CMSC 141 Automata and Language Theory

Context-Free Languages

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Closure Properties for CFL

Like regular languages, CFLs are close under union, concatenation, and Kleene star

Closure Under Union

Proof

Given two grammars for two context-free languages, with start symbols S and T. Rename the variables to ensure that the two grammars will not share any variable. Then construct a grammar for the union of the two languages by taking all the rules of both grammars and adding a new start state Z with rules $Z \to S \mid T$

Example

$L_1 \Rightarrow$

$$S \to aSb \mid \varepsilon$$

$L_2 \Rightarrow$

$$T \to aTa \mid bTb \mid a \mid b \mid \varepsilon$$

$L_1 \cup L_2 \Rightarrow$

$$\begin{aligned} Z &\to S \mid T \\ S &\to aSb \mid \varepsilon \\ T &\to aTa \mid bTb \mid a \mid b \mid \varepsilon \end{aligned}$$

Closure Under Concatenation

Proof

Same process as union, but instead, we have the rule for the start state $Z \rightarrow ST$

Example

$L_1 \Rightarrow$

$$S \rightarrow aSb \mid \varepsilon$$

$L_2 \Rightarrow$

$$T \rightarrow aTa \mid bTb \mid a \mid b \mid \varepsilon$$

$L_1 L_2 \Rightarrow$

$$\begin{aligned} Z &\rightarrow ST \\ S &\rightarrow aSb \mid \varepsilon \\ T &\rightarrow aTa \mid bTb \mid a \mid b \mid \varepsilon \end{aligned}$$

Closure Under Kleene Star

Proof
Given a grammar for a context-free language L with start symbol S, the grammar for L^*, with start symbol Z, contains all the rules of the original grammar along with the rules $Z \to ZS \mid \varepsilon$

Example

$L \Rightarrow$
$$S \to aSb \mid \varepsilon$$

$L^* \Rightarrow$
$$\begin{aligned} Z &\to ZS \mid \varepsilon \\ S &\to aSb \mid \varepsilon \end{aligned}$$

Other Closure Properties

Other closure properties for CFLs

- string reversal
- homomorphism (string substitutions)
- inverse homomorphisms

Proofs are left as exercise

Closure Properties

Also note, however, that CFLs are *not* closed under

- intersection
- set complement

Proofs are left as exercise

Pumping Lemma for CFLs

Pumping Lemma

If A is a context-free language that is infinite, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five parts $s = uvxyz$ satisfying:

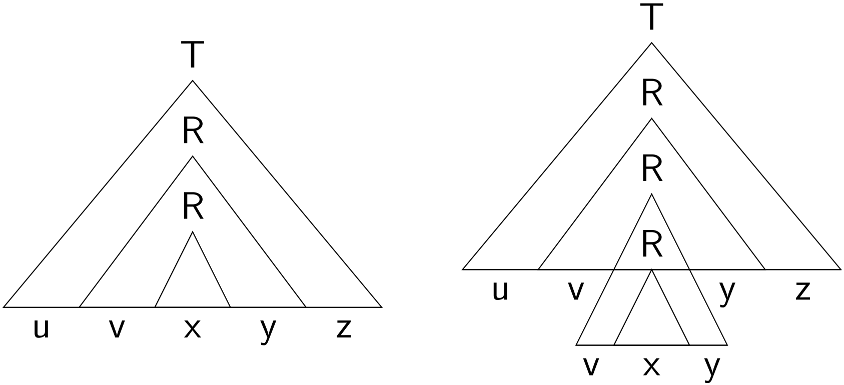
- for each $i \geq 0, uv^ixy^iz \in A$
- $|vy| > 0$ (v and y cannot be both empty)
- $|vxy| \leq p$

Idea of the Proof

- We use the pigeonhole principle on the nodes of the parse tree
- Given a very long string s, its parse tree would be very tall that there must exists some interior node (say R) that must be repeated
  - $R \to^* x \mid vRy$
- "Pumping" translates to expanding R any number of times

Pumping Lemma for CFLs

"Pumping" translates to expanding R any number of times



Non-Context Free Languages

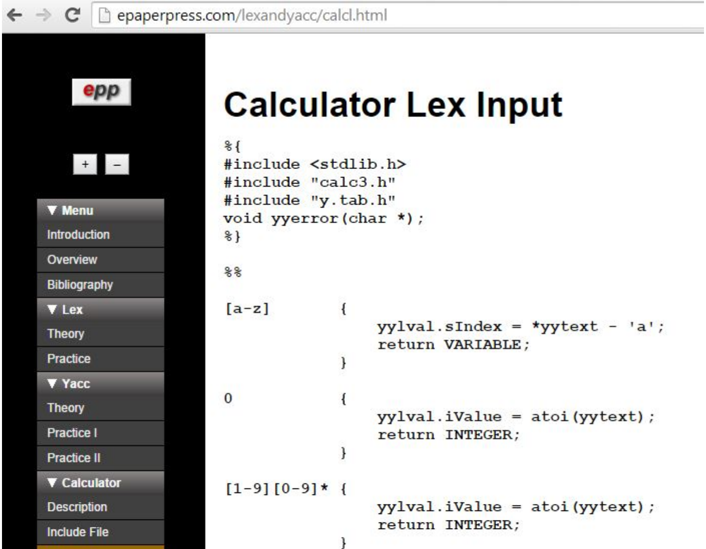
- Some languages cannot be recognized by a PDA or a CFG
- One of the simplest non-CFL is $L = \{a^n b^n c^n : n > 0\} = \{abc, aabbcc, ...\}$
  Can be proven using proof by contradiction and pumping lemma
- How can we still extend PDAs? 2 stacks??

