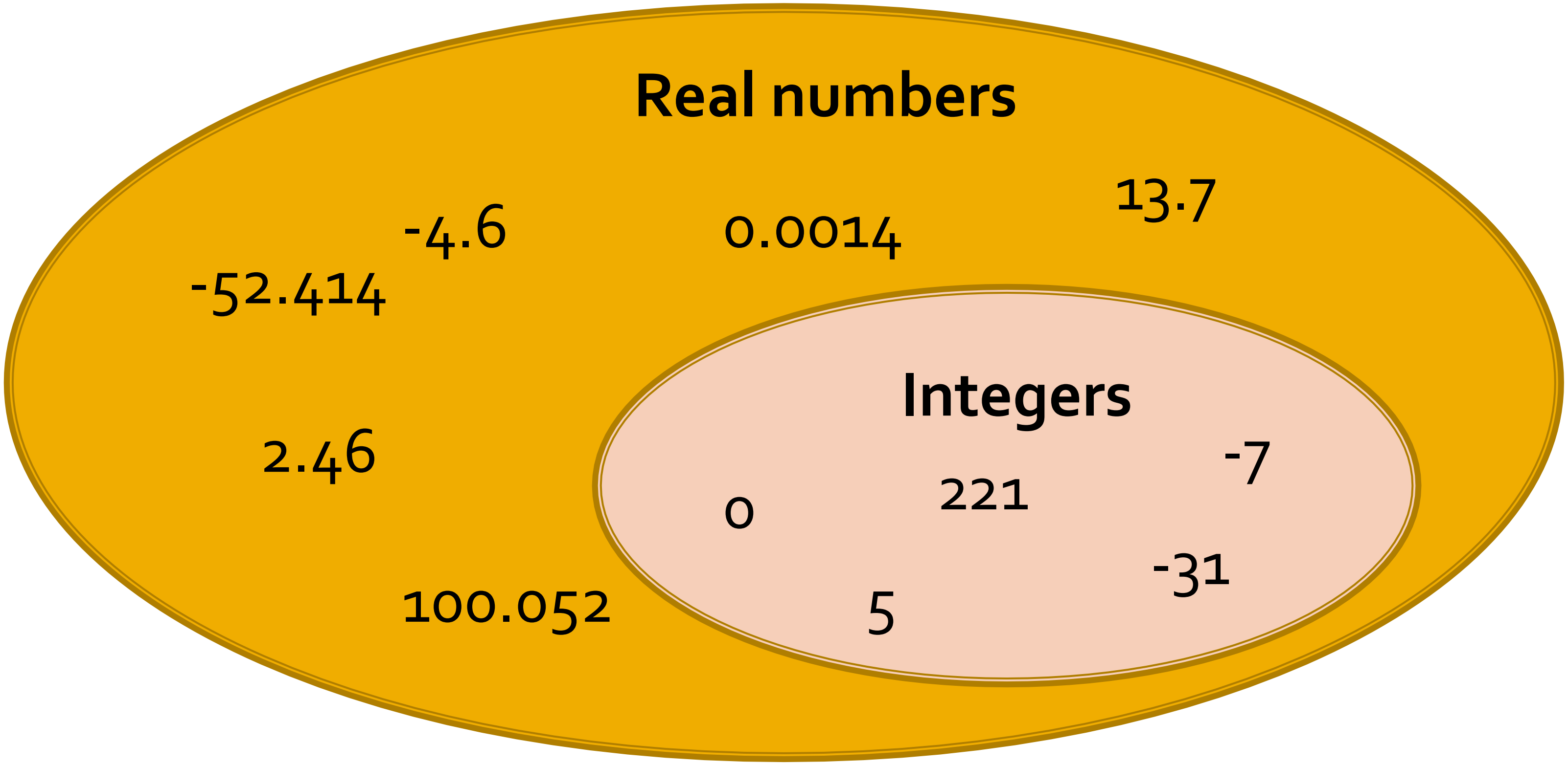
Lecture 2 – Negative Number Representation

CMSC 130

Real Numbers and Integers



Signed Magnitude

- Simplest method of representing negative numbers in binary format
- Uses an extra bit (the most significant bit) to represent the sign
  - "1" indicates a negative number
  - "0" indicates a positive number
  - remaining bits are for the magnitude of the number

