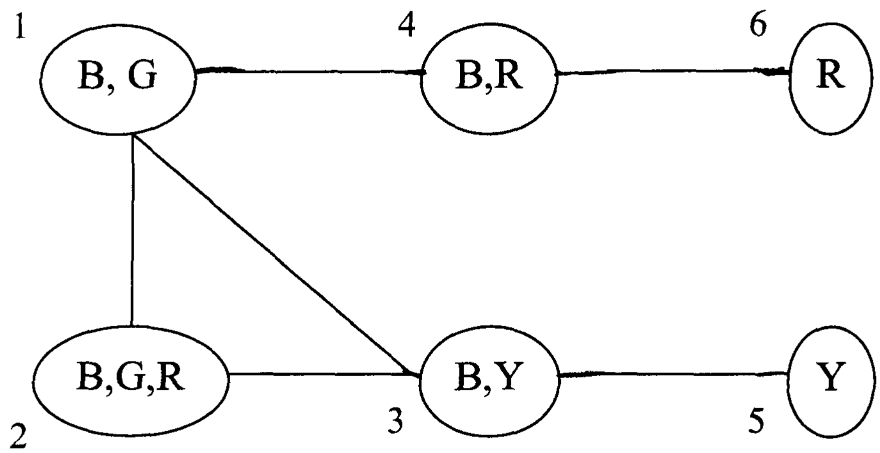
6.034 Quiz 2
November 16, 2005

Name	Solutions
Email	

Problem number	Maximum	Score	Grader	Understanding	Not Sure	insufficent Understanding
1	34			30 & up	29 to 25	24 and down
2	30			26 & up	25	24 and down
3	36			28 & up	27 to 24	23 and down
Total	100					

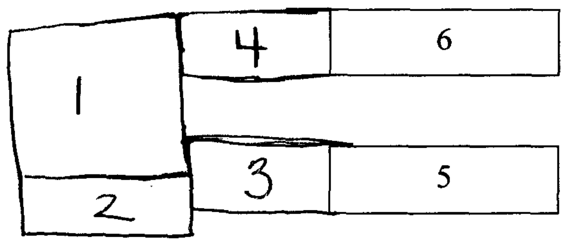
Problem 1. Constraint Propagation (34 pts)

The following constraint network represents a coloring problem with 6 variables (Var 1 though 6) and one or two or three colors in the domain of each variable (B, G, R, and Y). Each line represents a constraint that requires that the two variables have different colors.



Binary constraints between variables are due to touching regions, while the domain restrictions on each variable come from some other source.

Part A: (6 pts)
Draw a map containing 6 "countries" that exhibits the same constraints as the above constraint graph:



Two of the 6 countries are drawn for you. Draw 4 additional countries above such that if there is a constraint between two blocks, the two blocks share a boundary.

Part B (28 points)
Find one of the valid solutions for this problem using **backtracking with forward checking <u>AND propagating through singleton domains</u>**. Examine variables in numerical order and values in alphabetical order.

