Game Theory and Applications (博弈论及其应用)

Chapter 16: Signaling Games

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Recap on Previous chapter

- Extensive game with imperfect information $G = \{N, H, P, I, \{u_i\}\}$
- Behavior strategies
- Belief
- Sequential equilibrium

Signaling games (信 号 传 递 博 弈)

The most interesting class of games that are solved used the sequential Equilibrium concept are signaling games

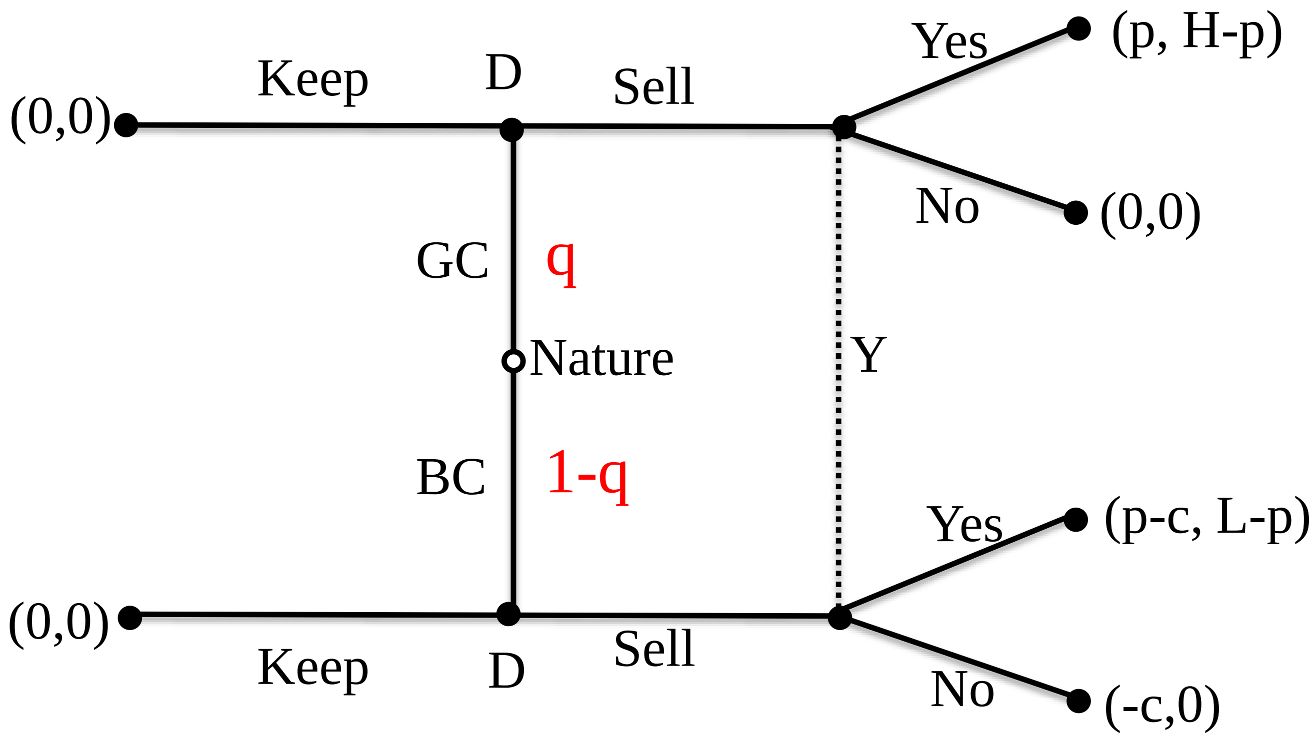
Michael Spence, 2001 Nobel Memorial Prize in economics: job-market signaling model

- A prospective employer can hire an applicant.
- The applicant has high or low ability, but the employer doesn't know which
- Applicant can give a signal about ability, e.g., education

