Game Theory and Applications

南京大学

高 尉

Course Information and Textbooks

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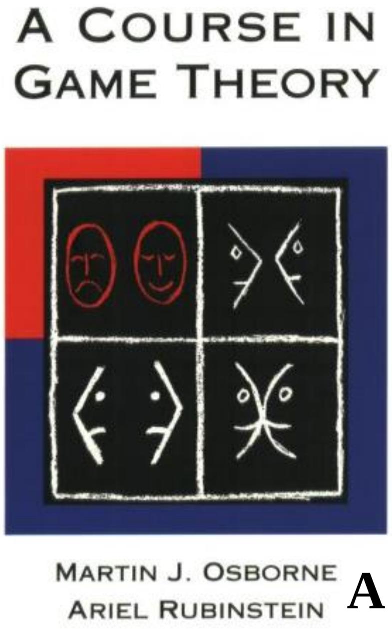
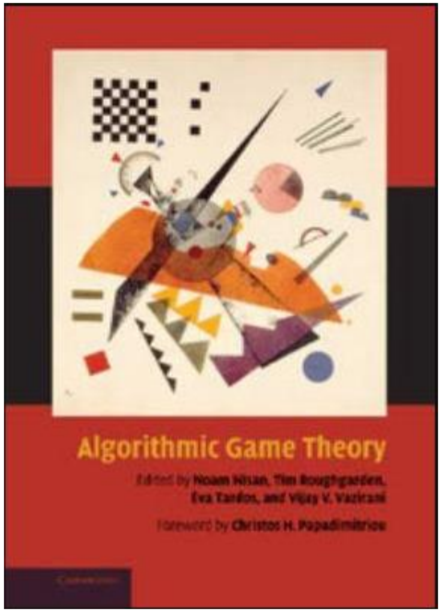


A Course in Game Theory

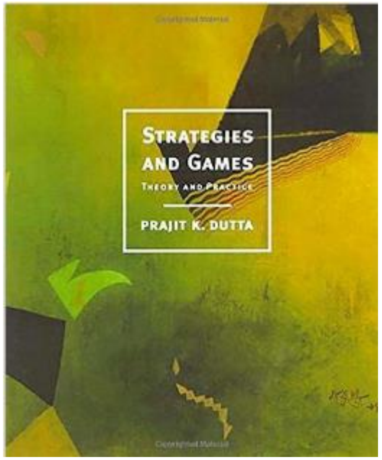
- Martin J. Osborne and Ariel Rubinstein
- MIT Press 1994



Algorithmic Game Theory

- Noam Nisan, Tim Roughgarden and Eva Tardos
- Cambridge University Press 2007

Textbooks

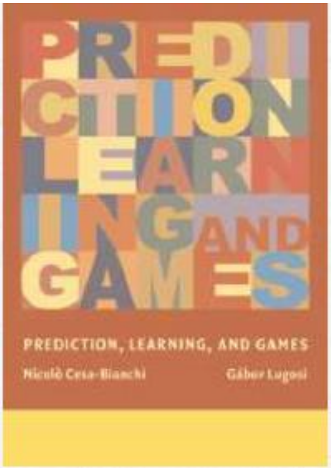


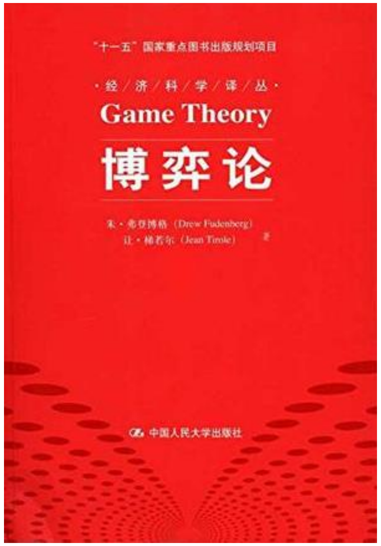
Strategies and Games: Theory and Applications

