Number systems

How data and programs are internally represented

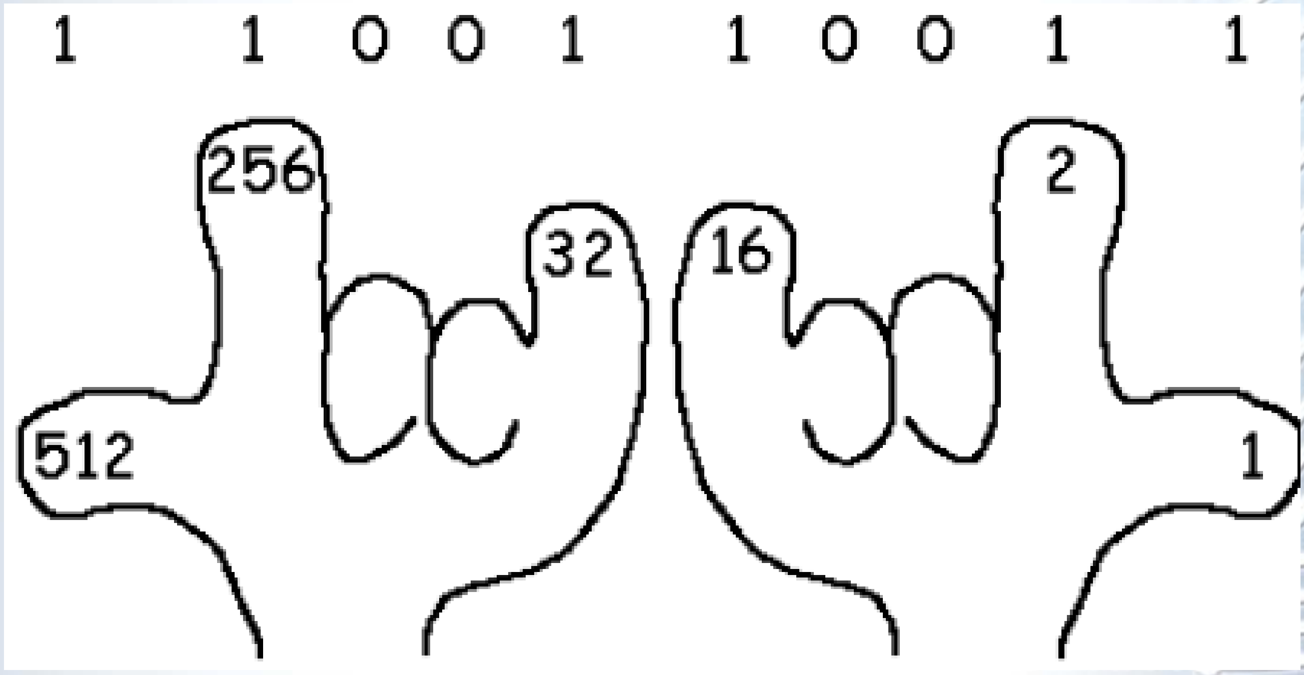
Objectives

- At the end of the meeting, students should be able to:
    - Enumerate the different number systems
    - Convert numbers into the other number systems
    - Perform arithmetic operations on the different number systems
    - Understand how some data are internally represented in the computer

Number systems

- Most people prefer the decimal number system (mainly because we have 10 fingers!)
  - 234.56 means 2*100 + 3*10 + 4*1 + 5/10 + 6/100
  - We have 10 digits {0..9} and we use powers of 10
- Computers have "on-off switches" instead of fingers and so they use the binary system
  - They use 2 digits {0,1} and they use powers of 2
  - 1101 means 1*8 + 1*4 + 0*2 + 1*1
  - 110.101 means 1*4 + 1*2 + 0*1 + 1/2 + 0/4 + 1/8

How high can you count with your ten fingers?



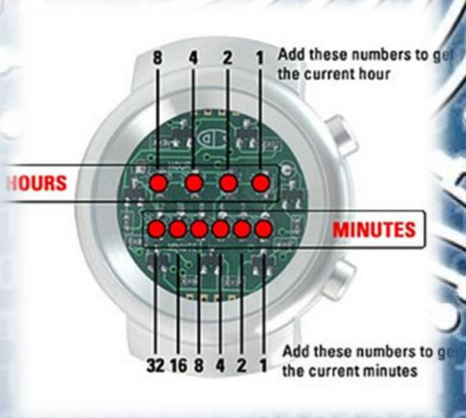
www.mathmaniacs.org/lessons/01binary/fingers.gif

Converting binary to decimal

- Converting a binary number to its decimal equivalent is performed by adding the successive powers of 2 where the bits (binary digits) are on

