Chapter 4 (Part 2)
Simplification of Logic Circuits

Simplification Example

- Simplify: **B’C’D’ + A’BC’D’ + A’B’CD + AB’CD + A’B’CD’ + A’BCD’ + ABCD’ + AB’CD’ + ABC’D’**

Simplification Example

- Simplify: B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'

AB \ CD	00	01	11	10
00				
01				
11				
10				

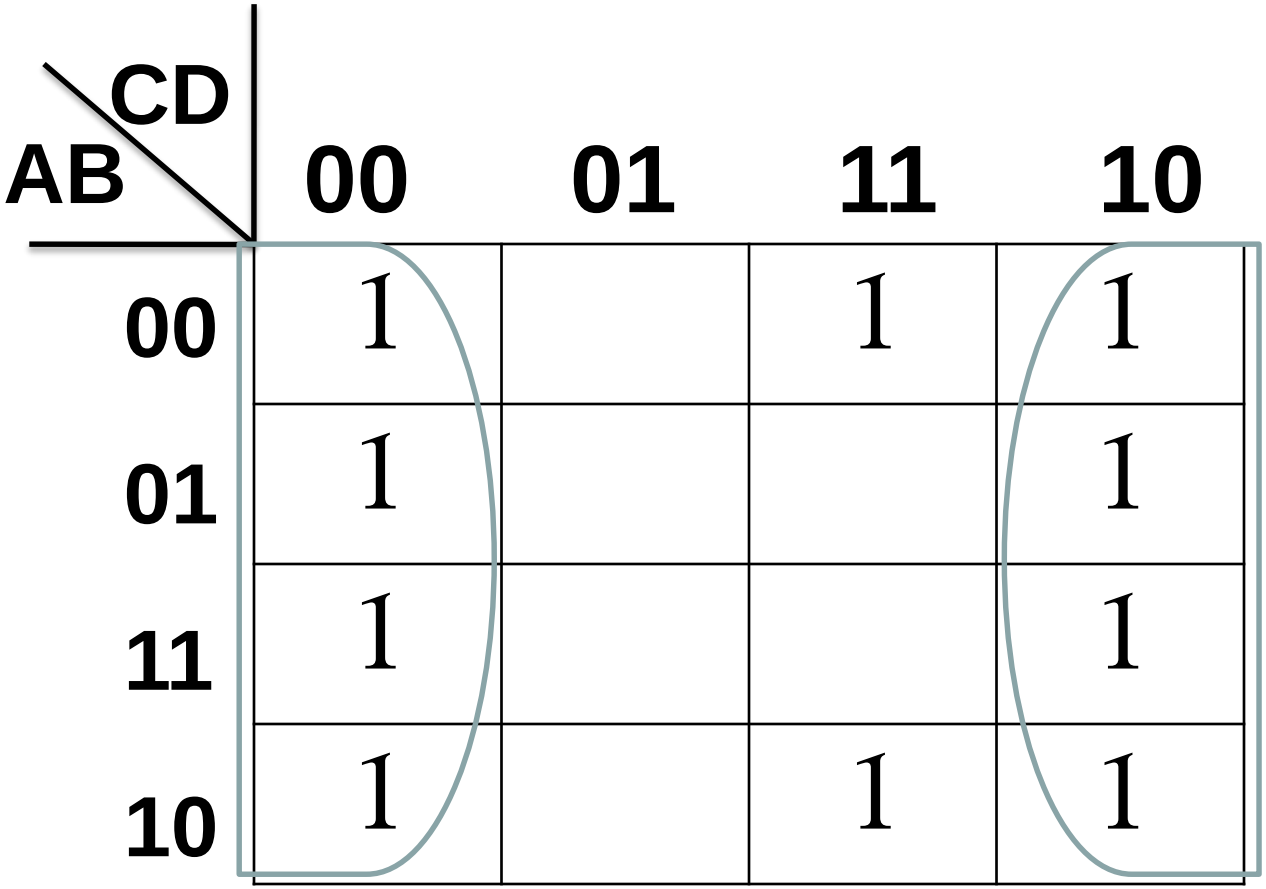
Simplification Example

- Simplify: B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'

