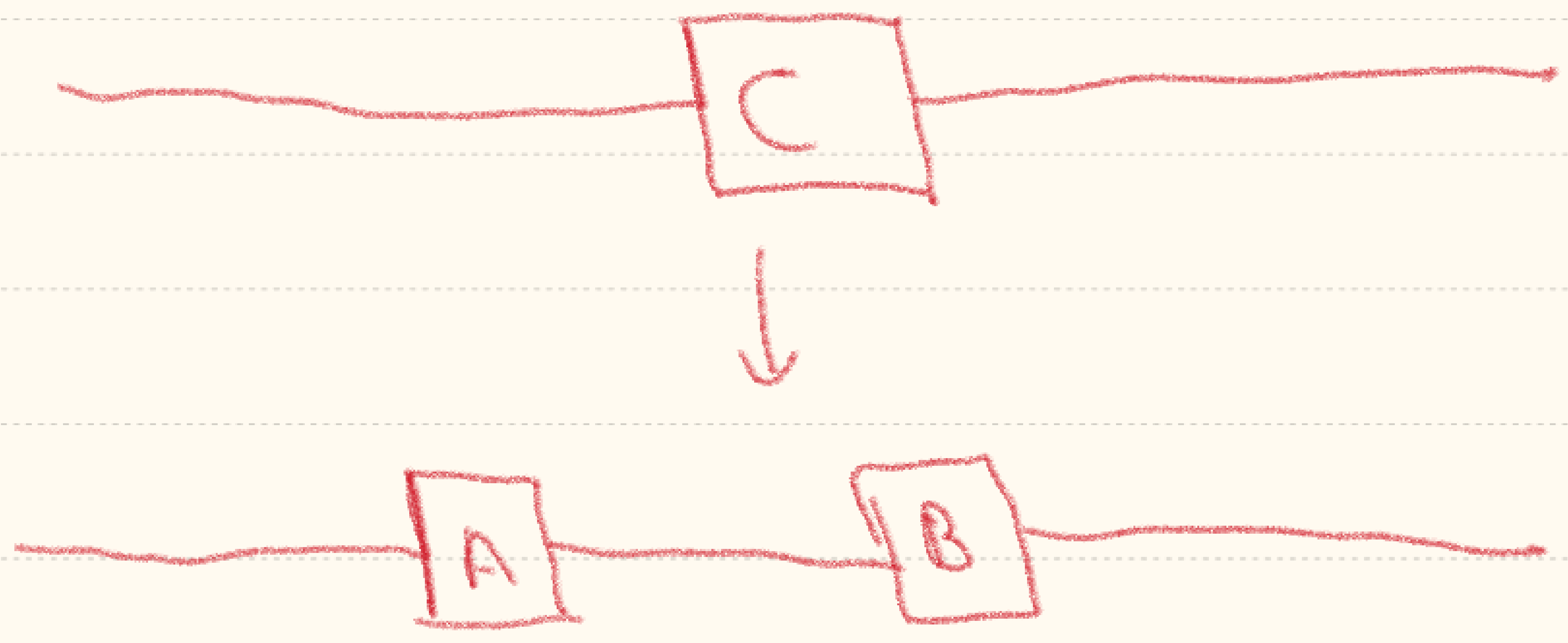
To check if any circuit decomposition is correct you can use 2 methods:

Suppose we are trying to implement an n (say, $n=1$ here) qubit gate C using its decomposition, i.e.



To check if its correct you can:

(Easier method & faster in some cases but not the best for beginners as using analytical method will help you learn more about gate algebra)

(i) See the output for all the 2^n basis vectors, if each gives the expected output then the circuit is correct as any state is a linear combination of these basis vectors & any gate is a linear operator.

(Analytical method, faster in some cases)

(ii) Calculate if $BA = C$ analytically.

For your question, the decomposition contains CNOT, H, T & S gate

Now, for 3 qubit circuit

$$CNOT_{1\to 1} \equiv |0\rangle\langle 0| \otimes I \otimes I + |1\rangle\langle 1| \otimes X \otimes I$$

$$\equiv |0\rangle\langle 0| \otimes \left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) \otimes \left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$|1\rangle\langle 1| \otimes \left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) \otimes \left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

(Try to reason why this is correct)

$$H_3 = I \otimes I \otimes \left(|+\rangle\langle 0| + |-\rangle\langle 1|\right)$$

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix} = |0\rangle\langle 0| \otimes I \otimes I + e^{i\frac{\pi}{4}} |1\rangle\langle 1| \otimes I \otimes I$$

$$S_2 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \equiv I \otimes \left(|0\rangle\langle 0| + i|1\rangle\langle 1|\right) \otimes I$$

The subscript A_n denotes A is acting only on n^{th} qubit. I have given representation for specific n's, adjust the repre -- as n changes yourself

Now that you know outer product representation of the required gates,

Do the $BA = C$ method & see if the the decomposition is correct.