CMSC 141 Automata and Language Theory

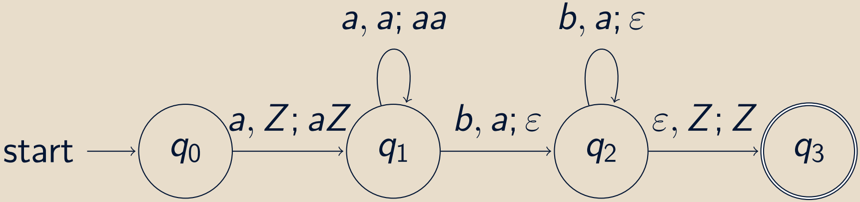
Context-Free Languages

Mark Froilan B. Tandoc

October 3, 2014

PDA FOR $\{a^n b^n : n > 0\}$

aabb



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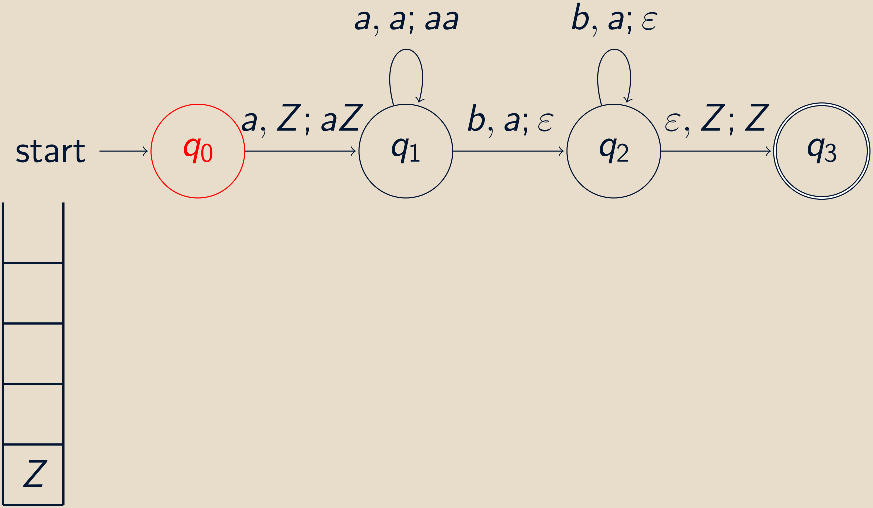

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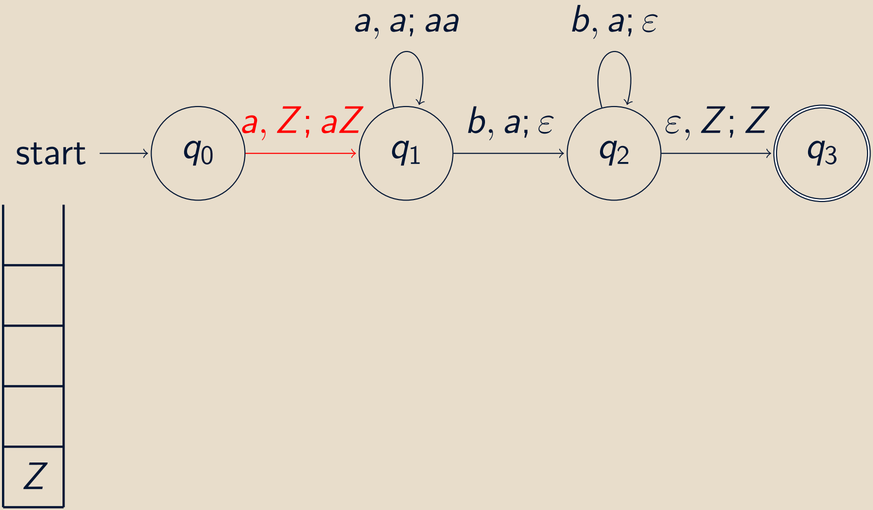

