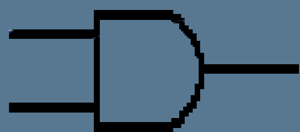
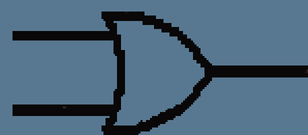
Chapter 3

BOOLEAN ALGEBRA, LOGIC FUNCTIONS and LOGIC GATES

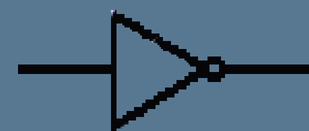
Digital Logic Gates



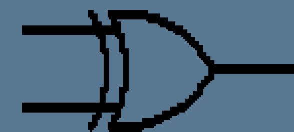
AND gate



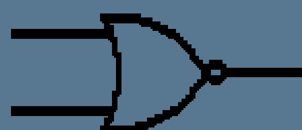
OR gate



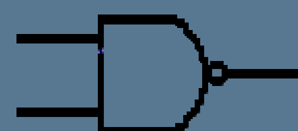
NOT gate



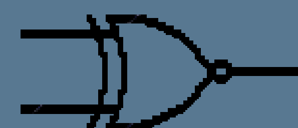
XOR gate



NOR gate



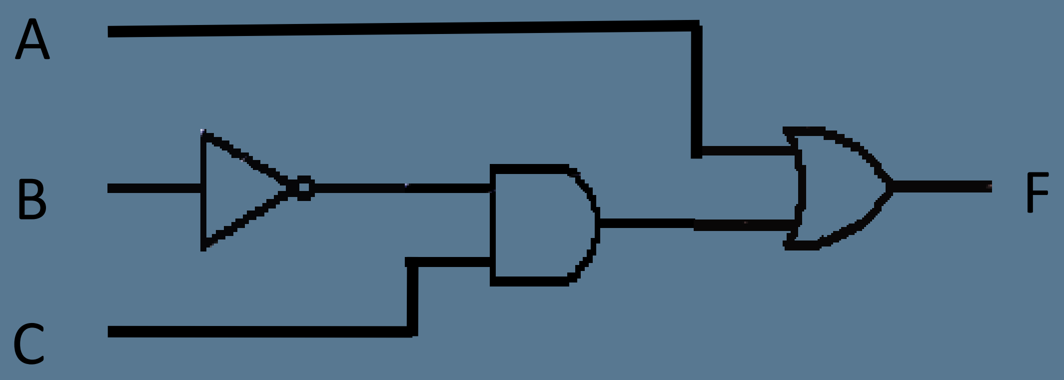
NAND gate



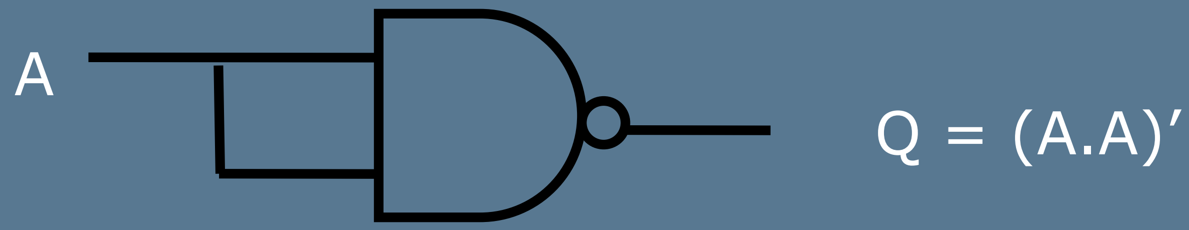
XNOR gate

Digital Logic gates

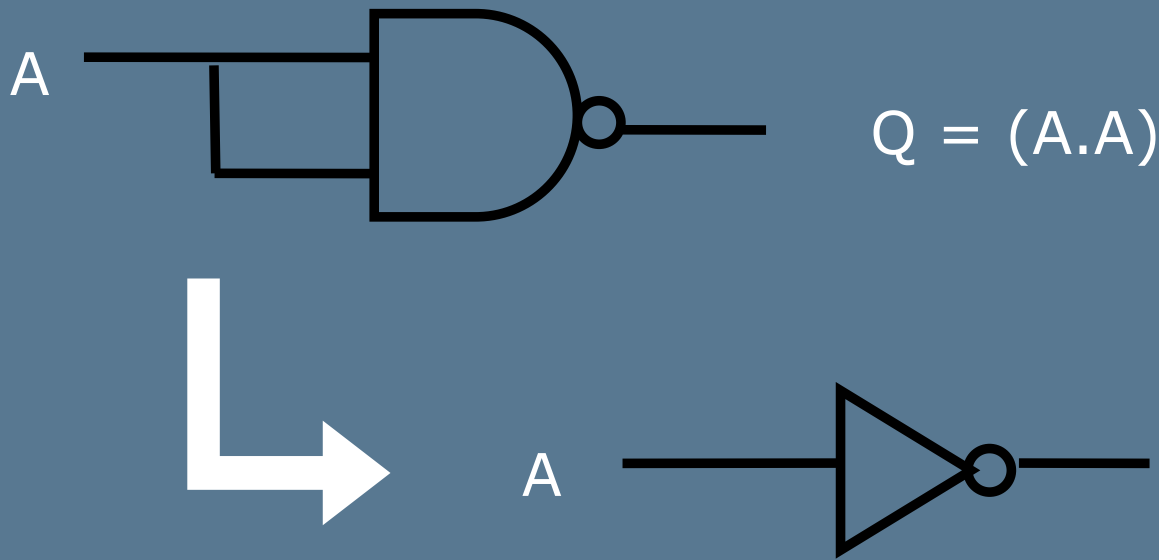
- Draw the logic diagram of the function
  $F = A+B'C$



Universality of NAND gate



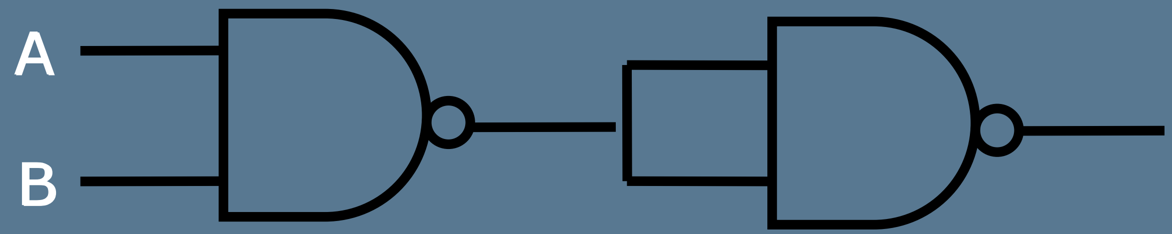
Universality of NAND gate



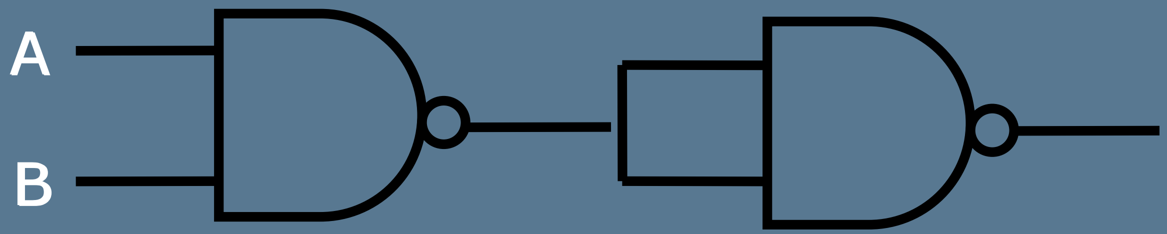
$Q = (A.A)'$

$Q = A'$

Universality of NAND Gate

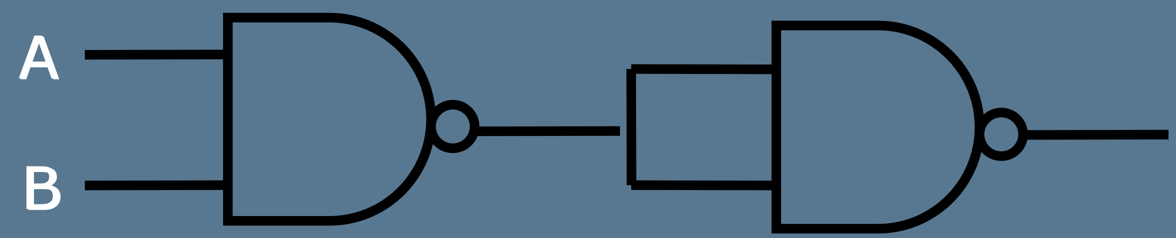


Universality of NAND Gate

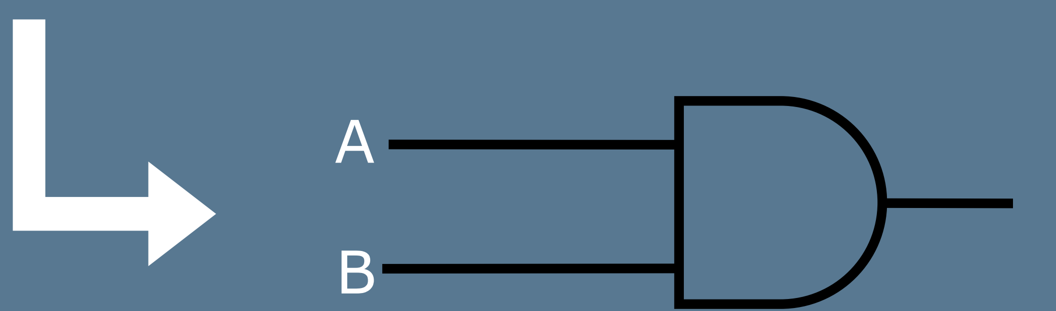


Q =((AB)' (AB)')'

