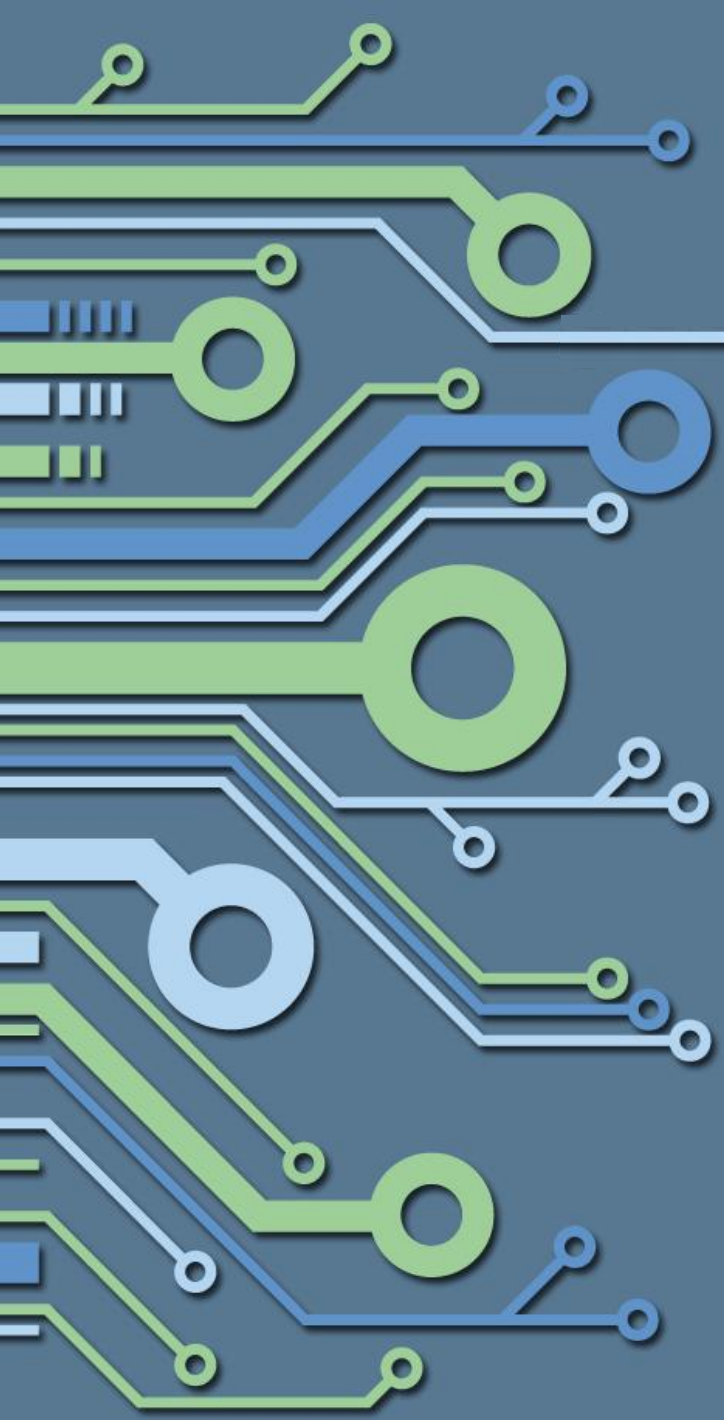


Chapter 3

BOOLEAN ALGEBRA, LOGIC FUNCTIONS and LOGIC GATES

Binary Variables

- Forms of variables
  - normal (x)
  - complement (x')
- Forms of terms (variables x and y)
  - Minterms m_i (or standard product)

    $x'y'$, $x'y$, xy', xy
  - Maxterms M_i (or standard sum)

    $x+y$, $x+y'$, $x'+y$, $x'+y'$

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0		m0		M0
0	0	1		m1		M1
0	1	0		m2		M2
0	1	1		m3		M3
1	0	0		m4		M4
1	0	1		m5		M5
1	1	0		m6		M6
1	1	1		m7		M7

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0	x'y'z'	m0		M0
0	0	1	x'y'z	m1		M1
0	1	0	x'yz'	m2		M2
0	1	1	x'yz	m3		M3
1	0	0	xy'z'	m4		M4
1	0	1	xy'z	m5		M5
1	1	0	xyz'	m6		M6
1	1	1	xyz	m7		M7

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0	x'y'z'	m0	x+y+z	M0
0	0	1	x'y'z	m1	x+y+z'	M1
0	1	0	x'yz'	m2	x+y'+z	M2
0	1	1	x'yz	m3	x+y'+z'	M3
1	0	0	xy'z'	m4	x'+y+z	M4
1	0	1	xy'z	m5	x'+y+z'	M5
1	1	0	xyz'	m6	x'+y'+z	M6
1	1	1	xyz	m7	x'+y'+z'	M7

Forms of Boolean Functions

- Canonical Form
  - Sum of minterms
  - Product of maxterms
- Standard Form
  - Sum of products
  - Product of sums

Forms of Boolean Functions

Examples

- F(a,b,c) = abc' + a’bc
- F(w,x,y,z) = (w+x'+y'+z)(x+y+z')
- F(x,y,z) = xz'+y
- F(a,b,c,d) = (a+b‘+c+d)(a+b+c’+d’)

Sum of Minterms

- Any Boolean function can be expressed as a sum of minterms
- "sum" means ORing the minterms that produces a 1 in the function
- Each minterm is obtained from an AND term of the *n* variables, with each variable being primed if the corresponding bit of the binary number is a 0 and unprimed if a 1.