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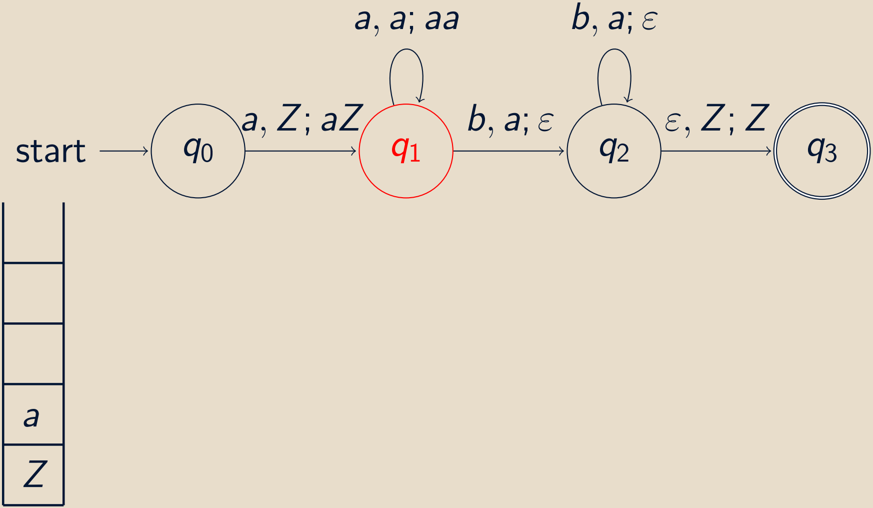

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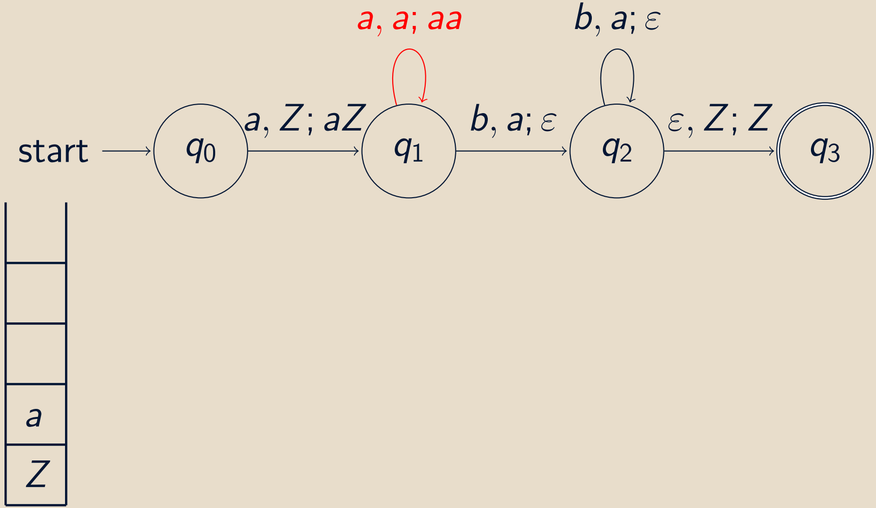

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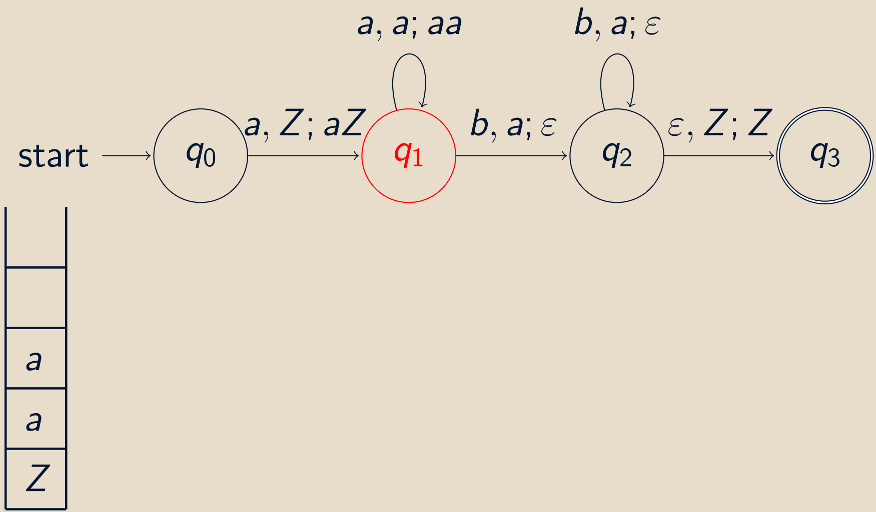

