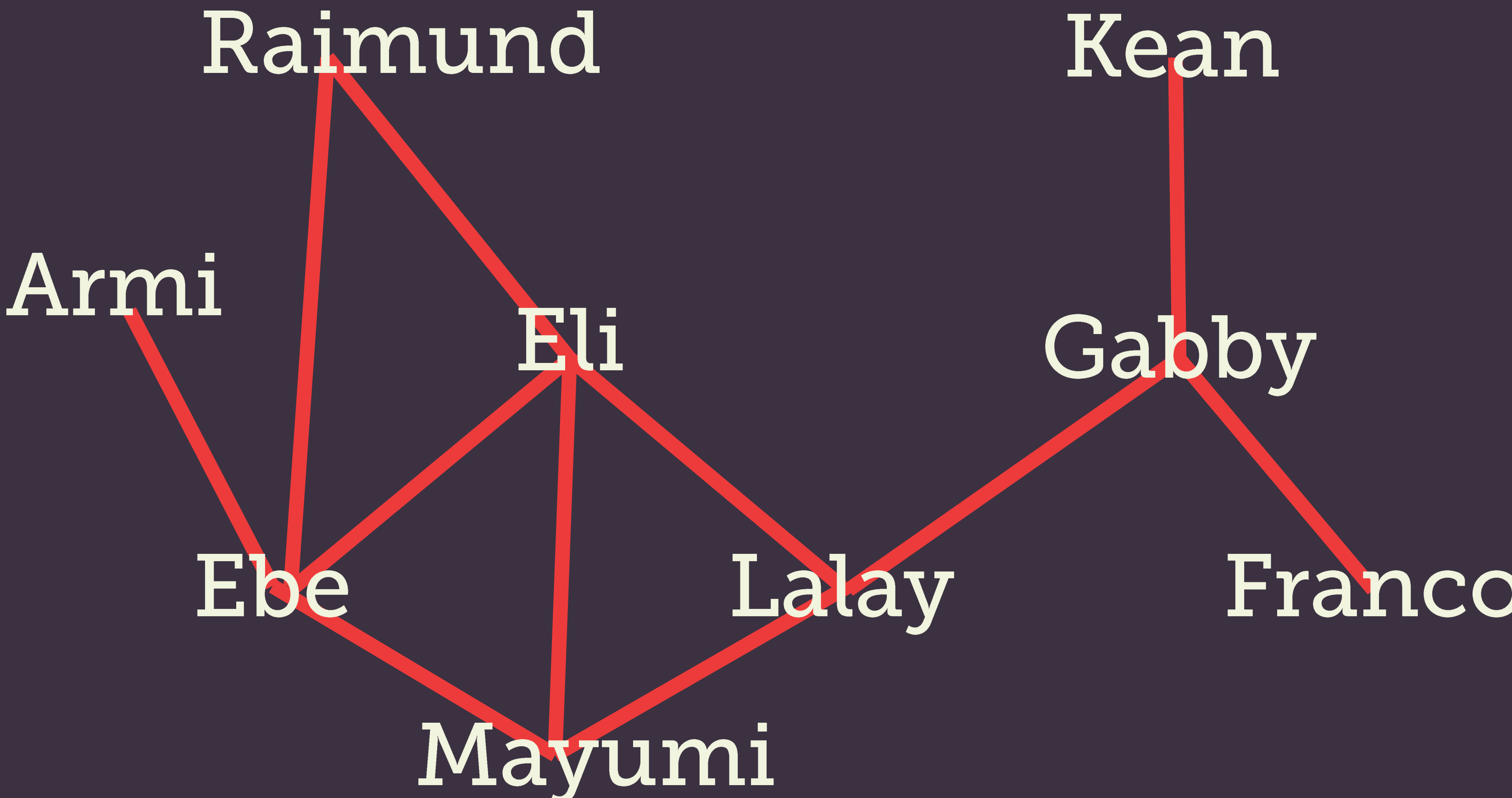
GRAPH ALGORITHMS

+ DATA STRUCTURES

GRAPH

Graph consists of a set of vertices and a set of edges.

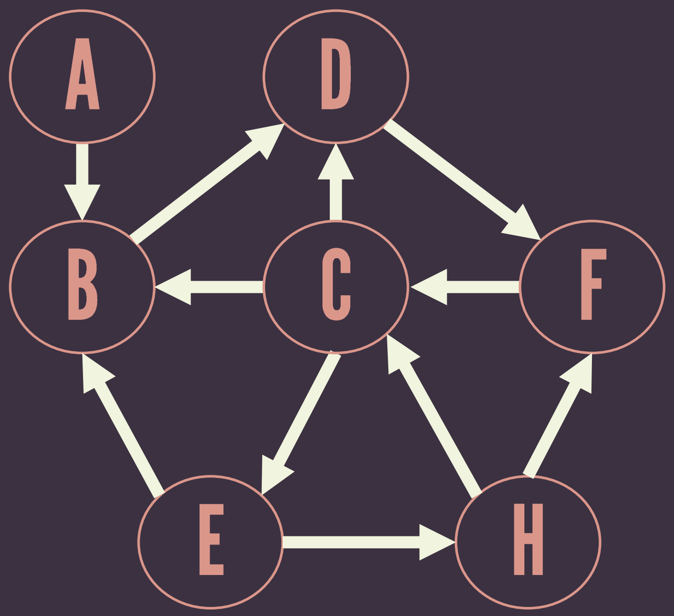
$$G = (V, E)$$

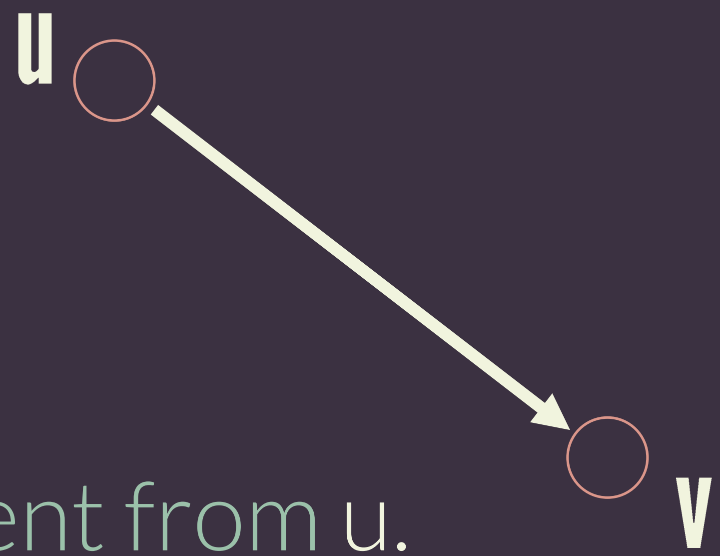

Raimund
Kean
Armi
Eli
Gabby
Ebe
Lalay
Franco
Mayumi

types of
GRAPHS

directed

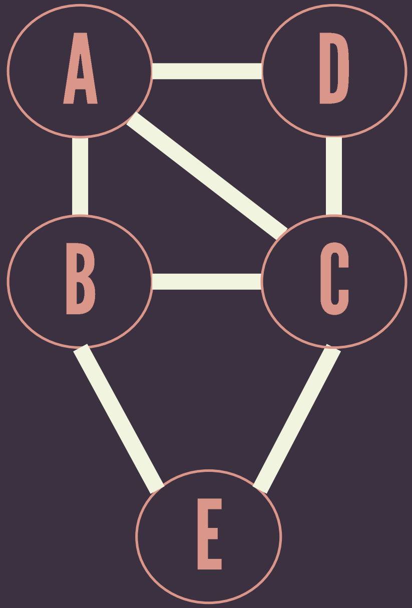
GRAPH

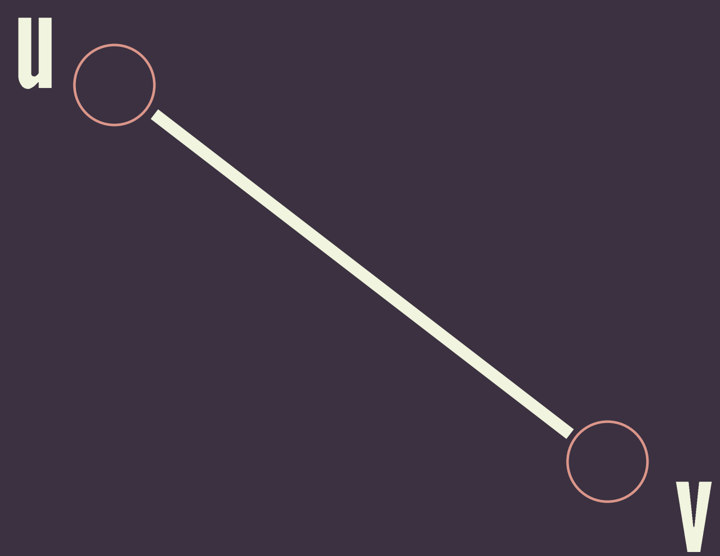




Edge (u,v) is incident from u.
Edge (u,v) is incident to v.
Vertex v is adjacent to vertex u.

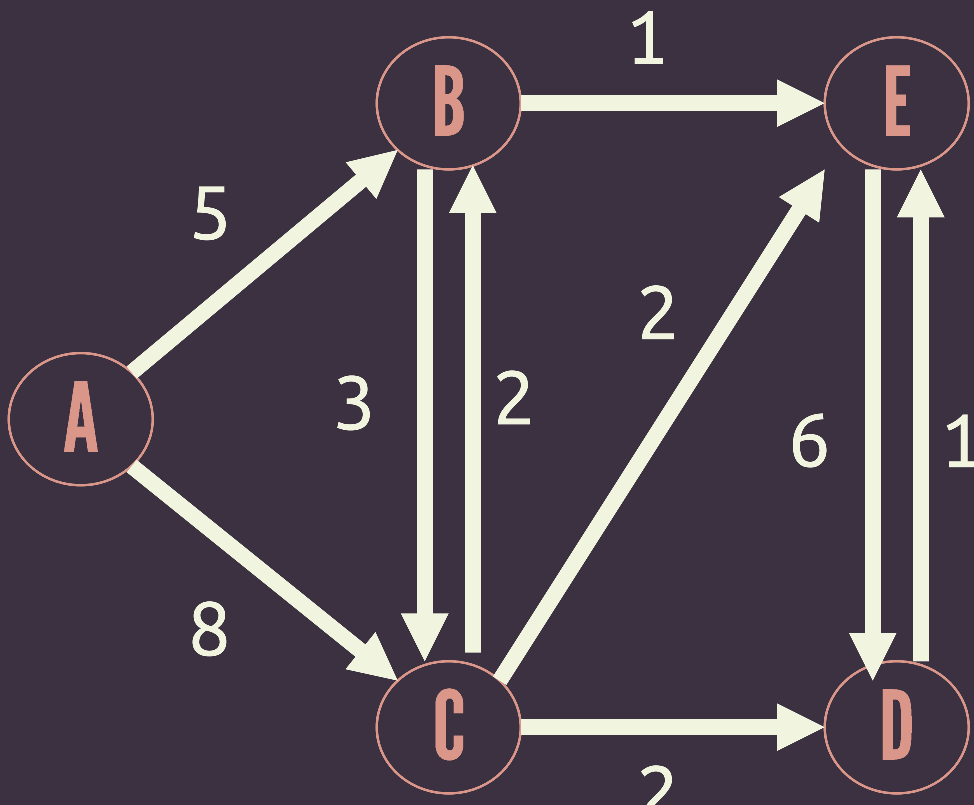
undirected
GRAPH





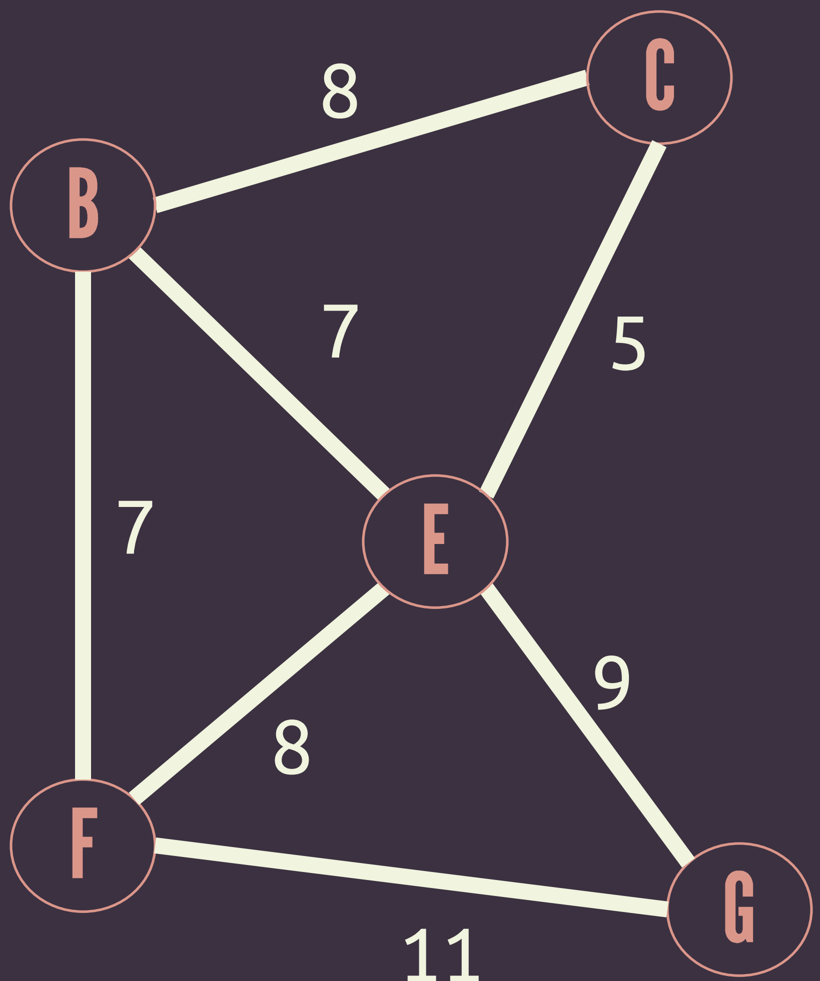
Edge (u,v) or (v,u) is incident on u and v.
Vertex v is adjacent to vertex u.
Vertex u is adjacent to vertex v.

weighted
GRAPH



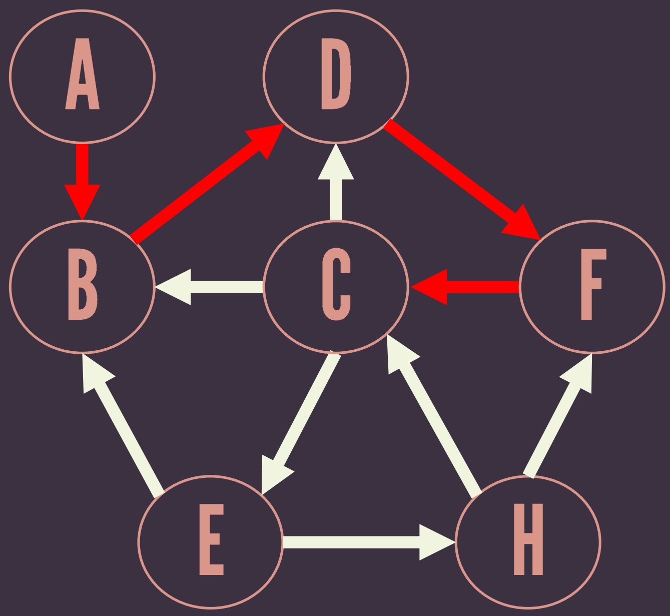
weighted

GRAPH

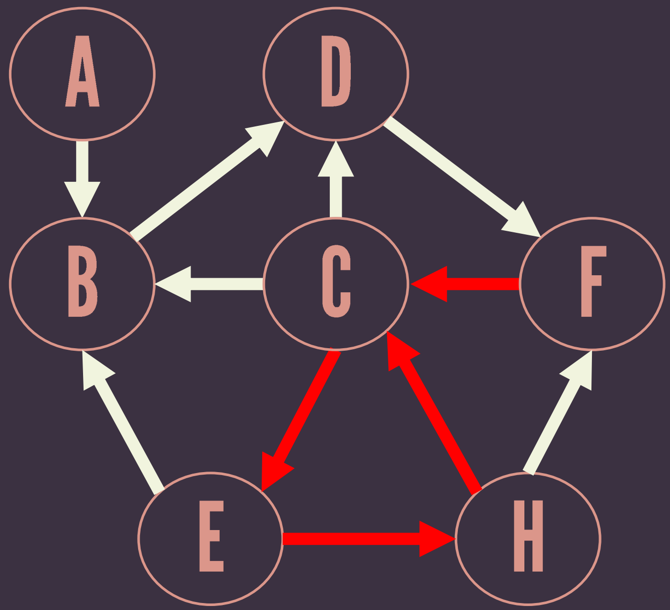


path in a

GRAPH



path in a
GRAPH



in-degree, $\rho^{+}(v)$

# of edges incident to v

out-degree, $\rho^{-}(v)$

# of edges incident from v

degree of vertex v, $\rho(v)$

of edges incident on v

$= \rho^-(v) + \rho^+(v)$

degree of vertex v, $\rho(v)$

# of edges incident on v.

(also applicable for undirected graphs)

GRAPH

Graph consists of a set of vertices and a set of edges.

$$G = (V, E)$$

GRAPH
Representations

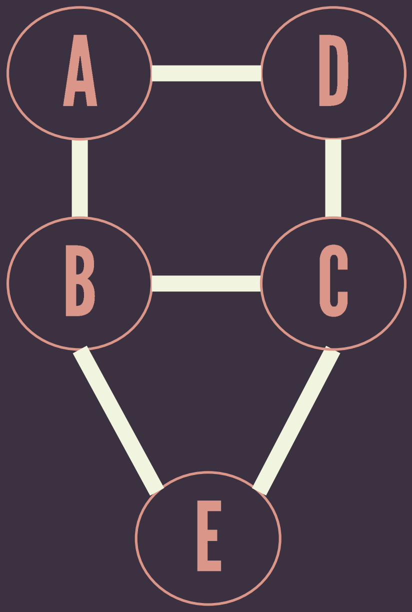
adjacency
MATRIX

2D
ARRAY

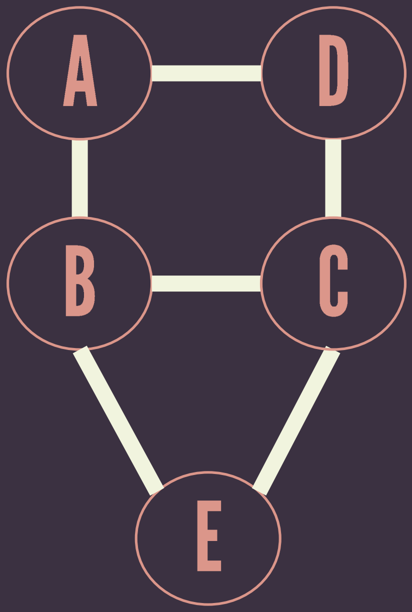
adjacency
LIST

list of
LISTS

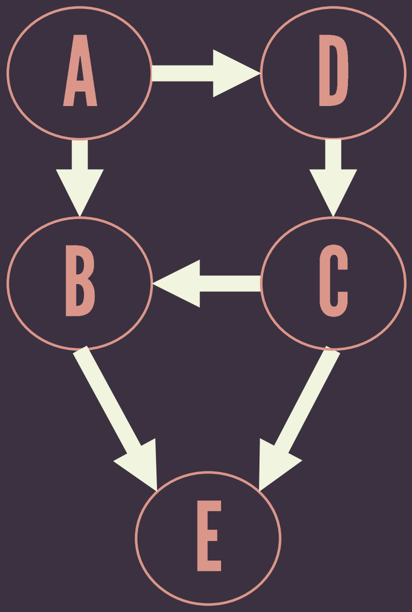
	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	0	1
C	1	0	0	1	1
D	1	0	1	0	0
E	0	1	1	0	0



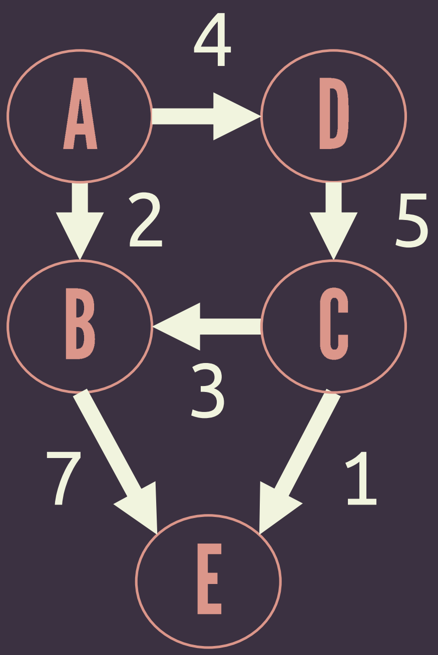
	A	B	C	D	E
A	∞	1	∞	1	∞
B	1	∞	1	∞	1
C	1	∞	∞	1	1
D	1	∞	1	∞	∞
E	∞	1	1	∞	∞

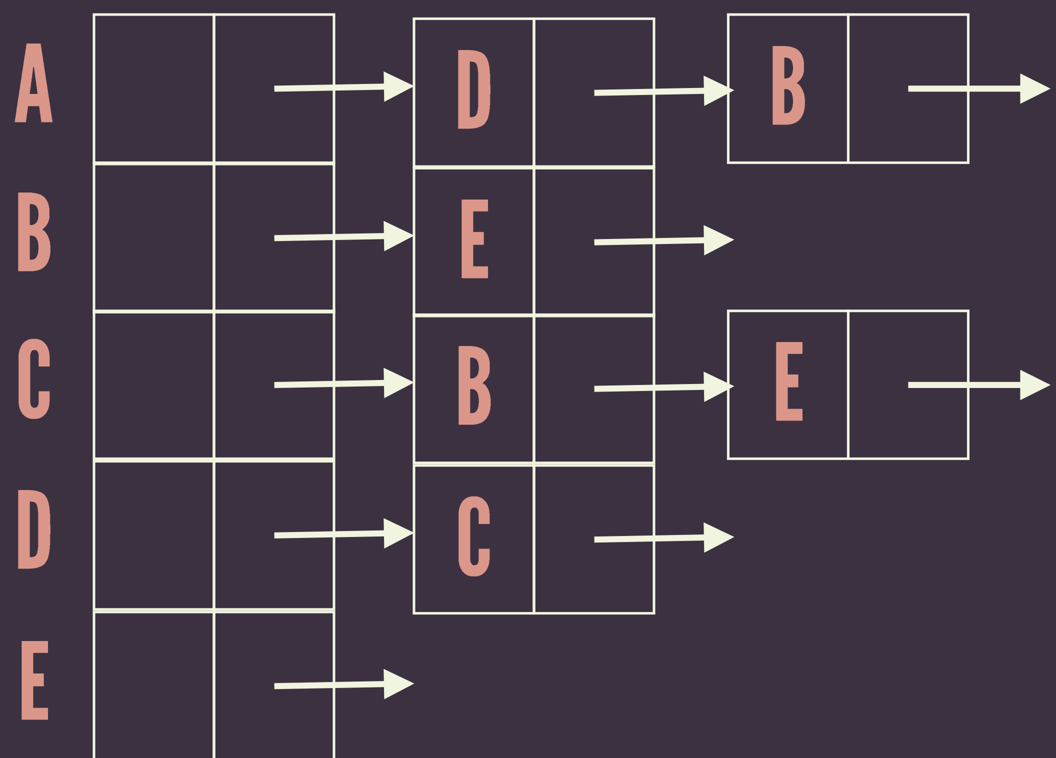


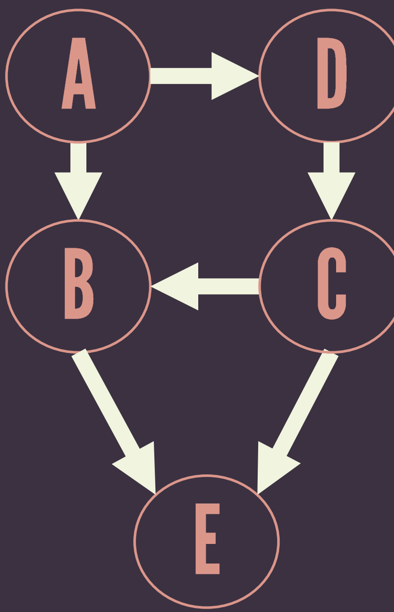
	A	B	C	D	E
A	0	1	0	1	0
B	0	0	0	0	1
C	0	1	0	0	1
D	0	0	1	0	0
E	0	0	0	0	0