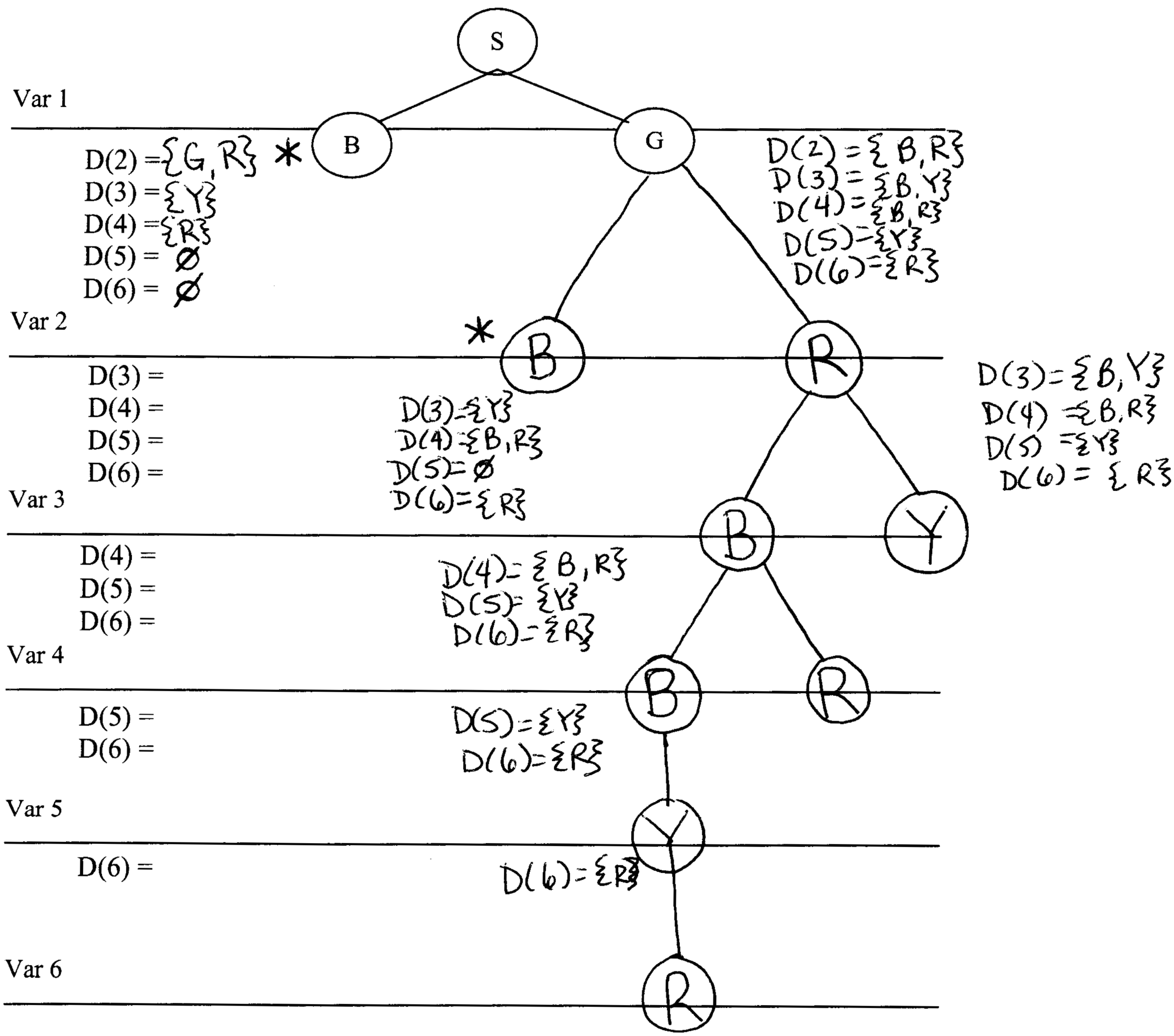
(1) Variables are explored in <u>numerical</u> order, starting with Variable 1.
(2) Values (Colors) are explored in <u>alphabetic</u> order (B first, then G, then R, then Y)

Draw a search tree for using **backtracking with forward checking <u>AND propagating through singleton domains</u>**.

In addition:

1. For every node in the tree draw *only* the valid descendants at that point *and*
2. For every node in the tree draw the domains at that point, assuming constraint propagation runs through **all singleton domains** until no further domain reduction is possible anywhere *and*
3. Mark with an ASTERISK (*) <u>ALL</u> nodes at which assignments initiate constraint propagation through singleton domains.

A portion of the tree is drawn for you



Part B1: (4 pts)

How many asterisks did you mark in your tree? $\boxed{2}$

Part B2: (8 pts)

Interpret your tree by filling in the blanks below:

The first time in the tree search that a variable assignment produces propagation *through singleton domains* is when variable ___1___ is assigned the value ___B___.

After propagation, the domain of variable(s) _5, 6_ is/are found to be empty, and backtracking occurs.

Part B3: (8 pts)

Circle either (I), (II) or (III), and fill in the corresponding blanks, if appropriate :

(I)
The second time that a variable assignment produces propagation through singleton domeains is when variable _2_ is assigned the value _B_.

After propagation, the domain of variable(s) _5_ is/are found to be empty, and backtracking occurs.

(II)
The second time constraints are propagated through singleton domains is when variable ____ is assigned the value ____.

After propagation, the search *continues* with another variable assignment.

