MODELS

OF COUNTING

SAMPLE

MODEL OF COUNTING

DISTRIBUTION

MODEL OF COUNTING

SAMPLE

MODEL OF COUNTING

SAMPLE

MODEL OF COUNTING

In how many ways can we **TAKE A SAMPLE** of k objects from a group of n distinct objects?

SAMPLE

MODEL OF COUNTING

Is the order of the objects chosen important?

Are repetitions of objects allowed?

SEQUENCE

Order matters
Repetitions allowed

PERMUTATION

Order matters
Repetitions not allowed

MULTISET

Order does not matter
Repetitions allowed

COMBINATION

Order does not matter
Repetitions not allowed

SAMPLE

MODEL OF COUNTING

Take a sample of 2 letters from the set

$\{ a, b, c \}$.

If the samples are
SEQUENCES

aa ab ac
ba bb bc
ca cb cc

If the samples are

PERMUTATIONS

ab ac

ba bc

ca cb

If the samples are
MULTISETS

aa ab ac
bb bc
cc

If the samples are

COMBINATIONS

ab ac
bc

SEQUENCE

Ordering of objects matters

Repetitions of objects are allowed

SEQUENCE

How many sequences of
n objects taken k at a time?

SEQUENCE

Select 1^{st} object	n possible objects
Select 2^{nd} object	n possible objects
:	:
Select k^{th} object	n possible objects

Using product rule,

$$n \cdot n \cdot \ldots \cdot n = n^k$$

SEQUENCE

How many sequences of
n objects taken k at a time?

$$S(n,k) = n^k$$

PERMUTATION

Ordering of objects matters

Repetitions of objects are not allowed

PERMUTATION

How many permutations of
n objects taken k at a time?

PERMUTATION

Select 1st object	n possible objects
Select 2nd object	n-1 possible objects
Select 3rd object	n-2 possible objects
:	:
Select kth object	n - (k-1) possible objects

PERMUTATION

Using product rule,

$$n \cdot (n-1) \cdot (n-2) \ldots \cdot n-(k-1)$$

$$= \frac{n!}{(n-k)!}$$

PERMUTATION

How many permutations of n objects taken k at a time?

$$P(n,k) = \frac{n!}{(n-k)!}$$

COMBINATION

Ordering of objects does not matter

Repetitions of objects are not allowed

COMBINATION

How many combinations of
n objects taken k at a time?

COMBINATION

Permutations are obtained by

SELECTING A COMBINATION

of k objects from n objects and then

ARRANGING THEM

COMBINATION

$P(n,k) = C(n,k)$ then, arrange them

COMBINATION

$$P(n,k) = C(n,k) \cdot k!$$

(arrange them)

COMBINATION

$$C(n,k) = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)!k!}$$

COMBINATION

How many combinations of
n objects taken k at a time?

$$C(n,k) = \frac{n!}{(n-k)!k!}$$

MULTISET

> Ordering of objects does not matter
>
> Repetitions of objects are allowed

MULTISET

How many multisets of
n objects taken k at a time?

$$M(n,k) = C(n-1+k,\ k) = \frac{(n-1+k)!}{(n-1)!k!}$$

SEQUENCE

$$S(n,k) = n^k$$

PERMUTATION

$$P(n,k) = \frac{n!}{(n-k)!}$$

MULTISET

$$M(n,k) = \frac{(n-1+k)!}{(n-1)!k!}$$

COMBINATION

$$C(n,k) = \frac{n!}{(n-k)!\,k!}$$

How many ways can we answer this 4-pics 1 word problem?

Answer: P(12, 5) = 95040

How many ways can we answer this 4-pics 1 word problem?

A certain PVZ mini game has the following defense towers:

fanpop.com

serbagunamarine.com

How many ways are there to chose defense towers if the game has a limit of 30 towers?

