

Chapter 1

DATA REPRESENTATION





Analog vs. Digital

Analog quantities

- can vary over a continuous range of values.
- Examples: voltage, room thermostat

Digital quantities

- represented by symbols called digits.
- Example: digital watch



Digital Number Systems

- Many number systems are in use in digital technology.
- The most common are the *decimal, binary, octal, and hexadecimal systems.*



Conversion: Base- r to Decimal

Procedure

- **Step 1:**

Multiply each coefficient with the corresponding power of r .

- **Step 2:**

Get the sum.



Conversion: Decimal to Base-r

Procedure

- **Step 1:**
Separate integer from fraction
- **Step 2:**
Convert integer to base-r
- **Step 3:**
Convert fraction to base-r

Integer to base-r

- Divide integer by r
- Accumulate remainders

Fraction to base-r

- Multiply fraction by r
- Accumulate integers



Conversion: Binary to octal or hexadecimal

Binary To Octal

Procedure:

- Partition binary number into groups of 3 digits



Conversion: Binary to octal or hexadecimal

Binary To Hexadecimal

Procedure:

- Partition binary number into groups of 4 digits



Conversion: Octal or hexadecimal to binary

Octal To Binary

Procedure:

- Each octal digit is converted to its 3-digit binary equivalent



Conversion: Octal or hexadecimal to binary

Hexadecimal to Binary

Procedure:

- Each hexadecimal digit is converted to its 4-digit binary equivalent



Fixed-Point Representation

Unsigned Number

- leftmost bit is the most significant bit
- Example:
 - $01001 = 9$
 - $11001 = 25$

Signed Number

- leftmost bit represents the sign
- Example:
 - $01001 = +9$
 - $11001 = -9$

Systems Used to Represent Negative Numbers

Signed-Magnitude representation

- A number consists of a magnitude and a symbol indicating whether the magnitude is positive or negative.

Examples:

$$+85 = 01010101_2$$

$$+127 = 01111111_2$$

$$-85 = 11010101_2$$

$$-127 = 11111111_2$$



Systems Used to Represent Negative Numbers

Signed-Complement System

- This system negates a number by taking its complement as defined by the system.
- Types of complements:
 - Radix-complement
 - Diminished Radix-complement

Chapter 2

COMPUTER ARITHMETIC





Arithmetic in various Number Systems

- Addition of numbers in any number system
 - Add numbers starting at the least significant digit.
 - Perform addition on numbers of the same number base.
- Subtraction of numbers
 - Must use complements



Binary Addition

- To add binary numbers: $(X + Y)$
 - Get the SCR of the negative numbers
 - Add the two numbers
 - If the SCR used is:
 - 2's C: Discard end carry
 - 1's C: Add the end carry to the sum

Binary Subtraction

- To subtract binary numbers: $(X - Y)$
 - Take the complement of the subtrahend.
 - Then, add the two numbers.
 - If the complement used is:
 - 1's C: Add the end carry to the sum
 - 2's C: Discard end carry

$$(X - Y) \ggg X + (\text{complement of } Y)$$



Error Detection

- Overflow
 - occurs when an arithmetic operation yields a result that is greater than the range's positive limit



Error Detection

- Underflow
 - occurs when an arithmetic operation yields a result that is lesser than the range's negative limit



BCD Addition

- Sum less than or equal to 9
 - Normal binary addition
- Sum greater than 9
 - Add the codes
 - Add a correction value of 0110 to any sum

Chapter 3

BOOLEAN ALGEBRA, LOGIC FUNCTIONS and LOGIC GATES



Boolean Operations

- AND

- represented by a dot or the absence of an operator
- 0 dominates

x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

Boolean Operations

- OR
 - represented by a plus sign
 - 1 dominates

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Operations

- NOT
 - represented by a prime
 - inversion or complementation

x	x'
0	1
1	0

Boolean Operations on constants

AND	OR	NOT
$0 \cdot 0 = 0$	$0 + 0 = 0$	$0' = 1$
$0 \cdot 1 = 0$	$0 + 1 = 1$	$1' = 0$
$1 \cdot 0 = 0$	$1 + 0 = 1$	
$1 \cdot 1 = 1$	$1 + 1 = 1$	

Boolean Operations on one variable

AND	OR	NOT
$A \cdot 0 = 0$	$A + 0 = A$	$A'' = A$
$A \cdot 1 = A$	$A + 1 = 1$	
$A \cdot A = A$	$A + A = A$	
$A \cdot A' = 0$	$A + A' = 1$	

Boolean Operations On Two or More Variables

- Commutative laws

$$A + B = B + A$$

$$AB = BA$$

- Associative laws

$$A + (B + C) = (A + B) + C$$

$$A(BC) = (AB)C$$

- Distributive laws

$$A(B + C) = AB + AC$$

$$A + (BC) = (A + B)(A + C)$$

- De Morgan's laws

$$(A + B)' = A'B'$$

$$(AB)' = A' + B'$$

- Laws of Absorption

$$A + AB = A$$

$$A(A + B) = A$$

A decorative graphic on the left side of the slide, consisting of a vertical arrangement of stylized circuit components. It includes green and blue circular nodes of various sizes, connected by thin white lines that branch out horizontally and vertically, resembling a circuit board or a network diagram.

Boolean Functions

- Boolean functions are expressions formed with binary variables and boolean operators
- Representations of boolean functions:
 - algebraic expression
 - truth table

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m0	$x+y+z$	M0
0	0	1	$x'y'z$	m1	$x+y+z'$	M1
0	1	0	$x'yz'$	m2	$x+y'+z$	M2
0	1	1	$x'yz$	m3	$x+y'+z'$	M3
1	0	0	$xy'z'$	m4	$x'+y+z$	M4
1	0	1	$xy'z$	m5	$x'+y+z'$	M5
1	1	0	xyz'	m6	$x'+y'+z$	M6
1	1	1	xyz	m7	$x'+y'+z'$	M7

Forms of Boolean Functions

- Canonical Form

- Sum of minterms

$$F(x,y,z) =$$

$$xyz' + x'yz$$

- Product of maxterms

$$F(x,y,z) =$$

$$(x' + y' + z)(x + y + z')$$

- Standard Form

- Sum of products

$$F(x,y,z) = xz' + y$$

- Product of sums

$$F(x,y,z) = (x + y')z$$

Chapter 4

SIMPLIFICATION of LOGIC CIRCUITS



A decorative graphic on the left side of the slide, consisting of a vertical arrangement of stylized circuit traces. These traces are in shades of green and blue, with some circular nodes and branching lines, resembling a printed circuit board (PCB) layout.

