COMBINATORICS

ALGEBRAIC STRUCTURES

DISCRETE PROBABILITY

GRAPH THEORY

# DISCRETE PROBABILITY

# APPLICATIONS

*Analysis of Algorithms*
*(Average Case Analysis)*

*Probabilistic Algorithms*

# RANDOM EXPERIMENT

*Any activity that (randomly) yields a result or an outcome.*

# SAMPLE SPACE

*Set of all possible outcomes of a*

**RANDOM EXPERIMENT.**

**SAMPLE POINT**

*An element of the*

**SAMPLE SPACE.**

# RANDOM EXPERIMENT

*Rolling a die*

# *EXAMPLE*

# RANDOM EXPERIMENT

*Taking a course until the course is passed*

# EXAMPLE

# RANDOM EXPERIMENT

*Answering three*
*TRUE or FALSE questions*

# *EXAMPLE*

# EVENT

*A collection of*
**SAMPLE POINTS**
*in the*
**SAMPLE SPACE**
*(subset)*

# EVENT SPACE

*Set of all possible*
**EVENTS**

*(Power set of the* **SAMPLE SPACE***)*

# RANDOM EXPERIMENT

*Tossing a 1 peso and a 5 peso coin*

# EXAMPLE

# EVENTS

*The event that both coins turn up tails*

# EXAMPLE

# EVENTS

*The event that at least one turns up heads*

# EXAMPLE

# RANDOM EXPERIMENT

*Drawing four chocolates from a box containing milk and dark chocolates with replacement*

# EXAMPLE

# EVENTS

*The event that exactly one dark chocolate is drawn*

# EXAMPLE

# EVENTS

*The event that exactly two milk chocolates are drawn*

# EXAMPLE

*The*

**PROBABILITY**

*that event A will occur*

$$P(A) = |A| / |S|$$

# GIVEN

*The sample space is finite*

*Each sample point is equally likely*

$A \subseteq S$

# RANDOM EXPERIMENT

*Drawing four chocolates from a box containing milk and dark chocolates with replacement*

# EXAMPLE

# PROBABILITY

*What is the probability that exactly two milk chocolates are drawn?*

# EXAMPLE

# PROBABILITY

*What is the probability that no dark chocolates are drawn?*

# EXAMPLE

# PROBABILITY

*In a 6/42 lottery , what is the probability of picking the winning combination?*

# EXAMPLE

# PROBABILITY

*In poker, what is the probability that a hand of five cards contains four cards of one kind?*

# EXAMPLE

# PROBABILITY

*In poker, what is the probability that a hand of five cards contains a full house?*

# EXAMPLE

# PROBABILITY

*Full house:*

*Three of one kind and two of another kind*

# EXAMPLE

# AXIOMS

## PROBABILITY

# AXIOMS

## PROBABILITY

$P(A) \geq 0$

$P(S) = 1$

# AXIOMS

## PROBABILITY

If $A_1$, $A_2$, $A_3$, ... are
**MUTUALLY EXCLUSIVE EVENTS**, then
$P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

# THEOREMS

## PROBABILITY

# THEOREMS

## PROBABILITY

$P(\varnothing) = 0$

$P(A') = 1-P(A)$

$P(A-B) = P(A) - P(AB)$

# THEOREMS

## PROBABILITY

$P(A \cup B) = P(A) + P(B) - P(AB)$

if $A \subseteq B$, then $P(A) \leq P(B)$

# PROBABILITY

*A sequence of 10 bits is randomly generated.*

# EXAMPLE

# PROBABILITY

*What is the probability that at least one of the bits is zero?*

# EXAMPLE

# PROBABILITY

*Consider a standard deck of 52 cards.*

# EXAMPLE

# PROBABILITY

*What is the probability of obtaining a face card or a diamond?*

# EXAMPLE

# MUTUALLY EXCLUSIVE EVENTS

*If the occurrence of one hinders/ prevents the occurrence of the other*

# MUTUALLY EXCLUSIVE EVENTS

if and only if $P(A \cup B) = P(A) + P(B)$

$P(AB) = 0$

# MUTUALLY EXCLUSIVE EVENTS

*Consider rolling a die four times.*

## *EXAMPLE*

# MUTUALLY EXCLUSIVE EVENTS

*What is the probability that the first roll results in a number less than 3 or greater than 3?*

## EXAMPLE

# AN EXPERIMENT WITH OUTCOMES THAT ARE NOT EQUALLY LIKELY

# A BIASED DIE

*Consider rolling a die such that any even number is twice as likely to show up as any odd number.*