(Given you max out the limit)

Ordering is not important

Repetitions are allowed

$$M(9,30)$$
$$= C(38,30)$$
$$= 48903492$$

McKinley High’s glee club is searching for

3 TENORS

5 BASS

3 ALTOS

AND 5 SOPRANOS

McKinley High's glee club is searching for

3 TENORS

5 BASS

3 ALTOS

AND 5 SOPRANOS

How many ways can Mr. Schuster screen the applicants if

10 TENORS

5 BASS

7 ALTOS

AND 12 SOPRANOS

show up?

Searching for

3 TENORS

5 BASS

3 ALTOS

5 SOPRANOS

Applicants

10 TENORS

5 BASS

7 ALTOS

12 SOPRANOS

Ordering is not important

Repetitions are not allowed

Searching for

3 TENORS

5 BASS

3 ALTOS

5 SOPRANOS

Applicants

10 TENORS

5 BASS

7 ALTOS

12 SOPRANOS

$$C(10,3) \cdot C(5,5) \cdot C(7,3) \cdot C(12,5)$$

How many ways can we answer this 4-pics 1 word problem?

SPECIAL CASE

Some letters are identical.

D D D

R R

S S

H

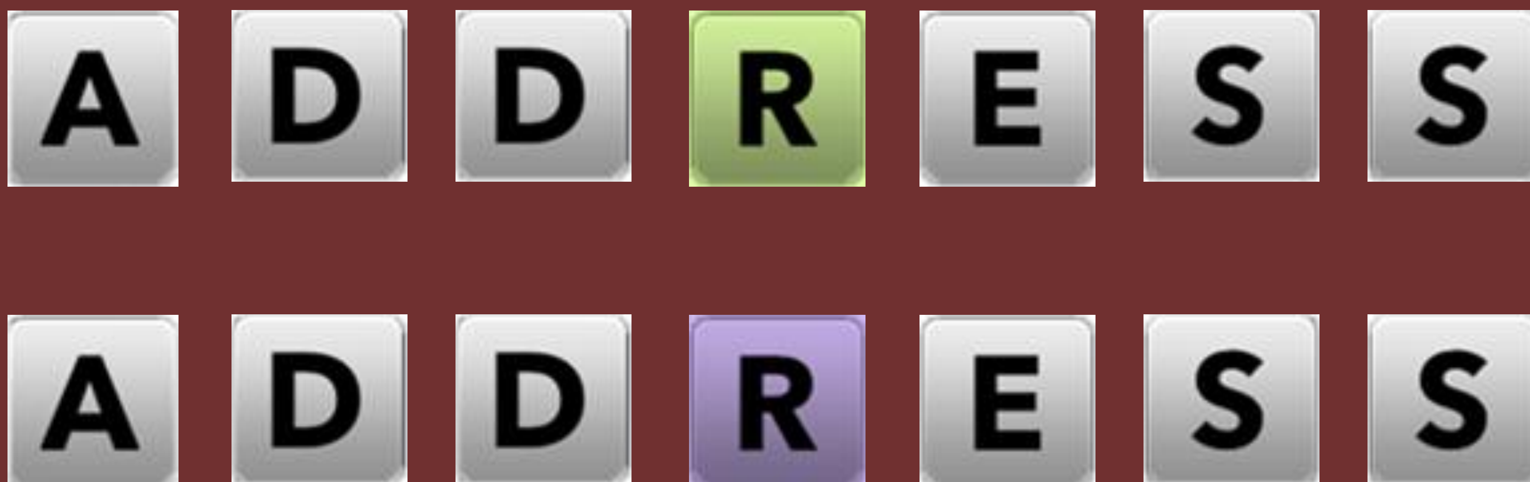
Q Q

A

E

SPECIAL CASE

Some permutations will be overcounted.



