



Chapter 4

SIMPLIFICATION of LOGIC CIRCUITS

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

		CD			
		0 0	0 1	1 1	1 0
AB	0 0				
	0 1				
	1 1				
	1 0				
	1 0				

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1		1	1
0 1	1			1
1 1	1			1
1 0	1		1	1

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

		CD			
		0 0	0 1	1 1	1 0
AB	0 0	1		1	1
	0 1	1			1
	1 1	1			1
	1 0	1		1	1

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1			1	1
0 1	1				1
1 1	1				1
1 0	1			1	1

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1			1	1
0 1	1				1
1 1	1				1
1 0	1			1	1

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function values for the variables A, B, C, and D. The variables A and B are the rows, and C and D are the columns. The map is divided into four groups of 1s, each enclosed by a curved line, representing the simplified terms: $B'C'D'$, $A'BC'D'$, $A'B'CD$, and $AB'CD$. A red arrow points to the 1s in the column where C=1 and D=0, labeled D' .

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

		CD			
		0 0	0 1	1 1	1 0
AB	0 0	1		1	1
	0 1	1			1
	1 1	1			1
	1 0	1		1	1

Groupings and Simplifications:

- Group 1 (Gray): $B'C$ (covers cells where $B=0, C=1$)
- Group 2 (Gray): D' (covers cells where $D=0$)
- Group 3 (Gray): $A'B'$ (covers cells where $A=0, B=0$)
- Group 4 (Gray): $A'B$ (covers cells where $A=0, B=1$)
- Group 5 (Gray): AB' (covers cells where $A=1, B=0$)
- Group 6 (Gray): AB (covers cells where $A=1, B=1$)

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABCD'$

$$= B'C + D'$$

AB \ CD	00	01	11	10	
00	1		1	1 B'C
01	1			1	
11	1			1 D'
10	1		1	1	

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0				
0 1				
1 1				
1 0				

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	0			
0 1				
1 1				
1 0	0			

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD	00	01	11	10
00	0			0
01				
11				
10	0			

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD	0 0	0 1	1 1	1 0
0 0	0			0
0 1				
1 1				
1 0	0	0		

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	0 0	0 1	1 1	1 0
0 0	0			0
0 1	0			
1 1				
1 0	0	0		

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	0 0	0 1	1 1	1 0
0 0	0			0
0 1	0			
1 1	0			
1 0	0	0		

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD	00	01	11	10
00	0			0
01	0			
11	0			
10	0	0		

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0				0
0 1	0				
1 1	0				
1 0	0	0			

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0	0			0
0 1	0				
1 1	0				
1 0	0	0			

Diagram illustrating a 4x4 Karnaugh map for the simplification of the Boolean expression. The map shows the values of the expression for all combinations of variables A, B, C, and D. The cells are labeled with 0 or 1. The map is divided into four groups of cells, each containing a 0, indicating that the expression is 0 for those combinations. The groups are: (0,0), (0,1), (1,1), (1,0) for CD=00; (0,0), (0,1), (1,1), (1,0) for CD=01; (0,0), (0,1), (1,1), (1,0) for CD=11; and (0,0), (0,1), (1,1), (1,0) for CD=10. The expression is 0 for all combinations of A, B, C, and D.

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0				0
0 1	0				
1 1	0				
1 0	0	0			

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function value (0 or 1) for each combination of variables A, B, C, and D. The variables A and B are listed on the left, and C and D are listed on the top. The map is divided into four groups of cells, each containing a 0, representing the function value for those combinations. The groups are: (A=0, B=0), (A=0, B=1), (A=1, B=1), and (A=1, B=0). The group (A=0, B=0) is highlighted with a red oval and labeled $A+B+D$. The group (A=0, B=1) is highlighted with a red oval. The group (A=1, B=1) is highlighted with a red oval. The group (A=1, B=0) is highlighted with a red oval. A dotted line connects the group (A=0, B=0) to the group (A=1, B=1).

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0				0
0 1	0				
1 1	0				
1 0	0	0			

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function value (0 or 1) for each combination of variables A, B, C, and D. The variables A and B are the row labels, and C and D are the column labels. The map is divided into four groups of cells, each representing a simplified term:

- Group 1: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $A+B+D$.
- Group 2: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $C+D$.
- Group 3: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $A+B+D$.
- Group 4: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $C+D$.

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0				0
0 1	0				
1 1	0				
1 0	0	0			

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function value (0 or 1) for each combination of variables A, B, C, and D. The variables are arranged in a 4x4 grid with rows labeled AB and columns labeled CD. The function value is 0 for the following cells: (0,0), (0,1), (1,1), (1,0), (1,0), and (1,0). The function value is 1 for the following cells: (0,1), (0,1), (1,0), and (1,0).