AB \ CD	00	01	11	10
00	1		1	1
01	1			1
11	1			1
10	1		1	1

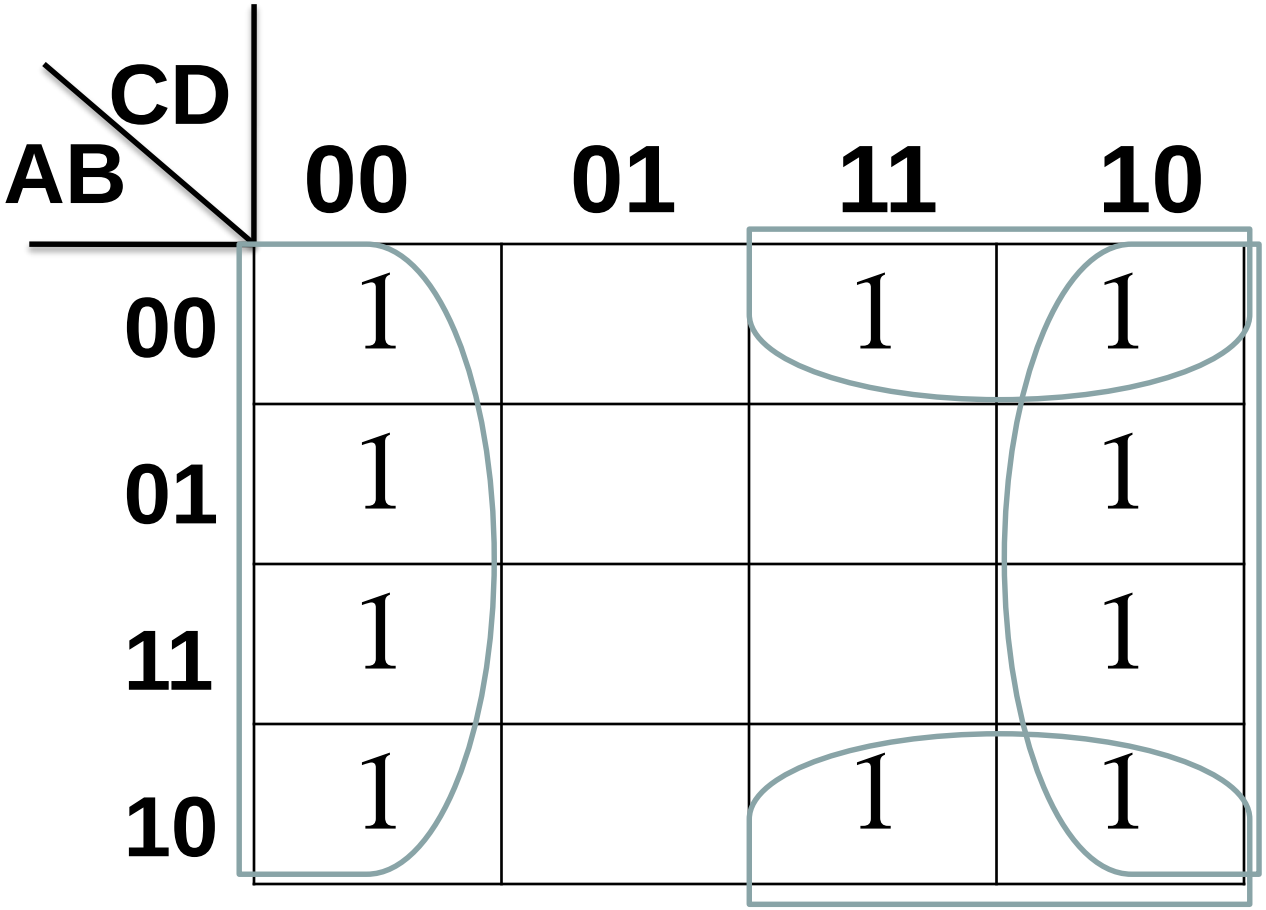
Simplification Example

- Simplify: B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



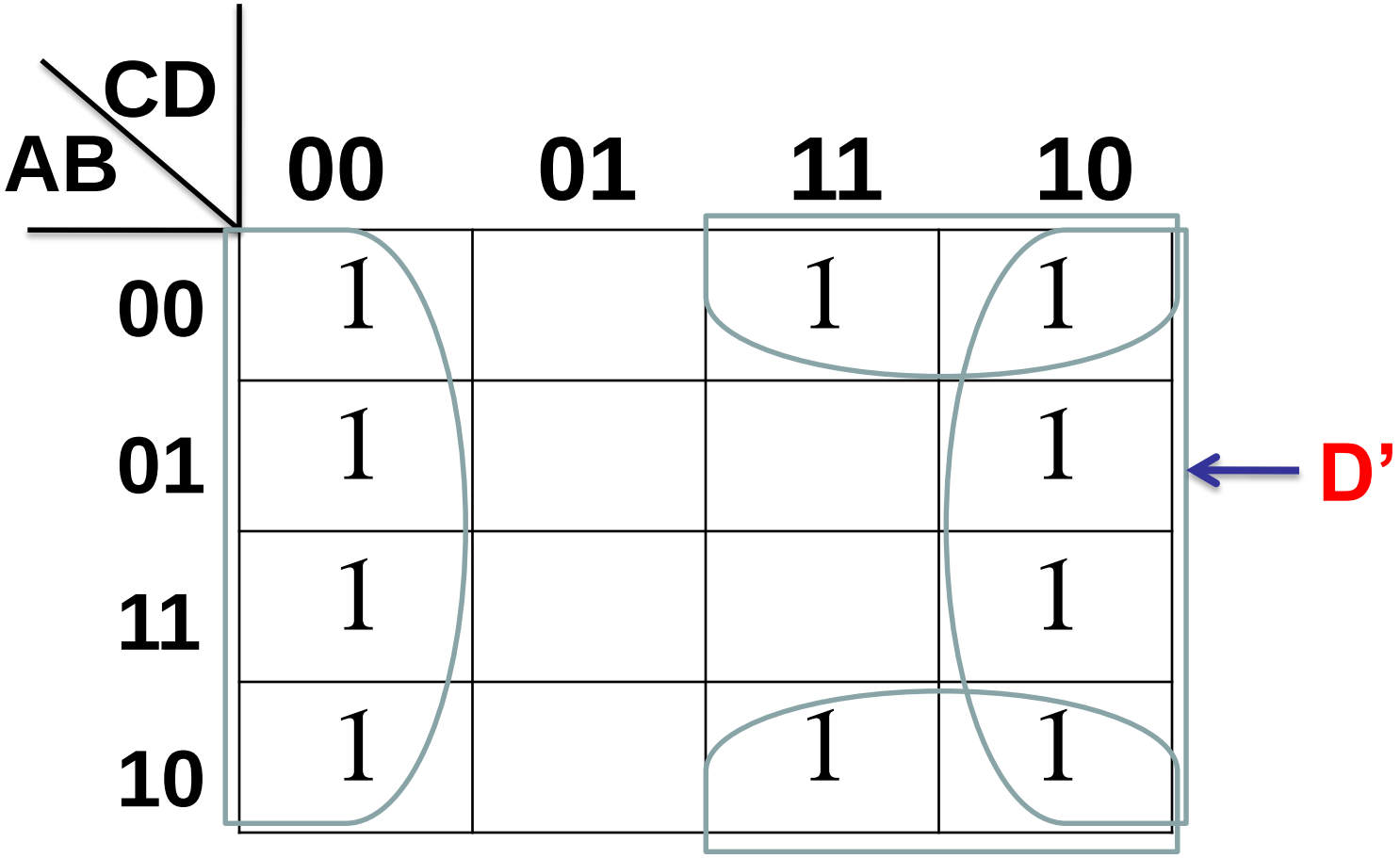
Simplification Example

- Simplify: B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



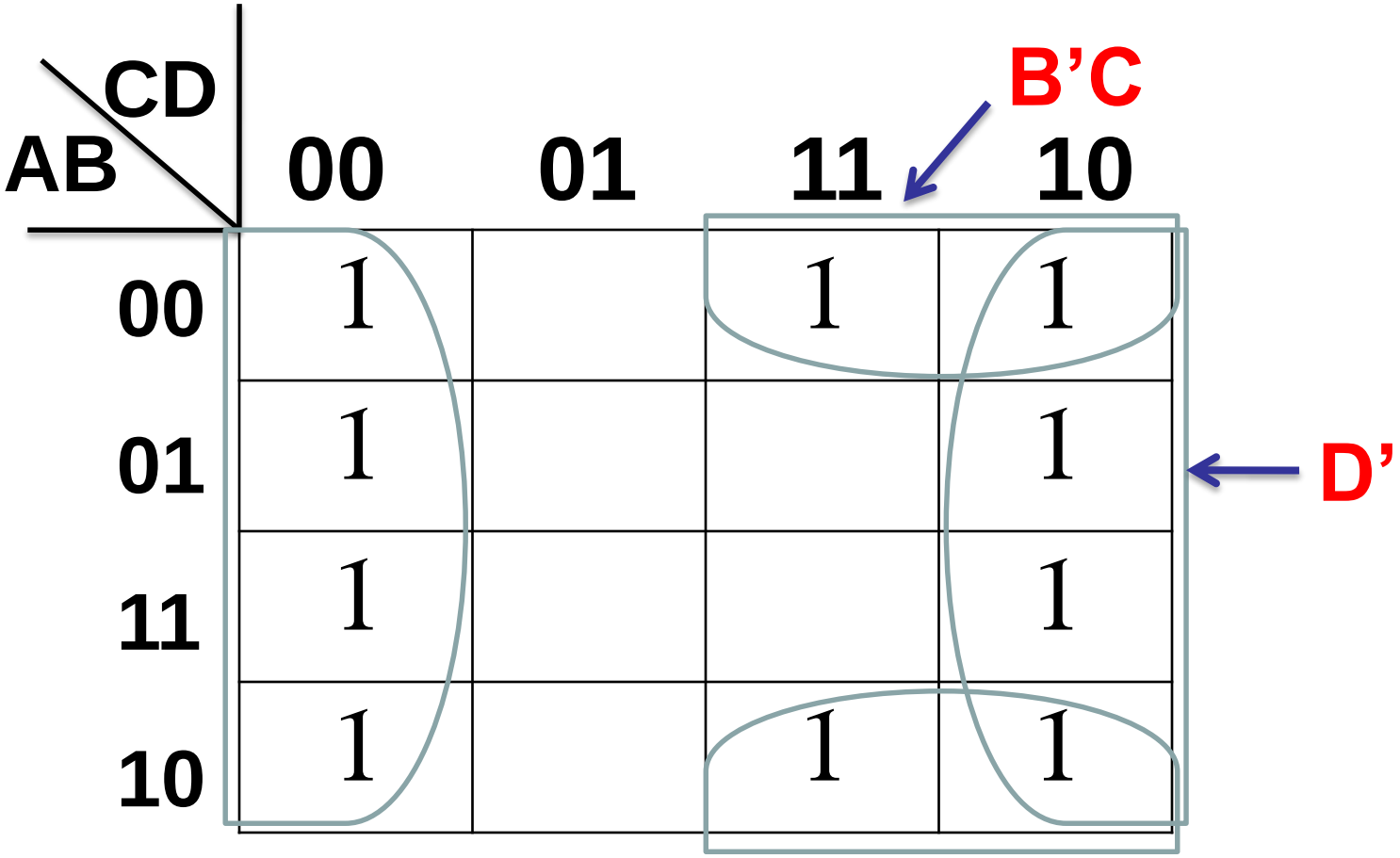
Simplification Example

- Simplify: B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



Simplification Example

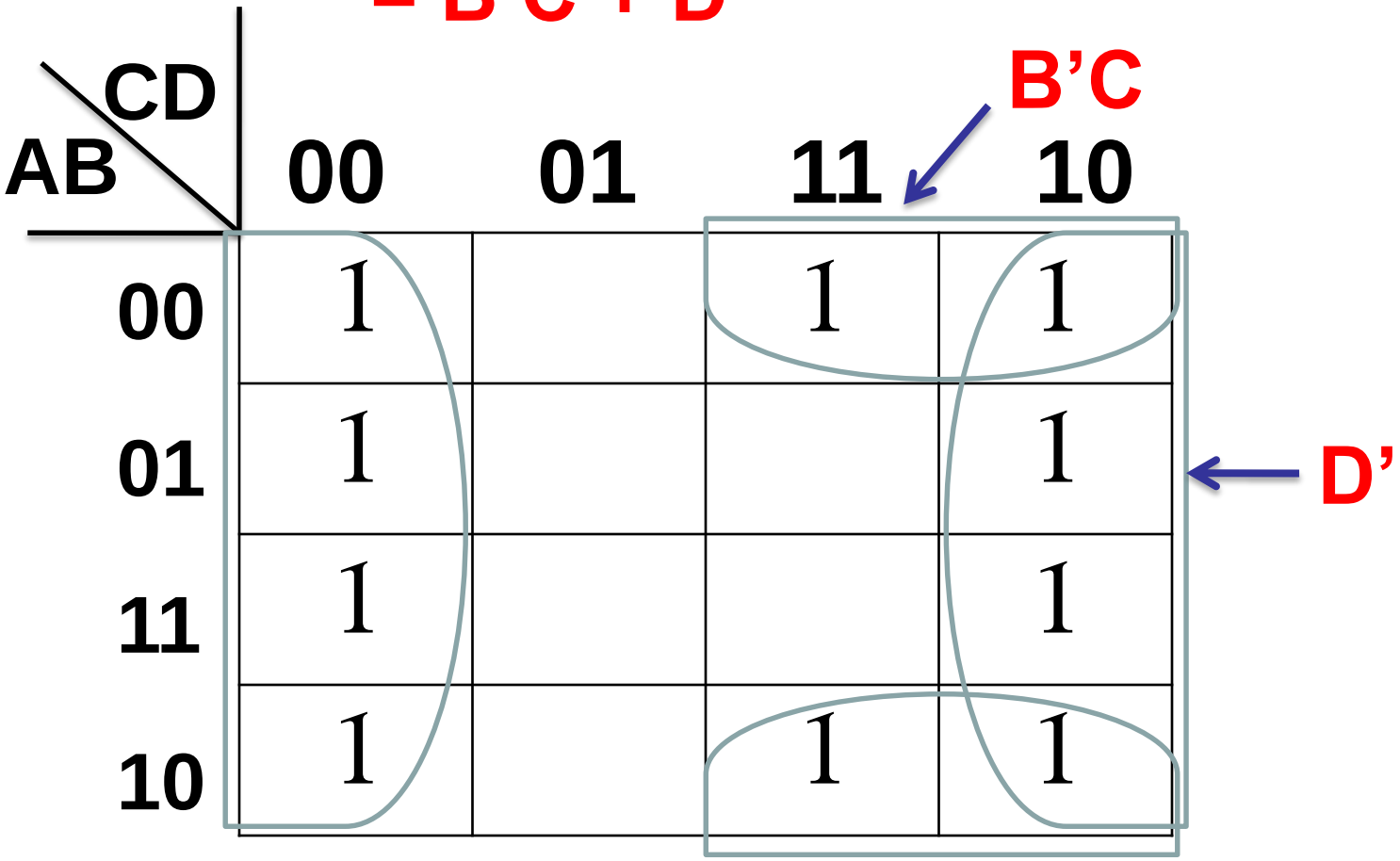
- Simplify: B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'



Simplification Example

- Simplify: B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'

= B'C + D'



Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	00	01	11	10
00				
01				
11				
10				

Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	00	01	11	10
00	0			
01				
11				
10	0			

Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	00	01	11	10
00	0			0
01				
11				
10	0			

Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	00	01	11	10
00	0			0
01				
11				
10	0	0		

Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	00	01	11	10
00	0			0
01	0			
11				
10	0	0		

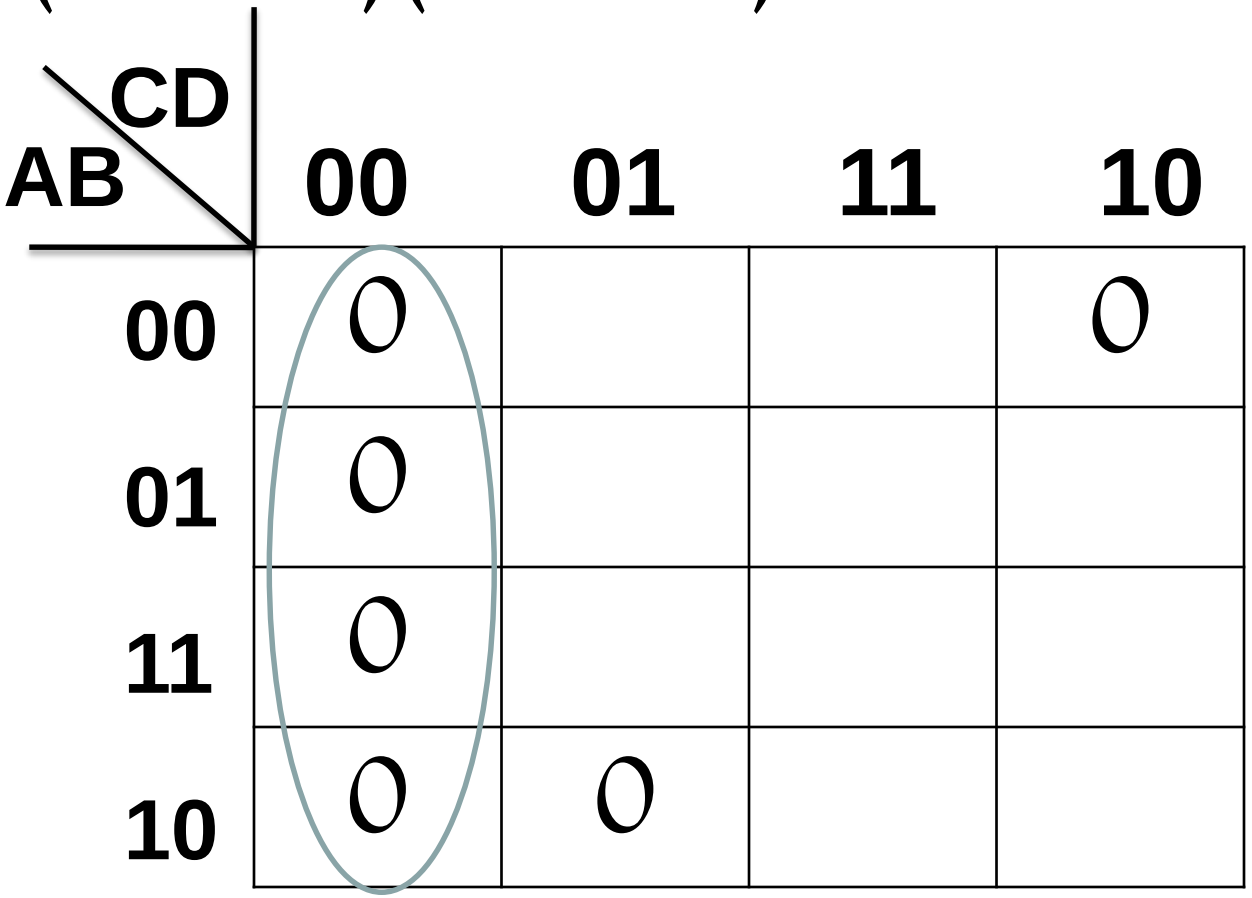
Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

AB \ CD	00	01	11	10
00	0			0
01	0			
11	0			
10	0	0		

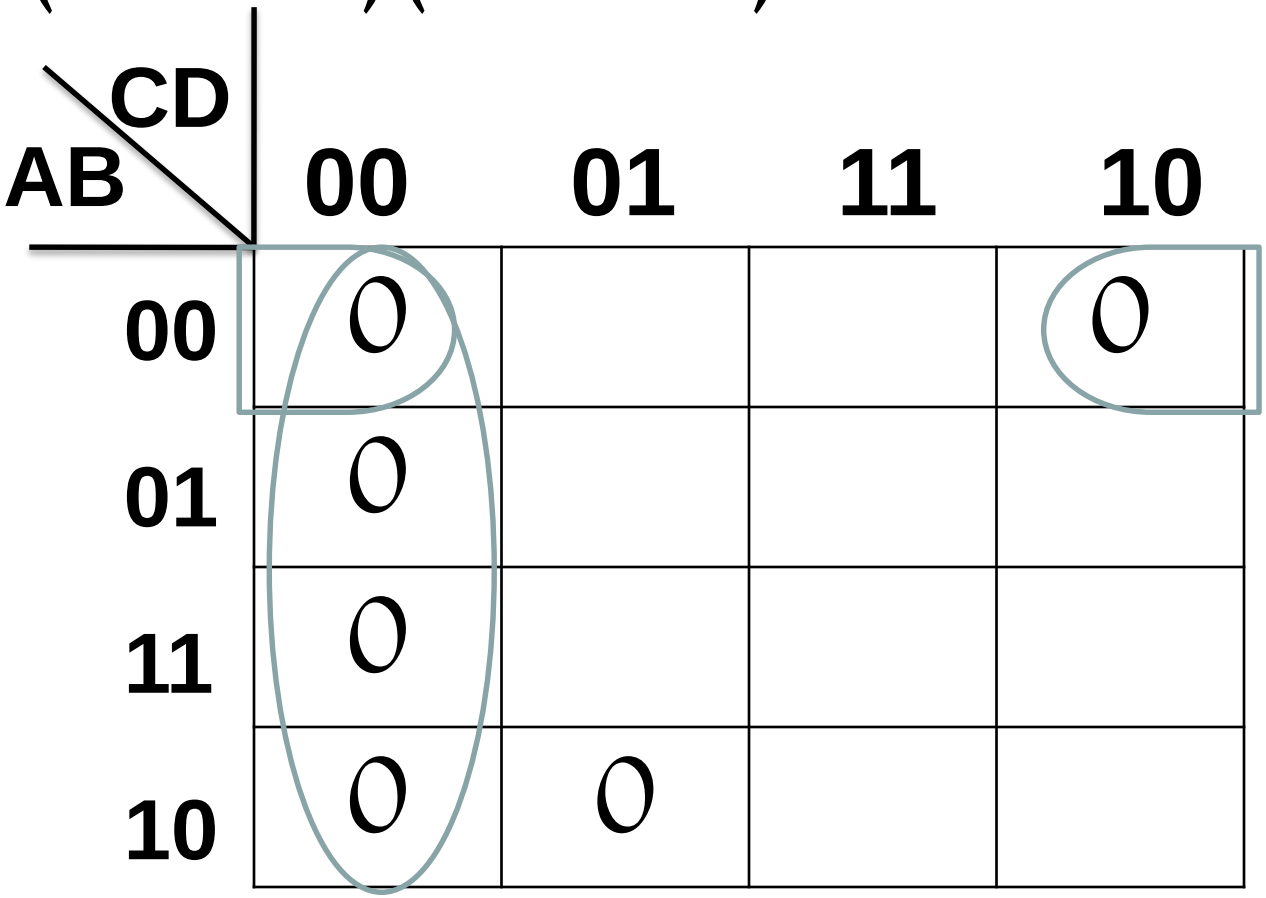
Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