Universality of NAND Gate

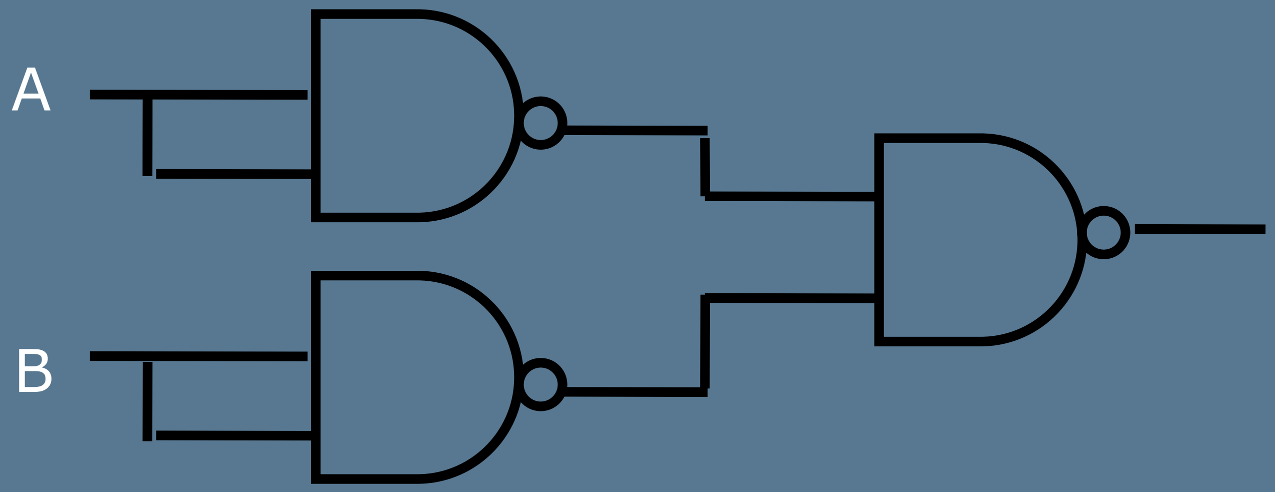


$$Q = ((AB)'\,(AB)')'$$

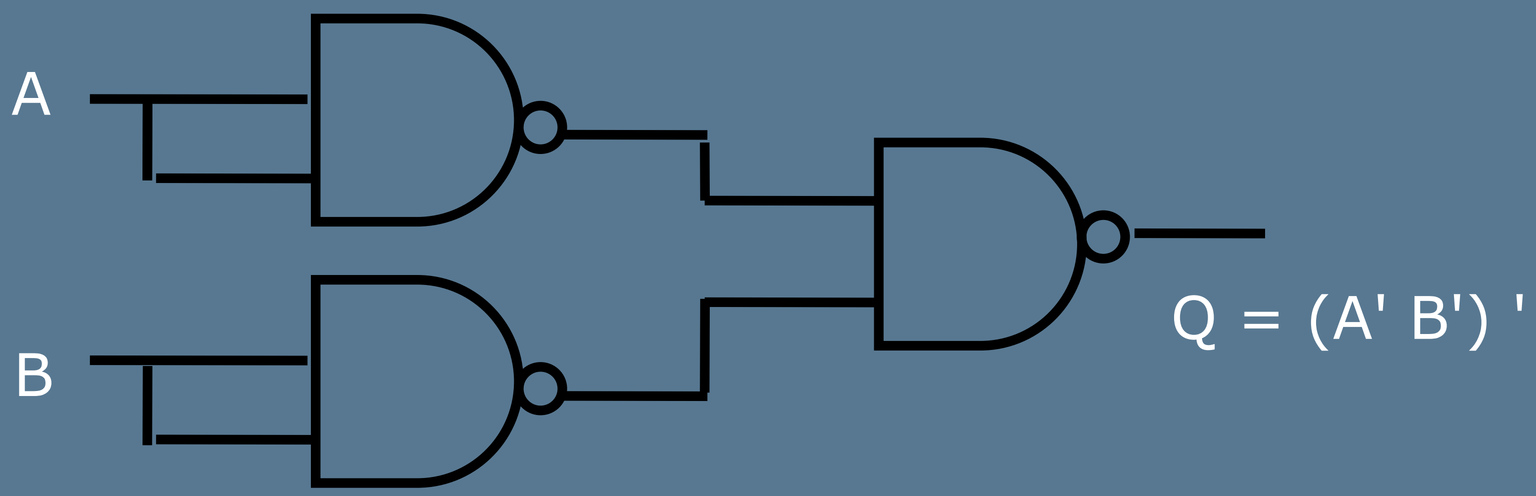


$$Q = AB$$

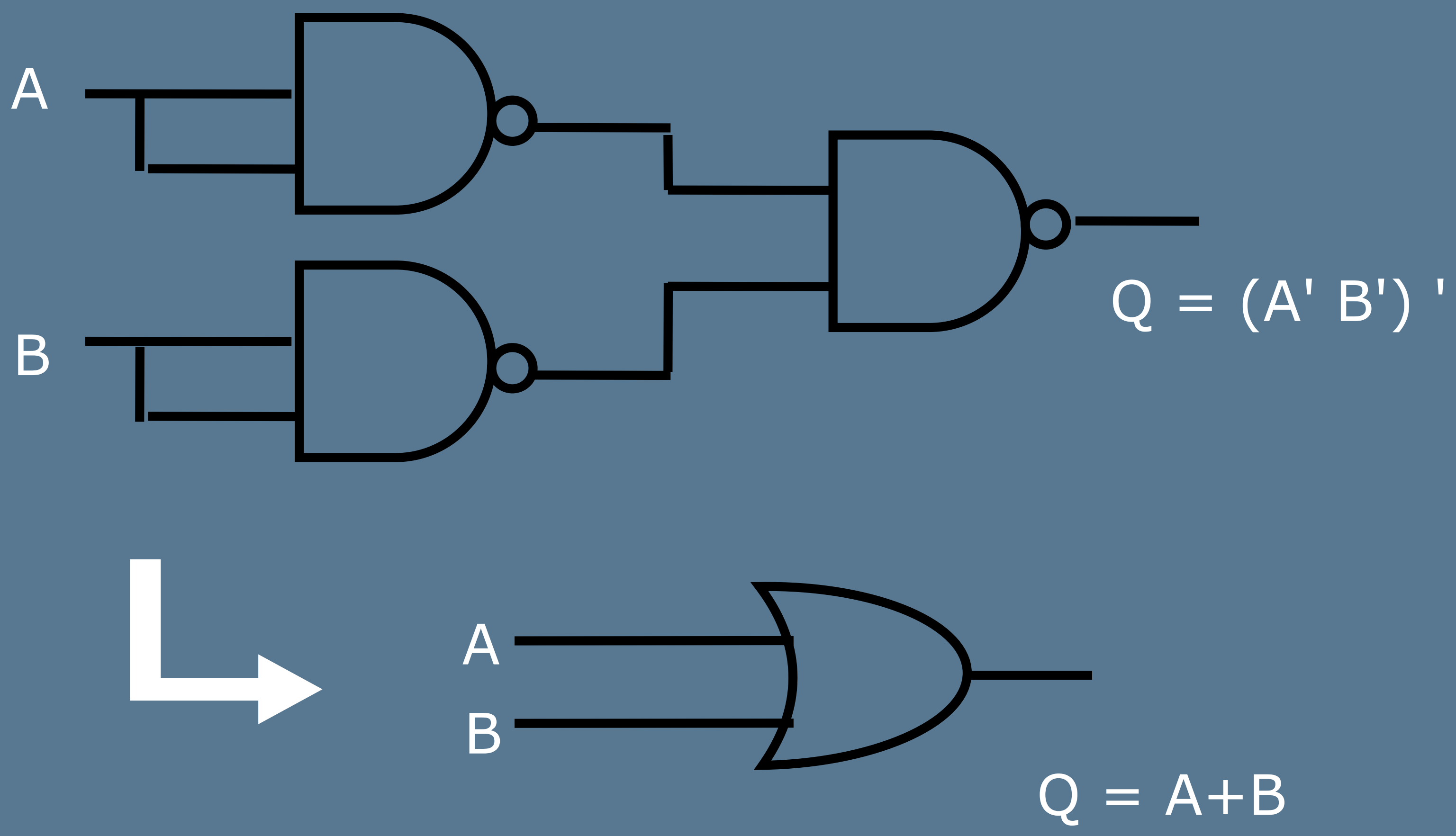
Universality of NAND Gate



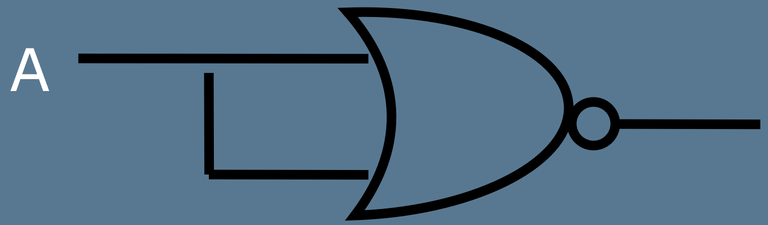
Universality of NAND Gate



Universality of NAND Gate

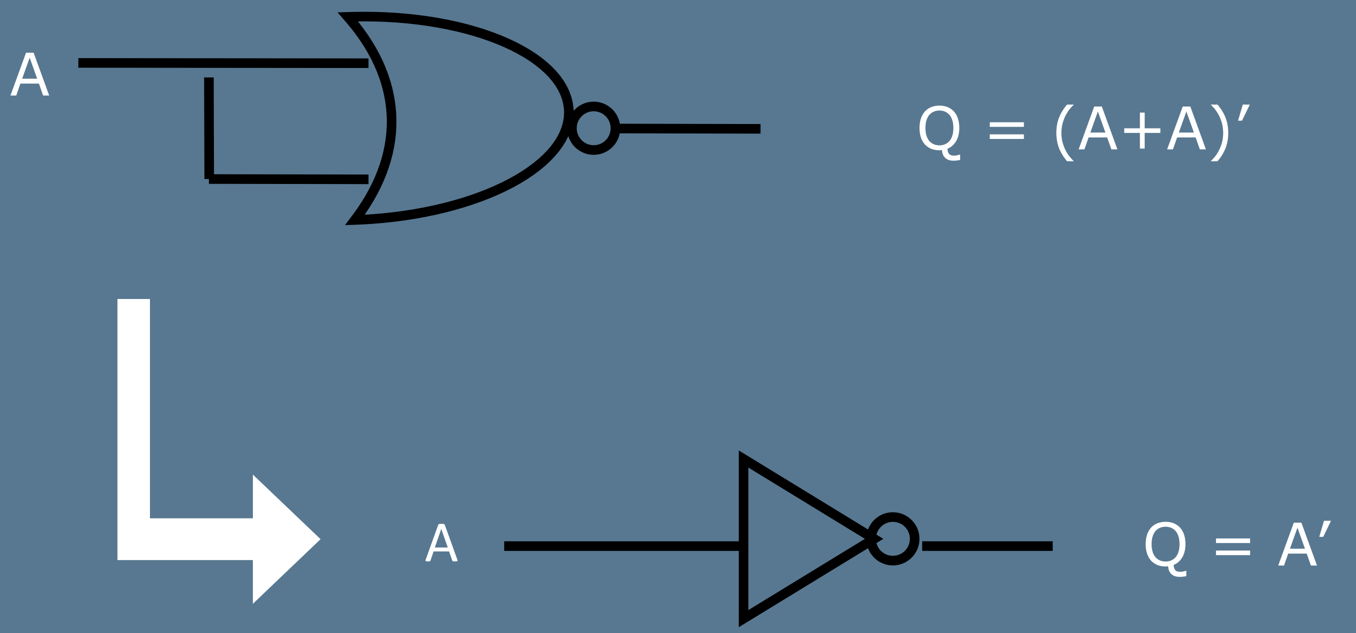


Universality of NOR Gate

