# GRAPH THEORY

## Problems and their Applications

MINIMUM SPANNING TREE PROBLEM

TRAVELLING SALESPERSON PROBLEM

GRAPH COLORING

SHORTEST PATH PROBLEM

GRAPH MATCHING

# MINIMUM SPANNING TREE PROBLEM

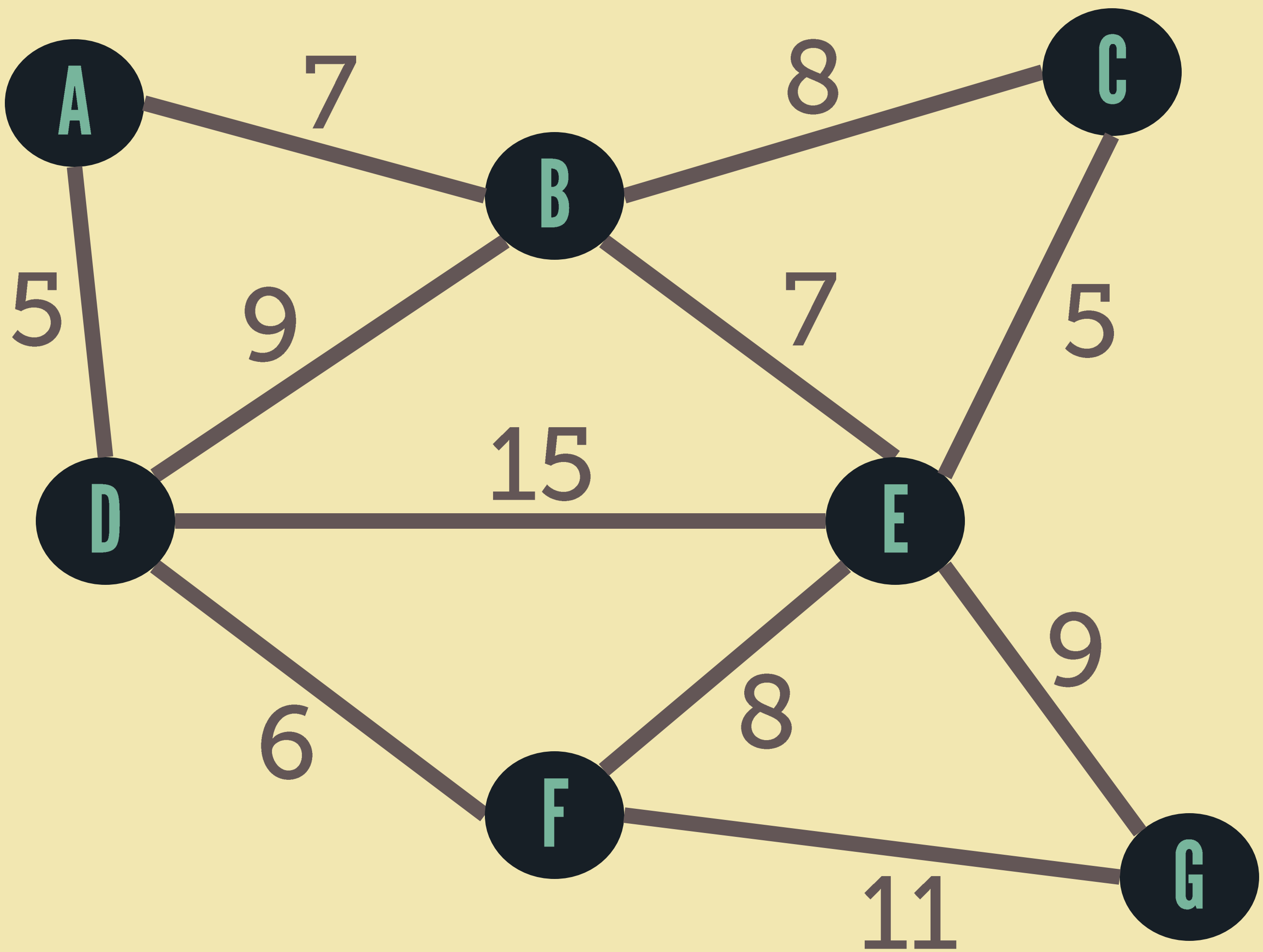
Given a weighted graph, find a **SPANNING TREE** such that the sum of the weights of the edges is the **SMALLEST** possible.

# KRUSKAL'S algorithm

- Start with a null graph T with vertices of G.