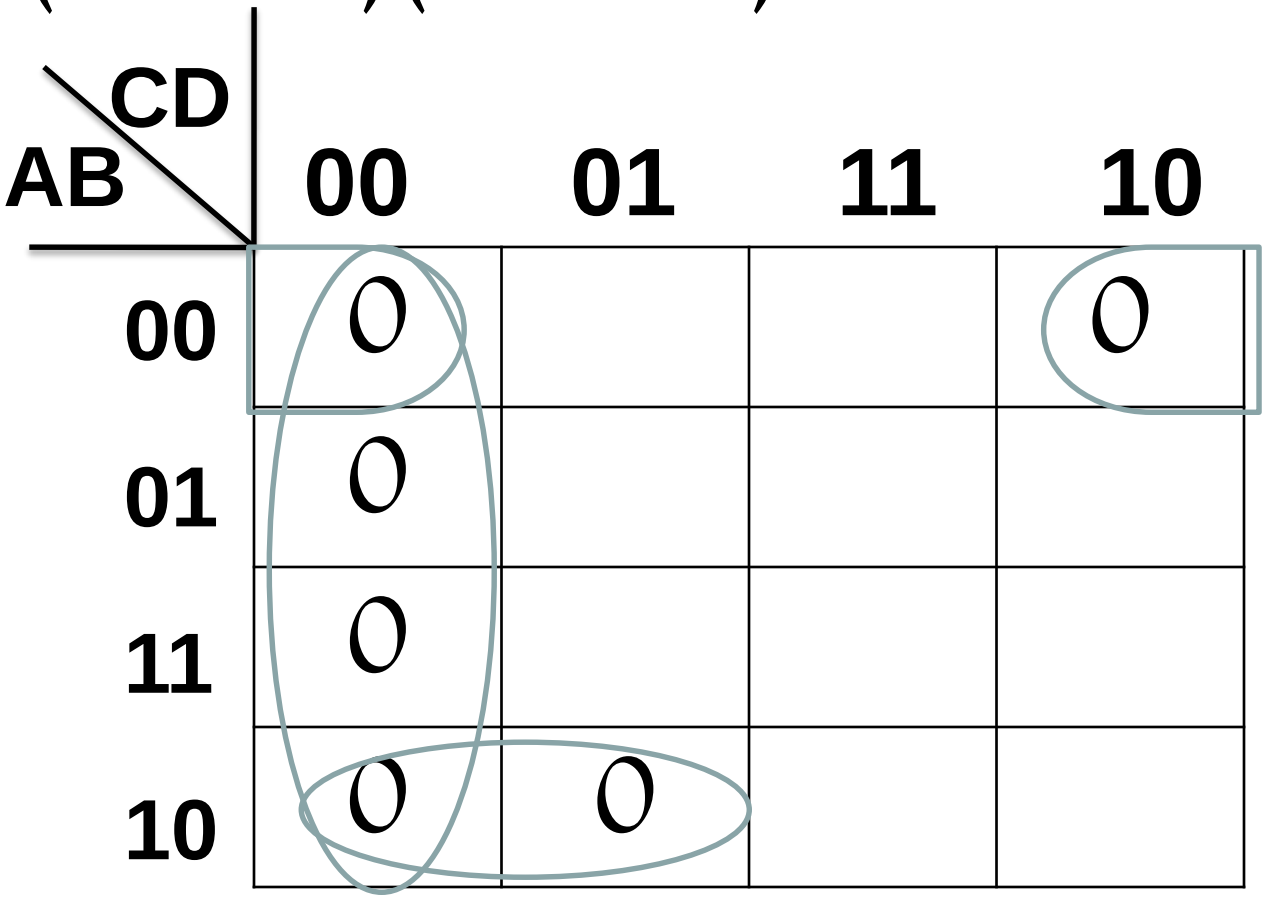
Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



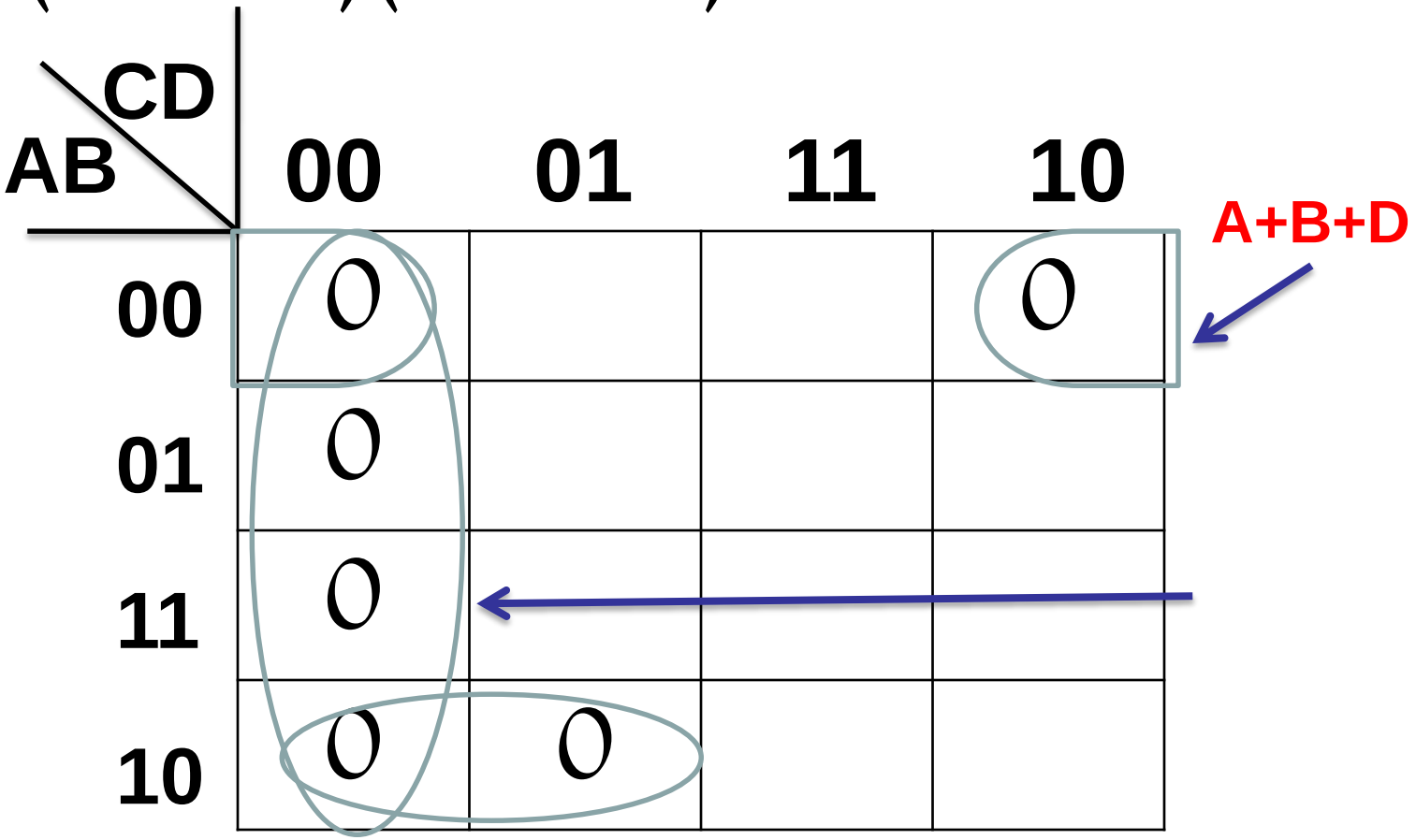
Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



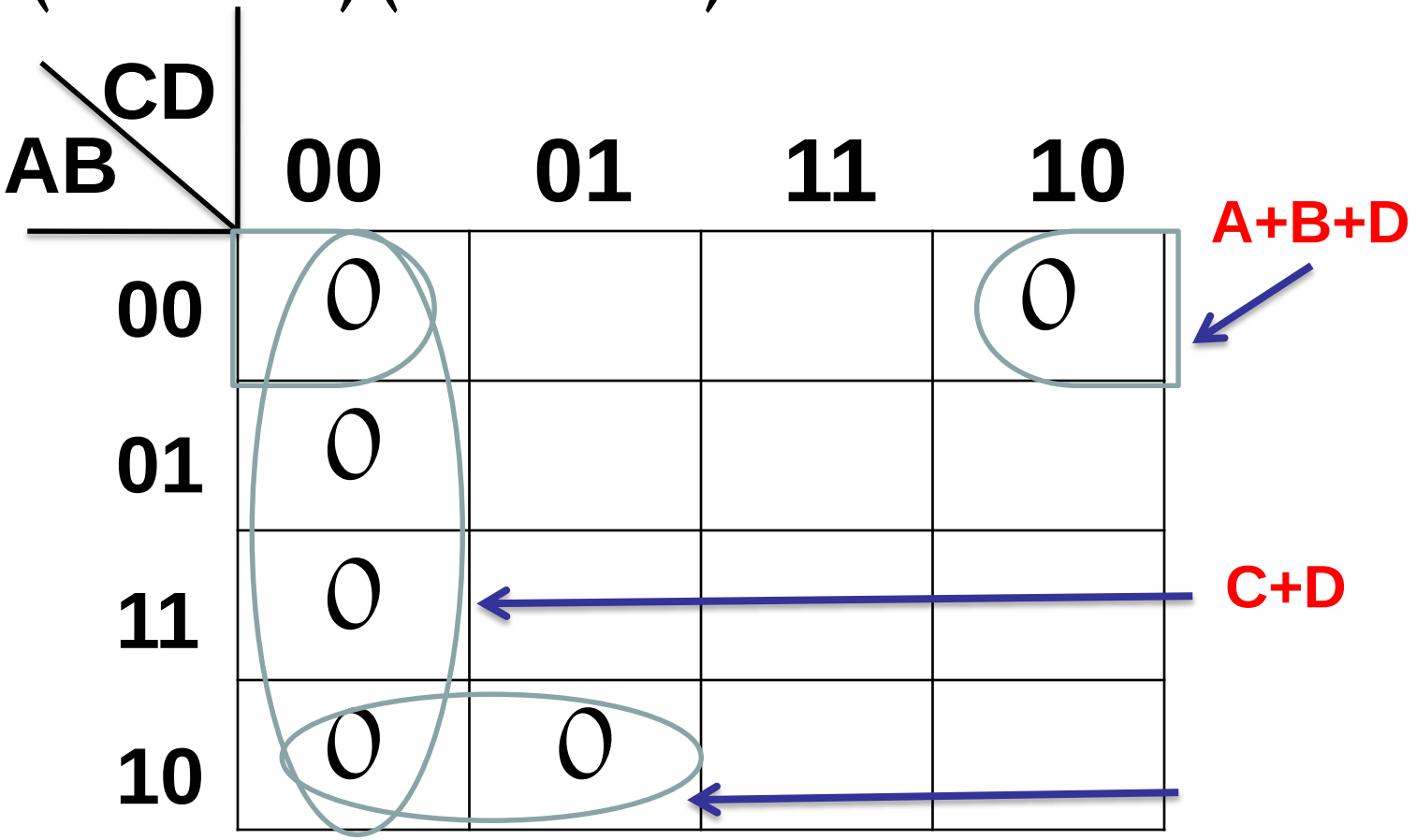
Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



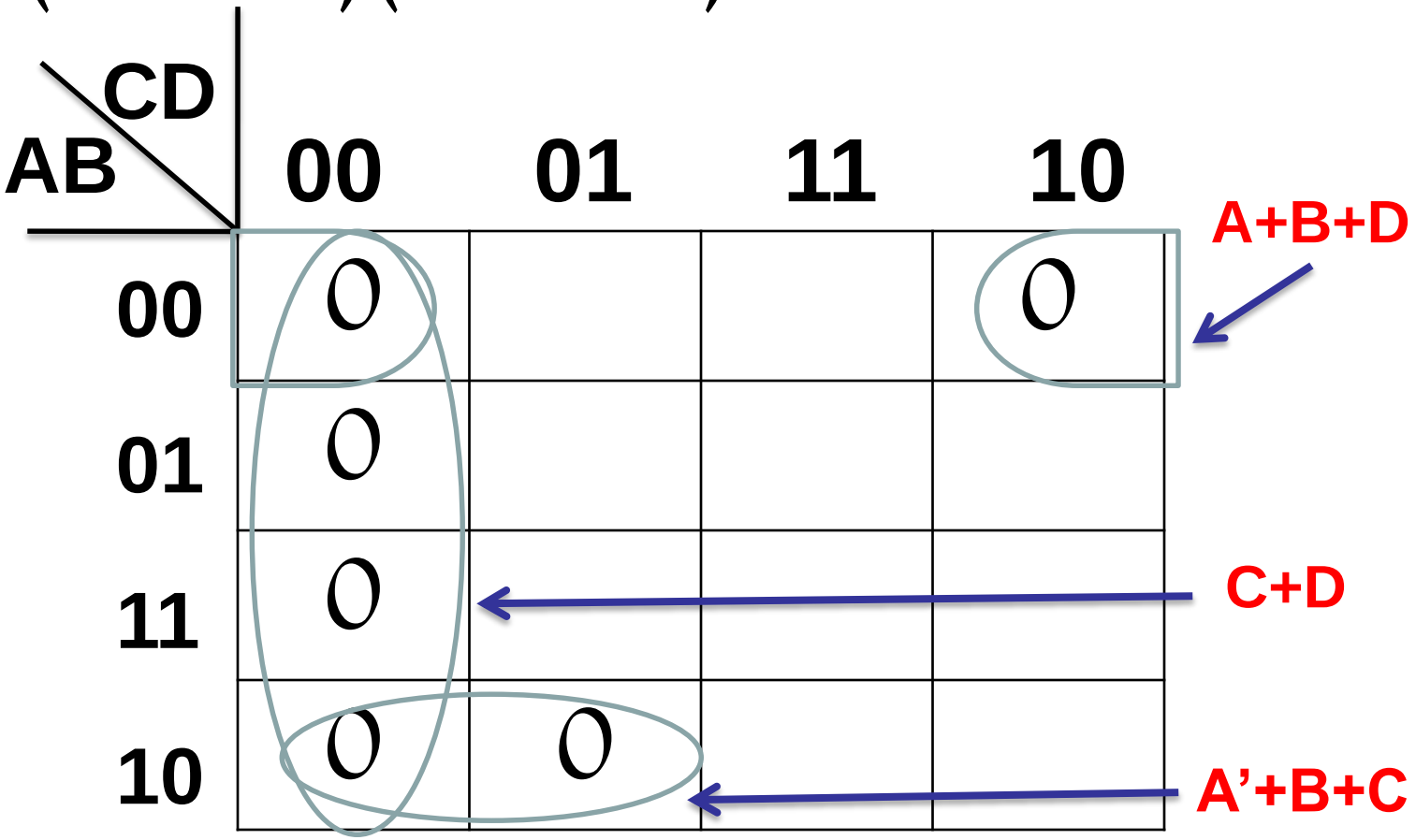
Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



