



Chapter 3

BOOLEAN ALGEBRA, LOGIC FUNCTIONS and LOGIC GATES

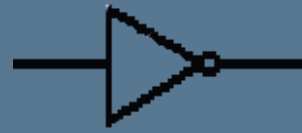
Digital Logic Gates



AND gate



OR gate



NOT gate



XOR gate



NOR gate



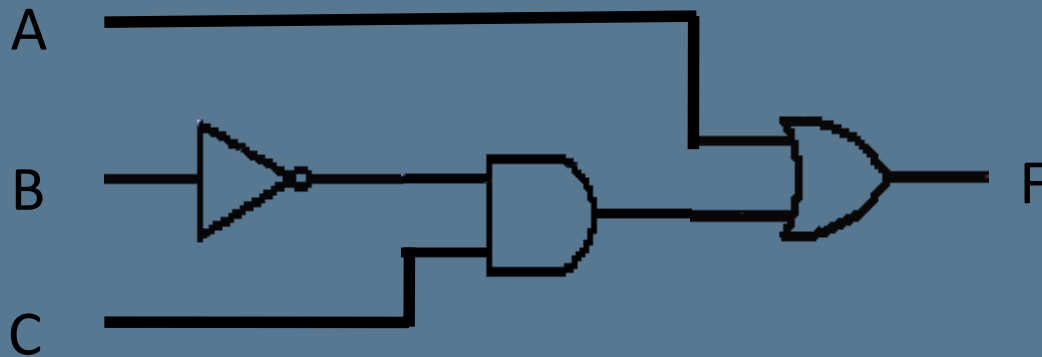
NAND gate



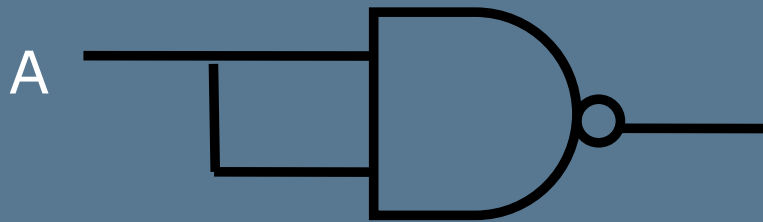
XNOR gate

Digital Logic gates

- Draw the logic diagram of the function $F = A + B'C$

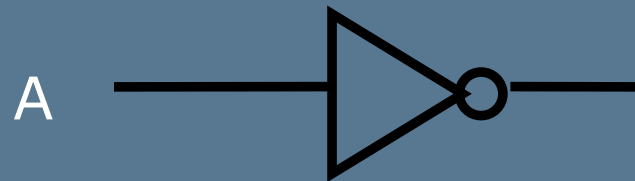
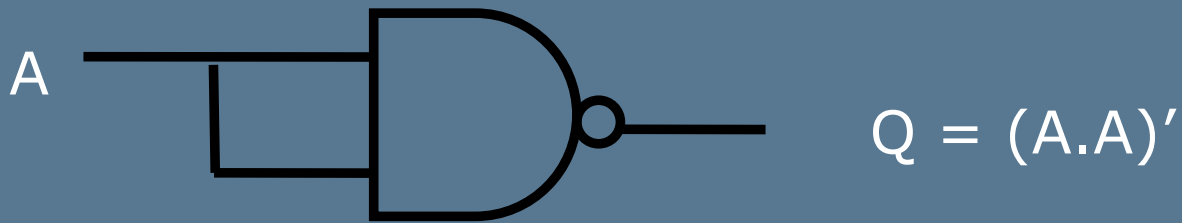


Universality of NAND gate



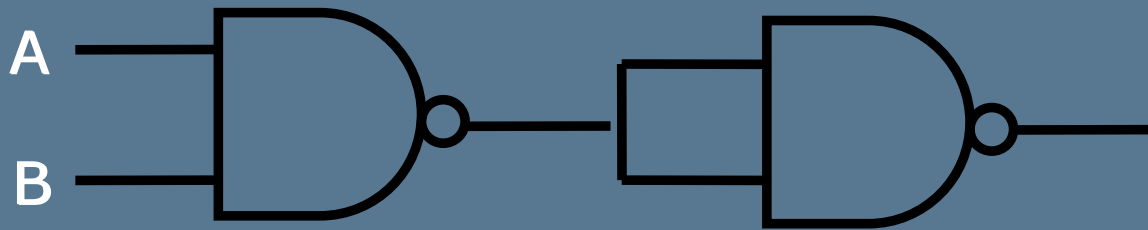
$$Q = (A.A)'$$

Universality of NAND gate

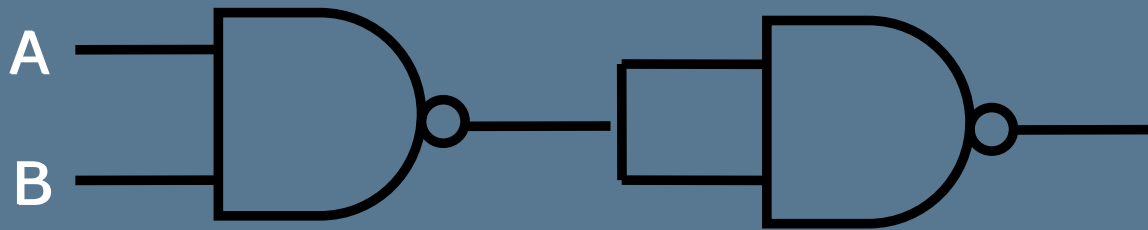


$$Q = A'$$

Universality of NAND Gate

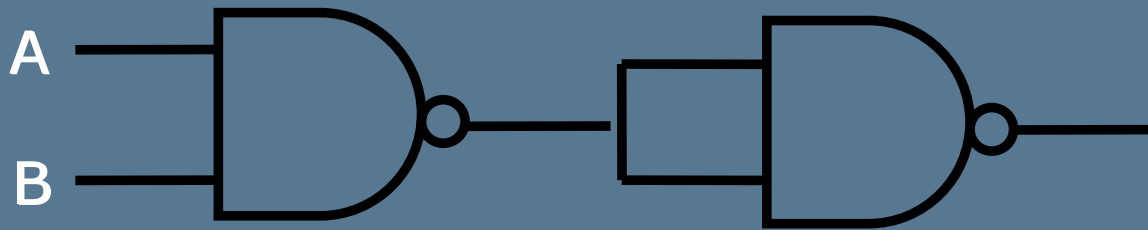


Universality of NAND Gate

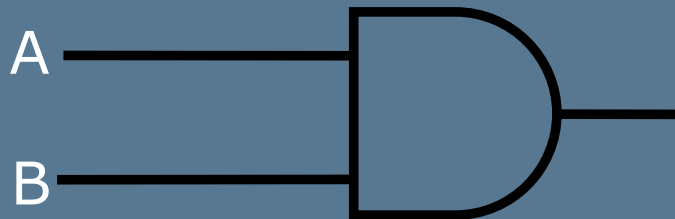
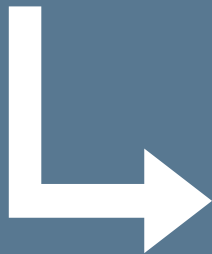