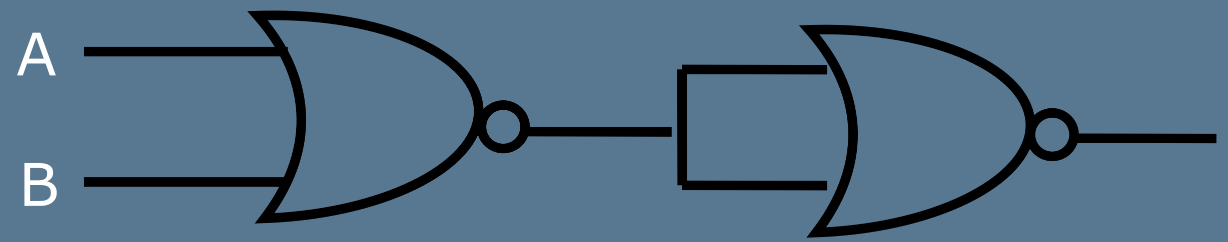


$Q = (A+A)'$

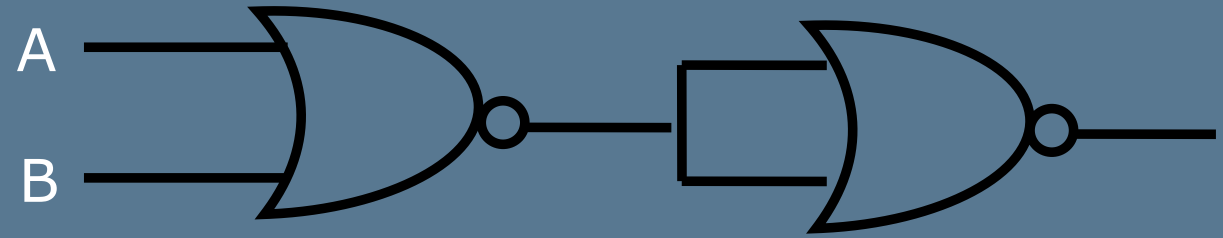
Universality of NOR Gate



Universality of NOR Gate

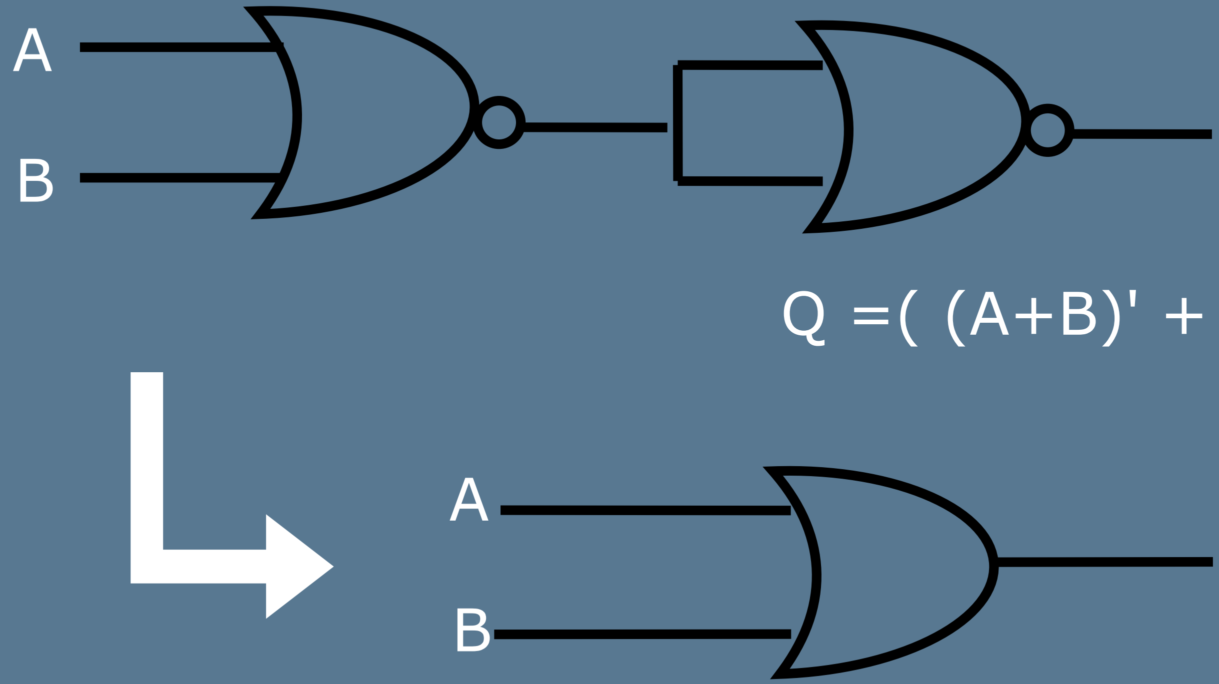


Universality of NOR Gate



Q =((A+B)' + (A+B)')'

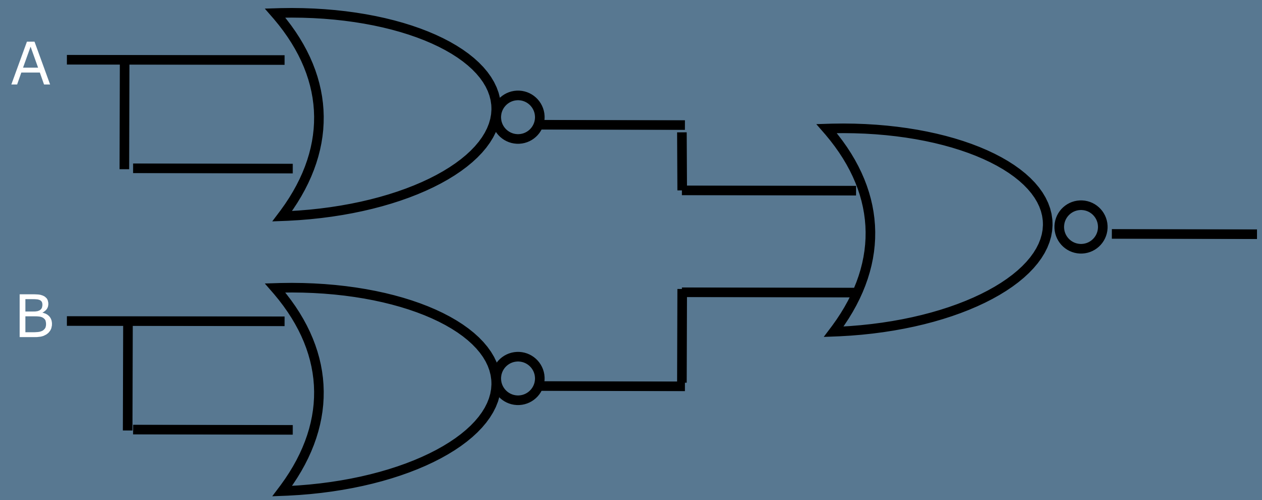
Universality of NOR Gate



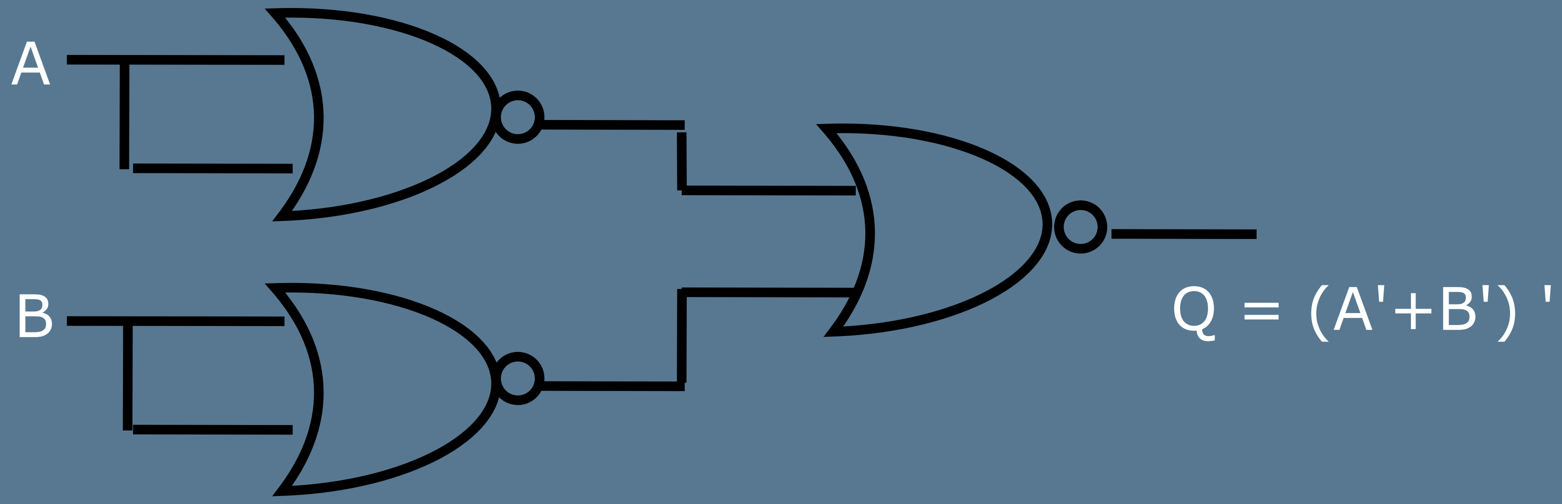
Q =((A+B)' + (A+B)')'