EXAMPLE

# A BIASED DIE

*What is the probability that a prime is obtained?*

# EXAMPLE

# CONDITIONAL PROBABILITY

# CONDITIONAL PROBABILITY

*Suppose that a coin is flipped three times.*

## EXAMPLE

# CONDITIONAL PROBABILITY

*Moreover, it is known that the first flip came up tails.*

# EXAMPLE

# CONDITIONAL PROBABILITY

*Given this information, what is the probability that an odd number of tails appears?*

# EXAMPLE

# CONDITIONAL PROBABILITY

*Let A = the first flip came up tails.*

*B = odd number of tails appears.*

# EXAMPLE

# CONDITIONAL PROBABILITY

*The probability of an event (B) occurring when it is known that some other event (A) has occurred.*

# CONDITIONAL PROBABILITY

$$P(B|A) = P(AB) / P(A)$$

$P(A) > 0$

# CONDITIONAL PROBABILITY

$$P(AB) = P(A) \cdot P(B|A)$$
$$P(AB) = P(B) \cdot P(A|B)$$

# CONDITIONAL PROBABILITY

*Consider a family with two children.*

## EXAMPLE

# CONDITIONAL PROBABILITY

*What is the probability that they have two boys, given they have at least one boy?*

# EXAMPLE

# CONDITIONAL PROBABILITY

## EXAMPLE

*Let A = they have least one boy.*

*B = they have two boys .*

# CONDITIONAL PROBABILITY

*Consider answering an exam with 10 all-or-nothing questions.*

## EXAMPLE

# CONDITIONAL PROBABILITY

*What is the probability of having exactly 6 correct answers given at least one of the answers is correct?*

# EXAMPLE

# CONDITIONAL PROBABILITY

*Let A = at least one is a correct answer.*

*B = exactly six are correct answers.*

# EXAMPLE

# CONDITIONAL PROBABILITY

*Suppose that the probability of a teacher having a fever is 0.03.*

# EXAMPLE

# CONDITIONAL PROBABILITY

*Moreover, the probability that a teacher will conduct no class given he/she has a fever is 0.5.*

# EXAMPLE

# CONDITIONAL PROBABILITY

*Also, the probability that a teacher will conduct no class is 0.02.*

# EXAMPLE

# CONDITIONAL PROBABILITY

*Find the probability that a teacher has a fever given that he/she conducted no class.*

# EXAMPLE

# PROPERTIES

## CONDITIONAL PROBABILITY

# PROPERTIES

## CONDITIONAL PROBABILITY

$P(\varnothing|B) = 0$

$P(A'|B) = 1 - P(A|B)$

# PROPERTIES

## CONDITIONAL PROBABILITY

$$P(A \cup B \mid C) = P(A|C) + P(B|C) - P(AB|C)$$

$$\text{if } A \subseteq B, \text{ then } P(A|C) \leq P(B|C)$$

# THEOREMS

## CONDITIONAL PROBABILITY

# THEOREMS

## CONDITIONAL PROBABILITY

$$P(A_1A_2A_3\ldots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1A_2) \ldots P(A_n|A_1A_2\ldots A_{n-1})$$

# CONDITIONAL PROBABILITY

*Three magic cards are picked without replacement from a deck with 21  , 22  and 19  cards.*

# EXAMPLE

# CONDITIONAL PROBABILITY

*What is the probability of obtaining a*  *card on the first two draws followed by a*  *card?*

# EXAMPLE

# INDEPENDENT EVENTS

*If the occurrence of event A does not affect the probability of the occurrence of event B and vice versa, then A and B are independent.*

# PROVING INDEPENDENCE

## INDEPENDENT EVENTS

# PROVING INDEPENDENCE

## INDEPENDENT EVENTS

*A and B are independent*

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$

# PROVING INDEPENDENCE

## INDEPENDENT EVENTS

*A and B are independent* $\Leftrightarrow$ $P(AB) = P(A) \cdot P(B)$

# PROVING INDEPENDENCE

## INDEPENDENT EVENTS

*A and B are independent* $\Leftrightarrow$ $P(A|B) = P(A|B^C)$

# INDEPENDENT EVENTS

*Suppose that a coin is flipped three times.*

## EXAMPLE

# INDEPENDENT EVENTS

*Let A = the first flip came up tails.*

*B = odd number of tails appears.*

# EXAMPLE

# INDEPENDENT EVENTS

*Does knowing that the first flip comes up tails (A) alter the probability that tails comes up and odd number of times (B)?*

# EXAMPLE

# INDEPENDENT EVENTS

*Are the events A and B independent?*

# EXAMPLE

# INDEPENDENT EVENTS

*Consider drawing a card from a deck of 52 cards.*

# EXAMPLE

# INDEPENDENT EVENTS

Let $F$ = a face card is drawn.

$C$ = a club card is drawn.

$B$ = A black card is drawn.

# EXAMPLE

# INDEPENDENT EVENTS

*Which pairs of events are independent?*

# EXAMPLE