- Prajit K. Duta
- MIT Press 1999

Prediction, Learning and Games

- Nicolo Cesa-Bianchi and Gabor Lugosi
- MIT Press 1999



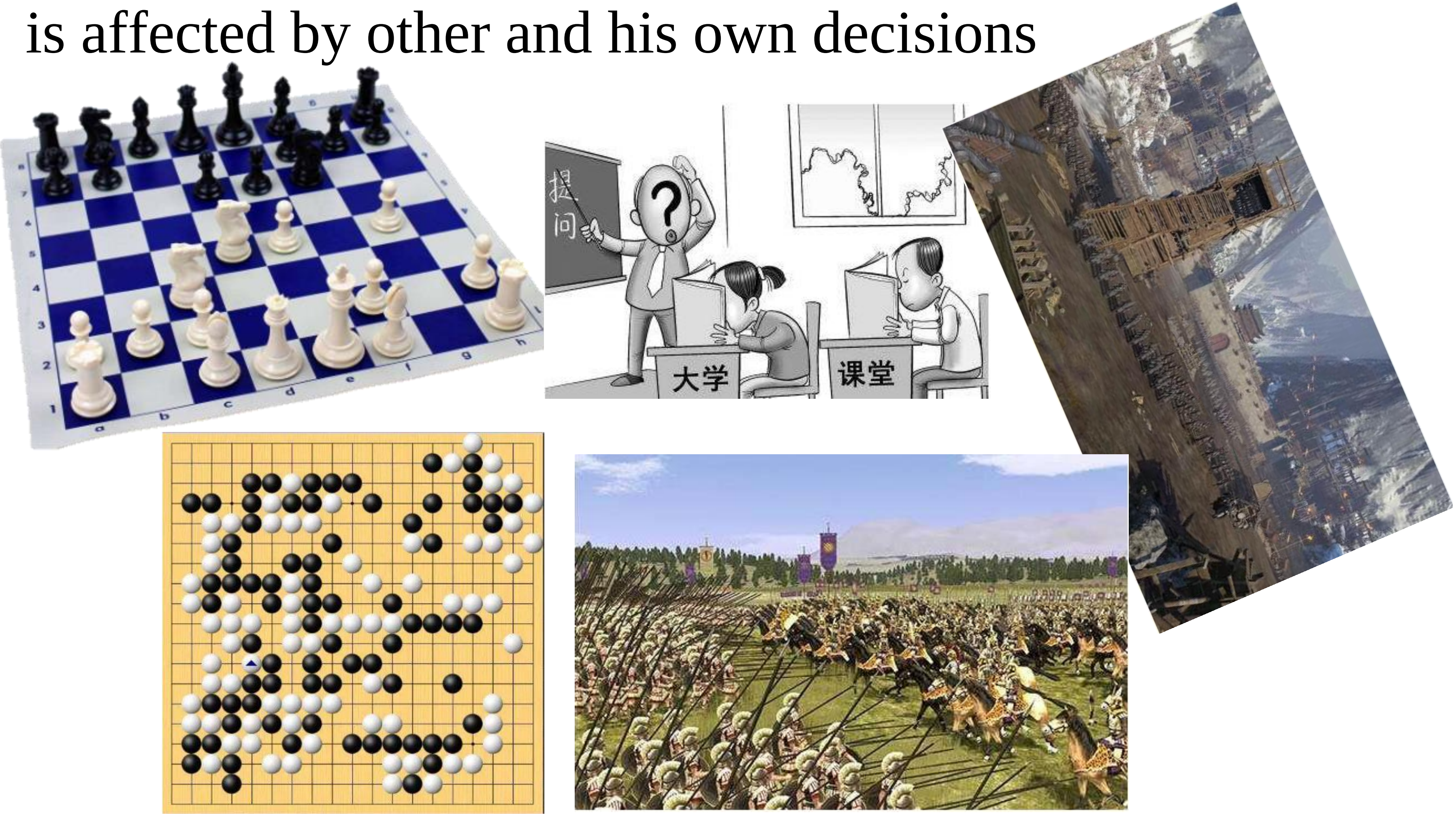


博弈论

- 人民大学出版社

What is Game

- A game: multi-person decisions/interacts, each outcome is affected by other and his own decisions



Key Elements for Game

- Players: Who is interacting (1, 2, multi-persons)
- Strategies/Decision: What are their options
- Payoff: What are their incentives
- Information: What do you know
- Rationality: How do you think

Two Players Strategy Game: Payoff Matrix

Player 2
(Column)

		Strategy C	Strategy D
Player 1 (Row)	Strategy A	P11 P21	P12 P22
	Strategy B	P13 P23	P14 P24

Note

- The strategies A and B many be similar/different from C and D
- P1i and P2j may be different

An Example: Prisoners’ Dilemma

		Prisoner B	
		Confess	Don’t confess
Prisoner A	Confess	-6 -6	0 -12
	Don’t confess	-12 0	-1 -1

