

fr. 18<sup>th</sup> Feb Stunt of  $\vdots \text{RG} \vdots$

1

[illegible]

$$= 1 \times \frac{1}{2}(1-\frac{1}{2})(1-2\frac{1}{2}) + \frac{1}{2} \times (1-\frac{1}{2})(1-2\frac{1}{2})(1-3\frac{1}{2}) + 4 \times \frac{1}{4} \times \frac{1}{2}(1-\frac{1}{2})(1-2\frac{1}{2})$$

Fri 18<sup>th</sup> Feb

(2)

$$P\left(\begin{smallmatrix} \circ & c & \circ \\ \circ & & \circ \end{smallmatrix} \middle| \begin{smallmatrix} \circ & & \circ \\ \circ & R & \circ \end{smallmatrix}\right) = P\left(\begin{smallmatrix} \circ & c & \circ \\ \circ & & \circ \end{smallmatrix} \middle| \begin{smallmatrix} \circ & P & \circ \\ \circ & & \circ \end{smallmatrix}, \begin{smallmatrix} \circ & & \circ \\ \circ & R & \circ \end{smallmatrix}\right) P\left(\begin{smallmatrix} \circ & P & \circ \\ \circ & & \circ \end{smallmatrix}\right)$$

$$\begin{matrix} u & y & v & y \\ v & y & u & y \end{matrix} = \frac{1}{2} y(1-y)(1-2f)$$

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$$P\left(\begin{smallmatrix} \circ & \overline{c} & \circ \\ \circ & & \circ \end{smallmatrix} \middle| \begin{smallmatrix} \circ & & \circ \\ \circ & R & \circ \end{smallmatrix}\right) = P\left(\begin{smallmatrix} \circ & \overline{c} & \circ \\ \circ & & \circ \end{smallmatrix} \middle| \begin{smallmatrix} \circ & P & \circ \\ \circ & & \circ \end{smallmatrix}, \begin{smallmatrix} \circ & & \circ \\ \circ & R & \circ \end{smallmatrix}\right) P\left(\begin{smallmatrix} \circ & P & \circ \\ \circ & & \circ \end{smallmatrix}\right) +$$

$$P\left(\begin{smallmatrix} \circ & \overline{c} & \circ \\ \circ & & \circ \end{smallmatrix} \middle| \begin{smallmatrix} \circ & / & \circ \\ \circ & & \circ \end{smallmatrix}, \begin{smallmatrix} \circ & & \circ \\ \circ & R & \circ \end{smallmatrix}\right) P\left(\begin{smallmatrix} \circ & / & \circ \\ \circ & & \circ \end{smallmatrix}\right) +$$

$$= \frac{1}{4} y(1-y)(1-2f) + \frac{1}{4} y(1-y)(1-2f)$$

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$$\begin{matrix} y & y & g_2 y & y \\ u & v & g_1 u & v \end{matrix}$$

$$P\left(\begin{smallmatrix} \circ & c & \circ \\ \circ & & \circ \end{smallmatrix} \middle| \begin{smallmatrix} \circ & & \circ \\ \circ & R & \circ \end{smallmatrix}\right) \begin{matrix} y-v & u-y & y-v & u-y \\ u-v & y-v & y-v & u-v \end{matrix}$$

(3)

$$\begin{aligned}
 P\left(\begin{array}{c} \circ \\ \vdots \\ \circ C \circ \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ P \circ, \circ R \circ \\ \vdots \\ \circ \end{array}\right) &= P\left(\begin{array}{c} \circ \\ \vdots \\ \circ C \circ \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ P \circ, \circ R \circ \\ \vdots \\ \circ \end{array}\right) P\left(\begin{array}{c} \circ \\ \vdots \\ \circ P \circ \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ R \circ \\ \vdots \\ \circ \end{array}\right) + \\
 &P\left(\begin{array}{c} \circ \\ \vdots \\ \circ C \circ \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ P \circ, \circ R \circ \\ \vdots \\ \circ \end{array}\right) P\left(\begin{array}{c} \circ \\ \vdots \\ \circ P \circ \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ R \circ \\ \vdots \\ \circ \end{array}\right) + \\
 &P\left(\begin{array}{c} \circ \\ \vdots \\ \circ C \circ \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ \overline{P} \circ, \circ R \circ \\ \vdots \\ \circ \end{array}\right) P\left(\begin{array}{c} \circ \\ \vdots \\ \circ \triangle \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ R \circ \\ \vdots \\ \circ \end{array}\right) + \\
 &P\left(\begin{array}{c} \circ \\ \vdots \\ \circ C \circ \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ \overline{P} \circ, \circ R \circ \\ \vdots \\ \circ \end{array}\right) P\left(\begin{array}{c} \circ \\ \vdots \\ \circ \triangle \\ \vdots \\ \circ \end{array} \middle| \begin{array}{c} \circ \\ \vdots \\ \circ R \circ \\ \vdots \\ \circ \end{array}\right)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \ell(1-\ell)(1-2\ell) + \\
 &\quad 1 \ell^2(1-\ell) + \\
 &\quad \frac{1}{4} (1-\ell)\ell^2 + \\
 &\quad \frac{1}{4} (1-\ell)\ell^2
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \ell(1-\ell)(1-2\ell) + \\
 &\quad 3/2 \ell^2(1-\ell)
 \end{aligned}$$

(4)

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & R \\ \text{---} & \end{array}\right) = P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right)$$

$$= 1 \times y^2(1-b)$$

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & R \\ \text{---} & \end{array}\right) = P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right) +$$

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right) +$$

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right)$$

$$= \frac{1}{4} y(1-b)(1-2y) + \frac{1}{4} y(1-b)(1-2y)$$

$$+ y^2(1-b)$$

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & R \\ \text{---} & \end{array}\right) = P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right)$$

$$+ y^2(1-b)$$

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & R \\ \text{---} & \end{array}\right) = P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right) +$$

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right) +$$

$$P\left(\begin{array}{c|c} \text{---} & \vdots \\ \text{C} & P, R \\ \text{---} & \end{array}\right) P\left(\begin{array}{c|c} \text{---} & \vdots \\ P & \\ \text{---} & \end{array}\right)$$

$$= y^3 + \frac{1}{4} y^2(1-b) + \frac{1}{4} y^2(1-b)$$

end of ...