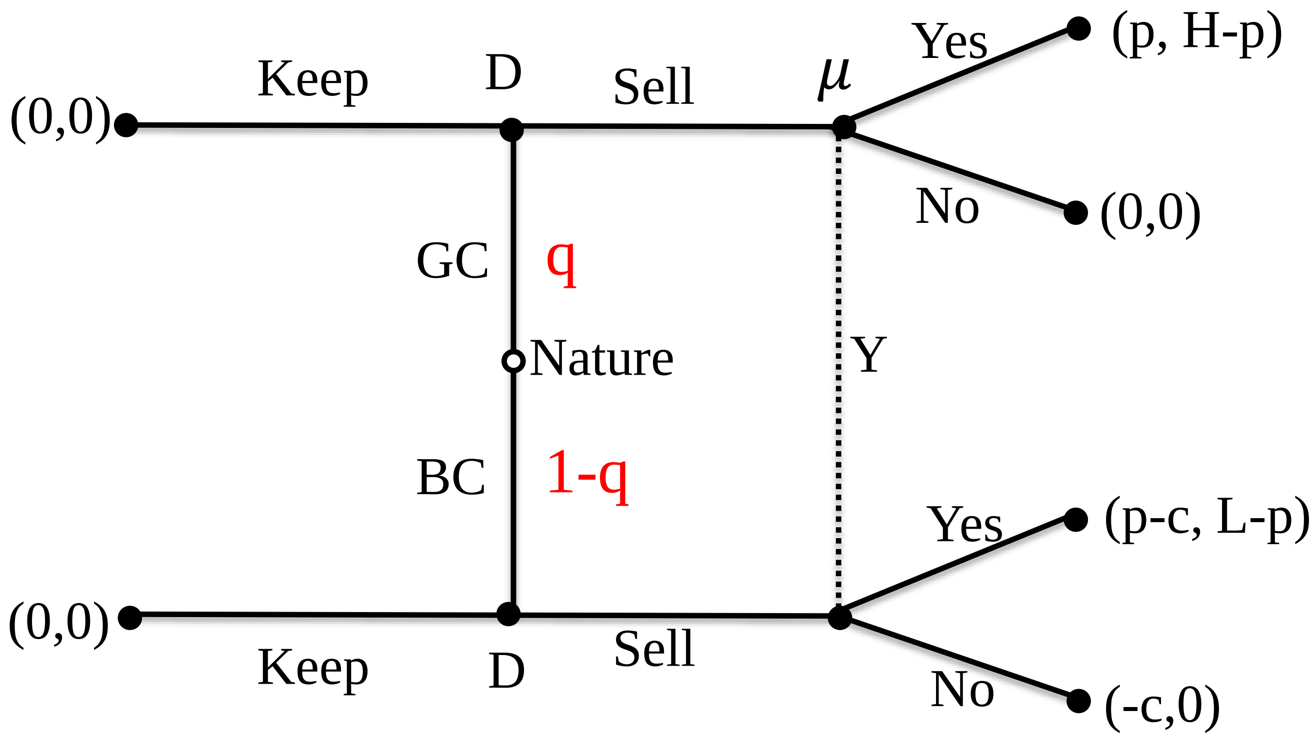
Signaling Games: Used-Car Market

- You want to buy a used-car which may be either good or bad
- A good car is worth H and a bad one L dollars
- You cannot tell a good car from a bad one but believe a proportion q of cars are good
- The car you are interested in has a price p
- The dealer knows quality but you don't
- The bad car needs additional costs c to make it look like good
- The dealer decides whether to put a given car on sale or keep
- You decide whether to buy or not
- Assume $H > p > L$

