# TREE ADT

# BST

# AVL

# TREE ADT

## MOTIVATIONS

Lists - Linear
Trees - Logarithmic

File Systems

Arithmetic Expressions

Compiler Designs

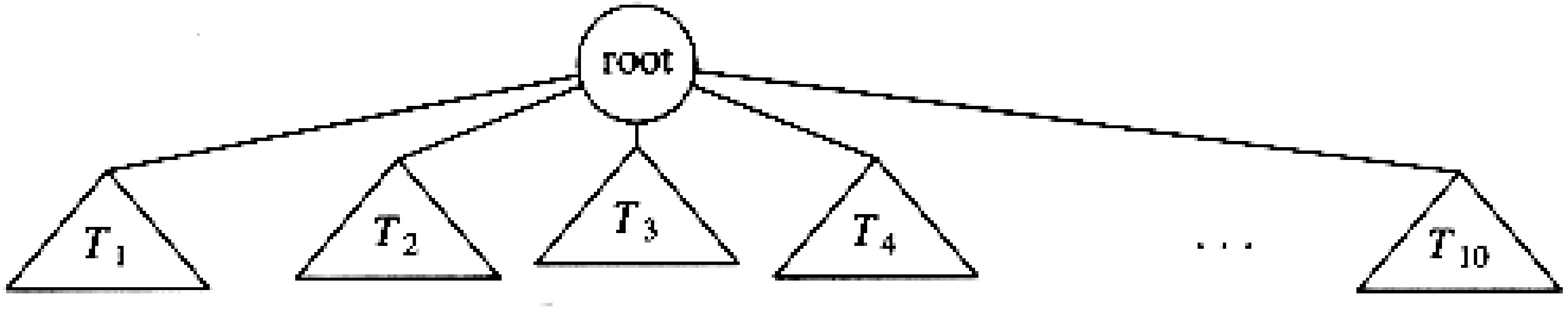
# TREE

A connected graph with no cycles.

# TREE

A tree consists of a distinguished node r (the root), and zero or more sub trees, $T_1$, $T_2$ …, $T_k$ each of whose roots are connected by a directed edge to r.

# TREES



# TREE

## THEOREMS

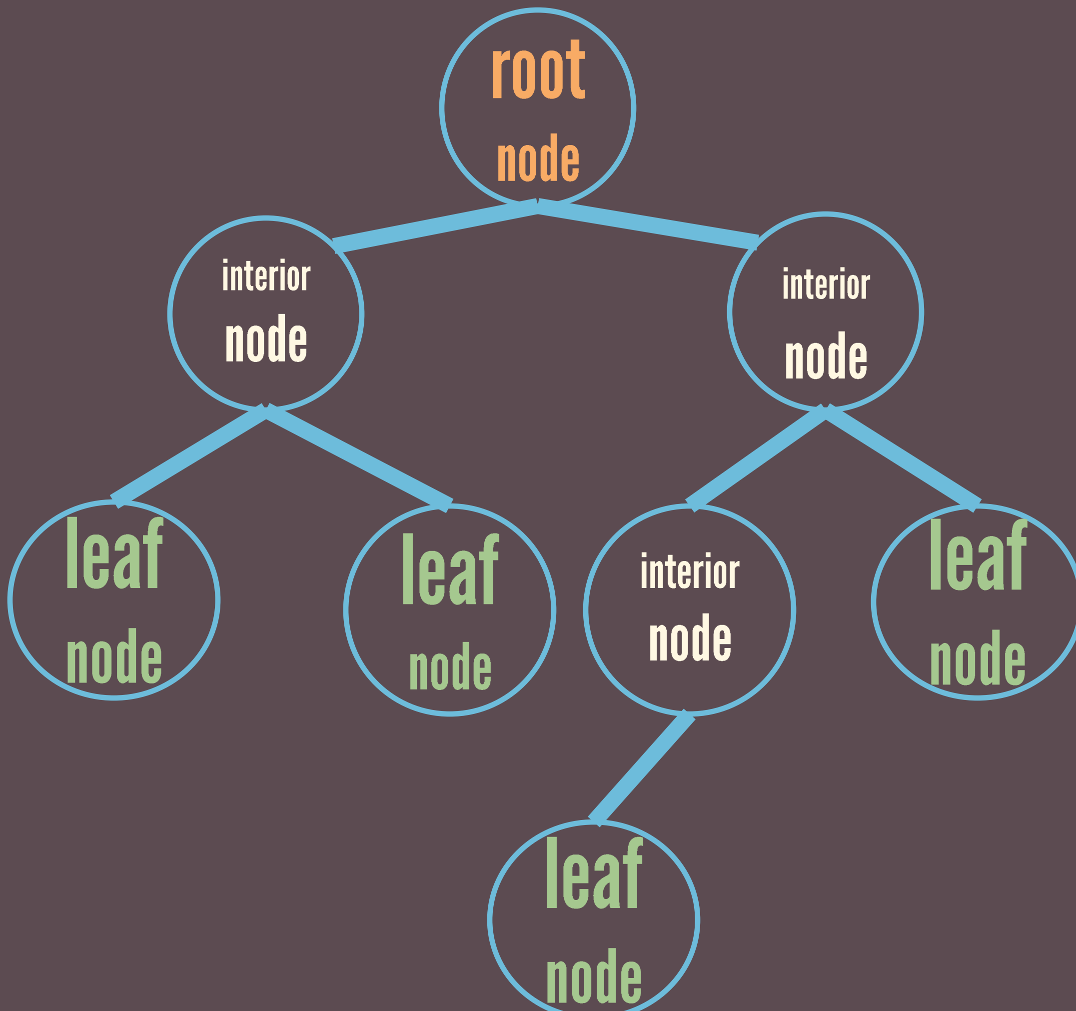
Any two vertices are connected by a unique path.

# TREE

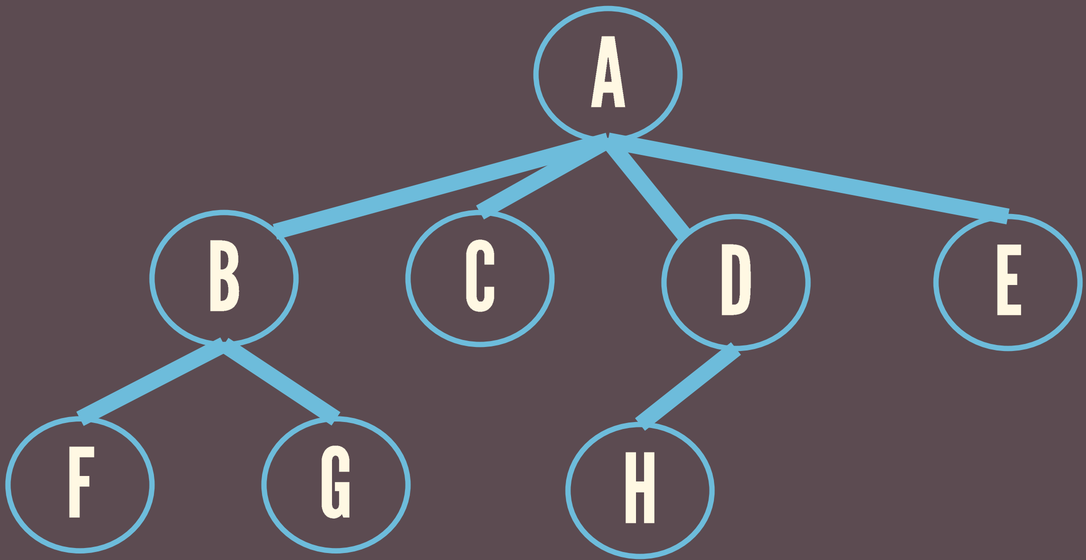
## THEOREMS

The number of edges in a tree is $|V(G)| - 1$

# ROOT INTERIOR NODE LEAF NODE



# SIBLINGS PARENT CHILD GRANDCHILD ANCESTORS DESCENDANTS



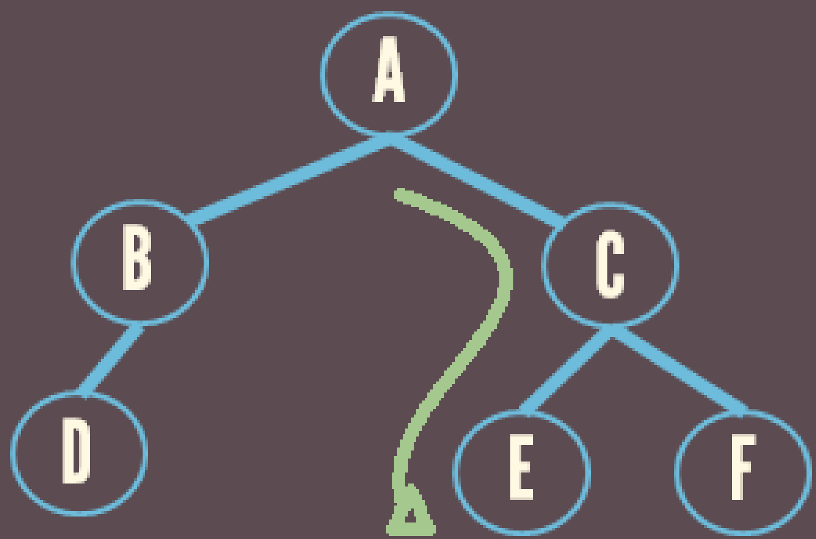
# PATH

from node $n_1$ to $n_k$

Sequence of nodes $n_1$, $n_2$, … $n_k$ such that $n_i$ is the parent of $n_{i+1}$

$1 \leq i \leq k$

# PATH




ACE

## length of the
# PATH

The number of edges on the path.

# HEIGHT

of node $n_i$

The height of node $n_i$ is the longest path from $n_i$ to a leaf.

# DEPTH

of node $n_i$

The length of the unique path from the root to $n_i$

Height of A = 2

Height of B = 0

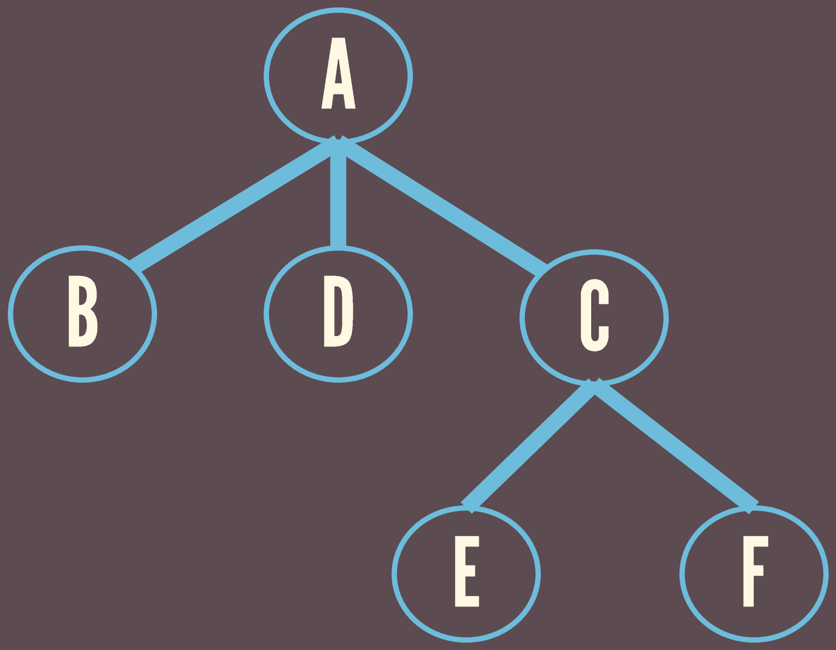
Height of D = 0

Height of C = 1

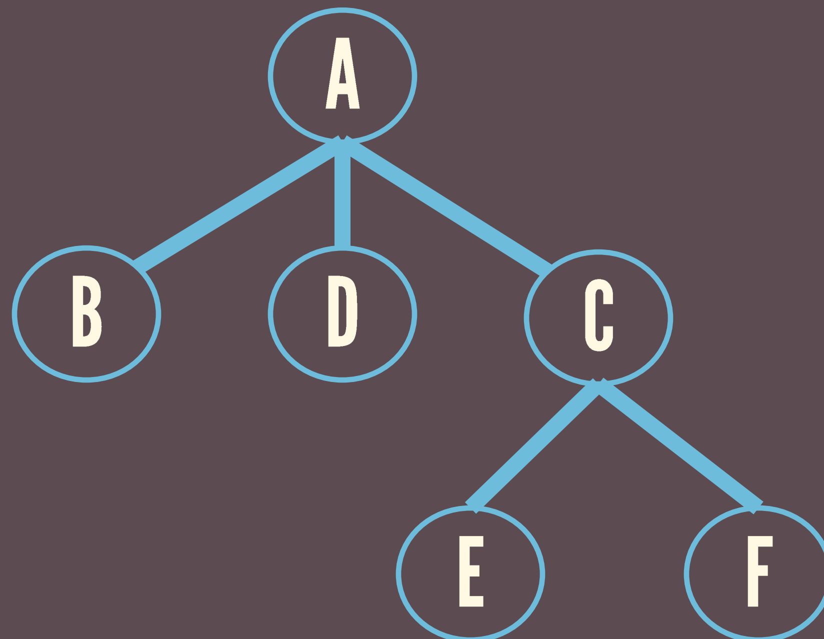
Height of E = 0

Height of F = 0

Height of the tree = 2



| DEPTH | LEVEL |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |



# IMPLEMENTATIONS

## LINKED
REPRESENTATION

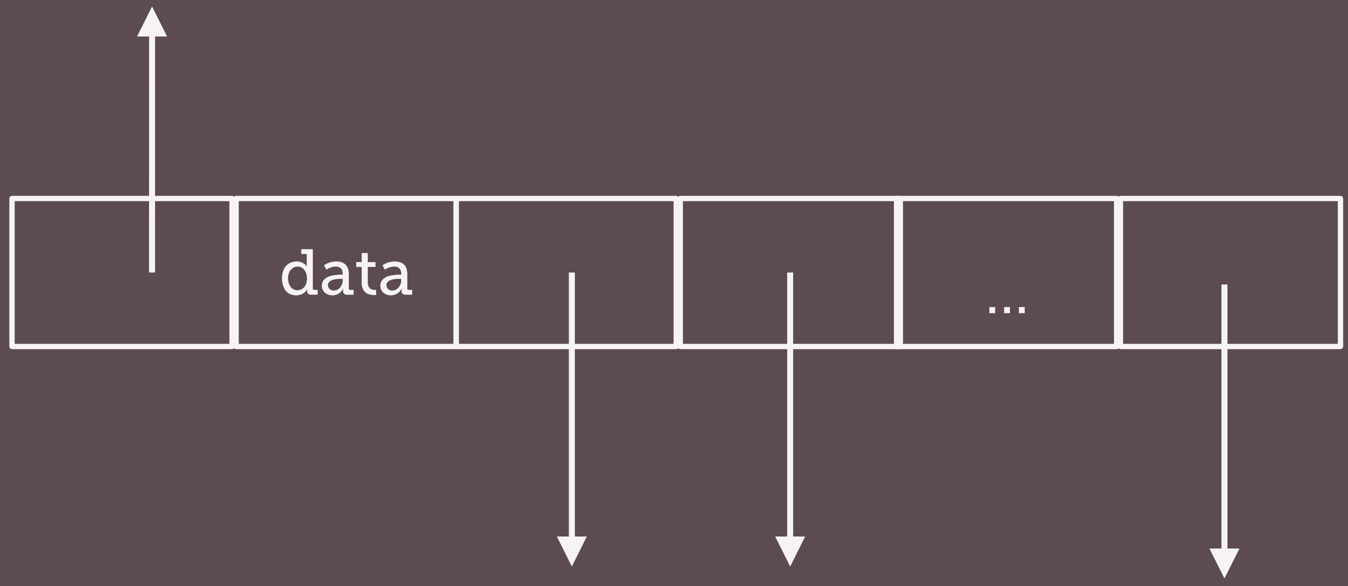
## FIRST CHILD, NEXT SIBLING
REPRESENTATION

# LINKED REPRESENTATION

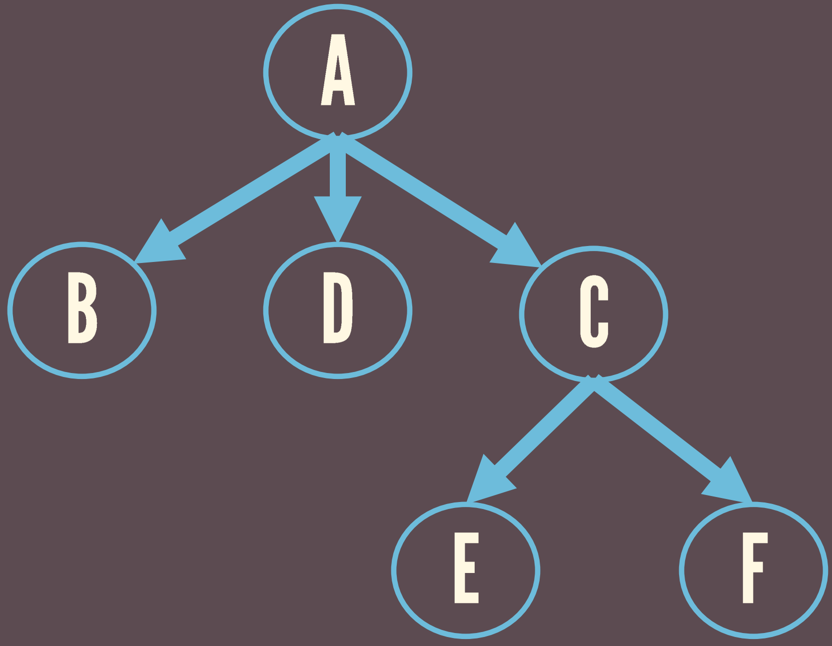
Each node besides its data has a pointer to each child of the node.

```
typedef struct node{
  int data;
  struct node *parent;
  struct node *child1;
  struct node *child2;
  ...
  struct node *childk;
}tree;
```

# LINKED REPRESENTATION



# LINKED REPRESENTATION



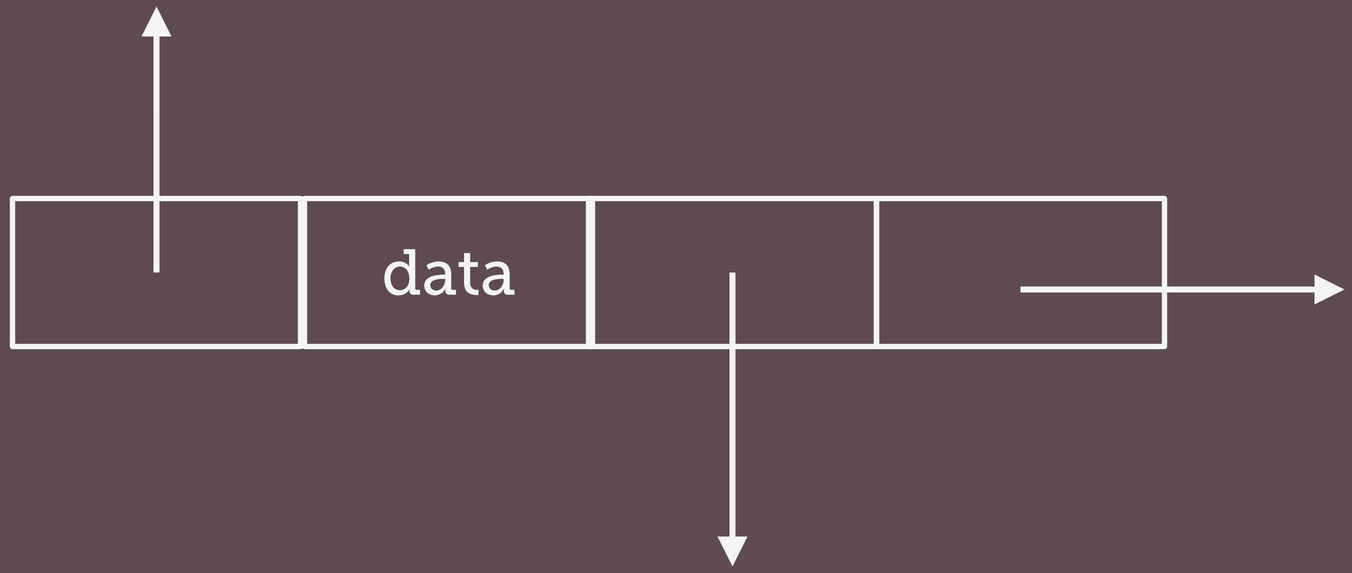
# FIRST CHILD, NEXT SIBLING REPRESENTATION

Keep the children of each node in a linked list of tree nodes.

```c
typedef struct node{
  int data;
  struct node *parent;
  struct node *first_child;
  struct node *next_sibling;
}tree;
```

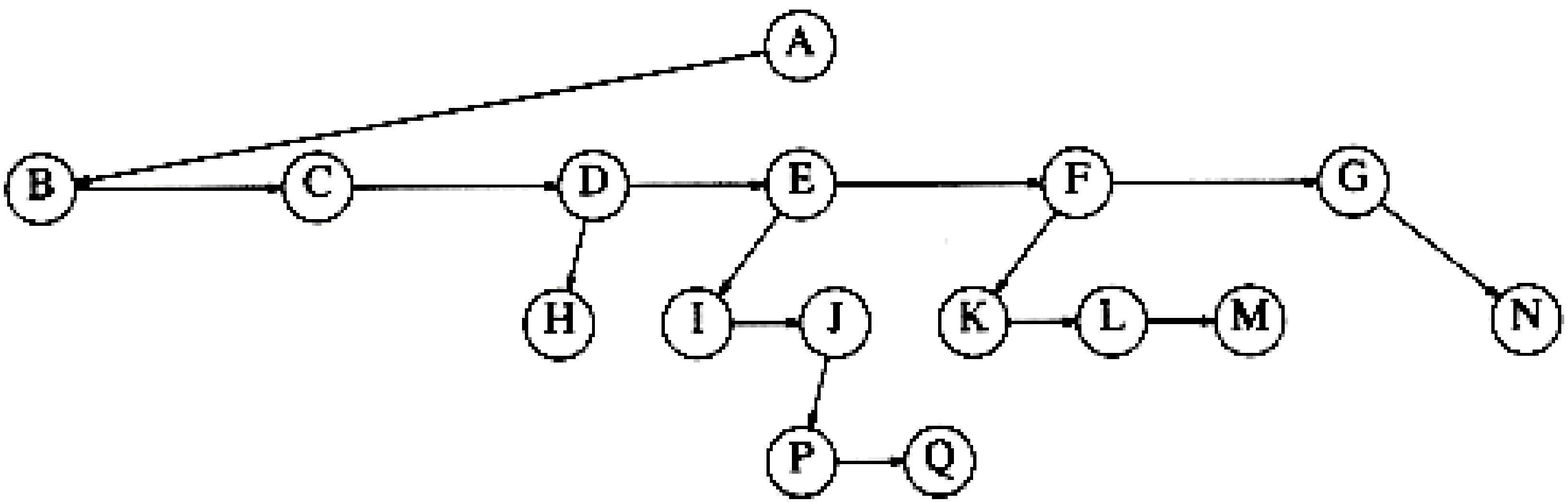
# FIRST CHILD, NEXT SIBLING

## REPRESENTATION



# FIRST CHILD, NEXT SIBLING

## REPRESENTATION



# BINARY TREES

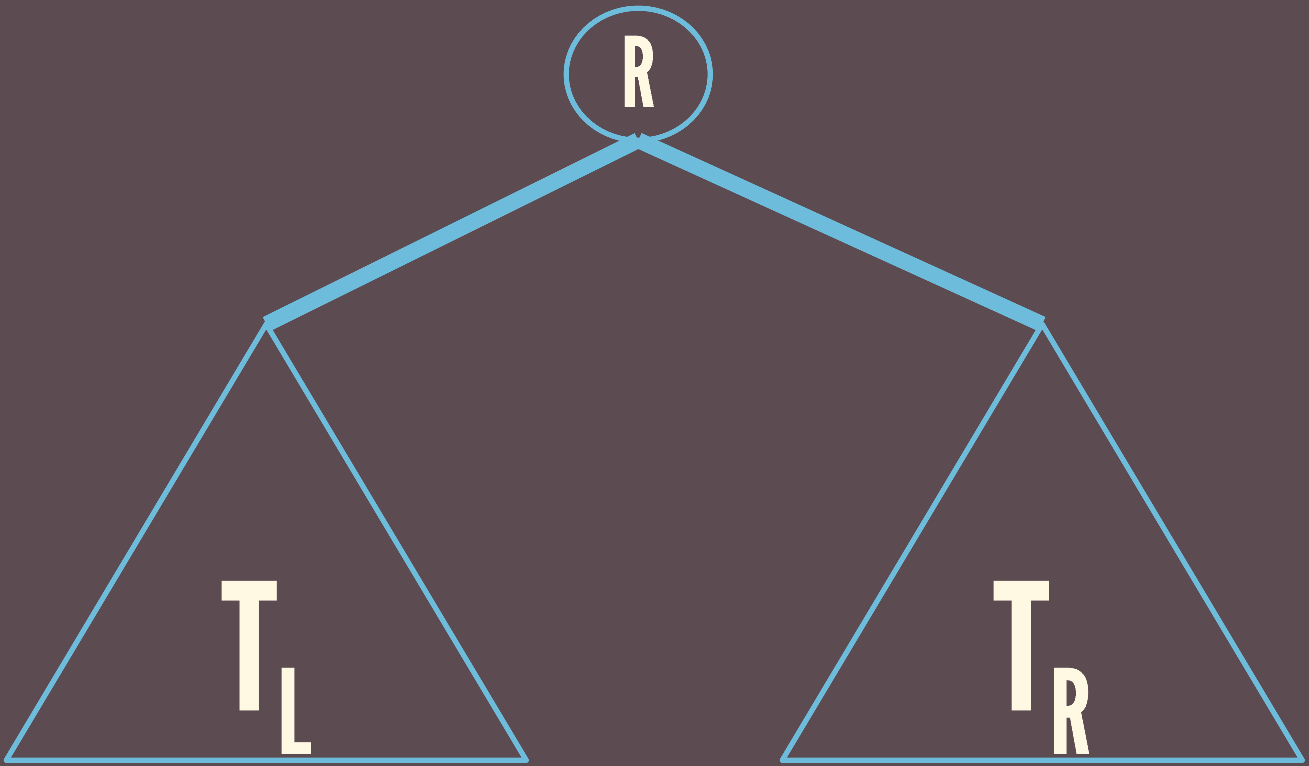
# BINARY TREE

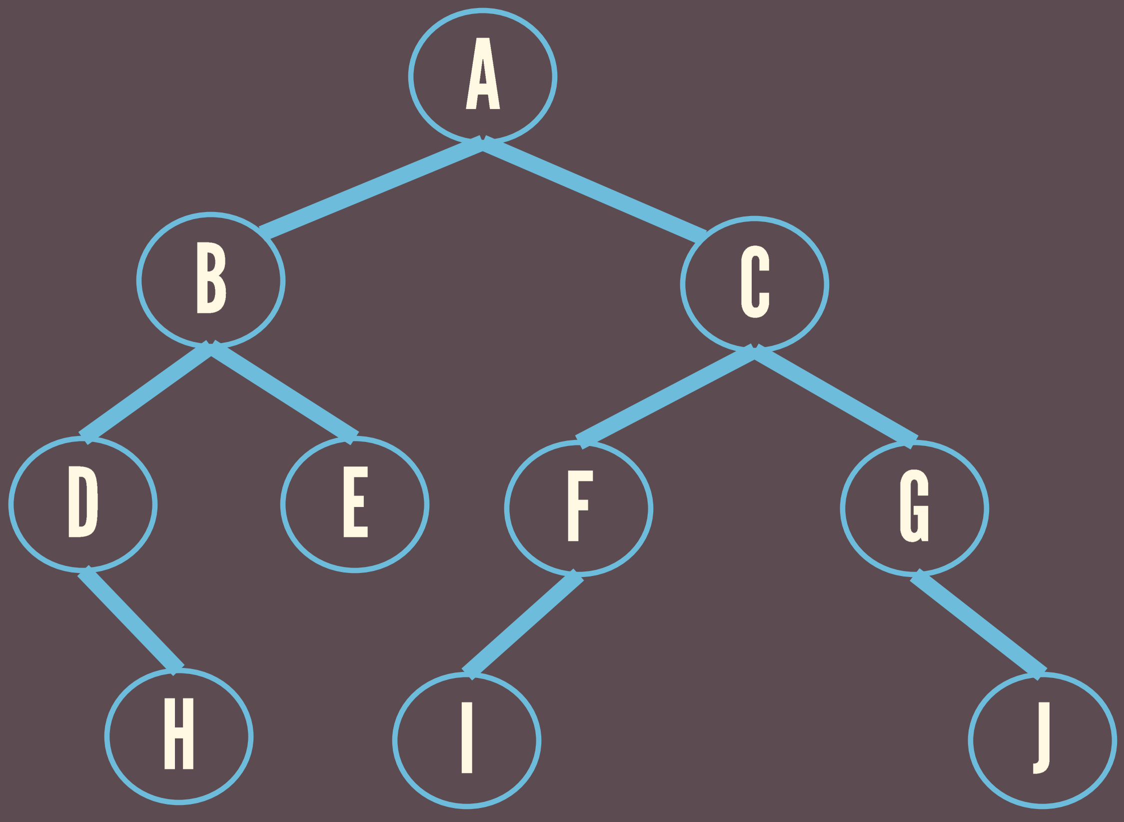
A tree in which no node can have more than two children.

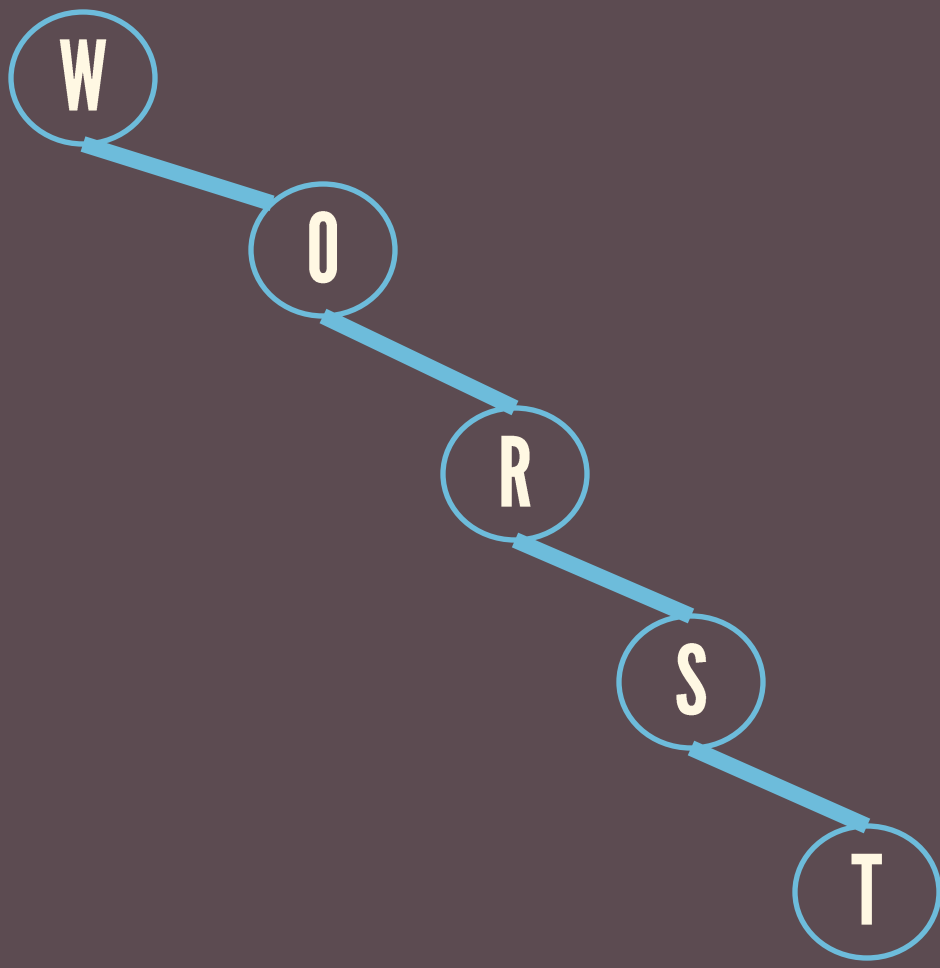
# BINARY TREE

A tree where each node has either

- no children
- a left child
- a right child
- both left and right child


R
$T_L$
$T_R$

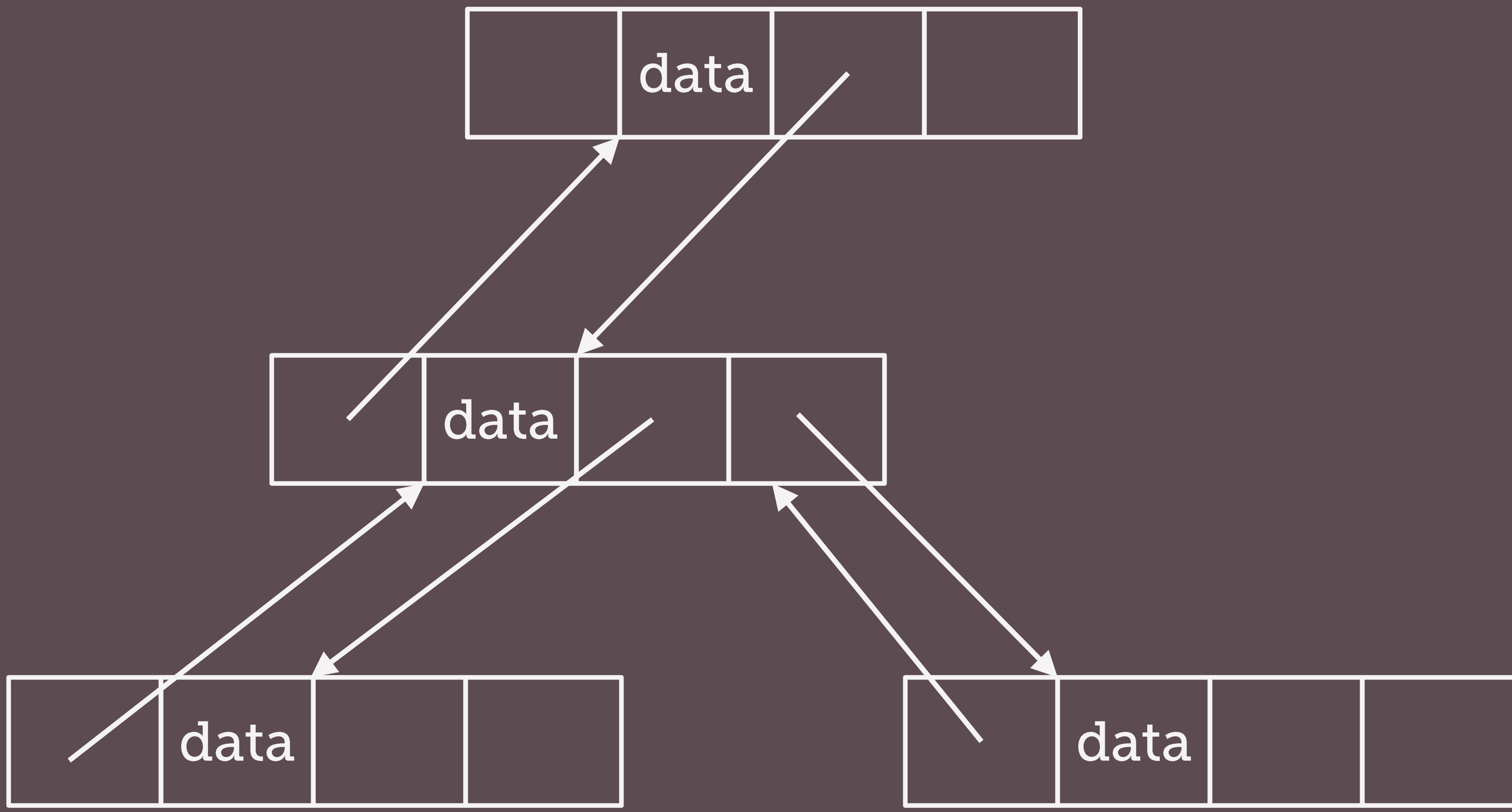

A
B
C
D
E
F
G
H
I
J


W
O
R
S
T

# IMPLEMENTATION

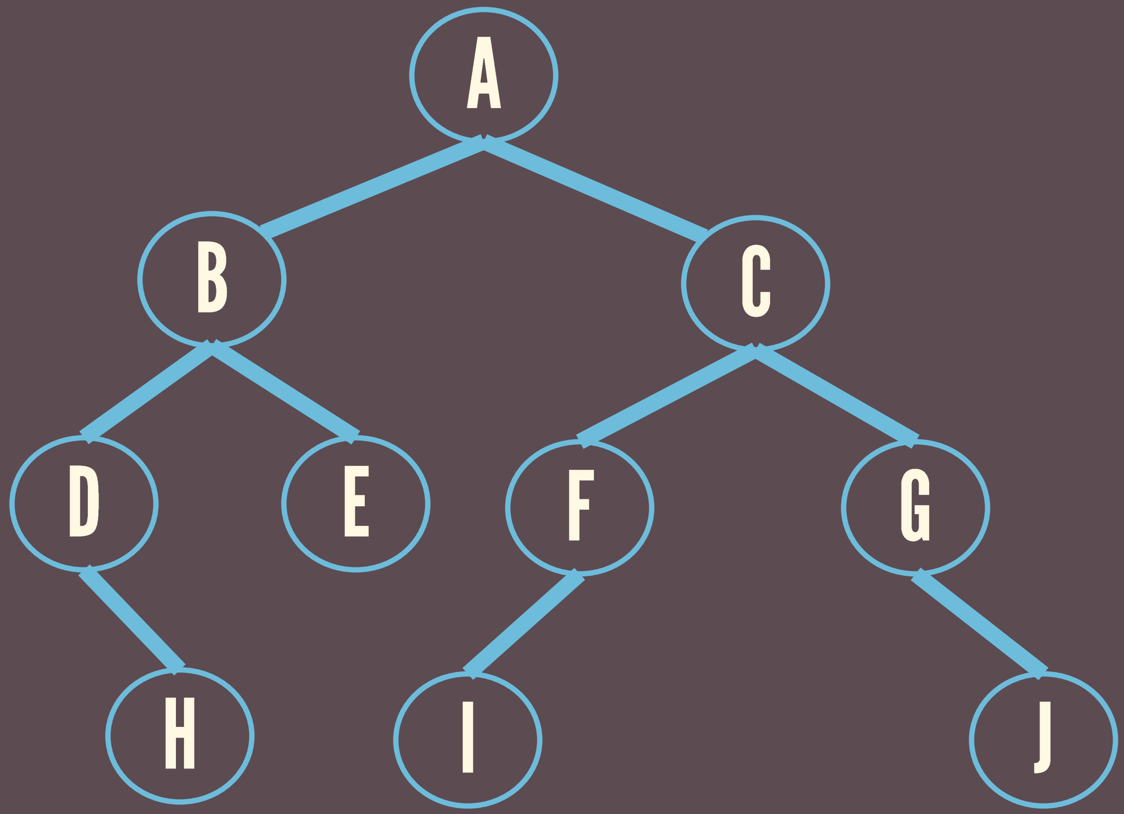
# LINKED REPRESENTATION

```c
typedef struct node{
   int data;
   struct node *parent;
   struct node *left;
   struct node *right;
}tree;
```


data
data
data
data

## full
# LEVEL

Level $i$ is full if there are exactly $2^i$ nodes at this level.