Q = A+B

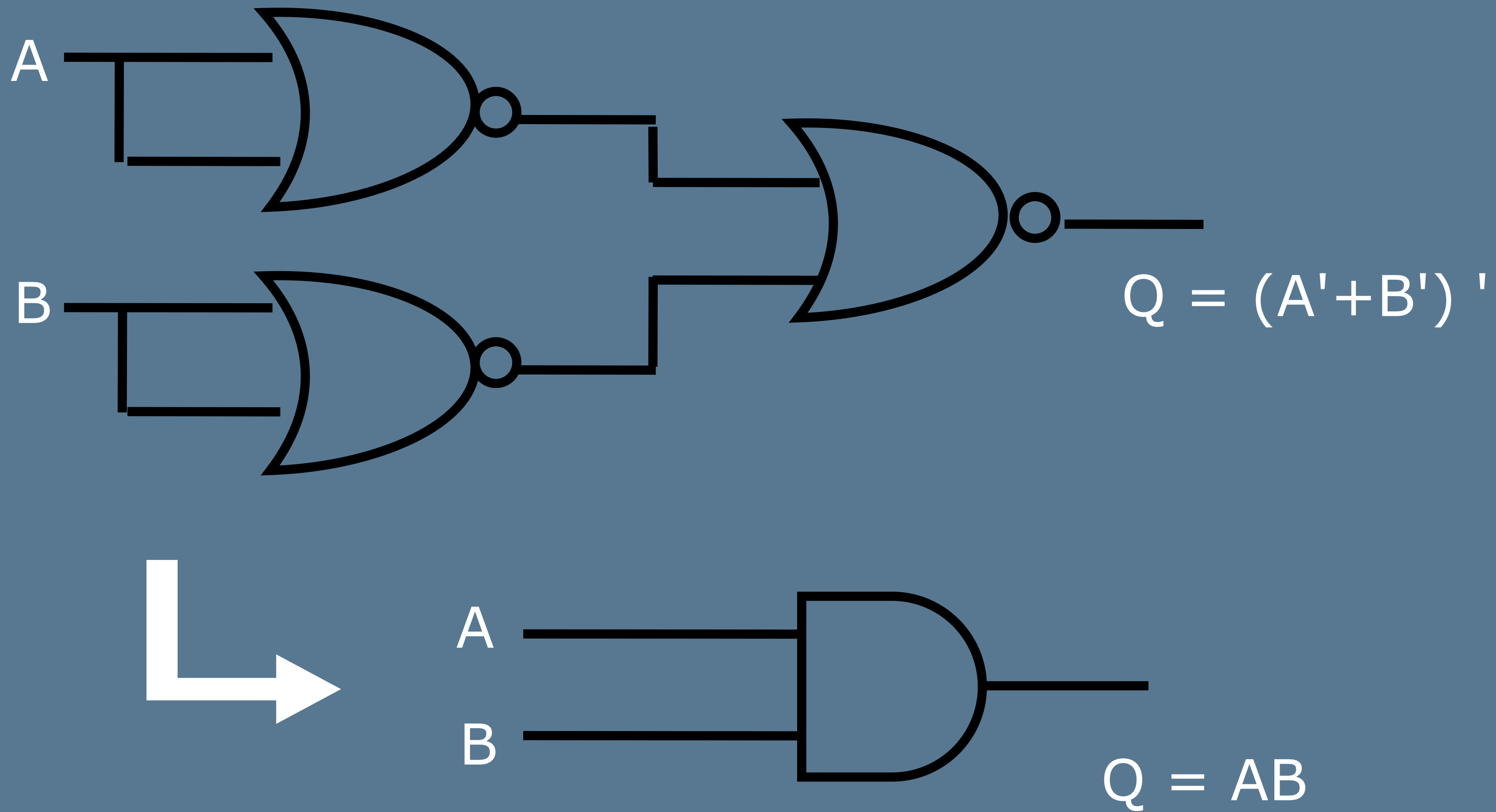
Universality of NOR Gate



Universality of NOR Gate



Universality of NOR Gate



Real Gates

- Logic gates are in integrated form
  - built within a solid piece of silicon called an IC (integrated circuit)

  - Several gates are included in a single plastic moulding

IC Families

- Transistor-Transistor Logic (TTL)
- Emitter Coupled Logic (ECL)
- Complementary Metal-Oxide-Semiconductor (CMOS)

Levels of IC

- Small-scale Integration
  - ICs with 1 to 10 gates
- Medium-scale Integration
  - ICs with 10 to 100 gates
- Large-scale Integration
  - ICs with 100 to 1000s of gates
- Very large-scale Integration
  - ICs with 1000s to millions of gates

Chapter 4

SIMPLIFICATION of LOGIC CIRCUITS

RECALL: Representations of Boolean Functions

- Truth table
  - unique

  Example:

x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

- Algebraic expression
  - not unique
  - convenient for manipulation

  Example: F = x + y
- Logic circuits
  - not unique
  - close to implementation

Simplification of Boolean functions

- Simpler circuit is faster
- Simpler circuit is less expensive
- Reduce complexity of the gate level implementation
- Reduce signal propagation delays

Ways to simplify Boolean functions

- Boolean Algebra
- Graphical method (Karnaugh Map)
- Tabular method (Quine-McCluskey)

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$= x'(y' + y) + xyz$ Comm / Dist.

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$= x'(y' + y) + xyz$ Comm / Dist.

$= x' + xyz$ Inv / Iden

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$= x'(y' + y) + xyz$ Comm / Dist.

$= x' + xyz$ Inv / Iden

$= (x' + x)(x' + yz)$ Dist.

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$= x'(y' + y) + xyz$	Comm / Dist.
$= x' + xyz$	Inv / Iden
$= (x' + x)(x' + yz)$	Dist.
$= (x' + yz)$	Inv / Iden

Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$

Simplification: Boolean Algebra

Simplify AB + A(B+C) + B(B+C)

= AB+AB+AC+BB+BC Dist.

Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$

$= AB+AB+AC+BB+BC$ Dist.

$= AB+AC+B+BC$ Idempotency

Simplification: Boolean Algebra

Simplify AB + A(B+C) + B(B+C)

= AB+AB+AC+BB+BC	Dist.
= AB+AC+B+BC	Idempotency
= AB+AC+B	Absorption

Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$

$= AB+AB+AC+BB+BC$	Dist.
$= AB+AC+B+BC$	Idempotency
$= AB+AC+B$	Absorption
$= B+AC$	Absorption

Simplification: Graphical method

- Karnaugh map (K-map)
  - alternate way of representing Boolean functions
  - a graphical tool for assisting in the general simplification procedure
  - a simpler way to handle most jobs of manipulating logic functions

General Steps of K-Map Simplification

- Express function in canonical form
- Map expression on a K-Map
- Group 1’s or 0's
- Determine the minimum expression

Step 1: Function in Canonical form

- Sum of minterms
  - $F(x,y) = x'y' + xy = \sum(0,3)$
  - $G(a,b,c) = a'b'c + abc' + a'bc = \sum(1,3,6)$
- Product of maxterms
  - $H(x,y) = (x+y')(x'+y) = \prod(1,2)$
  - $I(a,b,c) = (a+b+c)(a'+b+c') = \prod(0,5)$

Step 2: Map expression

X \ Y	0	1
0	m_0	m_1
1	m_2	m_3

Two-variable map

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Three-variable map

Step 2: Map expression

WX \ YZ	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Four-variable map

Mapping Example

- $F = x'y'z + x'yz' + xyz' + xyz$

X \ YZ	00	01	11	10
0		1		1
1			1	1

Mapping Example

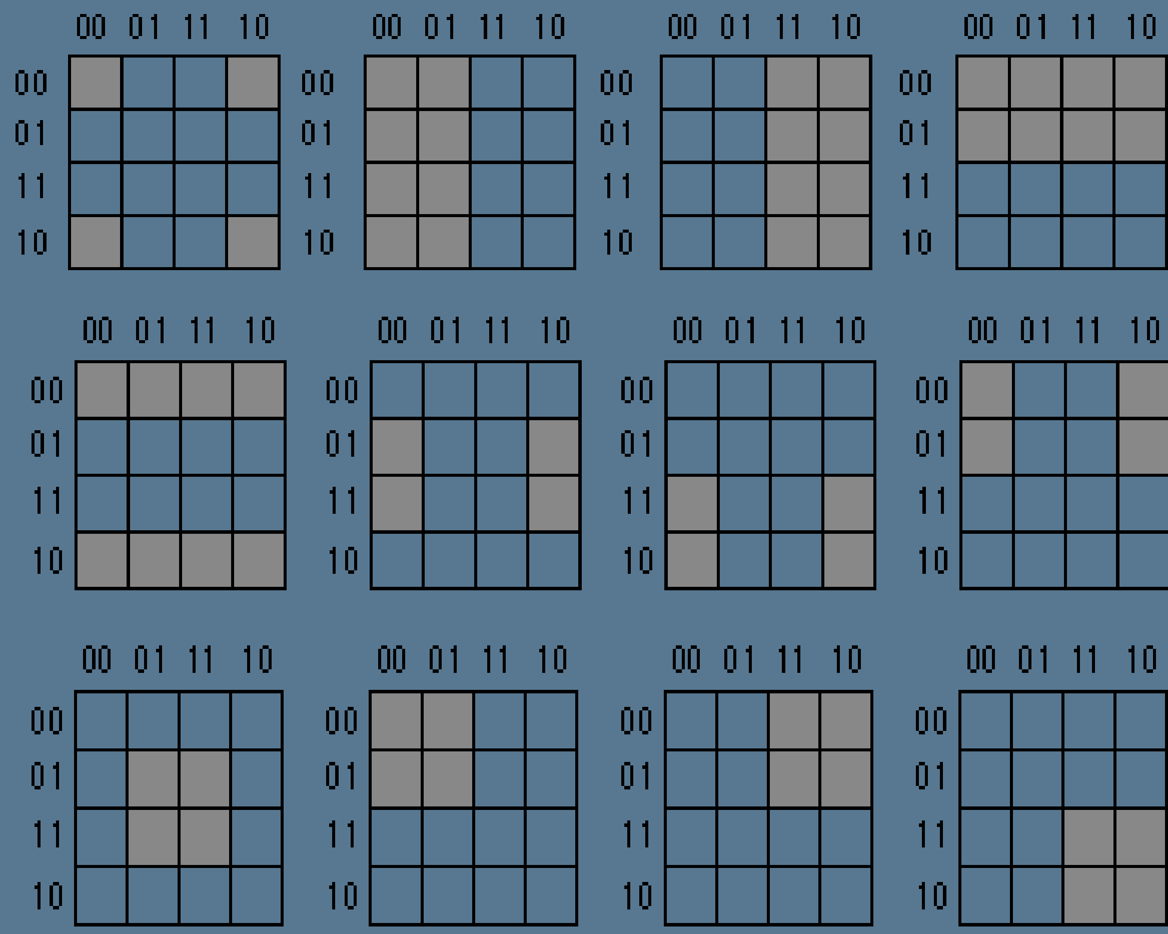
- $G = \sum(2,4,6,8,12,14,15)$

WX \ YZ	00	01	11	10
00				1
01	1			1
11	1		1	1
10	1			

Step 3: Group 1's (or 0's)

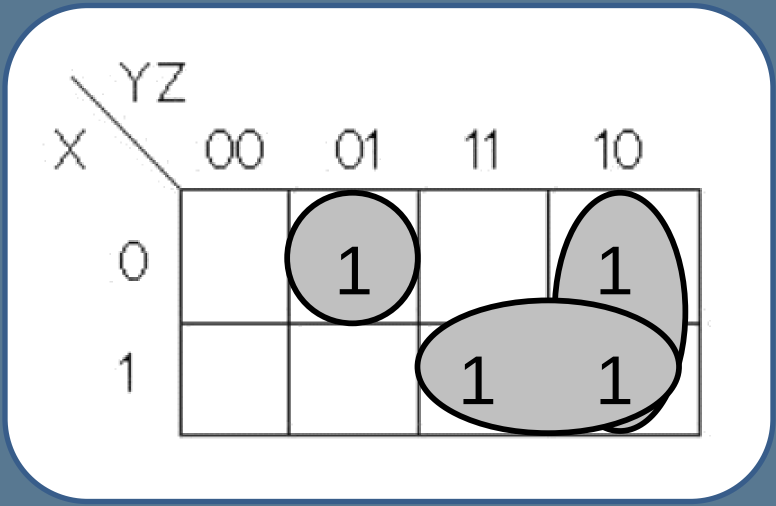
- Grouping Rules:
  - A group must contain either 1, 2,4, 8 and 16 cells
  - Each cell in a group must be adjacent to one or more cells in that same group.
  - Always include the largest possible group in accordance with the first rule.
  - Each element of a group must be included in at least one group.