Signaling Games: Used-Car Market



Belief

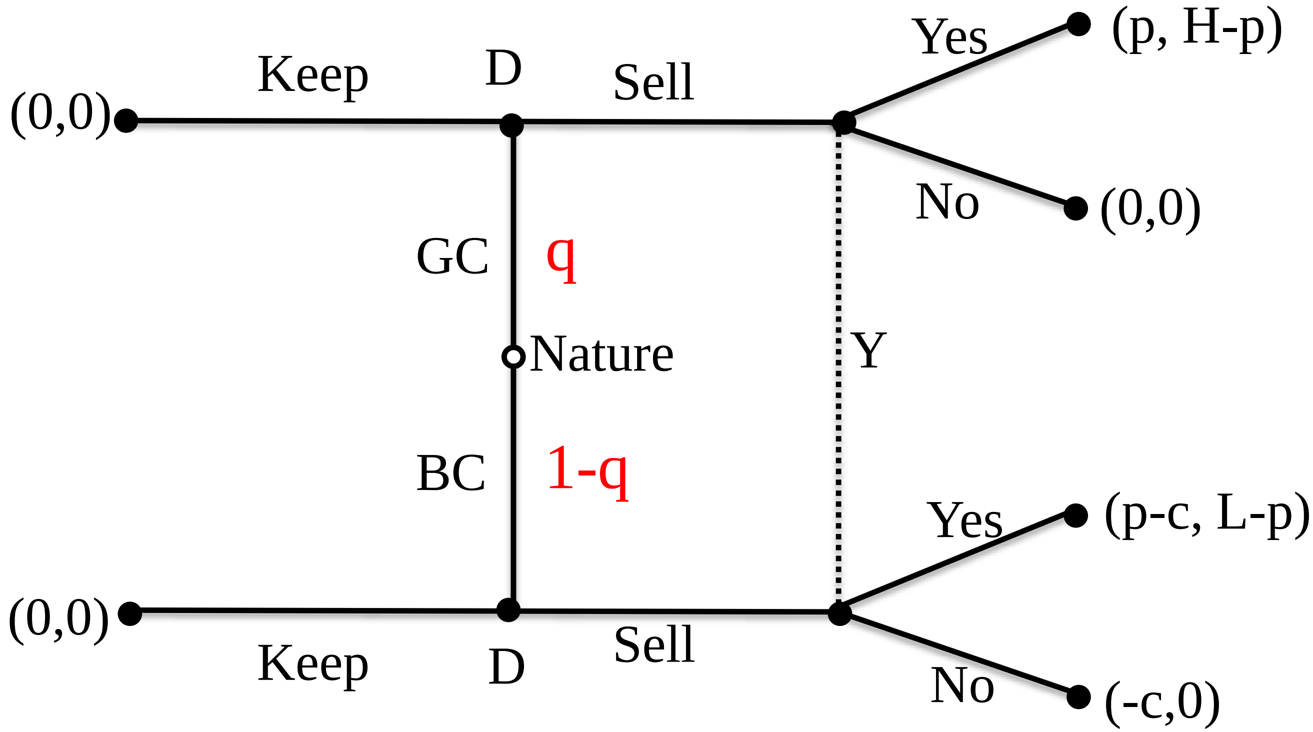


Dealer strategy: Offer if good; Hold if bad

What is your consistent belief if you observe the dealer sell a car?

$$\mu = \frac{P(\text{GC and sell})}{P(\text{sell})} = \frac{q \times 1}{q \times 1 + 0 \times (1 - q)} = 1$$

