

# Chapter 4 (Part 2)

## Simplification of Logic Circuits

# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>				
<b>01</b>				
<b>11</b>				
<b>10</b>				

# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

<b>AB \ CD</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	1		1	1
<b>01</b>	1			1
<b>11</b>	1			1
<b>10</b>	1		1	1

# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

<b>CD</b>					
<b>AB</b>		<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	1			1	1
<b>01</b>	1				1
<b>11</b>	1				1
<b>10</b>	1			1	1



# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

<b>CD</b>					
<b>AB</b>		<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	1			1	1
<b>01</b>	1				1
<b>11</b>	1				1
<b>10</b>	1			1	1

# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

<b>CD</b>					
<b>AB</b>		<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>		1		1	1
<b>01</b>		1			1
<b>11</b>		1			1
<b>10</b>		1		1	1

← **D'**

# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

CD \ AB		CD			
		00	01	11	10
AB	00	1		1	1
	01	1			1
	11	1			1
	10	1		1	1

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function  $B'C$  (indicated by a red label and a blue arrow pointing to the column CD=10) and the function  $D'$  (indicated by a red label and a blue arrow pointing to the rows AB=00 and AB=01). The simplified expression is  $B'C + D'$ .



# Simplification Example

- Simplify:  $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$   
 $= B'C + D'$

CD \ AB	00	01	11	10
00	1		1	1
01	1			1
11	1			1
10	1		1	1

Annotations:

- A blue arrow points from the label  $B'C$  to the group of 8 cells (B'C).
- A blue arrow points from the label  $D'$  to the group of 4 cells (D').

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>				
<b>01</b>				
<b>11</b>				
<b>10</b>				

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0			
<b>01</b>				
<b>11</b>				
<b>10</b>	0			

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0			0
<b>01</b>				
<b>11</b>				
<b>10</b>	0			

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0			0
<b>01</b>				
<b>11</b>				
<b>10</b>	0	0		



# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0			0
<b>01</b>	0			
<b>11</b>				
<b>10</b>	0	0		

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0			0
<b>01</b>	0			
<b>11</b>	0			
<b>10</b>	0	0		

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b> <b>AB</b>	<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0			0
<b>01</b>	0			
<b>11</b>	0			
<b>10</b>	0	0		

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b>					
<b>AB</b>		<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0				0
<b>01</b>	0				
<b>11</b>	0				
<b>10</b>	0	0			

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

<b>CD</b>					
<b>AB</b>		<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
<b>00</b>	0				0
<b>01</b>	0				
<b>11</b>	0				
<b>10</b>	0	0			



# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

CD \ AB		00	01	11	10
00	0				0
01	0				
11	0				
10	0	0			

$A+B+D$

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

CD \ AB		00	01	11	10	
AB	00	0			0	$A+B+D$ ←
	01	0				
	11	0				$C+D$ ←
	10	0	0			$C+D$ ←

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

CD \ AB		00	01	11	10	
AB	00	0			0	$A+B+D$
	01	0				
	11	0				$C+D$
	10	0	0			$A'+B+C$

# Simplification Example

- Simplify:  $(B+C+D) (A+B+C'+D) (A'+B+C+D')$   
 $(A+B'+C+D) (A'+B'+C+D)$

$= (A+B+D)(C+D)(A'+B+C)$

CD \ AB	00	01	11	10	
00	0			0	$A+B+D$ ←
01	0				
11	0				$C+D$ ←
10	0	0			$A'+B+C$ ←

# Advantages of using K-map

- Can be done more systematically
- Much simpler to find minimum solutions
- Easier to see what is happening



# Simplification: Quine–McCluskey

- **Advantages**

- Specific step by step procedure
- Can be applied to problems with many variables
- Suitable for machine computation

# Simplification: Quine–McCluskey

- **Steps**
  - Construct prime implicants table
  - Construct prime implicants chart
  - Select all essential prime implicants
  - Select a minimum cover from the remaining prime implicants

# Constructing Prime Implicants Table

- List terms in a column using their binary representation
  - Group terms so that each group contains minterms with the same number of 1's
  - Place groupings which differ by only one literal adjacent to one another

# Example

- Simplify:  $F = \Sigma m(1,3,7,8,14,15)$

1    0001

3    0011

7    0111

8    1000

14   1110

15   1111

# Example

- Simplify:  $F = \Sigma m(1,3,7,8,14,15)$

Column 1

1	0001	1	0001
3	0011	8	1000
7	0111		
8	1000	3	0011
14	1110		
15	1111	7	0111
		14	1110
		15	1111



