Lab 3: RC Circuits

Prelab

Deriving equations for the output voltage of the voltage dividers you constructed in lab 2 was fairly simple. Now we want to derive an equation for the output voltage of a circuit with both resistance and reactance, which will require a brief foray into the field of complex analysis.

Watch the Lab 3 video demonstrating the derivation for the output voltage of the circuit shown in figure 2 (On moodle or `www.youtube.com/watch?v=nAsnk1Yj4u8`). *Now derive the output voltage for the circuit shown in figure 3.*

In each case, what are the phase angle and magnitude of V_{out} (relative to V_in) when $\omega = 1/RC$ ($f = 1/(2\pi RC)$)?

Construct circuit 2 in EveryCircuit. Set values for the capacitor and resistor to match those in figure 2 and set the frequency to $1/(2\pi RC)$. Select the input and output voltage nodes (wires) and in each case click the "eye" icon to show oscilloscope plots. Click on the "t" to start the analysis on the clock icon to zoom on the time axis.

Export an image of the circuit to include in your lab report. Does the phase shift between the voltages in the simulation agree with the one you found above?

Part I: Capacitor Charging and Discharging

When charging or discharging a capacitor, the voltage difference across it has an exponential dependence on time, as shown in equations (1.1) and (1.2). The time constant τ is equal to the product of the series resistance and capacitance, ($\tau = RC$).

$$V_{out}(t) = V_{max}(1 - e^{-t/\tau}) \quad \text{(Charging)} \tag{1.1}$$

$$V_{out}(t) = V_{max}(e^{-t/\tau}) \quad \text{(Discharging)} \tag{1.2}$$

A charging capacitor ends at V_{max} and a discharging capacitor starts at V_{max}. *In each case, after a period of time equal to the time constant (that is $t = \tau$), what percent of V_{max} will the capacitor voltage be at?*

How many times constants will have to pass for a charging capacitor's voltage to be above 99% of the max voltage?

You are going to use your scope to measure the time constant for the circuit in figure 1.

1. Build the circuit shown in figure 1, drive it with a 7-8 kHz square wave.
2. Reset your scope by hitting the "DEFAULT SETUP" button

3. Be sure both the probes and each scope channel are in 10X mode
4. Connect channel 1 to view the square wave and channel 2 to view the capacitor voltage
5. To clean up the signal, set ACQUIRE to average 4 or 16 samples
6. Move the signals so that the "1" arrow and "2" arrow are both vertically centered on the screen
7. Adjust the zoom and position settings until you are nearly filling the screen with the charging cycle
8. Set the cursor to voltage mode and find delta V between the top and bottom. Using your calculations from above, determine what the delta V should be after 1 time constant and move the top cursor to that level.
9. Move the signals horizontally until the top cursor crossing point aligns with the center axis of the grid.
10. Switch cursors to time mode and measure the amount of time from when the signal begins to rise until it reaches the voltage level you determined.
11. Repeat this procedure for the discharge cycle.

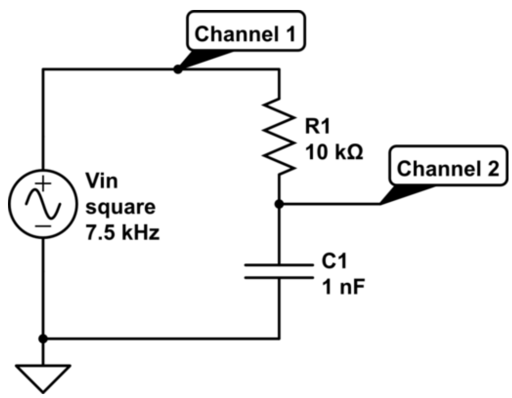

Figure 1: A voltage divider circuit with a load

Use your multimeter to find more exact values for the resistor and capacitor you used. *What value do you calculate for the time constant?*

What time constant did you measure on the charge cycle? On the discharge cycle?

Take a picture of a full charge and discharge cycle to include in your report.

Part II: Capacitor Voltage

Now that we've examined the step response of the RC circuit we are going to examine the response of the capacitor voltage to a sine wave.

