Chapter 3

Boolean Algebra, Logic Functions, and Logic Gates (Part 2)

Binary Variables

- Forms of variables
  - normal (x)
  - complement (x')
- Forms of terms (variables x and y)
  - Minterms m_i (or standard product)

    $x'y', x'y, xy', xy$
  - Maxterms M_i (or standard sum)

    $x+y, x+y', x'+y, x'+y'$

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0		m0		M0
0	0	1		m1		M1
0	1	0		m2		M2
0	1	1		m3		M3
1	0	0		m4		M4
1	0	1		m5		M5
1	1	0		m6		M6
1	1	1		m7		M7

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0	x‘y’z’	m0		M0
0	0	1	x‘y’z	m1		M1
0	1	0	x‘yz’	m2		M2
0	1	1	x‘yz	m3		M3
1	0	0	xy‘z’	m4		M4
1	0	1	xy‘z	m5		M5
1	1	0	xyz'	m6		M6
1	1	1	xyz	m7		M7

Minterms and Maxterms for 3 variables

			MINTERM		MAXTERM	
x	y	z	Term	Designation	Term	Designation
0	0	0	x‘y’z’	m0	x+y+z	M0
0	0	1	x‘y’z	m1	x+y+z’	M1
0	1	0	x‘yz’	m2	x+y’+z	M2
0	1	1	x‘yz	m3	x+y’+z’	M3
1	0	0	xy‘z’	m4	x‘+y+z	M4
1	0	1	xy‘z	m5	x‘+y+z’	M5
1	1	0	xyz'	m6	x‘+y’+z	M6
1	1	1	xyz	m7	x‘+y’+z’	M7

Forms of Boolean Functions

- Canonical Form
  - Sum of minterms
  - Product of maxterms
- Standard Form
  - Sum of products
  - Product of sums

Forms of Boolean Functions

Examples

- $F(a,b,c) = abc' + a'bc$
- $F(w,x,y,z) = (w+x'+y'+z)(x+y+z')$
- $F(x,y,z) = xz' + y$
- $F(a,b,c,d) = (a+b'+c+d)(a+b+c'+d')$

Sum of Minterms

- Any Boolean function can be expressed as a sum of minterms
- “sum” means Oring the minterms that produces a 1 in the function
- Each minterm is obtained from an AND term of the *n* variables, with each variable being primed if the corresponding bit of the binary number is a 0 and unprimed if a 1.

Example – Truth Table

Express $F(A,B,C) = A + B'C$ in sum of minterms

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Example – Truth Table

Express $F(A,B,C) = A + B'C$ in sum of minterms

$= A'B'C + AB'C' + AB'C + ABC' + ABC$

$= \Sigma\,(1, 4, 5, 6, 7)$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in sum of minterms

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in sum of minterms

$F = A(B+B') + B'C$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in sum of minterms

$F = A(B+B') + B'C$

$= AB + AB' + B'C$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in sum of minterms

$$F = A(B+B') + B'C$$
$$= AB + AB' + B'C$$
$$= AB(C+C') + AB'(C+C') + B'C(A+A')$$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in sum of minterms

$F = A(B+B') + B'C$

$= AB + AB' + B'C$

$= AB(C+C') + AB'(C+C') + B'C(A+A')$

$= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in sum of minterms

$F = A(B+B') + B'C$

$= AB + AB' + B'C$

$= AB(C+C') + AB'(C+C') + B'C(A+A')$

$= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$

$= A'B'C + AB'C' + AB'C + ABC' + ABC$

Example – Algebraic Manipulation

Express F(A,B,C) = A +B'C in sum of minterms

F = A(B+B') + B'C

= AB + AB' + B'C

= AB(C+C') + AB'(C+C') + B'C(A+A')

= ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C

= A'B'C + AB'C' + AB'C + ABC' + ABC

= $m_1 + m_4 + m_5 + m_6 + m_7$

= Σ (1, 4, 5, 6, 7)

Product of Maxterms

- Any Boolean function can be expressed as a product of maxterms
- "product" means ANDing the maxterms that produces a 0 in the function
- Each maxterm is obtained from OR term of the *n* variables, with each variable being primed if the corresponding bit of the binary number is a 1 and unprimed if a 0.

Example – Truth Table

Express $F(A,B,C) = A + B'C$ in product of maxterms

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Example – Truth Table

Express $F(A,B,C) = A + B'C$ in product of maxterms

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$= (A+B+C)(A+B'+C)(A+B'+C')$

$= \Pi(0, 2, 3)$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in product of maxterms

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in product of maxterms

$F = (A+B')(A+C)$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in product of maxterms

$F = (A+B')(A+C)$

$= (A+B'+CC')(A+C)$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in product of maxterms

$$F = (A+B')(A+C)$$
$$= (A+B'+CC')(A+C)$$
$$= (A+B'+C)(A+B'+C')(A+C+BB')$$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in product of maxterms

$$F = (A+B')(A+C)$$
$$= (A+B'+CC')(A+C)$$
$$= (A+B'+C)(A+B'+C')(A+C+BB')$$
$$= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C)$$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in product of maxterms

$$
\begin{aligned}
F &= (A+B')(A+C) \\
&= (A+B'+CC')(A+C) \\
&= (A+B'+C)(A+B'+C')(A+C+BB') \\
&= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C) \\
&= (A+B+C)(A+B'+C)(A+B'+C')
\end{aligned}
$$

Example – Algebraic Manipulation

Express $F(A,B,C) = A + B'C$ in product of maxterms

$F = (A+B')(A+C)$

$= (A+B'+CC')(A+C)$

$= (A+B'+C)(A+B'+C')(A+C+BB')$

$= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C)$

$= (A+B+C)(A+B'+C)(A+B'+C')$

$= M_0M_2M_3$

$= \Pi(0, 2, 3)$

Conversion between Canonical forms

- Consider the previous example: $F = A+B'C$

  $F(A,B,C) = \Sigma(1, 4, 5, 6, 7)$

  This has a complement that can be expressed as:

