COMBINATORICS

ALGEBRAIC STRUCTURES

DISCRETE PROBABILITY

GRAPH THEORY

DISCRETE PROBABILITY

APPLICATIONS

Analysis of Algorithms (Average Case Analysis)

Probabilistic Algorithms

Any activity that (randomly) yields a result or an outcome.

SAMPLE SPACE

Set of all possible outcomes of a RANDOM EXPERIMENT.

SAMPLE POINT

An element of the SAMPLE SPACE.

Rolling a die

Taking a course until the course is passed

Answering three
TRUE or FALSE questions

EVENT

A collection of

SAMPLE POINTS

in the

SAMPLE SPACE

(subset)

EVENT SPACE

Set of all possible **EVENTS**

(Power set of the SAMPLE SPACE)

Tossing a 1 peso and a 5 peso coin

EVENTS

The event that both coins turn up tails

EVENTS

The event that at least one turns up heads

Drawing four chocolates from a box containing milk and dark chocolates with replacement

EVENTS

The event that exactly one dark chocolate is drawn

EVENTS

The event that exactly two milk chocolates are drawn

The **PROBABILITY** that event A will occur

$$P(A) = |A| / |S|$$

GIVEN

The sample space is finite

Each sample point is equally likely

$$A \subseteq S$$

Drawing four chocolates from a box containing milk and dark chocolates with replacement

What is the probability that exactly two milk chocolates are drawn?

What is the probability that no dark chocolates are drawn?

In a 6/42 lottery, what is the probability of picking the winning combination?

In poker, what is the probability that a hand of five cards contains four cards of one kind?

In poker, what is the probability that a hand of five cards contains a full house?

Full house:
Three of one kind and two of another kind

AXIONS

AXIONS

$$P(A) \geq 0$$

$$P(S) = 1$$

AXIONS

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If A_1, A_2, A_3, ... are MUTUALLY EXCLUSIVE EVENTS, then P(A_1 \cup A_2 \cup A_3 \cup ...) = P(A_1) + P(A_2) + P(A_3) + ...
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THEOREMS

THEOREMS

$$P(\varnothing)=0$$

$$P(A') = 1 - P(A)$$

$$P(A-B) = P(A) - P(AB)$$

THEOREMS

PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

if $A\subseteq B$, then $P(A) \leq P(B)$

A sequence of 10 bits is randomly generated.

What is the probability that at least one of the bits is zero?

Consider a standard deck of 52 cards.

What is the probability of obtaining a face card or a diamond?

If the occurrence of one hinders/ prevents the occurrence of the other

if and only if $P(A \cup B) = P(A) + P(B)$

$$P(AB) = 0$$

Consider rolling a die four times.

What is the probability that the first roll results in a number less than 3 or greater than 3?

AN EXPERIMENT WITH OUTCOMES THAT ARE NOT EQUALLY LIKELY

A BIASED DIE

Consider rolling a die such that any even number is twice as likely to show up as any odd number.

A BIASED DIE

What is the probability that a prime is obtained?

Suppose that a coin is flipped three times.

Moreover, it is known that the first flip came up tails.

Given this information, what is the probability that an odd number of tails appears?

Let A =the first flip came up tails. B =odd number of tails appears.

The probability of an event (B) occurring when it is known that some other event (A) has occurred.

$$P(B|A) = P(AB) / P(A)$$

$$P(AB) = P(A) \cdot P(B|A)$$

$$P(AB) = P(B) \cdot P(A|B)$$

Consider a family with two children.

What is the probability that they have two boys, given they have at least one boy?

Let A =they have least one boy. B =they have two boys.

Consider answering an exam with 10 all-or-nothing questions.

What is the probability of having exactly 6 correct answers given at least one of the answers is correct?

Let A = at least one is a correct answer. B = exactly six are correct answers.

Suppose that the probability of a teacher having a fever is 0.03.

Moreover, the probability that a teacher will conduct no class given he/she has a fever is 0.5.

Also, the probability that a teacher will conduct no class is 0.02.

Find the probability that a teacher has a fever given that he/she conducted no class.

PROPERTIES

CONDITIONAL PROBABILITY

PROPERTIES

CONDITIONAL PROBABILITY

$$P(\varnothing|B)=0$$

$$P(A'|B) = 1 - P(A|B)$$

PROPERTIES

CONDITIONAL PROBABILITY

 $P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(AB \mid C)$

if $A \subseteq B$, then $P(A|C) \leq P(B|C)$

THEOREMS

CONDITIONAL PROBABILITY

THEOREMS

CONDITIONAL PROBABILITY

$$P(A_1A_2A_3...A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1A_2) ... P(A_n|A_1A_2...A_{n-1})$$

Three magic cards are picked without replacement from a deck with

21 (2), 22 (3) and 19 (3) cards.



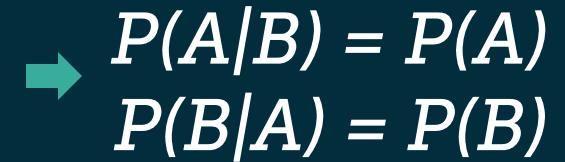
What is the probability of obtaining a card on the first two draws followed by a card?

If the occurrence of event A does not affect the probability of the occurrence of event B and vice versa, then A and B are independent.

INDEPENDENT EVENTS

INDEPENDENT EVENTS

A and B are independent



INDEPENDENT EVENTS

A and B are independent
$$\longrightarrow P(AB) = P(A) \cdot P(B)$$

INDEPENDENT EVENTS



A and B are \rightarrow $P(A|B) = P(A|B^C)$ independent

Suppose that a coin is flipped three times.

Let A =the first flip came up tails. B =odd number of tails appears.

Does knowing that the first flip comes up tails (A) alter the probability that tails comes up and odd number of times (B)?

Are the events A and B independent?

Consider drawing a card from a deck of 52 cards.

Let F = a face card is drawn.

C = a club card is drawn.

B = A black card is drawn.

Which pairs of events are independent?