Ways to simplify Boolean functions

- Boolean Algebra
- Graphical method (Karnaugh Map)
- Tabular method (Quine-McCluskey)



Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$



Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

$$= x' + xyz$$

Comm / Dist.

Inv / Iden

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

$$= x' + xyz$$

$$= (x' + x)(x' + yz)$$

Comm / Dist.

Inv / Iden

Dist.

Simplification: Boolean Algebra

Simplify $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.

$$= x' + xyz$$

Inv / Iden

$$= (x' + x)(x' + yz)$$

Dist.

$$= (x' + yz)$$

Inv / Iden



Simplification: Graphical method

- Karnaugh map (K-map)
 - alternate way of representing Boolean functions
 - a graphical tool for assisting in the general simplification procedure
 - a simpler way to handle most jobs of manipulating logic functions



General Steps of K-Map Simplification

- Express function in canonical form
- Map expression on a K-Map
- Group 1's or 0's
- Determine the minimum expression

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0				
0 1				
1 1				
1 0				

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1		1	1
0 1	1			1
1 1	1			1
1 0	1		1	1

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1			1	1
0 1	1				1
1 1	1				1
1 0	1			1	1

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

		CD			
		0 0	0 1	1 1	1 0
AB	0 0	1		1	1
	0 1	1			1
	1 1	1			1
	1 0	1		1	1

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1			1	1
0 1	1				1
1 1	1				1
1 0	1			1	1

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function value (1 or 0) for each combination of variables A, B, C, and D. The variables A and B are the rows, and C and D are the columns. The map is divided into four groups of 1s, each enclosed by a curved line, representing the simplified terms: $B'C'D'$, $A'BC'D'$, $A'B'CD$, and $AB'CD$. A red arrow points to the 1s in the column where C=1 and D=0, labeled D' .

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1			1	1
0 1	1				1
1 1	1				1
1 0	1			1	1

Groupings and Simplifications:

- B'C**: Grouped by a horizontal oval across the first two columns (CD = 00 and 01) for rows AB = 00 and 01.
- D'**: Grouped by a vertical oval across the first two rows (AB = 00 and 01) for columns CD = 10 and 11.
- B'D**: Grouped by a vertical oval across the last two columns (CD = 11 and 10) for rows AB = 00 and 10.
- AC**: Grouped by a horizontal oval across the last two columns (CD = 11 and 10) for rows AB = 11 and 10.

Simplification Example

- Simplify $B'C'D' + A'BC'D' + A'B'CD + AB'CD + A'B'CD' + A'BCD' + ABCD' + AB'CD' + ABC'D'$

$$= B'C + D'$$

AB \ CD	00	01	11	10	
00	1		1	1 B'C
01	1			1	
11	1			1 D'
10	1		1	1	

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0				
0 1				
1 1				
1 0				

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	0			
0 1				
1 1				
1 0	0			

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD	00	01	11	10
00	0			0
01				
11				
10	0			

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	0 0	0 1	1 1	1 0
0 0	0			0
0 1				
1 1				
1 0	0	0		

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD	0 0	0 1	1 1	1 0
0 0	0			0
0 1	0			
1 1				
1 0	0	0		

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	0 0	0 1	1 1	1 0
0 0	0			0
0 1	0			
1 1	0			
1 0	0	0		

Simplification Example

- Simplify $(B+C+D) (A+B+C'+D) (A'+B+C+D') (A+B'+C+D) (A'+B'+C+D)$

AB \ CD	00	01	11	10
00	0			0
01	0			
11	0			
10	0	0		

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0 0	0			0
	0 1	0			
	1 1	0			
	1 0	0	0		

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0	0			0
0 1	0				
1 1	0				
1 0	0	0			

Diagram illustrating a 4x4 Karnaugh map for the simplification of the Boolean expression. The map shows the values of the expression for all combinations of variables A, B, C, and D. The cells are labeled with 0 or 1. The map is divided into four groups of cells, each containing a 0, indicating that the expression is 0 for those combinations. The groups are: (0,0,0,0), (0,0,1,0), (1,1,0,0), and (1,0,0,0). The cell (0,0,1,0) is also circled, and an arrow points to it from the right.

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0	0			0
0 1	0				
1 1	0				
1 0	0	0			

... $A+B+D$

...

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0				0
0 1	0				
1 1	0				
1 0	0	0			

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function value (0 or 1) for each combination of variables A, B, C, and D. The variables A and B are the row labels, and C and D are the column labels. The map is divided into four groups of cells, each representing a simplified term:

- Group 1: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $A+B+D$.
- Group 2: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $C+D$.
- Group 3: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $A+B+D$.
- Group 4: Cells (0,0), (0,1), (1,1), (1,0) are grouped together, representing the term $C+D$.

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	0				0
0 1	0				
1 1	0				
1 0	0	0			

Diagram illustrating the simplification of the Boolean expression using a Karnaugh map. The map shows the function value (0 or 1) for each combination of variables A, B, C, and D. The variables are arranged in a 4x4 grid with rows labeled AB and columns labeled CD. The function value is 0 for the following cells: (0,0), (0,1), (1,1), (1,0), (1,0), and (1,0). The function value is 1 for the following cells: (0,1), (0,0), (1,0), (1,1), (1,1), and (1,0).