Example – Truth Table

Express F(A,B,C) = A + B'C in sum of minterms

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Example – Truth Table

Express F(A,B,C) = A + B'C in sum of minterms

= A'B'C + AB'C' + AB'C + ABC'

+ ABC

= Σ (1, 4, 5, 6, 7)

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Example – Algebraic Manipulation

Express F(A,B,C) = A + B'C in sum of minterms

Example – Algebraic Manipulation

Express F(A,B,C) = A + B'C in sum of minterms

F = A(B+B') + B‘C

Example – Algebraic Manipulation

Express F(A,B,C) = A + B'C in sum of minterms

F = A(B+B') + B'C

= AB + AB' + B‘C

Example – Algebraic Manipulation

Express F(A,B,C) = A + B'C in sum of minterms

F = A(B+B') + B'C

= AB + AB' + B'C

= AB(C+C') + AB'(C+C') + B'C(A+A‘)

Example – Algebraic Manipulation

Express F(A,B,C) = A + B'C in sum of minterms

F = A(B+B') + B'C

= AB + AB' + B'C

= AB(C+C') + AB'(C+C') + B'C(A+A')

= ABC + ABC' + AB'C + AB'C' + AB'C + A'B‘C

Example – Algebraic Manipulation

Express F(A,B,C) = A + B'C in sum of minterms

F = A(B+B') + B'C

= AB + AB' + B'C

= AB(C+C') + AB'(C+C') + B'C(A+A')

= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C

= A'B'C + AB'C' + AB'C + ABC' + ABC

Example – Algebraic Manipulation

Express F(A,B,C) = A + B'C in sum of minterms

F = A(B+B') + B'C

= AB + AB' + B'C

= AB(C+C') + AB'(C+C') + B'C(A+A')

= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C

= A'B'C + AB'C' + AB'C + ABC' + ABC

= $m_1 + m_4 + m_5 + m_6 + m_7$

= $\Sigma(1, 4, 5, 6, 7)$

Product of Maxterms

- Any Boolean function can be expressed as a product of maxterms
- "product" means ANDing the maxterms that produces a 0 in the function
- Each maxterm is obtained from an OR term of the *n* variables, with each variable being primed if the corresponding bit of the binary number is a 1 and unprimed if a 0.

Example – Truth Table

Express F(A,B,C) = A + B'C in product of maxterms

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Example – Truth Table

Express F(A,B,C) = A + B'C in product of maxterms

= (A+B+C)(A+B'+C) (A+B'+C')

= Π(0,2,3)

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Example – Algebraic Manipulation

Express F = A + B'C in product of maxterms

Example – Algebraic Manipulation

Express F = A + B'C in product of maxterms

F = (A+B')(A+C)

Example – Algebraic Manipulation

Express F = A + B'C in product of maxterms

F = (A+B')(A+C)

= (A+B'+CC')(A+C)

Example – Algebraic Manipulation

Express F = A + B'C in product of maxterms

F = (A+B')(A+C)

= (A+B'+CC')(A+C)

= (A+B'+C)(A+B'+C')(A+C+BB‘)

Example – Algebraic Manipulation

Express F = A + B'C in product of maxterms

F = (A+B')(A+C)

= (A+B'+CC')(A+C)

= (A+B'+C)(A+B'+C')(A+C+BB')

= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C)

Example – Algebraic Manipulation

Express F = A + B'C in product of maxterms

F = (A+B')(A+C)

= (A+B'+CC')(A+C)

= (A+B'+C)(A+B'+C')(A+C+BB')

= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C)

= (A+B+C)(A+B'+C)(A+B'+C‘)

Example – Algebraic Manipulation

Express F = A + B'C in product of maxterms

F = (A+B')(A+C)

= (A+B'+CC')(A+C)

= (A+B'+C)(A+B'+C')(A+C+BB')

= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C)

= (A+B+C)(A+B'+C)(A+B'+C')

= $M_0M_2M_3$

= Π(0,2,3)

Conversion between Canonical forms

- Consider the previous example: F = A+B'C

  $F(A,B,C) = \Sigma(1,4,5,6,7)$

  This has a complement that can be expressed as:

  $F'(A,B,C) = \Sigma(0,2,3) = m_0 + m_2 + m_3$

  Now, take the complement of F', we will obtain

  $F = (m_0+m_2+m_3)' = m_0'm_2'm_3' = M_0M_2M_3$

  $= \Pi(0,2,3)$

Digital Logic Gates

AND gate

OR gate

NOT gate

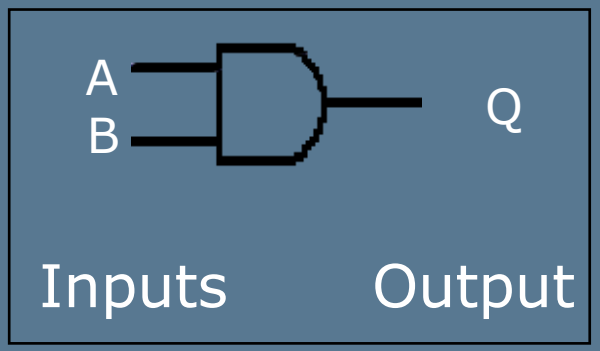
XOR gate

NOR gate

NAND gate

XNOR gate

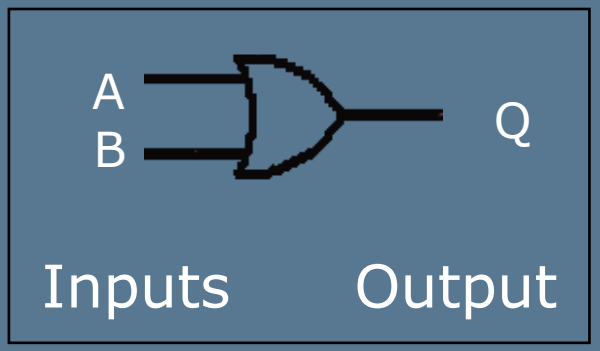
AND Gate



Truth Table

Inputs		Output
A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

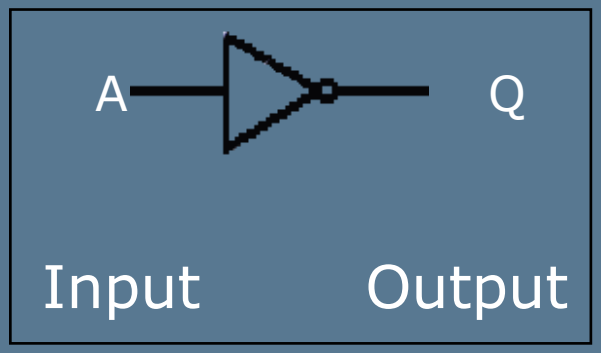
OR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

