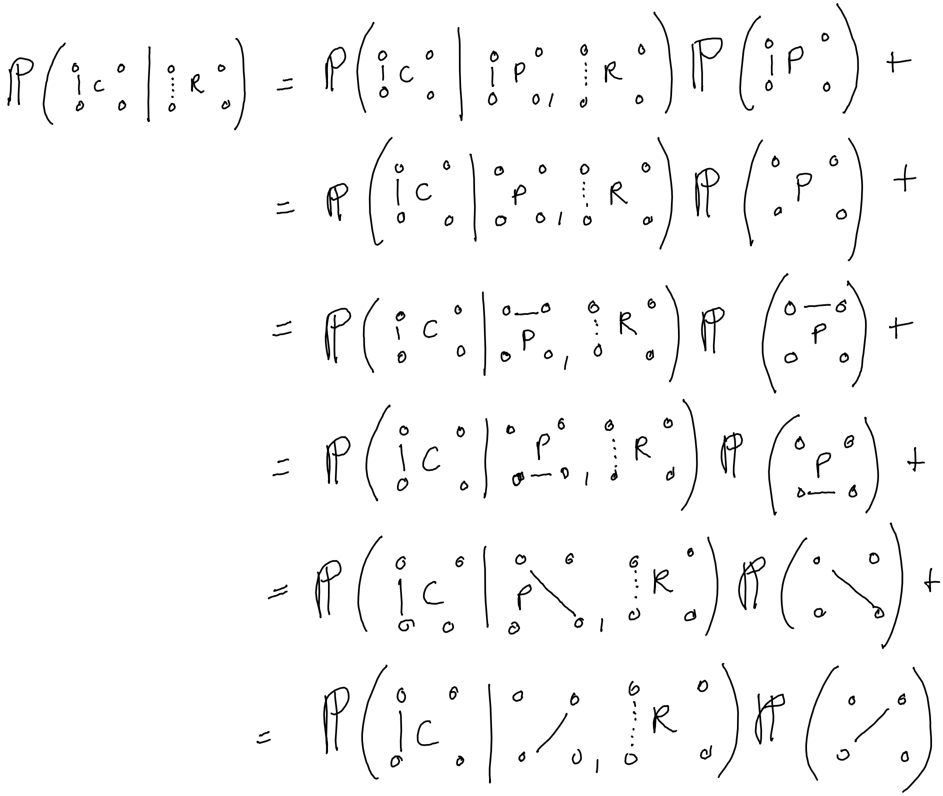
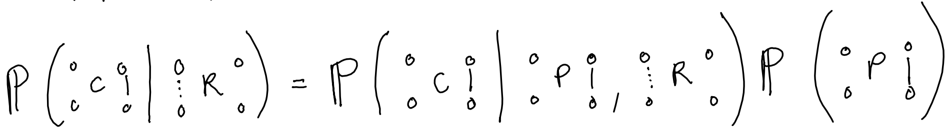
Fri 18th Feb **Start of RG**



$$= 1 \times f(1-f)(1-2f) \; +$$
$$\tfrac{1}{2} \times (1-f)(1-2f)(1-3f) \; +$$
$$4 \times \tfrac{1}{4} \times f(1-f)(1-2f)$$

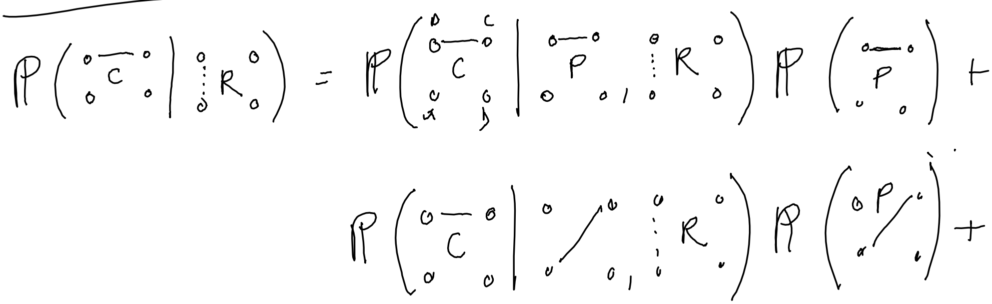
Fri 18th Feb



u y v y

v y u y

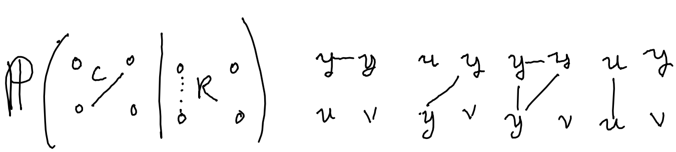
$$= \tfrac{1}{2} f(1-f)(1-2f)$$

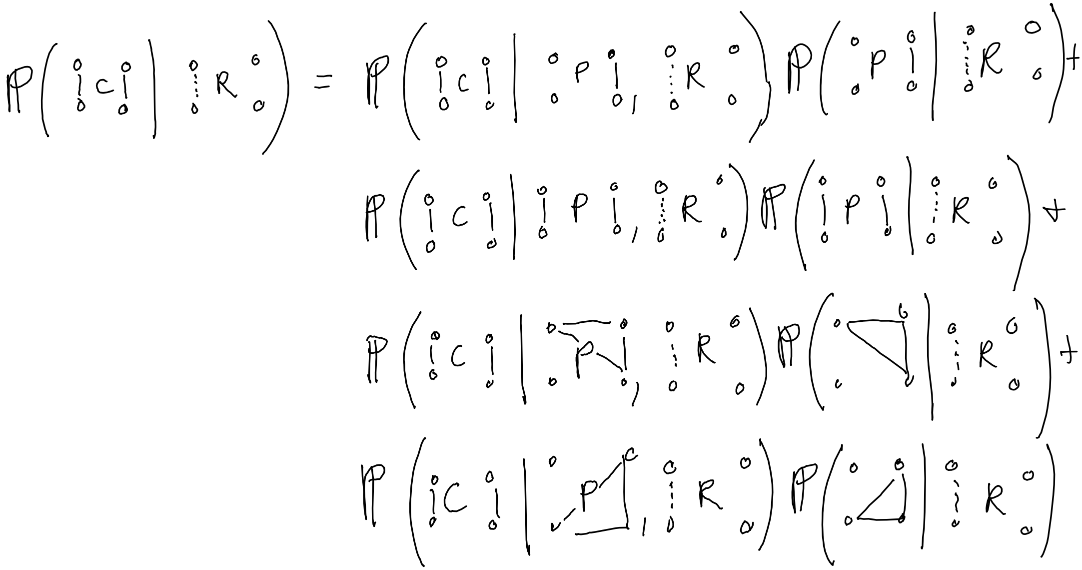


$$= \tfrac{1}{4} f(1-f)(1-2f) +$$
$$\tfrac{1}{4} f(1-f)(1-2f)$$

y y $\quad$ g_2 y y

u v $\quad$ g_1 u v





$$= \tfrac{1}{2} f(1-f)(1-2f) + 1\, f^2(1-f) + \tfrac{1}{4}(1-f)f^2 + \tfrac{1}{4}(1-f)f^2$$

$$= \tfrac{1}{2} f(1-f)(1-2f) + \tfrac{3}{2} f^2(1-f)$$

$$\mathbb{P}(C \mid R) = \mathbb{P}(C \mid P, R)\, \mathbb{P}(P)$$

$$= 1 \times f^2(1-f)$$

$$\mathbb{P}(C \mid R) = \mathbb{P}(C \mid P, R)\, \mathbb{P}(P \mid R) +$$

$$\mathbb{P}(C \mid P, R)\, \mathbb{P}(\ \mid R) +$$

$$\mathbb{P}(C \mid P, R)\, \mathbb{P}(P \mid R)$$

$$= \tfrac{1}{4} f(1-f)(1-2f) + \tfrac{1}{4} f(1-f)(1-2f)$$

$$1 \quad f^2(1-f)$$

$$\mathbb{P}(C \mid R) = \mathbb{P}(C \mid P, R)\, \mathbb{P}(P \mid R)$$

$$1 \qquad f^2(1-f)$$

$$\mathbb{P}(\ \mid R) = \mathbb{P}(\ \mid\ , R)\, \mathbb{P}(\ ) +$$

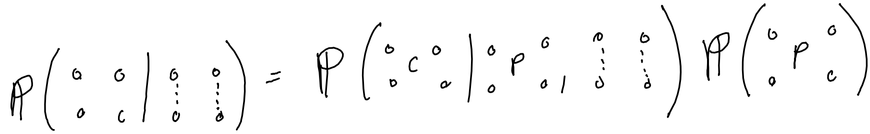
$$\mathbb{P}(\ \mid\ , R)\, \mathbb{P}(\ ) +$$

