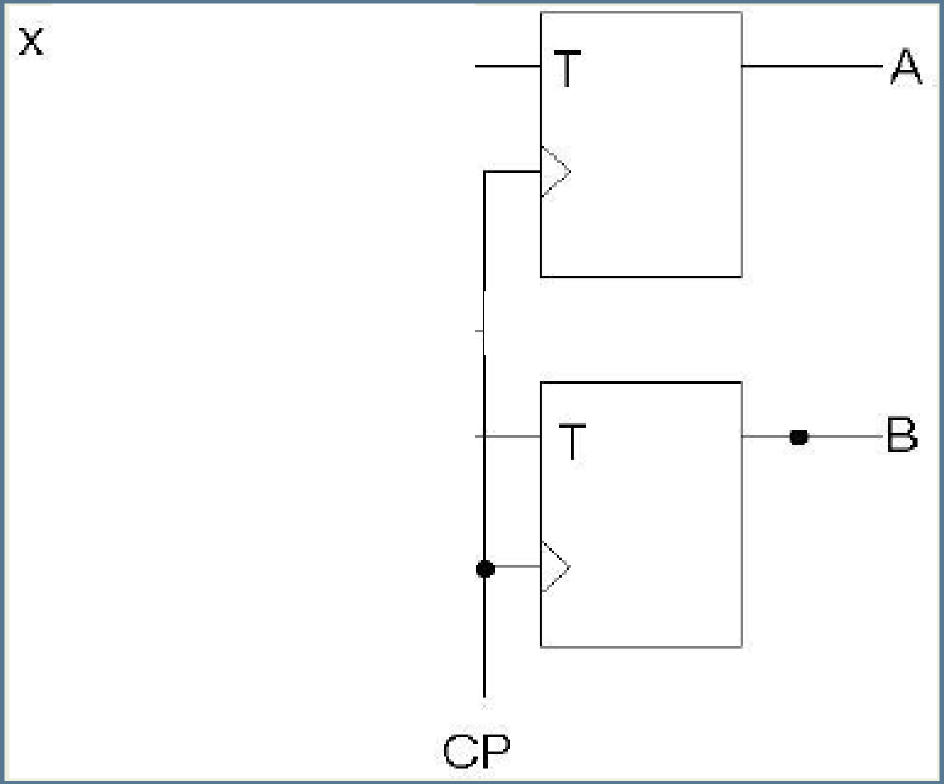
A \ Bx	00	01	11	10
0			1	
1			1	

TA = Bx

A \ Bx	00	01	11	10
0		1	1	
1		1	1	

TB = x

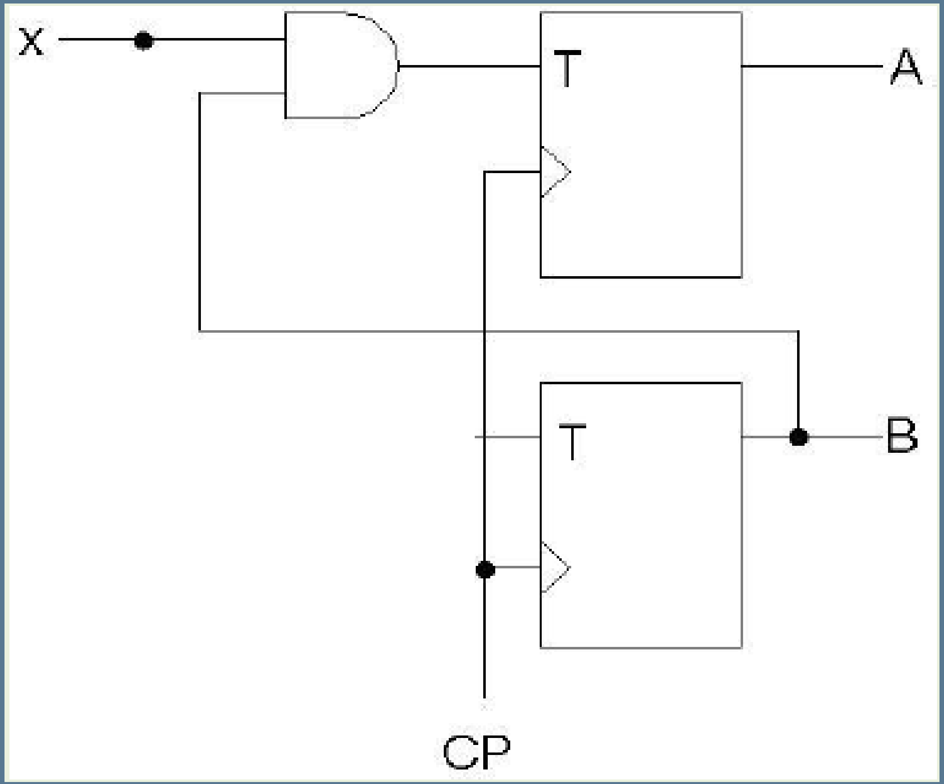
Step 8: Logic Diagram



TA = Bx

TB = x

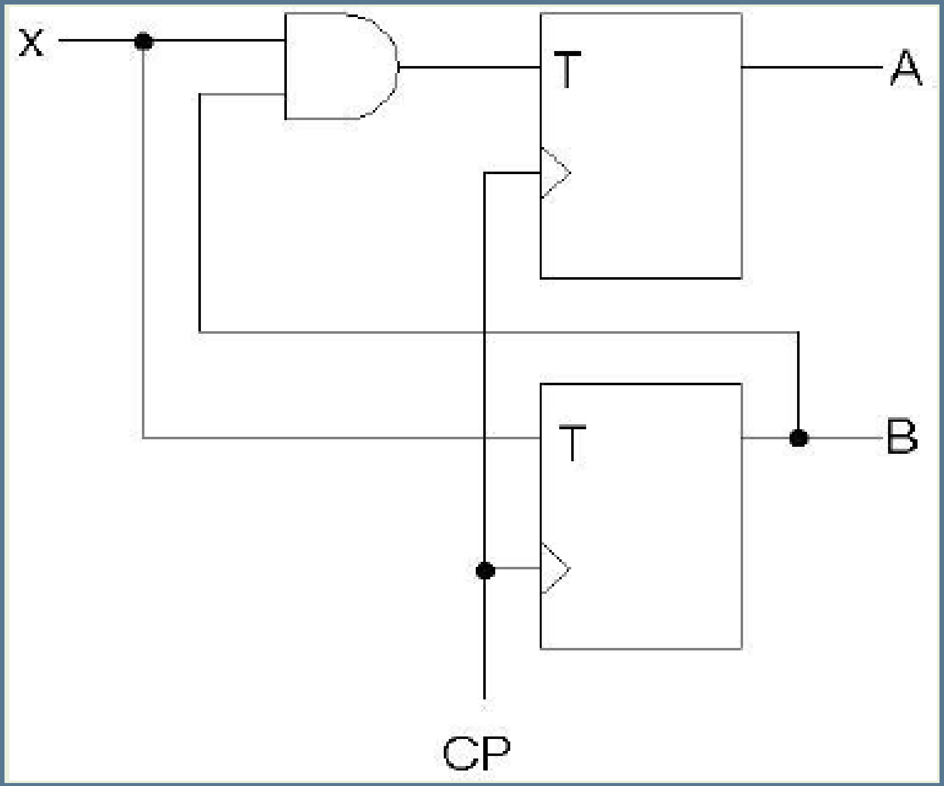
Step 8: Logic Diagram



TA = Bx

TB = x

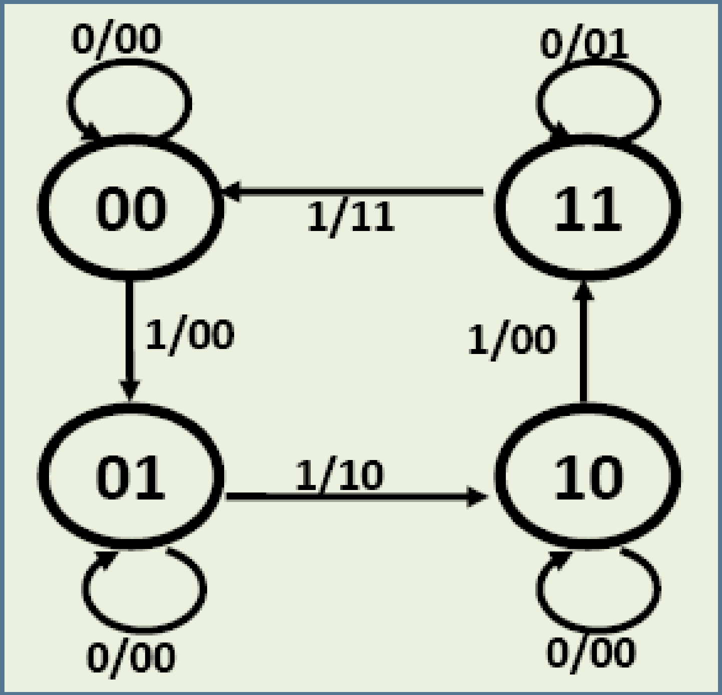
Step 8: Logic Diagram



TA = Bx

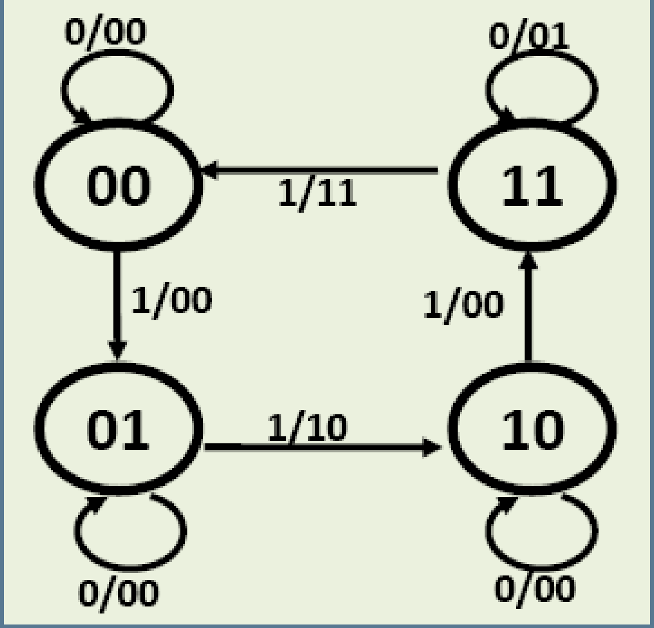
TB = x

Example 2



Design a circuit whose state diagram is shown above. Use JK flip-flop(s).

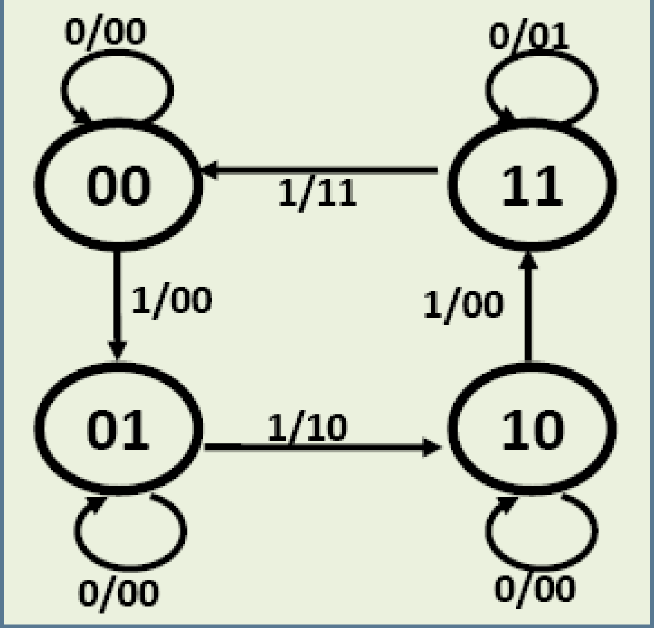
Step 2: State Table



State table will look like:

Present State		input	Next State		output	
A	B	x	A	B	Y	Z
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

Step 2: State Table



State table will look like:

Present State		input	Next State		output	
A	B	x	A	B	Y	Z
0	0	0	0	0	0	0
0	0	1	0	1	0	0
0	1	0	0	1	0	0
0	1	1	1	0	1	0
1	0	0	1	0	0	0
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	0	0	1	1

Steps 3-5:

- Step 3: The circuit is already in reduced form.
  - Form state reduction, use the second form of the state table.
- Step 4: The circuit needs two flip-flops (A & B)
- Step 5: Use JK flip-flops

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B					Y	Z
0	0	0	0	0					0	0
0	0	1	0	1					0	0
0	1	0	0	1					0	0
0	1	1	1	0					1	0
1	0	0	1	0					0	0
1	0	1	1	1					0	0
1	1	0	1	1					0	1
1	1	1	0	0					1	1