$$Q = ((AB)' (AB)')'$$

Universality of NAND Gate

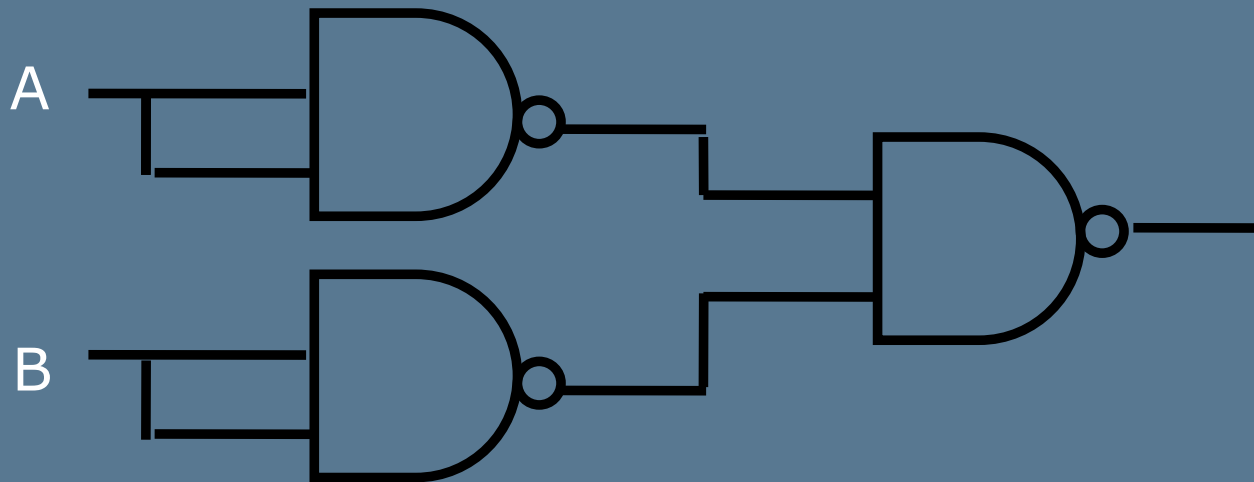


$$Q = ((AB)' (AB)')'$$

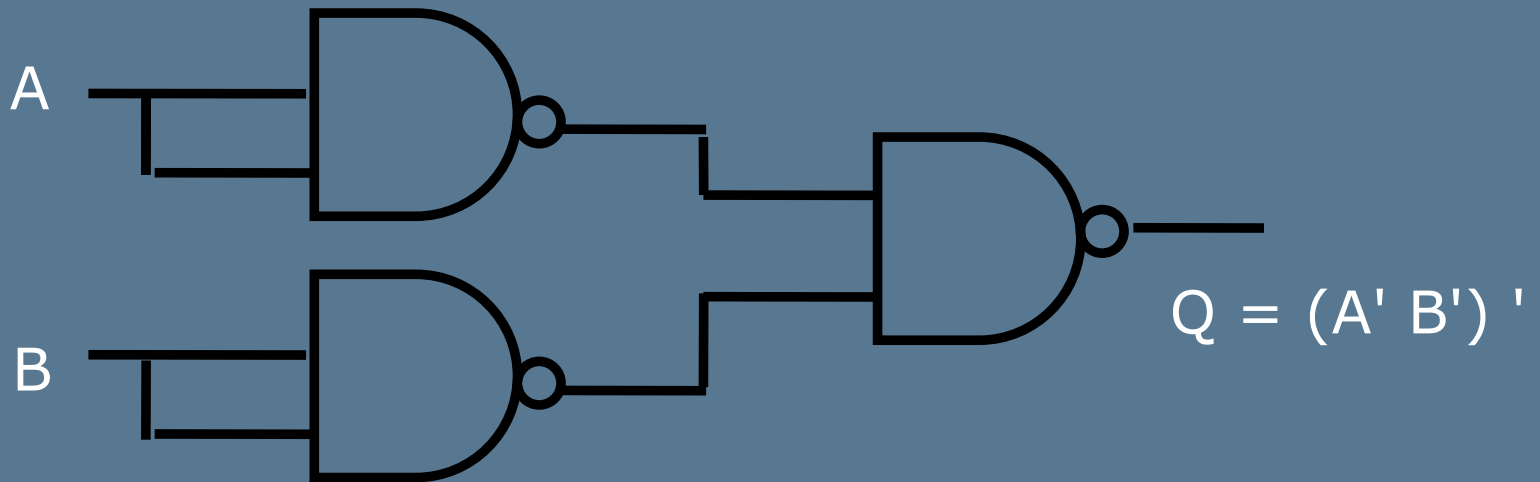


$$Q = AB$$

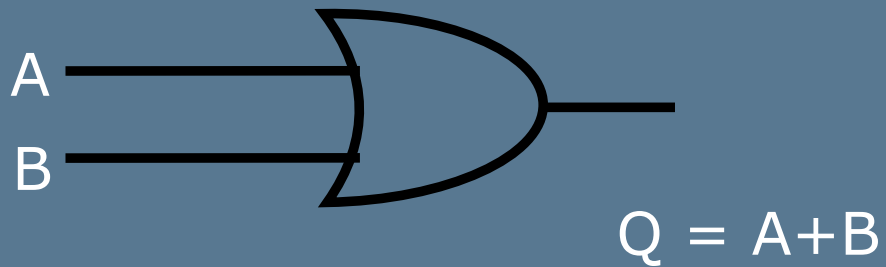
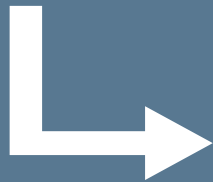
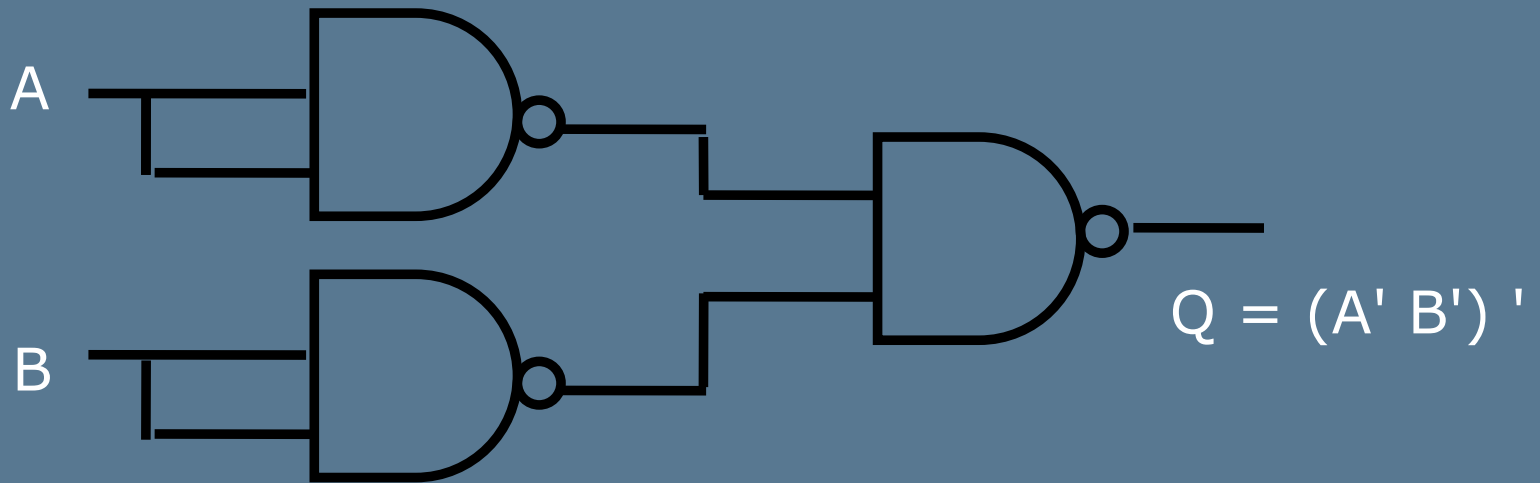
Universality of NAND Gate