$$\mathbb{P}(\ \mid\ , R)\, \mathbb{P}(\ )$$

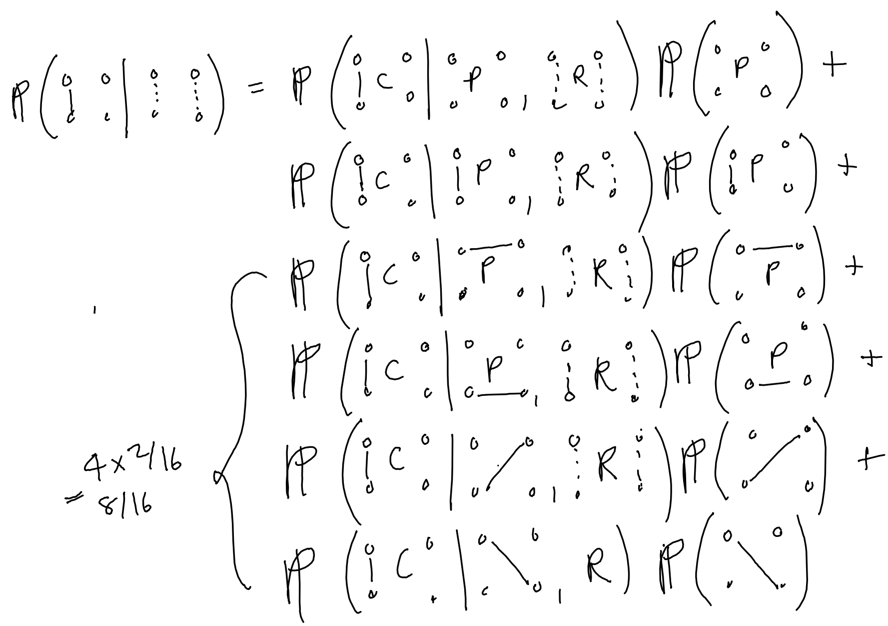
$$= f^3 + \tfrac{1}{4} f^2(1-f) + \tfrac{1}{4} f^2(1-f)$$

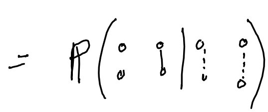
end of

Start of RG

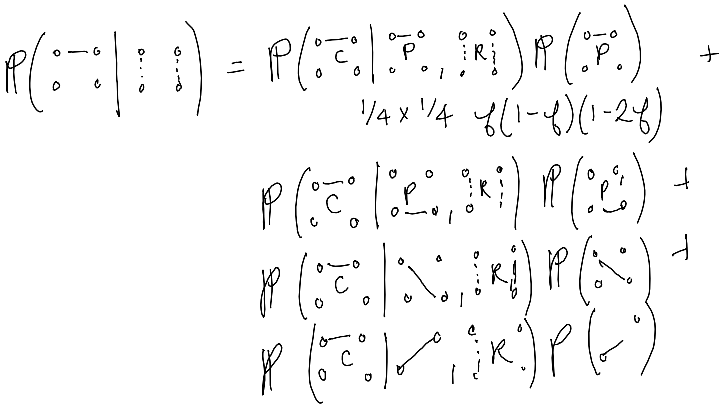


$$= \frac{1}{4}(1-f)(1-2f)(1-3f)$$



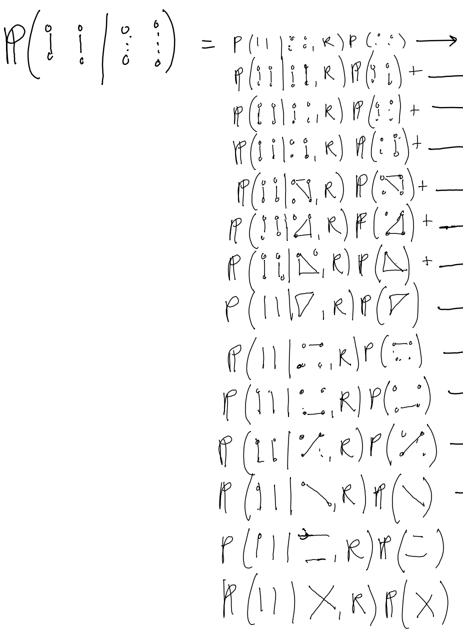


$$= \frac{1}{4}(1-f)(1-2f)(1-3f) + f(1-f)(1-2f)$$



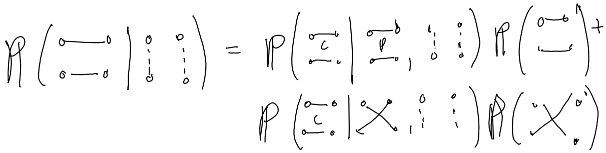
$$\frac{1}{4} \times \frac{1}{4}\, \gamma(1-\gamma)(1-2\gamma)$$

$$= \frac{4}{16}\, \gamma(1-\gamma)(1-2\gamma)$$



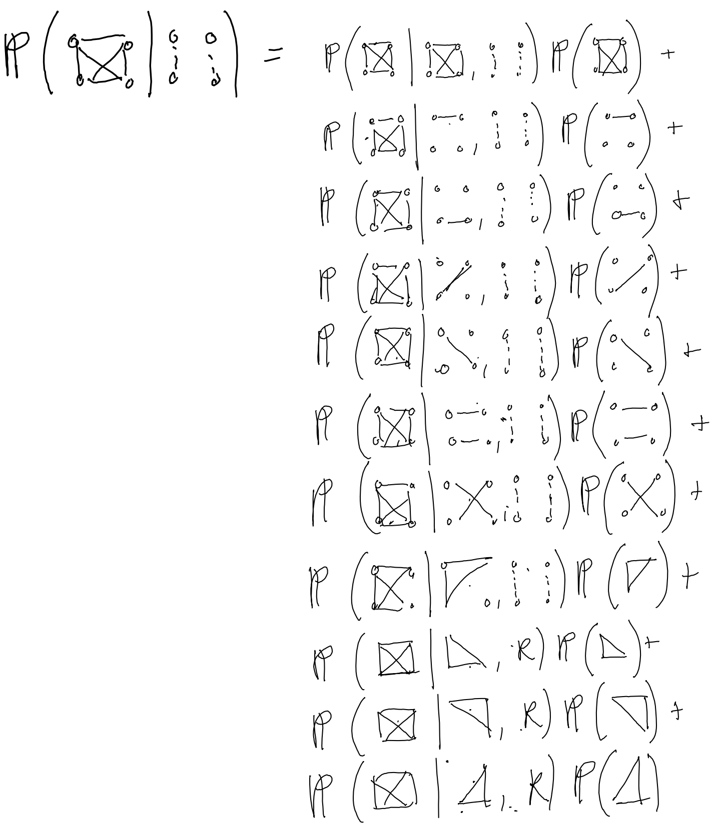
$$\to \frac{4}{16}\,(1-\gamma)(1-2\gamma)(1-3\gamma)$$
$$\to 1\ \gamma^2(1-\gamma)$$
$$\to \frac{1}{2}\ \gamma(1-\gamma)(1-2\gamma)$$
$$\to \frac{1}{2}\ \gamma(1-\gamma)(1-2\gamma)$$
$$\to \frac{1}{4}\ \gamma^2(1-\gamma)$$
$$\to \frac{1}{4}\ \gamma^2(1-\gamma)$$
$$\to \frac{1}{4}\ \gamma^2(1-\gamma)$$
$$\to \frac{1}{4}\ \gamma^2(1-\gamma)$$
$$\to \frac{3}{16}\ \gamma(1-\gamma)(1-2\gamma)$$
$$\to \frac{3}{16}\ \gamma(1-\gamma)(1-2\gamma)$$
$$\to \frac{3}{16}\ \gamma(1-\gamma)(1-2\gamma)$$
$$\to \frac{3}{16}\ \gamma(1-\gamma)(1-2\gamma)$$
$$\to \frac{1}{8}\ \gamma^2(1-\gamma)$$
$$\to \frac{1}{8}\ \gamma^2(1-\gamma)$$

$$\frac{1}{4}(1-\gamma)(1-2\gamma)(1-3\gamma) + \left(1+\frac{3}{4}\right)\gamma(1-\gamma)(1-2\gamma) + \left(2+\frac{1}{4}\right)\gamma^2(1-\gamma)$$