Groupings and Simplifications:

- Group 1: $A+B+D$ (Cells: (0,0), (0,1), (1,0), (1,0))
- Group 2: $C+D$ (Cells: (0,1), (1,1), (1,0), (1,0))
- Group 3: $A'+B+C$ (Cells: (0,0), (0,1), (1,0), (1,0))

Simplification Example

- Simplify $(B+C+D)(A+B+C'+D)(A'+B+C+D')(A+B'+C+D)(A'+B'+C+D)$ **$= (A+B+D)(C+D)(A'+B+C)$**

AB \ CD		CD			
		0 0	0 1	1 1	1 0
AB	0 0	0			0
	0 1	0			
	1 1	0			
	1 0	0	0		

... **$A+B+D$**

... **$C+D$**

... **$A'+B+C$**



Simplification: Quine-McCluskey

- Advantages
 - Specific step by step procedure
 - Can be applied to problems with many variables
 - Suitable for machine computation



Simplification: Quine-McCluskey

- Steps
 - Construct prime implicants table
 - Construct prime implicants chart
 - Select all essential prime implicants
 - Select a minimal cover from the remaining prime implicants



Constructing Prime Implicants Table

- List terms in a column using their binary representation
 - Group terms so that each group contains minterms with the same number of 1's
 - Place groupings which differ by only one literal adjacent to one another

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

1	0001
3	0011
7	0111
8	1000
14	1110
15	1111

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1
1	0001	1 0001
3	0011	8 1000
7	0111	
8	1000	3 0011
14	1110	
15	1111	7 0111
		14 1110
		15 1111



Constructing Prime Implicants Table

- Perform exhaustive search for logically adjacent terms between adjacent groups
 - Each term should be checked off
 - Combine each pair of terms into a single term replacing the differing literal with '-'
 - Repeat procedure until no further terms can be created
 - All unchecked terms are prime implicants

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1	0001
3	0011	8	1000
7	0111		
8	1000	3	0011
14	1110		
15	1111	7	0111
		14	1110
		15	1111

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1 0001 ✓	1,3 00-1
3	0011	8 1000	
7	0111		
8	1000	3 0011 ✓	
14	1110		
15	1111	7 0111	
		14 1110	
		15 1111	

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1		Column 2	
1	0001	1	0001 ✓	1,3	00-1
3	0011	8	1000		
7	0111			3,7	0-11
8	1000	3	0011 ✓		
14	1110				
15	1111	7	0111 ✓		
		14	1110		
		15	1111		

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1 0001 ✓	1,3 00-1
3	0011	8 1000	
7	0111		3,7 0-11
8	1000	3 0011 ✓	
14	1110		7,15 -111
15	1111	7 0111 ✓ 14 1110	
		15 1111 ✓	

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1 0001 ✓	1,3 00-1
3	0011	8 1000	
7	0111		3,7 0-11
8	1000	3 0011 ✓	
14	1110		7,15 -111
15	1111	7 0111 ✓	14,15 111-
		14 1110 ✓	
		15 1111 ✓	

Example

- Simplify $F = \sum m(1, 3, 7, 8, 14, 15)$

		Column 1	Column 2
1	0001	1 0001 ✓	1,3 00-1
3	0011	8 1000	
7	0111		3,7 0-11
8	1000	3 0011 ✓	
14	1110		7,15 -111
15	1111	7 0111 ✓	14,15 111-
		14 1110 ✓	
		15 1111 ✓	

Prime
implicants:
 $AB'C'D'$
 $A'B'D$
 $A'CD$
 BCD
 ABC



Construct Prime Implicants Chart

- Terms are listed horizontally
- Prime implicants are listed vertically
- Place an X whenever a prime implicant covers a minterm

Example

	1	3	7	8	14	15
8		1	0	0	0	
1,3	0	0	0	1		
3,7	0	1	1			
7,15	1	1	1			
14,15				1	1	1

Example

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X

Select Essential Prime Implicants

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
7,15	-111			X			X
14,15	111-					X	X

Select Essential Prime Implicants

	1	3	7	8	14	15
8 1000				X		
1,3 00-1	X	X				
3,7 0-11		X	X			
7,15 -111			X			X
14,15 111-					X	X

Select Essential Prime Implicants

	1	3	7	8	14	15
8 1000				X		
1,3 00-1	X	X				
3,7 0-11		X	X			
7,15 -111			X			X
14,15 111-					X	X
	✓			✓	✓	

Select Minimum Cover

	1	3	7	8	14	15
8 1000				X		
1,3 00-1	X	X				
3,7 0-11		X	X			
7,15 -111			X			X
14,15 111-					X	X
	✓	✓		✓	✓	✓



Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
 - $F = AB'C'D' + A'B'D + ABC$

Select Minimum Cover

		1	3	7	8	14	15
8	1000				X		
1,3	00-1	X	X				
3,7	0-11		X	X			
✓ 7,15	-111			X			X
14,15	111-					X	X
		✓	✓		✓	✓	✓

Minimum Expression

- Essential prime implicants + the prime implicants that cover the columns that were not removed
- Hence
 - $F = AB'C'D' + A'B'D + ABC + BCD$

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0				
0 1				
1 1				
1 0				

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$

AB \ CD	00	01	11	10
00		0	0	0
01	0			
11	0			
10		0		

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	0	1	1

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1	0	0	0	
0 1	0	1	1	1	
1 1	0	1	1	1	
1 0	1	0	1	1	

