

MODELS

OF COUNTING

SAMPLE

MODEL OF
COUNTING

DISTRIBUTION

MODEL OF
COUNTING

SAMPLE

MODEL OF
COUNTING

SAMPLE

MODEL OF
COUNTING

In how many ways can we
TAKE A SAMPLE
of k objects from a group
of n distinct objects?

SAMPLE

MODEL OF
COUNTING

Is the order of the objects
chosen **important**?

Are **repetitions** of objects
allowed?

SEQUENCE

Order matters
Repetitions allowed

PERMUTATION

Order matters
Repetitions not allowed

MULTISET

Order does not matter
Repetitions allowed

COMBINATION

Order does not matter
Repetitions not allowed

SAMPLE

MODEL OF
COUNTING

Take a sample of 2 letters
from the set

$\{a, b, c\}$.

If the samples are
SEQUENCES

aa ab ac
ba bb bc
ca cb cc

If the samples are
PERMUTATIONS

ab ac
ba bc
ca cb

If the samples are
MULTISSETS

aa ab ac
bb bc
cc

If the samples are
COMBINATIONS

ab ac
bc

SEQUENCE

Ordering of objects matters

Repetitions of objects are allowed

SEQUENCE

How many sequences of
 n objects taken k at a time?

SEQUENCE

Select 1 st object	n possible objects
Select 2 nd object	n possible objects
:	:
Select k^{th} object	n possible objects

Using product rule,
 $n \cdot n \cdot \dots \cdot n = n^k$

SEQUENCE

How many sequences of
n objects taken k at a time?

$$S(n,k) = n^k$$

PERMUTATION

Ordering of objects matters

Repetitions of objects are not allowed

PERMUTATION

How many permutations of
 n objects taken k at a time?

PERMUTATION

Select 1st object

Select 2nd object

Select 3rd object

:

Select kth object

n possible objects

n-1 possible objects

n-2 possible objects

:

n - (k-1) possible
objects

PERMUTATION

Using product rule,

$$\begin{aligned} & n \cdot (n-1) \cdot (n-2) \dots \cdot n-(k-1) \\ &= \frac{n!}{(n-k)!} \end{aligned}$$

PERMUTATION

How many permutations of
n objects taken k at a time?

$$P(n,k) = \frac{n!}{(n-k)!}$$

COMBINATION

Ordering of objects does not matter

Repetitions of objects are not allowed

COMBINATION

How many combinations of
 n objects taken k at a time?

COMBINATION

Permutations are obtained by

SELECTING A COMBINATION

of k objects from n objects and then

ARRANGING THEM

COMBINATION

$P(n,k) = C(n,k)$ then, arrange them

COMBINATION

$$P(n,k) = C(n,k) \cdot k!$$

(arrange them)

COMBINATION

$$C(n,k) = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)!k!}$$

COMBINATION

How many combinations of
n objects taken k at a time?

$$C(n,k) = \frac{n!}{(n-k)!k!}$$

MULTISET

Ordering of objects does not matter

Repetitions of objects are allowed

MULTISET

How many multisets of
n objects taken k at a time?

$$M(n,k) = C(n-1+k, k) = \frac{(n-1+k)!}{(n-1)!k!}$$

SEQUENCE

$$S(n,k) = n^k$$

PERMUTATION

$$P(n,k) = \frac{n!}{(n-k)!}$$

MULTISET

$$M(n,k) = \frac{(n-1+k)!}{(n-1)!k!}$$

COMBINATION

$$C(n,k) = \frac{n!}{(n-k)! \cdot k!}$$

How many ways
can we answer this
4-pics 1 word
problem?





Answer: $P(12, 5) = 95040$

How many ways
can we answer this
4-pics 1 word
problem?



A certain PVZ mini
game has the
following defense
towers:



fanpop.com



serbagunamarine.com



fanpop.com

How many ways are
there to chose
defense towers if the
game has a limit of 30
towers?

(Given you max out the limit)



fanpop.com

Ordering is not
important

Repetitions are
allowed



fanpop.com

$$\begin{aligned} M(9,30) \\ &= C(38, 30) \\ &= 48903492 \end{aligned}$$

McKinley High's glee
club is searching for

3 TENORS

5 BASS

3 ALTOS

AND 5 SOPRANOS

McKinley High's glee club is searching for

3 TENORS

5 BASS

3 ALTOS

AND 5 SOPRANOS

How many ways can
Mr. Schuster screen
the applicants if

10 TENORS

5 BASS

7 ALTOS

AND 12 SOPRANOS

show up?