Prisoners’ Dilemma： Prisoner A

		Prisoner A	
		Confess	Don’t confess
Prisoner B	Confess	-6	-12
	Don’t confess	0	-1

Prisoner A: choose ‘confess’

Prisoners’ Dilemma: Prisoner B

		Prisoner A	
		Confess	Don’t confess
Prisoner B	Confess	-6	0
	Don’t confess	-12	-1

Prisoner B: choose ‘confess’

Prisoners’ Dilemma （cont.）

		Prisoner A	
		Confess	Don’t confess
Prisoner B	Confess	-6 -6	0 -12
	Don’t confess	-12 0	-1 -1

Each single optimal decision is not global optimum

No-cooperative

Applications of Prisoners' Dilemma

- Lesson for military: consider the safety of two nations if they disarm (cooperate) or both heavily armed?

  二次打击

- Market Strategies: Two rival companies offer small discounts and retain a good market share, or offer huge discounts?

- Cooperation depend on morality, or the complicated dynamics of environment.

田忌赛马

			田 忌	
		上	中	下
齐威王	上	1 -1	1 -1	1 -1
	中	-1 1	1 -1	1 -1
	下	-1 1	-1 1	1 -1

1：齐威王（上）vs 田忌（下）
2：齐威王（中）vs 田忌（上）
3：齐威王（下）vs 田忌（中）

Imperfect information
Random Strategies

What is Game Theory (博 弈 论)

- Game theory = Multi-person decision theory

- Game theory: **study of mathematical models of conflict and cooperation between intelligent rational decision-makers** (Wikipedia)
  - Game theory is highly mathematical
  - Game theory assumes all human interactions can be understood and navigated by presumptions
  - Abstraction of real complex situation
  - Finding acceptable, if not optimal, strategies in conflict situations

The Importance of Game Theory

- All intelligent beings make decisions all the time.

- AI needs to perform these tasks as a result.

- Help to analyze situations more rationally, and formulate an acceptable alternative with respect to circumstance.

Key Elements of Games Theory

- Player
- Strategy/Decision
- Payoff
- Information
- Rationality

Players

- A player is a decision maker and can be anything from individuals to entire nations.
- Players have the ability to choose among a set of possible actions.
- Games are often characterized by the fixed number of players.

Strategies

- A strategy is a set of actions available to a player.
- Strategies may be simple or complex.
- In non-cooperative games each player is uncertain about what the other will do since players can not reach agreements among themselves.

Payoffs

- Payoffs are the final returns to the players at the conclusion of the game.
- Payoffs are usually measure in utility although sometimes measure monetarily.
- In general, players are able to rank the payoffs from most preferred to least preferred.
- Players seek the highest payoff available.

Information

- Various rules in game
- The set of strategies for each players
- The payoff matrix
- All information about the game

Rationality

Assumptions:

- humans are rational beings
- humans always seek the best alternative in a set of possible strategies
-

Why assume rationality?

- narrow down the range of possibilities
- predictability

History of Game Theory: Milestone I

John Von Neumann (mathematician)

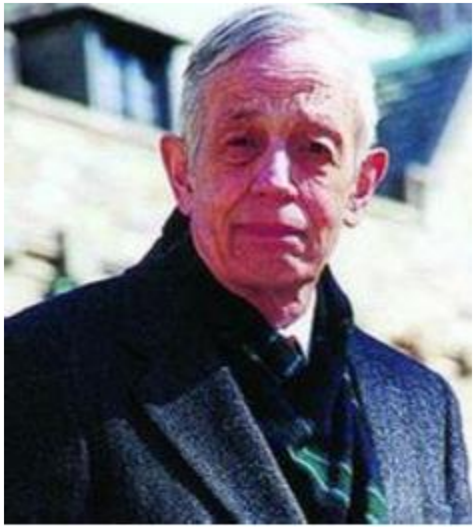
Oskar Morgenstern (economist)

“Theory of game and economic Behavior” Princeton University Press 1944

① Mathematical method to analyze games

② A new scientific approach to the study of economics

History of Game Theory: Milestone II



John Forbes Nash (1928-2015)

Main contribution: Nash Equilibrium

1) In non-cooperative games,

Neither player has an incentive to change strategy, given the other player’s choice

2) Proof of the existence of Nash Equilibrium

Nash Equilibrium of Prisoners' Dilemma

		Prisoner A	
		Confess	Don't confess
Prisoner B	Confess	-6 -6	0 -12
	Don't confess	-12 0	-1 -1

History of Game Theory: Prosperity

- Wide applications (after 1950s): economics, computer science, artificial intelligence…

- Nobel Prize in Economics
  - 1994, Nash, Selten and Harsanyi
  - 2005, Thomas Schelling and Robert Aumann
  - 2007, Leonid Hurwicz, Eric Maskin and Roger Myerson
  - 2012, Alvin E. Roth and Lloyd S. Shapley
  - 2014, Jean Tirole

Types of Games

- # of players:
  - 1, 2, multi-persons games
- Orders of players, time and repeat
  - Simultaneous and sequential
- Payoff
  - Zero sum and non-zero sum

Types of Games (cont.)