A
B
C
D
E
F
G
H
I
J

# BINARY TREE TRAVERSALS

PREORDER

INORDER

POSTORDER

# PREORDER

Visit root node, then left subtree and finally the right subtree.

```
preorder(tree *node){

  if(node!=NULL){
      print node->data
      preorder(node->left);
      preorder(node->right);
  }

}
```

# INORDER

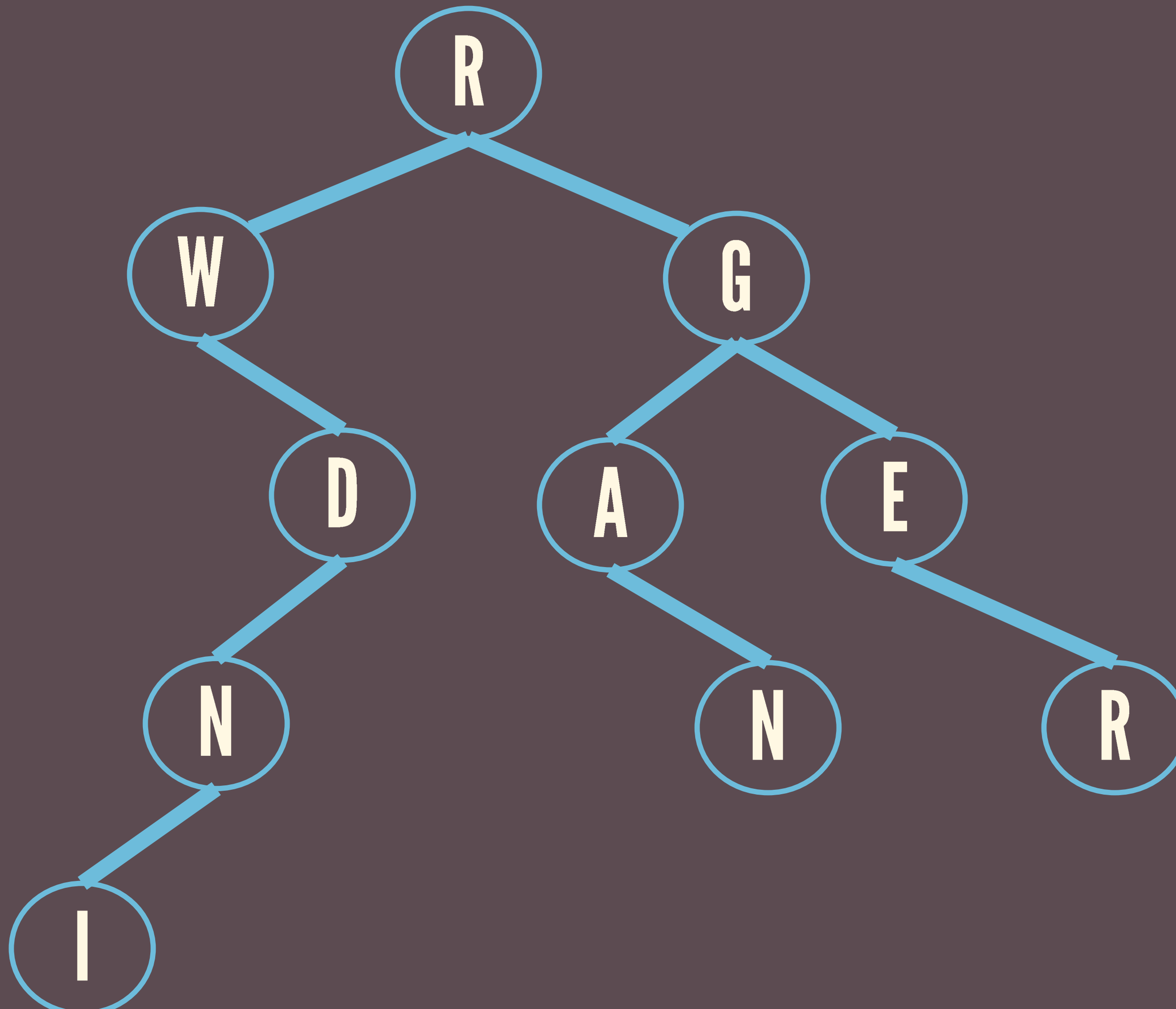
Visit  left subtree, then root node and finally the right subtree.

```
inorder(tree *node){

  if(node!=NULL){
      inorder(node->left);
      print node->data
      inorder(node->right);
  }

}
```

# POSTORDER

Visit  left subtree, then right subtree and finally the root node.

```
postorder(tree *node){

  if(node!=NULL){
      postorder(node->left);
      postorder(node->right);
      print node->data
  }

}
```


R
W
G
D
A
E
N
N
R
I

PREORDER: R W D N I G A N E R

INORDER: W I N D R A N G E R

POSTORDER: _ _ _ _ _ _ _ _ _ _

PREORDER: R W D N I G A N E R

INORDER: W I N D R A N G E R

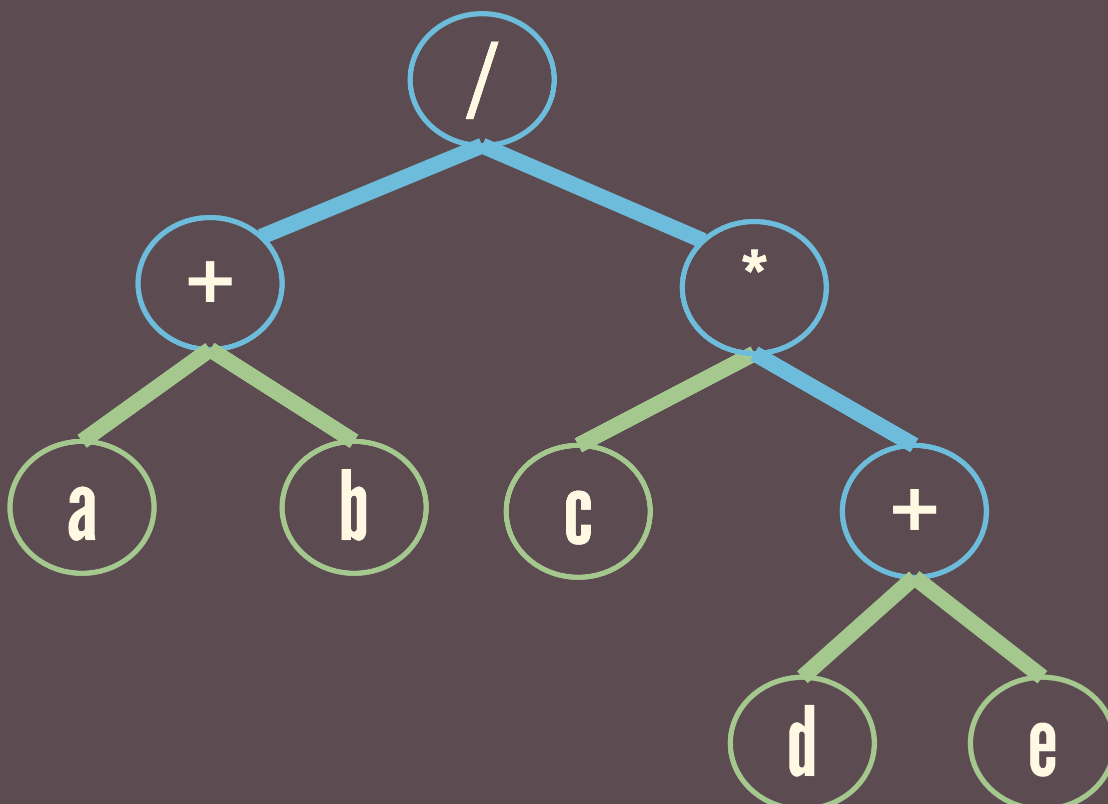
POSTORDER: I N D W N A R E G R

# EXPRESSION TREES

LEAVES
OPERANDS

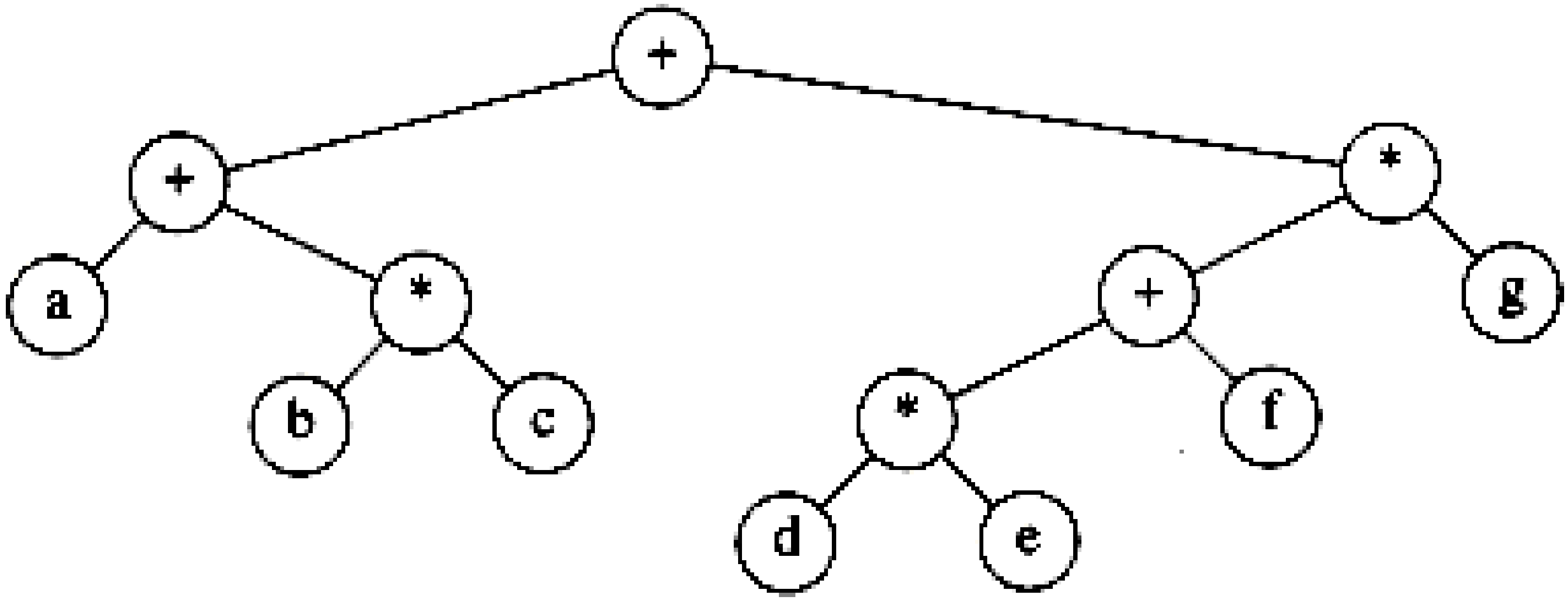
INTERNAL NODES
OPERATORS

(a + b) / (c * (d + e))

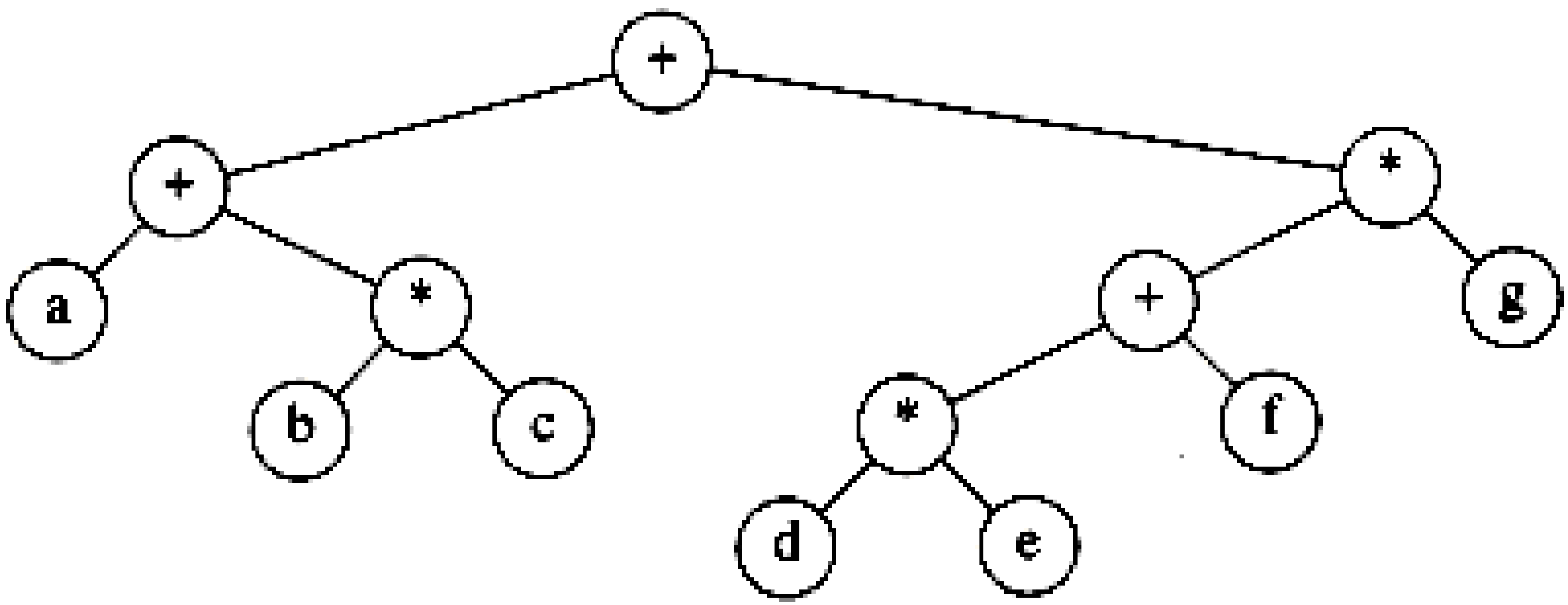


# EXPRESSION TREE TRAVERSALS

| PREORDER | INORDER | POSTORDER |
| --- | --- | --- |
| PREFIX | INFIX | POSFIX |

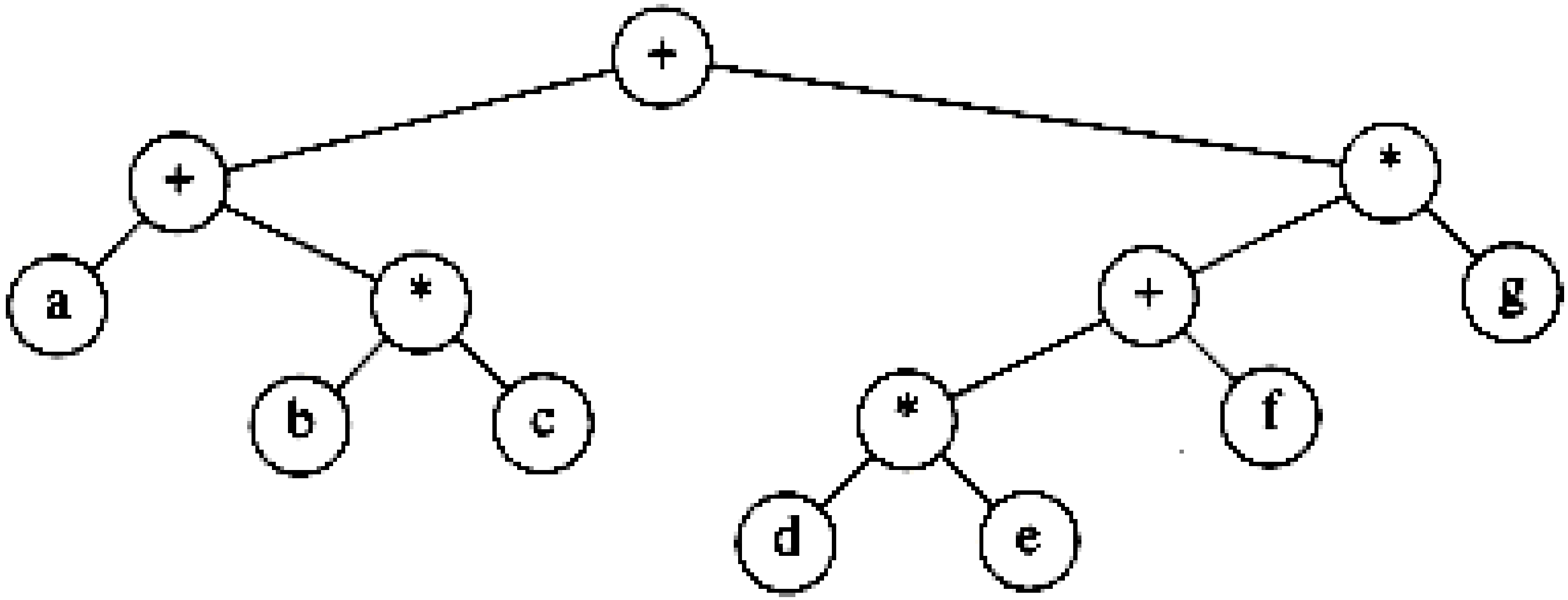



(a + b * c) + ((d * e + f) * g)



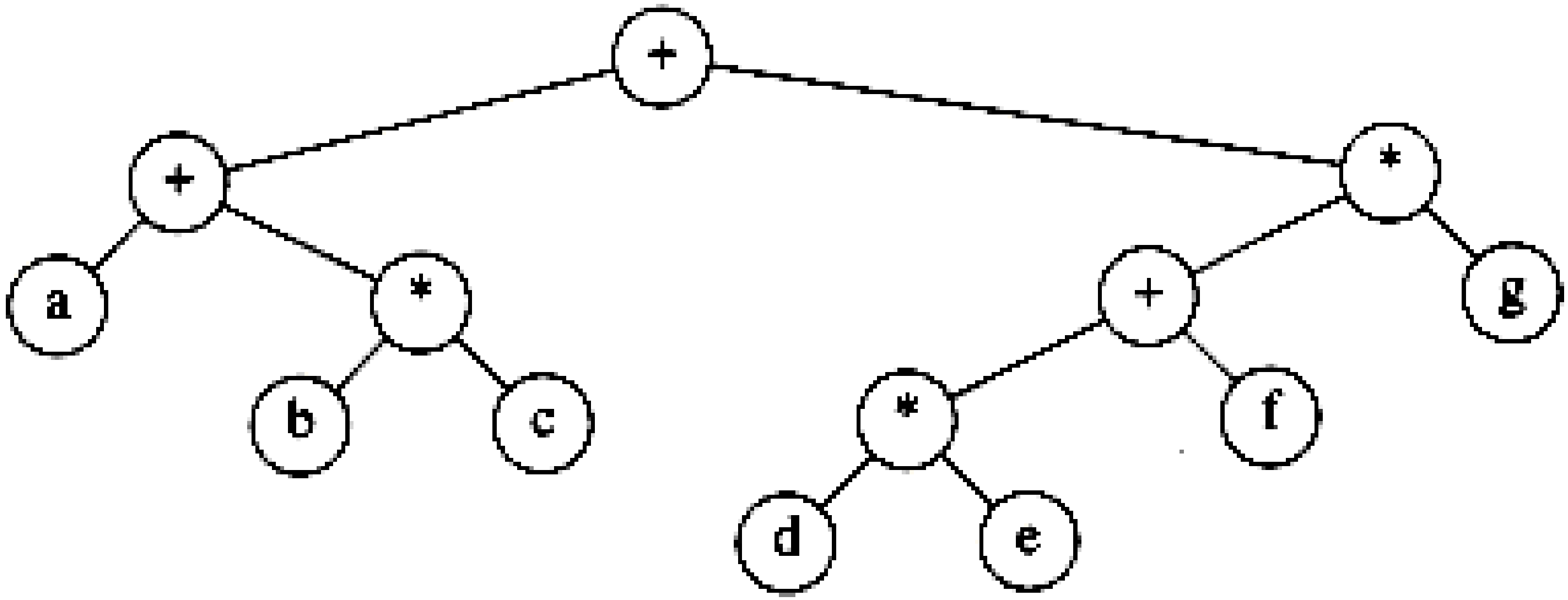

PREFIX

+ + a * b c * + * d e f g




POSTFIX

a b c * + d e * f + g * +




INFIX

a + b * c + d * e + f * g

# ALGORITHM
## TO CONSTRUCT
# EXPRESSION TREES

Convert the expression to postfix.

Use a stack.

Read the expression (postfix) one symbol at a time:

if the symbol is an operand,

- create a one-node tree
- push a pointer to it onto a stack

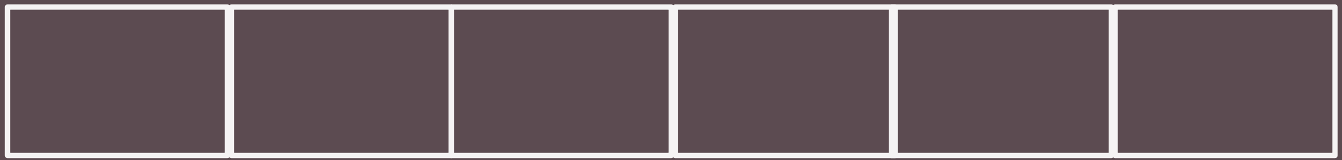
Read the expression (postfix) one symbol at a time:

if the symbol is an operator,

- pop two pointers to two trees ($T_1$ and $T_2$).
- Form a new tree whose root is the operator with left and right child pointing to $T_1$ and $T_2$ respectively.
- push onto the stack a pointer to this new tree.

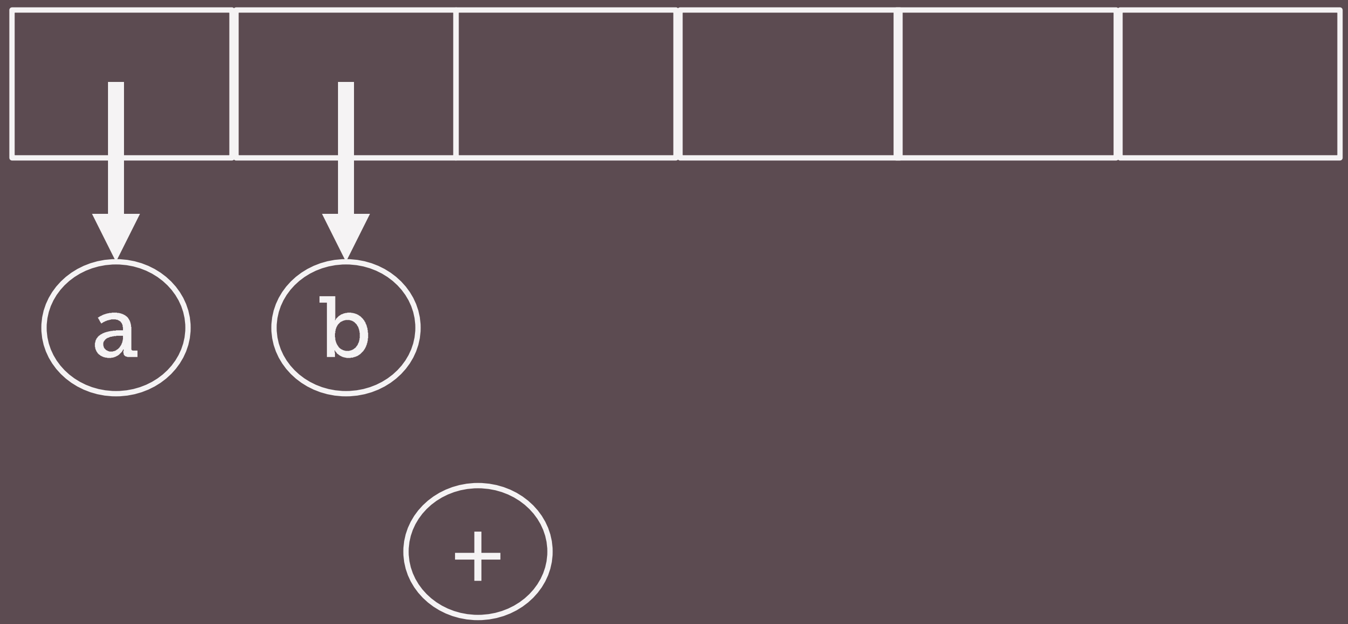
# EXAMPLE

a b + c d e + * *



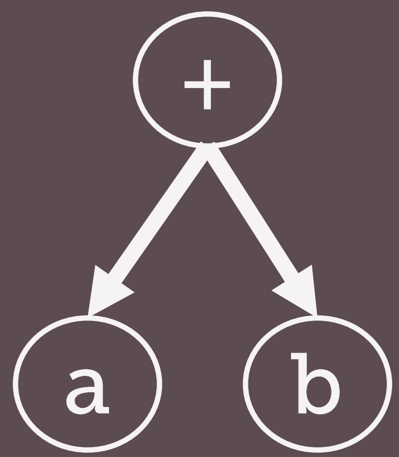
a b + c d e + * *

a b + c d e + * *



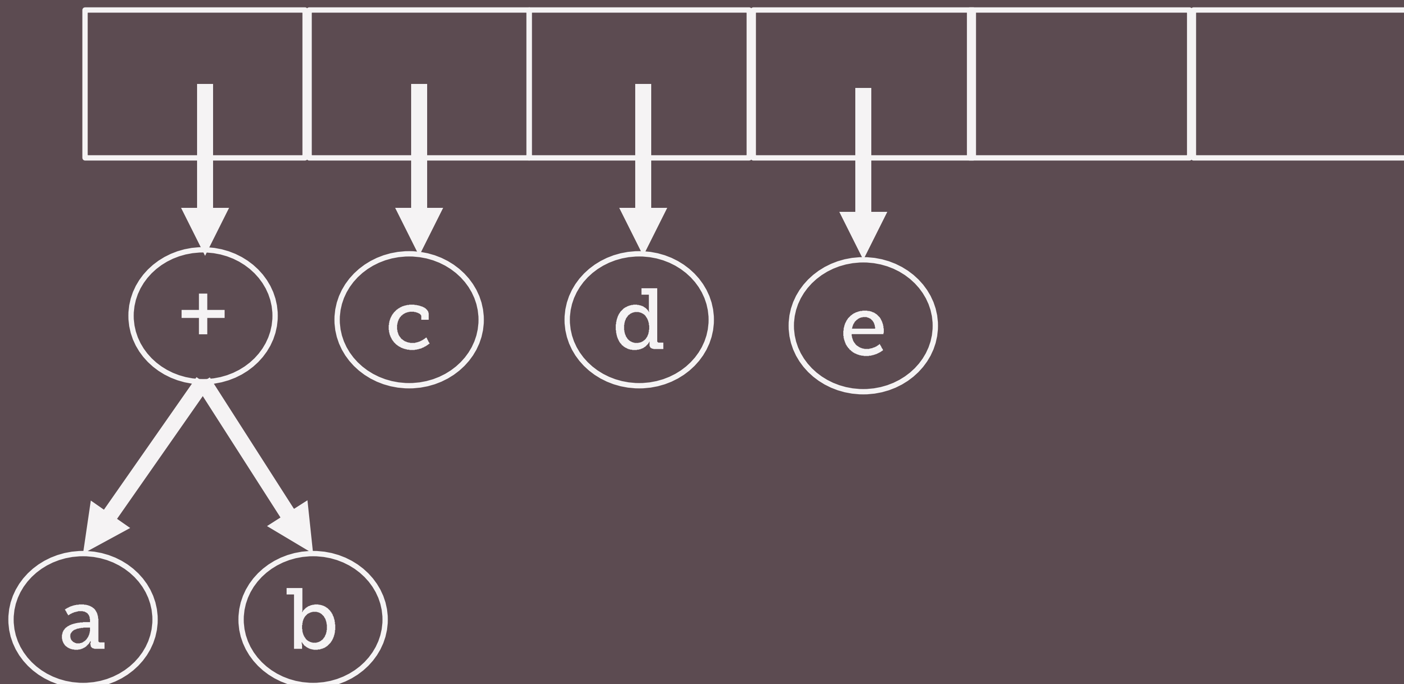
a b + c d e + * *

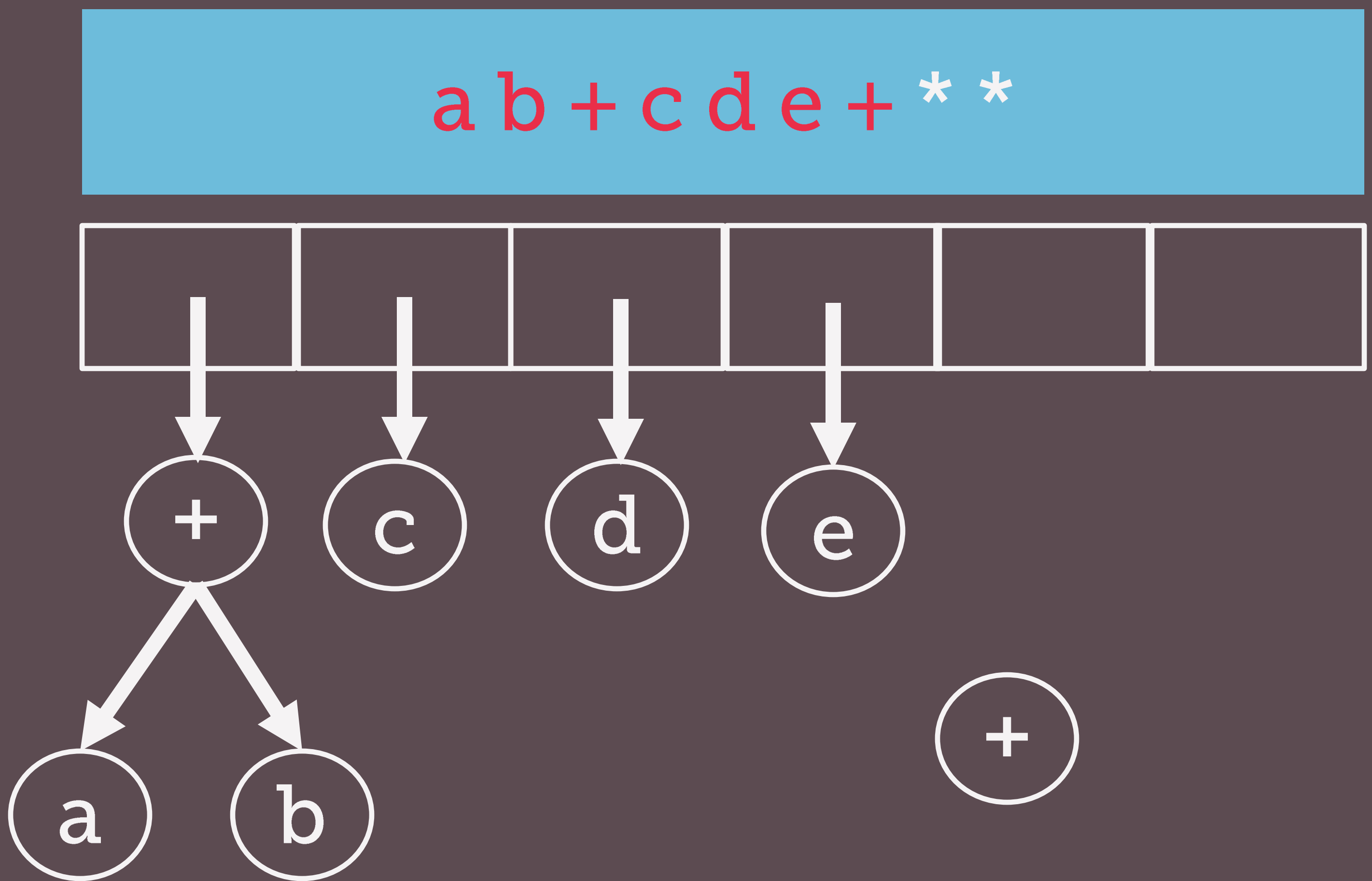
| | | | | | |
|---|---|---|---|---|---|
| | | | | | |



