CMSC 141 Automata and Language Theory

Context-Free Languages

Mark Froilan B. Tandoc

October 15, 2014

Simplifying Context Free Grammars

- Chomsky Normal Form

- Chomsky Normal Form
  - $V \to (T + VV)$

- Chomsky Normal Form
  - $V \rightarrow (T + VV)$
- Greibach Normal Form

Simplifying Context Free Grammars

- Chomsky Normal Form
  - $V \rightarrow (T + VV)$
- Greibach Normal Form
  - $V \rightarrow TV^*$

- Chomsky Normal Form
  - $V \rightarrow (T + VV)$
- Greibach Normal Form
  - $V \rightarrow TV^*$
- Elimination of unit productions ($V \rightarrow W$), and empty productions ($V \rightarrow \varepsilon$) except for the start state

Chomsky Normal Form

Chomsky Normal Form

A context-free grammar in Chomsky Normal Form (CNF) have rules of the form:

Chomsky Normal Form

A context-free grammar in Chomsky Normal Form (CNF) have rules of the form:

$A \to BC$
$A \to a$

Chomsky Normal Form

A context-free grammar in Chomsky Normal Form (CNF) have rules of the form:

$A \rightarrow BC$
$A \rightarrow a$

where a is any terminal and A, B, C are any variables - except that B and C may not be the start variable. Also, only the start variable (say S), can have the rule $S \rightarrow \varepsilon$

Chomsky Normal Form

Convert the grammar to CNF

Chomsky Normal Form

Convert the grammar to CNF

Grammar

$$
\begin{aligned}
S &\rightarrow ASA \mid aB \\
A &\rightarrow B \mid S \\
B &\rightarrow b \mid \varepsilon
\end{aligned}
$$

Chomsky Normal Form

Add a new start variable (say S_0) and have the rule $S_0 \rightarrow S$ where S is the original start state

Grammar

$$
\begin{aligned}
S &\rightarrow ASA \mid aB \\
A &\rightarrow B \mid S \\
B &\rightarrow b \mid \varepsilon
\end{aligned}
$$

Chomsky Normal Form

Add a new start variable (say S_0) and have the rule $S_0 \to S$ where S is the original start state

Grammar

$$
\begin{aligned}
S_0 &\to S \\
S &\to ASA \mid aB \\
A &\to B \mid S \\
B &\to b \mid \varepsilon
\end{aligned}
$$

Chomsky Normal Form

Remove the ε rules. $B \to \varepsilon$

Grammar

$$
\begin{aligned}
S_0 &\to S \\
S &\to ASA \mid aB \\
A &\to B \mid S \\
B &\to b \mid \varepsilon
\end{aligned}
$$

Chomsky Normal Form

Remove the ε rules. $B \rightarrow \varepsilon$

Grammar

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow ASA \mid aB \mid a \\
A &\rightarrow B \mid S \mid \varepsilon \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Remove the ε rules. $A \rightarrow \varepsilon$

Grammar

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow ASA \mid aB \mid a \\
A &\rightarrow B \mid S \mid \varepsilon \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Remove the ε rules. $A \to \varepsilon$

Grammar

$$
\begin{aligned}
S_0 &\to S \\
S &\to ASA \mid aB \mid a \mid SA \mid AS \mid S \\
A &\to B \mid S \\
B &\to b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $S \rightarrow S$

Grammar

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \mid S \\
A &\rightarrow B \mid S \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $S \to S$

Grammar

$$
\begin{aligned}
S_0 &\to S \\
S &\to ASA \mid aB \mid a \mid SA \mid AS \\
A &\to B \mid S \\
B &\to b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $S_0 \to S$

Grammar

$$
\begin{aligned}
S_0 &\to S \\
S &\to ASA \mid aB \mid a \mid SA \mid AS \\
A &\to B \mid S \\
B &\to b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $S_0 \to S$