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$

		CD			
		00	01	11	10
AB	00	1	0	0	0
	01	0	1	1	1
	11	0	1	1	1
	10	1	0	1	1

Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$

		CD			
		00	01	11	10
AB	00	1	0	0	0
	01	0	1	1	1
	11	0	1	1	1
	10	1	0	1	1

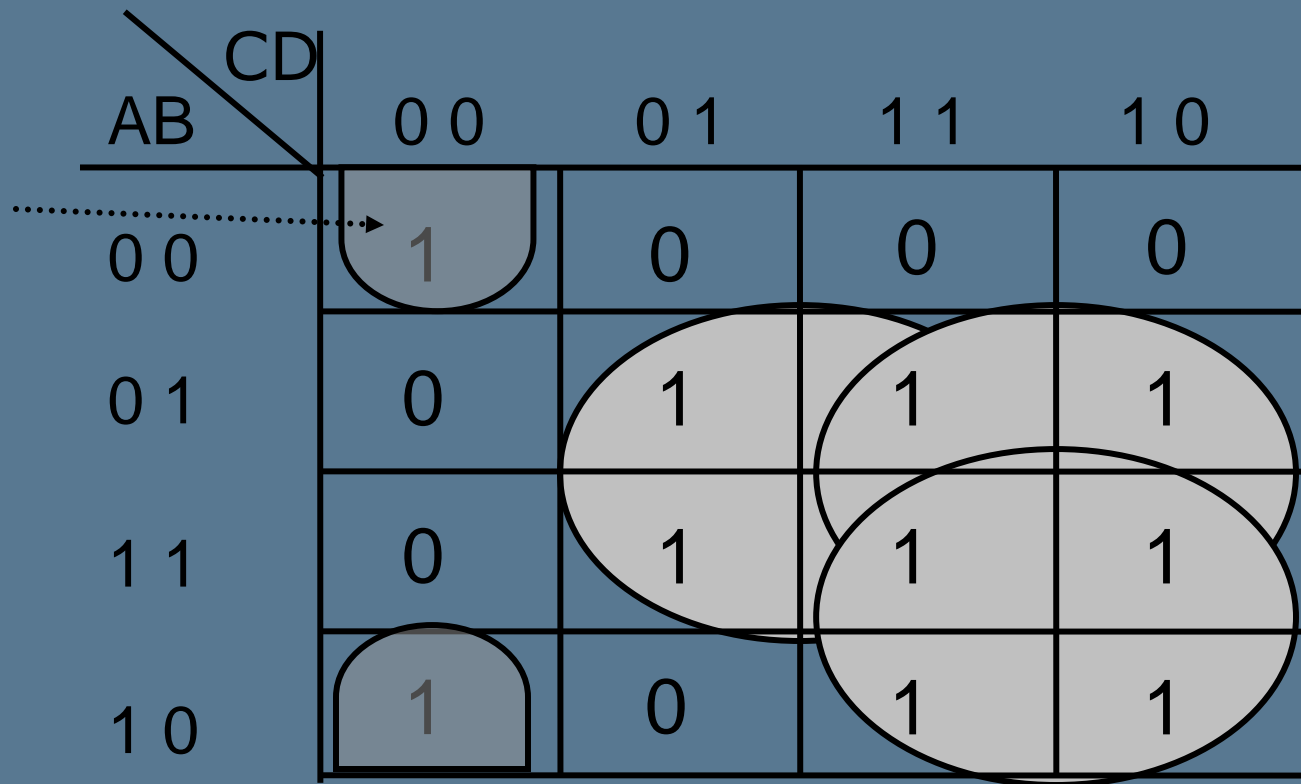
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$

		CD			
		00	01	11	10
AB	00	1	0	0	0
	01	0	1	1	1
	11	0	1	1	1
	10	1	0	1	1

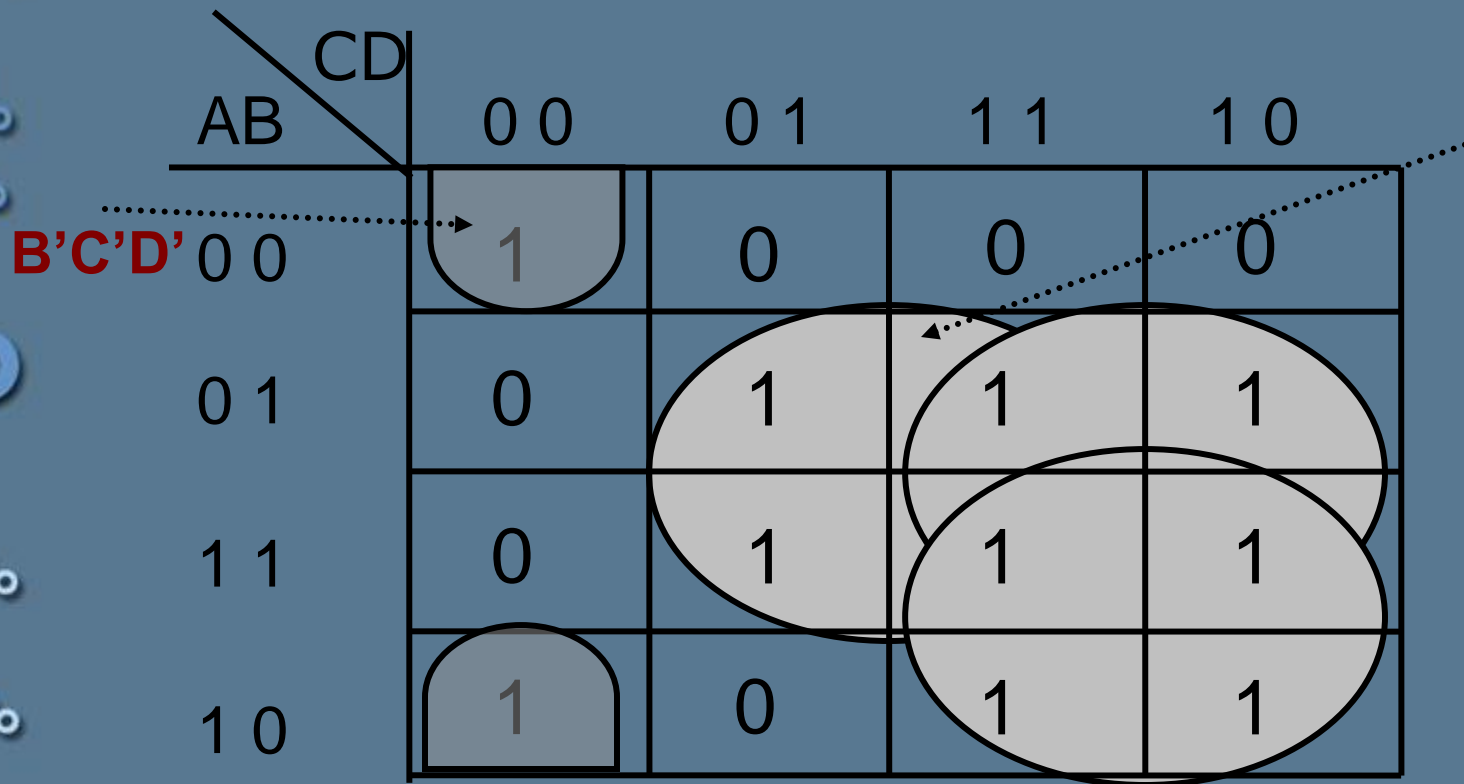
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$