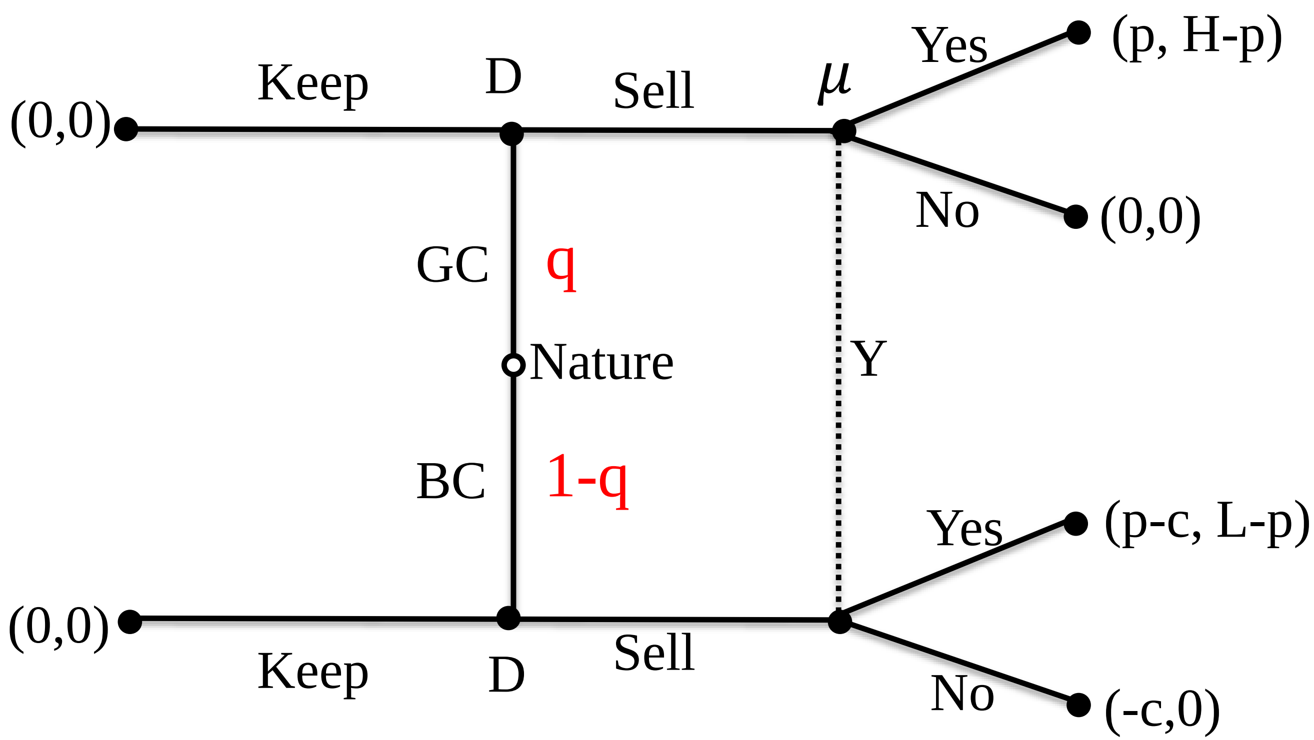
Signaling Games: Used-Car Market



We look for two types of equilibria

1) Pooling Equilibria: GC and BC dealer paly the same strategy
2) Separating Equilibria: GC and BC dealer paly different strategy

Pooling Strategy: Both Sell



Both strategies: Sell

Belief:

$$\mu = \frac{q}{1 \times q + 1 \times \ (1-q)} = q$$

Pooling Strategy: Both Sell

- If Y buys a car with your prior beliefs q your expected payoff is
$$V = q \times (H - p) + (1 - q) \times (L - p) \geq 0$$
- What does sequential rationality of seller imply?
- You must be buying and it must be the case that $p \geq c$

Pooling Equilibrium I

If $p \geq c$ and $V \geq 0$ the following is a PBE

Behavioral Strategy Profile: (GC: Sell, BC: Sell), (Y: Yes)

Belief System: $\mu = q$

Pooling Equilibria: Both Keep

You must be saying No

– Otherwise Good car dealer would offer

Under what conditions would Ysay No?

$$\mu \times (H - p) + (1 - \mu) \times (L - p) \leq 0$$

So we can set $\mu = 0$

The following is a PBE

Behavioral Strategy Profile: (Good: Hold, Bad: Hold),(You: No)

Belief System: $\mu = 0$

This is complete market failure: a few bad apples (well lemons) can ruin a market

Separating Equilibria - Good: Offer and Bad: Hold

- What about your beliefs?

$$\mu = 1$$

- What does you sequential rationality imply?
  - You say Yes
- Is Good car dealer’s sequential rationality satisfied?
  - Yes
- Is Bad car dealer’s sequential rationality satisfied?
  - Yes if $p \leq c$
- If $p \leq c$ the following is a PBE
  Behavioral Strategy Profile: (Good: Offer, Bad: Hold), (You: Yes)
  Belief System: $\mu = 1$