1	0	0	0	1	1	0	1
128	64	32	16	**8**	**4**	2	**1**

128 + 8 + 4 + 1 = 141 (in decimal)



Binary to decimal conversion powers-of-two algorithm

```
input binary integer;
decimal = 0;
power = 1;
for (each binary digit from right to left) {
    if (current bit == 1)
        decimal = decimal + power;
    power = power*2;
}
output decimal;
```

110**1**	power = 1, decimal = 1
11**0**1	power = 2, decimal = 1
1**1**01	power = 4, decimal = 5
1101	power = 8, decimal = 13

Binary to decimal conversion another algorithm

```
input binary integer;
decimal = 0;
for (each binary digit from left to right) {
    if (current bit == 0) decimal = decimal * 2;
    else decimal = decimal * 2 + 1;
}
output decimal;
```

1101	decimal = 0 * 2 + 1 = 1
1101	decimal = 1 * 2 + 1 = 3
1101	decimal = 3 * 2 = 6
1101	decimal = 6 * 2 + 1 = 13

How high can you count with your ten fingers?

With 1 bit, there are only 2 possible values: 0 and 1

With 2 bits, 4 possible values: 00, 01, 10, 11 (0 to 3)

With 3 bits, 8 possible values: 000, 001, 010, 011, 100, 101, 110, 111

With 4 bits, 16 possible values

With 5 bits, 32 possible values

With 6 bits, 64 possible values

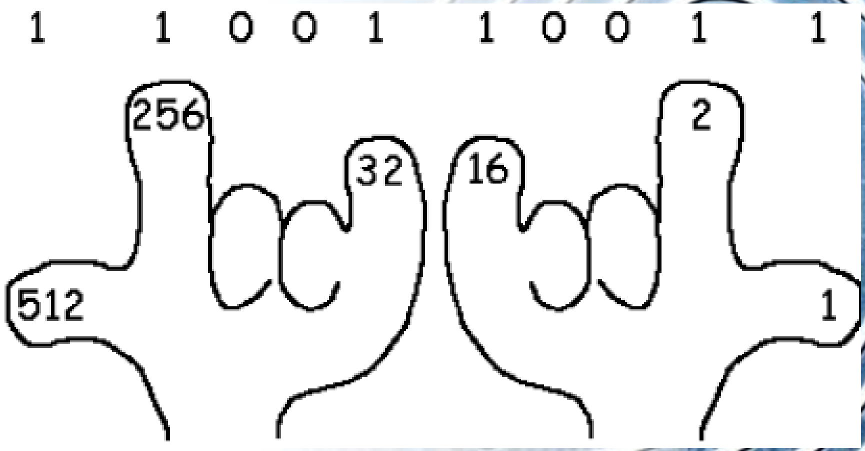
With 7 bits, 128 possible values

With 8 bits, 256 possible values

With 9 bits, 512 possible values

With 10 bits or fingers, there are 1024 possible values

from 00000 00000 to 11111 11111 (0 to 1023)



Decimal to binary conversion repeated subtraction

Example: What is 300 in binary?

One way is by repeated subtraction of powers of two

300 – **256** = 44	2^8
44 – **32** = 12	2^5
12 – **8** = 4	2^3
4 – **4** = 0	2^2

300 in decimal is equivalent to

1	0	0	1	0	1	1	0	0
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Decimal to binary conversion repeated integer division

Example: What is 300 in binary?

Another algorithm is by repeated division by 2

300 / 2 = 150 r **0**

150 / 2 = 75 r **0**

75 / 2 = 37 r **1**

37 / 2 = 18 r **1**

18 / 2 = 9 r **0**

9 / 2 = 4 r **1**

4 / 2 = 2 r **0**

2 / 2 = 1 r **0**

1 / 2 = 0 r 1

$\uparrow$

300 in decimal
is equivalent to
1 0010 1100 in binary

Base 8 (octal) and Base 16 (hexadecimal)

- 8 octal digits are

  0, 1, 2, 3, 4, 5, 6, 7

- 16 hexadecimal digits are

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Number systems in C programming

- `printf()` number formats

  “`%d`” print as a decimal number

  “`%o`” print as an octal number

  “`%x`” print as a hexadecimal number

Number systems in C programming

```
#include<stdio.h>

main(){
    int x = 28;
    printf("%d decimal = %o octal = %x hex\n", x, x, x);
}
```

Output:

```
    28 decimal = 34 octal = 1c hex
```

Number systems in C programming

- Constant numbers prefix

  0x for hex

  0 for octal

Number systems in C programming

```
#include<stdio.h>
main(){
    int p = 0x1a, q = 017;
    printf("%d decimal = %o octal = %x hex\n", p, p, p);
    printf("%d decimal = %o octal = %x hex\n", q, q, q);
}
```