Grammar

$$
\begin{aligned}
S_0 &\to ASA \mid aB \mid a \mid SA \mid AS \\
S &\to ASA \mid aB \mid a \mid SA \mid AS \\
A &\to B \mid S \\
B &\to b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $A \rightarrow B$

Grammar

$$
\begin{aligned}
S_0 &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
S &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
A &\rightarrow B \mid S \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $A \rightarrow B$

Grammar

$$
\begin{aligned}
S_0 &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
S &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
A &\rightarrow S \mid b \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $A \to S$

Grammar

$$
\begin{aligned}
S_0 &\to ASA \mid aB \mid a \mid SA \mid AS \\
S &\to ASA \mid aB \mid a \mid SA \mid AS \\
A &\to S \mid b \\
B &\to b
\end{aligned}
$$

Chomsky Normal Form

Remove unit rules. $A \rightarrow S$

Grammar

$$
\begin{aligned}
S_0 &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
S &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
A &\rightarrow b \mid ASA \mid aB \mid a \mid SA \mid AS \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Convert the remaining rules into proper form by adding additional variables and rules

Grammar

$$
\begin{aligned}
S_0 &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
S &\rightarrow ASA \mid aB \mid a \mid SA \mid AS \\
A &\rightarrow b \mid ASA \mid aB \mid a \mid SA \mid AS \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Convert the remaining rules into proper form by adding additional variables and rules

Grammar

$$
\begin{aligned}
S_0 &\rightarrow AA_1 \mid UB \mid a \mid SA \mid AS \\
S &\rightarrow AA_1 \mid UB \mid a \mid SA \mid AS \\
A &\rightarrow b \mid AA_1 \mid UB \mid a \mid SA \mid AS \\
A_1 &\rightarrow SA \\
U &\rightarrow a \\
B &\rightarrow b
\end{aligned}
$$

Chomsky Normal Form

Chomsky Normal Form

Convert to CNF

$$S \to ab$$
$$S \to aSb$$

Chomsky Normal Form

Convert to CNF

$$\begin{aligned} S &\to ab \\ S &\to aSb \end{aligned}$$

CNF

$$\begin{aligned} S &\to AB \mid XB \\ X &\to AY \\ Y &\to AB \mid XB \\ A &\to a \\ B &\to b \end{aligned}$$

Greibach Normal Form

Greibach Normal Form

A context-free grammar in Greibach Normal Form (GNF) have rules of the form:

Greibach Normal Form

A context-free grammar in Greibach Normal Form (GNF) have rules of the form:

$$V \rightarrow TV^*$$

Greibach Normal Form

A context-free grammar in Greibach Normal Form (GNF) have rules of the form:

$$V \to TV^*$$

where V can be any variable and T can be any terminal. Only the start variable (say S), can have the rule $S \to \varepsilon$

Greibach Normal Form

Greibach Normal Form

Convert the grammar to GNF

$S \rightarrow a \mid S + S$

Greibach Normal Form

Convert the grammar to GNF

$$S \rightarrow a \mid S + S$$

GNF

$$\begin{aligned} S &\rightarrow a \\ S &\rightarrow aPS \quad \text{(this makes + right-associative)} \\ P &\rightarrow + \end{aligned}$$

Greibach Normal Form

Convert the grammar to GNF

$$S \rightarrow a \mid S + S$$

GNF

$$\begin{aligned} S &\rightarrow a \\ S &\rightarrow aPS \quad \text{(this makes + right-associative)} \\ P &\rightarrow + \end{aligned}$$

When in GNF, an input string of length n can always be derived in n steps

Equivalence of PDAs and CFGs

Equivalence of PDAs and CFGs

Theorem

Every CFG can be converted into an equivalent PDA, and vice versa

Equivalence of PDAs and CFGs

Theorem

Every CFG can be converted into an equivalent PDA, and vice versa

Note that we are referring to non-deterministic PDA (NPDA) because deterministic PDA are weaker than NPDA