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B	JA	KA	JB	KB	Y	Z
0	0	0	0	0					0	0
0	0	1	0	1					0	0
0	1	0	0	1					0	0
0	1	1	1	0					1	0
1	0	0	1	0					0	0
1	0	1	1	1					0	0
1	1	0	1	1					0	1
1	1	1	0	0					1	1

Step 6: Excitation equation(s)

Q(t)	Q(t+1)	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

Q(t)	Q(t+1)	D
0	0	0
0	1	1
1	0	0
1	1	1

Q(t)	Q(t+1)	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Q(t)	Q(t+1)	T
0	0	0
0	1	1
1	0	1
1	1	0

Flip-flop Excitation Tables

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B	JA	KA	JB	KB	Y	Z
0	0	0	0	0					0	0
0	0	1	0	1					0	0
0	1	0	0	1					0	0
0	1	1	1	0					1	0
1	0	0	1	0					0	0
1	0	1	1	1					0	0
1	1	0	1	1					0	1
1	1	1	0	0					1	1

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B	JA	KA	JB	KB	Y	Z
0	0	0	0	0	0	X			0	0
0	0	1	0	1	0	X			0	0
0	1	0	0	1	0	X			0	0
0	1	1	1	0					1	0
1	0	0	1	0					0	0
1	0	1	1	1					0	0
1	1	0	1	1					0	1
1	1	1	0	0					1	1

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B	JA	KA	JB	KB	Y	Z
0	0	0	0	0	0	X			0	0
0	0	1	0	1	0	X			0	0
0	1	0	0	1	0	X			0	0
0	1	1	1	0	1	X			1	0
1	0	0	1	0					0	0
1	0	1	1	1					0	0
1	1	0	1	1					0	1
1	1	1	0	0					1	1

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B	JA	KA	JB	KB	Y	Z
0	0	0	0	0	0	X			0	0
0	0	1	0	1	0	X			0	0
0	1	0	0	1	0	X			0	0
0	1	1	1	0	1	X			1	0
1	0	0	1	0	X	0			0	0
1	0	1	1	1	X	0			0	0
1	1	0	1	1	X	0			0	1
1	1	1	0	0					1	1

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B	JA	KA	JB	KB	Y	Z
0	0	0	0	0	0	X			0	0
0	0	1	0	1	0	X			0	0
0	1	0	0	1	0	X			0	0
0	1	1	1	0	1	X			1	0
1	0	0	1	0	X	0			0	0
1	0	1	1	1	X	0			0	0
1	1	0	1	1	X	0			0	1
1	1	1	0	0	X	1			1	1

Step 6: Excitation equation(s)

Present State		input	Next State		FF input functions				output	
A	B	x	A	B	JA	KA	JB	KB	Y	Z
0	0	0	0	0	0	X	0	X	0	0
0	0	1	0	1	0	X	1	X	0	0
0	1	0	0	1	0	X	X	0	0	0
0	1	1	1	0	1	X	X	1	1	0
1	0	0	1	0	X	0	0	X	0	0
1	0	1	1	1	X	0	1	X	0	0
1	1	0	1	1	X	0	X	0	0	1
1	1	1	0	0	X	1	X	1	1	1

