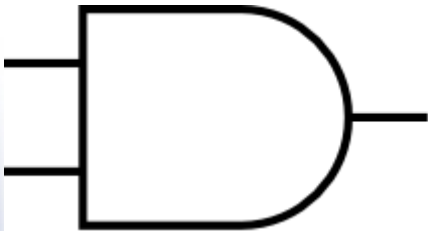


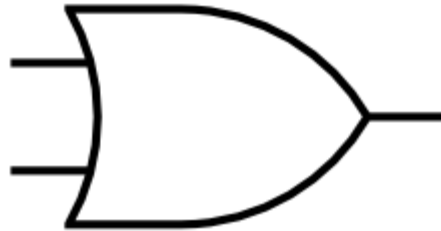
Chapter 3

Boolean Algebra, Logic Functions, and Logic Gates

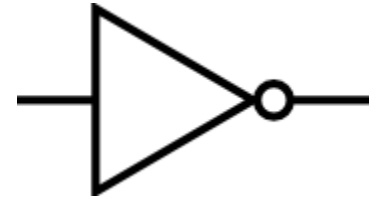
Digital Logic Gates



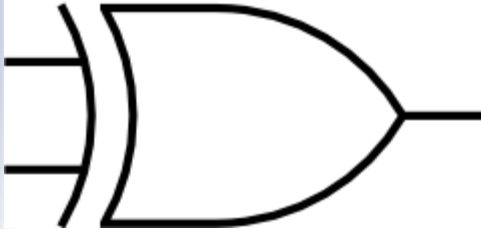
AND Gate



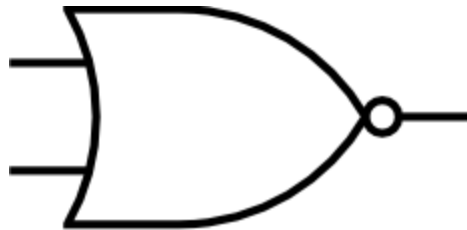
OR Gate



NOT Gate



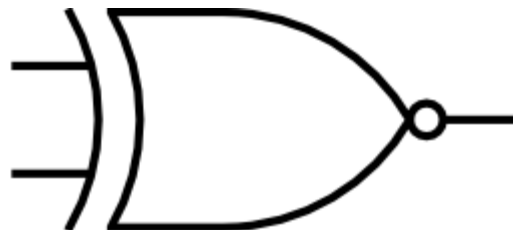
XOR Gate



NOR Gate



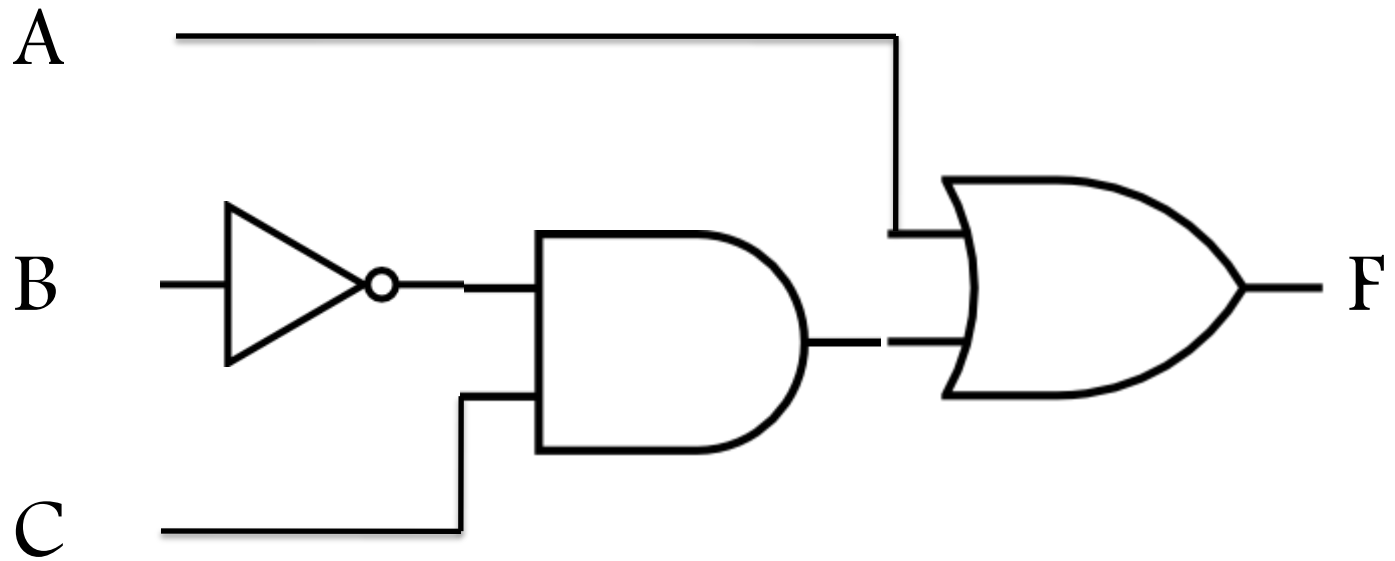
NAND Gate



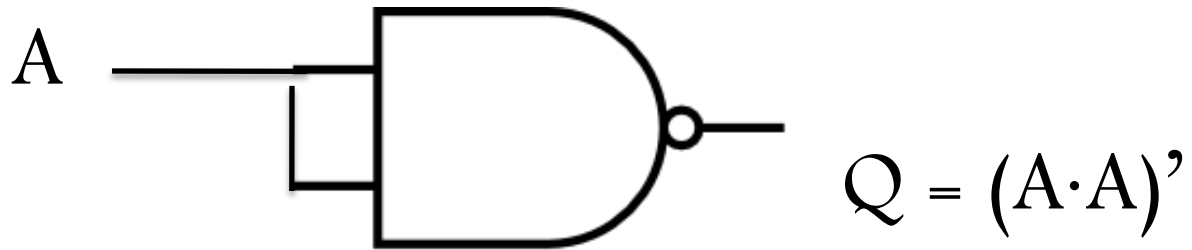
XNOR Gate

Digital Logic gates

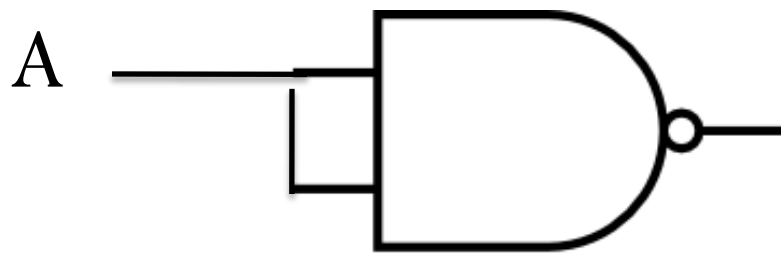
- Draw the logic diagram of the function $F = A + B'C$



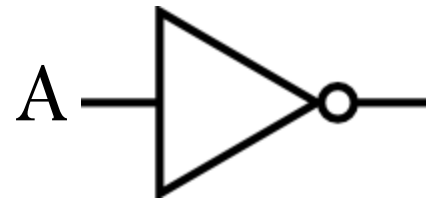
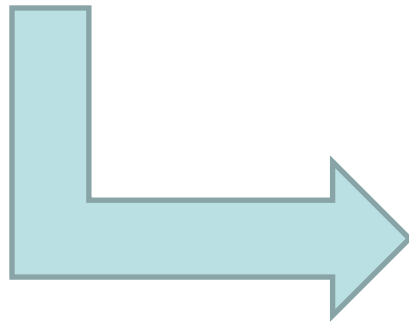
Universality of a NAND gate