Simplification Example

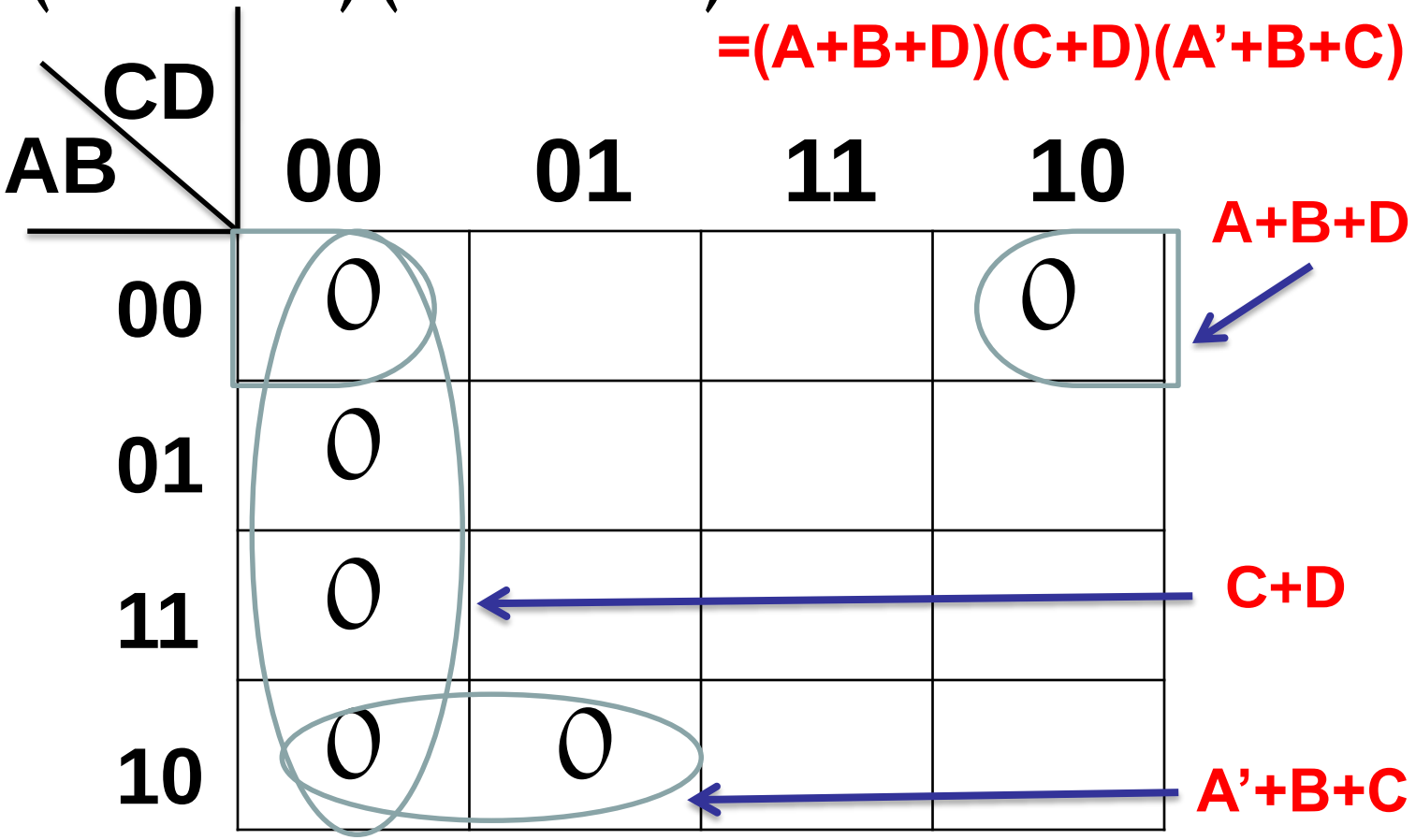
- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)



Simplification Example

- Simplify: (B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)

=(A+B+D)(C+D)(A'+B+C)



Advantages of using K-map

- Can be done more systematically
- Much simpler to find minimum solutions
- Easier to see what is happening

Simplification: Quine-McCluskey

- **Advantages**
  - Specific step by step procedure
  - Can be applied to problems with many variables
  - Suitable for machine computation

Simplification: Quine-McCluskey

- **Steps**
  - Construct prime implicants table
  - Construct prime implicants chart
  - Select all essential prime implicants
  - Select a minimum cover from the remaining prime implicants

Constructing Prime Implicants Table

- List terms in a column using their binary representation
  - Group terms so that each group contains minterms with the same number of 1's
  - Place groupings which differ by only one literal adjacent to one another

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

1	0001
3	0011
7	0111
8	1000
14	1110
15	1111

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

Column 1

1	0001	1	0001
3	0011	8	1000
7	0111		
8	1000	3	0011
14	1110		
15	1111	7	0111
		14	1110
		15	1111

Constructing Prime Implicants Table

- Perform exhaustive search for logically adjacent terms between adjacent groups
  - Each term should be checked off
  - Combine each pair of terms into a single term replacing the differing literal with '-'
  - Repeat procedure until no further terms can be created
  - All unchecked terms are prime implicants

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

		Column 1		Column 2
1	0001	1	0001	
3	0011	8	1000	
7	0111			
8	1000	3	0011	
14	1110			
15	1111	7	0111	
		14	1110	
		15	1111	

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111					
8	1000	3	0011	√		
14	1110					
15	1111	7	0111			
		14	1110			
		15	1111			

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111				3,7	0-11
8	1000	3	0011	√		
14	1110					
15	1111	7	0111	√		
		14	1110			
		15	1111			

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111				3,7	0-11
8	1000	3	0011	√		
14	1110				7,15	-111
15	1111	7	0111	√		
		14	1110			
		15	1111	√		

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00–1
3	0011	8	1000			
7	0111				3,7	0–11
8	1000	3	0011	√		
14	1110				7,15	–111
15	1111	7	0111	√	14,15	111-
		14	1110	√		
		15	1111	√		

Example

- Simplify: $F = \Sigma m(1,3,7,8,14,15)$

		Column 1			Column 2	
1	0001	1	0001	√	1,3	00-1
3	0011	8	1000			
7	0111				3,7	0-11
8	1000	3	0011	√		
14	1110				7,15	-111
15	1111	7	0111	√	14,15	111-
		14	1110	√		
		15	1111	√		

Prime Implicants:

AB'C'D'

A'B'D

A'CD

BCD

ABC

Construct Prime Implicants Chart

- Terms are listed horizontally
- Prime implicants are listed vertically
- Place an X whenever a prime implicant covers a minterm

Example

		1	3	7	8	14	15
8	1000						
1,3	00-1						
3,7	0-11						
7,15	-111						
14,15	111-						

Example

		1	3	7	8	14	15
8	1000				**x**		
1,3	00–1	**x**	**x**				
3,7	0–11		**x**	**x**			
7,15	–111			**x**			**x**
14,15	111–					**x**	**x**

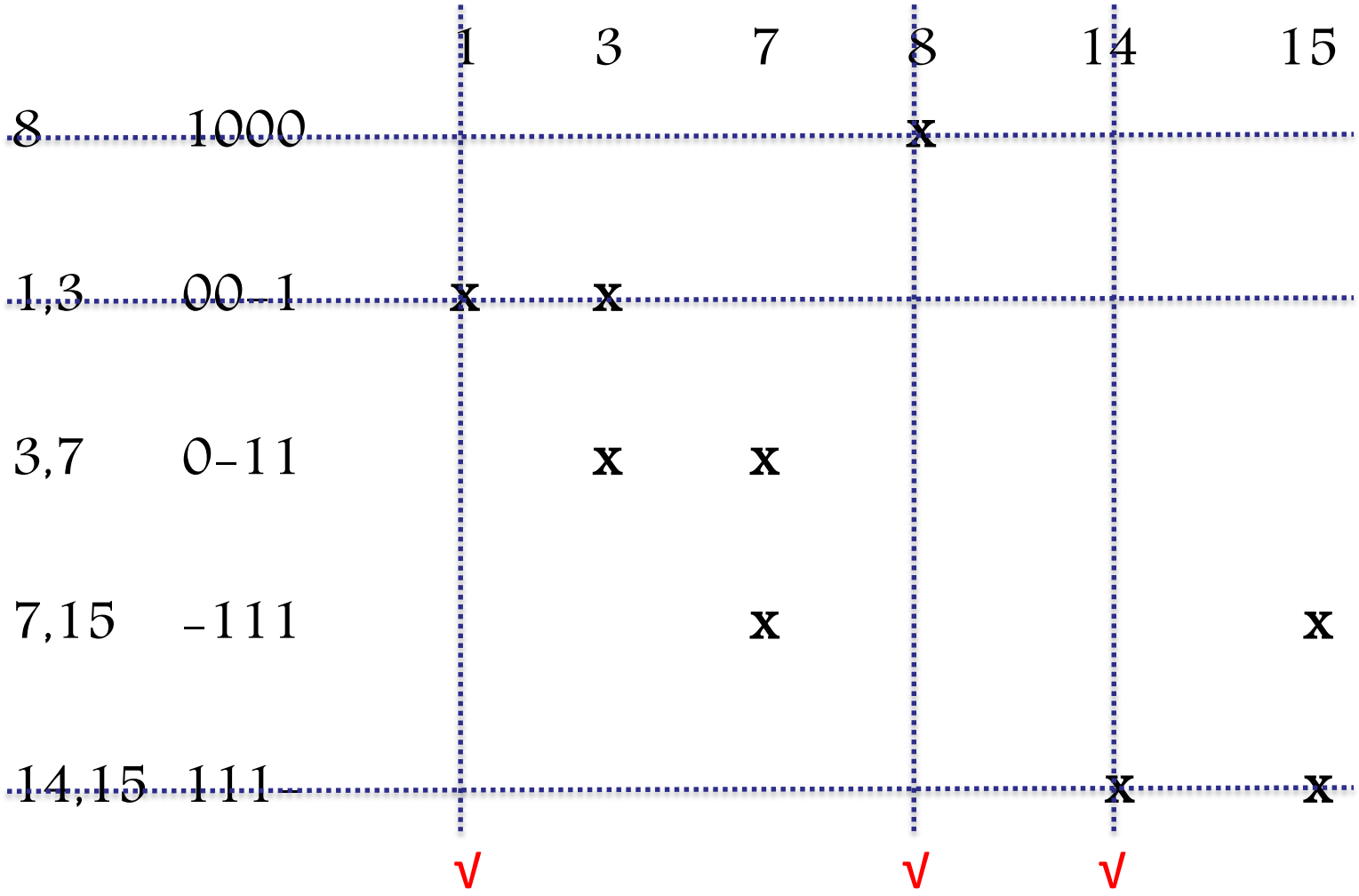
Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				**x**		
1,3	00–1	**x**	**x**				
3,7	0–11		**x**	**x**			
7,15	–111			**x**			**x**
14,15	111–					**x**	**x**