----

output:

- 26 decimal = 32 octal = 1a hex
- 15 decimal = 17 octal = f hex

Binary to Octal and Hexadecimal

- What's special about base 8 and base 16?
- Being powers of 2, conversions between binary, octal, and hex number systems are fairly easy
- Idea is to **group the bits by 3's** for octal numbers, or to **group the bits by 4's** for hex

Binary to Octal

100110111 (binary)

100	110	111
4	6	7

100 110 111 (binary)
is equivalent to
467 (octal)

Binary to Hexadecimal

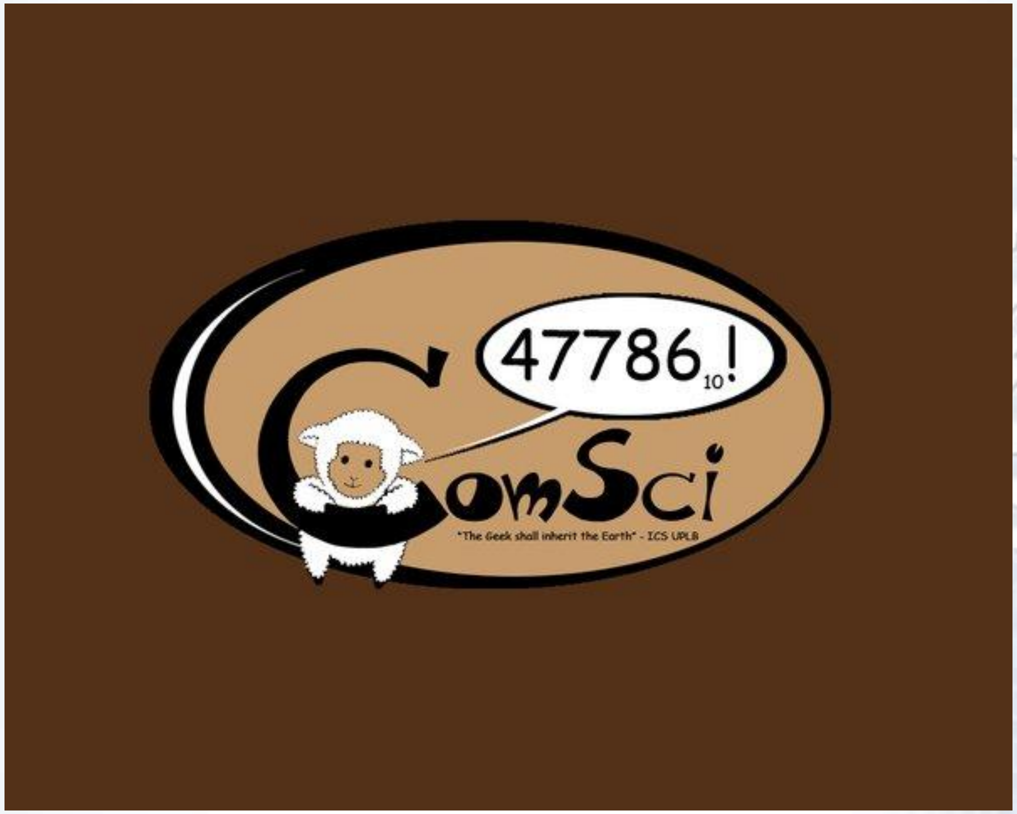
100110111 (binary)

0001 0011 0111

1 3 7

100 110 111 (binary)
is equivalent to
137 (hexadecimal)

BAAA!



Basic conversions

Dec	Bin	Oct	Hex
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Dec	Bin	Oct	Hex
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11

Example

Convert 34 (octal) into the other number systems

- **Octal to binary:** form the binary digits by 3's

  34 (octal) = 011 100 (binary)

- **Binary to hex:** regroup the bits into 4's, then translate to hex

  0001 1100 = 1C (hex)

- **Hex to decimal:** use powers of 16's

  1C (hex) = 1*16 + 12 = **28 (decimal)**

- **Check:** 34 (octal) = 3*8 + 4 = 28 (decimal)

Example

Convert **A4 (hex)** into the other number systems

- **Hex to binary:** form the binary digits by 4's

  A4 (hex) = 1010 0100 (binary)

- **Binary to octal:** regroup the bits into 3's, then translate to octal

  10 100 100 = 244 (octal)

- **Octal to decimal:** use powers of 8's

  244 (octal) = 2*64 + 4*8 + 4 = **164 (decimal)**

- **Check:** A4 (hex) = 10*16 + 4 = 164 (decimal)

Arithmetic operations

- Binary arithmetic is easy

+	0	1
0	0	1
1	1	10

x	0	1
0	0	0
1	0	1

- **Exercise:**

  Write C programs for similar addition and multiplication tables for the octal and hex number systems