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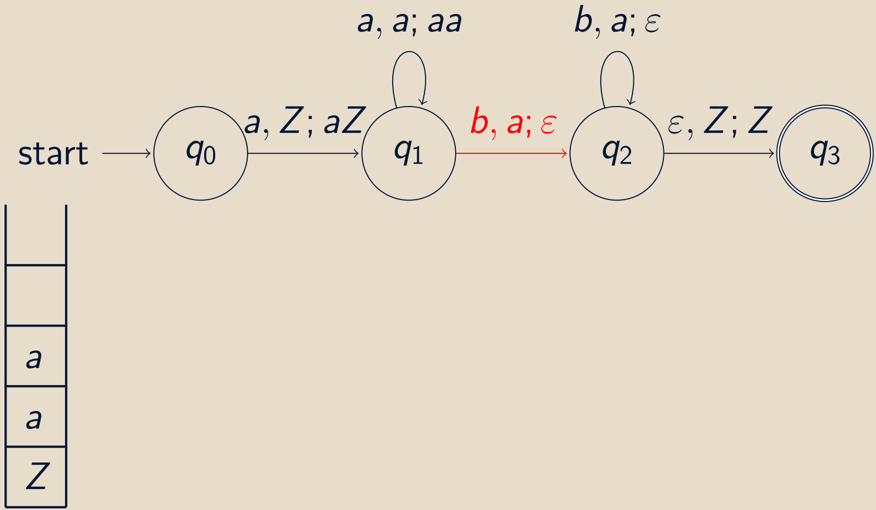

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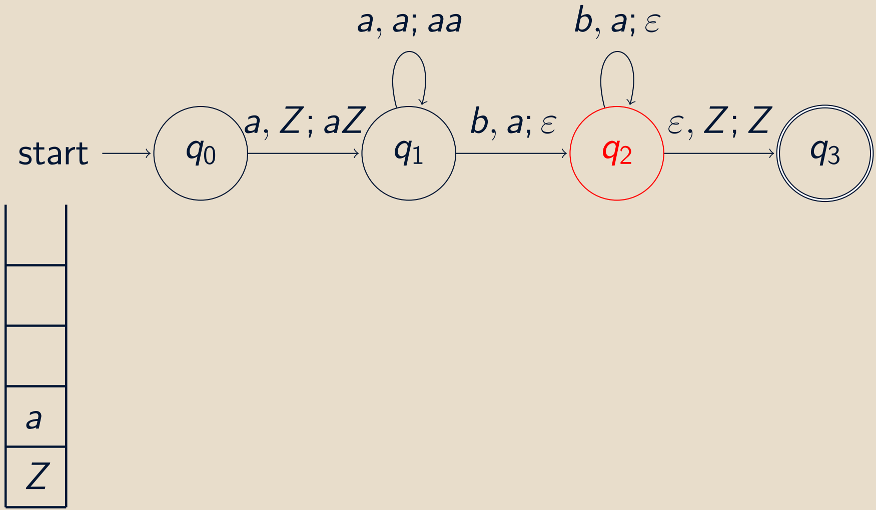