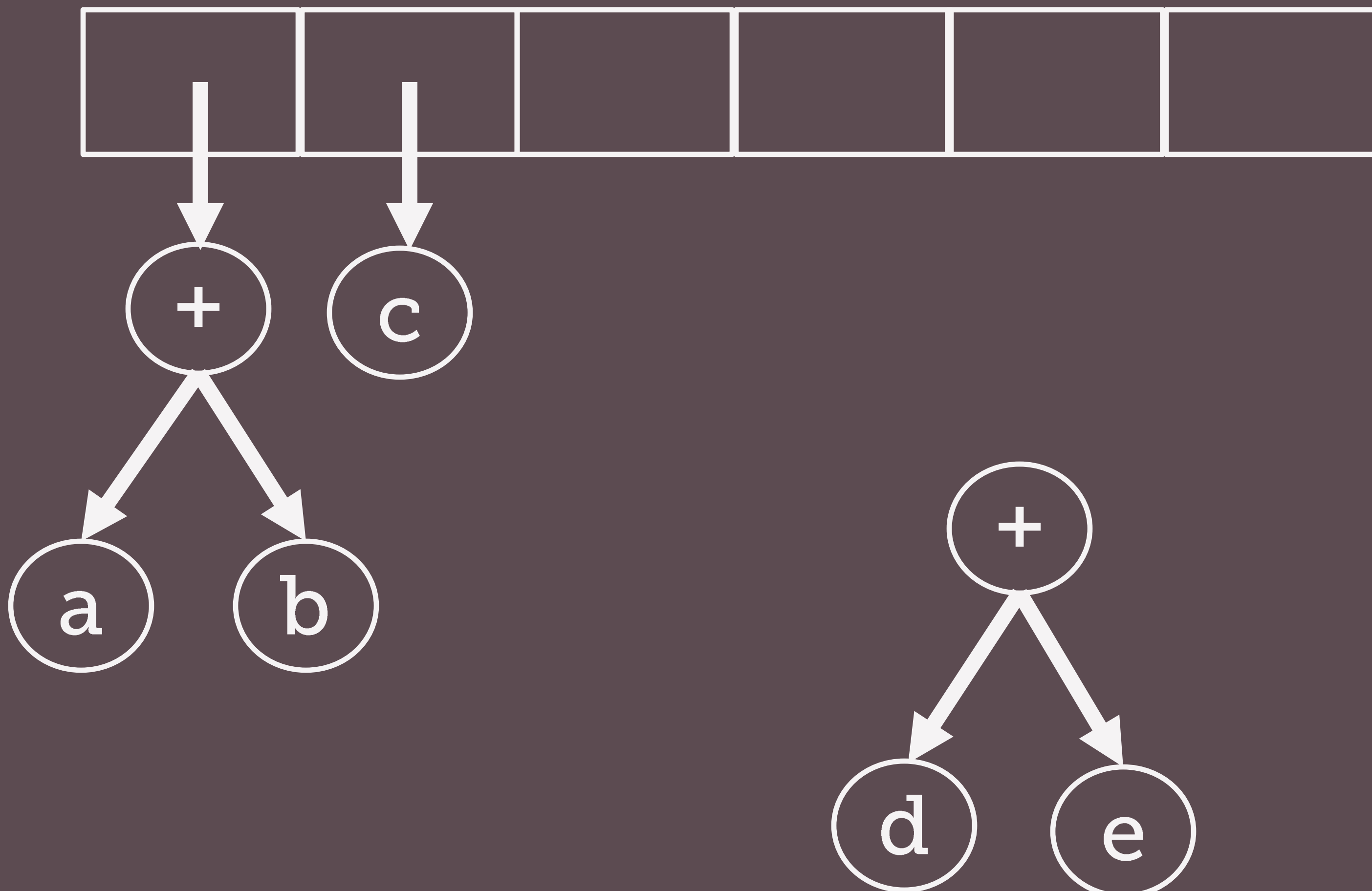
a b + c d e + * *

a b + c d e + * *

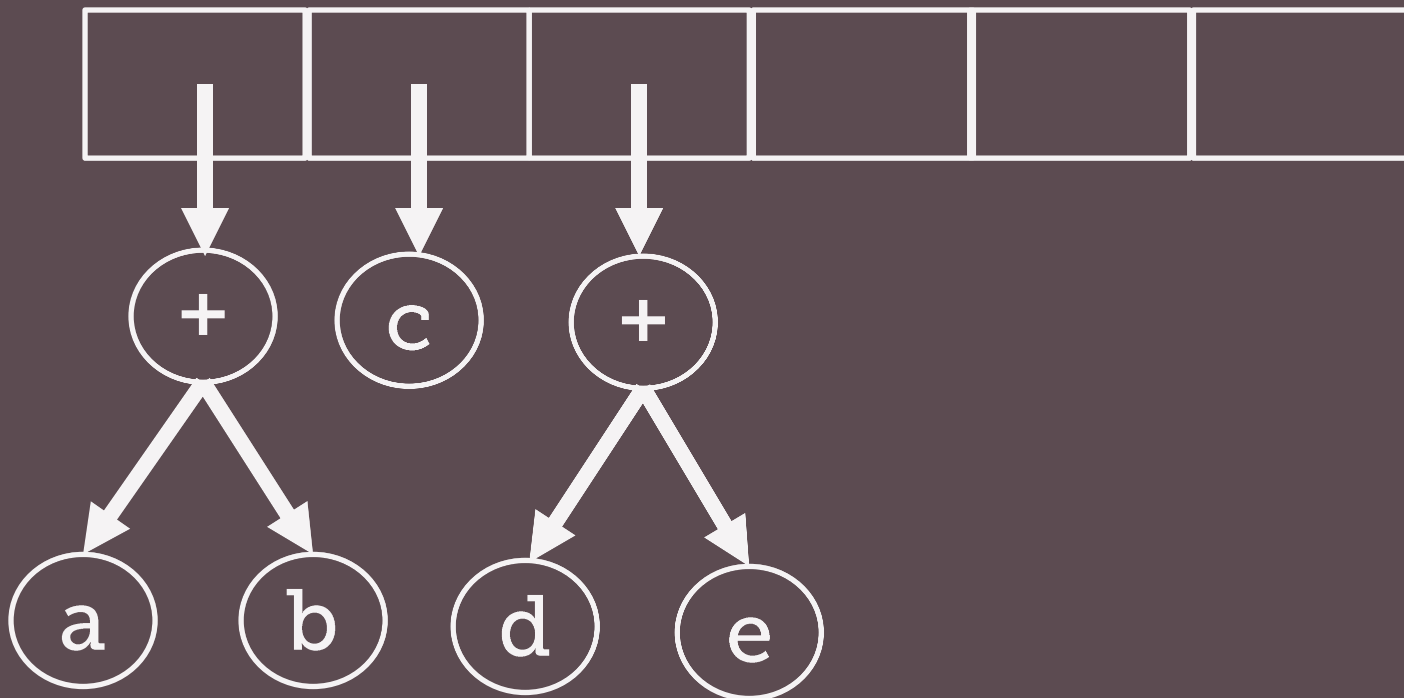



a b + c d e + * *
+
c
d
e
a
b
+

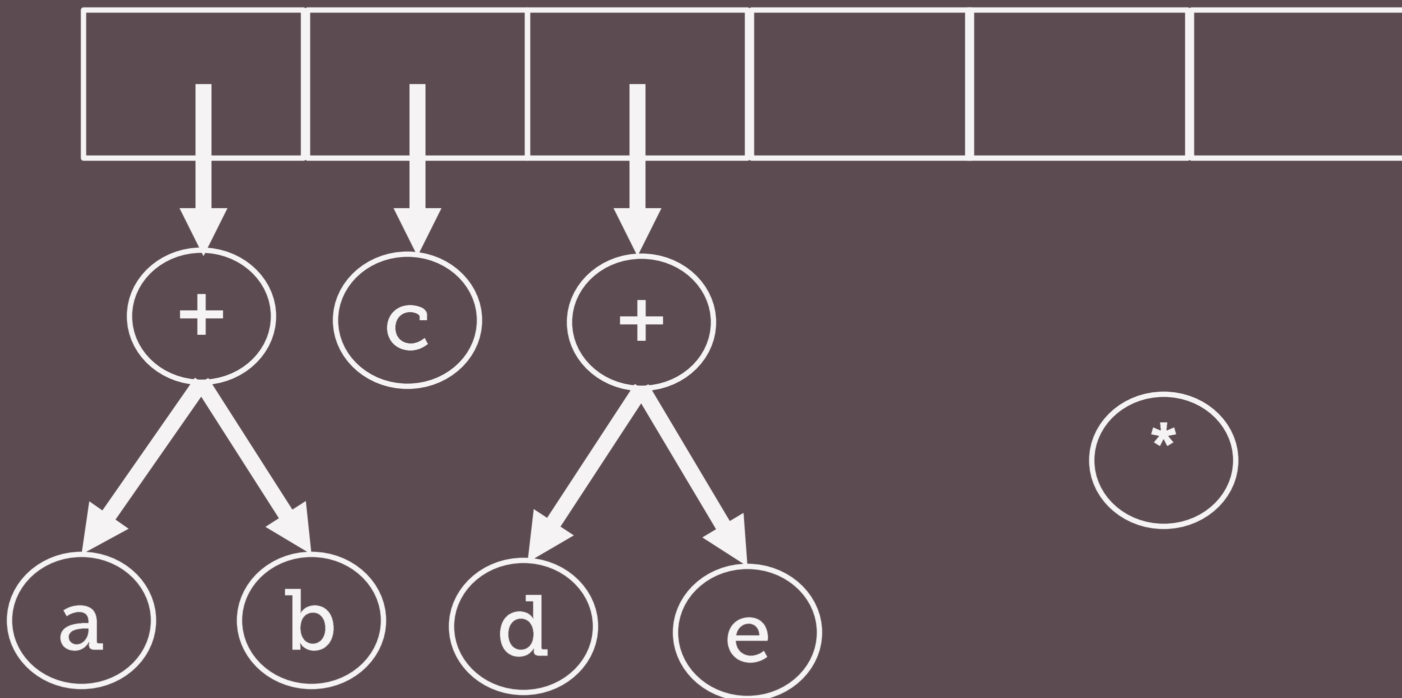
a b + c d e + * *



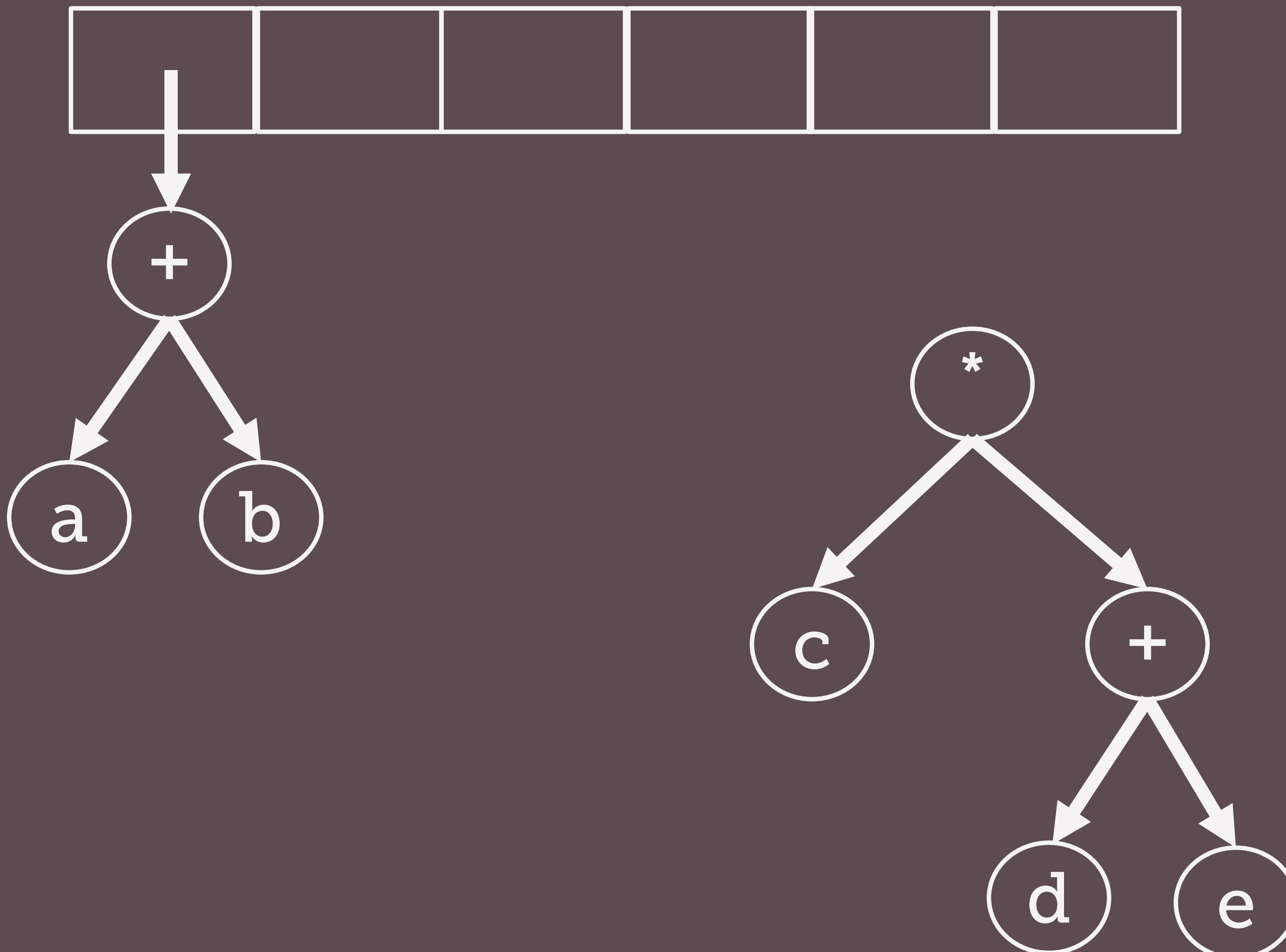
a b + c d e + * *



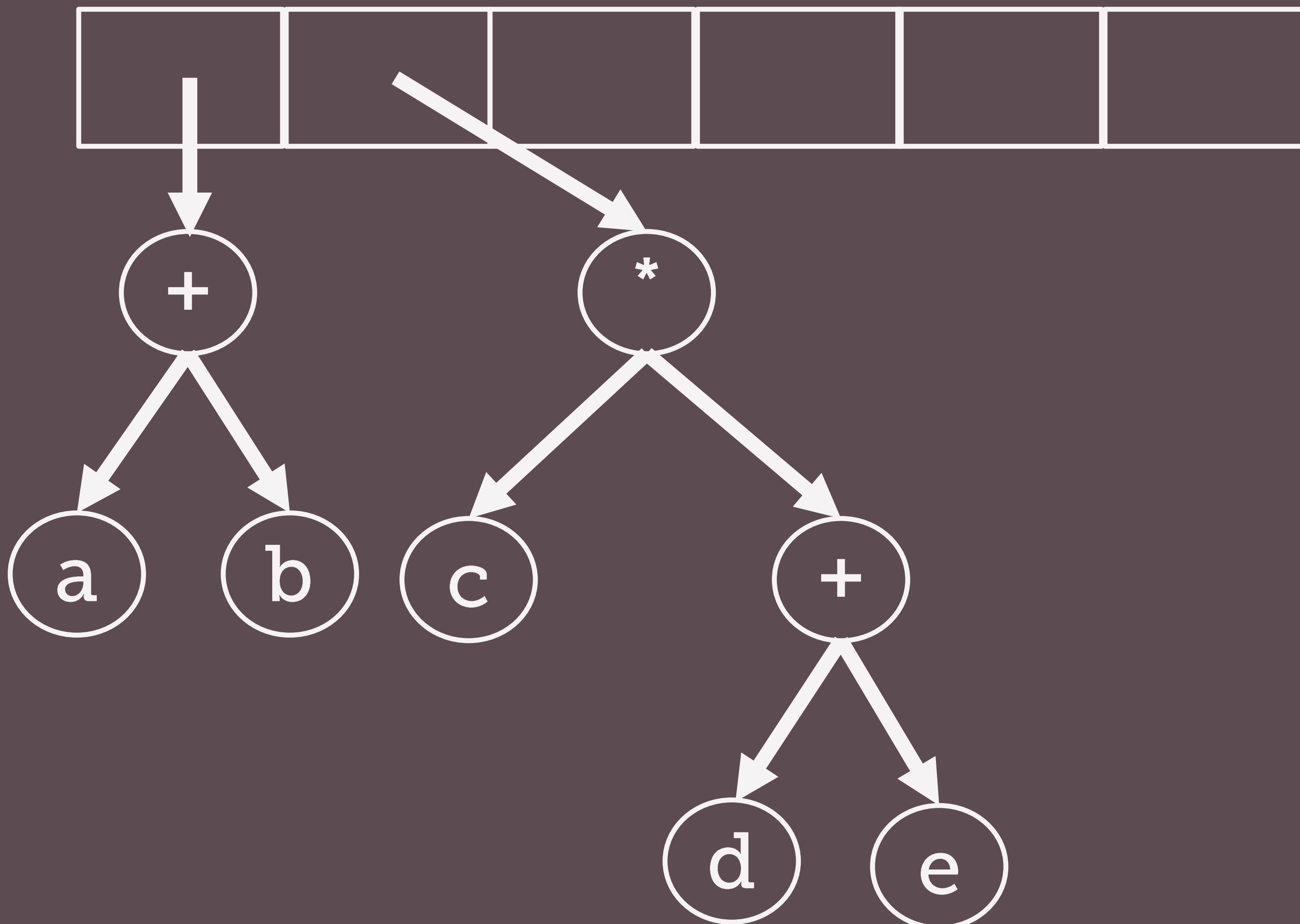
a b + c d e + * *



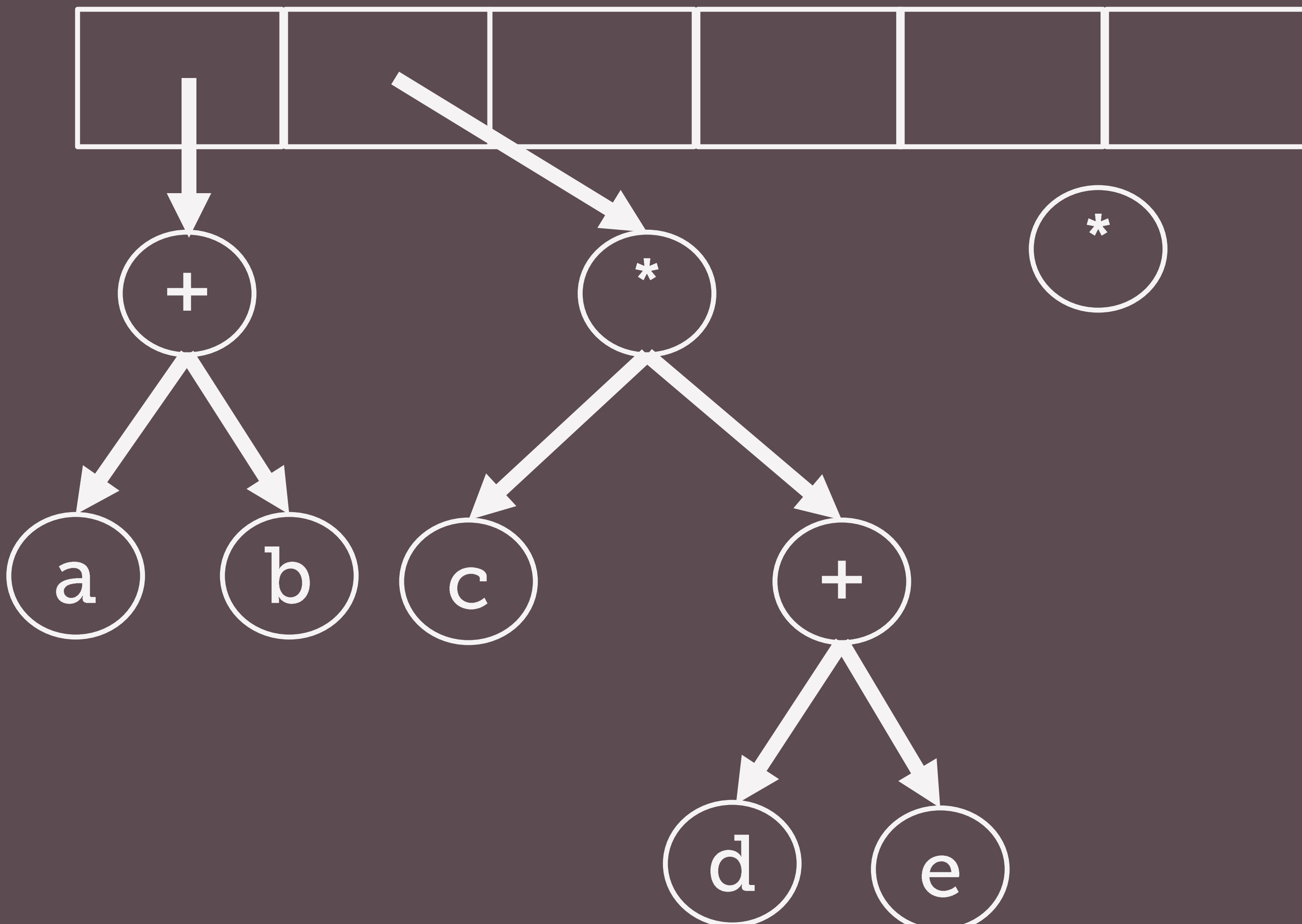
a b + c d e + * *



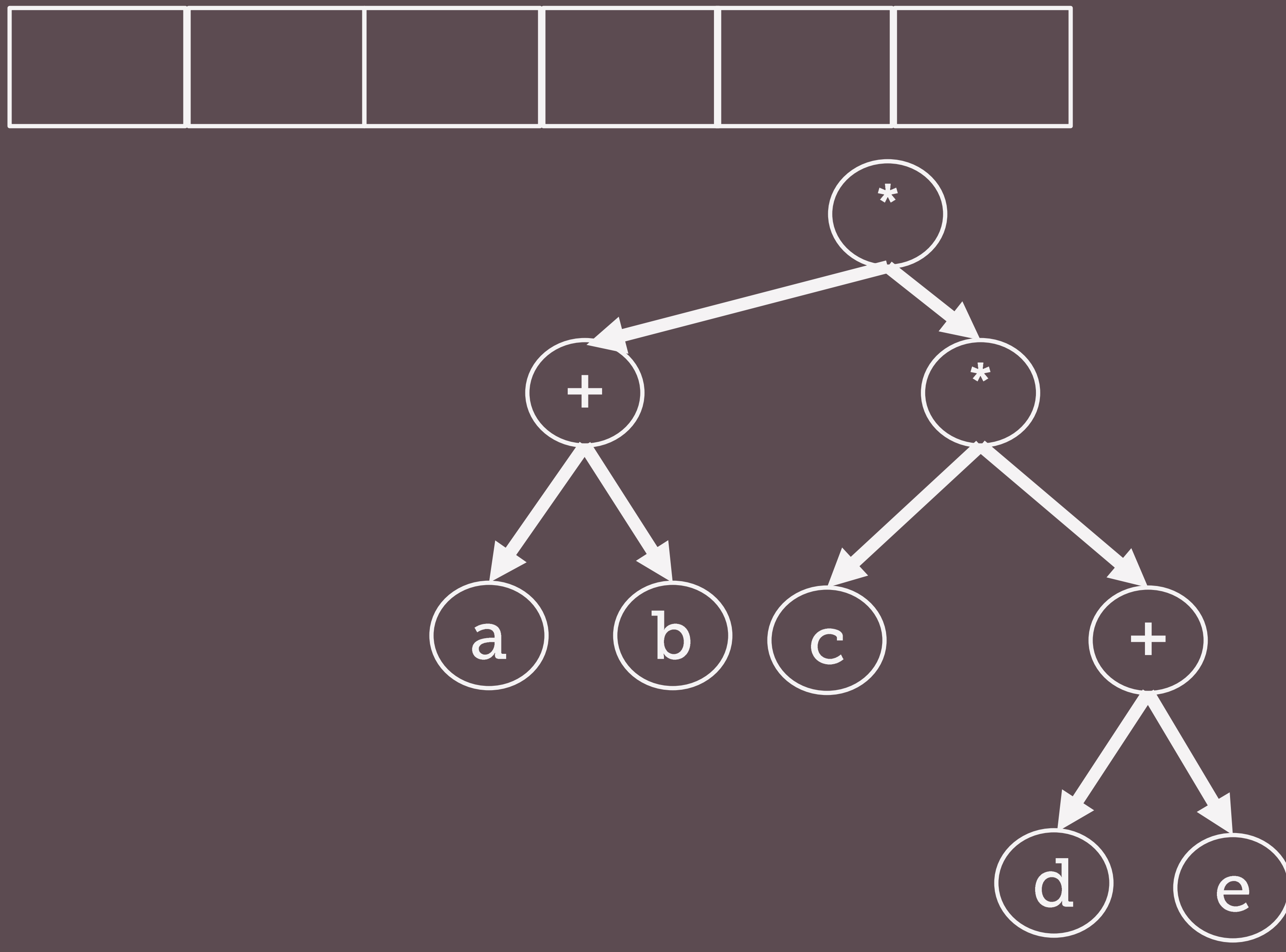
a b + c d e + * *



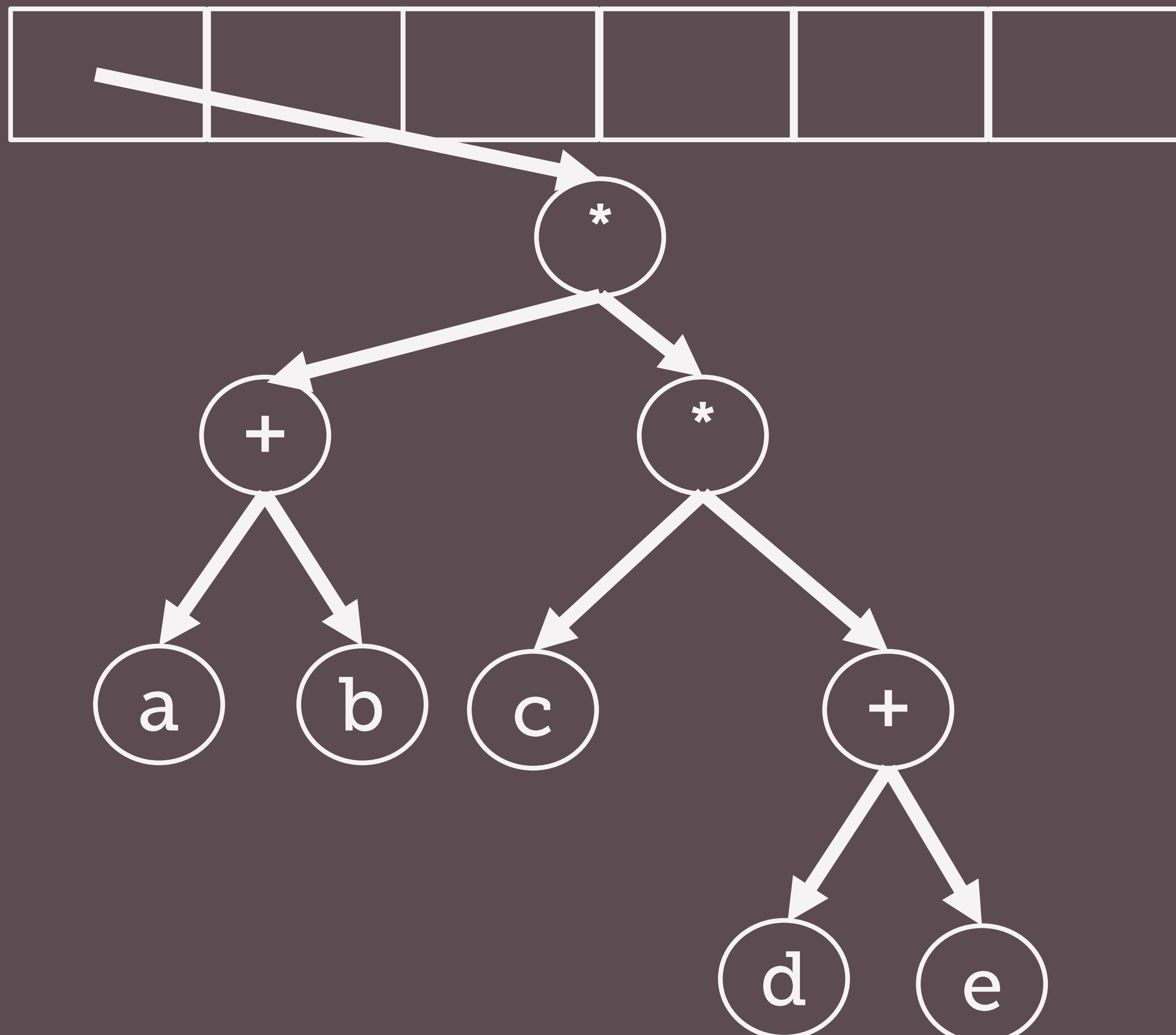
a b + c d e + * *



a b + c d e + * *



a b + c d e + * *



# SEARCH TREE ADT

BST

AVL

# BST

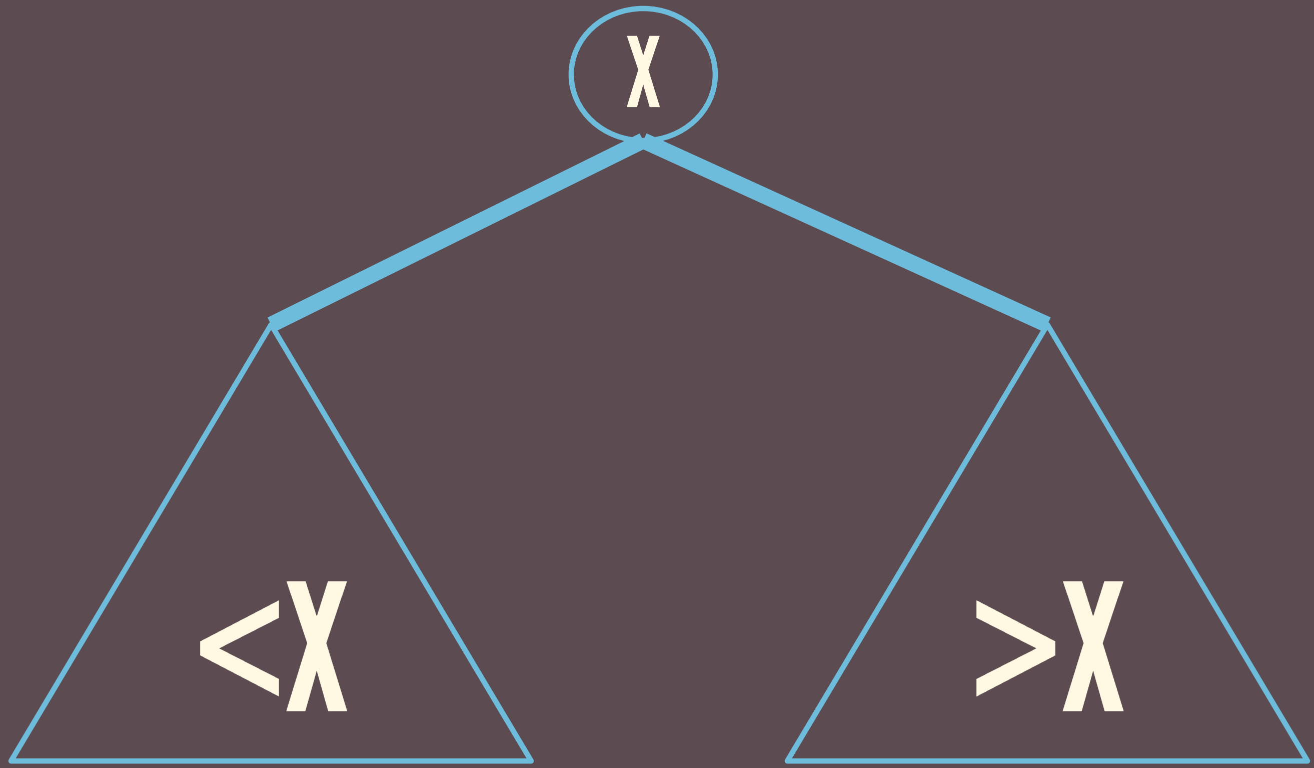
# BINARY SEARCH TREE

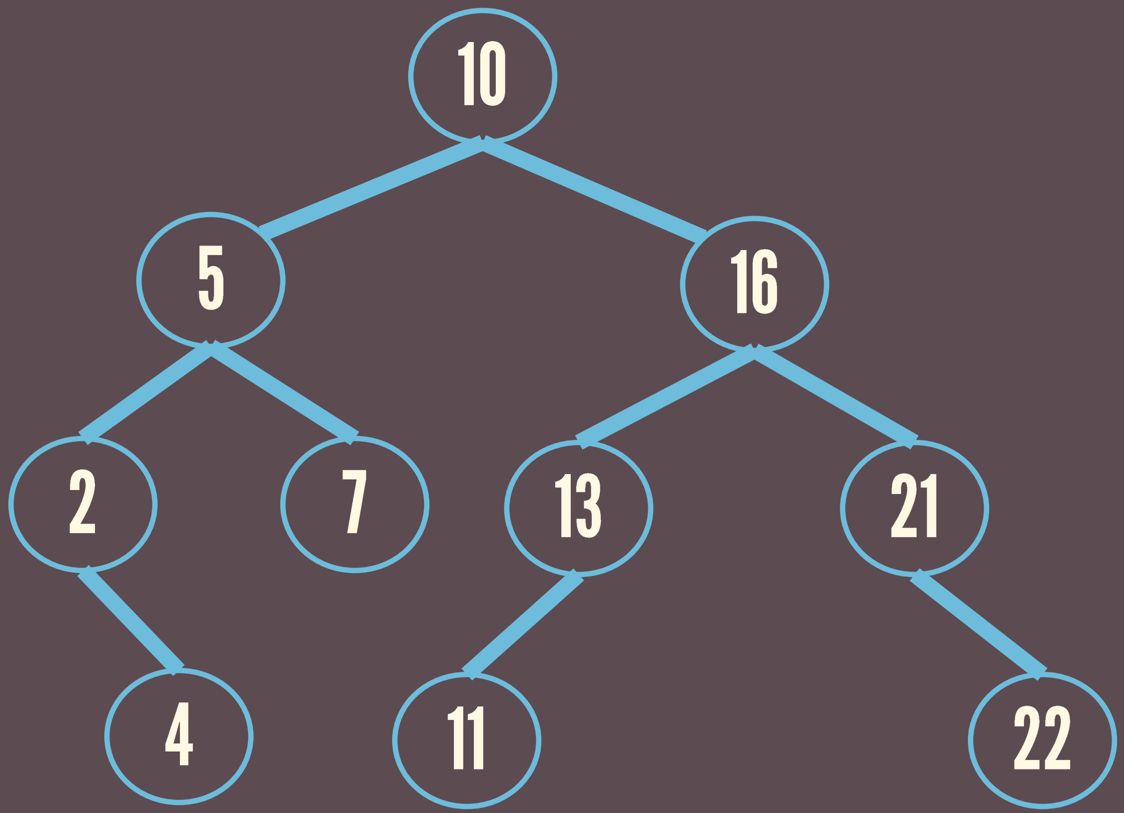
# BST

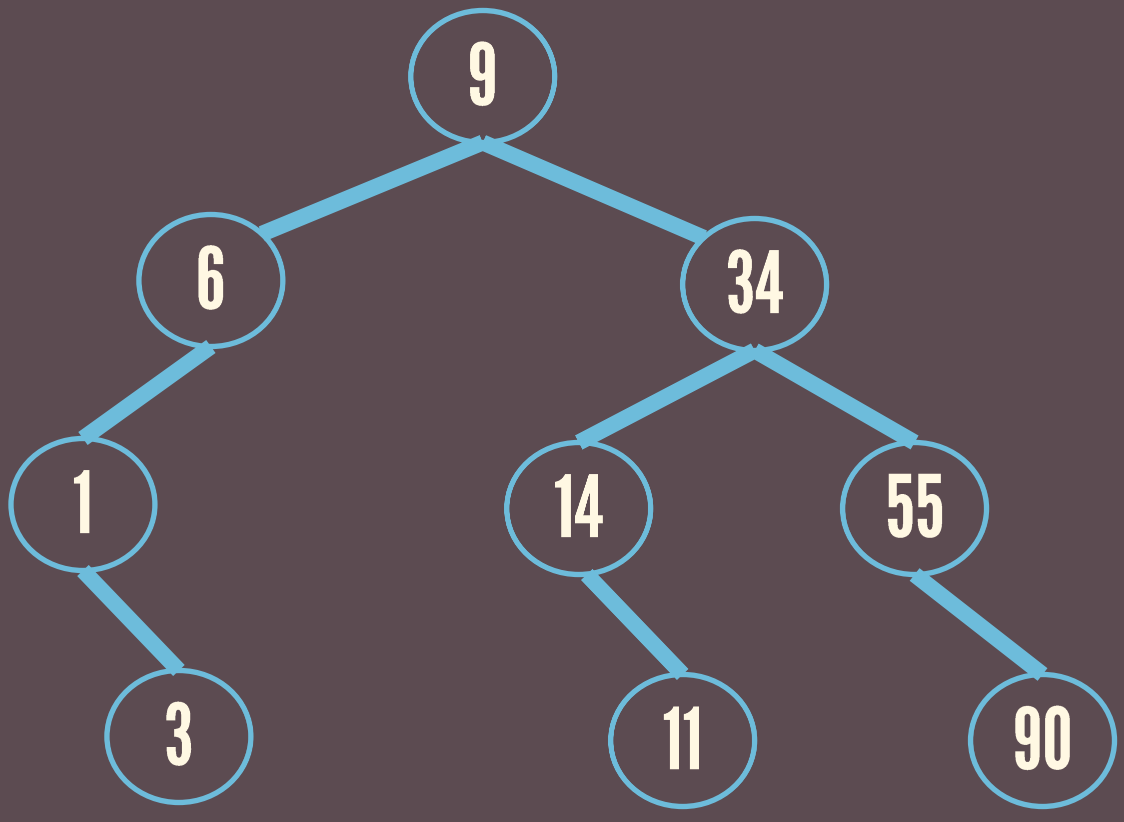
For every node, X in the tree,

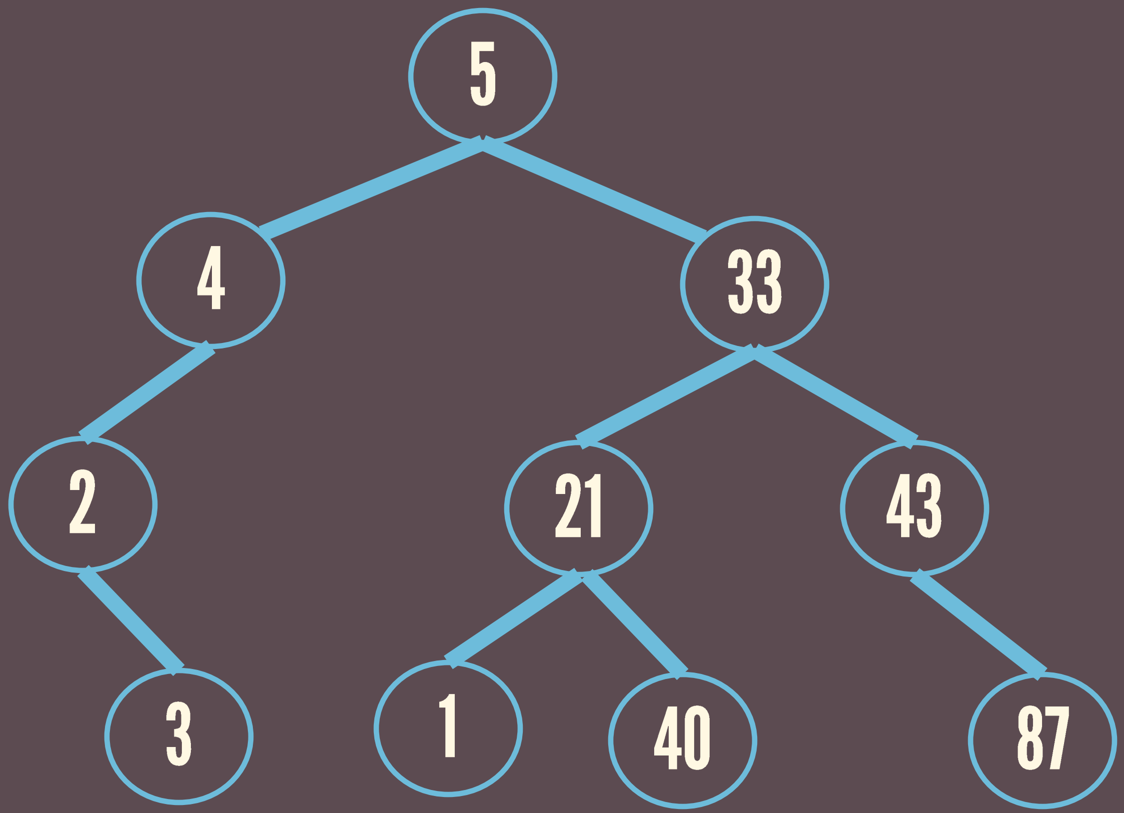
the values of all the keys in the left subtree are less than the key value in X; and