Universality of a NAND gate

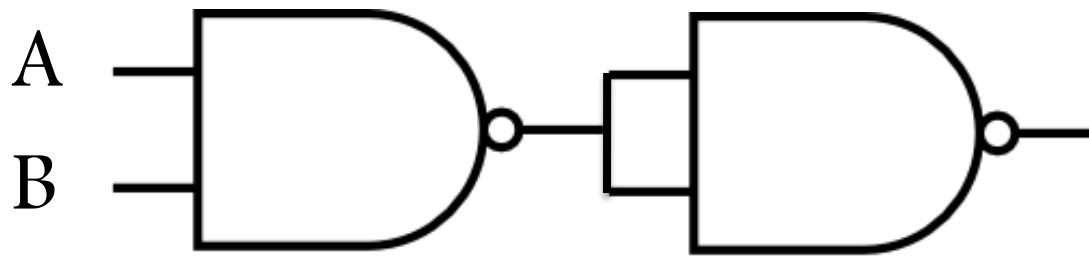


$$Q = (A \cdot A)'$$

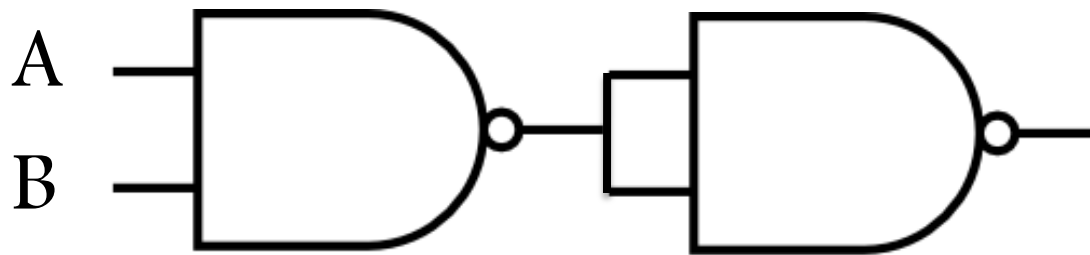


$$Q = A'$$

Universality of a NAND gate

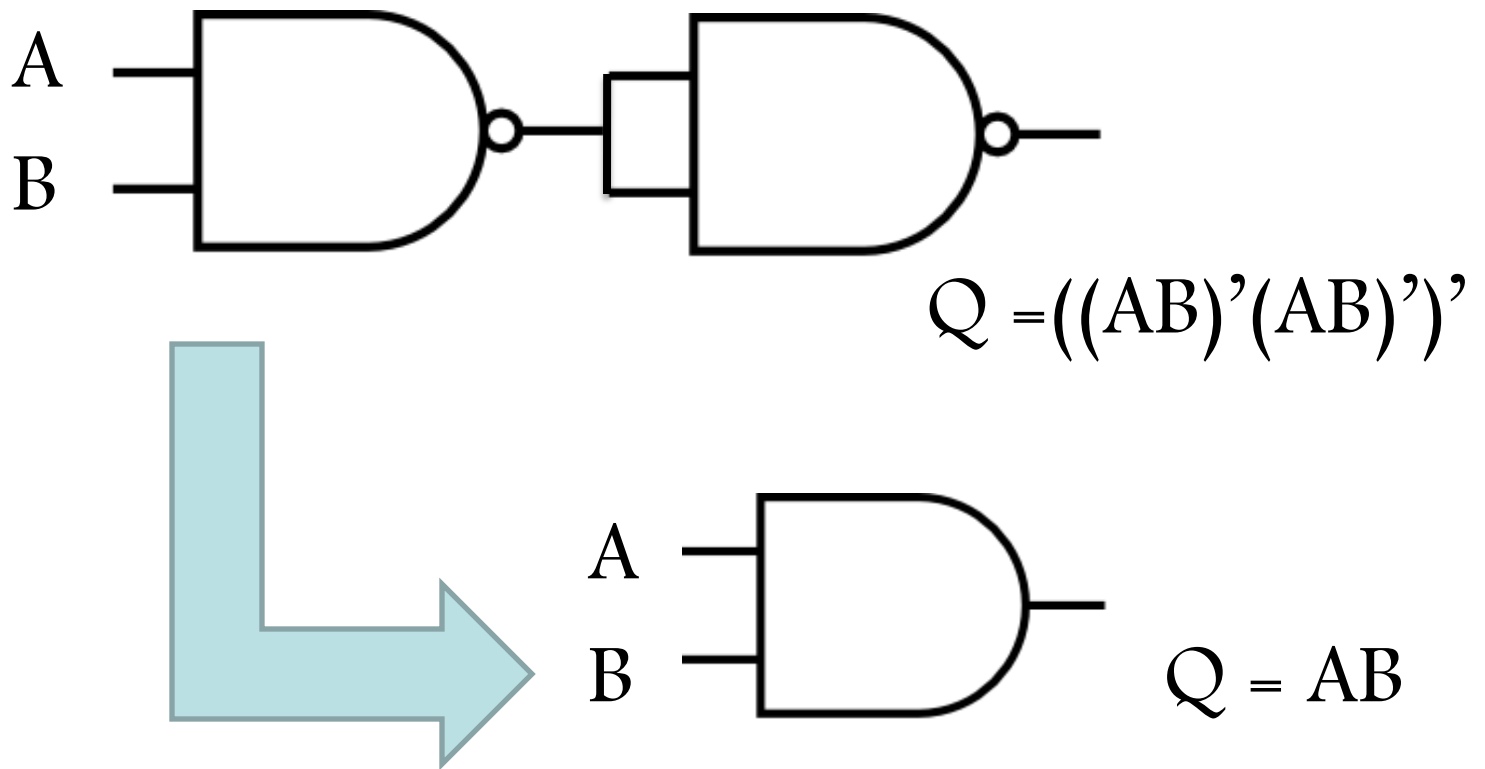


Universality of a NAND gate

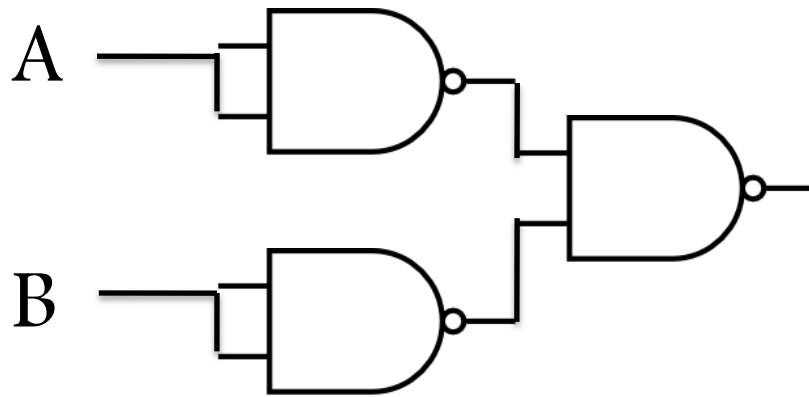


$$Q = ((AB)'(AB)')'$$

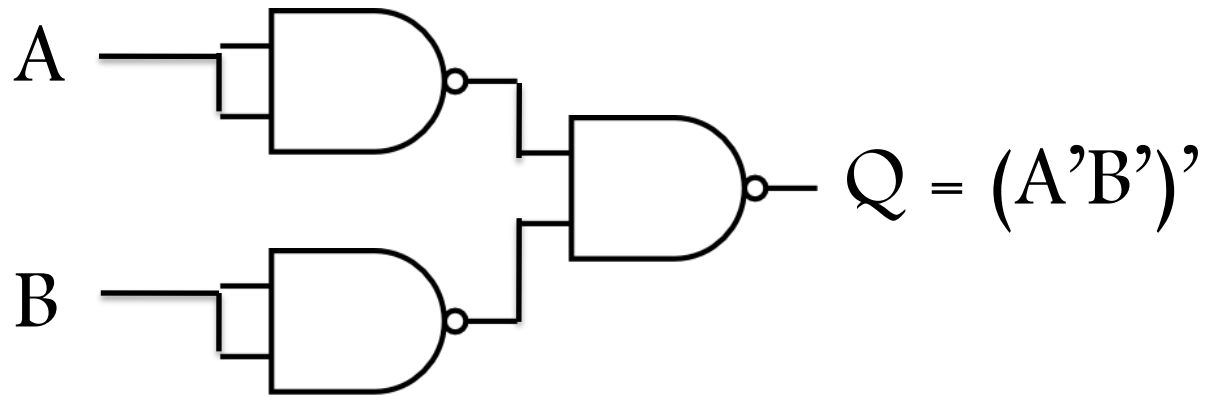
Universality of a NAND gate