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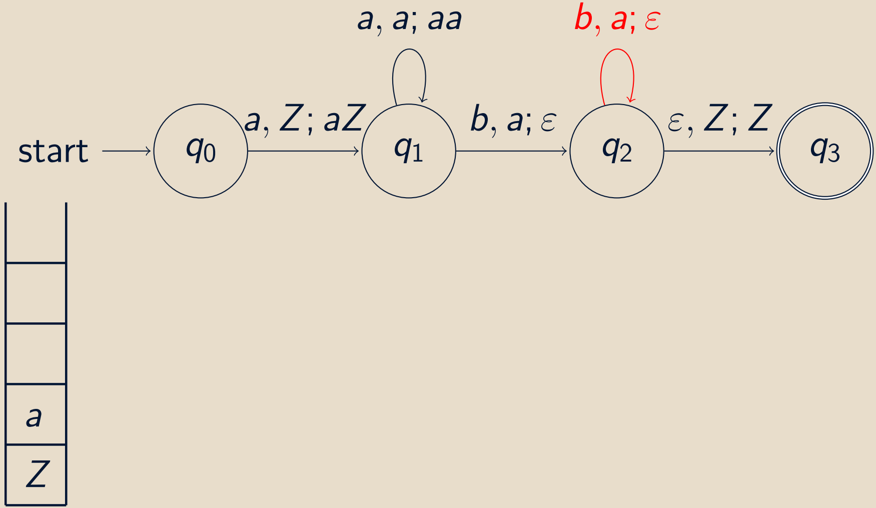

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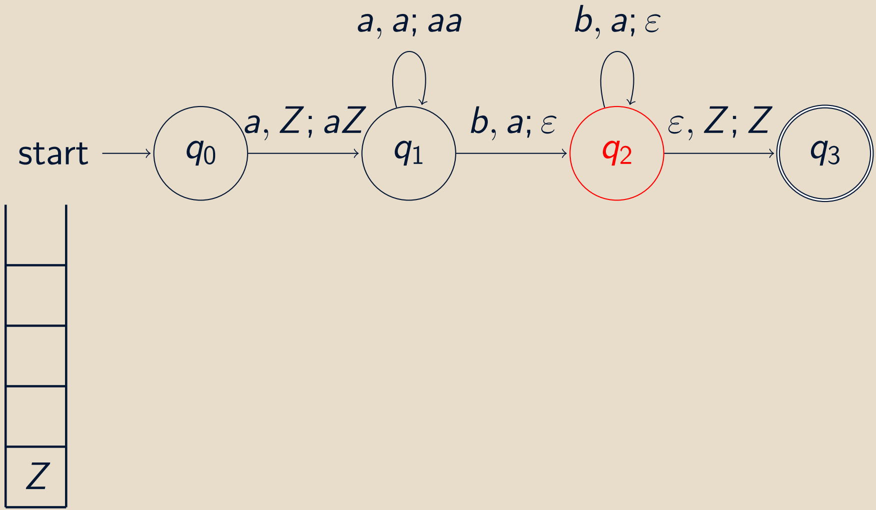

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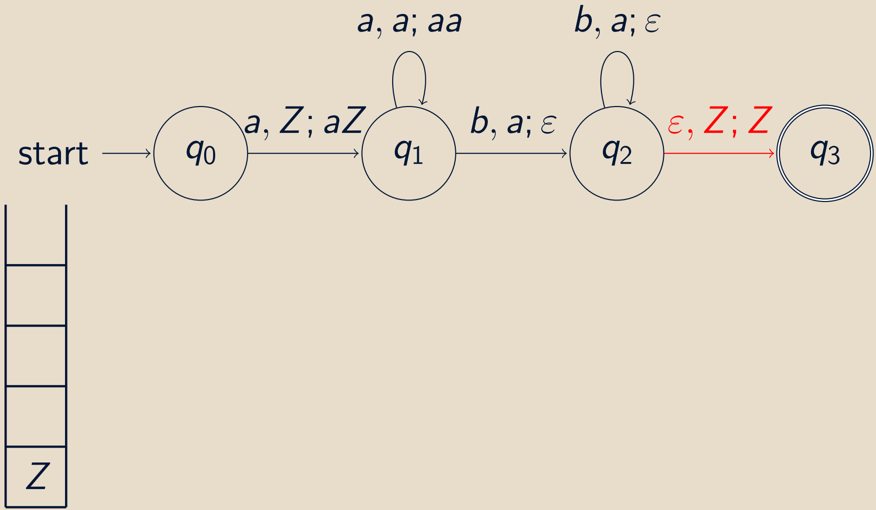