Start of  $\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} R_A \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

$$P\left(\begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix} \middle| \begin{pmatrix} 0 & 0 \\ 0 & d \end{pmatrix}\right) = P\left(\begin{pmatrix} 0 & c & 0 \\ 0 & 0 & 0 \end{pmatrix} \middle| \begin{pmatrix} 0 & p & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) P\left(\begin{pmatrix} 0 & p & 0 \\ 0 & 0 & c \end{pmatrix}\right)$$

$$= \frac{1}{4} (1-y)(1-2y)(1-3b)$$

$$P\left(\begin{array}{c|cc} \circ & \circ & \\ \circ & \vdots & \\ \circ & \vdots & \end{array}\right) = P\left(\begin{array}{c|cc} \circ & \circ & \\ \circ & \circ & \\ \circ & \circ & \end{array}\right) P\left(\begin{array}{c} \circ \\ \circ \end{array}\right) +$$
  

$$P\left(\begin{array}{c|cc} \circ & \circ & \\ \circ & \circ & \\ \circ & \circ & \end{array}\right) P\left(\begin{array}{c} \circ \\ \circ \end{array}\right) +$$
  

$$\left. \begin{aligned} &P\left(\begin{array}{c|cc} \circ & \circ & \\ \circ & \circ & \\ \circ & \circ & \end{array}\right) P\left(\begin{array}{c} \circ \\ \circ \end{array}\right) + \\ &P\left(\begin{array}{c|cc} \circ & \circ & \\ \circ & \circ & \\ \circ & \circ & \end{array}\right) P\left(\begin{array}{c} \circ \\ \circ \end{array}\right) + \\ &P\left(\begin{array}{c|cc} \circ & \circ & \\ \circ & \circ & \\ \circ & \circ & \end{array}\right) P\left(\begin{array}{c} \circ \\ \circ \end{array}\right) + \\ &P\left(\begin{array}{c|cc} \circ & \circ & \\ \circ & \circ & \\ \circ & \circ & \end{array}\right) P\left(\begin{array}{c} \circ \\ \circ \end{array}\right) \end{aligned} \right\} 4 \times 2 / 6 = 8 / 6$$

$$= P\left(\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right)$$



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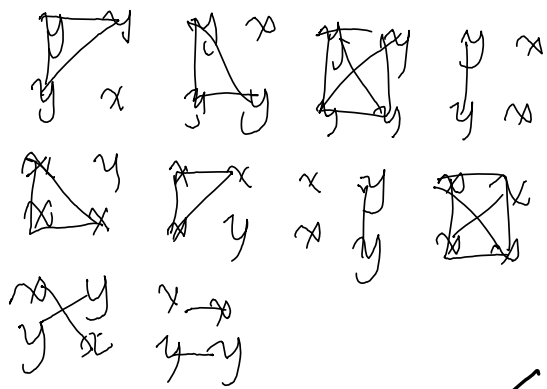
$$= \frac{1}{4} b^2 (1-b)$$

$$= y^3 + (1 + 1/4)y^2(1-y) + 1/4 y(1-y)(1-2y)$$

$$\begin{aligned}
 P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) &= P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{16} f(1-f)(1-2f) \\
 &\quad P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{16} f(1-f)(1-2f) \\
 &\quad P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{16} f(1-f)(1-2f) \\
 &\quad P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{16} f(1-f)(1-2f) \\
 &\quad P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{8} f^2(1-f) \\
 &\quad P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{8} f^2(1-f) \\
 &\quad P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{4} f^2(1-f) \\
 &\quad P\left(\begin{array}{c} \circ \nearrow \circ \\ \vdots \vdots \vdots \end{array} \middle| \begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}, R\right) P\left(\begin{array}{c} \circ \text{---} \circ \\ \vdots \vdots \vdots \end{array}\right) + \frac{1}{4} f^2(1-f)
 \end{aligned}$$

$$= \frac{1}{4} f(1-f)(1-2f) + \frac{3}{4} f^2(1-f)$$

yy  
xx



y  
x

end of ...



9

Children.

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$$= P\left(\begin{array}{c|c} \begin{array}{ccc} \circ & & \circ \\ & \diagdown & \diagup \\ \circ & C & \circ \\ & \diagup & \diagdown \\ \circ & & \circ \end{array} & \begin{array}{ccc} \circ & \cdots & \circ \\ & \diagdown & \diagup \\ \circ & X & \circ \\ & \diagup & \diagdown \\ \circ & & \circ \end{array} \end{array}\right) = P\left(\begin{array}{c|c} \begin{array}{ccc} \circ & & \circ \\ & \diagdown & \diagup \\ \circ & C & \circ \\ & \diagup & \diagdown \\ \circ & & \circ \end{array} & \begin{array}{ccc} \circ & \cdots & \circ \\ & \diagdown & \diagup \\ \circ & \times & \circ \\ & \diagup & \diagdown \\ \circ & & \circ \end{array} \end{array}\right)$$

$$= \frac{2}{16} \times (1 - f)$$

$$2/16 \times (1-f)$$

end of

$$P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & c & \circ \\ \circ & \circ & c \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ R \end{array}\right) = P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & c & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \text{anc}(ABC)_t \\ \circ \\ \text{anc}(ABC)_b \\ \circ \\ \text{anc}(B) \end{array}\right) P\left(\begin{array}{c} \circ \\ c \circ \end{array}\right)$$

$\underbrace{\hspace{10em}}_0 \qquad (1-\theta)(1-2\theta)$

$$P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & c & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ R \end{array}\right) = P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & c & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ \circ \circ \end{array}\right) P\left(\begin{array}{c} \circ \\ c \circ \end{array}\right)$$

$$= 2/8 \qquad (1-\theta)(1-2\theta)$$

$$= P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & c & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ \circ \circ \end{array}\right) = P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ \circ \circ \end{array}\right)$$

$$P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ \circ \circ \end{array}\right) = 0$$

$$P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ \circ \circ \end{array}\right) = 0$$

$$P\left(\begin{array}{cc|c} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{array} \middle| \begin{array}{c} \circ \circ \\ \circ \circ \\ \circ \circ \end{array}\right) = 0$$

$$P\left(\begin{array}{c|c} \begin{array}{cc} \circ & \circ \\ \circ & \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array}\right) = \underbrace{P\left(\begin{array}{c|c} \begin{array}{cc} \circ & \circ \\ \circ & \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array}\right)}_O \underbrace{P\left(\begin{array}{c} \circ \\ \vdots \\ \circ \end{array}\right)}_{NA} +$$

$$\underbrace{P\left(\begin{array}{c|c} \begin{array}{cc} \begin{array}{c} b \\ \circ \end{array} & \begin{array}{c} c \\ \circ \end{array} \\ \begin{array}{c} a \\ \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array} & \begin{array}{c} \text{anc}(ABC)_1 \\ \circ \\ \vdots \\ \circ \end{array} \end{array}\right)}_O \underbrace{P\left(\begin{array}{c} \circ \\ \vdots \\ \circ \end{array}\right)}_{NA} +$$

$$P\left(\begin{array}{c|c} \begin{array}{cc} \begin{array}{c} b \\ \circ \end{array} & \begin{array}{c} c \\ \circ \end{array} \\ \begin{array}{c} a \\ \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array} & \begin{array}{c} \text{anc}(ABC)_1 \\ \circ \\ \vdots \\ \circ \end{array} \end{array}\right) P\left(\begin{array}{c} \circ \\ \vdots \\ \circ \end{array}\right) +$$