NOT Gate



Truth Table

Input	Output
A	Q
0	1
1	0

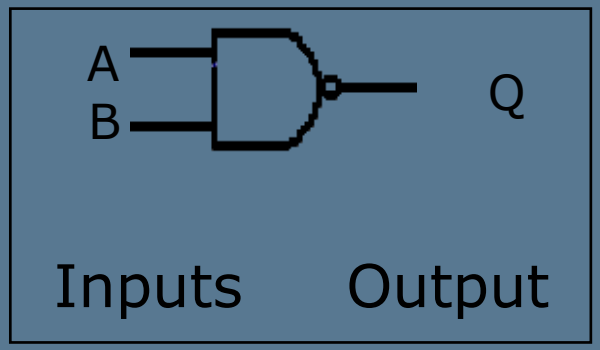
Other Gates

- Why use?
  - Saves cost
  - Saves space
  - Saves time

Other Gates

- Why use?
  - Saves cost
  - Saves space
  - Saves time
- Types
  - NAND gate
  - NOR gate
  - XOR gate
  - XNOR gate

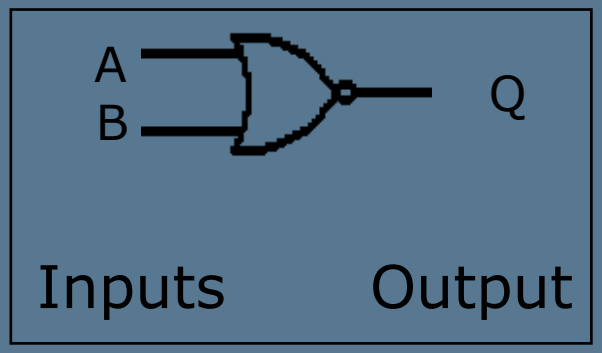
NAND Gate



Truth Table

Inputs		Output
A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

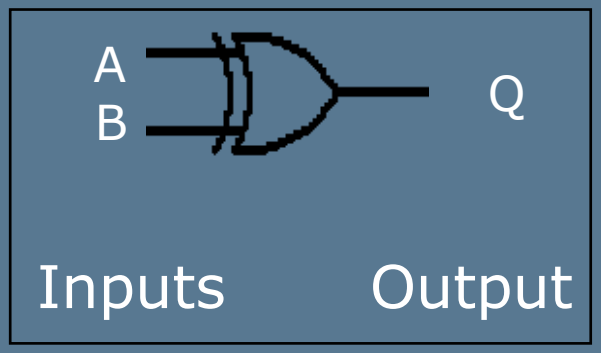
NOR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

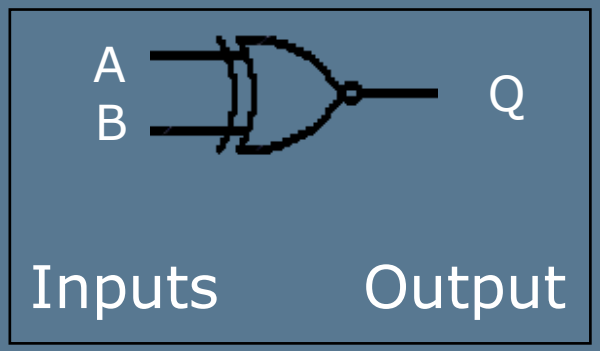
Exclusive-OR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