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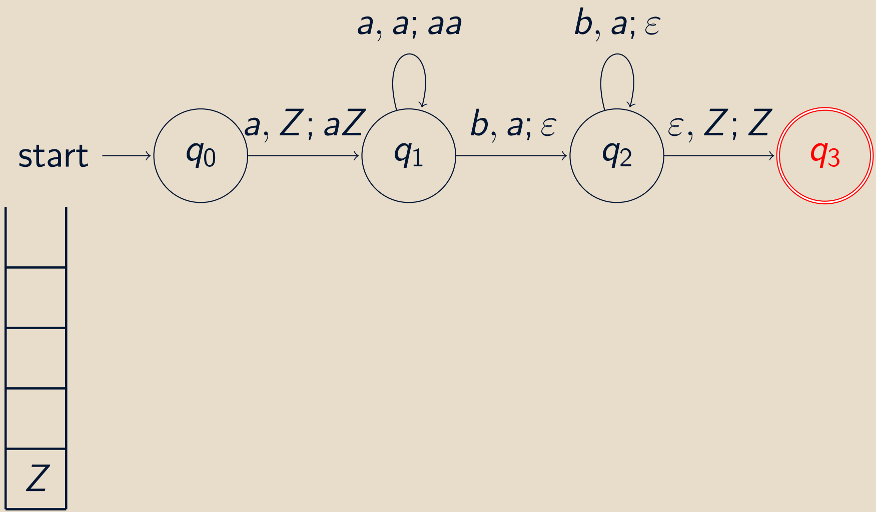

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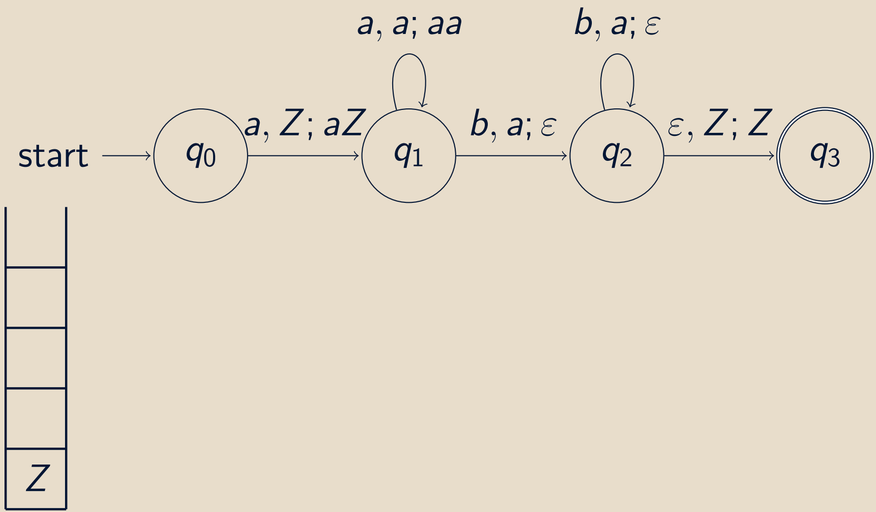

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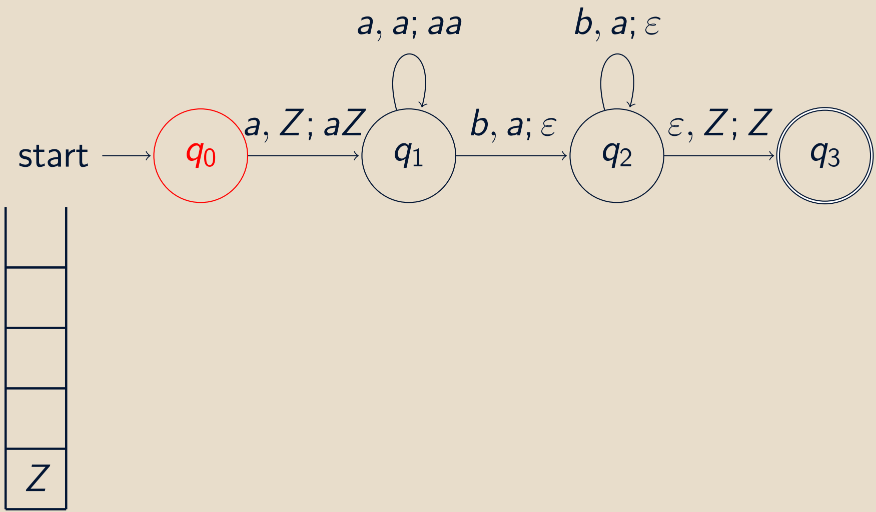