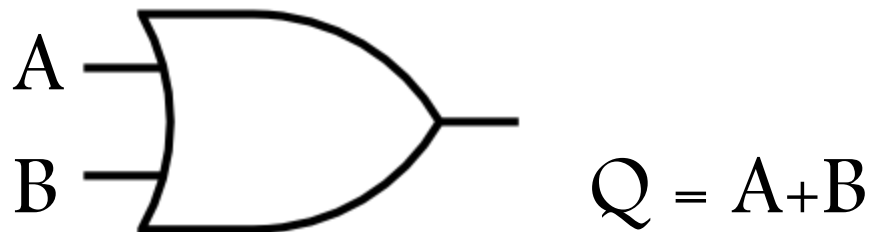
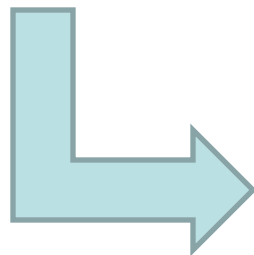
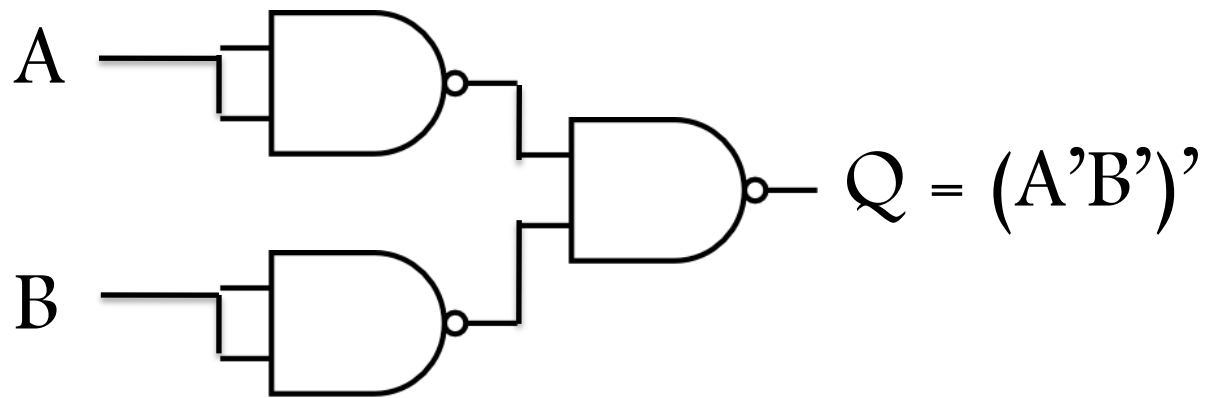
Universality of a NAND gate



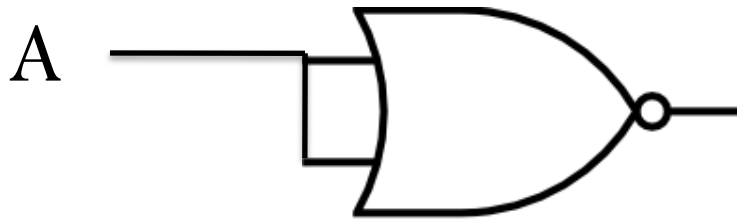
Universality of a NAND gate



Universality of a NAND gate

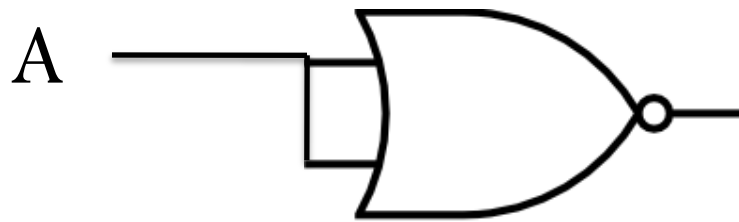


Universality of a NOR gate

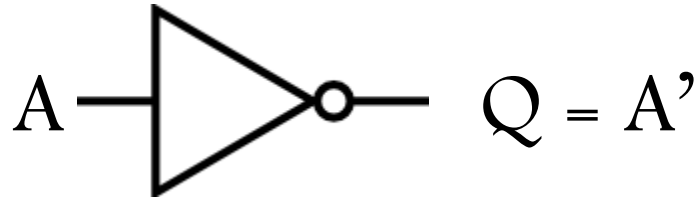
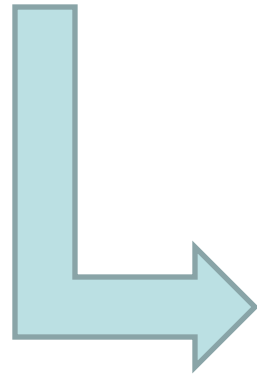


