EXER 7: Independent and Conditional Probability

1. Consider three identical boxes that we shall refer to as box1, box2, and box3. Each box contains 100 fruits, one of which is devil fruit. You randomly choose one fruit from each box.

a. What is the probability of selecting 3 non-devil fruits?
b. What is the probability of selecting a non-devil fruit from box1, a devil fruit from box2, and a non-devil fruit from box3?
c. What is the probability of selecting 1 devil fruit and 2 non-devil fruit?

Solution:
Devil fruit: P(D) = 0.01 and
Non-Devil fruit: P(D') =1− P(D) =0.99

a. P(D') P(D') P(D') = 0.99*0.99*0.99 = 0.9703
b. P(D') P(D) P(D') = 0.99*0.01*0.99 = 0.0098
c. We can select one devil fruit and 2 non-devil fruit in 3 different ways:
   $D1 \cap D2' \cap D3'$ (the devil fruit comes from box 1)
   $D1' \cap D2 \cap D3'$(the devil fruit comes from box 2)
   $D1' \cap D2' \cap D3$ (the devil fruit comes from box 3)

   P(D1) P(D2') P(D3') **U** P(D1') P(D2) P(D3') **U** P(D1') P(D2') P(D3)
   =P(D) P(D') P(D')+ P(D') P(D) P(D')+ P(D') P(D') P(D)
   =0.01*0.99*0.99 + 0.99*0.01*0.99 + 0.99*0.99*0.01
   =0.0294

2. Islands in the new world has atmospheric conditions classified into a finite number of categories. In category 1 atmospheric condition, there is a 40% chance of rain, a 30% chance of gale-force winds, and a 50% chance that it either rains or has gale-force winds. Suppose Luffy and his crew visited the island with category 1 condition and find that it is raining. What is the probability of gale-force winds? (Assuming that they know the all of the categories and can classify them correctly.)

Solution:

Let W = the event of gale-force winds, R = the event of rain

Given:
$P(R) = 0.4$, $P(W) = 0.3$, $P(R \cup W) = 0.5$

We are told that it is raining. Thus the event R has occurred. This means that we want to know the probability of gale-force winds given that it is raining. In symbols, we want $P(W | R)$.

Using the definition of conditional probability and P(R) = 0.4 leads to the following equation:
$P(W | R) = P(RW) / P(R)$
$= P(RW) / 0.4$

Compute for P(RW)
$P(R \cup W) = P(R) + P(W) – P(RW)$
$0.5 = 0.4 + 0.3 – P(RW)$
$P(RW) = 0.7-0.5$
$P(RW) = 0.2$

$P(W | R) = P(RW) / P(R)$
$= 0.2 / 0.4$
$=1/2 = 0.5$

4. According to statistics, 7 in 15 most wanted pirates become shichibukais. If the marines are to invite 8 most wanted pirates out of a hundred to be a shichibukai, what is the probability that at least five will be shichibukais and the remaining will not?

Solution:

$P(\text{will be}) = 7/15 = 0.4667$
$P(\text{will not}) = 0.5333$

$P(A) = C(8,5)(7/15)^5(8/15)^3 + C(8,6)(7/15)^6(8/15)^2 + C(8,7)(7/15)^7(8/15)^1 + C(8,8) \quad (7/15)^8(8/15)^0$

5. Suppose that the probability of Sanji getting along with Zoro is 0.01, and that the probability of Sanji finding All Blue given that he will get along with Zoro is 0.4 and that the probability of Sanji finding All Blue is 0.02. Find the probability that Sanji will get along with Zoro given that he find All Blue.

Solution:

Given: $P(G) = 0.01$ $P(B) = 0.02$ $P(B|G) = 0.04$
$P(G|B) = ?$

$P(B|G) = P(BG)/P(G)$
$P(BG) = P(B|G)*P(G)$
$P(BG) = 0.04(0.01) = 0.004$

$P(G|B) = P(BG)/P(B)$
$=0.004/0.02 = 0.2$

6. In 95% of all manned lunar flights, a midcourse trajectory correction is required. This is done by sending a fire signal from ground control to ignite small correction thrusters. For technical reasons, this fire signal is sometimes not executed by the thrusters. Tests show that the probability is .0001 that a fire signal will not be executed when it is required. If the correction is required and not executed, then the rocket will be not be able to escape the sun's gravitational field and will be sucked into the sun, which will make the astronauts unhappy. What is the probability that this will happen?

Solution:

Let R= the event correction is required
E = the event correction is not executed

Given:
$P(R) = 0.95$, P(correction is not executed | correction is required) = $P(E|R) = 0.0001$

We want to know the probability that a correction is required and that the correction is not executed. -> $P(E \cap R)$

$P(E \cap R) = P(R)P(E|R) = (.95)(.0001) = .000095$.

7. You have invested in Home-Clone Inc stocks, as you suspect that the company's "Clone-a-Sibling" kit will shortly be approved by the FDA. There is an 80% chance that FDA approval will be given, and a 95% chance that the value of the stock you hold will double if FDA approval is given. What is the probability that the FDA will approve the product and the value of the stock you hold will double?

Soln:
The events in question are as follows.

E is the event that the value of your stock will double
F is the event that FDA will approve the "Clone-a-Sibling" kit

Given:

$P(F) = 0.8$
$P(E|F) = 0.95$

Find P(EF):

$P(EF) = P(F)*P(E|F) = (0.8)*(0.95) = 0.76$