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

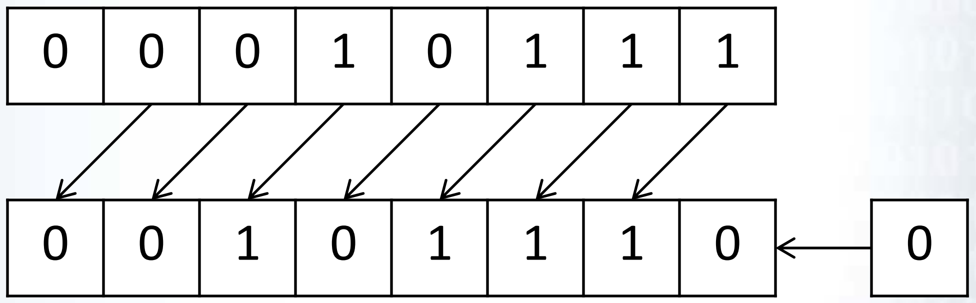
Addition examples

```
    10 101  (bin)         25  (oct)          21  (dec)
+   10 100            +   24             +   20
---------------       ---------          ---------
   101 001                51                 41
```

```
    1 0110  (bin)         16  (hex)          22  (dec)
+   1 0111            +   17             +   23
---------------       ---------          ---------
   10 1101                2D                 45
```

Bitwise logical operators

• Bitwise 1's complement ~	Ex: ~1100 = 0011
• Bitwise AND &	Ex: 1100 & 1010 = 1000
• Bitwise OR |	Ex: 1100 | 1010 = 1110
• Bitwise eXclusive OR ^	Ex: 1100 ^ 1010 = 0110
• Left shift <<	Ex: 10111 << 1 = 101110



Bitwise logical operators

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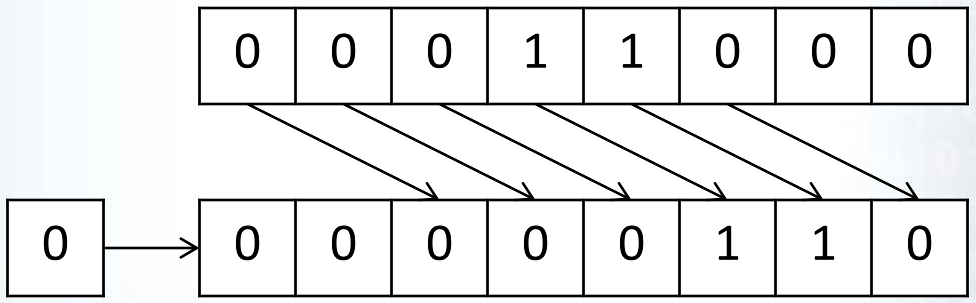
• Bitwise eXclusive OR ^	Ex: 1100 ^ 1010 = 0110
• Left shift <<	Ex: 10111 << 1 = 101110
• Right shift >>	Ex: 11000 >> 2 = 110



Bitwise logical operators

Run the ff. code fragment and try to explain its output

```
int j, p = 1;

for (j = 0; j < 32; j++) {
    printf(“%d %d\n”, j, p);
    p = p << 1;
}
```

Bits, nibbles and bytes

- Many processors now represent integers as 32-bit numbers. This is equivalent to 4 groups of 8-bit bytes, or 8 groups of 4-bit nibbles.

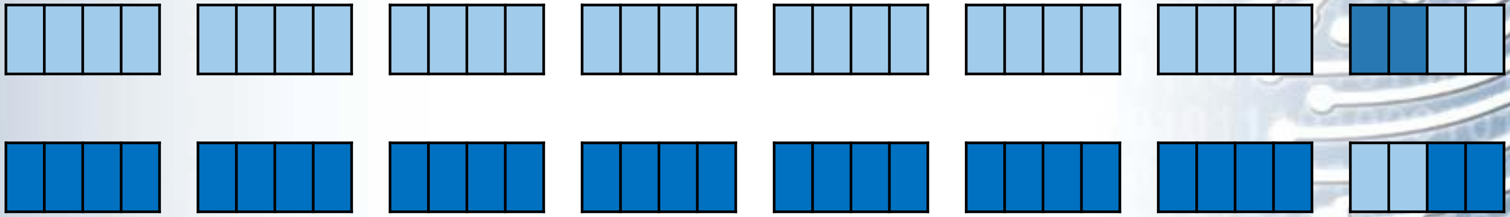
`sizeof``(int)` = 4 bytes = 32 bits

Bits, nibbles and bytes

Can you now explain the ff. code and its output?

```
printf("Complement of %x is %x", 0xC, ~0xC);
```

Complement of **c** is **fffffff3**



Representation of negative numbers

- Negative numbers are often stored in the computer's memory using the so-called 2's complement representation
- This is obtained by taking the 1's complement (flip all the bits) and then adding 1.