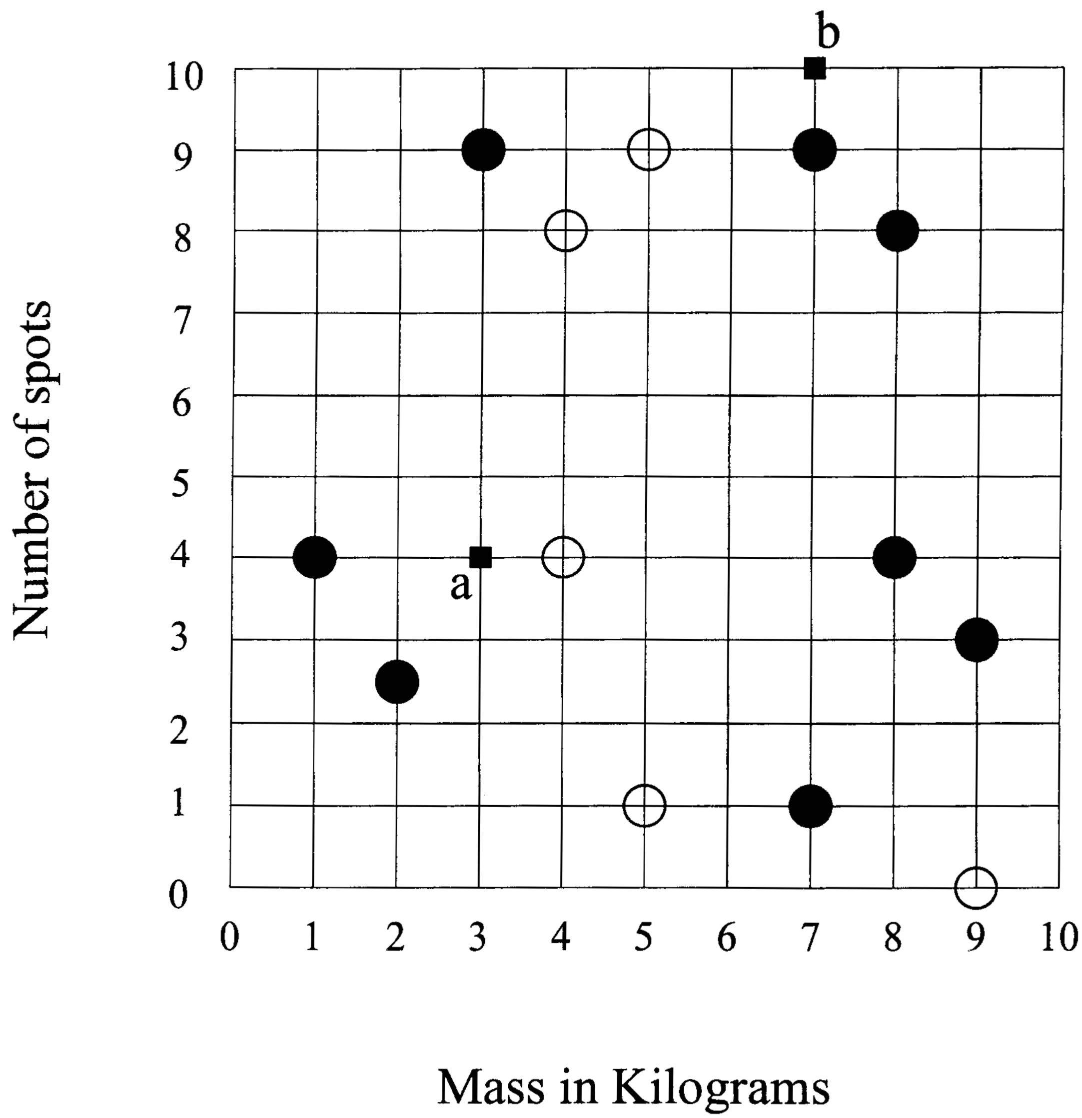
(III)
Constraints are not propagated through singleton domains a second time.

Part B4: (8 pts)

After the search is finished, the resulting variable assignments are:

Var 1	Var 2	Var 3	Var 4	Var 5	Var 6
G	R	B	B	Y	R

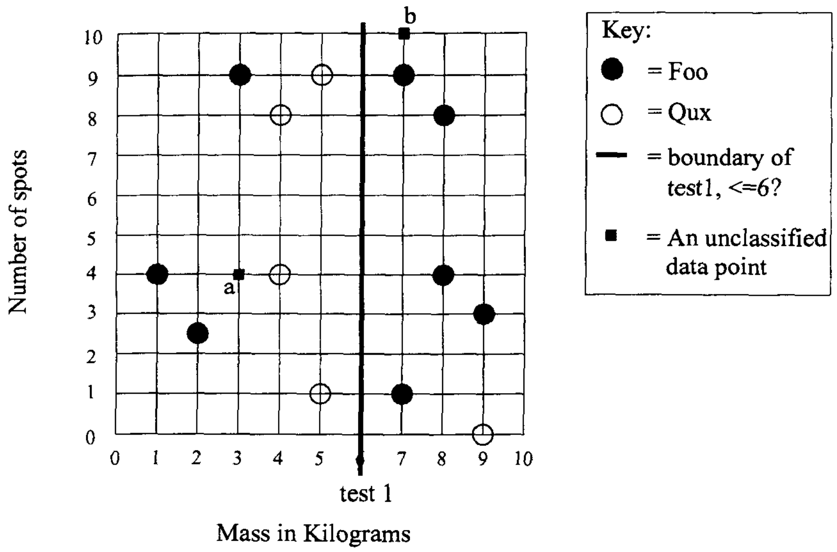
Question 2: Nearest Neighbors and ID Trees (30 points)



Here is a set of data on Foos and Quxes. You want to classify **a** and **b** as a Foo or a Qux. (Note: if you need a straightedge, the side of a sheet of paper works well.)

Part A: ID Trees (14 points)

First, use the ID Tree idea. We have provided you with the first test: "Is the mass less than or equal to 6?" and drawn it below:



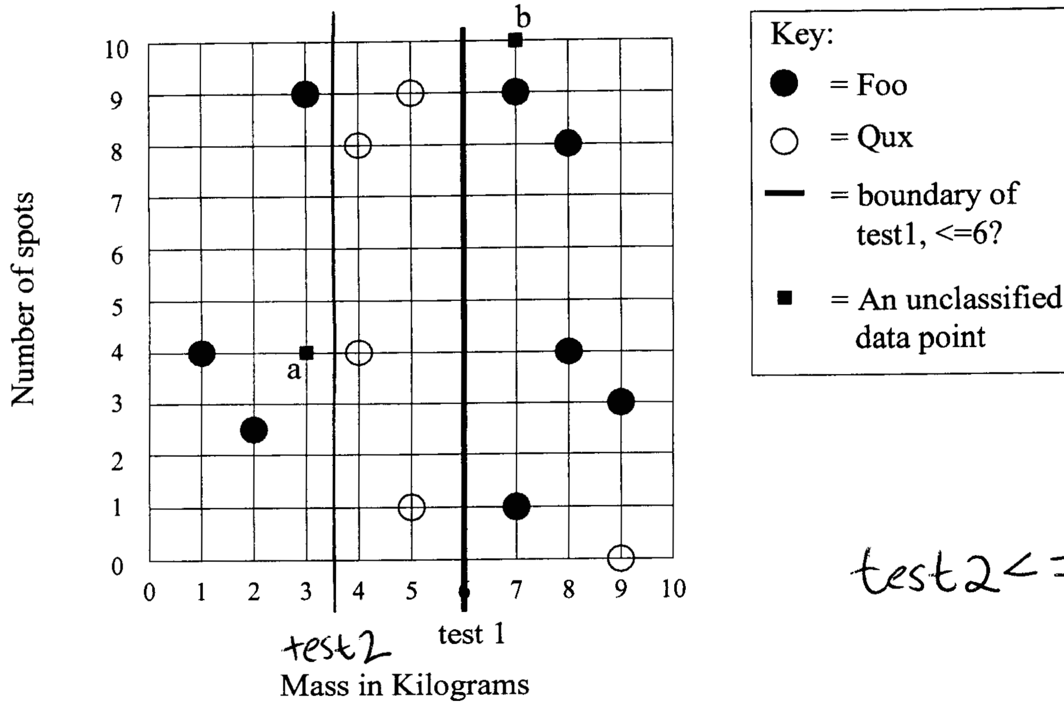
Part A1 (4 points)

What is the disorder of the first test? Write your answer in terms of fractions and logs. You need not compute a numerical answer with a calculator.

$$\frac{7}{13}\left(-\frac{3}{7}\log_2\frac{3}{7}-\frac{4}{7}\log_2\frac{4}{7}\right)+\frac{6}{13}\left(-\frac{5}{6}\log_2\frac{5}{6}-\frac{1}{6}\log_2\frac{1}{6}\right)$$

