4.1 Triple Integral in Rectangular Coordinates

Let $w = f(x,y,z)$ be defined (and continuous) over a solid S.

The triple integral of f over a solid in R^3 is given by

$$\iiint_S f(x,y,z)\,dV$$

where dV has six possible orders of integration.

Evaluating Triple Integrals

$$\int_1^2\int_0^2\int_0^1 (4z - xy)\,dx\,dy\,dz$$

$$= \int_1^2\int_0^2 \left[4xz - \frac{1}{2}x^2 y\right]_0^1 dy\,dz$$

$$= \int_1^2\int_0^2 \left[4z - \frac{1}{2}y\right] dy\,dz$$

$$= \int_1^2 \left(4yz - \frac{y^2}{4}\right)_0^2 dz$$

$$\int_1^2 \left(4yz - \frac{y^2}{4}\right)_0^2 dz = \int_1^2 (8z - 1)\,dz$$

$$= \left(4z^2 - z\right)_1^2$$

$$= \left(4(2)^2 - 2\right) - \left(4(1)^2 - 1\right)$$

$$= 14 - 3 = 11$$

$$\int_1^2\int_0^2\int_0^1 (4z - xy)\,dx\,dy\,dz = 11$$

END