- Add currently cheapest edge from G to T that will not form a cycle in T.

- Repeat previous step until a spanning tree is obtained.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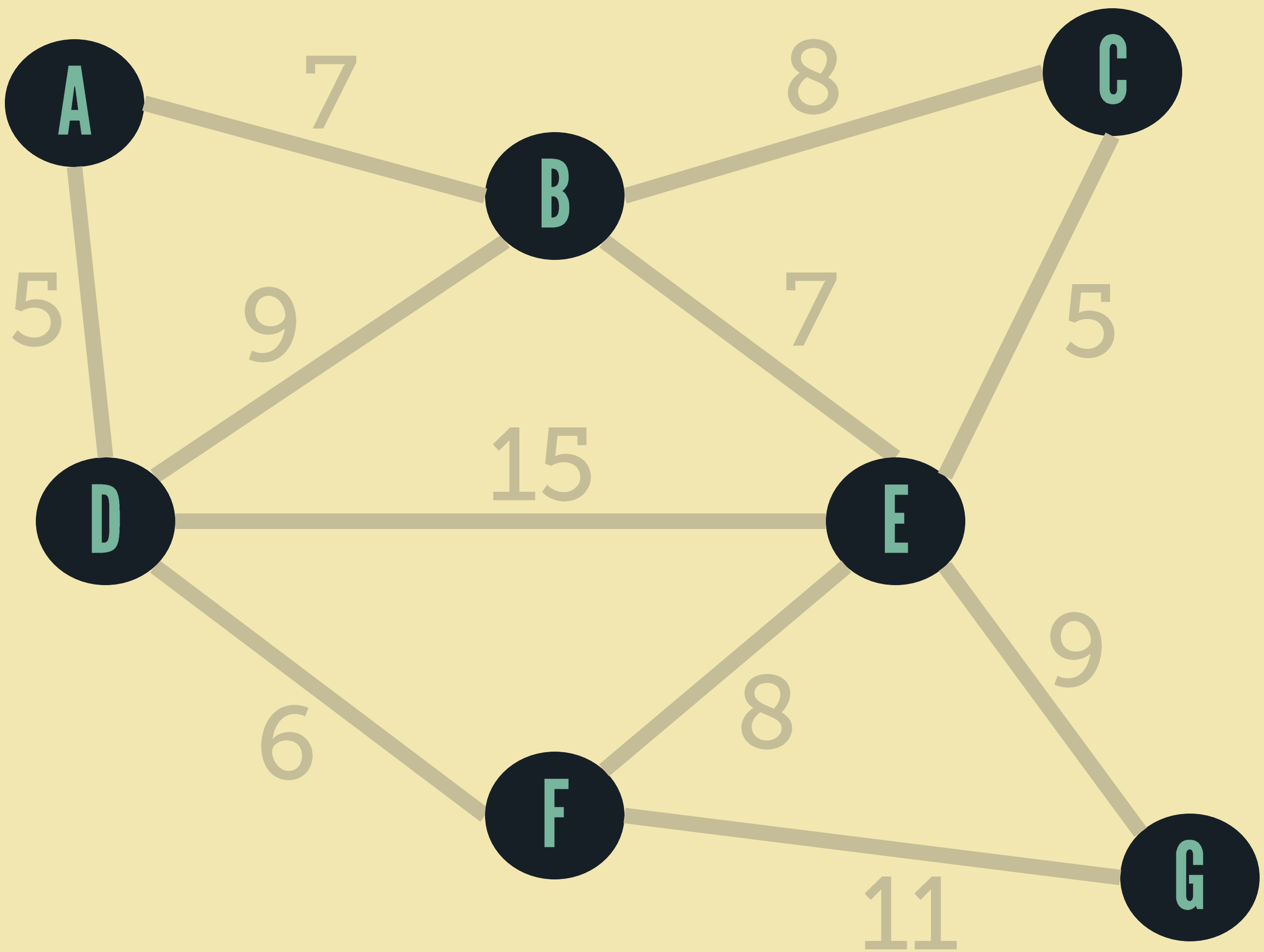
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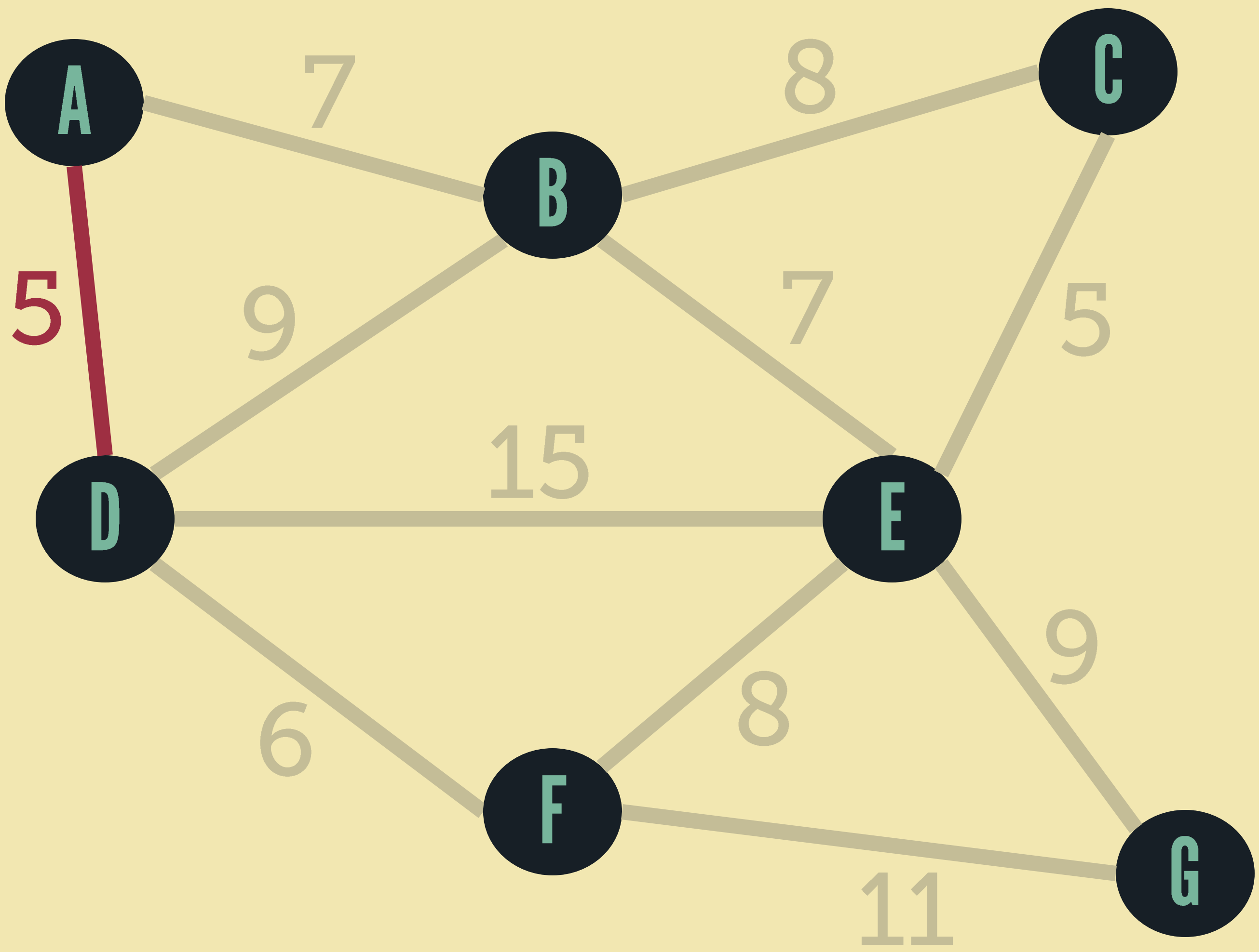
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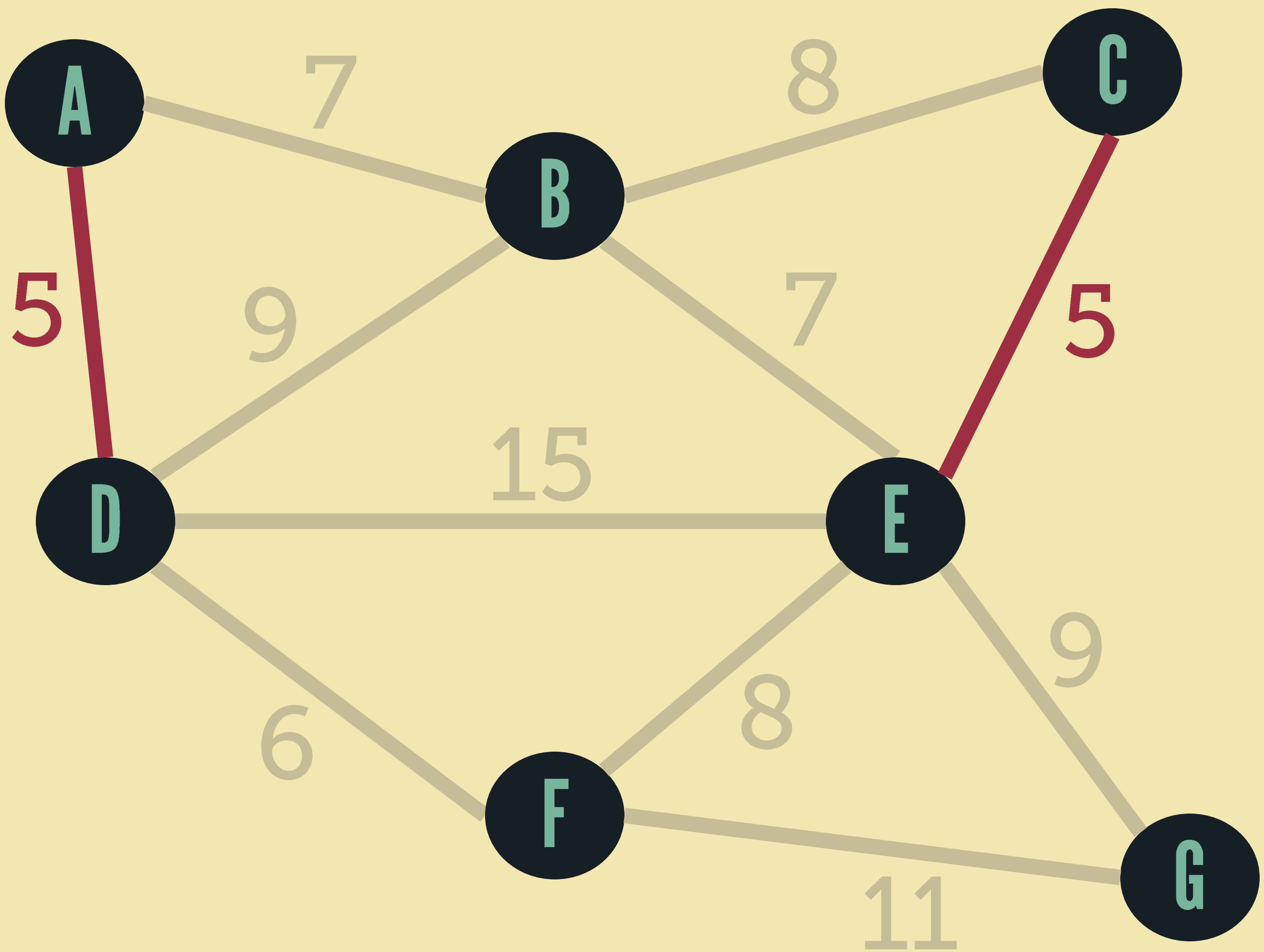
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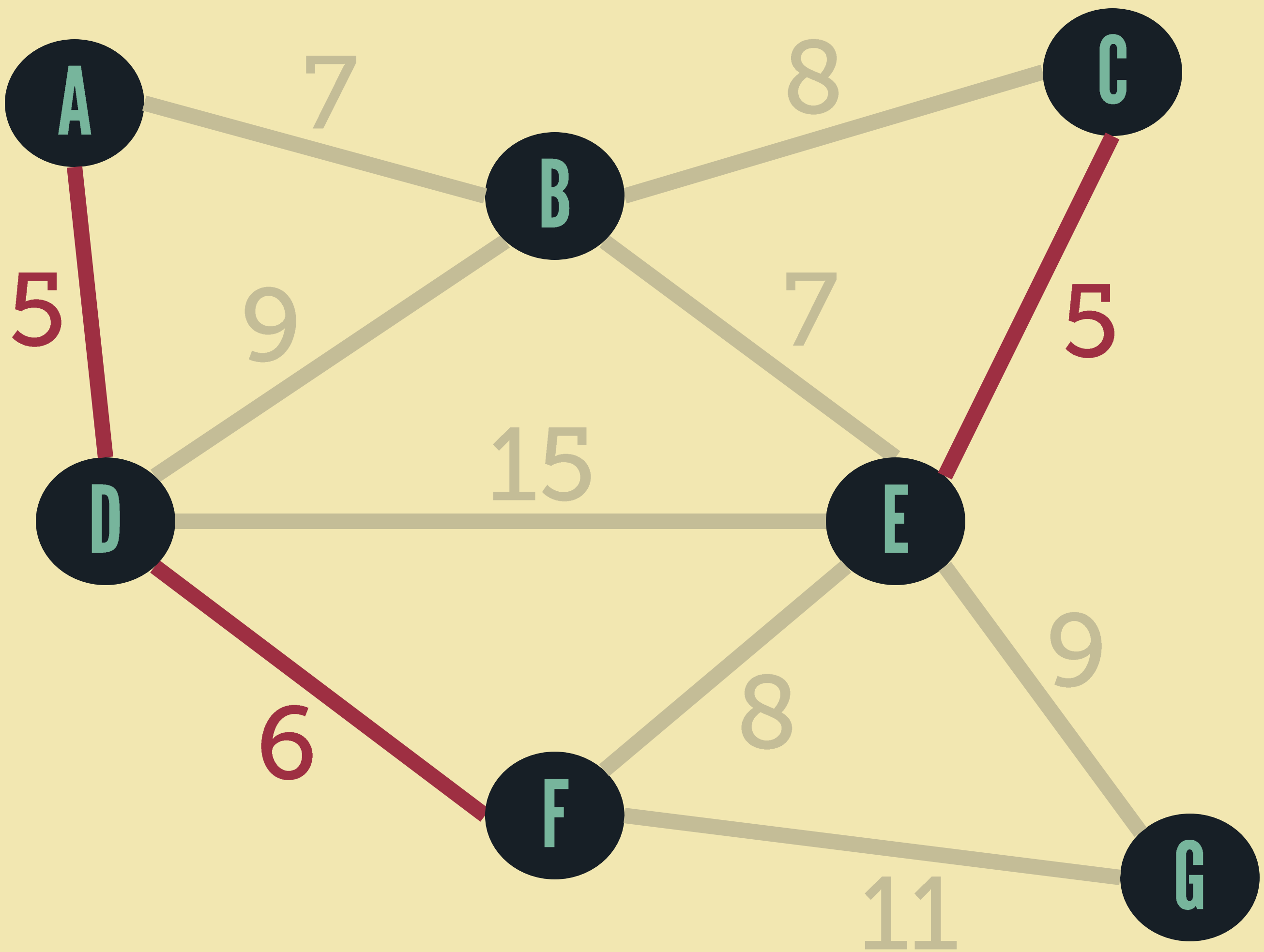

