Lab 4: Equivalent Circuits and Filters

Prelab

Read sections 2.31 – 2.33 in your textbook.

Q. What are the four categories of filters and how do they differ? (You will be building one of each in lab.)

Q. The vertical scale on a Bode plot is generally given in decibels. How is the attenuation in dB related to the input and output voltage?

Be sure that you or your partner bring the textbook and a laptop to lab.

Supplies

- Multimeter
- DC Power Supply
- Function Generator (x2)
- Oscilloscope (and probes)
- Breadboard

- Minigrabber probes w/ adapters
- 100Ω, 1kΩ, 6.2kΩ, and 7.5 kΩ resistors
- 22nF and 1nF capacitors
- 1000μH inductor

Figure 1: https://xkcd.com/567/

Part I: Thevenin's Theorem

Q. Following the guidelines in section 2.19 of your textbook, calculate the Thevenin voltage and resistance for the circuit in figure 2, between points A and B.

- Construct the circuit in figure 2 and measure the voltage between A and B.

- Replace the voltage source with a wire and measure the resistance between A and B.

Q. Compare these values with those you calculated.

- Connect a 100 Ω load resistor between points A and B and measure the voltage across it.

- Draw the Thevenin equivalent of circuit 2 in your lab notebook and then build it.

- Connect a 100 Ω resistor to the output of the Thevenin equivalent circuit and measure the voltage across it.

Q. Report and compare the voltage across the load for the full circuit and the Thevenin equivalent. Be sure to include a schematic of the Thevenin equivalent circuit in your lab report.
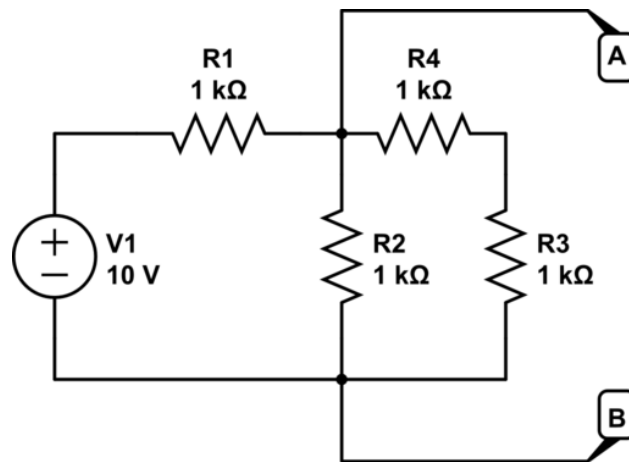

Figure 2: Resistor Network

Part II: Filters

2.1 Background info

Last week you examined the voltage across both the capacitor and the resistor in an RC circuit and found that the output amplitude and the phase angle were frequency dependent. These circuits let through some frequencies preferentially while filtering out others. You also looked at a special frequency at which the phase angle was $45°$ and the output voltage was $V_{out} = V_{in}/\sqrt{2}$. This frequency is known as the "cutoff frequency" of a filter. Equations (2.1) and (2.2) are the same as the ones you found in lab last week, but they have been recast in terms of the cutoff frequency, as defined in equation (2.3).

Equation (2.1) gives the voltage output across the capacitor and equation (2.2) gives the voltage out across the resistor.

$$V_{out} = \left(\frac{Vin}{\sqrt{1 + (f/f_c)^2}} \right) \text{ at an angle of } \left(\arctan\left(\frac{f_c}{f} \right) - 90° \right) \tag{2.1}$$

$$V_{out} = \left(\frac{Vin}{\sqrt{1 + (f/f_c)^{-2}}} \right) \text{ at an angle of } \left(\arctan\left(\frac{f_c}{f} \right) \right) \tag{2.2}$$

$$f_c = 1/(2\pi RC) \tag{2.3}$$

While last week you mainly focused on the phase angle, this week you will be focusing on the amplitude. The most common way to represent the effect a filter circuit has on the input amplitude is to calculate the decibels of attenuation over a range of frequencies and then create a plot of decibels on the vertical axis (with a linear scale) and frequency on the horizontal axis (with a log scale). These are known as Bode plots and there are several examples in your book.