XNOR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	1

Digital Logic gates

- Draw the logic diagram of the function
  F = A+B'C

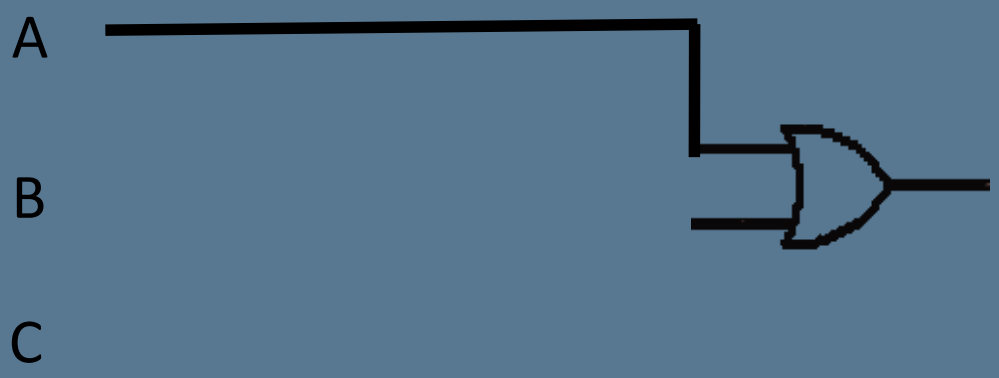
A

B

C

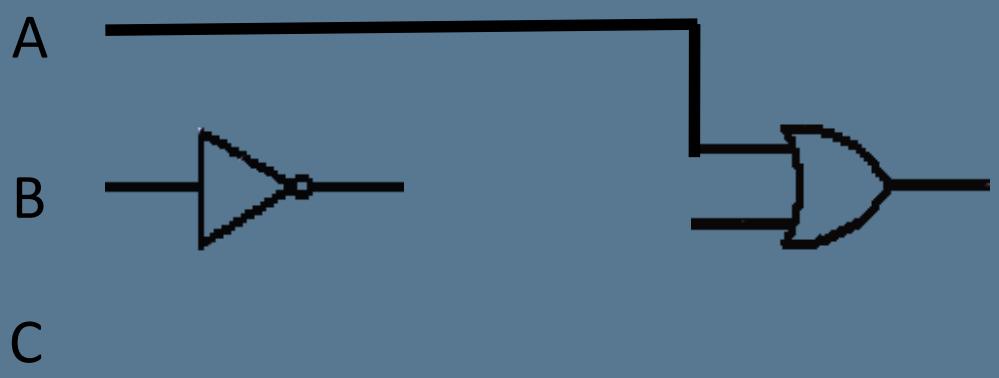
Digital Logic gates

- Draw the logic diagram of the function
  F = A+B'C



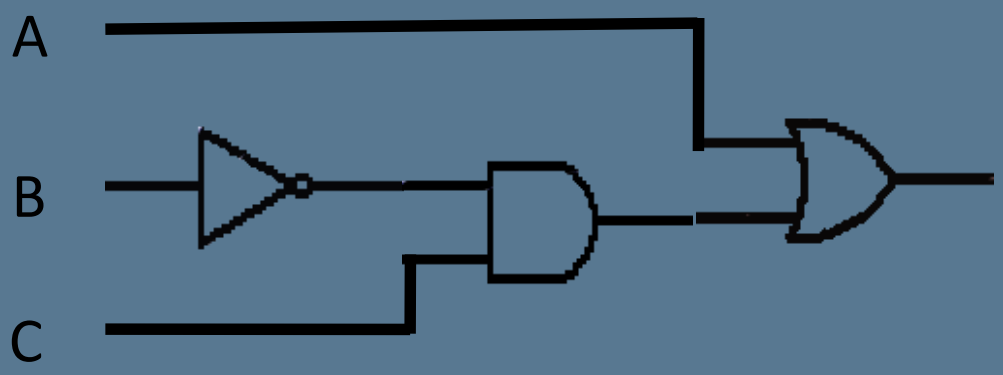
Digital Logic gates

- Draw the logic diagram of the function
  F = A+B'C



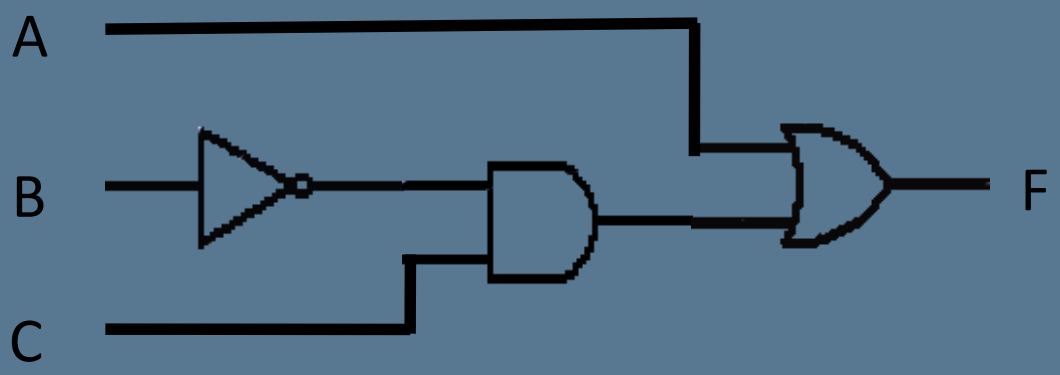
Digital Logic gates

- Draw the logic diagram of the function
  F = A+B'C

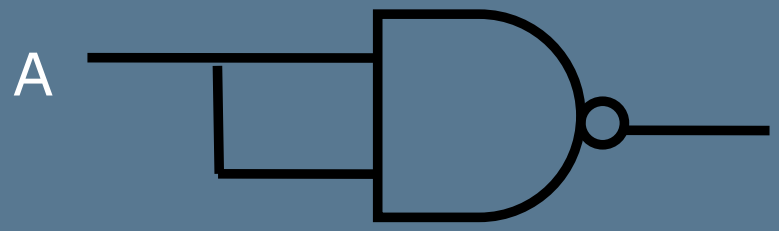


Digital Logic gates

- Draw the logic diagram of the function
  F = A+B'C

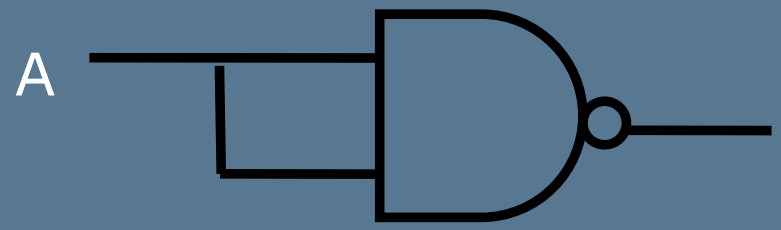


Universality of NAND gate



Q = (A.A)’

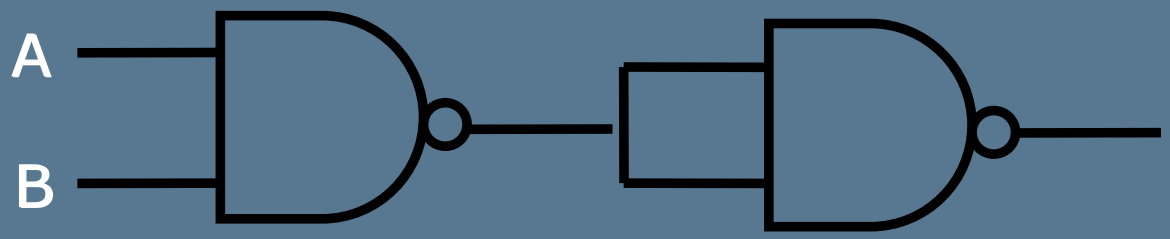
Universality of NAND gate



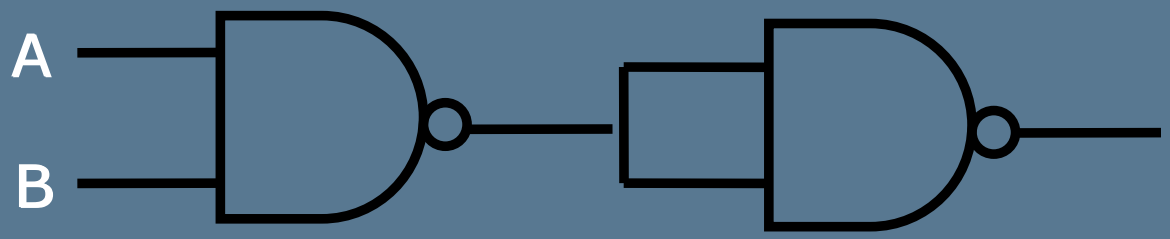
Q = (A.A)’