Sample Groupings: Four-variable map



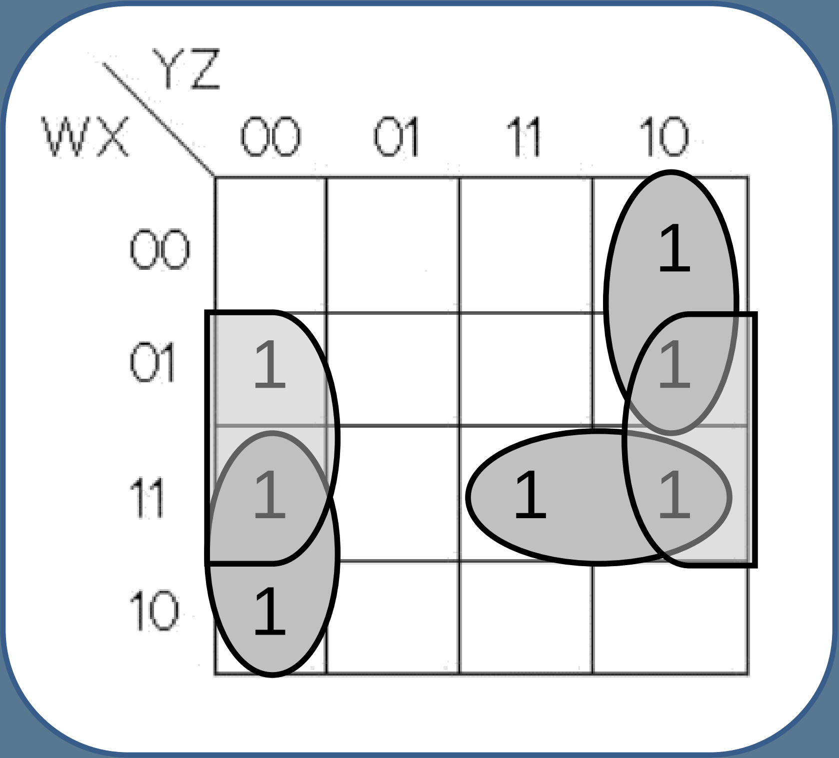
Mapping Example

- $F = x'y'z + x'yz' + xyz' + xyz$



Mapping Example

- $G = \sum(2,4,6,8,12,14,15)$



Grouping Examples

A \ BC	00	01	11	10
0	1		1	
1			1	1

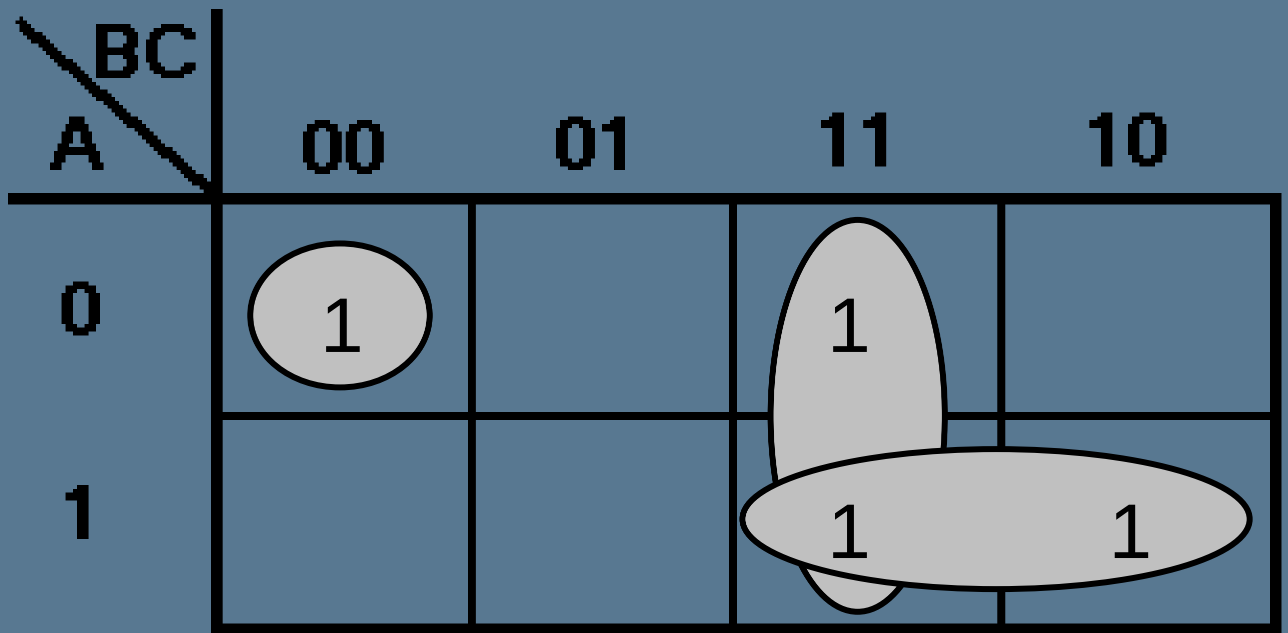
Grouping Examples

A \ BC	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC	00	01	11	10
0	1		1	
1			1	1

Grouping Examples



Grouping Examples

A \ BC	00	01	11	10
0	1	1		1
1	1	1	1	

Grouping Examples

A \ BC	00	01	11	10
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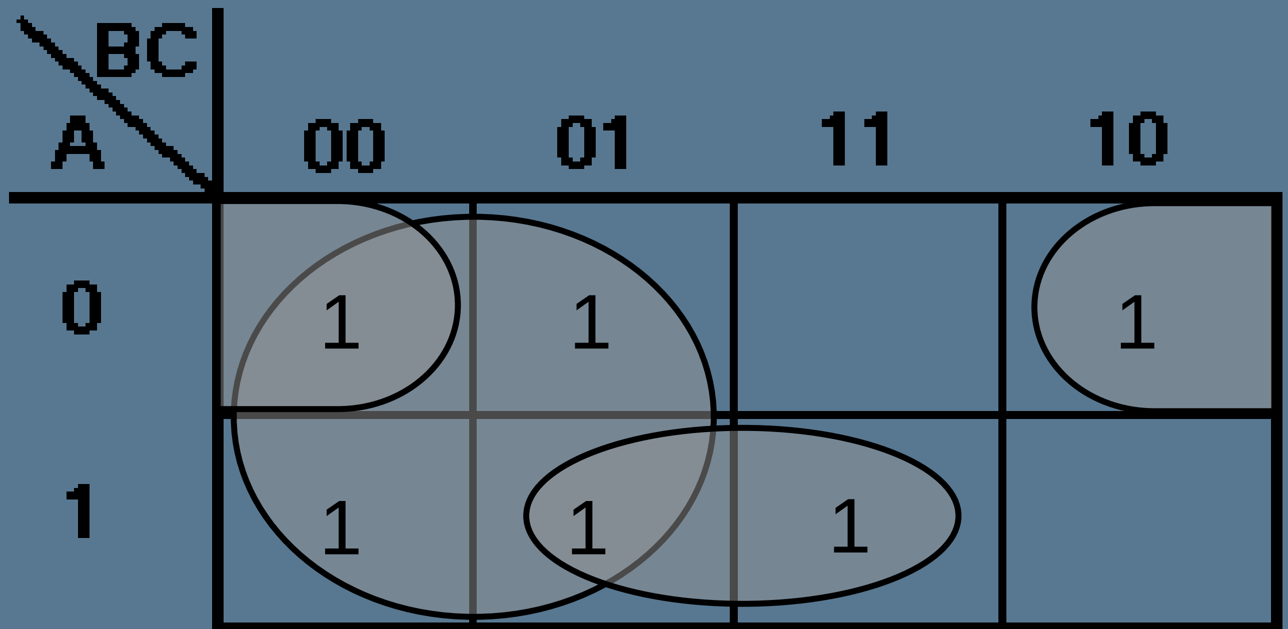
Grouping Examples