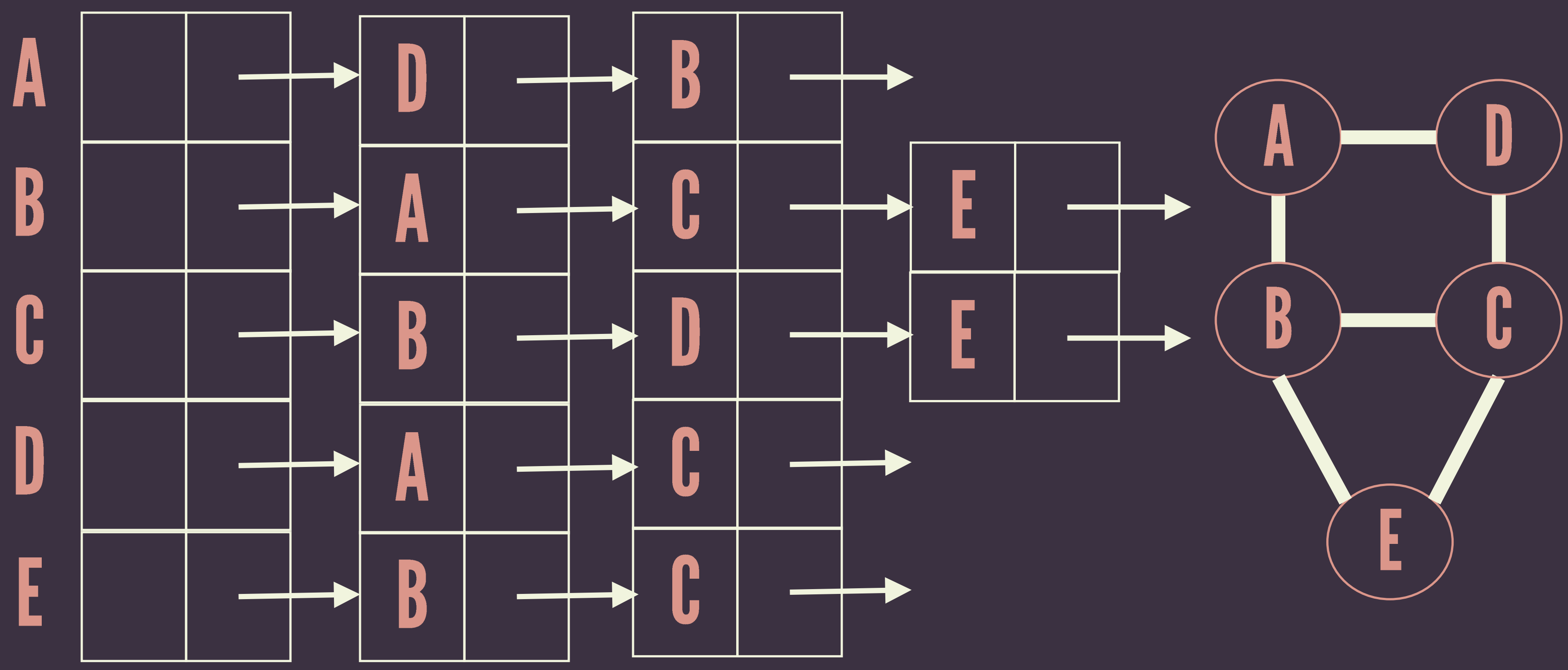


	A	B	C	D	E
A	0	2	0	4	0
B	0	0	0	0	7
C	0	3	0	0	1
D	0	0	5	0	0
E	0	0	0	0	0




A D B
B E
C B E
D C
E

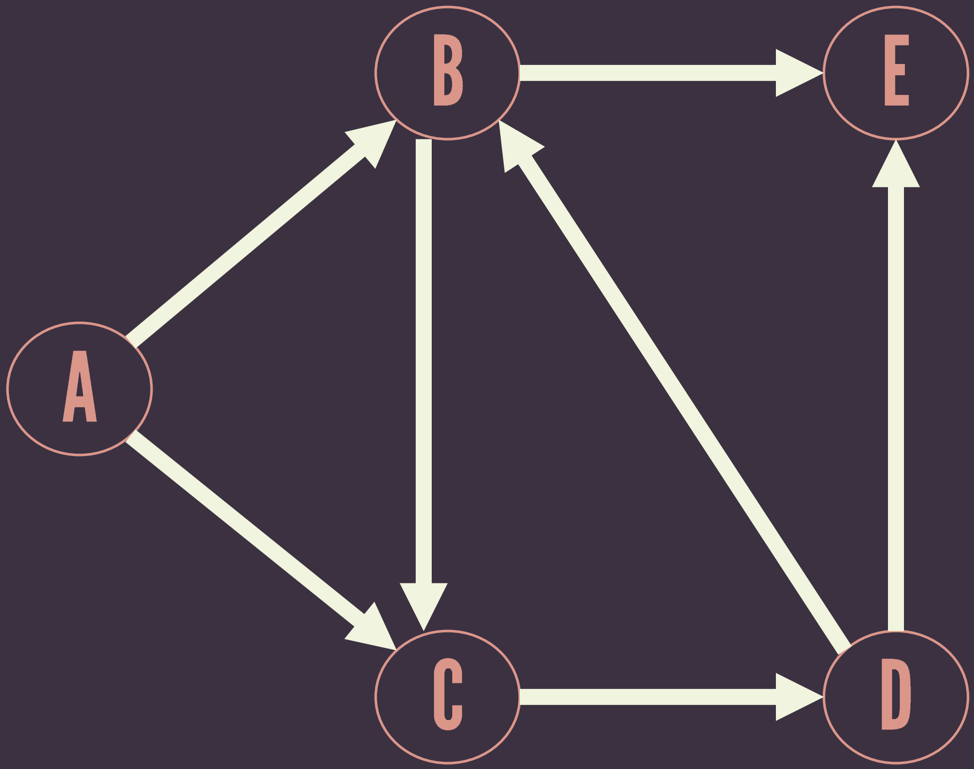

A D
B C
E


A
B
C
D
E
D
A
B
A
B
B
C
D
C
C
E
E
A
D
B
C
E

GRAPH ALGORITHMS

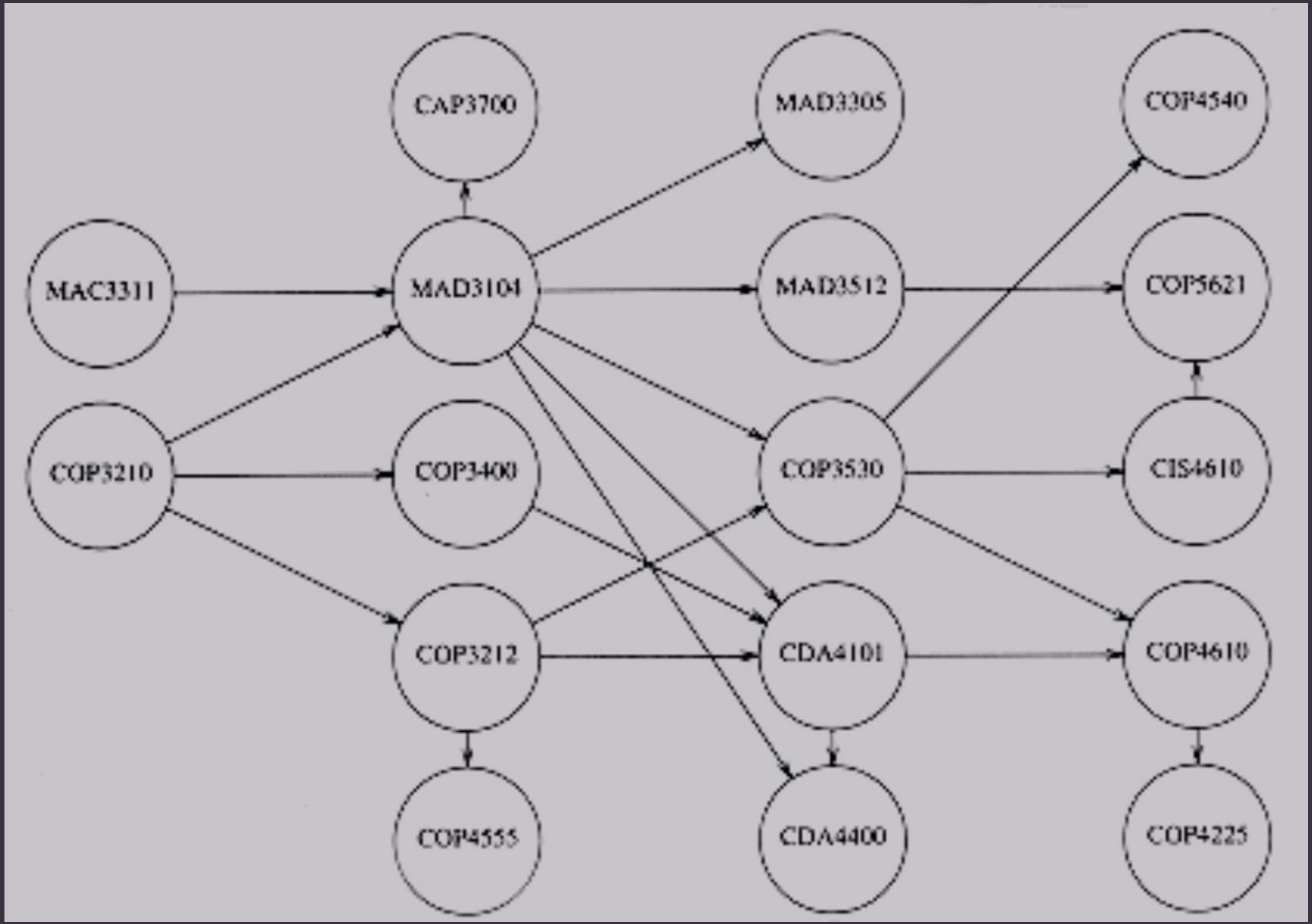
topological
SORT

An ordering of vertices in a directed acyclic graph such that if there is a path from v_i to v_j, then v_j appears after vi in the ordering.



A, B, C, D, E

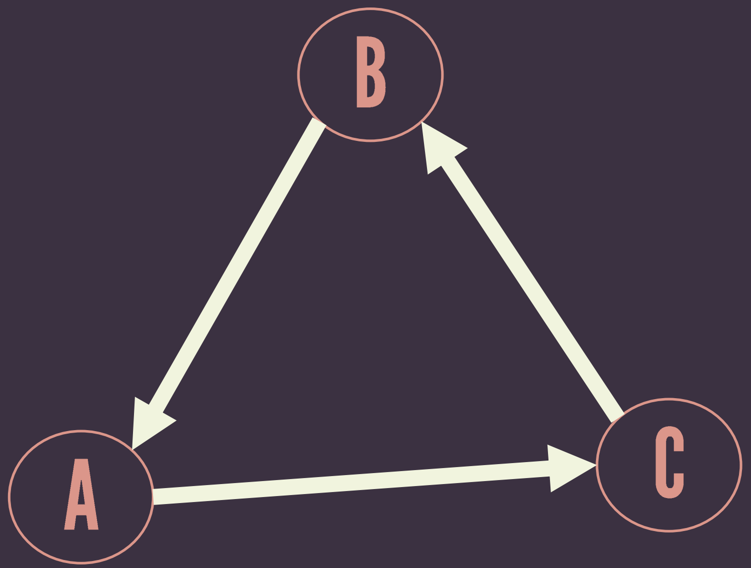
A, C, D, B, E


CAP3700
MAD3305
COP4540
MAC3311
MAD3104
MAD3512
COP5621
COP3210
COP3400
COP3530
CIS4610
COP3212
CDA4101
COP4610
COP4555
CDA4400
COP4225

directed
acyclic graph
DAG

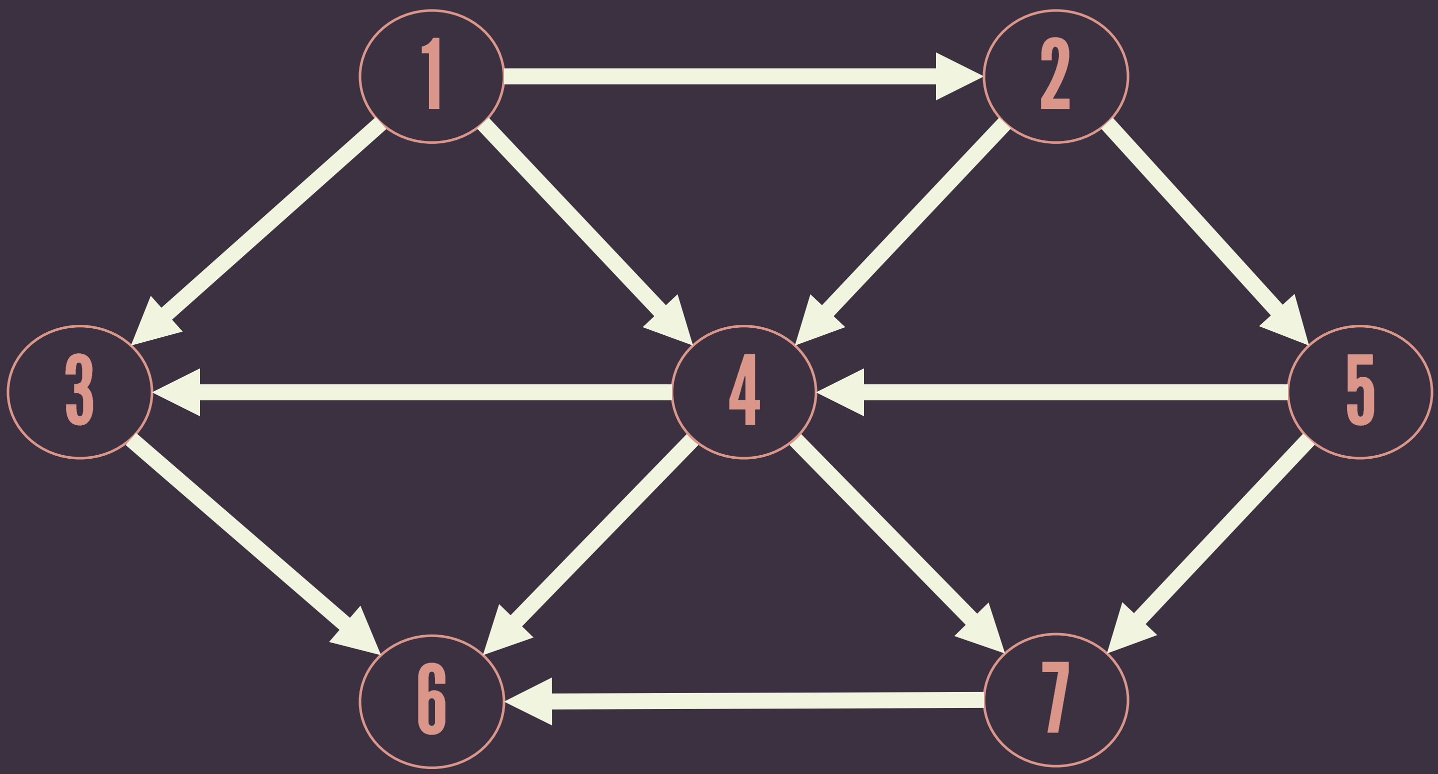
A directed graph with no directed cycles.

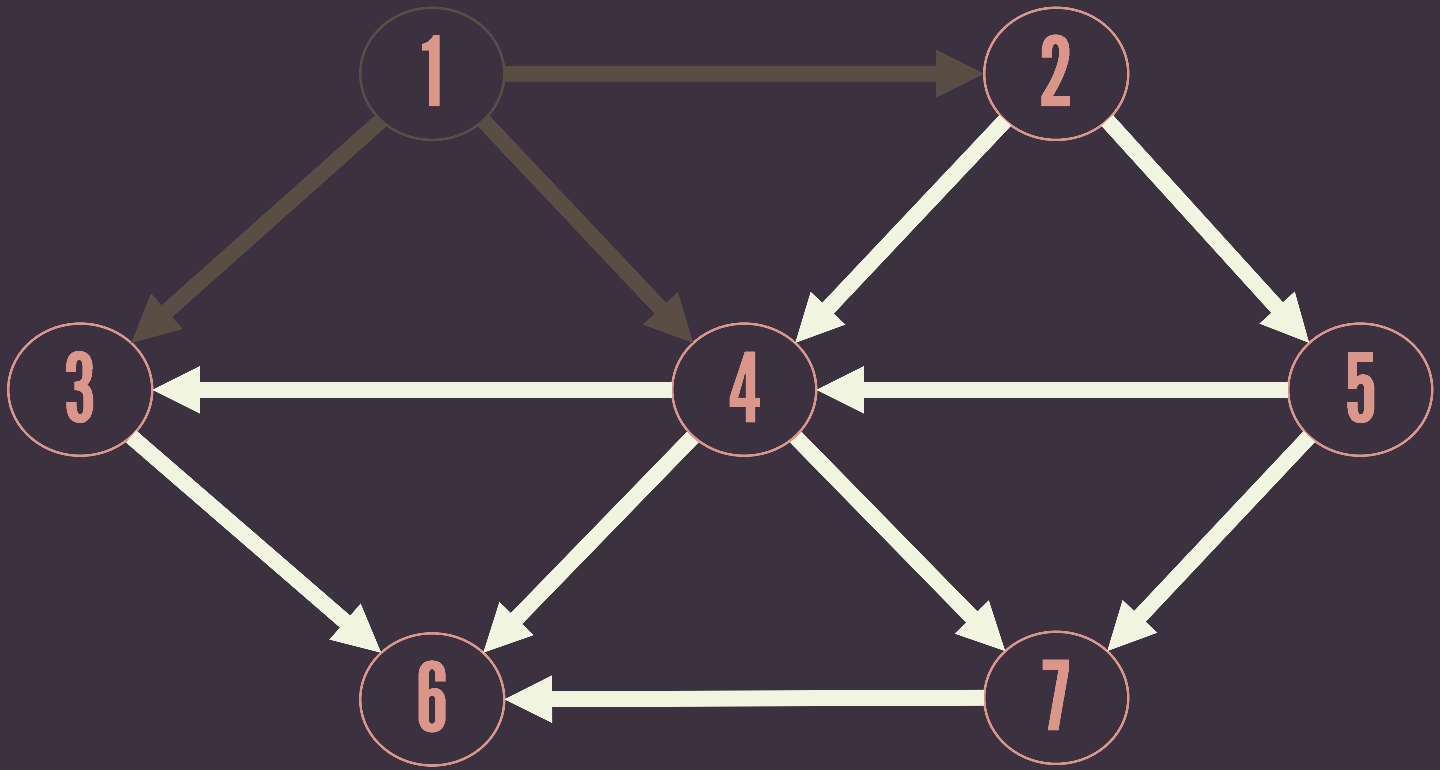
directed
acyclic graph
DAG



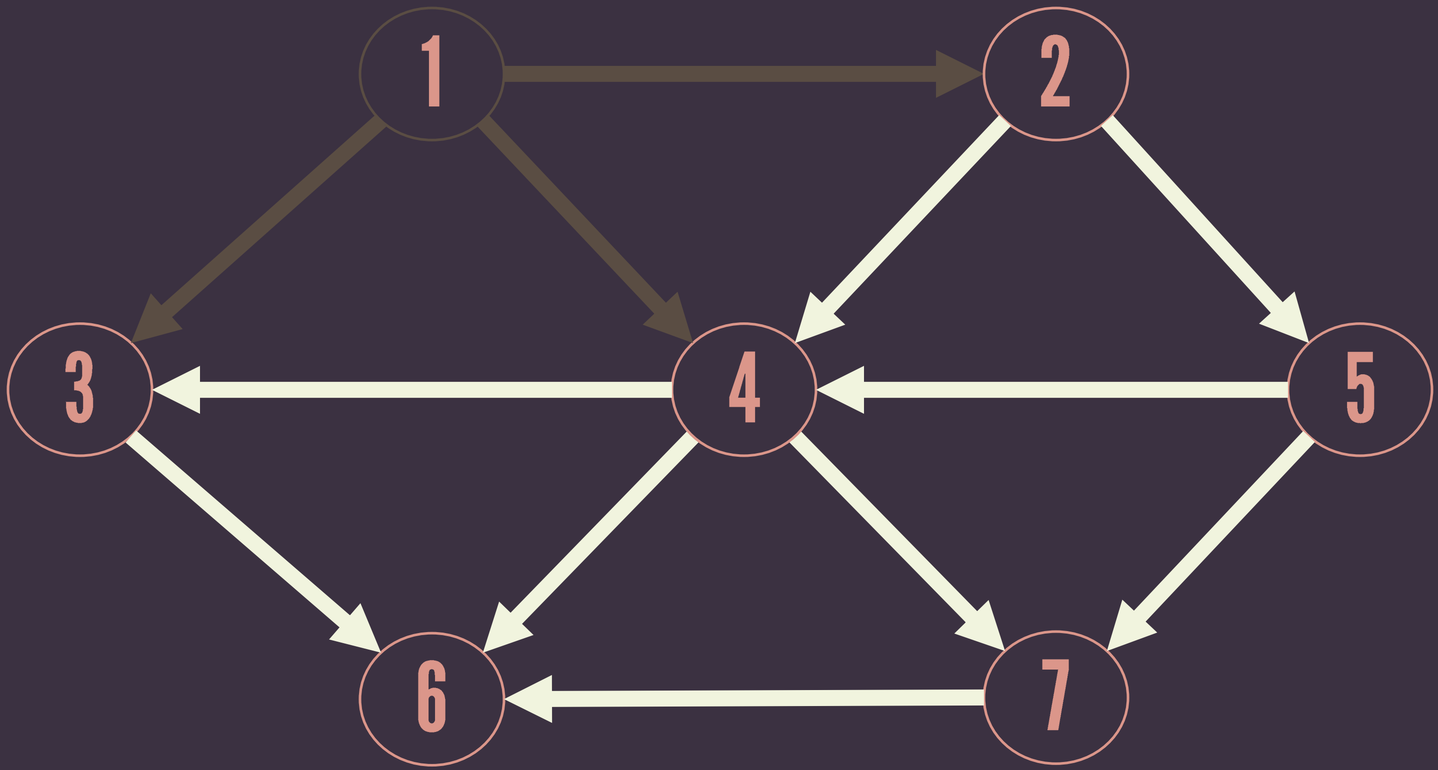
Algorithm:

- Find any vertex with no incoming edges (in-degree is 0).
- Print this vertex and remove it along with its edges from the graph.
- Repeat steps above.