$$\underbrace{\frac{1}{8}}_{\frac{1}{8}} \underbrace{P\left(\begin{array}{c|c} \begin{array}{cc} \begin{array}{c} b \\ \circ \end{array} & \begin{array}{c} c \\ \circ \end{array} \\ \begin{array}{c} a \\ \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array} & \begin{array}{c} \text{anc}(ABC)_1 \\ \circ \\ \vdots \\ \circ \end{array} \end{array}\right)}_{\frac{1}{8}} \underbrace{P\left(\begin{array}{c} \circ \\ \vdots \\ \circ \end{array}\right)}_{f(1-f)}$$

$$\underbrace{P\left(\begin{array}{c|c} \begin{array}{cc} \begin{array}{c} b \\ \circ \end{array} & \begin{array}{c} c \\ \circ \end{array} \\ \begin{array}{c} a \\ \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array} & \begin{array}{c} \text{anc}(ABC)_1 \\ \circ \\ \vdots \\ \circ \end{array} \end{array}\right)}_O \underbrace{P\left(\begin{array}{c} \circ \\ \vdots \\ \circ \end{array}\right)}_N$$

$$= \frac{1}{4} f(1-f)$$

$$= P\left(\begin{array}{c|c} \begin{array}{cc} \circ & \times \\ \circ & \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array}\right) = P\left(\begin{array}{c|c} \begin{array}{cc} \circ & \circ \\ \circ & \circ \end{array} & \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} \end{array}\right)$$

$$\begin{aligned}
 P\left(\begin{array}{c|c} \text{diag} & \text{diag} \end{array}\right) &= P\left(\begin{array}{c|c} \text{diag} & \text{diag} \end{array}\right) P\left(\begin{array}{c} \text{diag} \end{array}\right) \\
 &\quad \frac{2}{8} (1-b)(1-2b) + \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \text{diag} & \text{diag} \end{array}\right)}_1 \underbrace{P\left(\begin{array}{c} \text{diag} \end{array}\right)}_{b(1-b)} + \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \text{diag} & \text{diag} \end{array}\right)}_{1/8} \underbrace{P\left(\begin{array}{c} \text{diag} \end{array}\right)}_{b(1-b)} + \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \text{diag} & \text{diag} \end{array}\right)}_{1/8} \underbrace{P\left(\begin{array}{c} \text{diag} \end{array}\right)}_{b(1-b)} \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \text{diag} & \text{diag} \end{array}\right)}_G \underbrace{P\left(\begin{array}{c} \text{diag} \end{array}\right)}_{NA}
 \end{aligned}$$

$$(1 + 1/4)b(1-b) + 1/4(1-b)(1-2b)$$

$$\begin{aligned}
 P\left(\begin{array}{c|c} \circ & \nabla \\ \circ & \circ \end{array} \middle| \begin{array}{c} \circ \\ \circ \end{array}\right) &= P\left(\begin{array}{c|c} \circ & \nabla \\ \circ & \circ \end{array} \middle| \begin{array}{c} \circ \\ \circ \end{array}\right) \underbrace{P\left(\begin{array}{c} \circ \\ \circ \end{array}\right)}_{\text{NA}} + \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \circ & \nabla \\ \circ & \circ \end{array} \middle| \begin{array}{c} \Delta \\ \circ \end{array}\right)}_{\circ} \underbrace{P\left(\begin{array}{c} \Delta \\ \circ \end{array}\right)}_{\text{NA}} + \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \circ & \nabla \\ \circ & \circ \end{array} \middle| \begin{array}{c} \circ \\ \circ \end{array}\right)}_{1/8} \underbrace{P\left(\begin{array}{c} \circ \\ \circ \end{array}\right)}_{4(1-b)} + \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \circ & \nabla \\ \circ & \circ \end{array} \middle| \begin{array}{c} / \\ \circ \end{array}\right)}_{\circ} \underbrace{P\left(\begin{array}{c} / \\ \circ \end{array}\right)}_{\text{NA}} + \\
 &\quad \underbrace{P\left(\begin{array}{c|c} \circ & \nabla \\ \circ & \circ \end{array} \middle| \begin{array}{c} - \\ \circ \end{array}\right)}_{1/8} \underbrace{P\left(\begin{array}{c} - \\ \circ \end{array}\right)}_{4(1-b)}
 \end{aligned}$$

$$= 1/4 \cdot 4(1-b)$$

$$= P\left(\begin{array}{c} \Delta \\ \circ \end{array} \middle| \begin{array}{c} \nabla \\ \circ \end{array}\right) \left(\begin{array}{c} \nabla \\ \circ \end{array}\right)$$

$$= \left(\begin{array}{c} \Delta \\ \circ \end{array} \middle| \begin{array}{c} \nabla \\ \circ \end{array}\right) \left(\begin{array}{c} \nabla \\ \circ \end{array}\right)$$

$$P(\boxed{\times} | \boxed{\cdot}) = \frac{1}{4} \frac{1}{6} (1 - \frac{1}{6}) + \frac{1}{6}^2$$

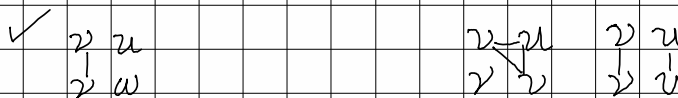
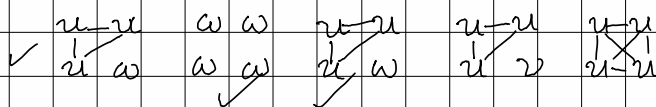
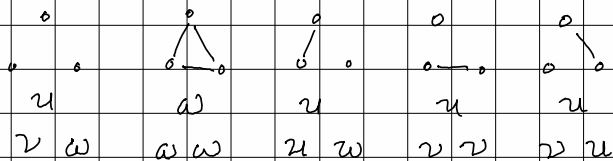
$$\left( \boxed{\times} | \boxed{\cdot} \right) = \underbrace{\left( \boxed{\times} | \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \right)}_0 \underbrace{\left( \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \right)}_{NA} +$$

$$\underbrace{\left( \boxed{\times} | \begin{array}{c} / \\ \cdot \end{array} \right)}_0 \underbrace{\left( \begin{array}{c} / \\ \cdot \end{array} \right)}_{NA} +$$

$$\underbrace{\left( \boxed{\times} | \begin{array}{c} \cdot \\ \backslash \end{array} \right)}_{\frac{1}{8}} \underbrace{\left( \begin{array}{c} \cdot \\ \backslash \end{array} \right)}_{\frac{1}{6}(1-\frac{1}{6})} +$$