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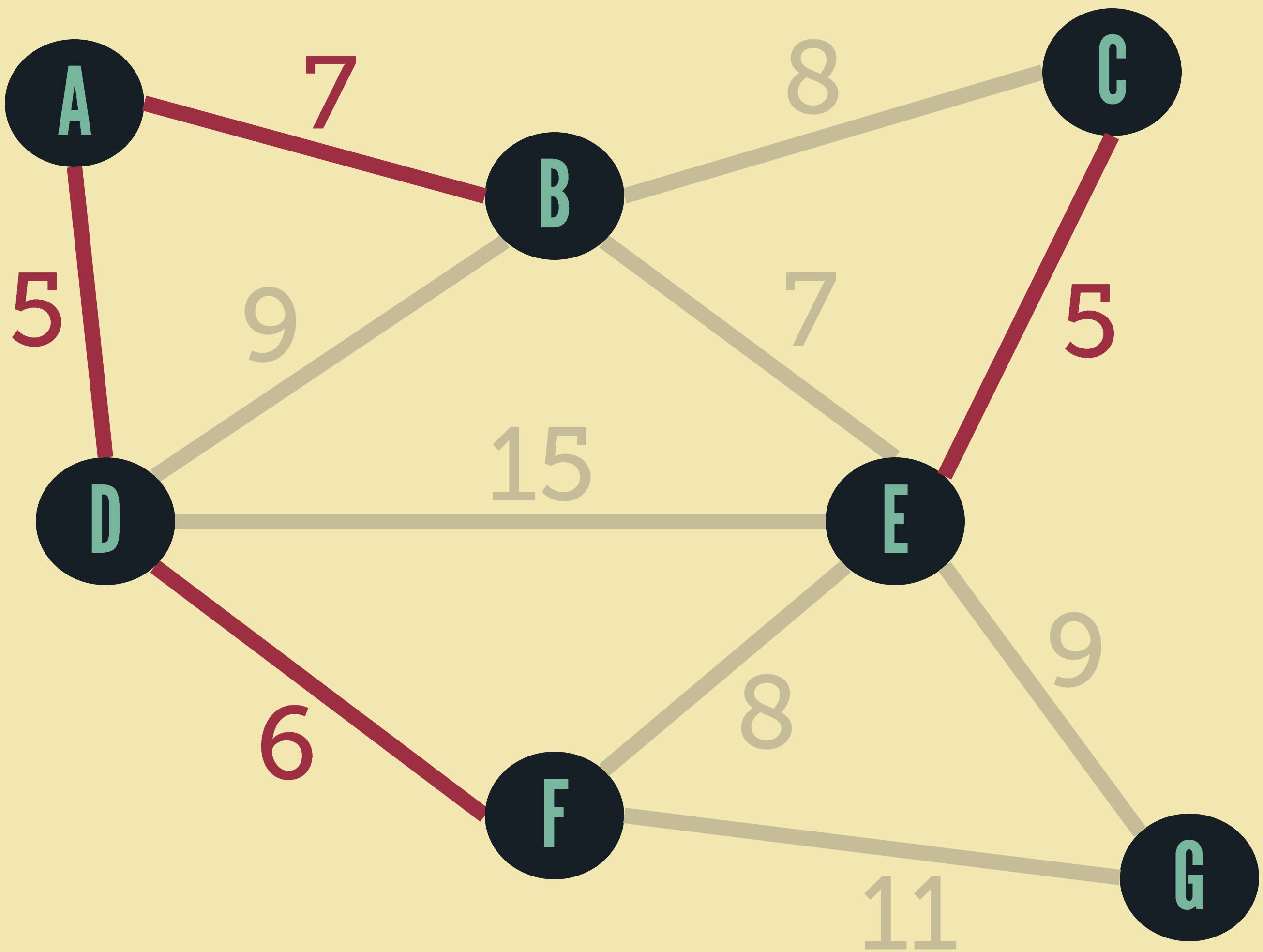
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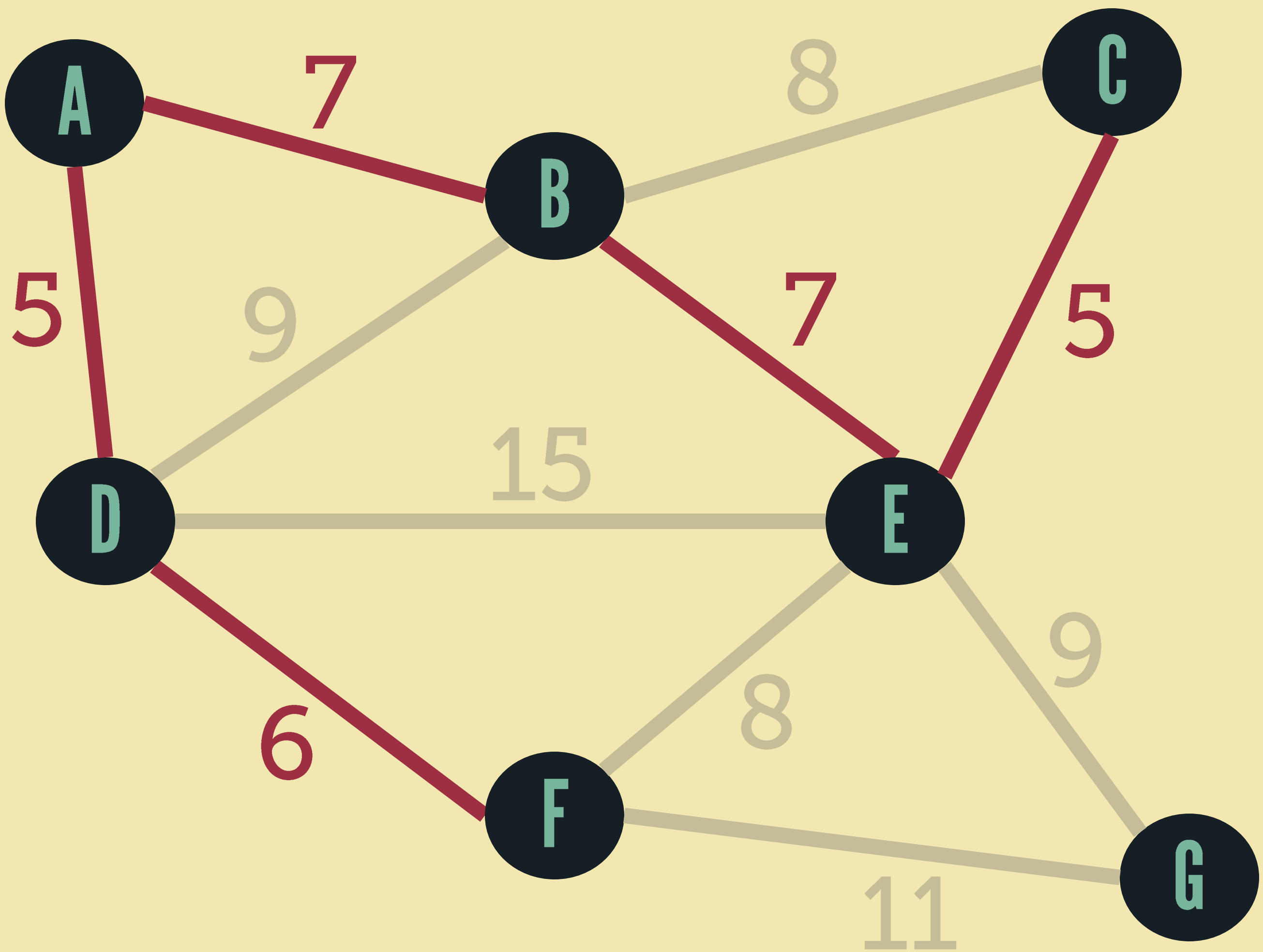
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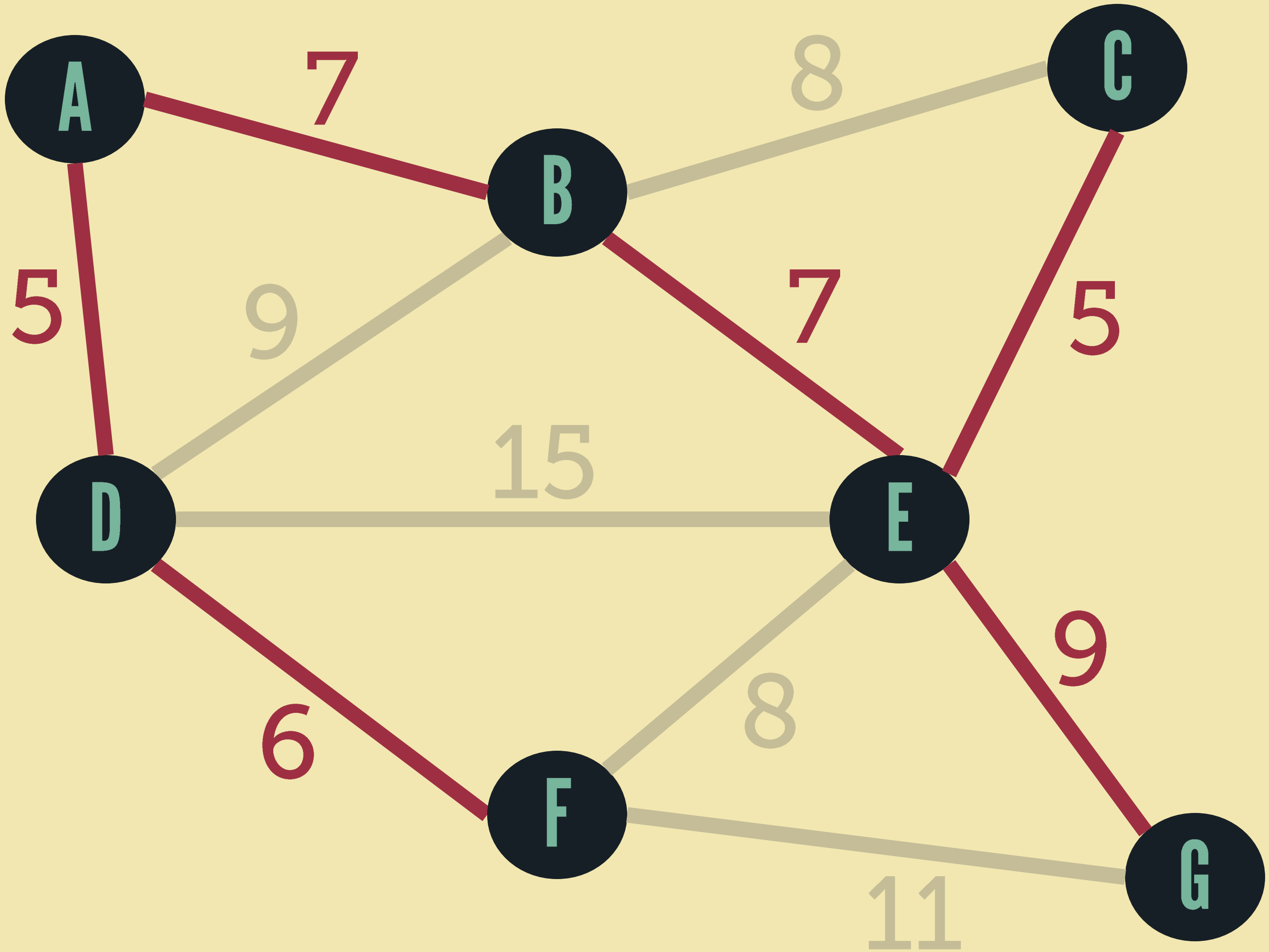
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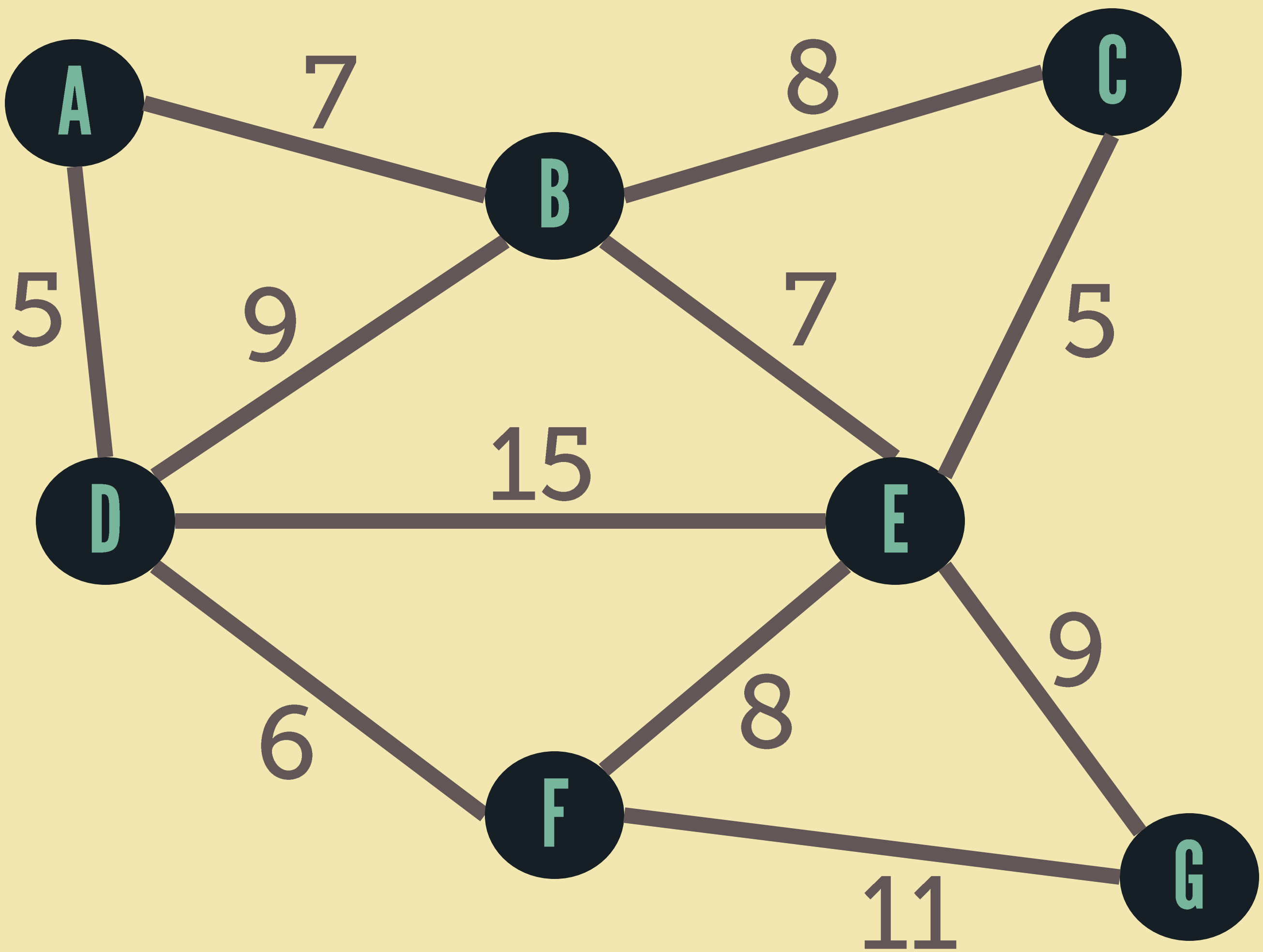
# PRIM'S algorithm