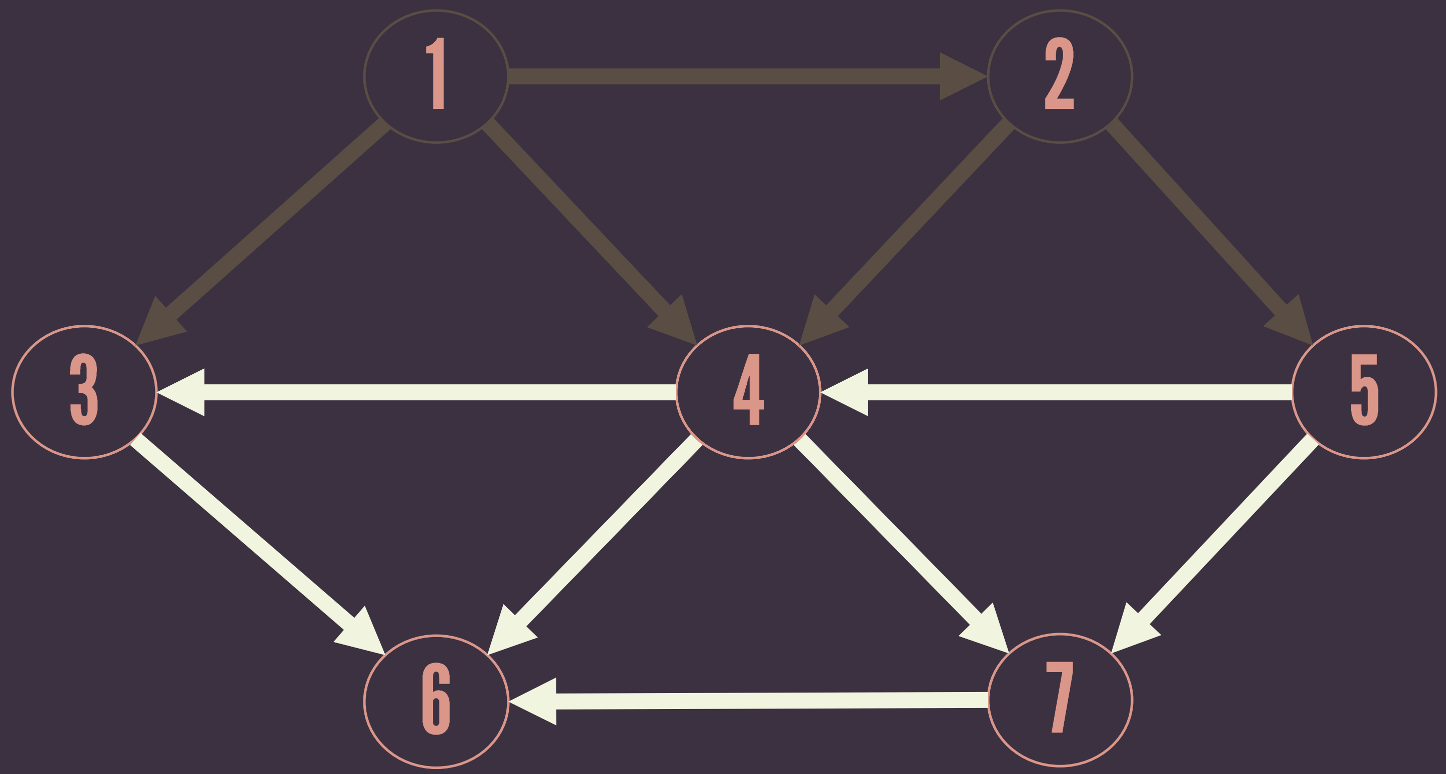

1
2
3
4
5
6
7



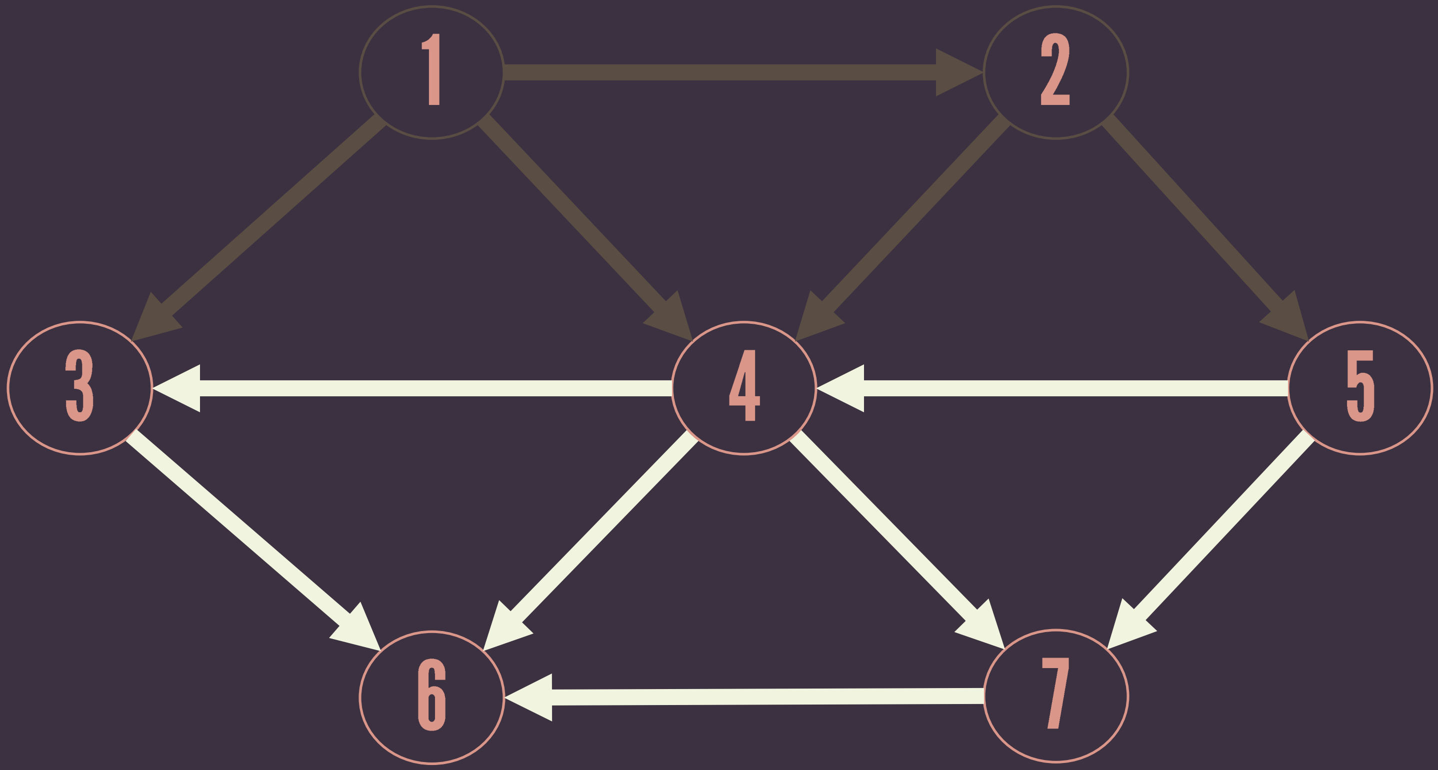
1



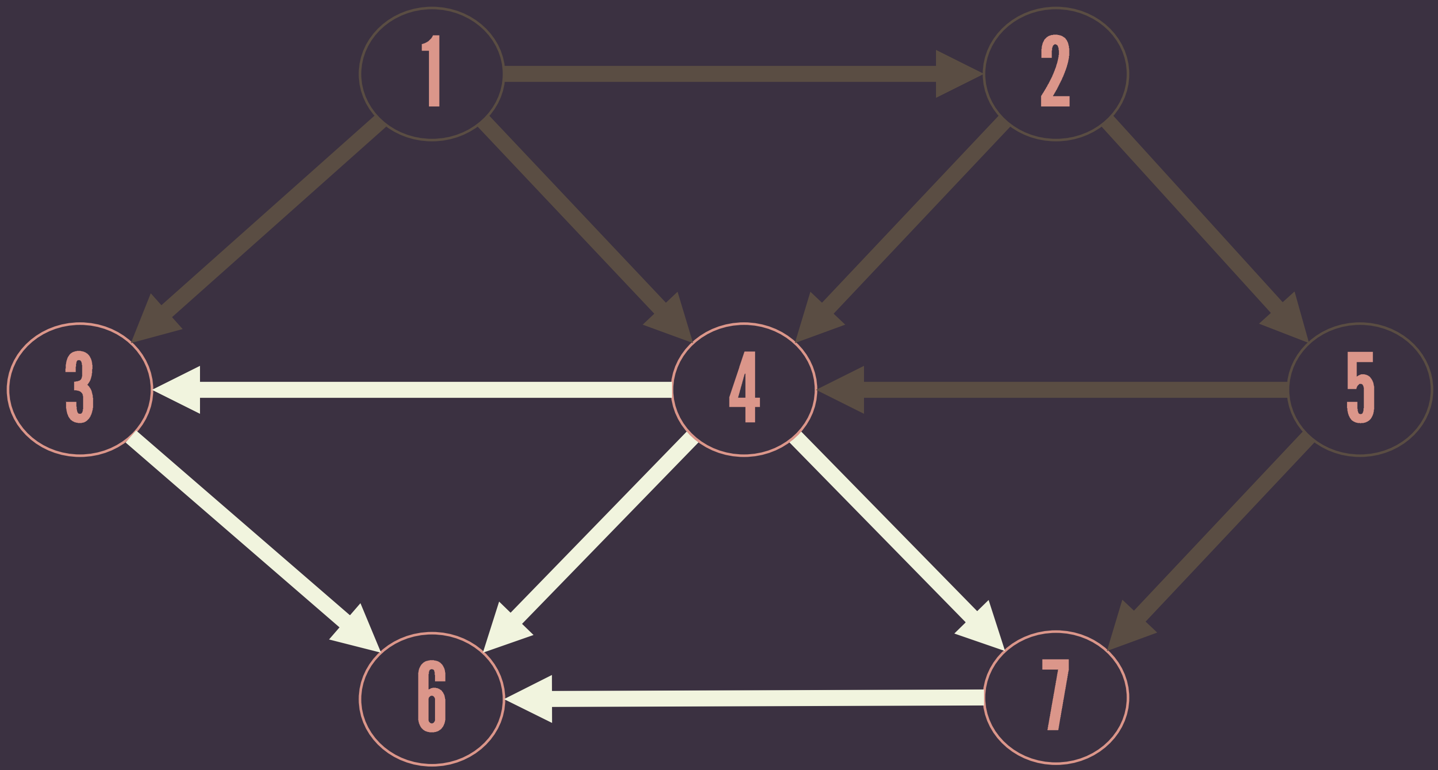
1 2



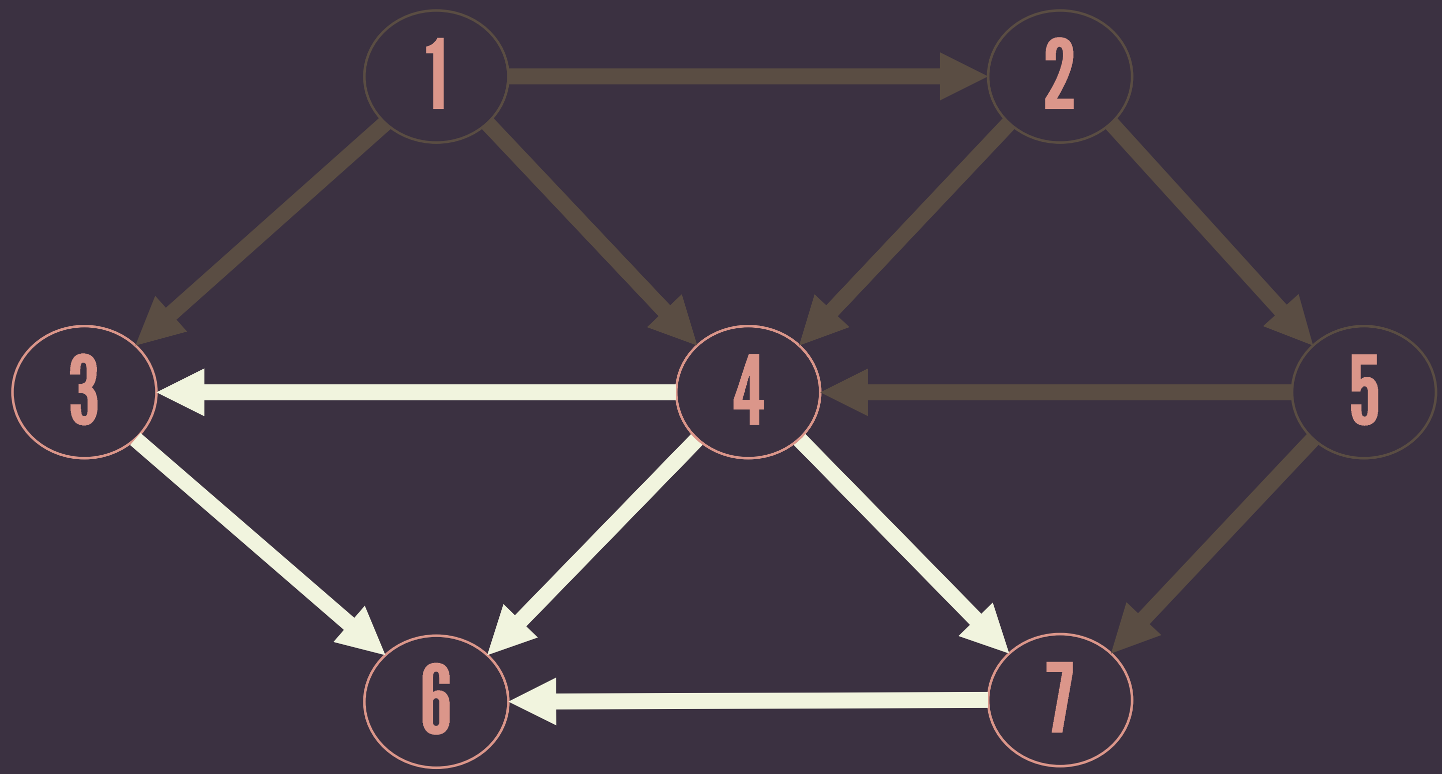
1 2



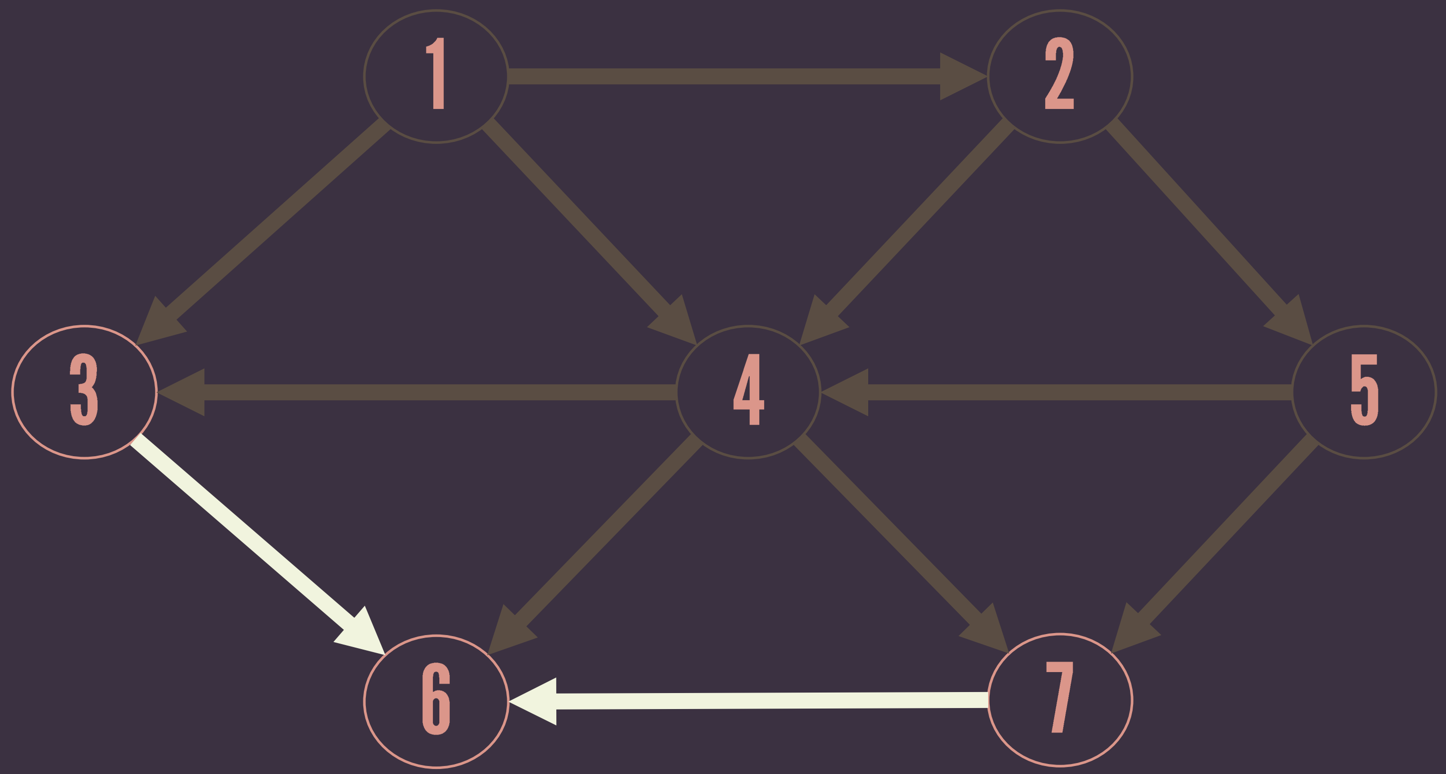
1 2 5



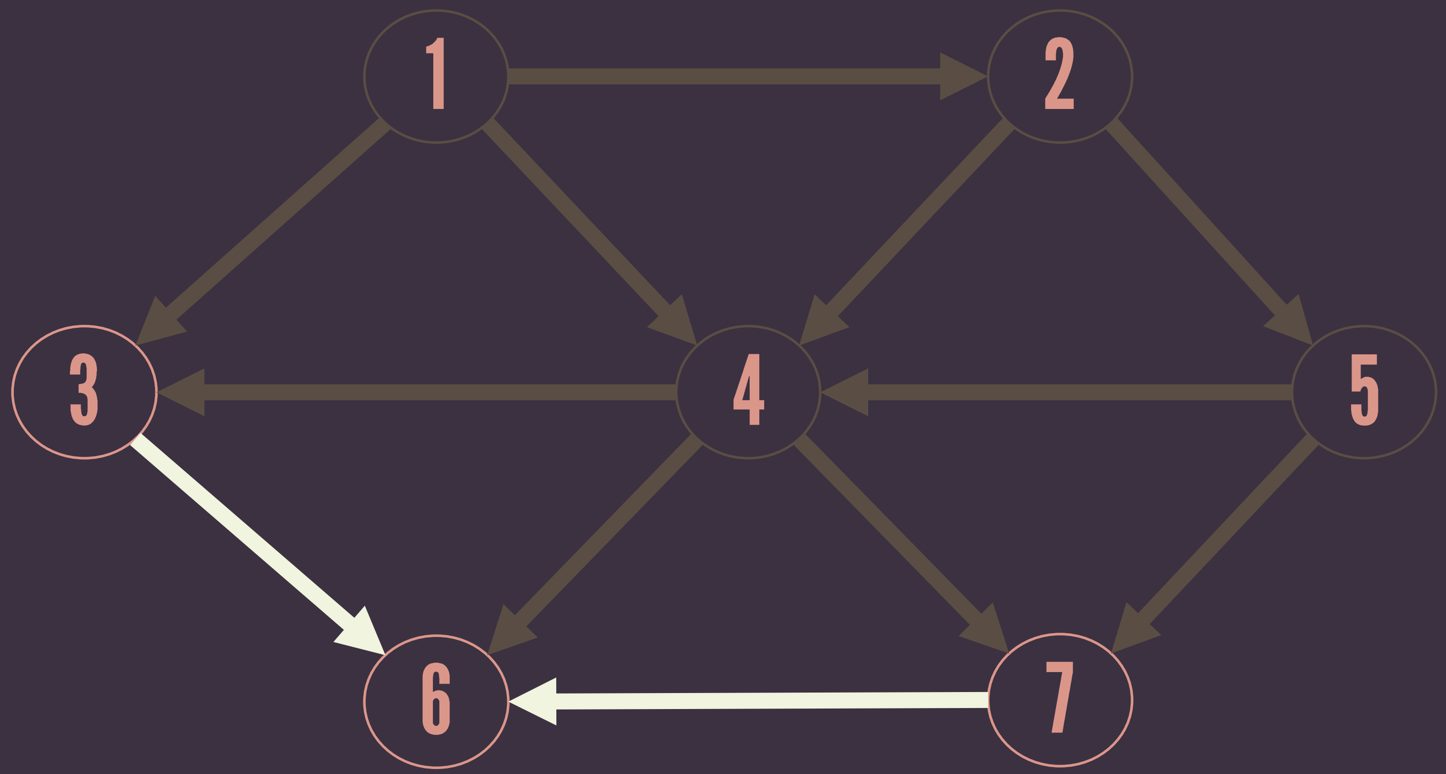
1 2 5



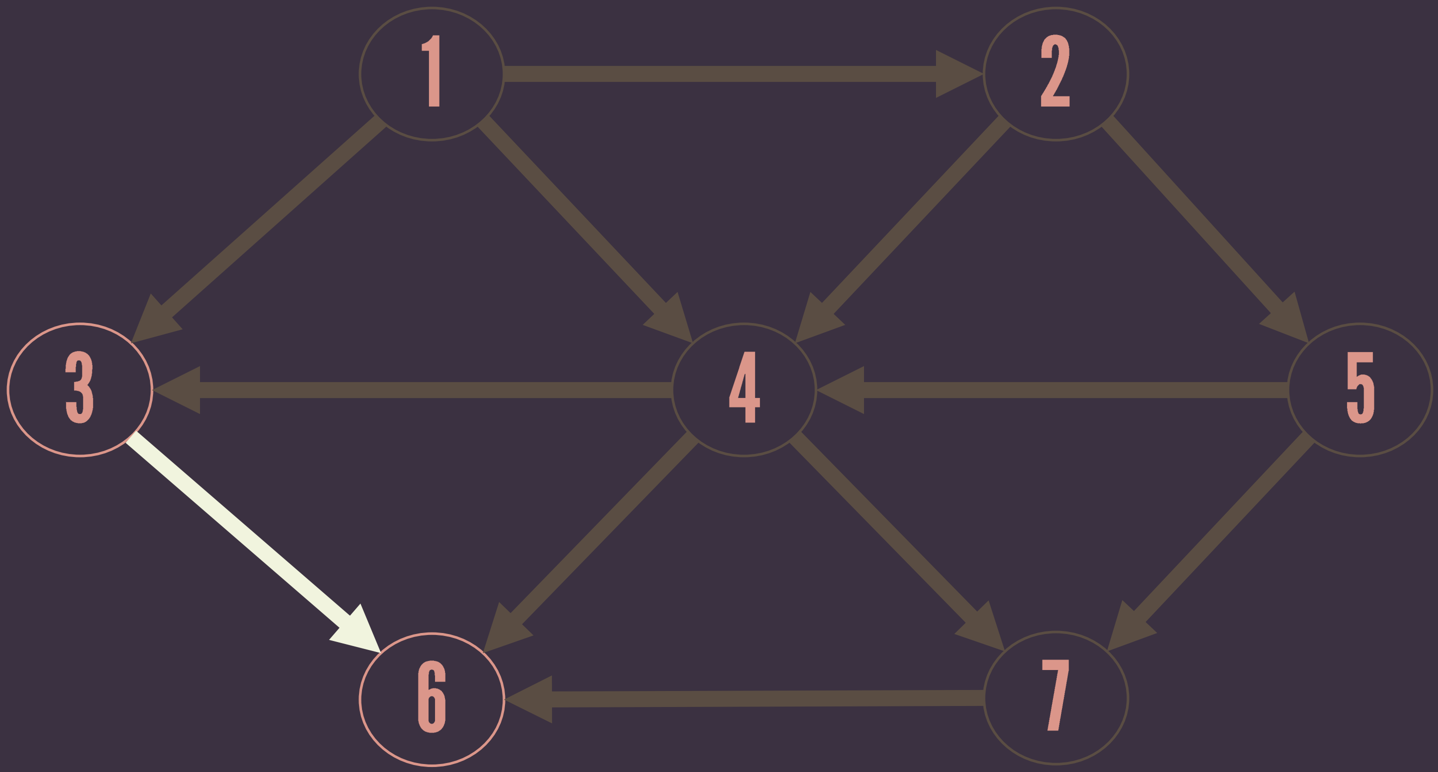
1 2 5 4



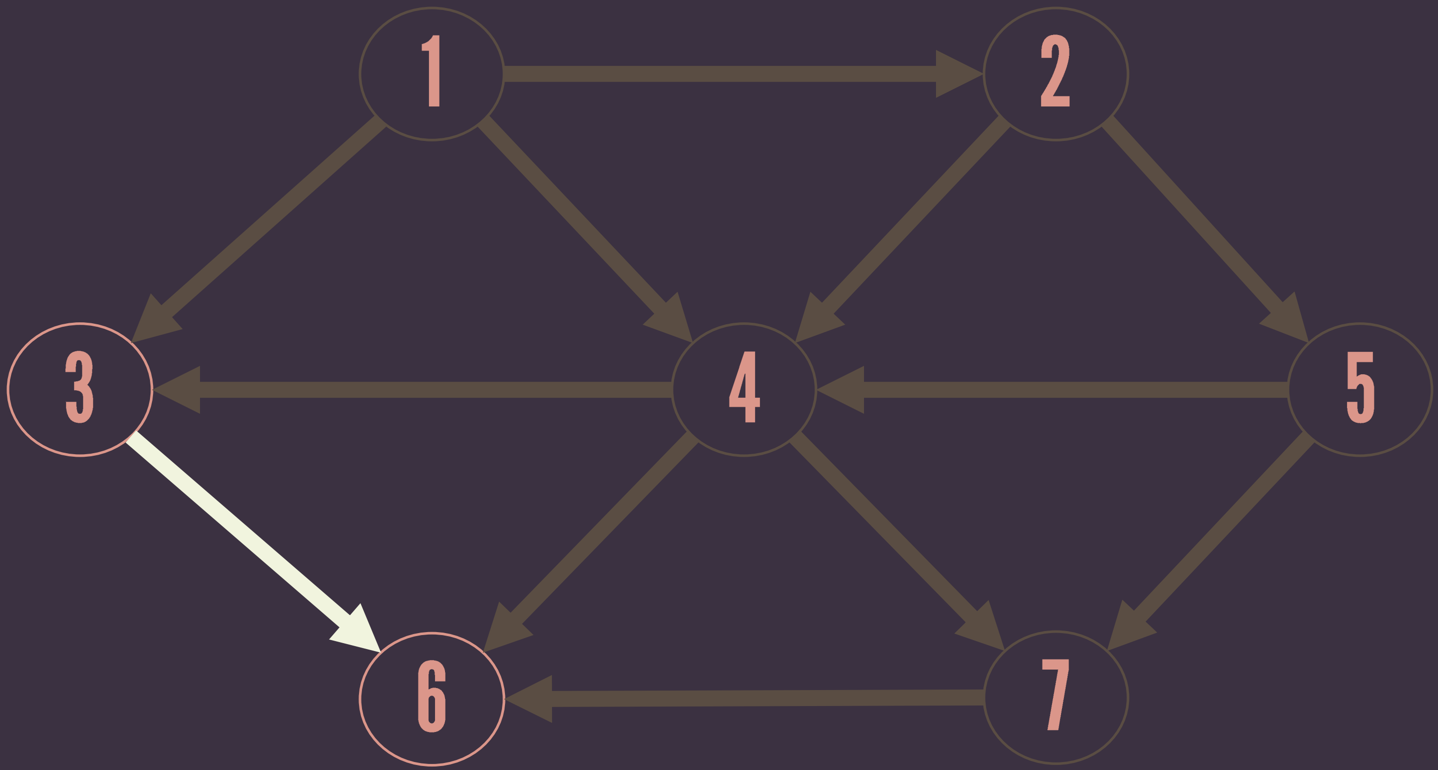
1 2 5 4



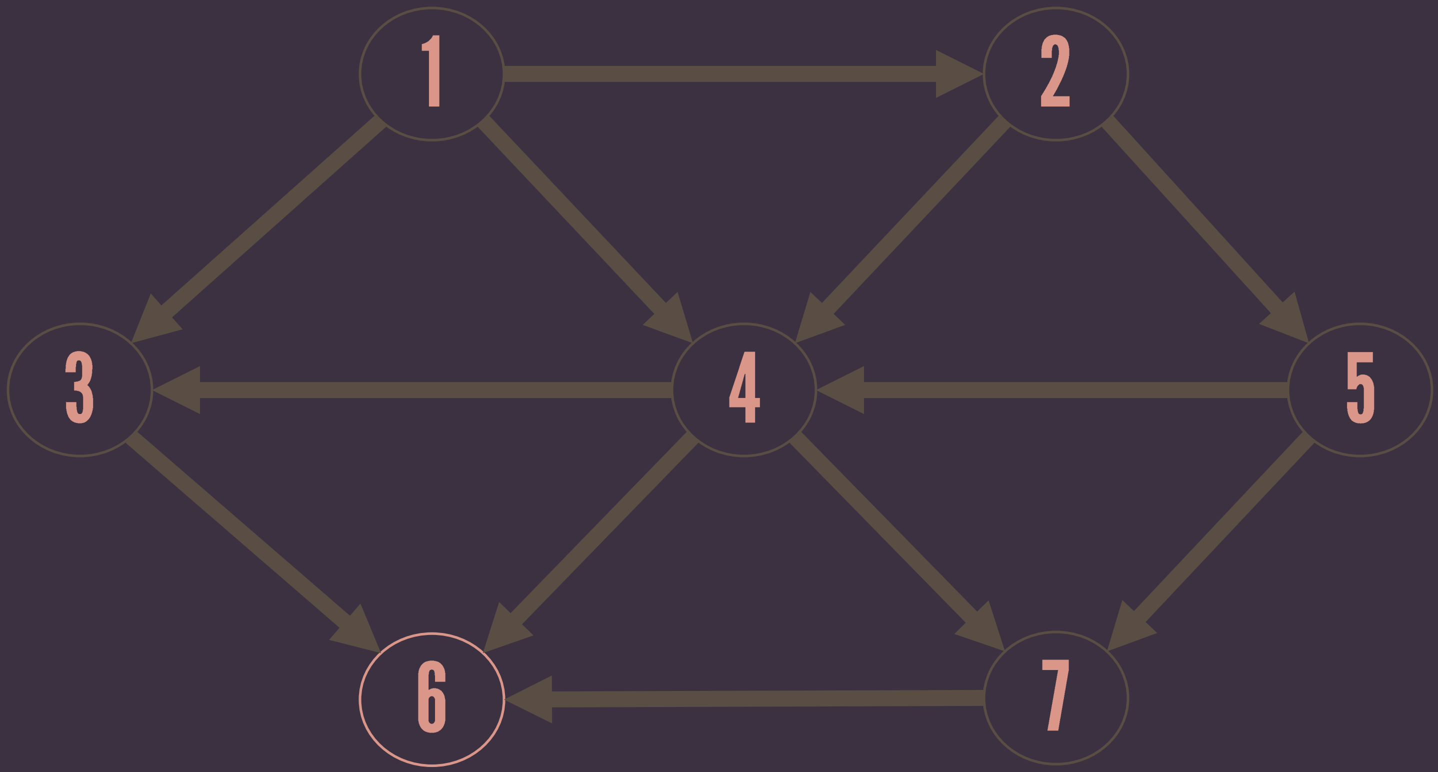
1 2 5 4 7



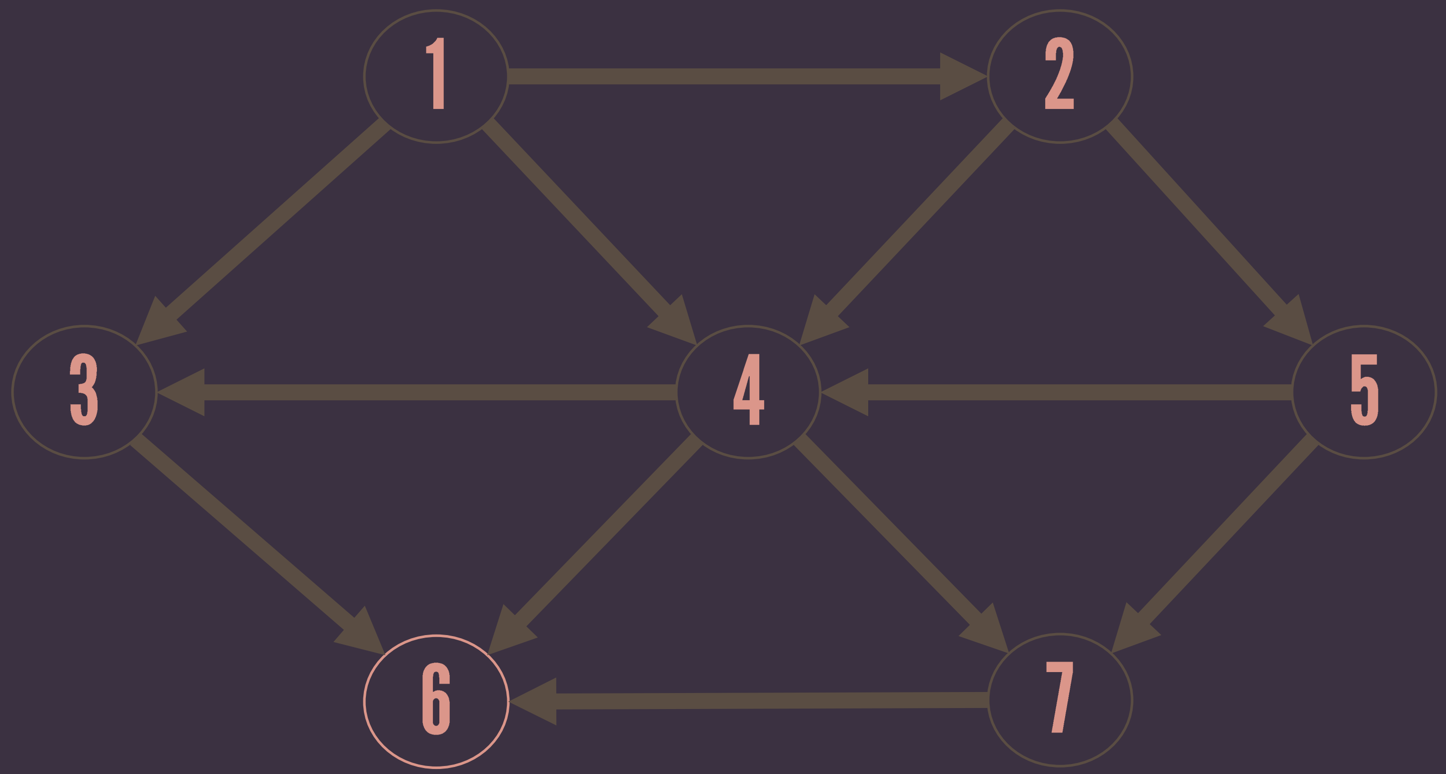
1 2 5 4 7



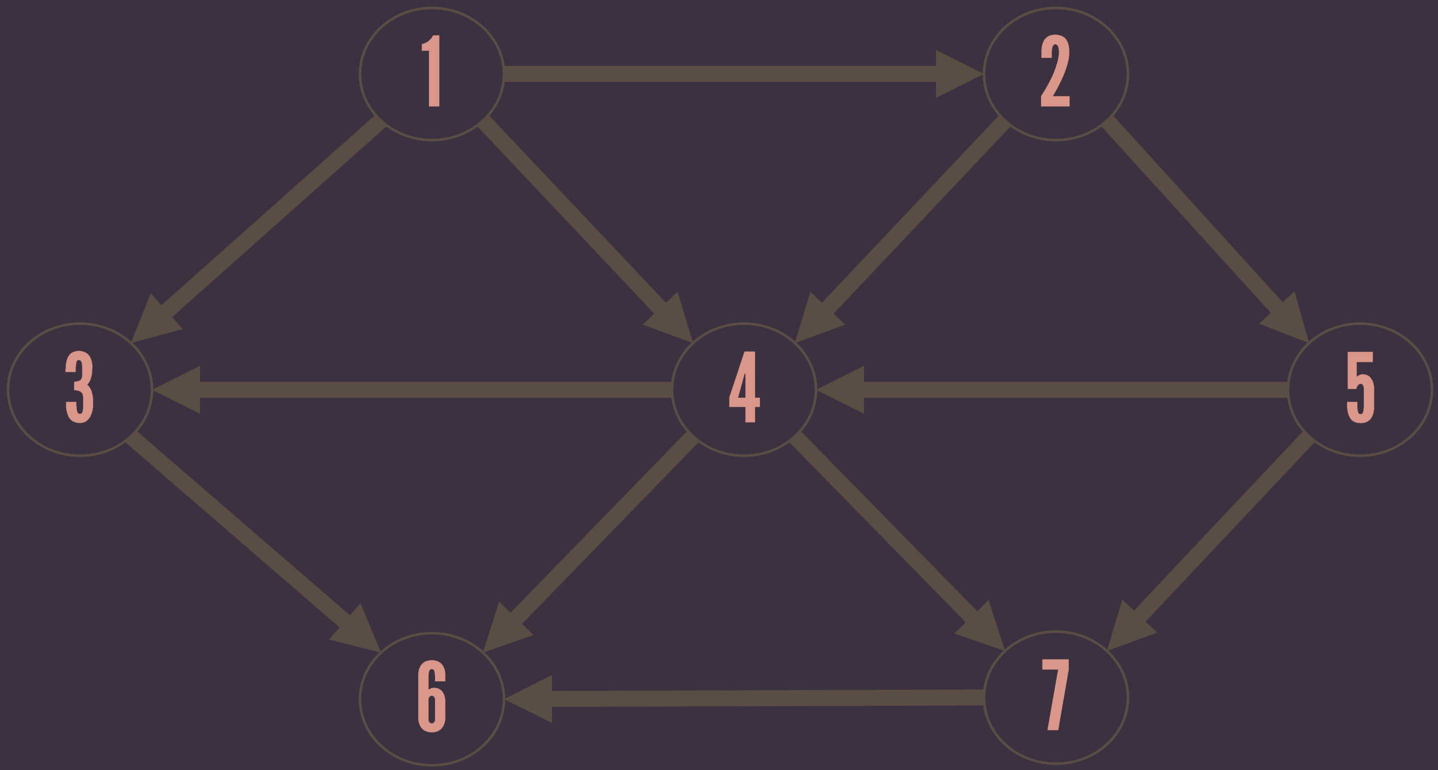
1 2 5 4 7 3



1 2 5 4 7 3



1 2 5 4 7 3 6



1 2 5 4 7 3 6

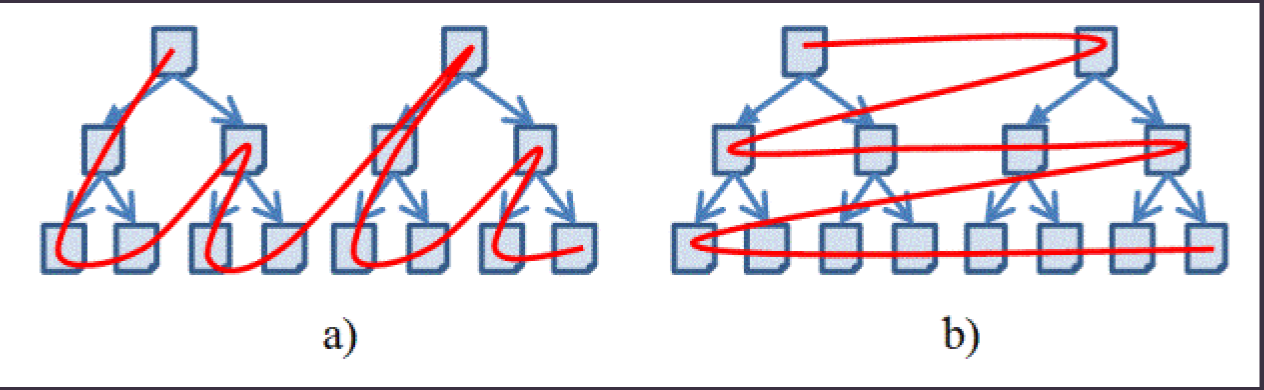
graph

TRAVERSALS

DEPTH FIRST
search

BREADTH FIRST

search



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DEPTH FIRST search

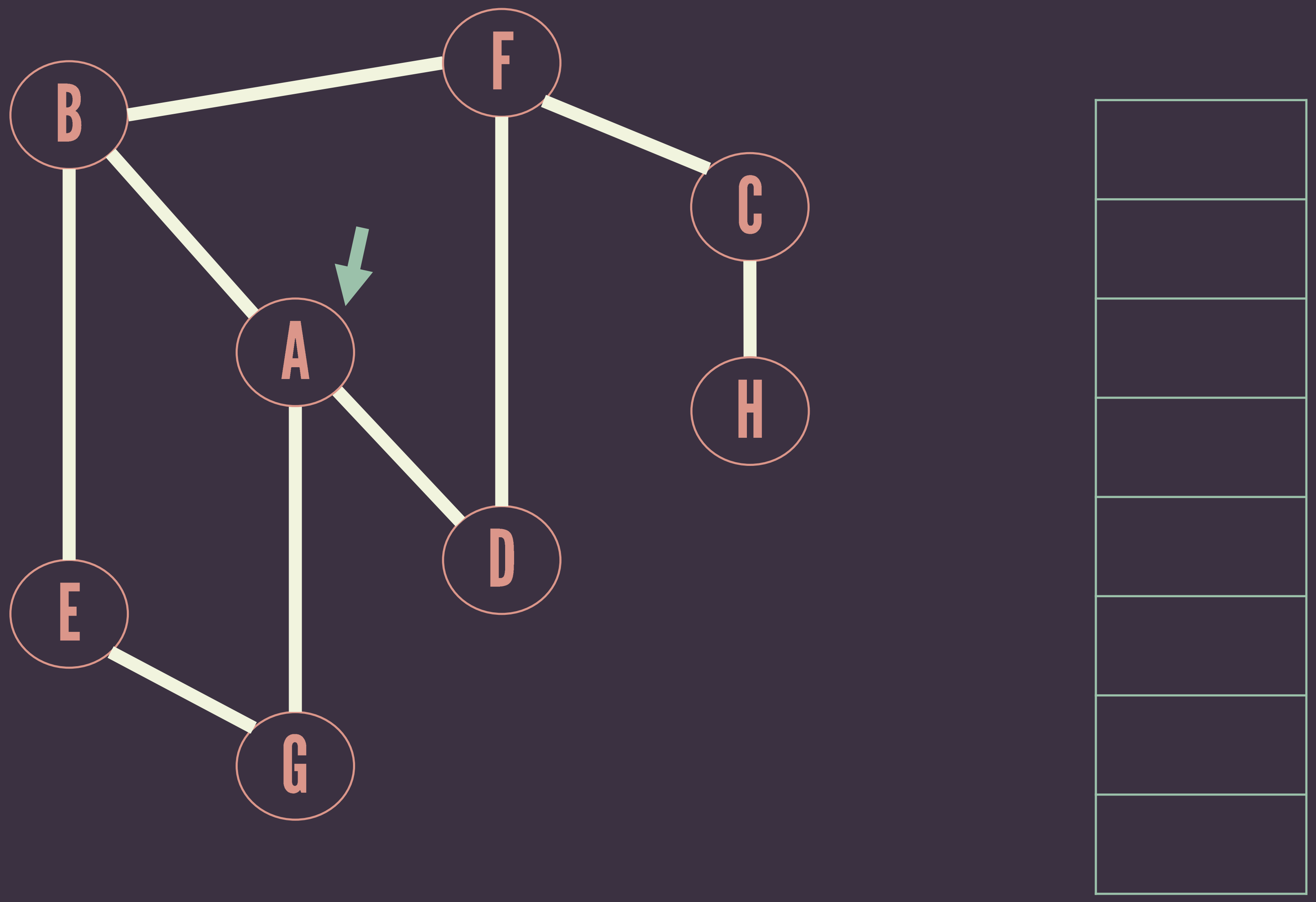
Generalization of preorder traversal.