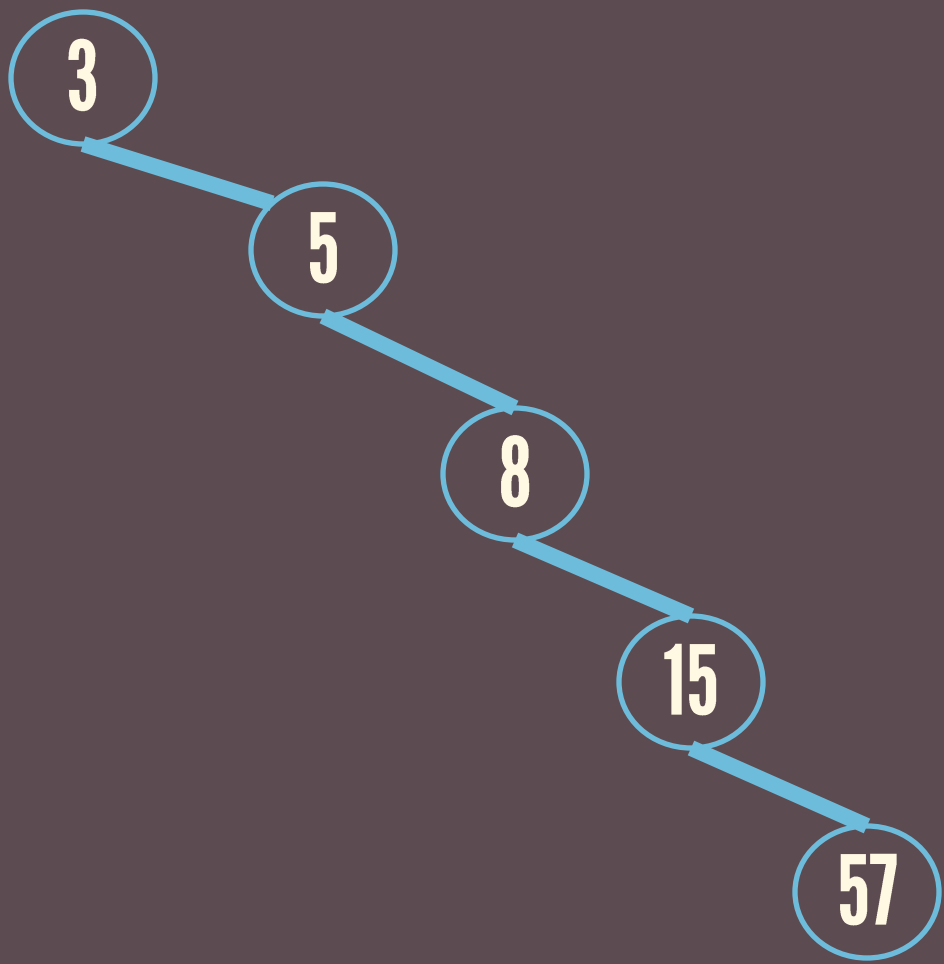
the values  of all the keys in the right subtree are larger than the key value in X.


x
<x
>x


10
5
16
2
7
13
21
4
11
22


9
6
34
1
14
55
3
11
90


5
4
33
2
21
43
3
1
40
87


3
5
8
15
57

# BST

## OPERATIONS

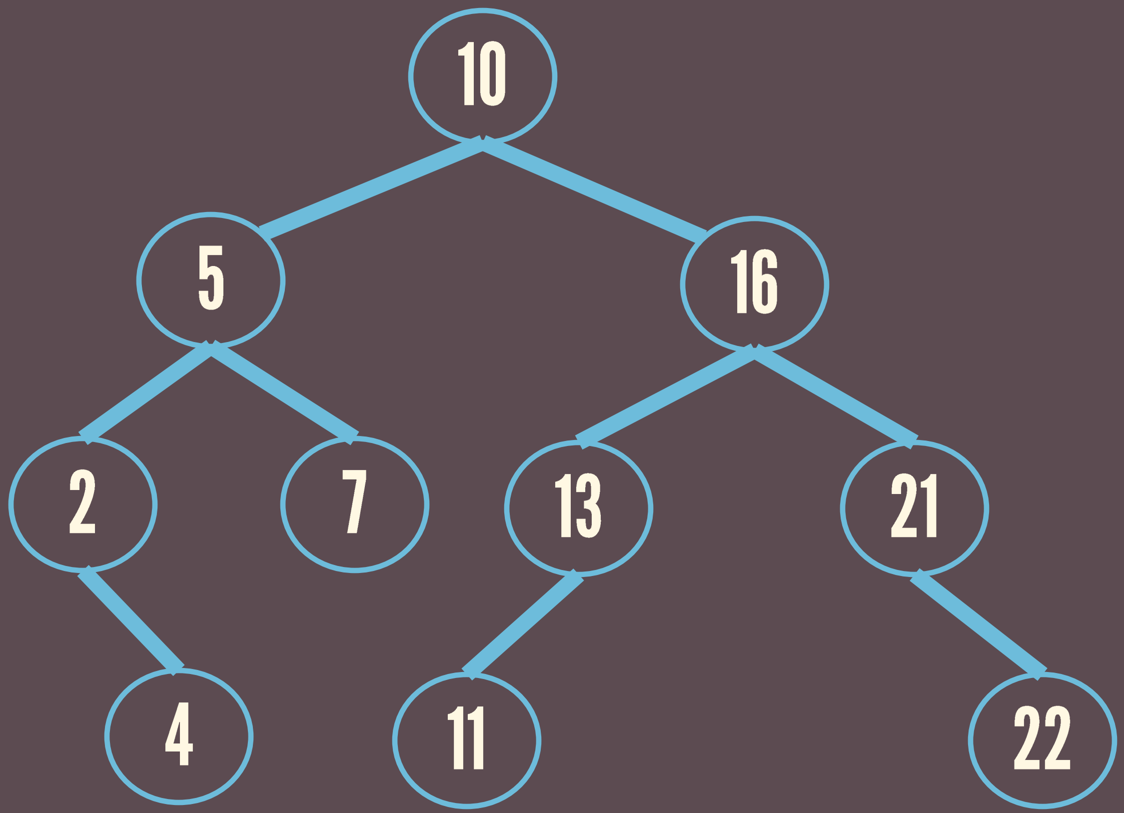
find
insert
delete
minimum
maximum
successor
predecessor

# IMPLEMENTATIONS

recursive
non-recursive

# IMPLEMENTATIONS

`find()`


10
5
16
2
7
13
21
4
11
22

```
typedef struct node{
  int data;
  struct node *left;
  struct node *right;
}BST;

BST *t;
```

```
BST find(int x, BST *t){

   if(t==NULL)
        return NULL;
   if(x < t->data)
        return ( ____________________ );
   if(x > t->data)
        return ( ____________________ );
   else
        return t;
}
```

```
BST find(int x, BST *t){

   if(t==NULL)
         return NULL;
   if(x < t->data)
         return (find(x, t->left) );
   if(x > t->data)
         return (find(x, t->right));
   else
         return t;
}
```

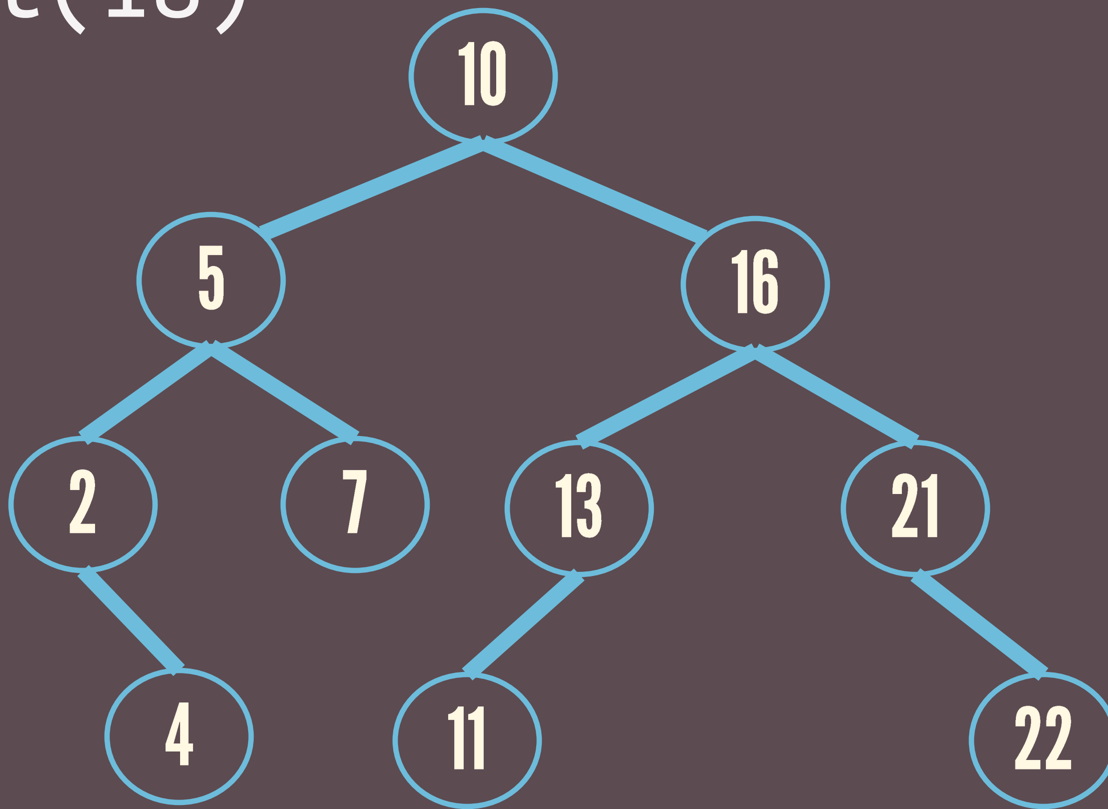
# IMPLEMENTATIONS

`find()`

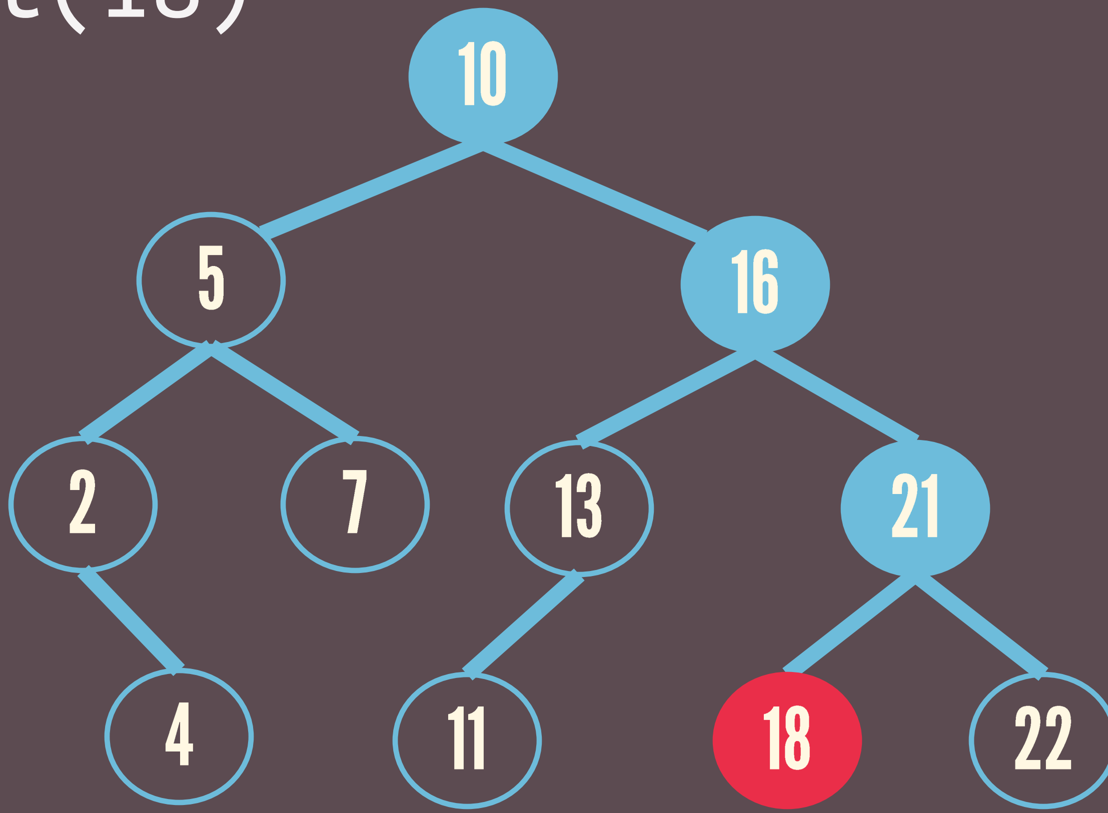
# IMPLEMENTATIONS

insert()

insert(18)



insert(18)



# IMPLEMENTATIONS

`minimum()`

IMPLEMENTATIONS

maximum()

# IMPLEMENTATIONS

`predecessor()`

# IMPLEMENTATIONS

`successor()`

# IMPLEMENTATIONS

`printBST()`

```
void printBST(BST *root,int tabs){
  int i;
  if(root!=NULL){
    printBST(root->right,tabs+1);
    for(i=0;i<tabs;i++)
        printf("\t");
    printf("%3i\n",root->value);
    printBST(root->left,tabs+1);


  }
}
```

# IMPLEMENTATIONS

`delete()`

**delete()**

3 cases

Node is a leaf

Node has one child

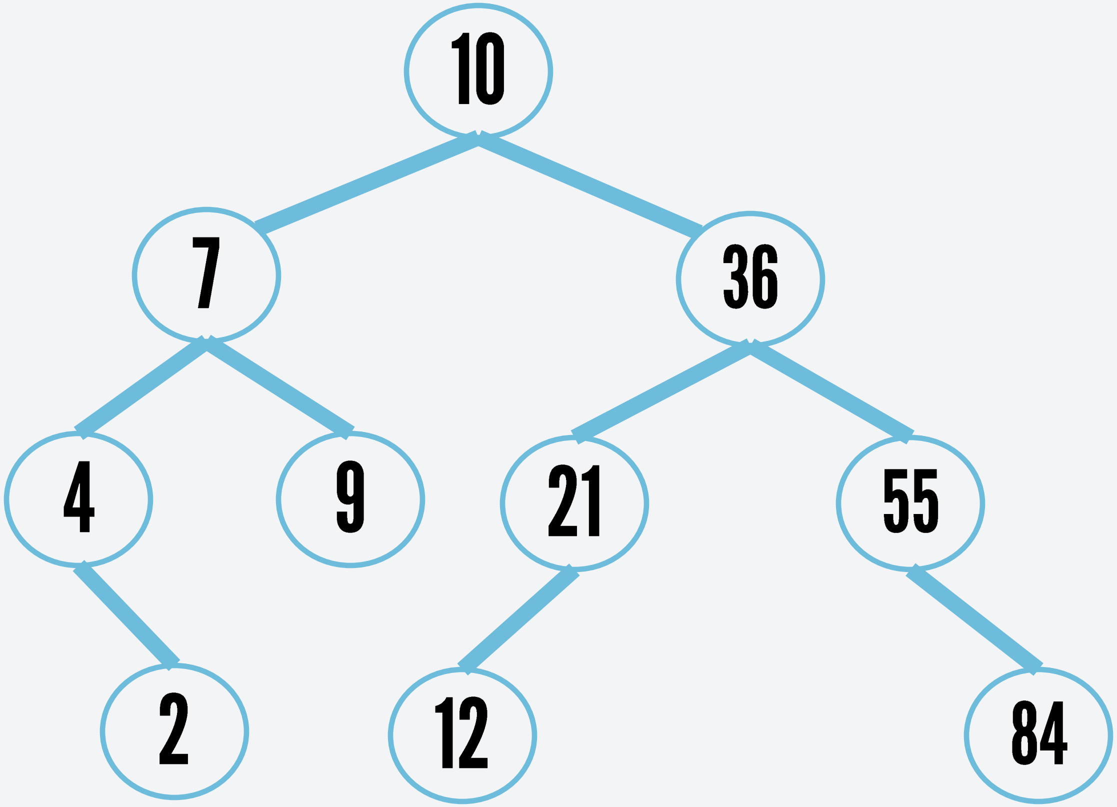
Node has two children

Node is a leaf

The node can be deleted immediately.

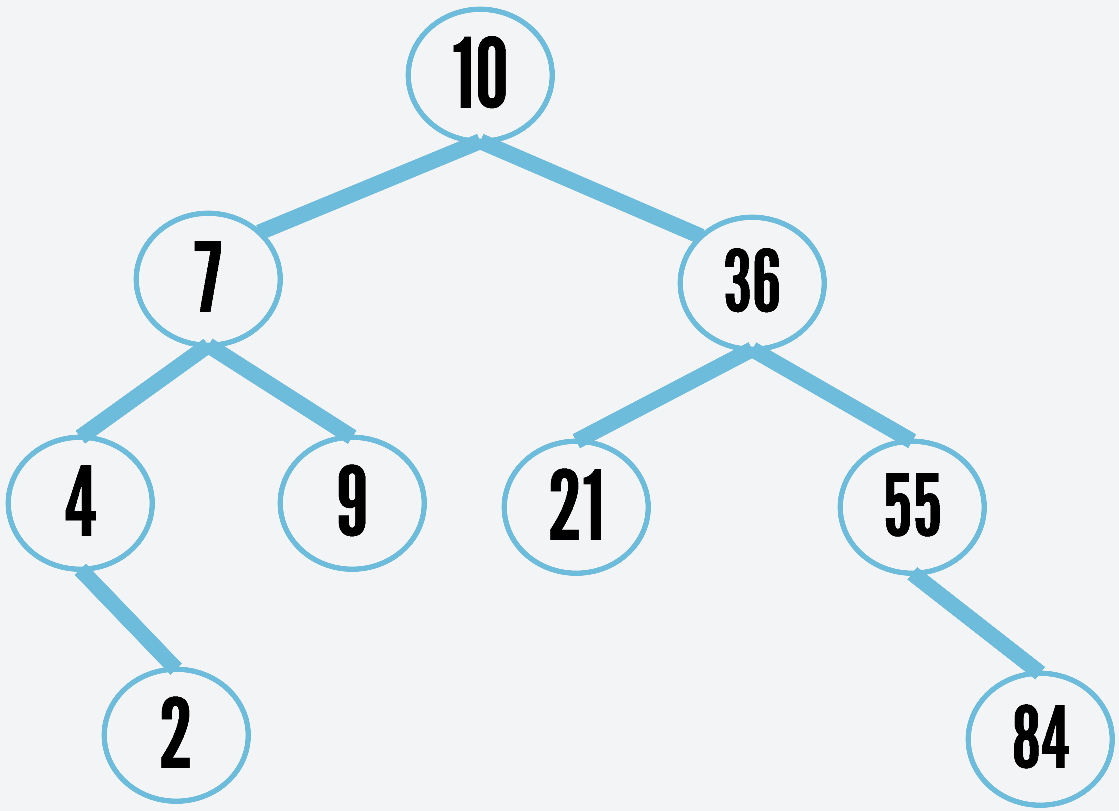
Node is a leaf

delete(12)



`delete(12)`

Node is a leaf

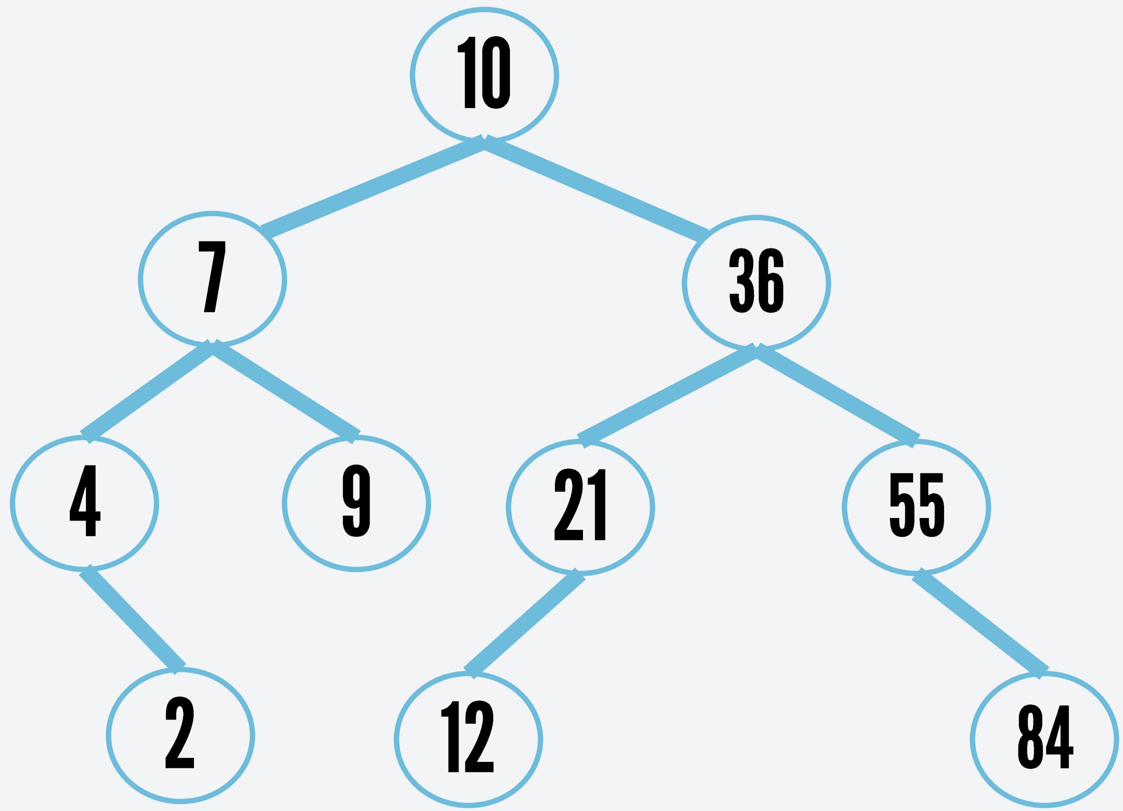


Node has one child

Its parent adjusts a pointer to bypass the node.

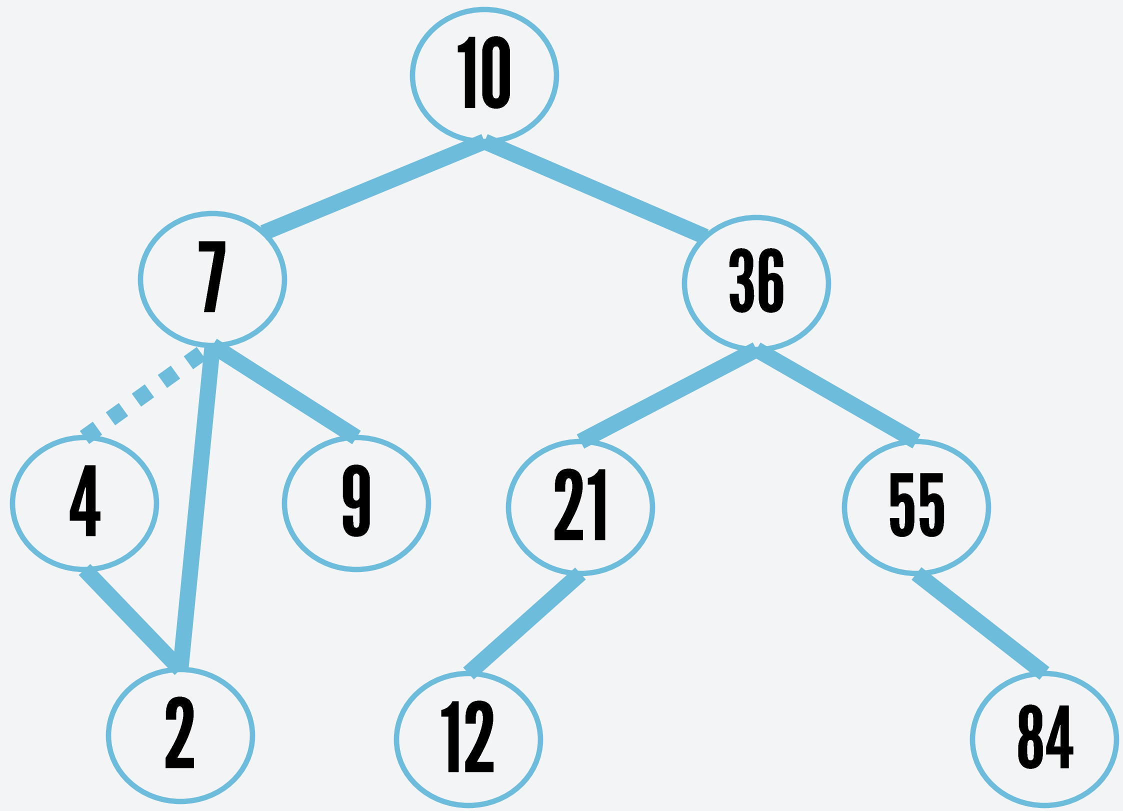
Node has one child

delete(4)



Node has one child

```
delete(4)
```



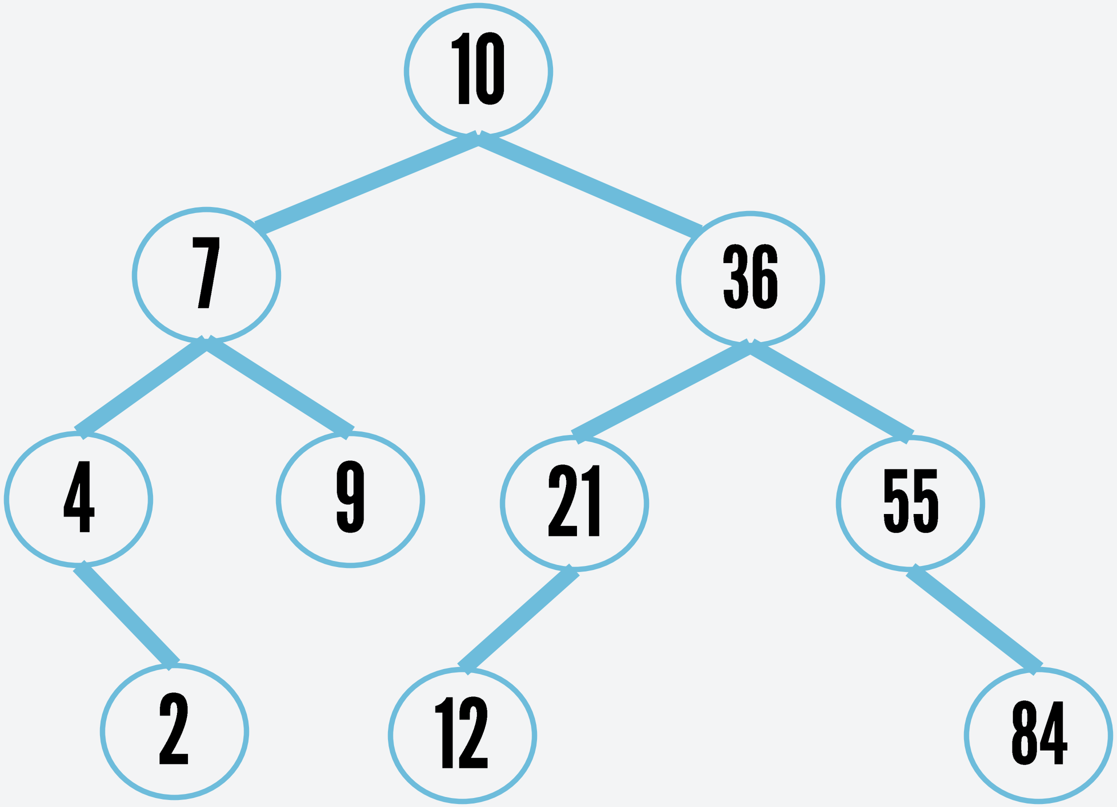
Node has two children

Replace this node with the smallest key of the right subtree.

Recursively delete this node.

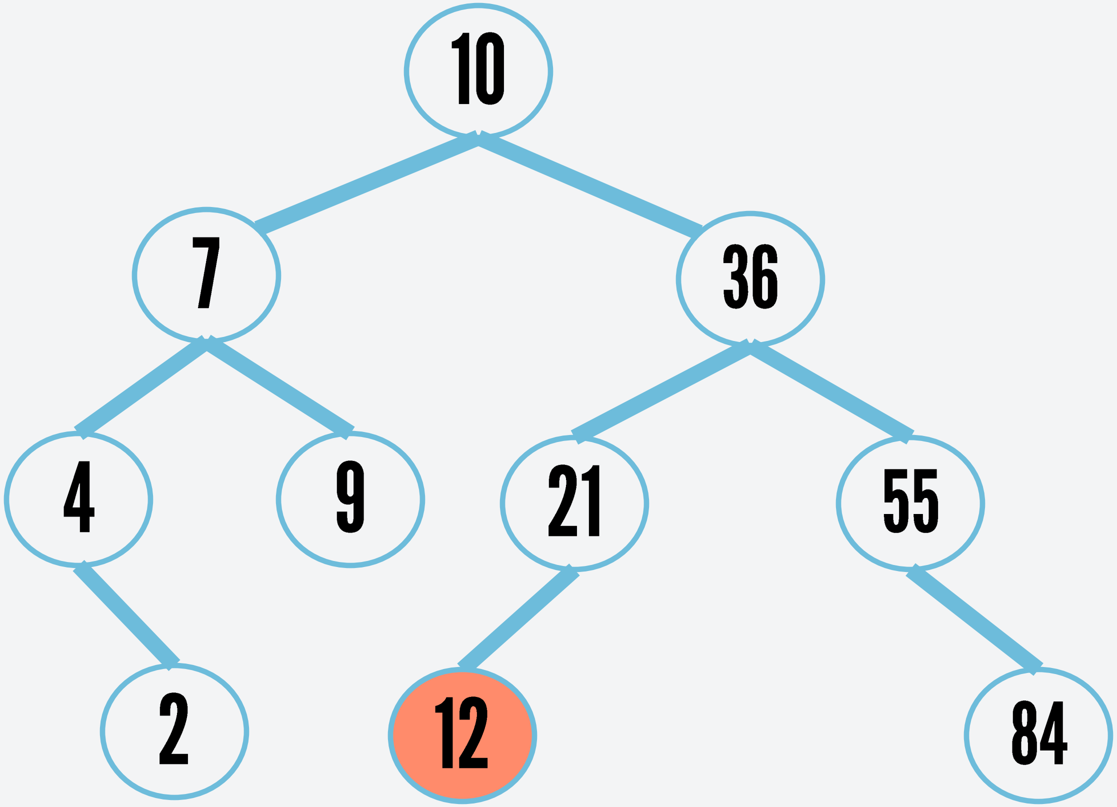
Node has two children

`delete(10)`



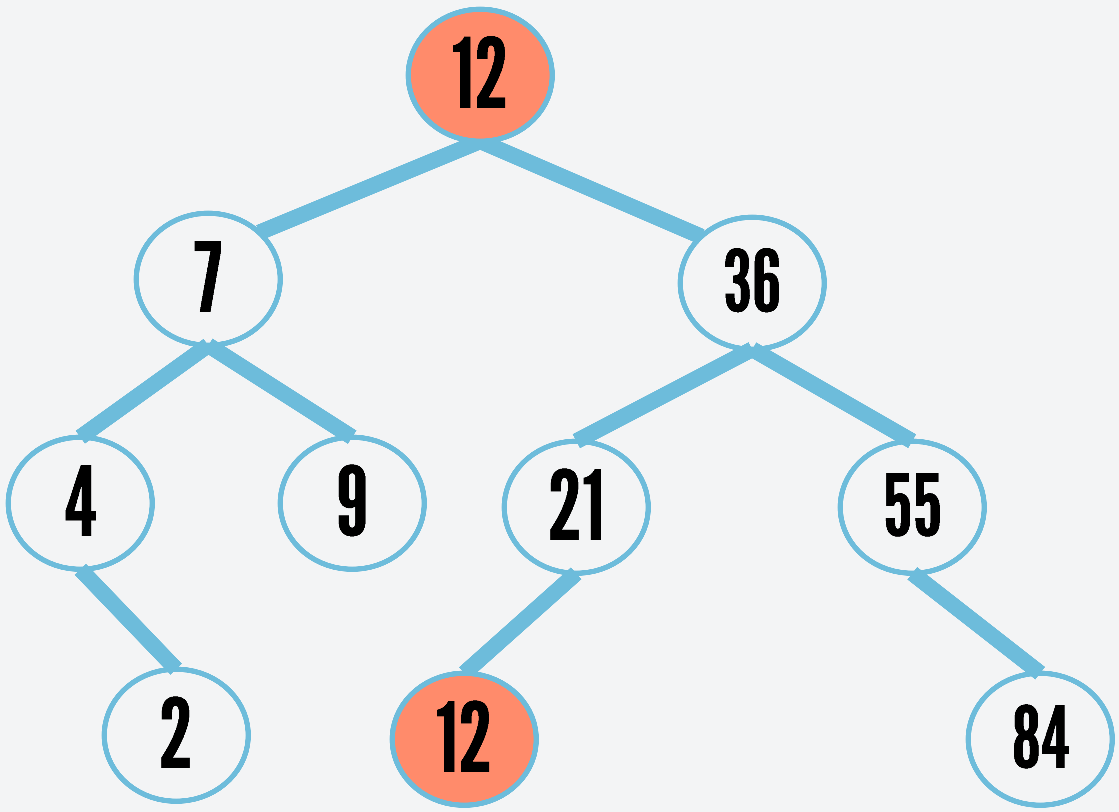
Node has two children

delete(10)



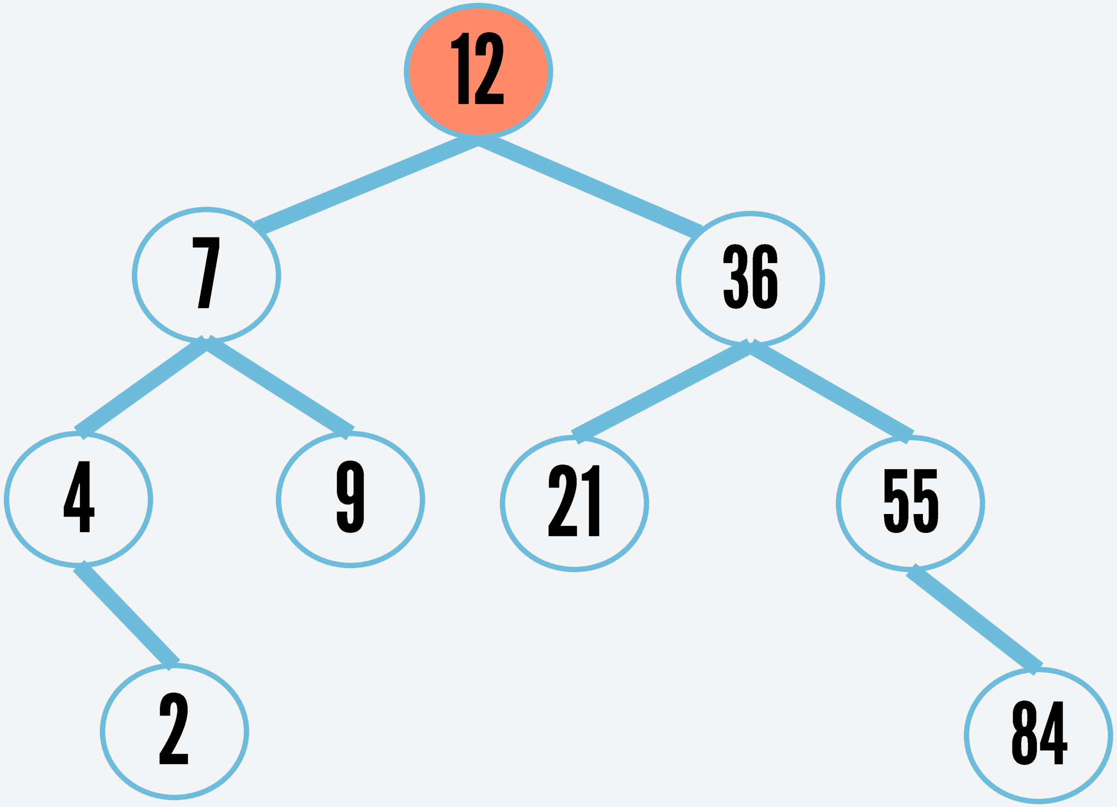
Node has two children

`delete(10)`



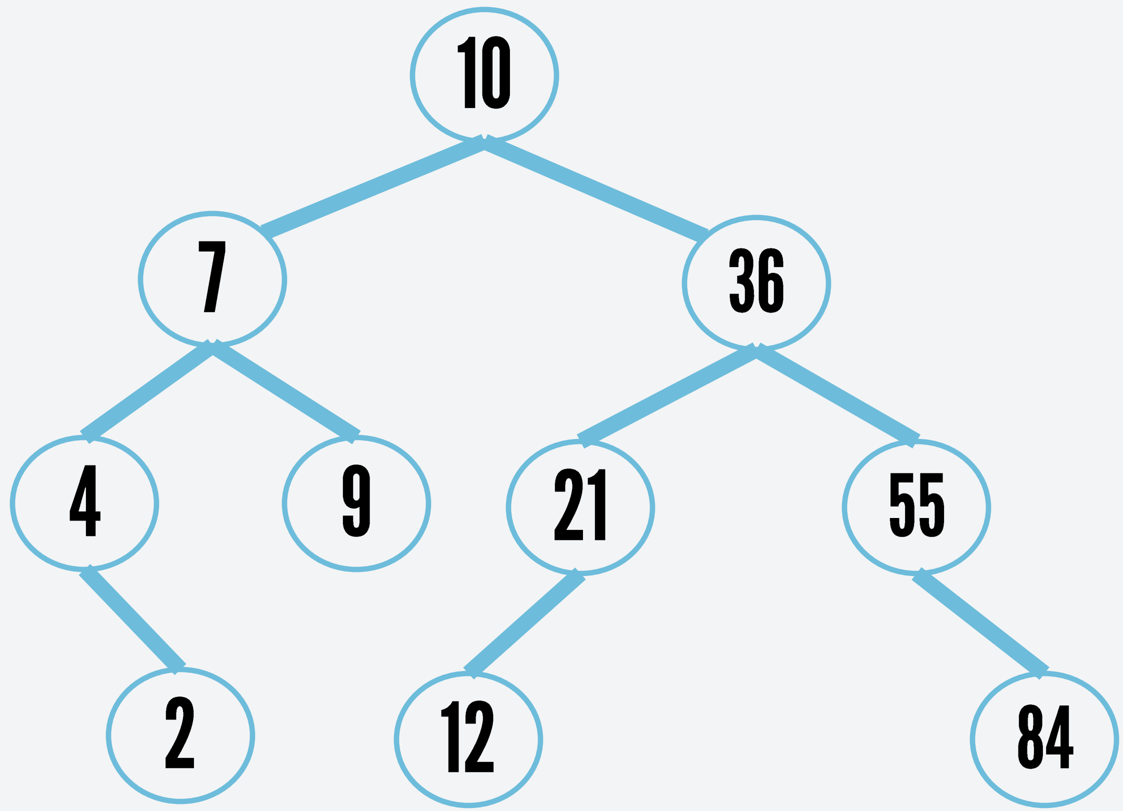
Node has two children

delete(10)