The simplified expression is derived from the prime implicants (groups of 1s) shown in the map:

- $A+B+D$ (Group 1: (0,0), (0,1), (1,0), (1,0))
- $C+D$ (Group 2: (0,1), (1,1), (1,0), (1,0))
- $A'+B+C$ (Group 3: (0,1), (1,0), (1,0), (1,0))

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$ **$= (A+B+D)(C+D)(A'+B+C)$**

AB \ CD		CD			
		0 0	0 1	1 1	1 0
AB	0 0	0			0
	0 1	0			
	1 1	0			
	1 0	0	0		

... **$A+B+D$**

... **$C+D$**

... **$A'+B+C$**

A decorative graphic on the left side of the slide, consisting of a vertical arrangement of stylized circuit traces. These traces are in shades of green and blue, with some circular nodes and branching lines, resembling a printed circuit board (PCB) layout. The pattern is more dense on the left and tapers off towards the right.

Advantages of using K-map

- Can be done more systematically
- Much simpler to find minimum solutions
- Easier to see what is happening



Simplification: Quine-McCluskey

- Advantages
 - Specific step by step procedure
 - Can be applied to problems with many variables
 - Suitable for machine computation



Simplification: Quine-McCluskey

- Steps
 - Construct prime implicants table
 - Construct prime implicants chart
 - Select all essential prime implicants
 - Select a minimal cover from the remaining prime implicants



Constructing Prime Implicants Table

- List terms in a column using their binary representation
 - Group terms so that each group contains minterms with the same number of 1's
 - Place groupings which differ by only one literal adjacent to one another

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

1	0001
3	0011
7	0111
8	1000
14	1110
15	1111

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1
1	0001	1 0001
3	0011	8 1000
7	0111	
8	1000	3 0011
14	1110	
15	1111	7 0111
		14 1110
		15 1111



Constructing Prime Implicants Table

- Perform exhaustive search for logically adjacent terms between adjacent groups
 - Each term should be checked off
 - Combine each pair of terms into a single term replacing the differing literal with '-'
 - Repeat procedure until no further terms can be created
 - All unchecked terms are prime implicants

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1	0001
3	0011	8	1000
7	0111		
8	1000	3	0011
14	1110		
15	1111	7	0111
		14	1110
		15	1111

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1		Column 2	
1	0001	1	0001 ✓	1,3	00-1
3	0011	8	1000		
7	0111				
8	1000	3	0011 ✓		
14	1110				
15	1111	7	0111		
		14	1110		
		15	1111		

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1		Column 2	
1	0001	1	0001 ✓	1,3	00-1
3	0011	8	1000		
7	0111			3,7	0-11
8	1000	3	0011 ✓		
14	1110				
15	1111	7	0111 ✓		
		14	1110		
		15	1111		

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1 0001 ✓	1,3 00-1
3	0011	8 1000	
7	0111		3,7 0-11
8	1000	3 0011 ✓	
14	1110		7,15 -111
15	1111	7 0111 ✓ 14 1110	
		15 1111 ✓	

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1 0001 ✓	1,3 00-1
3	0011	8 1000	
7	0111		3,7 0-11
8	1000	3 0011 ✓	
14	1110		7,15 -111
15	1111	7 0111 ✓	14,15 111-
		14 1110 ✓	
		15 1111 ✓	

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1 0001 ✓	1,3 00-1
3	0011	8 1000	
7	0111		3,7 0-11
8	1000	3 0011 ✓	
14	1110		7,15 -111
15	1111	7 0111 ✓	14,15 111-
		14 1110 ✓	
		15 1111 ✓	

Prime
implicants:
 $AB'C'D'$
 $A'B'D$
 $A'CD$
 BCD
 ABC



Construct Prime Implicants Chart

- Terms are listed horizontally
- Prime implicants are listed vertically
- Place an X whenever a prime implicant covers a minterm

Example

	1	3	7	8	14	15
8	1	0	0	0		
1,3	0	0	0	1		
3,7	0	1	1			
7,15	1	1	1			
14,15				1	1	1

Example

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X

Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X

Select Essential Prime Implicants

	1	3	7	8	14	15
8 1000				X		
1,3 00-1	X	X				
3,7 0-11		X	X			
7,15 -111			X			X
14,15 111-					X	X

Select Essential Prime Implicants

	1	3	7	8	14	15
8 1000				X		
1,3 00-1	X	X				
3,7 0-11		X	X			
7,15 -111			X			X
14,15 111-					X	X
	✓			✓	✓	

Select Minimum Cover

	1	3	7	8	14	15
8 1000				X		
1,3 00-1	X	X				
3,7 0-11		X	X			
7,15 -111			X			X
14,15 111-					X	X
	✓	✓		✓	✓	✓

Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
 - $F = AB'C'D' + A'B'D + ABC$

Select Minimum Cover

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
✓ 7,15	-111			X			X
14,15	111-					X	X
		✓	✓		✓	✓	✓

Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
 - $F = AB'C'D' + A'B'D + ABC + BCD$