$$= 2\left(\tfrac{1}{8}\right) f^2(1-f)$$

$$= \tfrac{1}{4} f^2(1-f)$$



$$1 \times f^3 \; +$$

$$\tfrac{1}{16} \times f(1-f)(1-2f) \; +$$

$$\tfrac{1}{16} \times f(1-f)(1-2f) \; +$$

$$\tfrac{1}{16} \times f(1-f)(1-2f) \; +$$

$$\tfrac{1}{16} \times f(1-f)(1-2f) \; +$$

$$\tfrac{2}{16} \times f^2(1-f) \; +$$

$$\tfrac{2}{16} \times f^2(1-f) \; +$$

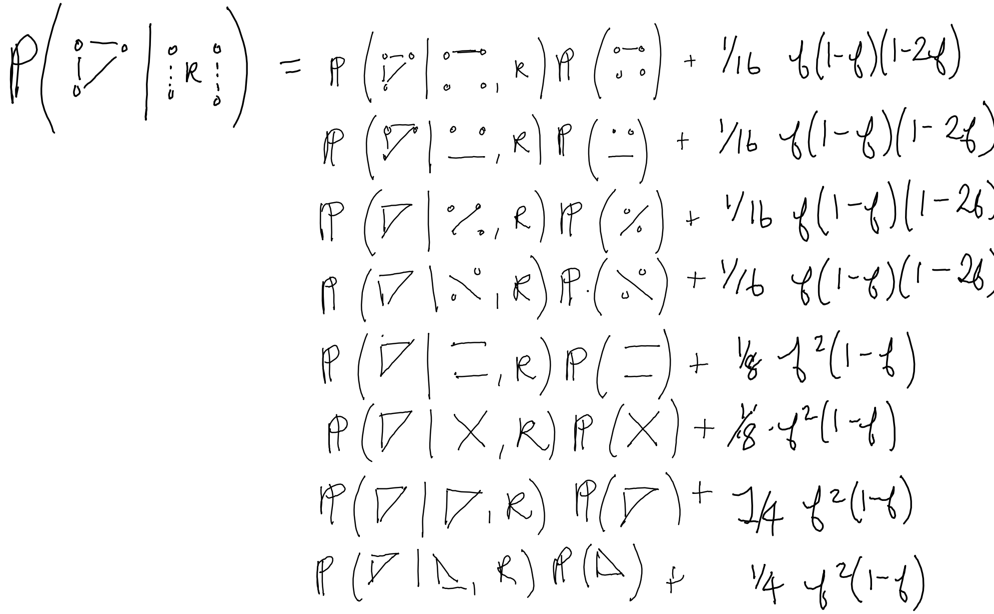
$$\tfrac{1}{4} \times f^2(1-f) \; +$$

$$\tfrac{1}{4} \times f^2(1-f) \; +$$

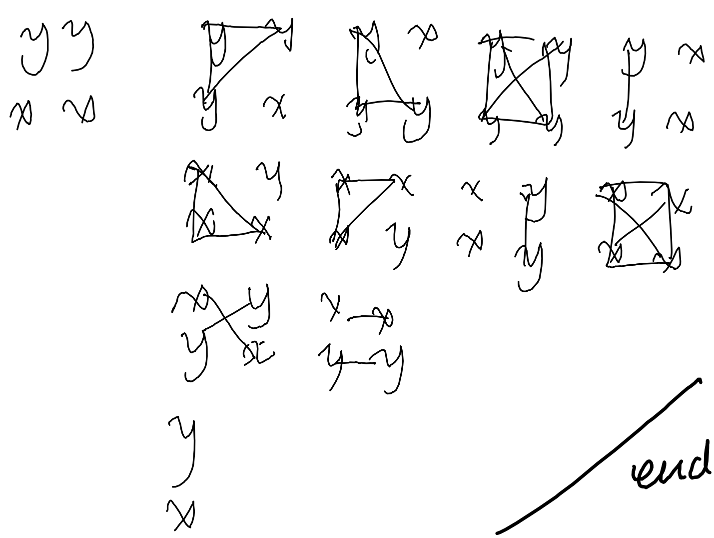
$$\tfrac{1}{4} \times f^2(1-f) \; +$$

$$\tfrac{1}{4} \times f^2(1-f)$$

$$= f^3 + \left(1 + \tfrac{1}{4}\right) f^2(1-f) + \tfrac{1}{4} f(1-f)(1-2f)$$

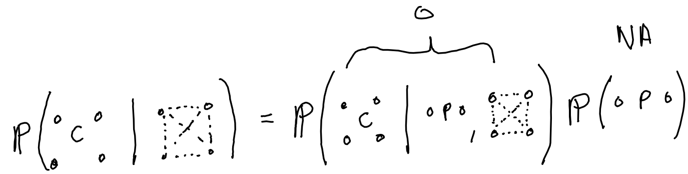


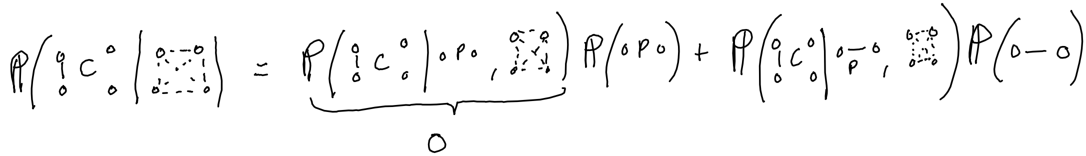
$$= 1/4\, f(1-f)(1-2f) + \tfrac{3}{4} f^2(1-f)$$

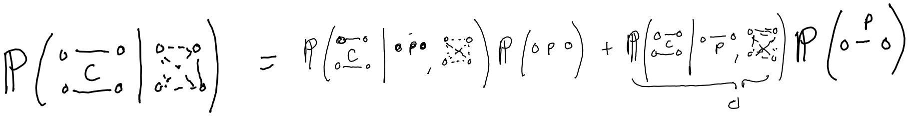


end of

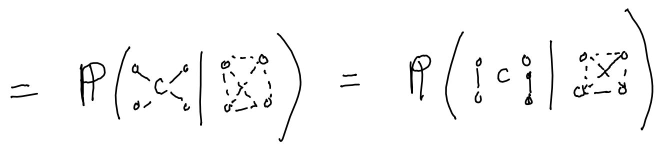
Parents Relationships $\rightarrow$ Children.