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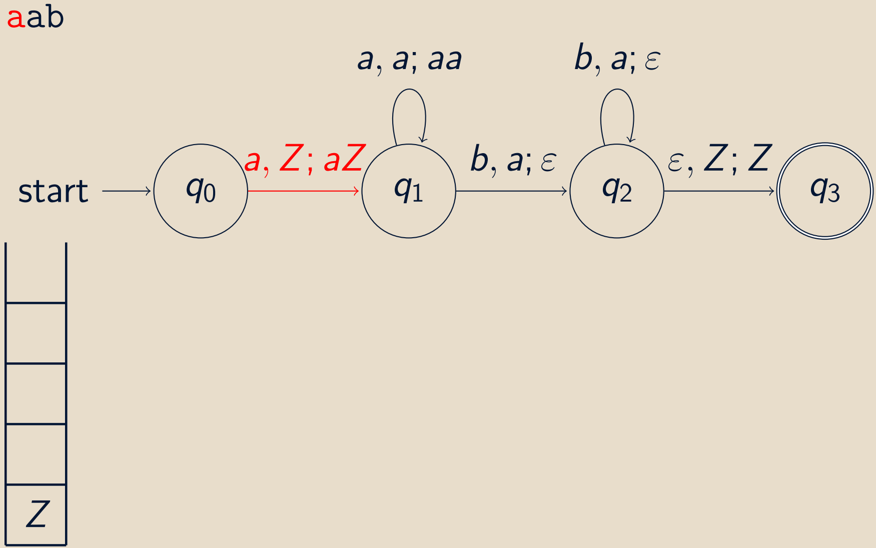

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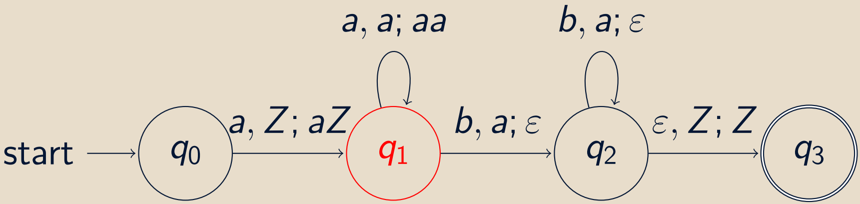

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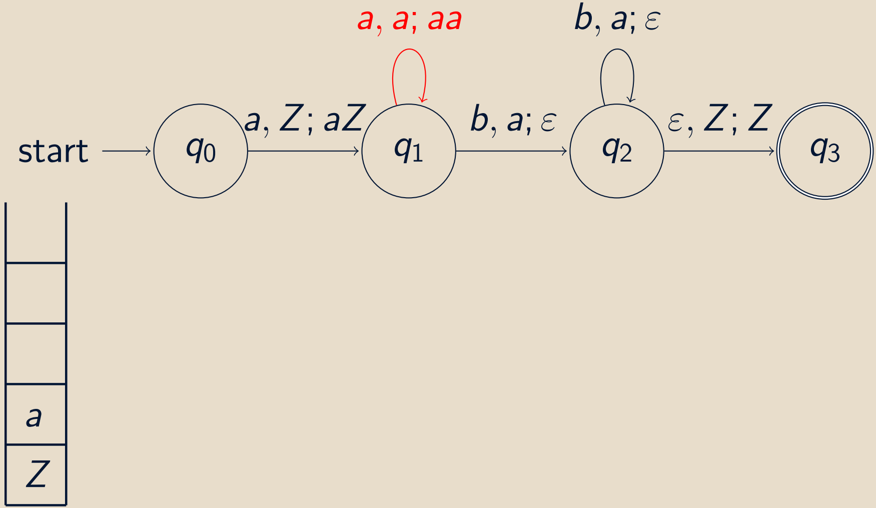