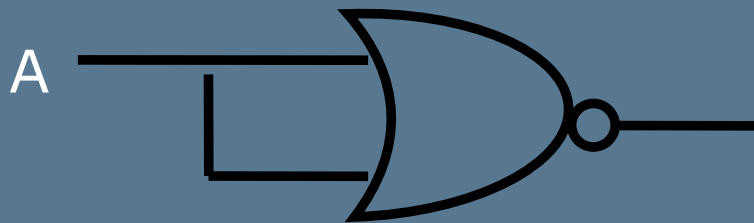
Universality of NAND Gate



Universality of NAND Gate

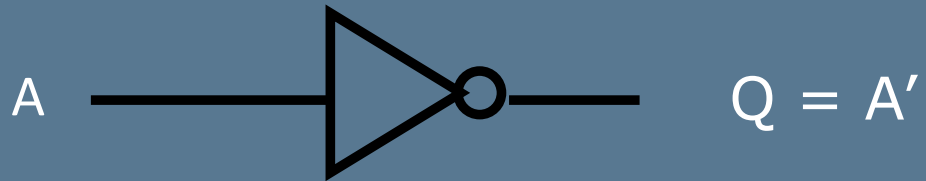
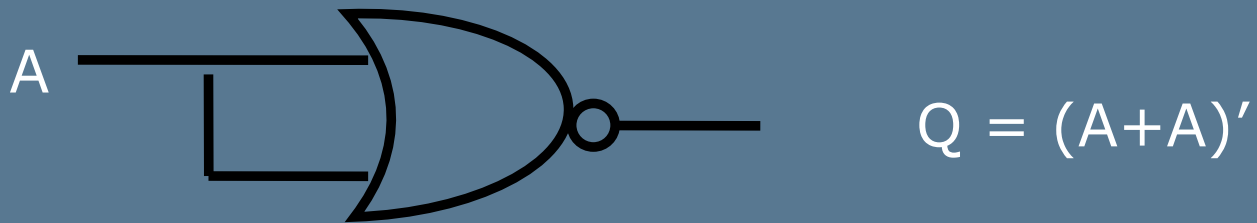