$$Q = (A+A)'$$

Universality of a NOR gate

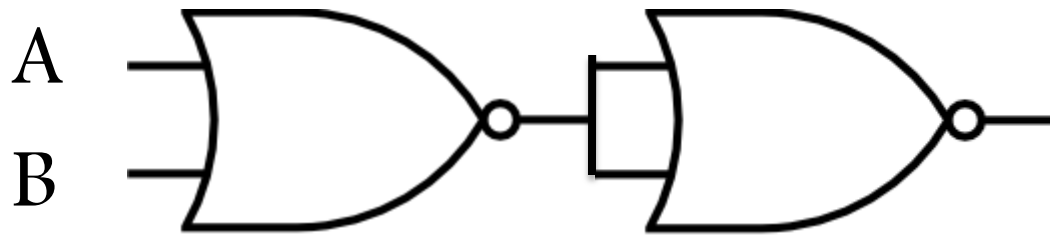


$$Q = (A + A)'$$

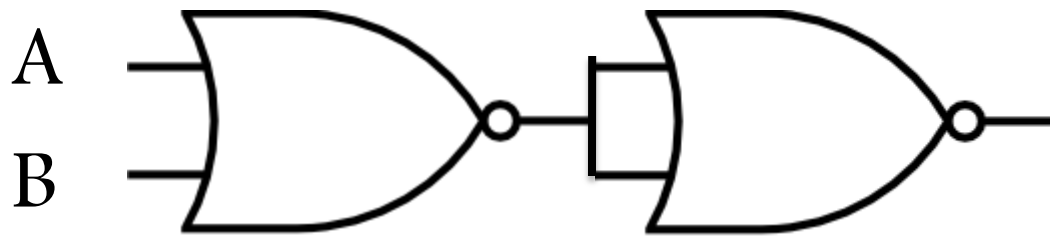


$$Q = A'$$

Universality of a NOR gate

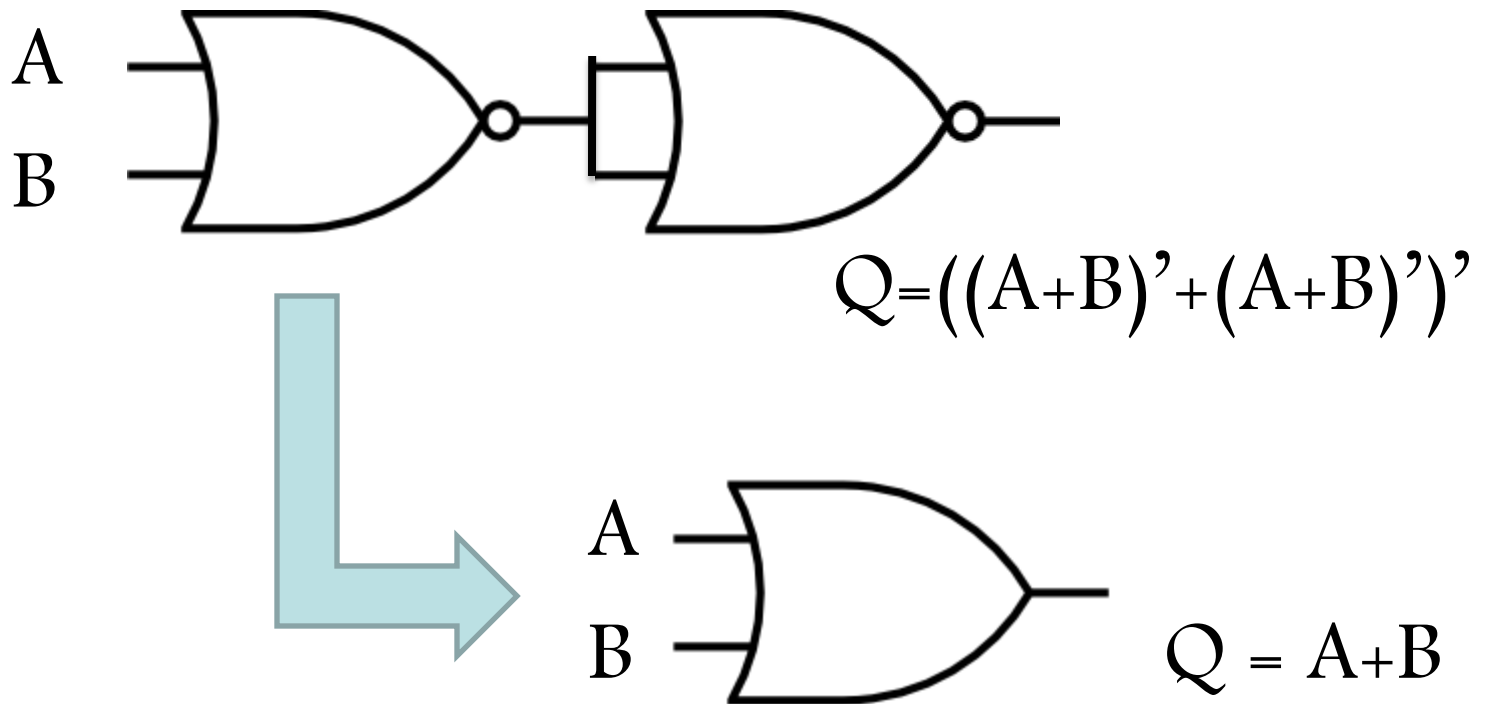


Universality of a NOR gate

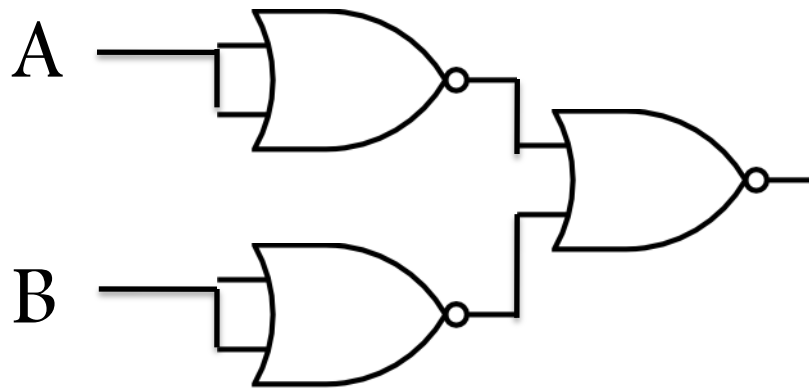


$$Q = ((A+B)' + (A+B)')'$$

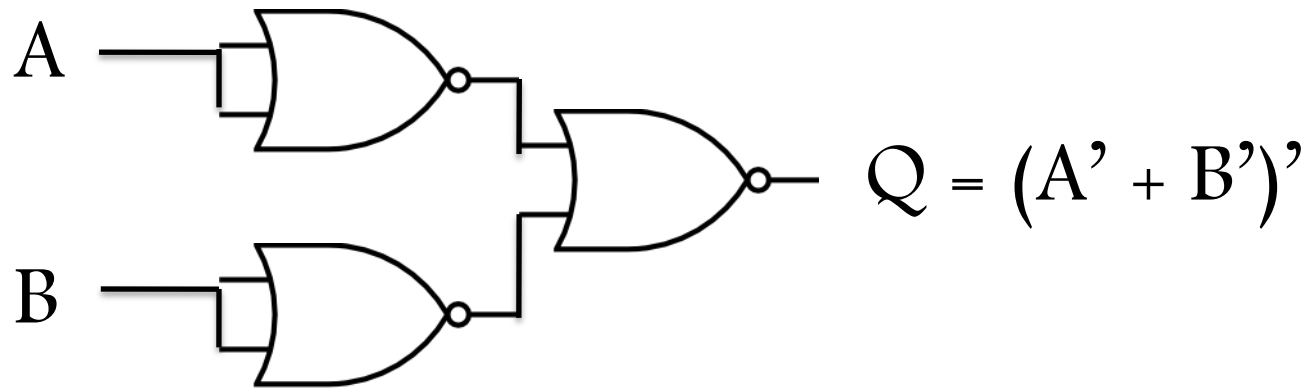
Universality of a NOR gate



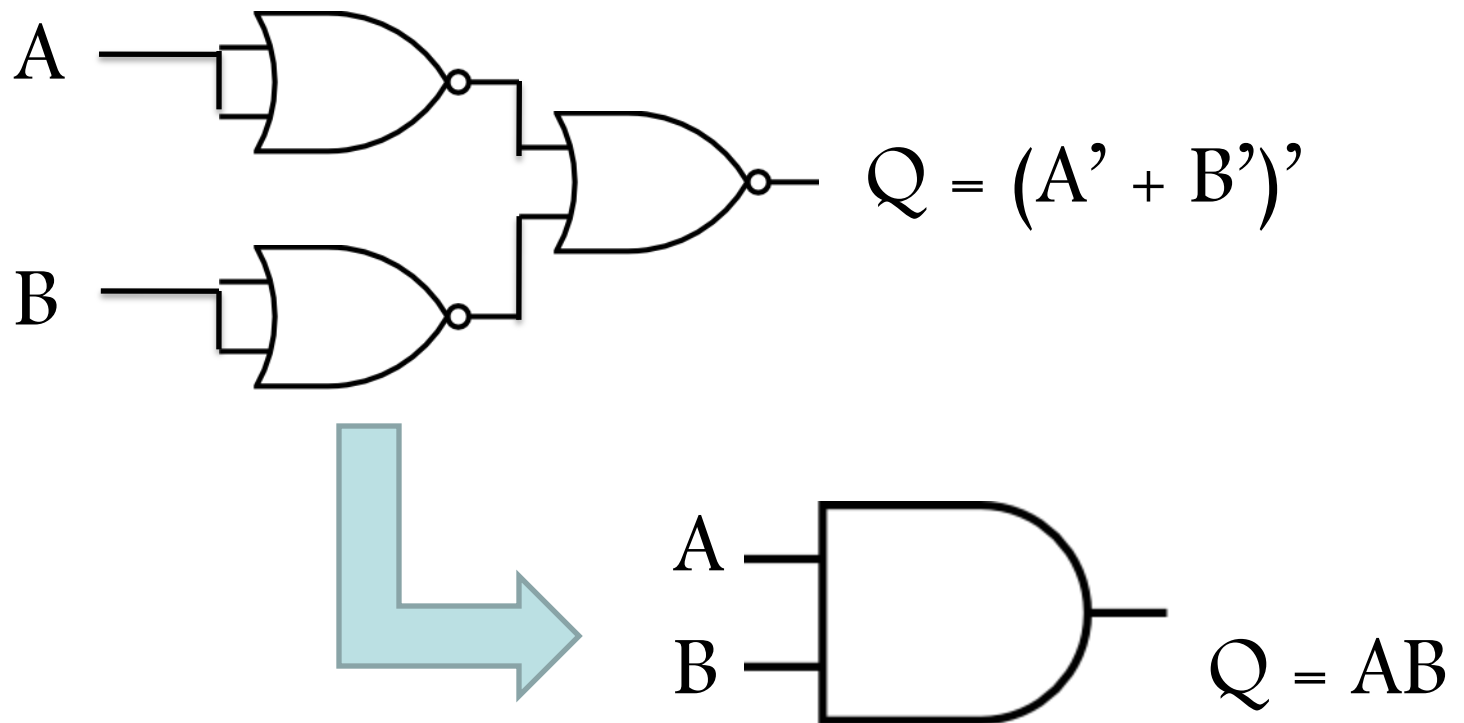
Universality of a NOR gate



Universality of a NOR gate



Universality of a NOR gate



Real Gates

- Logic gates are integrated form
 - Built within a solid piece of silicon called IC (Integrated Circuit)



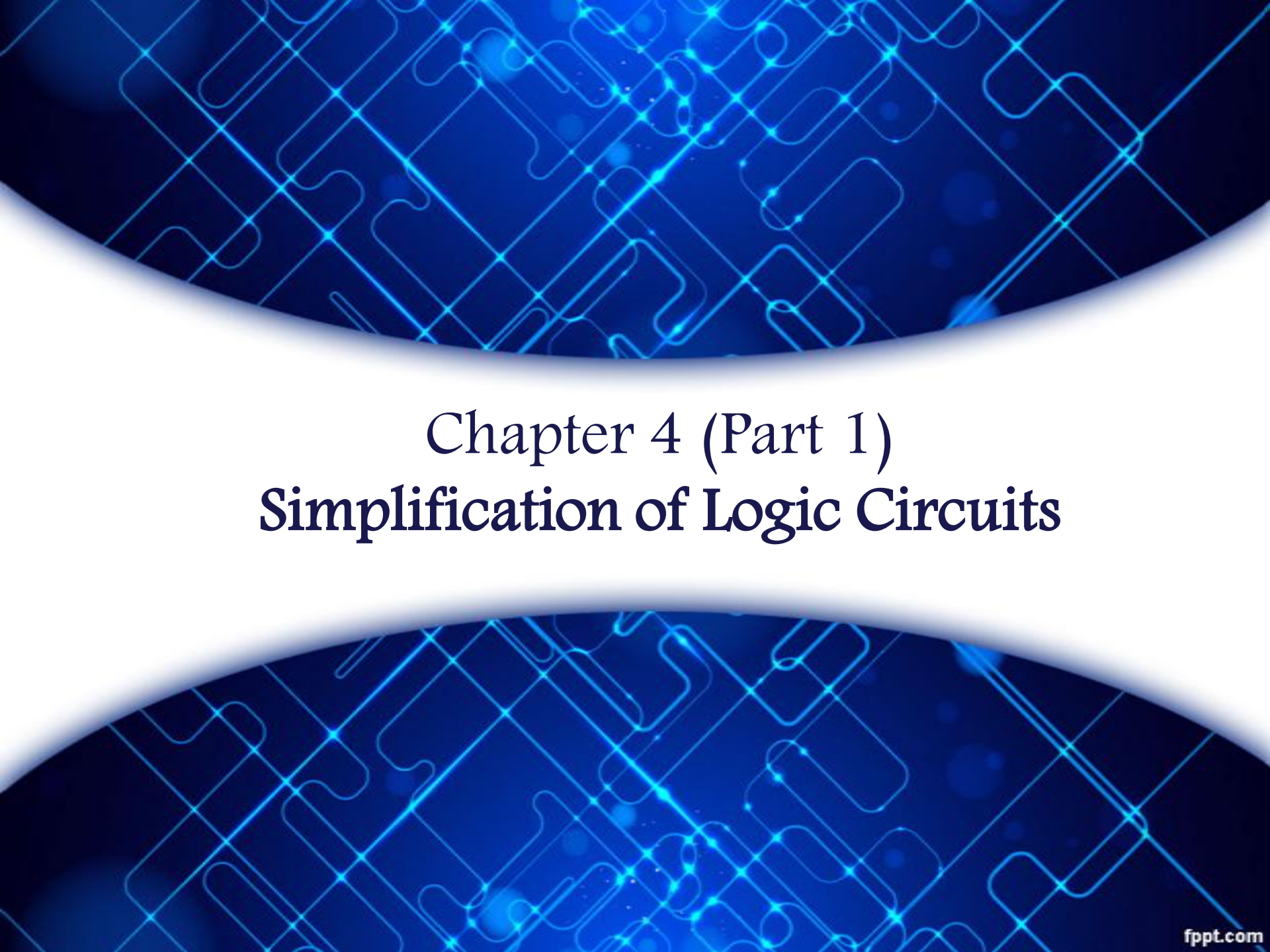
- Several gates are included in a single plastic moulding

IC Families

- Transistor–Transistor Logic (TTL)
- Emitter Coupled Logic (ECL)
- Complementary Metal–Oxide–Semiconductor (CMOS)

Levels of IC

- Small-scale Integration
 - ICs with 1 to 10 gates
- Medium-scale Integration
 - ICs with 10 to 100 gates
- Large-scale Integration
 - ICs with 100 to 1000s of gates
- Very large-scale Integration
 - ICs with 1000s to millions of gates



Chapter 4 (Part 1)

Simplification of Logic Circuits

Recall: Representations of Boolean Functions

- Truth Table
 - unique

Example:

x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

- Algebraic expression
 - not unique
 - convenient for manipulation

Example: $F = x + y$

- Logic circuits
 - not unique
 - close to implementation

Simplification of Boolean Functions

- Simpler circuit is faster
- Simpler circuit is less expensive
- Reduce complexity of the gate level implementation
- Reduce signal propagation delays

Ways to simplify Boolean functions

- Boolean Algebra
- Graphical method (Karnaugh Map)
- Tabular method (Quine–McCluskey)

Simplification: Boolean Algebra

Simplify: $x'y' + xyz + x'y$

Simplification: Boolean Algebra

Simplify: $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.