PERMUTATIONS
with
CONSTRAINED REPETITIONS

SPECIAL CASE

Among the n objects,

n_1 are of 1^{st} type
n_2 are of 2^{nd} type
:
n_t are of t^{th} type

SPECIAL CASE

$$P(n;n_1,n_2,\ldots,n_t) = \frac{P(n,k)}{n_1!n_2! \ldots nt!}$$

SPECIAL CASE

type 1 D D D

type 2 R R

type 3 S S

type 4 H

type 5 Q Q

type 6 A

type 7 E

SPECIAL CASE

P(12; 3, 2, 2, 1, 2 , 1, 1)

$$= \frac{\frac{12!}{5!}}{3!2!2!1!2!1!1!}$$

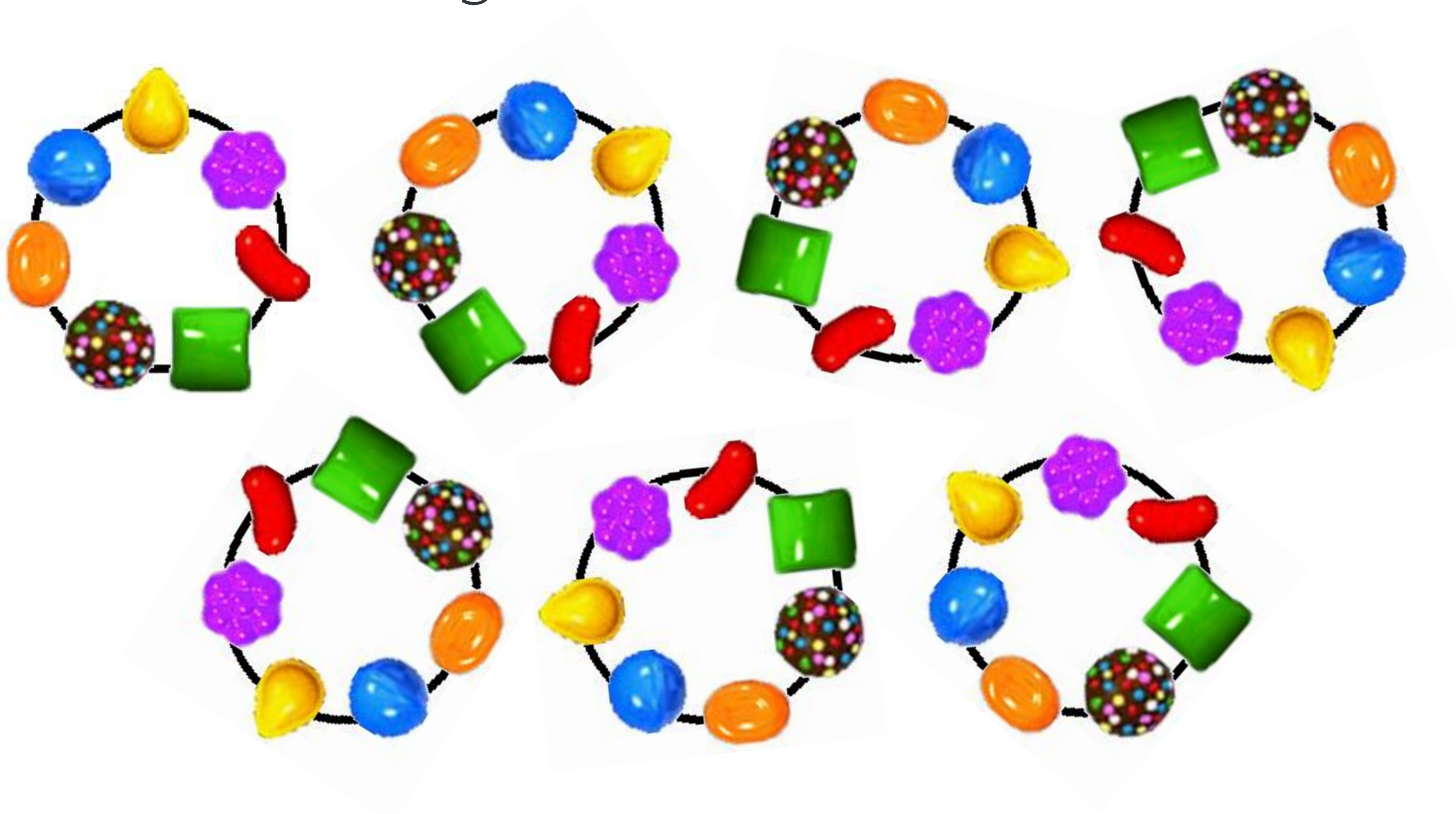
$$= 83160$$

How many ways are there of arranging

7 DIFFERENT CANDIES AROUND A BRACELET?

A product on ebay.uk by **shop2day83**

The following are
SIMILAR arrangements



Circular
PERMUTATIONS

SPECIAL CASE

The number of ways
n DISTINCT objects
can be arranged in a
CIRCLE is $(n-1)!$

SPECIAL CASE

How many ways are there of arranging

7 DIFFERENT CANDIES AROUND A BRACELET?

Answer: 6!