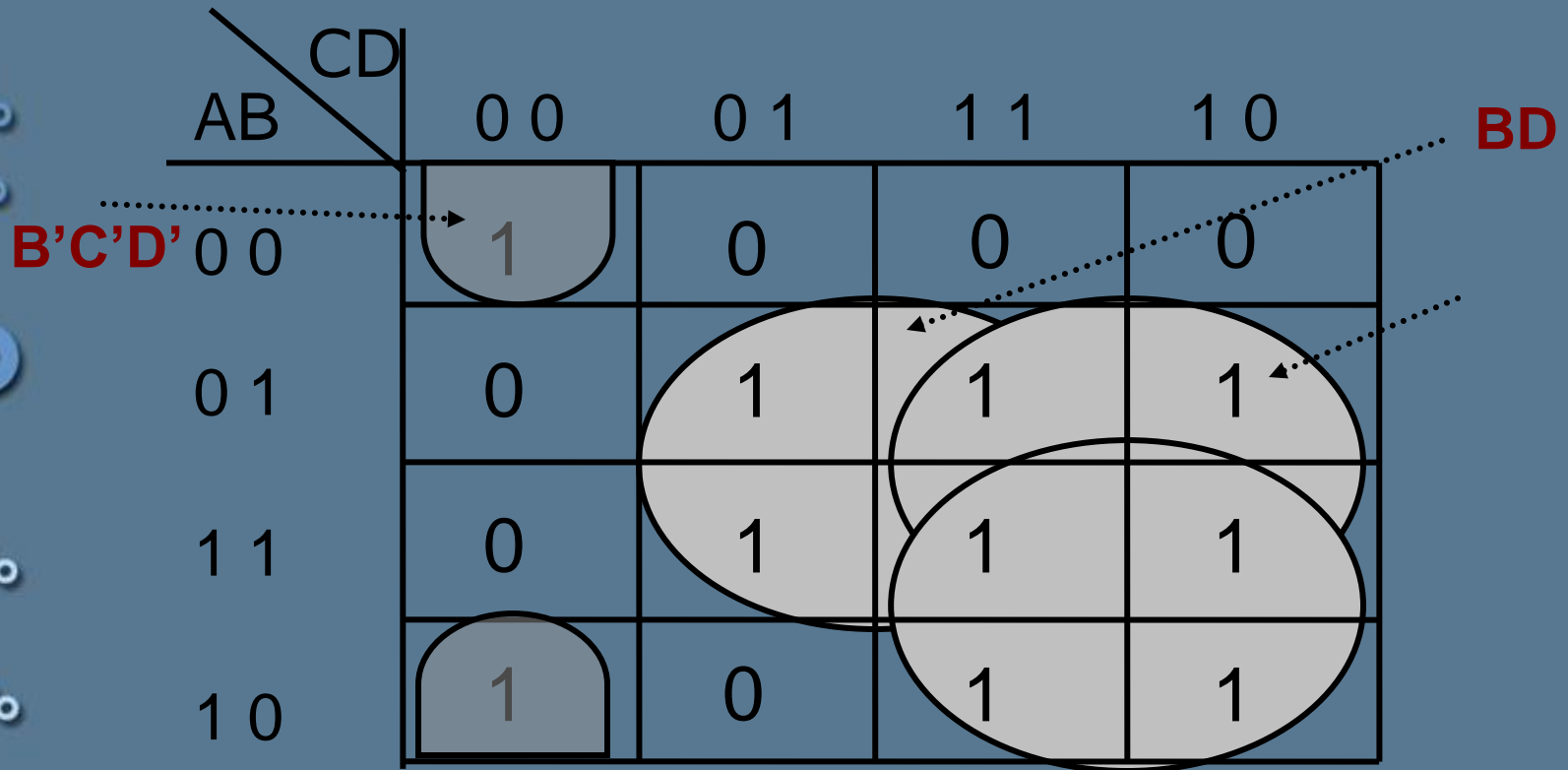
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$