Q = A’

Universality of NAND Gate

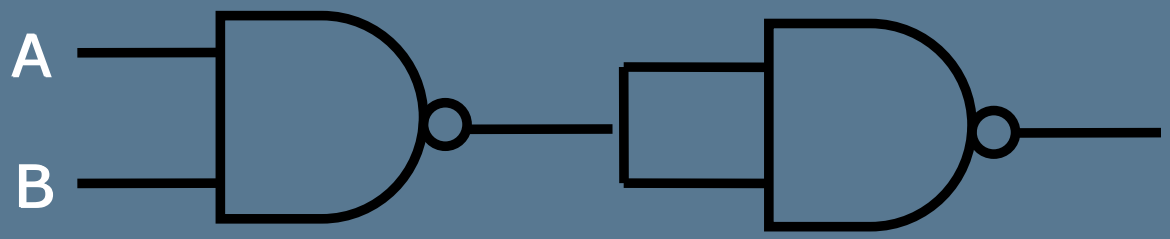


Universality of NAND Gate

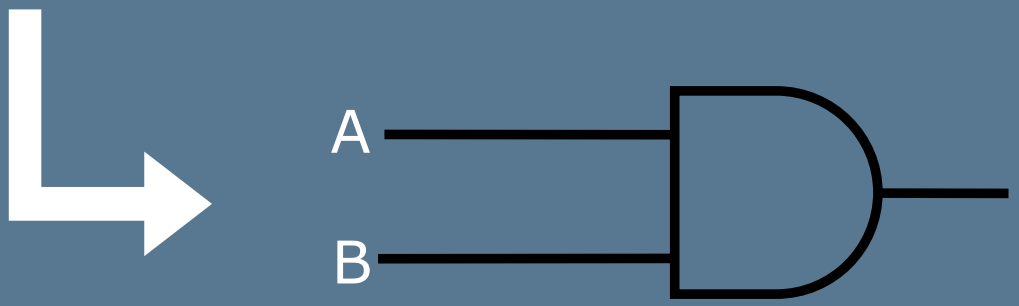


Q =((AB)' (AB)')'

Universality of NAND Gate

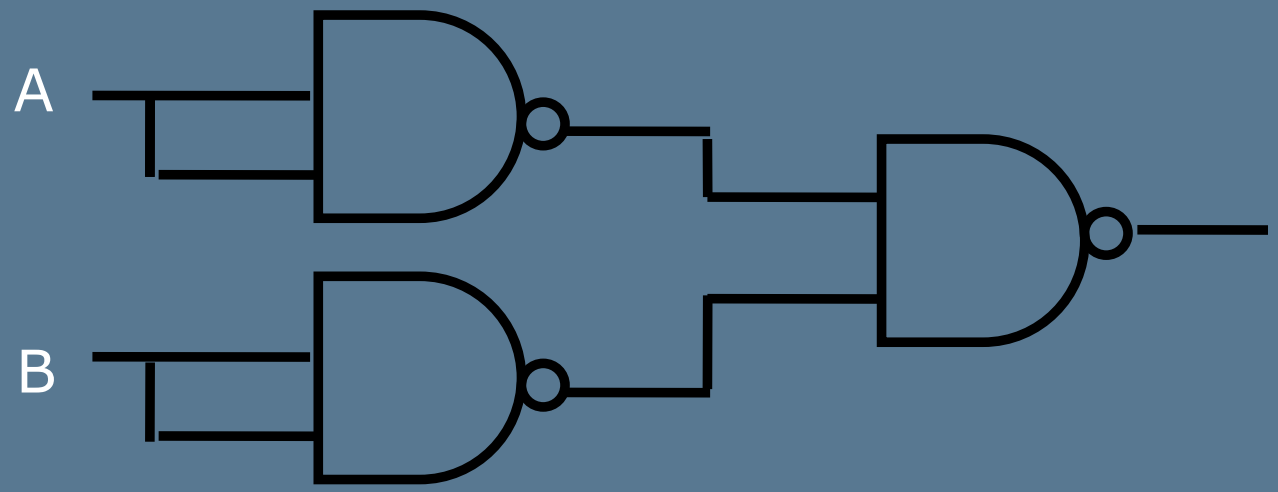


Q =((AB)' (AB)')'