DISTRIBUTION

MODEL OF COUNTING

DISTRIBUTION

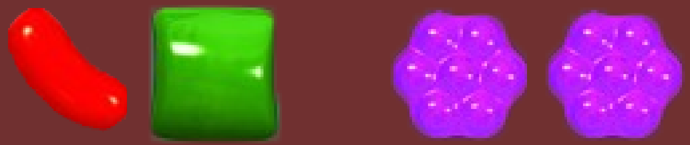
MODEL OF COUNTING

In how many ways can we **DISTRIBUTE** k objects into n distinct cells?

DISTRIBUTION

MODEL OF COUNTING

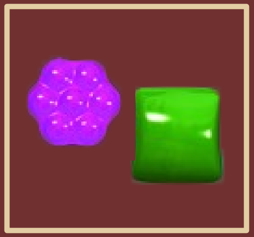
Are the objects distinct or identical?



DISTRIBUTION

MODEL OF COUNTING

Are the cells exclusive or non-exclusive?



SEQUENCE

Distinct objects
Non-exclusive cells

PERMUTATION

Distinct objects
Exclusive cells

MULTISET

Identical objects
Non-exclusive cells

COMBINATION

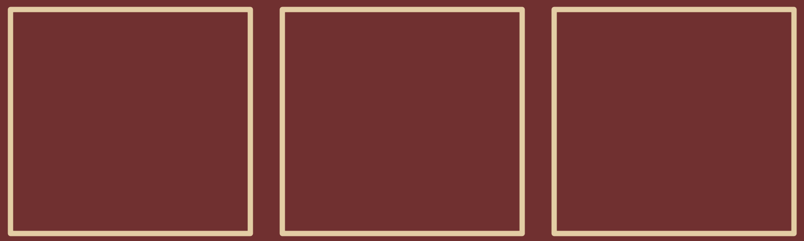
Identical objects
Exclusive cells

DISTRIBUTION

MODEL OF COUNTING

Distribute
two candies

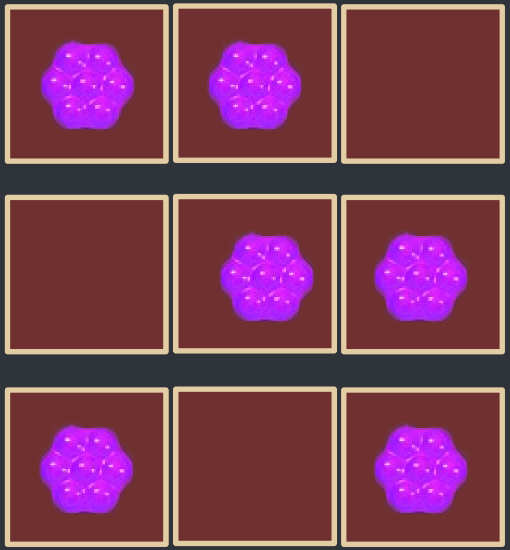
into three cells



Distribute **DISTINCT** objects to **EXCLUSIVE** cells

Distribute **DISTINCT** objects to **NON-EXCLUSIVE** cells

Distribute
IDENTICAL objects to
EXCLUSIVE cells



Distribute **IDENTICAL** objects to **NON-EXCLUSIVE** cells

SEQUENCE

Distinct objects
Non-exclusive cells

PERMUTATION

Distinct objects
Exclusive cells

MULTISET

Identical objects
Non-exclusive cells

COMBINATION

Identical objects
Exclusive cells

SEQUENCE

Distinct objects
Non-exclusive cells

PERMUTATION

Distinct objects
Exclusive cells

WHY?