$$\underbrace{\left( \boxed{\times} | \begin{array}{c} \cdot \\ \text{—} \end{array} \right)}_{\frac{1}{8}} \underbrace{\left( \begin{array}{c} \cdot \\ \text{—} \end{array} \right)}_{\frac{1}{6}(1-\frac{1}{6})} +$$

$$\underbrace{\left( \boxed{\times} | \triangle \right)}_1 \underbrace{\left( \triangle \right)}_{\frac{1}{6}^2}$$



$$P\left(\begin{pmatrix} c \\ c \\ c \end{pmatrix} \middle| \begin{pmatrix} r \\ r \\ r \end{pmatrix}\right) = \underbrace{P\left(\begin{pmatrix} c \\ c \\ c \end{pmatrix} \middle| \begin{pmatrix} o \\ o \\ o \end{pmatrix}\right)}_O \underbrace{P\left(\begin{pmatrix} o \\ o \\ o \end{pmatrix} \middle| \begin{pmatrix} r \\ r \\ r \end{pmatrix}\right)}_{NA} + O_S$$

$$P\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix} \middle| \begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right) = \underbrace{P\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix} \middle| \begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right)}_1 \underbrace{P\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right)}_{(1-f)(1-2f)} + \text{etc all zero}$$

$$P\left(\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \middle| \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right) = 0$$

$$P\left(\begin{smallmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{smallmatrix}\right) = \underbrace{P\left(\begin{smallmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{smallmatrix}\right)}_0 P\left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right) +$$

$$\underbrace{\left(\begin{smallmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{smallmatrix}\right)}_1 \underbrace{\left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)}_{1/(1-\phi)} + \text{etc all zero}$$

$$P\left(\begin{smallmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{smallmatrix}\right) =$$

$$P\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mid \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\right) = 0$$

$$P(X=1) = 0$$

$$P(\text{D} \mid \text{I}) = \underbrace{P(\text{D} \mid \text{I})}_{0} \underbrace{P(\text{I})}_{\text{NA}} + \underbrace{P(\text{D} \mid \text{I})}_{1} \underbrace{P(\text{I})}_{1(1-l)}$$

$$P(\Delta | 1:) = \binom{0}{1} \binom{NA}{f(1-f)}$$

$$P\left(\begin{smallmatrix} \square \\ \bullet \end{smallmatrix} \middle| \begin{smallmatrix} 0 \\ a \end{smallmatrix}\right) = P\left(\begin{smallmatrix} \triangle \\ \bullet \end{smallmatrix} \middle| \begin{smallmatrix} 0 \\ a \end{smallmatrix}\right) = 0$$

$$P(\boxed{\times} | 1 : :) = P(\boxed{\times} | \triangle) P(\triangle | \frac{1}{2})$$



$$P(\begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0$$

$$P(1 \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0 \text{ for all } 2,1,1$$

$$P(11 \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = \underbrace{P(11 | \circ \circ)}_1 \underbrace{P(\circ \circ)}_{1-\frac{1}{6}} + \underbrace{P(11 | \circ - \circ)}_0 \underbrace{P(-)}_{\frac{1}{6}}$$

$$P(X \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0$$

$$P(\equiv \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0$$

$$P(\overline{\nabla} \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0$$

$$P(\nabla \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0$$

$$P(\triangle \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0$$

$$P(\nabla^\circ \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = 0$$

$$P(\boxtimes \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | 11) = \underbrace{P(\boxtimes | \circ - \circ)}_1 \underbrace{P(\circ - \circ)}_{\frac{1}{6}} + \underbrace{P(\boxtimes | \circ \circ)}_0 \underbrace{P(\circ \circ)}_{NA}$$

$$\begin{aligned}
 P(\cdot \cdot \cdot | D_0) &= 0 \\
 &= P(\cdot \cdot \cdot | D_0) \text{ etc.} \\
 &= P(11 | \cdot \cdot \cdot) P(\cdot \cdot \cdot) + P(11 | \cdot \cdot \cdot) P(\cdot \cdot \cdot) \\
 &= P(X | D_0) = P(= | D_0) \\
 &= P(2 | D_0) = P(\Delta | D_0) = P(\Delta | D_0)
 \end{aligned}$$

$$P(D_0 | D_0) = \underbrace{P(D_0 | \cdot \cdot \cdot)}_1 \underbrace{P(\cdot \cdot \cdot)}_{1-\frac{1}{6}} + \underbrace{P(D_0 | \cdot \cdot \cdot)}_0 \underbrace{P(\cdot \cdot \cdot)}_{NA}$$

$$P(\boxtimes | D_0) = \underbrace{P(\boxtimes | \cdot \cdot \cdot)}_0 \underbrace{P(\cdot \cdot \cdot)}_{NA} + \underbrace{P(\boxtimes | \cdot \cdot \cdot)}_1 \underbrace{P(\cdot \cdot \cdot)}_{\frac{1}{6}}$$

$$P(\cdot \cdot \cdot | \cdot \cdot \cdot) = \underbrace{P(\cdot \cdot \cdot | \cdot \cdot \cdot)}_0 P(\cdot \cdot \cdot) + 0s$$

$$\begin{aligned}
 P(\cdot \cdot \cdot | \cdot \cdot \cdot) &= 0 \\
 P(\cdot \cdot \cdot | \cdot \cdot \cdot) &= 0 \\
 P(\cdot \cdot \cdot | \cdot \cdot \cdot) &= 0 \\
 P(\cdot \cdot \cdot | \cdot \cdot \cdot) &= 0 \\
 P(\cdot \cdot \cdot | \cdot \cdot \cdot) &= 0
 \end{aligned}$$

$$P(\cdot \cdot \cdot | \cdot \cdot \cdot) = P(\cdot \cdot \cdot | \cdot \cdot \cdot) P(\cdot \cdot \cdot) \\ \frac{1}{2} (1-\frac{1}{6})(1-\frac{2}{6})$$

$$\begin{aligned}
 P(= | \cdot \cdot \cdot) &= 0 \\
 P(X | \cdot \cdot \cdot) &= 0
 \end{aligned}$$

$$P(11 | \cdot \cdot \cdot) = \underbrace{P(11 | \cdot \cdot \cdot)}_{\frac{1}{2} (1-\frac{1}{6})(1-\frac{2}{6})} P(\cdot \cdot \cdot) + \underbrace{P(11 | \cdot \cdot \cdot)}_1 \underbrace{P(\cdot \cdot \cdot)}_{\frac{1}{6}(1-\frac{1}{6})} + \underbrace{P(11 | \cdot \cdot \cdot)}_{\frac{1}{4} \frac{1}{6}(1-\frac{1}{6})} P(\cdot \cdot \cdot) + \underbrace{P(11 | \cdot \cdot \cdot)}_{\frac{1}{4} \frac{1}{6}(1-\frac{1}{6})} P(\cdot \cdot \cdot)$$