- Information
  - Perfect information and imperfect information
- Rationality
  - Cooperative or non-cooperative
- Strategies/Decision
  - Finite and infinite strategies
- …

1-Person Game

固定费用　　被抢劫的花费以及可能性

	T. Cost	wicked w.	p. of w. w.
land	1000	200	5%
waterway	600	1200	20%

How to choose?

Expected expense of land = 1000+200*5% =1010

Expected expense of waterway = 600+1200*20% =840

How about only one time?

2-Persons Game: Simple Nim

- Rule
  - Two players carry coins in turn
  - A player remove exactly 1 or 2 coins/turn
  - The winner is the one taking the last coin.

Lemma: Suppose that player A and B are playing the simple Nim game, where at each round, a player can remove between 1 and k coins, then a player has a winning strategy if he can take coins so as to leave i(k+1) coins.

Proof by induction I: For i=1, A leaves k+1 coins, then B selects x coins ($1\leq x\leq k$). A takes the leaves and wins.

2-Persons Game: Simple Nim

Proof by induction II. Assume the statement is true for i=n i.e., if A leaves n(k+1) coins, then A wins.

Suppose A leaves (n+1)(k+1) coins. If B select x: $1\leq x\leq k$, then A selects k+1-x, and leaves n(k+1). By induction, A wins.

This lemma holds by induction for all i

Multiple-Persons Game: Pirate Game

Five Pirates A > B > C > D > E have 1000 gold coins, and decide how to distribute them

Pirate Rules

- The most senior pirate first proposes a plan of distribution. All pirates vote on whether to accept this distribution
- If the majority (including tie vote) accepts the plan, the coins are dispersed and the game ends
- If the majority rejects the plan, the proposer is thrown overboard from the pirate ship and dies, and the next most senior pirate makes a new proposal to begin the system again
- The process repeats until a plan is accepted or if one pirate lefts

Multiple-Persons Game: Pirate Game (cont.)

Five Pirates A > B > C > D > E have 1000 gold coins, and decide how to distribute them

Four decision factors

- Each pirate wants to survive
- Each pirate tries to maximize the number of gold coins if survival
- Each pirate would prefer to throw another overboard, if all other results would be equal
- The pirates do not trust each other, no cooperation

How to play?

-Average distributed

Multiple-Persons Game: Pirate Game (cont.)

- For D and E

decisions: D:1000 E:0

- For C, D and E

decisions: C: 999 D:0 E:1

- For B, C, D and E

decisions: B:999 C:0 D:1 E:0

- For A, B, C, D and E

decisions: A:998 B:0 C:1 D:0 E:1

Cooperative vs Non-Cooperative Game

Cooperation often leads to higher payoffs

- Prisoners’ Dilemma

		Prisoner A	
		Confess	Don’t confess
Prisoner B	Confess	-6 -6	0 -12
	Don’t confess	-12 0	-1 -1

- More examples
  - Countries cooperation on trade
  - Cartel: formation of monopoly by multiple organizations

Zero vs Non-Zero Sum Game

- Zero-Sum game: the total payoff among players is zero, i.e., neither create nor destroy in playing game

Rock-Paper-Scissors

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	0 0	-1 1	1 -1
	Paper	1 -1	0 0	-1 1
	Scissors	-1 1	1 -1	0 0

Many zero-sum games in our daily lives

- War, resources, trade …

Zero vs Non-Zero Sum Game

- Non-Zero-Sum game: the total payoff among players is not zero, may increase or decrease in playing game

Battle of sexes
性别战

		Man	
		Boxing	Ballet
Woman	Boxing	2 3	0 0
	Ballet	1 1	3 2

情侣分别去自己喜欢的 1 v 1
分别去自己不喜欢的 0 v 0
一起去对方喜欢的，比单独好

Most real-life games are non-zero-sum:

- China-vs-American trade
- Create an organization/company
- …

Simultaneous and Sequential Game

- Simultaneous Game: make actions simultaneously

  Rock-Paper-Scissors

  Battle of sexes

- Sequential/Dynamic Game: make actions one by one

Simultaneous and Sequential Game (cont.)