Searching for

3 TENORS

5 BASS

3 ALTOS

5 SOPRANOS

Applicants

10 TENORS

5 BASS

7 ALTOS

12 SOPRANOS

Ordering is not
important

Repetitions are not
allowed

Searching for

3 TENORS

5 BASS

3 ALTOS

5 SOPRANOS

Applicants

10 TENORS

5 BASS

7 ALTOS

12 SOPRANOS

$$C(10,3) \cdot C(5,5) \cdot C(7,3) \\ \cdot C(12,5)$$

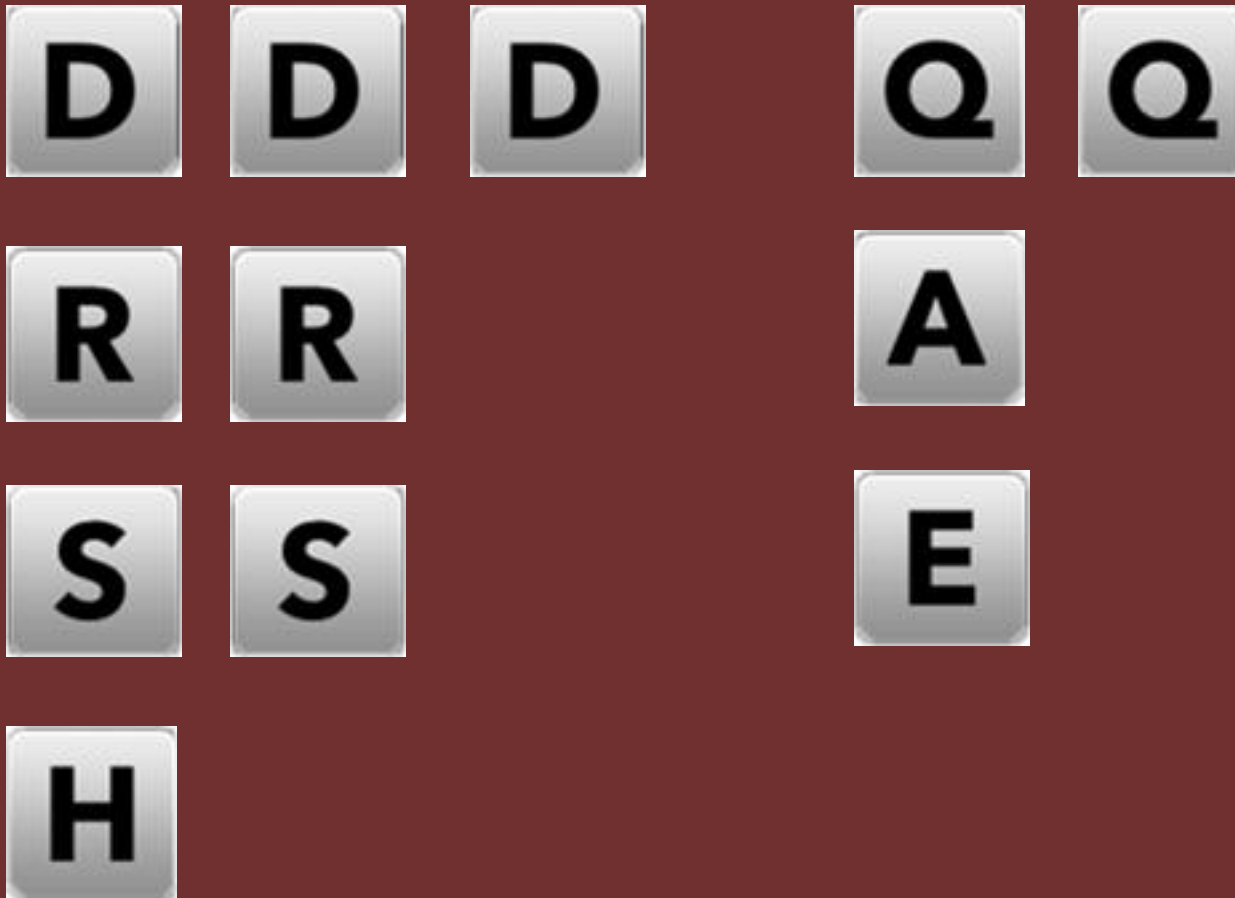
How many ways
can we answer this
4-pics 1 word
problem?