# Constructing Prime Implicants Table

- Perform exhaustive search for logically adjacent terms between adjacent groups
  - Each term should be checked off
  - Combine each pair of terms into a single term replacing the differing literal with ‘–’
  - Repeat procedure until no further terms can be created
  - All unchecked terms are prime implicants

# Example

- Simplify:  $F = \Sigma m(1,3,7,8,14,15)$

		Column 1	Column 2
1	0001	1	0001
3	0011	8	1000
7	0111		
8	1000	3	0011
14	1110		
15	1111	7	0111
		14	1110
		15	1111

# Example

- Simplify:  $F = \Sigma m(1,3,7,8,14,15)$

		Column 1		Column 2
1	0001	1	0001 ✓	1,3 00-1
3	0011	8	1000	
7	0111			
8	1000	3	0011 ✓	
14	1110			
15	1111	7	0111	
		14	1110	
		15	1111	

# Example

- Simplify:  $F = \Sigma m(1,3,7,8,14,15)$

		Column 1		Column 2	
1	0001	1	0001 ✓	1,3	00-1
3	0011	8	1000		
7	0111			3,7	0-11
8	1000	3	0011 ✓		
14	1110				
15	1111	7	0111 ✓		
		14	1110		
		15	1111		

# Example

- Simplify:  $F = \Sigma m(1,3,7,8,14,15)$

		Column 1		Column 2
1	0001	1	0001 ✓	1,3 00-1
3	0011	8	1000	
7	0111			3,7 0-11
8	1000	3	0011 ✓	
14	1110			7,15 -111
15	1111	7	0111 ✓	
		14	1110	
		15	1111 ✓	

# Example

- Simplify:  $F = \Sigma m(1,3,7,8,14,15)$

		Column 1		Column 2
1	0001	1	0001 ✓	1,3 00-1
3	0011	8	1000	
7	0111			3,7 0-11
8	1000	3	0011 ✓	
14	1110			7,15 -111
15	1111	7	0111 ✓	14,15 111-
		14	1110 ✓	
		15	1111 ✓	



# Example

- Simplify:  $F = \sum m(1,3,7,8,14,15)$

		Column 1		Column 2
1	0001	1	0001 ✓	1,3 00-1
3	0011	8	1000	
7	0111			3,7 0-11
8	1000	3	0011 ✓	
14	1110			7,15 -111
15	1111	7	0111 ✓	14,15 111-
		14	1110 ✓	
		15	1111 ✓	

Prime  
Implicants:  
 **$AB'C'D'$**   
 **$A'B'D$**   
 **$A'CD$**   
 **$BCD$**   
 **$ABC$**

# Construct Prime Implicants Chart

- Terms are listed horizontally
- Prime implicants are listed vertically
- Place an X whenever a prime implicant covers a minterm

# Example

		1	3	7	8	14	15
8	1000						
1,3	00-1						
3,7	0-11						
7,15	-111						
14,15	111-						

# Example

		1	3	7	8	14	15
8	1000				<b>x</b>		
1,3	00-1	<b>x</b>	<b>x</b>				
3,7	0-11		<b>x</b>	<b>x</b>			
7,15	-111			<b>x</b>			<b>x</b>
14,15	111-					<b>x</b>	<b>x</b>

# Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				<b>x</b>		
1,3	00-1	<b>x</b>	<b>x</b>				
3,7	0-11		<b>x</b>	<b>x</b>			
7,15	-111			<b>x</b>			<b>x</b>
14,15	111-					<b>x</b>	<b>x</b>

# Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				<b>x</b>		
1,3	00-1	<b>x</b>	<b>x</b>				
3,7	0-11		<b>x</b>	<b>x</b>			
7,15	-111			<b>x</b>			<b>x</b>
14,15	111-					<b>x</b>	<b>x</b>



# Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				x		
1,3	00-1	x	x				
3,7	0-11		x	x			
7,15	-111			x			x
14,15	111-					x	x
		✓			✓	✓	

# Select Minimum Cover

		1	3	7	8	14	15
8	1000				x		
1,3	00-1	x	x				
3,7	0-11		x	x			
7,15	-111			x			x
14,15	111-					x	x
		✓	✓		✓	✓	✓

# Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
  - $F = AB'C'D' + A'B'D + ABC$