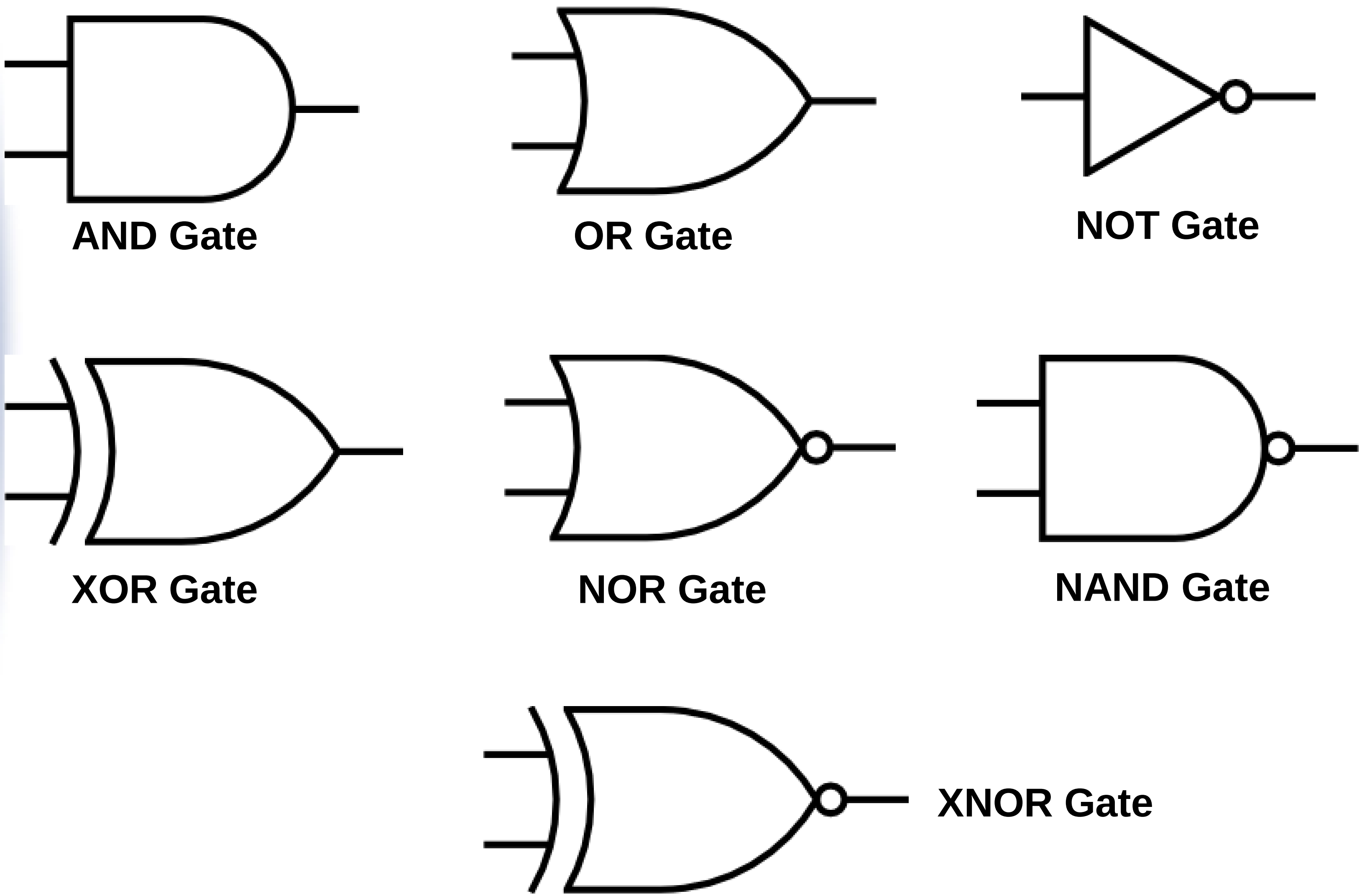
  $F'(A,B,C) = \Sigma(0, 2, 3) = m_0 + m_2 + m_3$

  Now, take the complement of F', we will obtain

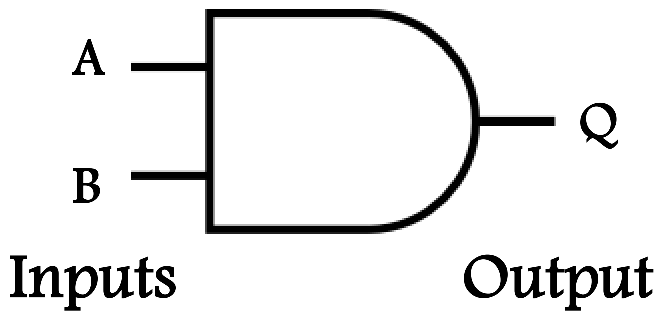
  $F = (m_0 + m_2 + m_3)' = m_0' \, m_2' \, m_3' = M_0M_2M_3$

  $= \Pi(0, 2, 3)$

Digital Logic Gates



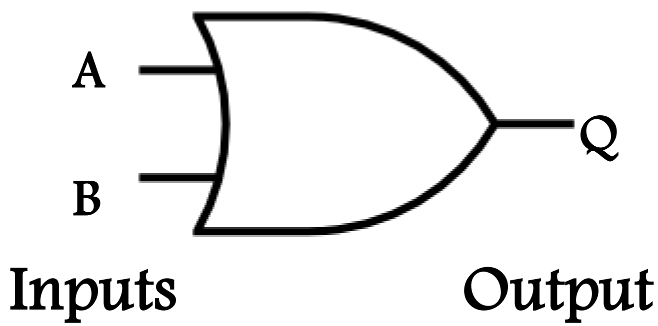
AND Gate



Truth Table

Inputs		Output
A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

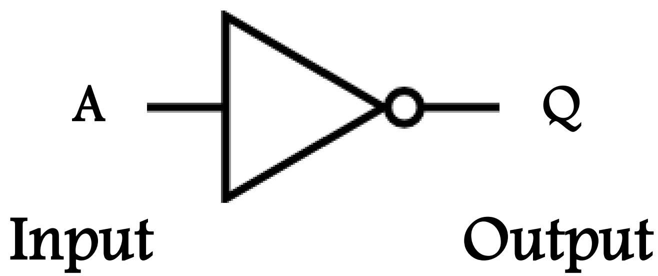
OR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

NOT Gate



Truth Table

Input	Output
A	Q
0	1
1	0

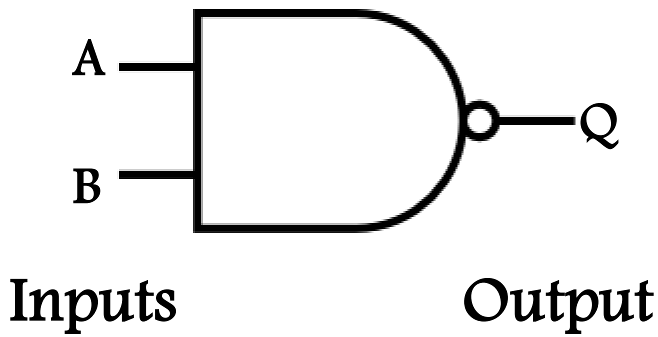
Other Gates

- Why use?
  - Saves cost
  - Saves space
  - Saves time

Other Gates

- Why use?
  - Saves cost
  - Saves space
  - Saves time
- Types
  - NAND gate
  - NOR gate
  - XOR gate
  - XNOR gate

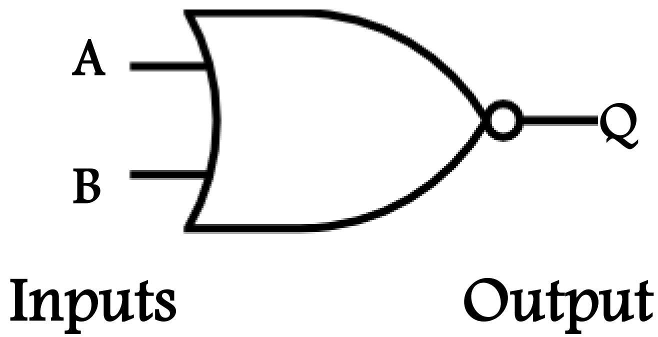
NAND Gate



Truth Table

Inputs		Output
A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

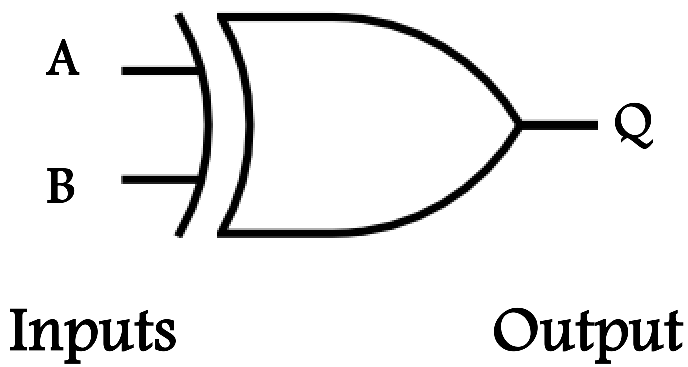
NOR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