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1	X	X	X	X

JA = Bx

A \ Bx	00	01	11	10
0	X	X	X	X
1			1	

KA = Bx

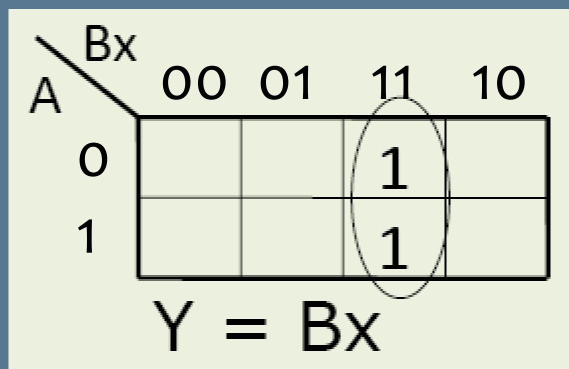
A \ Bx	00	01	11	10
0		1	X	X
1		1	X	X

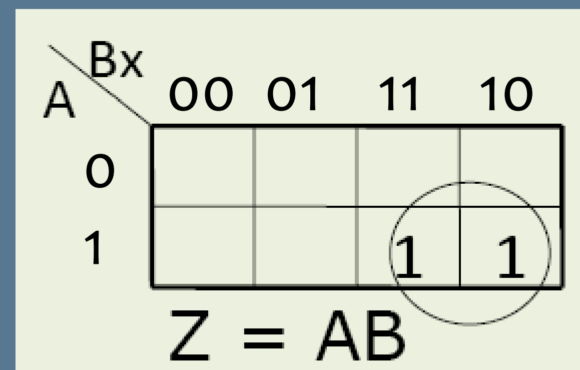
JB = x

A \ Bx	00	01	11	10
0	X	X	1	
1	X	X	1	

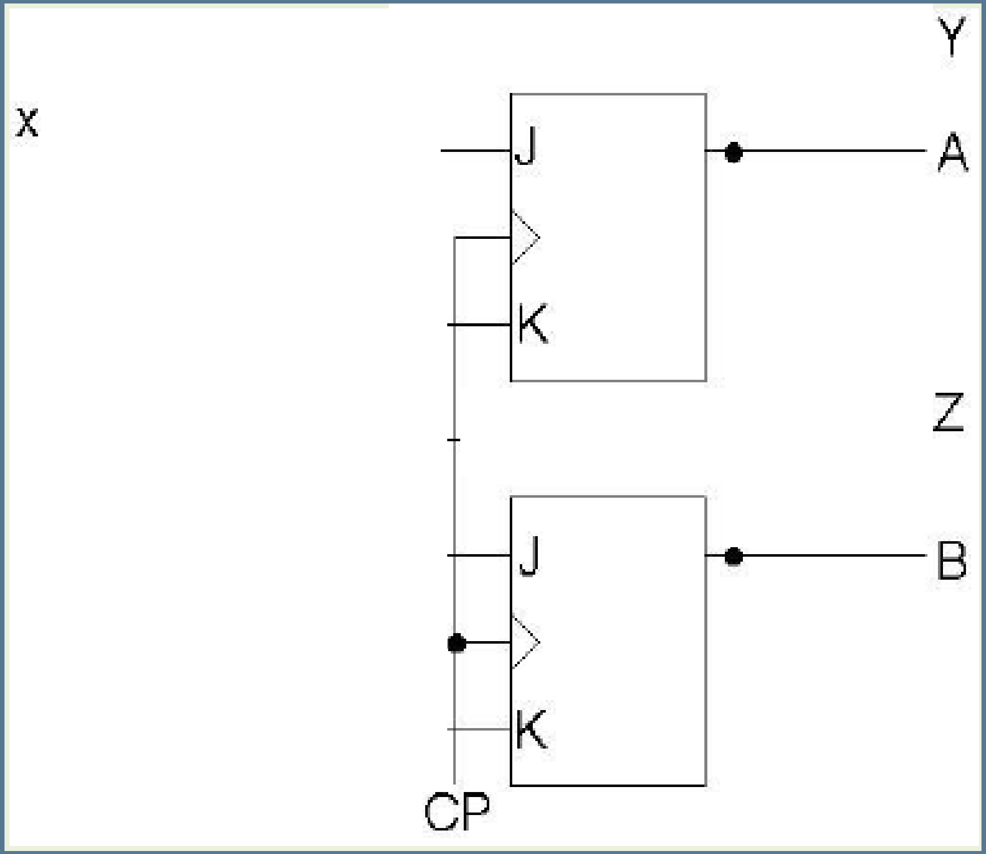
KB = x

Step 7: Simplification of Expressions





Step 8: Logic Diagram



JA = Bx

KA = Bx

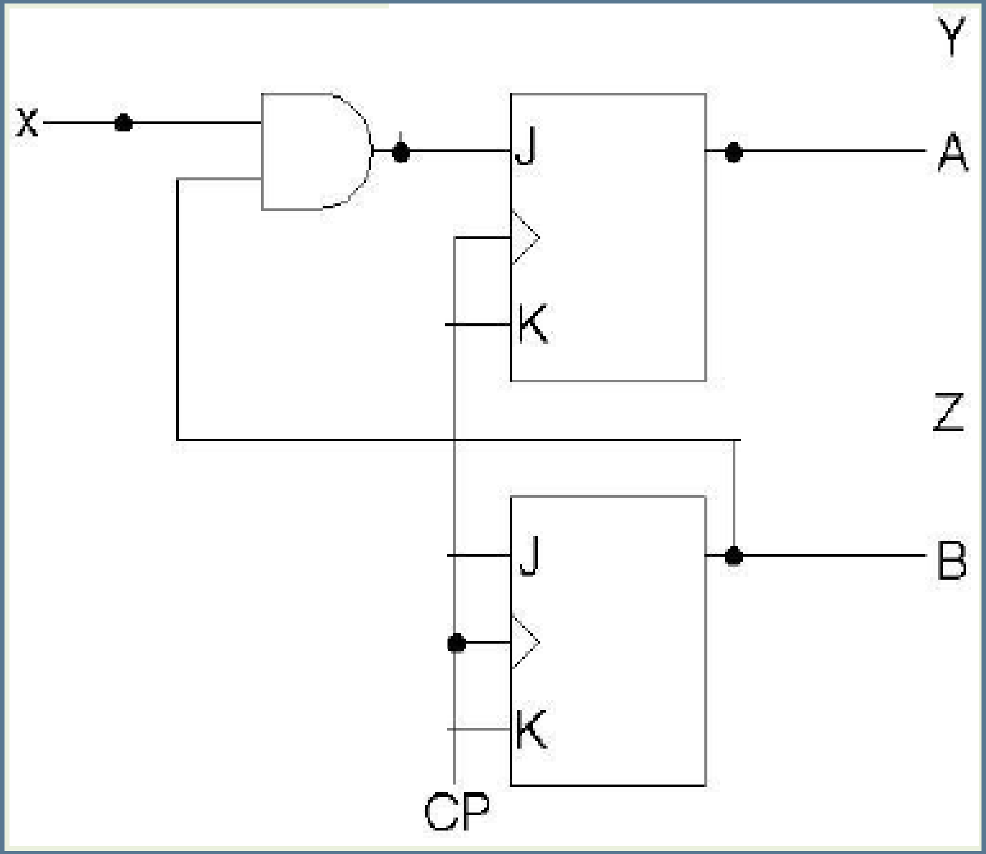
JB = x

KB = x

Y = Bx

Z = AB

Step 8: Logic Diagram



$JA = Bx$

$KA = Bx$

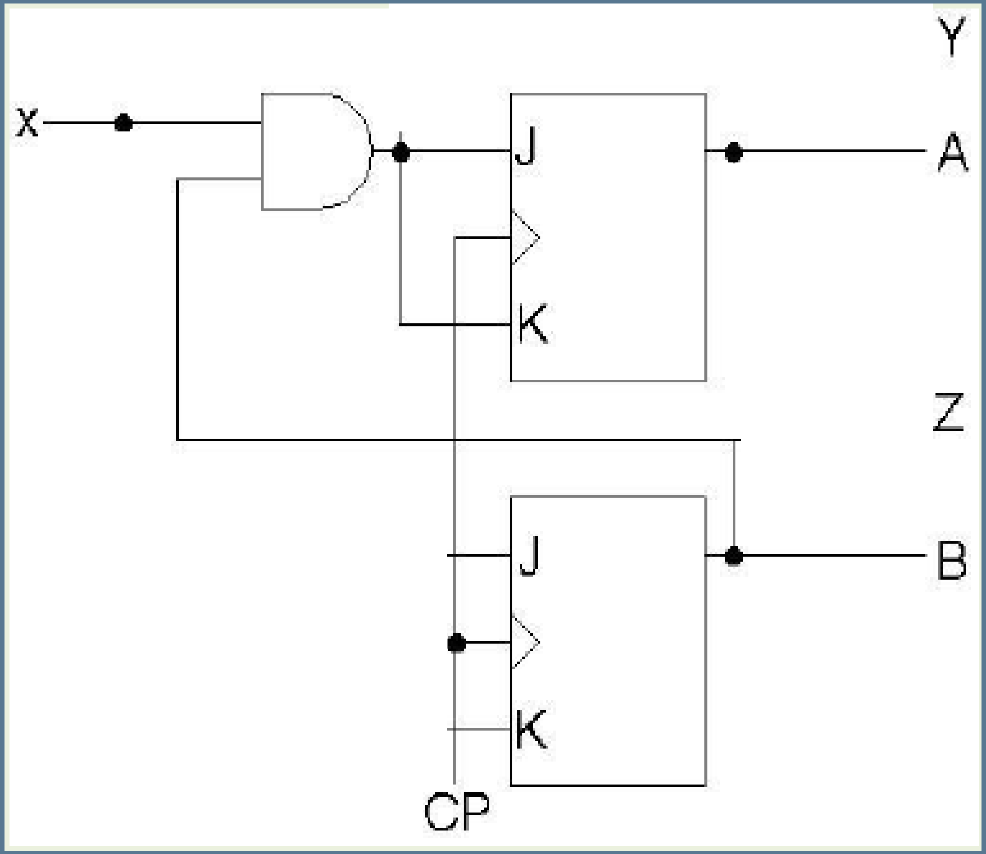
$JB = x$

$KB = x$

$Y = Bx$

$Z = AB$