Simplification: Boolean Algebra

Simplify: $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.

$$= x' + xyz$$

Inv / Identity

Simplification: Boolean Algebra

Simplify: $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.

$$= x' + xyz$$

Inv / Identity

$$= (x' + x)(x' + yz)$$

Dist.

Simplification: Boolean Algebra

Simplify: $x'y' + xyz + x'y$

$$= x'(y' + y) + xyz$$

Comm / Dist.

$$= x' + xyz$$

Inv / Identity

$$= (x' + x)(x' + yz)$$

Dist.

$$= x' + yz$$

Inv / Identity

Simplification: Boolean Algebra

Simplify: $AB + A(B+C) + B(B+C)$

Simplification: Boolean Algebra

Simplify: $AB + A(B+C) + B(B+C)$
 $= AB + AB + AC + BB + BC$ *Dist.*

Simplification: Boolean Algebra

Simplify: $AB + A(B+C) + B(B+C)$

$= AB + AB + AC + BB + BC$ *Dist.*

$= AB + AC + B + BC$ *Idempotency*

Simplification: Boolean Algebra

Simplify: $AB + A(B+C) + B(B+C)$

$= AB + AB + AC + BB + BC$ *Dist.*

$= AB + AC + B + BC$ *Idempotency*

$= AB + AC + B$ *Absorption*

Simplification: Boolean Algebra

Simplify: $AB + A(B+C) + B(B+C)$

$= AB + AB + AC + BB + BC$ *Dist.*

$= AB + AC + B + BC$ *Idempotency*

$= AB + AC + B$ *Absorption*

$= B + AC$ *Absorption*

Simplification: Graphical method

- Karnaugh map (K-map)
 - alternate way of representing Boolean functions
 - a graphical tool for assisting in the general simplification procedure
 - A simpler way to handle most jobs of manipulating logic functions

General Steps of K-Map Simplification

- Express function in canonical form
- Map expression on a K-Map
- Group 1's or 0's
- Determine the minimum expression

Step 1: Function in Canonical form

- Sum of minterms
 - $F(x,y) = x'y' + xy = \Sigma(0,3)$
 - $G(a,b,c) = a'b'c + abc' + a'bc = \Sigma(1,3,6)$
- Product of maxterms
 - $H(x,y) = (x+y')(x'+y) = \Pi(1,2)$
 - $I(a,b,c) = (a+b+c)(a'+b+c') = \Pi(0,5)$

Step 2: Map Expression

X \ Y	0	1
0	m_0	m_1
1	m_2	m_3

Two-variable map

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Three-variable map

Step 2: Map Expression

WX \ YZ		YZ			
		00	01	11	10
WX	00	m_0	m_1	m_3	m_2
	01	m_4	m_5	m_7	m_6
	11	m_{12}	m_{13}	m_{15}	m_{14}
	10	m_8	m_9	m_{11}	m_{10}

Four-variable map

Mapping Example

- $F = x'y'z + x'yz' + xyz' + xyz$

X \ YZ		YZ			
		00	01	11	10
X	0		1		1
	1			1	1

Mapping Example

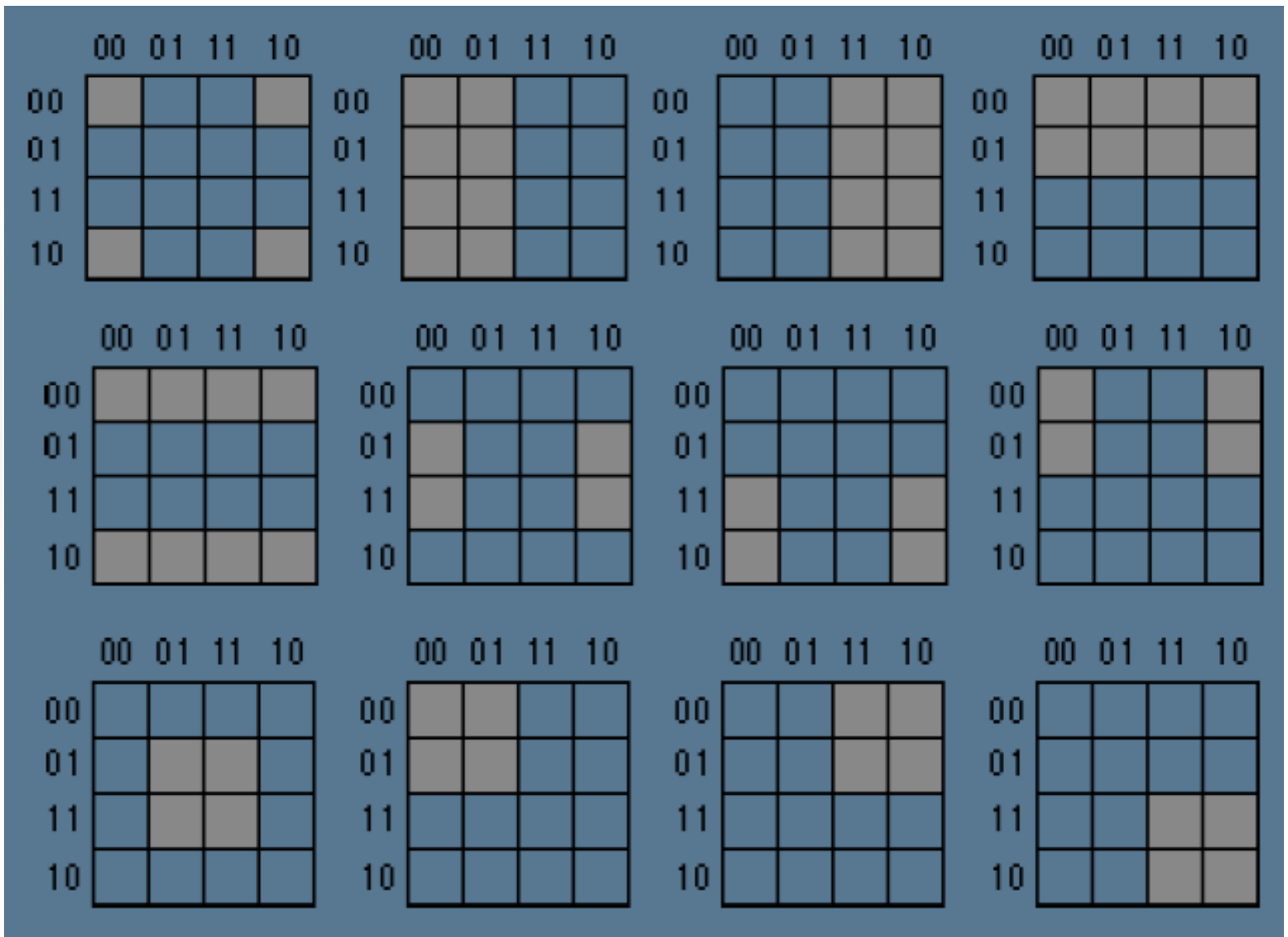
- $G = \Sigma (2,4,6,8,12,14,15)$

WX \ YZ		YZ			
		00	01	11	10
00					1
01	1				1
11	1			1	1
10	1				

Step 3: Group 1's (or 0's)

- Grouping Rules:
 - A group must contain either 1,2,4,8 and 16 cells.
 - Each cell in a group must be adjacent to one or more cells in that same group.
 - Always include the largest possible group in accordance with the first rule.
 - Each element of a group must be included in at least one group.