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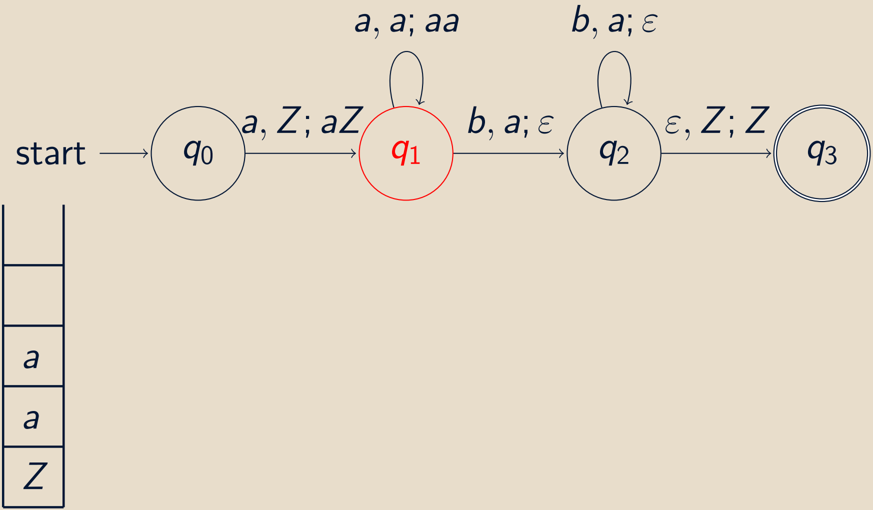

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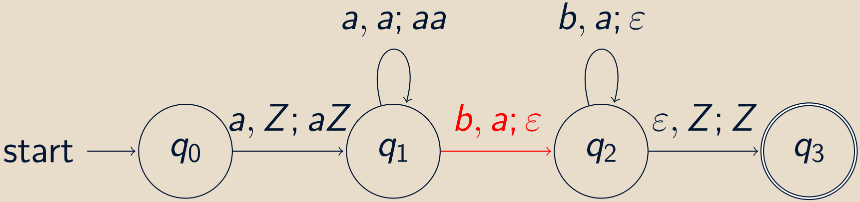

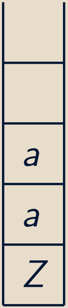

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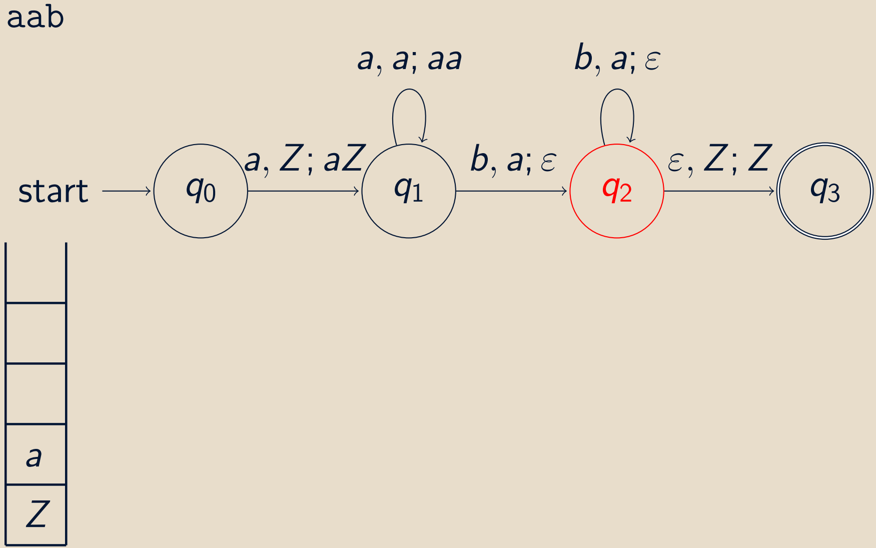

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$$\begin{aligned} S &\rightarrow ab && \text{(base case)} \\ S &\rightarrow aSb && \text{(recursive rule)} \end{aligned}$$

- The grammar can also be shorten by combining rules with the same left-hand side and using "|"
$S \rightarrow ab \mid aSb$

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A string x can be derived from a grammar if x can be generated by successive applications of the production rules starting from the start symbol.

