# 1.7

# TAYLOR SERIES EXPANSION

## Definition.

Let $f$ be a function with derivatives of all orders in some interval containing $a$.

Then the **TAYLOR SERIES** expansion of $f$ at $a$ is

$$f(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots +$$

# Definition.

The **MACLAURIN SERIES** expansion of $f$ at $a$ is

$$f(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \ldots +$$

*the **TAYLOR SERIES** expansion of* $f$ *at* $x = 0$ .

**Examples.** Obtain the Taylor series expansion of the following and determine its interval of convergence:

a. $f(x) = e^{x}$ *about* $a = 0$

**VERIFY:**

$$e^{x} = \sum_{n=0}^{+\infty} \frac{x^{n}}{n!} \quad , \; x \in (-\infty, +\infty)$$

**Examples.** Obtain the Taylor series expansion of the following and determine its interval of convergence:

b. $f(x) = \dfrac{1}{x}$ *about* $a = 2$

**VERIFY:**

$$\frac{1}{x} = \sum_{n=0}^{+\infty} (-1)^n \frac{(x-2)^n}{2^{n+1}}, \quad x \in (0,4)$$

**Examples.** Obtain the Taylor series expansion of the following and determine its interval of convergence:

c. $f(x) = \ln(1+x)$ *about* $a = 0$

**VERIFY:**

$$\ln(1+x) = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} x^n, \; x \in (-1, 1]$$

**Examples.** Obtain the Taylor series expansion of the following and determine its interval of convergence:

d. $f(x)=\sin x$ *about* $a=\dfrac{\pi}{3}$

**VERIFY:**

$$\sin x=\frac{\sqrt{3}}{2}+\frac{1}{2}\left(x-\frac{\pi}{3}\right)-\frac{\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{3}\right)^2}{2!}-\frac{\frac{1}{2}\left(x-\frac{\pi}{3}\right)^3}{3!}+\ldots\quad,\ x\in R$$

# END