Universality of NOR Gate

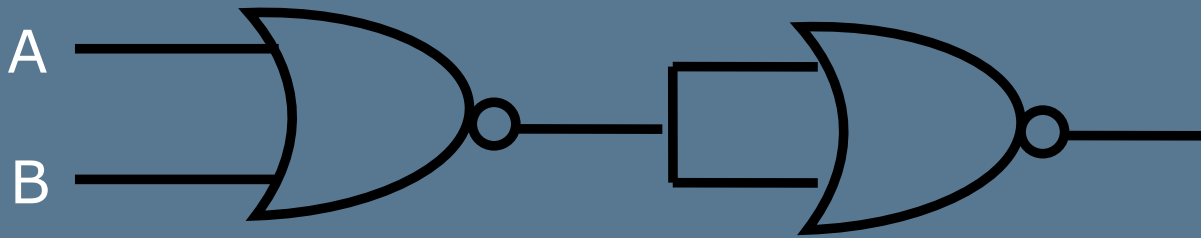


$$Q = (A + A)'$$

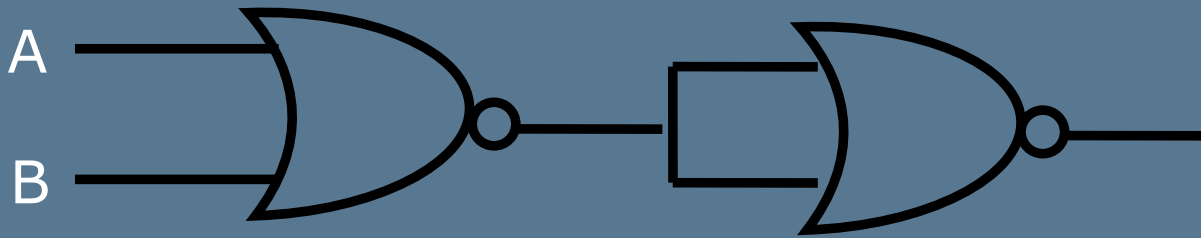
Universality of NOR Gate



Universality of NOR Gate

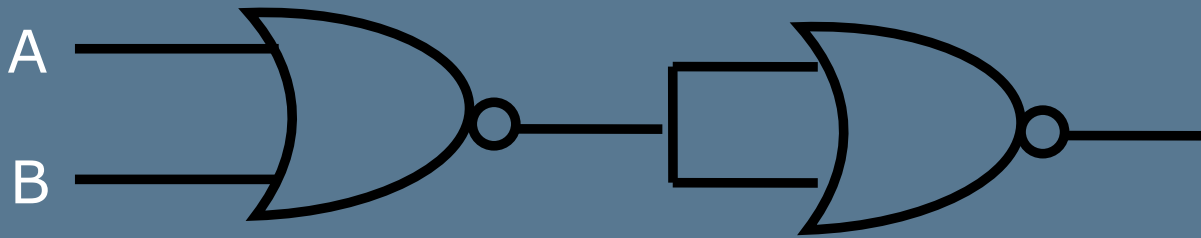


Universality of NOR Gate

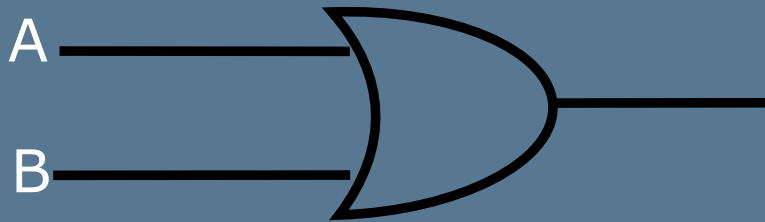


$$Q = ((A+B)' + (A+B)')'$$

Universality of NOR Gate

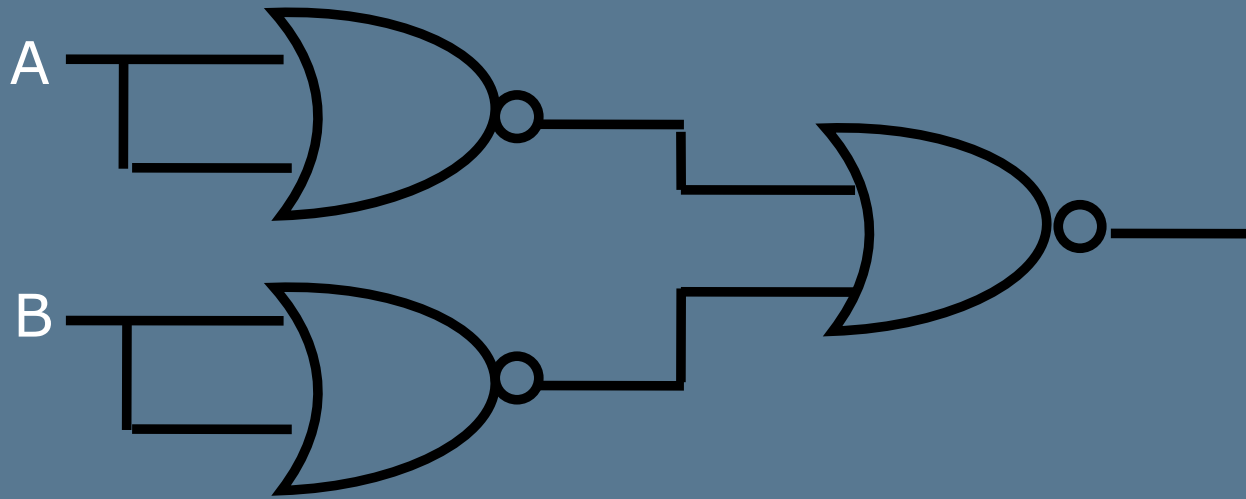


$$Q = ((A+B)' + (A+B)')'$$

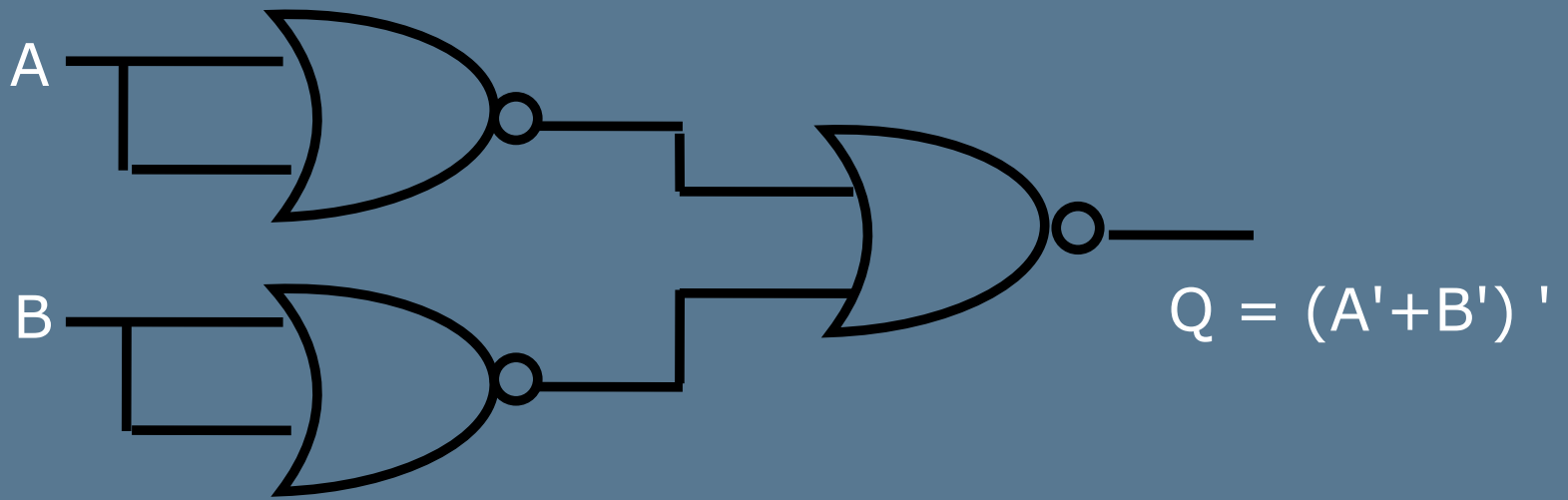


$$Q = A+B$$

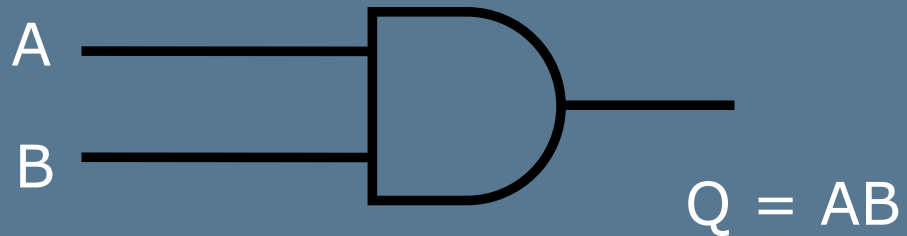
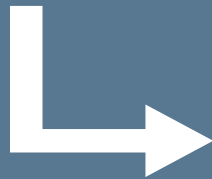
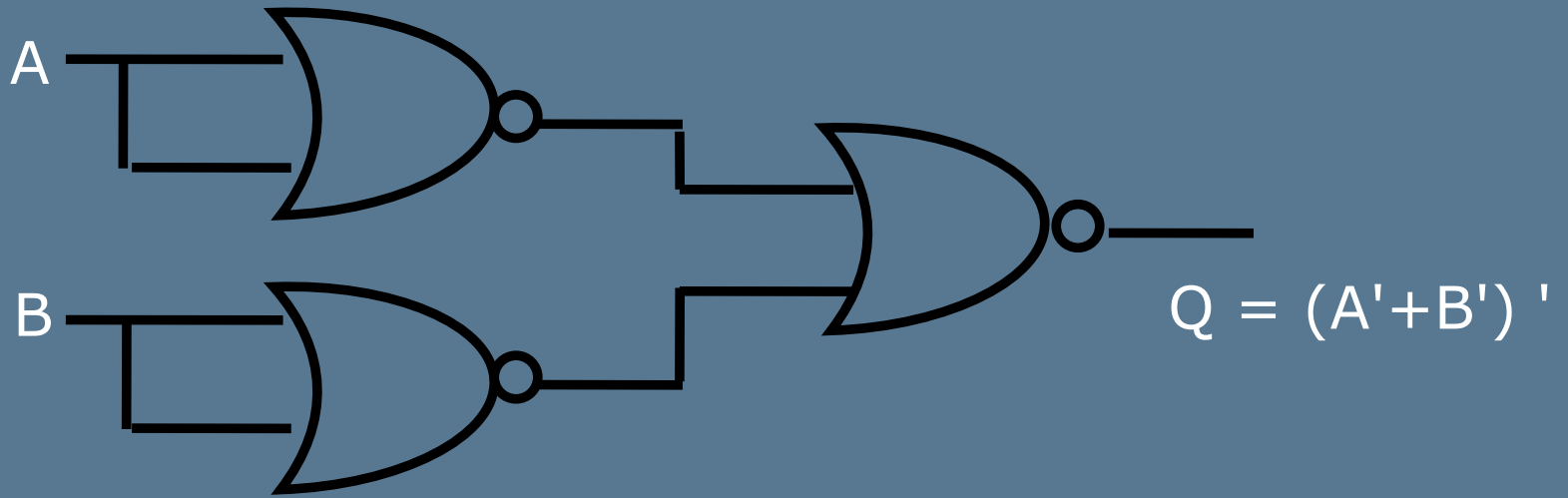
Universality of NOR Gate



Universality of NOR Gate



Universality of NOR Gate



Real Gates

- Logic gates are in integrated form
 - built within a solid piece of silicon called an IC (integrated circuit)



- Several gates are included in a single plastic moulding



IC Families

- Transistor-Transistor Logic (TTL)
- Emitter Coupled Logic (ECL)
- Complementary Metal-Oxide-Semiconductor (CMOS)



Levels of IC

- Small-scale Integration

- ICs with 1 to 10 gates

- Medium-scale Integration

- ICs with 10 to 100 gates

- Large-scale Integration

- ICs with 100 to 1000s of gates

- Very large-scale Integration

- ICs with 1000s to millions of gates



Chapter 4

SIMPLIFICATION of LOGIC CIRCUITS

RECALL: Representations of Boolean Functions

- Truth table
 - unique

Example:

x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

- Algebraic expression
 - not unique
 - convenient for manipulation

Example: $F = x + y$

- Logic circuits
 - not unique
 - close to implementation



Simplification of Boolean functions

- Simpler circuit is faster
- Simpler circuit is less expensive
- Reduce complexity of the gate level implementation
- Reduce signal propagation delays

A decorative graphic on the left side of the slide, consisting of a vertical arrangement of stylized circuit traces. These traces are in shades of green and blue, with some circular nodes and branching lines, resembling a printed circuit board (PCB) layout.

