CMSC 141 Automata and Language Theory

Context-Free Languages

Mark Froilan B. Tandoc

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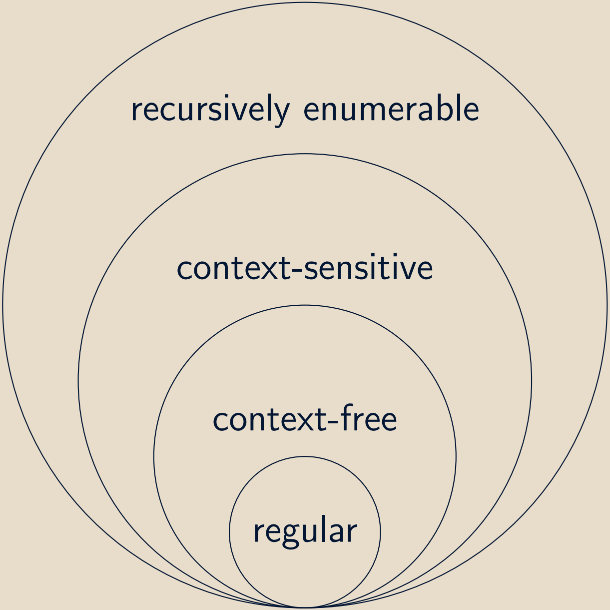
Chomsky Hierarchy



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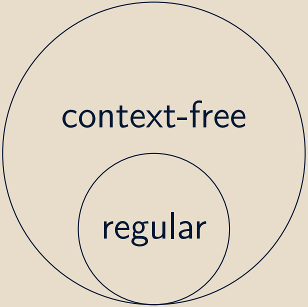

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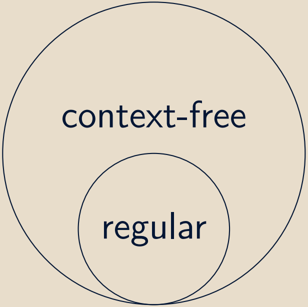

- Regular Languages $V \rightarrow T^*(V + \varepsilon)$
  - $S \rightarrow abS \mid a \mid \varepsilon$
  - $S \rightarrow 0S \mid 1S \mid 11T$
    $T \rightarrow 0T \mid 1T \mid \varepsilon$

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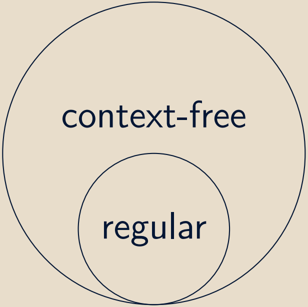

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  - $S \rightarrow 0S \mid 1S \mid 11T$
    $T \rightarrow 0T \mid 1T \mid \varepsilon$
- Context-Free Languages $V \rightarrow (V+T)^*$
  - $S \rightarrow ab \mid aSb$
  - $S \rightarrow () \mid SS \mid (S) \mid \varepsilon$

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- $S \rightarrow a \mid S + S$ is ambiguous

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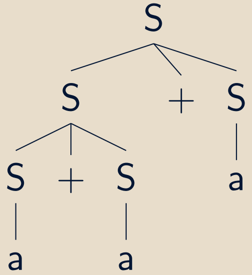

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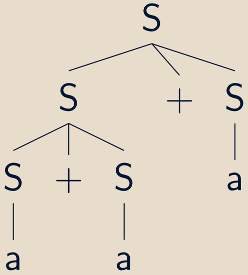

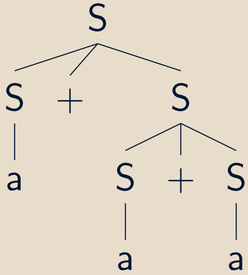

Ambiguity in Natural Languages

- The man on the hill saw the boy with a telescope.

Ambiguity in Natural Languages

- The man on the hill saw the boy with a telescope.
- Look at the dog with one eye.

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- G_2 is better because it is non-ambiguous by forcing the rule on left-associativity

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- Consider: $S \to 0 \mid 1 \mid S + S \mid S^{*}S \mid (S)$

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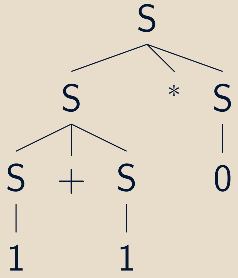

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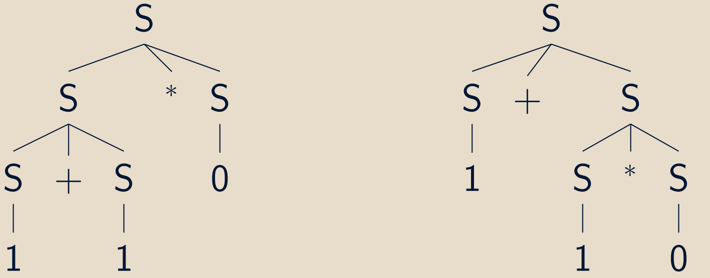

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(enforcing operator precedence)

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\begin{aligned}
S &\rightarrow E \\
E &\rightarrow E + T \mid T && \text{(expressions)} \\
T &\rightarrow T^{*} F \mid F && \text{(terms)} \\
F &\rightarrow 0 \mid 1 \mid (E) && \text{(factors)}
\end{aligned}
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Exercise: Try parsing strings like $1 + 1{*}0$ and $(1 + 1){*}0$

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$B \rightarrow \textit{block}$

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Exercise: Design/Create a non-ambiguous version of the grammar that associates an else-clause to the nearest if

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S &\rightarrow XC \mid AY \\
X &\rightarrow ab \mid aXb \\
A &\rightarrow a \mid aA \\
C &\rightarrow c \mid cC \\
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- Show that aabbcc has 2 different parse trees
- Why?

References

- Previous slides on CMSC 141
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- J.E. Hopcroft, R. Motwani and J.D. Ullman. Introduction to Automata Theory, Languages and Computation. 2nd ed, Addison-Wesley, 2001.
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- JFLAP, www.jflap.org
- Various online LaTeX and Beamer tutorials