CMSC 141 Automata and Language Theory

Regular Languages

Mark Froilan B. Tandoc

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Finite Automata and Regular Expressions

Finite automata and regular expressions describe exactly the same class of languages - the class of *regular languages*

What's next?

Simplification of regular expressions and Minimization of finite automata

Simplification of Regular Expressions
Exercise: proofs

- **Identity elements for union and concatenation**
  $\emptyset + x = x + \emptyset = x$
  and
  $\varepsilon x = x\varepsilon = x$
- **Annihilation element for concatenation**
  $\emptyset x = x\emptyset = \emptyset$
- **Commutativity of union**
  $x + y = y + x$
- **Associativity of union, concatenation**
  $(x + y) + z = x + (y + z)$
  and
  $(xy)z = x(yz)$

More Identities
Exercise: proofs

- **Distributive properties**
  $x(y+z) = xy + xz$
  and
  $(x+y)z = xz + yz$
  but
  $(x+y)^* \neq x^* + y^*$
- **Idempotency of union and Kleene closure**
  $x + x = x$
  and
  $(x^*)^* = x^*$
  and
  $(x^+)^+ = x^+$

More Identities
Exercise: proofs

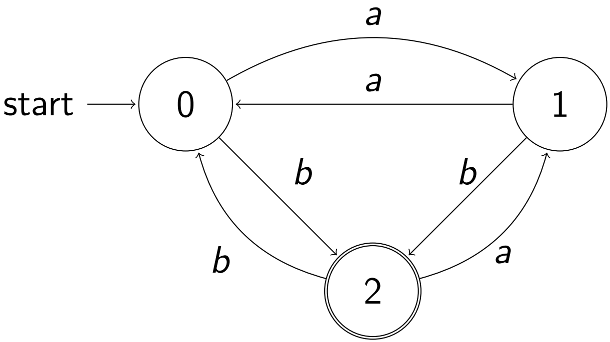
- **Absorption property**
  If $x \subseteq y$ then $x + y = y$
- **Kleene star properties**
  $\varepsilon^* = \varepsilon^+ = \varepsilon$
  and
  $\emptyset^* = \varepsilon$
  and
  $(x^* y^*)^* = (x + y)^*$

Minimization of Finite Automata

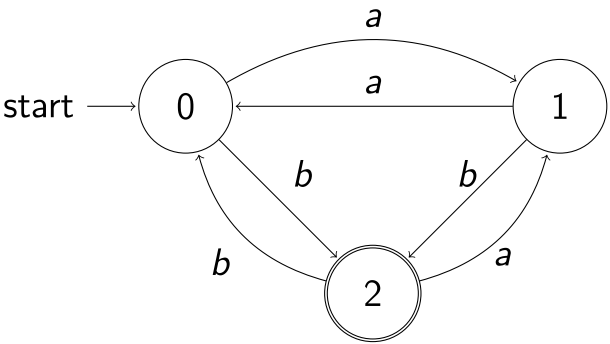
- Idea is to identify states which are essentially the same, or *indistinguishable*, and merge them into a single state.
- Easy to do with graphical tools like JFLAP where we can drag states around.

Distinguishable States

- States p and q are **distinguishable** if
  - one is a final state and the other is a non-final state, or
  - there is some string $x \in \Sigma^*$, such that $\delta(p, x)$ and $\delta(q, x)$ are distinguishable



Minimization of FA

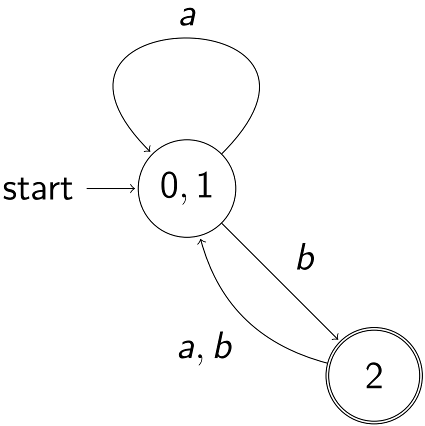


	0	1	2
0	-		X
1		-	X
2	X	X	-

States 0 and 1 are indistinguishable

for any string x, $\delta(0, x)$ and $\delta(1, x)$ are both non-final states.