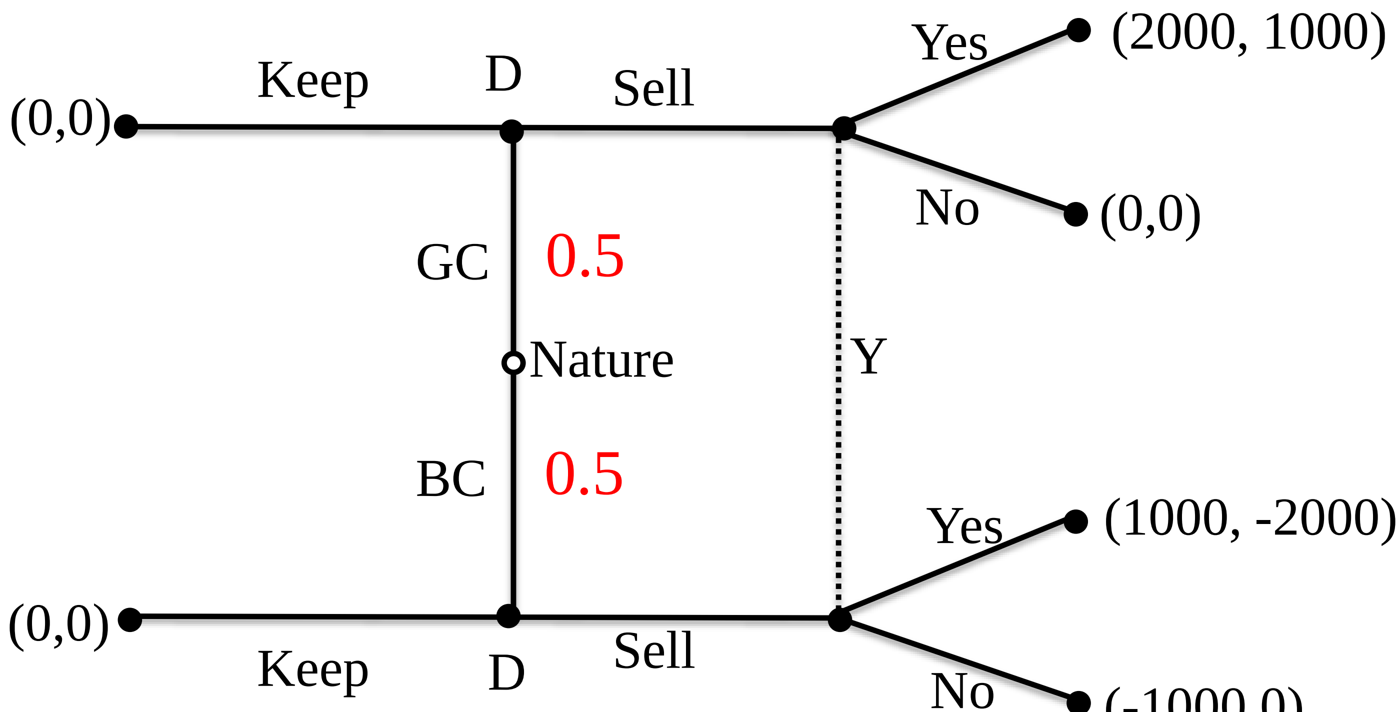
Separating Equilibria - Good: Keep and Bad: Sell

- What does Bayes Law imply about your beliefs?

$$\mu = 0$$

- What does you sequential rationality imply?
  - You say No
- Is Good car dealer's sequential rationality satisfied?
  - Yes
- Is Bad car dealer's sequential rationality satisfied?
  - No
- There is no PBE in which Good dealer Holds and Bad dealer Offers

