DISCRETE PROBABILITY

Axioms and Theorems on Probability

Mutually Exclusive Events

Conditional Probability

Independent Events

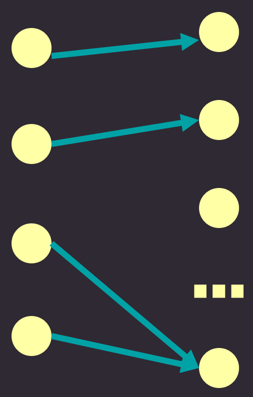
OUTLINE

Random Variables

Discrete Probability Distributions

OUTLINE

RANDOM VARIABLES



A **FUNCTION** that assigns a real number to each and every sample point of the sample space.

EXAMPLE

Consider rolling a die twice.

EXAMPLE

Consider rolling a die twice.

X = sum of the two rolls.

N = number of 6s obtained.

EXAMPLE

Consider rolling a die twice.

$$B = \begin{cases} 0 & \text{if a 6 appears.} \\ 1 & \text{if the sum of the rolls is even.} \end{cases}$$

EXAMPLE

Consider rolling a die twice.

EXAMPLE

Consider drawing 10 tablets from a box with 100 tablets and counting the number of defectives from these.

$$D = \begin{cases} 1 & \text{if } 1 \le \#\text{ defectives} < 5 \\ 2 & \text{if } \#\text{ defectives} \ge 5 \end{cases}$$

EXAMPLE

Consider drawing 10 tablets from a box with 100 tablets and counting the number of defectives from these.

DISCRETE PROBABILITY DISTRIBUTION

on a random variable X.

A function $f(x)$ that satisfies the following:

(for each possible outcome x)

- $f(x) \geq 0$
- $\sum_{\text{all } x} f(x) = 1$
- $P(X=x) = f(x)$

EXAMPLE

Consider rolling a single die.

$$X = \begin{cases} 1 \text{ if the result is even.} \\ 2 \text{ if the result is odd.} \end{cases}$$

N = number of 6s obtained.

EXAMPLE

Consider rolling a die twice.

F = number of 5s obtained.

EXAMPLE

Consider rolling a die twice.

DISCRETE PROBABILITY DISTRIBUTIONS

DISCRETE PROBABILITY DISTRIBUTIONS

- Binomial
- Multinomial
- Geometric
- Hypergeometric

BINOMIAL PROBABILITY DISTRIBUTION

BINOMIAL EXPERIMENT

Consists of n repeated identical and independent trials.

BINOMIAL EXPERIMENT

BERNOULLI TRIAL

Each trial results in two outcomes:

SUCCESS

FAILURE

BINOMIAL EXPERIMENT

BERNOULLI TRIAL

The probability of success is fixed for the whole experiment.

SUCCESS

FAILURE

BINOMIAL EXPERIMENT

EXAMPLE

Tossing a fair coin n times.

SUCCESS (Head)

FAILURE (Tail)

BINOMIAL PROBABILITY DISTRIBUTION

Given that a Bernoulli trial can result in

SUCCESS	FAILURE
With probability p	With probability 1-p

BINOMIAL PROBABILITY DISTRIBUTION

X = number of successes obtained out of the n trials.

BINOMIAL PROBABILITY DISTRIBUTION

Probability that X=k is

$$B(X=k\ ;\ n,\ p) = C(n,k)p^{k}(1-p)^{n-k}$$

BINOMIAL PROBABILITY DISTRIBUTION

EXAMPLE

Consider rolling a die seven times.

BINOMIAL PROBABILITY DISTRIBUTION

EXAMPLE

What is the probability of obtaining a number greater than four three times?

BINOMIAL PROBABILITY DISTRIBUTION

EXAMPLE

Success: obtained a number > 4

P(number>4) = 2/6

$$B(X=3) = C(7,3)(2/6)^3(4/6)^4$$
$$= 0.256059$$

MULTINOMIAL PROBABILITY DISTRIBUTION

MULTINOMIAL EXPERIMENT

Consists of n repeated identical and independent trials.

MULTINOMIAL EXPERIMENT

Each trial can result in at least two possible outcomes.

E_1 E_2 ... E_t

MULTINOMIAL EXPERIMENT

EXAMPLE

Consider rolling a fair die four times and checking if the results are:

<3 =3 or >3

MULTINOMIAL EXPERIMENT

EXAMPLE

Consider rolling a fair die four times and checking if the results are:

<3	=3	or	>3
2/6	1/6		3/6

MULTINOMIAL PROBABILITY DISTRIBUTION

Given:

- a trial can result in t outcomes E_1, E_2, ... E_t.

MULTINOMIAL PROBABILITY DISTRIBUTION

Given:

- Outcome E_i has fixed probability p_i.

MULTINOMIAL PROBABILITY DISTRIBUTION

Given:

- n trials are performed.

MULTINOMIAL PROBABILITY DISTRIBUTION

What is the probability that E_i will occur n_i times?