$$P(D_0 | \cdot \cdot \cdot) = P(\Delta | \cdot \cdot \cdot) = 0$$

$$P(\Delta | \cdot \cdot \cdot) = \underbrace{P(\Delta | \cdot \cdot \cdot)}_{\frac{1}{4}} \underbrace{P(\cdot \cdot \cdot)}_{\frac{1}{6}(1-\frac{1}{6})} + \underbrace{P(\Delta | \cdot \cdot \cdot)}_{\frac{1}{4}} \underbrace{P(\cdot \cdot \cdot)}_{\frac{1}{6}(1-\frac{1}{6})} = P(\Delta | \cdot \cdot \cdot)$$

$$\begin{aligned}
 P(\boxtimes | \cdot \cdot \cdot) &= \underbrace{P(\boxtimes | \cdot \cdot \cdot)}_0 \underbrace{P(\cdot \cdot \cdot)}_{NA} + \underbrace{P(\boxtimes | \cdot \cdot \cdot)}_1 \underbrace{P(\cdot \cdot \cdot)}_{\frac{1}{6}^2} + \underbrace{P(\boxtimes | \cdot \cdot \cdot)}_{\frac{1}{4}} \underbrace{P(\cdot \cdot \cdot)}_{\frac{1}{6}(1-\frac{1}{6})} + \underbrace{P(\boxtimes | \cdot \cdot \cdot)}_{\frac{1}{4}} \underbrace{P(\cdot \cdot \cdot)}_{\frac{1}{6}(1-\frac{1}{6})} \\
 &= \frac{1}{6}^2 + \frac{1}{2} \frac{1}{6}(1-\frac{1}{6})
 \end{aligned}$$

$$P\left(\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right) = 0$$

— Marnie follow up

$$P\left(\begin{smallmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{smallmatrix} \middle| \begin{smallmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right) = \underbrace{P\left(\begin{smallmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{smallmatrix} \middle| \begin{smallmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right)}_{\frac{2}{4} \cdot \frac{1}{2} \cdot \frac{1}{2}} P\left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{smallmatrix}\right) + 0_s$$

$$P\left(\begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix} \middle| \begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix}\right) = P\left(\begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix} \middle| \begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix}\right) P\left(\begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix}\right) + P\left(\begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix} \middle| \begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix}\right) P\left(\begin{smallmatrix} | & | & | \\ \vdots & \vdots & \vdots \\ | & | & | \end{smallmatrix}\right)$$

$$P(\begin{smallmatrix} \square & | & \begin{smallmatrix} \square \\ \cdot \end{smallmatrix} \end{smallmatrix}) = P(\begin{smallmatrix} \square & | & \begin{smallmatrix} \cdot \\ \cdot \end{smallmatrix} \end{smallmatrix}) P(\begin{smallmatrix} \cdot \\ \cdot \end{smallmatrix}) + P(\begin{smallmatrix} \square & | & \begin{smallmatrix} \square \\ \cdot \end{smallmatrix} \end{smallmatrix}) P(\begin{smallmatrix} \cdot \\ \cdot \end{smallmatrix}) +$$

$$P(\Delta \mid \cdot) = P(\Delta \mid \circ)P(\cdot) + P(\Delta \mid \ominus)P(\cdot)$$

$$P(\text{Box} | \text{Dot}) = P(\text{Box} | \text{Dot}) \frac{P(\text{Dot})}{\frac{1}{4}} + P(\text{Box} | \text{Dot}) \frac{P(\text{Dot})}{\frac{1}{4}}$$

$$Y_P \left( \begin{array}{c|c} \boxed{\times} & \triangle \\ \hline 1 & y^2 \end{array} \right)$$

$$P(IG \mid RG) = \sum_{IG_p} P(IG \mid RG, IG_p) P(IG_p)$$

graphs  $\left( \begin{array}{c} \bullet \quad \bullet \quad | \quad \boxed{\times} \\ \bullet \quad \bullet \quad | \quad \boxed{\times} \end{array} \right) = \left( \begin{array}{c} \bullet \quad \bullet \quad | \quad \boxed{\times} \\ \bullet \quad \bullet \quad | \quad \boxed{\times} \end{array}, \bullet \bullet \right) (\bullet \bullet) + \left( \begin{array}{c} \bullet \quad \bullet \quad | \quad \boxed{\times} \\ \bullet \quad \bullet \quad | \quad \boxed{\times} \end{array}, - \right) (-) = 0$

sets  $(1234 | RG) = (1234 | RG, 12)(12) + (1234 | RG, 11)(11)$

$$\left( \begin{array}{c|c} \bullet & \square \\ \hline \diagup & \diagdown \\ \hline \bullet & \bullet \end{array} \right) = \left( \begin{array}{c|c} \bullet & \square \\ \hline \diagup & \diagdown \\ \hline \bullet & \bullet \end{array}, 0 \right) \left( \begin{array}{c} \bullet \\ \bullet \end{array} \right) + \left( \begin{array}{c|c} \bullet & \square \\ \hline \diagup & \diagdown \\ \hline \bullet & \bullet \end{array}, - \right) \left( \begin{array}{c} \bullet \\ - \end{array} \right) = 0$$

$$\left( \begin{pmatrix} \times & | & \boxed{\times} \end{pmatrix} \right) = \left( \begin{pmatrix} \times & | & \circ & \circ \end{pmatrix} \right) \begin{pmatrix} \circ & \circ \\ 1 & -1 \end{pmatrix} \quad (1213 | 12)(12) + (1213 | 11)(11)$$

$$\begin{pmatrix} \triangle & \square \end{pmatrix} = \begin{pmatrix} \triangle & \square \end{pmatrix}_{2/8} \begin{pmatrix} \circ & \circ \end{pmatrix}_{(1-b)}$$

$$\left( \begin{array}{c|c} \nabla & \boxtimes \\ \hline \bullet & \end{array} \right) = \underbrace{\left( \begin{array}{c|c} \nabla & \boxtimes \\ \hline \bullet & \end{array} \right)}_{218} \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}$$

$$\left( \begin{array}{c|c} \square & \square \\ \hline \square & \square \end{array} \right) = \left( \begin{array}{c|c} \square & \square \\ \hline \square & \square \end{array} \right)_{2/8} \begin{pmatrix} 0 & 0 \\ 1 & -6 \end{pmatrix} + \left( \begin{array}{c|c} \square & \square \\ \hline \square & \square \end{array} \right)_1 \begin{pmatrix} - & - \\ & - \end{pmatrix}_y$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{\times} \\ \boxed{\times} \\ \boxed{\times} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