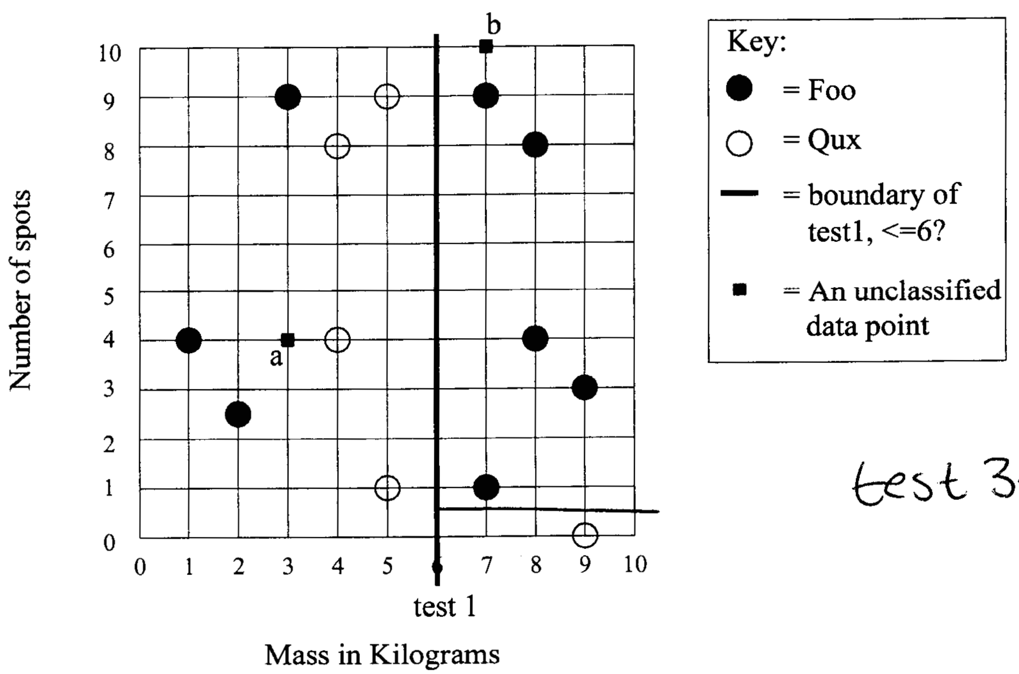
Part A2 (6 points)

Test2, which is to test just one variable, will occur if the answer to test1 is **true.** Draw test2 below:



test2<=3.5?

Test3, which is to test just one variable, will occur if the answer to test1 is **false.** Draw test3 below:



test 3<= .5?

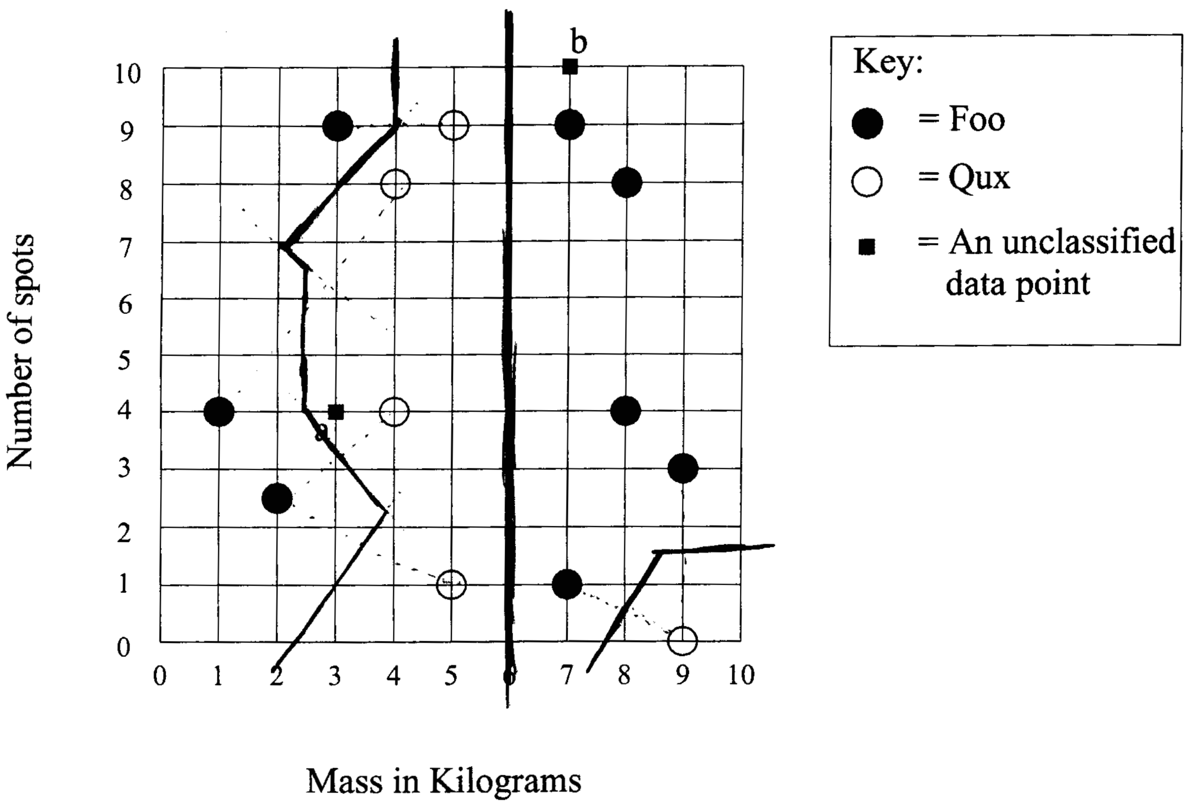
Part A4 (4 points)