D	R	S	Q	D	D	
S	A	Q	H	R	E	

SPECIAL CASE

Some letters are identical.



SPECIAL CASE

Some permutations will be overcounted.

A D D R E S S

A D D R E S S

SPECIAL CASE

PERMUTATIONS

with

CONSTRAINED REPETITIONS

SPECIAL CASE

Among the n objects,

n_1 are of 1st type

n_2 are of 2nd type

:

n_t are of t^{th} type

SPECIAL CASE

$$P(n; n_1, n_2, \dots, n_t) = \frac{P(n, k)}{n_1! n_2! \dots n_t!}$$

SPECIAL CASE

type 1



type 5



type 2



type 6



type 3



type 7



type 4



SPECIAL CASE

$$P(12; 3, 2, 2, 1, 2, 1, 1)$$

$$= \frac{\frac{12!}{5!}}{3!2!2!1!2!1!1!}$$

$$= 83160$$

How many
ways are there
of arranging
**7 DIFFERENT
CANDIES
AROUND A
BRACELET?**



A product on [ebay.uk](https://www.ebay.co.uk) by [shop2day83](#)

The following are
SIMILAR arrangements



Circular PERMUTATIONS

SPECIAL CASE

The number of ways
n DISTINCT objects
can be arranged in a
CIRCLE is $(n-1)!$

SPECIAL CASE

How many
ways are there
of arranging
**7 DIFFERENT
CANDIES
AROUND A
BRACELET?**



Answer: 6!

DISTRIBUTION

MODEL OF
COUNTING

DISTRIBUTION

MODEL OF
COUNTING

In how many ways
can we
DISTRIBUTE
 k objects into
 n distinct cells?

DISTRIBUTION

MODEL OF
COUNTING

Are the objects
distinct or **identical**?



DISTRIBUTION

MODEL OF
COUNTING

Are the cells
exclusive or
non-exclusive?



SEQUENCE

Distinct objects
Non-exclusive cells

PERMUTATION

Distinct objects
Exclusive cells

MULTISET

Identical objects
Non-exclusive cells

COMBINATION

Identical objects
Exclusive cells

DISTRIBUTION

MODEL OF
COUNTING

Distribute
two candies



into three cells



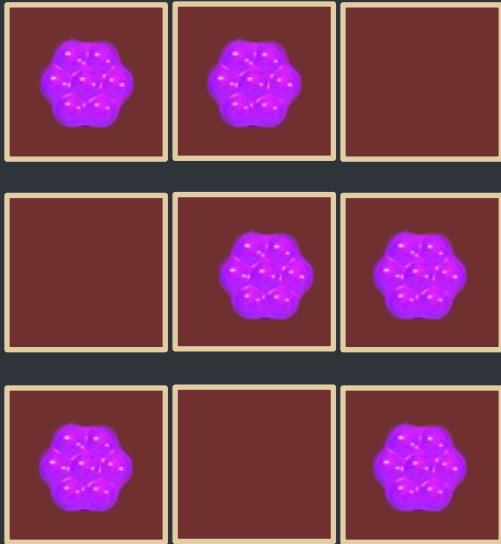
Distribute
DISTINCT objects to
EXCLUSIVE cells



Distribute
DISTINCT objects to
NON-EXCLUSIVE cells



Distribute
IDENTICAL objects to
EXCLUSIVE cells



Distribute
IDENTICAL objects to
NON-EXCLUSIVE cells



SEQUENCE

Distinct objects
Non-exclusive cells

PERMUTATION

Distinct objects
Exclusive cells

MULTISET

Identical objects
Non-exclusive cells

COMBINATION

Identical objects
Exclusive cells

SEQUENCE

Distinct objects
Non-exclusive cells

PERMUTATION

Distinct objects
Exclusive cells

WHY?