MULTISET

Identical objects
Non-exclusive cells

COMBINATION

Identical objects
Exclusive cells

DISTRIBUTE

k objects into the n cells

SELECT A CELL NUMBER

for each object

SELECT k CELL NUMBERS
From the n cell numbers

SAMPLE
model of
counting

DISTINCT
objects

ORDER
in selecting cell numbers is
IMPORTANT

IDENTICAL

objects

ORDER

in selecting cell numbers is

NOT IMPORTANT

NON-EXCLUSIVE
cells

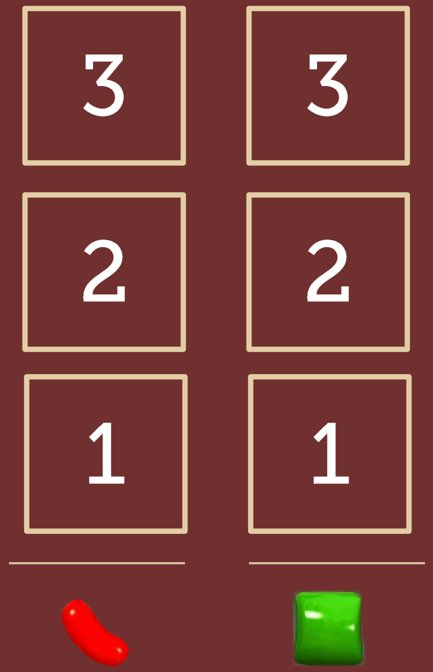
cell’s number
CAN BE REPEATED

EXCLUSIVE
cells

cell’s number
CANNOT BE
REPEATED

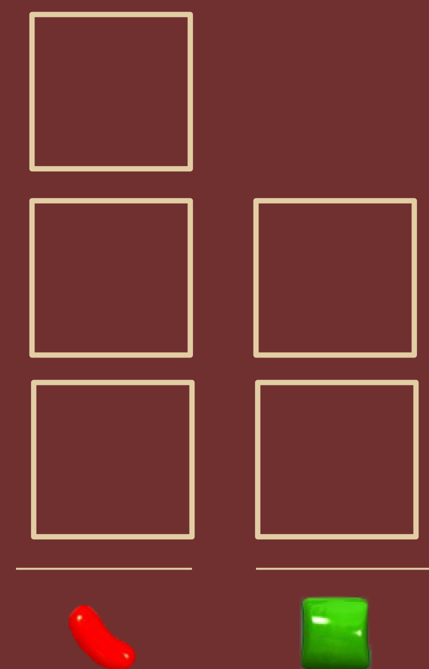
Distribute **DISTINCT** objects to **NON-EXCLUSIVE** cells

SEQUENCE



Distribute **DISTINCT** objects to **EXCLUSIVE** cells

PERMUTATION



Distribute **IDENTICAL** objects to **EXCLUSIVE** cells

COMBINATION

Order in selecting cell numbers is not important

Cell numbers may not repeat

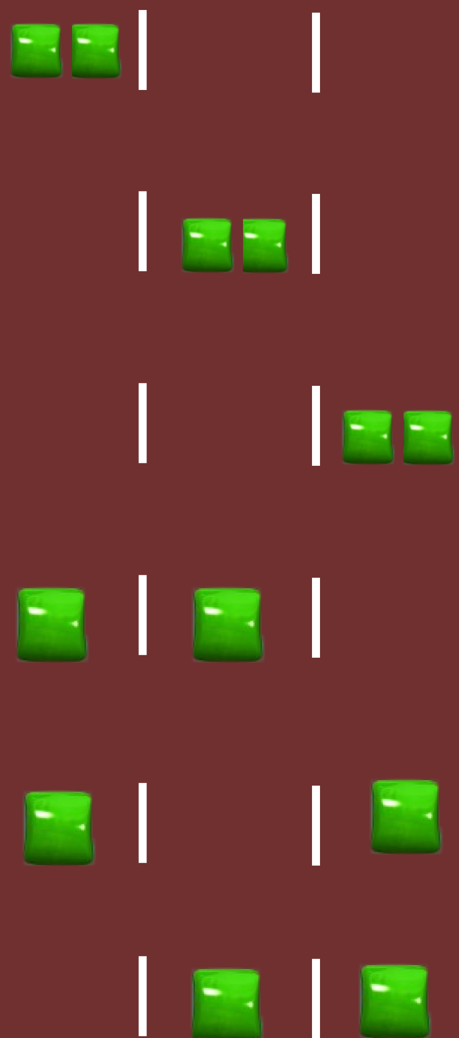
Distribute
IDENTICAL objects to
NON-EXCLUSIVE cells