Behavior Strategy



Behavior strategy: Yes Prob. $x \in (0,1)$

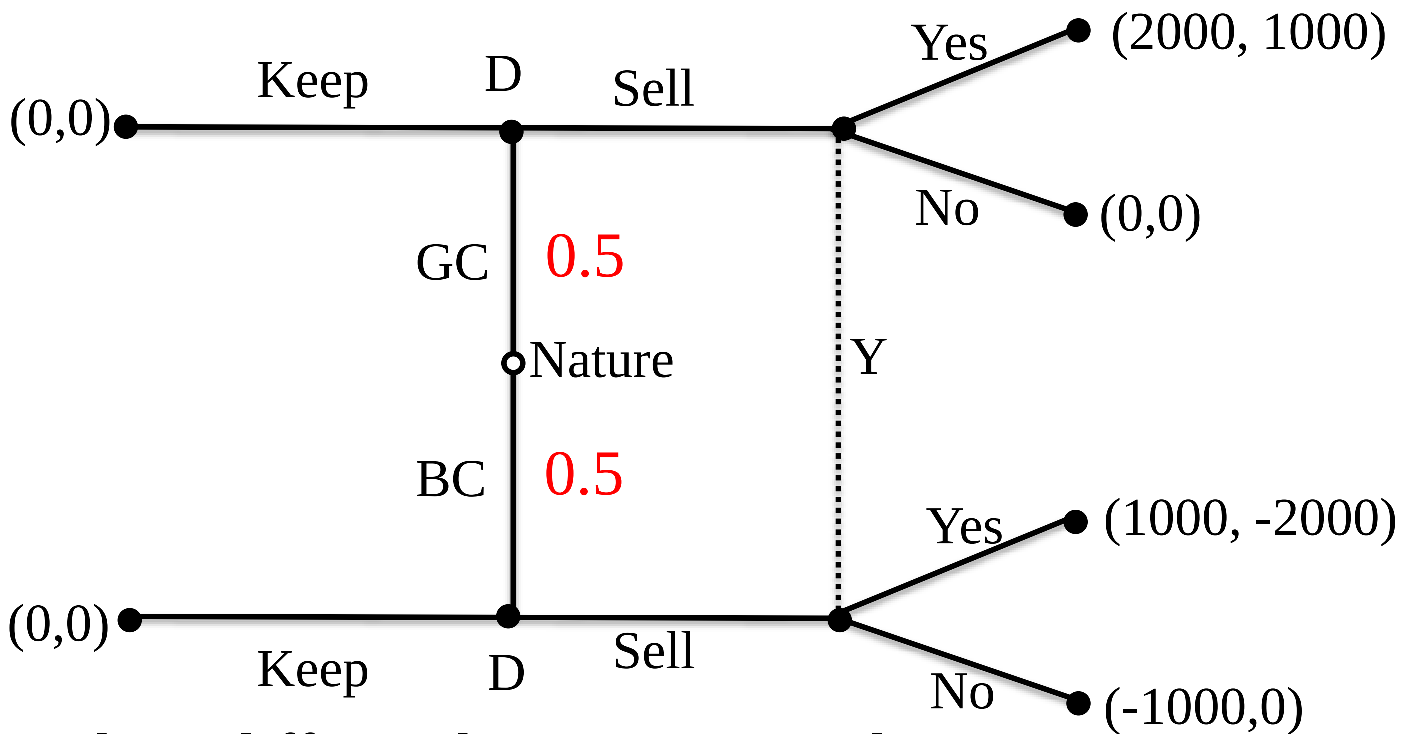
Behavior strategy: BC – sell Prob. y

Belief: GC – sell Prob. μ

You must be indifferent between Yes and No

$1000\mu - (1-\mu)2000 = 0$ implies μ=2/3

Behavior Strategy



You must be indifferent between Yes and No

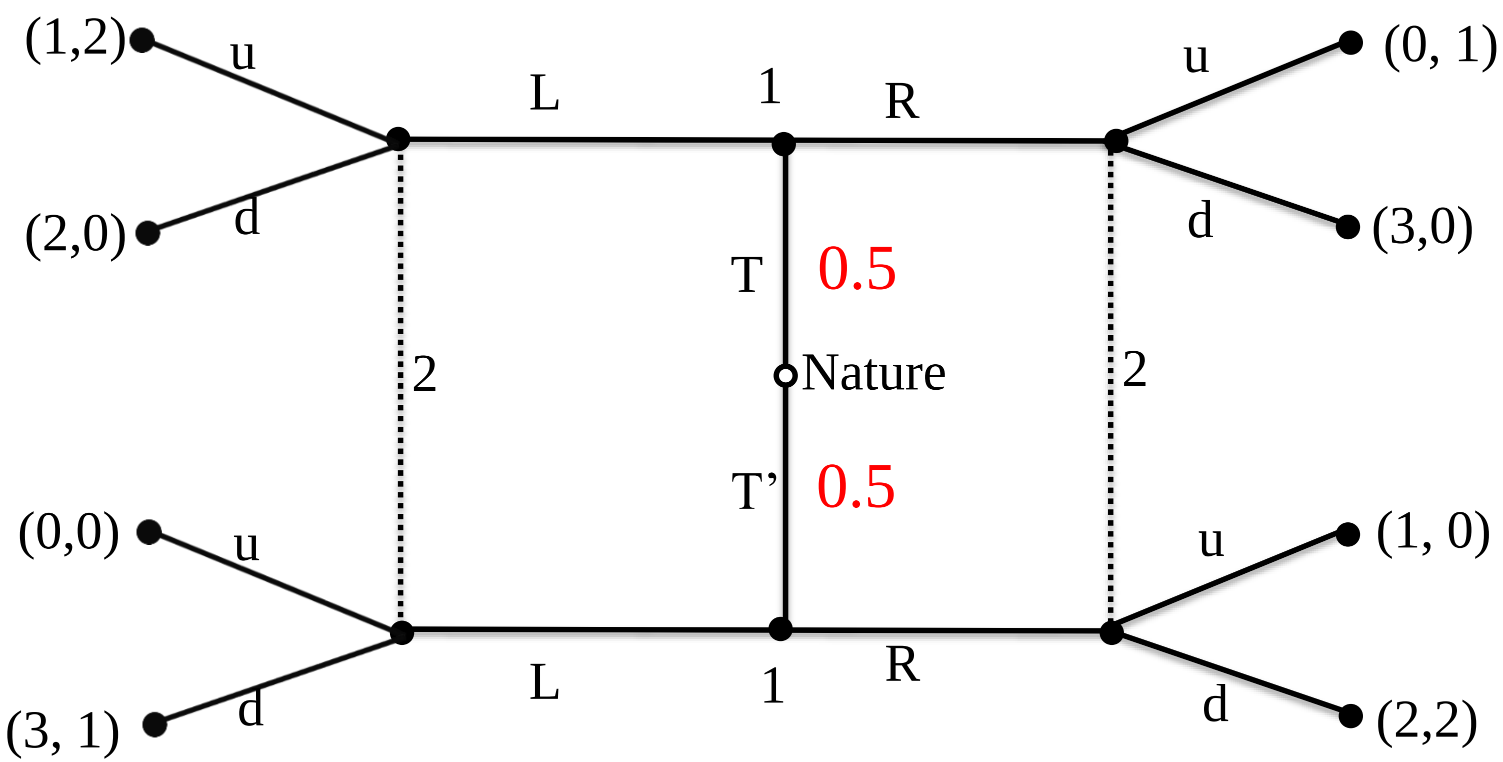
$$1000\mu - (1-\mu)2000 = 0 \text{ implies } \mu\text{=2/3}$$

$$\frac{0.5}{0.5+0.5y} = \frac{2}{3} \text{ implies } y = 0.5$$

Bad car dealers must be indifferent between Keep and Sell

$$0 = 1000x - 1000(1-x) \text{ implies } x = 0.5$$

Signaling Game: Another Example



(a) Find the corresponding strategic form game and its pure-strategy Nash equilibria.

(b) Determine (if any) the game's separating equilibria.

(c) Determine (if any) the game's pooling equilibria.

Signaling Game: Another Example

a) P1's pure strategies are pairs in LL, LR, RL, RR

P2's pure strategies are pairs in uu, ud, du, dd

	uu		ud		Du		dd	
LL	0.5	1	0.5	1	2.5	0.5	2.5	0.5
LR	1	1	1.5	2	1.5	0	2	1
RL	0	0.5	1.5	0	1.5	1	3	0.5
RR	0.5	0.5	2.5	1	0.5	0.5	2.5	1

Nash equilibrium ((R; R); (u; d)).

(b): Separating equilibria must be Nash equilibria:

((R; R); (u; d))

Pooling equilibria, no separating equilibria.