MULTINOMIAL PROBABILITY DISTRIBUTION

$$M(n_1, n_2, \dots, n_t; n; p_1, p_2, \dots, p_t) =$$

$$\frac{n!}{n_1! n_2! \dots n_t!} p_1 n_1 p_2 n_2 \dots p_t n_t$$

MULTINOMIAL PROBABILITY DISTRIBUTION

EXAMPLE

MULTINOMIAL PROBABILITY DISTRIBUTION

EXAMPLE

In Jay's playlist, there are 5 artists and each artist have several songs as listed:

MULTINOMIAL PROBABILITY DISTRIBUTION

EXAMPLE

6 songs by Franco,
7 songs by Fun,
5 songs by Demi,
11 songs by Maroon 5
and 2 songs by Pink.

MULTINOMIAL PROBABILITY DISTRIBUTION

EXAMPLE

Find the probability of randomly playing 16 songs where

5 are Franco's songs,

7 are Fun's songs

and 4 are Demi's songs

(Assume that all song may be played more than once).

HYPERGEOMETRIC PROBABILITY DISTRIBUTION

HYPERGEOMETRIC EXPERIMENT

A random sample of n objects is selected **without replacement** from N objects.

HYPERGEOMETRIC EXPERIMENT

Each object can be classified in one of two types:

SUCCESS	FAILURE
s out of N	N-s out of N

HYPERGEOMETRIC EXPERIMENT

EXAMPLE

A box contains 12 chocolates with almonds although only 5 actually contain at least one almond.

HYPERGEOMETRIC EXPERIMENT

EXAMPLE

Consider picking up 10 chocolates randomly from the box and counting how many has at least one almond.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION

HYPERGEOMETRIC PROBABILITY DISTRIBUTION

A sample of n objects selected from a set of N objects:

SUCCESS

s out of N

FAILURE

N-s out of N

HYPERGEOMETRIC PROBABILITY DISTRIBUTION

X = number of objects with success type.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION

Probability that X = k is

$$H(X=k\ ;\ N,\ n,\ s) = \frac{C(s,\ k)\ C(N-s,\ n-k)}{C(N,\ n)}$$

HYPERGEOMETRIC PROBABILITY DISTRIBUTION

EXAMPLE

A box contains 12 chocolates with almonds although only 5 actually contain at least one almond.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION

EXAMPLE

Consider picking up 10 chocolates randomly from the box. What is the probability that 5 have at least one almond?

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

A sample of n objects selected from a set of N objects of which there are t types.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

s_i of the N objects are of i^{th} type.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

Obtain x_i of the i^{th} type for all t types.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

Probability of obtaining x_i of the i^{th} type for all t types:

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

$$H(x_1, x_2, \ldots, x_t; N, n, s_1, s_2, \ldots, s_t) =$$

$$\frac{C(s_1,x_1)C(s_2,x_2)\ldots C(s_t,x_t)}{C(N, n)}$$

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

EXAMPLE

A noob player joined an online game and wants to form a group of 5 players from an online game with 6000 players including him.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

EXAMPLE

On this game, it is also known that:
the number of pros is 435,
number of amateurs is 600,
and the rest are noobs.

HYPERGEOMETRIC PROBABILITY DISTRIBUTION (MULTIVARIATE)

EXAMPLE

What is the probability that his team has players wherein one is pro, one is an amateur and three are noobs?

GEOMETRIC PROBABILITY DISTRIBUTION

GEOMETRIC EXPERIMENT

Consists of identical and independent trials.

GEOMETRIC EXPERIMENT

Each trial results in two outcomes:

SUCCESS **FAILURE**

GEOMETRIC EXPERIMENT

The probability of success is fixed for the whole experiment.

SUCCESS	FAILURE
With probability p	With probability $1-p$

GEOMETRIC EXPERIMENT

Each trial is repeated until a success is obtained.

GEOMETRIC EXPERIMENT

EXAMPLE

Rolling a fair die until a six is obtained.

GEOMETRIC PROBABILITY DISTRIBUTION

GEOMETRIC PROBABILITY DISTRIBUTION

Given that a Bernoulli trial can result in

SUCCESS

With probability p

FAILURE

With probability 1-p

GEOMETRIC PROBABILITY DISTRIBUTION

A Bernoulli trial is repeated until a success occurs.

GEOMETRIC PROBABILITY DISTRIBUTION

X = number of trials needed to obtain a success.

GEOMETRIC PROBABILITY DISTRIBUTION

Probability that X=k is

$$G(X=k\ ;\ p) = (1-p)^{k-1}p$$

GEOMETRIC PROBABILITY DISTRIBUTION

EXAMPLE

In Dota 2, the Radiant side has 51.78% chance of winning.

GEOMETRIC PROBABILITY DISTRIBUTION

EXAMPLE

What is the probability that Dendi plays Dota 2 (on the radiant side) and only won at the 5th game?

GEOMETRIC PROBABILITY DISTRIBUTION

EXAMPLE

Suppose that you want to win in a raffle with a 3% chance of winning by sending an email.

GEOMETRIC PROBABILITY DISTRIBUTION

EXAMPLE

What is the probability of winning the raffle by sending email at most four tries?