Sample Groupings: Four-variable map



Mapping Example

- $F = x'y'z + x'yz' + xyz' + xyz$

		YZ			
		00	01	11	10
X	0		1		1
	1			1	1

Mapping Example

- $G = \Sigma (2,4,6,8,12,14,15)$

A 4x4 Karnaugh map for a 4-variable function. The horizontal axis is labeled 'YZ' with values 00, 01, 11, 10. The vertical axis is labeled 'WX' with values 00, 01, 11, 10. The map contains 1s in the following cells: (WX=00, YZ=10), (WX=01, YZ=00), (WX=01, YZ=10), (WX=11, YZ=00), (WX=11, YZ=11), (WX=10, YZ=00), and (WX=10, YZ=11). There are four groupings: a vertical group of four 1s in the YZ=10 column, a horizontal group of four 1s in the WX=01 row, a horizontal group of two 1s in the WX=11 row (YZ=00 and YZ=11), and a vertical group of two 1s in the WX=10 column (YZ=00 and YZ=11).

WX \ YZ	00	01	11	10
00				1
01	1			1
11	1		1	1
10	1			

Grouping Examples

A \ BC				
	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC				
	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC	BC			
	00	01	11	10
0	1		1	
1			1	1

Grouping Examples

A \ BC		BC			
A		00	01	11	10
0		1		1	
				1	1
1					

Grouping Examples

A \ BC				
	00	01	11	10
0	1	1		1
1	1	1	1	

Grouping Examples

<div><div>BC</div><div>A</div></div>					
		00	01	11	10
A	0	1	1		1
	1	1	1	1	

Grouping Examples

A \ BC		00	01	11	10
0	1	1			1
1	1	1	1		

The Karnaugh map shows the function F(A,B,C) = A + B + C. The groupings are as follows:

- A circle groups the four 1s in the first two columns (BC = 00 and 01), representing the term B.
- A circle groups the four 1s in the first two rows (A = 0 and 1), representing the term A.
- A circle groups the two 1s in the last column (C = 10), representing the term C.

Grouping Examples

$A \backslash BC$		BC			
		00	01	11	10
A	0	1	1		1
	1	1	1	1	

The Karnaugh map shows the following groupings:

- A group of four 1s in the first two rows (A=0 and A=1) and the first two columns (BC=00 and BC=01), forming a 2x2 square.
- A group of two 1s in the first row (A=0) and the first and third columns (BC=00 and BC=10), forming a horizontal group.
- A group of two 1s in the second row (A=1) and the second and third columns (BC=01 and BC=11), forming a horizontal group.

Grouping Examples

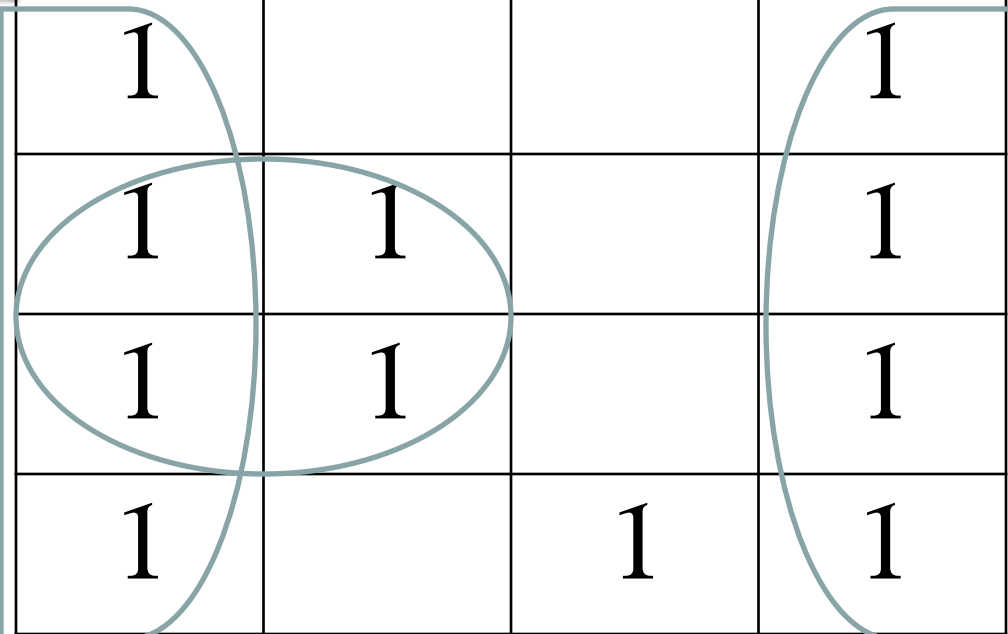
AB \ CD	CD			
	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

Grouping Examples

AB \ CD	CD			
	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

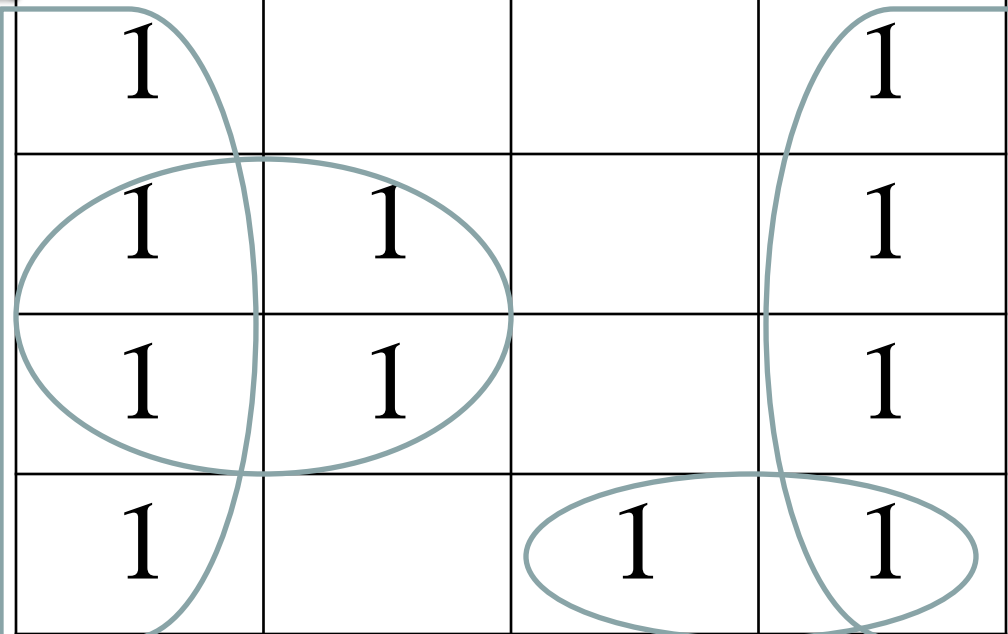
Grouping Examples

AB \ CD	CD			
	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

A Karnaugh map for a 4-variable function with variables AB and CD. The map is a 4x4 grid. The columns are labeled 00, 01, 11, and 10. The rows are labeled 00, 01, 11, and 10. The cells contain 1s at (00,00), (00,10), (01,00), (01,01), (01,11), (01,10), (11,10), and (10,00), (10,10). There are three groupings: a vertical group of four 1s in the first column (00) from row 00 to 10, a horizontal group of four 1s in the second column (01) from row 00 to 10, and a vertical group of four 1s in the fourth column (10) from row 00 to 10.

