Lab 0: Are your results meaningful?

1 Theory

1.1 Why do we care?

The result of an experiment is often almost, if not entirely, meaningless without an understanding of the uncertainty associated with that result. If I tell you that I measured two objects to be 13.2 meters and 13.25 meters in length then you might think those objects are very similar in size. But if I then tell you that my measuring was done in a manner that gave each value an uncertainty of ± 3 meters, you would realize you can assume no such thing.

Note: Remember that in the context of statistics "uncertainty" and "error" are exactly the same thing. A "statistical error" is NOT a mistake.

You should be familiar with the terms "relative" and "absolute" uncertainty. The absolute uncertainty is simply the plus or minus range, like the example above. Relative uncertainty is the absolute uncertainty divided by the value. So if the absolute is $\pm\Delta x$ then the relative is $\Delta x/x$. To get the percent, just multiply this by 100%. Relative uncertainty is often useful when dealing with number ranges outside of our intuitive experience. For example, let's say I measured the distance to the moon with an absolute uncertainty of ± 3000 km. Is that good or not? It turns out that would be about a 0.7% error.

There is another value often used and that is the "percent difference" or "percent change" between two values. This should not be confused with percent uncertainty and ought to be used with caution. Again, think about the first example which would have a percent difference of $|3.2 - 3.25|/3.2 * 100 =$ 1.6% which seems very small, but we know that number is irrelevant since our absolute uncertainty is so large. One place where percent difference (or percent change) is useful is when reporting stock market changes. Saying the DOW was down 13 points doesn't mean much if you don't know the starting value. However if I say the DOW dropped 5% then you would know that was a fairly significant drop. This works well since stock market prices have no uncertainty. In general we would only want to use percent difference when the absolute uncertainty is much smaller than the difference between the two numbers.

1.1.1 Assumptions

You've all had it beaten into you that "everything has uncertainty," but figuring out how to account for that uncertainty in real life can get tricky. We're going to cover just the most basic knowledge you need to effectively produce an answer at the end of a experiment which has the appropriate error bars. In the vein of simplification we are going to make two assumptions which will serve us well in most cases. (If you want to know why these are helpful, go read a statistics book.)

1. All input variables are *independent*.

2. All uncertainties follow a Gaussian distribution.

1.1.2 Where does uncertainty come from?

Often one of the first quantitative things students learn about uncertainty is the idea of analog and digital precision, often stated as

1. Digital "uncertainty" is ± 1 in the smallest digit. (Rarely useful in practice.)
2. Analog "uncertainty" is ± 0.5 of the smallest division. (Occasionally useful in practice.)

These rules are mostly useless. The first thing one should realize about these rules is that they are a MINIMUM level of uncertainty. A digital display is *incapable* of displaying a number with any greater precision, this is the *resolution* of the display.[1] HOWEVER, there are many things that can increase this uncertainty! So, what are these things?

One reason that the uncertainty would be higher than this minimum level is how the instrument is being used or what is being measured. Imagine measuring the radius of a soccer ball by using a meter stick and trying to line up the markings by eye. This would clearly have a vastly larger uncertainty than the ± 0.5mm analog precision of the meter stick. A more subtle example might be using a vernier scale caliper to measure a rough surface where the unevenness of the surface is on the order of (or larger than) the uncertainty of the caliper.

There are a whole host of ways uncertainty can creep into our measurements (such as analog to digital conversion, temperature dependence, component specs, etc). Fortunately, much of the time we don't have to worry about those details because instruments such as these always have manuals with uncertainty specs which we can use. The answer to the question "what is the uncertainty of my measurement?" will nearly always be "**READ THE MANUAL**."

Think of the uncertainty as the range of numbers that you are sure the "true" value falls between.

Examples

- In one experiment I was measuring the change in pressure of a chamber with a mercury barometer by measuring the height of the column of mercury with a meter stick. We know from our analog uncertainty rule that the minimum uncertainty would be $\pm 0.5mm$. However, the vacuum was changing fairly rapidly and so I had to estimate where the top of the column was while it was moving. For the uncertainty, I looked at it and determined what two points I was *sure* the height of the mercury column was between and I decided I was confident that at a given time point the height was in a range of about $4mm$, or an uncertainty of $\pm 2mm$.
- Something many of you have experienced (or will experience) in lab is having the value your multimeter displays fluctuate over time. In this case I would base my uncertainty on the range of variation. Let's say after looking at the readout for long enough to get a good sense of the fluctuations, the lowest number you have seen is 1.382 Amps and the highest is 1.397 Amps, then a good approximation of the uncertainty would be $(1.397 - 1.281)/2 = \pm .008$ Amps

Caveat: There may be multiple sources of uncertainty and one must always use largest. So say your multimeter was fluctuating as in the example above, but from looking at the manual you have

[1] Often DMMs and the like will be specified as being $3\frac{1}{2}$ digits or similar, don't confuse this with with the resolution. For more info see www.edn.com/electronics-news/4389451/What-s-a-half-digit-anyway

determined that the measurement uncertainty is $\pm .01 Amps$, then that would override the $\pm .008$ Amps.

1.2 The Math

Now we know how to determine the uncertainty of a measurement, but how do we translate those into uncertainties for our final calculated value? This is where *propagation of uncertainty* comes into play. The most general equation for uncertainty propagation is the total partial differential with respect to all variables which contain uncertainty. So for a value F which is a function of multiple variables, $F = f(x \pm \Delta x, y \pm \Delta y, z \pm \Delta z, \ldots)$, then the uncertainty of F is given by:

$$(\Delta F)^2 = \left(\frac{\partial F}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial F}{\partial y}\right)^2 (\Delta y)^2 + \left(\frac{\partial F}{\partial z}\right)^2 (\Delta z)^2 + \ldots \tag{1.1}$$

This allows us to propagate uncertainty through any differentiable equation. It simplifies down for basic arithmetic operations to a few equations you may be more familiar with.

For addition and subtraction

$$F(x, y) = (x \pm \Delta x) \pm (y \pm \Delta y) \qquad \Delta F = \sqrt{(\Delta x)^2 + (\Delta y)^2} \tag{1.2}$$

For multiplication and division

$$F(x, y) = (x \pm \Delta x) \times (y \pm \Delta y)^{\pm 1} \qquad \Delta F = F \times \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \tag{1.3}$$

For exponents (with uncertainty in the base)

$$F(x) = (x \pm \Delta x)^n \qquad \Delta F = F \times n \times \left(\frac{\Delta x}{x}\right) \tag{1.4}$$

2 Application

Question 1

In your lab report derive the formula for multiplicative uncertainty (1.3) (That is, ΔF when $F = x \times y$) using the general uncertainty equation (1.1).

Question 2

If a resistor has the color bands brown-black-red-gold this translates to $10 \times 10^2\ \Omega \pm 5\%$ (a pretty standard $1k\Omega$ resistor).

- Based on the color bands what is the absolute uncertainty of this resistor?
- What would be the resulting value and uncertainty of a series combination of two such resistors?
- What would be the resulting value and uncertainty of a parallel combination of two such resistors?

Question 3

It is often more precise to measure a component's value (such as a resistor) with a multimeter than to go off the uncertainty marked on the component. Use a multimeter to measure the resistance of 5 different $1k\Omega$ resistors. The uncertainty in the multimeter's measurement can be found in the multimeter's manual.

- Record the values you measured, and the absolute uncertainty in each value based on the uncertainty you found in the manual.
- Calculate the average resistance and uncertainty. *Be sure to propagate the uncertainty of each value through the average equation!*