Step 8: Logic Diagram



JA = Bx

KA = Bx

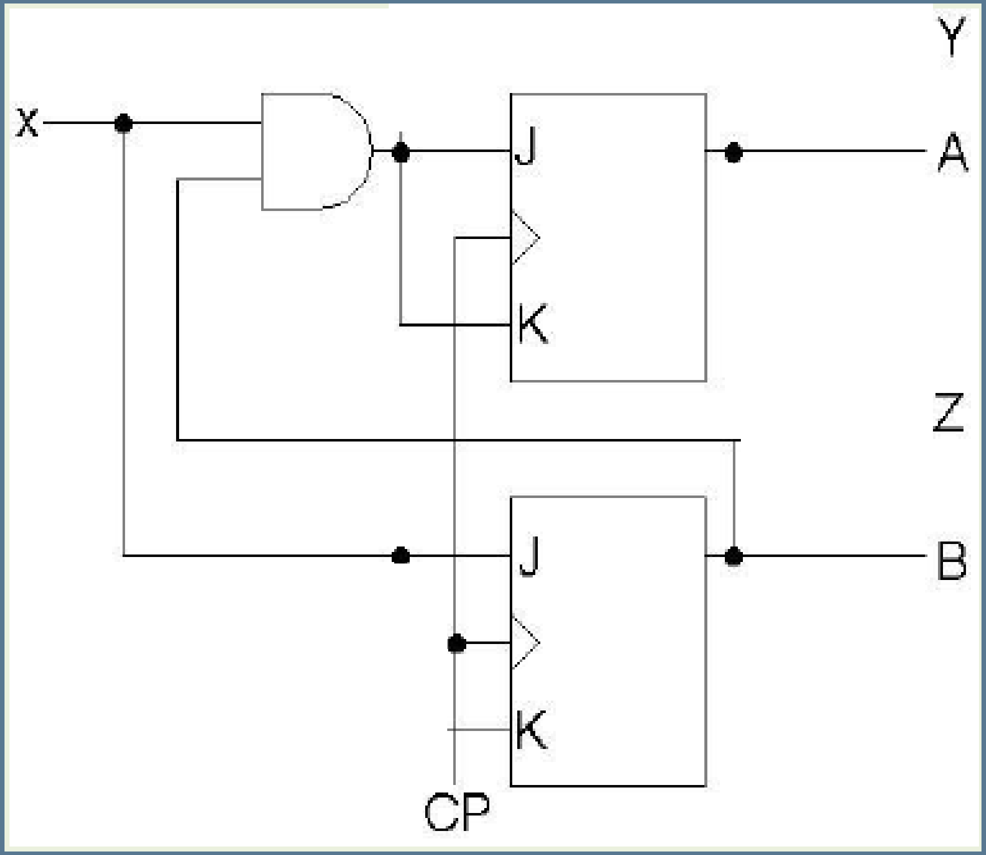
JB = x

KB = x

Y = Bx

Z = AB

Step 8: Logic Diagram



JA = Bx

KA = Bx

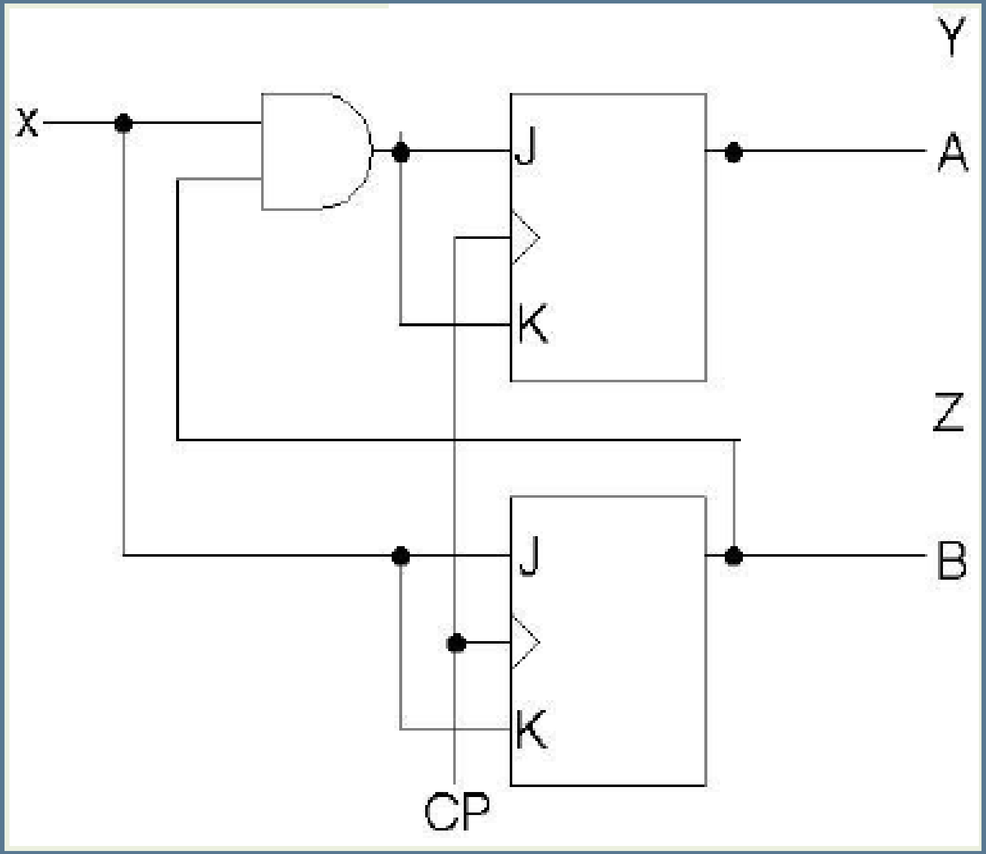
JB = x

KB = x

Y = Bx

Z = AB

Step 8: Logic Diagram



$JA = Bx$

$KA = Bx$

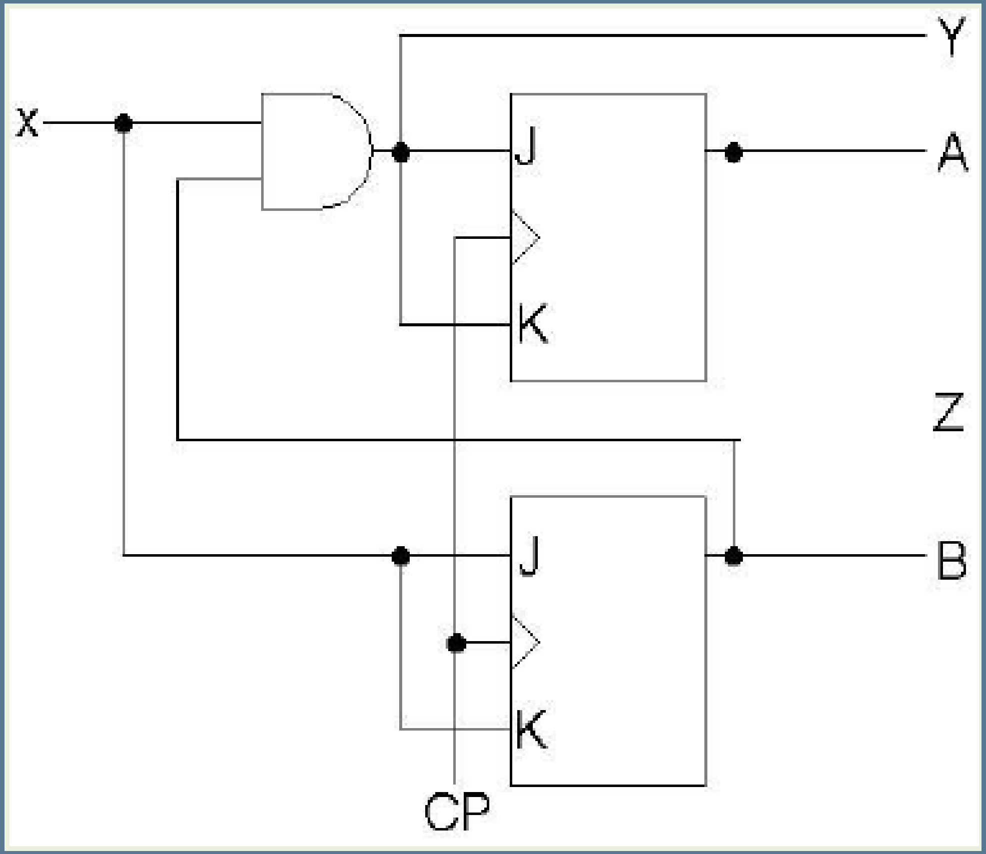
$JB = x$

$KB = x$

$Y = Bx$

$Z = AB$

Step 8: Logic Diagram



$JA = Bx$

$KA = Bx$

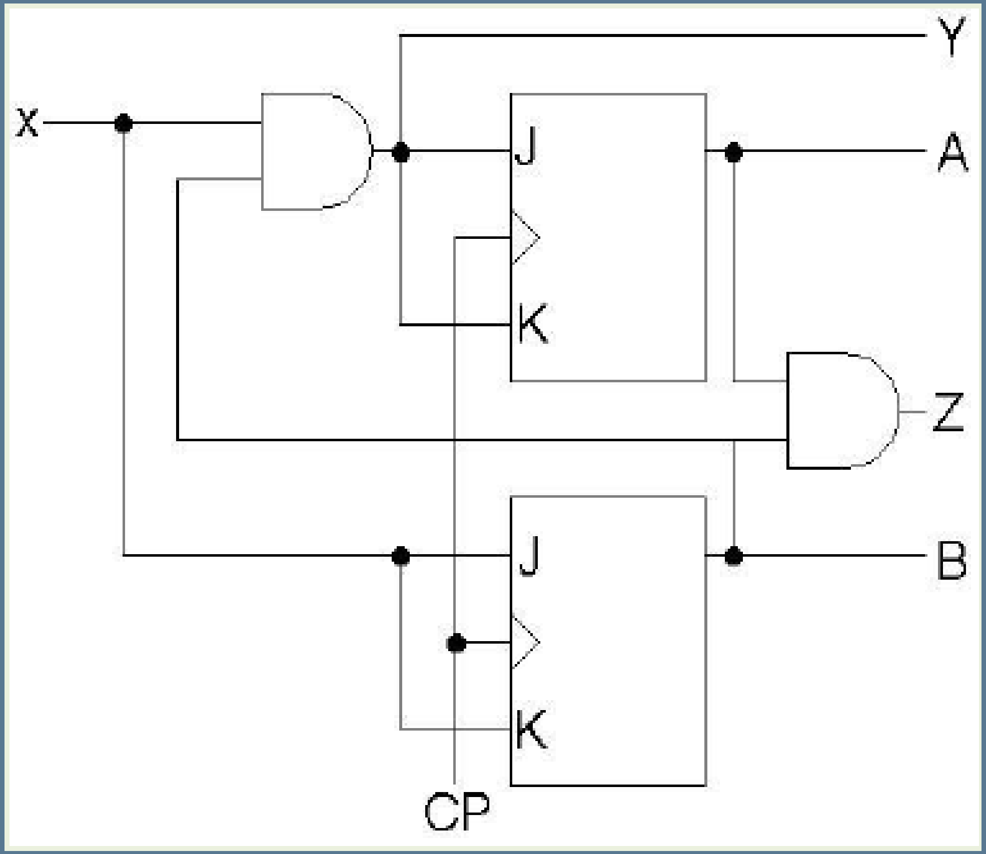
$JB = x$

$KB = x$

$Y = Bx$

$Z = AB$

Step 8: Logic Diagram



JA = Bx

KA = Bx

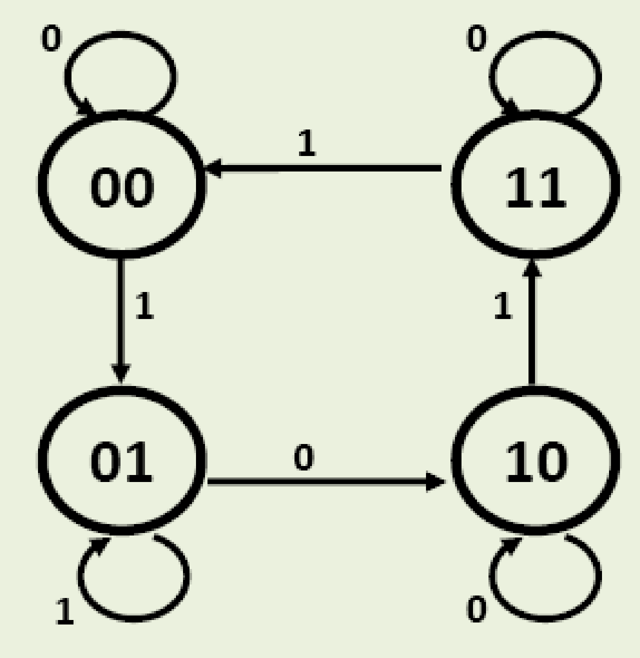
JB = x

KB = x

Y = Bx

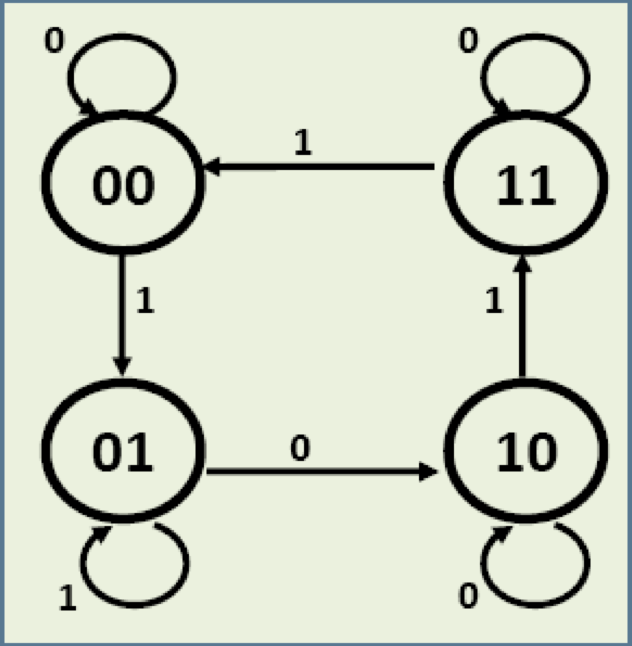
