Graph Theory

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DEFINITION OF TERMS

Graph

A *graph* G = (V, E) consists of a set of *vertices* V and a set of *edges* E. Each edge is a pair (v, w) where v, w ∈ V. Edges is sometimes referred to as arcs. If the pair of (v, w) is ordered, then the graph is a directed graph (also referred to as digraphs). A vertex w is adjacent to v if and only if (v, w) ∈ E. Thus in an undirected graph, v is adjacent to w, and w is adjacent to v. A *cost* or *weight* is sometimes added in the graph as its third component.

Vertex and Edge

A vertex is a "point" or "node" of the graph while an edge is a line that connects two "points" or nodes.

Path

A path in a graph is a sequence of vertices w_1, w_2, w_3,…,w_n such that (w_i, w_{i+1}) ∈ E for 1 ≤ i ≤ n. The length of the path is simply the number of edges on that path equal to n-1. A path length of 0 is possible if and only if the path contains no edges.

REPRESENTATION OF GRAPHS

Consider the two graphs below

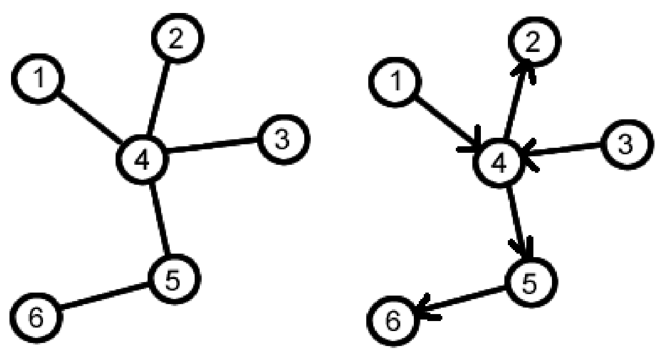

Figure 1. Graph on the left is an undirected graph; on the right is a directed graph.

Based on the graph above (they are the same except the fact that the graph on left is the undirected version of the graph in the right), V = {1, 2, 3, 4, 5, 6}.

The set of edges E for the undirected graph is E= {(1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)}.
For the directed graph, the set of edges E is E= {(1, 4), (4, 2), (3, 4), (4, 5), (5, 6)}.

Graphs can be represented using two ways:

1. Adjacency Lists Representation
2. Adjacency Matrix

Adjacency Lists Representation
The adjacency Lists representation of a graph G consists of an array of pointers. Each entry in the array is a pointer to a list of nodes adjacent to the node being considered.

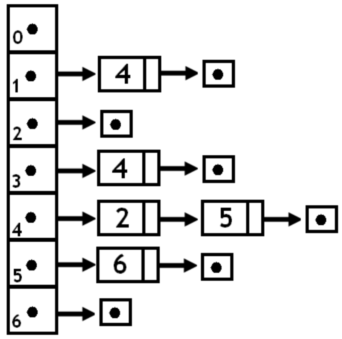

Figure 2. Adjacency List representation of directed graph in Figure 1. Pointer of index zero is unused.

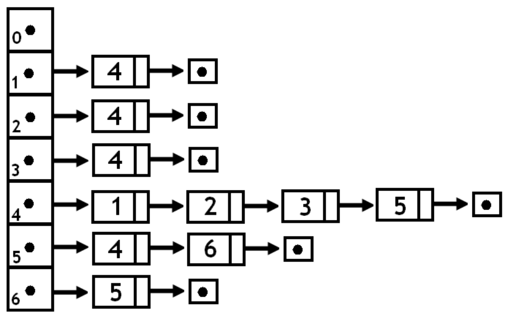

Figure 3. Adjacency List representation of undirected graph in Figure 1. Pointer of index zero is unused.

Adjacency Matrix Representation
The adjacency matrix representation of a graph G = (V, E) consists of |V| x |V| matrix A = a_{ij}, where a_{ij} = 1 if (i, j) ∈ E and 0 otherwise.

	0	1	2	3	4	5	6
0							
1		0	0	0	1	0	0
2		0	0	0	0	0	0
3		0	0	0	1	0	0
4		0	1	0	0	1	0
5		0	0	0	0	0	1
6		0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		0	0	0	1	0	0
2		0	0	0	1	0	0
3		0	0	0	1	0	0
4		1	1	1	0	1	0
5		0	0	0	1	0	1
6		0	0	0	0	1	0

Figure 4. Adjacency Matrix Representation of the directed (L) and undirected (R) graph in Figure 1.