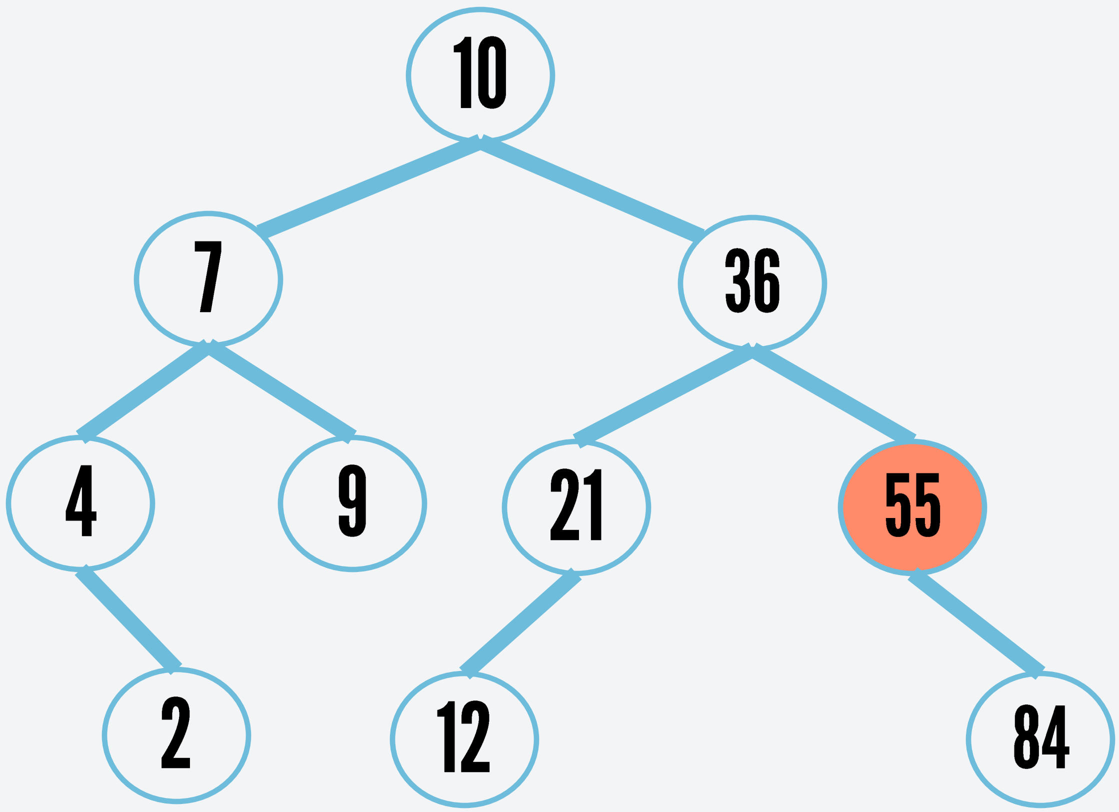
Node has two children

`delete(36)`



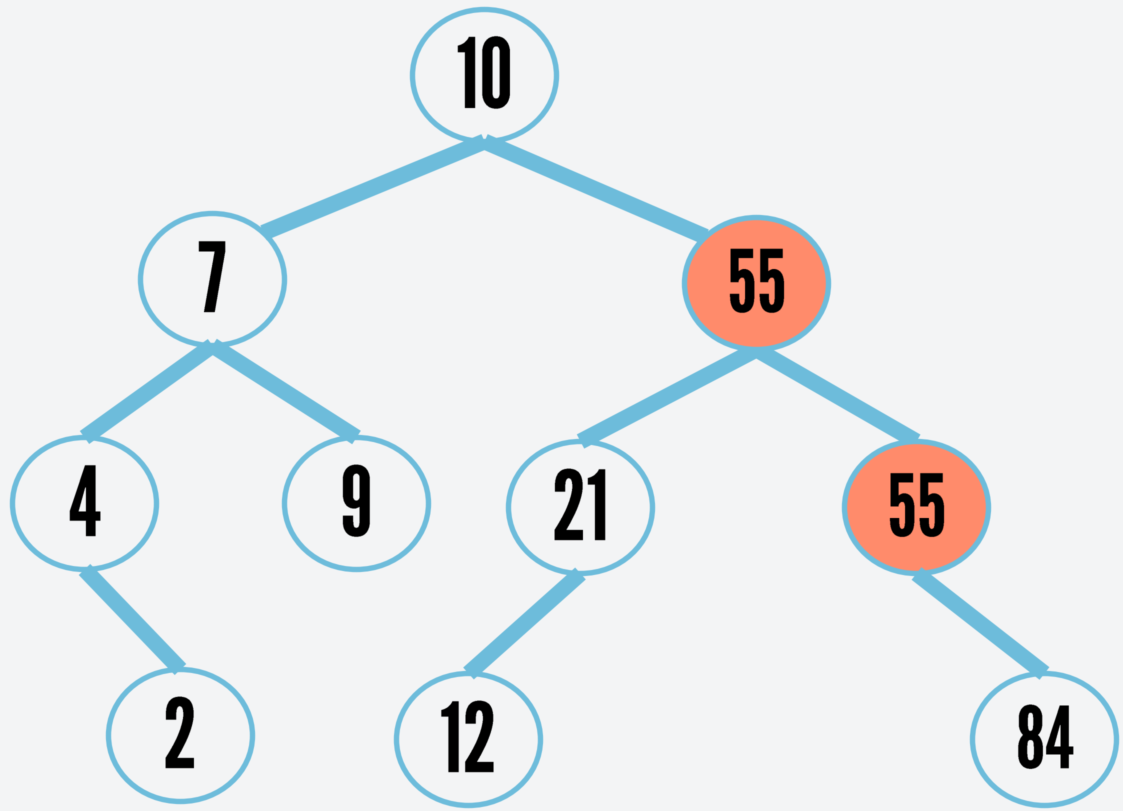
Node has two children

`delete(36)`



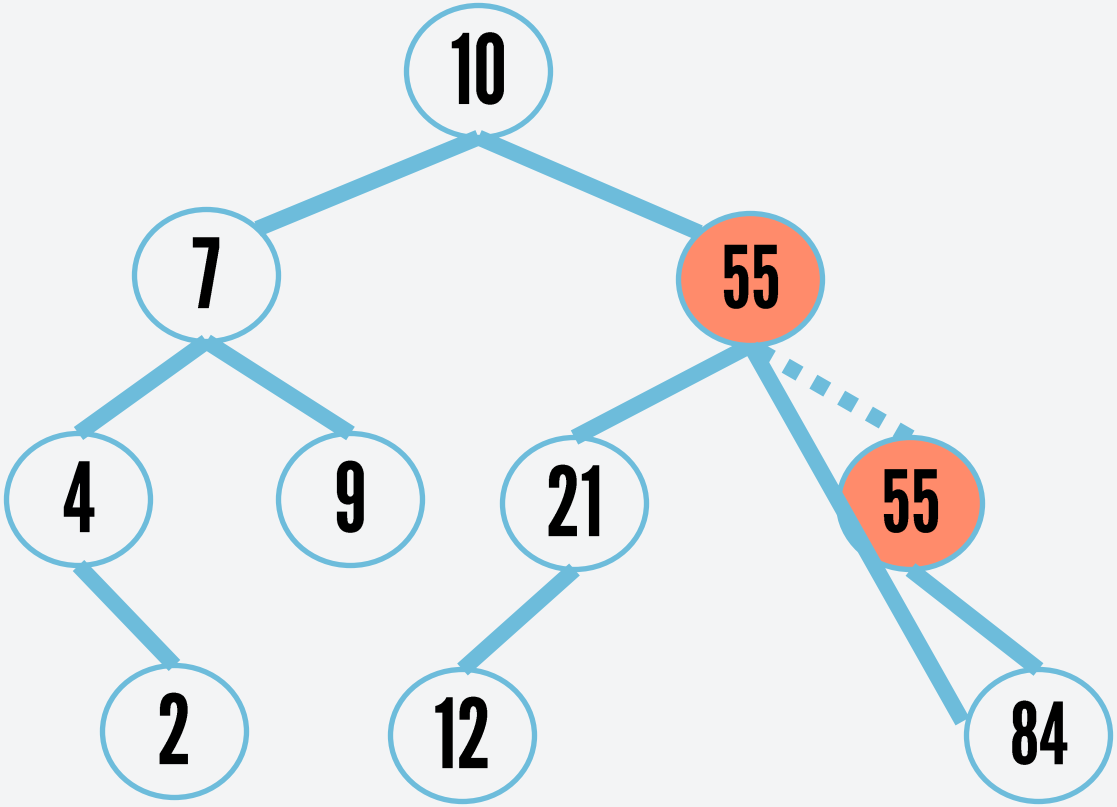
delete(36)

Node has two children



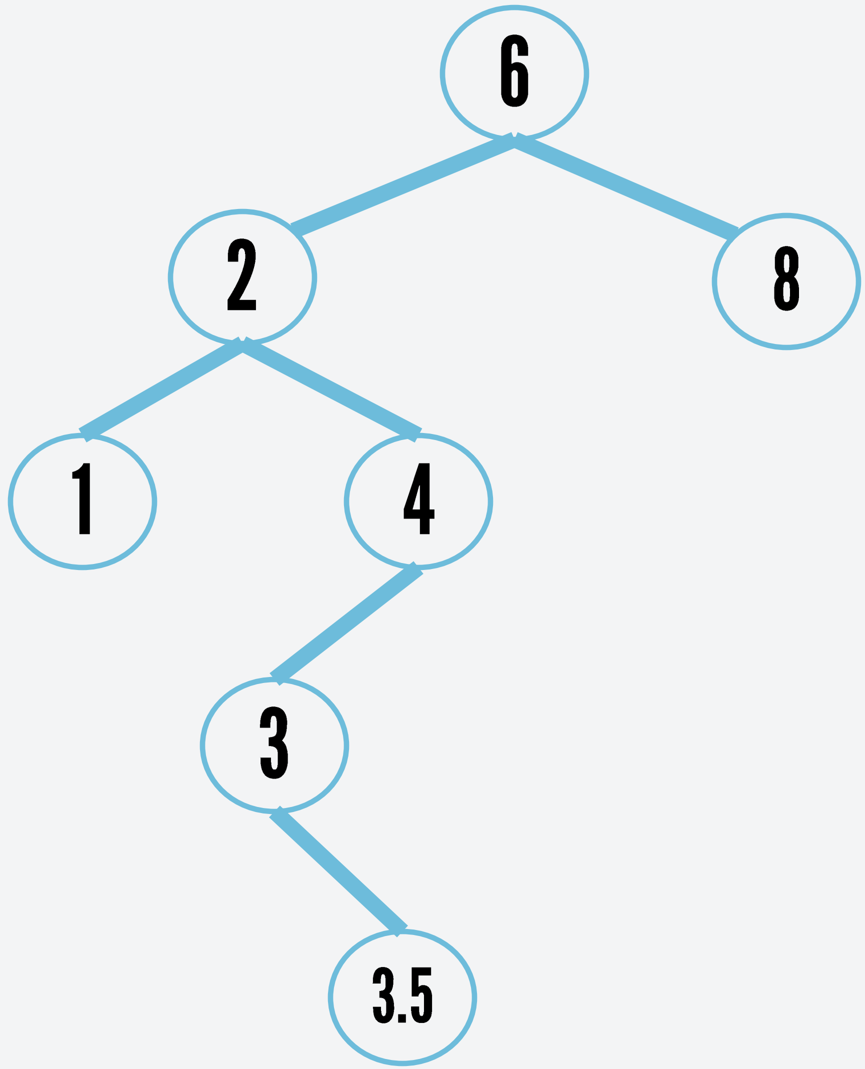
Node has two children

delete(36)



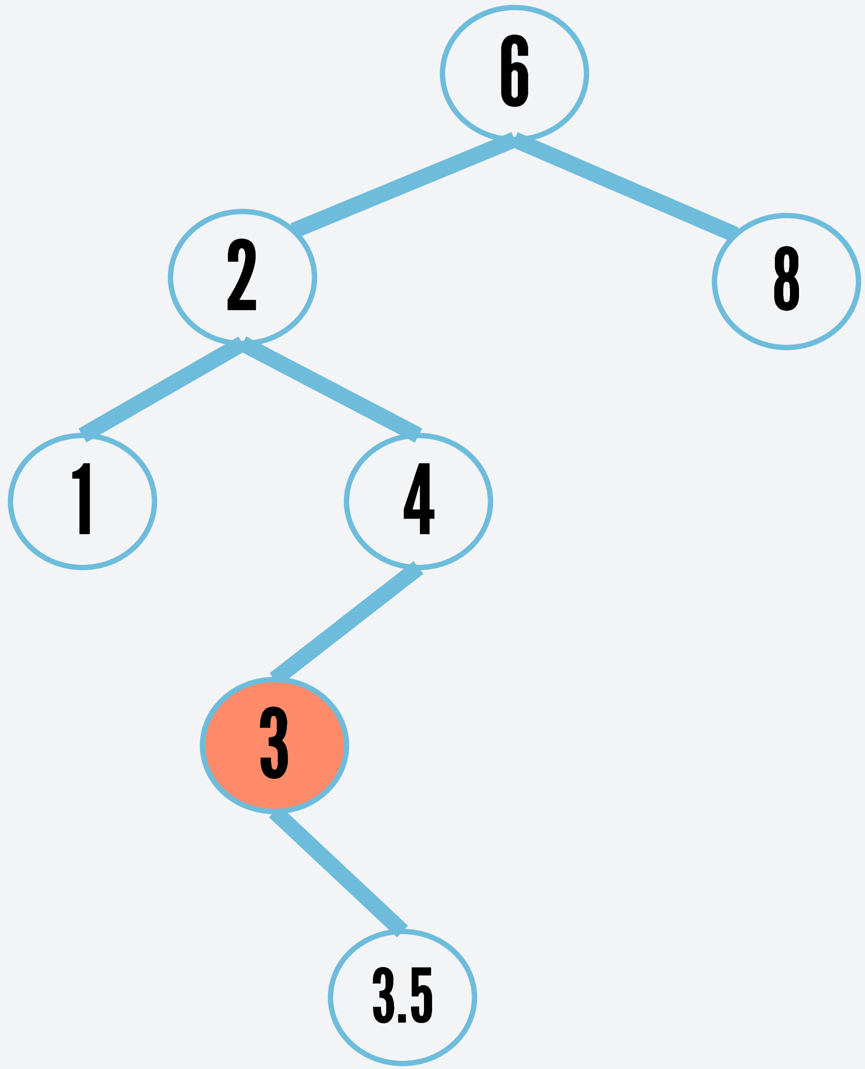
`delete(2)`

Node has two children



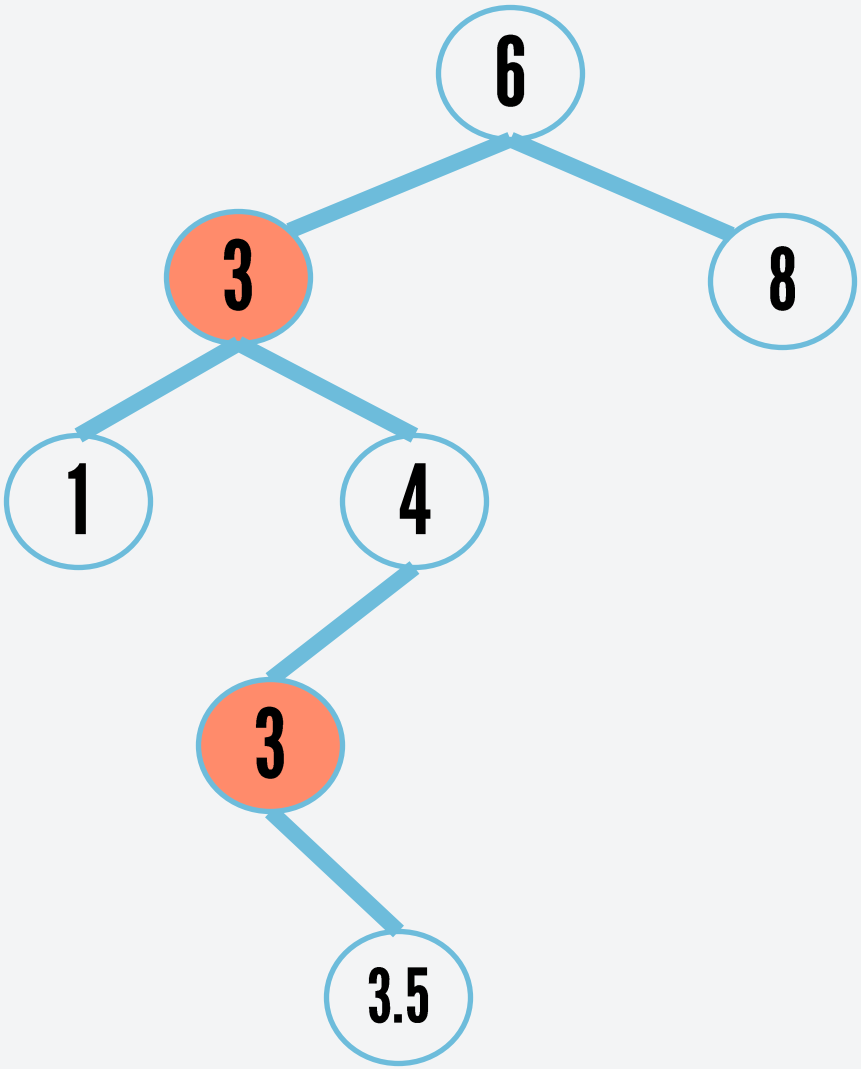
Node has two children

`delete(2)`



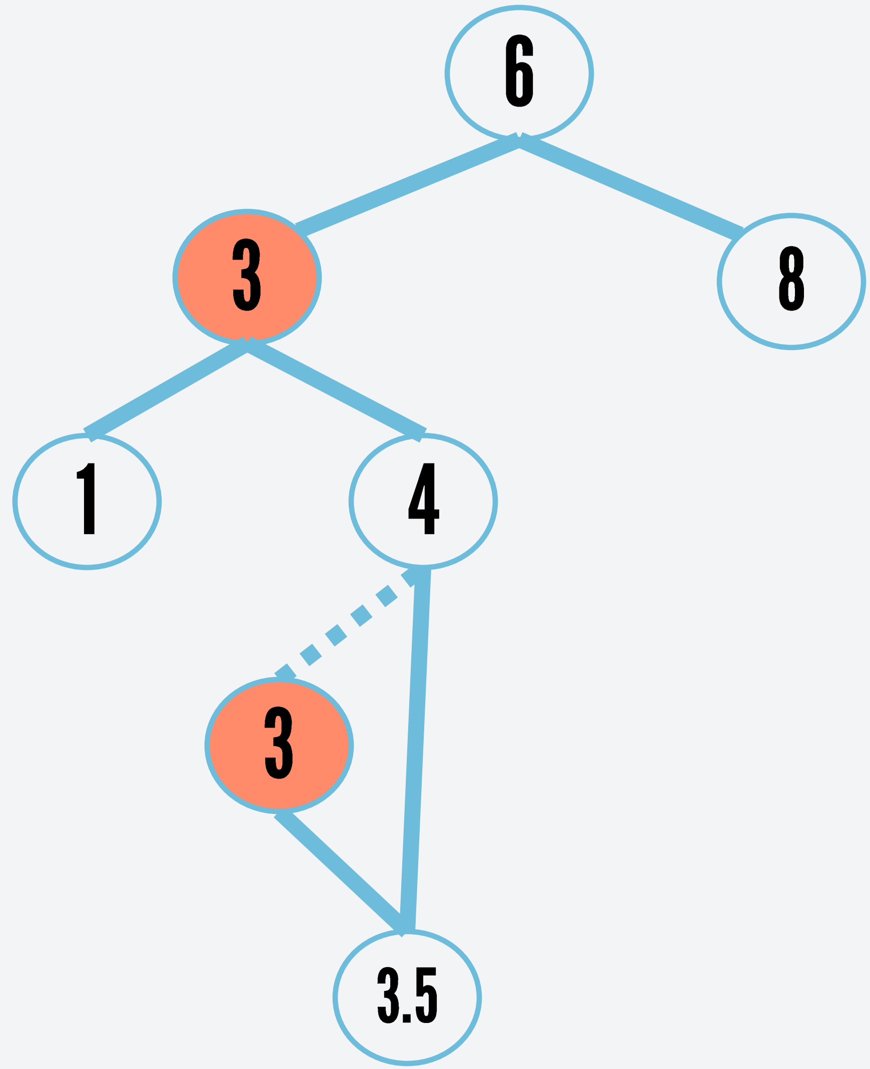
delete(2)

Node has two children



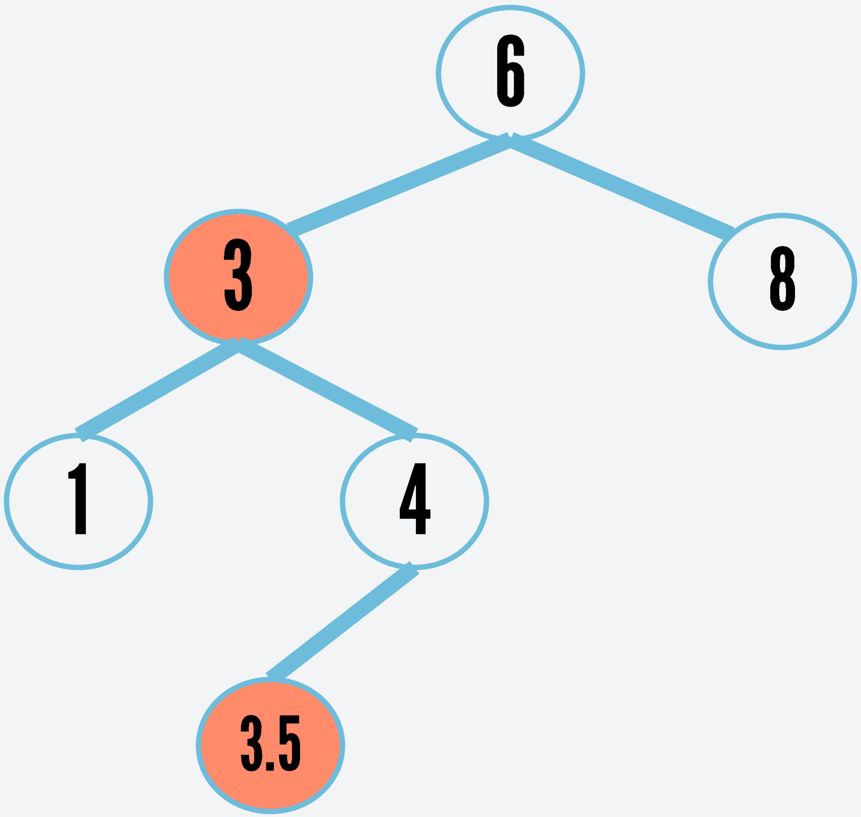
Node has two children

delete(2)



Node has two children

`delete(2)`



# AVL TREE

# ADELSON-VELSKII AND LANDIS' TREE

# AVL TREE

A binary search tree with a balance condition.

# AVL TREE

For every node in the tree, the height of its left and right subtrees can differ by at most 1.

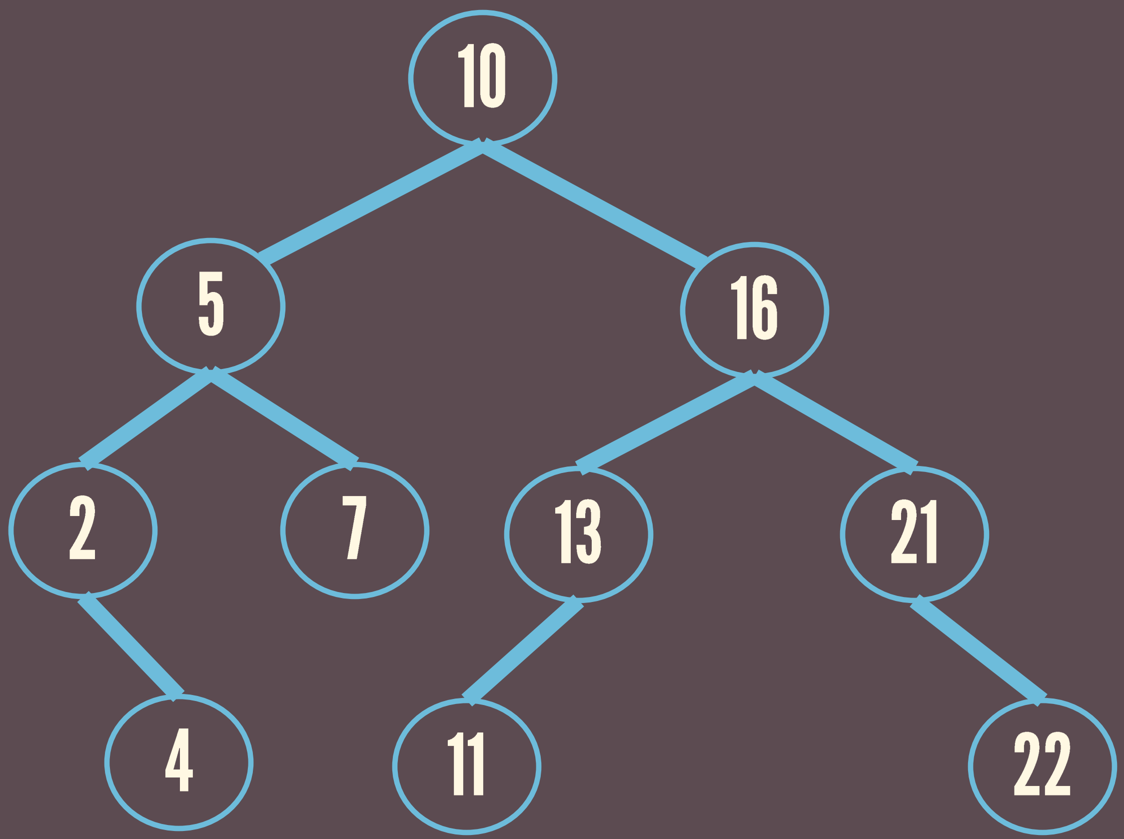
# AVL TREE

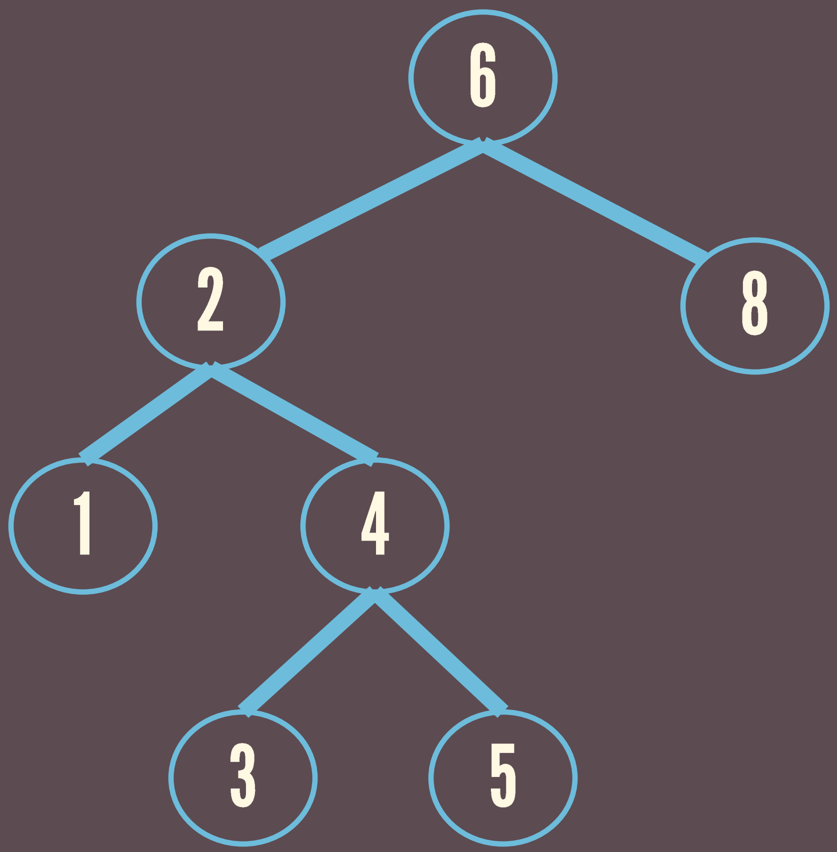
If anytime they differ by more than one, rebalancing is done to restore this property.

**NOTE**

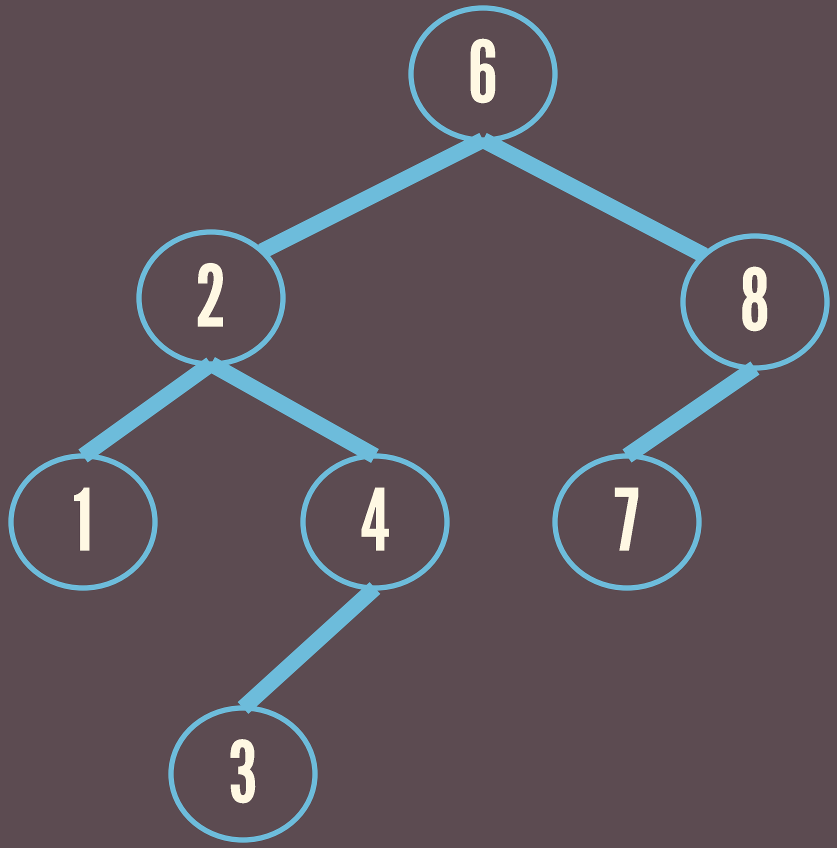
The height of an empty tree is -1.

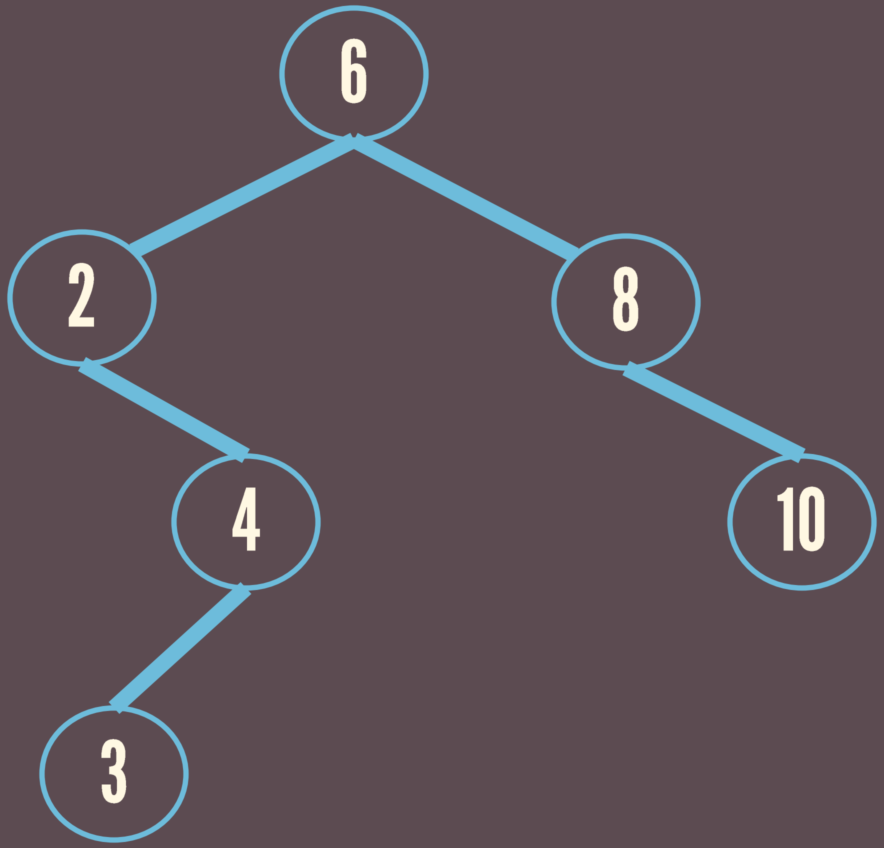
```c
typedef struct node{
  int data;
  struct node *left;
  struct node *right;
  int height;
}BST;

BST *t;
```