- Start with a trivial graph T with a single arbitrary vertex from G.

- Add the vertex y and edge e = (x,y) such that e is the cheapest edge connecting y to some vertex x already in T (and will not form a cycle)

- Repeat previous step until a spanning tree is obtained.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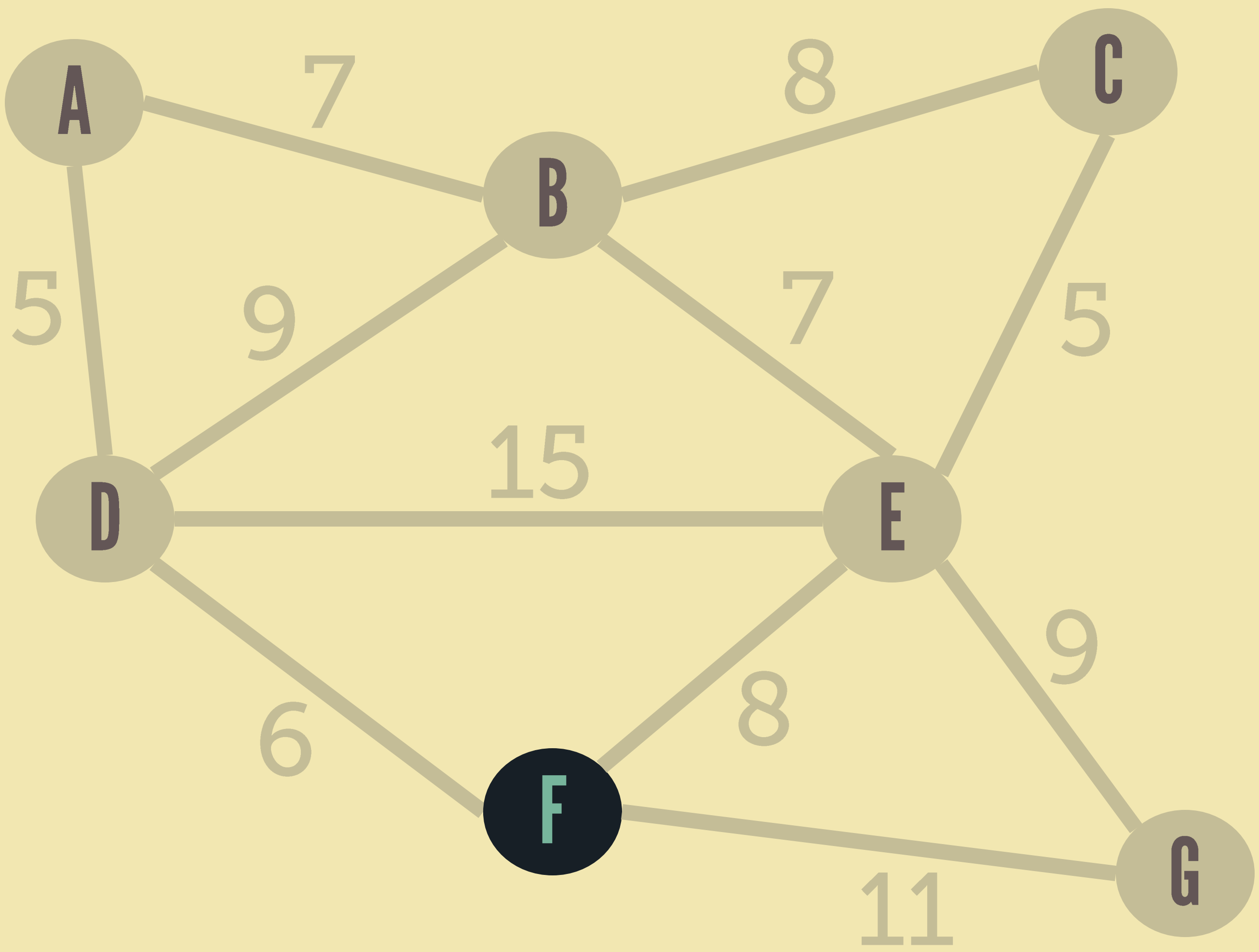
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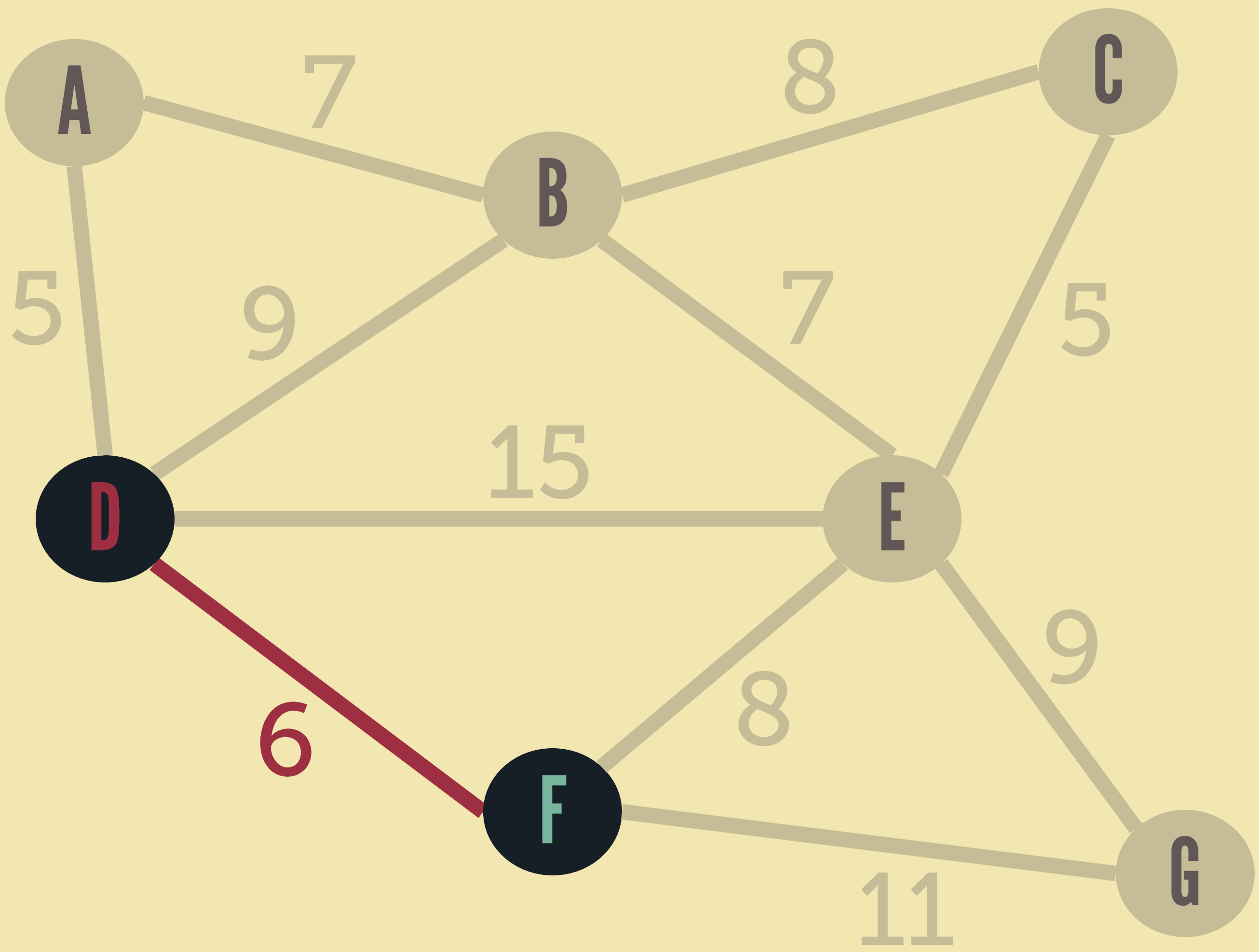