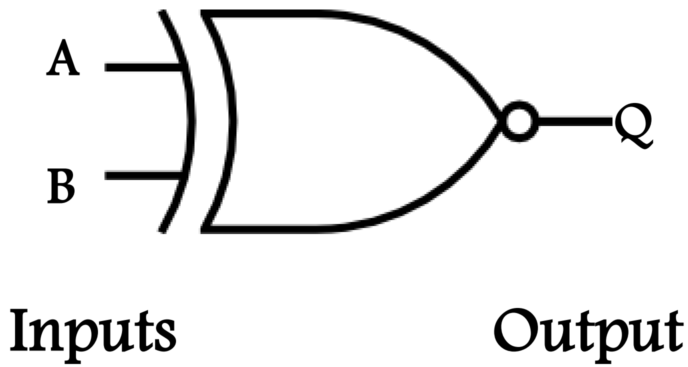
Exclusive-OR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

XNOR Gate



Truth Table

Inputs		Output
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	1

Digital Logic gates

- Draw the logic diagram of the function
  $F = A + B'C$

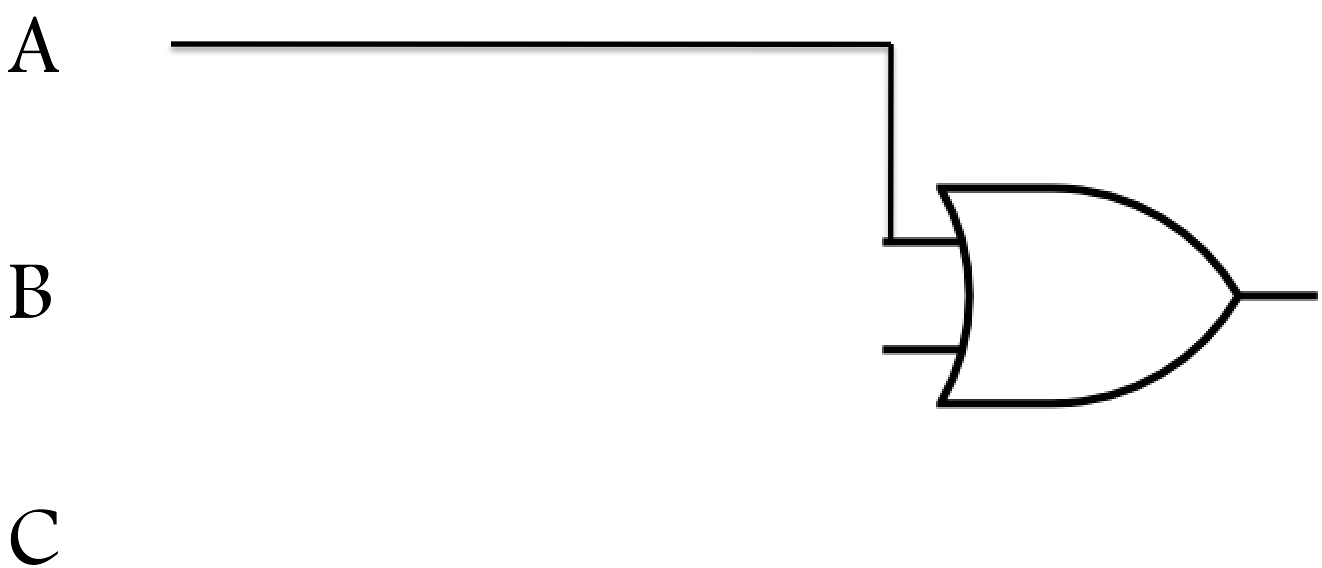
A

B

C

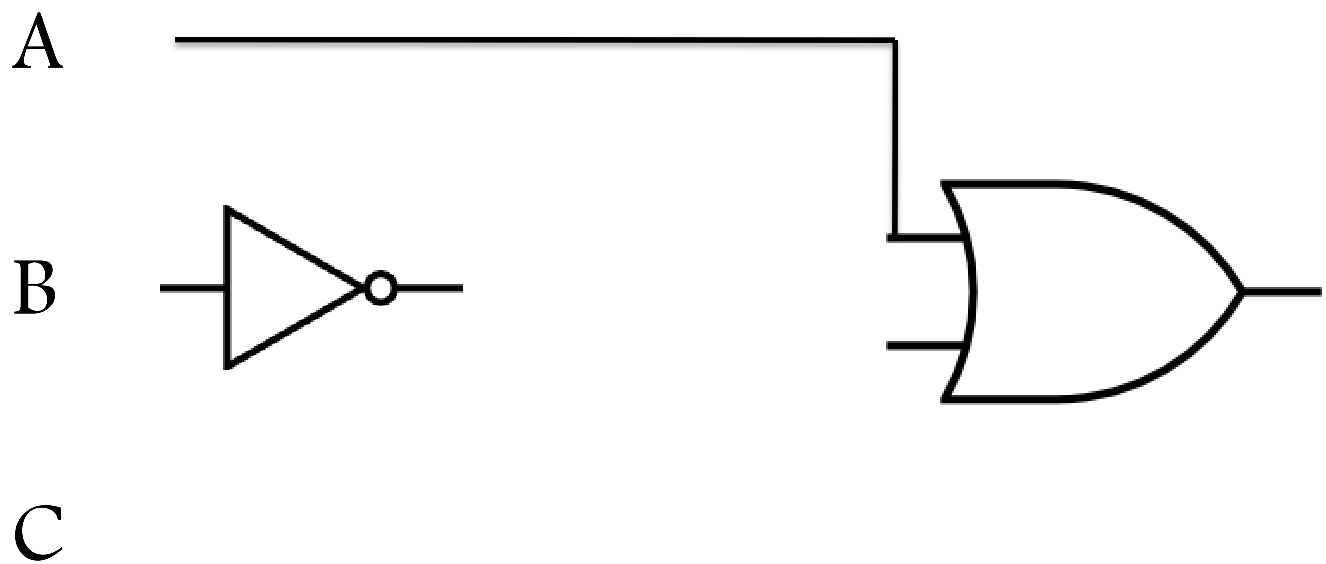
Digital Logic gates