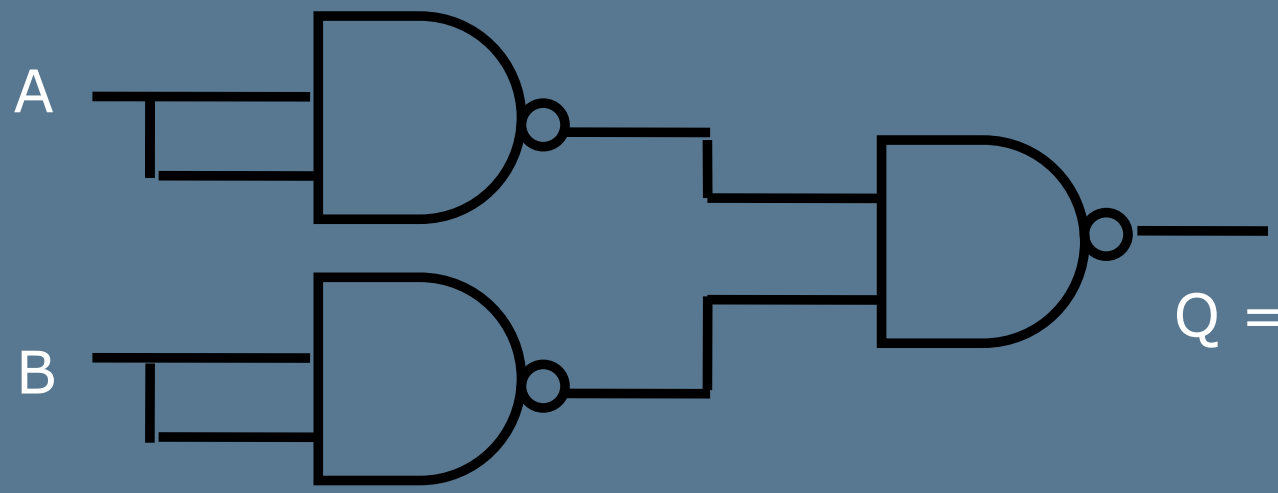


Q = AB

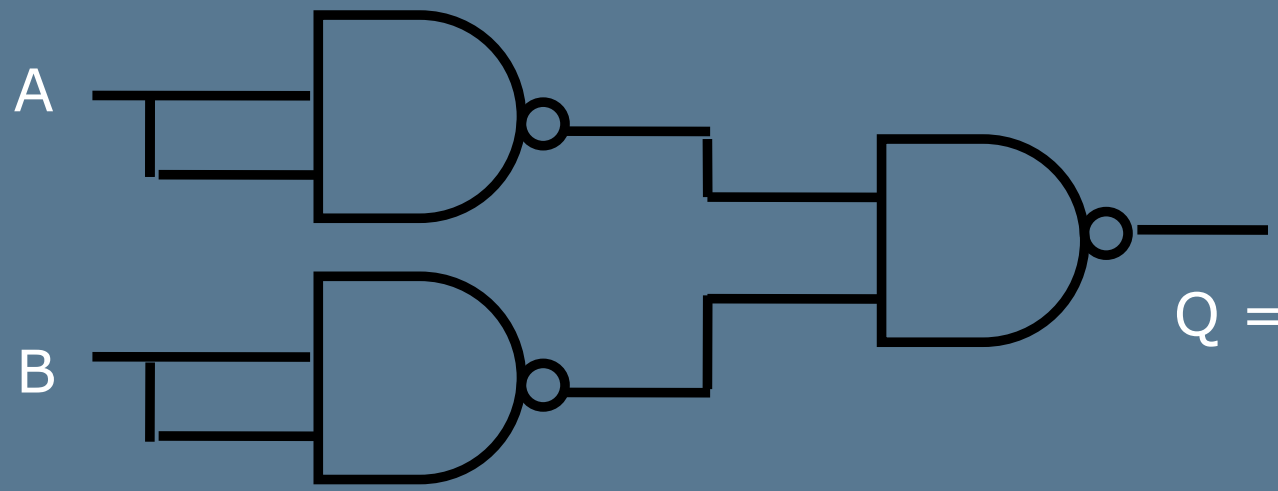
Universality of NAND Gate



Universality of NAND Gate



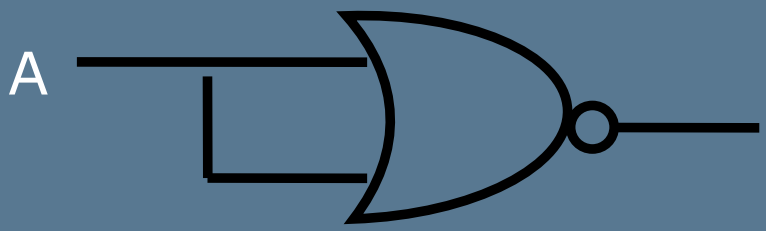
Universality of NAND Gate



Q = (A' B') '

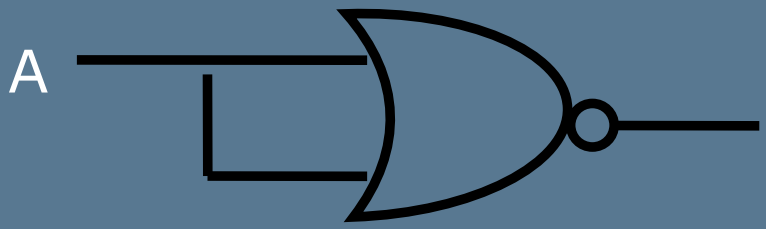
Q = A+B

Universality of NOR Gate

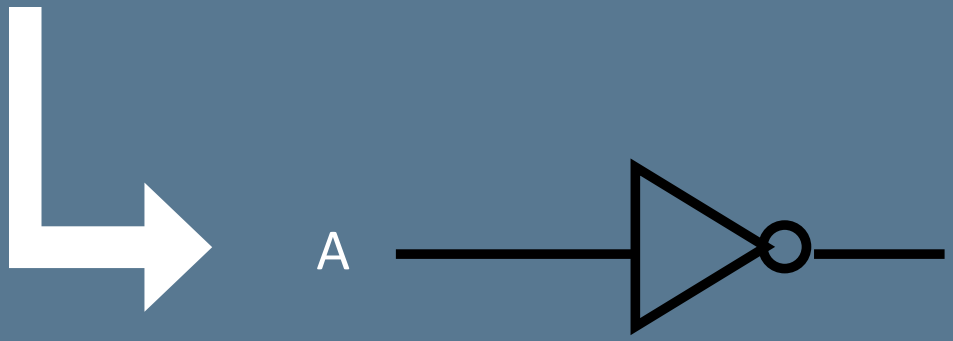


Q = (A+A)’

Universality of NOR Gate

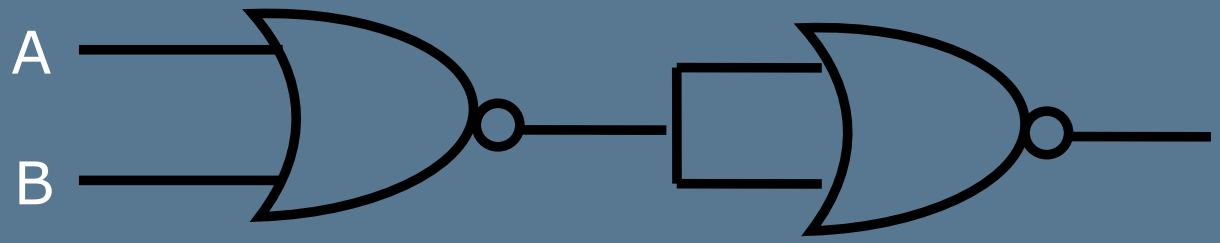


Q = (A+A)’