Z = AB

Example 3



Design a circuit whose state diagram is shown above. Use JK flip-flop(s).

Step 2: State Table



State table will look like:

Present State		input	Next State	
A	B	x	A	B
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

Steps 3-5:

- Step 3: State reduction
  - Goal: to design a circuit with less number of flip-flops
- Step 4: Number of flip-flops
  - Binary numbers inside each circle identifies the state of the flip-flops
- Step 5: Type of flip-flop to be used in the design
  - In the example, use JK flip-flops

Step 6: Excitation equation

Q(t)	Q(t+1)	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

Q(t)	Q(t+1)	D
0	0	0
0	1	1
1	0	0
1	1	1

Q(t)	Q(t+1)	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Q(t)	Q(t+1)	T
0	0	0
0	1	1
1	0	1
1	1	0

Flip-flop Excitation Tables

Step 6: Excitation equation for JK flip-flops

Present State		input	Next State		FF input functions			
A	B	x	A	B	JA	KA	JB	KB
0	0	0	0	0				
0	0	1	0	1				
0	1	0	1	0				
0	1	1	0	1				
1	0	0	1	0				
1	0	1	1	1				
1	1	0	1	1				
1	1	1	0	0				

Step 6: Excitation equation for JK flip-flops

Present State		input	Next State		FF input functions			
A	B	x	A	B	JA	KA	JB	KB
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1