2.1 AC analysis of an RC circuit

1. Switch your function generator to output a sine wave with the same frequency as in the previous section. You should see two sine waves (input and output) with a phase shift
2. *Using the equations from the prelab, calculate what this phase shift should be (in degrees)*
3. Convert the phase shift to a fraction of the period. Use the MEASURE menu to find the period of the signal and then *calculate the corresponding Δt between the waveforms due to this phase shift.*
4. *Using the cursors, measure the time shift between the input and output waveforms and compare to the value you calculated.*

2.2 RC circuit as an integrator

For positive arguments, inverse tangent varies from zero to 90 degrees. From the phase angle equation for the voltage across the capacitor, we can see that this means the angle of the output voltage can vary from -90 degrees to zero degrees. When the output has a phase angle close to -90 degrees, the output is very nearly the integral of the input (multiplied by an amplitude factor).

In terms of the frequency $f_0 = 1/(2\pi RC)$, what frequency range will give a phase shift of at least -89 degrees? (Hint: the tangent term would need to equal what? Now solve that for frequency.) (Example: A frequency of $x * f_0$ or lower/higher.)

At this frequency, what will the amplitude of the output be (as a fraction of V_{in})? How does the amplitude depend on frequency?

1. Build the circuit shown in figure 2. Plug in R and C to what you just did and use the result as your frequency (once again use your multimeter to obtain more accurate values for R and C)
2. Set each scope channel to AC coupling (this strips off DC offsets which sometimes creep into the signal)
3. Set the output amplitude of the function generator to maximum

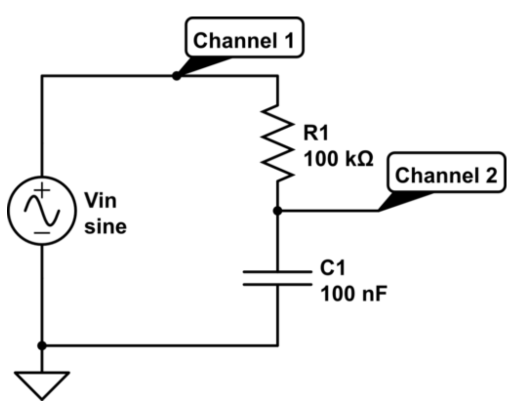

Figure 2: An RC circuit which outputs the integral of the input.

If you did everything correctly, the voltages should have a phase difference of 89 degrees with the output lagging the input.

Measure the Δt between the voltage waveforms with the cursors, and the period with the MEASURE menu. Convert Δt to degrees. Did you get a reasonable result?

Measure the peak to peak voltage of the input and output wavesforms. Does the output amplitude agree with what you predicted above?

I claimed that this would form a decent integrator. Let's test that.

Take pictures or sketch the output for sine, square, and triangle waves. What is the integral of a sine wave, a square wave, and a triangle wave? (Think about what functions describe the parts of the later two). Do your outputs make sense as the integral of the input?

Part III: Resistor Voltage

Now that we've looked at the voltage across the capacitor in an RC circuit, we are going to examine the voltage across the resistor.

3.1 RC circuit as a differentiator

We once again want a phase angle near 90 degrees, but this time it will be near positive 90 degrees.

In terms of the frequency $f_0 = 1/(2\pi RC)$, what frequency range will give a phase shift of at least +89 degrees? At this frequency, what will the amplitude of the output be (as a fraction of V_{in})?

Set the function generator to this frequency and follow a procedure similar to that in part II.

Determine the experimental phase angle and output voltage peak to peak amplitude. Compare your results to your predictions. How does the output amplitude depend on frequency?

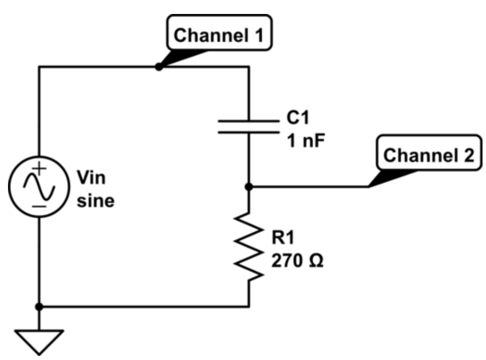

Figure 3: An RC circuit which output the derivative of the input.

Examine the output of each wave type at this frequency. *Take a picture of, or sketch, each output. What is the derivative of a sine wave, a triangle wave, and a square wave? Do your outputs make sense as the derivative of the input waveform?*