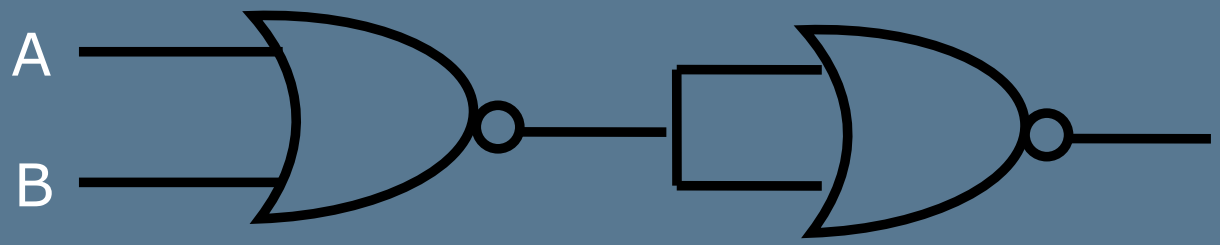


Q = A’

Universality of NOR Gate

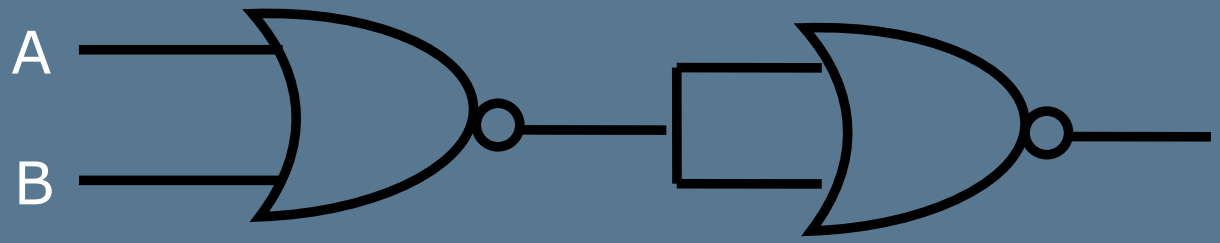


Universality of NOR Gate

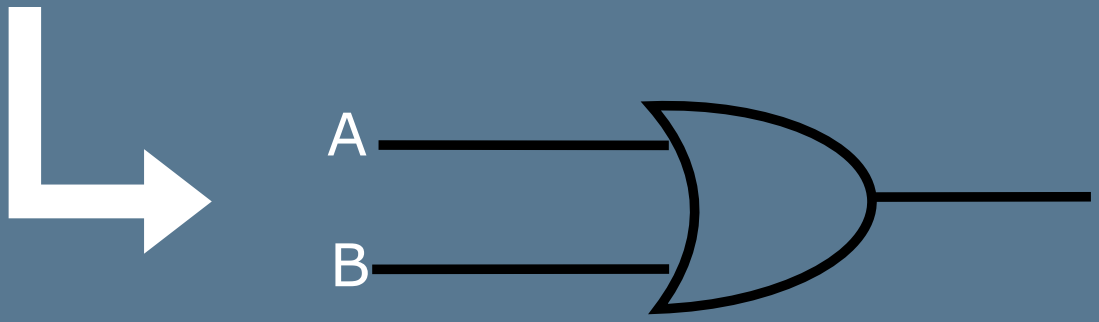


Q =((A+B)' + (A+B)')'

Universality of NOR Gate

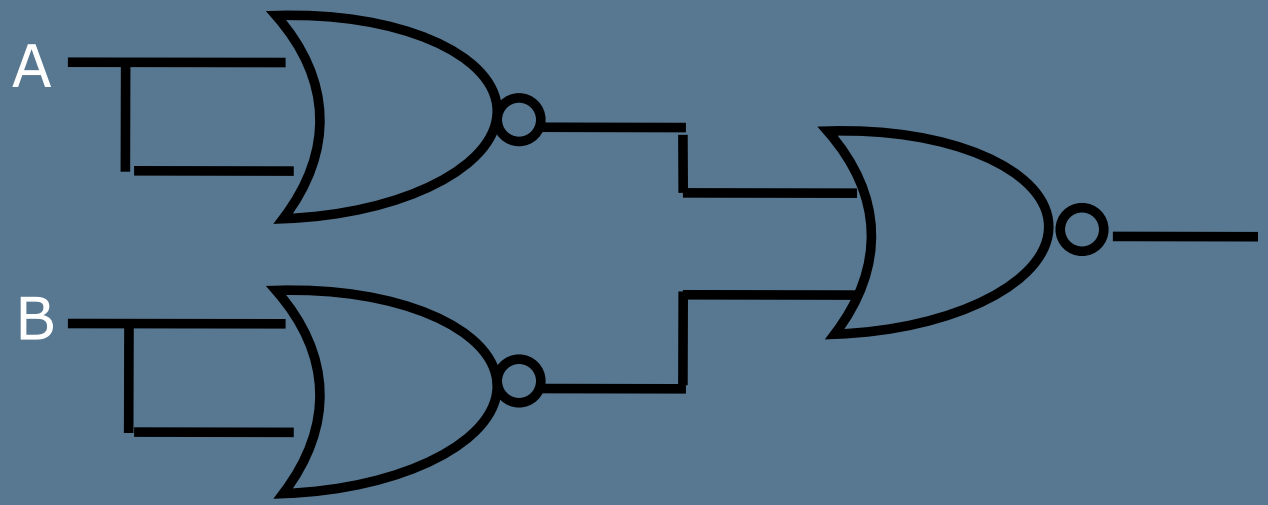


Q =((A+B)' + (A+B)')'