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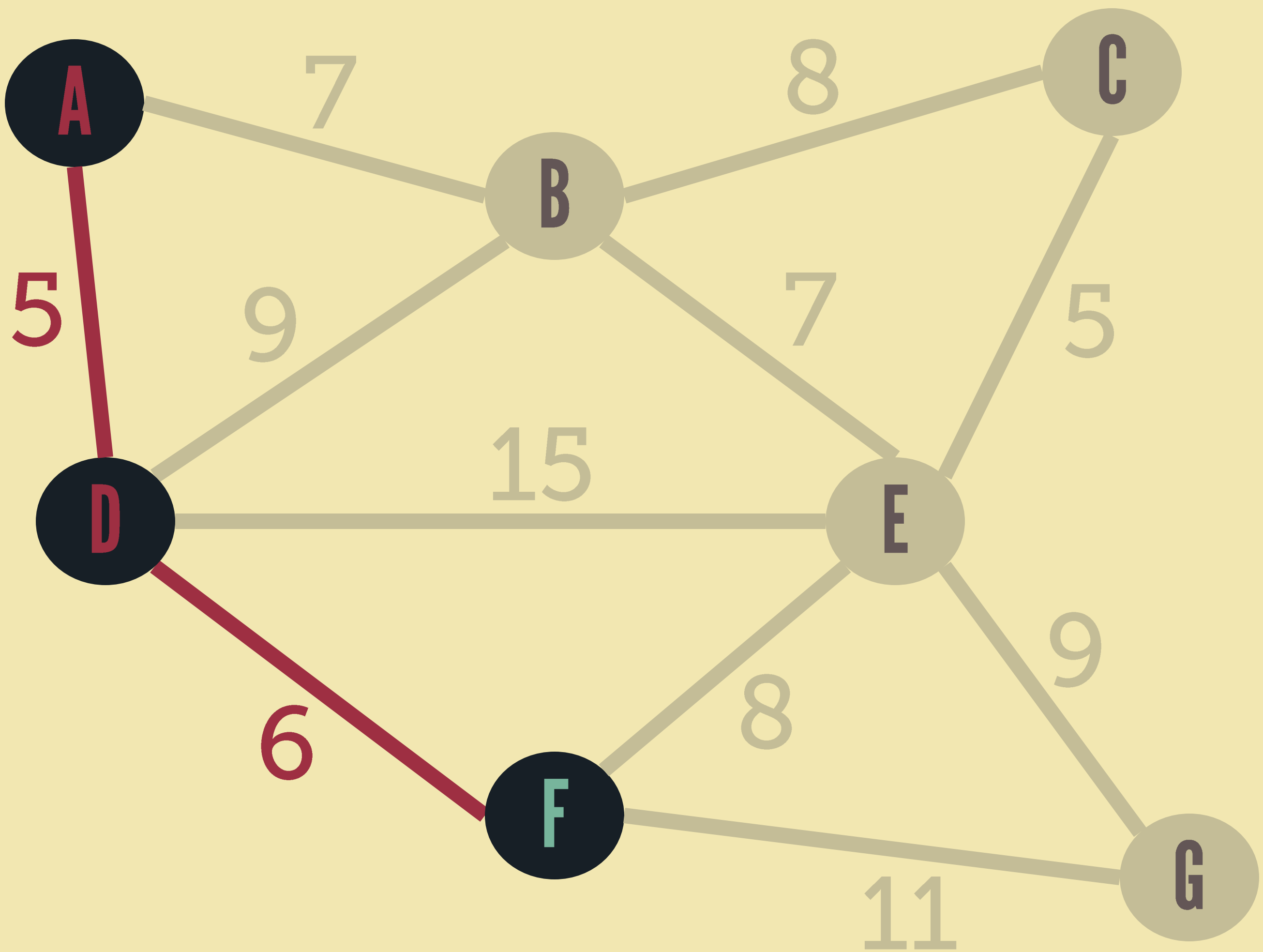
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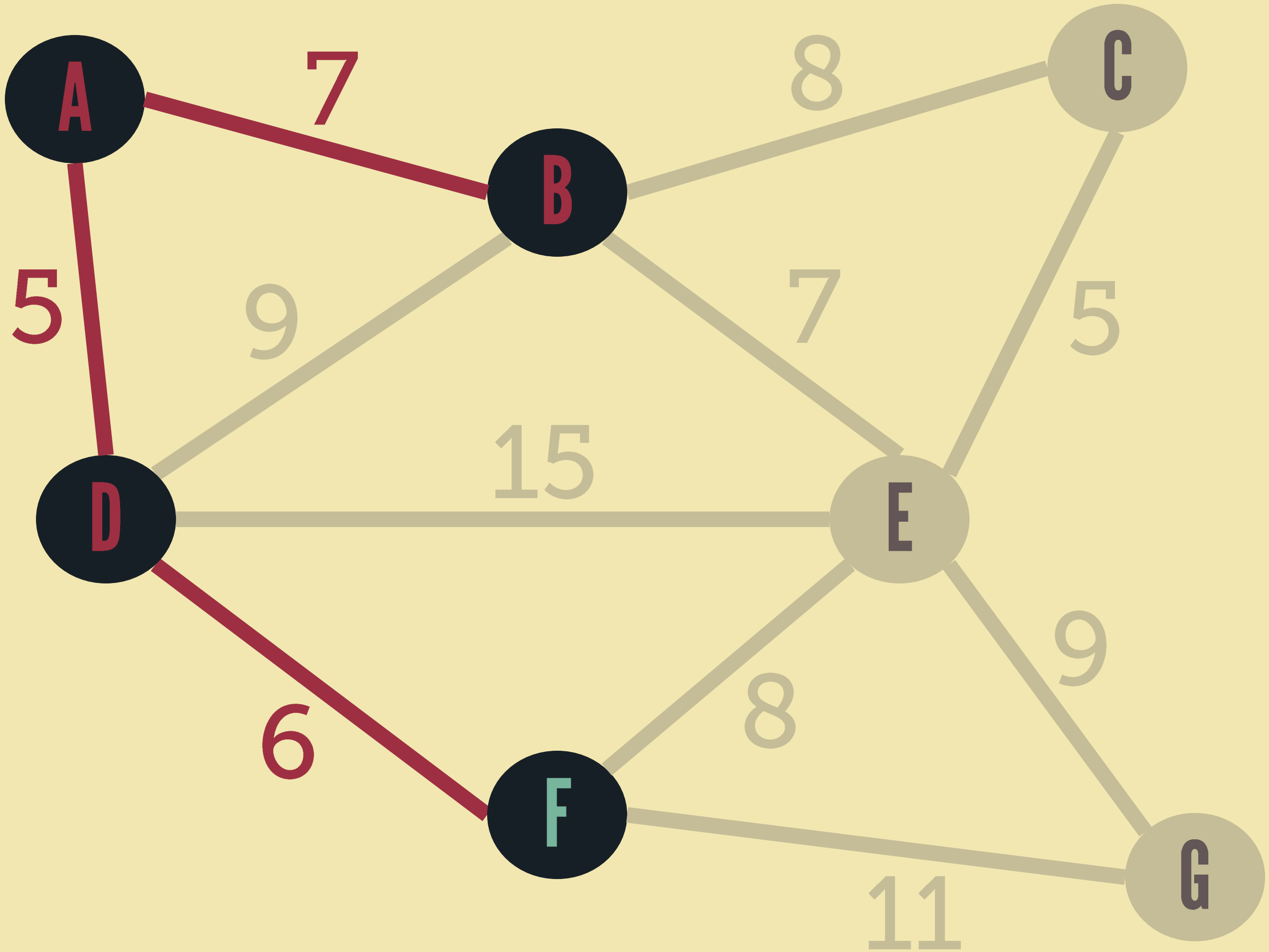
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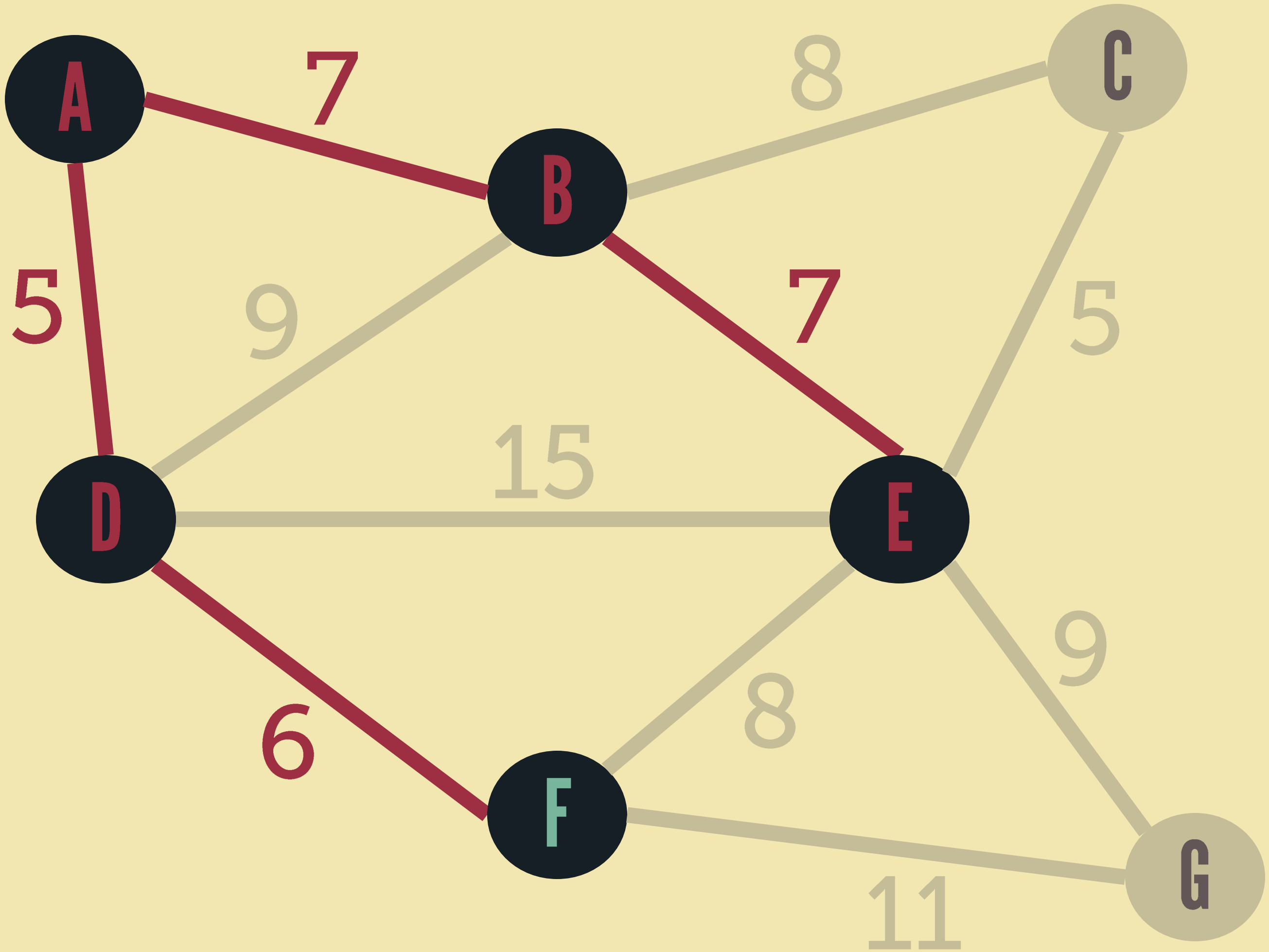
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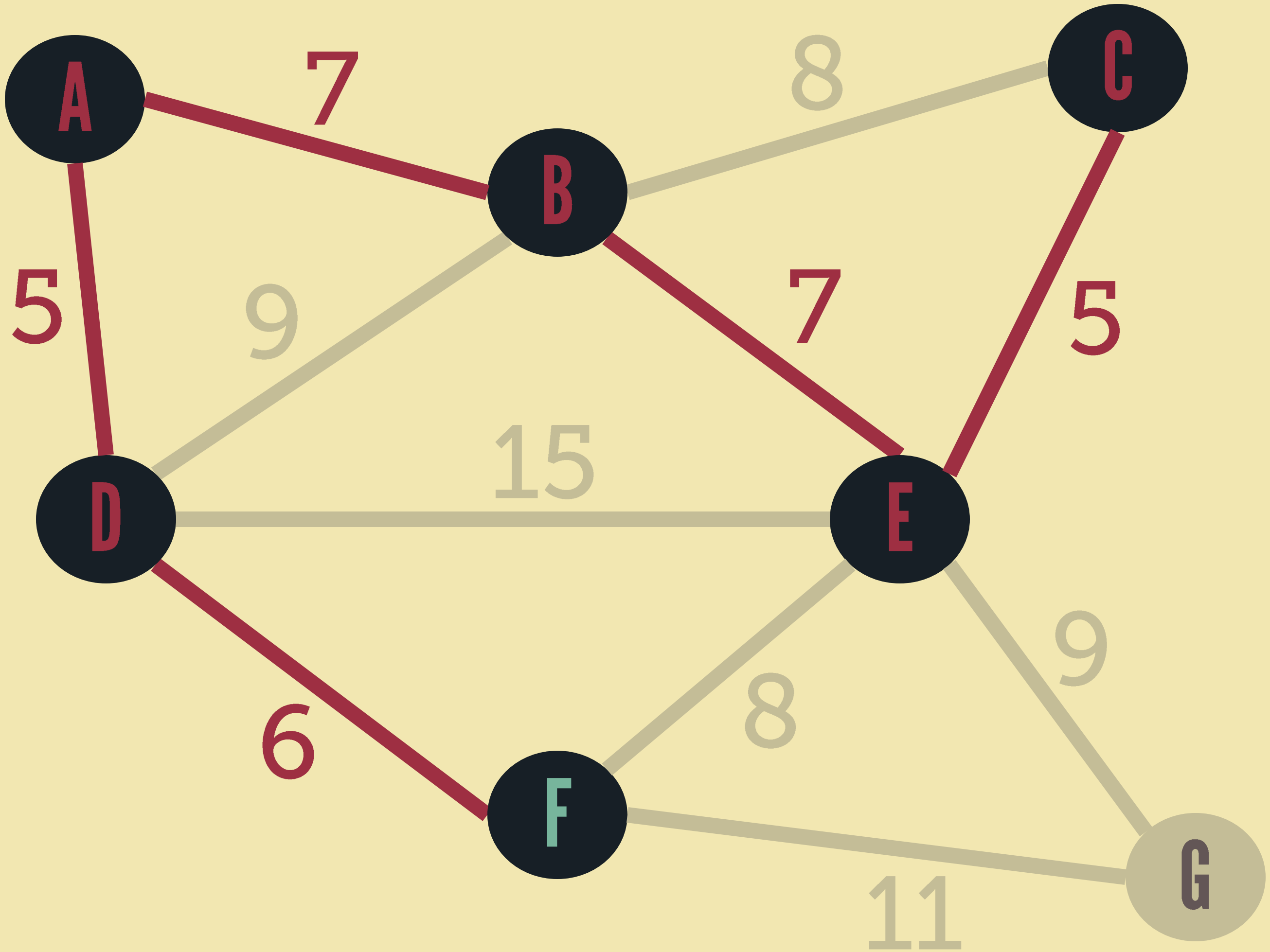
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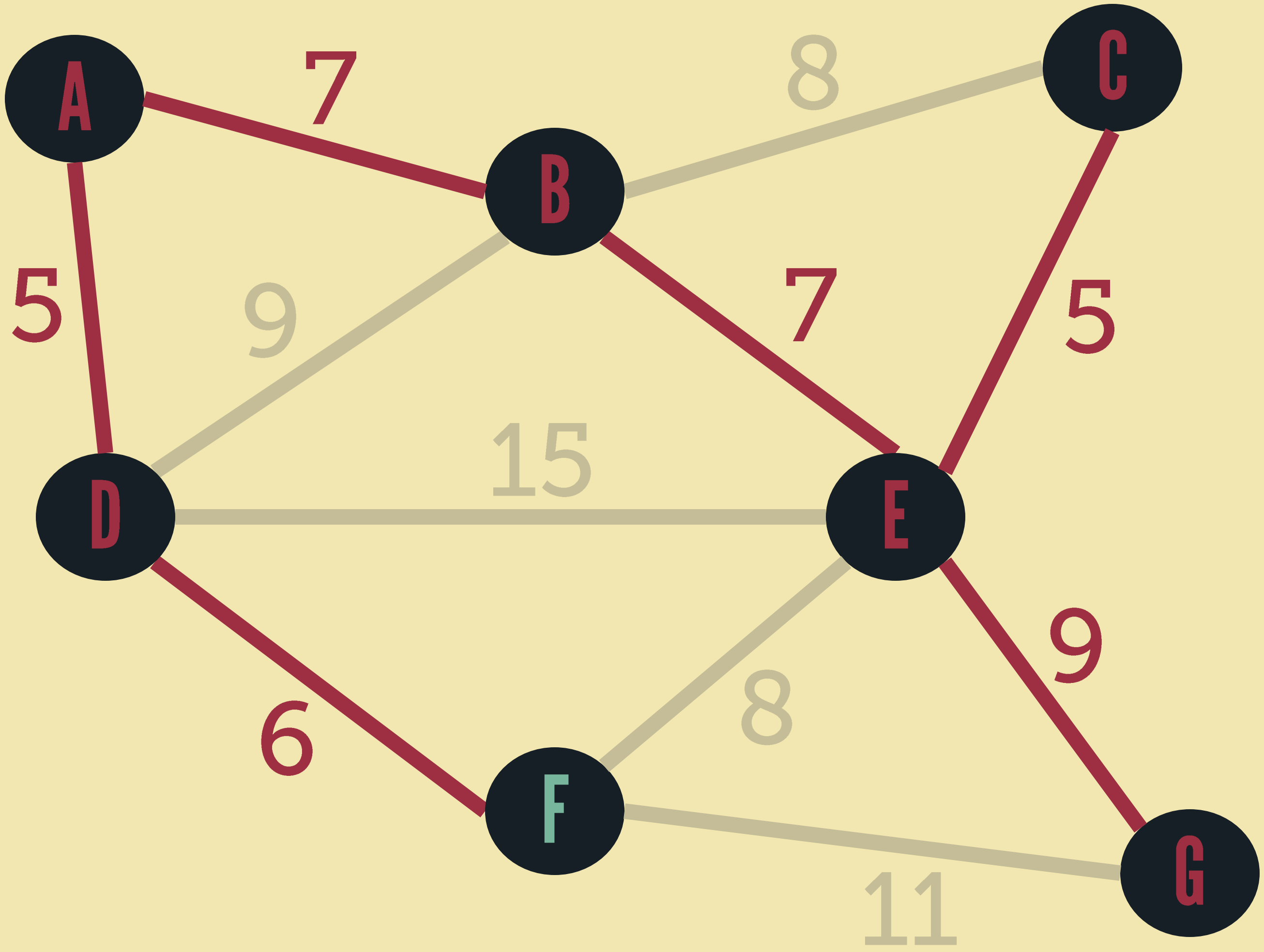