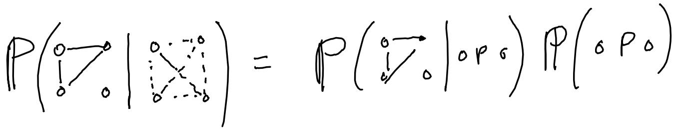




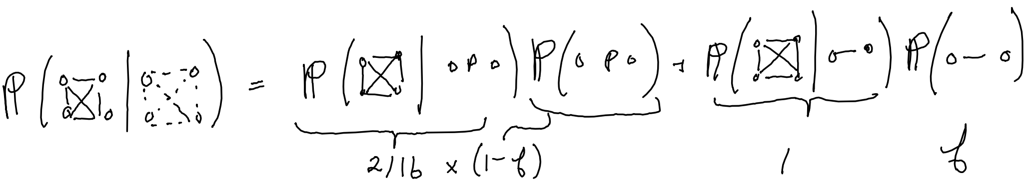


$= (1/8) \times (1-f)$



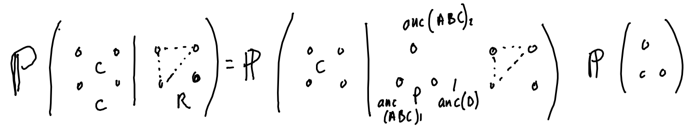


$= 2/16 \times (1-f)$



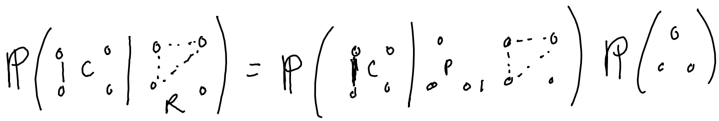
$= 1/8(1-f) + f$

end of 



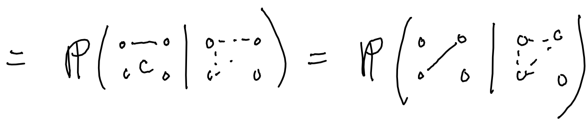
0

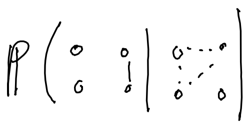
$(1-f)(1-2f)$



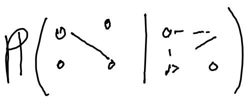
$= 2/8$

$(1-f)(1-2f)$

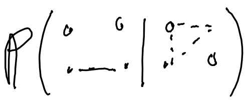




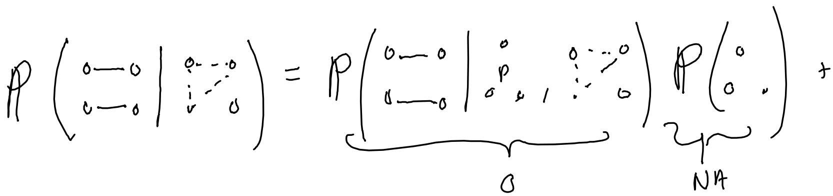
$= 0$

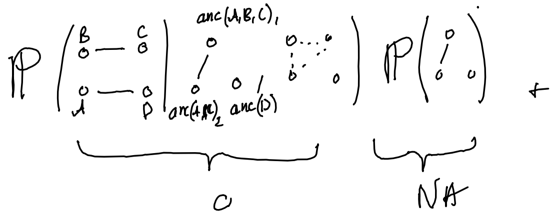


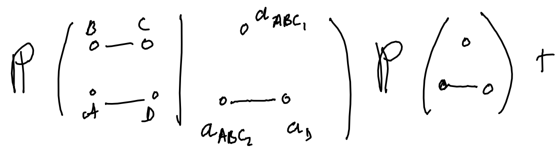
$= 0$