Ways to simplify Boolean functions

- Boolean Algebra
- Graphical method (Karnaugh Map)
- Tabular method (Quine-McCluskey)

A decorative graphic on the left side of the slide, consisting of a vertical arrangement of stylized circuit traces. These traces are colored in shades of green and blue, with some circular nodes and branching lines, resembling a printed circuit board (PCB) layout.

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$



Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.



Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

$$= x' + xyz$$

Comm / Dist.

Inv / Iden

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

$$= x' + xyz$$

$$= (x' + x)(x' + yz)$$

Comm / Dist.

Inv / Iden

Dist.

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.

$$= x' + xyz$$

Inv / Iden

$$= (x' + x)(x' + yz)$$

Dist.

$$= (x' + yz)$$

Inv / Iden



Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$



Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$

$$= AB + AB + AC + BB + BC$$

Dist.

Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$

$$= AB + AB + AC + BB + BC$$

Dist.

$$= AB + AC + B + BC$$

Idempotency



Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$

$$= AB + AB + AC + BB + BC$$

Dist.

$$= AB + AC + B + BC$$

Idempotency

$$= AB + AC + B$$

Absorption

Simplification: Boolean Algebra

Simplify $AB + A(B+C) + B(B+C)$

$$= AB + AB + AC + BB + BC$$

Dist.

$$= AB + AC + B + BC$$

Idempotency

$$= AB + AC + B$$

Absorption

$$= B + AC$$

Absorption



Simplification: Graphical method

- Karnaugh map (K-map)
 - alternate way of representing Boolean functions
 - a graphical tool for assisting in the general simplification procedure
 - a simpler way to handle most jobs of manipulating logic functions



General Steps of K-Map Simplification

- Express function in canonical form
- Map expression on a K-Map
- Group 1's or 0's
- Determine the minimum expression

Step 1: Function in Canonical form

- Sum of minterms

- $F(x,y) = x'y' + xy = \sum(0,3)$

- $G(a,b,c) = a'b'c + abc' + a'bc = \sum(1,3,6)$

- Product of maxterms

- $H(x,y) = (x+y')(x'+y) = \prod(1,2)$

- $I(a,b,c) = (a+b+c)(a'+b+c') = \prod(0,5)$

Step 2: Map expression

X \ Y	0	1
	0	1
0	m_0	m_1
1	m_2	m_3

Two-variable
map

X \ YZ	00	01	11	10
	0	1	1	0
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Three-variable
map

Step 2: Map expression

WX \ YZ	YZ			
	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Four-variable map

Mapping Example

- $F = x'y'z + x'yz' + xyz' + xyz$

X \ YZ	YZ			
	00	01	11	10
0		1		1
1			1	1

Mapping Example

- $G = \Sigma(2, 4, 6, 8, 12, 14, 15)$

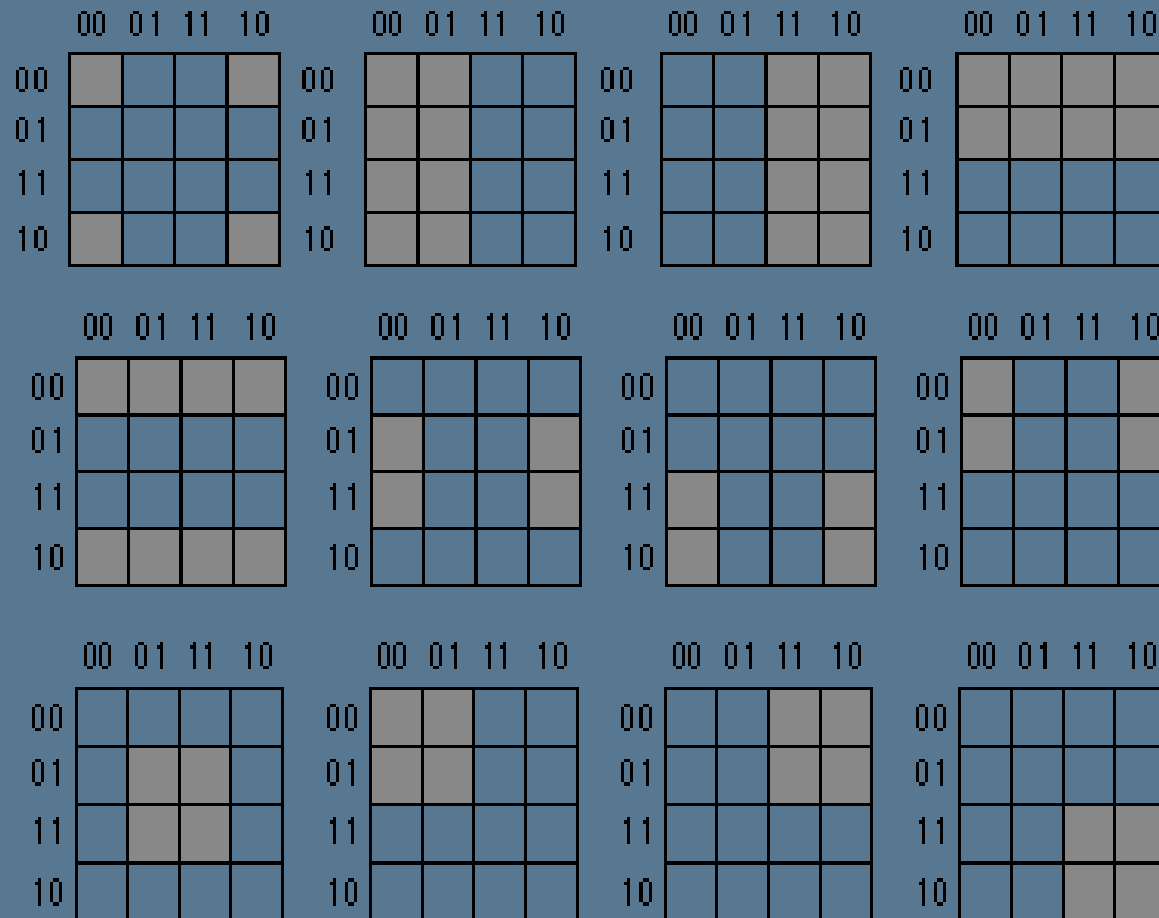
WX \ YZ	YZ			
	00	01	11	10
00				1
01	1			1
11	1		1	1
10	1			



Step 3: Group 1's (or 0's)

- Grouping Rules:
 - A group must contain either 1, 2, 4, 8 and 16 cells
 - Each cell in a group must be adjacent to one or more cells in that same group.
 - Always include the largest possible group in accordance with the first rule.
 - Each element of a group must be included in at least one group.