- Draw the logic diagram of the function $F = A+B'C$



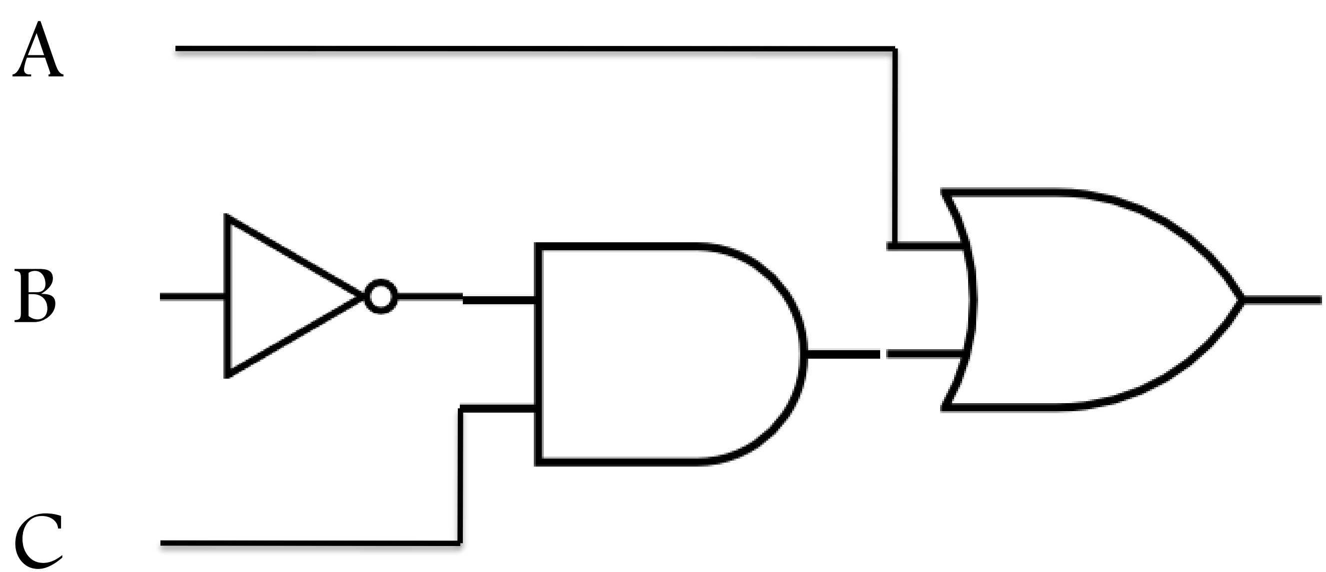
Digital Logic gates

- Draw the logic diagram of the function
  F = A+B'C



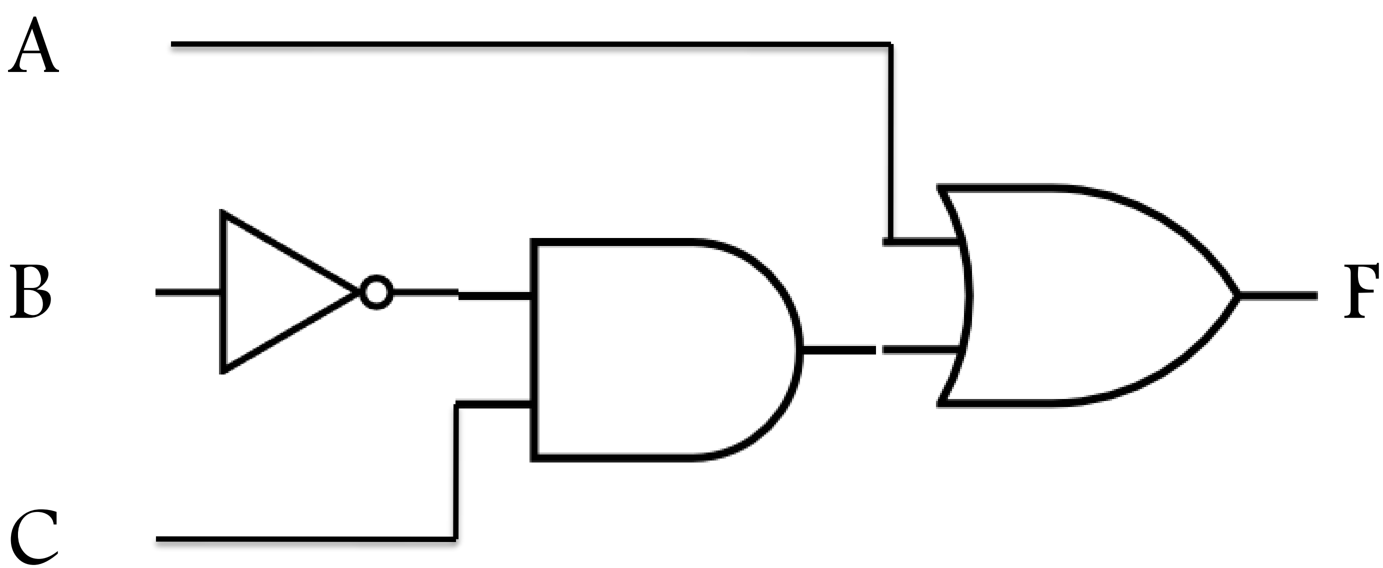
Digital Logic gates

- Draw the logic diagram of the function $F = A+B'C$

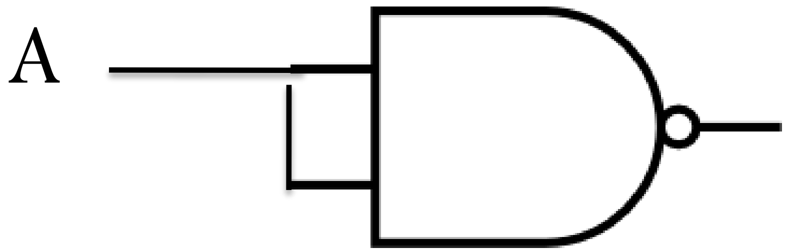


Digital Logic gates

- Draw the logic diagram of the function
  F = A+B'C

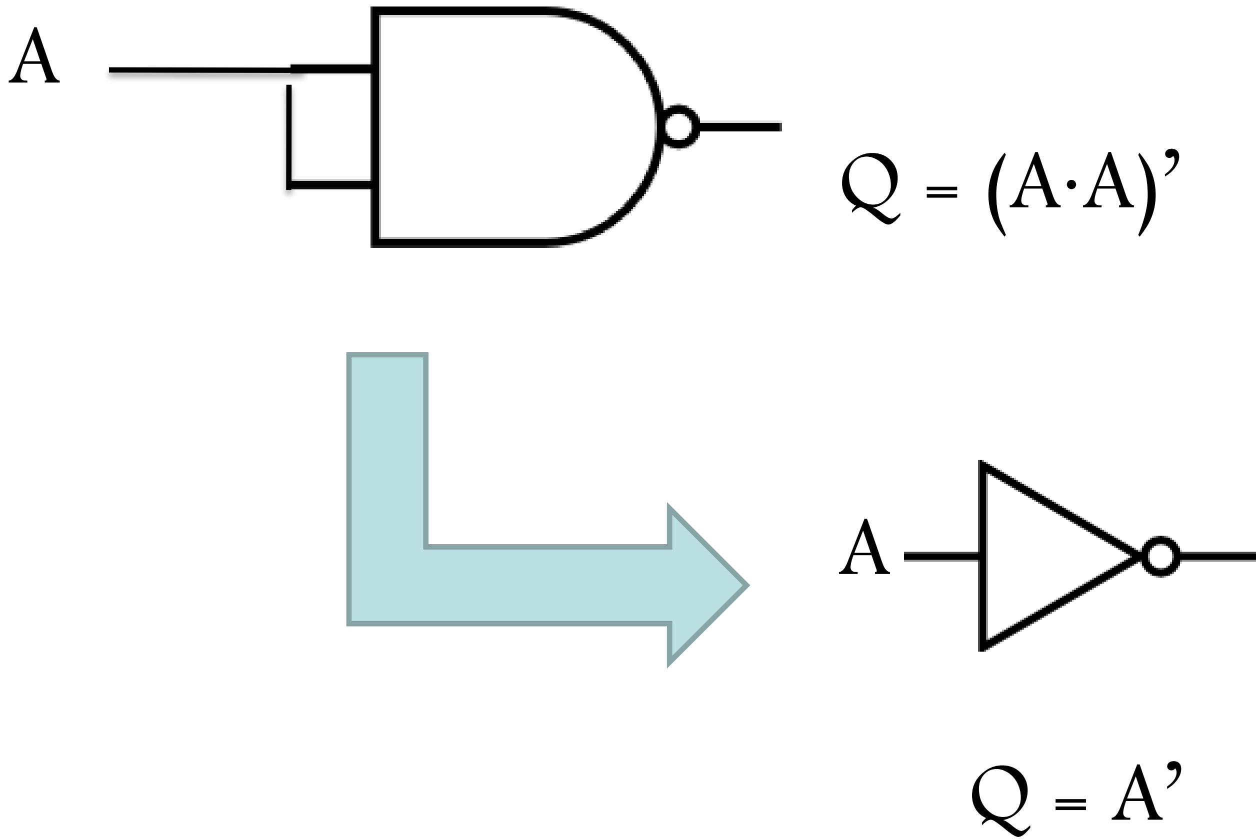