Grouping Examples

CD \ AB		00	01	11	10
00	1				1
01	1	1			1
11	1	1			1
10	1			1	1



The Karnaugh map shows four groupings of 1s:

- A vertical group of four 1s in the first column (AB=00), spanning CD values 00, 01, 11, and 10.
- A horizontal group of three 1s in the first row (CD=00), spanning AB values 00, 01, and 11.
- A horizontal group of three 1s in the first row (CD=00), spanning AB values 01, 11, and 10.
- A horizontal group of two 1s in the last row (CD=10), spanning AB values 11 and 10.

Step 4: Determine Minimum Expression

- Determine minimum terms

# of cells	# of vars
1	3
2	2
4	1
8	Fn's value is 1

three-variable map

# of cells	# of vars
1	4
2	3
3	2
8	1
16	Fn's value is 1

four-variable map

Determining Minimum Expression

Example

A \ BC	BC			
	00	01	11	10
0	1		1	
1			1	1

Determining Minimum Expression

Example

$A'B'C'$

$A \backslash BC$	00	01	11	10
0	1		1	
1			1	1

The Karnaugh map shows the function $A'B'C'$ with the following groupings and annotations:

- A blue arrow points from the label $A'B'C'$ to the cell at $A=0, BC=00$.
- A blue circle groups the cell at $A=0, BC=00$.
- A blue circle groups the cells at $A=0, BC=11$ and $A=1, BC=11$.
- A blue circle groups the cells at $A=1, BC=11$ and $A=1, BC=10$.
- A blue arrow points from the right edge of the map to the cell at $A=0, BC=11$.

Determining Minimum Expression

Example

$A'B'C'$

$A \backslash BC$	00	01	11	10
0	1		1	
1			1	1

BC

The Karnaugh map shows the function $A'B'C' + BC$. The first group, $A'B'C'$, consists of the cells where $A=0$ and $BC=00$ and $BC=11$. The second group, BC , consists of the cells where $BC=11$ and $BC=10$ and $A=1$.

Determining Minimum Expression

Example

$A'B'C'$

$A \backslash BC$	00	01	11	10
0	1		1	
1			1	1

BC

AB

Determining Minimum Expression

Example

<div>A \ BC</div>					
		00	01	11	10
0	1			1	
1				1	1

$A'B'C'$

BC

AB

$$= A'B'C' + BC + AB$$

Determining Minimum Expression

Example

<div><div>BC</div><div>A</div></div>		BC			
		00	01	11	10
A	0	1	1		1
	1	1	1	1	

Determining Minimum Expression

Example

BC					
A		00	01	11	10
0	1	1			1
1	1	1	1		

Determining Minimum Expression

Example

$\begin{array}{c} \text{BC} \\ \diagdown \\ \text{A} \end{array}$		BC			
		00	01	11	10
A	0	1	1		1
	1	1	1	1	

B'

$A'C'$

Determining Minimum Expression

Example

$A \backslash BC$					
		00	01	11	10
0	1	1			1
1	1	1	1		

B'

$A'C'$

AC

Determining Minimum Expression

Example

$A \backslash BC$					
		00	01	11	10
0	1	1			1
1	1	1	1		

Diagram illustrating the Karnaugh map for the function $F(A, B, C)$. The map shows the following groupings:

- Group 1 (Red): $A'C'$ (Circled 1s at $A=0, BC=00$ and $A=0, BC=10$)
- Group 2 (Red): AC (Circled 1s at $A=1, BC=00$ and $A=1, BC=01$)
- Group 3 (Red): B' (Circled 1s at $BC=00$ and $BC=01$)

$$= A'C' + AC + B'$$

Determining Minimum Expression

Example

CD \ AB		00	01	11	10
00	1				1
01	1	1			1
11	1	1			1
10	1			1	1

A Karnaugh map for variables A, B, C, and D. The map is a 4x4 grid with rows labeled AB (00, 01, 11, 10) and columns labeled CD (00, 01, 11, 10). The cells containing 1s are at (00,00), (00,10), (01,00), (01,01), (01,11), (10,00), (10,01), (10,11), and (10,10). The groups of 1s are: a vertical group of four 1s in the first column (AB=00), a horizontal group of four 1s in the first two columns (CD=01), a horizontal group of two 1s in the last two columns (CD=10), and a vertical group of two 1s in the last column (AB=10). A blue arrow points to the first column group.

Determining Minimum Expression

Example

CD \ AB		00	01	11	10
00	1				1
01	1	1			1
11	1	1			1
10	1			1	1

Diagram illustrating the Karnaugh map for determining the minimum expression. The map shows the following groupings:

- A vertical group of four 1s in the first column (AB=00), circled in light blue.
- A horizontal group of four 1s in the first row (CD=00), circled in light blue.
- A horizontal group of four 1s in the second row (CD=01), circled in light blue.
- A horizontal group of four 1s in the third row (CD=11), circled in light blue.
- A horizontal group of four 1s in the fourth row (CD=10), circled in light blue.
- A vertical group of four 1s in the last column (AB=10), circled in light blue.
- A horizontal group of four 1s in the first column (AB=00), circled in light blue.
- A horizontal group of four 1s in the last column (AB=10), circled in light blue.

Annotations:

- A blue arrow points from the label **D'** to the cell (AB=00, CD=00).
- A blue arrow points from the label **D'** to the cell (AB=01, CD=01).

Determining Minimum Expression

Example

CD \ AB		00	01	11	10
00	1				1
01	1	1			1
11	1	1			1
10	1			1	1

Diagram illustrating the Karnaugh map for determining the minimum expression. The map shows four groups of 1s circled in green:

- Group 1: D' (Covers cells where $D=0$)
- Group 2: BC' (Covers cells where $B=1$ and $C=0$)
- Group 3: CD (Covers cells where $C=1$ and $D=1$)
- Group 4: AB (Covers cells where $A=1$ and $B=1$)

Determining Minimum Expression

Example

CD \ AB		00	01	11	10
00	1				1
01	1	1			1
11	1	1			1
10	1			1	1

Diagram illustrating the Karnaugh map for the function $F(A, B, C, D)$ with the minimum expression $D' + BC' + AB'C$.

The Karnaugh map shows the function values for all combinations of A, B, C, D . The prime implicants are circled and labeled:

- D' (Covers all cells where $D=0$)
- BC' (Covers all cells where $B=1$ and $C=0$)
- $AB'C$ (Covers all cells where $A=1$ and $B=0$ and $C=1$)

Determining Minimum Expression

Example

CD \ AB		00	01	11	10
00	1				1
01	1	1			1
11	1	1			1
10	1			1	1

Diagram illustrating the Karnaugh map for the function $F(A, B, C, D)$. The map shows the following groupings and their corresponding prime implicants:

- Group 1 (Red): D' (Covers cells where $D=0$)
- Group 2 (Blue): BC' (Covers cells where $B=1$ and $C=0$)
- Group 3 (Green): $AB'C$ (Covers cells where $A=1$ and $B=0$ and $C=1$)

$$= D' + BC' + AB'C$$