CFG to NPDA

CFG to NPDA

Idea:

CFG to NPDA

Idea:

- Use a single state q, with stack alphabet $\Gamma = V \cup T$, and the PDA is accepting by empty stack

CFG to NPDA

Idea:

- Use a single state q, with stack alphabet $\Gamma = V \cup T$, and the PDA is accepting by empty stack
- Initial stack symbol is the start variable

CFG TO NPDA

Idea:

- Use a single state q, with stack alphabet $\Gamma = V \cup T$, and the PDA is accepting by empty stack
- Initial stack symbol is the start variable
- For every terminal symbol a in Σ, add the transition $\delta(q, a, a) = (q, pop)$

CFG TO NPDA

Idea:

- Use a single state q, with stack alphabet $\Gamma = V \cup T$, and the PDA is accepting by empty stack
- Initial stack symbol is the start variable
- For every terminal symbol a in Σ, add the transition $\delta(q, a, a) = (q, pop)$
- For every empty production $A \to \varepsilon$, add the transition $\delta(q, \varepsilon, A) = (q, pop)$

CFG to NPDA

Idea:

- Use a single state q, with stack alphabet $\Gamma = V \cup T$, and the PDA is accepting by empty stack
- Initial stack symbol is the start variable
- For every terminal symbol a in Σ, add the transition $\delta(q, a, a) = (q, pop)$
- For every empty production $A \to \varepsilon$, add the transition $\delta(q, \varepsilon, A) = (q, pop)$
- For every rule $A \to B_1 B_2 \dots B_n$, add the transition $\delta(q, \varepsilon, A) = (q, \{pop; pushB_n; pushB_{n-1}; \dots; pushB_1\})$

CFG to NPDA

$$S \to \varepsilon \mid aSb$$

CFG to NPDA

$$S \to \varepsilon \mid aSb \implies L(G) = \{a^n b^n : n \geq 0\}$$

CFG to NPDA

$$S \to \varepsilon \mid aSb \Longrightarrow L(G) = \{a^n b^n : n \geq 0\}$$

Stack alphabet:
$\Gamma = \{S, a, b\}$

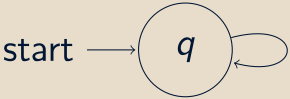
Initial stack symbol: S

CFG TO NPDA

$$S \to \varepsilon \mid aSb \Longrightarrow L(G) = \{a^n b^n : n \geq 0\}$$

Stack alphabet:
$\Gamma = \{S, a, b\}$

Initial stack symbol: S

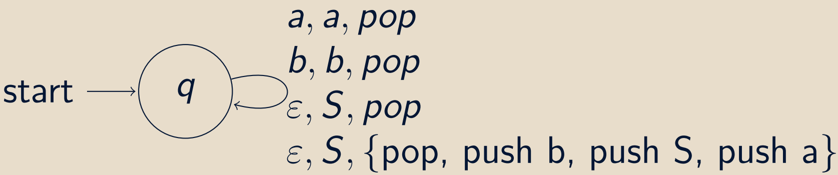


CFG to NPDA

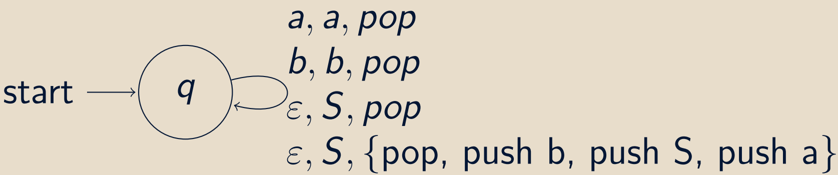
$$S \to \varepsilon \mid aSb \implies L(G) = \{a^n b^n : n \geq 0\}$$