Sample Groupings: Four-variable map



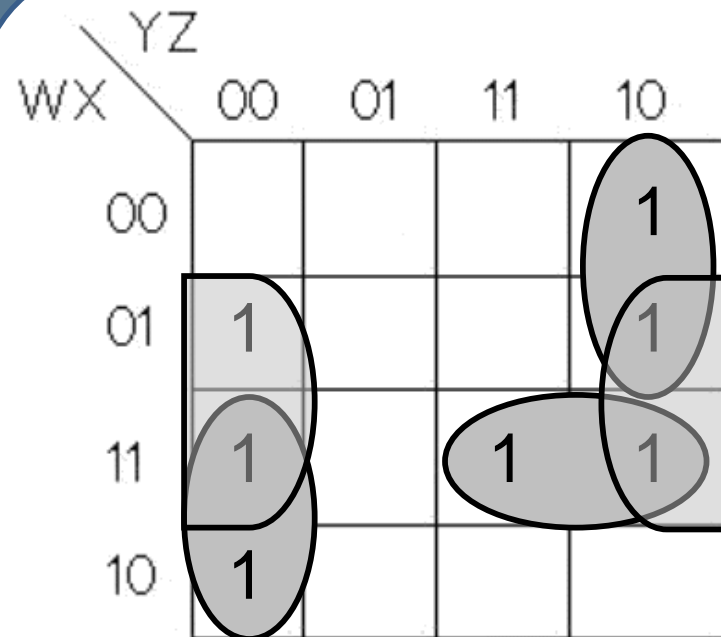
Mapping Example

- $F = x'y'z + x'yz' + xyz' + xyz$

X \ YZ	00	01	11	10
0		1		1
1			1	1

Mapping Example

- $G = \Sigma(2, 4, 6, 8, 12, 14, 15)$



Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1	1		1
1	1	1	1	

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1	1		1
1	1	1	1	

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1	1		1
1	1	1	1	

The Karnaugh map shows a 2x4 grid of cells. The columns are labeled 00, 01, 11, and 10 from left to right. The rows are labeled 0 and 1 from top to bottom. The cells containing '1' are at (0,00), (0,01), (0,10), (1,00), (1,01), and (1,11). There are three groupings: a circle grouping the four cells (0,00), (0,01), (1,00), and (1,01); a circle grouping the two cells (0,00) and (0,10); and a circle grouping the two cells (0,10) and (0,11).

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1	1		1
1	1	1	1	

The Karnaugh map shows the following groupings:

- A group of four cells (A=0, B=0, C=0; A=0, B=0, C=1; A=1, B=0, C=0; A=1, B=0, C=1) is circled, representing the term $\bar{B}\bar{C}$.
- A group of two cells (A=0, B=0, C=1; A=1, B=0, C=1) is circled, representing the term $\bar{B}C$.
- A group of two cells (A=0, B=0, C=1; A=1, B=1, C=1) is circled, representing the term $\bar{A}C$.
- A group of two cells (A=0, B=1, C=0; A=1, B=1, C=0) is circled, representing the term BC .

Grouping Examples

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1			1
0 1	1	1		1
1 1	1	1		1
1 0	1		1	1

Grouping Examples

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1	1			1
1 1	1	1			1
1 0	1			1	1

Grouping Examples

		CD			
		0 0	0 1	1 1	1 0
AB	0 0	1			1
	0 1	1	1		1
	1 1	1	1		1
	1 0	1		1	1

Grouping Examples

		CD			
		0 0	0 1	1 1	1 0
AB	0 0	1			1
	0 1	1	1		1
	1 1	1	1		1
	1 0	1		1	1

Step 4: Determine Minimum Expression

- Determine minimum terms

Cell	# Variables
------	-------------

1	3
---	---

2	2
---	---

4	1
---	---

8	fn's value is 1
---	-----------------

Cell	# Variables
------	-------------

1	4
---	---

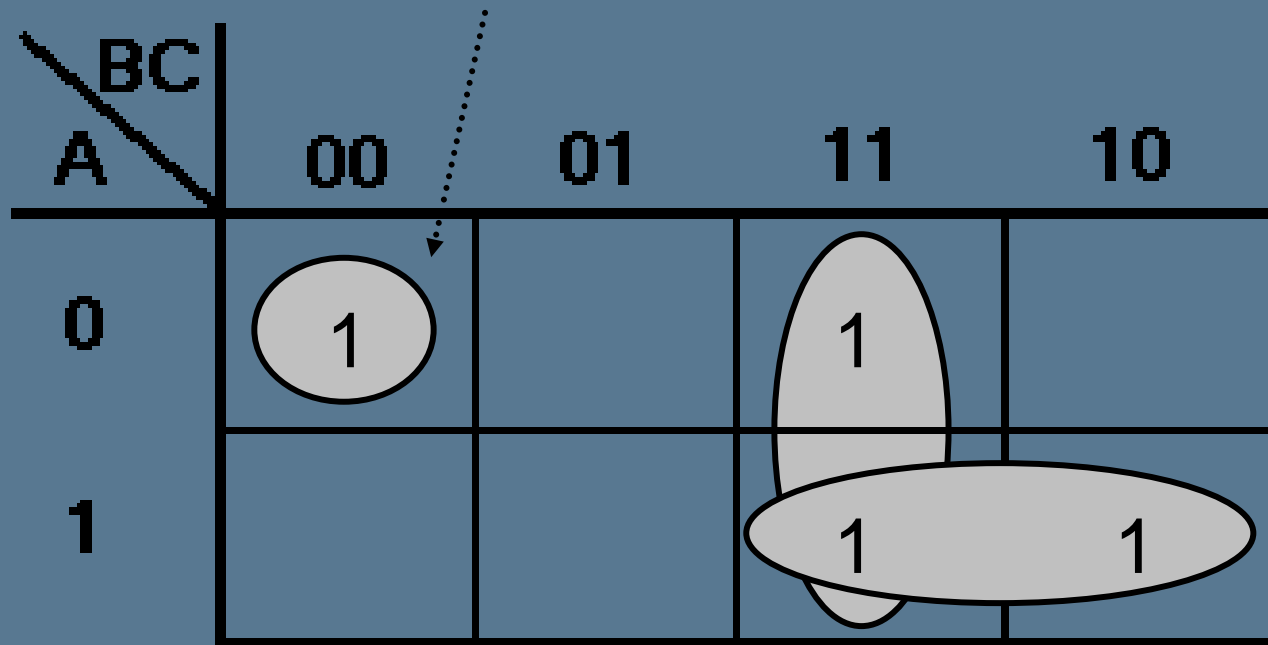
2	3
---	---

4	2
---	---

8	1
---	---

16	fn's value is 1
----	-----------------

Determining Minimum Expression Example



A Karnaugh map (K-map) for a function of three variables, A, B, and C. The map is a 2x4 grid. The rows are labeled A=0 and A=1. The columns are labeled BC=00, 01, 11, and 10. The cell at (A=0, BC=00) contains a 1 and is circled. The cell at (A=0, BC=11) contains a 1 and is part of a vertical oval group. The cells at (A=1, BC=11) and (A=1, BC=10) both contain 1s and are part of a horizontal oval group. A dotted arrow points to the cell at (A=0, BC=00).