A \ BC	00	01	11	10
0	1	1		1
1	1	1	1	

Grouping Examples



Grouping Examples

AB \ CD	0 0	0 1	1 1	10
0 0	1			1
0 1	1	1		1
1 1	1	1		1
1 0	1		1	1

Grouping Examples

AB \ CD	0 0	0 1	1 1	10
0 0	1			1
0 1	1	1		1
1 1	1	1		1
1 0	1		1	1

Grouping Examples

AB \ CD	0 0	0 1	1 1	10
0 0	1			1
0 1	1	1		1
1 1	1	1		1
1 0	1		1	1

Grouping Examples

AB \ CD	0 0	0 1	1 1	10
0 0	1			1
0 1	1	1		1
1 1	1	1		1
1 0	1		1	1

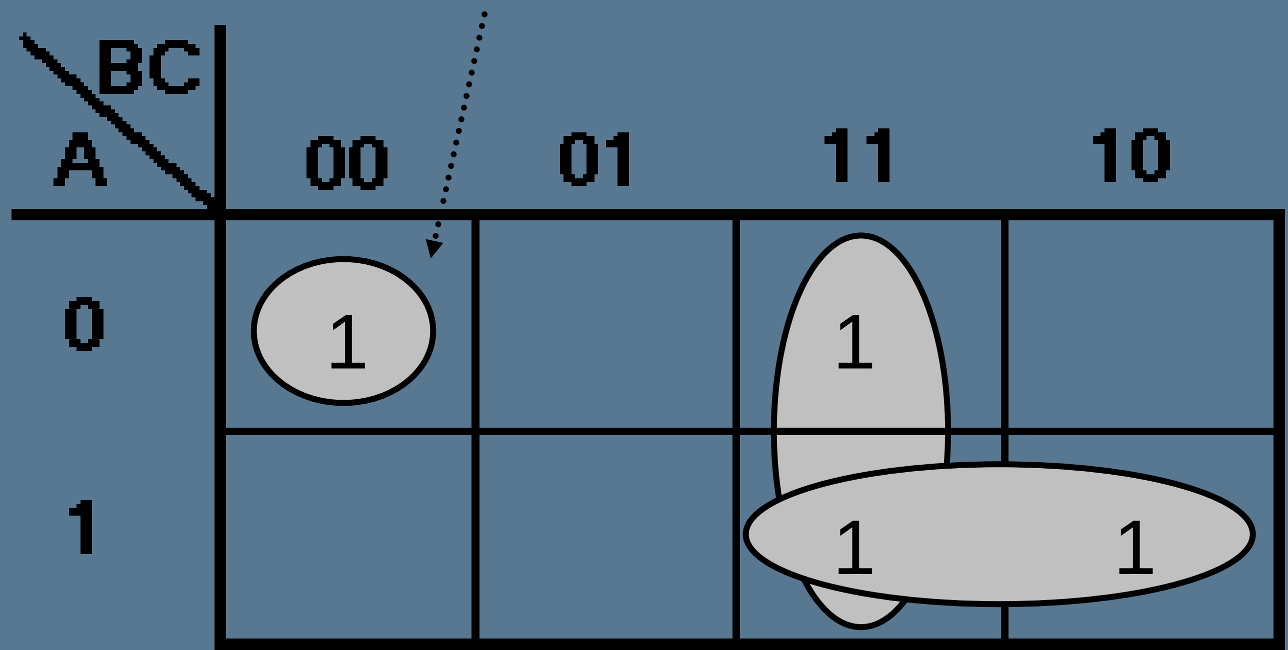
Step 4: Determine Minimum Expression

- Determine minimum terms

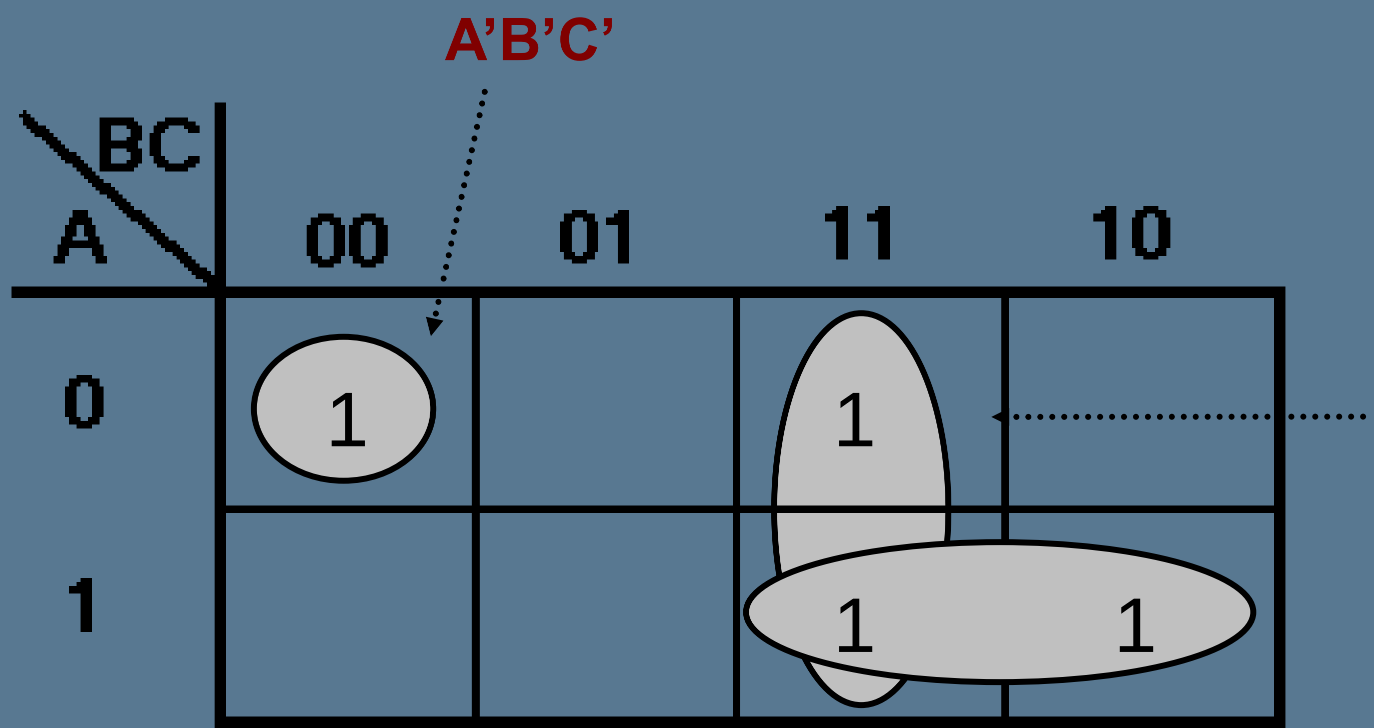
Cell	# Variables
1	3
2	2
4	1
8	fn's value is 1