Signaling Game: Another Example

The candidate strategy ((R; R); (u; d))

But what should the belief system be? Let $\alpha_1, \alpha_2 \in [0,1]$ denote the prob. assigned to the top

Bayesian consistency: requires that $\alpha_2 = 1/2$, $\alpha_1 \in [0,1]$

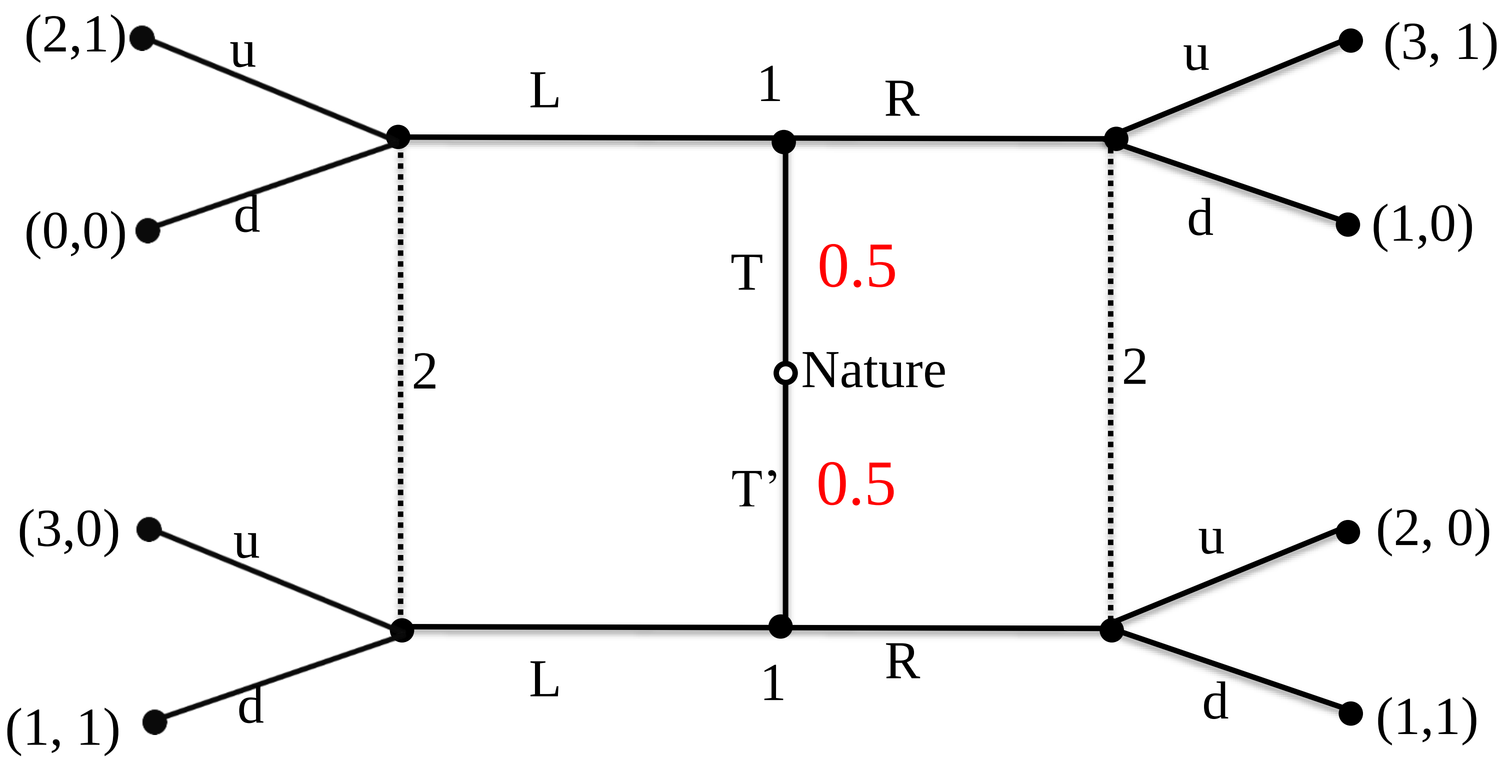
Sequential rationality:

- ((R; R); (u; d)) is a NE
- P2's payoff from u is $2\alpha_1 + 0(1 - \alpha_1)$ and from d is $0\alpha_1 + 1(1 - \alpha_1)$, so requires $\alpha_1 \geq \frac{1}{3}$

Conclude: Assessments (s1; s2; β) with strategies

- (s1; s2) = ((R; R); (u; d)) and belief system
- $\beta = (\alpha_1, \alpha_2), \alpha_1 \in [1/3,1]$ $\alpha_2 = 1/2$ are pooling equilibria

Exercise



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