Example:	0001 1011	(under an 8-bit system)
1's complement	1110 0100	
2's complement	1110 0101	

Representation of negative numbers

Positive and negative numbers under a 4-bit system

	decimal		decimal
0111	7	1000	-8
0110	6	1001	-7
0101	5	1010	-6
0100	4	1011	-5
0011	3	1100	-4
0010	2	1101	-3
0001	1	1110	-2
0000	0	1111	-1

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0011	3	1100	-4
0010	2	1101	-3
0001	1	1110	-2
0000	0	1111	-1

- Note that the **left-most bit acts a sign bit (1 = negative)**

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0100	4	1011	-5
0011	3	1100	-4
0010	2	1101	-3
0001	1	1110	-2
0000	0	1111	-1

- Note that the left-most bit acts a sign bit (1 = negative), and that ***n* bits can represent all integers in [-2$^{n\text{-}1}$....2$^{n\text{-}1}$-1]**

If **n = 4**, this can represent all integers in
[-2$^{n\text{-}1}$....2$^{n\text{-}1}$-1]
[-2$^{4\text{-}1}$....2$^{4\text{-}1}$-1]
[-2^{3}....2^{3}-1]
[-8....7]

Representation of negative numbers

Positive and negative numbers under a 4-bit system

	decimal		decimal
0111	7	1000	-8
0110	6	1001	-7
0101	5	1010	-6
0100	4	1011	-5
0011	3	1100	-4
0010	2	1101	-3
0001	1	1110	-2
0000	0	1111	-1

Addition examples

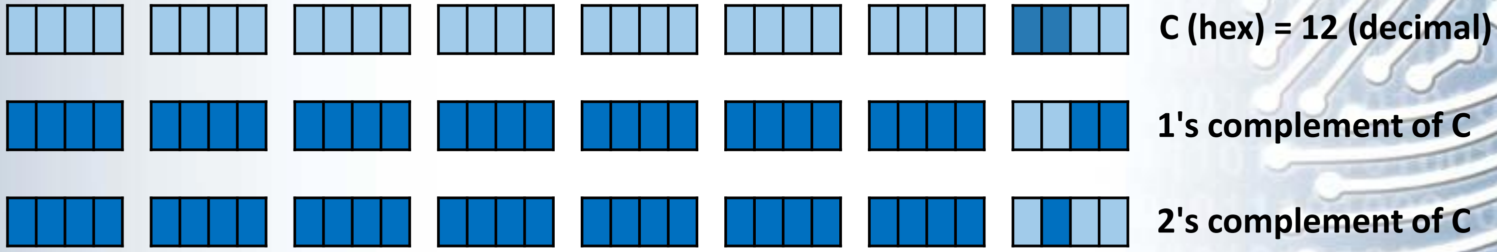
```
  0011            3
+ 1101         + -3
-----------    --------
 10000            0
(with overflow)

  0010            2
+ 1011         + -5
-----------    -----
  1101           -3
```

Representation of negative numbers

Can you now explain the ff. code and its output?

```
printf("Negative of %d is %d\n", 0xc, ~0xc + 1);
```

```
Negative of 12 is -12
```



Representing chars

- **Plain ASCII code is a 7-bit code** to represent the most common characters
- **Extended ASCII uses 8 bits** to include certain additional chars like ñ, arrows, and lines
- **Unicode uses 16 bits** in order to represent practically all character sets (including Japanese, Korean, Arabic, etc)