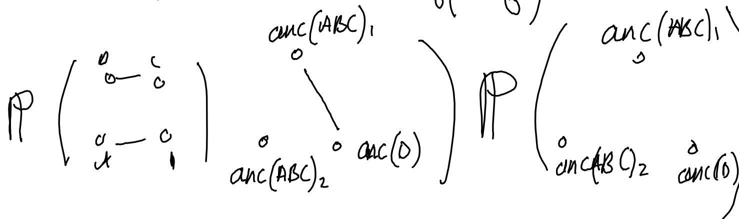
$= 0$



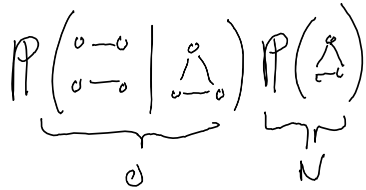




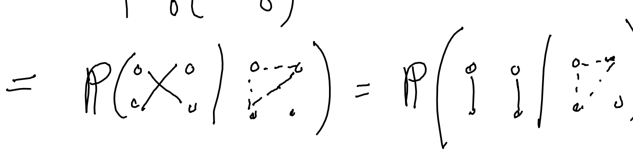
$1/8 \qquad f(1-f)$

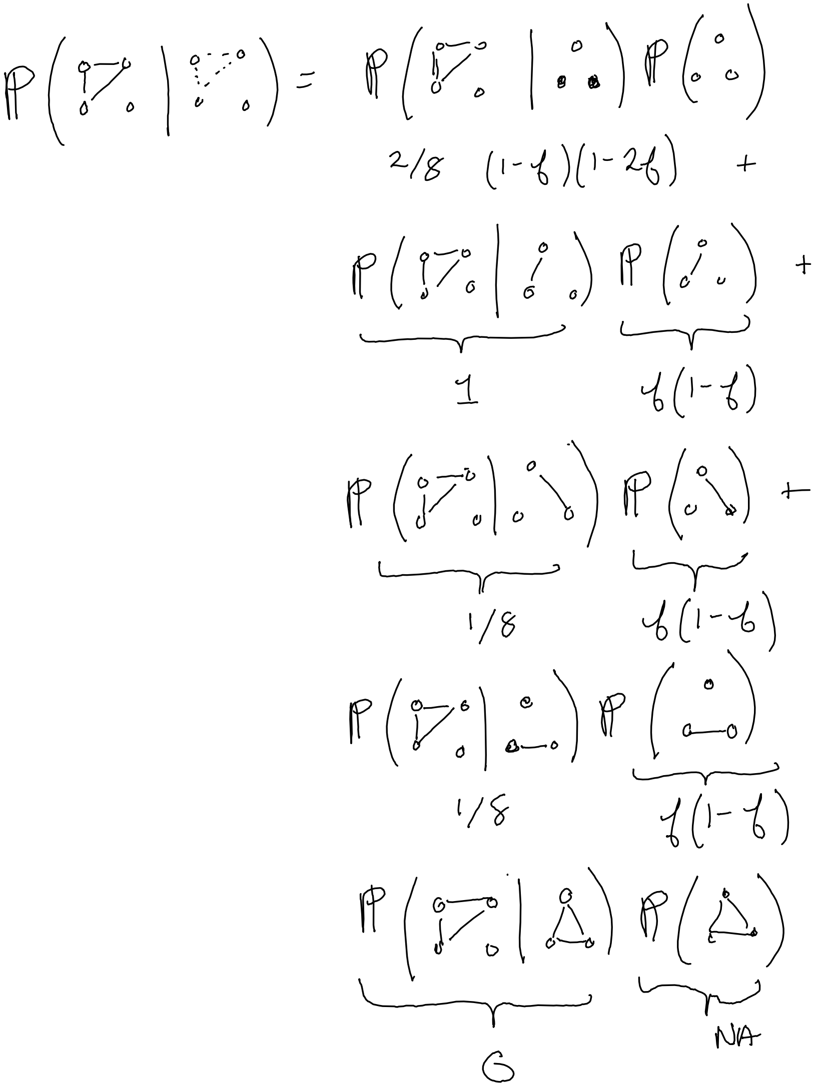


$1/8 \qquad f(1-f)$

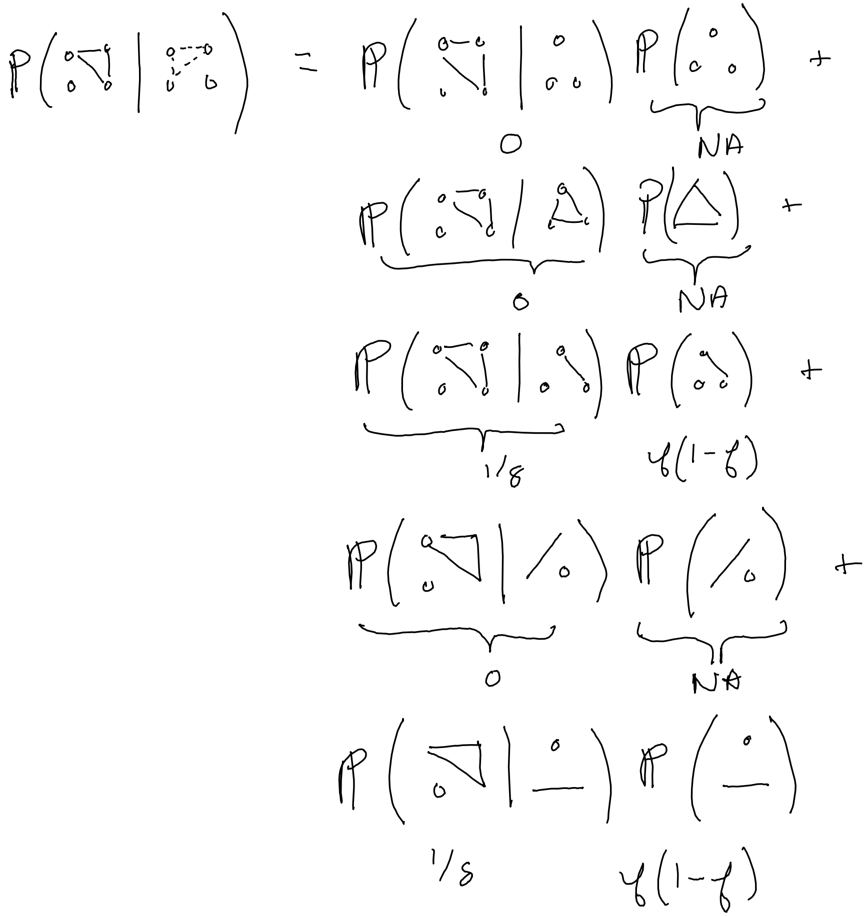


$= 1/4\, f(1-f)$

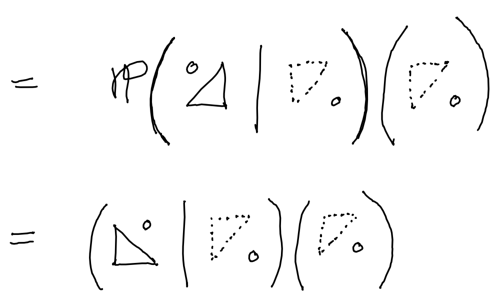


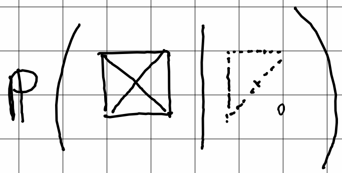


$$\left(1+\tfrac{1}{4}\right) f(1-f) + \tfrac{1}{4}(1-f)(1-2f)$$

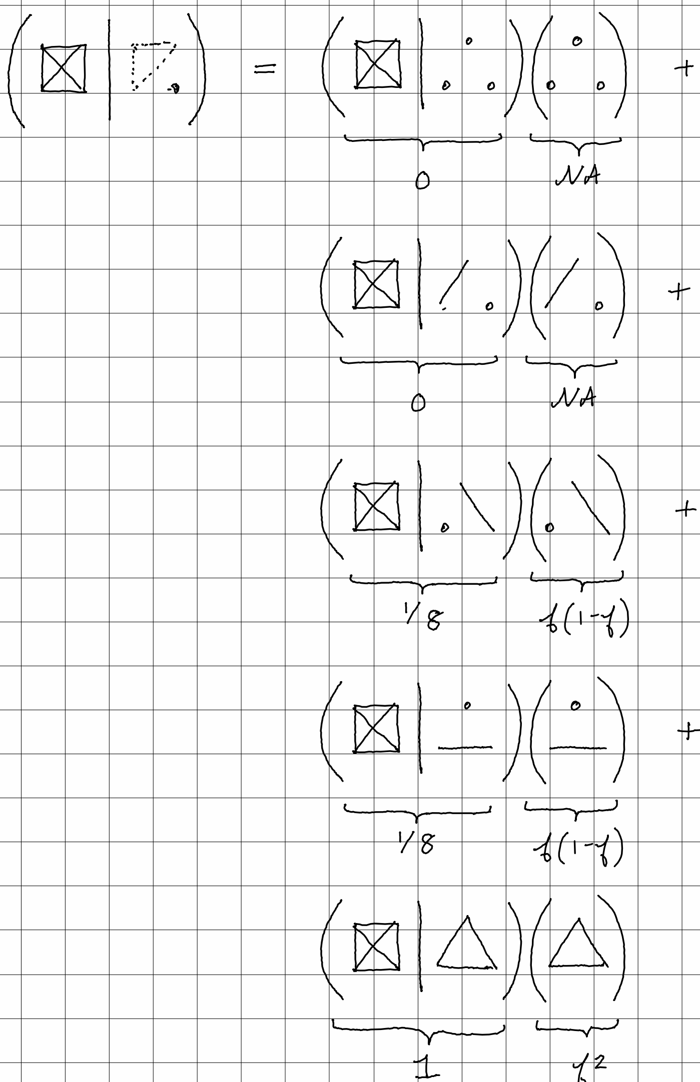


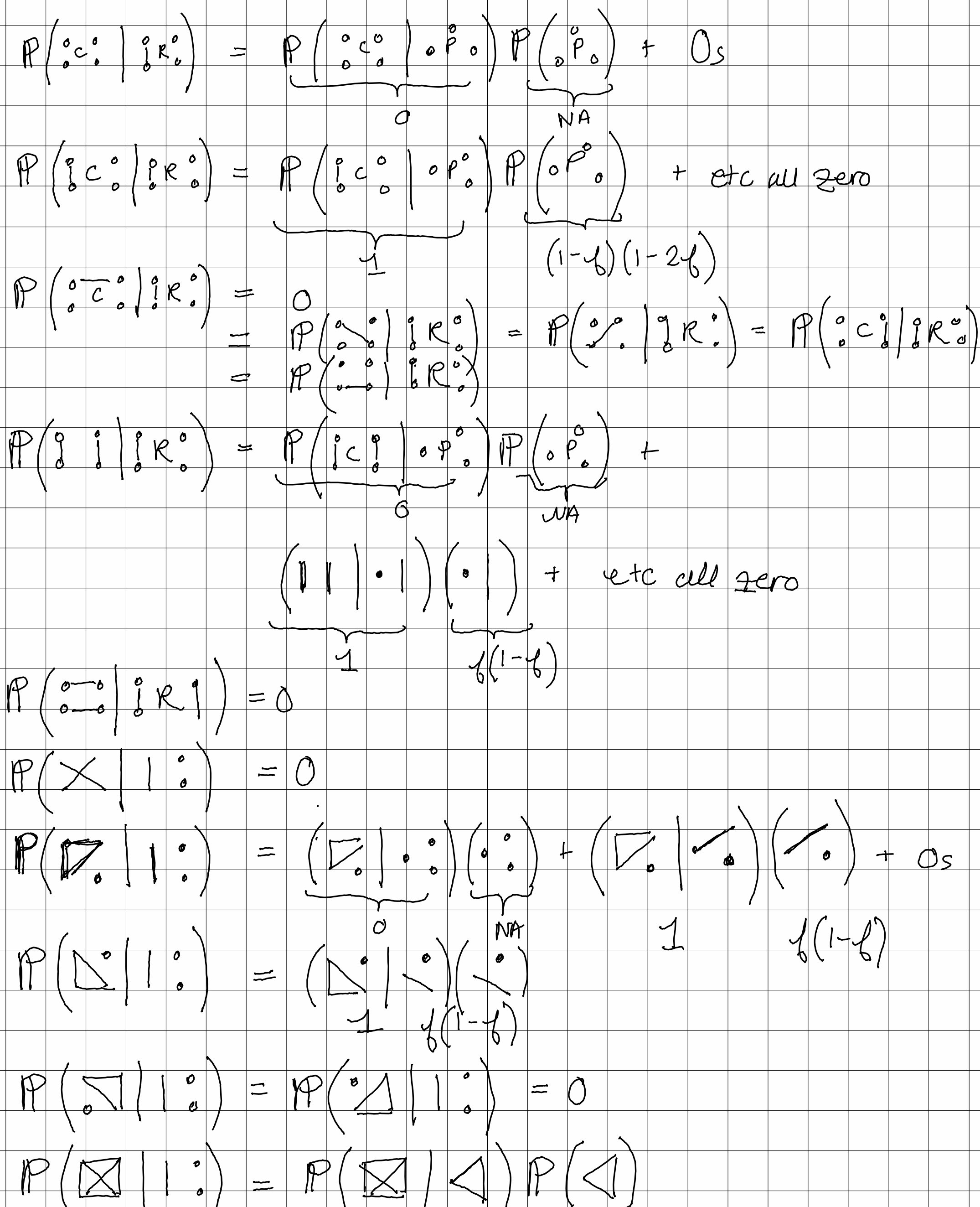
$= 1/4\, b(1-b)$



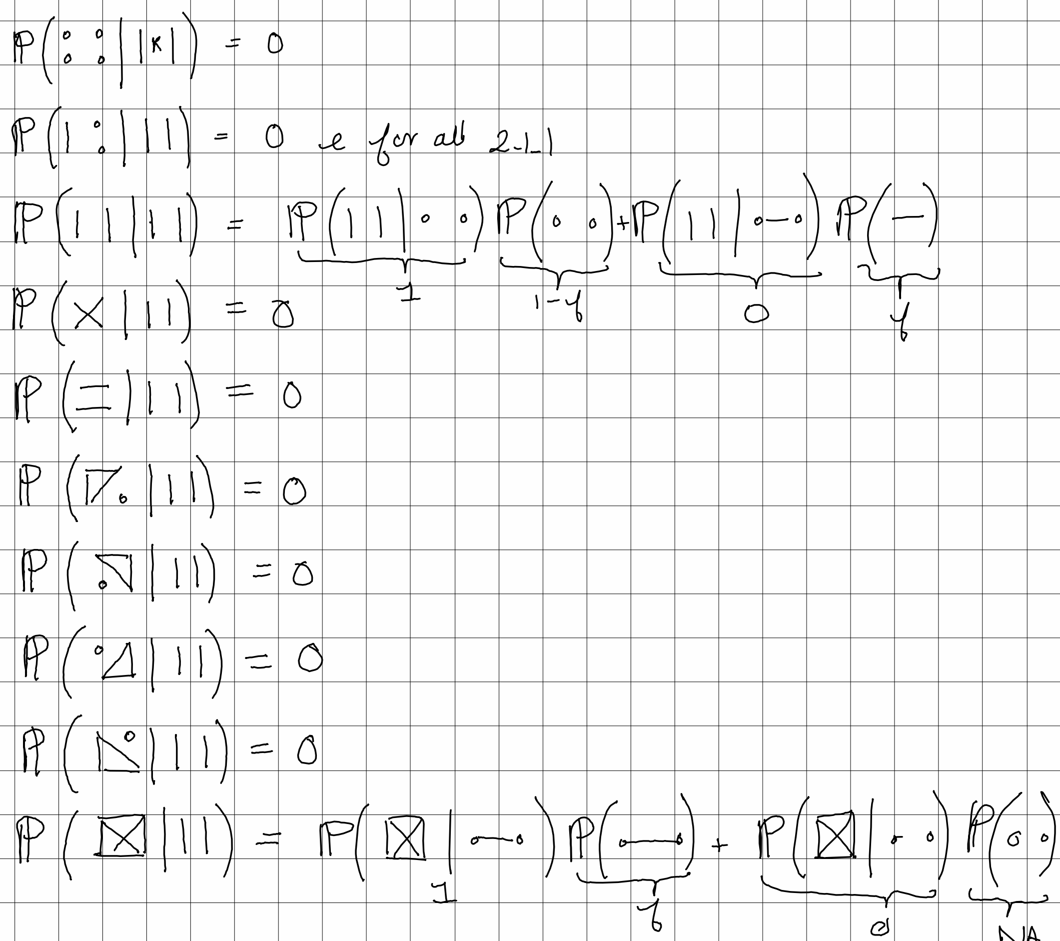


$$= \tfrac{1}{4} f(1-f) + f^2$$