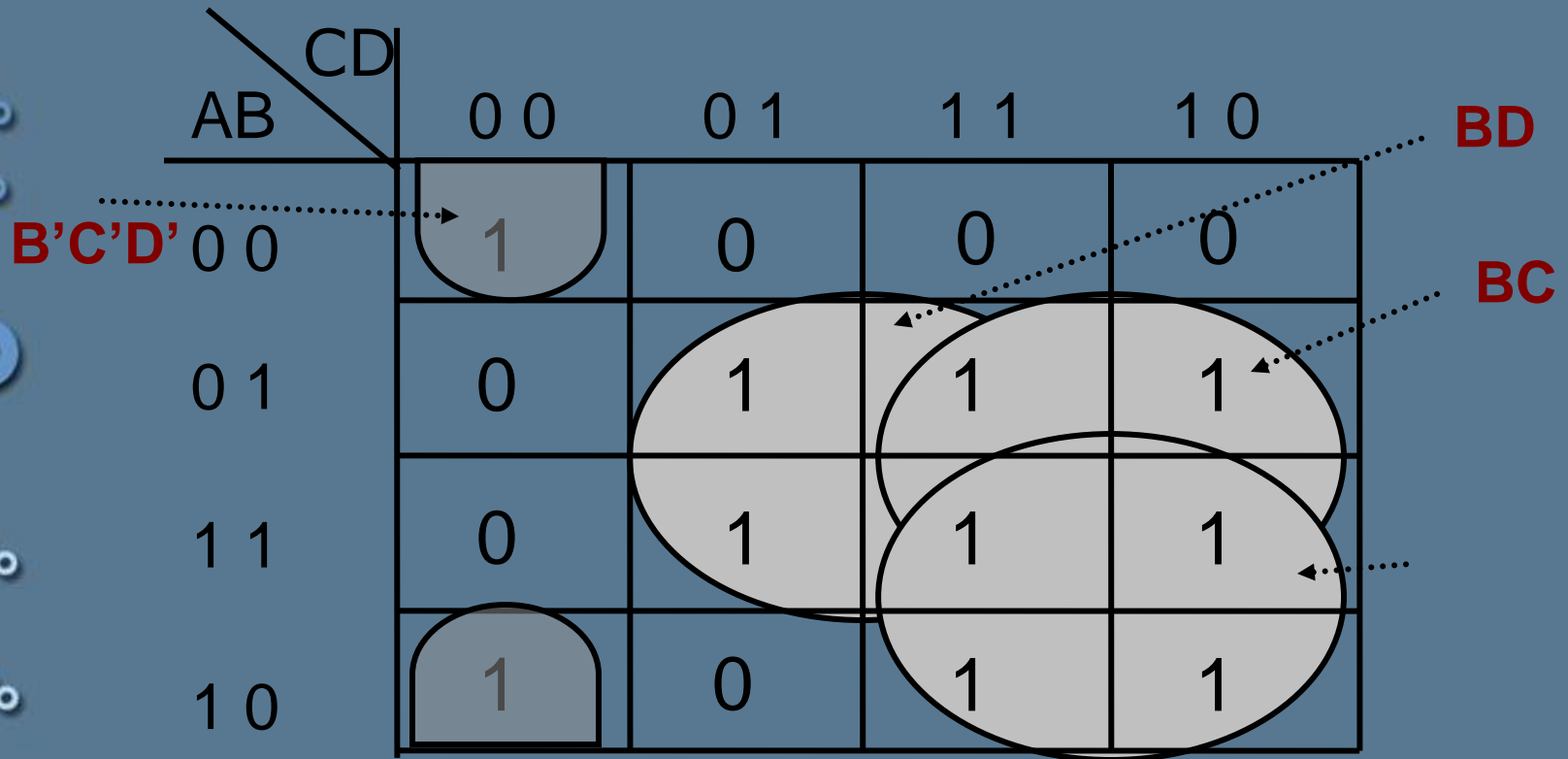
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$



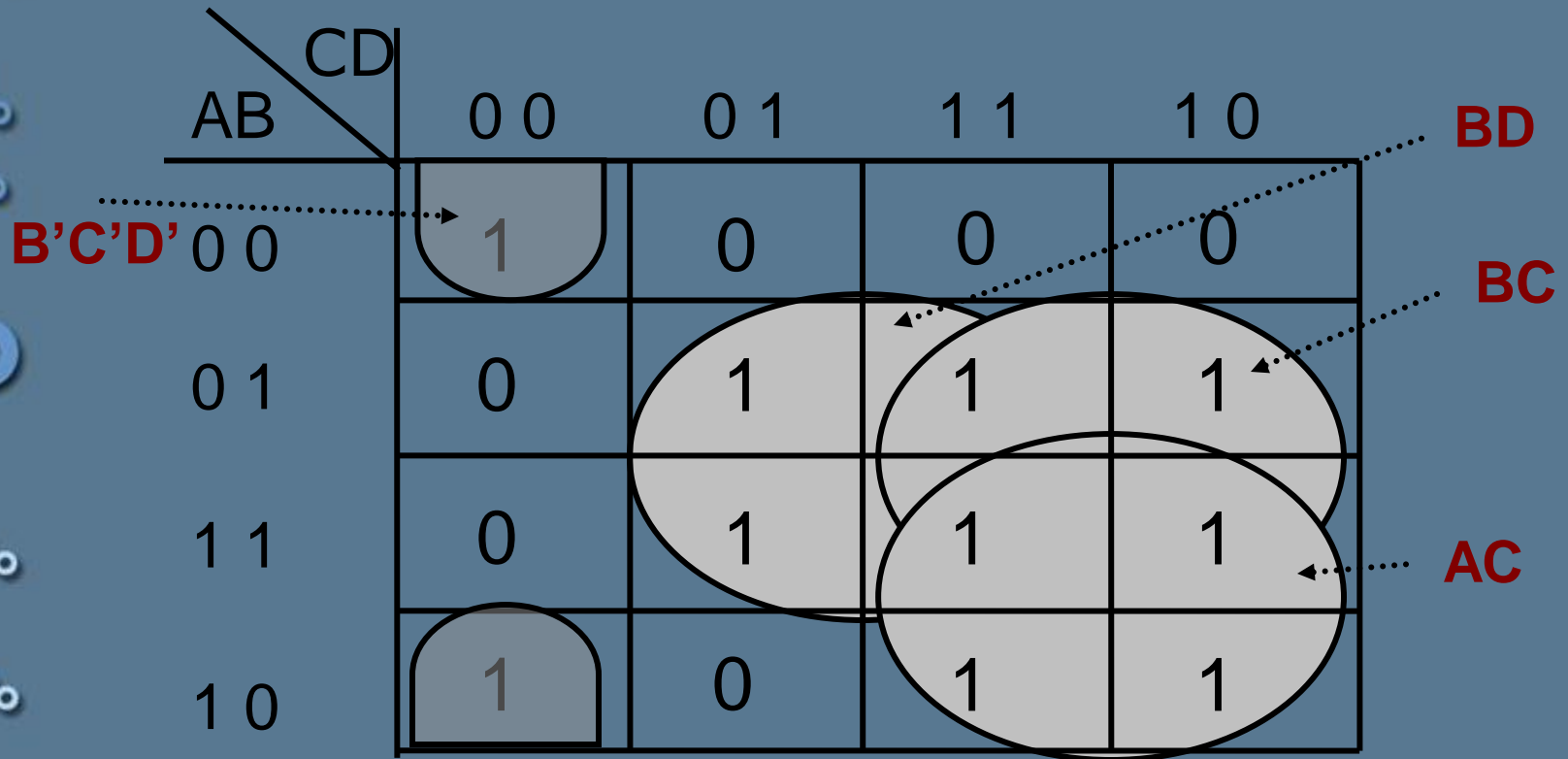
Converting Between POM & SOM Using K-Map