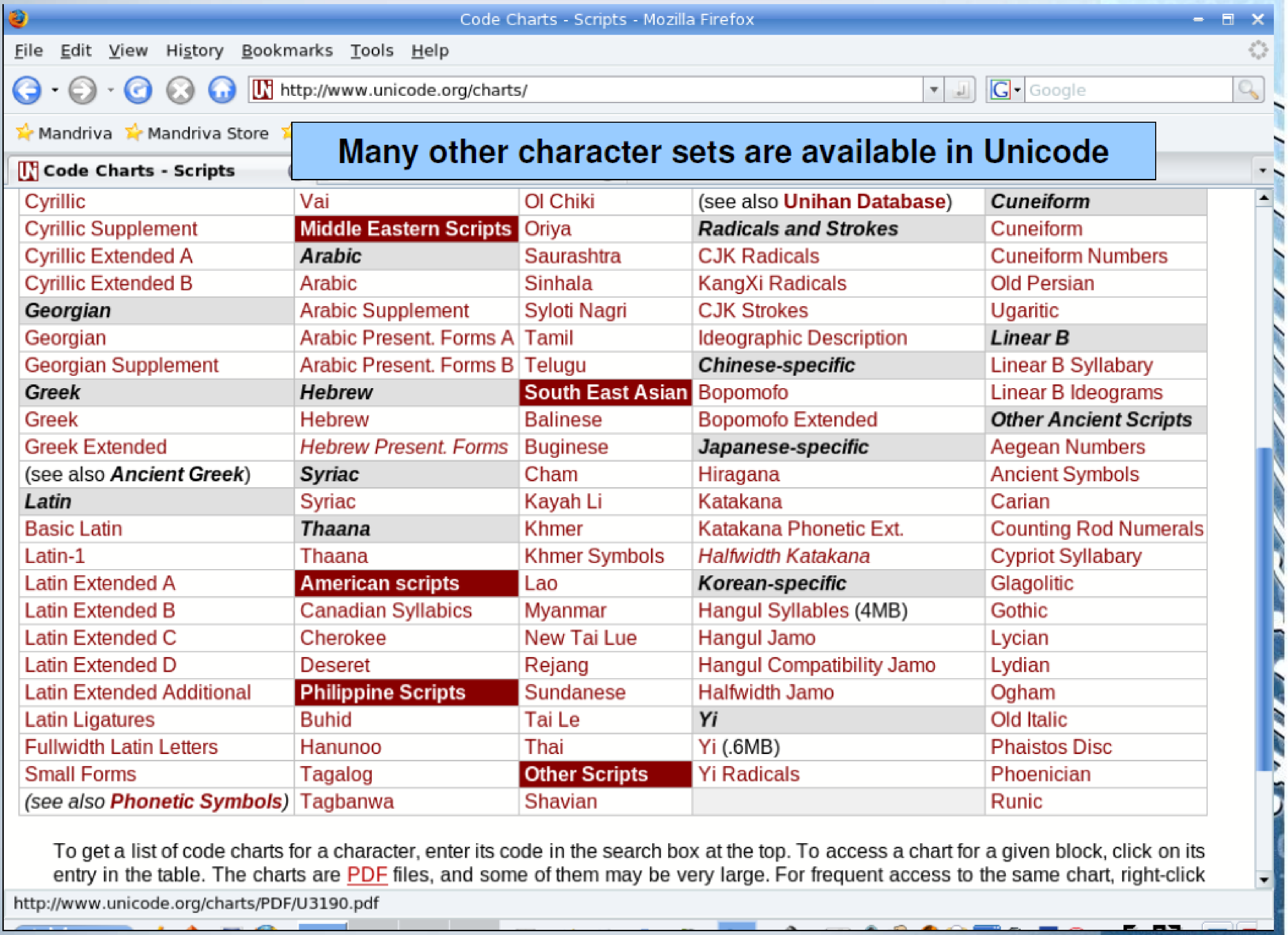
```
int c;
for (c = 0; c < 128; c++) {
  printf("char %c = decimal %d = hex %x\n", c, c, c);
}
```

Part of the ASCII character set

Chr	Ctrl	Dec	Hex	Chr	Dec	Hex	Chr	Dec	Hex	Chr	Dec	Hex
NUL	^@	0	0	SP	32	20	@	64	40	`	96	60
SOH	^A	1	1	!	33	21	A	65	41	a	97	61
STX	^B	2	2	"	34	22	B	66	42	b	98	62
ETX	^C	3	3	#	35	23	C	67	43	c	99	63
EOT	^D	4	4	$	36	24	D	68	44	d	100	64
ENQ	^E	5	5	%	37	25	E	69	45	e	101	65
ACK	^F	6	6	&	38	26	F	70	46	f	102	66
BEL	^G	7	7	'	39	27	G	71	47	g	103	67
BS	^H	8	8	(	40	28	H	72	48	h	104	68
HT	^I	9	9	)	41	29	I	73	49	i	105	69
LF	^J	10	A	*	42	2A	J	74	4A	j	106	6A
.....				.....			.....			.....		
CAN	^X	24	18	8	56	38	X	88	58	x	120	78
EM	^Y	25	19	9	57	39	Y	89	59	y	121	79
SUB	^Z	26	1A	:	58	3A	Z	90	5A	z	122	7A
ESC	^[	27	1B	;	59	3B	[	91	5B	{	123	7B
FS	^\	28	1C	<	60	3C	\	92	5C	|	124	7C
GS	^]	29	1D	=	61	3D	]	93	5D	}	125	7D
RS	^^	30	1E	>	62	3E	^	94	5E	~	126	7E
US	^_	31	1F	?	63	3F	_	95	5F	DEL	127	7F