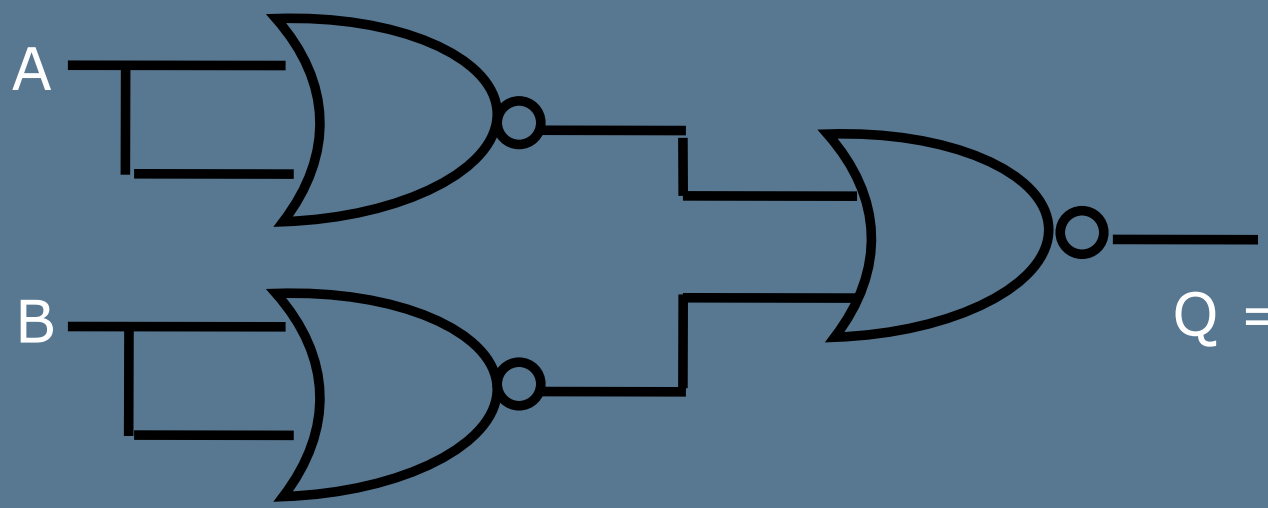


Q = A+B

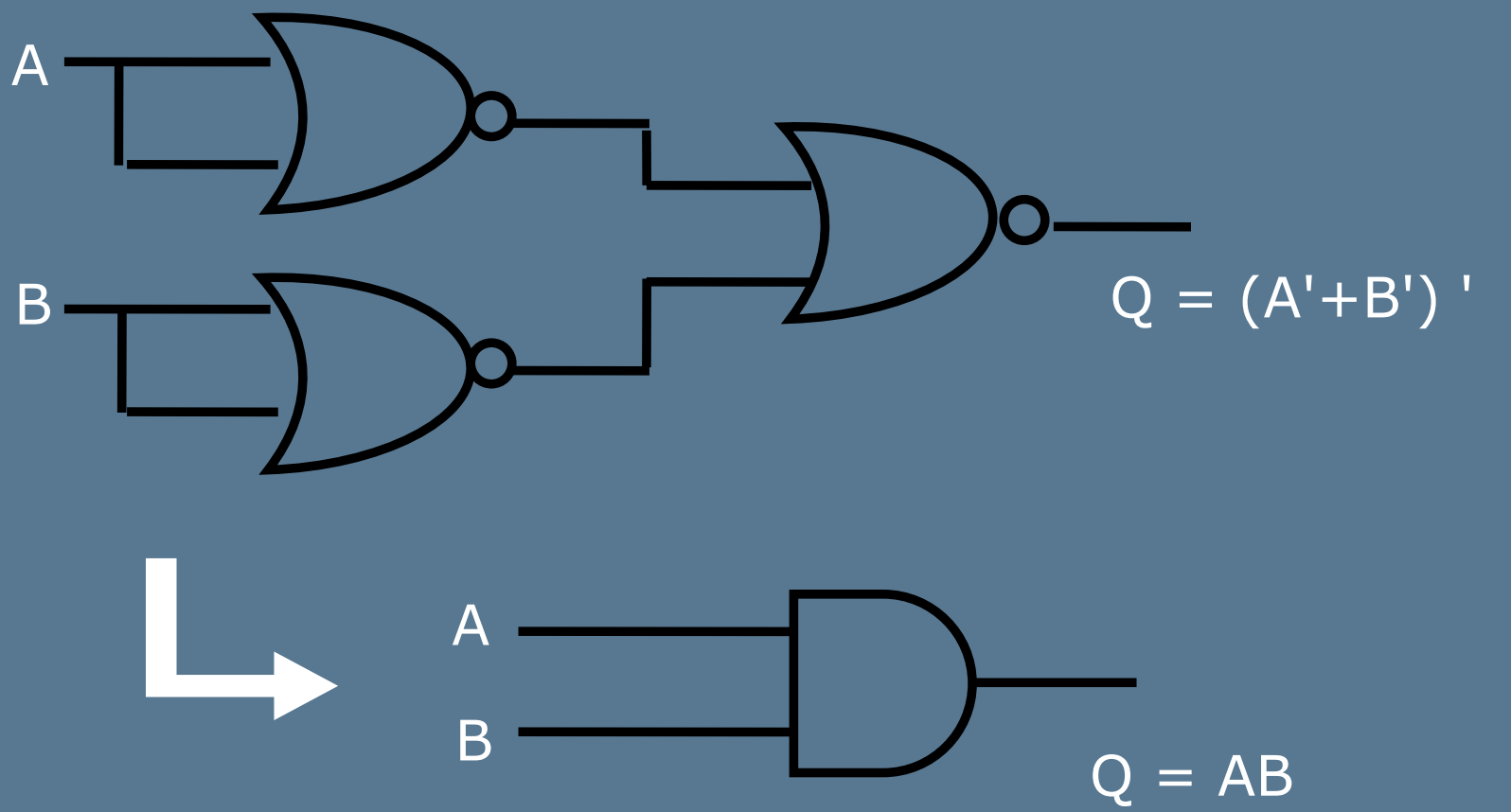
Universality of NOR Gate



Universality of NOR Gate



Universality of NOR Gate



Real Gates

- Logic gates are in integrated form
  - built within a solid piece of silicon called an IC (integrated circuit)

  - Several gates are included in a single plastic moulding

IC Families

- Transistor-Transistor Logic (TTL)
- Emitter Coupled Logic (ECL)
- Complementary Metal-Oxide-Semiconductor (CMOS)

Levels of IC

- Small-scale Integration
  - ICs with 1 to 10 gates
- Medium-scale Integration
  - ICs with 10 to 100 gates
- Large-scale Integration
  - ICs with 100 to 1000s of gates
- Very large-scale Integration
  - ICs with 1000s to millions of gates