DEPTH FIRST search

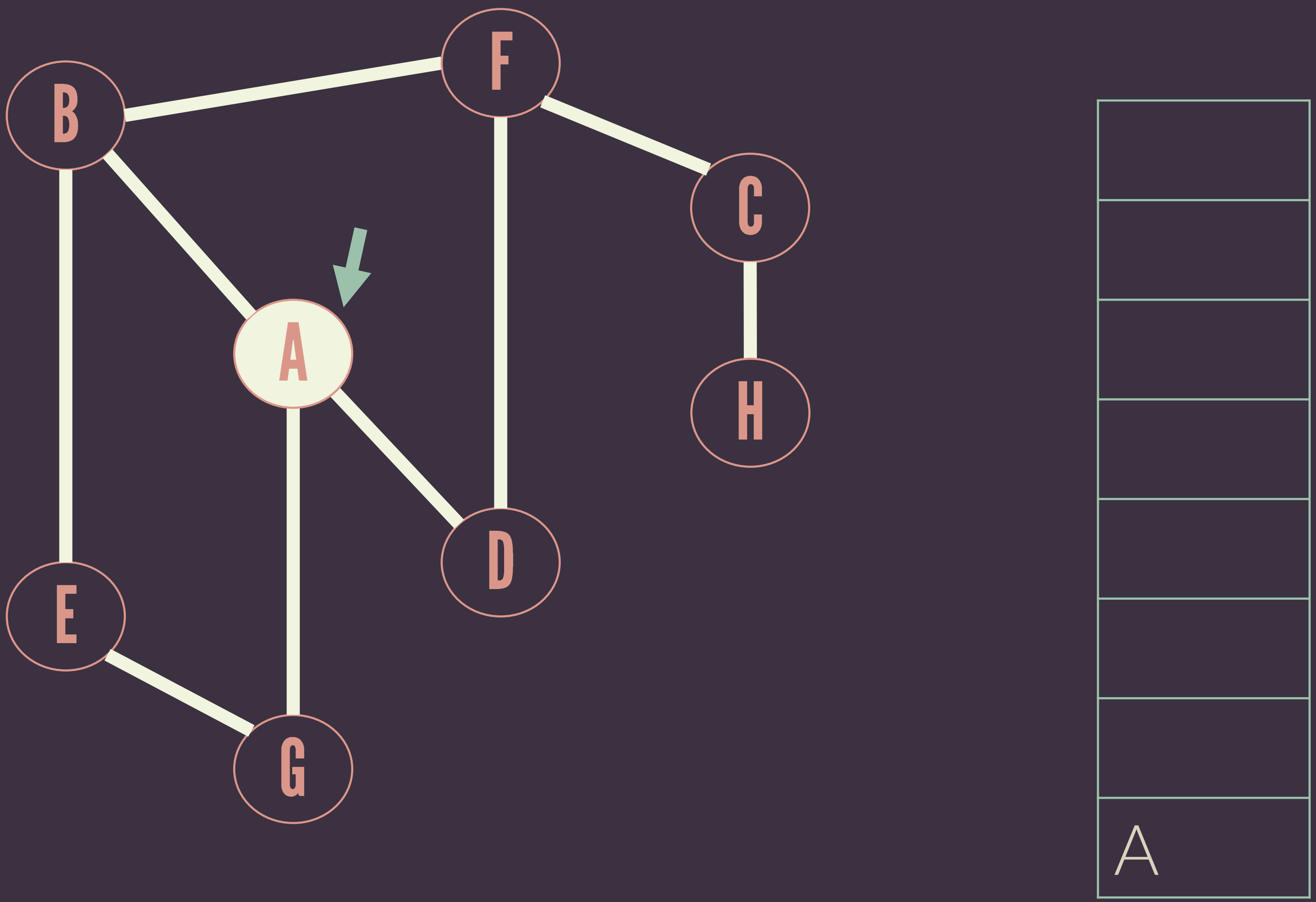
Starting at some vertex v, process v and recursively traverse all vertices adjacent to v.

```
void DFS( vertex v, graph G ){
  print v;
  visited[v] = TRUE;
  for each w adjacent to v:
    if( !visited[w] )
      DFS(w);
}
```

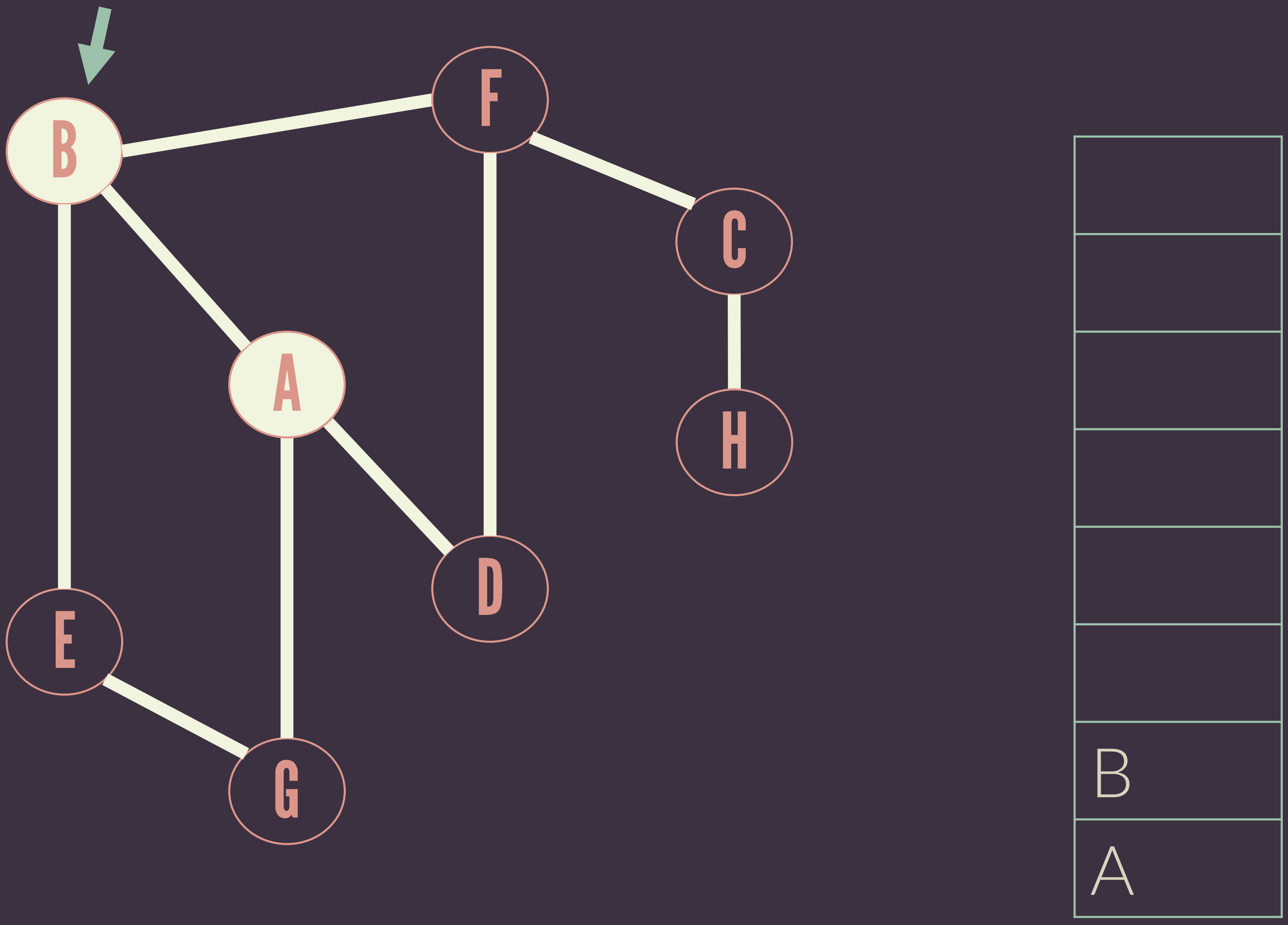
```
void DFS( vertex v, graph G ){
  stack S;
  push(v,S);
  while stack S is not empty{
    v = pop(S);
    if ( !visited[v] ){
      print v;
      visited[v] = TRUE;
      for each w adjacent to v:
        if( !visited[w] )
          push(w, S);
    }
  }
}
```



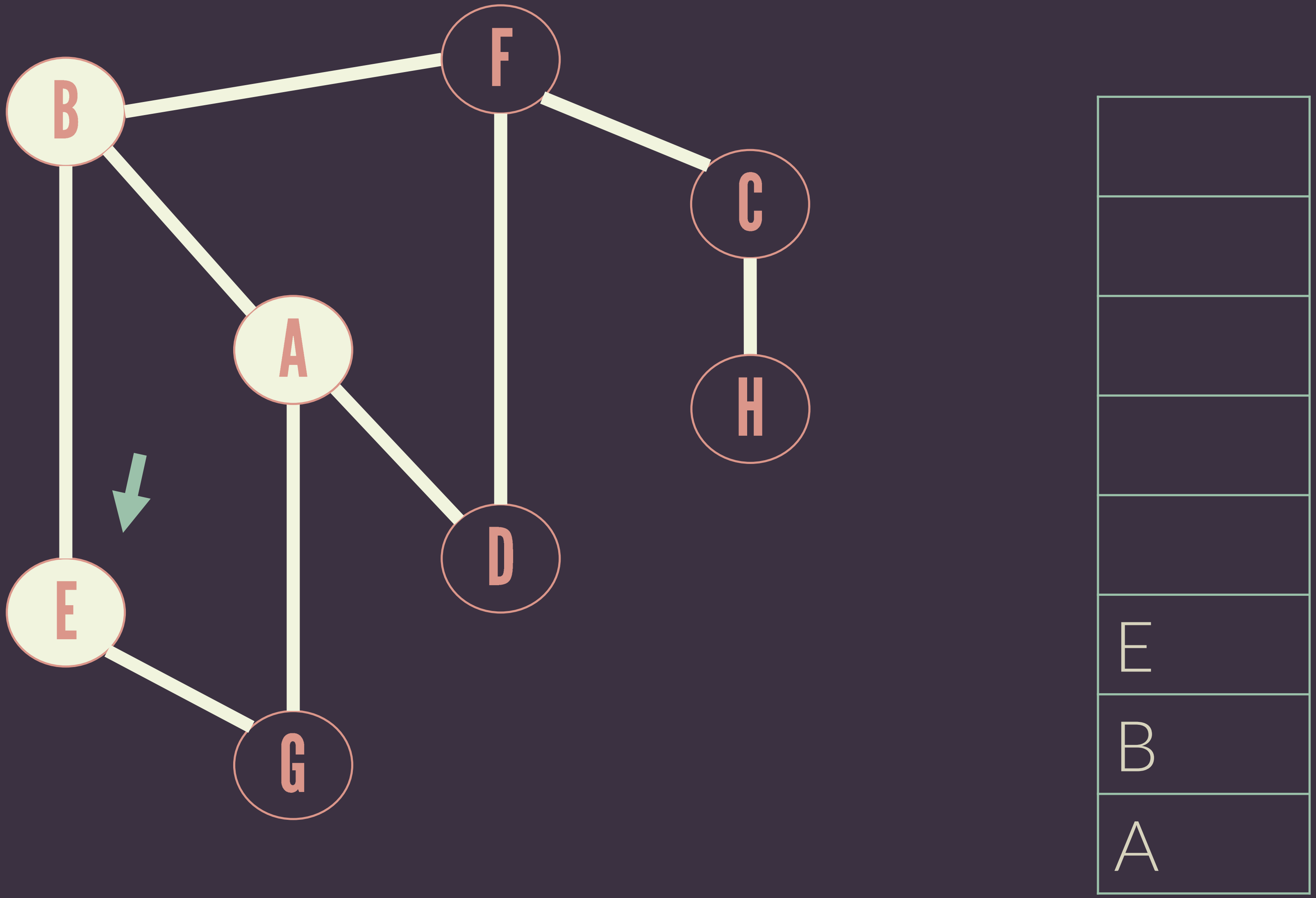
Result:



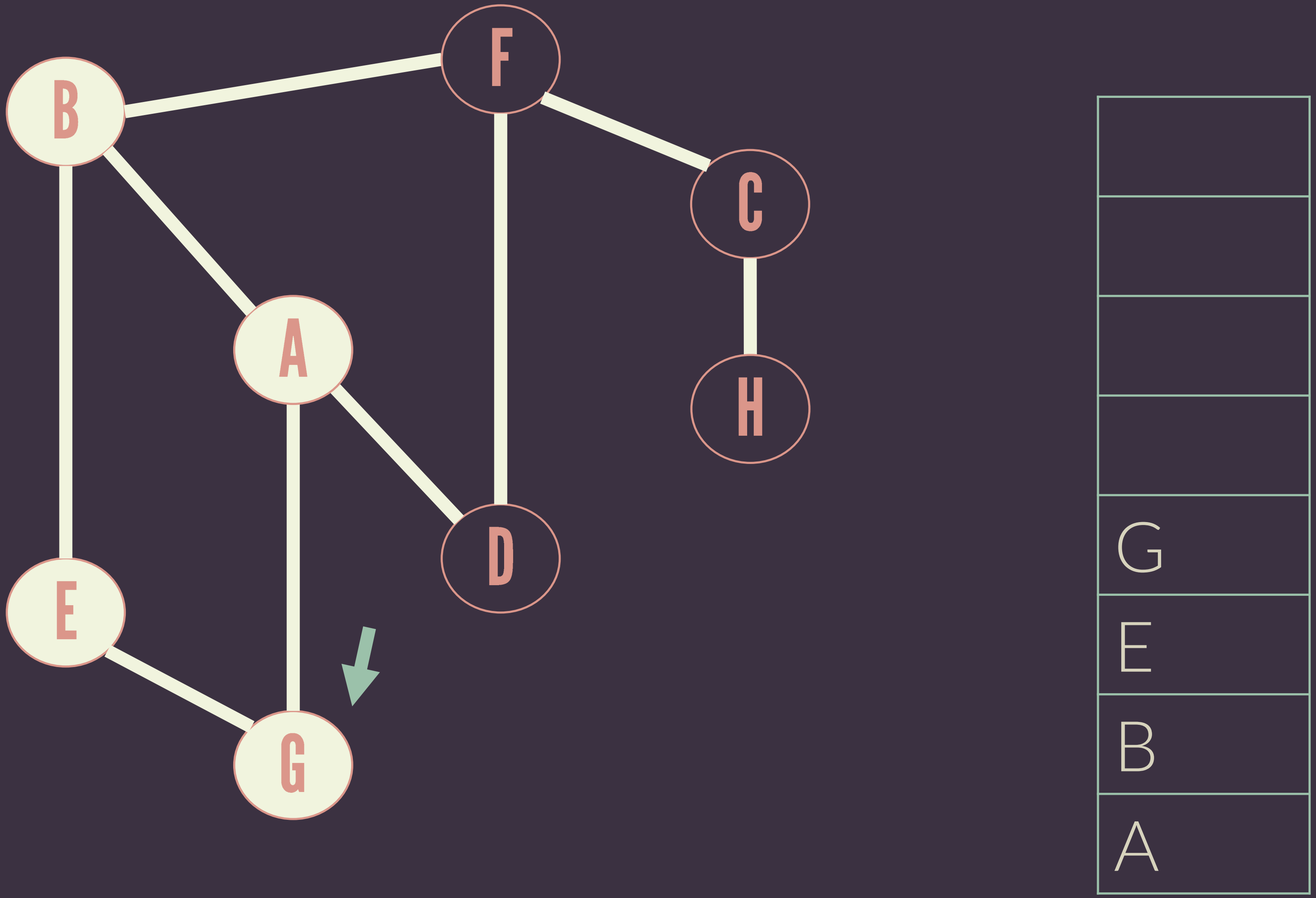
Result: A



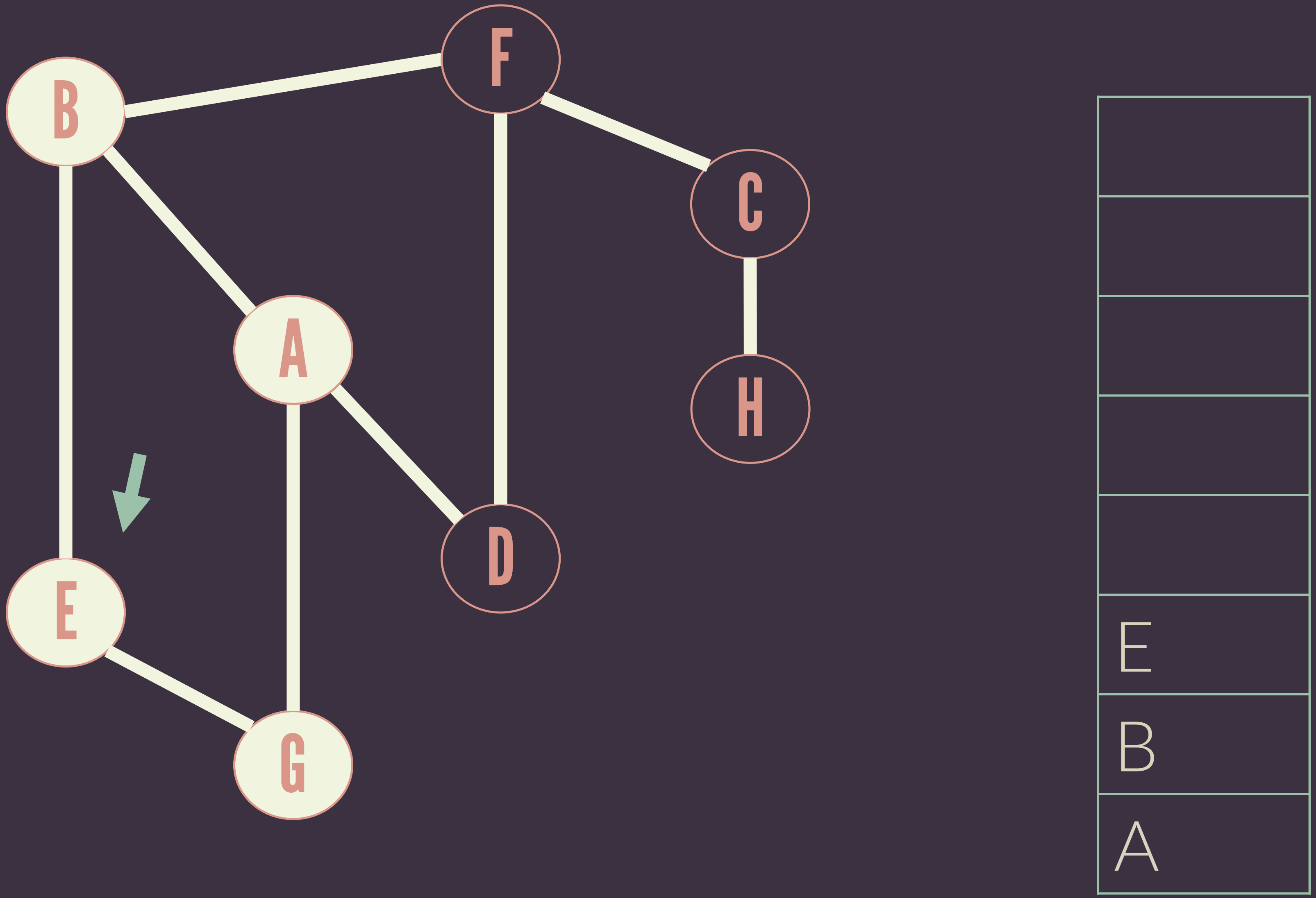
Result: A B



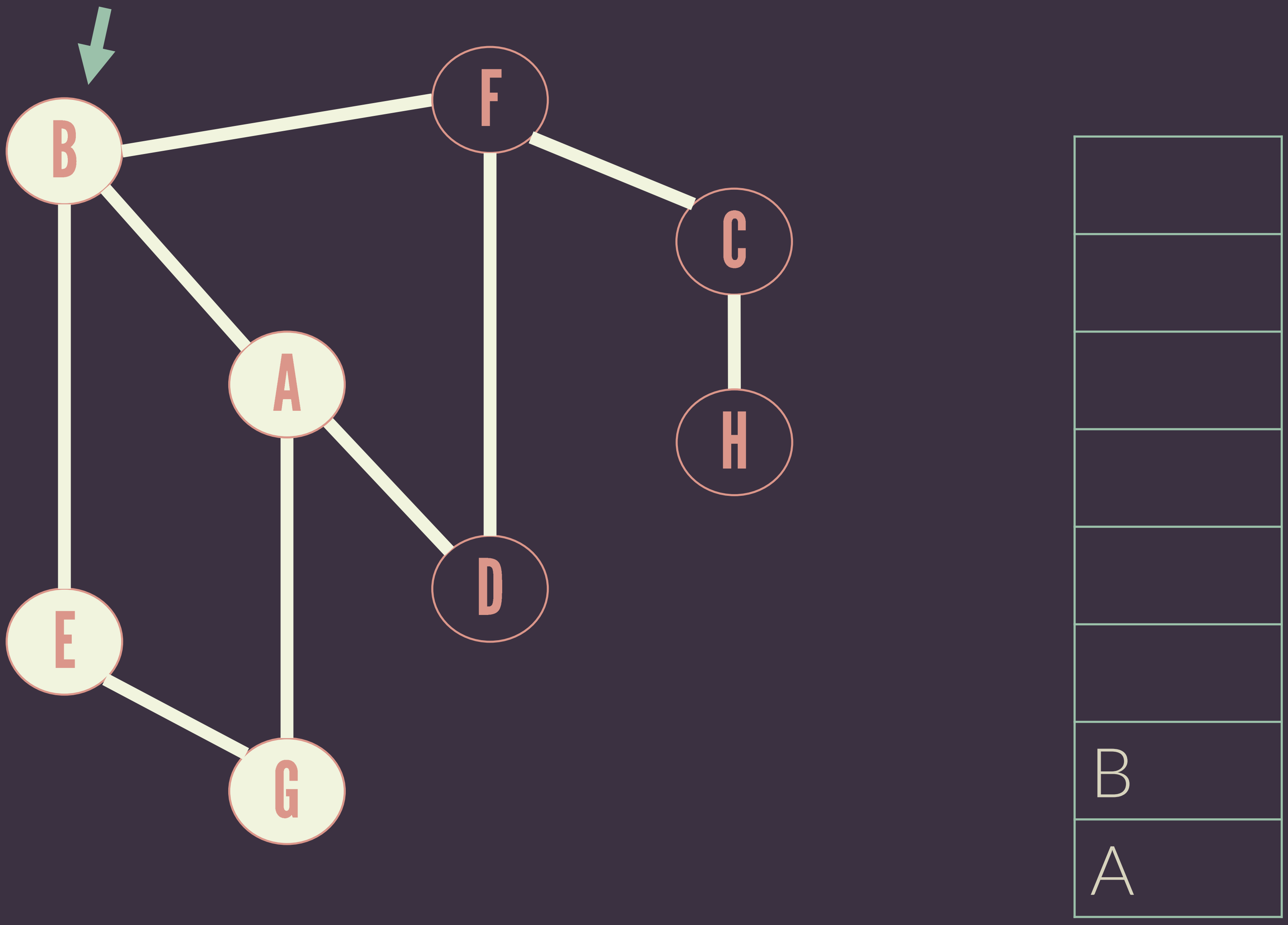
Result: A B E



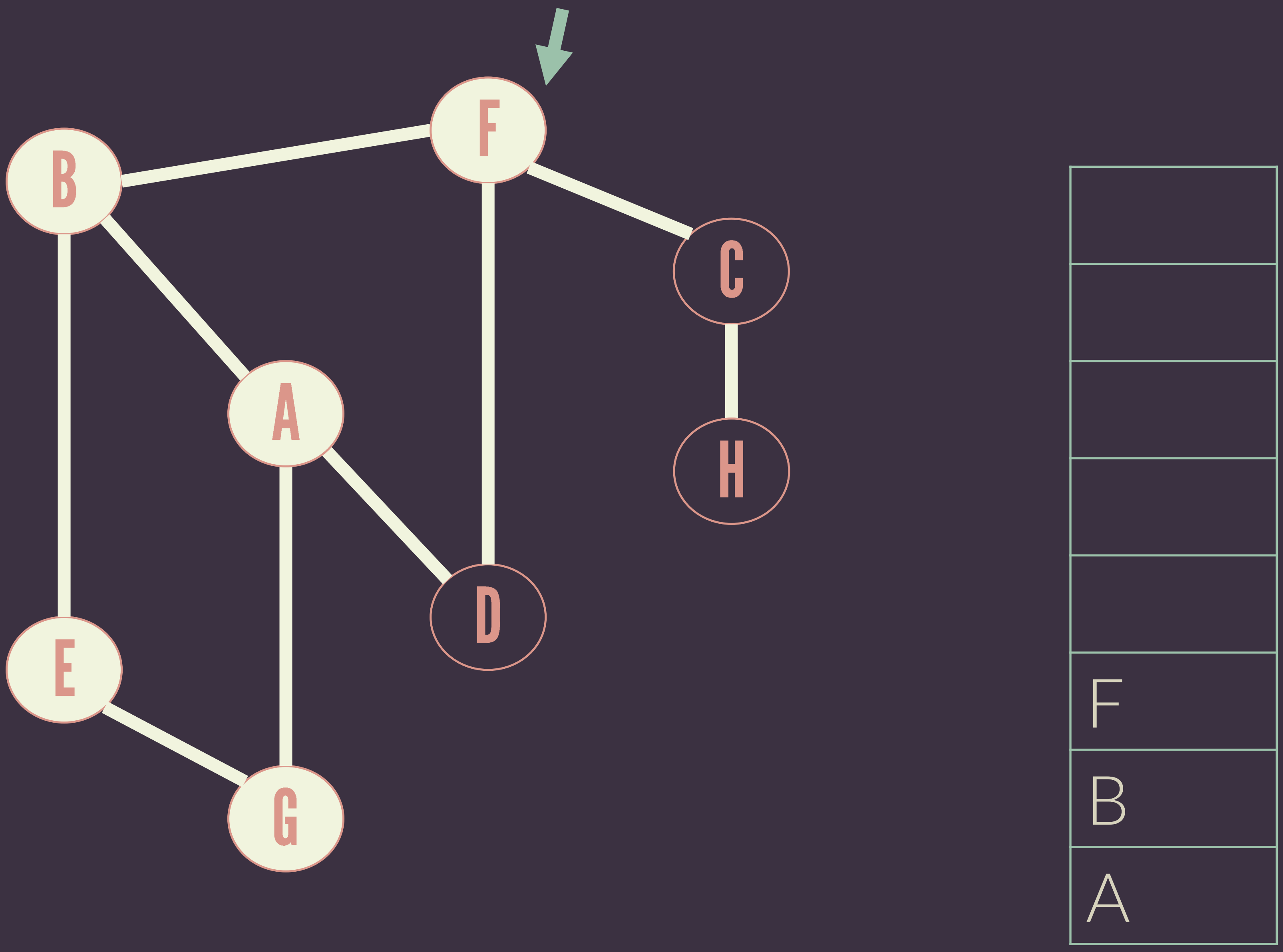
Result: A B E G



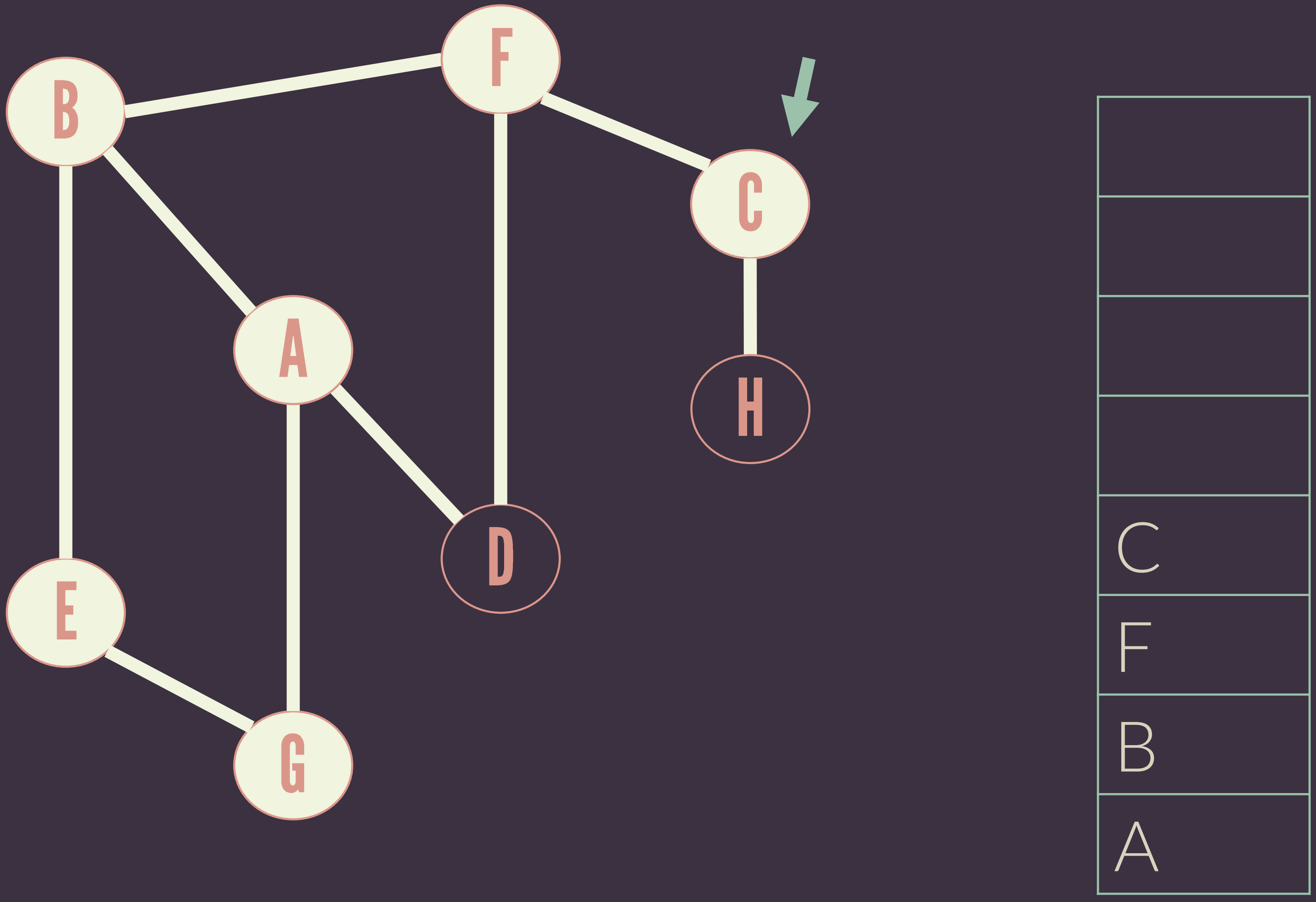
Result: A B E G



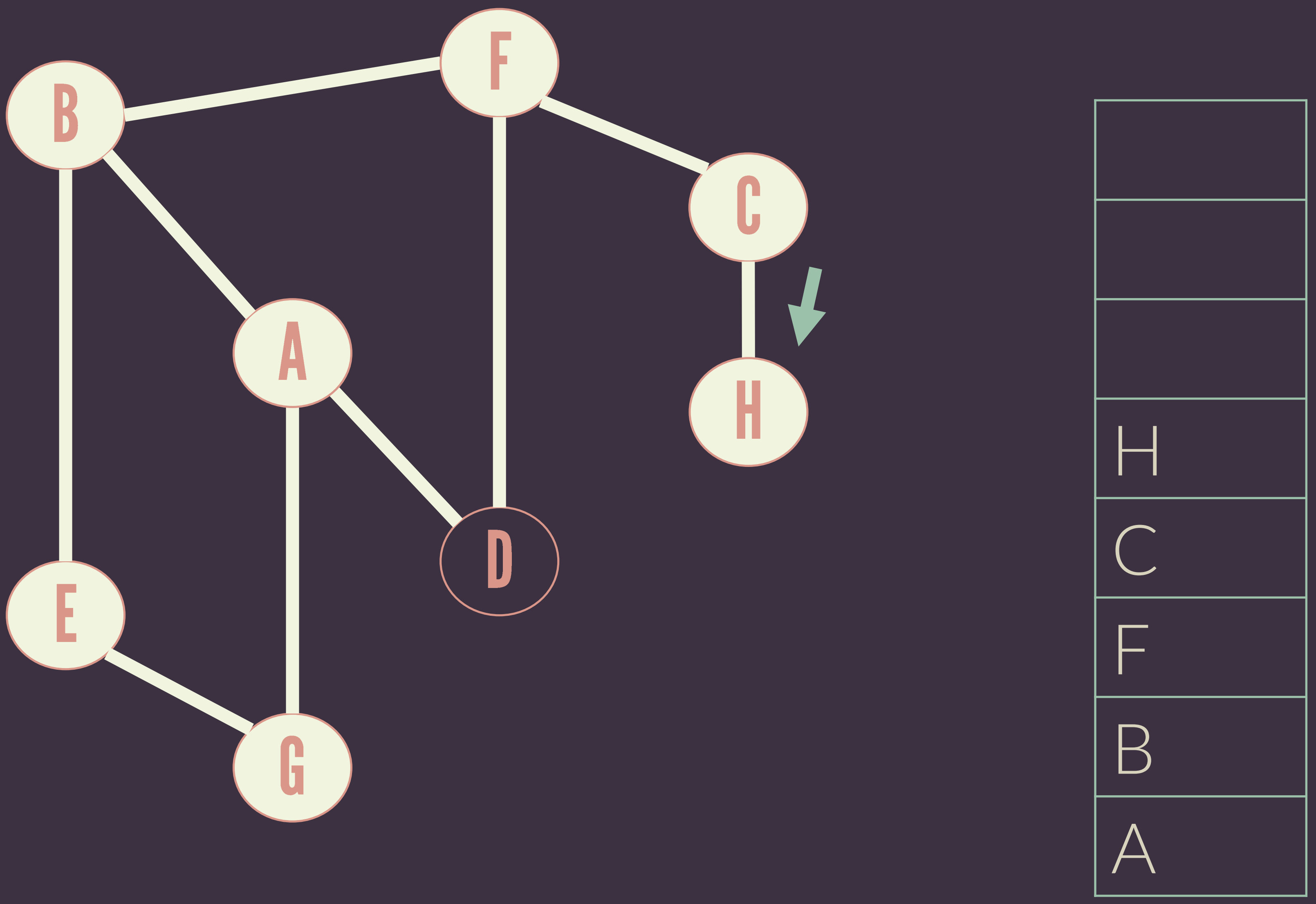
Result: A B E G



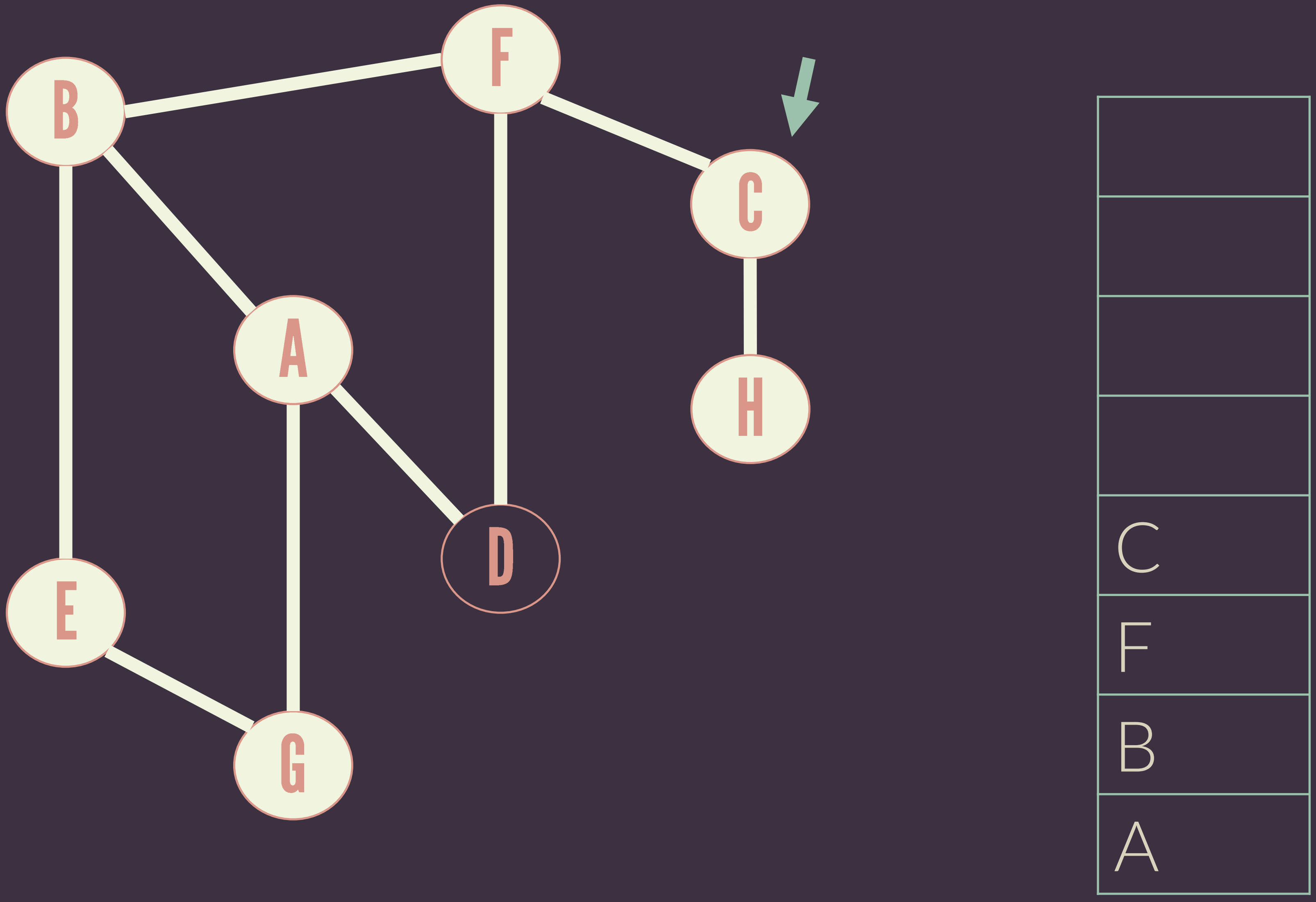
Result: A B E G F



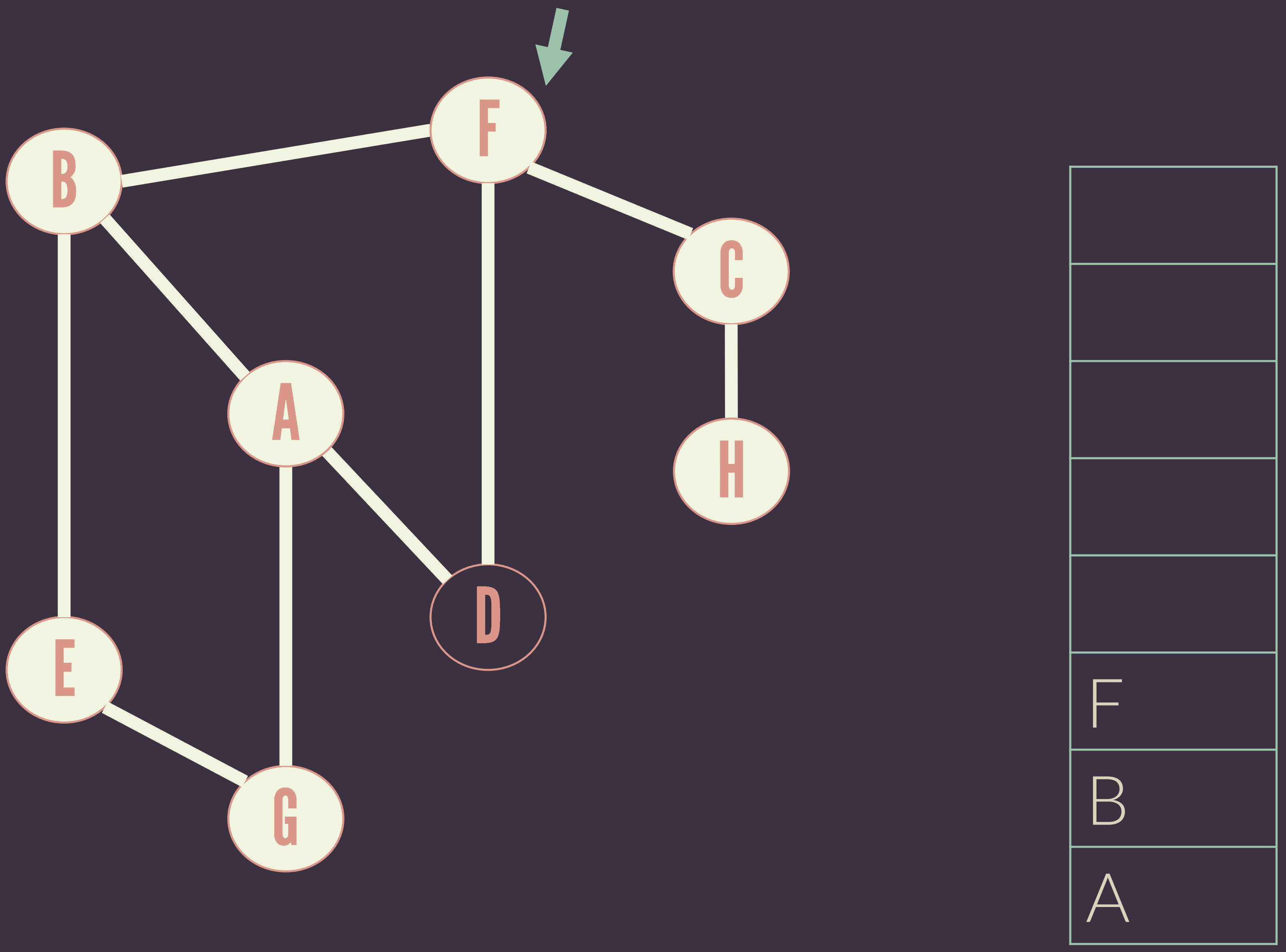
Result: A B E G F C



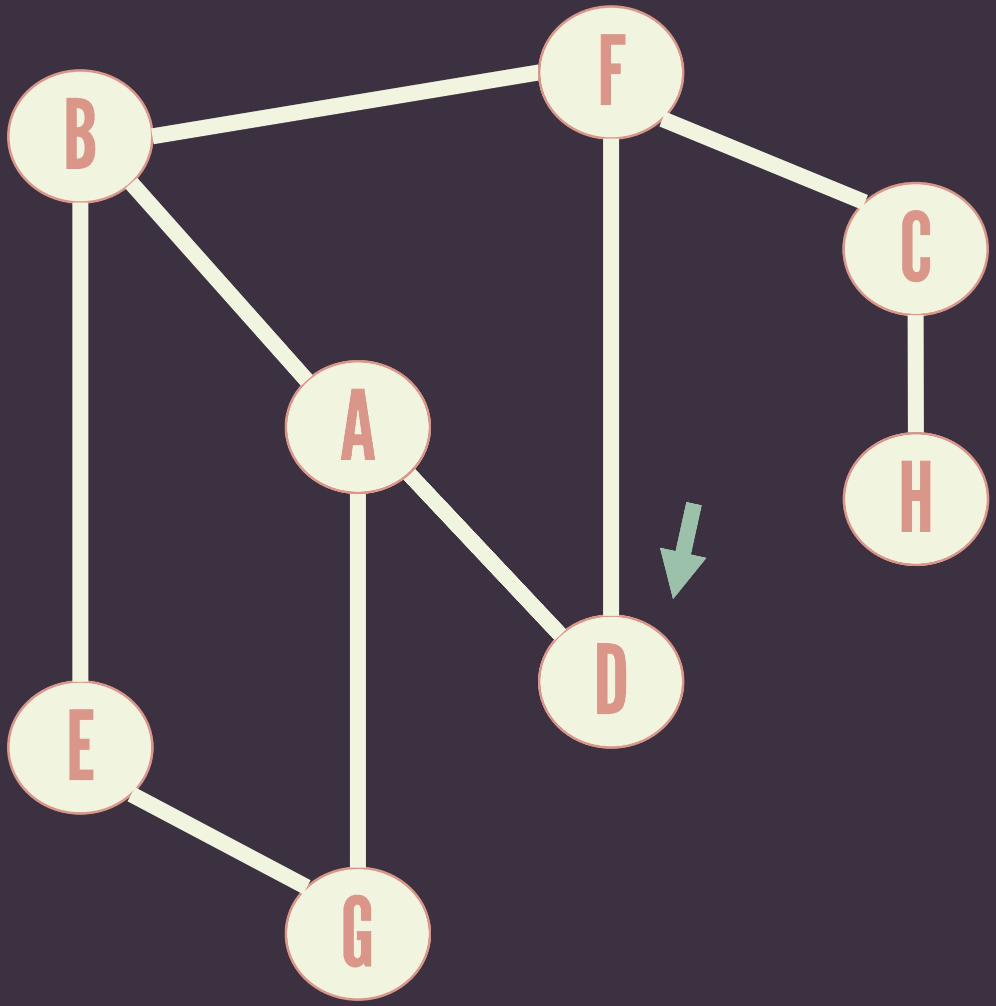
Result: A B E G F C H



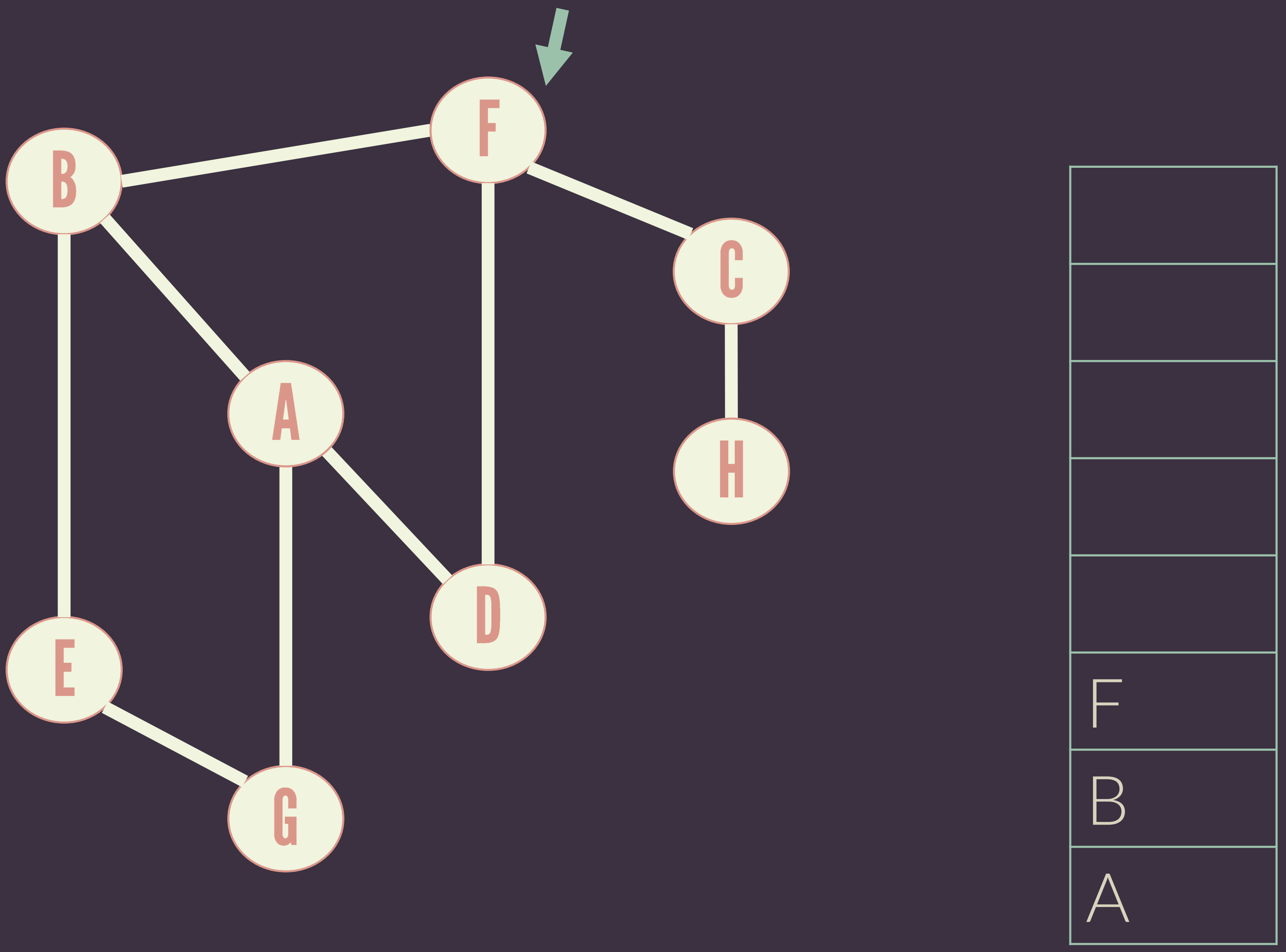
Result: A B E G F C H



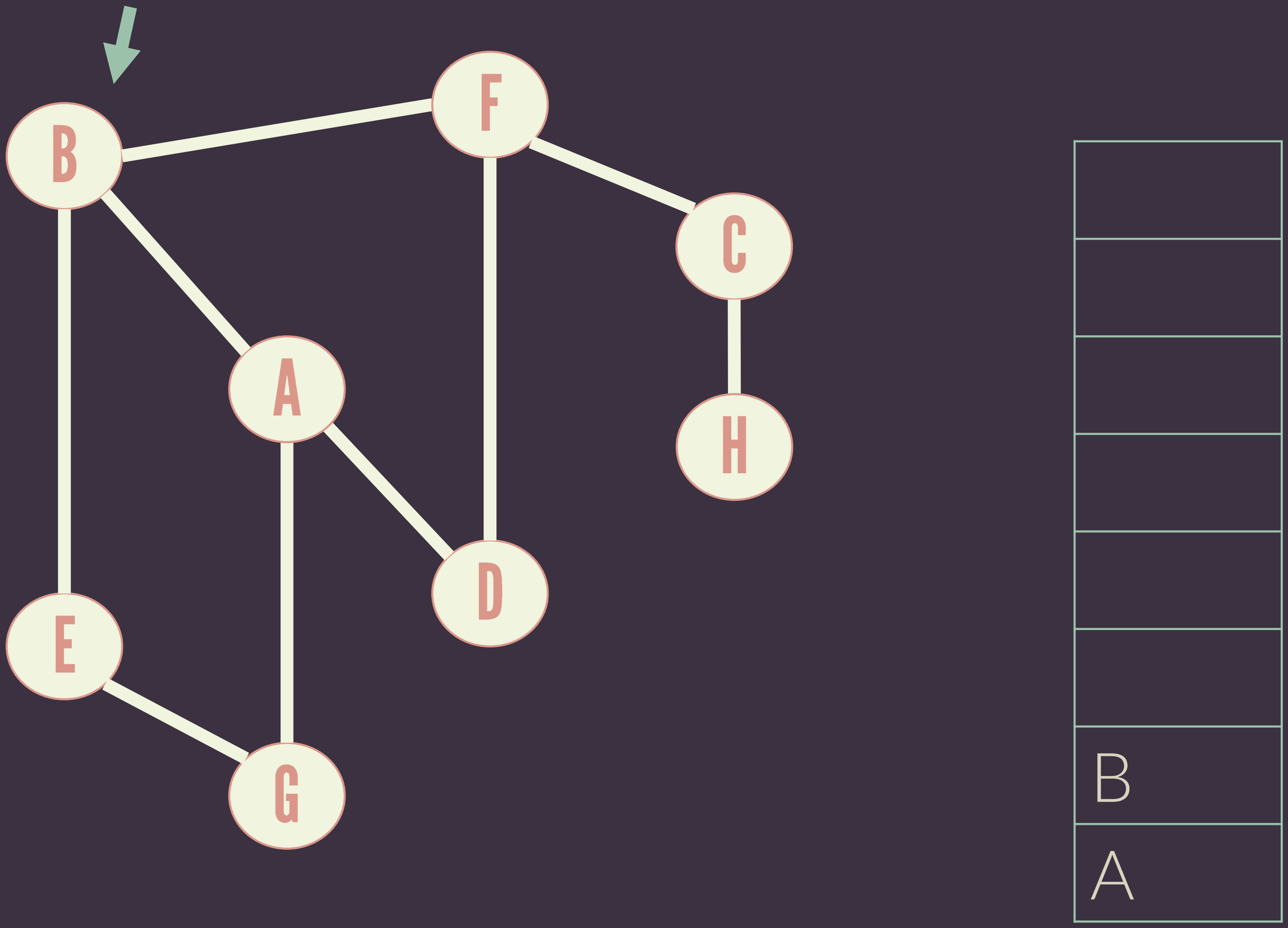
Result: A B E G F C H



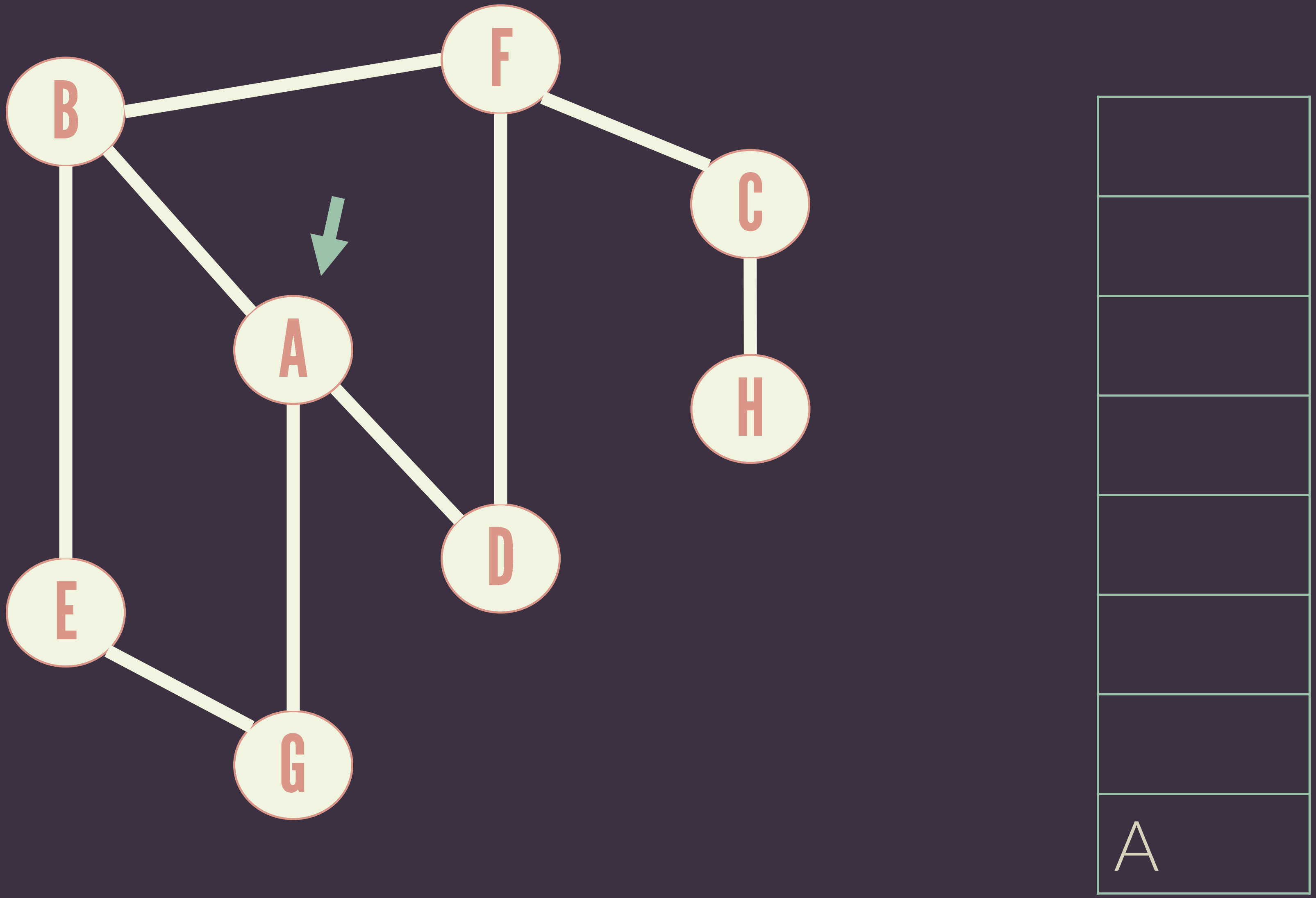
Result: A B E G F C H D



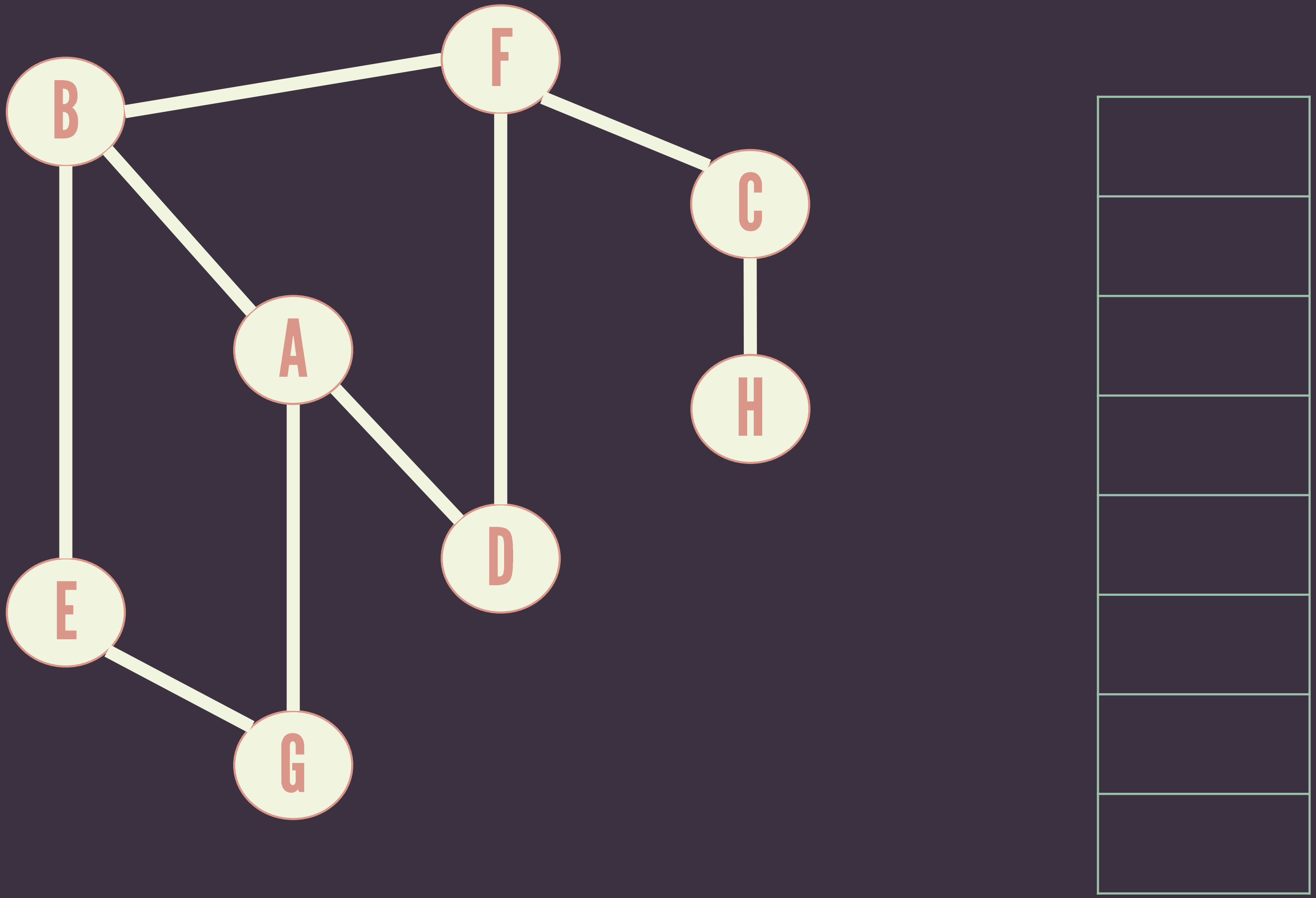
Result: A B E G F C H D



Result: A B E G F C H D



Result: A B E G F C H D



Result: A B E G F C H D

DFS
applications

Detecting cycle in a graph.

