1.4

Some Theorems on Convergence of Infinite Series

GOAL

Determine whether an infinite series is CONVERGENT or DIVERGENT (by comparison).

Theorem 1

$$\sum_{n=1}^{+\infty} a_n \text{ and } \sum_{n=1}^{+\infty} b_n \text{ differ only}$$

by a finite number of terms,

$\Rightarrow$ both series converges or both series diverges

Example. Convergent or divergent?

$$\sum_{n=1}^{+\infty} \left(\frac{1}{2}\right)^{n+2}$$

Convergent

Example. Convergent or divergent?

$$\sum_{n=1}^{+\infty} \frac{1}{n+4}$$

Divergent

Theorem 2

Let c be a constant.

$\sum_{n=1}^{+\infty} u_n$ is convergent with a sum of S

$\Rightarrow \sum_{n=1}^{+\infty} c \cdot u_n$ is convergent with a sum of $c \cdot S$.

Theorem 3

Let c be a non-zero constant.

$\sum_{n=1}^{+\infty} u_n$ is divergent

$\Rightarrow \sum_{n=1}^{+\infty} c \cdot u_n$ is divergent.

Example. Convergent or divergent?

$$\sum_{n=1}^{+\infty} \frac{4}{n(n+1)}$$

Convergent

Example. Convergent or divergent?

$$\sum_{n=1}^{+\infty} \frac{-5}{n^{1/2}}$$

Divergent

Theorem 4.a

$\sum_{n=1}^{+\infty} a_n$ is convergent with sum S.

$\sum_{n=1}^{+\infty} b_n$ is convergent with sum R.

$\Rightarrow \sum_{n=1}^{+\infty} (a_n \pm b_n)$ are also convergent with a sum of $S \pm R$

Theorem 4.b

$\sum_{n=1}^{+\infty} a_n$ is convergent with sum S.

$\sum_{n=1}^{+\infty} b_n$ is divergent.

$\Rightarrow \sum_{n=1}^{+\infty} (a_n \pm b_n)$ are divergent.

Example 3. Convergent or divergent?

$$\sum_{n=1}^{+\infty} \left(\frac{1}{2^{n-1}} - \frac{6}{3^{n-1}} \right)$$

Convergent

Example 4. Convergent or divergent?

$$\sum_{n=1}^{+\infty} \left(\frac{1}{n(n+1)} + \frac{1}{n} \right)$$

Divergent

CAUTION ! ! !

$\sum_{n=1}^{+\infty} a_n$ and $\sum_{n=1}^{+\infty} b_n$ are divergent.

NO (GENERAL) CONCLUSION on convergence/divergence of

$$\sum_{n=1}^{+\infty} (a_n \pm b_n)$$