Minimized FA



Regular Grammars

- **Grammars** are rule-based systems for describing languages.

  Regular grammars is for regular languages
- Example: The regular grammar below consists of a single variable $\{S\}$, two terminals $\{rose, red\}$ and two production rules $\{S \to rose, S \to red\ S\}$
  (can also be written as $\{S \to rose | red\ S\}$)
- This grammar generates the language $\{rose, red\ rose, red\ red\ rose, ...\}$

Formal Definition of Regular Grammars

A **regular grammar** is a 4-tuple (V, T, P, S) where

- V is a finite set of variables
- T is a finite set of terminal symbols $= \Sigma$
- P is a finite set of production rules, each of the form

  $< variable > \rightarrow < terminal >^*$

  or

  $< variable > \rightarrow < terminal >^* < variable >$
- S is the start variable, $S \in V$

Derivation

- A grammar G is said to generate a string x, if x can be derived from the start variable S, by finite sequence of variable replacements based on the production rules

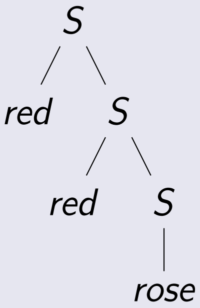
Example Derivation

Production rules: $\{S \rightarrow rose | red\ S\}$
String: "red red rose"

linear derivation

$$
\begin{aligned}
S &\rightarrow red\ S \\
&\rightarrow red\ red\ S \\
&\rightarrow red\ red\ rose
\end{aligned}
$$

parse tree



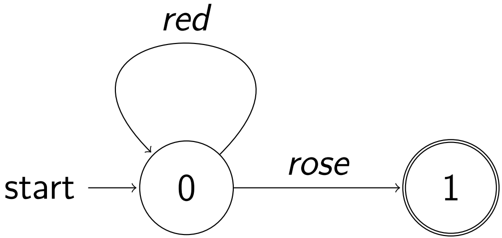
Note that $L(G) = (red)^* rose$

Regular Grammars = FA

Idea of proof:

- FA *states* are *variables* in the regular grammar
- The *start state* is the *start symbol*
- $\delta(X, a) = Y$ if and only if the rule $X \rightarrow aY$ is present
- *X is a final state* if and only if the rule $X \rightarrow \varepsilon$ is present

$S \rightarrow \mathit{red}\ S | \mathit{rose}\ T$
$T \rightarrow \varepsilon$



Example

Regular Expression

$$(0+1)^*11(0+1)^*$$

Regular Grammar

$$S \to 0S \mid 1S \mid 11T$$
$$T \to 0T \mid 1T \mid \varepsilon$$

Derivation Example

Regular Grammar

$$S \to 0S \mid 1S \mid 11T$$
$$T \to 0T \mid 1T \mid \varepsilon$$

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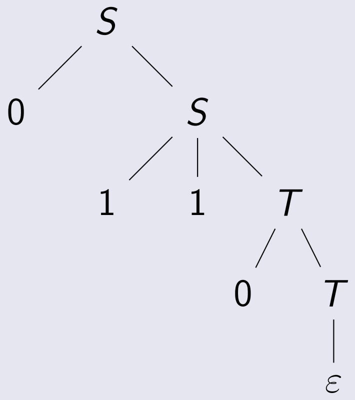
$$\begin{aligned} S &\to 0S \\ &\to 011T \\ &\to 0110T \\ &\to 0110 \end{aligned}$$

parse tree



References

- Previous slides on CMSC 141
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- J.E. Hopcroft, R. Motwani and J.D. Ullman. Introduction to Automata Theory, Languages and Computation. 2nd ed, Addison-Wesley, 2001.
- E.A. Albacea. Automata, Formal Languages and Computations, UPLB Foundation, Inc. 2005
- JFLAP, www.jflap.org