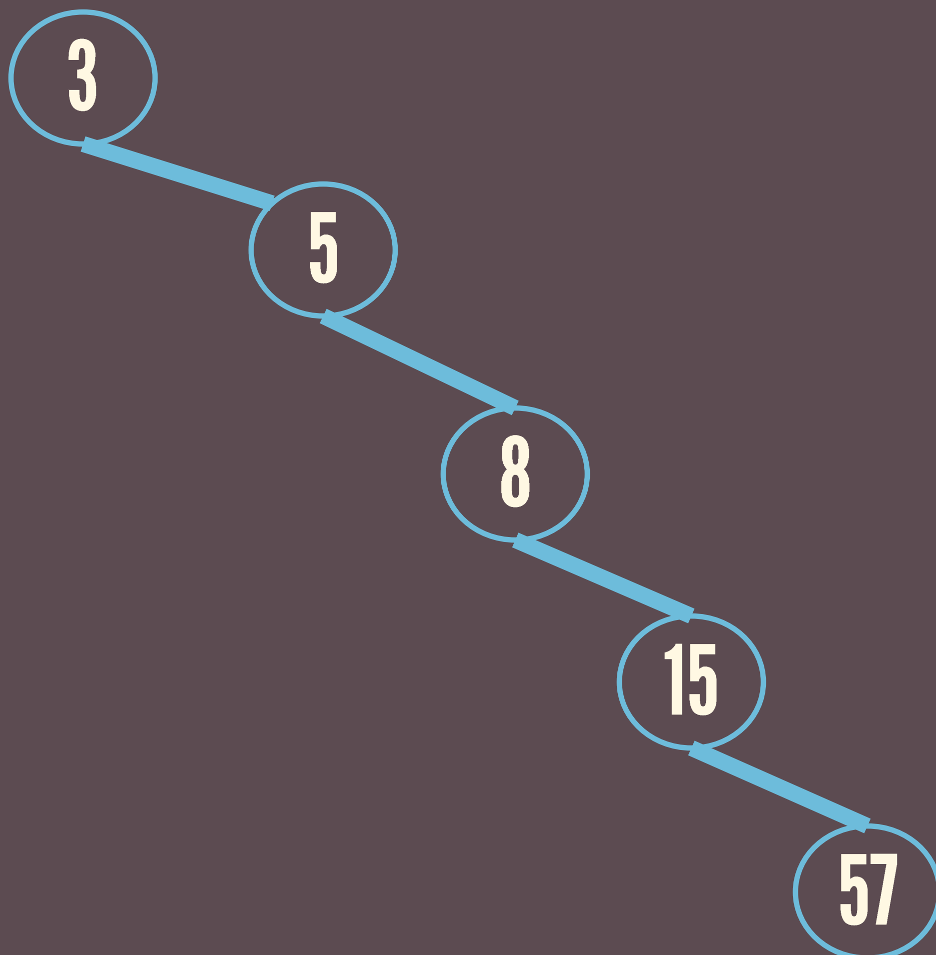

10
5
16
2
7
13
21
4
11
22

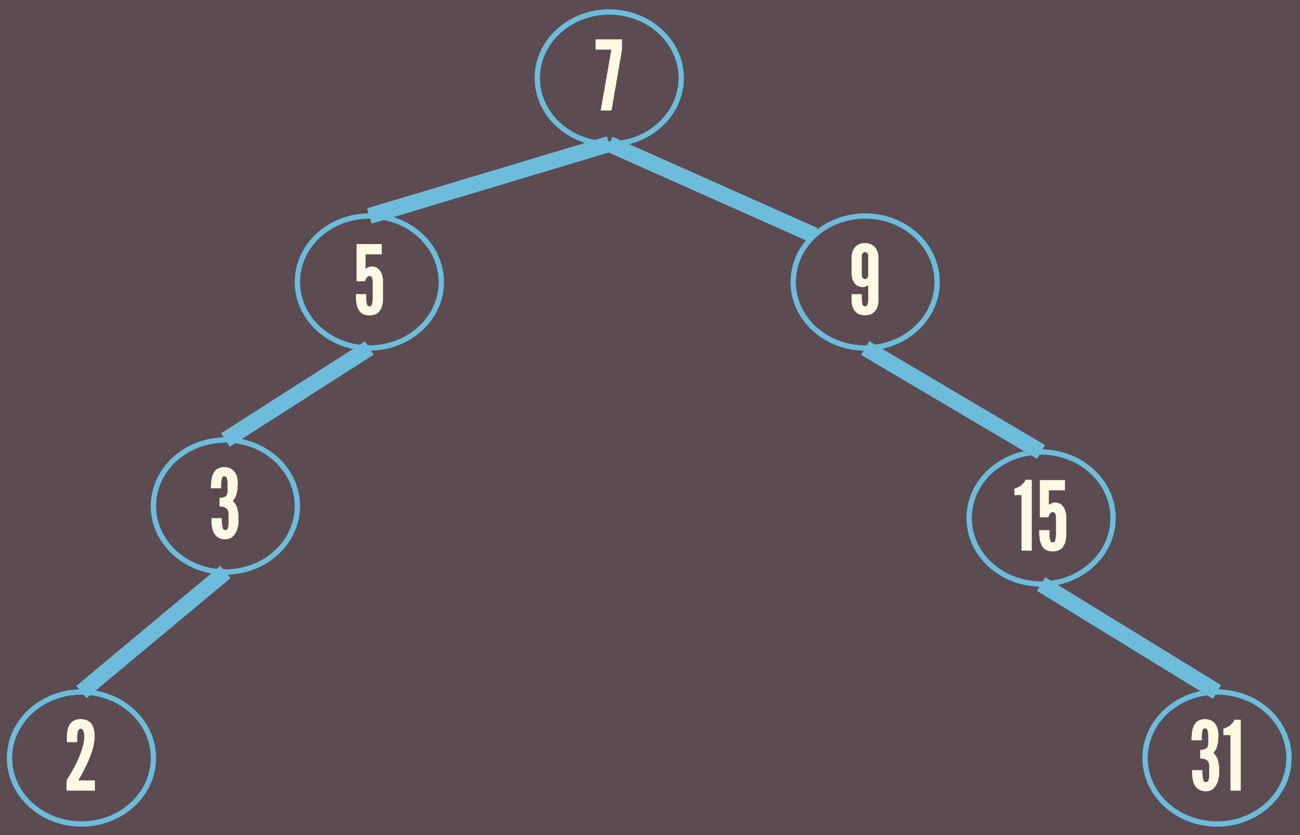


6
2
8
1
4
3
5


6
2
8
1
4
7
3


6
2
8
4
10
3


3
5
8
15
57


7
5
9
3
15
2
31

# AVL

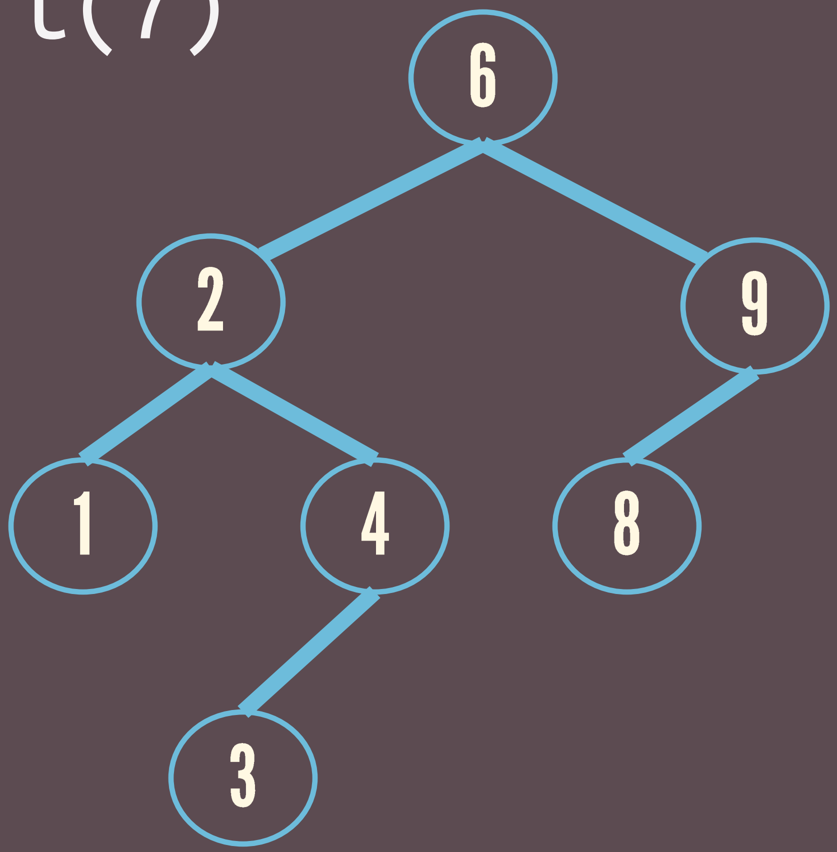
## OPERATIONS

find
insert (with rotations)
delete (with rotations)
minimum
maximum
successor
predecessor

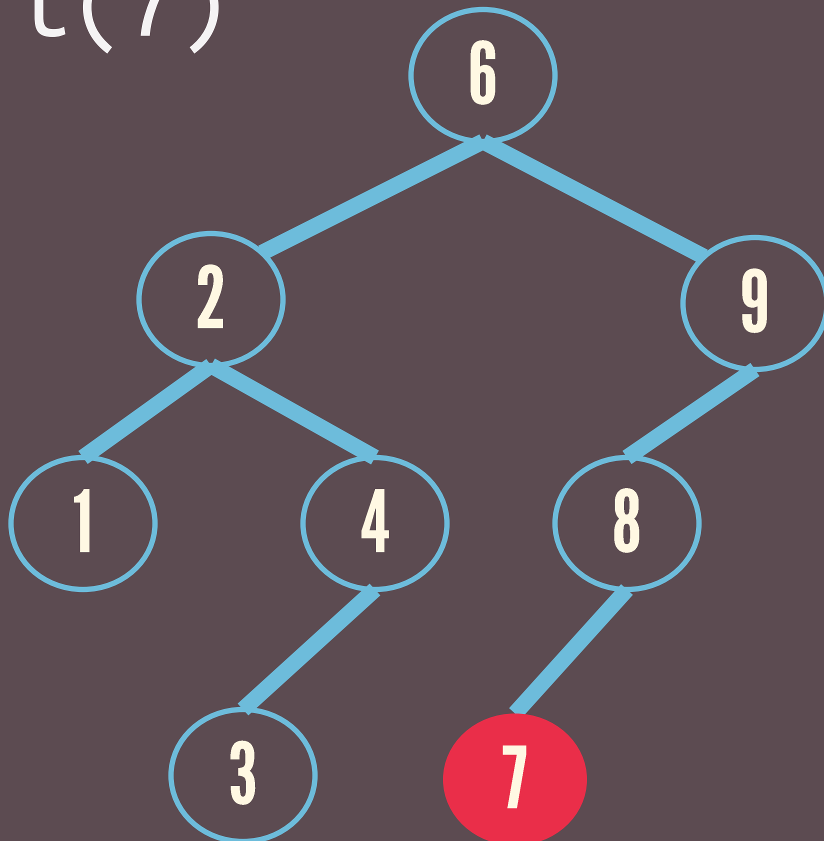
# ROTATIONS

Rotations are done to maintain the AVL property.

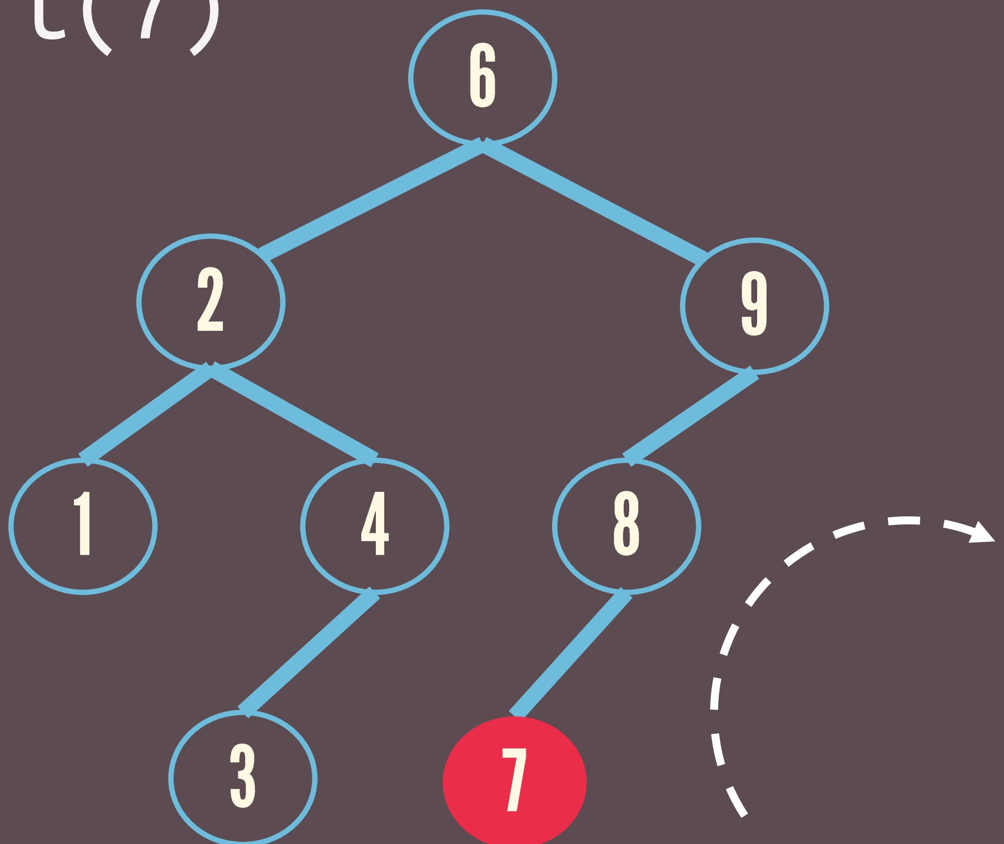
`insert(7)`



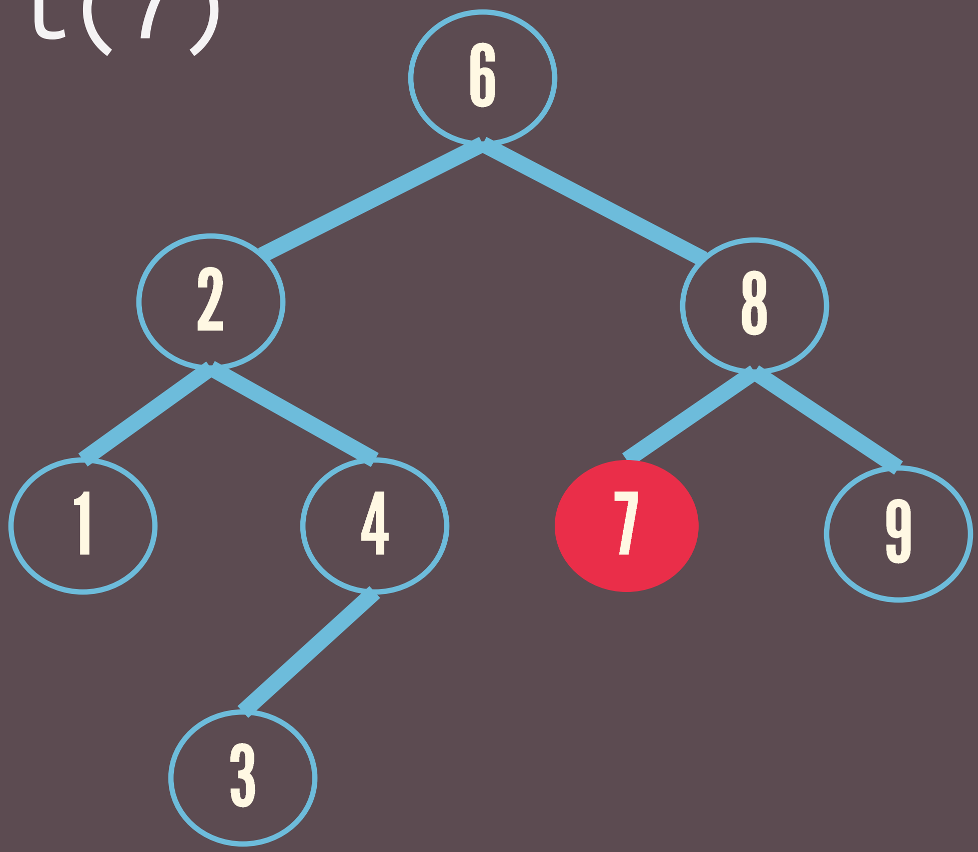
insert(7)



insert(7)



insert(7)



# ROTATIONS

## INSERT OPERATION

Single:

Left Rotate
Right Rotate

# ROTATIONS

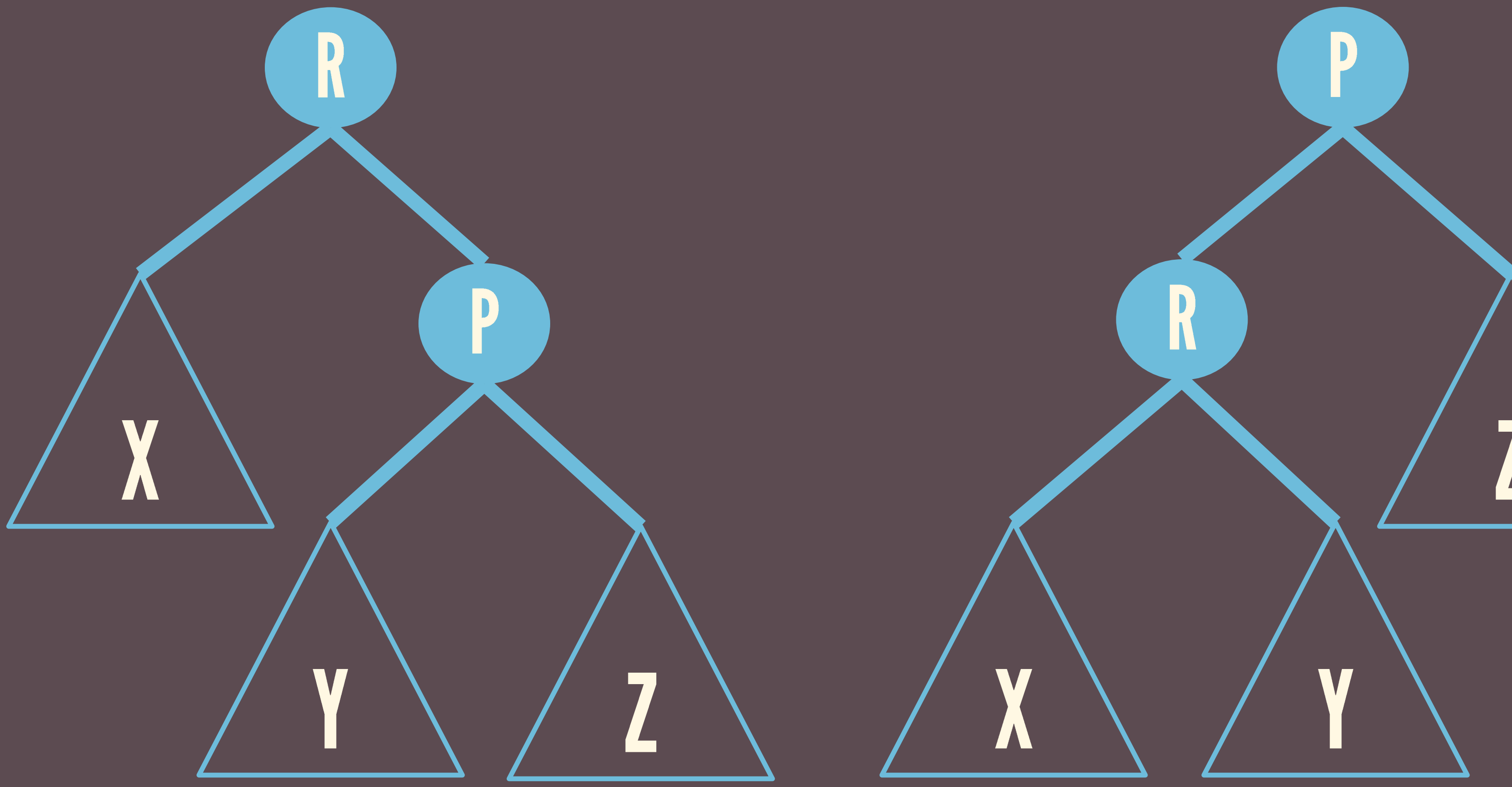
## INSERT OPERATION

Double:

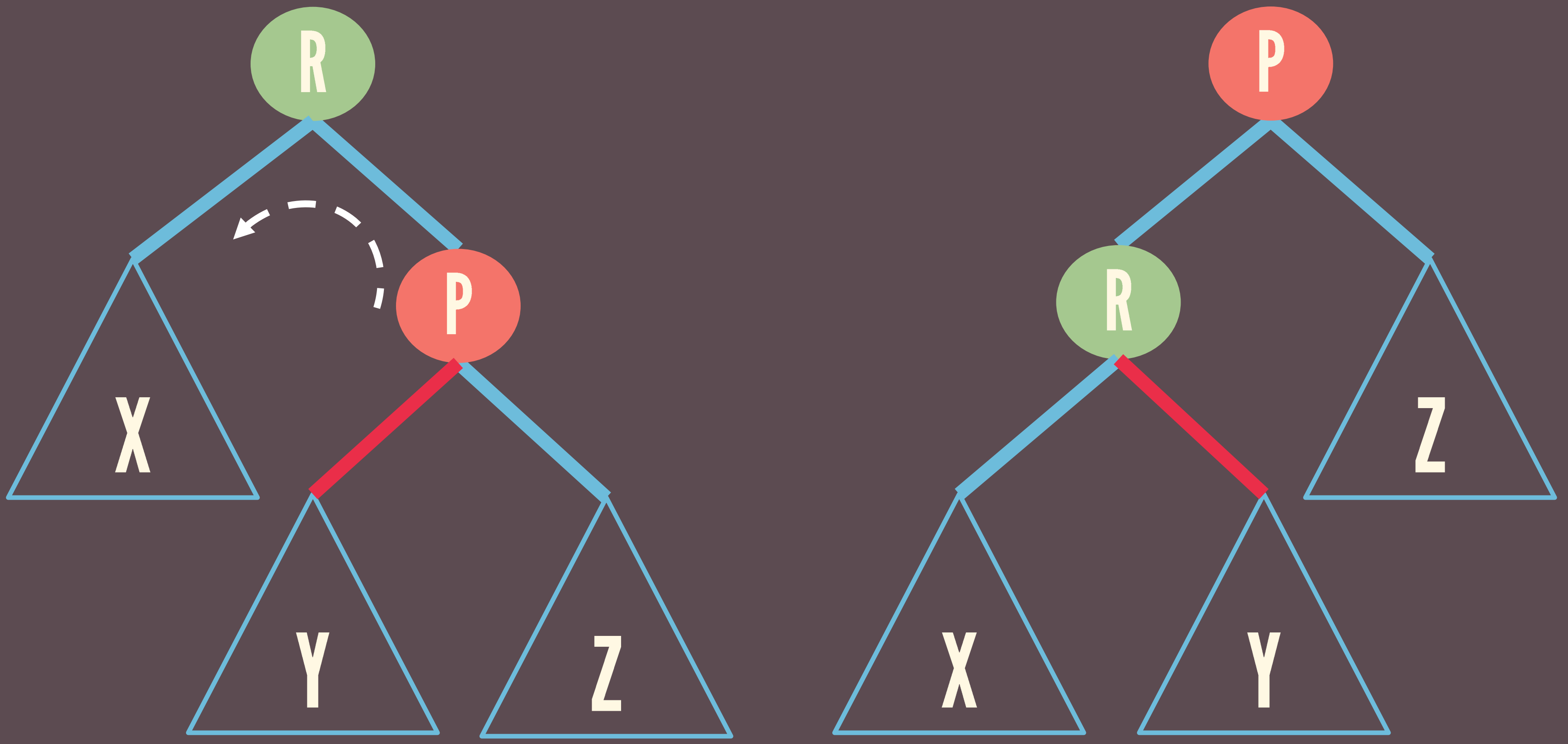
Left-right Rotate
Right-left Rotate

# ROTATIONS

# ILLUSTRATED




LEFT ROTATE

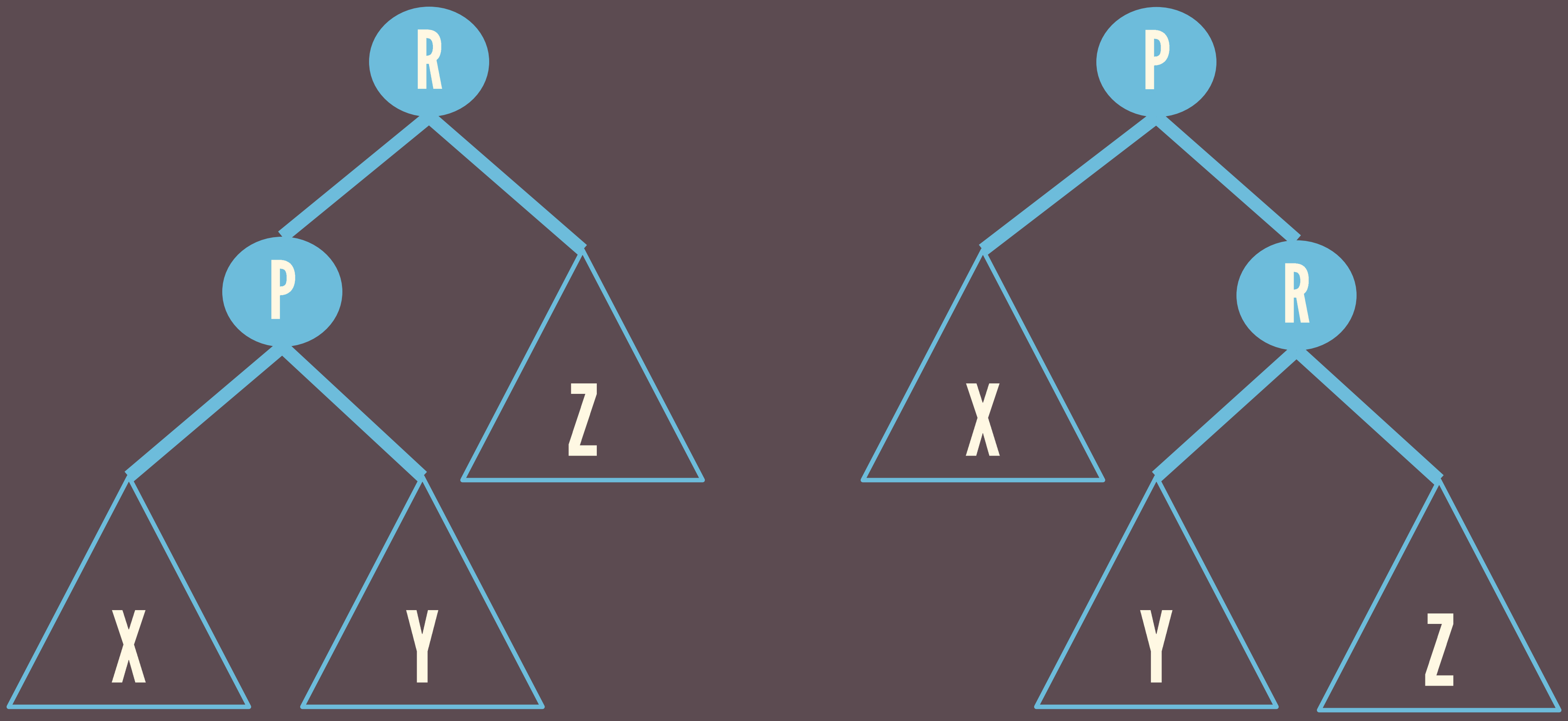



LEFT ROTATE

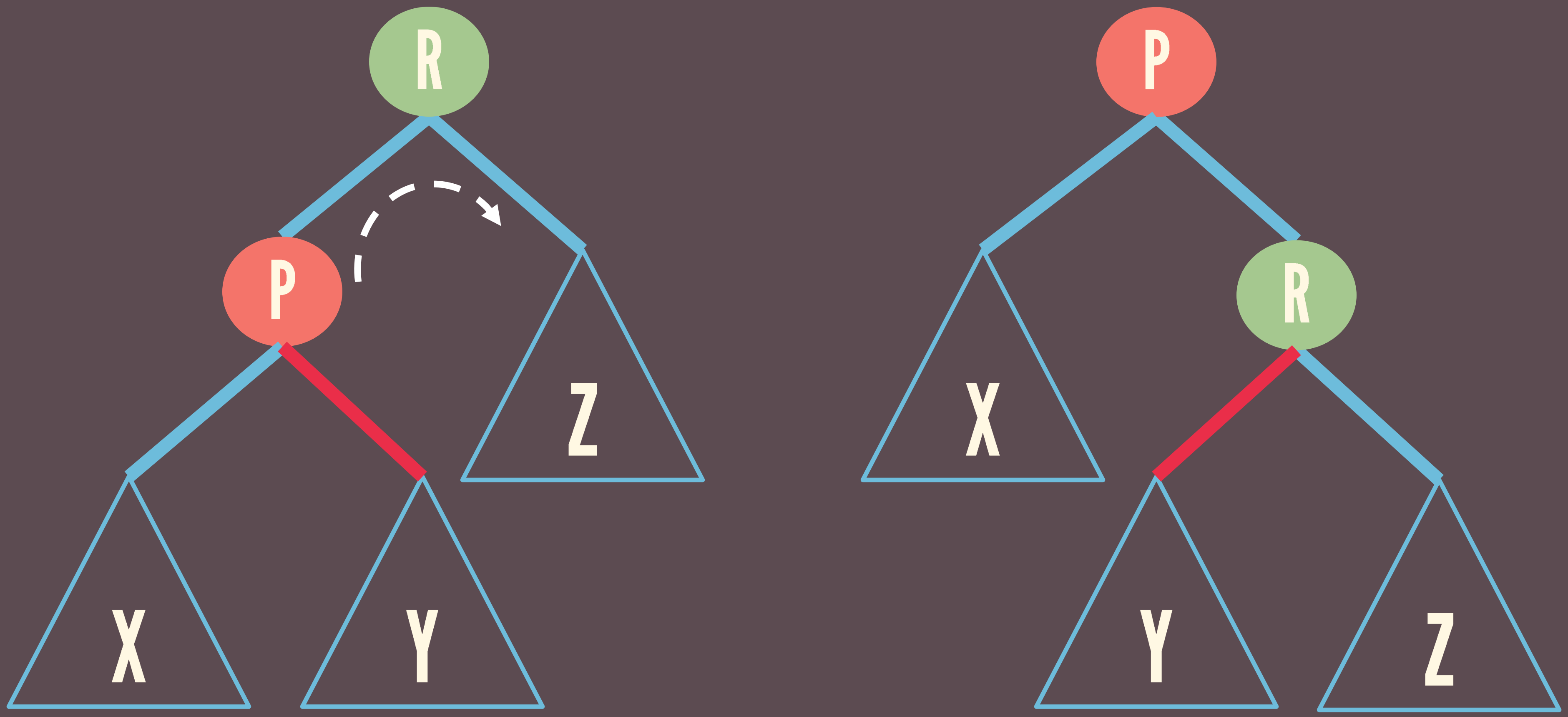
# LEFT ROTATE

R becomes the left child of P

Y becomes the right child of R




RIGHT ROTATE

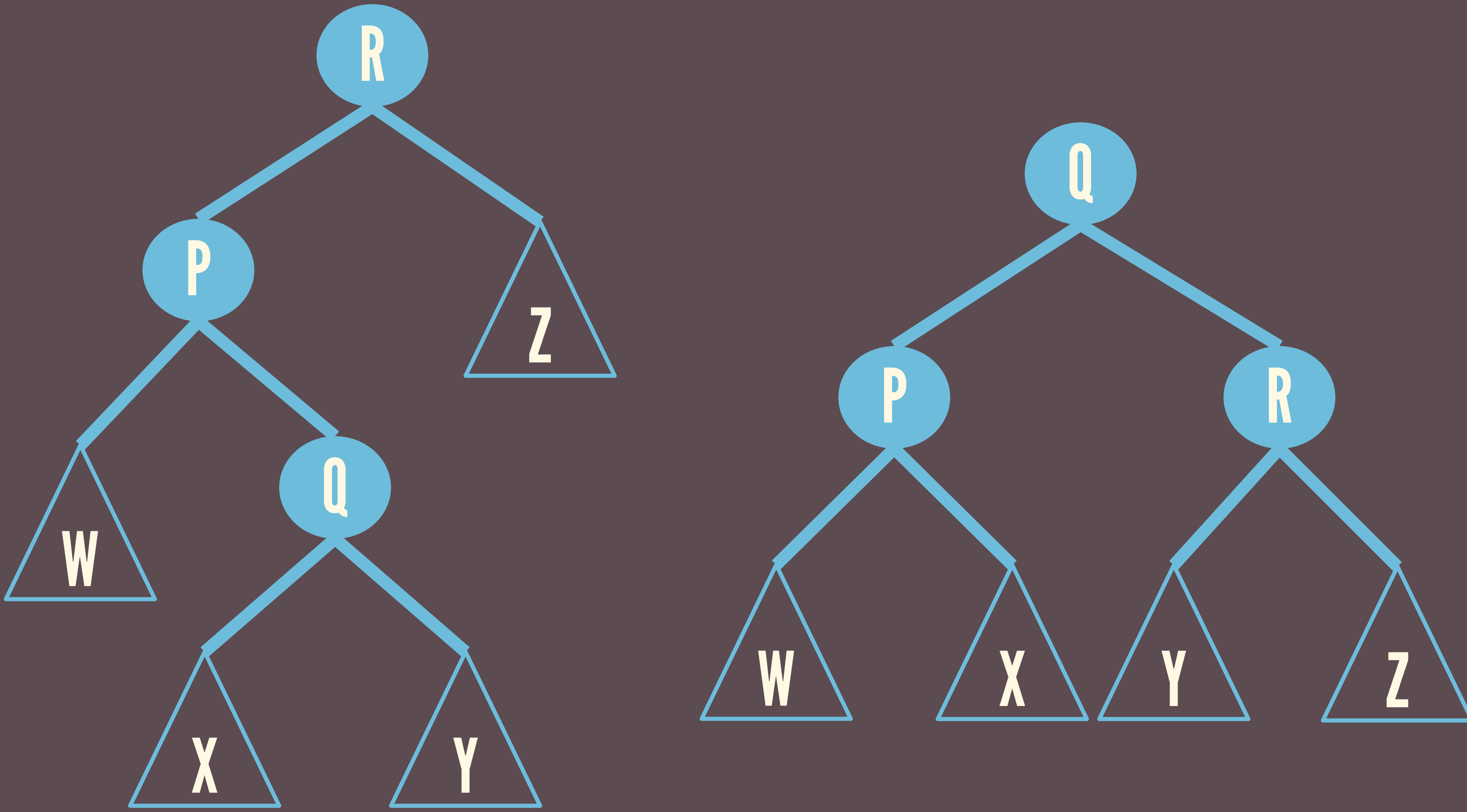



RIGHT ROTATE

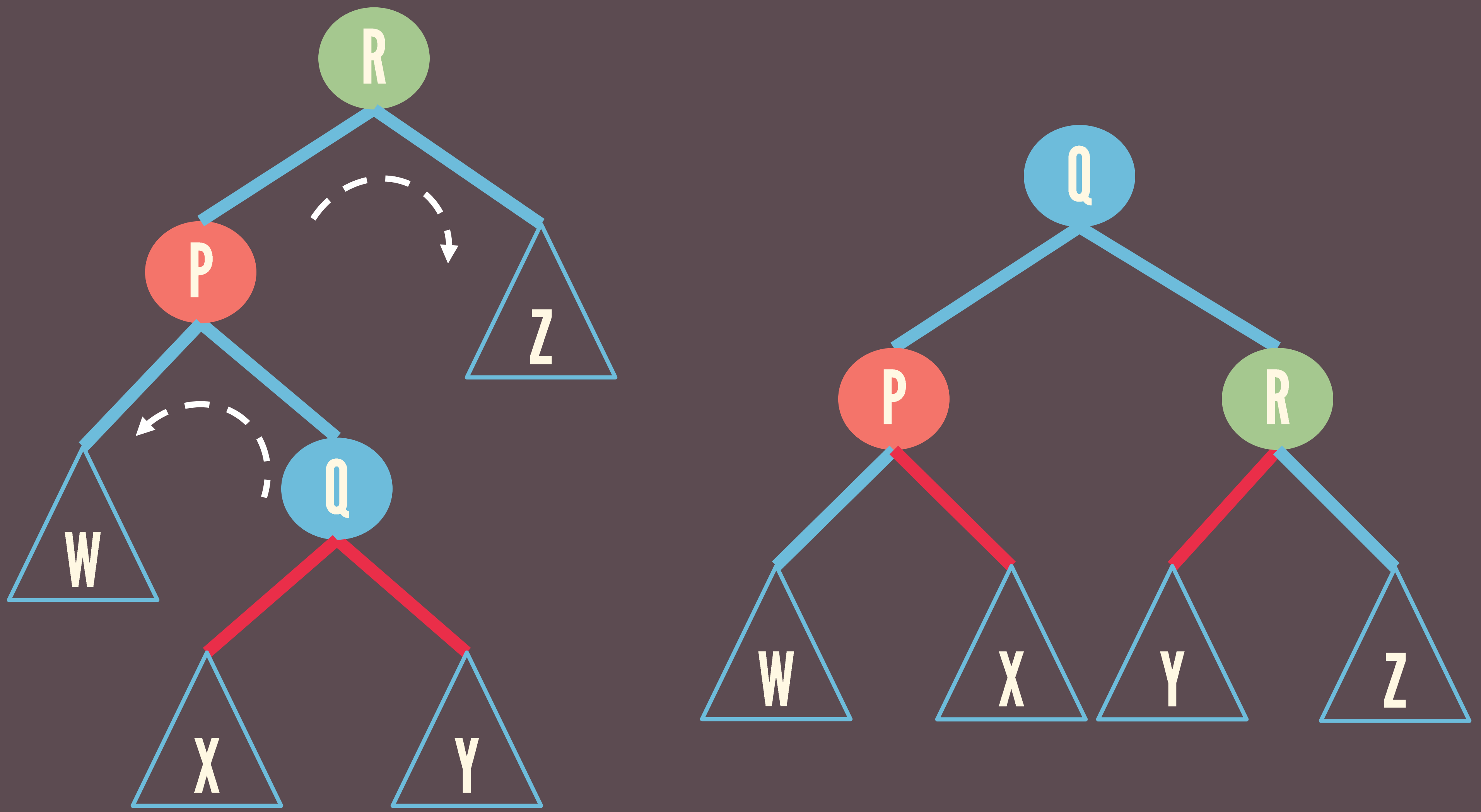
# RIGHT ROTATE

R becomes the right child of P

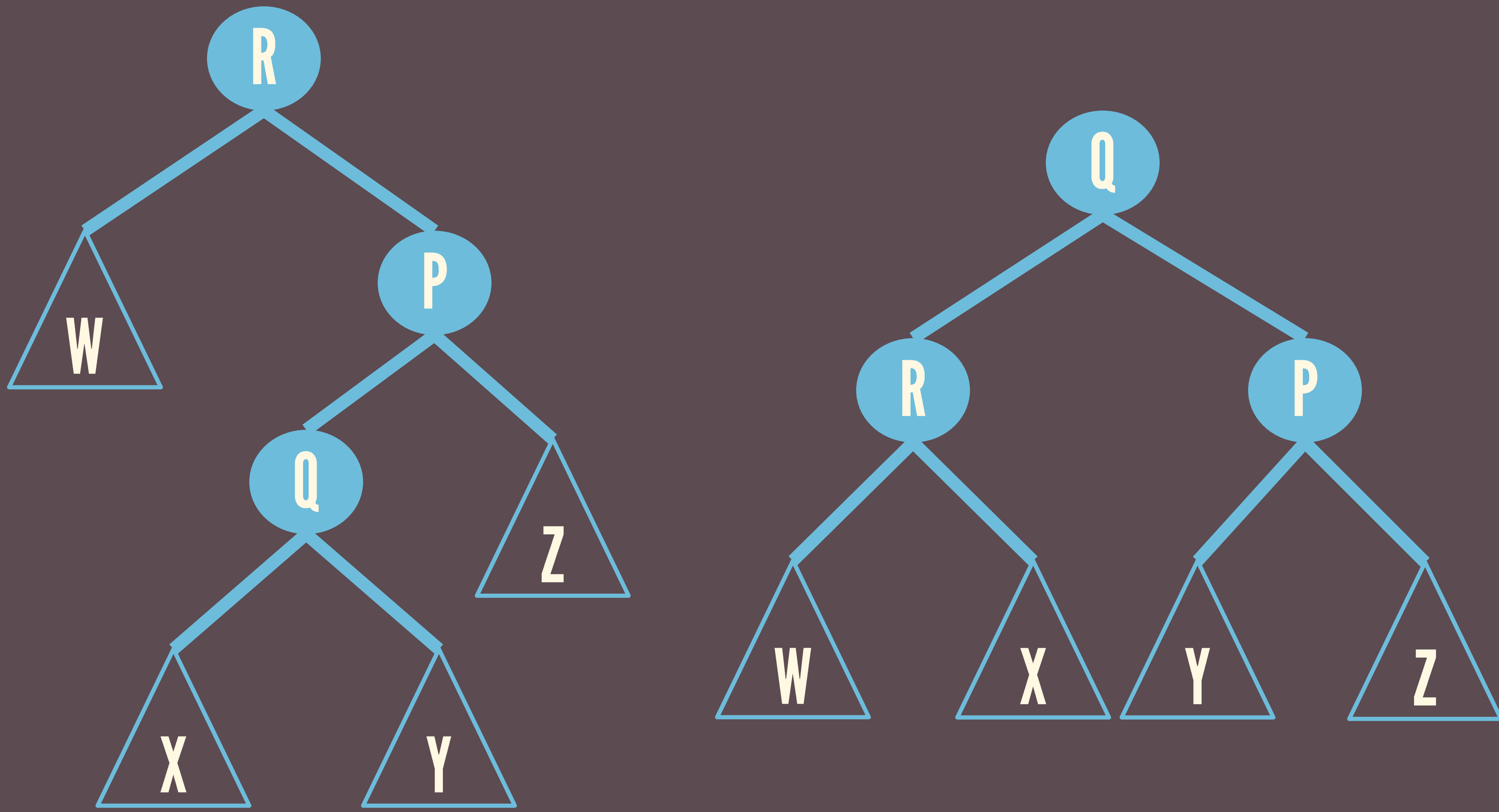
Y becomes the left child of R



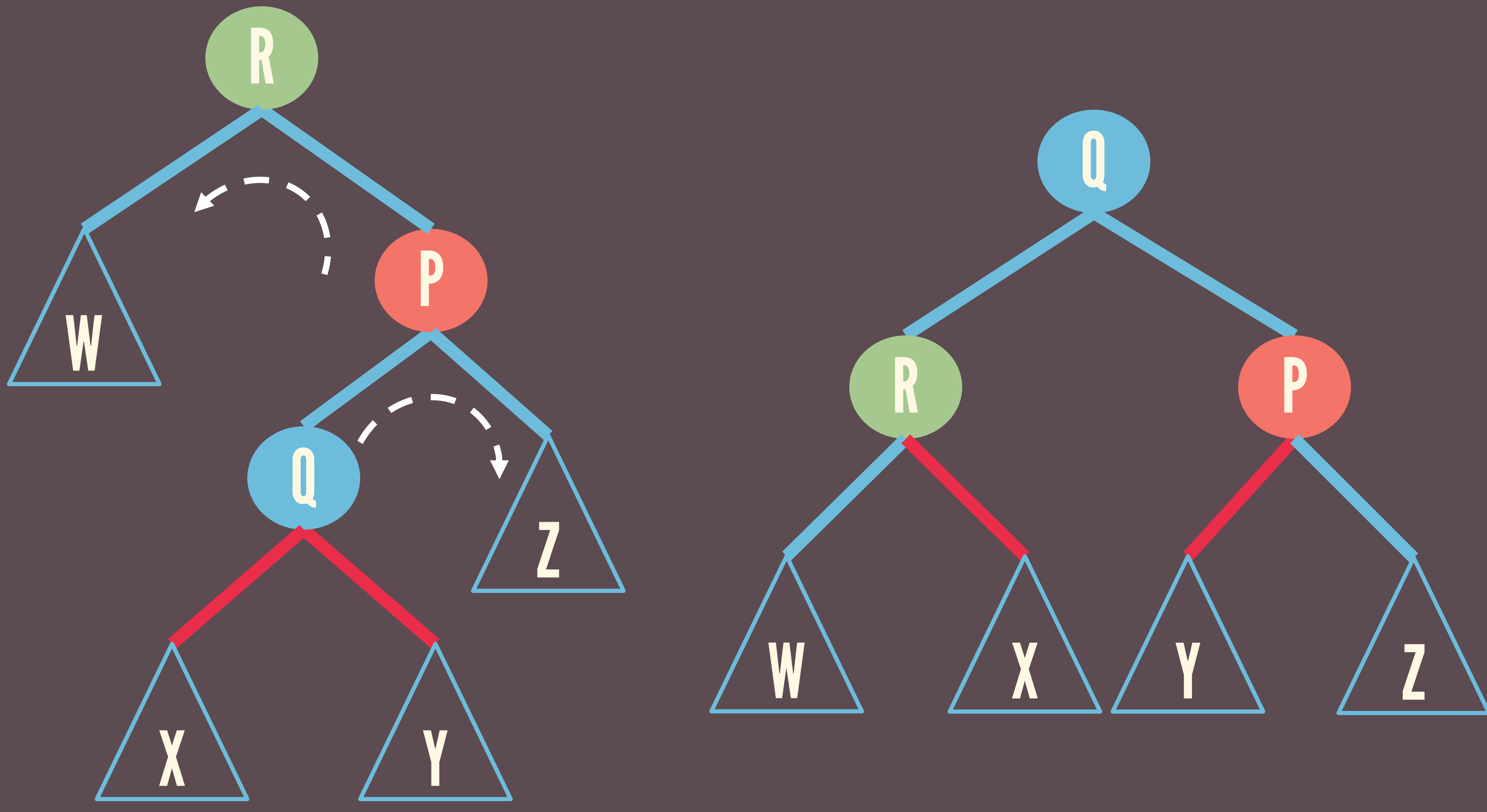

# LEFT-RIGHT ROTATE




LEFT-RIGHT ROTATE




RIGHT-LEFT ROTATE

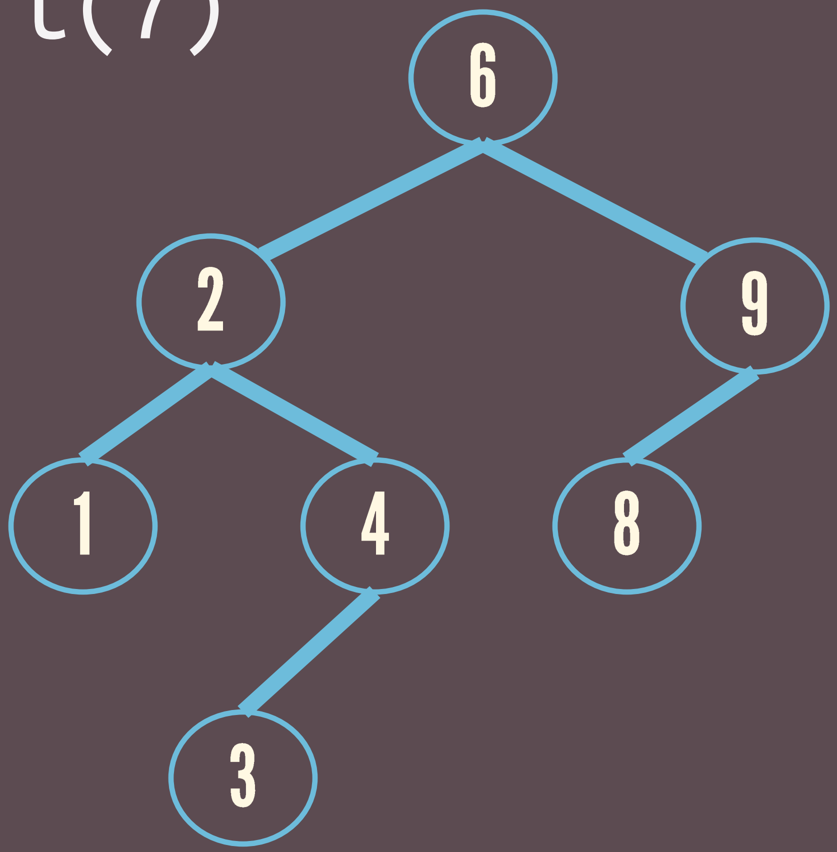



RIGHT-LEFT ROTATE

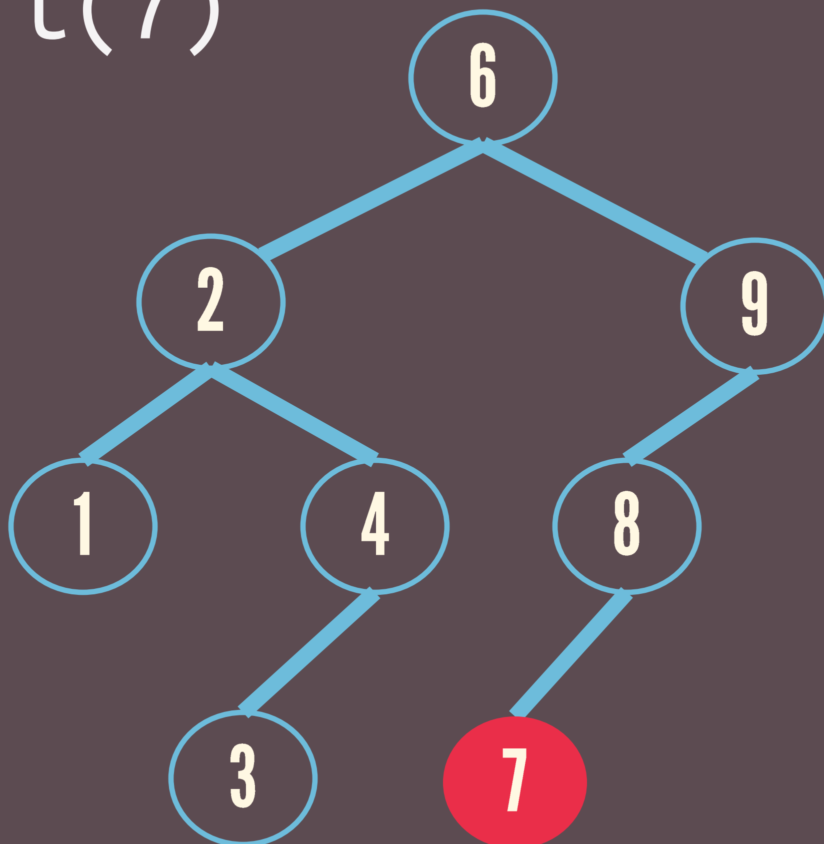
# WHEN TO USE WHAT ROTATION

## 4 CASES

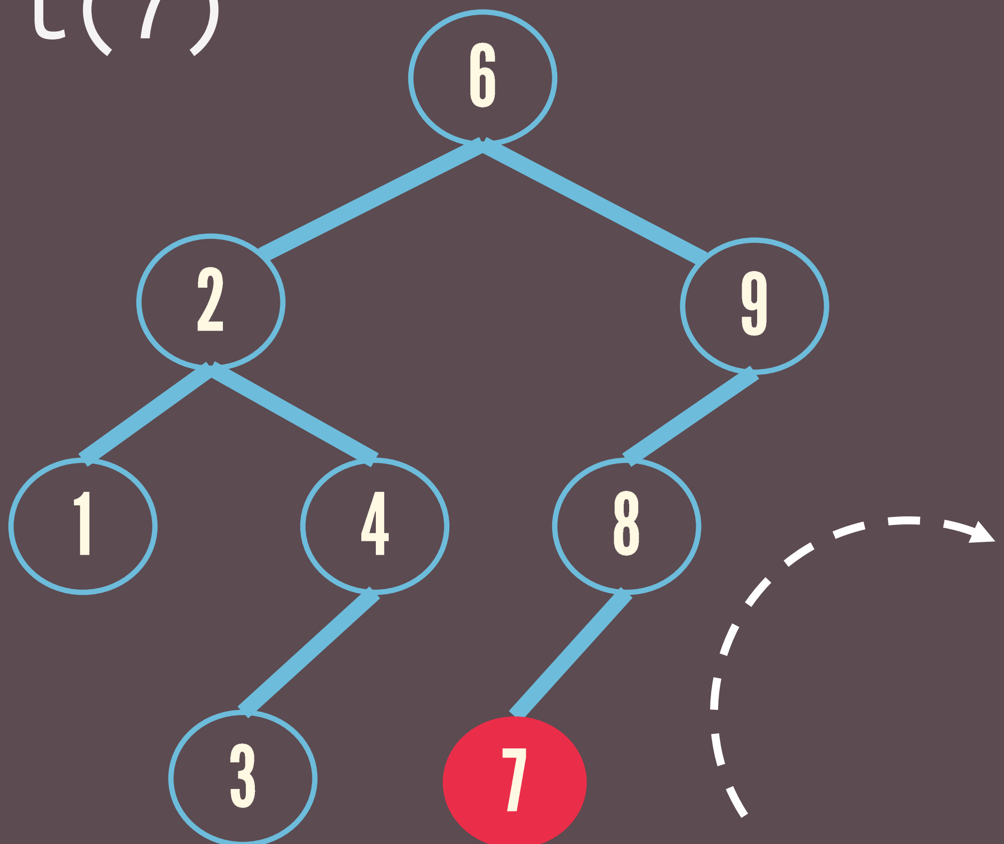
```
insert(7)
```



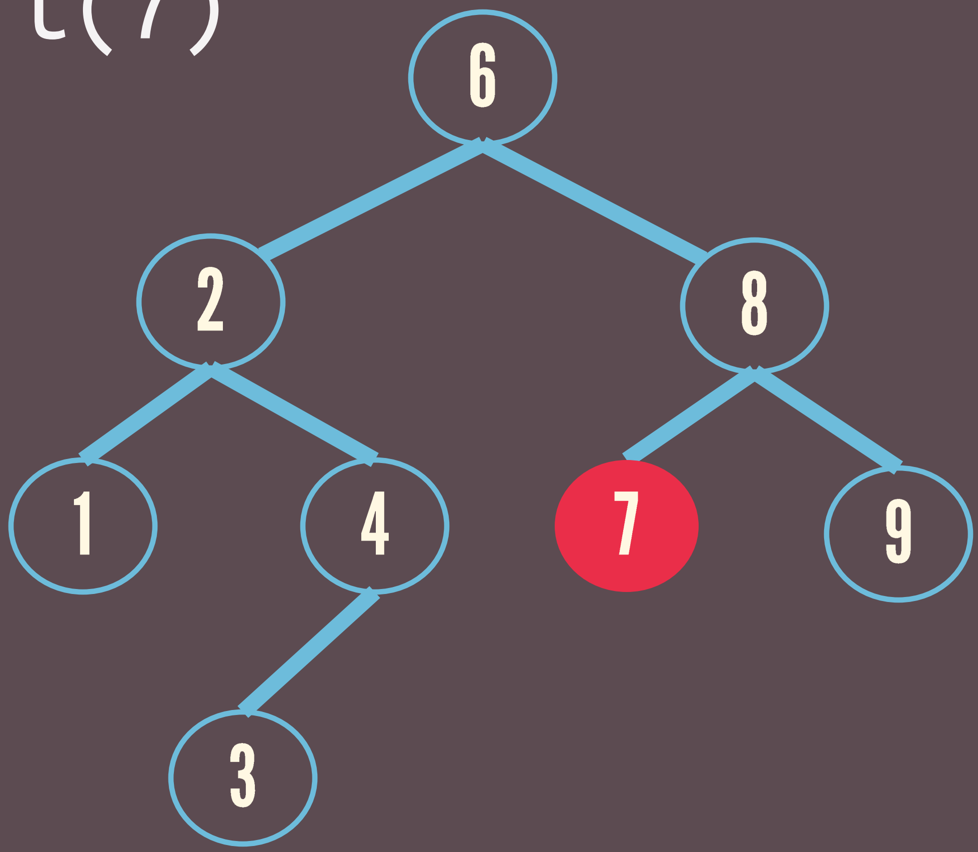
`insert(7)`



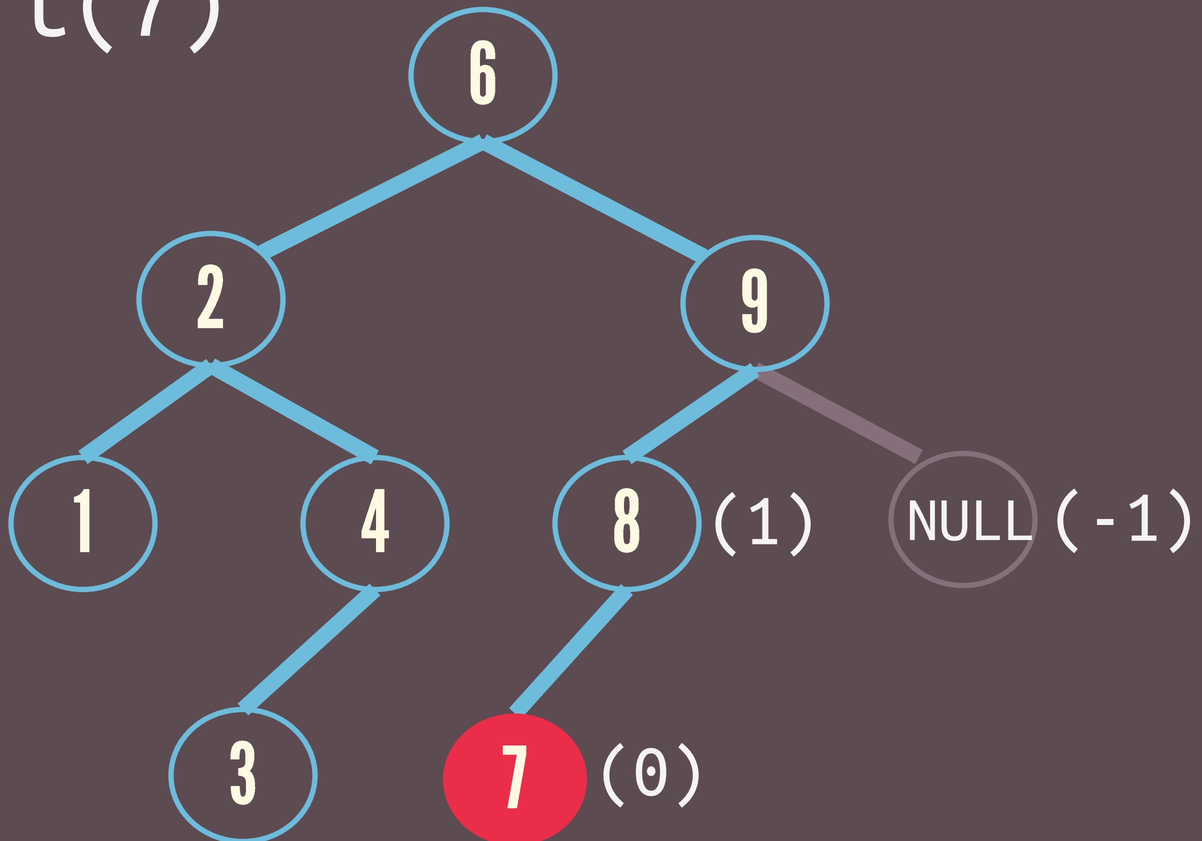
insert(7)



insert(7)



insert(7)



```
insertNode(){

   algorithm for inserting a node.
   update height of nodes.
   fixUp()

}
```

```
fixUp(){

   start at the node inserted and travel
   up the tree:
      if an imbalance is found,
         check the four cases and do the
         appropriate rotation.
      update height of the nodes.

}
```

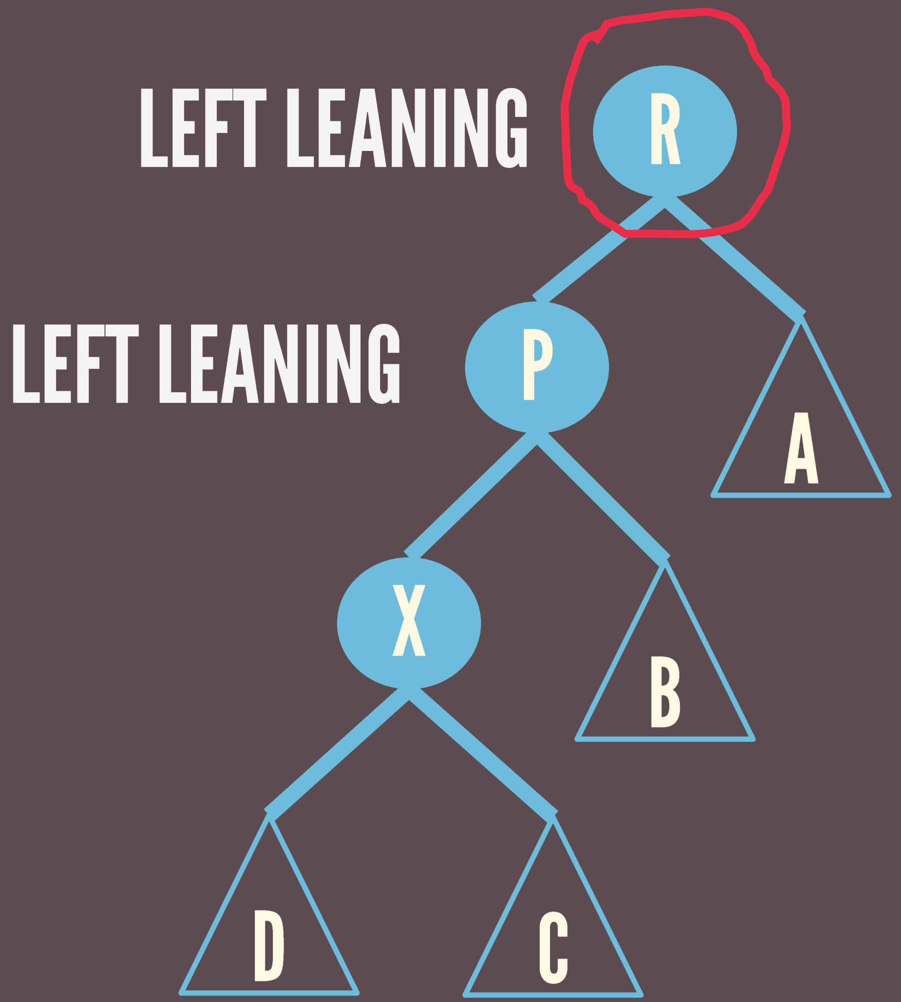
fixUp()

rotation is made where the imbalance is found

#1

LEFT LEFT CASE

RIGHT ROTATE



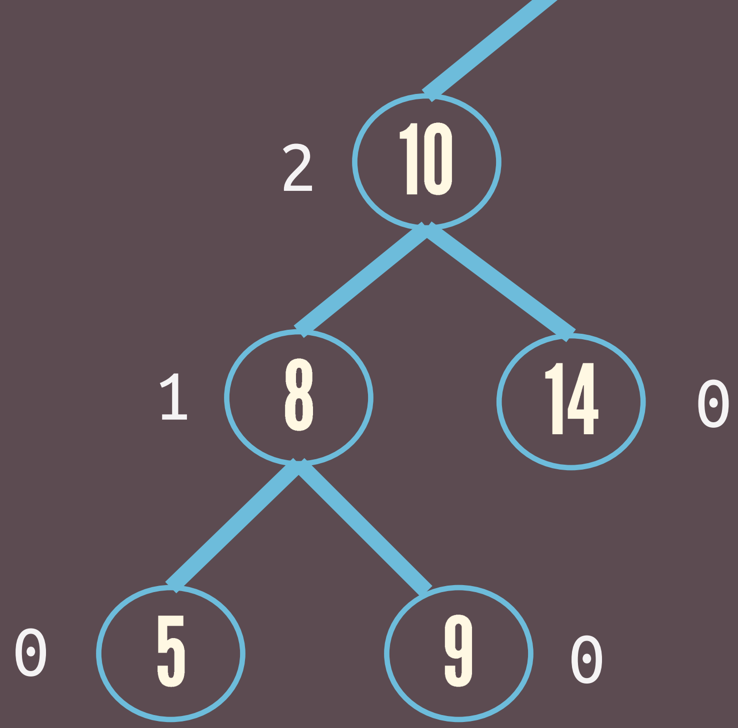

R – root

P – pivot

# #1

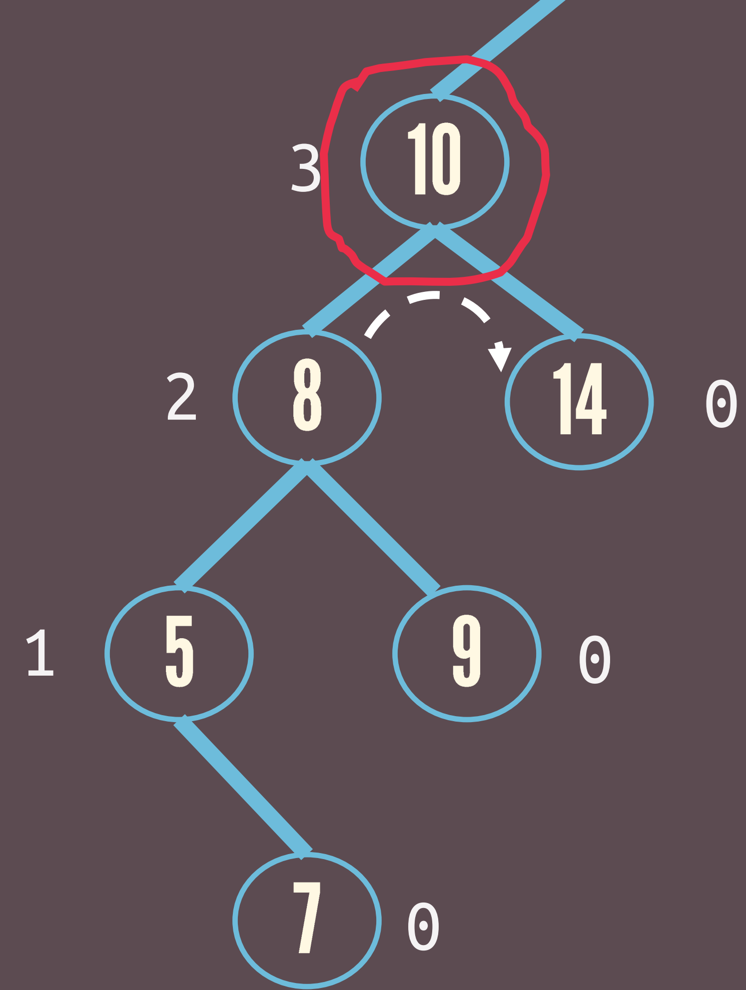
```
fixUp(){

  start at the node inserted and travel
  up the tree:
    if an imbalance is found,
      if pivot is left leaning and
      root is left leaning
        do a left rotation on root.
    update height of the nodes.

}
```

# #1



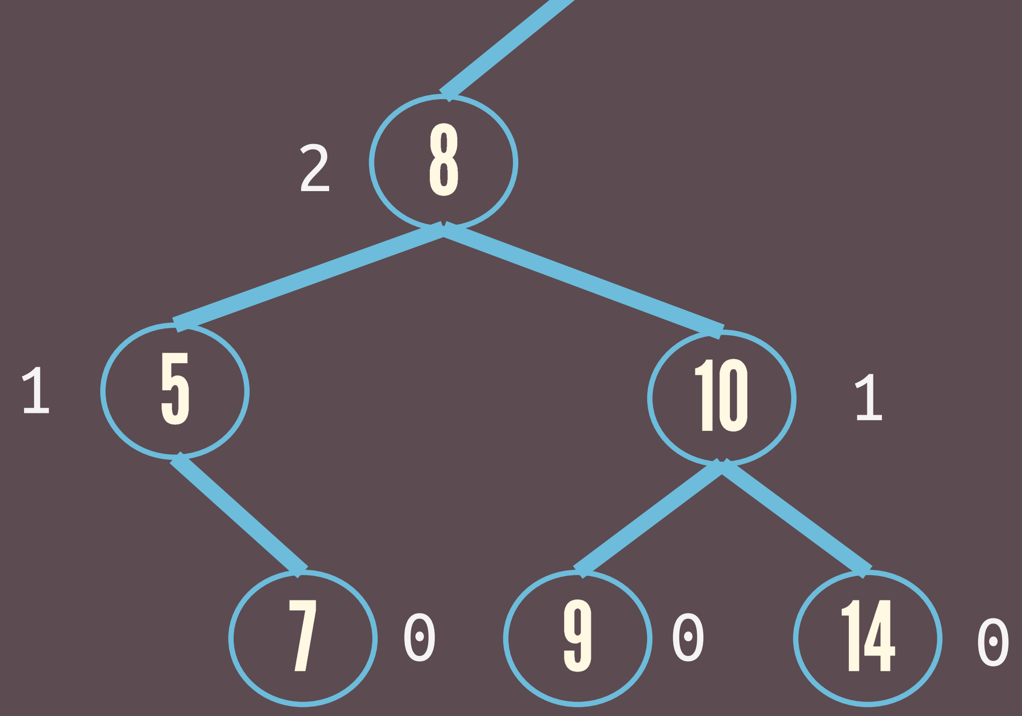

insert(7)

#1




`insert(7)`

#1



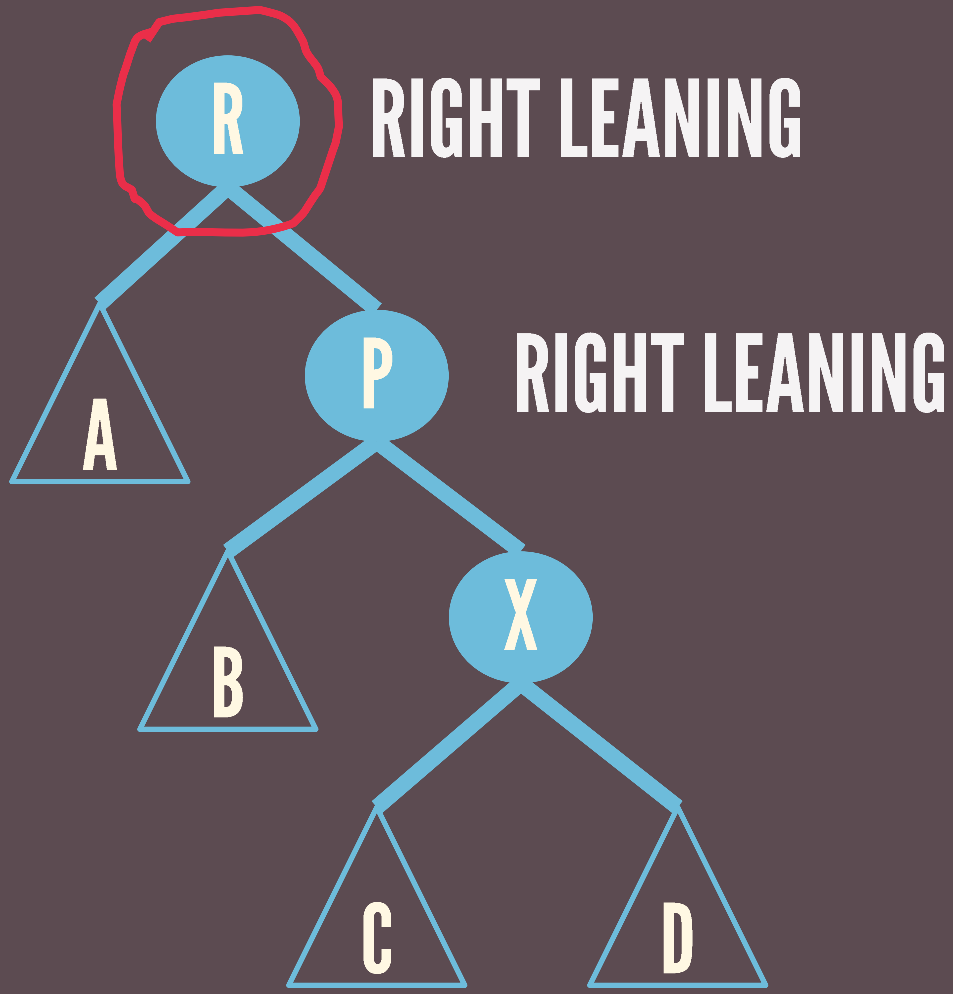

insert(7)

#2

RIGHT RIGHT CASE

LEFT ROTATE

# #2



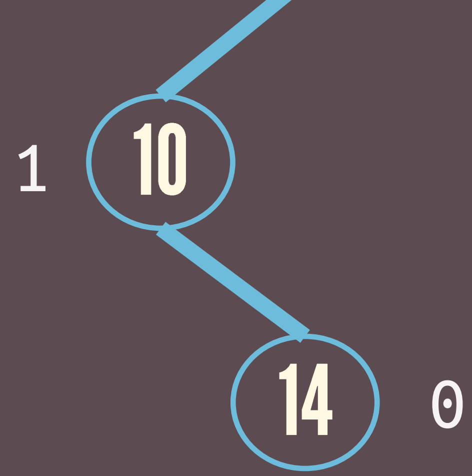

R – root

P – pivot

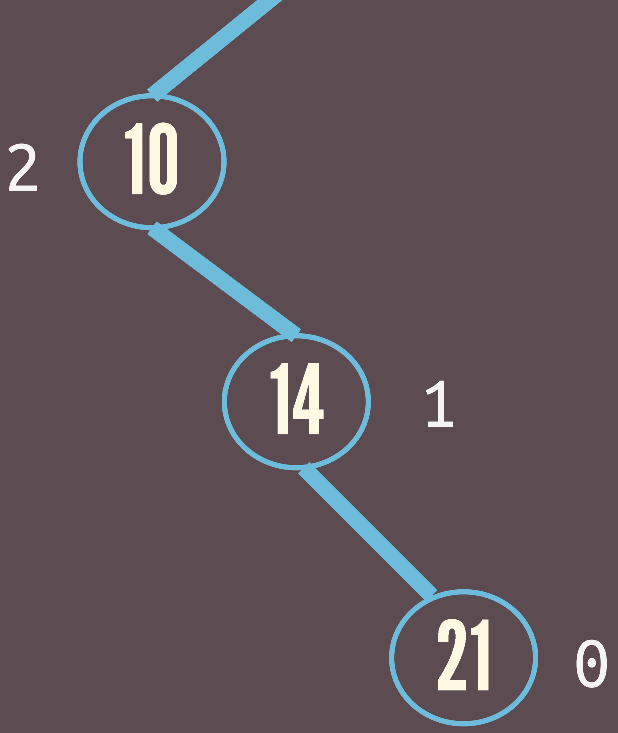
#2

```
fixUp(){

  start at the node inserted and travel
  up the tree:
    if an imbalance is found,
      if pivot is right leaning and
      root is right leaning
        do a left rotation on root.
    update height of the nodes.

}
```

#2

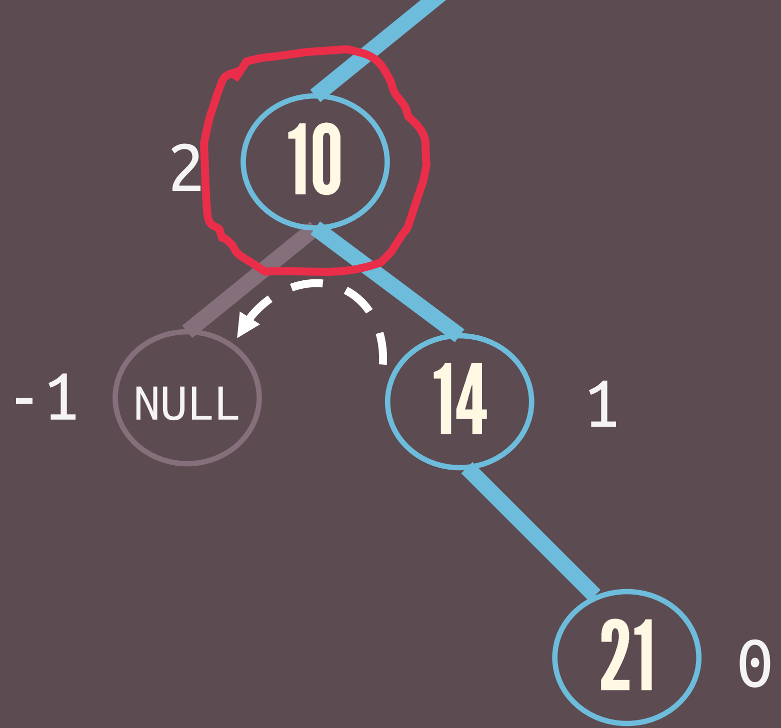



```
insert(21)
```

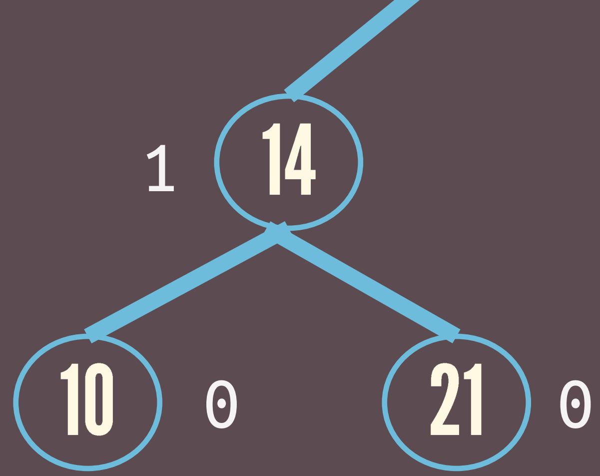



insert(21)

#2




insert(21)