Signed Magnitude

- Examples,
  - -7 = **1**111
  - -1 = **1**001
- Use of signed magnitude is complicated when performing arithmetic operations
- 2 values for zero
  - 0000 = 0
  - 1000 = -0

Complements

- Used in digital computers for simplifying the subtraction operation and for logical manipulations
- For base r numbers,
  - Radix Complement (r's complement)
  - Diminished Radix Complement ((r-1)'s complement)

Complements

- For binary numbers (r=2),
  - Radix Complement (2's complement)
  - Diminished Radix Complement (1's complement)
- For decimal numbers (r=10),
  - Radix Complement (10's complement)
  - Diminished Radix Complement (9's complement)

Diminished Radix Complement

- Number representation
  - Positive – same way as in signed magnitude
  - Negative – represented using (r-1)'s complement (for base r numbers)
- (r-1)'s complement: a positive number N in base r with n digits of integer part and m digits of fraction part,
  - $\mathbf{r^n – r^{-m} – N}$

Diminished Radix Complement

- Examples, ($\mathbf{r^n - r^{-m} - N}$)
  - 9's complement for $(24)_{10}$
  - 9's complement for $(3257)_{10}$
  - 9's complement for $(7.636)_{10}$
  - 1's complement for $(100110)_2$
  - 1's complement for $(0.1010)_2$

Radix Complement

- Number representation
  - Positive – same way as in signed magnitude
  - Negative – represented using r's complement (for base r numbers)
- r's complement: a positive number N in base r with n digits integer part,
  - $N \neq 0$, $\mathbf{r^n - N}$
  - $N = 0$, $\mathbf{0}$

Radix Complement

- Examples, (**r^n – N**)
  - 10's complement for $(012372)_{10}$
  - 10's complement for $(131200)_{10}$

Radix Complement

- Answers, ($\mathbf{r^n - N}$)
  - $(10\text{^}6)_{10} - (012372)_{10}$

    $= (1000000)_{10} - (012372)_{10}$

    $= (987628)_{10}$
  - $(131200)_{10} = (868800)_{10}$

Radix Complement

- More examples, ($\mathbf{r^n - N}$)
  - 10's complement for $(24)_{10}$
  - 10's complement for $(3257)_{10}$
  - 10's complement for $(7.636)_{10}$
  - 2's complement for $(100110)_2$
  - 2's complement for $(0.0110)_2$

Radix Complement

- Answers, ($\mathbf{r^n - N}$)
  - $(76)_{10}$ - 10's complement for $(24)_{10}$
  - $(6743)_{10}$ - 10's complement for $(3257)_{10}$
  - $(2.364)_{10}$ - 10's complement for $(7.636)_{10}$
  - $(011010)_2$ - 2's complement for $(100110)_2$
  - $(0.1010)_2$ - 2's complement for $(0.0110)_2$

Subtracting with Complement

- Using r's complement to $M — N$:
  - Add minuend M to r's complement of subtrahend N:

    $M + (r^n — N) = M — N + r^n$
  - Check result for an end carry
    - If an end carry occurs, discard it
    - If no end carry, take r's complement of result and place a negative sign in front

Subtracting with Complement

- r's complement: examples,
  - $(72533 - 3250)_{10}$
  - $(3250 - 72533)_{10}$
  - $(1110100 - 1001100)_{2}$
  - $(1000100 - 1010100)_{2}$

Subtracting with Complement

- (r-1)'s complement
  - Add minuend M to (r-1)'s complement of subtrahend N: $M + (r^n - 1 - N) = M - N + r^n - 1$
  - Check result for an end carry
    - If end carry occurs, add 1 to the least significant bit (**end-around carry**) and ignore the end carry
    - If no end carry, take (r-1)'s complement of result and place a negative sign in front

Subtracting with Complement

- (r-1)'s complement: examples,
  - $(72533 - 3250)_{10}$
  - $(3250 - 72533)_{10}$
  - $(1110100 - 1001100)_2$
  - $(1000100 - 1010100)_2$

...done...

- Prepare for a quiz next meeting (before lecture starts)

Reference

- Mano, M. M. and M. D. Ciletti. Digital design, fourth edition. Prentice Hall.