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# TRAVELLING SALESPERSON PROBLEM

Given a weighed graph, find a **HAMILTONIAN CYCLE** such that the sum of the weights of the edges is the **SMALLEST** possible.

# BRUTE FORCE algorithm

- Generate all possible Hamiltonian cycles

- Select the Hamiltonian cycle with the least cost.

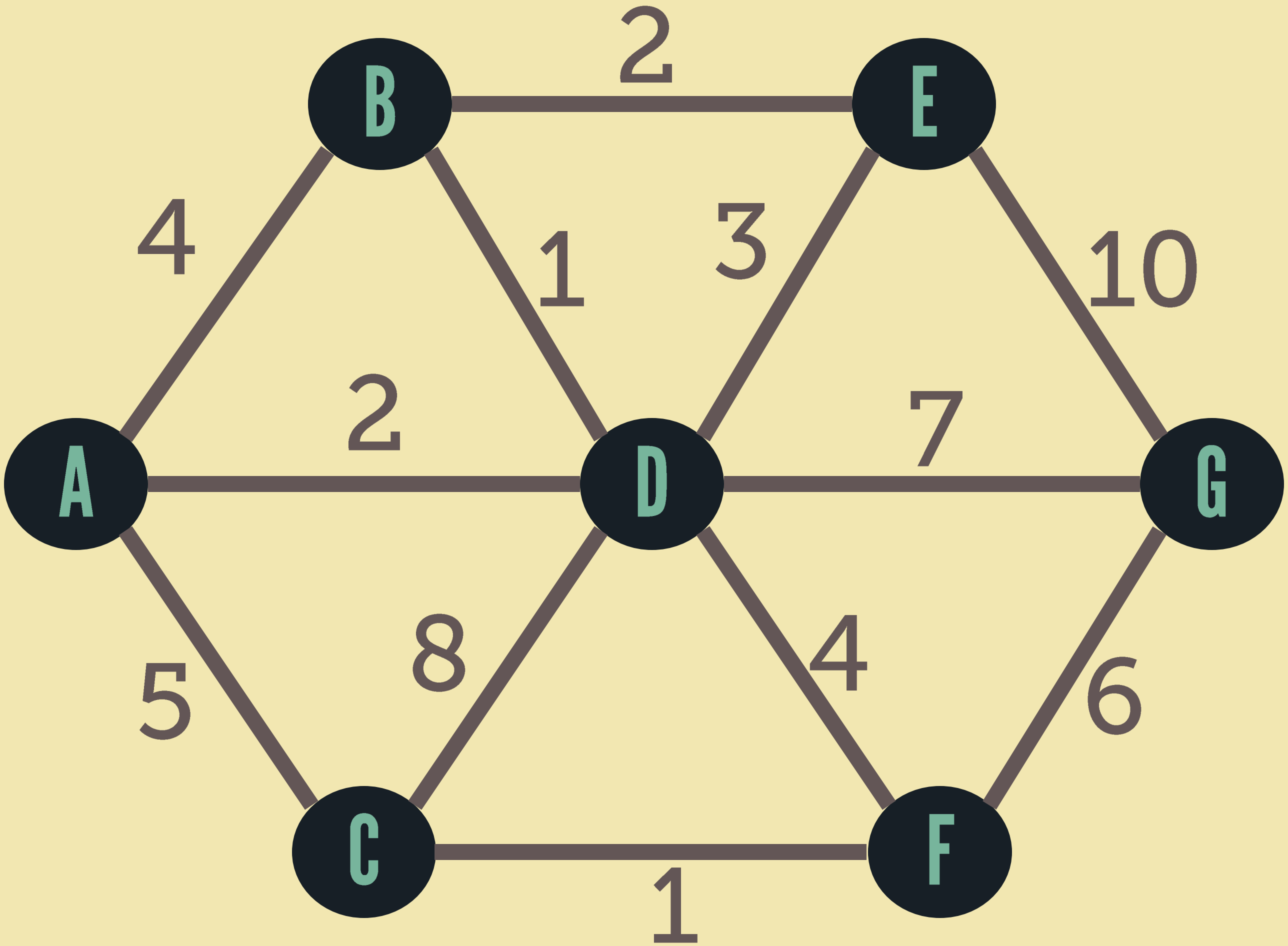
# GREEDY algorithm

Modified Kruskal's algorithm

- Start with a null graph T with vertices of G.

- Add currently cheapest edge from G to T that
    - will not form a cycle in T
    - will not cause a vertex in T to have a degree of 3 or more.

- Repeat previous step until # edges = # vertices.



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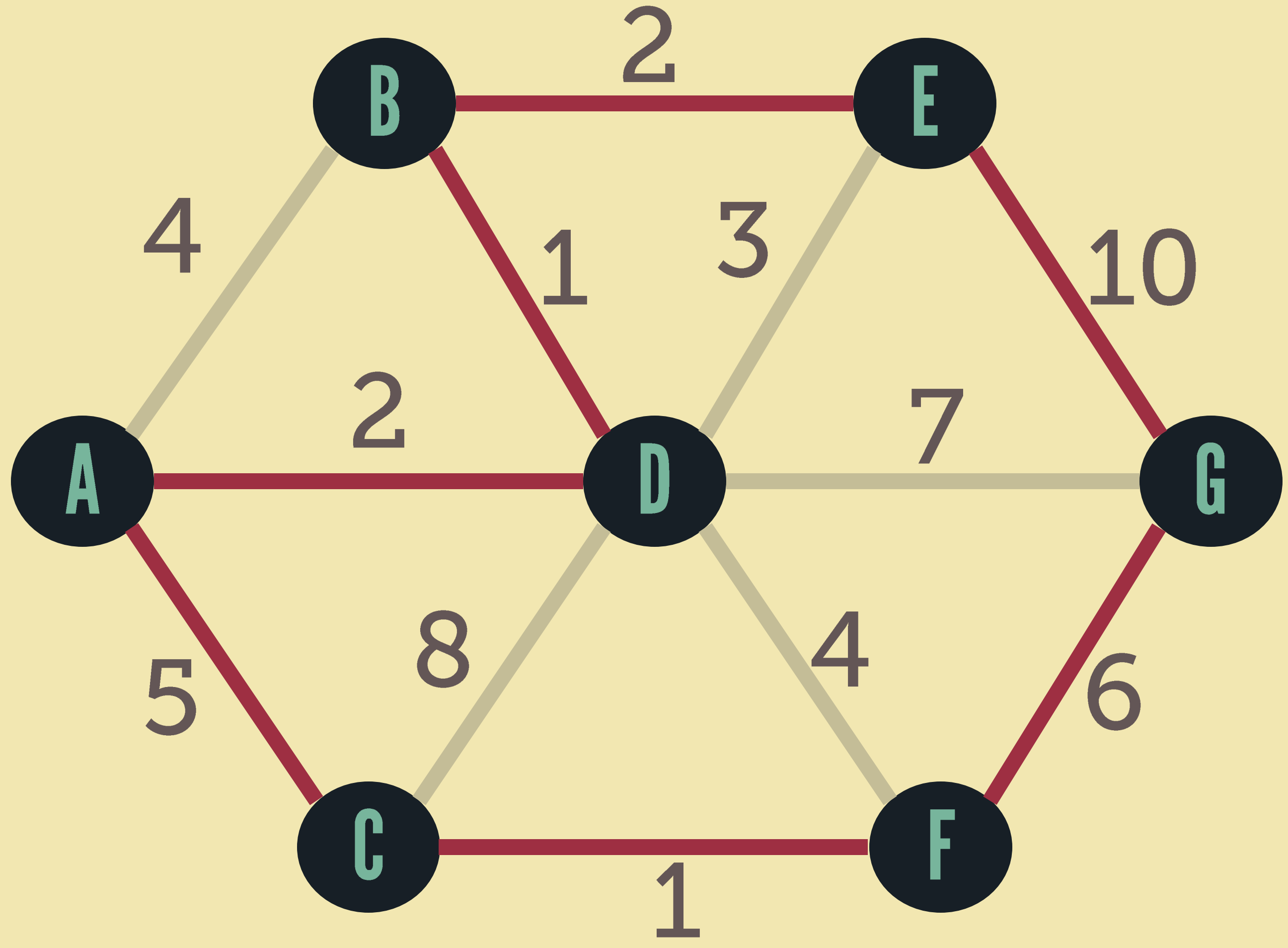

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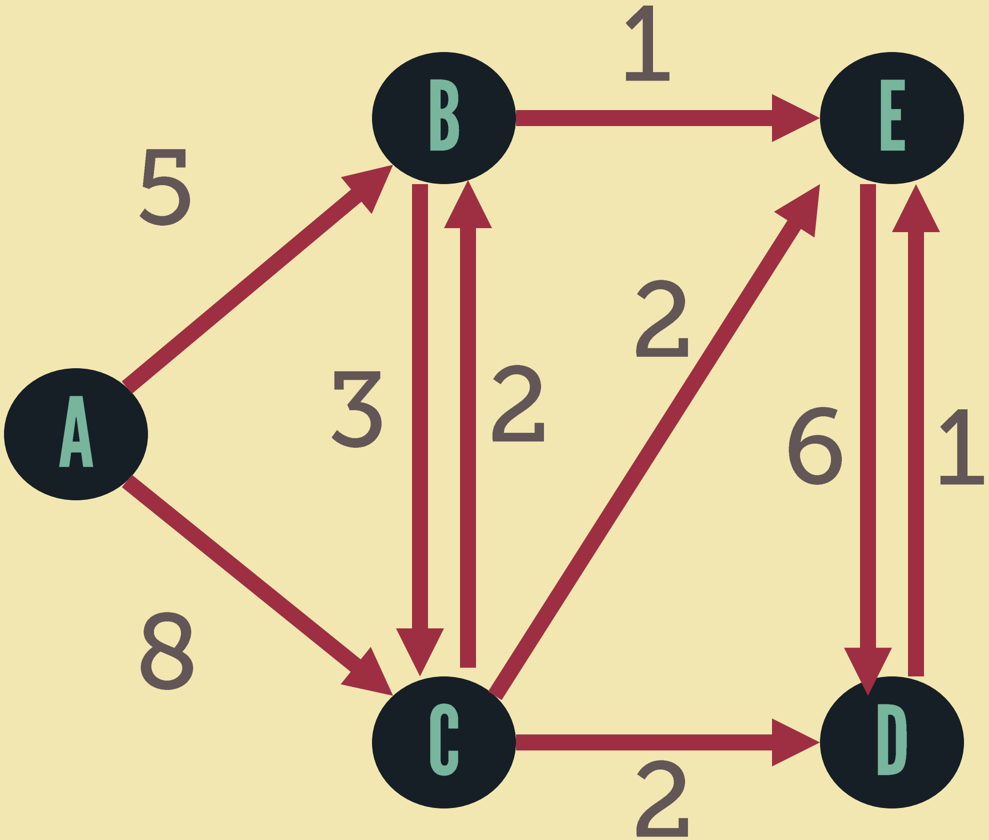
# SHORTEST PATH PROBLEM

Given a weighed (directed) graph, find the **SHORTEST PATH** from vertex u to v.

DIJKSTRA'S algorithm

FLOYD'S algorithm

- Find the shortest paths from a specified source vertex to every other vertices in the graph.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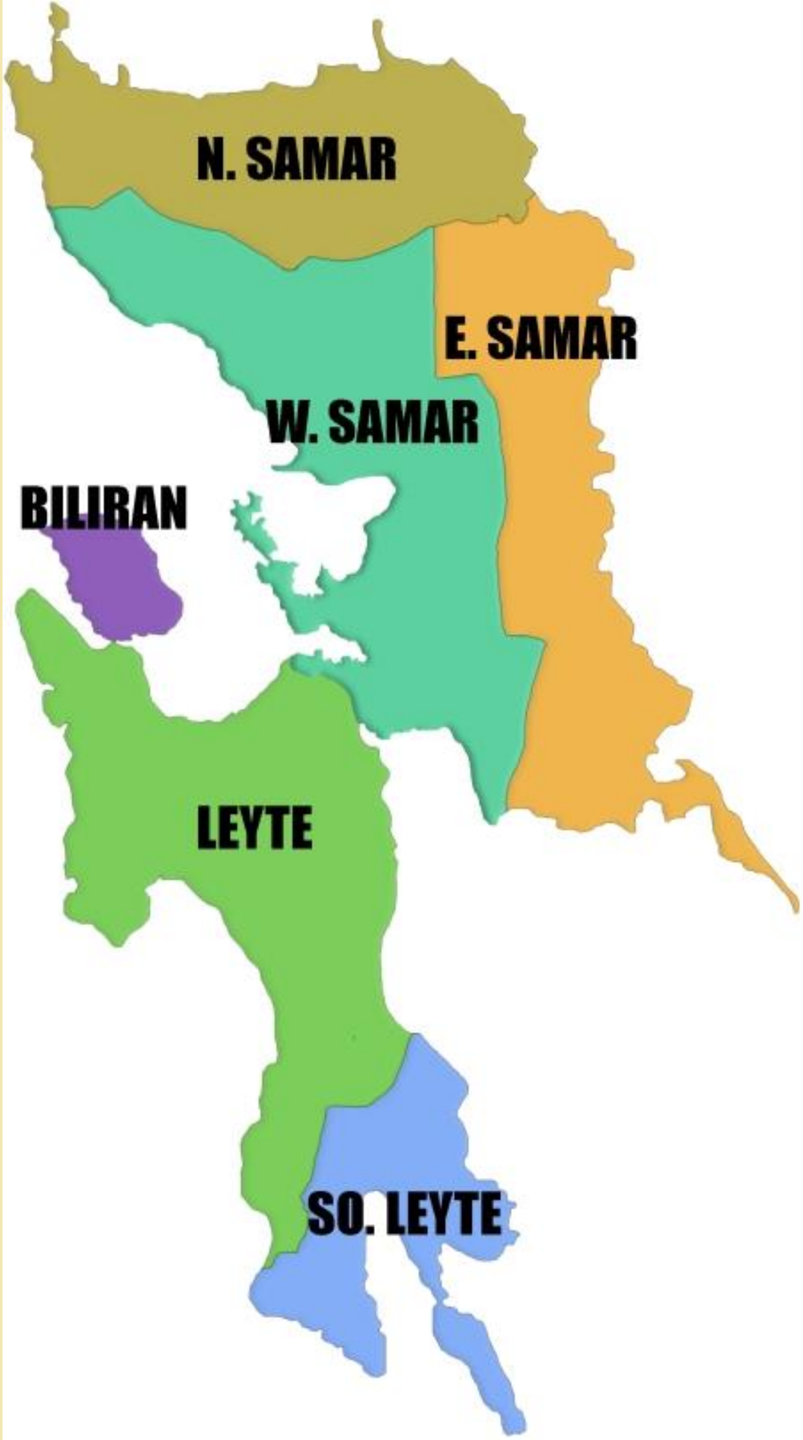
# GRAPH COLORING