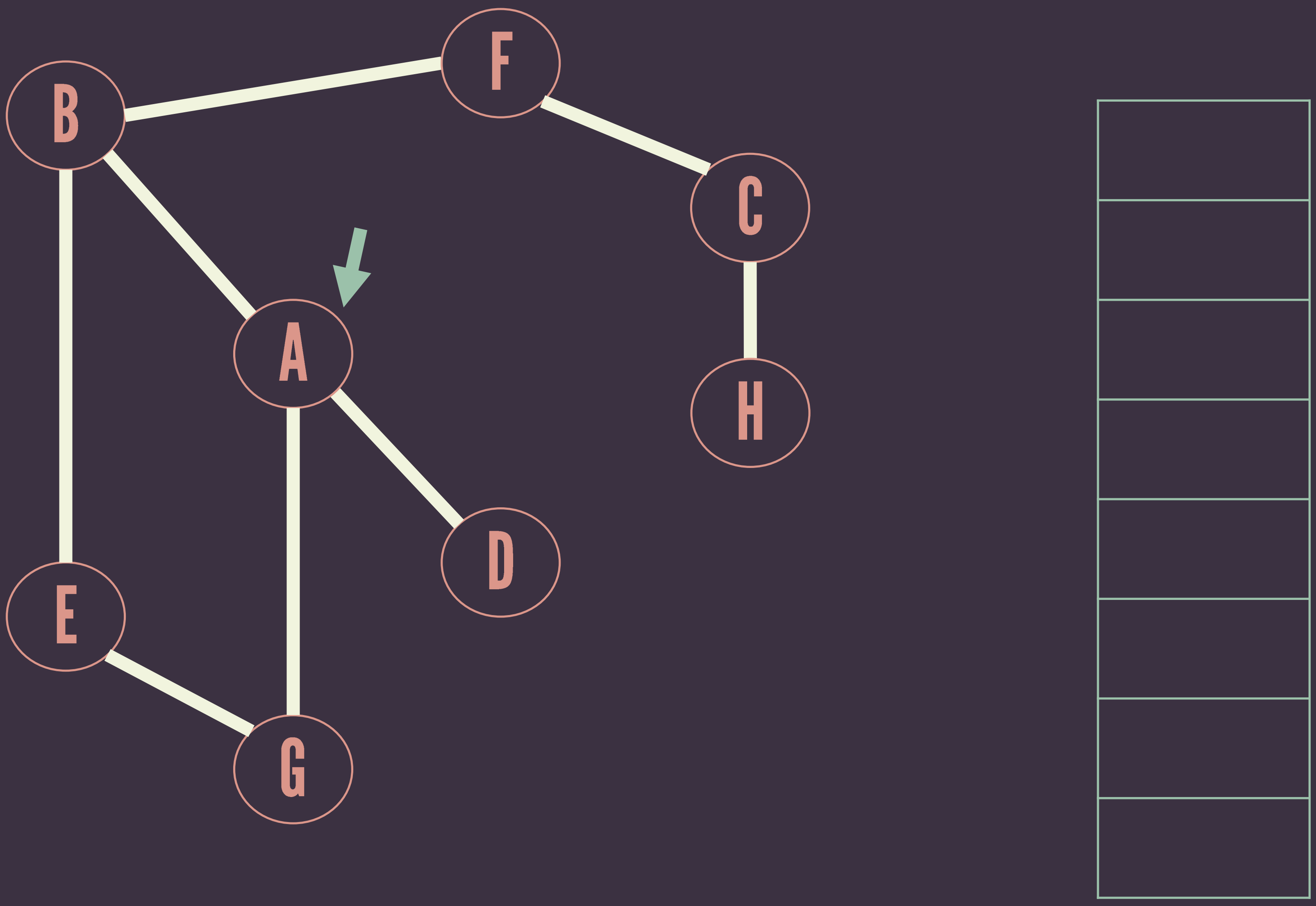
Topological sorting.

Path finding.

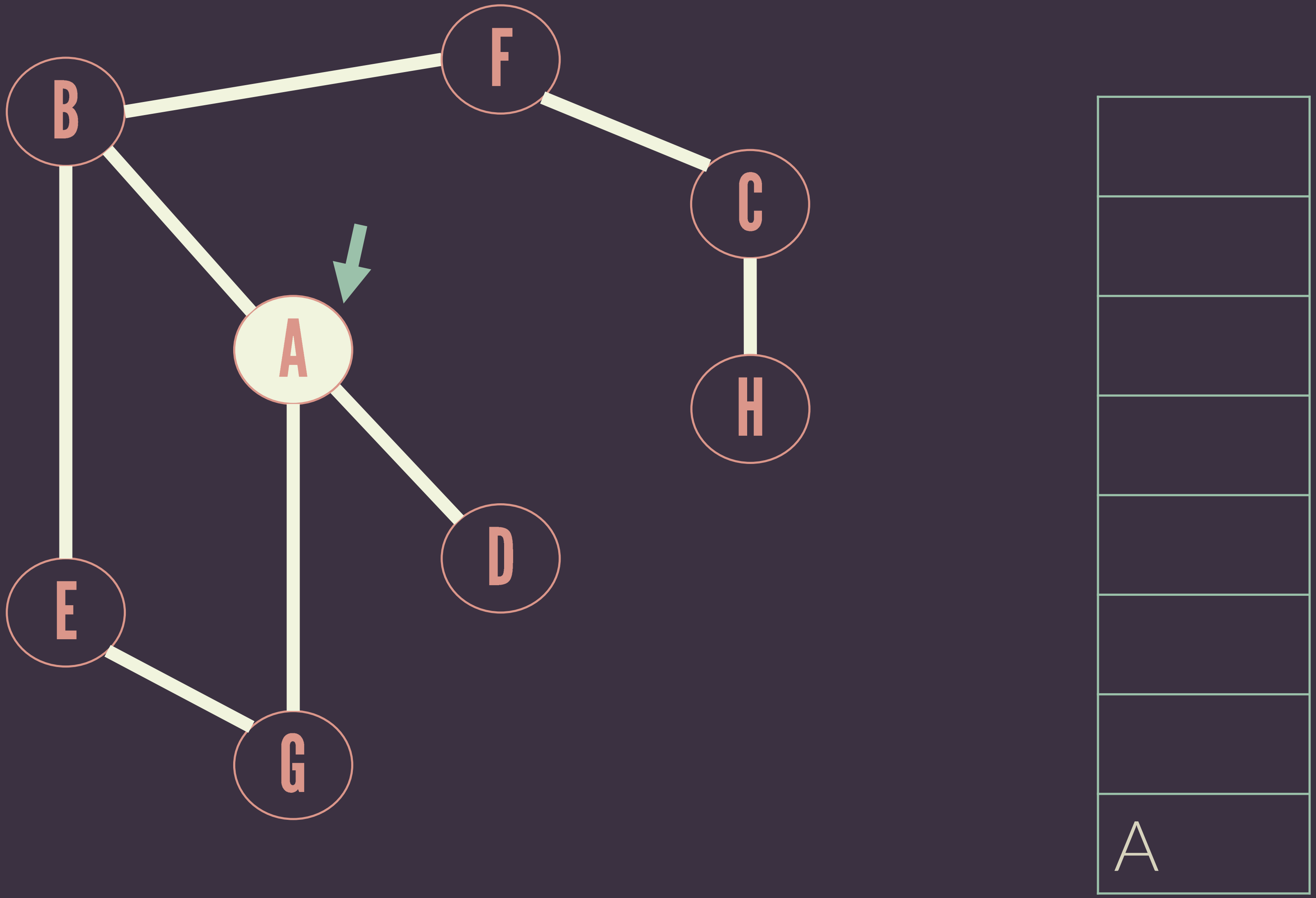
Spanning trees.

DFS
detecting cycle in a graph

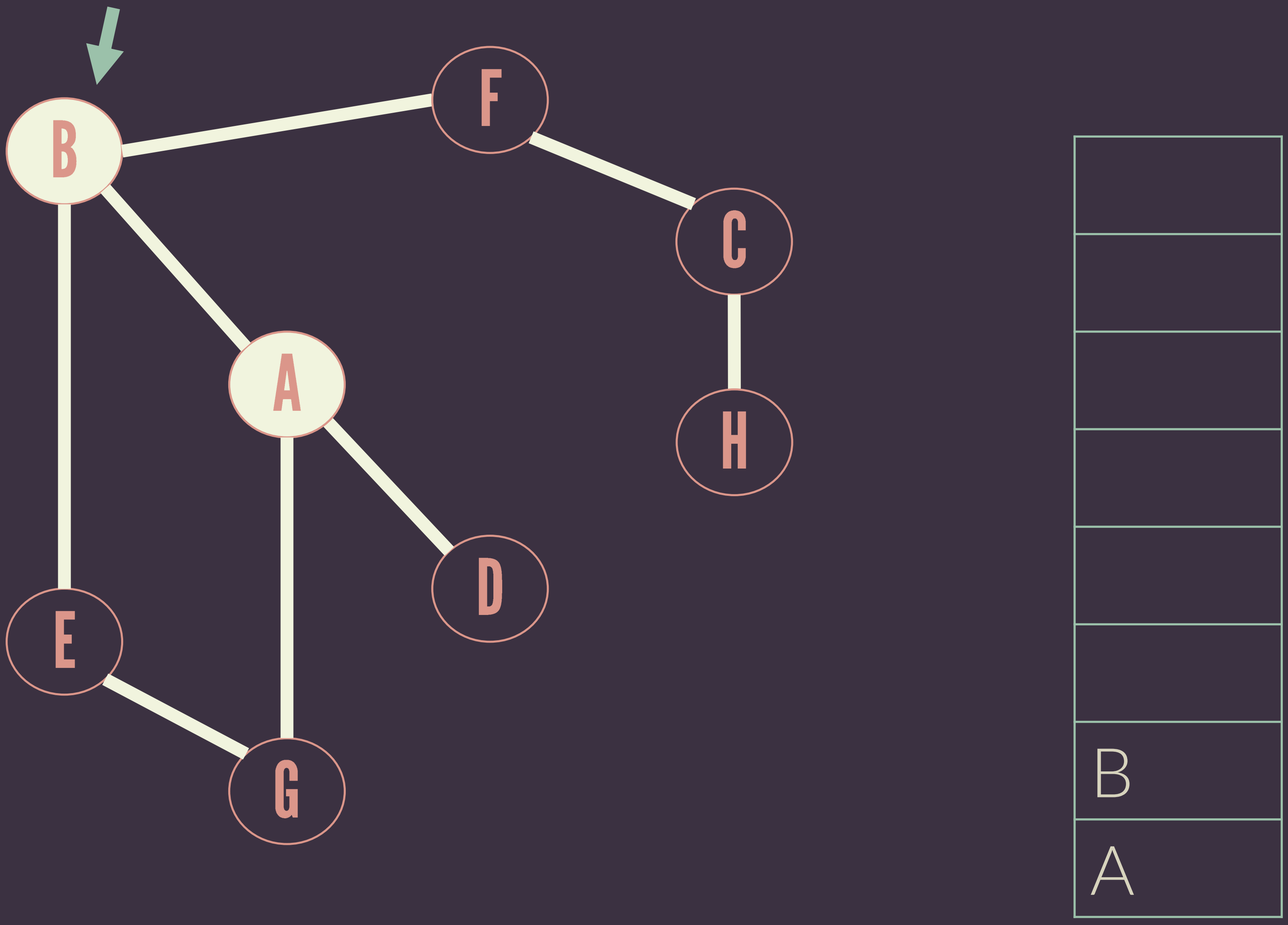
A graph has a cycle if and only if we can find a back edge during DFS.



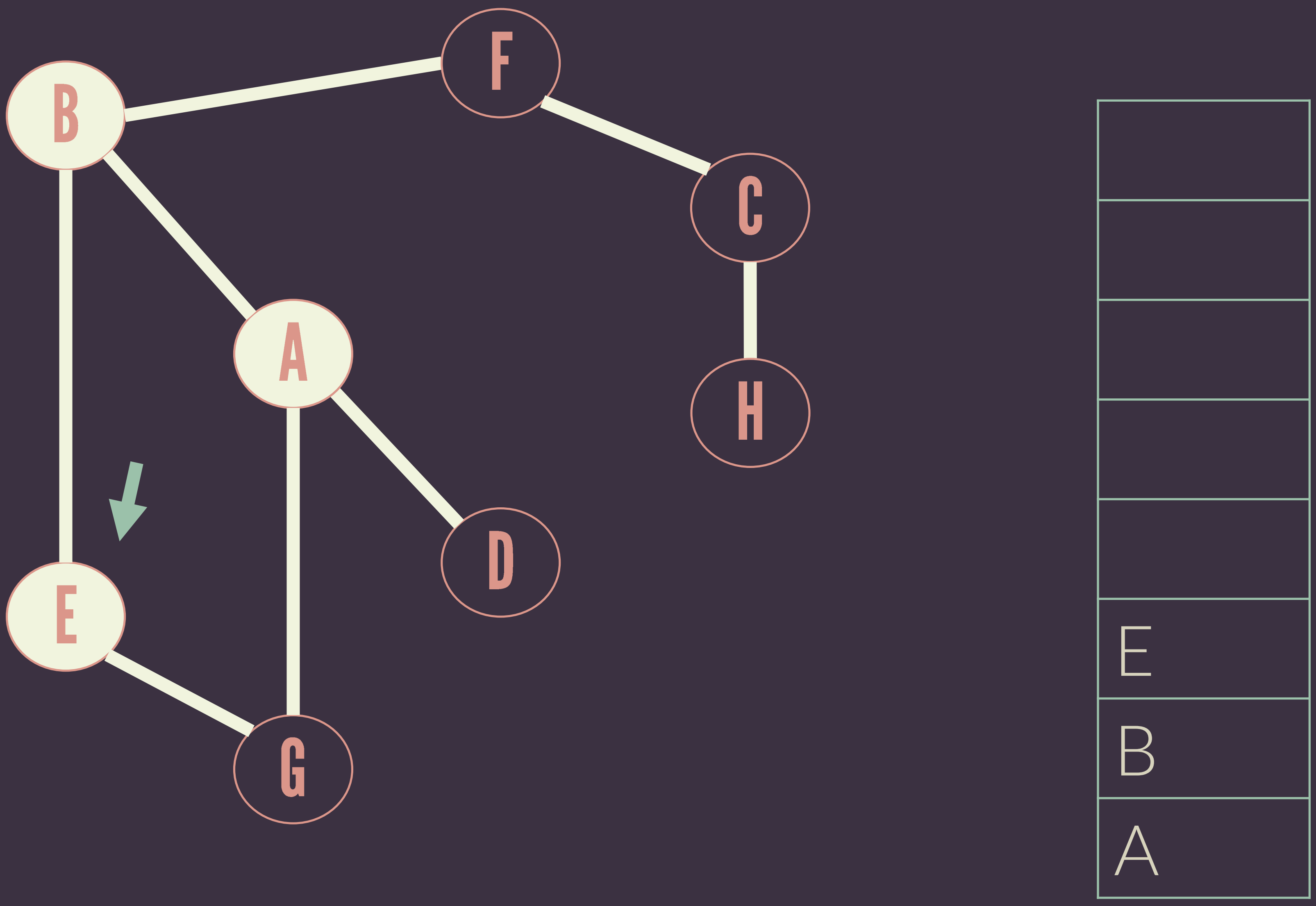
Result/Visited:



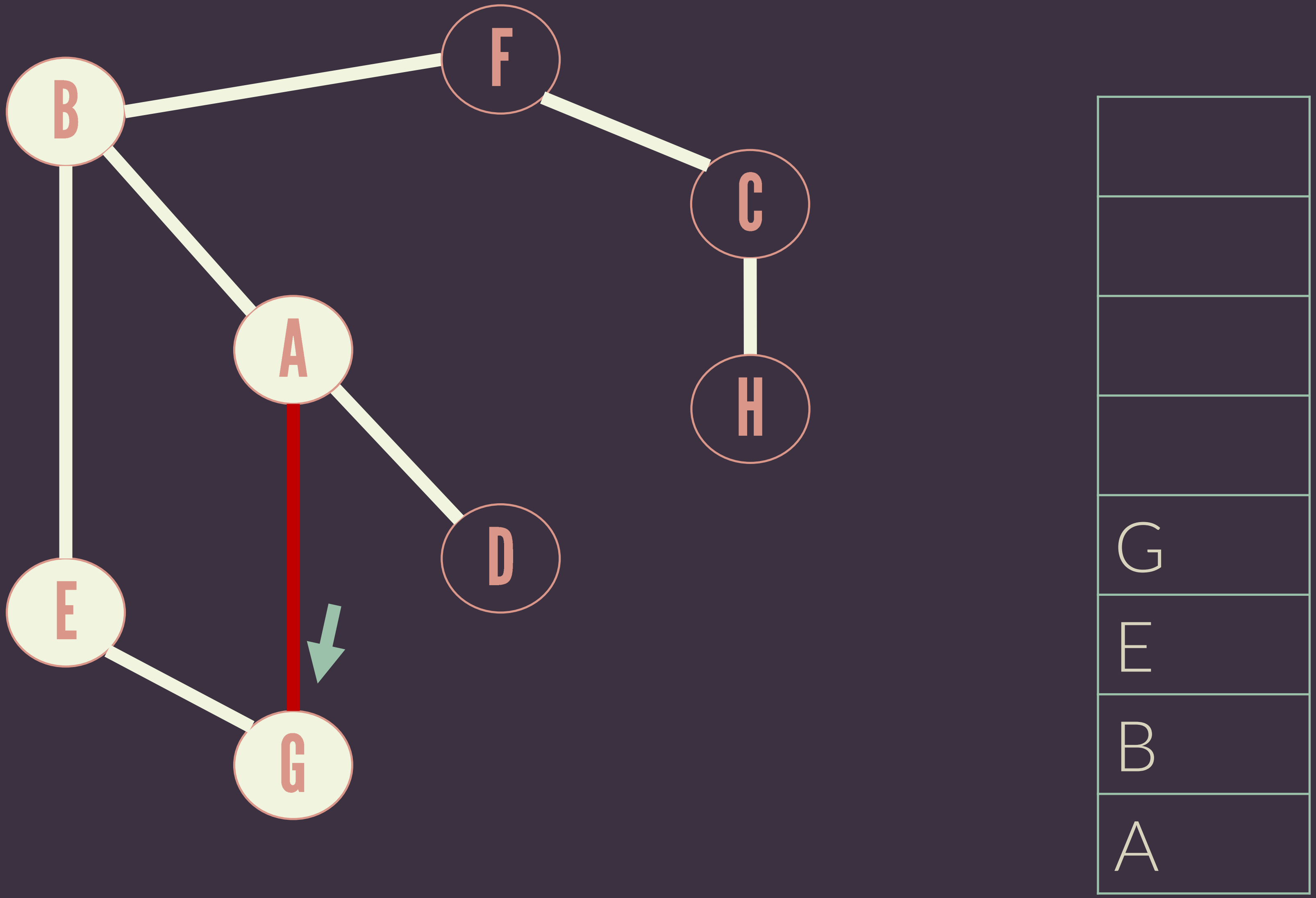
Result/Visited: A



Result/Visited: A B



Result/Visited: A B E

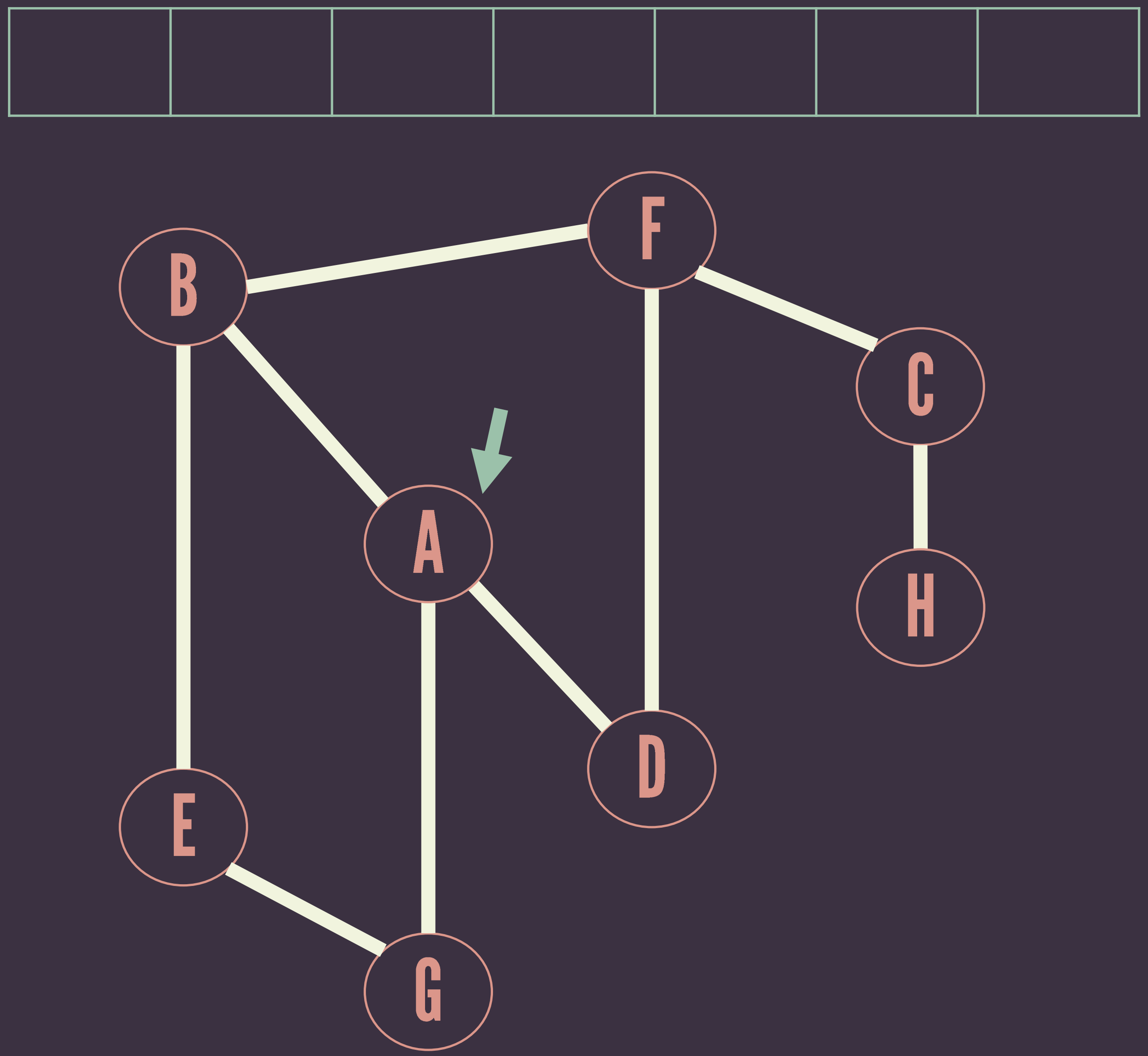


Result/Visited: A B E G

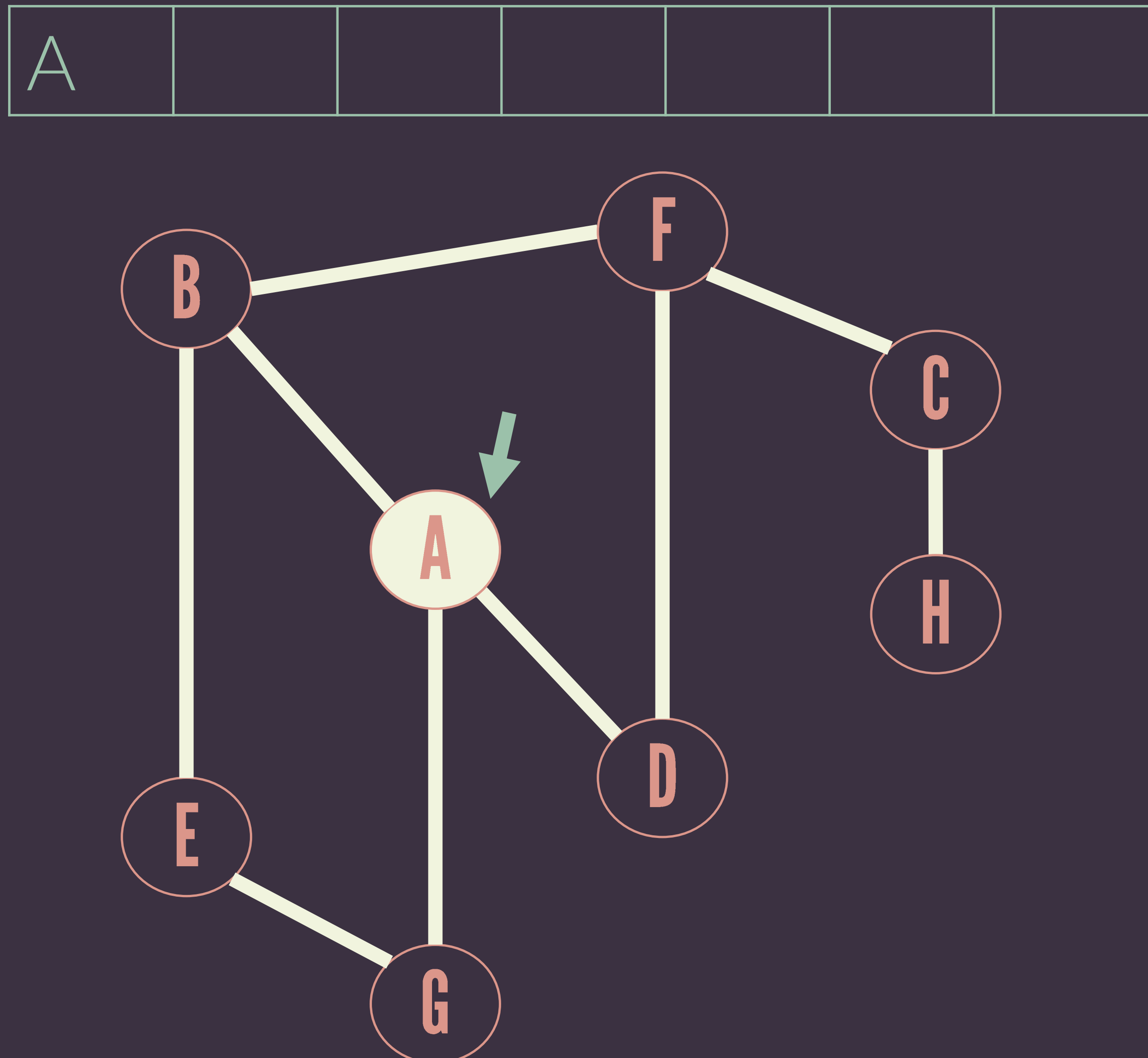
BREADTH FIRST search

Generalization of level-order traversal of trees.

```
void BFS( vertex v, graph G ){
  queue Q;
  enqueue(v,Q);
  while queue Q is not empty{
    v = dequeue(Q);
    if ( !visited[v] ){
      print v;
      visited[v] = TRUE;
      for each w adjacent to v:
        if( !visited[w] )
          enqueue(w, Q);
    }
  }
}
```