# Select Minimum Cover

		1	3	7	8	14	15
8	1000				x		
1,3	00-1	x	x				
3,7	0-11		x	x			
✓ 7,15	-111			x			x
14,15	111-					x	x
		✓	✓		✓	✓	✓

# Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
  - $F = AB'C'D' + A'B'D + ABC + BCD$

## Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$   
 $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$

AB \ CD	00	01	11	10
00				
01				
11				
10				



## Converting Between POM & SOM Using K-Map

- $(A' + B' + C + D)(A + B' + C + D)(A + B + C + D')$   
 $(A + B + C' + D')(A' + B + C + D')(A + B + C' + D)$

AB \ CD	00	01	11	10
00		0	0	0
01	0			
11	0			
10		0		

## Converting Between POM & SOM Using K-Map

- $(A' + B' + C + D)(A + B' + C + D)(A + B + C + D')$   
 $(A + B + C' + D')(A' + B + C + D')(A + B + C' + D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

## Converting Between POM & SOM Using K-Map

- $(A' + B' + C + D)(A + B' + C + D)(A + B + C + D')$   
 $(A + B + C' + D')(A' + B + C + D')(A + B + C' + D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

## Converting Between POM & SOM Using K-Map

- $(A' + B' + C + D)(A + B' + C + D)(A + B + C + D')$   
 $(A + B + C' + D')(A' + B + C + D')(A + B + C' + D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

## Converting Between POM & SOM Using K-Map

- $(A' + B' + C + D)(A + B' + C + D)(A + B + C + D')$   
 $(A + B + C' + D')(A' + B + C + D')(A + B + C' + D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

## Converting Between POM & SOM Using K-Map

- $(A' + B' + C + D)(A + B' + C + D)(A + B + C + D')$   
 $(A + B + C' + D')(A' + B + C + D')(A + B + C' + D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1



## Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$   
 $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$

**B'C'D'**

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

## Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$   
 $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

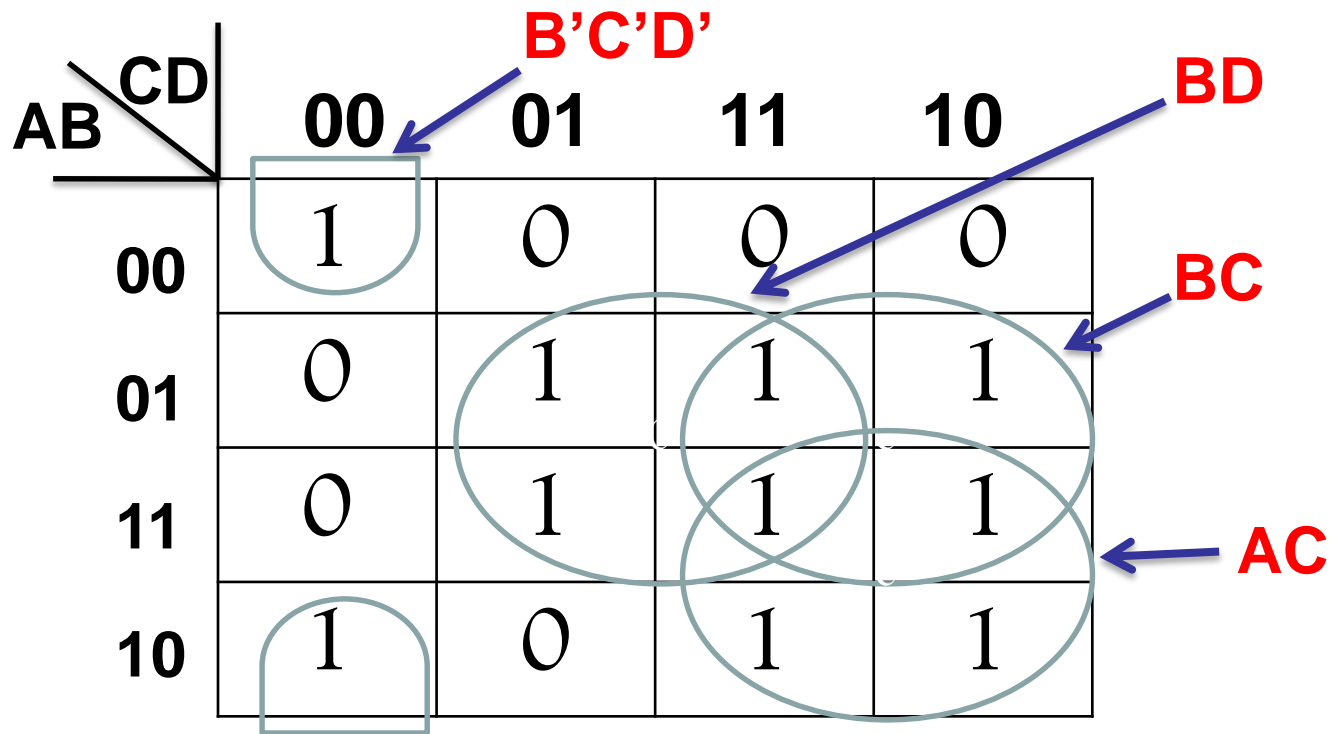
## Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$   
 $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

## Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D)(A+B'+C+D)(A+B+C+D')$   
 $(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$



# Converting Between POM & SOM Using K-Map

$$= B'C'D' + BD + BC + AC$$

- $(A' + B' + C + D)(A + B' + C + D)(A + B + C + D')$   
 $(A + B + C' + D')(A' + B + C + D')(A + B + C' + D)$

Karnaugh Map (K-Map) for a 4-variable function (A, B, C, D). The map shows the following minterms (1s) and groupings:

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

Groupings and Simplifications:

- $B'C'D'$** : Group of 1s at (00,00) and (10,00).
- $BD$** : Group of 1s at (01,00), (11,00), (01,01), (11,01), (01,11), (11,11), (01,10), and (11,10).
- $BC$** : Group of 1s at (01,01), (11,01), (01,11), and (11,11).
- $AC$** : Group of 1s at (01,01), (11,01), (01,11), (11,11), (01,10), and (11,10).

# Don't Care Conditions

- The unspecified minterms (maxterms) of an incompletely specified function
- An **X** inside a map represents a don't care condition



# Don't Care Conditions

- There are 2 cases when this occurs.
  - The input combination never occurs
    - e.g. the BCD code does not use the 6 remaining codes.
  - The input combinations are expected to occur, but we do not care what the outputs are



# Representation of Don't Cares

- e.g.

$$F(W,X,Y,Z) = \Sigma m(0,1,2,4,6,7,8,10)$$

$$d(W,X,Y,Z) = \Sigma d(12,13,14,15)$$

- It could also be represented as:

$$F(W,X,Y,Z) = \Sigma m(0,1,2,4,6,7,8,10) \\ + \Sigma d(12,13,14,15)$$

# Example

- Simplify  $F = \sum m(1, 3, 7, 14, 15) + d(8)$

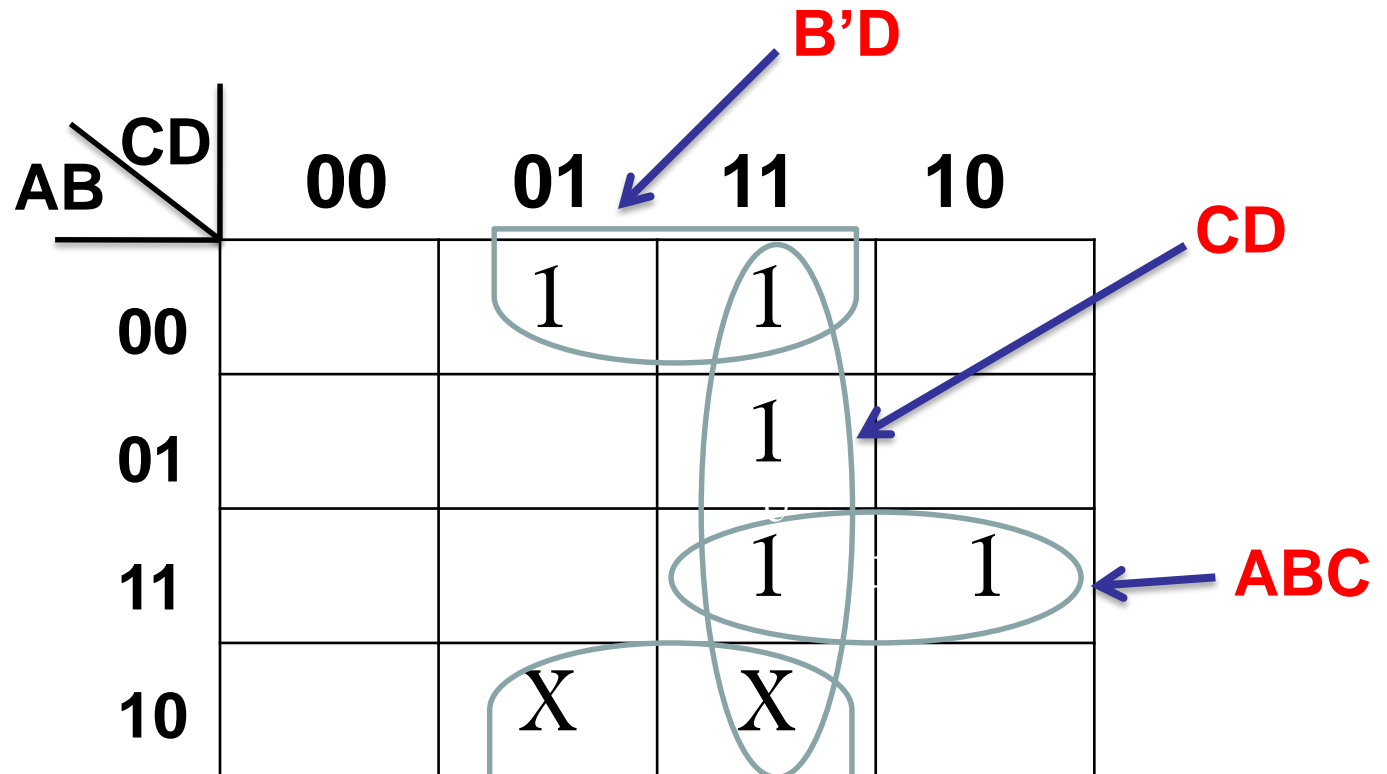
AB \ CD	00	01	11	10
00		1	1	
01			1	
11			1	1
10	X			

Diagram illustrating the Karnaugh map for the function  $F = \sum m(1, 3, 7, 14, 15) + d(8)$ . The map shows the following groupings:

- A'B'D**: Grouped cells (0,1), (0,3), (1,3), (1,4) corresponding to minterms 1, 3, 7, 15.
- BCD**: Grouped cells (1,3), (1,4), (2,3), (2,4) corresponding to minterms 3, 7, 11, 15.
- ABC**: Grouped cells (2,3), (2,4) corresponding to minterms 11, 15.

# Example

- Simplify  $F = \sum m(1, 3, 7, 14, 15) + d(9, 11)$



# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	00	01	11	10
00				
01				
11				
10				

# Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$

AB \ CD	00	01	11	10
00	1			1
01	1			
11	1			
10	1	1		

# Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$

AB \ CD	00	01	11	10
00	1			1
01	1			X
11	1	X	X	
10	1	1		

# Example

- $F(A,B,C,D) = \Sigma m(0,2,4,8,9,12) + \Sigma d(6,13,15)$

AB \ CD	00	01	11	10
00	1			1
01	1			X
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00	1			1
01	1			X
11	1	X	X	
10	1	1		

Diagram illustrating the Karnaugh map for the function  $F(A,B,C,D)$ . The map shows the function value (1 or X) for each combination of A, B, C, and D. The variables A and B are the row labels, and C and D are the column labels. The function is defined as  $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$ .

Two groups are highlighted with blue circles and arrows:

- A group of four cells (00, 01, 11, 10) in the first column (CD=00) is circled, labeled **A'D'**.
- A group of four cells (00, 01, 11, 10) in the second column (CD=01) is circled, labeled **A'D**.

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	00	01	11	10	
00	1			1	← <b>A'D'</b>
01	1			X	
11	1	X	X		← <b>AC'</b>
10	1	1			

# Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15) = A'D' + AC'$

AB \ CD	00	01	11	10
00	1			1
01	1			X
11	1	X	X	
10	1	1		

Diagram illustrating the Karnaugh map for the function  $F(A,B,C,D)$ . The map shows the function value for each combination of variables A, B, C, and D. The function is 1 for minterms 0, 2, 4, 8, 9, and 12, and is a don't care (X) for minterms 6, 13, and 15. The map is simplified to the expression  $A'D' + AC'$ .

Groupings shown:

- A group of four cells (minterms 0, 2, 4, 6) is circled, corresponding to the term  $A'D'$ .
- A group of four cells (minterms 4, 5, 12, 13) is circled, corresponding to the term  $AC'$ .