# VERTEX COLORING

Color the vertices of a graph such that no two adjacent vertices have the same color.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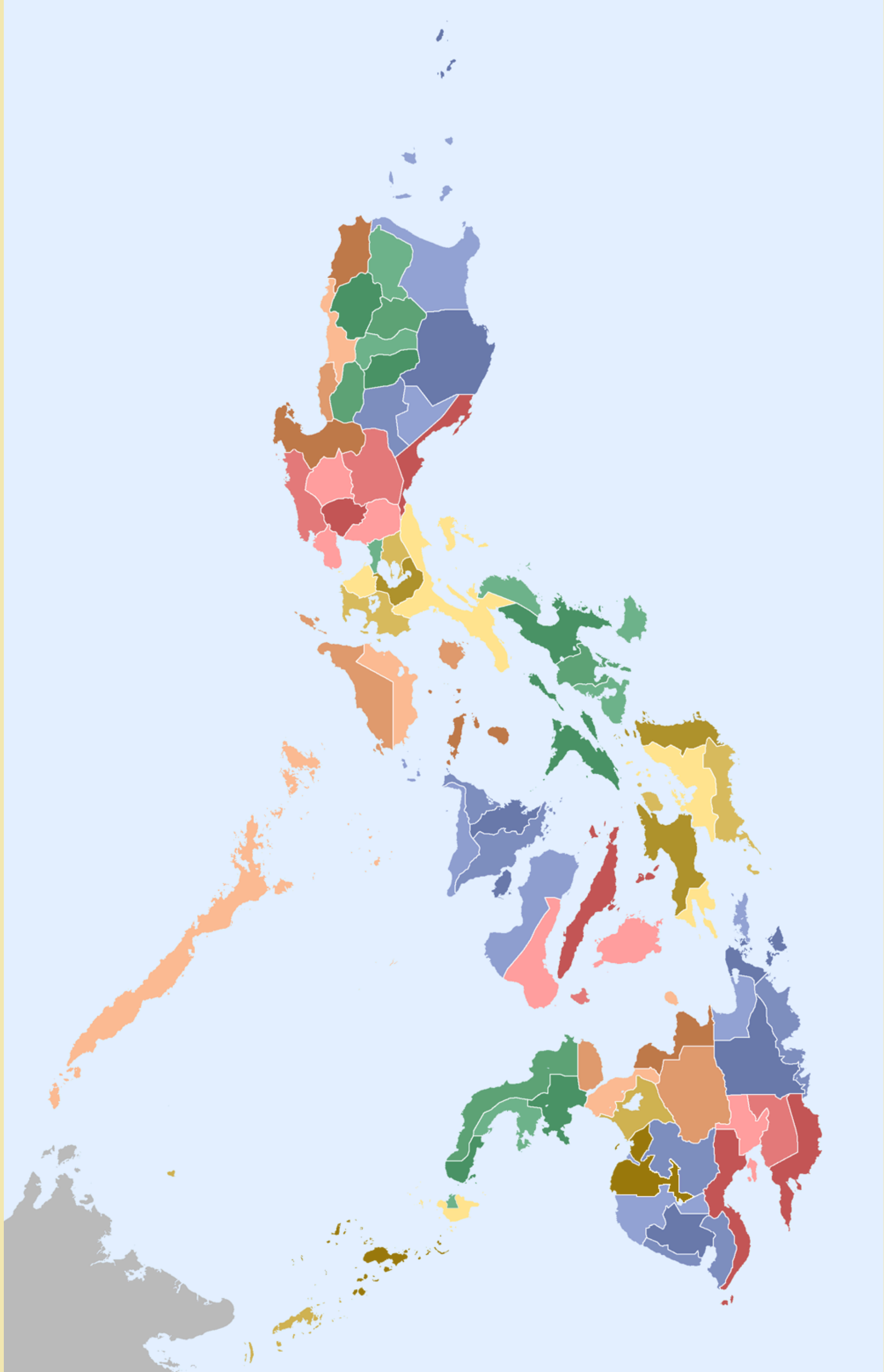
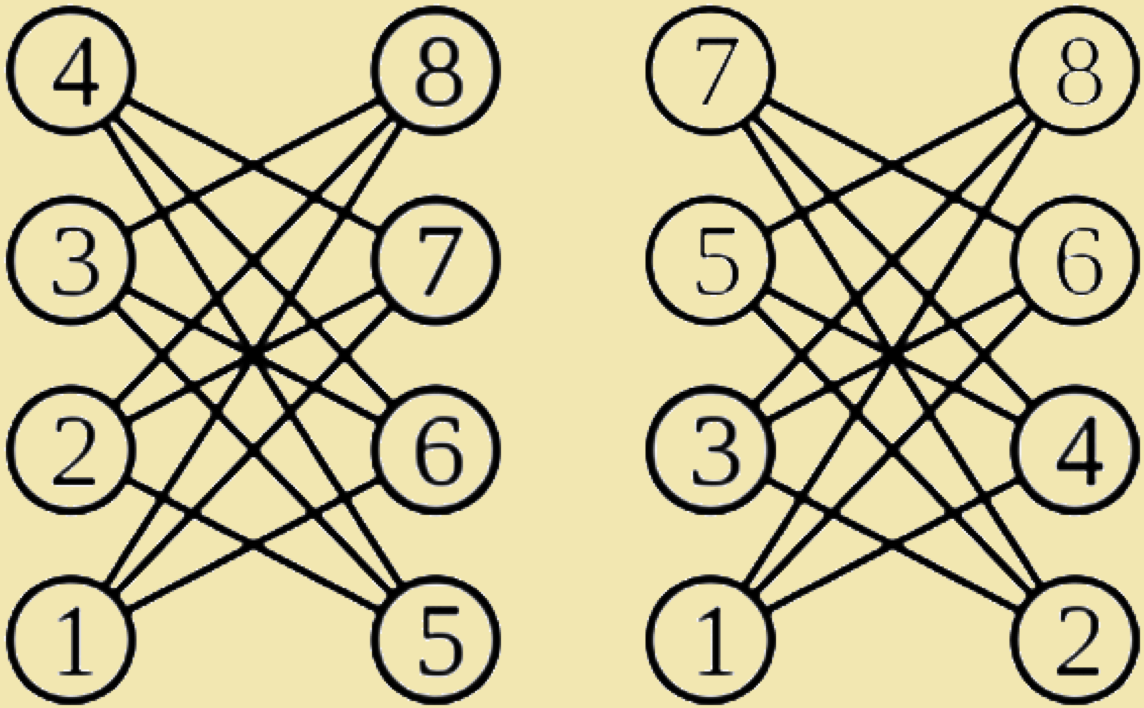
BILIRAN
LEYTE
SO. LEYTE


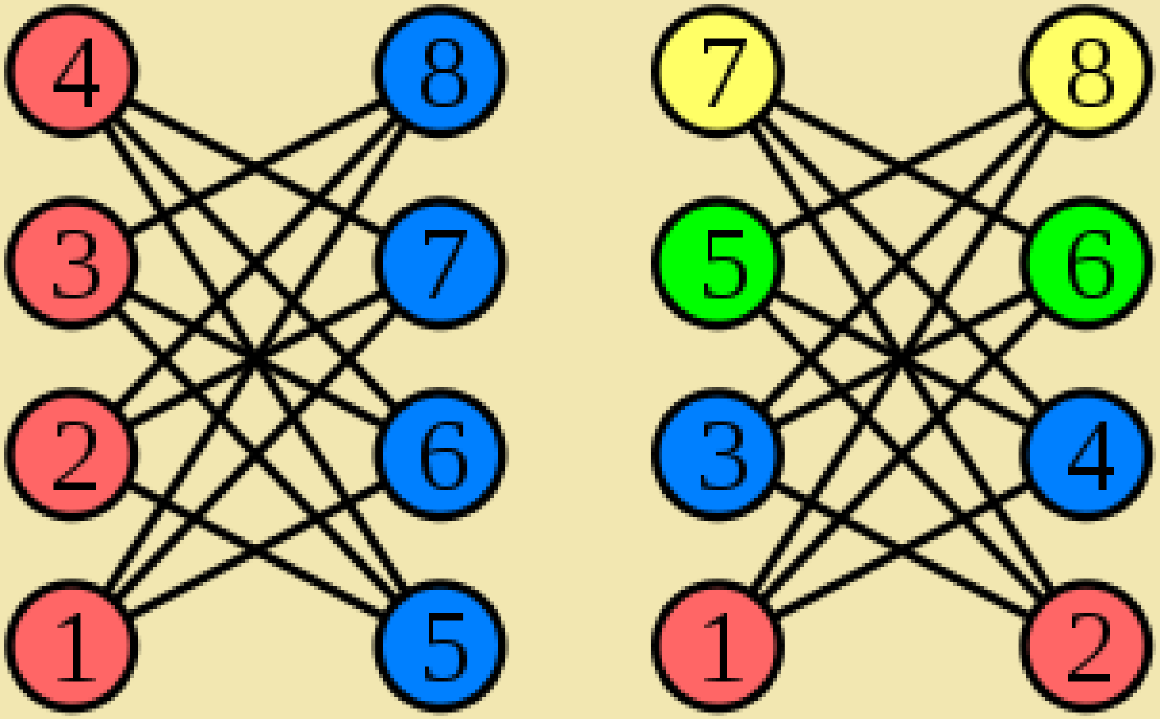
# GREEDY algorithm

- Consider the vertices in a specific order $v_1, \ldots v_n$.