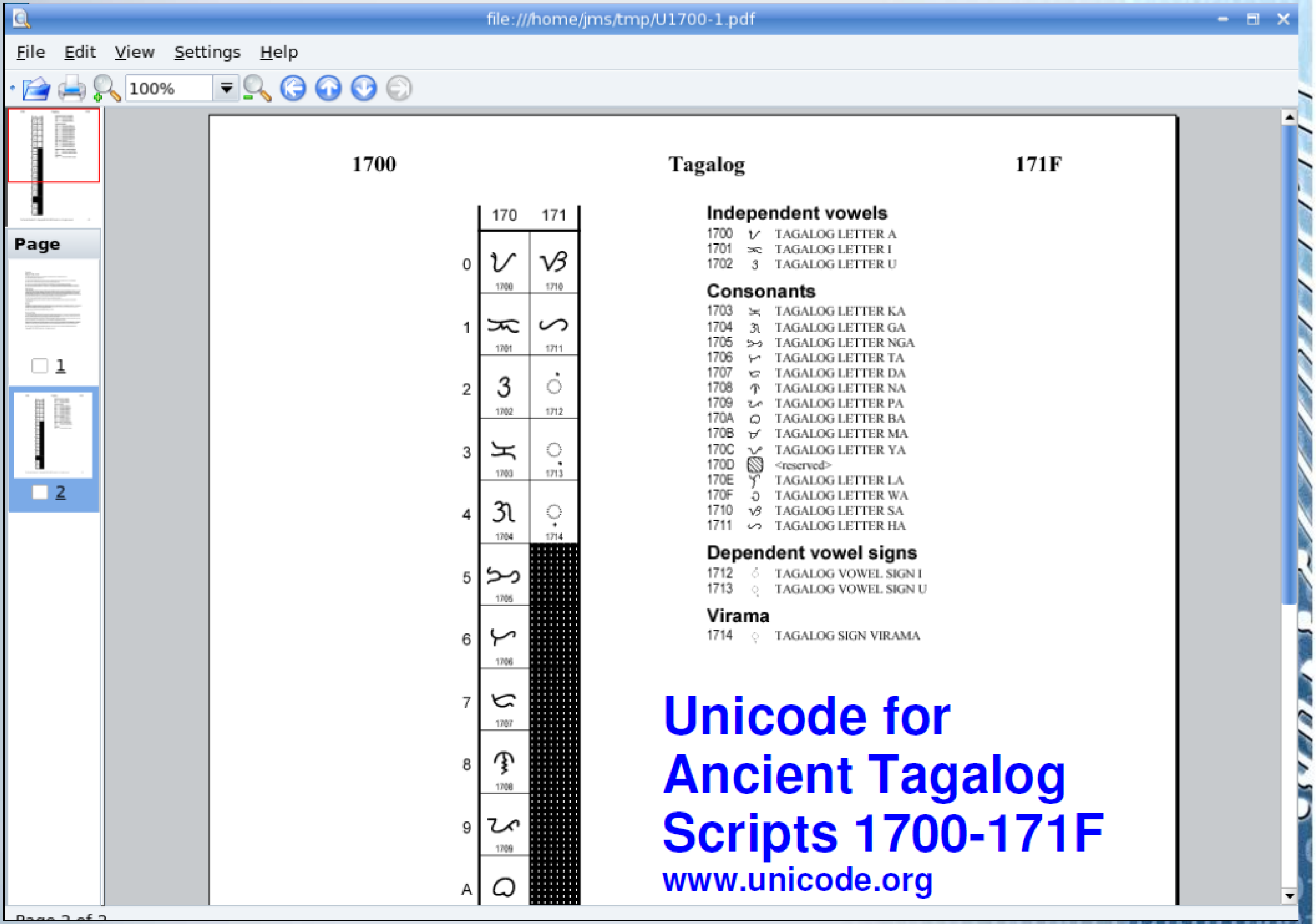
http://www.mhuffman.com/notes/numbers/numrep.htm

Many other character sets are available in Unicode


Code Charts - Scripts - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://www.unicode.org/charts/
Google
Mandriva
Mandriva Store
Code Charts - Scripts
http://www.unicode.org/charts/PDF/U3190.pdf

