REVIEW ITEMS FOR MIDTERM EXAM

PART 1. FILL THE BLANKS WITH CORRECT EXPRESSIONS OR WORDS.

1. If $\vec{A} = \langle 2,-1,-2 \rangle$, then $\underline{\|\vec{A}\| = 3}$
2. The unit vector in the same direction as $\vec{A} = \langle 2,-1,-2 \rangle$ is $\underline{\vec{u}_{\vec{A}} = \left\langle \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3} \right\rangle}$
3. If $\vec{C} = \langle 3,-4,1 \rangle$ and $\vec{D} = \langle -8,6,3 \rangle$, then $\underline{3\vec{C} + 2\vec{D} = \langle -7,0,9 \rangle}$
4. The direction angles of $\langle 0,0,-3 \rangle$ are $\alpha = \frac{\pi}{2}, \beta = \frac{\pi}{2}$ and $\underline{\gamma = \pi}$
5. If $\vec{A} = \left\langle \frac{-\sqrt{3}}{2}, \frac{1}{2} \right\rangle$ and $\vec{B} = \langle 0,2 \rangle$, then $\underline{\vec{A} \cdot \vec{B} = 1}$
6. In problem no.5, the radian measure of the angle between $\vec{A}$ and $\vec{B}$ is $\underline{\frac{\pi}{3} = Arc\cos\left(\frac{1}{2}\right)}$
7. In problem no.5, the scalar projection of $\vec{A}$ onto $\vec{B}$ is $\underline{\frac{1}{2}}$
8. In problem no.5, the vector projection of $\vec{A}$ onto $\vec{B}$ is $\underline{\left\langle 0, \frac{1}{2} \right\rangle}$
9. If the direction angle of a vector $\vec{G}$ is $\frac{5\pi}{4}$ and its magnitude is 4, then $\underline{\vec{G} = \langle -2\sqrt{2}, -2\sqrt{2} \rangle}$
10. Consider the points $C(4,-5)$ and $D(-3,2)$. If $\overrightarrow{DC}$ is a representation of $\vec{E}$, then $\underline{\vec{E} = \langle 7,-7 \rangle}$
11. An equation of a plane that is parallel to the xz-plane and which passes through the point $(1,2,3)$ is $\underline{y = 2}$
12. The distance between $A(1,2,3)$ and $B(-2,3,-4)$ is $\underline{\sqrt{59}}$.
13. The midpoint of the segment whose endpoints are $A(1,2,3)$ and $B(-2,3,-4)$ is $\underline{\left(\frac{-1}{2}, \frac{5}{2}, \frac{-1}{2} \right)}$
14. The standard equation of the sphere with $A(1,2,3)$ and $B(-2,3,-4)$ as endpoints of a diameter is $\underline{\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 + \left(z + \frac{1}{2}\right)^2 = \frac{59}{4}}$
15. The point $(1,2,3)$ lies **<u>outside</u>** the sphere given by $x^2 + y^2 + (z-1)^2 = 5$.
16. The graph of $x^2 + 4x + y^2 - 6y + z^2 - 2z - 10 = 0$ is a **<u>sphere</u>**.
17. A standard equation of the plane passing through $(1,2,3)$ and having $\langle -2,3,-4 \rangle$ as a normal vector is given by $\underline{-2(x-1) + 3(y-2) - 4(z-3) = 0}$
18. The distance between the parallel planes given by $2x - 2y + z + 5 = 0$ and $4x - 4y + 2z + 6 = 0$ is $\underline{\frac{2}{3}}$
19. The distance from the point $(1,2,3)$ to the plane given by $2x - 2y + z + 5 = 0$ is $\underline{2}$
20. The parametric equations of the line passing through $(1,2,3)$ and is parallel to $\langle 4,5,6 \rangle$ are given by $\underline{x = 1 + 4t,\ y = 2 + 5t,\ z = 3 + 6t}$
21. If $\vec{A} = \langle 1,2,3 \rangle$ and $\vec{B}(-2,3,-4)$, then $\underline{\vec{A} \times \vec{B} = -17i - 2j + 7k}$
22. In R^3, the graph of $x^2 - 4y = 1$ is called a **<u>parabolic</u>** cylinder.
23. The trace of $\frac{x^2}{2} - \frac{y^2}{9} - z^2 = 1$ on the **xz-plane** is called a **<u>hyperbola</u>**
24. The limit of the sequence 1,-1,1,-1,1,-1,… **<u>does not exist</u>**
25. The limit of the sequence $\left\{ \frac{\sin n}{n} \right\}$ as $n \to \infty$ is $\underline{0}$
26. $\lim\limits_{n \to \infty} \frac{2n+1}{1-3n^2}$ is equal to $\underline{0}$

27. The k-th partial sum of the geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ is $\underline{\dfrac{a\left(1-r^k\right)}{1-r}}$

28. The series $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^n$ is **absolutely convergent**.

29. The sum $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n}+\ldots$ is equal to $\underline{+\infty}$.

30. The sequence $\left\{\frac{(-1)^n}{n}\right\}$ is **convergent**

31. If $f'(x)<0$ for all $x \geq 1$, then $\{f(n)\}$ is **decreasing**

32. The sum of the infinite series $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{n+1}$ is $\underline{\frac{1}{6}}$