MULTISET

Why?

$$M(n,k) = C(n-1+k,\ k) = \frac{(n-1+k)!}{(n-1)!k!}$$

Distribute two **IDENTICAL** candies into three **NON-EXCLUSIVE** cells



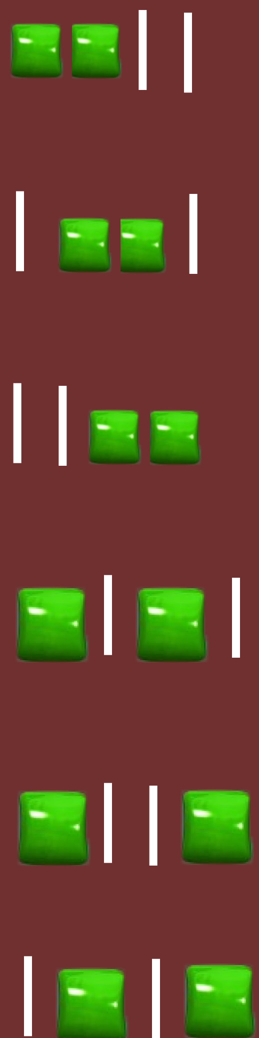
Similar to

ARRANGING

2 

and

2 |



Similar to

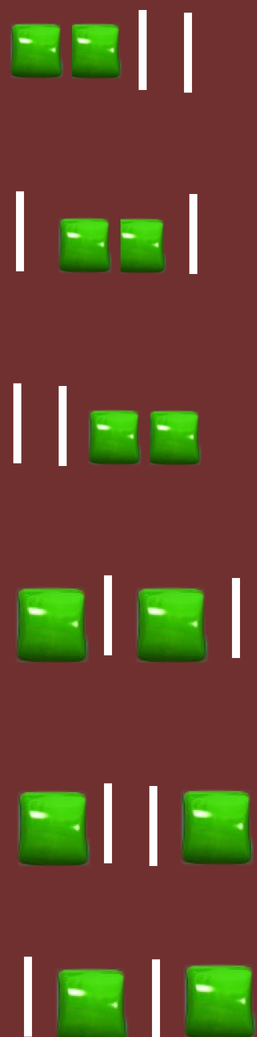
ARRANGING

2 

and

2 |

PERMUTATIONS with CONSTRAINED REPETITIONS



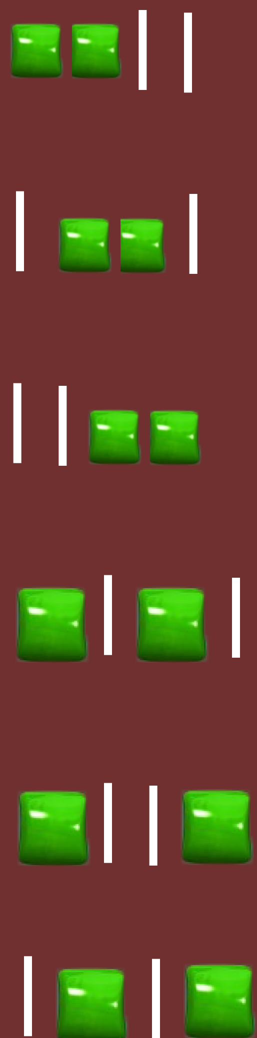
n = 4 objects

2 + 2 |

k = 4

$P(4; 2, 2) = 4!/2!2!$

PERMUTATIONS with CONSTRAINED REPETITIONS



n = 4 objects

2  + 2 |

k = 4

P(4; 2, 2) = 4!/2!2!