Cyrillic	Vai	Ol Chiki	(see also **Unihan Database**)	***Cuneiform***
Cyrillic Supplement	**Middle Eastern Scripts**	Oriya	***Radicals and Strokes***	Cuneiform
Cyrillic Extended A	***Arabic***	Saurashtra	CJK Radicals	Cuneiform Numbers
Cyrillic Extended B	Arabic	Sinhala	KangXi Radicals	Old Persian
Georgian	Arabic Supplement	Syloti Nagri	CJK Strokes	Ugaritic
Georgian	Arabic Present. Forms A	Tamil	Ideographic Description	***Linear B***
Georgian Supplement	Arabic Present. Forms B	Telugu	***Chinese-specific***	Linear B Syllabary
Greek	***Hebrew***	**South East Asian**	Bopomofo	Linear B Ideograms
Greek	Hebrew	Balinese	Bopomofo Extended	***Other Ancient Scripts***
Greek Extended	*Hebrew Present. Forms*	Buginese	***Japanese-specific***	Aegean Numbers
(see also ***Ancient Greek***)	***Syriac***	Cham	Hiragana	Ancient Symbols
Latin	Syriac	Kayah Li	Katakana	Carian
Basic Latin	***Thaana***	Khmer	Katakana Phonetic Ext.	Counting Rod Numerals
Latin-1	Thaana	Khmer Symbols	*Halfwidth Katakana*	Cypriot Syllabary
Latin Extended A	**American scripts**	Lao	***Korean-specific***	Glagolitic
Latin Extended B	Canadian Syllabics	Myanmar	Hangul Syllables (4MB)	Gothic
Latin Extended C	Cherokee	New Tai Lue	Hangul Jamo	Lycian
Latin Extended D	Deseret	Rejang	Hangul Compatibility Jamo	Lydian
Latin Extended Additional	**Philippine Scripts**	Sundanese	Halfwidth Jamo	Ogham
Latin Ligatures	Buhid	Tai Le	***Yi***	Old Italic
Fullwidth Latin Letters	Hanunoo	Thai	Yi (.6MB)	Phaistos Disc
Small Forms	Tagalog	**Other Scripts**	Yi Radicals	Phoenician
*(see also **Phonetic Symbols**)*	Tagbanwa	Shavian		Runic

To get a list of code charts for a character, enter its code in the search box at the top. To access a chart for a given block, click on its entry in the table. The charts are PDF files, and some of them may be very large. For frequent access to the same chart, right-click



1700 Tagalog 171F

	170	171
0	ᜀ 1700	ᜐ 1710
1	ᜁ 1701	ᜑ 1711
2	ᜂ 1702	◌ᜒ 1712
3	ᜃ 1703	◌ᜓ 1713
4	ᜄ 1704	◌᜔ 1714
5	ᜅ 1705	
6	ᜆ 1706	
7	ᜇ 1707	
8	ᜈ 1708	
9	ᜉ 1709	
A	ᜊ	

Independent vowels

1700	ᜀ	TAGALOG LETTER A
1701	ᜁ	TAGALOG LETTER I
1702	ᜂ	TAGALOG LETTER U

Consonants

1703	ᜃ	TAGALOG LETTER KA
1704	ᜄ	TAGALOG LETTER GA
1705	ᜅ	TAGALOG LETTER NGA
1706	ᜆ	TAGALOG LETTER TA
1707	ᜇ	TAGALOG LETTER DA
1708	ᜈ	TAGALOG LETTER NA
1709	ᜉ	TAGALOG LETTER PA
170A	ᜊ	TAGALOG LETTER BA
170B	ᜋ	TAGALOG LETTER MA
170C	ᜌ	TAGALOG LETTER YA
170D	▨	<reserved>
170E	ᜎ	TAGALOG LETTER LA
170F	ᜏ	TAGALOG LETTER WA
1710	ᜐ	TAGALOG LETTER SA
1711	ᜑ	TAGALOG LETTER HA

Dependent vowel signs

1712	◌ᜒ	TAGALOG VOWEL SIGN I
1713	◌ᜓ	TAGALOG VOWEL SIGN U

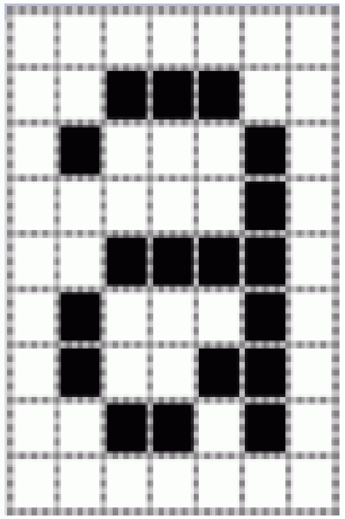
Virama

1714	◌᜔	TAGALOG SIGN VIRAMA

Unicode for Ancient Tagalog Scripts 1700-171F

www.unicode.org

Representing images



0000000
0011100
0100010
0000010
0011110
0100010
0100110
0011010
0000000

- A black & white image 7 pixels wide and 9 pixels high can be presented as a sequence of 63 bits.
- How many bits per pixel do we need if we want 16 shades of gray? Or if we want 256 different colors?