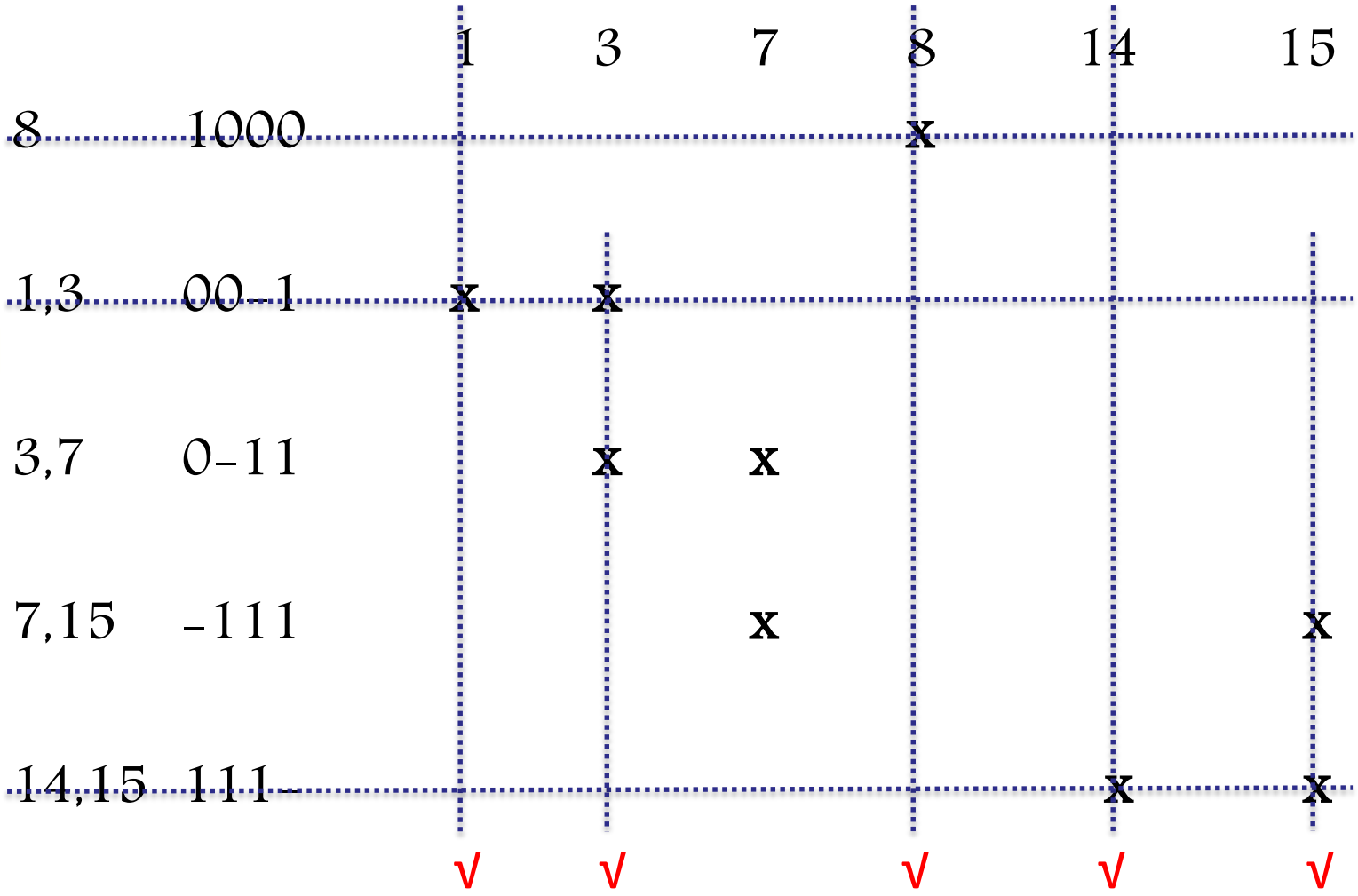
Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				x		
1,3	00–1	x	x				
3,7	0–11		x	x			
7,15	–111			x			x
14,15	111–					x	x

Select Essential Prime Implicants



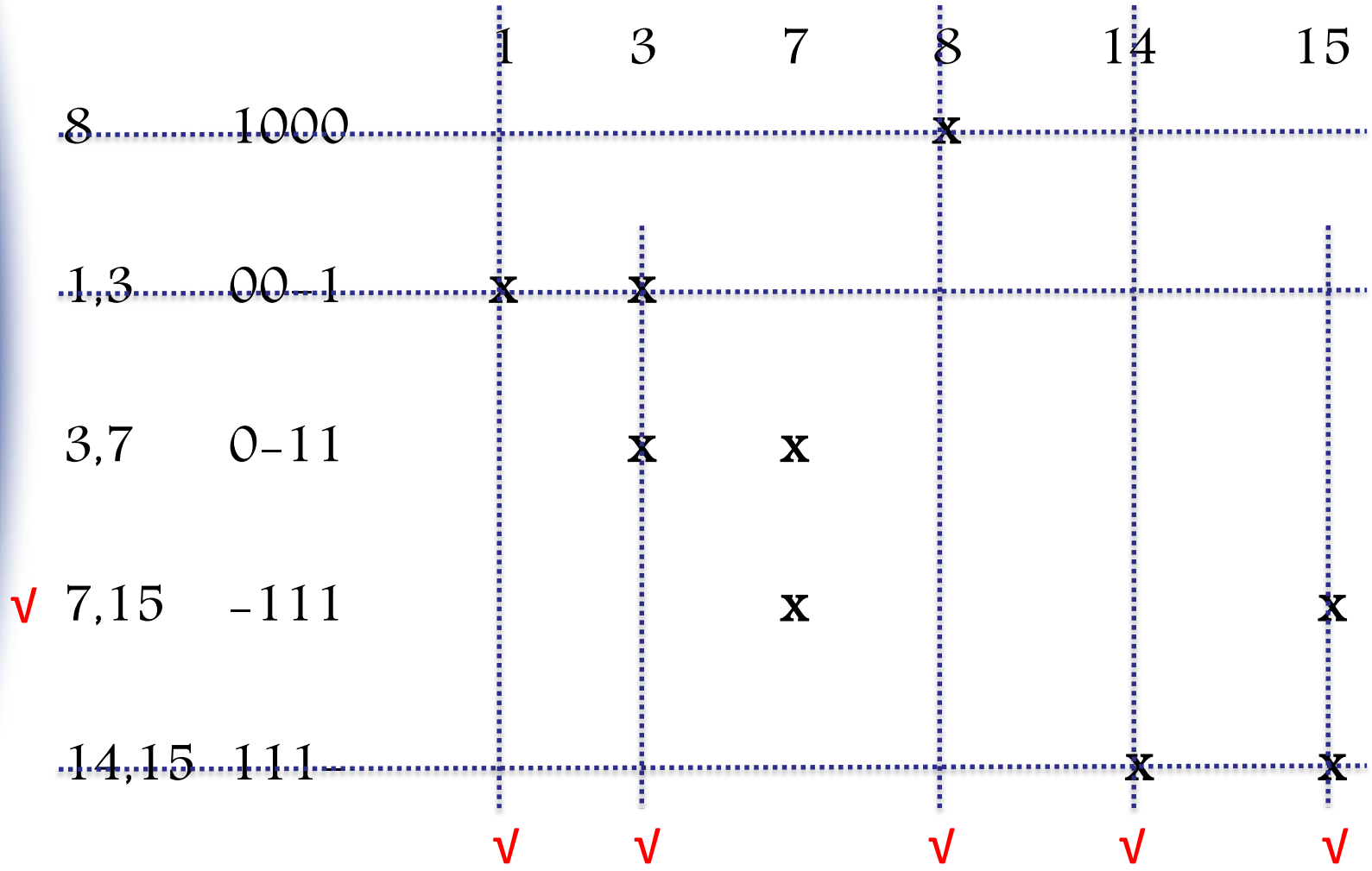
Select Minimum Cover



Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
  - F = AB'C'D' + A'B'D + ABC

Select Minimum Cover



Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
  - F = AB'C'D' + A'B'D + ABC + BCD

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$
  $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$

AB \ CD	00	01	11	10
00				
01				
11				
10				

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$
  $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$

AB \ CD	00	01	11	10
00		0	0	0
01	0			
11	0			
10		0		

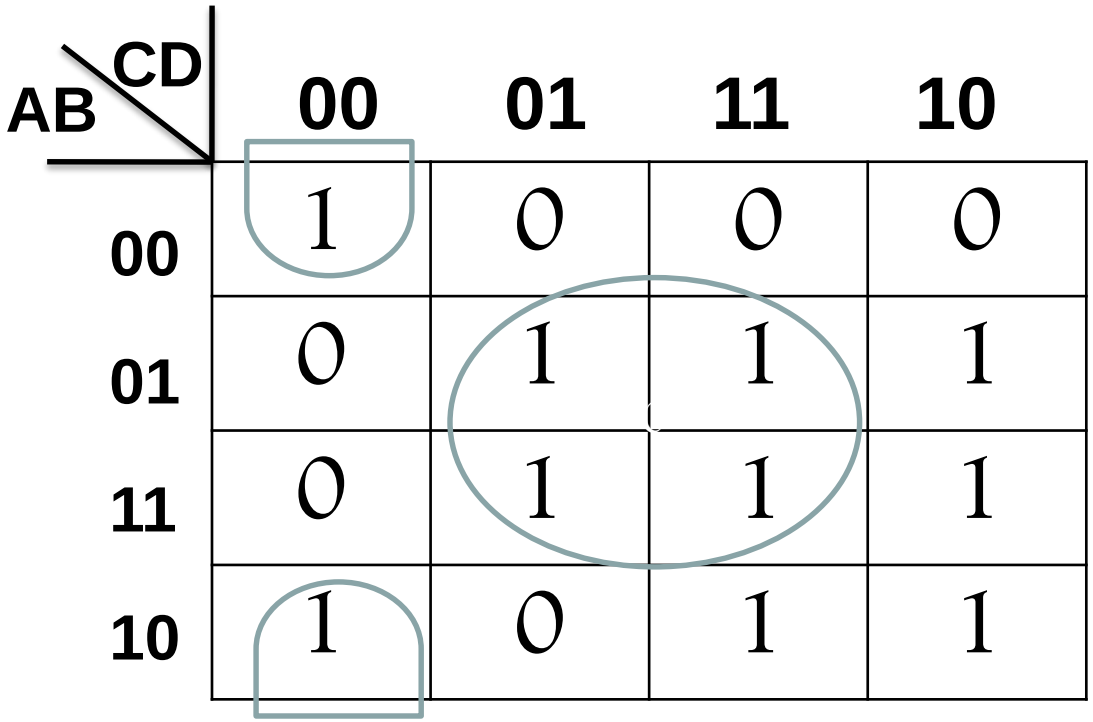
Converting Between POM & SOM Using K-Map

- (A'+B'+C+D)(A+B'+C+D)(A+B+C+D')
  (A+B+C'+D')(A'+B+C+D')(A+B+C'+D)

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

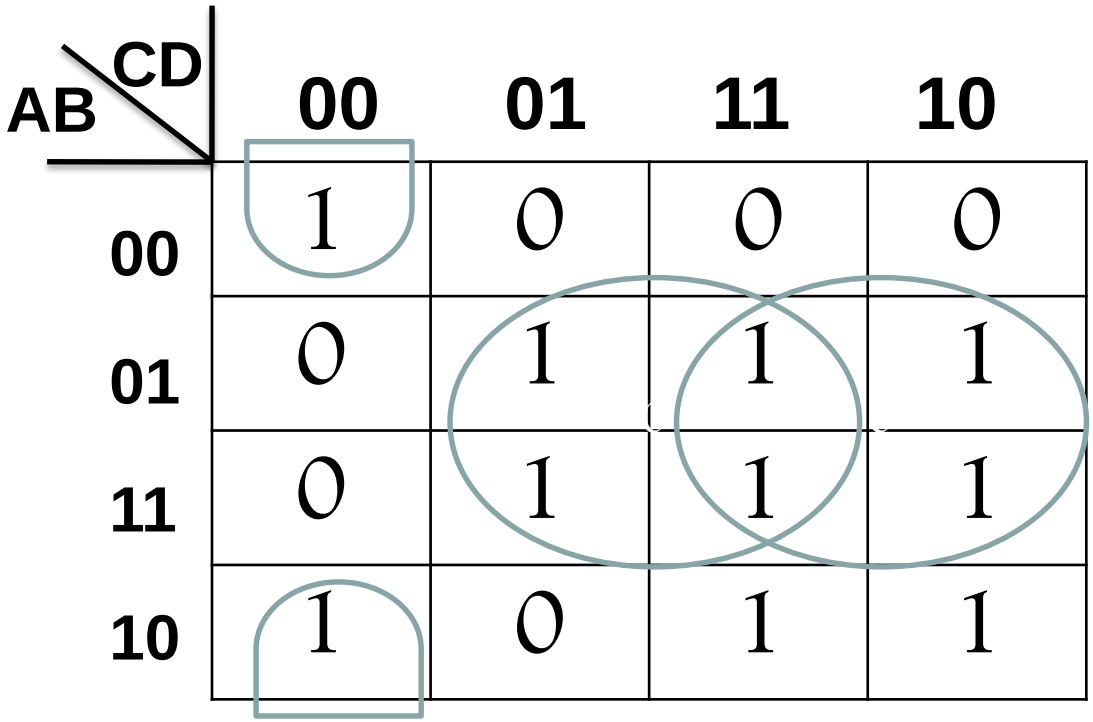
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$
  $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$



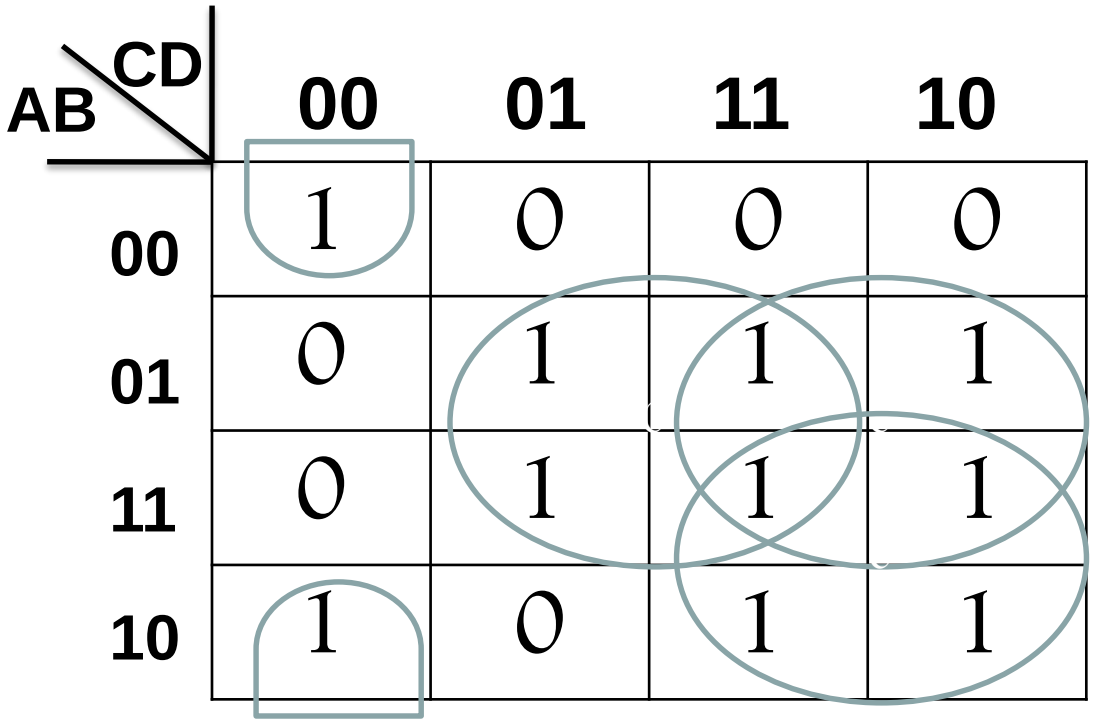
Converting Between POM & SOM Using K-Map