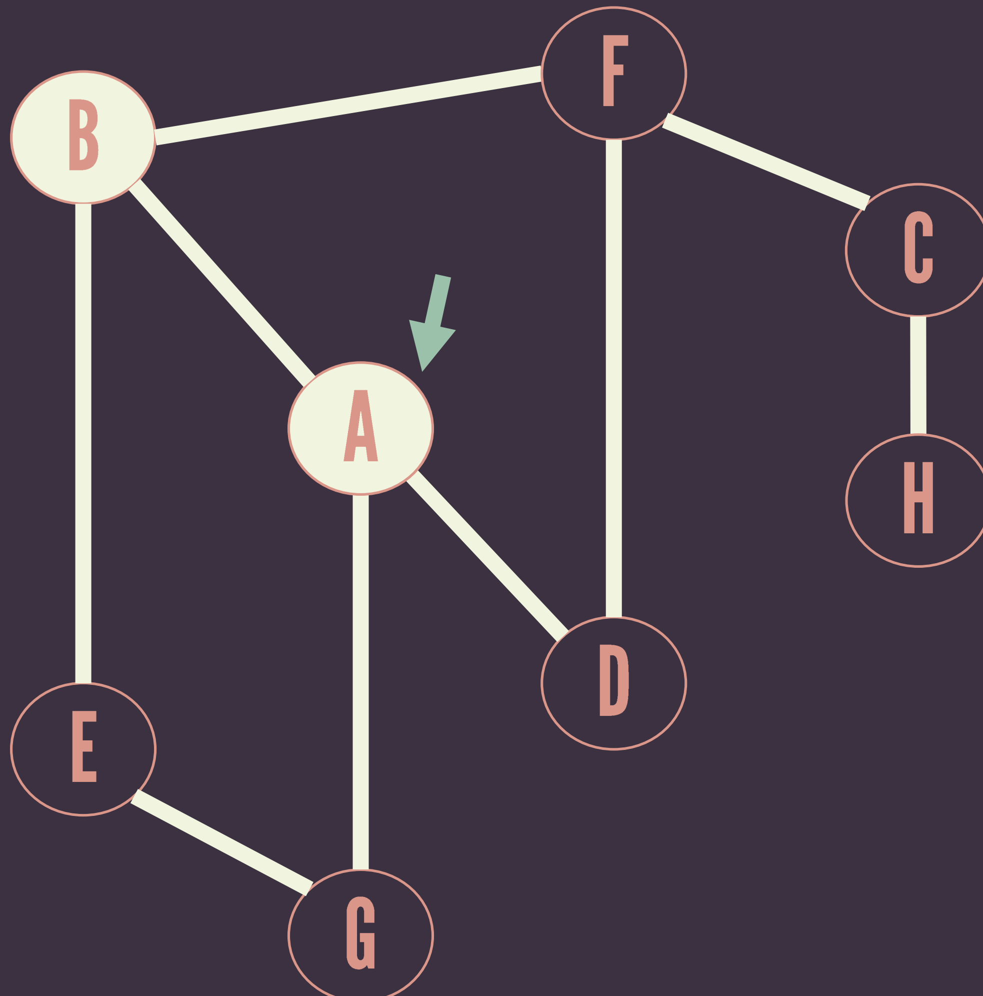


Result: A



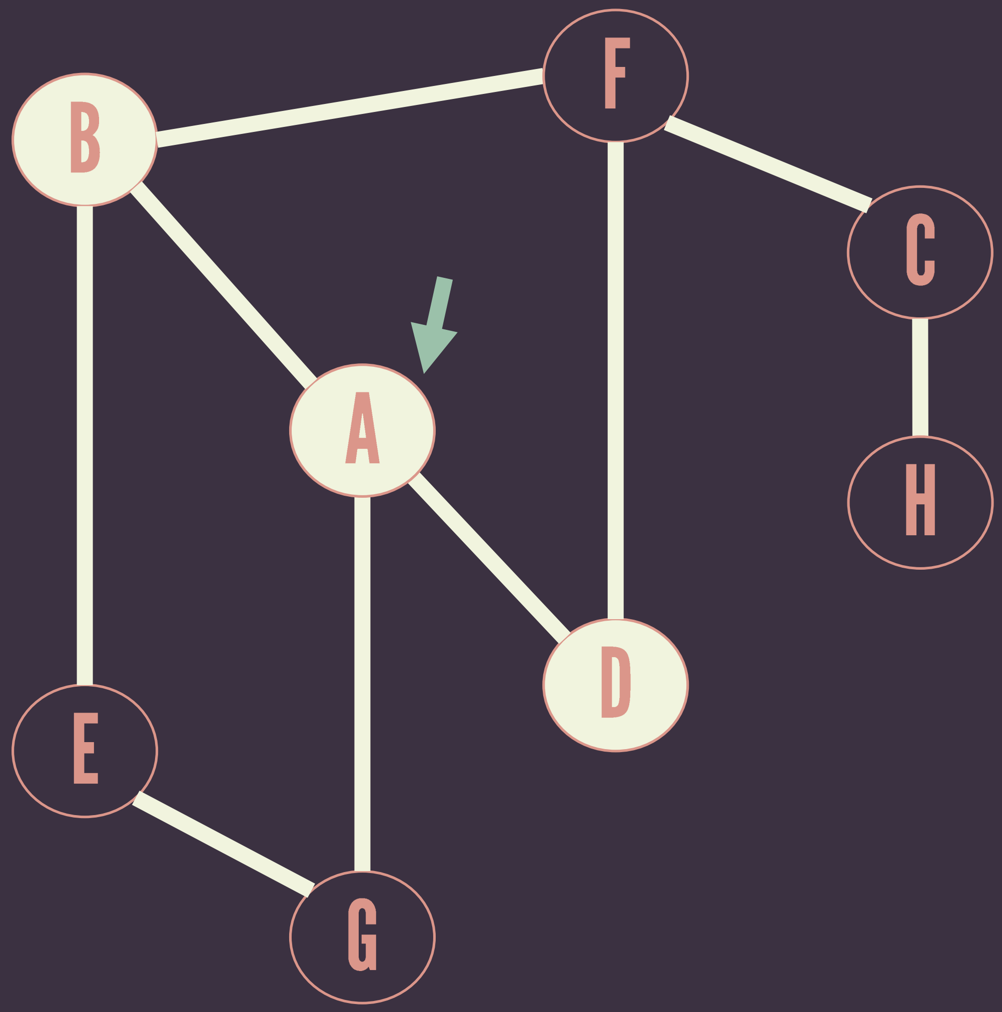
Result: A

A	B					



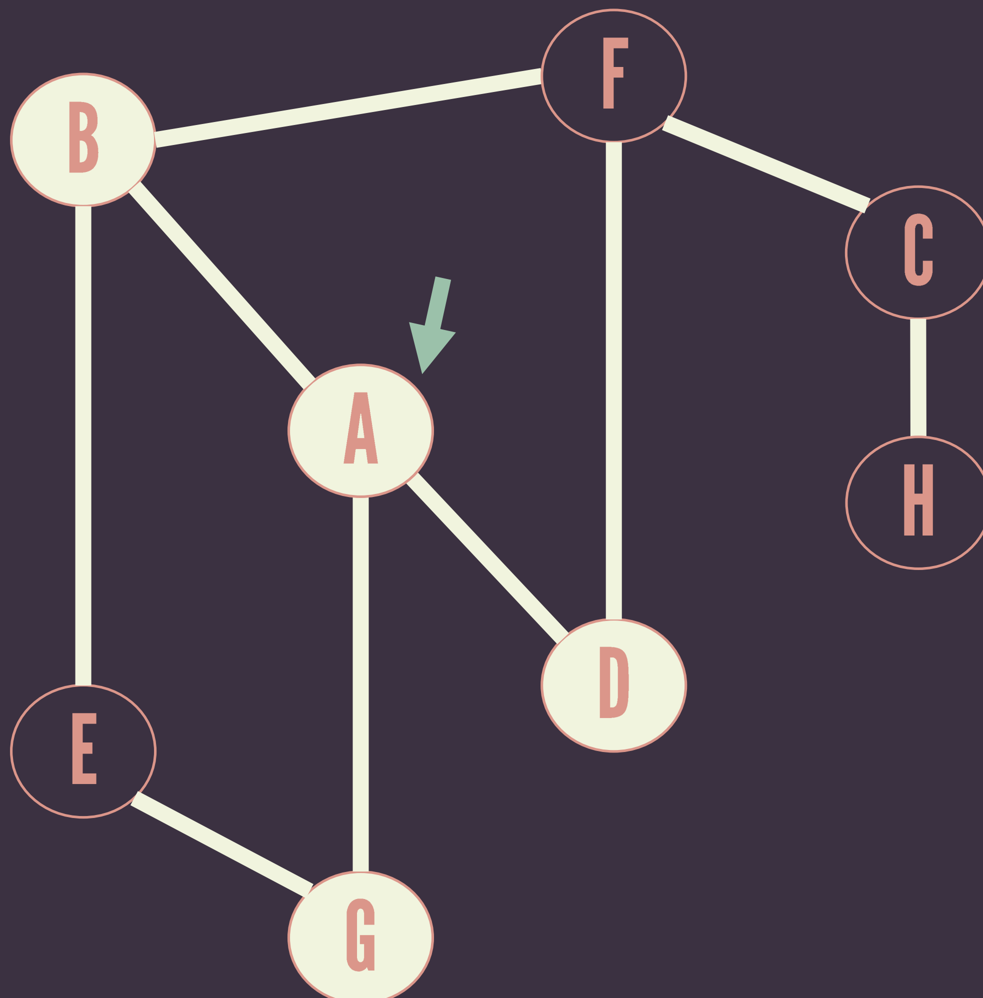
Result: A B

A	B	D				

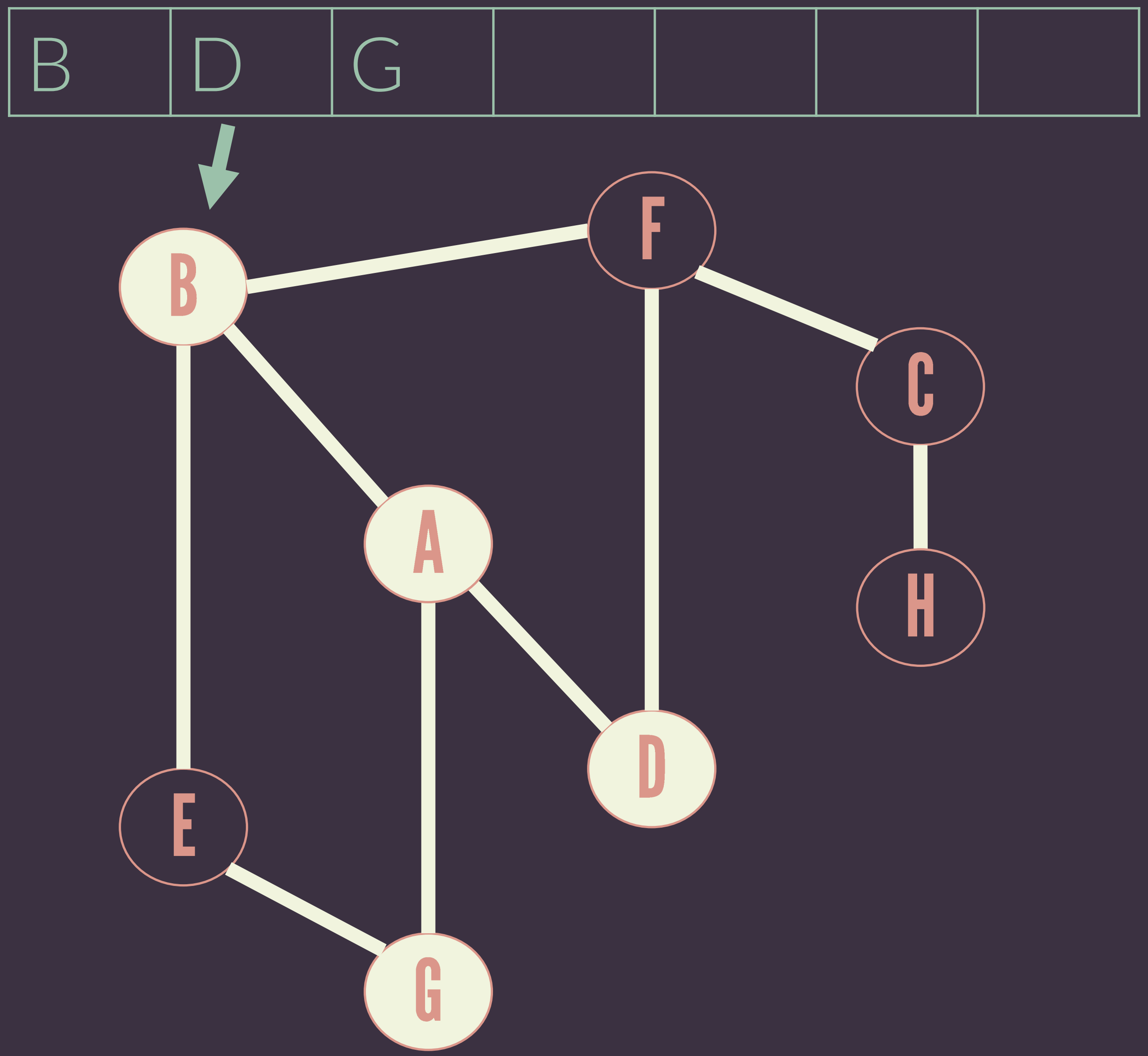


Result: A B D

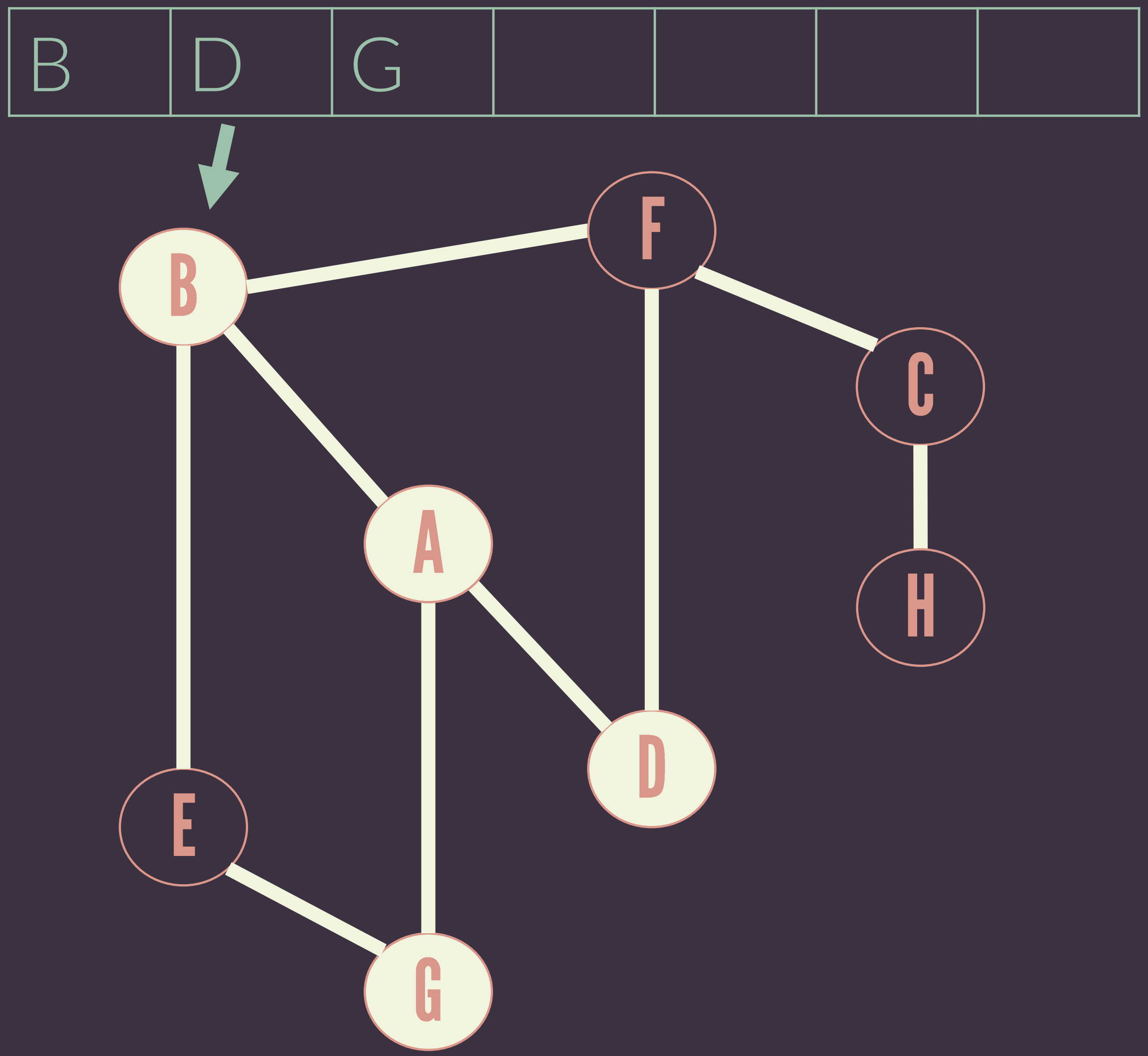
A	B	D	G			



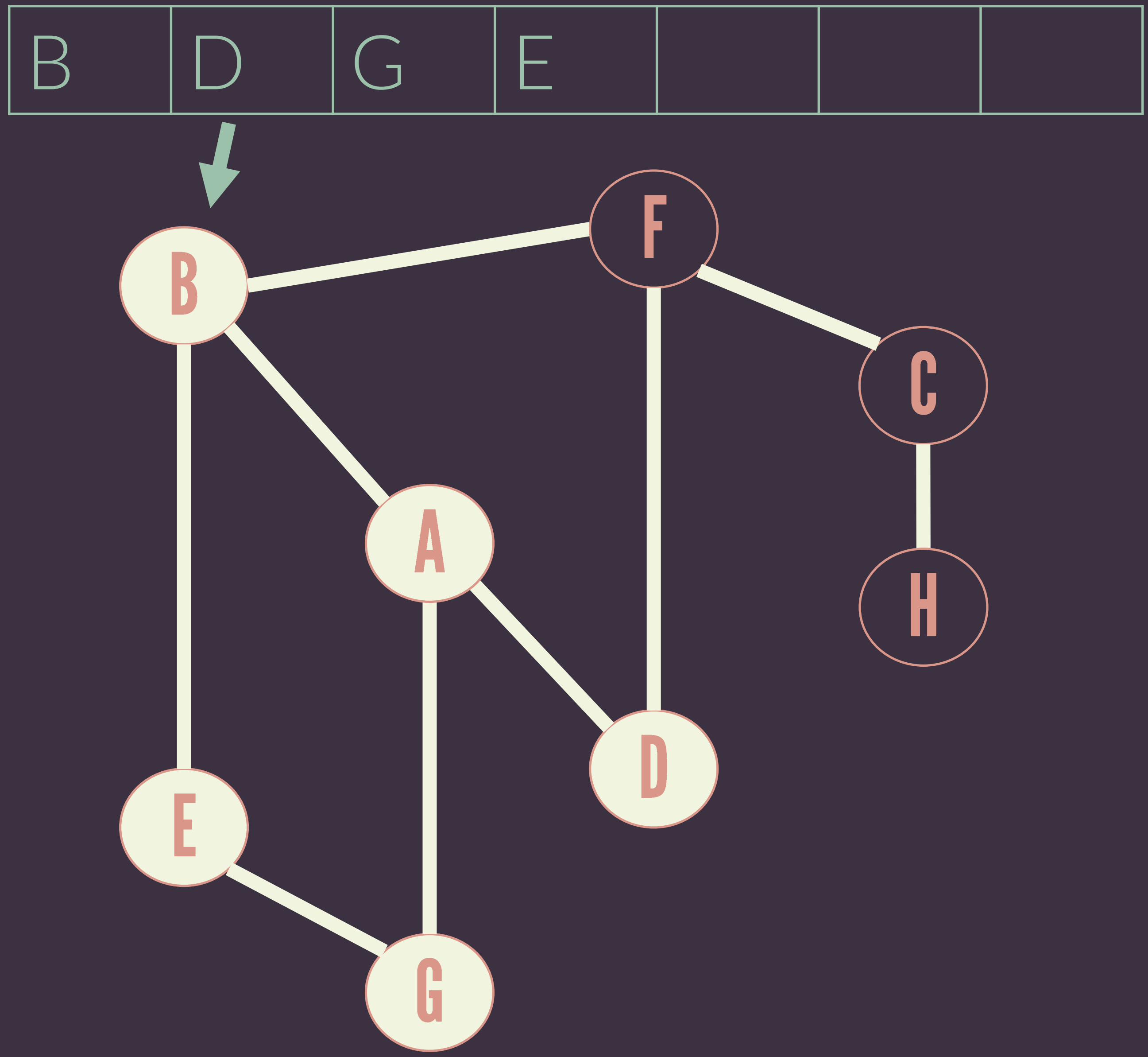
Result: A B D G



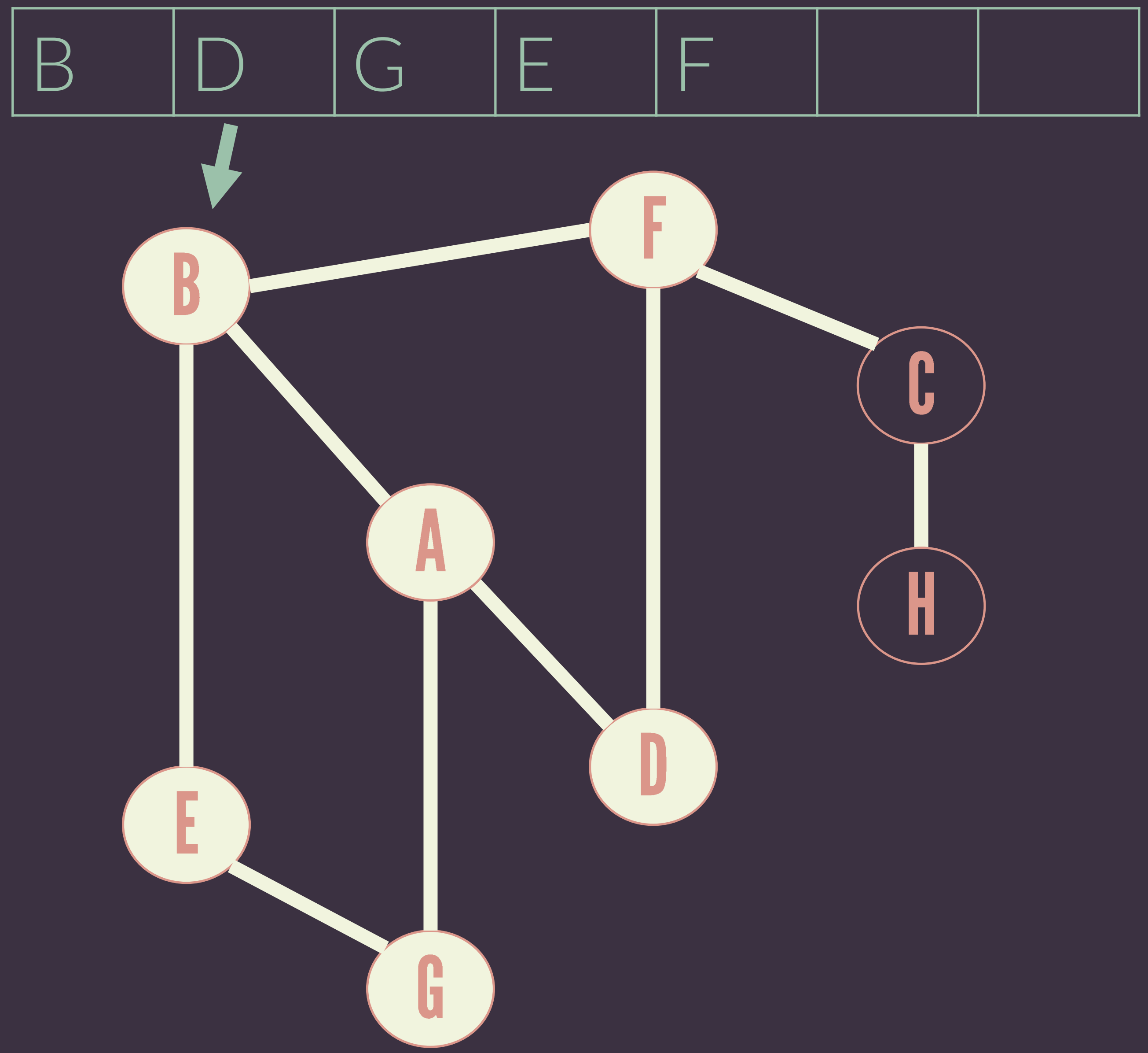
Result: A B D G



Result: A B D G

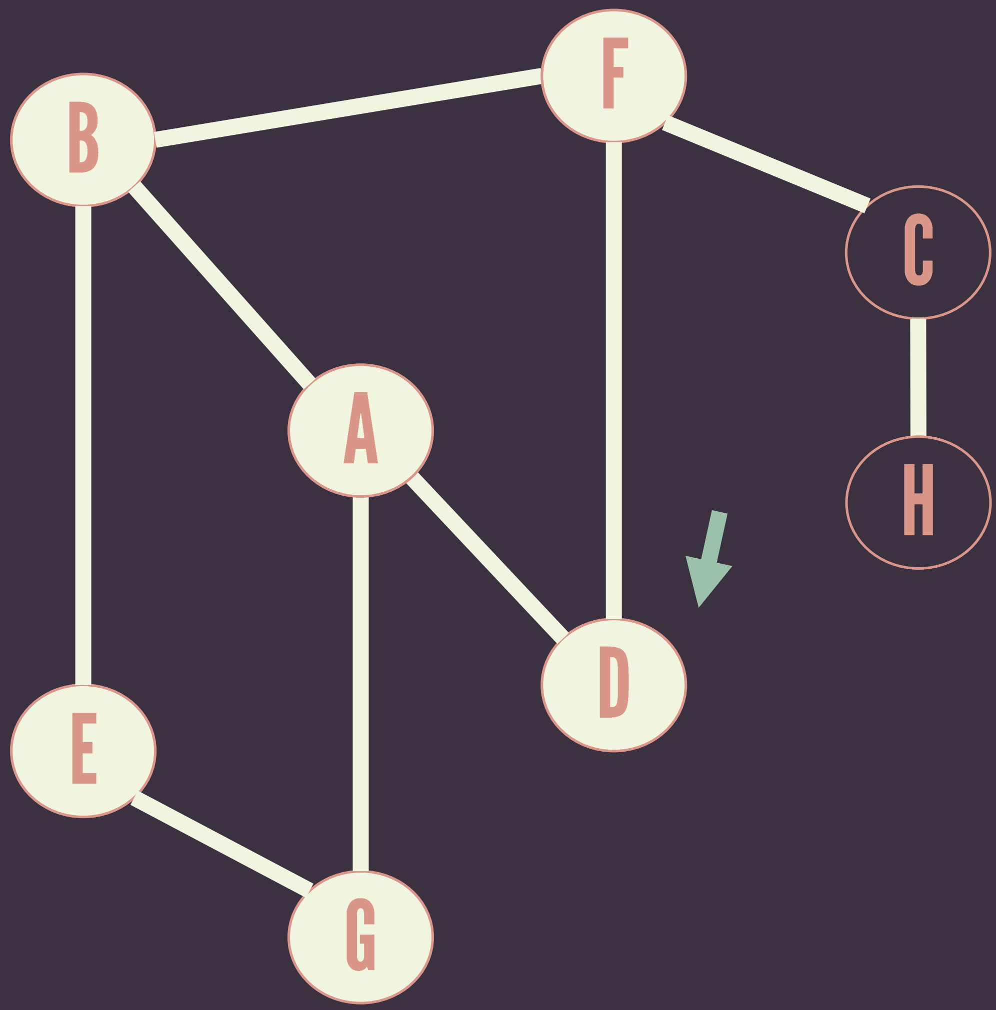


Result: A B D G E



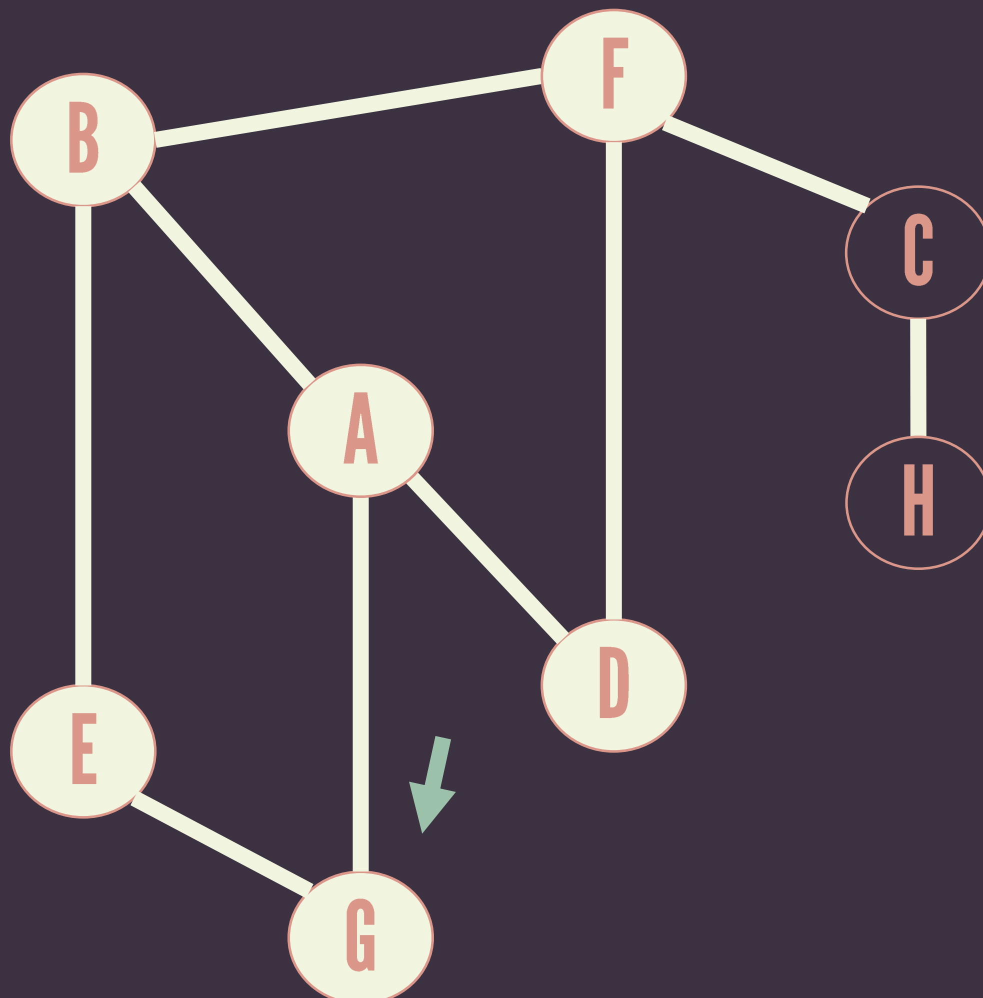
Result: A B D G E F

D	G	E	F			

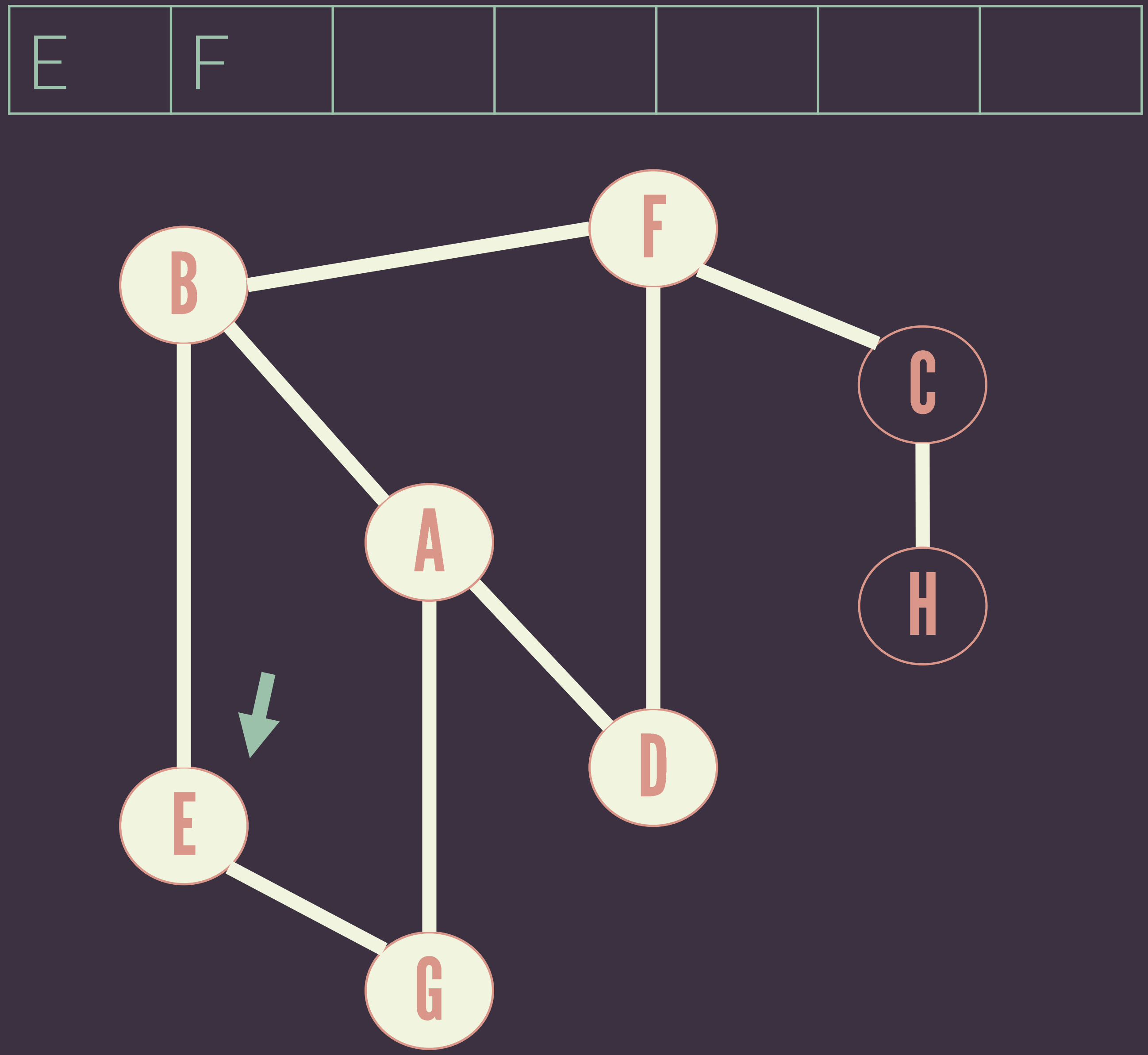


Result: A B D G E F

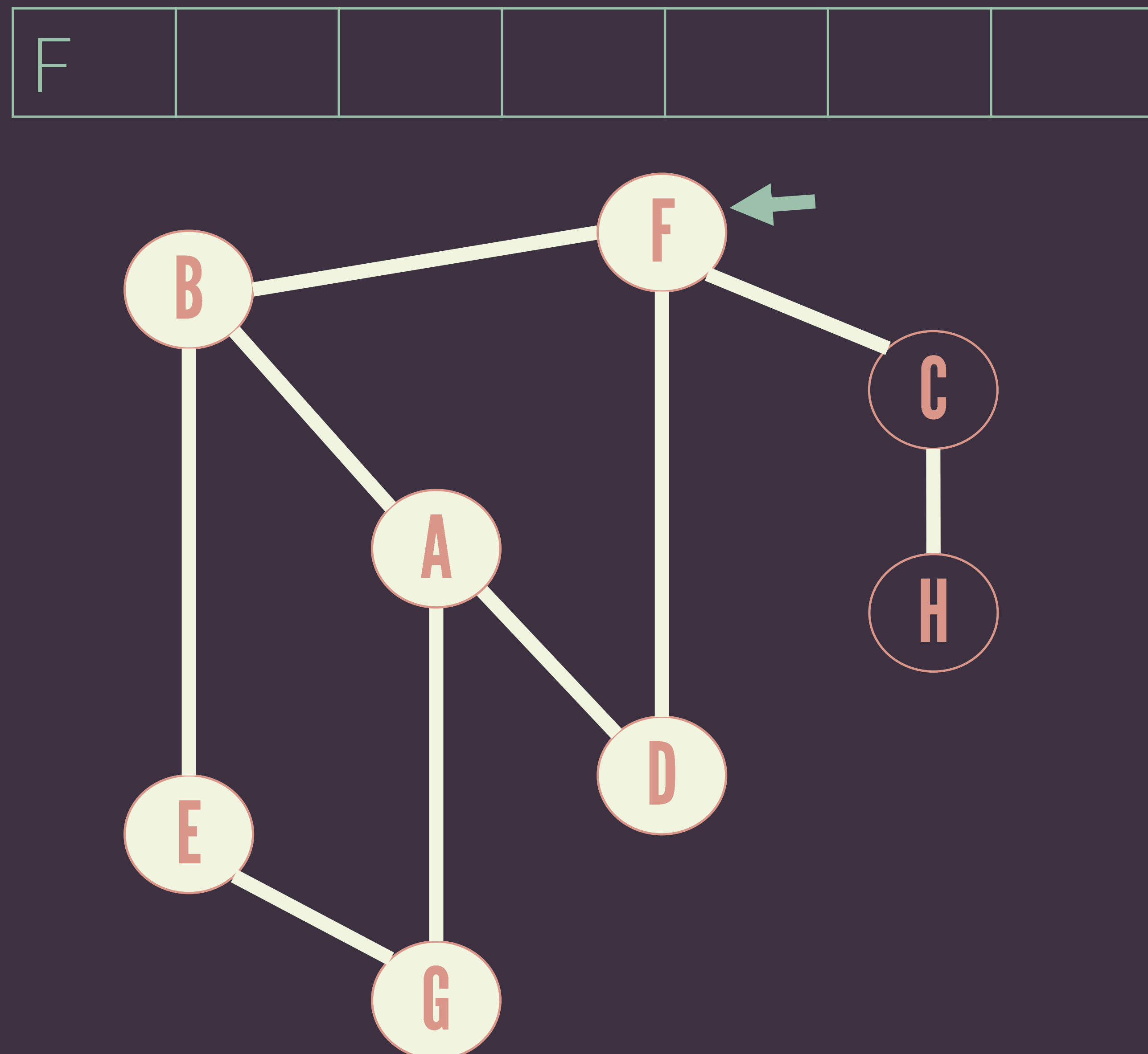
G	E	F				



Result: A B D G E F

