Doubts :

1. $\big(|v\rangle\langle w|\big)|v\rangle$

$\Rightarrow \quad |v\rangle \langle w|v\rangle$

(As we know $\langle w|v\rangle$ is a complex number (let it be c))

$\Rightarrow \quad |v\rangle \times c$

$\Rightarrow \quad c\,|v\rangle$

(In scalar & vector multiplication order doesn't matter)

2. As we know,

$$\langle \psi|\phi\rangle^* = \langle \phi|\psi\rangle$$

$$\therefore \langle \psi|A\psi\rangle^* = \langle A\psi|\psi\rangle$$

(Here, $\phi = A|\psi\rangle$)

This exchange of arguments of inner product is a inherent property of inner product & is valid even when A may not be hermitian.

I assumed hermitian A to prove the following :

$$\langle \psi|A\psi\rangle^* = \langle A\psi|\psi\rangle$$

$$= \langle \psi|A^\dagger\psi\rangle$$

$$= \langle \psi|A\psi\rangle$$

$$\therefore \langle \psi|A\psi\rangle^* = \langle \psi|A\psi\rangle$$

$\therefore$ It is real (can be used to prove A has real eigenvalue)