A \ BC	00	01	11	10
0	1		1	
1			1	1

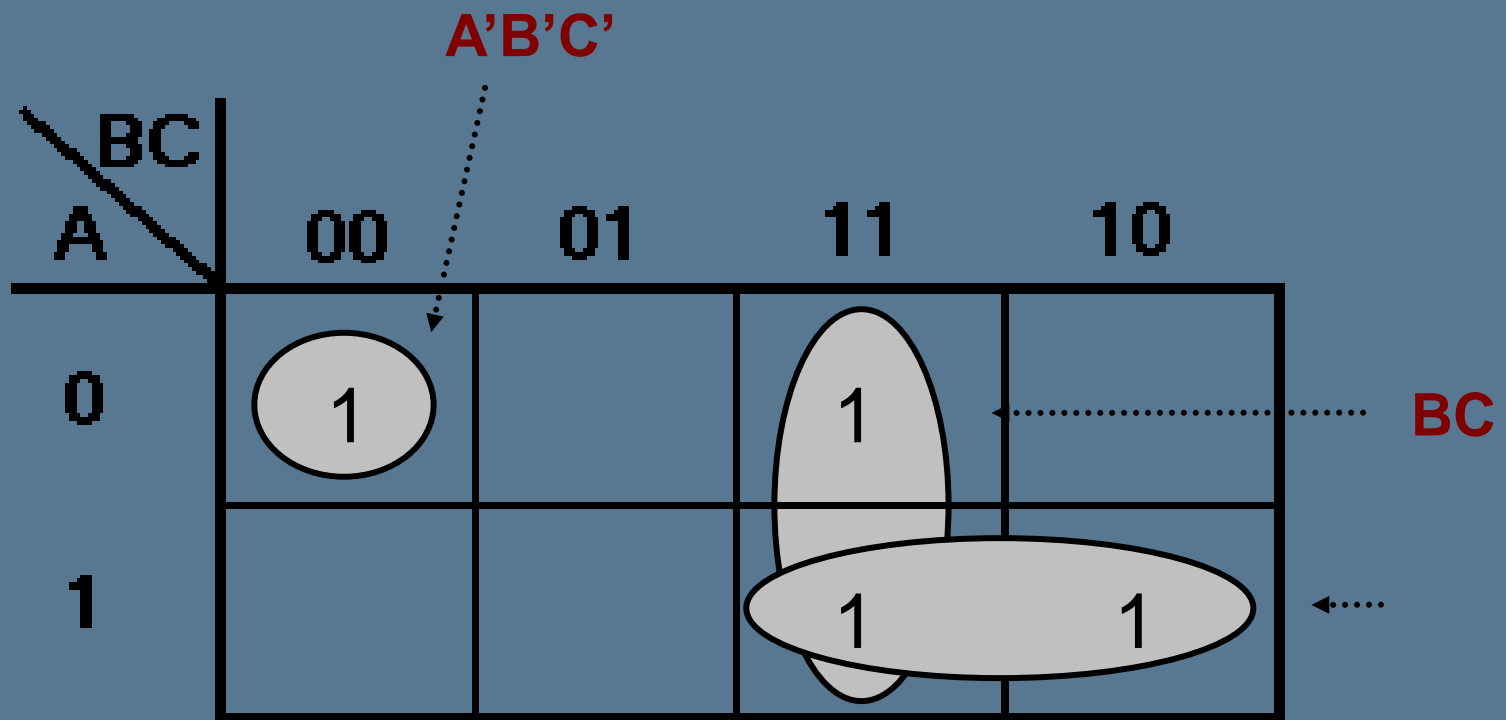
Determining Minimum Expression Example

$A'B'C'$

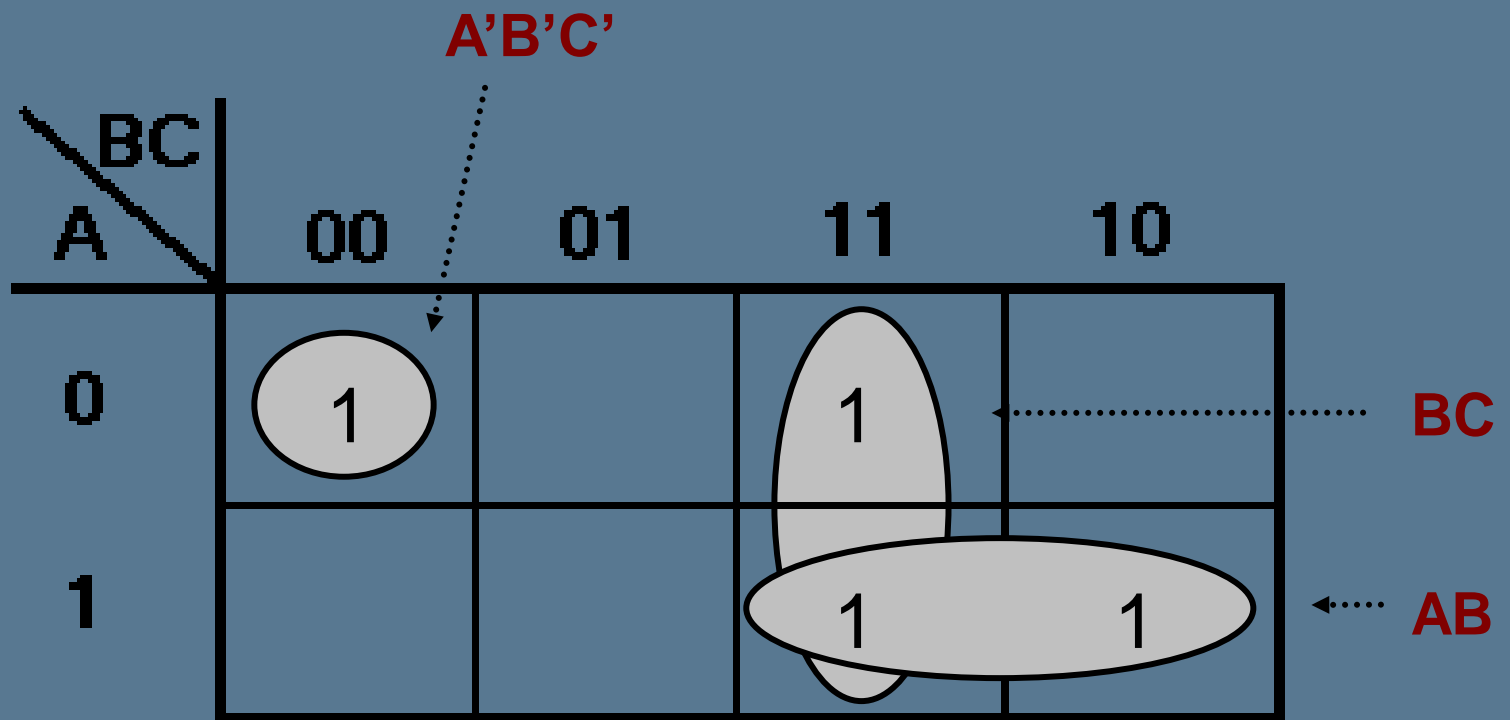
\swarrow BC A	00	01	11	10
0	1		1	
1			1	1

The image shows a Karnaugh map for a three-variable function with variables A, B, and C. The map is a 2x4 grid. The columns are labeled with BC values (00, 01, 11, 10) and the rows are labeled with A values (0, 1). The cells containing 1s are at (A=0, BC=00), (A=0, BC=11), (A=1, BC=11), and (A=1, BC=10). Three groups are identified: a circle around the cell (0, 00) labeled $A'B'C'$, a vertical oval around the cells (0, 11) and (1, 11), and a horizontal oval around the cells (1, 11) and (1, 10). Dotted lines indicate the groupings.