MULTISET

Identical objects
Non-exclusive cells

COMBINATION

Identical objects
Exclusive cells

DISTRIBUTE

k objects into
the n cells

**SELECT A
CELL NUMBER**

for each object

SELECT k CELL NUMBERS

From the n cell numbers

SAMPLE

model of
counting

DISTINCT
objects

ORDER
in selecting cell
numbers is
IMPORTANT

IDENTICAL
objects

ORDER
in selecting cell
numbers is
NOT IMPORTANT

NON-EXCLUSIVE
cells

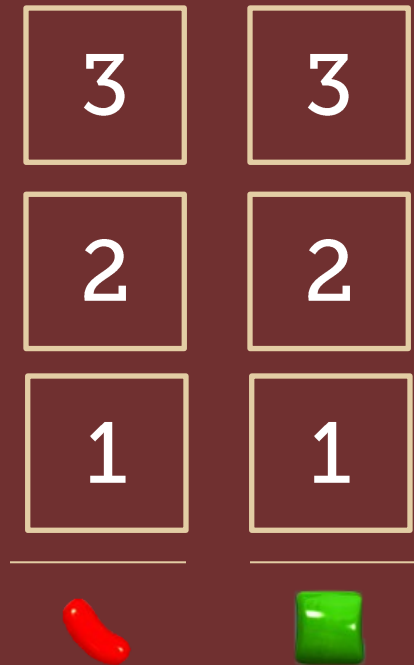
cell's number
CAN BE REPEATED

EXCLUSIVE
cells

cell's number
CANNOT BE
REPEATED

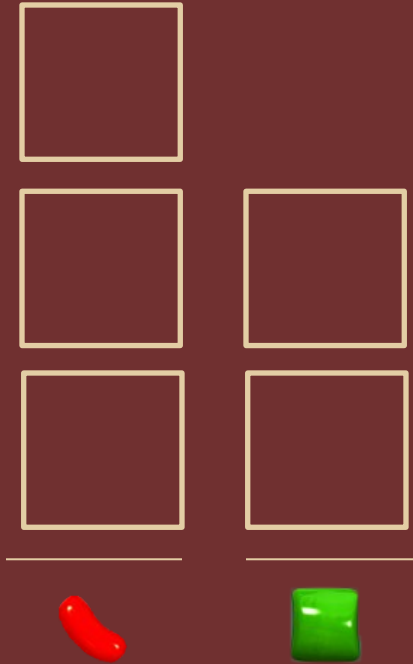
Distribute
DISTINCT objects to
NON-EXCLUSIVE cells

SEQUENCE



Distribute
DISTINCT objects to
EXCLUSIVE cells

PERMUTATION



Distribute
IDENTICAL objects to
EXCLUSIVE cells

COMBINATION

Order in selecting cell numbers
is not important

Cell numbers may not repeat

Distribute
IDENTICAL objects to
NON-EXCLUSIVE cells

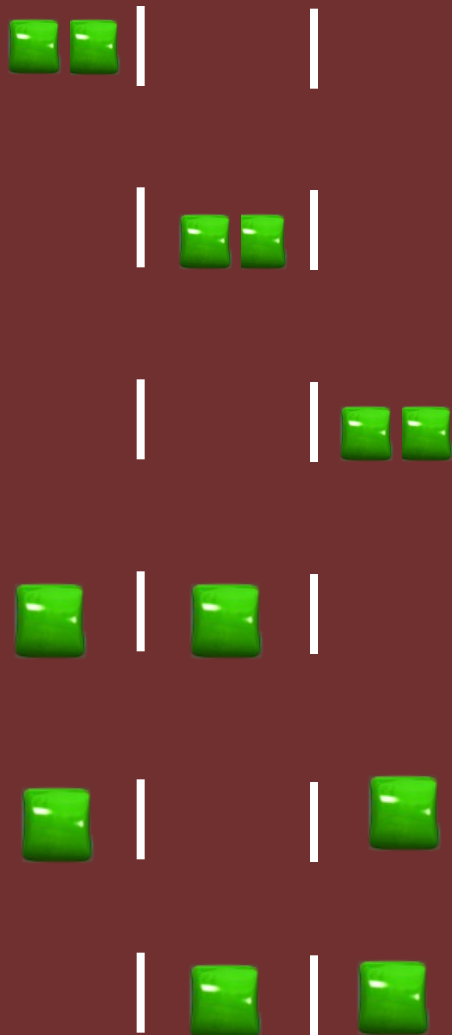
MULTISET

Why?