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Derive: aaabbb

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaabbb$

Parse Trees (Derivation Tree)

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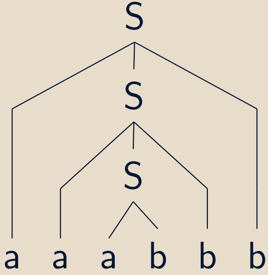

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- We write $L(G)$ for the language of grammar G
- Any language that can be generated by some context-free grammar is called a *context-free language*

English Language Example

$$
\begin{aligned}
\text{SENTENCE} &\rightarrow \text{NOUN-PHRASE VERB-PHRASE} \\
\text{NOUN-PHRASE} &\rightarrow \text{CMPLX-NOUN} \\
&\rightarrow \text{CMPLX-NOUN PREP-PHRASE} \\
\text{VERB-PHRASE} &\rightarrow \text{CMPLX-VERB} \\
&\rightarrow \text{CMPLX-VERB PREP-PHRASE} \\
\text{PREP-PHRASE} &\rightarrow \text{PREP CMPLX-NOUN} \\
\text{CMPLX-NOUN} &\rightarrow \text{ARTICLE NOUN} \\
\text{CMPLX-VERB} &\rightarrow \text{VERB} \mid \text{VERB NOUN PHRASE} \\
\text{ARTICLE} &\rightarrow \text{a} \mid \text{the} \\
\text{NOUN} &\rightarrow \text{boy} \mid \text{girl} \mid \text{flower} \\
\text{VERB} &\rightarrow \text{touches} \mid \text{likes} \mid \text{sees} \\
\text{PREP} &\rightarrow \text{with}
\end{aligned}
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Sample strings we can derive from the grammar are:

- `a boy sees`
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Try deriving them using the grammar

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SENTENCE $\Rightarrow$ NOUN-PHRASE VERB-PHRASE

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\end{aligned}
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- A *context-free grammar* is a 4-tuple (V, Σ, R, S), where
  - V is a finite set of *variables* (or non-terminals)
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- The rule for the *rules* is $V \rightarrow (V + T)^*$
- Previous grammar is more formally defined as $G = (\{S\}, \{a, b\}), \{S \rightarrow ab, S \rightarrow aSb\}, S$

Chomsky Hierarchy

by Noam Chomsky

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Containment hierarchy of classes of formal grammars

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- Regular grammars (simplest, weakest)
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- Context-sensitive grammars

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  - $V \rightarrow (V + T)^*$
- Context-sensitive grammars
- Unrestricted grammars/Recursively enumerable grammars
  (most expressive)

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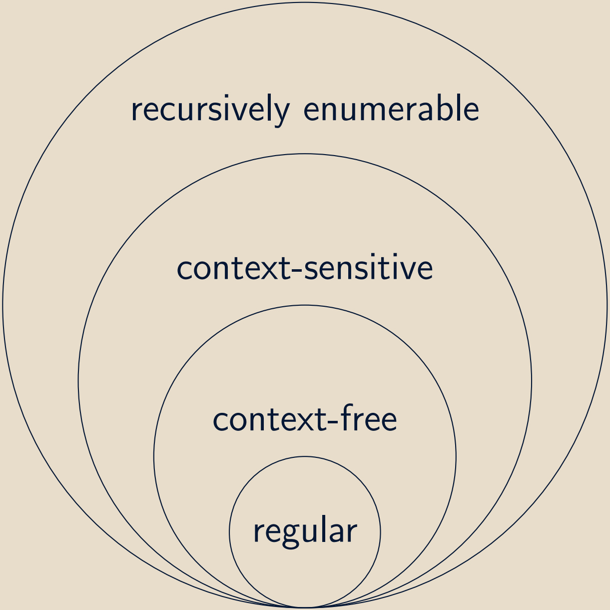

References

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- E.A. Albacea. Automata, Formal Languages and Computations, UPLB Foundation, Inc. 2005
- JFLAP, www.jflap.org
- Various online LaTeX and Beamer tutorials