Equation (2.4) gives the relationship between the voltage attenuation fraction and the attenuation level in decibels. When at the cutoff frequency, the attenuation level is almost exactly -3dB. For this reason, the cutoff frequency is also often referred to as the "3dB point" or f_{3dB}. (Note the lack of negative sign, that is simply convention, which is just the fancy way of saying we're all too lazy to keep writing the negative sign.)

   Q. Show how the -3dB value is arrived at and what the exact value is.

$$[dB] = (20dB)\log(V_{out}/V_{in}) \tag{2.4}$$

When at a frequency much higher than the cutoff frequency for a low pass filter, the "1" under the square root in equation (2.1) becomes nearly irrelevant. In this range, a doubling of frequency results to a halving of voltage. We refer to this "doubling" of the frequency as an "octave".

   Q. Examine the decibel attenuation equation. Every time the voltage decreases by a factor of 2, what is the increase in attenuation in decibels?

   Q. A change in frequency by an order of magnitude is known as a "decade". For eq 2.1, when $f \gg f_c$, how much does the voltage decrease per "decade"? What change in attenuation (in dB) does this result in, or, in other words, what is the increase in attenuation per decade?

2.2 Low Pass Filter

You will be constructing and characterizing the low pass filter circuit in figure 3

- Measure the components with your multimeter to obtain more accurate values. (Use the "RELΔ" button when measuring small capacitors to zero out the capacitance of the probes.)

Q. Using these measured values, what do you expect the cutoff frequency to be? **Report the value with properly propagated uncertainty.**

- Reset the scope using DEFAULT SETUP. Be sure probes and scope are in 10X mode and probes are properly compensated. Set ACQUIRE to average 4 or 16 samples.

- Set the function generator to 50 Hz and overlay the resulting waveforms.

- The input and output voltages should both be right at 15 volts peak-to-peak. If they are slightly off, adjust the amplitude until the output (channel 2) is right at 15 volts.

Q. Taking the input voltage to be 15 volts, what output voltage do you expect to measure when at the cutoff frequency you calculated above?

- Measure the output peak-to-peak voltage starting at 50 Hz and doubling the frequency for each data point until you reach 204.8 kHz. In addition, include a data point at the cutoff frequency you calculated.

Q. Compare the V_{pp} you measured at the cutoff frequency to the value you calculated.

Q. Using Excel (or similar), calculate the attenuation in decibels for each data point.

Q. Make an attenuation Bode plot of the results as described earlier in this writeup, and as shown in figure 2.189 in your textbook. Clearly title the plot and label the axes for inclusion in your report.

Q. Far when at a frequency far from the passband, what do you obtain for the slope of the line (in units of decibels per octave)? What did you expect the slope to be?
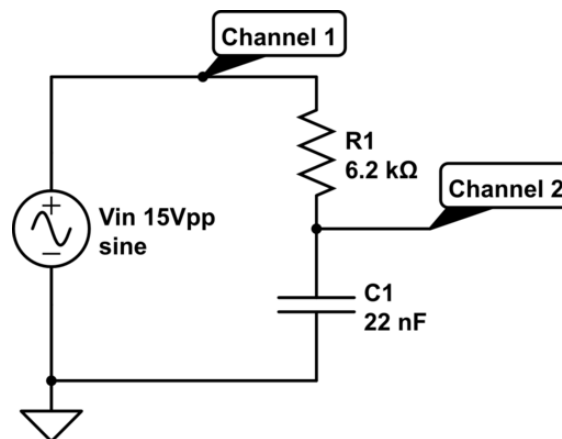

Figure 3: A low pass RC filter

2.3 High Pass Filter

Now you will construct and characterize a high pass filter.

- Obtain the components in figure 4 (Use two 1 nF caps in parallel to obtain a value of 2 nF).

- Follow the same procedure as in the previous section, answering all the same questions and making the same plots, except this time, start taking data at 204.8 kHz and work your way down to 50 Hz.