Universality of a NAND gate

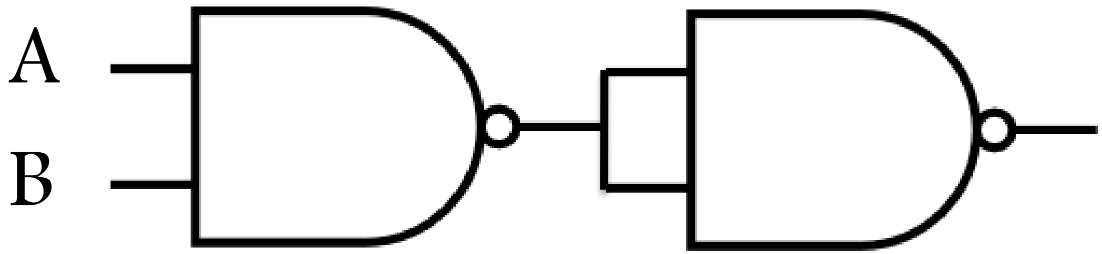


Q = (A·A)’

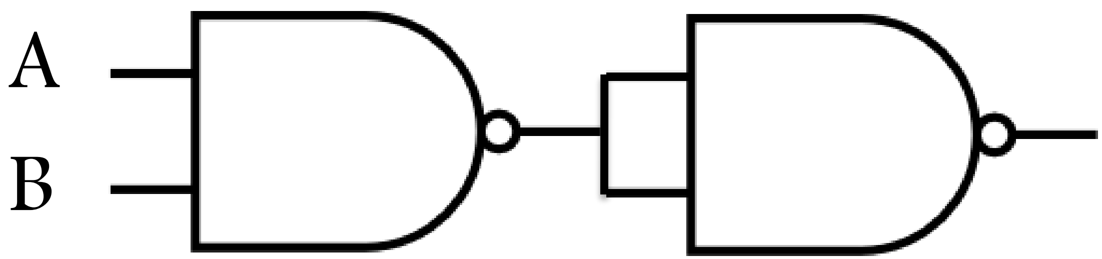
Universality of a NAND gate



Universality of a NAND gate

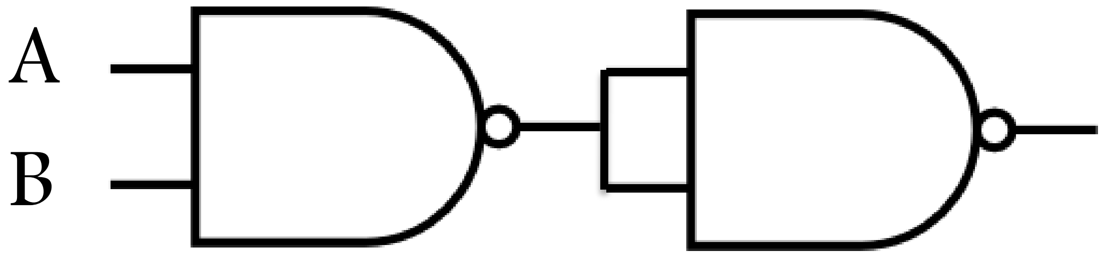


Universality of a NAND gate

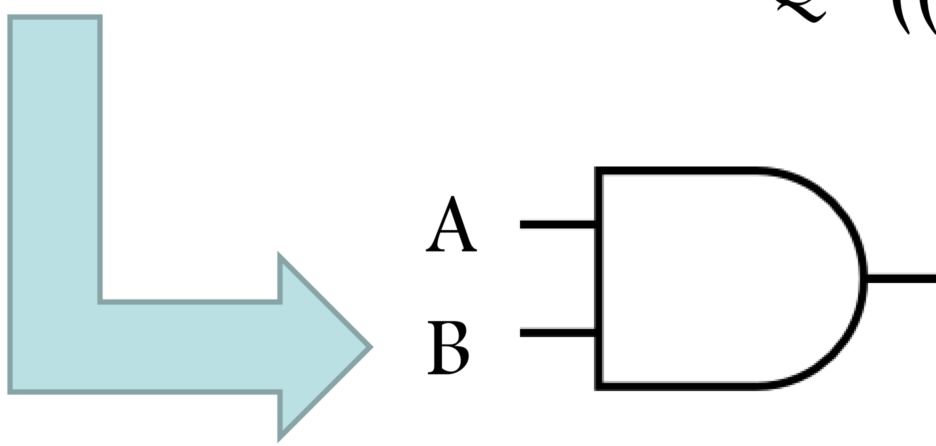


Q =((AB)’(AB)’)’

Universality of a NAND gate

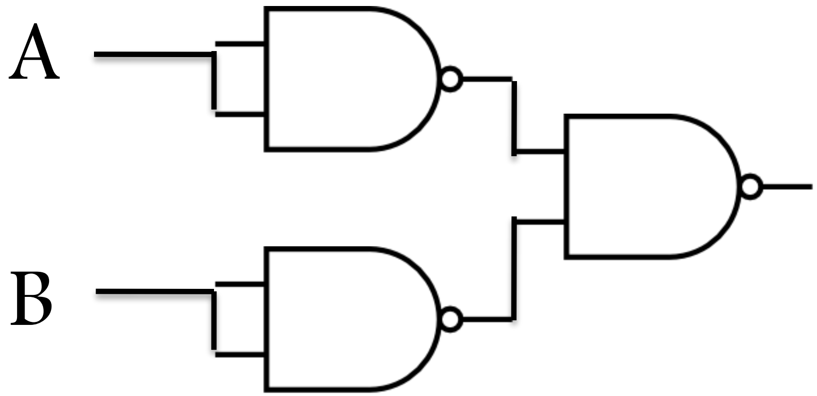


Q =((AB)'(AB)')'

