CMSC 141 Automata and Language Theory

Context-Free Languages

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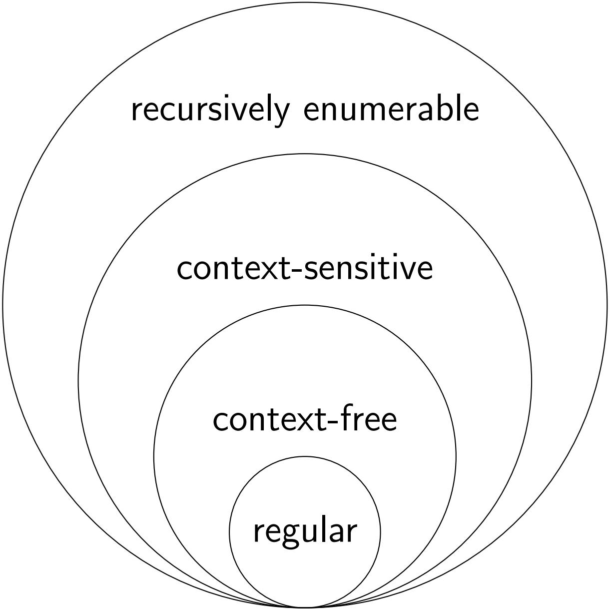
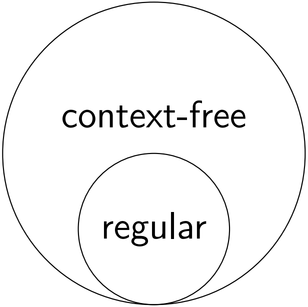
Chomsky Hierarchy



Chomsky Hierarchy



- Regular Languages $V \rightarrow T^*(V + \varepsilon)$
  - $S \rightarrow abS \mid a \mid \varepsilon$
  - $S \rightarrow 0S \mid 1S \mid 11T$
    $T \rightarrow 0T \mid 1T \mid \varepsilon$
- Context-Free Languages $V \rightarrow (V + T)^*$
  - $S \rightarrow ab \mid aSb$
  - $S \rightarrow () \mid SS \mid (S) \mid \varepsilon$

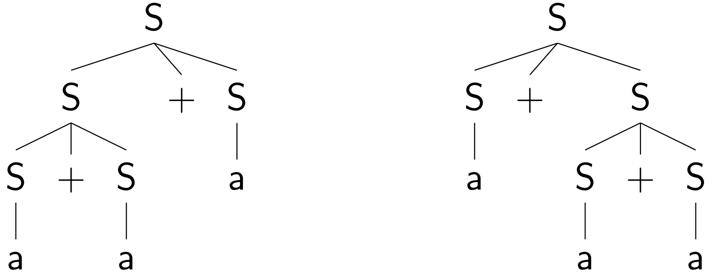
Ambiguous Grammars

- A grammar is *ambiguous* if there is more than one parse tree for any string x in the language
- A grammar is *non-ambiguous* if every string in a language has a unique parse tree
- $S \rightarrow ab \mid aSb$ is non-ambiguous
- $S \rightarrow a \mid S + S$ is ambiguous

Ambiguous Grammars

Grammar: $S \rightarrow a \mid S + S$

Derive: `a + a + a`



Ambiguity in Natural Languages

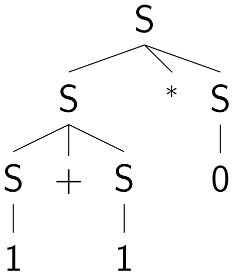
- The man on the hill saw the boy with a telescope.
- Look at the dog with one eye.

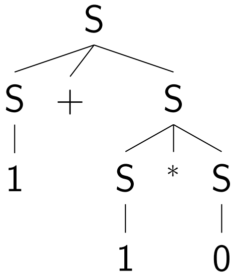
Removing Ambiguity

- $G_1 : S \rightarrow a \mid S + S$ is ambiguous
- $G_2 : S \rightarrow a \mid S + a$ is non-ambiguous
- The two grammars generate the same language $L(G_1) = L(G_2)$
- G_2 is better because it is non-ambiguous by forcing the rule on left-associativity

Removing Ambiguity

- Consider: $S \to 0 \mid 1 \mid S + S \mid S^* S \mid (S)$
- How many ways can we parse $1 + 1^* 0$





Removing Ambiguity

A non-ambiguous grammar
(enforcing operator precedence)

$$
\begin{array}{lll}
S & \to E & \\
E & \to E + T \mid T & \text{(expressions)} \\
T & \to T{*}F \mid F & \text{(terms)} \\
F & \to 0 \mid 1 \mid (E) & \text{(factors)}
\end{array}
$$

Exercise: Try parsing strings like $1 + 1{*}0$ and $(1 + 1){*}0$

Dangling-Else Problem

Consider:

$S \rightarrow B$ | if C then S | if C then S else S

$B \rightarrow block$

$C \rightarrow (cond)$

```
if (cond1) then
  if (cond2) then block1
else block2
```

```
if (cond1) then
  if (cond2) then block1
  else block2
```

Exercise: Design/Create a non-ambiguous version of the grammar that associates an else-clause to the nearest if

Inherently-Ambiguous Languages

- A language L is inherently ambiguous if every grammar for L is ambiguous
- Example: $L = \{a^m b^m c^n\} \bigcup \{a^m b^n c^n\}; m, n > 0$
- Sample strings are aabbcccc, abbbccc, abc
- A CFG for L is given by

$$\begin{aligned} S &\to XC \mid AY \\ X &\to ab \mid aXb \\ A &\to a \mid aA \\ C &\to c \mid cC \\ Y &\to bc \mid bYc \end{aligned}$$

- Show that aabbcc has 2 different parse trees
- Why?

References

- Previous slides on CMSC 141
- M. Sipser. Introduction to the Theory of Computation. Thomson, 2007.
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- E.A. Albacea. Automata, Formal Languages and Computations, UPLB Foundation, Inc. 2005
- JFLAP, www.jflap.org
- Various online LaTeX and Beamer tutorials