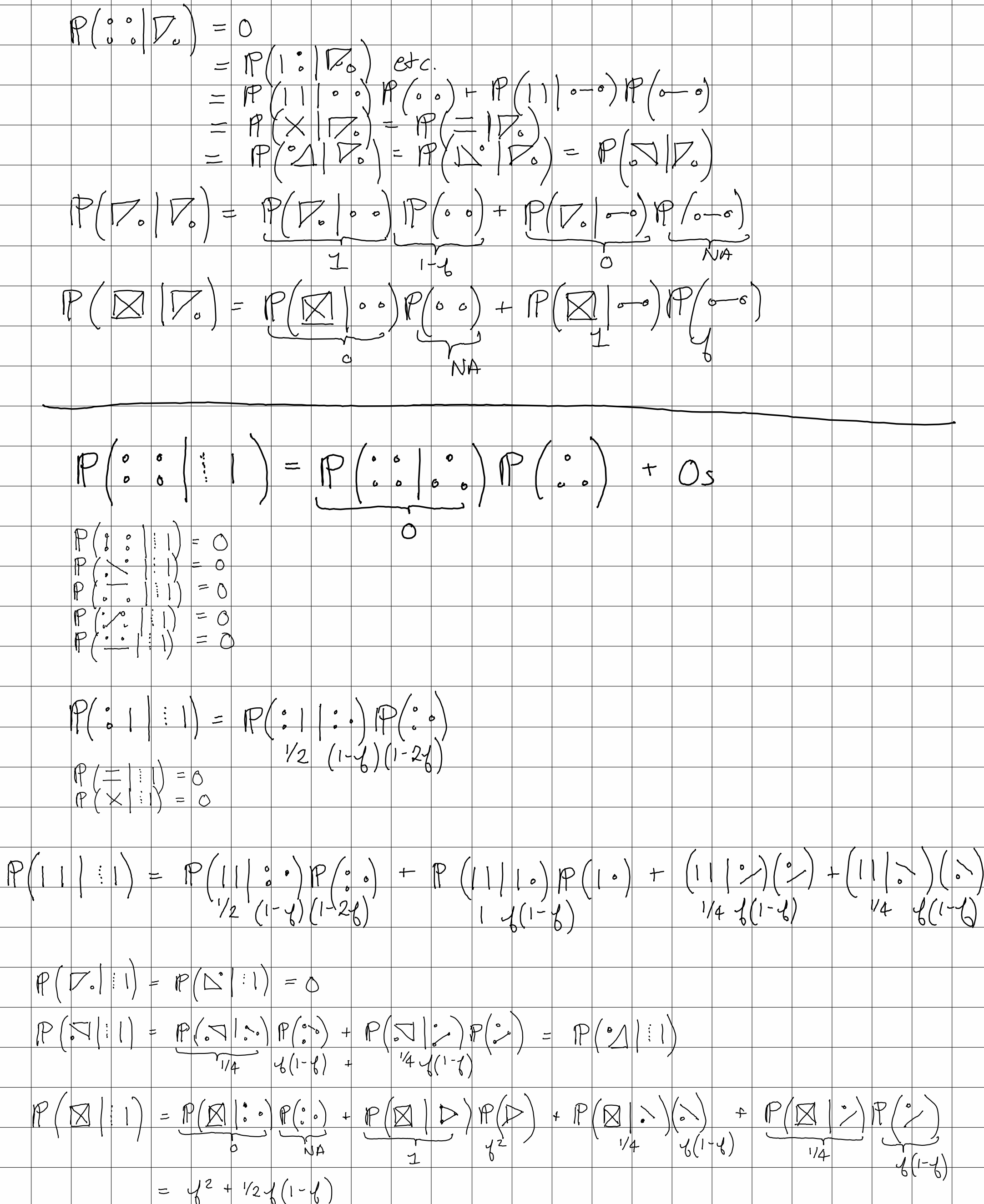




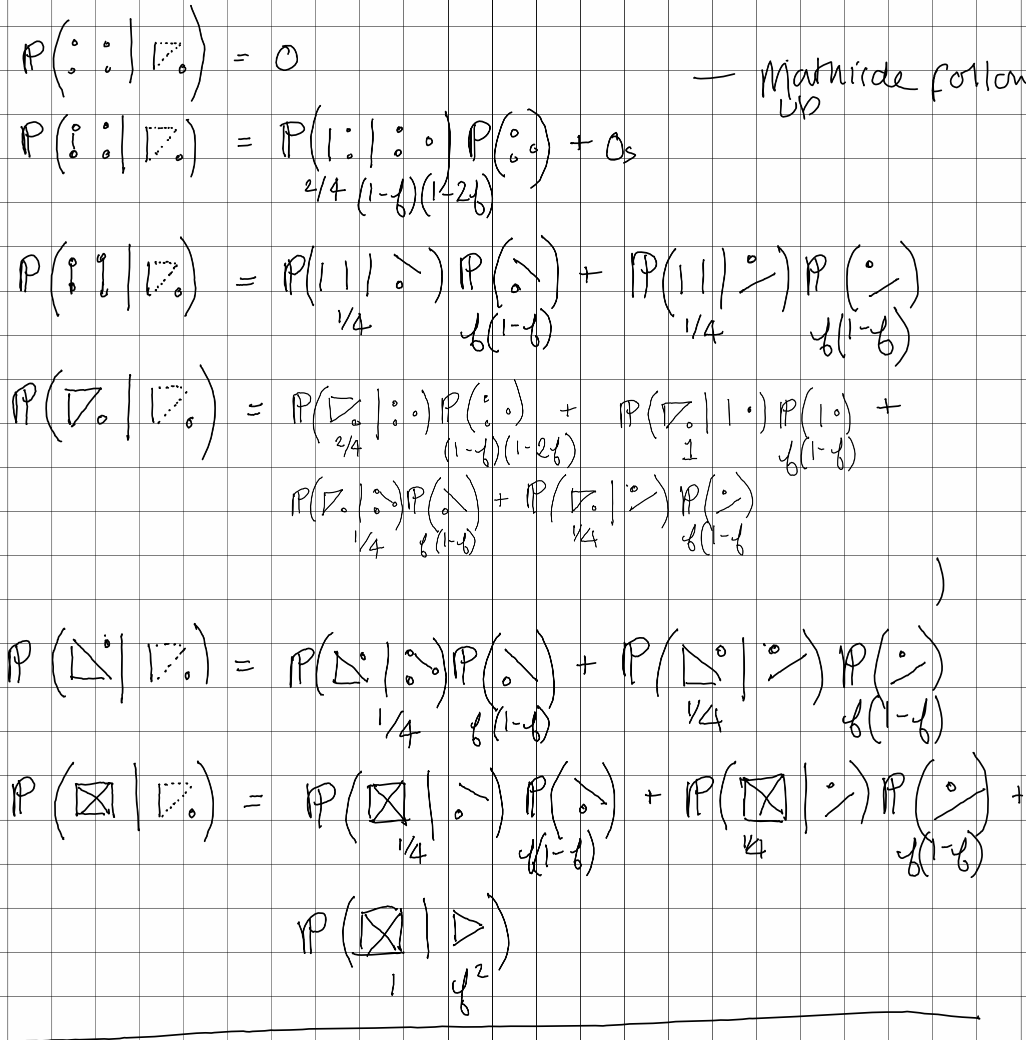
P
C
R
P
O
NA
+ Os
etc all zero
1
(1-f)(1-2f)
= 0
f(1-f)
= 0
f²



= 0
= 0 for all 2-1-1
1
1 − q
0
q
= 0
= 0
= 0
= 0
= 0
= 0
1
q
d
NA

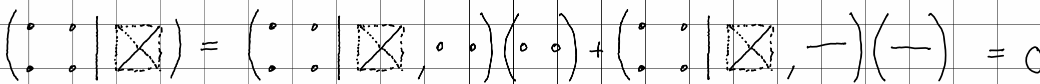


= 0
etc.
1
1-f
0
NA
f
+ 0s
1/2
(1-f)(1-2f)
1/4
f(1-f)
f²
= f² + 1/2 f(1-f)


— Mathilde follow up
P = 0
+ 0s
2/4 (1-f)(1-2f)
1/4 f(1-f) 1/4 f(1-f)
2/4 (1-f)(1-2f) 1 f(1-f)
1/4 f(1-f) 1/4 f(1-f)
1/4 f(1-f) 1/4 f(1-f)
1/4 f(1-f) 1/4 f(1-f)
1 f²

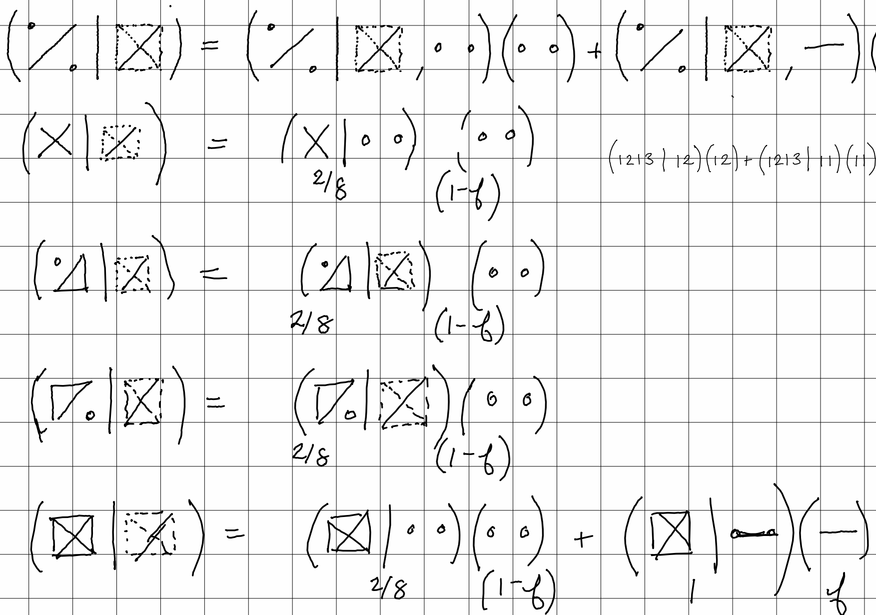
$\mathbb{P}(1G \mid RG) = \sum_{1Gp} \mathbb{P}(1G \mid RG, 1Gp)\,\mathbb{P}(1Gp)$