- Assign to $v_i$ the smallest available color not used by $v_i$'s neighbors (add a new color if needed).




by Thore Husfeldt

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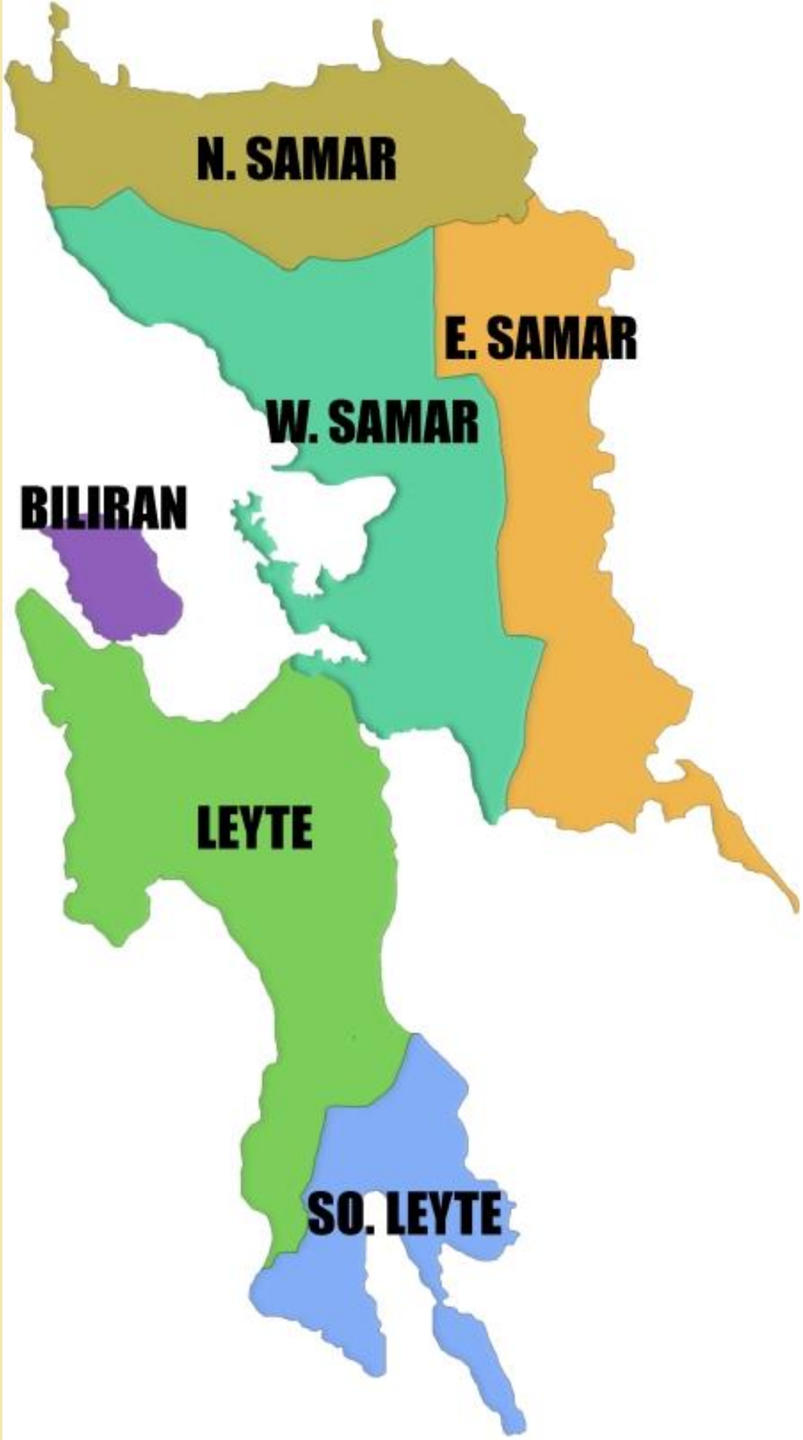
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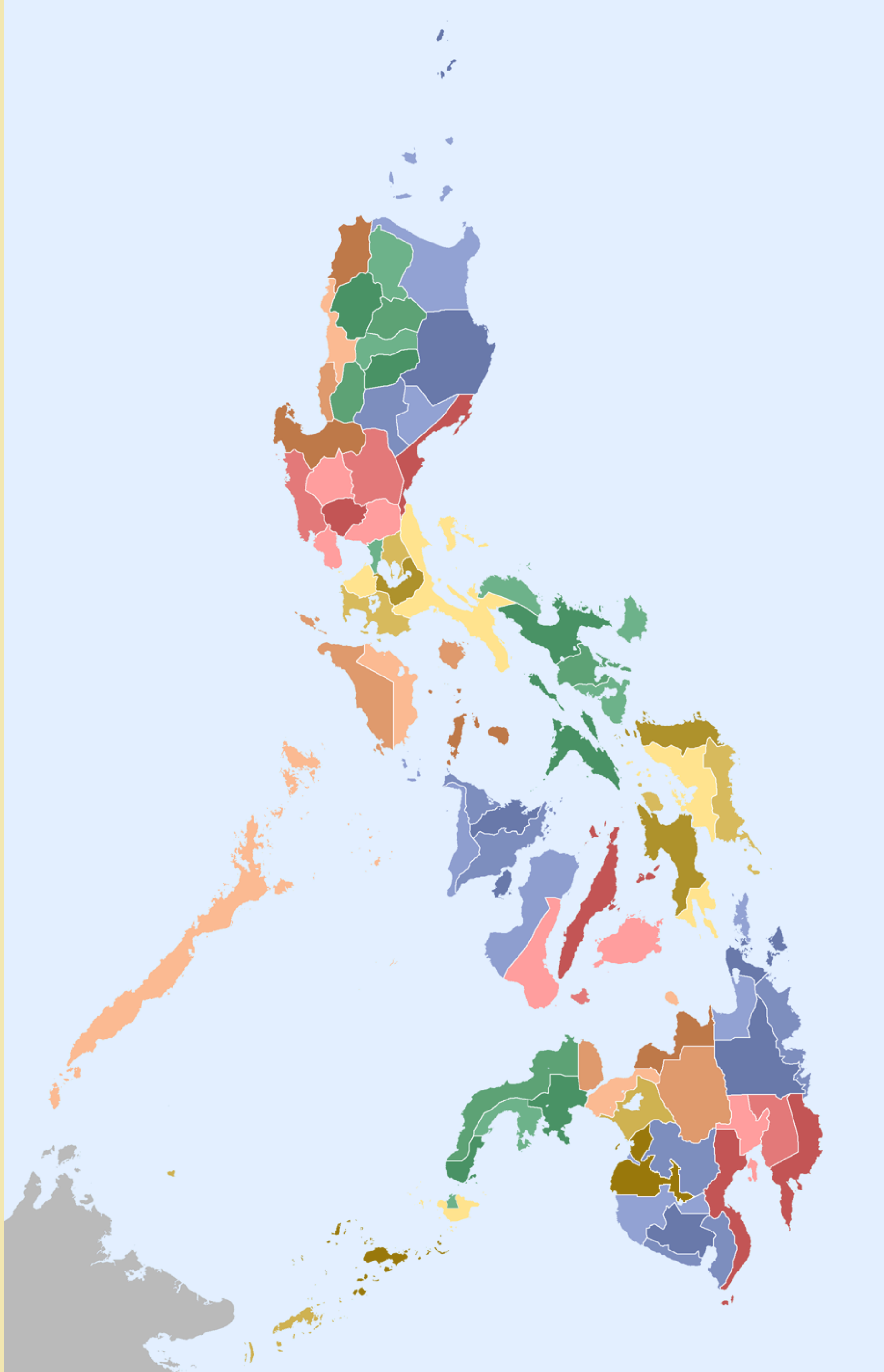
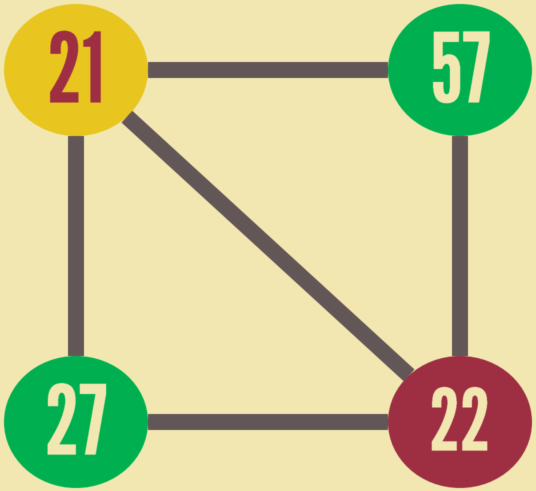


What is the minimum number of time slots needed to schedule 4 exams given the following:

CMSC 21: Annie, Armin
CMSC 57: Annie, Mikasa, Eren
MATH 27: Armin
CMSC 22: Annie, Armin, Eren, Jean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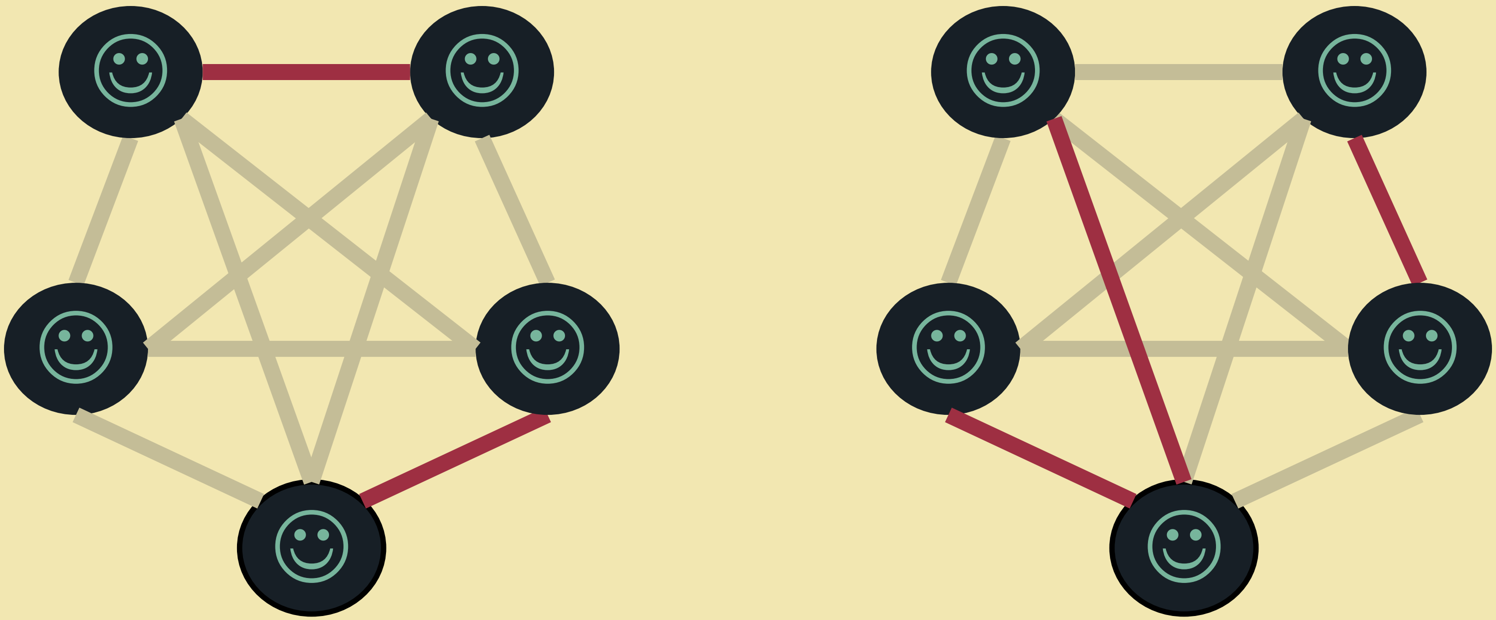
# GRAPH MATCHING

Pair off as many vertices as possible, that is, find a **MAXIMAL MATCHING** for a given graph.

# MATCHING

Set of edges where no two edges are adjacent

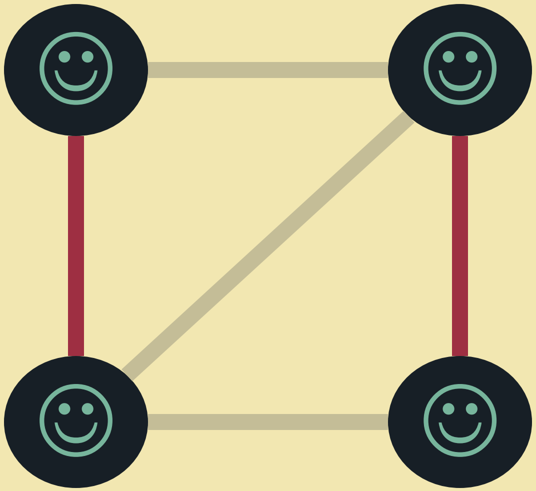


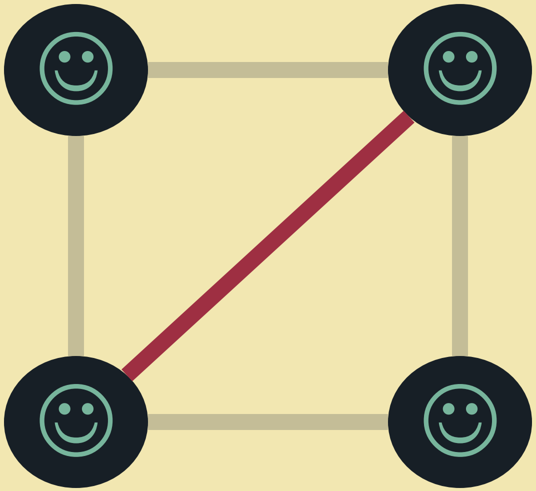
# MAXIMAL MATCHING

Matching with the largest number of edges.

# PERFECT MATCHING

A matching where every vertex in G is matched.





# AUGMENTING PATHS

method