Supplementary Exercise

Finding Local Extrema
Find all local extrema, and saddle points of the given functions.
1. $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$
2. $f(x,y) = 4xy - x^4 - y^4$
3. $f(x,y) = \dfrac{1}{x^2 + y^2 - 1}$
4. $f(x,y) = y \sin x$
5. $f(x,y) = \dfrac{1}{x} + xy + \dfrac{1}{y}$

Finding Absolute Extrema
Find the absolute extrema of the functions on the given domain.
6. $f(x,y) = x^2 + y^2$ on the closed triangular plate bounded by the lines
   $x = 0, y = 0, y + 2x = 2$ in the first quadrant
7. $f(x,y) = 48xy - 32x^3 - 24y^2$ on the rectangular plate $0 \le x \le 1, 0 \le y \le 1$

Lagrange Method
8. Find the points on the ellipse $x^2 + 2y^2 = 1$ where $f(x,y) = xy$ has its extreme value.
9. Find he points on the curve $x^2 y = 2$ nearest the origin.
10. Find the radius and height of the open right circular cylinder of largest surface area that can be inscribed in a sphere of radius a. What is the largest surface area?
11. A closed rectangular box is to have volume V cm^3. The cost of the material used in the box is a $cents/cm^2$ for top and bottom, b $cents/cm^2$ for front and back, and c $cents/cm^2$ for the remaining sides. What dimensions minimize the total cost of materials?