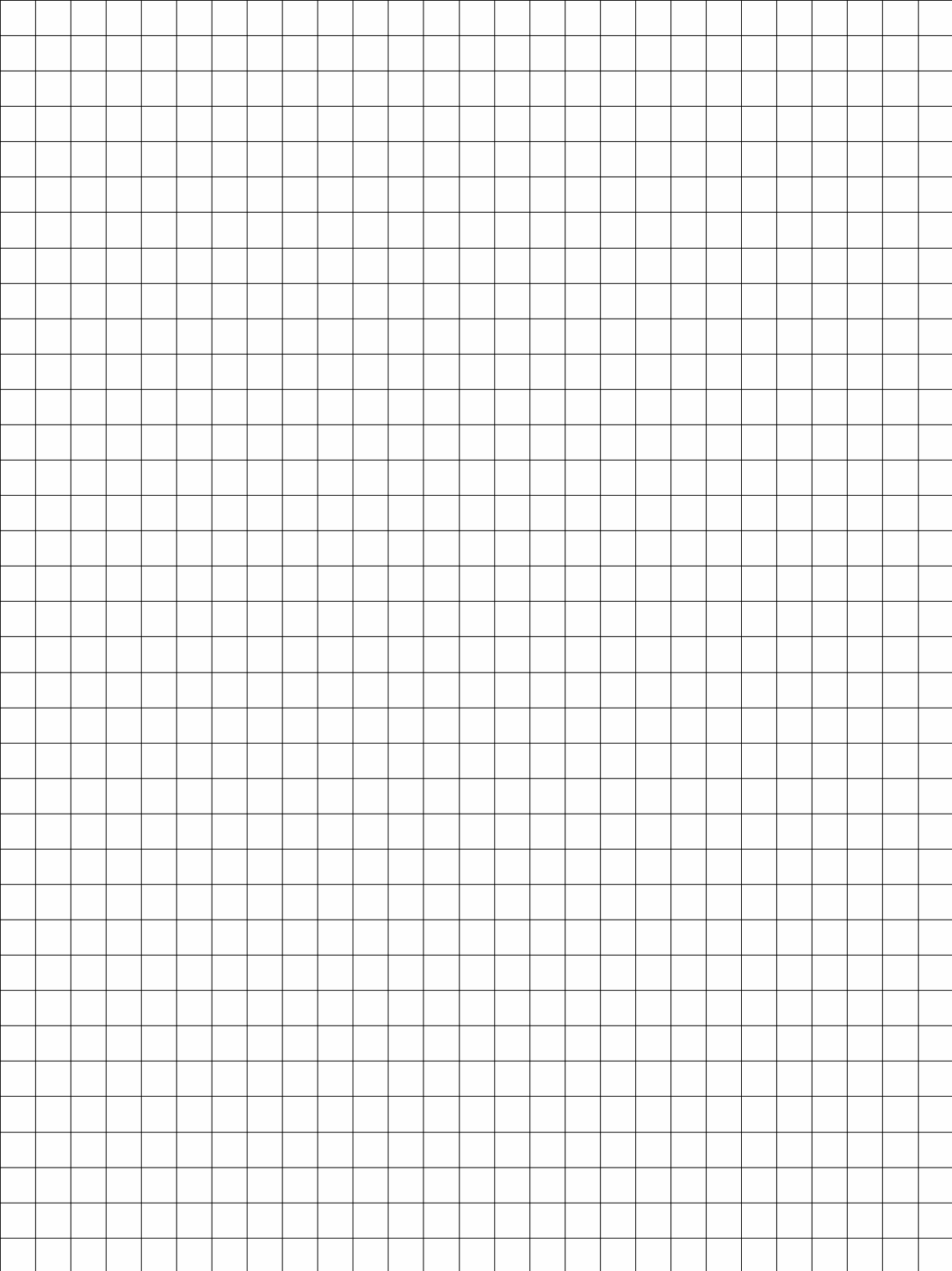
$$(X | \tilde{X}) = \underbrace{(X | \bullet \bullet)}_{1/2} \underbrace{(\bullet \bullet)}_{(1-\delta)}$$

$$\begin{pmatrix} \square & \boxtimes \end{pmatrix} = 0$$

$$\left( \begin{array}{c|c} \boxed{\times} & \boxed{\times} \\ \hline \end{array} \right) = \underbrace{\left( \begin{array}{c|c} \boxed{\times} & \cdot \cdot \end{array} \right)}_{1/2} \begin{pmatrix} 0 & 0 \\ 1 & -6 \end{pmatrix} + \underbrace{\left( \begin{array}{c|c} \boxed{\times} & - \end{array} \right)}_7 \underbrace{\begin{pmatrix} - \\ 7 \end{pmatrix}}_7$$

u	x		x	u
x	u		u	x

u	u		x	x
u	u		x	x





0000



1111



1001



0110



0011

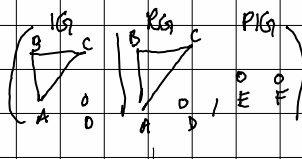


1100

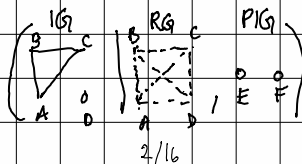


0010

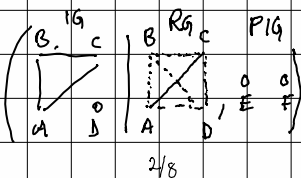




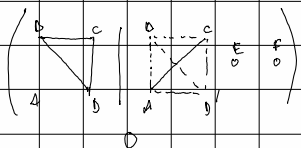
$$\begin{array}{r|l}
 ABCD & ABCD \\
 \hline
 1112 & 12 \\
 \hline
 ABCD & ABCD \\
 2111 & 12
 \end{array}$$



$$\begin{array}{r|l}
 ABCD & ABCD \\
 \hline
 1112 & 12 \\
 \hline
 \downarrow &
 \end{array}$$



$$\begin{array}{r|l}
 ABCD & ABCD \\
 \hline
 1112 & 1101, 12 \\
 \hline
 \end{array}$$

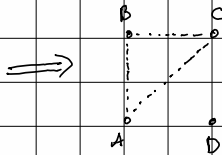
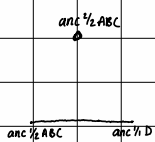


$$\begin{array}{r|l}
 ABCD & ABCD \\
 \hline
 2111 & ABAD \quad 1 \quad 2
 \end{array}$$

$\begin{array}{c} \swarrow \searrow \\ \swarrow \searrow \end{array}$ 
 $\begin{array}{c} \swarrow \searrow \\ \swarrow \searrow \end{array}$

How to compute without example? Once solved, algorithm should follow.