33. The sum of the infinite series $\sum_{n=1}^{\infty}\frac{2}{n(n+1)}$ is $\underline{2}$

34. The p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $\underline{p>1}$

35. The series $\sum_{n=1}^{\infty}\left(\frac{1}{n}+\frac{1}{2^n}\right)$ is **divergent**

36. If $\sum_{n=1}^{\infty}\frac{k}{n}$ converges, then k = **0**.

37. The series $\sum_{n=1}^{\infty}\frac{n^2}{2^{n^2}}$ is absolutely convergent. Using the *ratio* test, the value of $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|$ is $\underline{0}$

38. In its interval of convergence, the sum of the power series $\sum_{n=0}^{\infty}(x-1)^n$ is expressed by $\underline{\frac{1}{2-x}}$

39. The interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ is $\underline{(-1,1]}$

40. The Maclaurin series expansion of the function $f(x)=\sin x$ is $\underline{\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}}$

PART 2. PROBLEM SOLVING. WRITE YOUR SOLUTIONS NEATLY, COMPLETELY AND LOGICALLY.

1. Determine the general equation of the plane through the point $P(0,2,-1)$ and parallel to the plane $2x-y+3z+8=0$.
   Solution:
   Consider point $P_0=(0,2,-1)$ and a normal vector $N=\langle 2,-1,3\rangle$ which is a vector orthogonal to the plane $2x-y+3z+8=0$
   Thus, the needed equation of the plane is given by $2x-y+3z+5=0$.

2. Find an equation of a plane containing the point $(3,1,-1)$ and parallel to the lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$.
   Solution:
   We need to find a vector perpendicular both to the lines, which is also a normal vector to the plane. The needed vector is given by
   $N=\langle 1,3,1\rangle\times\langle 1,4,2\rangle=\langle 2,-1,1\rangle$.
   An equation of the plane is given by $2(x-3)-(y-1)+(z+1)=0$.

In general, we have $2x - y + z - 4 = 0$.

3. Identify and sketch the graph of the following surfaces in R^3 :
   a. $4x^2 - 9z^2 = 36$
   b. $\frac{x^2}{4} + \frac{y^2}{16} - \frac{z^2}{2} = 1$
   c. $4y^2 + z^2 = 4x$

   Solution:

   a. Hyperbolic cylinder
   b. Elliptic hyperboloid of one sheet
   c. Elliptic paraboloid

4. Given the series $\sum_{n=0}^{\infty} u_n = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots$
   a. Find u_n .

      Solution: $u_n = \frac{1}{(2n+1)(2n+3)} = \frac{1}{2(2n+1)} - \frac{1}{2(2n+3)}$

      (Use method of partial fractions)
   b. Let $S_n = u_1 + u_2 + \ldots + u_n$. Find a formula for S_n .

      Solution: $S_n = \left(\frac{1}{2} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{10}\right) + \ldots + \left(\frac{1}{2(2n+1)} - \frac{1}{2(2n+3)}\right)$

      $$S_n = \frac{1}{2} - \frac{1}{2(2n+3)} = \frac{1}{2}\left(1 - \frac{1}{2n+3}\right)$$
   c. Find $\lim_{n\to\infty} S_n$, if it exists.

      Solution: $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{2n+3}\right) = \frac{1}{2}$
   d. Is the series $\sum_{n=0}^{\infty} u_n$ convergent? Why?

      Solution: Since $\lim_{n\to\infty} S_n$ exists, then the series $\sum_{n=0}^{\infty} u_n$ is convergent.

5. Use Ratio Test to determine whether the series $\sum_{n=1}^{+\infty} \frac{3^n}{n^2}$ is convergent or divergent.

   Solution: Let $u_n = \frac{3^n}{n^2}$. Consider $\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{3^{n+1}}{(n+1)^2} \cdot \frac{n^2}{3^n}\right| = \frac{3n^2}{(n+1)^2}$

   Using ratio test, $L = \lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \frac{3n^2}{(n+1)^2} = 3$ **.** Since $L > 1$, then the series is divergent.

6. Consider the power series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{2^{2n+1}}$. Find its radius of convergence and determine its interval of convergence.

   Solution: Applying the ratio test, let $\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{(x-3)^{n+1}}{2^{2n+3}} \cdot \frac{2^{2n+1}}{(x-3)^n}\right| = \frac{|x-3|}{4}$

   $L = \lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \frac{|x-3|}{4} < 1 \Rightarrow |x-3| < 4 \Leftrightarrow -1 < x < 7$

   At $x = 7$: $\sum_{n=0}^{\infty} \frac{(-1)^n}{2}$ which is a divergent series since $\lim_{n\to\infty} \frac{(-1)^n}{2}$ is not equal to zero (***nth term test for divergence***)

   At $x = -1$: $\sum_{n=0}^{\infty} \frac{1}{2}$ which is a divergent series since $\lim_{n\to\infty} \frac{1}{2}$ is not equal to zero (***nth term test for divergence***). Thus, IOC is $(-1,7)$

End of items