Some applications of CFL

- Compiler Design/Programming Language Design
- Lindenmayer Systems (L-Systems)

Can be used in tools like Lex/Flex and Yacc/Bison

From http://epaperpress.com/lexandyacc



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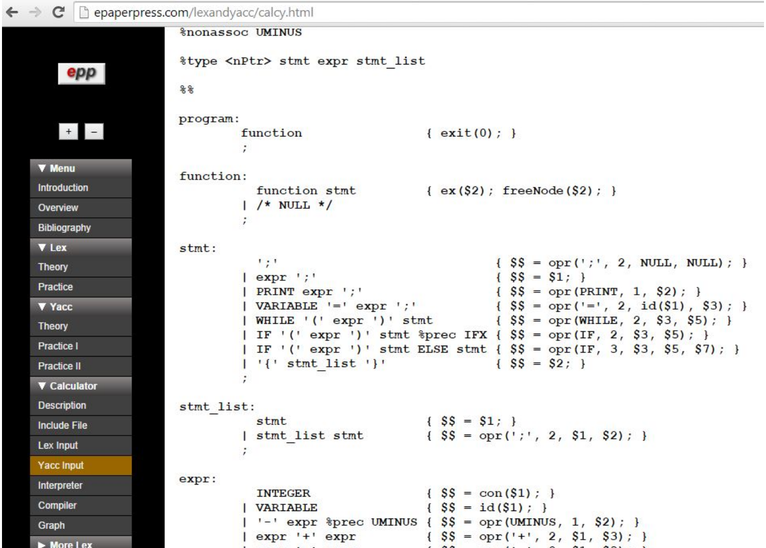
Include File

Calculator Lex Input

```
%{
#include <stdlib.h>
#include "calc3.h"
#include "y.tab.h"
void yyerror(char *);
%}

%%

[a-z]       {
                yylval.sIndex = *yytext - 'a';
                return VARIABLE;
            }

0           {
                yylval.iValue = atoi(yytext);
                return INTEGER;
            }

[1-9][0-9]* {
                yylval.iValue = atoi(yytext);
                return INTEGER;
            }
```

Can be used in tools like Lex/Flex and Yacc/Bison

From http://epaperpress.com/lexandyacc




```
%nonassoc UMINUS

%type <nPtr> stmt expr stmt_list

%%

program:
        function                { exit(0); }
        ;

function:
          function stmt         { ex($2); freeNode($2); }
        | /* NULL */
        ;

stmt:
          ';'                            { $$ = opr(';', 2, NULL, NULL); }
        | expr ';'                       { $$ = $1; }
        | PRINT expr ';'                 { $$ = opr(PRINT, 1, $2); }
        | VARIABLE '=' expr ';'          { $$ = opr('=', 2, id($1), $3); }
        | WHILE '(' expr ')' stmt        { $$ = opr(WHILE, 2, $3, $5); }
        | IF '(' expr ')' stmt %prec IFX { $$ = opr(IF, 2, $3, $5); }
        | IF '(' expr ')' stmt ELSE stmt { $$ = opr(IF, 3, $3, $5, $7); }
        | '{' stmt_list '}'              { $$ = $2; }
        ;

stmt_list:
          stmt                  { $$ = $1; }
        | stmt_list stmt        { $$ = opr(';', 2, $1, $2); }
        ;

expr:
          INTEGER               { $$ = con($1); }
        | VARIABLE              { $$ = id($1); }
        | '-' expr %prec UMINUS { $$ = opr(UMINUS, 1, $2); }
        | expr '+' expr         { $$ = opr('+', 2, $1, $3); }
```

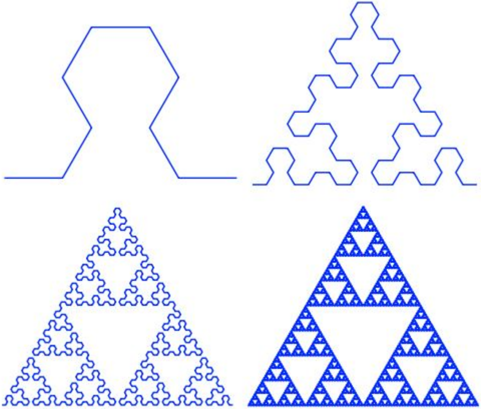
Lindenmayer Systems

grammar-like structures for drawing fractals

Example 5: Sierpinski triangle [edit]

The Sierpinski triangle drawn using an L-system.

- **variables** : A B
- **constants** : + −
- **start** : A
- **rules** : (A → B−A−B), (B → A+B+A)
- **angle** : 60°



Evolution for $n = 2$, $n = 4$, $n = 6$, $n = 8$

From Wikipedia

References

- Previous slides on CMSC 141
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- JFLAP, www.jflap.org
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