121  $\Rightarrow$  # choices to pick from = 2



and

$$\begin{matrix} A & B & C & D \\ 2 & 2 & 2 & 1 \end{matrix} = 8 \text{ configurations} \\ \underbrace{\hspace{1.5cm}}_{d_b} \quad 1 \text{ per metric}$$

eg. if  $\text{anc}^{1/2} ABC = x$  and  $\text{anc}^{2/2} ABC = u$

$x \ x$	$x \ u$	$u \ x$	$u \ u$
$x \ x$	$x \ x$	$x \ x$	$x \ x$
1111	1121	1211	1221
$u \ u$	$x \ x$	$u \ x$	$x \ u$
$u \ x$	$u \ x$	$u \ x$	$u \ x$
1112	2111	1122	1212

of which one corresponds to 1121  
one corresponds to 1211

s.t.

$$\left( \begin{array}{c|c} \begin{array}{c} A \quad C \\ \diagdown \quad \diagup \\ A \quad D \end{array} & \begin{array}{c} B \quad C \\ \diagdown \quad \diagup \\ A \quad D \end{array} \end{array} \middle| \begin{array}{c} \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} D \end{array} \right) = \left( \begin{array}{c|c} \begin{array}{c} A \quad C \\ \diagdown \quad \diagup \\ A \quad D \end{array} & \begin{array}{c} \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} D \end{array} \end{array} \right) = 1/8$$

$$\left( \begin{array}{c|c} \begin{array}{c} B \quad C \\ \diagdown \quad \diagup \\ A \quad D \end{array} & \begin{array}{c} B \quad C \\ \diagdown \quad \diagup \\ A \quad D \end{array} \end{array} \middle| \begin{array}{c} \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} D \end{array} \right) = \left( \begin{array}{c|c} \begin{array}{c} B \quad C \\ \diagdown \quad \diagup \\ A \quad D \end{array} & \begin{array}{c} \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} ABC \\ \text{anc}^{1/2} D \end{array} \end{array} \right) = 1/8$$

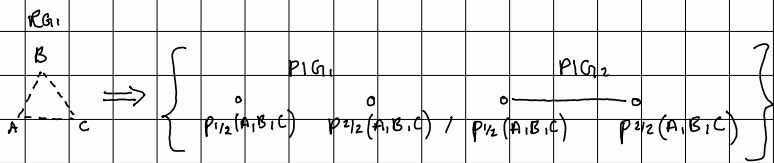
You can only pick two things from 2



at most

$$\begin{aligned}
 & \overset{IG}{\left( \begin{array}{c|cc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)} \xrightarrow{RG} \overset{1df}{\left( \begin{array}{c|cc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)} = \left( \begin{array}{c|cc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} 0 \\ 0 \end{array} \right) = (1-b)(1-2b) \\
 & \left( \begin{array}{c|cc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) = \left( \begin{array}{c|cc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) = b(1-b) \\
 & \left( \begin{array}{c|cc} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right) = \left( \begin{array}{c|cc} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} 0 \\ 1 \end{array} \right) = b(1-b) \\
 & \left( \begin{array}{c|cc} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right) = \left( \begin{array}{c|cc} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right) = b^2 \\
 & \left( \begin{array}{c|cc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) = \left( \begin{array}{c|cc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right) = b(1-b) \\
 & \left( \begin{array}{c|cc} \Delta & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) = \left( \begin{array}{c|cc} \Delta & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} \Delta \\ 0 \\ 0 \end{array} \right) = b^2
 \end{aligned}$$


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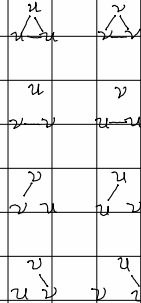
lineage count given  $P(G)$ ,  $n$

lineage graph set size given  $P(G)$

lineage graph set example

2 e.g.  $uv$   
 $\overset{u}{\underset{u}{\overset{v}{\underset{v}{\circ}}}} \times \overset{v}{\underset{v}{\overset{u}{\underset{u}{\circ}}}} = 2^3 = 8$

1 e.g.  $u$   
 $\overset{u}{\underset{u}{\overset{u}{\underset{u}{\circ}}}} = 1^3 = 1$



$$\begin{array}{c} I_{G_1} \quad R_{G_1} \\ \left( \begin{array}{c|c} \triangle & \triangle \end{array} \right) = \left( \begin{array}{c|c} \triangle & \circ \circ \end{array} \right) \left( \begin{array}{c} \circ \circ \\ 2/8 \end{array} \right) + \left( \begin{array}{c|c} \triangle & - \end{array} \right) \left( \begin{array}{c} - \\ 1/1 \end{array} \right) = \frac{1}{4}(1-\frac{1}{6}) + \frac{1}{6}
 \end{array}$$

$$\begin{array}{c} I_{G_1} \quad P_{G_1} \\ \left( \begin{array}{c|c} \circ \circ & \triangle \end{array} \right) = \left( \begin{array}{c|c} \circ \circ & \circ \circ \end{array} \right) \left( \begin{array}{c} \circ \circ \\ 2 \times 1 \times 0/8 \end{array} \right) + \left( \begin{array}{c|c} \circ \circ & - \end{array} \right) \left( \begin{array}{c} - \\ 1 \times 0 \times 0/8 \end{array} \right) = 0
 \end{array}$$

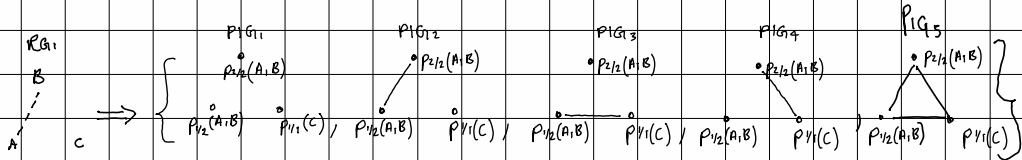
$$\begin{array}{c} I_{G_1} \quad P_{G_2} \\ \left( \begin{array}{c|c} / \circ & \triangle \end{array} \right) = \left( \begin{array}{c|c} / \circ & \circ \circ \end{array} \right) \left( \begin{array}{c} \circ \circ \\ 2 \times 1/8 \end{array} \right) + \left( \begin{array}{c|c} / \circ & - \end{array} \right) \left( \begin{array}{c} - \\ 1 \times 0 \end{array} \right) = \frac{1}{4}(1-\frac{1}{6})
 \end{array}$$

$$\begin{array}{c} I_{G_1} \quad P_{G_2} \\ \left( \begin{array}{c|c} \circ \backslash & \triangle \end{array} \right) = \frac{1}{4}(1-\frac{1}{6}) \text{ by symmetry}
 \end{array}$$

$$\begin{array}{c} I_{G_1} \quad P_{G_2} \\ \left( \begin{array}{c|c} \circ & \triangle \end{array} \right) = \frac{1}{4}(1-\frac{1}{6}) \text{ by symmetry}
 \end{array}$$