$$M(n,k) = C(n-1+k, k) = \frac{(n-1+k)!}{(n-1)!k!}$$



Distribute
two **IDENTICAL** candies
into three **NON-EXCLUSIVE**
cells



Similar to
ARRANGING

2 
and
2 |



Similar to
ARRANGING

2 

and

2 |

PERMUTATIONS with **CONSTRAINED REPETITIONS**



$n = 4$ objects

$2 \text{ } \blacksquare + 2 \text{ } |$

$k = 4$

$P(4; 2, 2) = 4!/2!2!$

PERMUTATIONS *with* CONSTRAINED REPETITIONS



$n = 4$ objects

$2 \text{ [green square]} + 2 |$

$k = 4$

$P(4; 2, 2) = 4!/2!2!$

PERMUTATIONS *with* CONSTRAINED REPETITIONS

$$P(4; 2, 2) = 4!/2!2!$$

Using formula for combination,

$$C(\text{\#of objects} + \text{\# of boxes} - 1, \text{\# of boxes})$$

$$C(n-1+k, k)$$

$$\frac{(n-1+k)!}{(n-1)!k!}$$

In how many ways can you distribute twelve bananas to five minions if each minion can hold as many bananas as they like?



IDENTICAL
bananas

NON-EXCLUSIVE
minions



$$M(5 \text{ minions}, 12 \text{ bananas}) = C(16, 12)$$



If there are only three bananas and each minion can only hold at most one banana, how many ways can you distribute them to five minions?



IDENTICAL
bananas

EXCLUSIVE
minions



$C(5 \text{ minions}, 3 \text{ bananas})$



In how many ways
can you lock up
twelve minions in
five labs if each
labs can hold as
many minions as
you like?



In how many ways
can you lock up
twelve minions in
five labs if each
labs can hold as
many minions as
you like?

DISTINCT
minions

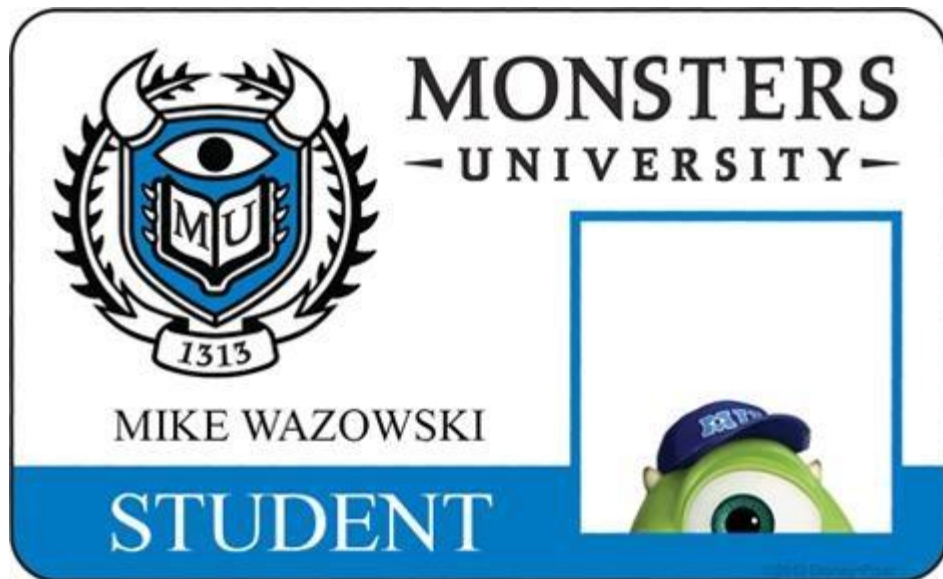
NON-EXCLUSIVE
labs



In how many ways
can you lock up
twelve minions in
five labs if each
labs can hold as
many minions as
you like?