- Sometimes the max output voltage will be a few volts lower than the input, even in the "pass band" frequencies. If so, adjust the amplitude until the output (channel 2) is right at 15 volts (which will make the input a bit higher than that). Compare the rest of your measurements to the 15 volts.
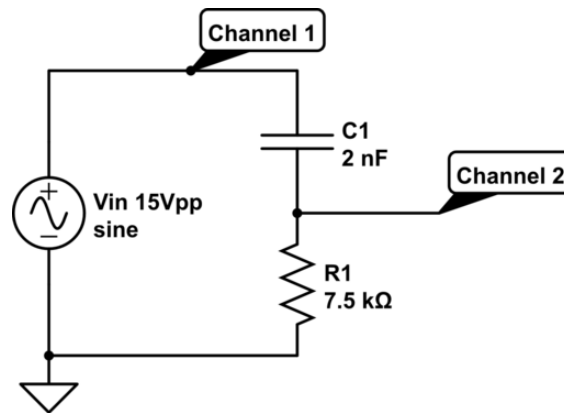

Figure 4: A high pass RC filter

2.4 Notch Filter

Next week we will look more in depth at notch and band pass filters. For now we can quickly get a feel for what they do.

- Add a 1000 μH inductor in parallel with the capacitors in the high pass filter circuit. (This configuration is often called a "tank circuit".)

In lab 1 we mentioned that the VCF input on the function generator is used to control the frequency via an external voltage. You are going to use a ramp (triangle wave) signal from one generator to control the voltage of the second generator. We'll call the generator outputting the ramp signal "gen1" and the generator outputting the sine wave into the filter "gen2." I suggest using the 5MHz generator as gen1 and the 2MHz as gen2.

You can get a good idea of the central frequency of the the notch filter using the following formula.

$$f_0 = 1/(2\pi\sqrt{LC}) \tag{2.5}$$

Q. What value does this give you for the central frequency?

Read the following steps very carefully and do exactly as they say.

- Set gen1 to a triangle wave and the frequency to between 50 and 100 Hz. This function generator tends to act up, if it's behaving strangely try turning it off and back on.

- Via a coax cable, connect the output of gen1 to channel 1 on the scope. Be sure channel 1 is in 1X mode with DC coupling. Turn off averaging.

- Adjust the peak to peak voltage of the triangle wave to 10 V. Use the DC offset knob to have the signal vary from 0 to 10 V instead of -5 to +5 V. Now that the voltage parameters are set properly, drop the frequency down to roughly 1 Hz.

- Use a second coax cable and a T-splitter to send the ramp signal to the VCF input of gen2 in addition to channel 1 on the scope. Check that the parameters are still as set previously.

- Connect the output of gen2 to the filter and the output of the filter to channel 2 on the scope just as in the previous section. Channel 2 should be still in X10 mode.

- Set the course frequency knob on gen2 to maximum and select the 100k range. As the voltage into the VCF input goes from 0 to 10 V, the frequency will go from the f_{max} to approximately $f_{max}/100$.

- You should see some fairly crazy signals on the scope. Go to the DISPLAY menu and change the format from YT to XY and increase the persistence to 5 seconds. Adjust the signal using the zoom and scroll knobs until it is close to filling the screen, but isn't cut off on any edge.

- Q. What is being plotted in XY mode on each axis?

- Q. Take a picture of the scope output to include in your report (setting persistence in to infinite will make for a better picture).

In this mode you can't make any direct measurements using the scope. Instead, remove the T-splitter from gen1 and connect it to your DC power supply (set to 0 Volts). Now you can slowly vary the frequency of gen2 using the DC supply. (Stay under 10 volts.)

- Q. Use this method to approximate the location of the center of the notch. Compare this to your calculation from earlier.

2.5   Band Pass Filter

Changing the current setup to a band pass filter is as simple as switching the input and the ground locations so that the scope is measuring the voltage across the capacitor/inductor combo instead of the resistor.

- Modify the circuit as described above and setup the sweep as you had before.

- Q. Take a picture to include in your report.

- Q. Once again, measure the center location using your DC power supply. Where is the center of the peak?