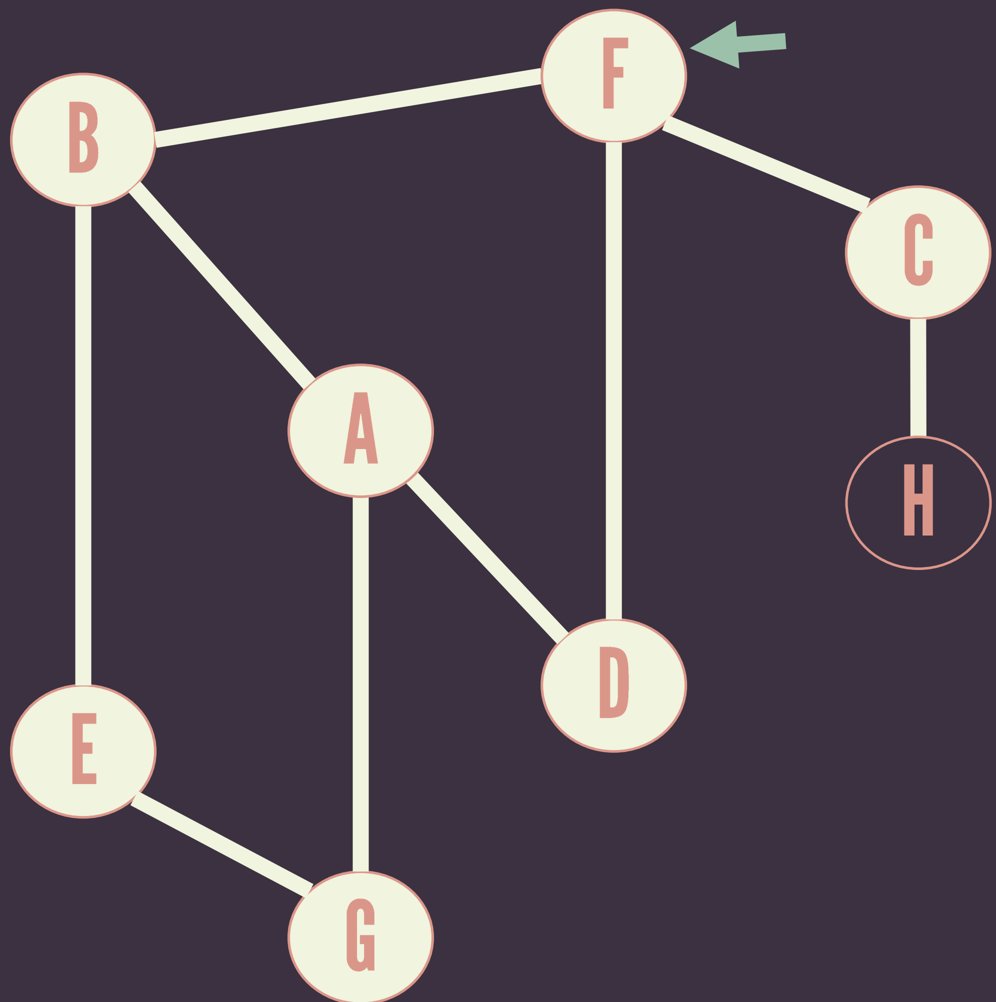


Result: A B D G E F

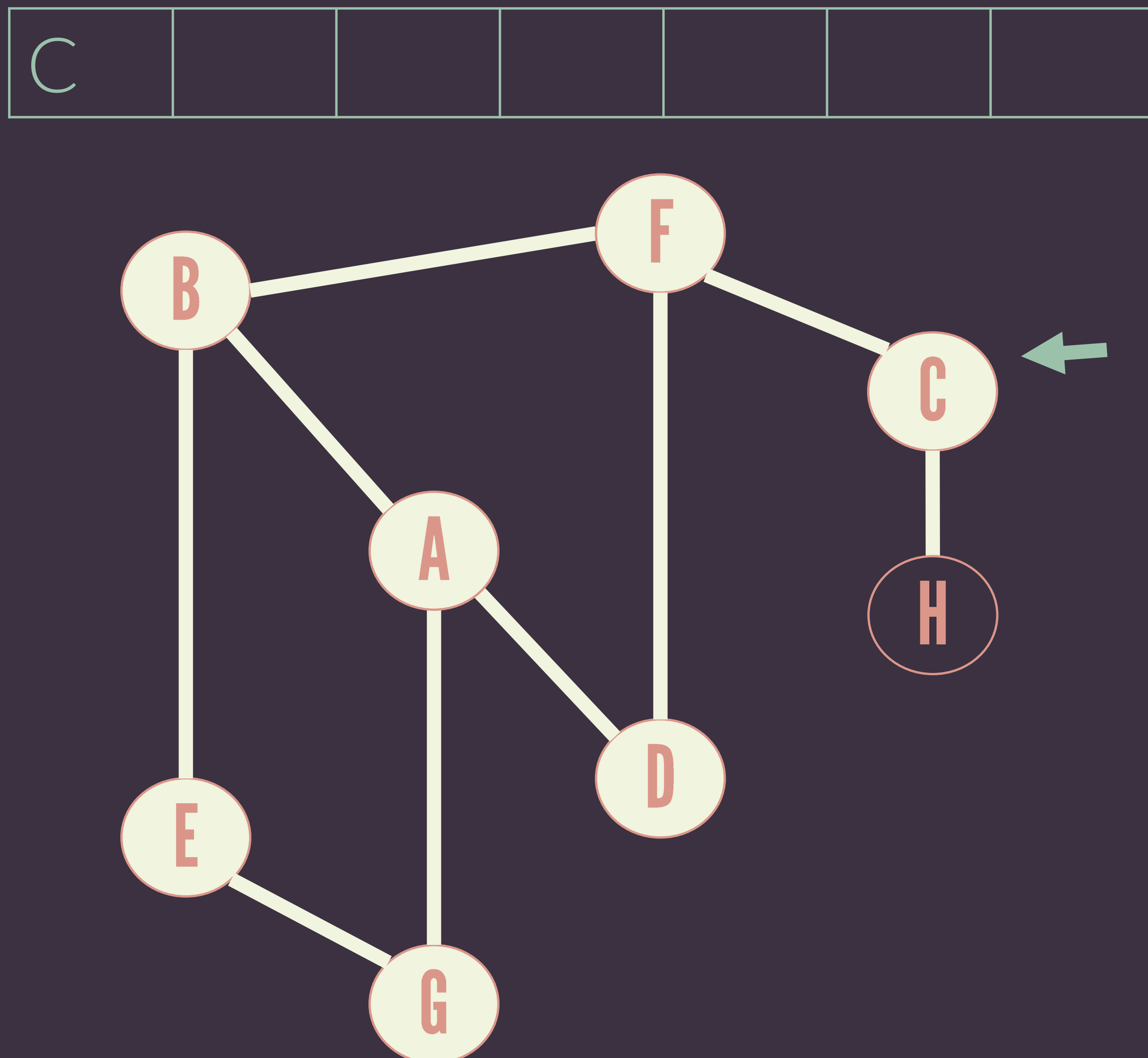


Result: A B D G E F

F	C					

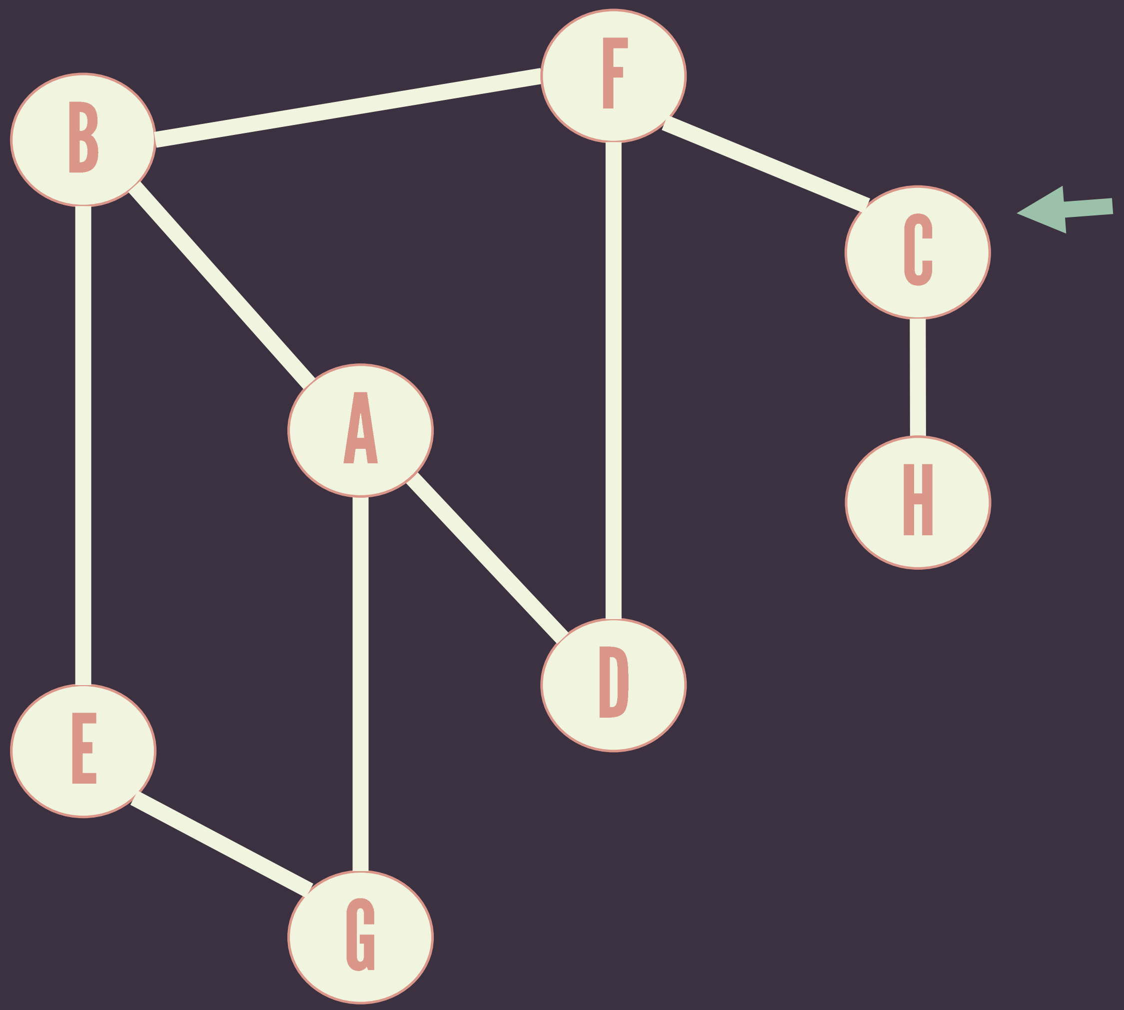


Result: A B D G E F C

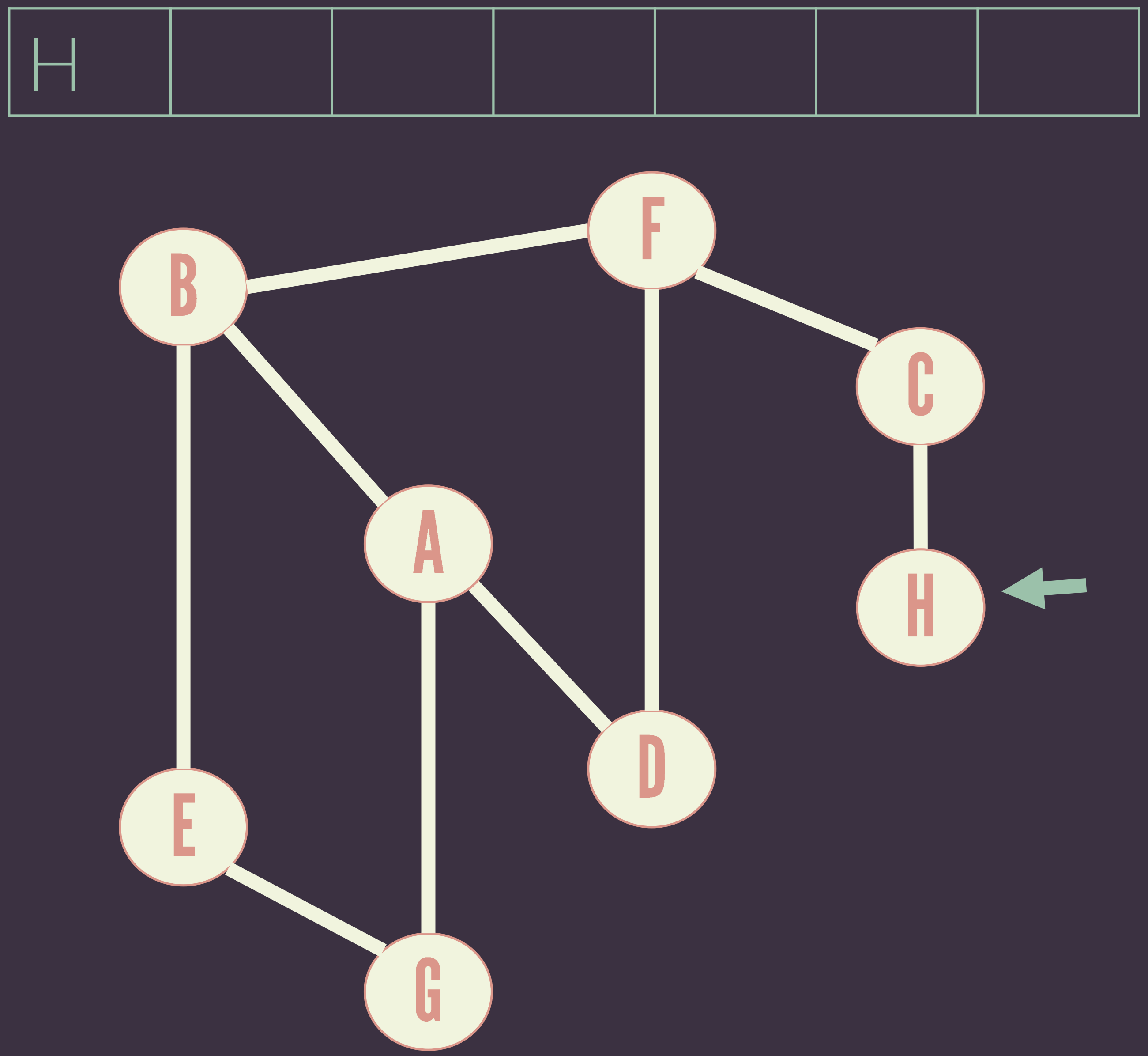


Result: A B D G E F C

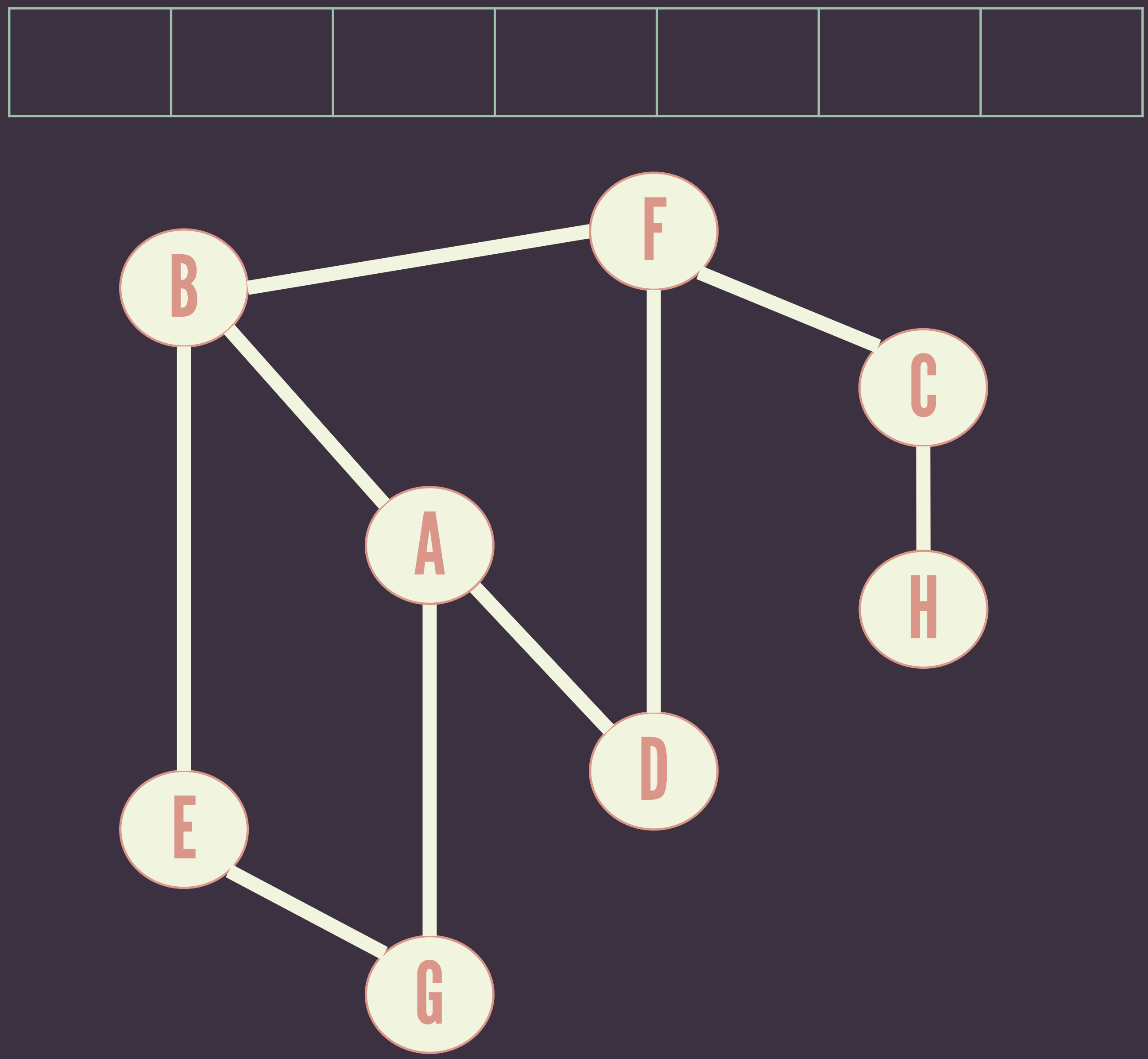
C	H					



Result: A B D G E F C H



Result: A B D G E F C H



Result: A B D G E F C H

BFS applications

Finding nodes within one. connected components.

Testing for bipartiteness.

Finding shortest paths.

minimum

SPANNING TREES

minimum
SPANNING TREE

A tree formed from graph edges that connects all the vertices of the graph at lowest cost.

ALGORITHMS

Prim's Algorithm
Kruskal's Algorithm