- (A'+B'+C+D)(A+B'+C+D)(A+B+C+D')
  (A+B+C'+D')(A'+B+C+D')(A+B+C'+D)



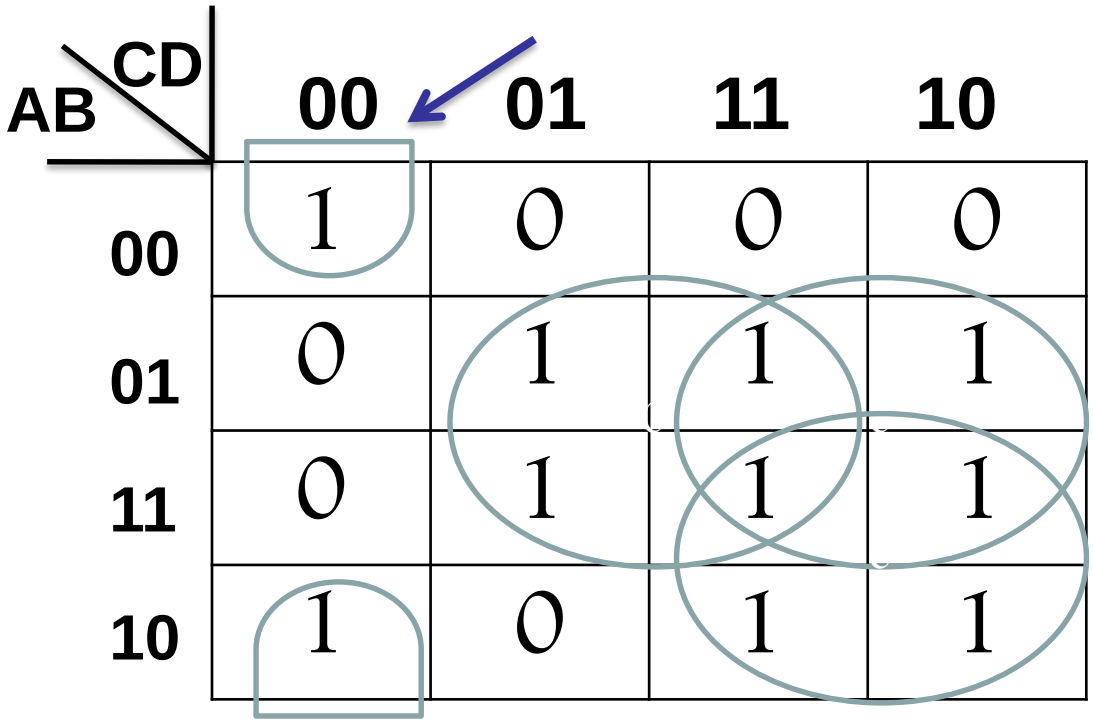
Converting Between POM & SOM Using K-Map

- (A'+B'+C+D)(A+B'+C+D)(A+B+C+D')
  (A+B+C'+D')(A'+B+C+D')(A+B+C'+D)



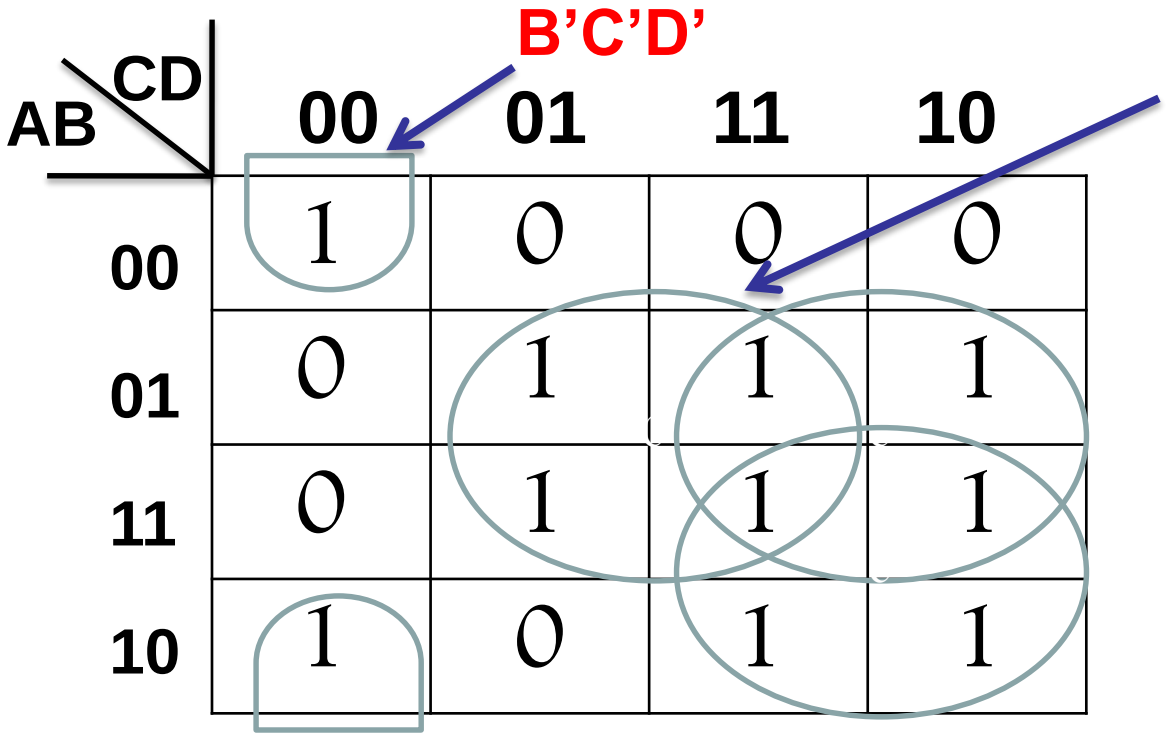
Converting Between POM & SOM Using K-Map

- (A'+B'+C+D)(A+B'+C+D)(A+B+C+D')
  (A+B+C'+D')(A'+B+C+D')(A+B+C'+D)



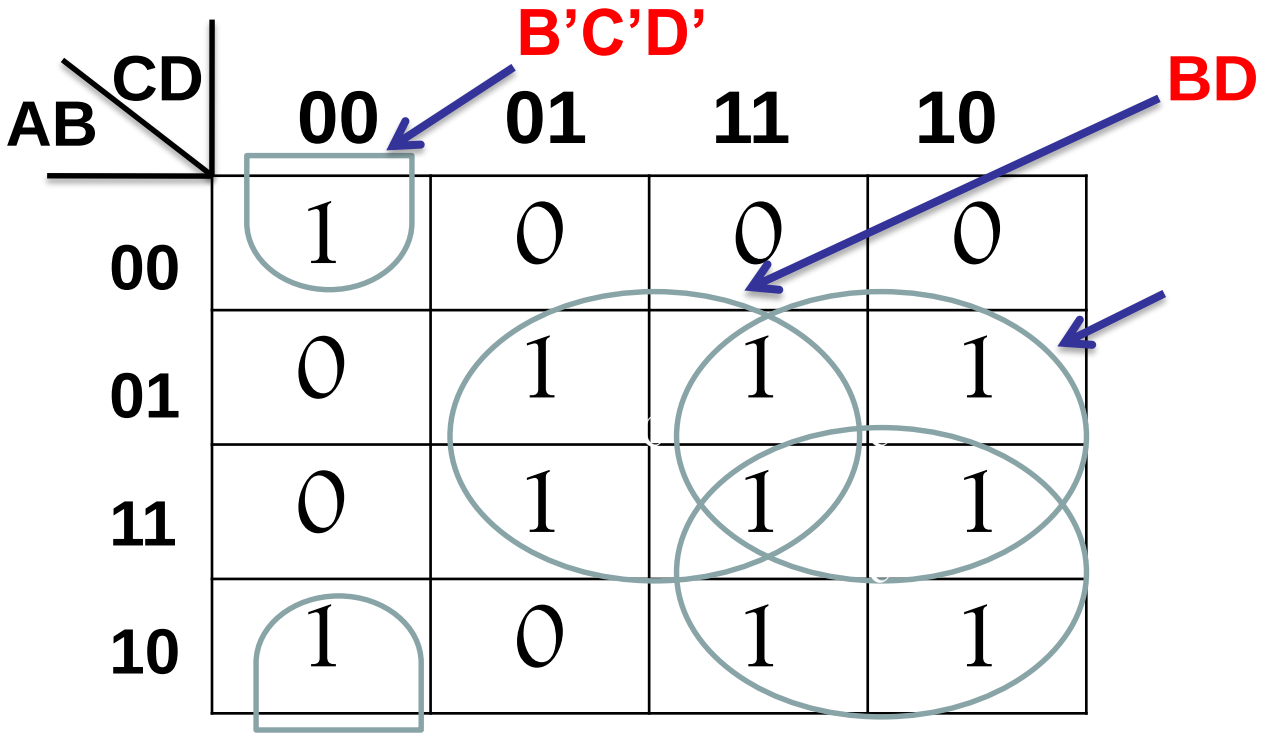
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$
  $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$



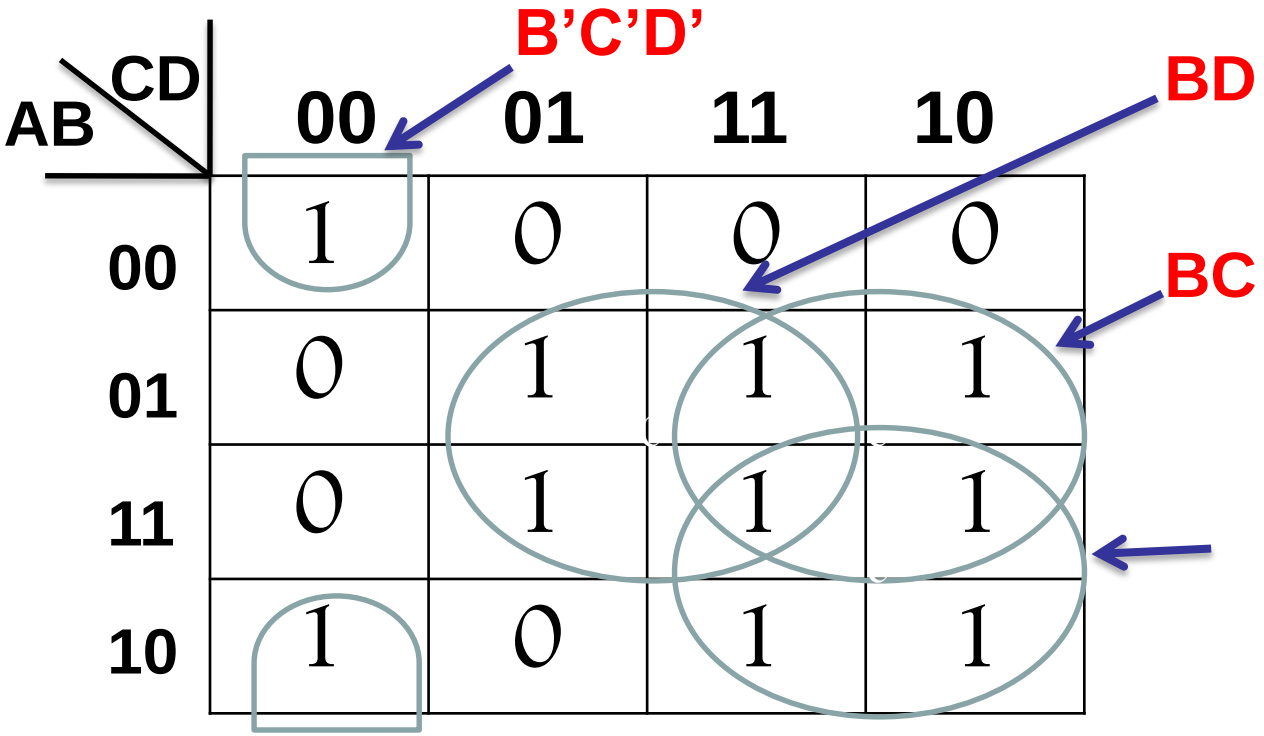
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$
  $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$