Using the ID tree you have completed, how would you classify **a** and **b**?

a is a foo **b** is a foo

Part B1 Nearest Neighbors (6 points)

Now you classify **a** and **b** using nearest neighbors. Draw the decision boundaries for nearest neighbor decisions, assuming 1 nearest neighbor is all that matters, on the graph below.



Part B2 (4 points)

Using 1 nearest neighbor, **a** is a Qux and **b** is a foo

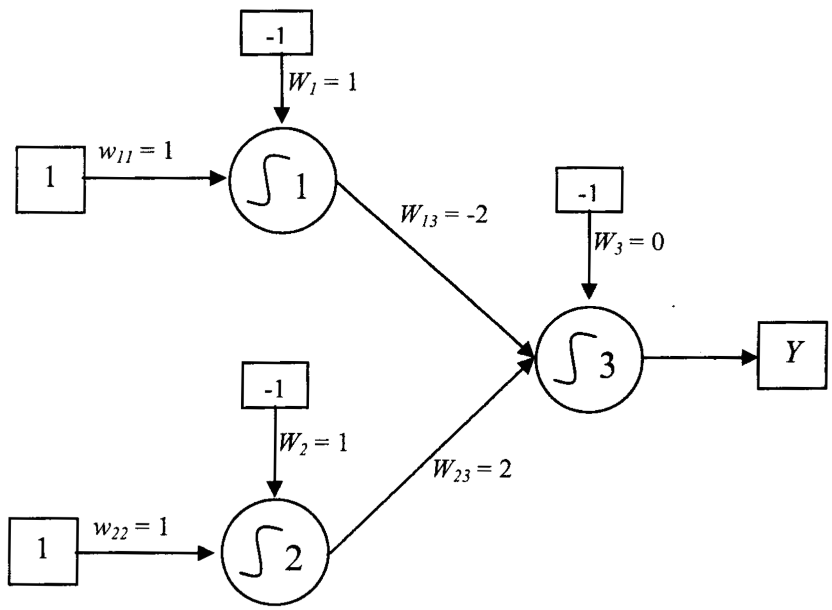
Part C (6 points)

Using 3 nearest neighbors, **a** is a foo and **b** is a foo

Question 3: Neural Networks (36 points)

Part A: Backward Propagation (24 points)

For this part of the problem, you may find the backpropagation notes attached at the end of the quiz to be helpful. Considering the following network, with **sigmoid** threshold functions.



Below are the initial weights, repeated in this table for your convenience. Note that w_{11} does not originate from sigmoid unit 1, but from x_1, and w_{22} does not originate from sigmoid unit 2, but from x_2.

w_1	w_{11}	w_{13}	w_2	w_{22}	w_{23}	w_3
1	1	-2	1	1	2	0

The actual outputs of the units are as follows:

y_1 (output of unit 1) = 1/2
y_2 (output of unit 2) = 1/2
y_3 (output of unit 3) = 1/2

Using a learning rate of 2, and desired output of $Y = 0$, and an error function $E = \frac{1}{2}(Y - Y^*)^2$ for the input $(x_1, x_2) = [1, 1]$, backpropagate the network for one iteration, by computing the new values for the weights w_{23} and w_{22} and filling out the values below. Provide numerical answers and show your work for partial credit.