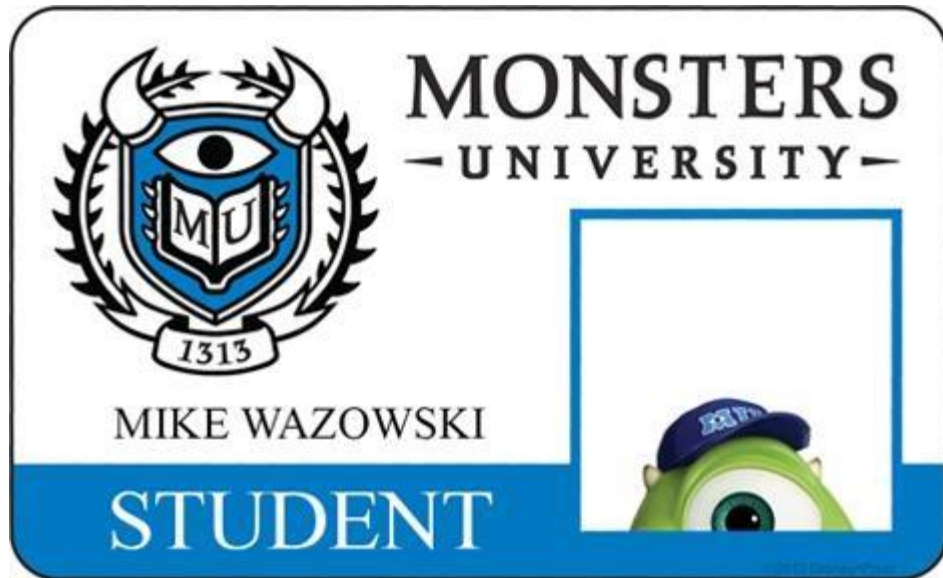
$S(5 \text{ labs}, 12 \text{ minions})$





In how many ways can you assign an ID number to twenty-five monsters if each ID number consists of an uppercase letter followed by a two digit number?

DISTINCT monsters



EXCLUSIVE ID numbers

In how many ways can you assign an ID number to twenty-five monsters if each ID number consists of an uppercase letter followed by a two digit number?

DISTINCT
monsters

$P(\text{ID numbers, monsters})$

$P(2600, 25)$



EXCLUSIVE
ID numbers

Professor Knight has prepared 10 questions on
SCAR 120 – HISTORY OF SCARING
written exam.

Faculty Profile



Professor Knight

Title/Department:

Professor
MU School of Scaring

Tenured Professor of Fright, School of Scaring
Just released the 15th edition of his book "A Study of Scaring," complete with new data and techniques directly from MU classroom experiences

How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points?

Faculty Profile



Professor Knight

Title/Department:

Professor
MU School of Scaring

Tenured Professor of Fright, School of Scaring
Just released the 15th edition of his book "A Study of Scaring," complete with new data and techniques directly from MU classroom experiences

IDENTICAL
points

NON-EXCLUSIVE
questions

Faculty Profile



Professor Knight

Title/Department:

Professor
MU School of Scaring

Tenured Professor of Fright, School of Scaring
Just released the 15th edition of his book "A Study
of Scaring," complete with new data and
techniques directly from MU classroom
experiences

$M(1,5)^{10}$

Distribute 5 points to
all 10 questions

Faculty Profile



Professor Knight

Title/Department:

Professor

MU School of Scaring

Tenured Professor of Fright, School of Scaring
Just released the 15th edition of his book "A Study
of Scaring," complete with new data and
techniques directly from MU classroom
experiences

$M(1,5)^{10} \cdot M(10,50)$

Distribute the
remaining points

Faculty Profile



Professor Knight

Title/Department:

Professor

MU School of Scaring

Tenured Professor of Fright, School of Scaring
Just released the 15th edition of his book "A Study
of Scaring," complete with new data and
techniques directly from MU classroom
experiences

How many nonnegative integral solutions
for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

if $x_1 = 2$, $x_2 = 5$?

How many nonnegative integral solutions
for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2, x_2 = 5 ?$$

IDENTICAL 1s

NON-EXCLUSIVE x_i s

How many nonnegative integral solutions
for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2, x_2 = 5 ?$$

$M(1,2)$.

Distribute 2 1s to x_1

How many nonnegative integral solutions
for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2, x_2 = 5 ?$$

$$M(1,2) \cdot M(1,5)$$

Distribute 5 1s to x_2

How many nonnegative integral solutions
for the equation

$$x_1 + x_2 + x_3 + x_4 = 16,$$

$$\text{if } x_1 = 2, x_2 = 5 ?$$

$$M(1,2) \cdot M(1,5) \cdot M(2,9)$$

Distribute remaining 9 1s to x_3 and x_4

QUIZ

In how many ways
can 36 different
cadets be distributed
to the 13 Scouting
Legion groups if each
can receive as many
cadets as possible?

PRINCIPLES OF COUNTING

INDIRECT METHOD OF COUNTING

MUTUAL INCLUSION-EXCLUSION PRINCIPLE

SAMPLE MODEL OF COUNTING

DISTRIBUTION MODEL OF COUNTING

DERANGEMENTS

PIGEONHOLE PRINCIPLE

RECURRENCE RELATIONS