Cell	# Variables
1	4
2	3
4	2
8	1
16	fn's value is 1

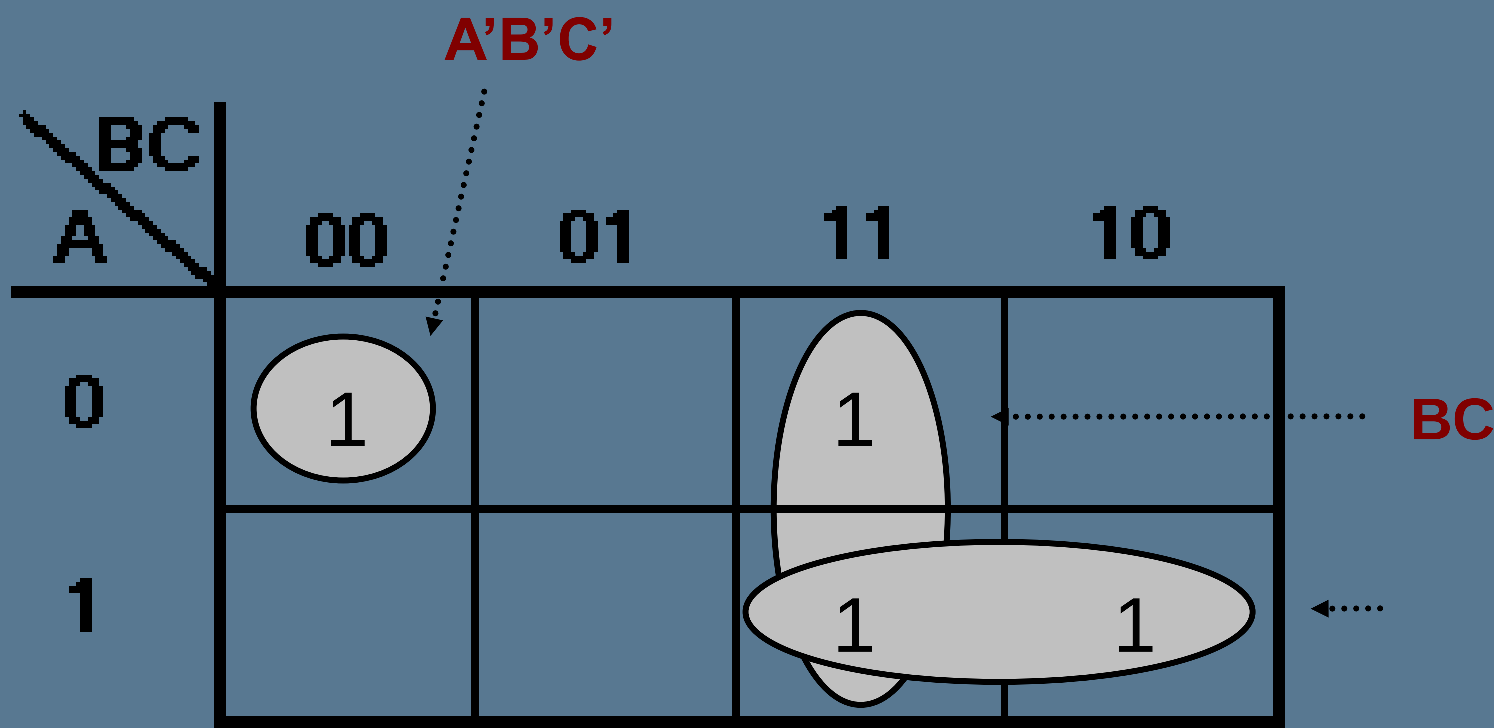
Determining Minimum Expression Example



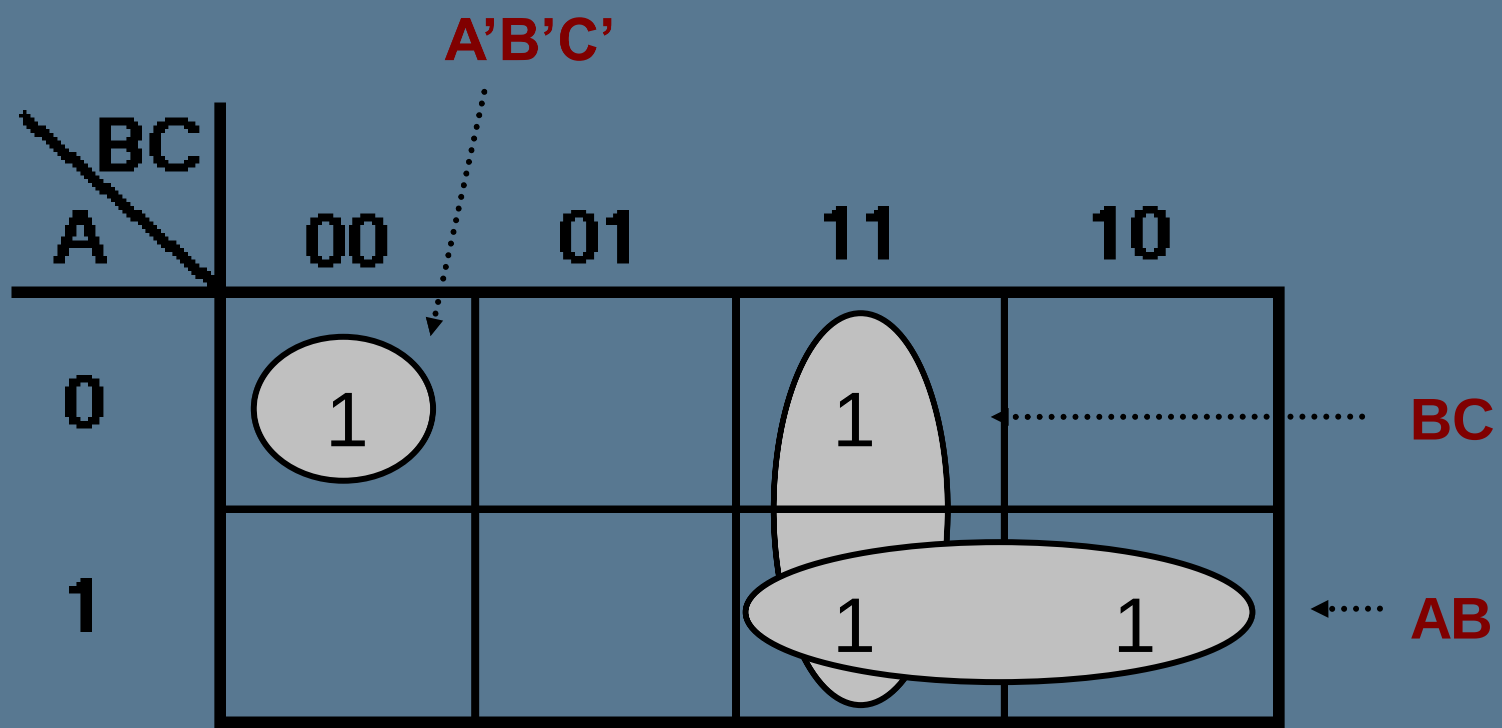
Determining Minimum Expression Example



Determining Minimum Expression Example

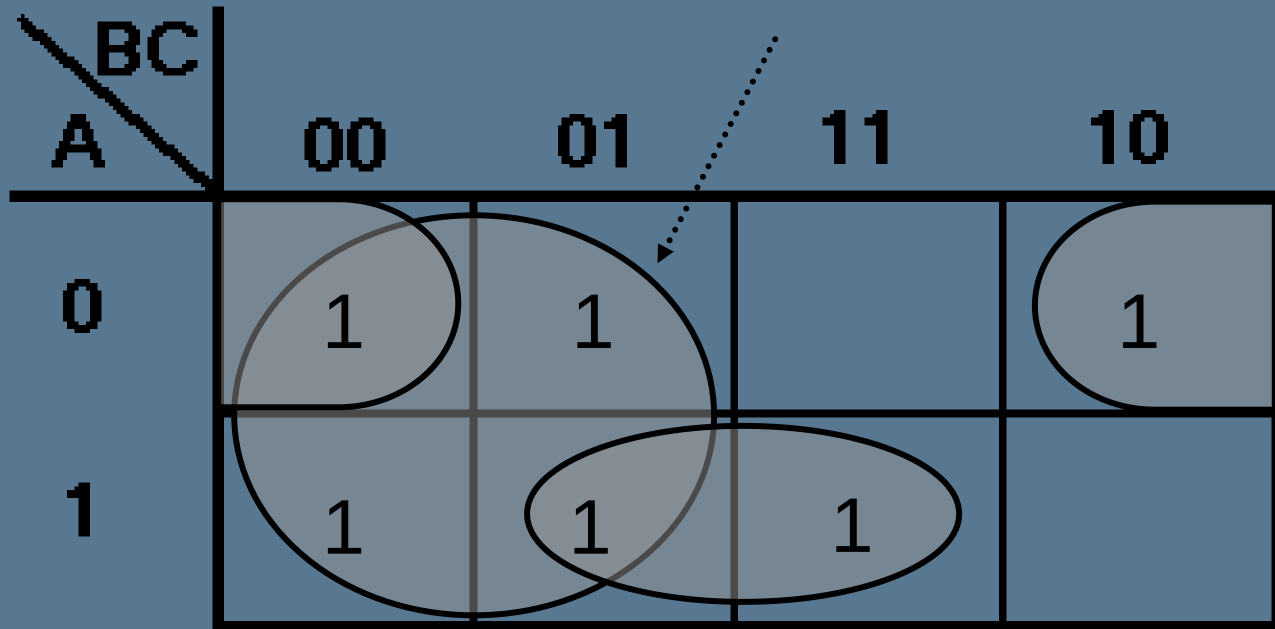


Determining Minimum Expression Example

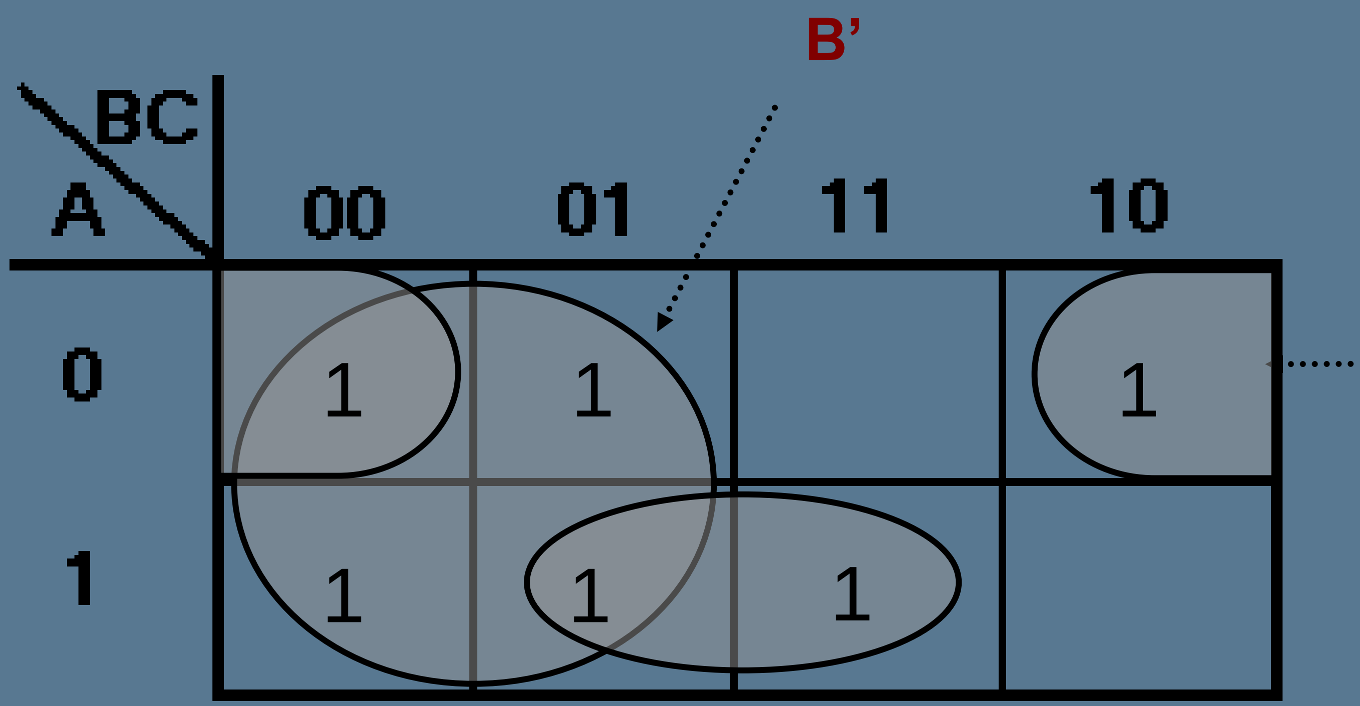


= A'B'C' + BC + AB

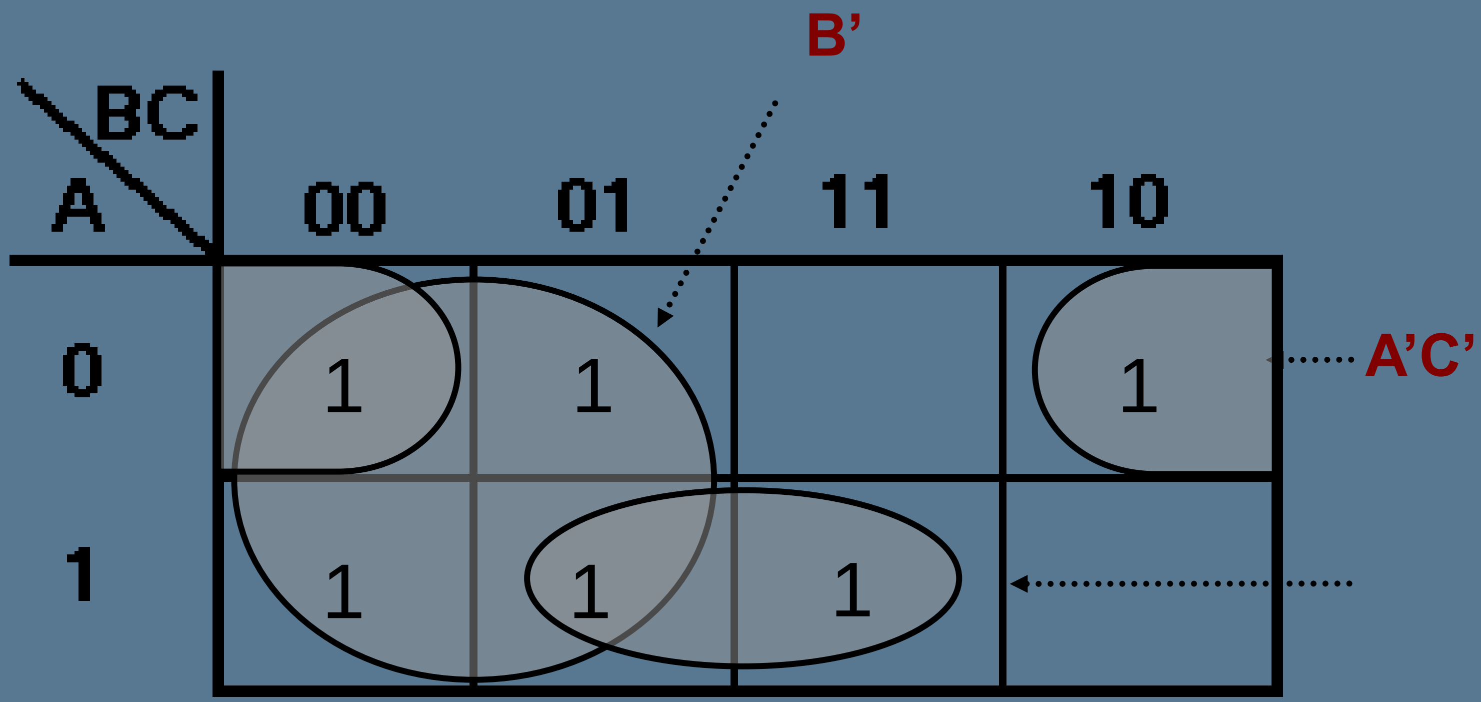
Determining Minimum Expression Example



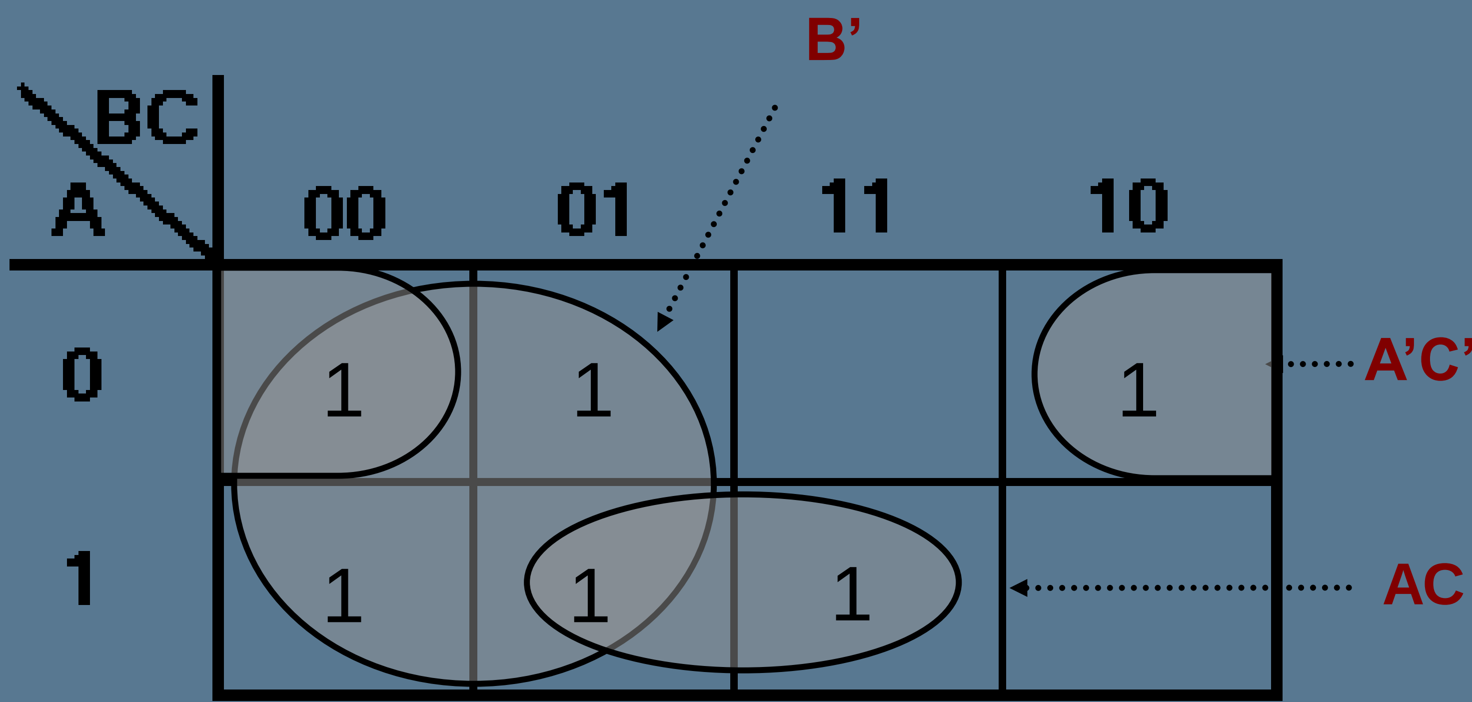