Determining Minimum Expression Example



Determining Minimum Expression Example

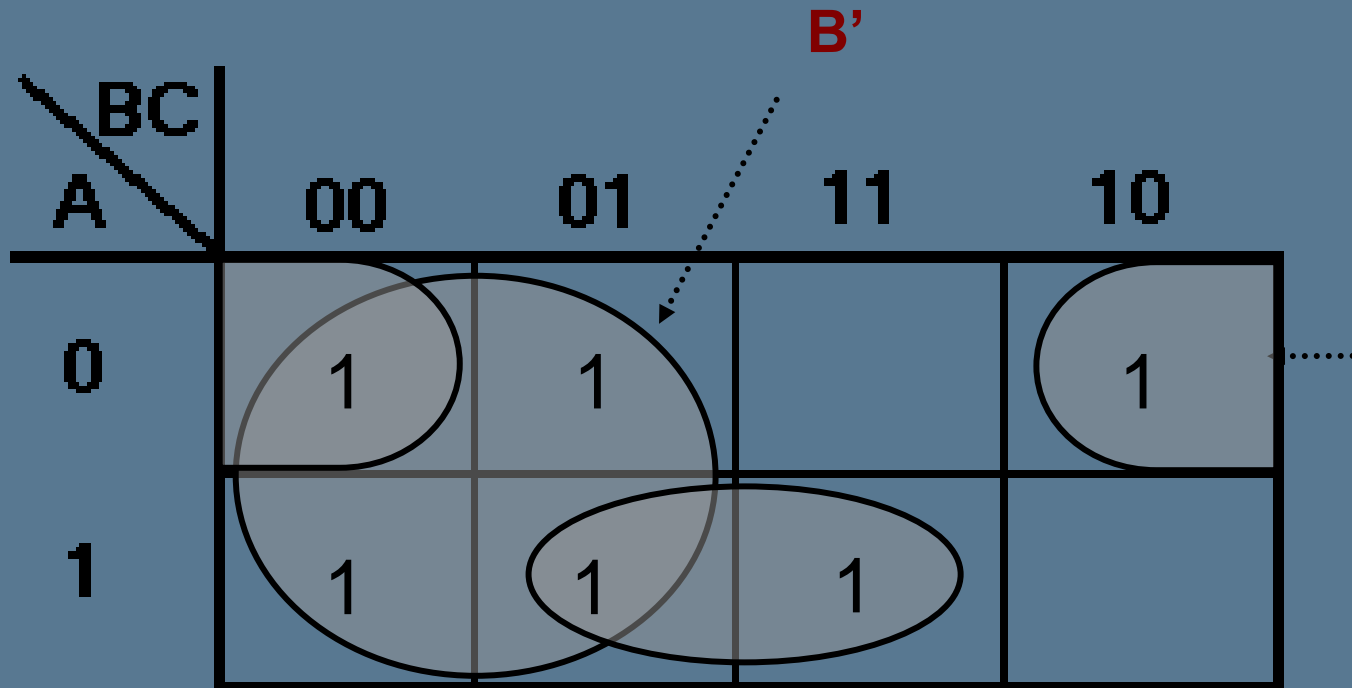


$$= A'B'C' + BC + AB$$

Determining Minimum Expression Example

A \ BC				
	00	01	11	10
0	1	1		1
1	1	1	1	

Determining Minimum Expression Example



BC		00	01	11	10
A					
0	1	1		1
1	1	1	1		

Determining Minimum Expression Example

BC		00	01	11	10
A	0	1	1		1
	1	1	1	1	

B'

$A'C'$

Determining Minimum Expression Example

BC A					
		00	01	11	10
0	1	1	1		1
1	1	1	1	1	

B'

$A'C'$

AC

$$= A'C' + AC + B'$$

Determining Minimum Expression Example

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1	1			1
1 1	1	1			1
1 0	1			1	1

.....

Determining Minimum Expression Example

		CD			
		0 0	0 1	1 1	1 0
AB	0 0	1			1
	0 1	1	1		1
	1 1	1	1		1
	1 0	1		1	1

Diagram illustrating a Karnaugh map for variables A, B, C, and D. The map shows four groups of 1s, each representing a prime implicant:

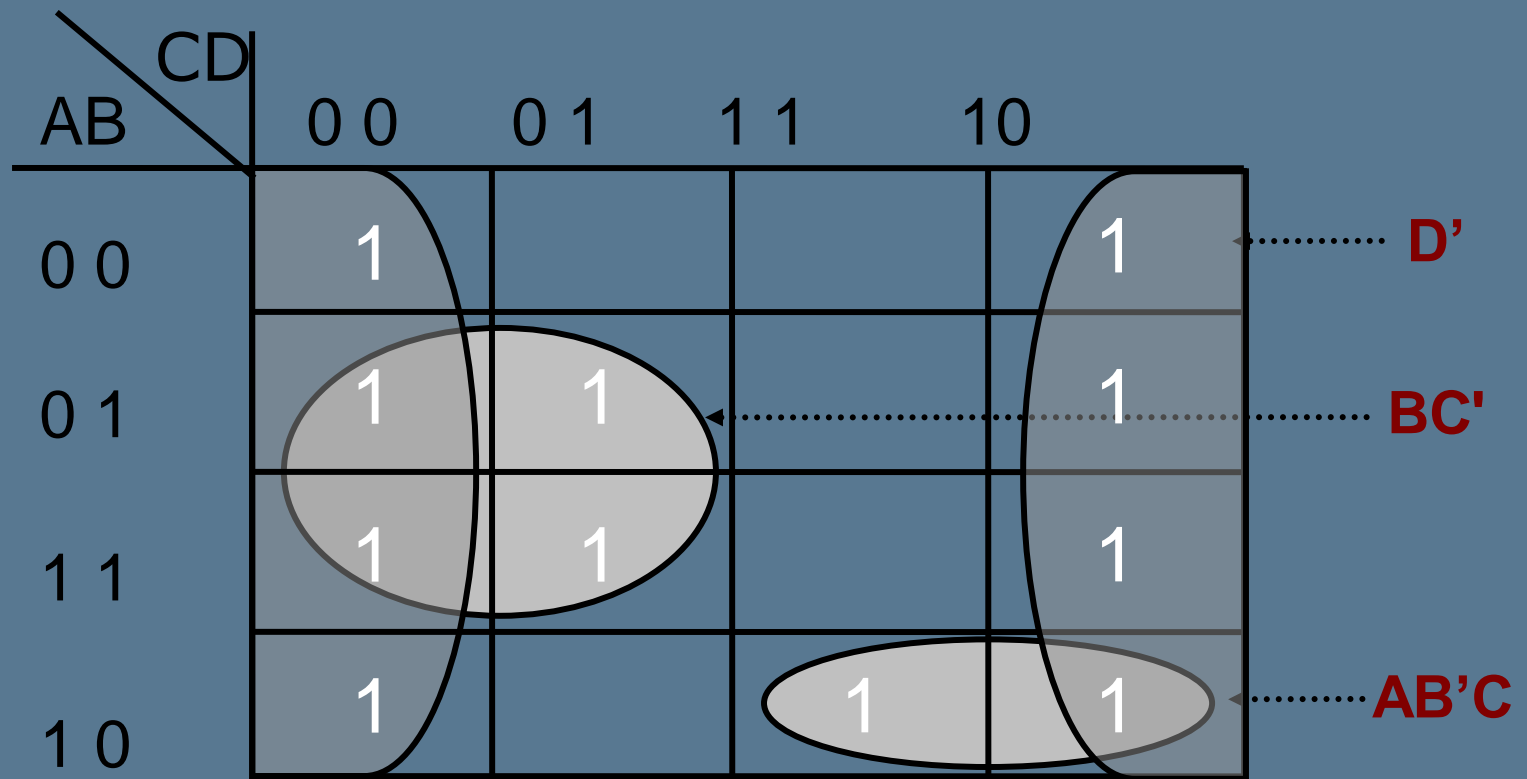
- Group 1 (Vertical): $A'B$ (Covers cells where A=0, B=1)
- Group 2 (Horizontal): $A'B$ (Covers cells where A=0, B=1)
- Group 3 (Vertical): $A'D$ (Covers cells where A=1, D=1)
- Group 4 (Horizontal): $AB'D$ (Covers cells where A=1, B=0, D=1)

The group $A'D$ is labeled D' with a red arrow.

Determining Minimum Expression Example

		CD				
		0 0	0 1	1 1	1 0	
AB	0 0	1			1 D'
	0 1	1	1		1 BC'
	1 1	1	1		1	
	1 0	1		1	1

Determining Minimum Expression Example



$$= D' + BC' + AB'C$$