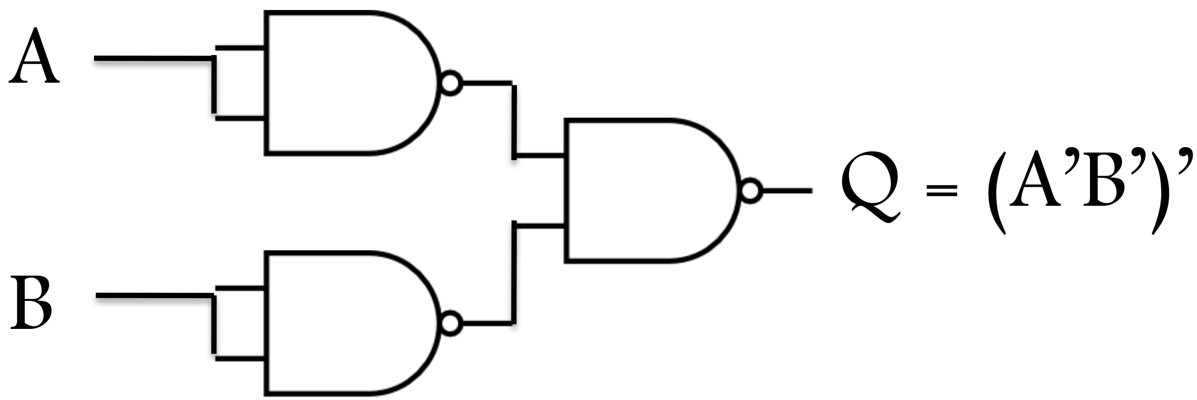


Q = AB

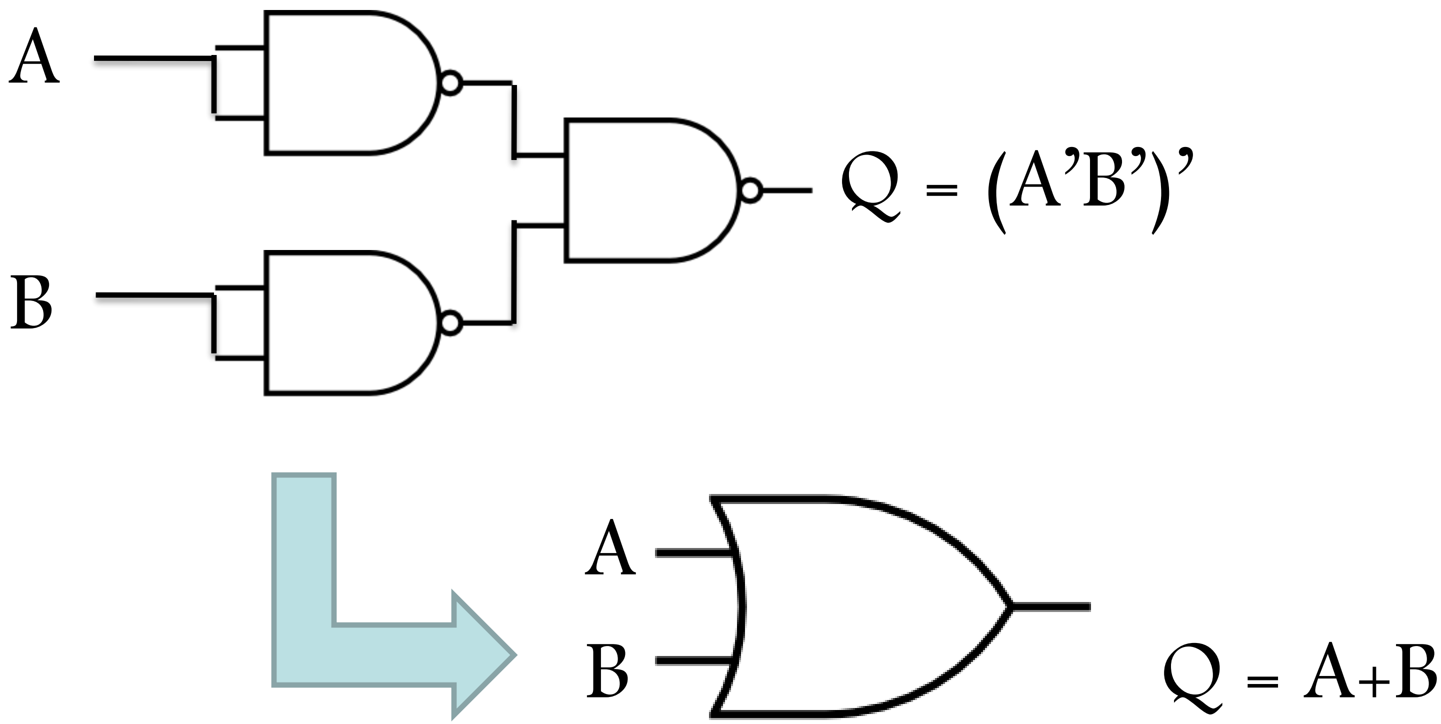
Universality of a NAND gate



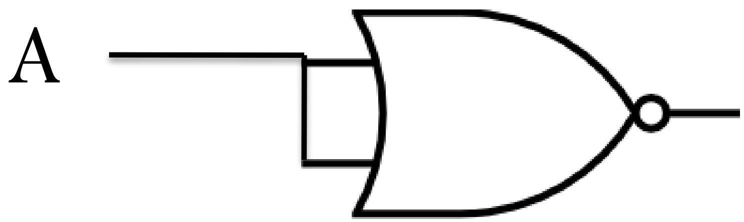
Universality of a NAND gate



Universality of a NAND gate

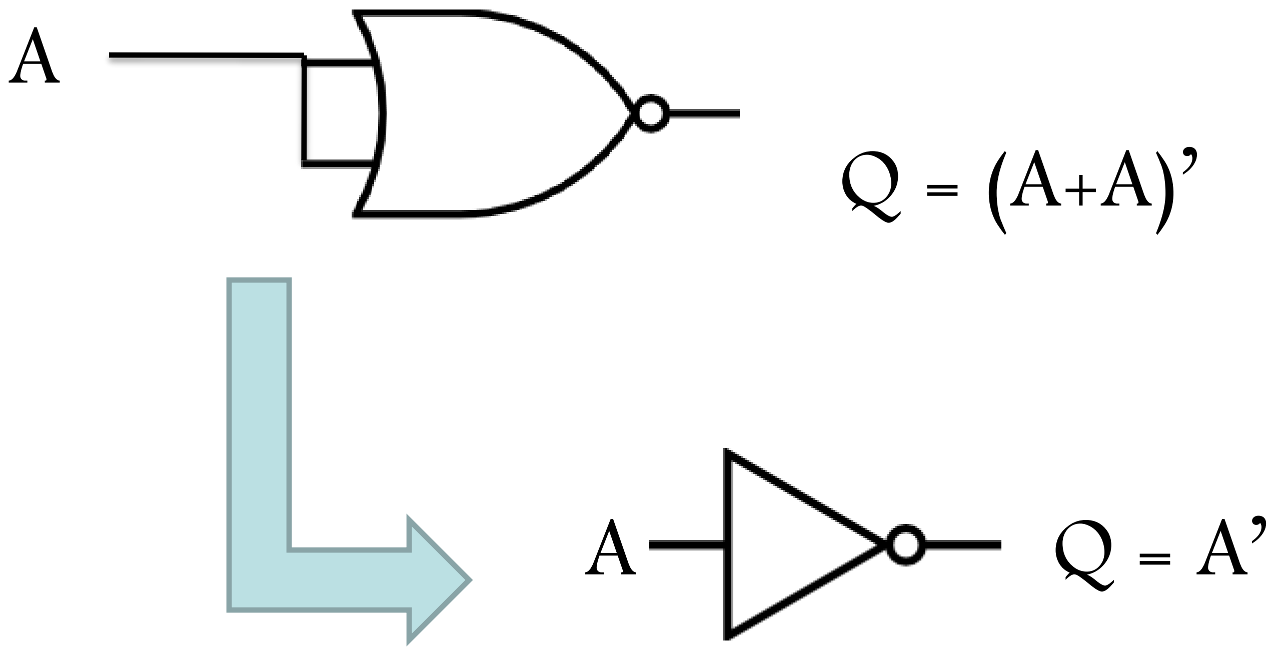


Universality of a NOR gate

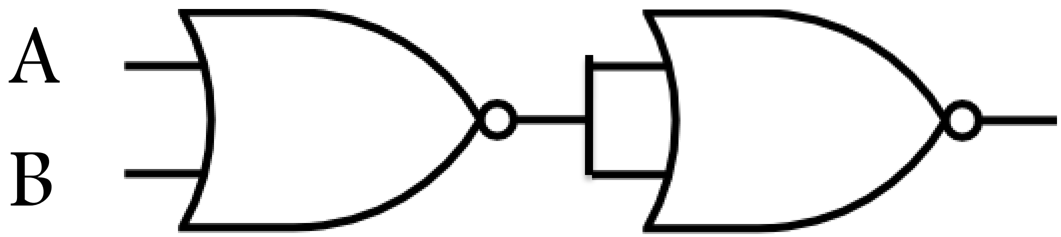


$Q = (A+A)'$

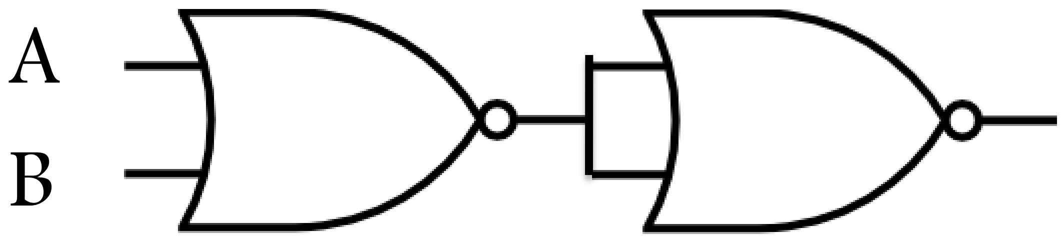
Universality of a NOR gate



Universality of a NOR gate

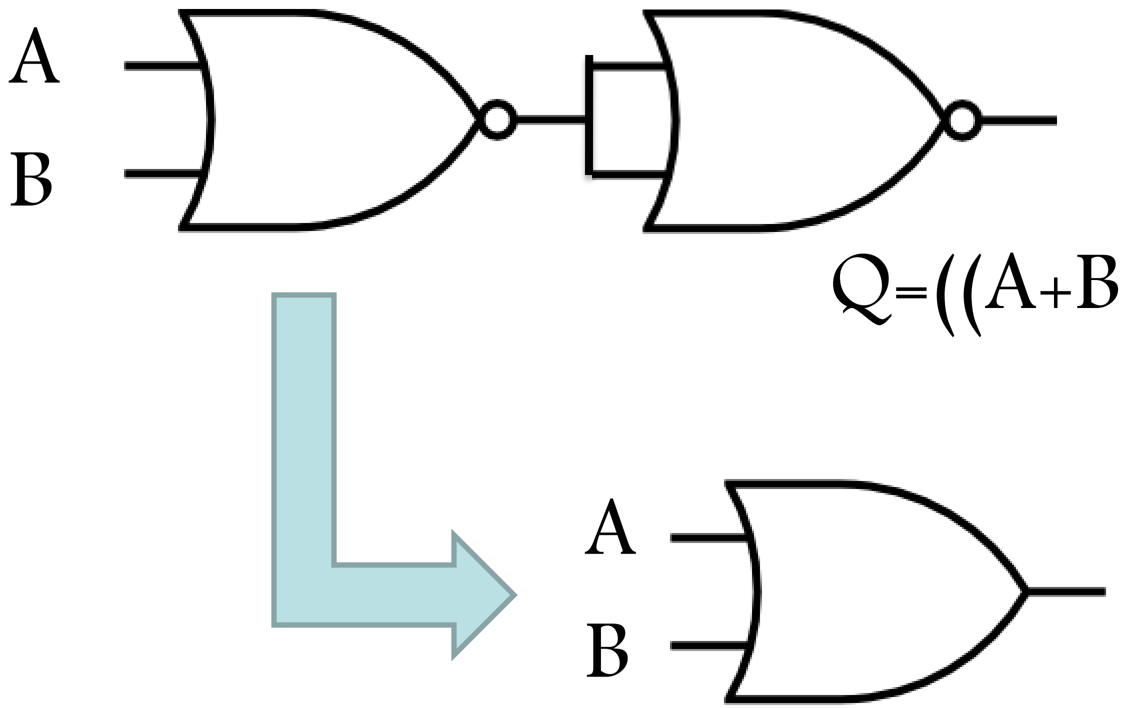