Determining Minimum Expression Example



Determining Minimum Expression Example



Determining Minimum Expression Example

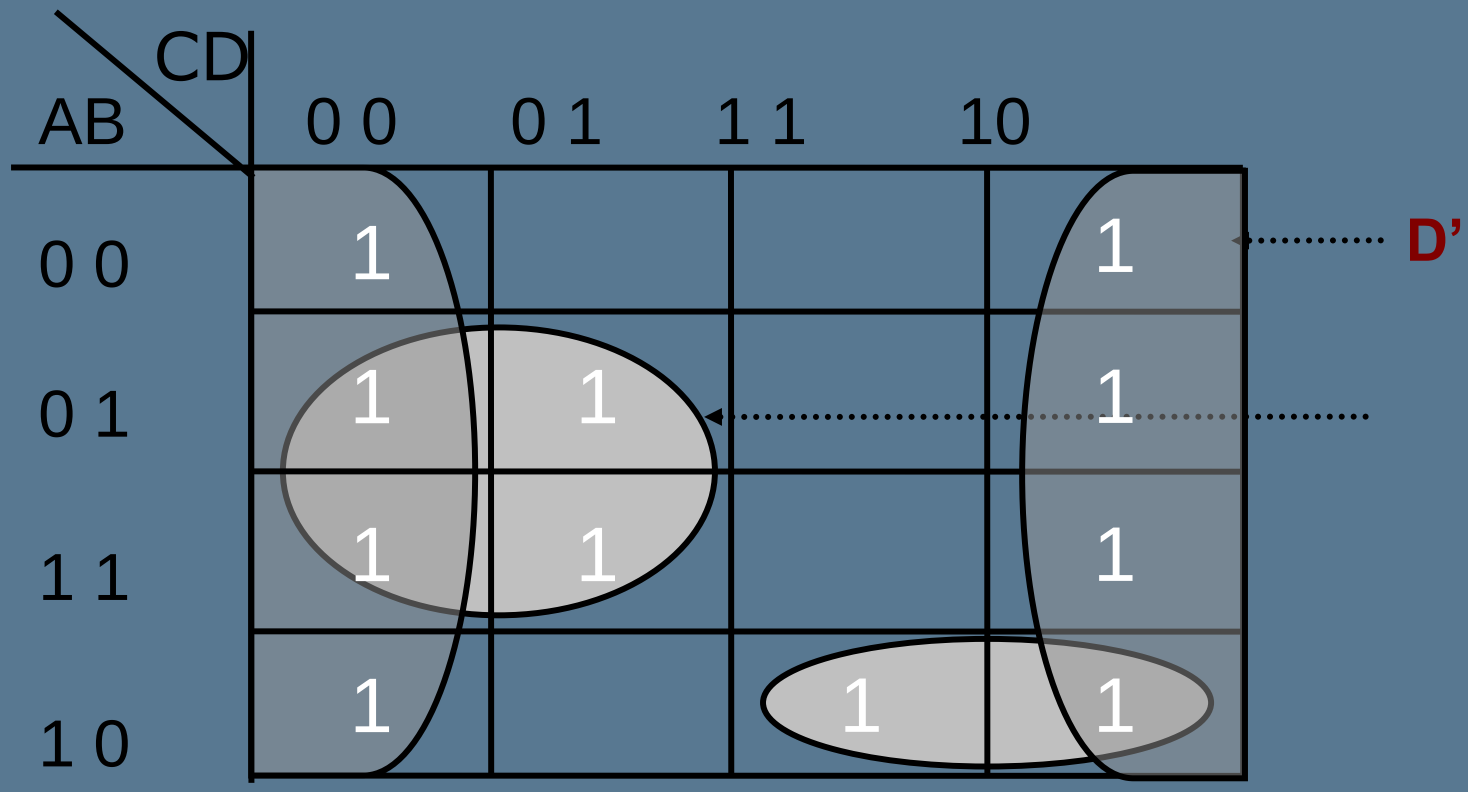


= A’C' + AC + B’

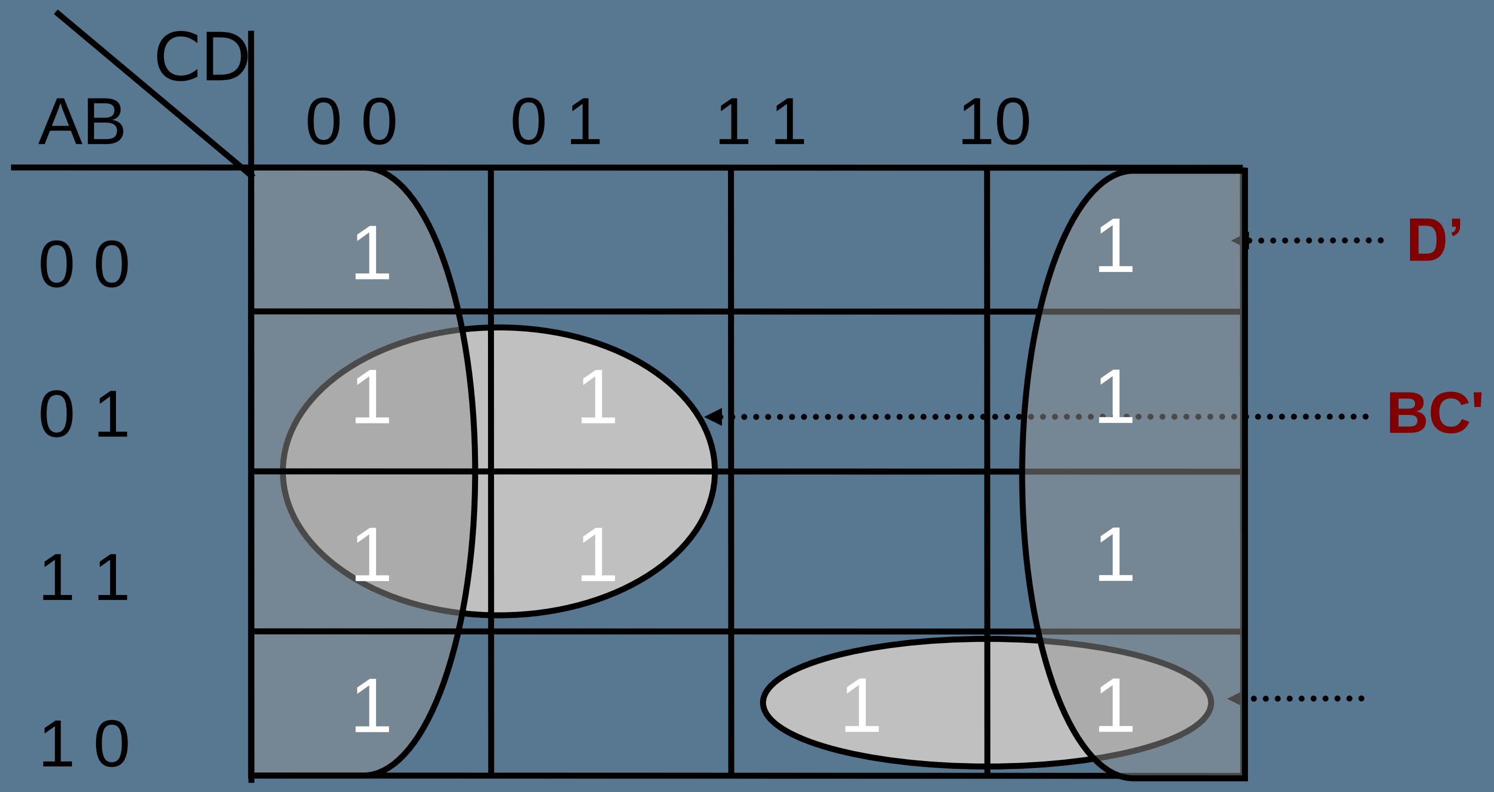
Determining Minimum Expression Example

AB \ CD	0 0	0 1	1 1	10
0 0	1			1
0 1	1	1		1
1 1	1	1		1
1 0	1		1	1

Determining Minimum Expression Example



Determining Minimum Expression Example



Determining Minimum Expression Example

AB \ CD	0 0	0 1	1 1	10
0 0	1			1
0 1	1	1		1
1 1	1	1		1
1 0	1		1	1

D’

BC'

AB’C

= D’ + BC' + AB’C