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$



Converting Between POM & SOM Using K-Map

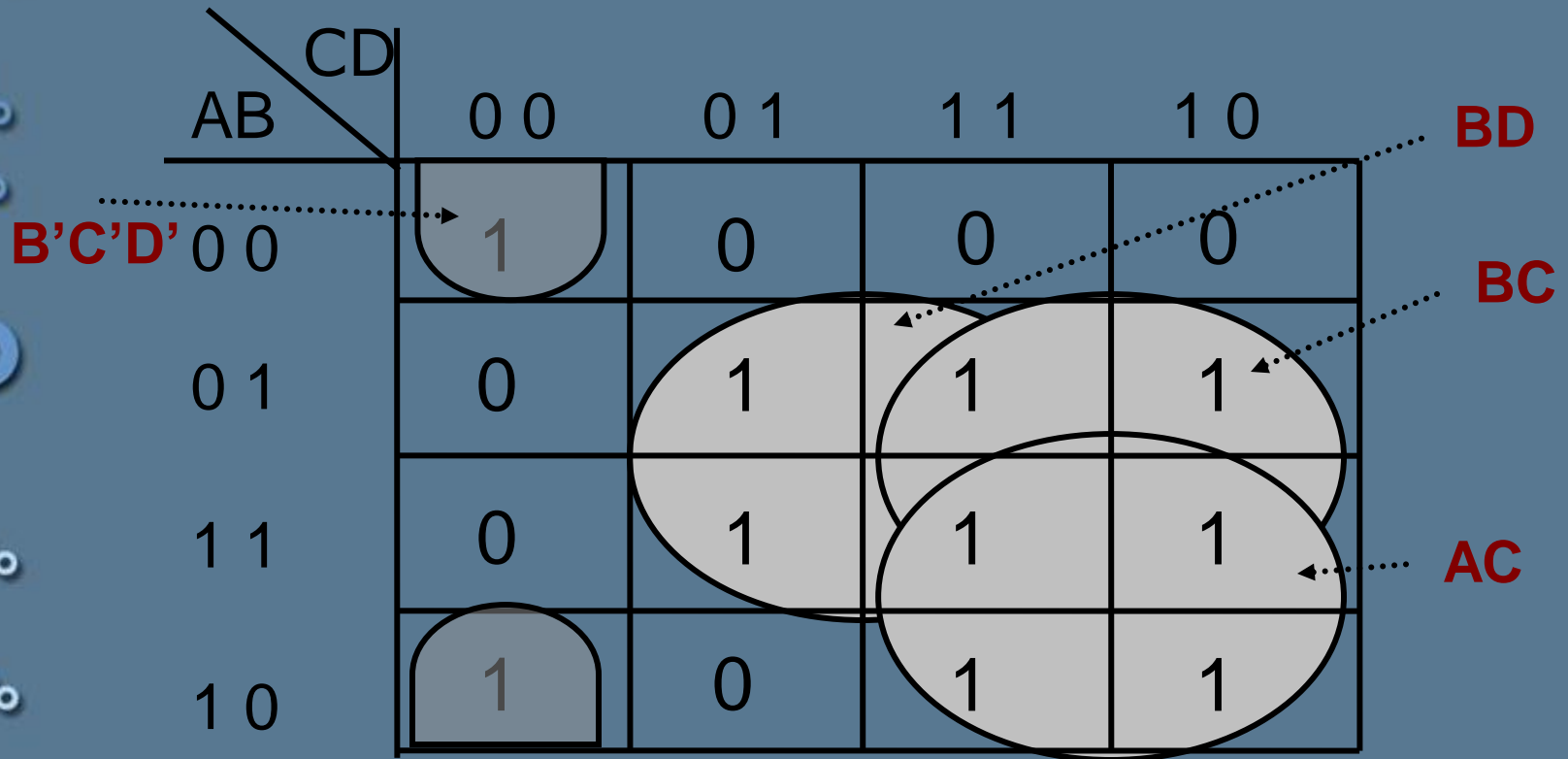
- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$



Converting Between POM & SOM Using K-Map

$$= B'C'D' + BD + BC + AC$$

- $(A'+B'+C+D) (A+B'+C+D) (A+B+C+D')$
 $(A+B+C'+D') (A'+B+C+D') (A+B+C'+D)$





Don't Care conditions

- The unspecified minterms (maxterms) of an incompletely specified function
- An **X** inside a map represents a don't care condition



Don't Care conditions

- There are 2 cases when this occurs.
 - The input combination never occurs
 - E.g. The BCD code does not use the 6 remaining codes.
 - The input combinations are expected to occur, but we do not care what the outputs are