graphs

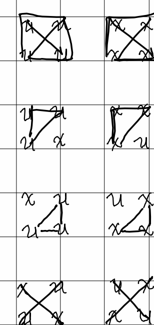


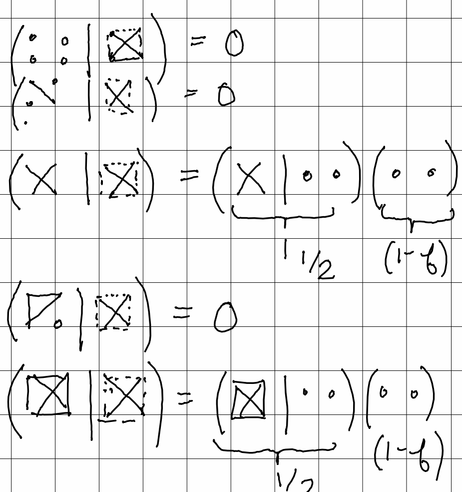
sets

$(1234 \mid RG) = (1234 \mid RG, 12)(12) + (1234 \mid RG, 11)(11)$



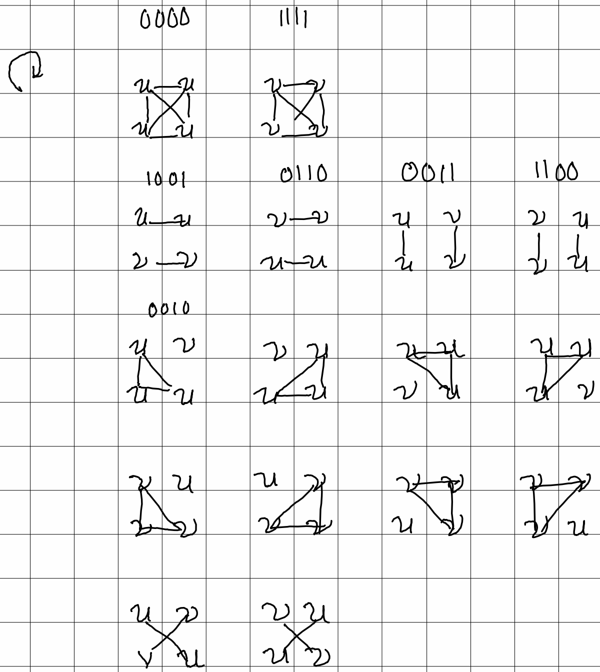
$(1213 \mid 12)(12) + (1213 \mid 11)(11)$

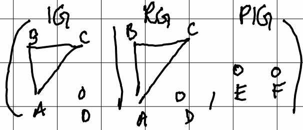




u x   x u
x u   u x

u u   x x
u u   x x

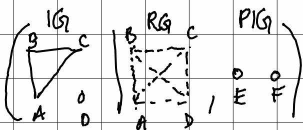

0000
1111
1001
0110
0011
1100
0010



1

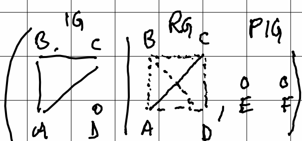
= ABCD 1112 | ABCD 12

ABCD 2111 | ABCD 12



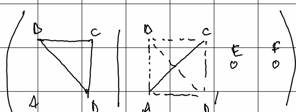
2/16

ABCD 1112 | ABCD 12



2/8

ABCD 1112 | ABCD 1101, ABCD 12



0

ABCD 2111

ABAD

ABCD 1

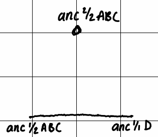
ABCD 2

2 21 21 2

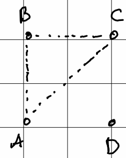
21 2 21 2

How to compute without example? Once solved, algorithm should follow.

121 $\Longrightarrow$ # lineages to pick from = 2



$\Longrightarrow$

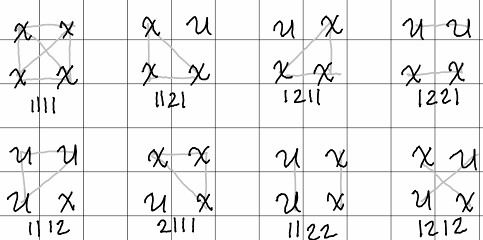


and

A	B	C	D

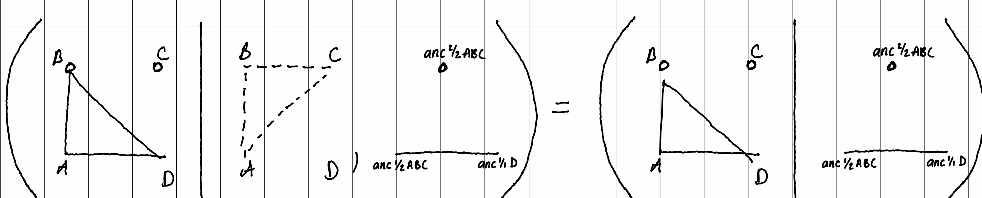
$\underbrace{2 \times 2 \times 2 \times 1}_{df} = 8$ configurations
1 per metric

e.g. if anc 1/2 ABC = x and anc 2/2 ABC = u

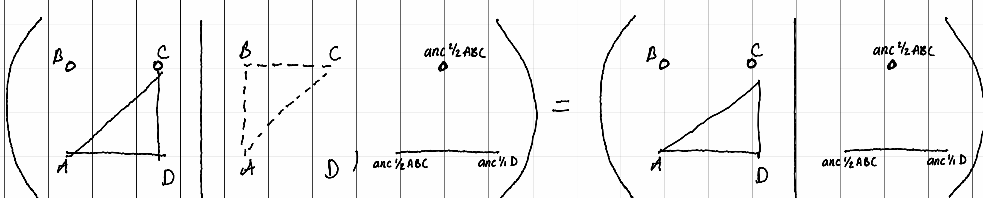


of which one corresponds to 1121
one corresponds to 1211

s.t.

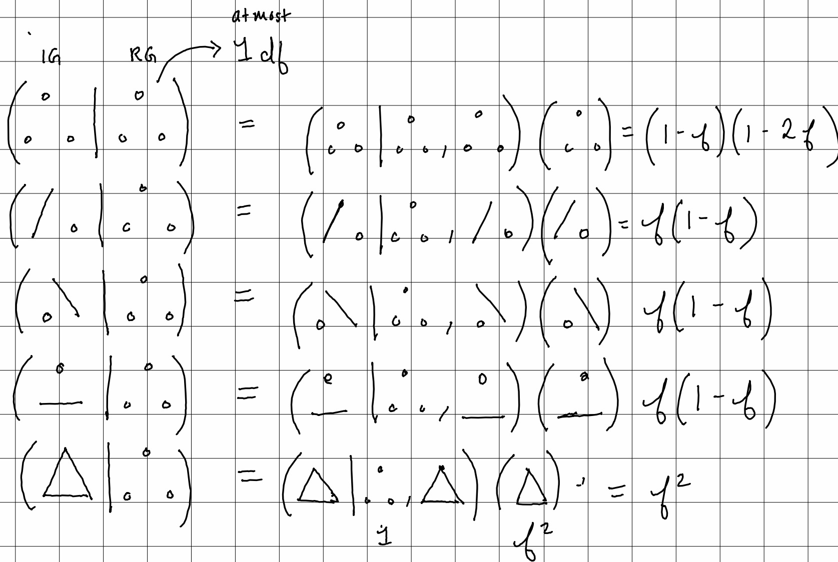


= 1/8

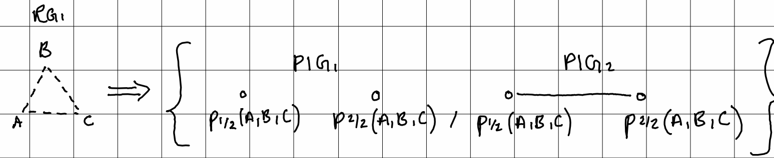


= 1/8

You can on pick two things from 2



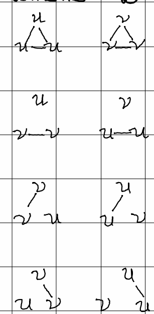
IG
RG
at most
1 df
$(1-f)(1-2f)$
$f(1-f)$
$f(1-f)$
$f(1-f)$
$= f^2$
1
f^2

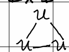


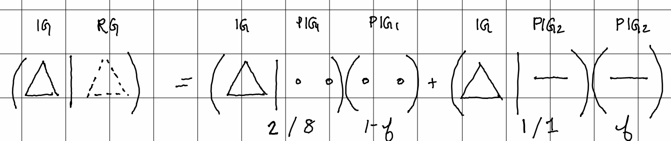
lineage count given PIG, n — 2 e.g. uv — 1 e.g. u

lineage graph set size given PIG — $2 \times 2 \times 2 = 2^3 = 8$ — $1 \times 1 \times 1 = 1^3 = 1$