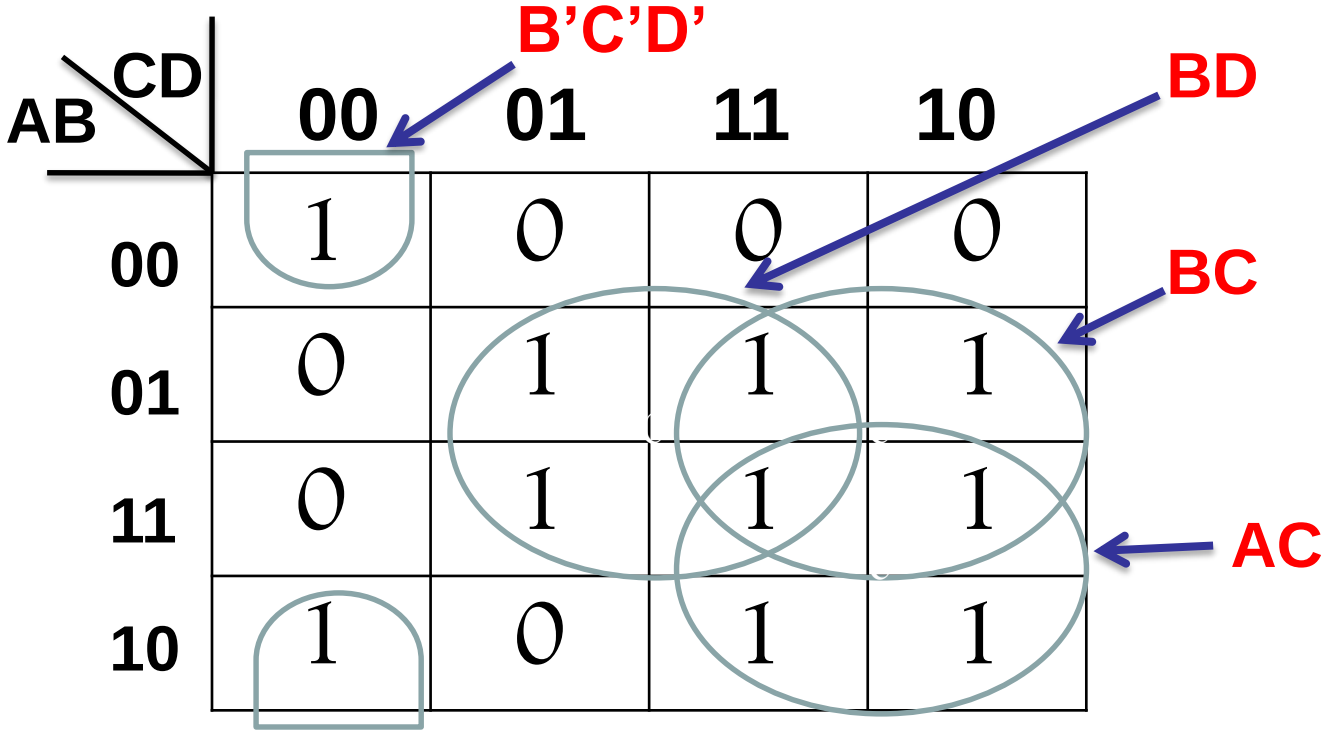
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$
  $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$



Converting Between POM & SOM Using K-Map

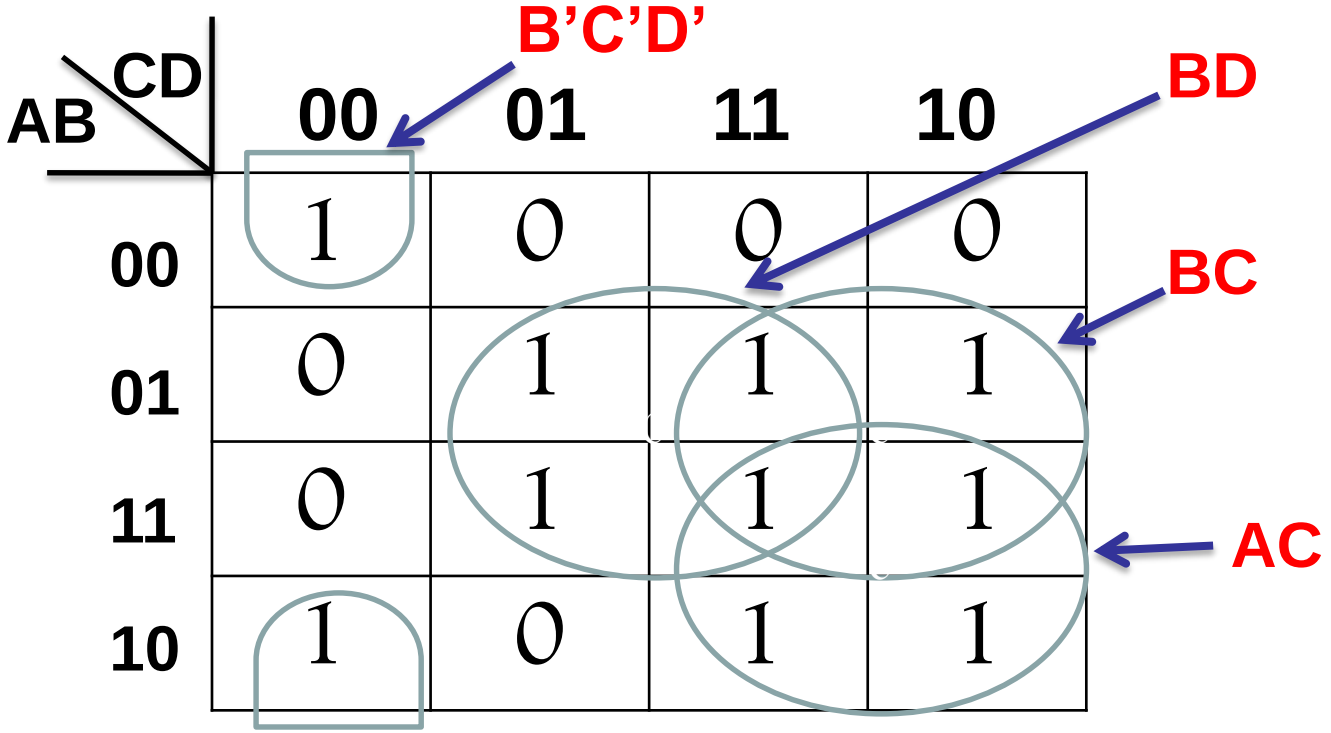
- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$
  $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$



Converting Between POM & SOM Using K-Map

=B'C'D'+ BD + BC + AC

- (A'+B'+C+D)(A+B'+C+D)(A+B+C+D')
  (A+B+C'+D')(A'+B+C+D')(A+B+C'+D)



Don’t Care Conditions

- The unspecified minterms (maxterms) of an incompletely specified function
- An **X** inside a map represents a don’t care condition

Don't Care Conditions

- There are 2 cases when this occurs.
  - The input combination never occurs
    - e.g. the BCD code does not use the 6 remaining codes.
  - The input combinations are expected to occur, but we do not care what the outputs are

Representation of Don't Cares

- e.g.

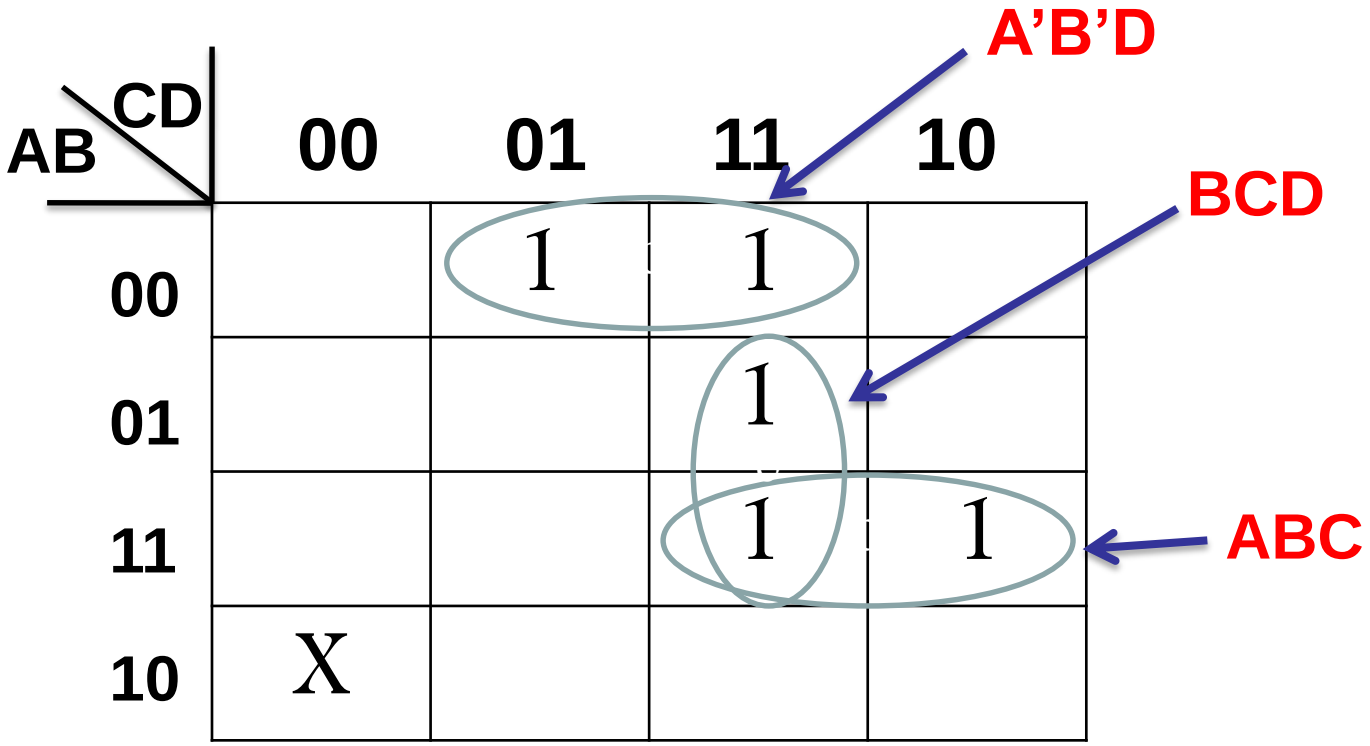
$$F(W,X,Y,Z) = \Sigma m(0,1,2,4,6,7,8,10)$$
$$d(W,X,Y,Z) = \Sigma d(12,13,14,15)$$

- It could also be represented as:

$$F(W,X,Y,Z) = \Sigma m(0,1,2,4,6,7,8,10) + \Sigma d(12,13,14,15)$$

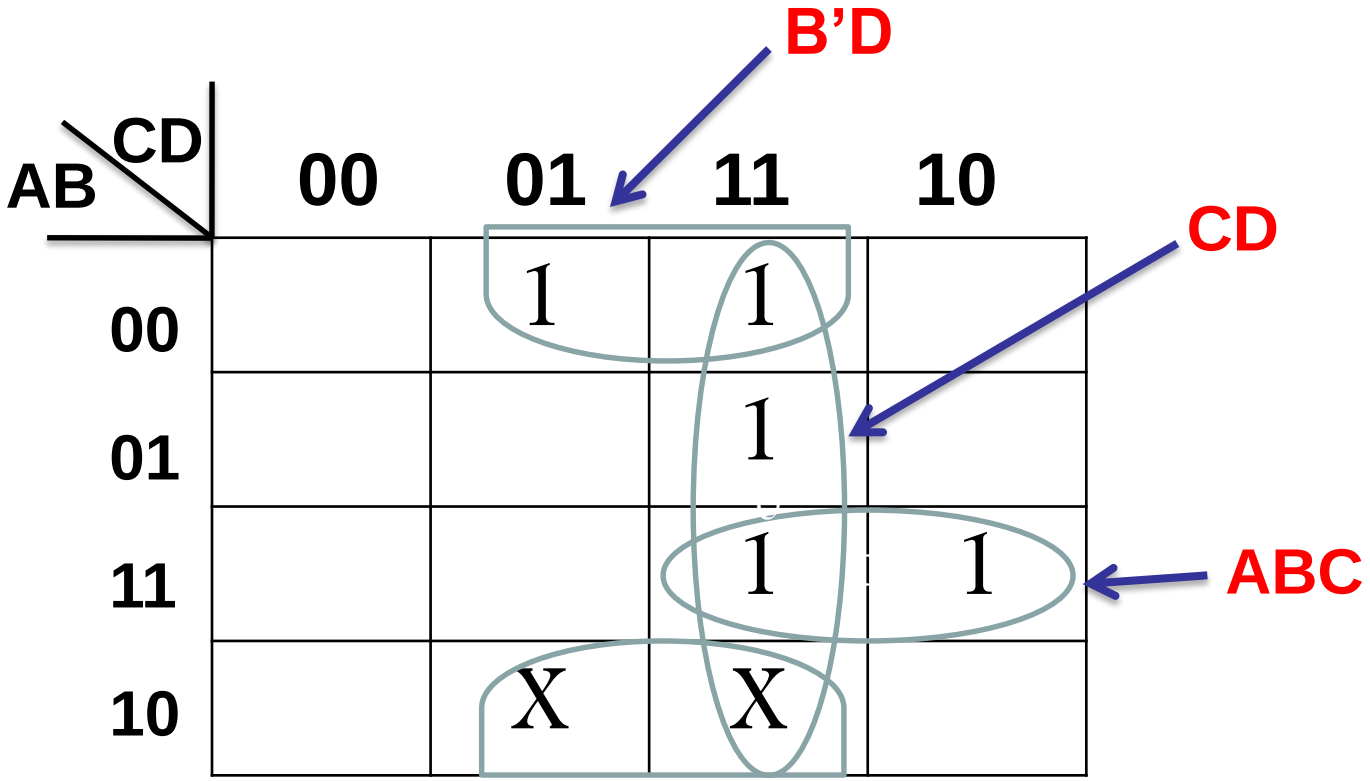
Example

- Simplify F= $\Sigma m(1,3,7,14,15) + d(8)$



Example

- Simplify F= $\Sigma m(1,3,7,14,15) + d(9,11)$



Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$

AB \ CD	00	01	11	10
00				
01				
11				
10				

Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$

AB \ CD	00	01	11	10
00	1			1
01	1			
11	1			
10	1	1		

Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$

AB \ CD	00	01	11	10
00	1			1
01	1			X
11	1	X	X	
10	1	1		

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AB \ CD	00	01	11	10
00	1			1
01	1			X
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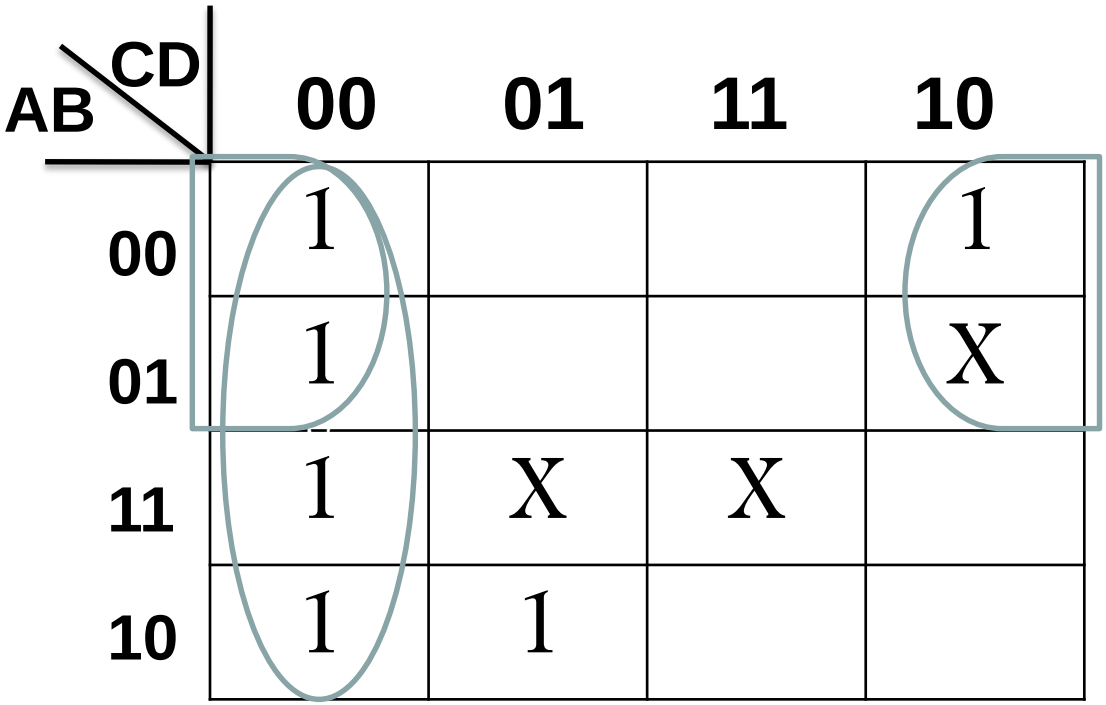
10	1	1		

Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$



Example

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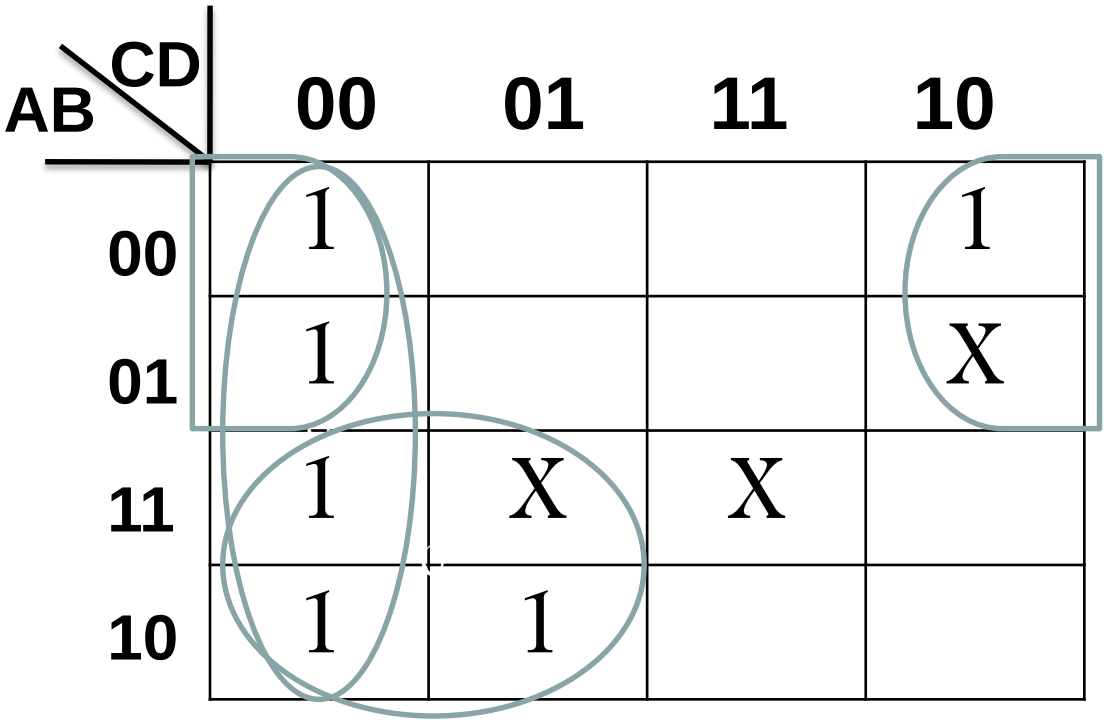

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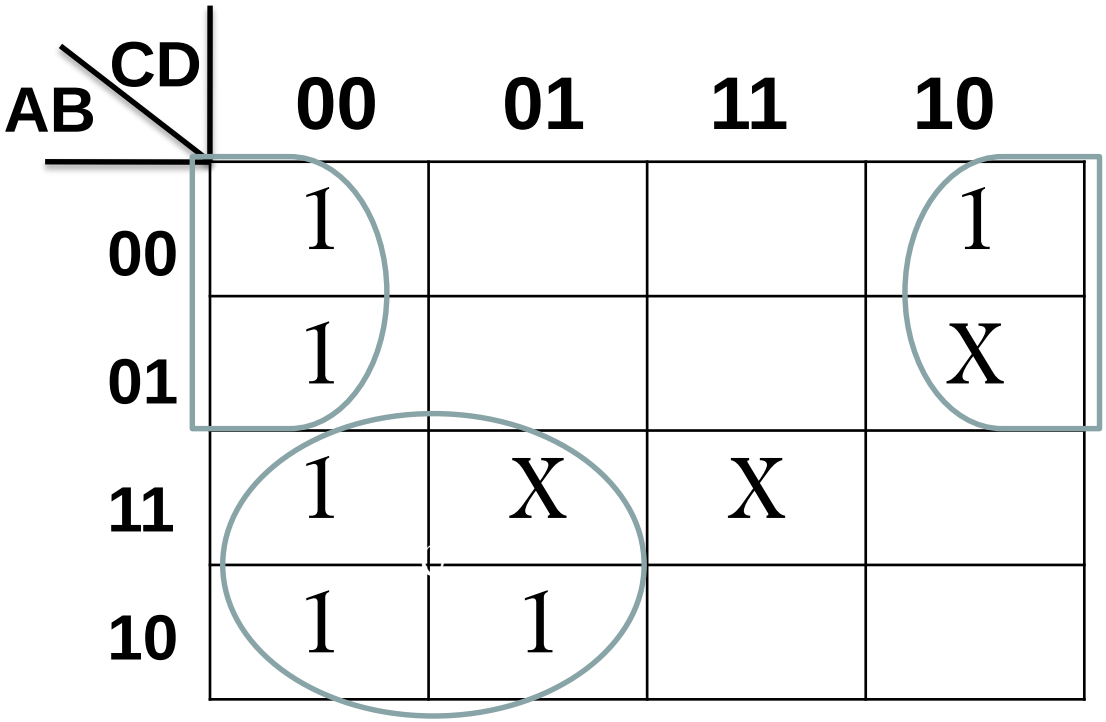

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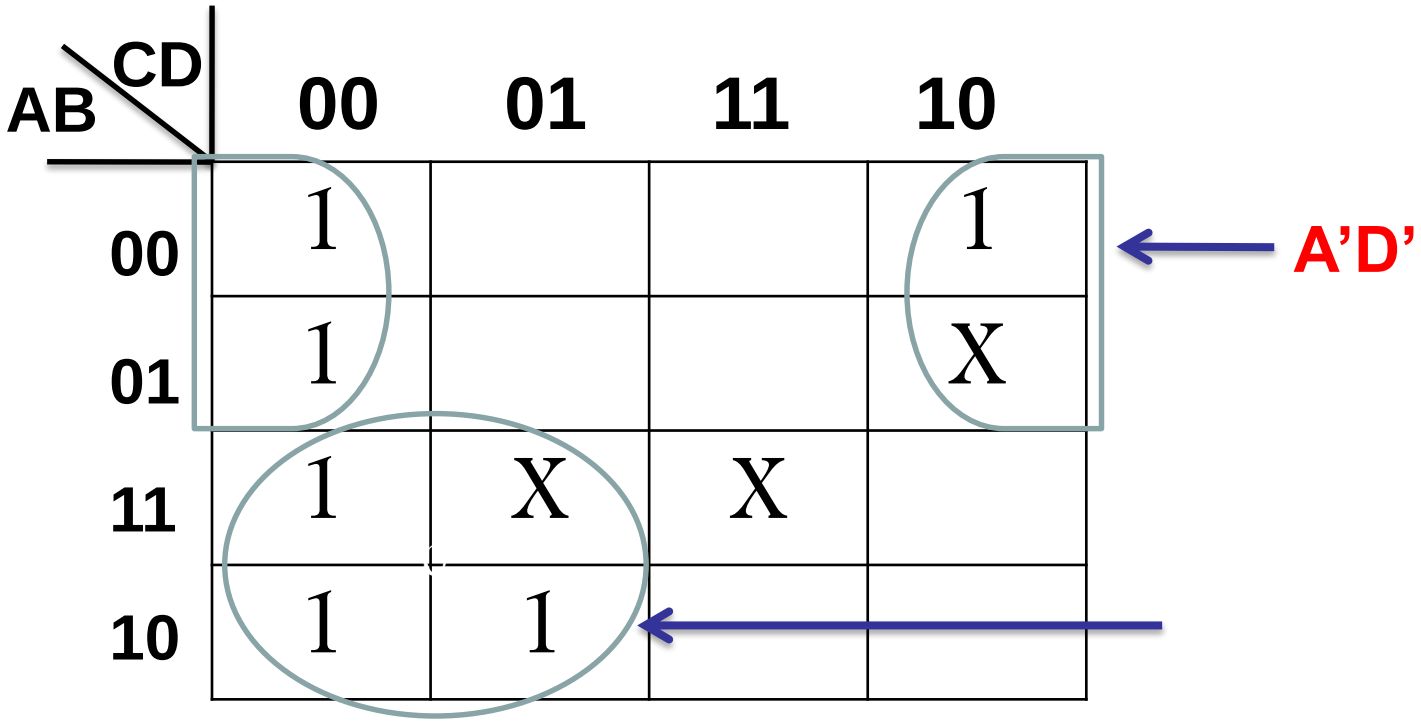

Example

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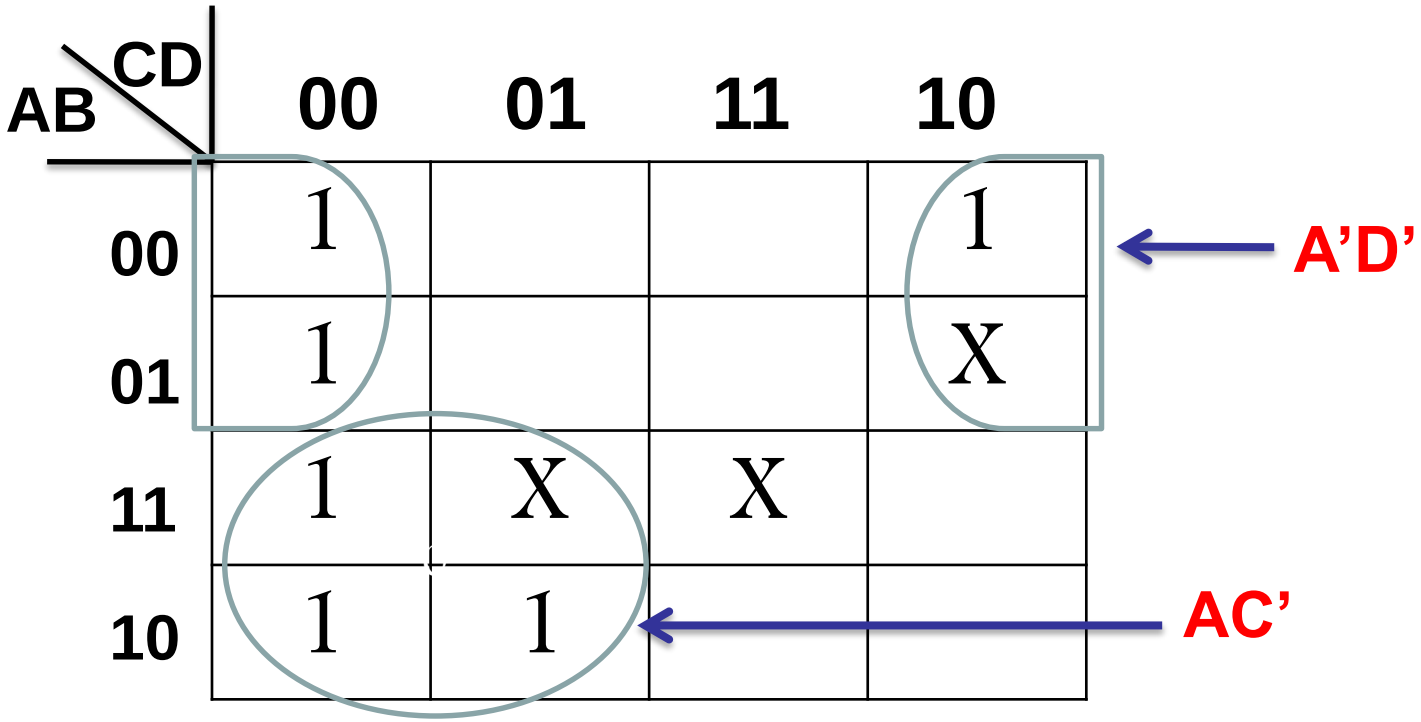

Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$ **= A'D' + AC'**