Representation of Don't Cares

- e.g:

$$F(W,X,Y,Z) = \sum m(0,1,2,4,6,7,8,10)$$

$$d(W,X,Y,Z) = \sum d(12,13,14,15)$$

- It could also be represented as:

$$F(W,X,Y,Z) = \sum m(0,1,2,4,6,7,8,10) \\ + \sum d(12,13,14,15)$$

Example

- Simplify $F = \sum m(1, 3, 7, 14, 15) + d(8)$

AB \ CD	CD			
	00	01	11	10
00		1	1	
01			1	
11			1	1
10	X			

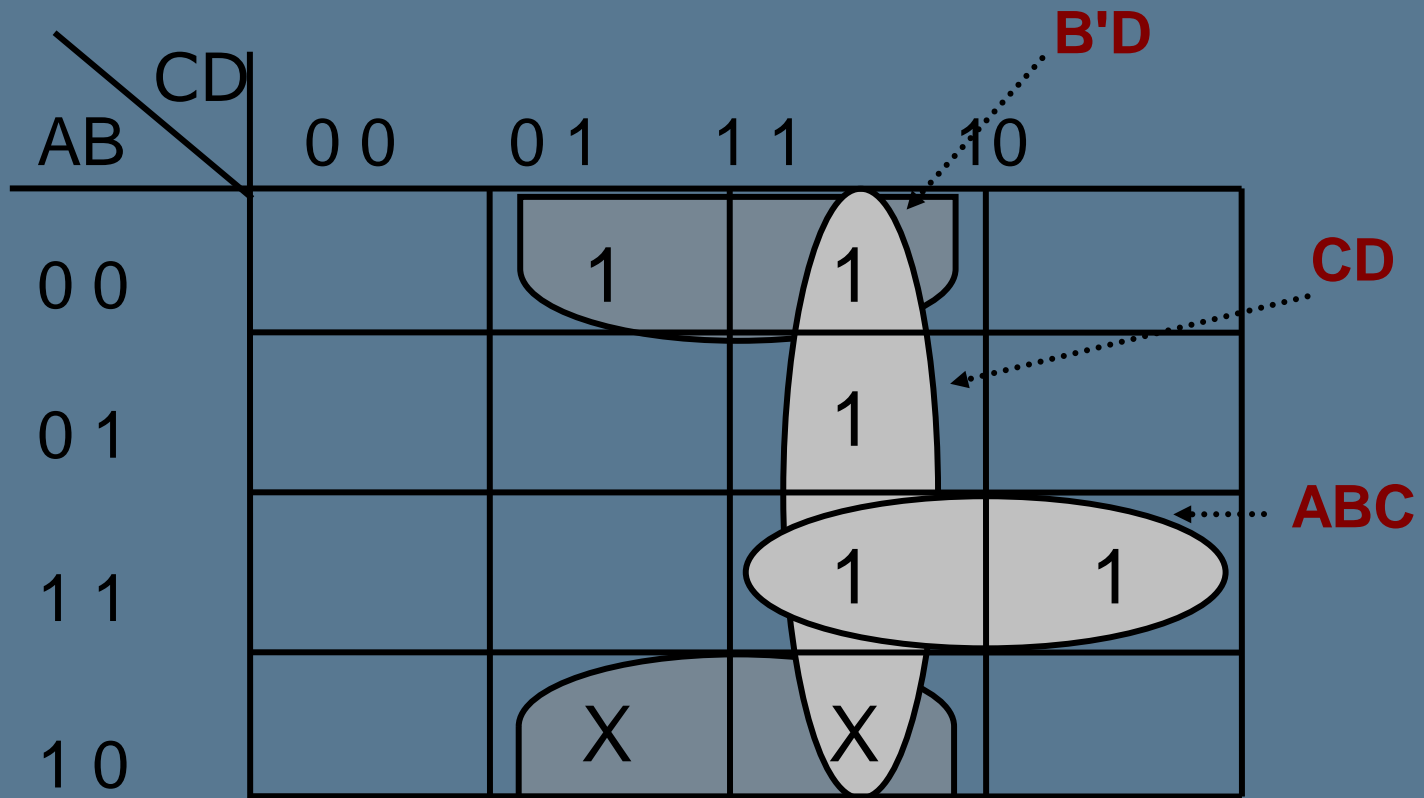
A'B'D (points to the group of 1s in row 00, columns 01 and 11)

BCD (points to the group of 1s in column 11, rows 01 and 11)

ABC (points to the group of 1s in row 11, columns 11 and 10)

Example

- Simplify $F = \sum m(1, 3, 7, 14, 15) + d(9, 11)$



Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0				
0 1				
1 1				
1 0				

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1			1
0 1	1			
1 1	1			
1 0	1	1		

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1			1
0 1	1			X
1 1	1	X	X	
1 0	1	1		

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD			
	0 0	0 1	1 1	1 0
0 0	1			1
0 1	1			X
1 1	1	X	X	
1 0	1	1		

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

CD \ AB		00	01	11	10
AB	00	1			1
	01	1			X
	11	1	X	X	
	10	1	1		

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

CD \ AB		00	01	11	10
AB	00	1			1
	01	1			X
	11	1	X	X	
	10	1	1		

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1				X
1 1	1	X	X		
1 0	1	1			

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1				X
1 1	1	X	X		
1 0	1	1			

Diagram illustrating the Karnaugh map for the function $F(A,B,C,D)$. The map shows minterms (1) and don't care terms (X) for the function $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$.

The map is a 4x4 grid with rows labeled AB and columns labeled CD. The cells contain the values 1 or X.

Groupings shown:

- A group of four cells (1s) is circled, corresponding to the term $A'D'$.
- A group of four cells (1s and Xs) is circled, corresponding to the term AB' .
- A group of four cells (1s and Xs) is circled, corresponding to the term BC .
- A group of two cells (1s) is circled, corresponding to the term CD .

Example

- $F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15)$

AB \ CD	CD				
	0 0	0 1	1 1	1 0	
0 0	1			1	←... A'D'
0 1	1			X	
1 1	1	X	X		←..... AC'
1 0	1	1			

Example

- $$F(A,B,C,D) = \sum m(0,2,4,8,9,12) + \sum d(6,13,15) = A'D' + AC'$$

AB \ CD		CD			
		0 0	0 1	1 1	1 0
0 0	1				1
0 1	1				X
1 1	1	X	X		
1 0	1	1			

Diagram illustrating the Karnaugh map for the function $F(A,B,C,D)$. The map shows the minterms (1s) and don't care terms (Xs) for the function. The prime implicants are highlighted by circles:

- The prime implicant $A'D'$ is highlighted by a circle around the minterms 00 and 01.
- The prime implicant AC' is highlighted by a circle around the minterms 11 and 10.