

## CMSC 57 Discrete Mathematical Structures in Computer Science II

### Exercise 2. Sample Models of Counting

On your papers, write your name, student number, an emoticon of how you feel at the moment (you may add in some words), section, and the date today. Answer each item as neatly as possible. Minimize erasures. Review your solutions before submitting your papers. Do not discuss your work with any other group.

1. Suppose that a district court summons 10 men and 15 women. How many ways are there to form a jury of eight if it must have the same number of men and women?
2. A professor writes 40 discrete mathematics True/False questions of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?
3. Thirteen model wannabes show up for a modeling reality show. How many ways are there to cast 10 contestants for the said reality show?
4. Referring to item#4, 3 are minors. How many ways are there to choose 10 contestants if at least one of those who will be casted should be a minor?
5. A candy store has the following flavors of jelly beans: cinnamon, ginger, cola, vanilla, tutti frutti, and licorice. How many ways are there to choose a dozen jelly beans?
6. A candy store has the following flavors of jelly beans: cinnamon, ginger, cola, vanilla, tutti frutti, and licorice. How many ways are there to choose a dozen jelly beans with at least two of each kind?
7. A candy store has the following flavors of jelly beans: cinnamon, ginger, cola, vanilla, tutti frutti, and licorice. How many ways are there to choose a dozen jelly beans with at least five cinnamon and at least three licorice jelly beans?
8. How many strings of 10 ternary digits (0,1, or 2) are there that contain exactly two 0's, three 1's and five 2's?
9. In how many ways can five distinct Republicans and five distinct Democrats can be seated at a circular table if no two Republicans sit together?
10. In how many ways can five distinct Republicans and eight distinct Democrats can be seated at a circular table if no two Republicans sit together?

## CMSC 57 Discrete Mathematical Structures in Computer Science II

### Exercise 2. Sample Models of Counting (Solutions)

1. If we are to form a jury of eight, with the same number of men and women, then we may only select 4 from the men and 4 from the women. Thus, there are  $C(10,4)*C(15,4)$  ways to do that.
2. In this problem we need to count the possible answer keys. To do this, we need to know the number of ways we can arrange the 40 answers and **not** the questions. Since we need to arrange the objects which can be classified into types (True or False) and those of the same type are identical, we have  $P(40;17,23)$  possible answer keys.
3. Since the order of casting is not important, we only need to select which ones will be casted. There are  $C(13,10)$  ways to do this.
4. We need to check all the possible cases where there is at least one minor. First, the case with 1 minor and 9 non-minors in the cast. Second, the case with 2 minors and 8 non-minors. Third, the case with 3 minors and 7 non-minors in the cast. There are  $C(3,1)*C(10,9)+C(3,2)*C(10,8)+C(3,3)*C(10,7)$  possible ways choose 10 contestants.  
Another way to solve this is with the indirect method of counting. There are  $C(13,10)$  ways to select 10 contestants and  $C(10,10)$  ways to select 10 contestants without minors. So there are  $C(13,10)-C(10,10)$  ways to select 10 contestants with at least 1 minor.
5. In this problem, we are simply selecting 6 items (jelly bean varieties) with repetition. There are  $M(6,12)$  or  $C(17,12)$  ways to do this.
6. There is only  $M(1,2)$  or  $C(2,2)$  or 1 way to select a variety of jelly beans. We do that for every variety, and using the product rule, we come up with only 1 way to do this.
7. Similar to the previous problem, we first make sure there are 5 cinnamon and 3 licorice jelly beans. There is only 1 way to do this. Next we choose 4 more from the 6 available varieties. There are  $M(6,4)$  ways to do this.
8. This is like selecting in which of the three boxes 0, 1 and 2 will we place our distinct string character positions. Again, there are set numbers of each position for each box. So there are  $P(10;2,3,5)$  or  $C(10,2)*C(8,3)*C(5,5)$  possible strings.
9. First seat the Democrats. There are  $4!$  ways to do this. Then seat the Republicans, placing each between two Democrats. There are  $5!$  ways to do this. By the product rule, there are  $4!5!$  ways to seat them.
10. Again, seat the Democrats first. There are  $7!$  ways to do this. Then seat the 5 Republicans, placing each between two Democrats, selecting 5 from the 8 available places. There are  $P(8,5)$  ways to do this. By product rule, there are  $7!P(8,5)$  ways to do this.