Stack alphabet:
$\Gamma = \{S, a, b\}$

Initial stack symbol: S



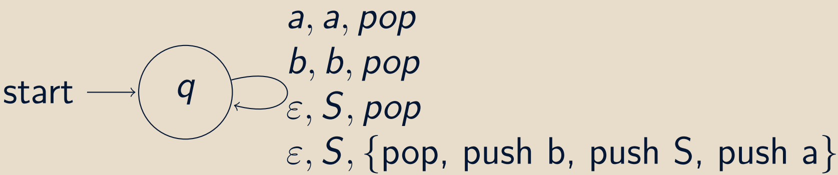
CFG TO NPDA



Trace for aabb:

______ ______ ______ ______ ______ ______ ______ ______

CFG TO NPDA

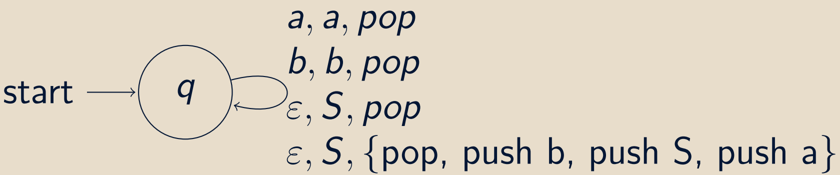


Trace for `aabb`:

$\boxed{\text{S}}$ _____ _____ _____ _____ _____ _____ _____

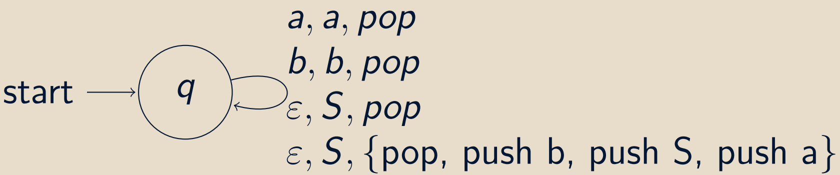
`aabb`

CFG to NPDA



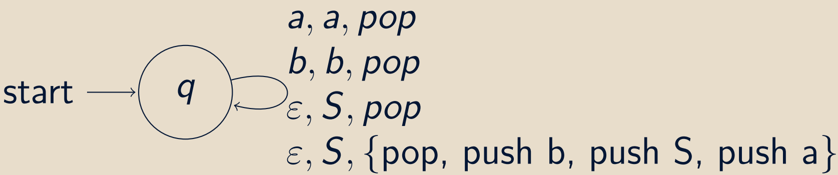
Trace for `aabb`:

CFG TO NPDA

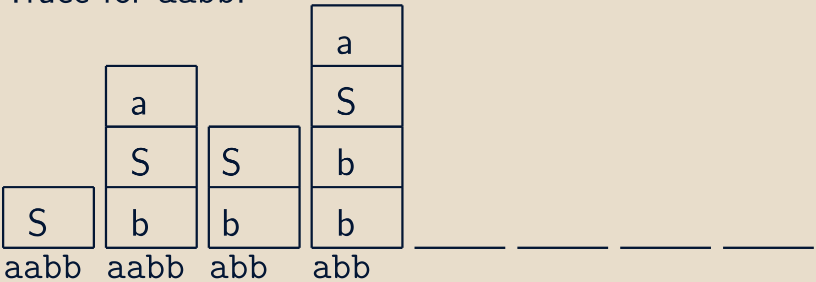


Trace for aabb:

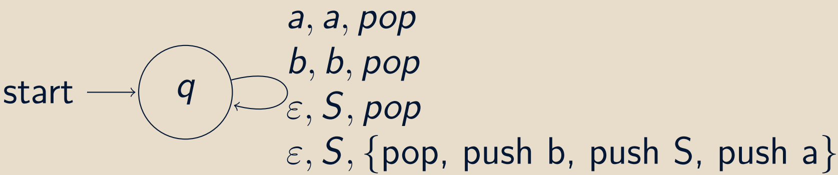
CFG to NPDA



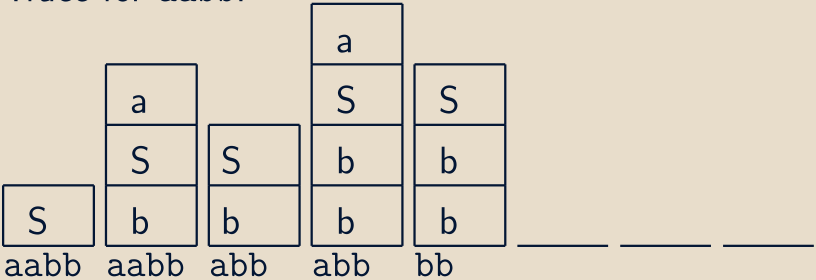
Trace for aabb:



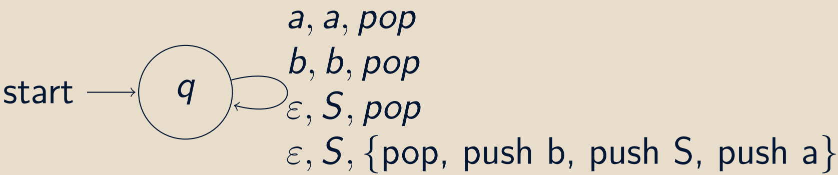
CFG to NPDA



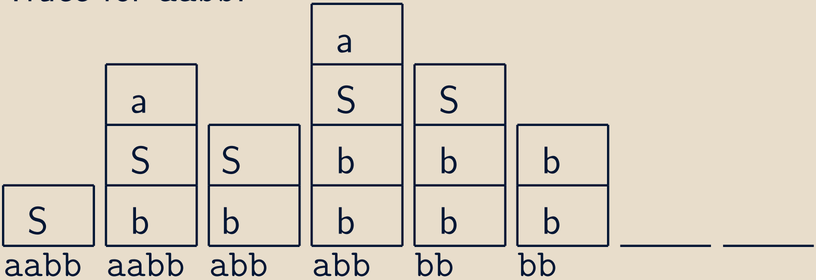
Trace for aabb:



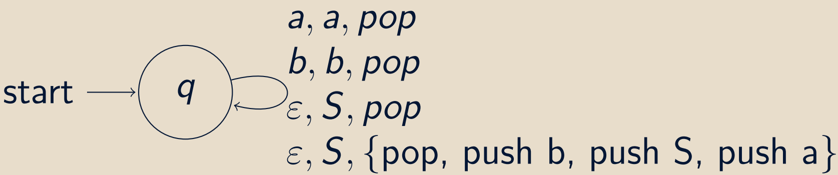
CFG to NPDA



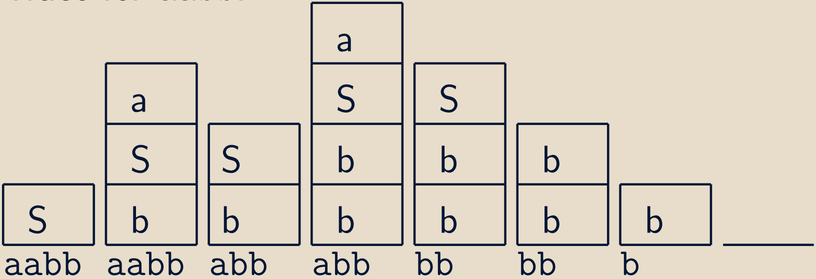
Trace for aabb:



CFG TO NPDA



Trace for aabb:



References

- Previous slides on CMSC 141
- M. Sipser. Introduction to the Theory of Computation. Thomson, 2007.
- J.E. Hopcroft, R. Motwani and J.D. Ullman. Introduction to Automata Theory, Languages and Computation. 2nd ed, Addison-Wesley, 2001.
- E.A. Albacea. Automata, Formal Languages and Computations, UPLB Foundation, Inc. 2005
- JFLAP, www.jflap.org
- Various online $\LaTeX$ and Beamer tutorials