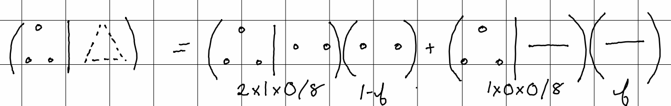
lineage graph set example



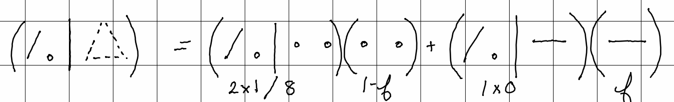




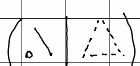
$= 1/4(1-f) + f$



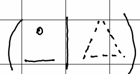
$= 0$



$= 1/4(1-f)$



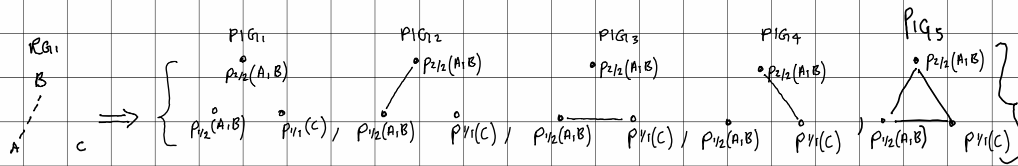
$= 1/4(1-f)$ by symmetry



$= 1/4(1-f)$ by symmetry

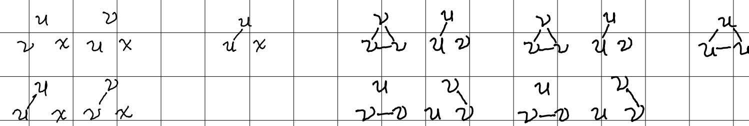
In summary, $\mathbb{P}(IG \mid RG) = \sum \mathbb{P}(IG \mid PIG)\,\mathbb{P}(PIG)$

where $\mathbb{P}(IG \mid PIG) = n \times \max(0, n-1) \times \max(0, n-2) \,/\, n \times n \times n$

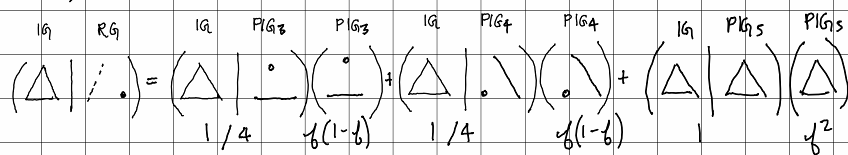


	PIG1	PIG2	PIG3	PIG4	PIG5
lineage count given PIG, n	3 e.g. uvx	2 e.g ux	2 e.g uv	2 eg uv	1 e.g. u
lineage graph set size given PIG	$2\times2\times1=4$	$1\times1\times1=1$	$2\times2\times1=4$	$2\times2\times1=4$	$1\times1\times1=1$

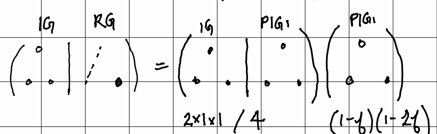
lineage graph set example



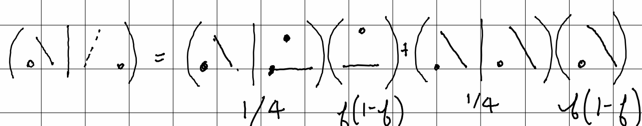
Dropping summation over incompatible PIG...



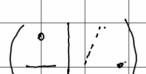
$$= f^2 + \tfrac{1}{2} f(1-f)$$



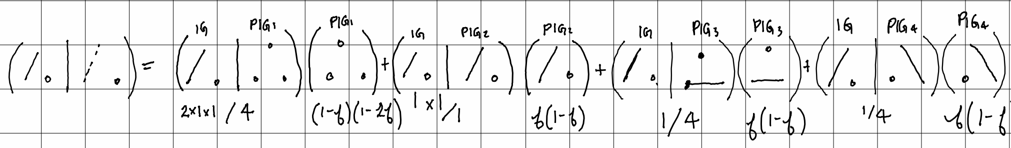
$$= \tfrac{1}{2}(1-f)(1-2f)$$



$$= \tfrac{1}{2} f(1-f)$$



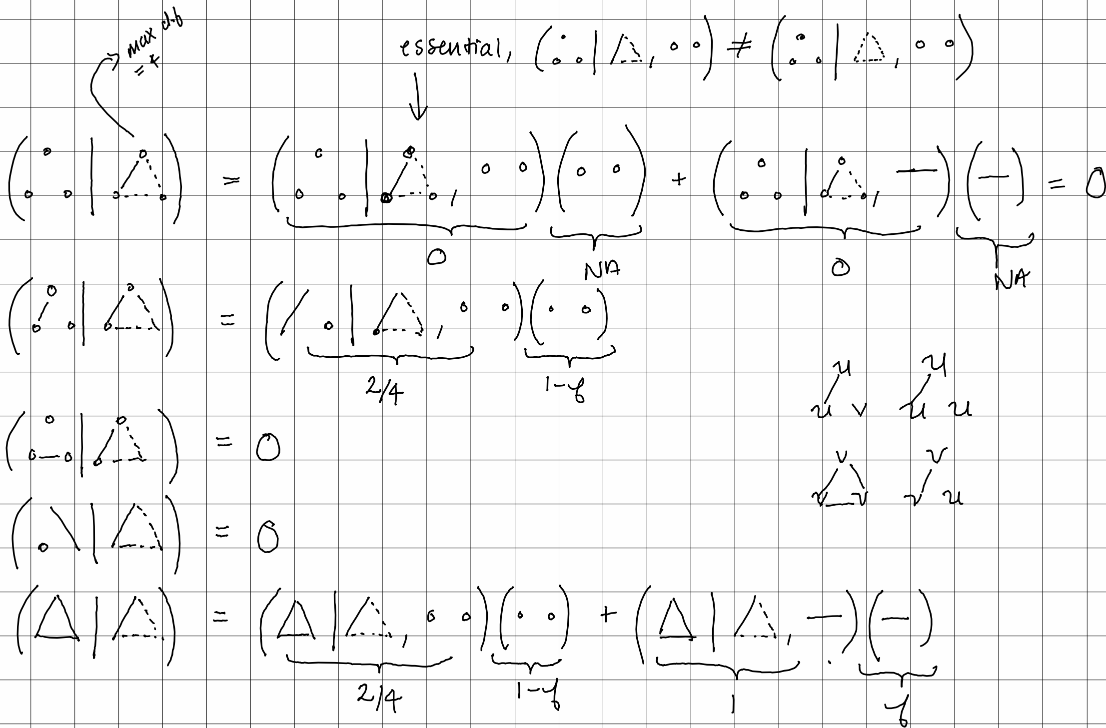
$= \tfrac{1}{2} f(1-f)$ by symmetry



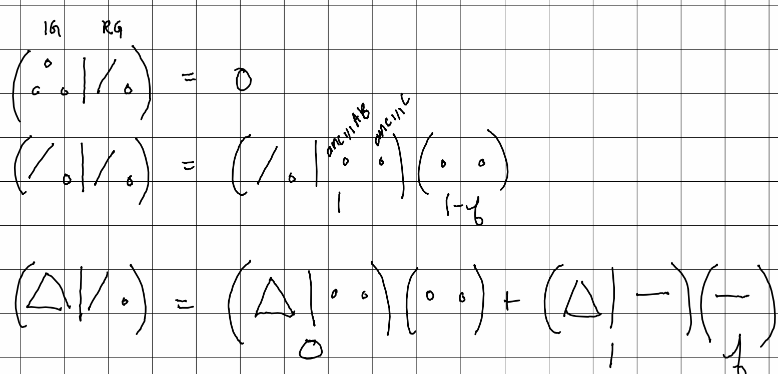
$$= \tfrac{1}{2}(1-f)(1-2f) + \tfrac{3}{2} f(1-f)$$

! check matches !

$P(IG \mid PIG) \neq n \times \max(0, n-1) \times \max(0, n-2) \,/\, n \times n \times n$ because of c.



$$= \gamma + 0.5(1-\gamma)$$



Extension beyond equifrequent lineages

~~The model~~ The limited holds only for an idealised population in which F results from 1/F equifrequent lineages

Höhle

Klotz 1979, Mase 1992 → approximate solⁿ

Exact solⁿ w smaller sum over r nts of singletons, doublets, triplets etc.

$$\Pr\left(\begin{array}{cc|cc} \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \end{array}\right) \;\therefore\; \sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} p_{i_1} p_{i_2} p_{i_3} p_{i_4}$$

IG RG

N = no. line probabilities $p_1, \ldots, p_N$

→ su over 3,921,225 terms for $N=100$.

No. of Singlets ublets, triplets, $t_1, \ldots t_n$

For exac d for approximate, need $p_1, \ldots, p_N$

Birthday problem : at least one IBD

Q: Do solⁿ extend to "exactly one IBD", "exactly two IBD" etc.?

See ?birthdayproblem : pbirthday-up for refs.

Read Das Gupta 2005

Read Diaconis