- Simultaneous Game: Payoff matrices

Battle of sexes

		Man			
		Boxing		Ballet	
Woman	Boxing	2	3	0	0
	Ballet	0	0	3	2

- Sequential/Dynamic game: tree

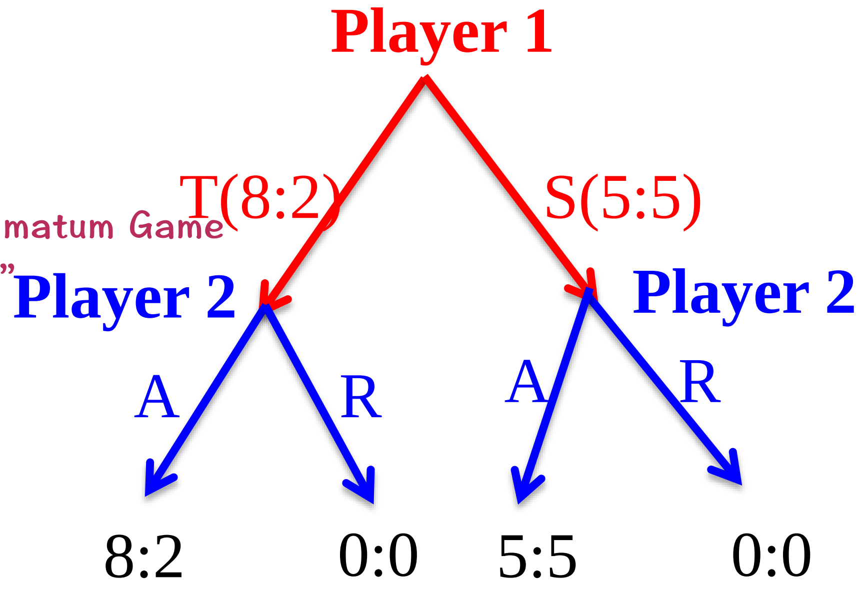
Perfect vs Imperfect Information

Sequential games

Perfect information game: all players know the actions previously made by all other players

Ultimatum Game
最后通牒博弈

海盗分金就是有多名响应者时的Ultimatum Game
最后通牒：谈判破裂前的“最后的话”



Perfect vs Imperfect Information (cont.)

Sequential games

Imperfect information game: New players do not know some actions previously made by other players

Applications of Game Theory

- **Mathematics**
- **Computer Science**
- **Economics**
- **Biology**
- Political Science
- International Relations
- Philosophy
- Psychology
- Law
- **War**
- **Management**
- Sport
- **Game playing**

Limitations & Problems

- Assumes players always maximize their outcomes
- Some outcomes are difficult to provide a utility
- Not all of the payoffs can be quantified
- Not applicable to all problems

Contents

- Strategic game with perfect information
- Strategic game with imperfect information
- Extensive game with perfect information
- Extensive game with imperfect information

  拓展式博弈（有先后）
- Repeated game

Chapter

Chapter

10. One Deviation, Back Induction
11. Repeated Game
12. Analysis of Repeated Game
13. Extensive Game with Incomplete Information I
14. Extensive Game with Incomplete Information II
15. Prediction with Experts Games
16. Randomized Prediction Games
17. Applications II

考核方式

- Home work: 20% (4-6次作业)
- Mid-Term exam: 20% (平时作业中两次最高分)
- Final exam: 60%

Preliminary Courses

- Calculus
- Linear algebra
- Probability

Exercises

- Let $\{a_n\}$ be a sequence of positive real number. Denote by $S_n = \sum_{i=1}^{n} a_i$. If $S_{n+1} \geq 2S_n$, then there exists a constant $c > 0$, such that $a_n \geq 2^n c$ for every positive n.

- Suppose that $(1,1,-1)$ is an eigenvector of matrix

$$\begin{bmatrix} 2 & -1 & b \\ 5 & a & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

Solve $a, b,$ and the corresponding eigenvalue.

- For $\epsilon \in [0,1]$, prove that

$$\frac{1}{2}\left(1 + \sqrt{1 + 4\epsilon^2}\right) e^{1-\sqrt{1+4\epsilon^2}} \leq \mathrm{e}^{-(\epsilon^2-\epsilon^3)/2}$$