Step 7: Simplification of Expressions

A \ Bx	00	01	11	10
0				1
1	X	X	X	X

$JA = Bx'$

A \ Bx	00	01	11	10
0	X	X	X	X
1			1	

$KA = Bx$

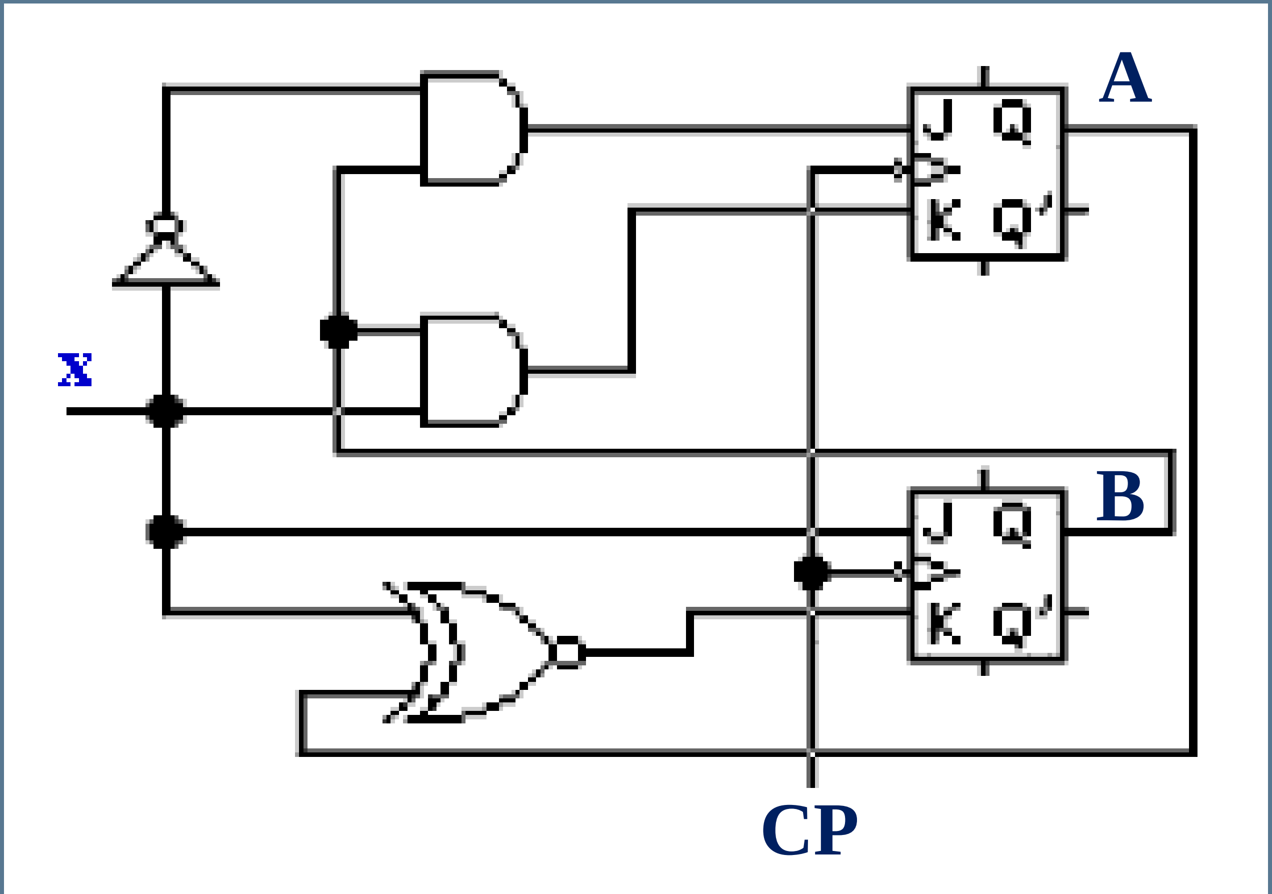
A \ Bx	00	01	11	10
0		1	X	X
1		1	X	X

$JB = x$

A \ Bx	00	01	11	10
0	X	X		1
1	X	X	1	

$KB = A \odot x$

Step 8: Logic Diagram



Example 4

- Design using D Flip-flops

Present State		input	Next State		FF input functions		output
A	B	x	A	B	DA	DB	Z
0	0	0	0	0			0
0	0	1	0	1			0
0	1	0	0	0			0
0	1	1	1	0			0
1	0	0	1	1			0
1	0	1	1	0			0
1	1	0	0	0			0
1	1	1	0	1			1

Example 4

- Design using D Flip-flops

Present State		input	Next State		FF input functions		output
A	B	x	A	B	DA	DB	Z
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	0	0	0	0	0
0	1	1	1	0	1	0	0
1	0	0	1	1	1	1	0
1	0	1	1	0	1	0	0
1	1	0	0	0	0	0	0
1	1	1	0	1	0	1	1

Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1	1	1		

DA=

A \ Bx	00	01	11	10
0		1		
1	1		1	

DB=

A \ Bx	00	01	11	10
0				
1			1	

Z=

Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1	1	1		

DA=

A \ Bx	00	01	11	10
0		1		
1	1		1	

DB=

A \ Bx	00	01	11	10
0				
1			1	

Z=

Simplification of Expressions

A \ Bx	00	01	11	10
0			1	
1	1	1		

DA=AB’ + A’Bx

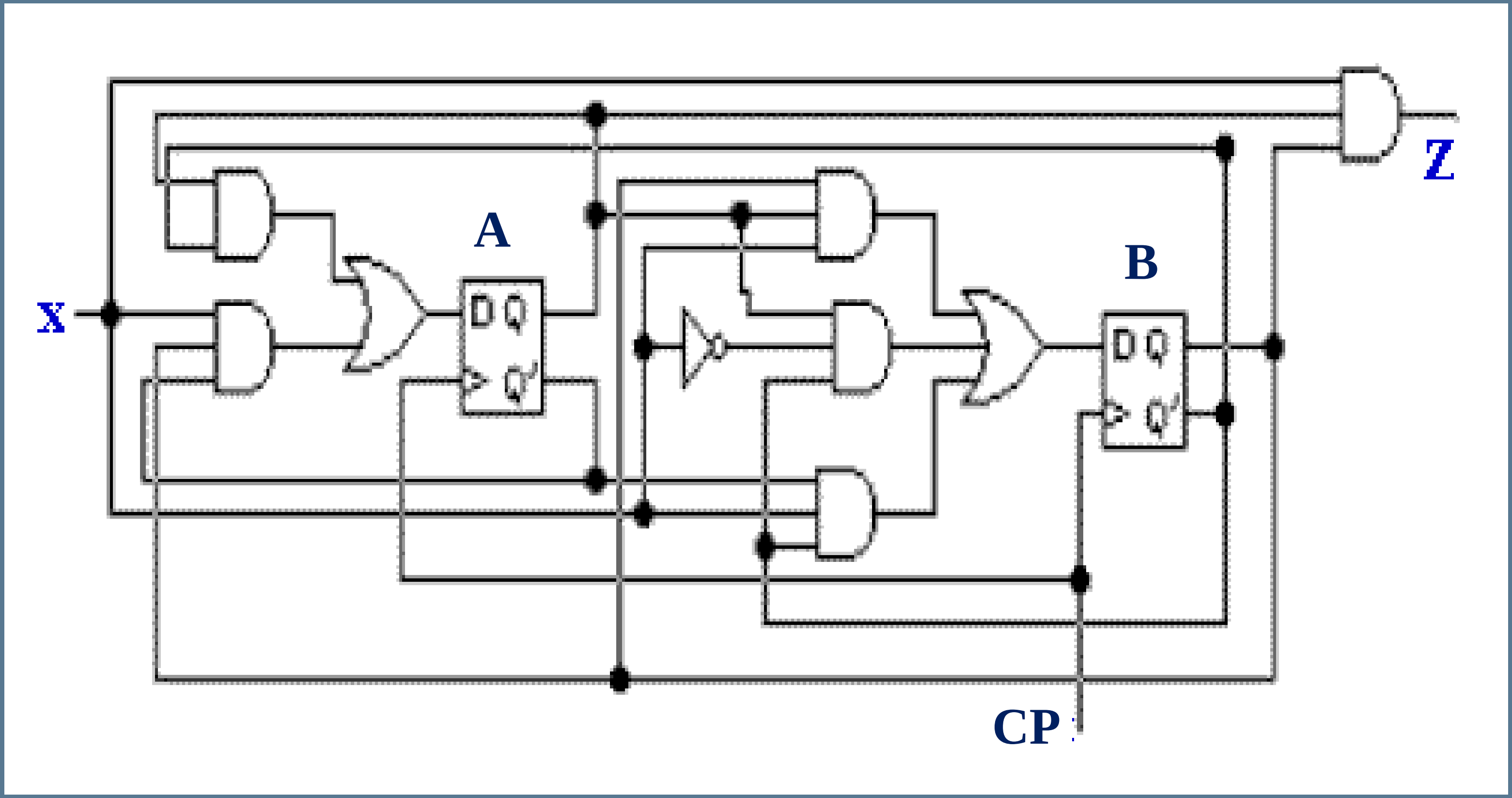
A \ Bx	00	01	11	10
0		1		
1	1		1	

DB=A’B’x+AB’x’+ABx

A \ Bx	00	01	11	10
0				
1			1	

Z=ABx

Logic Diagram



Quiz

Quiz

Given the state diagram, how many ...

1. rows in the state table?
2. input variables?
3. output variables?
4. FF input functions (if SR FFs will be used)?
5. SR flip-flops?