$R = 2$

$w_{23} = w_{23} - R\delta_3 y_2 = 2 - 2(1/8)(1/2) = \boxed{15/8 \text{ or } 1.875}$

$w_{22} = w_{22} - R\delta_2 x_2 = 1 - 2(1/16)(1) = \boxed{7/8 \text{ or } 0.875}$

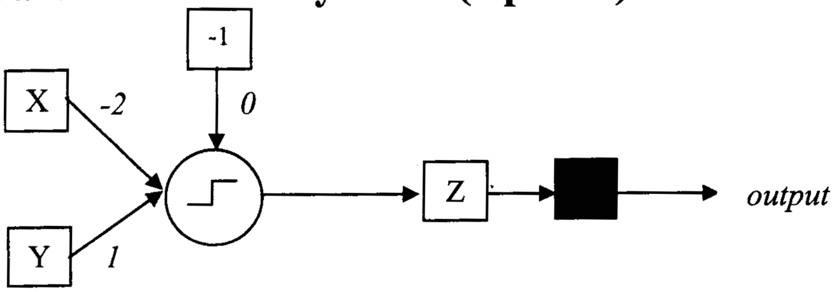
$\delta_3 = y_3(1-y_3)(y_3 - y_3^*)$

$\delta_3 = 1/2\,(1/2)(1/2) = 1/8$

$\delta_2 = y_2(1-y_2)(w_{23}\delta_3)$

$\delta_2 = 1/2\,(1/2)(2 \cdot 1/8) = 1/16$

Part B1: Boundary Lines (8 points)



All units have step function thresholds, not sigmoids!!

That is, output = 1 for input greater than 0; output = 0 for input equal to or less than 0. The black box in the diagram is a transmogrifier that converts its input into the following outputs:

Z	*Output*
1	△
0	□

$y - 2x > 0 \xrightarrow{z=1} \triangle$

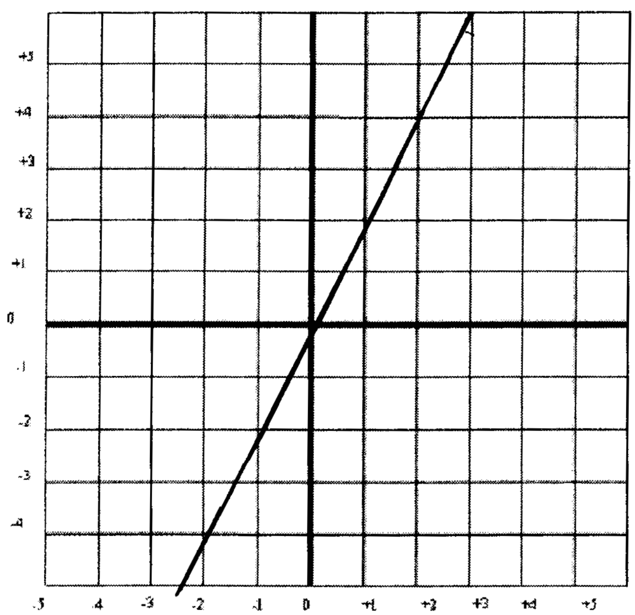
$y - 2x \leq 0 \xrightarrow{z=0} \square$

boundary line @

$y - 2x = 0$

$y = 2x$

Draw the boundary line this unit produces on the following graph:



Part B2 (4 points)

What will our system output if we feed it any point above the boundary line? Circle your answer.

△ □

Above the line:

$y - 2x > 0$

Backprop notes

$$E = \frac{1}{2} \sum_k (o_k - d_k)^2$$

$$w_{i \to j} = w_{i \to j} - \Delta w_{i \to j}$$

$$\Delta w_{i \to j} = R \times o_{l_i} \times \delta_{r_j}$$

where R is a rate constant and the δs are computed with the following formulas:

$$\delta_k = o_k(1 - o_k) \times (o_k - d_k)$$

$$\delta_{l_i} = o_{l_i}(1 - o_{l_i}) \times \sum_j w_{i \to j} \times \delta_{r_j}$$

where

o_k	is output k of the output layer
d_k	is the desired output k of the output layer
δ_k	is a delta associated with the output layer
o_{l_i}	is output i of left layer in a left-right pair
δ_{l_i}	is a delta associated with the layer l
δ_{r_j}	is a delta associated with the adjacent layer to the right, layer r