PERMUTATIONS with CONSTRAINED REPETITIONS

$P(4; 2, 2) = 4!/2!2!$

Using formula for combination,

C(#of objects + # of boxes – 1, # of boxes)

$$C(n-1+k, k)$$

$$\frac{(n-1+k)!}{(n-1)!k!}$$

In how many ways can you distribute twelve bananas to five minions if each minion can hold as many bananas as they like?

IDENTICAL
bananas

NON-EXCLUSIVE
minions

$$M(5_{\text{ minions}}, 12_{\text{ bananas}}) = C(16,12)$$

If there are only three bananas and each minion can only hold at most one banana, how many ways can you distribute them to five minions?

IDENTICAL
bananas

EXCLUSIVE
minions

C(5 minions,3 bananas)

In how many ways can you lock up twelve minions in five labs if each labs can hold as many minions as you like?

In how many ways can you lock up twelve minions in five labs if each labs can hold as many minions as you like?

DISTINCT
minions

NON-EXCLUSIVE
labs

In how many ways can you lock up twelve minions in five labs if each labs can hold as many minions as you like?

S(5 labs,12 minions)

In how many ways can you assign an ID number to twenty-five monsters if each ID number consists of an uppercase letter followed by a two digit number?

DISTINCT
monsters

EXCLUSIVE
ID numbers

In how many ways can you assign an ID number to twenty-five monsters if each ID number consists of an uppercase letter followed by a two digit number?

DISTINCT
monsters

EXCLUSIVE
ID numbers

P(ID numbers, monsters)

P(2600, 25)

Professor Knight has prepared 10 questions on

SCAR 120 – HISTORY OF SCARING

written exam.

Faculty Profile

Professor Knight

Title/Department:

Professor

MU School of Scaring

Tenured Professor of Fright, School of Scaring

Just released the 15th edition of his book "A Study of Scaring," complete with new data and techniques directly from MU classroom experiences

How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points?

IDENTICAL
points

NON-EXCLUSIVE
questions

Faculty Profile

Professor Knight

Title/Department:
Professor
MU School of Scaring

Tenured Professor of Fright, School of Scaring
Just released the 15th edition of his book "A Study of Scaring," complete with new data and techniques directly from MU classroom experiences

$M(1,5)^{10}$

Distribute 5 points to all 10 questions

Faculty Profile

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MU School of Scaring

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$$M(1,5)^{10} \cdot M(10,50)$$

Distribute the remaining points

Faculty Profile

Professor Knight

Title/Department:
Professor
MU School of Scaring

Tenured Professor of Fright, School of Scaring
Just released the 15th edition of his book "A Study of Scaring," complete with new data and techniques directly from MU classroom experiences

How many nonnegative integral solutions for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2\ ,\ x_2 = 5\ ?$$

How many nonnegative integral solutions for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2\ ,\ x_2 = 5\ ?$$

IDENTICAL 1s

NON-EXCLUSIVE x_is

How many nonnegative integral solutions for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2\ ,\ x_2 = 5\ ?$$

M(1,2)·

Distribute 2 1s to x_1

How many nonnegative integral solutions for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2\ ,\ x_2 = 5\ ?$$

$M(1,2) \cdot M(1,5)$

Distribute 5 1s to x_2

How many nonnegative integral solutions for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2\,,\ x_2 = 5\ ?$$

$M(1,2)\cdot M(1,5)\cdot M(4,9)$

Distribute remaining 9 1s to any x_is

QUIZ

In how many ways can 36 different cadets be distributed to the 13 Scouting Legion groups if each can receive as many cadets as possible?

PRINCIPLES OF COUNTING

INDIRECT METHOD OF COUNTING

MUTUAL INCLUSION-EXCLUSION PRINCIPLE

SAMPLE MODEL OF COUNTING

DISTRIBUTION MODEL OF COUNTING

DERANGEMENTS

PIGEONHOLE PRINCIPLE

RECURRENCE RELATIONS