In summary,  $P(I_G | R_G) = \sum P(I_G | P_G) P(P_G)$

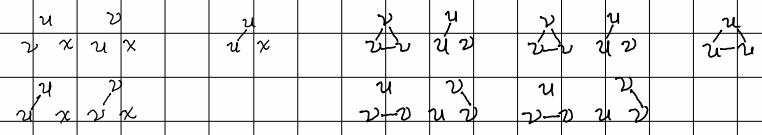
where  $P(I_G | P_G) = n \times \max(0, n-1) \times \max(0, n-2) / n \times n \times n$



Lineage cannot given  $PIG, n$

Lineage graph set size given  $PIG$       3 e.g.  $uwx$       2 e.g.  $ux$       2 e.g.  $uw$       2 e.g.  $uw$       1 e.g.  $u$

Lineage graph set examples



Dropping summation over incompatible  $PIG...$

$$\begin{matrix} IG_1 & RG_1 & IG_1 & PIG_2 & PIG_3 & IG_1 & PIG_4 & PIG_4 & IG_1 & PIG_5 & PIG_5 \\ \left( \begin{array}{c|c} \Delta & \vdots \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) = \left( \begin{array}{c|c} \Delta & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \Delta & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \Delta & \Delta \\ \hline \end{array} \middle| \begin{array}{c} \Delta \\ \hline \end{array} \right) = f^2 + \frac{1}{2} f(1-f) \\ \frac{1}{4} & & \frac{1}{4} & f(1-f) & \frac{1}{4} & f(1-f) & 1 & f^2
 \end{matrix}$$

$$\begin{matrix} IG_1 & RG_1 & IG_1 & PIG_1 & PIG_1 \\ \left( \begin{array}{c|c} \bullet & \vdots \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) = \left( \begin{array}{c|c} \bullet & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) = \frac{1}{2} (1-f)(1-2f) \\ \frac{2 \times 1 \times 1}{4} & & (1-f)(1-2f)
 \end{matrix}$$

$$\begin{matrix} IG_1 & RG_1 & IG_1 & PIG_1 & PIG_1 \\ \left( \begin{array}{c|c} \bullet & \vdots \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) = \left( \begin{array}{c|c} \bullet & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) + \left( \begin{array}{c|c} \bullet & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) = \frac{1}{2} f(1-f) \\ \frac{1}{4} & & \frac{1}{4} & f(1-f) & f(1-f)
 \end{matrix}$$

$$\left( \begin{array}{c|c} \bullet & \vdots \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) = \frac{1}{2} f(1-f) \text{ by symmetry}$$

$$\begin{matrix} IG_1 & PIG_1 & PIG_1 & IG_1 & PIG_2 & PIG_2 & IG_1 & PIG_3 & PIG_3 & IG_1 & PIG_4 & PIG_4 \\ \left( \begin{array}{c|c} / & \vdots \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) = \left( \begin{array}{c|c} / & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) + \left( \begin{array}{c|c} / & / \\ \hline \end{array} \middle| \begin{array}{c} / \\ \hline \end{array} \right) + \left( \begin{array}{c|c} / & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) + \left( \begin{array}{c|c} / & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) + \left( \begin{array}{c|c} / & \bullet \\ \hline \end{array} \middle| \begin{array}{c} \bullet \\ \hline \end{array} \right) \\ \frac{2 \times 1 \times 1}{4} & & (1-f)(1-2f) & \frac{1 \times 1}{1} & f(1-f) & \frac{1}{4} & f(1-f) & \frac{1}{4} & f(1-f) & \frac{1}{4} & f(1-f) \\ \\ = \frac{1}{2} (1-f)(1-2f) + \frac{3}{2} f(1-f)
 \end{matrix}$$

! check matches!

$$P(G|PIG) \neq n \times \max(0, n-1) \times \max(0, n-2) / n \times n \times n \text{ because of } c.$$

$$\text{essential, } (\begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle, \circ \circ) \neq (\begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle, \circ \circ)$$

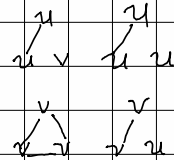
max  $\Delta_f$   
 $\Rightarrow +$

$$\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle \right) = \underbrace{\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle, \circ \circ \right)}_0 \underbrace{\left( \begin{smallmatrix} \circ & \circ \end{smallmatrix} \right)}_{NA} + \underbrace{\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle, - \right)}_0 \underbrace{\left( - \right)}_{NA} = 0$$

$$\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle \right) = \underbrace{\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle, \circ \circ \right)}_{2/4} \underbrace{\left( \begin{smallmatrix} \circ & \circ \end{smallmatrix} \right)}_{1-\gamma}$$

$$\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle \right) = 0$$

$$\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \triangle \right) = 0$$



$$\left( \triangle | \triangle \right) = \underbrace{\left( \triangle | \triangle, \circ \circ \right)}_{2/4} \underbrace{\left( \begin{smallmatrix} \circ & \circ \end{smallmatrix} \right)}_{1-\gamma} + \underbrace{\left( \triangle | \triangle, - \right)}_1 \underbrace{\left( - \right)}_{\gamma}$$

$$= \gamma + 0.5(1-\gamma)$$

$$\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} \right) = 0$$

$$\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} \right) = \underbrace{\left( \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} | \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} \right)}_{1} \underbrace{\left( \begin{smallmatrix} \circ & \circ \end{smallmatrix} \right)}_{1-\gamma}$$

$$\left( \triangle | \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} \right) = \underbrace{\left( \triangle | \begin{smallmatrix} \circ & \circ \\ \circ & \circ \end{smallmatrix} \right)}_0 \underbrace{\left( \begin{smallmatrix} \circ & \circ \end{smallmatrix} \right)}_{1-\gamma} + \underbrace{\left( \triangle | - \right)}_1 \underbrace{\left( - \right)}_{\gamma}$$

# Extension beyond equiprocent lineages

The model we limited holds only for an idealised population in which  $F$  results from  $1/F$  equiprocent lineages

Höhle

Klotz 1979, Mase 1992  $\rightarrow$  approximate sol<sup>n</sup>

Exact sol<sup>n</sup> is smaller sum over c<sup>ts</sup> of singlets, doublets, triplets etc.

$$P\left( \begin{array}{c|c} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array} \right) \quad \therefore \sum_{1 \leq i_1 < i_2 < i_3 < i_4 \leq N} p_{i_1} p_{i_2} p_{i_3} p_{i_4}$$

$L_R \quad R_R$

$N$  = no. line

probabilities  $p_1, \dots, p_N$

$\rightarrow$  sum

over 3,921,225 terms for  $N=100$ .

No. of  
Singlets

doublets, triplets,  $t_1, \dots, t_n$

For exact

for approximate, need  $p_1, \dots, p_N$

$\rightarrow$  Birthday problem: at least one 1BD

Q: Do sol<sup>n</sup> extend to "exactly one 1BD", "exactly two 1BD" etc.?

See ? birthday problem: birthday - up for refs,

Read Das Gupta 2005

Read Diaconis