Universality of a NOR gate



$$Q=((A+B)'+(A+B)')'$$

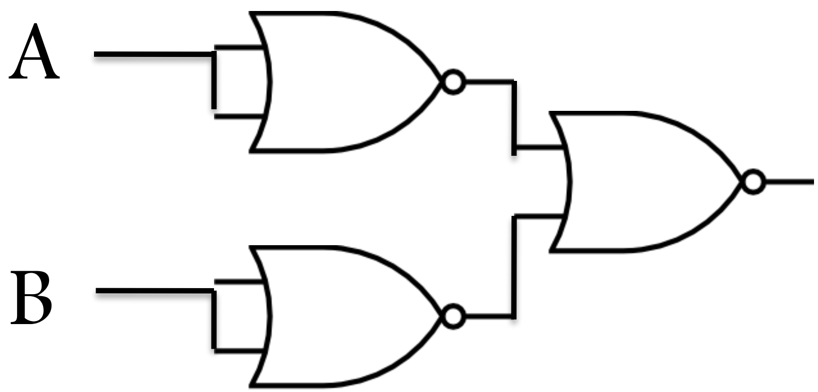
Universality of a NOR gate



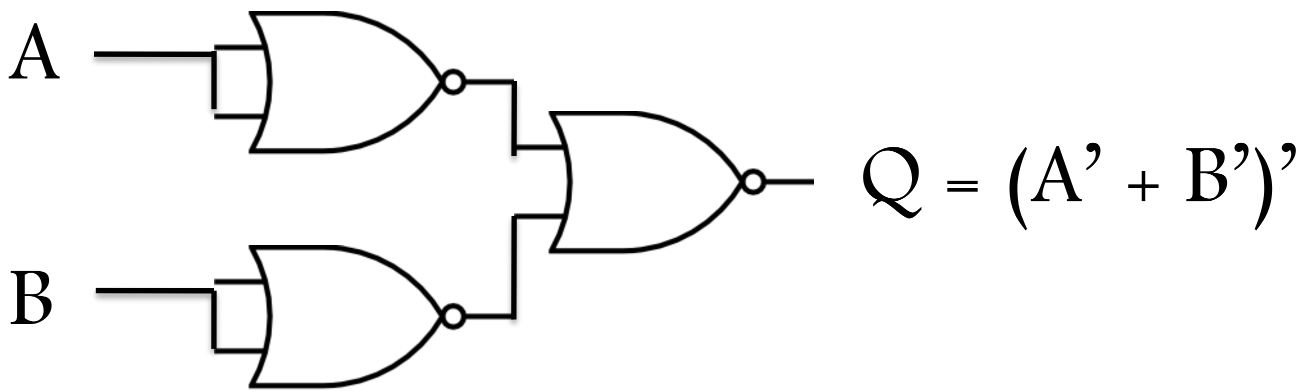
$Q=((A+B)'+(A+B)')'$

$Q = A+B$

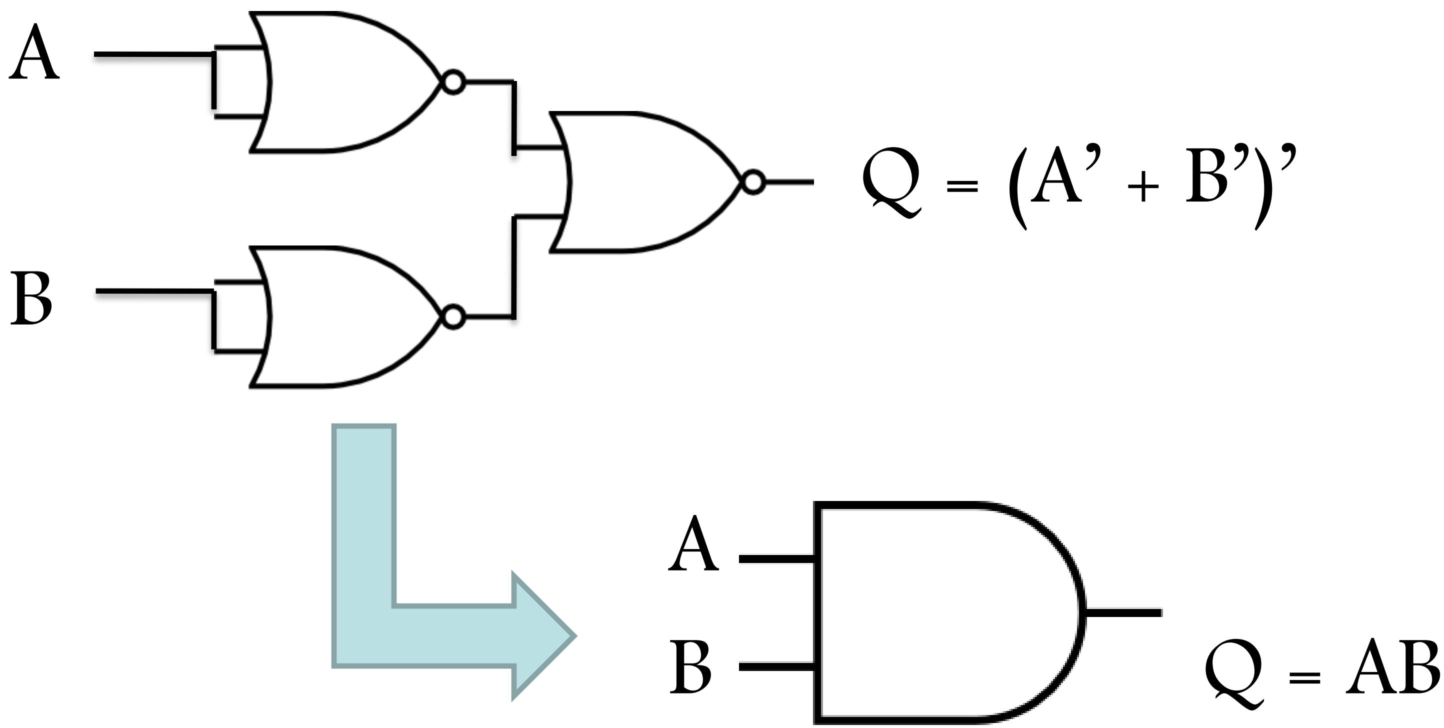
Universality of a NOR gate



Universality of a NOR gate

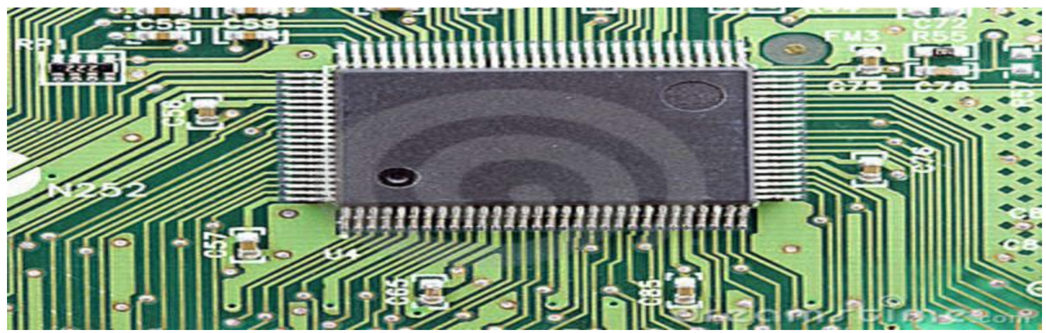


Universality of a NOR gate



Real Gates

- Logic gates are integrated form
  – Built within a solid piece of silicon called IC (Integrated Circuit)



  – Several gates are included in a single plastic moulding

IC Families

- Transistor-Transistor Logic (TTL)
- Emitter Coupled Logic (ECL)
- Complementary Metal-Oxide-Semiconductor (CMOS)

Levels of IC

- Small-scale Integration
  - ICs with 1 to 10 gates
- Medium-scale Integration
  - ICs with 10 to 100 gates
- Large-scale Integration
  - ICs with 100 to 1000s of gates
- Very large-scale Integration
  - ICs with 1000s to millions of gates