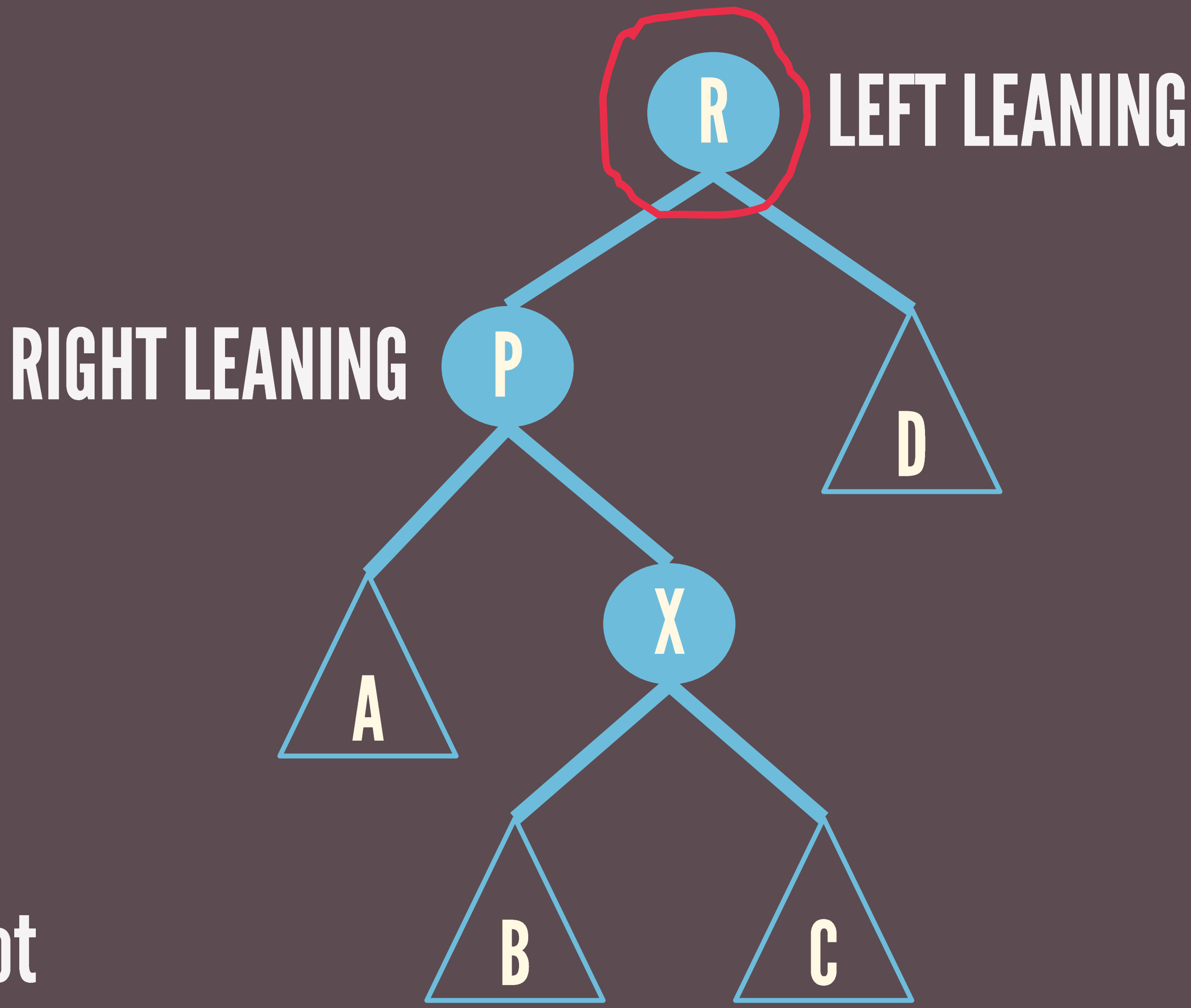



#2

insert(21)

LEFT RIGHT CASE

LEFT RIGHT ROTATE



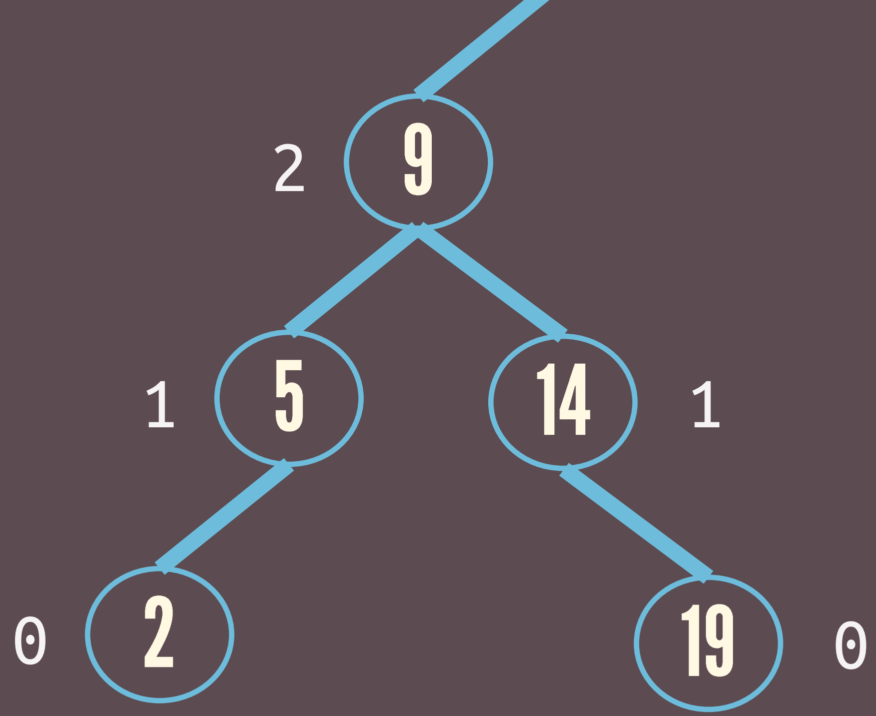

R – root

P – pivot

#3

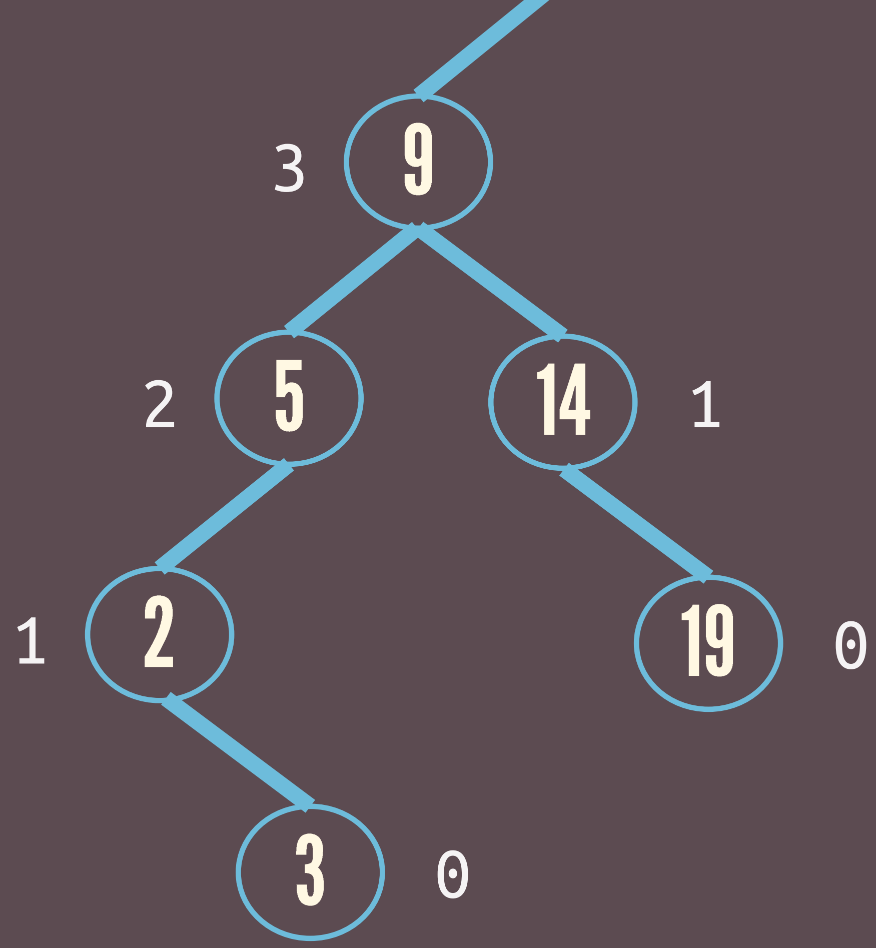
```
fixUp(){

   start at the node inserted and travel
   up the tree:
      if an imbalance is found,
         if pivot is right leaning and
         root is left leaning
            do a left rotation on pivot.
            do a right rotation on root.
      update height of the nodes.

}
```

#3

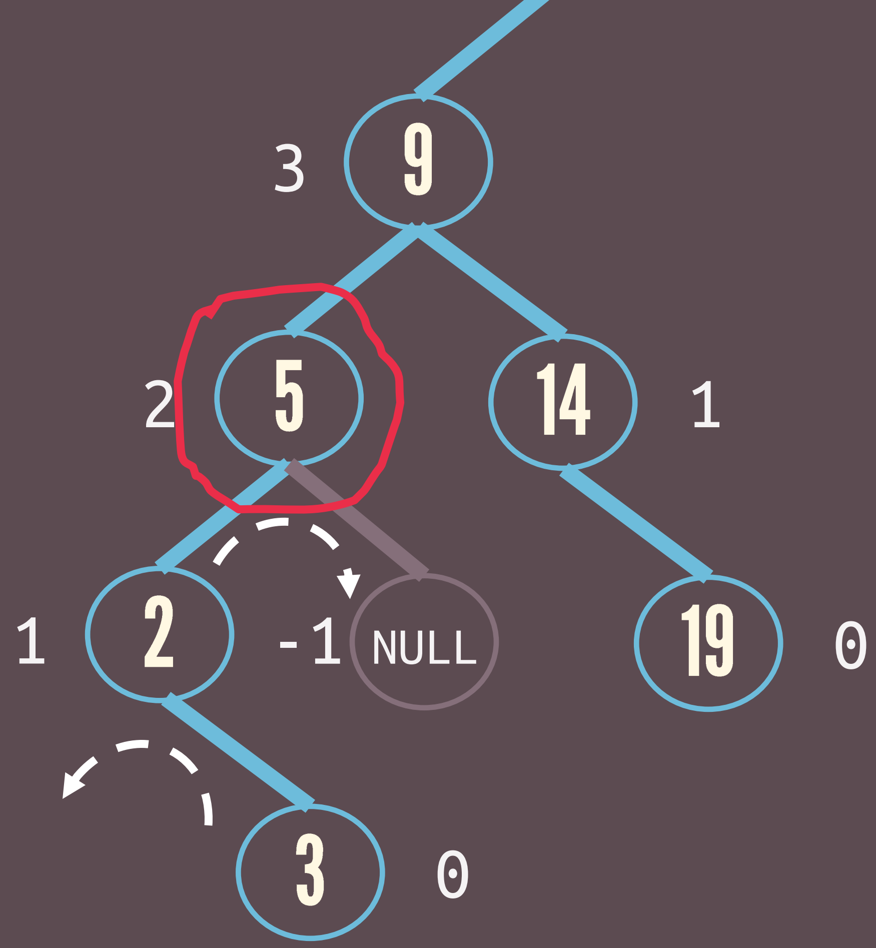



insert(3)

# #3



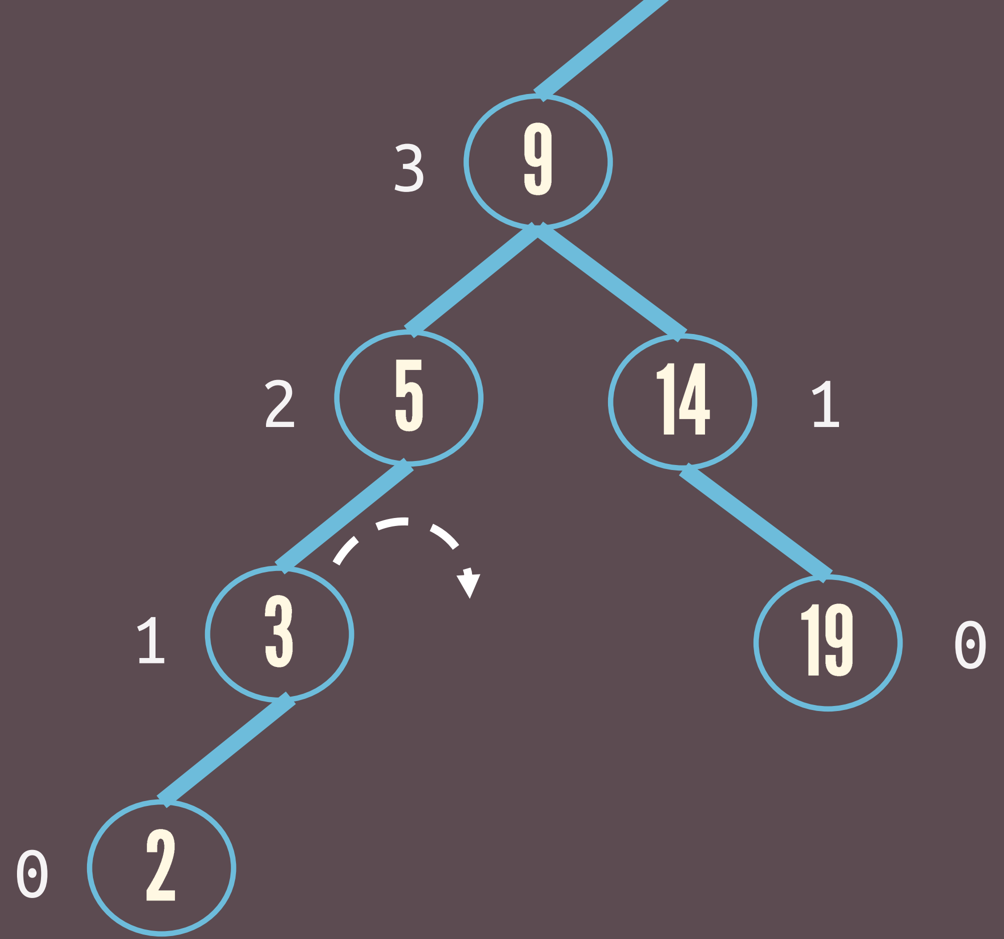

insert(3)




#3

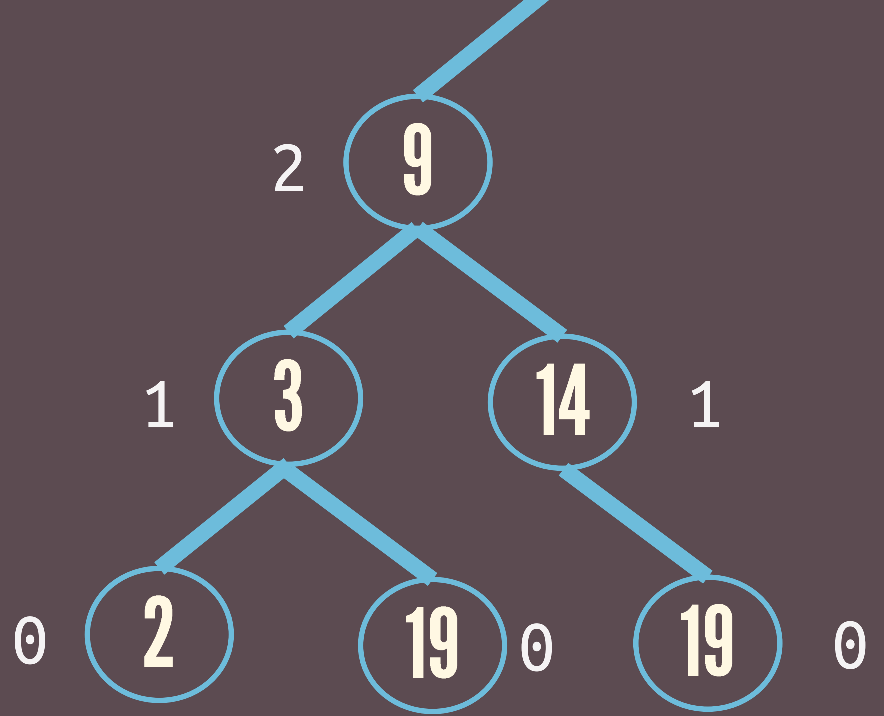
insert(3)

#3




insert(3)

#3

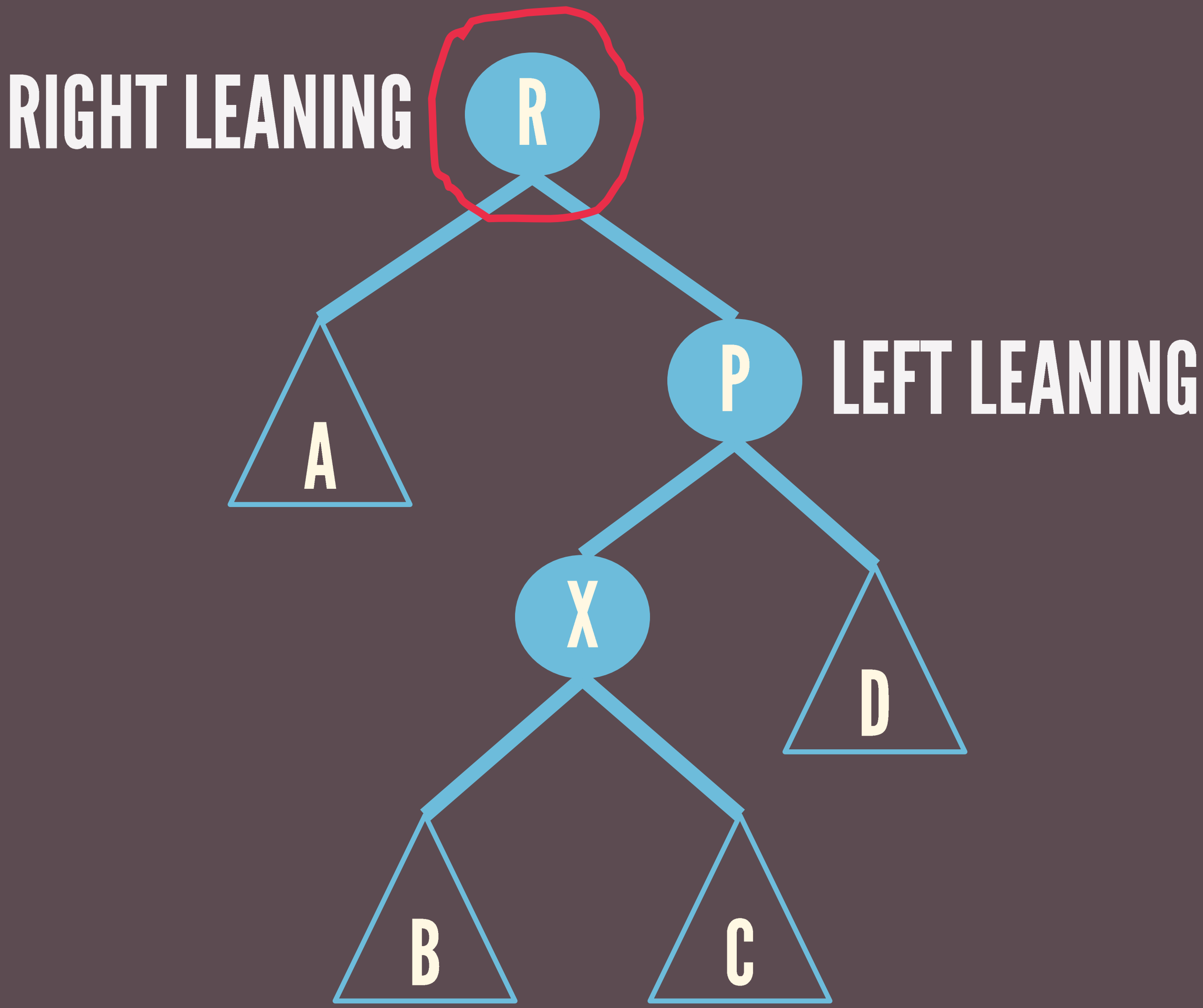



insert(3)

#4

RIGHT LEFT CASE

RIGHT LEFT ROTATE



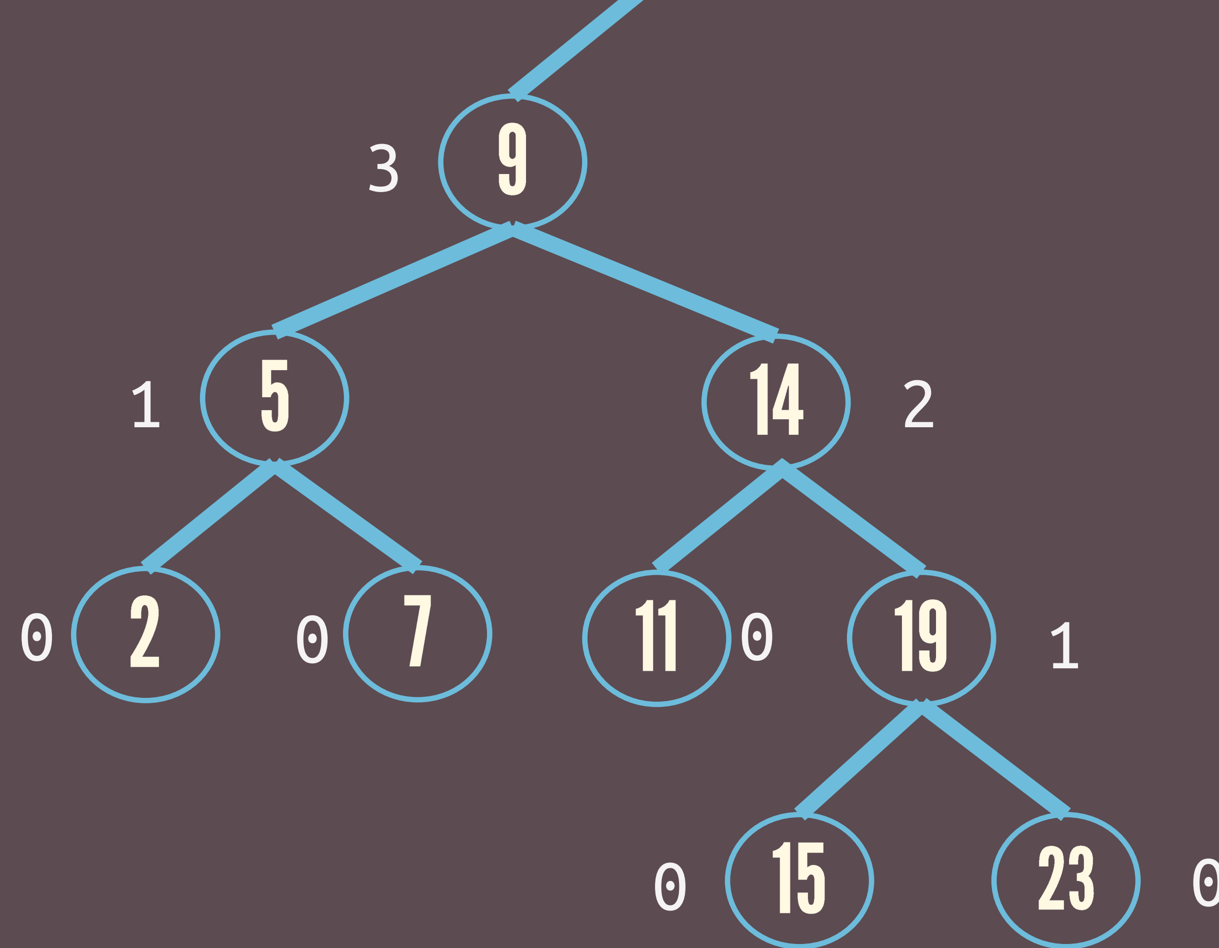

R – root

P – pivot

#4

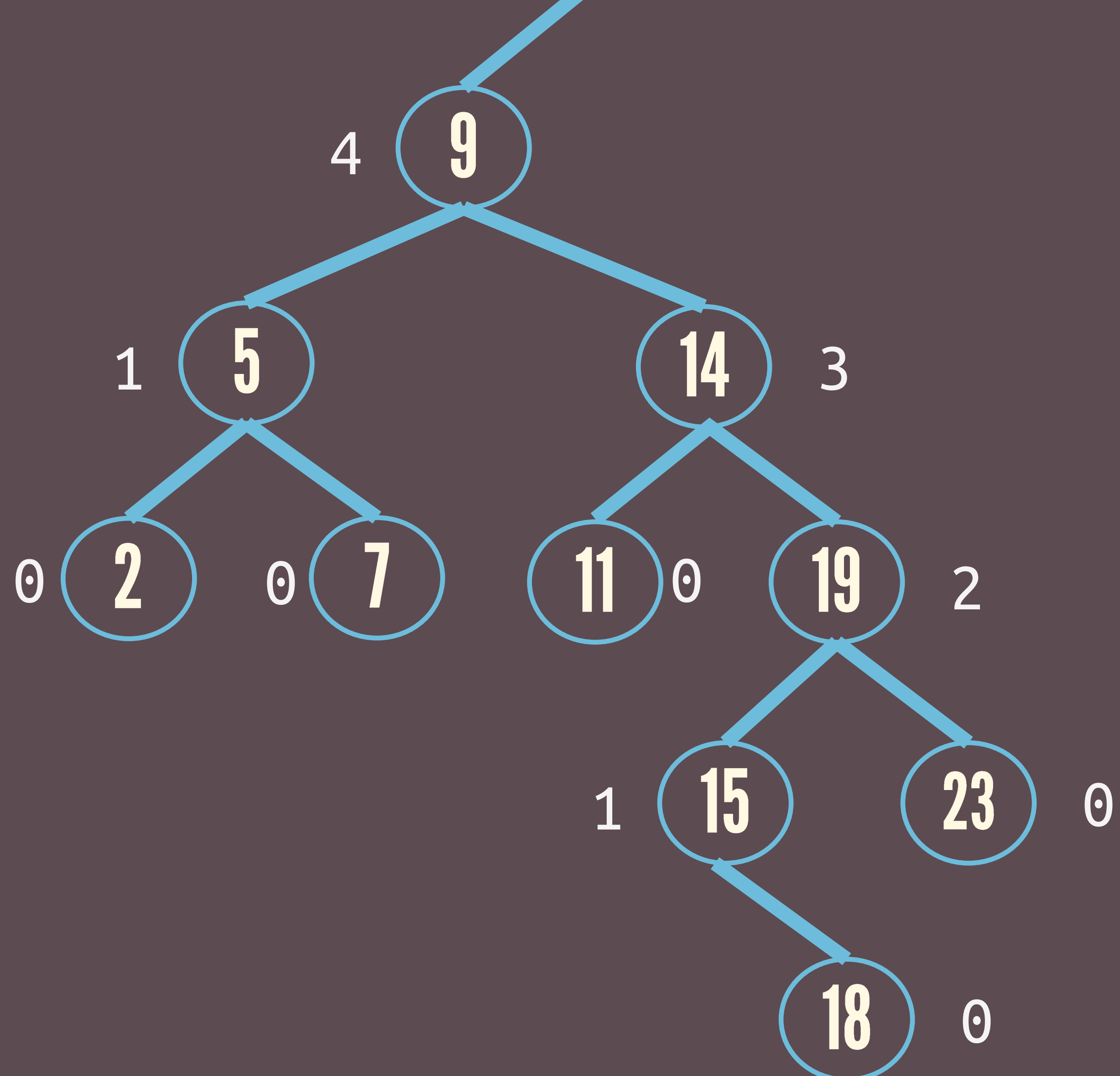
```
fixUp(){

  start at the node inserted and travel
  up the tree:
    if an imbalance is found,
       if pivot is left leaning and
       root is right leaning
          do a right rotation on pivot.
          do a left rotation on root.
    update height of the nodes.

}
```

#4

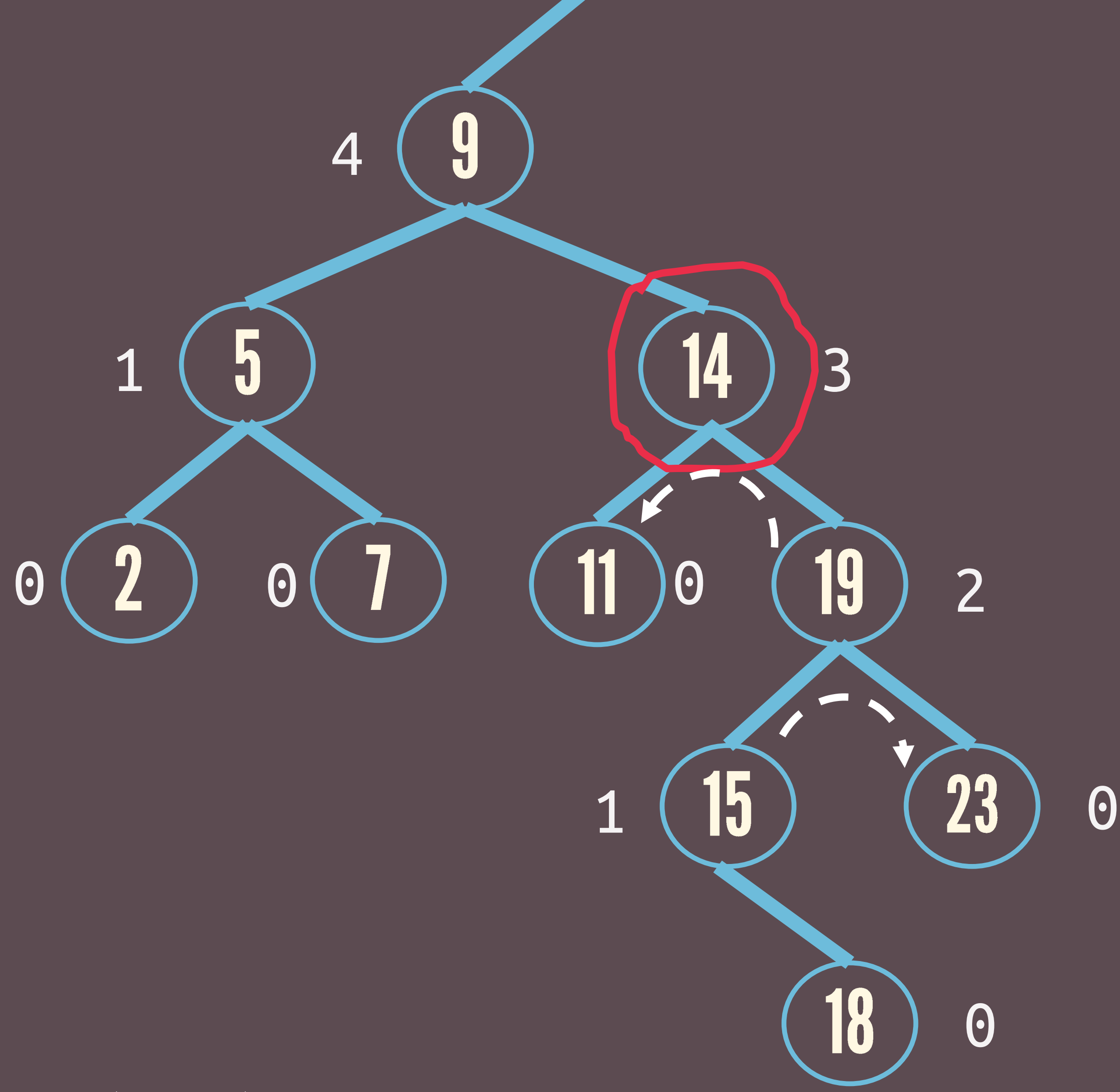



insert(18)

#4

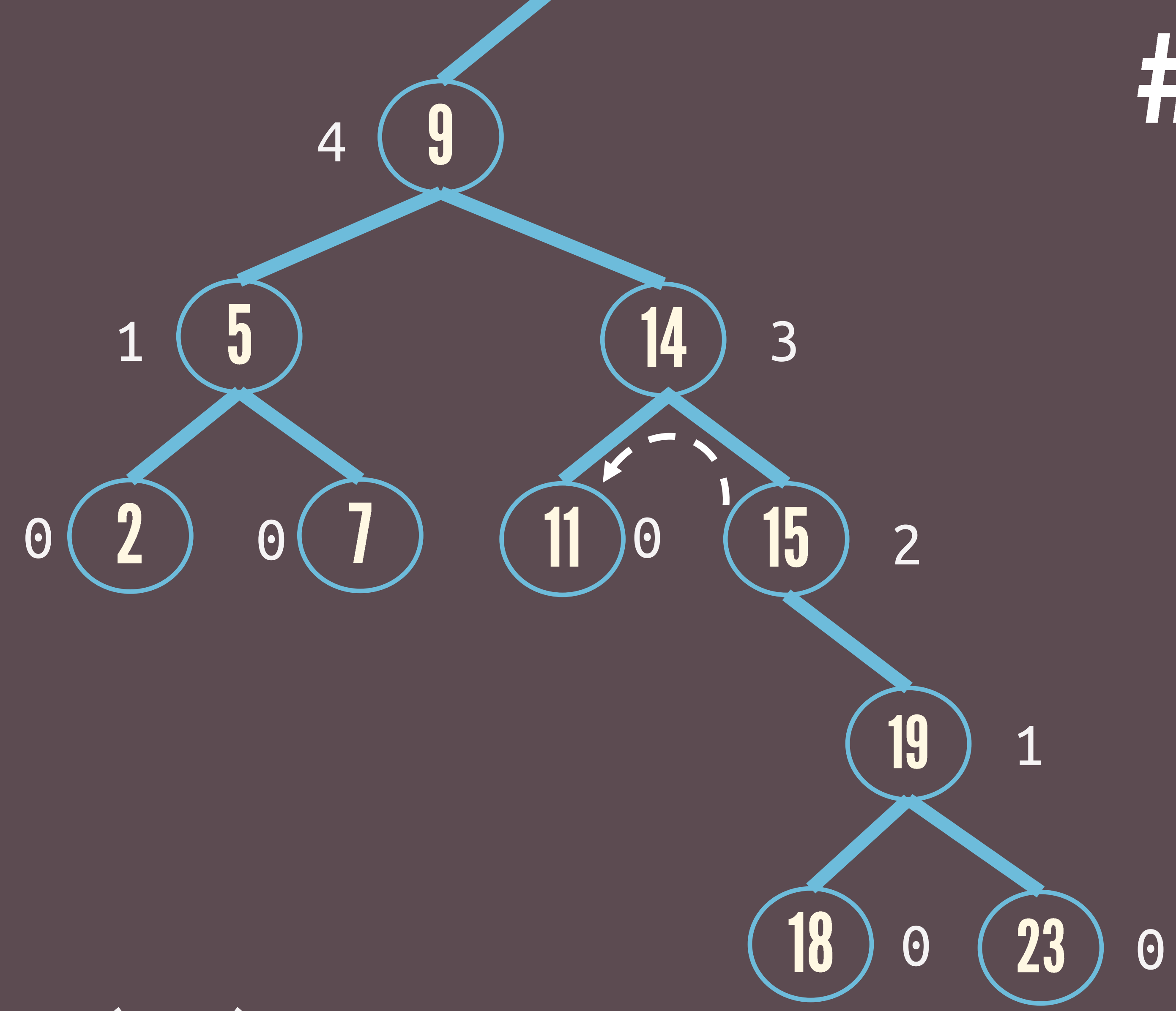



insert(18)



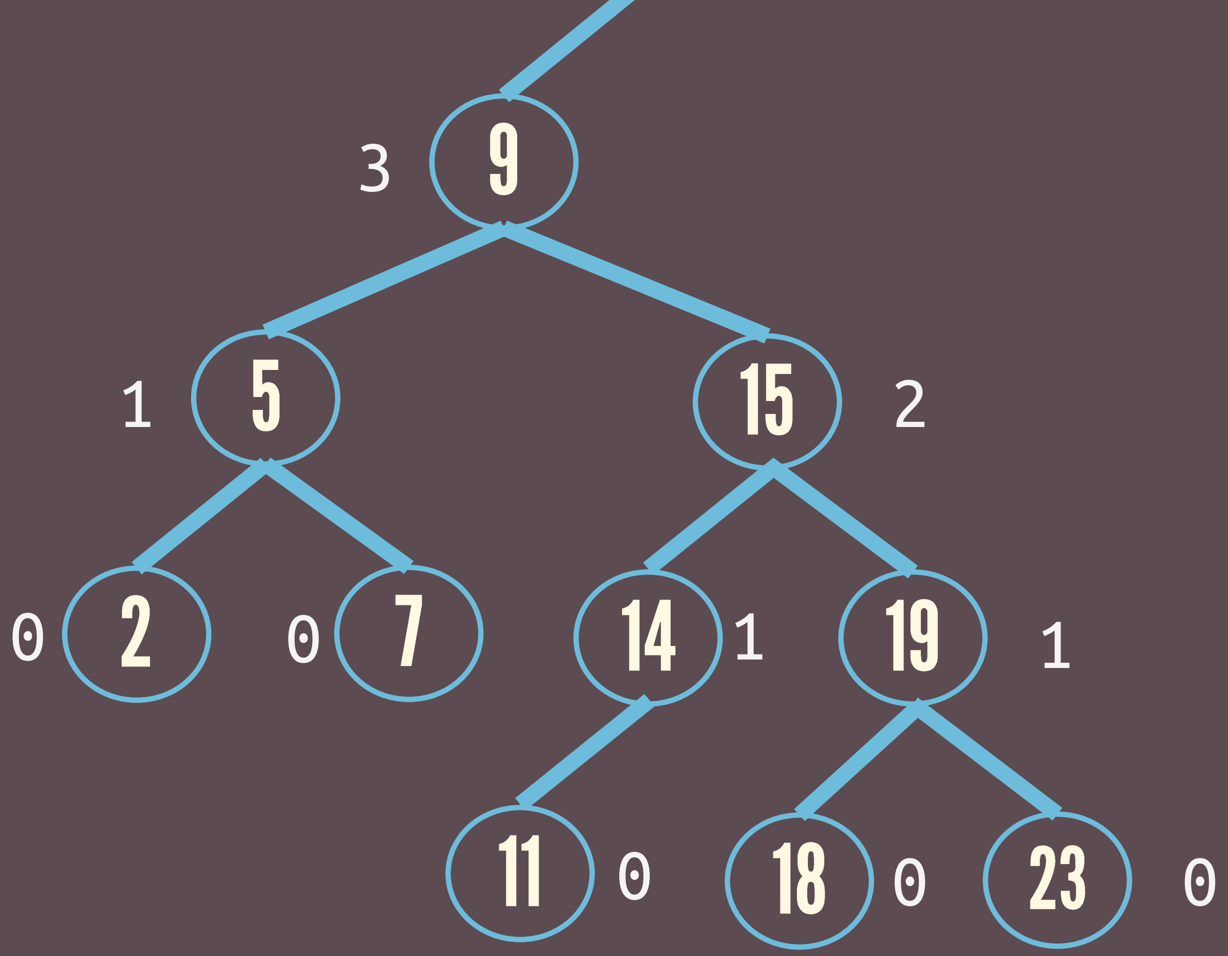

insert(18)

#4




insert(18)

#4




insert(18)