Chapter 3

Boolean Algebra, Logic Functions and Logic Gates (Part 1)

Binary Logic

- consists of binary variables and logical operations
- resembles binary arithmetic
- use and application of binary logic are demonstrated by switching cicuits
- Equivalent to Boolean Algebra

Boolean Algebra

- a set of elements, a set of operators, and a number of unproven axioms or postulates
- developed by an English mathematician named George Boole

Boolean Operations

- AND
  - represented by a dot or the absence of an operator.
  - 0 dominates

x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

Boolean Operations

- OR
  - represented by a plus sign.
  - 1 dominates

x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Operations

- NOT
  - represented by a prime
  - Inversion or complementation

x	x'
0	1
1	0

Boolean Theorems

- Boolean operations on constants
- Boolean operations on one variable
- Boolean operations on two or more variables

Boolean Operations on constants

AND	OR	NOT
$0 \cdot 0 = 0$	$0 + 0 = 0$	$0' = 1$
$0 \cdot 1 = 0$	$0 + 1 = 1$	$1' = 0$
$1 \cdot 0 = 0$	$1 + 0 = 1$	
$1 \cdot 1 = 1$	$1 + 1 = 1$	

Boolean Operations on one variable

AND	OR	NOT
$A \cdot 0 = 0$	$A + 0 = A$	$A'' = A$
$A \cdot 1 = A$	$A + 1 = 1$	
$A \cdot A = A$	$A + A = A$	
$A \cdot A' = 0$	$A + A' = 1$	

Boolean Operations On Two or More Variables

- Commutative laws

  $A + B = B + A$

  $AB = BA$

- Associative laws

  $A+(B+C) = (A+B)+C$

  $A(BC) = (AB)C$

- Distributive laws

  $A(B+C) = AB + AC$

  $A+(BC) = (A+B)(A+C)$

- De Morgan's laws

  $(A+B)' = A'B'$

  $(AB)' = A' + B'$

Boolean Operations On Two or More Variables

- Laws of Absorption

  $A + AB = A$

  $A(A+B) = A$

Boolean Functions

- Boolean functions are expressions formed with binary variables and boolean operators
- Representations of boolean functions:
  - Algebraic expression
  - Truth table

Algebraic Expression Examples

- $F_1 = xyz'$
- $F_2 = x + y'z$
- $F_3 = x'z + xy'$
- $F_4 = x'$
- $F_5 = 1$

Truth table examples

x	y	z	F_1	F_2	F_3	F_4	F_5
0	0	0	0	0	0	1	1
0	0	1	0	1	1	1	1
0	1	0	0	0	0	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	1	0	1
1	0	1	0	1	1	0	1
1	1	0	1	1	0	0	1
1	1	1	0	1	0	0	1

Simplification of Boolean functions

- $F_1 = x + x'y$

Simplification of Boolean functions

- $F_1 = x + x'y$

  $= (x + x')(x + y)$

Simplification of Boolean functions

- $F_1 = x + x'y$

  $= (x + x')(x + y)$

  $= 1(x + y)$

Simplification of Boolean functions

- $F_1 = x + x'y$
  $= (x + x')(x + y)$
  $= 1(x + y)$
  $= x + y$

Simplification of Boolean functions

- $F_2 = x(x'+y)$

Simplification of Boolean functions

- $F_2 = x(x'+y)$
  $= xx' + xy$

Simplification of Boolean functions

- $F_2 = x(x'+y)$

  $= xx' + xy$

  $= 0 + xy$

Simplification of Boolean functions

- $F_2 = x(x'+y)$

  $= xx' + xy$

  $= 0 + xy$

  $= xy$

Simplification of Boolean functions

- $F_3 = xy + xy'$

Simplification of Boolean functions

- $F_3 = xy + xy'$

  $= x(y + y')$

Simplification of Boolean functions

- $F_3 = xy + xy'$

  $= x(y + y')$

  $= x$

Simplification of Boolean functions

- $F_4 = x'y'z + x'yz + xy'$

Simplification of Boolean functions

- $F_4 = x'y'z + x'yz + xy'$

  $= x'z(y'+y) + xy'$

Simplification of Boolean functions

- $F_4 = x'y'z + x'yz + xy'$

  $= x'z(y'+y) + xy'$

  $= x'z + xy'$

Binary Variables

- Forms of variables
  - normal (x)
  - complement (x')
- Forms of terms (variables x and y)
  - Minterms m_i (or standard product)

    $x'y', x'y, xy', xy$
  - Maxterms M_i (or standard sum)

    $x+y, x+y', x'+y, x'+y'$

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0	x‘y’z’	m0	x+y+z	M0
0	0	1	x‘y’z	m1	x+y+z’	M1
0	1	0	x‘yz’	m2	x+y’+z	M2
0	1	1	x‘yz	m3	x+y’+z’	M3
1	0	0	xy‘z’	m4	x‘+y+z	M4
1	0	1	xy‘z	m5	x‘+y+z’	M5
1	1	0	xyz'	m6	x‘+y’+z	M6
1	1	1	xyz	m7	x‘+y’+z’	M7

Forms of Boolean Functions

- Canonical Form
  - Sum of minterms

$$F(x,y,z) = xyz' + x'yz$$

  - Product of maxterms

$$F(x,y,z) = (x'+y'+z)(x+y+z')$$

Forms of Boolean Functions

- Standard Form
  - Sum of products

$$F(x,y,z) = xz' + y$$

  - Product of sums

$$F(x,y,z) = (x+y')z$$